Flux Tubes Interacting with Superfluids

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Superconductor Recap

Type I: Zero resistance, complete expulsion of magnetic fields: Meissner effect
Destroyed by: critical current, critical temperature T_c ,
critical magnetic field H_c.

Type II: Zero resistance, Meissner effect only below first critical field H_{c1}
Magnetic flux enters above H_{c1} in form of flux tubes
Normal conducting state reached via T_c or second critical field H_{c2}

Fluxtube: Above H_{c1} magnetic fields enters SC in tubes, core of flux tube is normal conducting,
field only enters in ***n*** units of fundamental flux quantum

$$\Phi = B/A = n\Phi_0$$

Ginzburg-Landau parameter κ : Material parameter determining type of SC

Motivation

Protons and neutrons in compact stars form a Cooper pair condensate
→ interacting multi fluid system of a superconductor (SC) and a superfluid (SF)

"Common wisdom": Protons form type II SC (simple model calculations: $\kappa > \frac{1}{\sqrt{2}}$)

But: some observations support type I (long period precession)

In general: type I → II transition as function of density expected

? What does this transition look like **?**

? How is this simple picture modified by the presence of the superfluid **?**

? How does the phase diagram of the system look **?**

K. Glampedakis, N. Andersson, L. Samuelsson, MNRAS, 410, 802-829, arXiv:1001.4046
K. Buckley, M. Metlitski, A. Zhitnitsky, PRC 69, 055803

Model

Ginzburg-Landau Theory from Microscopic Model

Effective field-theoretical model: two coupled, complex, **gauged** scalar fields with self interaction and interaction between the fields.

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_i + \mathcal{L}_{Int}$$

$$\mathcal{L}_i = \sum_{i=1,2} \left[D_{i,\mu}\varphi_i \left(D_i^\mu \varphi_i\right)^* - m_i^2|\varphi_i|^2 - \lambda_i|\varphi_i|^4 \right]$$

$$\mathcal{L}_{Int} = 2h|\varphi_1|^2|\varphi_2|^2 - \frac{G}{2}\left(\varphi_1\varphi_2\left(D_{1,\mu}\varphi_1\right)^*\left(D_2^\mu\varphi_2\right)^* + perm.\right)$$

tree-level potential = Ginzburg-Landau free energy + entrainment term

G: derivative/entrainment coupling (Andreev-Bashkin effect)
h: non-entrainment coupling

Ungauged: Haber, Schmitt, Stetina, Phys.Rev. D93 (2016) no.2, 025011
Non-relativisitc: Alford, Good, Phys.Rev. B78 (2008) 024510

