NEUTRON STAR RADII AND CORE-CRUST TRANSITION

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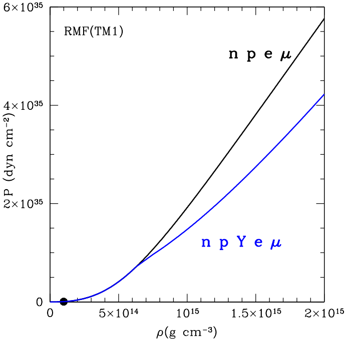
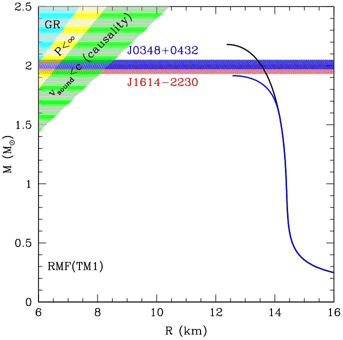
Context: equation of state (EoSs) and hyperons (Y)

Equation of state



$M - R$ plot



Hyperons

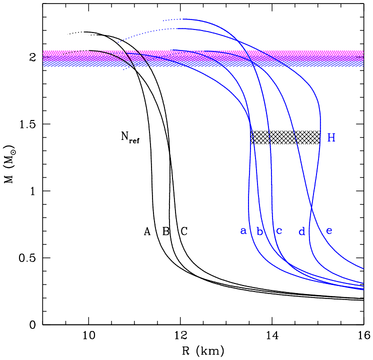
- reduce the pressure in the inner core. ie. softening of the EoS;
- reduce the maximum mass.

Claims that $M_{\rm max} \geq 2\,M_{\odot}$ rules out hyperonic equations of state ...

Hyperonic equations of state and radii

Fortin, Zdunik, Haensel and Bejger, A&A (2015)

$M - R$ plot



Nucleonic EoSs
Hyperonic EoSs

- 14 hyperonic EoSs (Y.EoSs), all consistent with $M_{\rm max} \geq 2$ M$_\odot$, all but one (Yamamoto et al. PRC 2014) are RMF models;
- 3 nucleonic ones (N.EoSs) as reference.

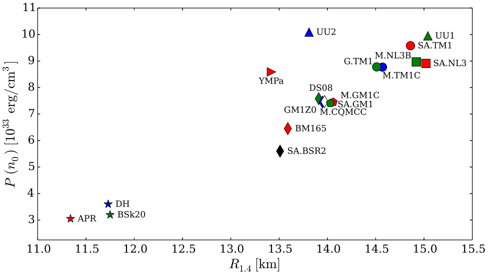
Y.EoSs

for $M \in [1.0 - 1.6]$ M$_\odot$, $R > 13$ km.

Hyperonic equations of state and radii

Fortin, Zdunik, Haensel and, Bejger, A&A (2015)

Pressure at $n_0 = 0.16$ fm^{-3} near the core-crust transition



- large radius for Y.EoSs correlated with a large pressure at n_0.

→ $M_{\rm max} \geq 2\,M_\odot$ possible if the decrease in the pressure at high density due to Y is compensated by a large pressure at low density.

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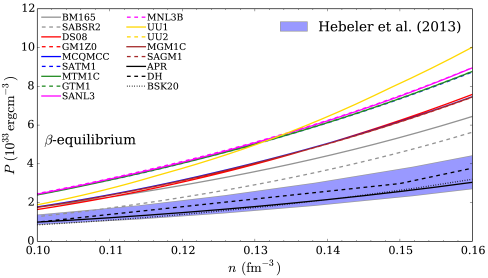

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→ $M_{\max} \geq 2\,M_\odot$ possible if the decrease in the pressure at high density due to Y is compensated by a large pressure at low density.

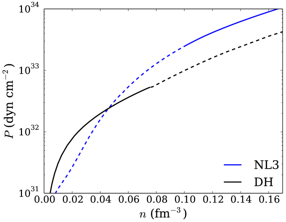
- blue strip: chiral effective field theory calculations up to n_0 (Hebeler et al. ApJ 2013).
- over-pressure at n_0 for hyperonic EOS inconsistent with this constraint.