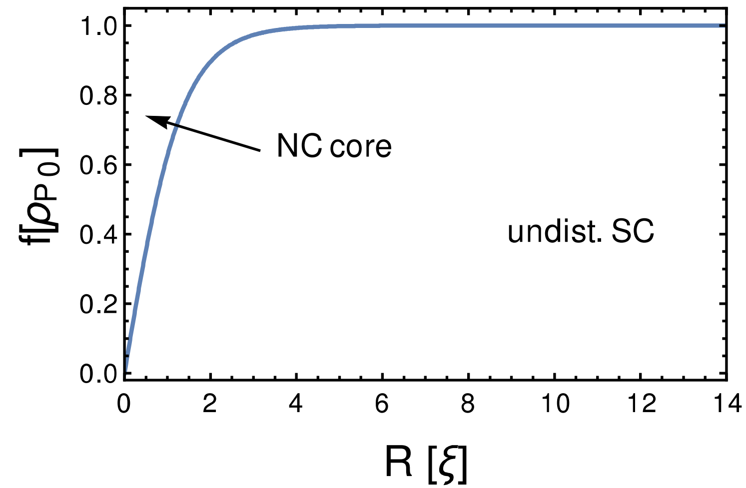
Profile Functions n=1

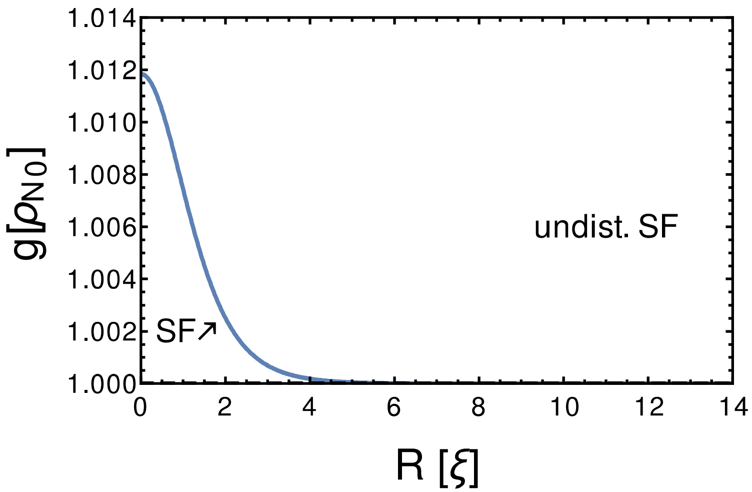
Solve EL-EOM numerically (relaxation method) with radially symmetric ansatz for flux tubes

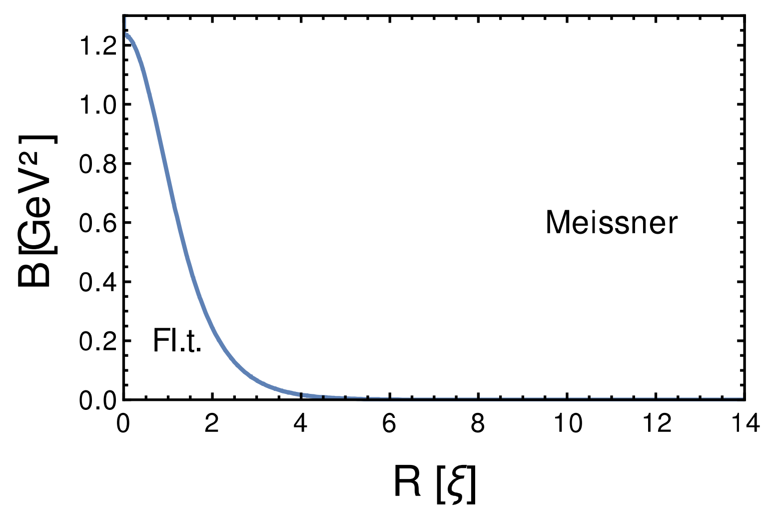
Assumptions: $q_2=0 \rightarrow$ (SC+SF), G=0, h<0, T=0, exterior B-Field

Results:

- No superfluid vortices induced
- Superfluid density enhanced/diminished in fluxtube depending on sgn(h)
- Multi flux quantum configurations can be computed