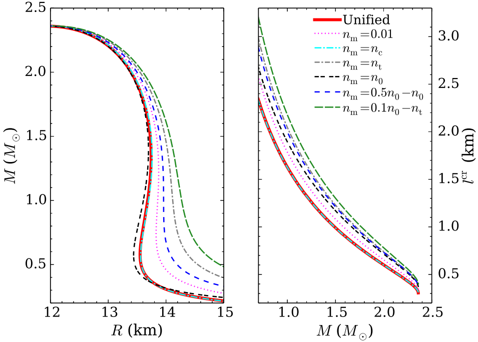
Recent work

Oertel et al. JPG (2015): hyperonic EoS consistent with Hebeler et al. constraint and with $M_{\max} \geq 2\,M_\odot$.

How to glue core and crust: NL3 & DH?

Fortin, Providência, Raduta, Gulminelli, Zdunik, Haensel, & Bejger, arXiv:1604.01944





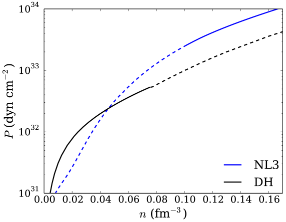
- core glued to BPS+BBP EOS at 0.01 fm^{-3};
- transition at the crossing density between the 2 EoSs;
- transition at the core-crust transition density n_t;
- transition at $n_0 = 0.16$ fm^{-3};
- crust below $0.5n_0$ and core above n_0;
- crust below $0.1n_0$ and core above n_t;
- reference: unified EoS.

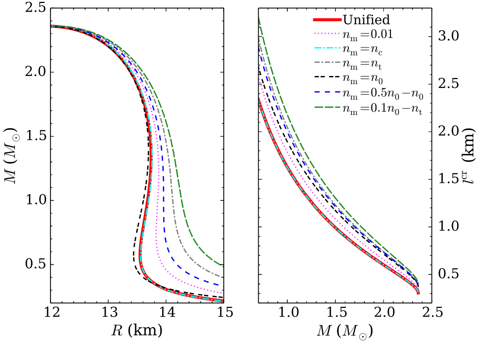
Uncertainty on R

- due to the treatment of the core-crust transition: up $\sim 4\%$ (up to $\sim 30\%$ on the crust thickness),
- decreases if crust and core EOS with similar saturation properties.

How to glue core and crust: NL3 & DH?

Fortin, Providência, Raduta, Gulminelli, Zdunik, Haensel, & Bejger, arXiv:1604.01944





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Uncertainty on R

- due to the treatment of the core-crust transition: up $\sim 4\%$
- with NICER, Athena or LOFT(?): expected precision $\sim 5\%$ . . . .
- how to, if not solve, at least handle this problem?

1. Thermodynamically consistent 'gluing'

In principle one should match P, ρ, n.

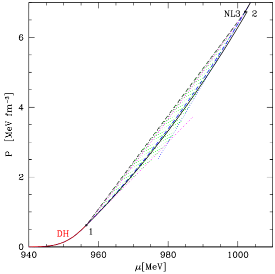
$n = \left(\frac{\mathrm{d}P}{\mathrm{d}\mu}\right)$, $\rho(\mu) = n(\mu)\mu - P(\mu)$.

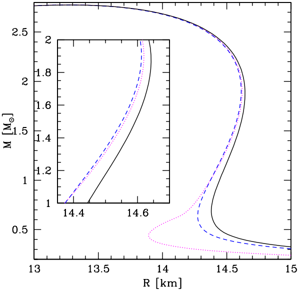
- thermodynamic consistency: n is an increasing function of $P \leftrightarrow P(\mu)$ is increasing and convex:

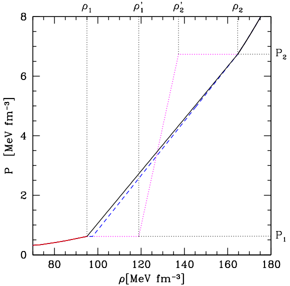
$$n_1 < \frac{P_2 - P_1}{\mu_2 - \mu_1} < n_2$$

- causality:

$$(\mathrm{d}P/\mathrm{d}\rho)^{1/2} = v_{\mathrm{sound}}/c \leq 1$$







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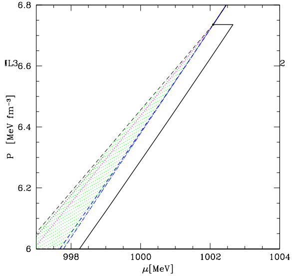

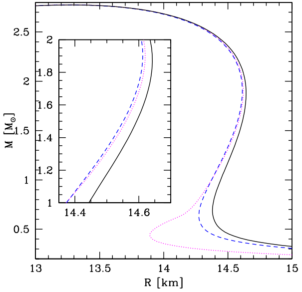

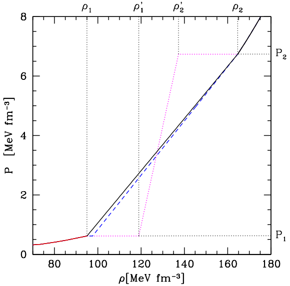

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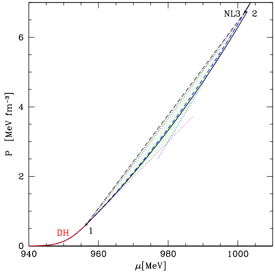
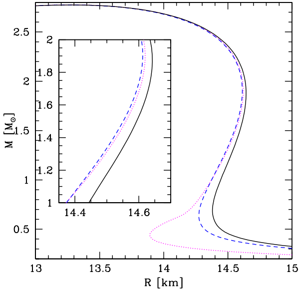
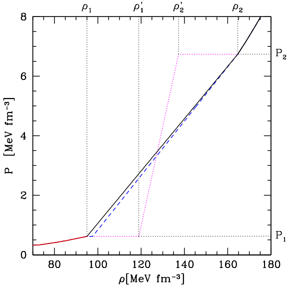






2. Unified equations of state

Very few unified EoSs for NSs exist eg. DH (Douchin & Haensel 2001), BSk (Brussels Uni.)

Fortin, Providência, Raduta, Gulminelli, Zdunik, Haensel, & Bejger, arXiv:1604.01944

9 RMF models

NL3, $NL3_{\omega\rho}$, DDME2, GM1, TM1, DDHδ, DD2, BSR2, and BSR6 with

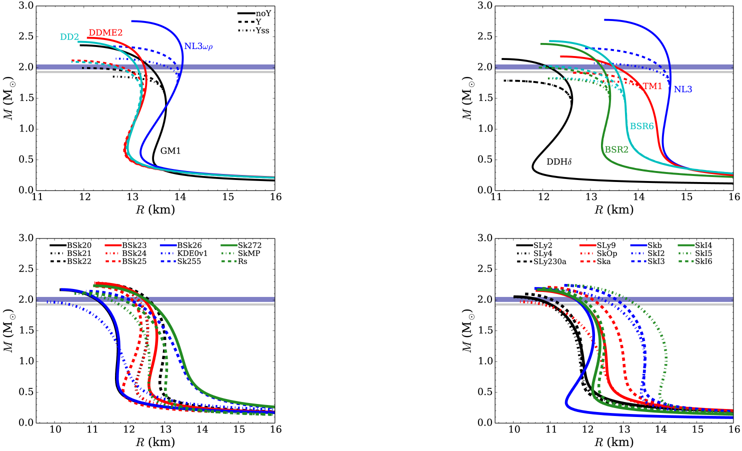
- outer-crust non consistently calculated but hardly affect the $M - R$ relations
- inner-crust with pasta phase from Thomas-Fermi calculations
- noY: a purely nucleonic core
- Y: transition to hyperonic matter in the core: SU(6) with the ϕ meson; $U_\Lambda^N(n_0) = -28$ MeV, $U_\Sigma^N(n_0) = 30$ MeV, $U_\Xi^N(n_0) = -18$ MeV
- Yss: transition to hyperonic matter in the core: SU(6) with the ϕ and σ^* mesons; $U_\Lambda^\Lambda(n_0) = -5$ MeV, $U_\Xi^\Xi \simeq 2U_\Lambda^\Lambda$, $g_{\sigma^*\Sigma} = g_{\sigma^*\Lambda}$