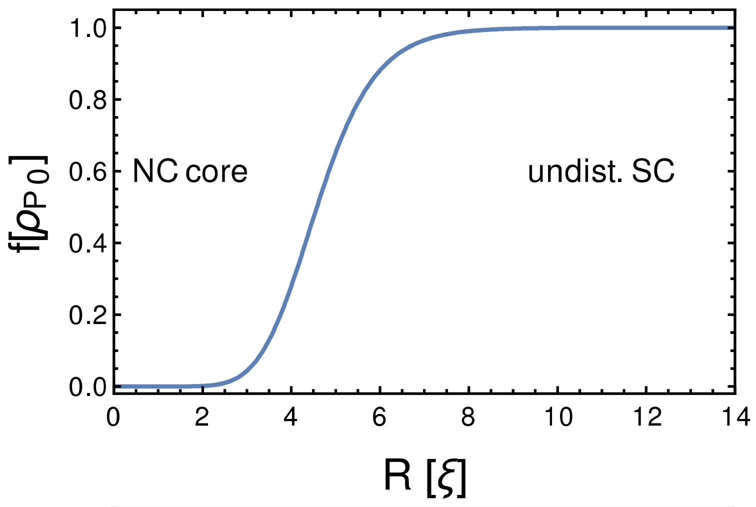


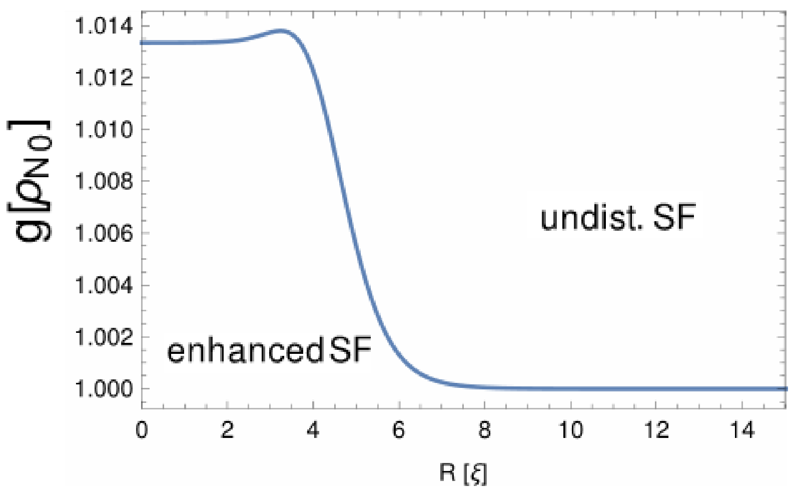
Profile Functions n=11

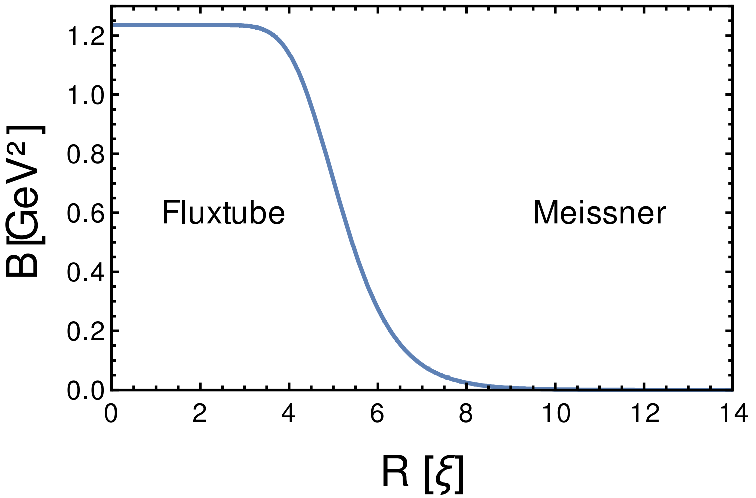
Assumptions: q_2=0→(SC+SF), **G<0, h<0**, T=0, exterior B-Field

Results:

- Superfluid density enhanced/diminished in fluxtube depending on sgn(h)
- "Bump" of the neutron condensate at the beginning of the descent due to gradient term







Critical Magnetic Fields I

Goal: calculate critical magnetic fields

H_c : compare free energy of Meissner phase with free energy of normal phase

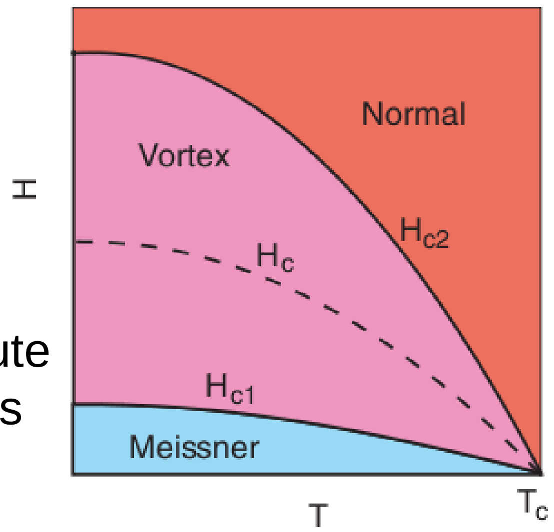
$$H_c = \sqrt{2\pi\lambda_1}\sqrt{1-\frac{h^2}{\lambda_1\lambda_2}\rho_{P_0}^2}$$