24 Skyrme models

Ska, Skb, SkI2, SkI3, SkI4, SkI5, SkI6, SLy2, SLy230a, SLy9, SkMP, SkOp, KDE0V, KDE0V1, Sk255, Sk272, Rs, BSk20, BSk21, BSk22, BSk23, BSk24, BSk25, and BSk26 with

- purely nucleonic core, causal up to 2 $M_\odot$
- compressible liquid drop model
- no shell effect and curvature terms

2. Unified equations of state



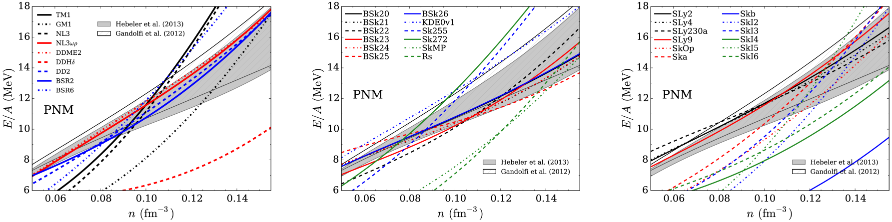
33 nucleonic EoSs and 15 hyperonic EoSs

- tables with n, ρ, P as supplemental material to the paper
- available on the open-source CompOSE database: `http://compose.obspm.fr/`

Comparison with nuclear constraints

1. Low-density

Hebeler et al. ApJ (2013): chiral effective field theory;
Gandolfi et al. PRC (2012): Quantum Monte Carlo technique



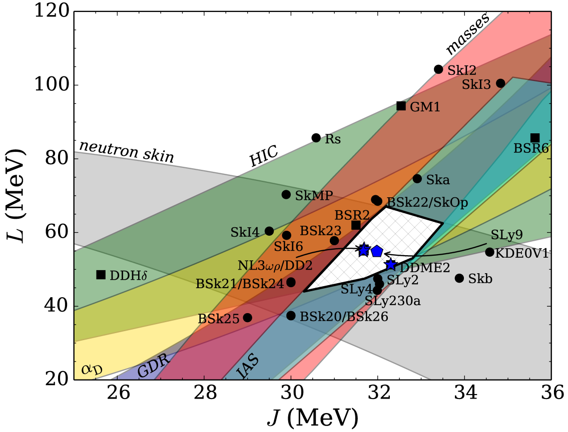
- RMF: DDME2; $\pm 10\%$: NL3$\omega\rho$, DD2
- Skyrme: SLy2, KDE0v, KDE0v1; $\pm 10\%$: SLy9

2. Incompressibility

$K = 230 \pm 40$ MeV: all but GM1 and TM1

Comparison with nuclear constraints

3. $L - J$ plane



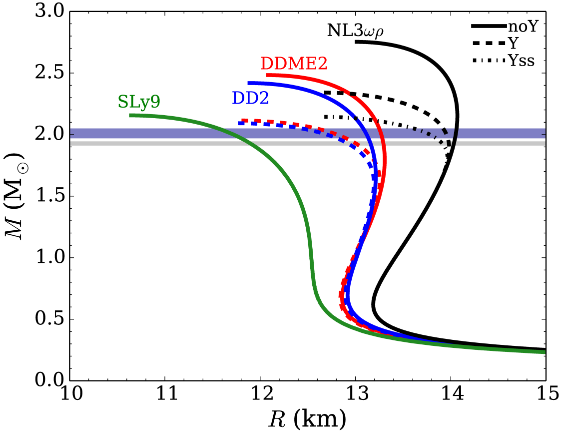
- neutron skin thickness of ^{208}Pb
- heavy ion collisions (HIC)
- electric dipole polarizalibility α_D
- giant dipole resonance (GDR) of ^{208}Pb
- measured nuclear masses
- isobaric analog states (IAS)

References: see eg. Lattimer & Steiner, EPJA (2015)

Comparison with nuclear constraints

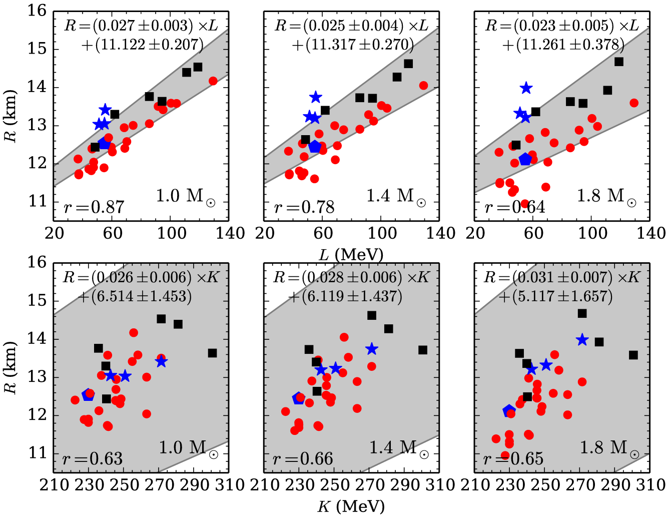
EOS fulfilling:

- all constraints 1.+2.+3.: DDME2
- constraint 1.$\pm$10%+ constraints 2.+3.: DD2, NL3$\omega\rho$ and SLy9.



$R_{1.4} = 13.10 \pm 0.65$ km.

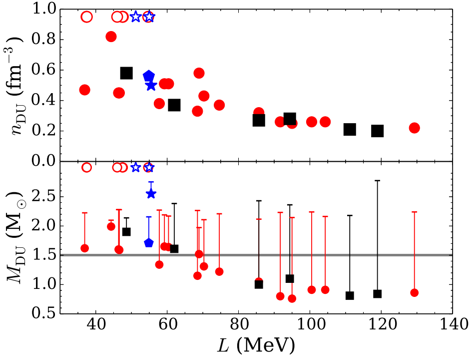
Correlations



- R vs L: dispersion larger for higher masses due to higher order terms
- R vs K: for isoscalar properties higher order terms can not be neglected

Nucleonic DUrca process

- $n \to p + e^- + \bar{\nu}_e$ and $p + e^- \to n + \nu_e$
- momentum conservation $\to$ density $n_{\rm DU}$ and mass $M_{\rm DU}$ threshold



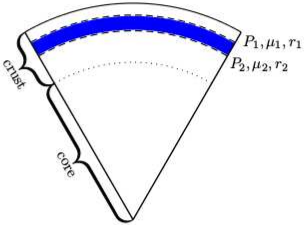
Additional DUrca processes for hyperonic EOS.

- Beznogov & Yakovlev MNRAS (2015): DUrca process needed to explain the thermal emission of isolated and accreting NS.
- Popov et al. A&A (2006): population synthesis of isolated NS requires $M_{\rm DU} > 1.5\, M_\odot$.
- For $L \gtrsim 70$ MeV, DUrca process always on for $M > 1.5\, M_\odot$.
- For $L \lesssim 70$ MeV, EOS with DUrca and others without.
- $L - J$ plane: the intersection of all constraints gives $L \lesssim 70$ MeV.

3. Approximate formula for the radius and crust thickness

Zdunik, Fortin, and Haensel, in prep.