H_{c2} : linearize equations of motion (= assume 2nd order phase transition), compute maximal magnetic field which allows solutions
Note independence of G

$$H_{c_2} = \sqrt{2}\kappa\sqrt{1-\frac{h^2}{\lambda_1\lambda_2}}H_c$$

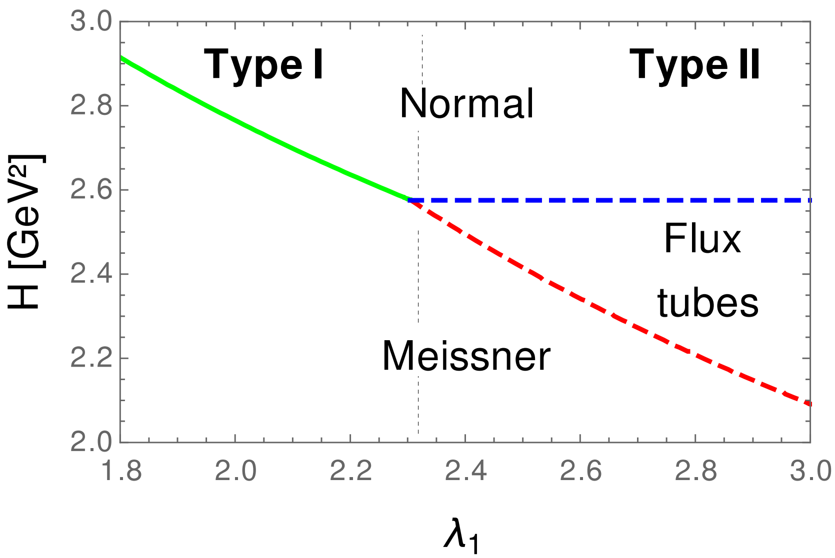
H_{c1} : compare Gibbs free energy of a **single** flux tube with **winding number n** with the Meissner phase
demands full numerical calculation of the flux tube profiles

finite T sketch



Critical Magnetic Fields II

Uncoupled Superconductor



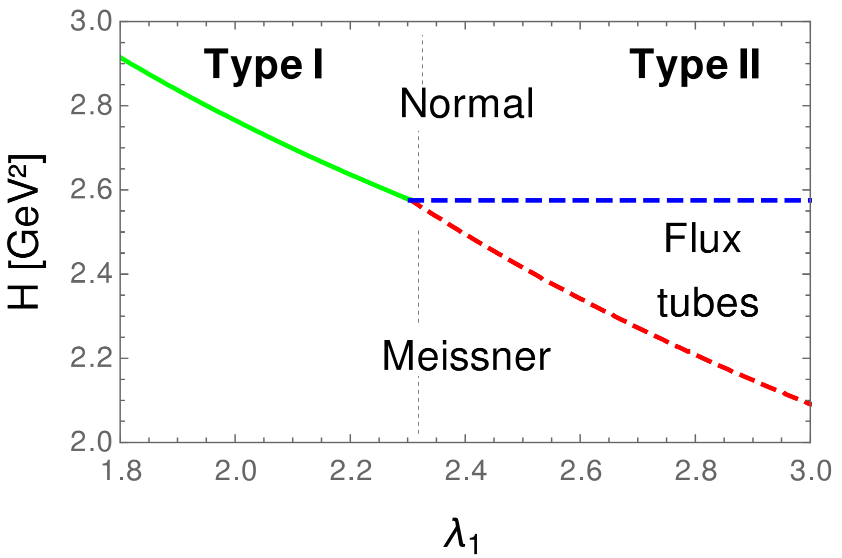
$$\kappa = \sqrt{\frac{\lambda_1}{4\pi q^2}}$$

Normal SC without interaction: consistent phase structure

Analytical result: SC phase is preferred directly below H_{c2} if phase transition is 2^{nd} order

Critical Magnetic Fields II

Uncoupled Superconductor



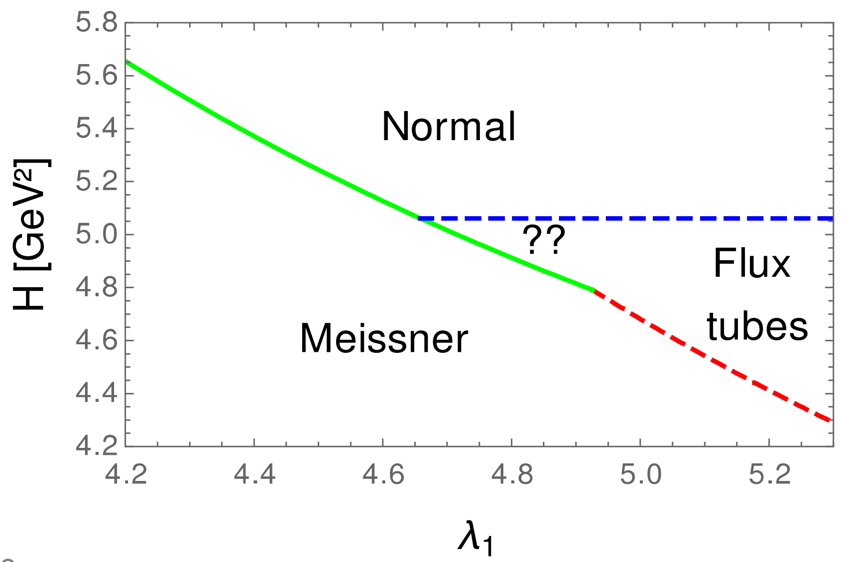
Normal SC without interaction: consistent phase structure