Thickness of a shell in a catalyzed crust



Assuming that in the crust $m \approx M$ and $4\pi r^3 P/mc^2 \ll 1$ in the TOV equation one obtains:

$$\frac{\mathrm{d}P}{\rho + P/c^2} = -GM \frac{\mathrm{d}r}{r^2(1 - 2GM/rc^2)} \ .$$

With

$$\frac{\mathrm{d}P}{\rho c^2 + P} = \frac{\mathrm{d}\mu}{\mu} \qquad \text{one gets} \qquad \frac{\sqrt{1 - 2GM/r_2 c^2}}{\sqrt{1 - 2GM/r_1 c^2}} = \frac{\mu_2}{\mu_1}$$

valid for no jump in the chemical potential.

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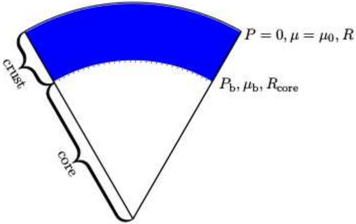

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valid for no jump in the chemical potential.
Taking $r_1 = R$ and $r_2 = R_{\rm core}$

$$\frac{\sqrt{1-2GM/Rc^2}}{\sqrt{1-2GM/R_{\rm core}c^2}} = \frac{\mu_{\rm b}}{\mu_0}$$

with $\mu_0 = \mu(P=0) = 930.4$ MeV - minimum energy per nucleon of a bcc lattice of ^{56}Fe and $\mu_{\rm b}$ at the core-crust transition.

3. Approximate formula for the radius and crust thickness

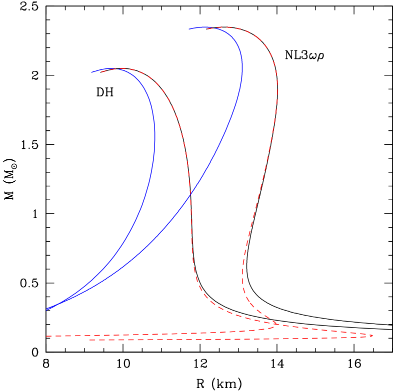
Zdunik, Fortin, and Haensel, in prep.

- All you need is ...: the core EOS down to a chosen density $n_{\rm b}$ with $\mu(n_{\rm b}) = \mu_{\rm b}$.
- Obtain the $M(R_{\rm core})$ relation solving the TOV equations.
- Obtain $M(R)$ with

$$R = R_{\rm core} / \left(1 - \left(\frac{\mu_{\rm b}^2}{\mu_0^2} - 1\right)\left(\frac{R_{\rm core} c^2}{2GM} - 1\right) \right).$$

Results

- uncertainty in the radius: $\lesssim 0.2\%$ for $M > 1\ M_\odot$
- uncertainty in the crust thickness: $\lesssim 1\%$ for $M > 1\ M_\odot$



Solution of the TOV equation with a unified EoS
TOV solution for the core $M(R_{\rm core})$
Approximate $M(R)$ for $n_{\rm b} = 0.077\ {\rm fm}^{-3}$

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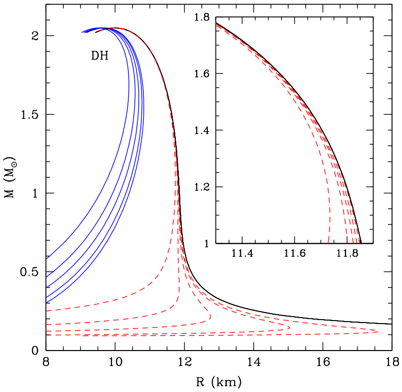
  $R =$
  $R_{\rm core} / \left(1 - (\frac{\mu_{\rm b}^2}{\mu_0^2} - 1)(\frac{R_{\rm core} c^2}{2GM} - 1)\right).$

How to choose the core-crust transition density $n_{\rm b}$?

- inversely proportional to L (Horowitz & Piekarewicz 2001)
- Ducoin et al. (2011): for EOS with $30 \leq L \leq 120$ MeV, obtain: $0.06 \lesssim n_{\rm b} \lesssim 0.10\ {\rm fm}^{-3}$

$\Rightarrow$ $n_{\rm b} \simeq n_0/2 = 0.08\ {\rm fm}^{-3}$



Solution of the TOV equation with a unified EoS
TOV solution for the core $M(R_{\rm core})$
Approximate $M(R)$ for $n_{\rm b} = 0.16, 0.13, 0.11, 0.09, 0.077\ {\rm fm}^{-3}$ from left to right.