Analytical result: SC phase is preferred directly below H_{c2} if phase transition is 2nd order

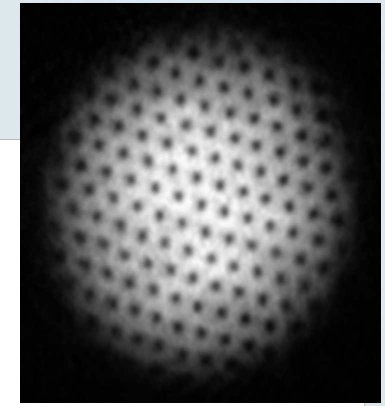
Coupled System

SC coupled to SF: phase structure more complicated

Exact phase structure depends on interaction between flux tubes



Flux Tube Interaction I



BEC vortex lattice by MIT

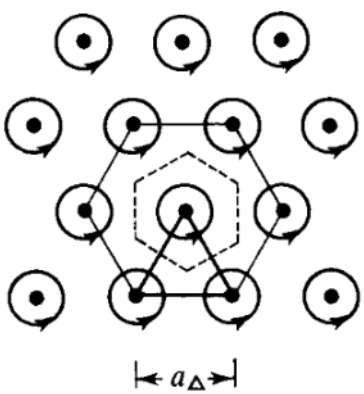
Two main contributions:

1) Repulsive part due to Lorentz force: magnetic field of one vortex reaches its neighbor $\vec{F}_L = q\vec{v}_2 \times \vec{h}_1\left(r_0\right)$

2) Attractive part: flux tubes want to overlap to gain condensation energy

$$\frac{F_{int}\left(r_0\right)}{L} = 2\rho_{P0}^2 r_0 \int_{r_0/2}^{\infty} \frac{dR}{\sqrt{R^2 - \left(\frac{r_0}{2}\right)^2}} \left[\frac{\kappa^2 n^2 a'\left(1-a\right)}{R^2} - \left(1-f\right)f' - x^2\left(1-g\right)g'\right]$$

K. Buckley, M. Metlitski, A. Zhitnitsky, PRC 69, 055803



Tinkham, Introduction to SC, Dover Press

Assumptions:
- Nearest neighbor approximation
- Large distance between tubes → linearization of EOM
- Hexagonal lattice

Flux Tube Interaction II

Integral can be solved analytically using asymptotic solutions in form of modified Bessel functions

$$F_{Gibbs} = U_{COE} + \frac{n\nu}{2q}\left[H_{c_1}(n) - H\right] + \frac{\#_{NN}\nu}{2}\frac{F_{int}(r_0)}{L}$$

with the flux tube area density $\nu = \frac{2}{\sqrt{3}r_0^2}$ and $\#_{NN} = 6$

→ dynamically compute lattice spacing by minimization of free energy

Possible effect: attractive term allows for flux tubes **below** H_{c1}

Analog: Baryon onset
Energy of single flux tube ↔ baryon mass
Attractive flux tube interaction ↔ binding energy

Summary & Outlook

Summary

- Behavior of a superconductor is altered by interaction with superfluid
- Effective parameter kappa due to interaction with SF
- Topology of phase diagrams complicated, flux tube interactions crucial
- Attractive interaction might lead to earlier 1st order onset of flux tube phase

Outlook

- Relate proton Cooper pair self-coupling to actual density profile of compact stars
- Fit other parameters to observational values
- Investigate transition at finite temperature by starting from Lagrangian with entrainment (results not shown here)