3. Approximate formula for the radius and crust thickness

Zdunik, Fortin, and Haensel, in prep.

Thickness of a shell in an accreted crust

For a catalyzed crust

$$\frac{\sqrt{1-2GM/r_2c^2}}{\sqrt{1-2GM/r_1c^2}} = \frac{\mu_2}{\mu_1}$$

For an accreted crust

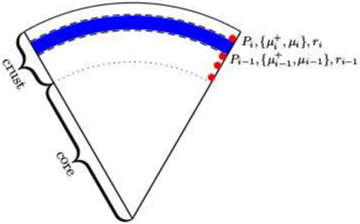
$$\frac{\sqrt{1-\frac{2GM}{Rc^2}}}{\sqrt{1-\frac{2GM}{R_1c^2}}} = \frac{\mu_1^+}{\mu_0}$$

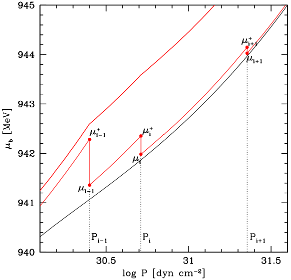
$$\ldots$$

$$\frac{\sqrt{1-\frac{2GM}{R_ic^2}}}{\sqrt{1-\frac{2GM}{R_{i+1}c^2}}} = \frac{\mu_{i+1}^+}{\mu_i}$$

$$\ldots$$

$$\frac{\sqrt{1-\frac{2GM}{R_nc^2}}}{\sqrt{1-\frac{2GM}{R_{\rm core}c^2}}} = \frac{\mu_{\rm b}}{\mu_n}$$





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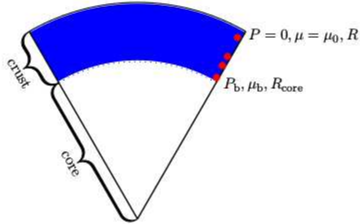


For a catalyzed crust

$$\frac{\sqrt{1 - 2GM/r_2c^2}}{\sqrt{1 - 2GM/r_1c^2}} = \frac{\mu_2}{\mu_1}$$

For an accreted crust

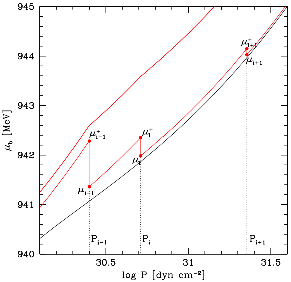
$$\frac{\sqrt{1 - \frac{2GM}{Rc^2}}}{\sqrt{1 - \frac{2GM}{R_{\rm core}c^2}}} = \frac{\mu_1^+}{\mu_1} \cdot \frac{\mu_2^+}{\mu_2} \cdots \frac{\mu_i^+}{\mu_i} \cdots \frac{\mu_n^+}{\mu_n} \cdot \frac{\mu_{\rm b}}{\mu_0}$$

$$= \frac{\mu_{\rm b}}{\mu_0} \cdot \prod_{i=1}^{n} \frac{\mu_i^+}{\mu_i}$$

Energy release at P_i: $Q_i = \mu_i^+ - \mu_i$.

$$\frac{\sqrt{1 - \frac{2GM}{Rc^2}}}{\sqrt{1 - \frac{2GM}{R_{\rm core}c^2}}} \simeq \frac{\mu_{\rm b}}{\mu_0} \cdot (1 + \frac{Q^{\rm tot}}{\mu_{\rm IC}})$$

with $Q^{\rm tot} = \sum_{i=1}^{n} Q_i$ the total energy release in the crust and the mean chemical potential in the inner-crust $\mu_{\rm IC} \simeq 941\ {\rm MeV}$.



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Catalyzed vs. accreted crusts

$$\frac{\sqrt{1-\frac{2GM}{Rc^2}}}{\sqrt{1-\frac{2GM}{R_{\rm core}c^2}}} \simeq \frac{\mu_{\rm b}}{\mu_0}\cdot\left(1+\frac{Q^{\rm tot}}{\mu_{\rm IC}}\right)$$

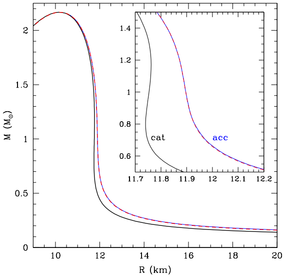
Radius of a star with a catalyzed crust: $R_{\rm cat}$
with an accreted crust $R_{\rm acc}$.

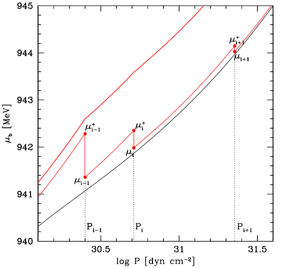
$$R_{\rm acc} = \frac{R_{\rm cat}}{1-(\alpha-1)\left(\frac{R_{\rm cat}c^2}{2GM}-1\right)}$$

with $\sqrt{\alpha} \equiv \prod_{i=1}^{n}\frac{\mu_i^+}{\mu_i} = \left(1+\frac{Q^{\rm tot}}{\mu_{\rm IC}}\right)$.

Difference in the radius between a NS with an accreted crust and a catalyzed crust

$$\Delta R \simeq 144\,{\rm m}\cdot\left(\frac{Q^{\rm tot}}{2\,{\rm MeV}}\right)\left(\frac{R_{\rm cat}}{10\,{\rm km}}\right)^2\left(\frac{M}{M_\odot}\right)\left(1-\frac{2GM}{R_{\rm cat}c^2}\right)$$





Conclusions

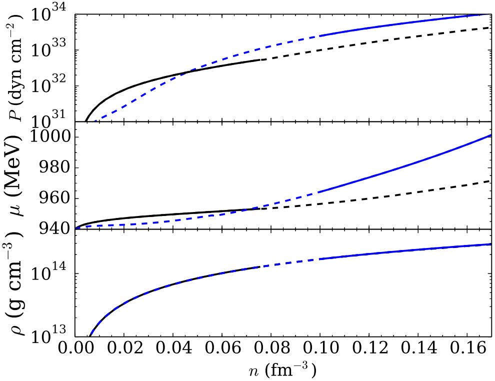
- Most hyperonic EoSs consistent with 2 $M_\odot$ have a large $R_{1.4}$ and overpressure close to saturation density (Fortin+, A&A 2015)
- Treatment of the gluing of non-unified core and crust EoSs introduces an uncertainty on the radius that can be as large as the expected precision from NICER, Athena or LOFT(?) (Fortin+, arXiv:1604.01944)
- Development of unified nucleonic and hyperonic EoSs based on 9 RMF and 24 Skyrme models (Fortin+, arXiv:1604.01944);
- available on the CompOSE database: `http://compose.obspm.fr` and as supplemental material to the paper;
- confrontation with nuclear constraints and selection of 4 EoS (Fortin+, arXiv:1604.01944).
- Approximate formula for $M(R)$ as a function of $M(R_{\rm core})$ for catalyzed and accreted crusts. (Zdunik+, in prep.)

Perspectives

- Fits by piecewise polytropes of the unified EOS for various applications.
- Study of rotating NS (Keplerian frequency, minimum mass, ...) with `LORENE` and of the surface gravitational redshift (spectral lines).
- Development of more EOS consistent with Hebeler et al. constraint and 2 $M_\odot$.

1. Thermodynamically consistent ‘gluing’

Example: as a function of n



Matched EOS

For $n_1 < n < n_2$:

- crust: SLy4 for $n \leq n_1 = 0.076$ fm^{-3},
- core: NL3 for $n \geq n_2 = 0.1$ fm^{-3}

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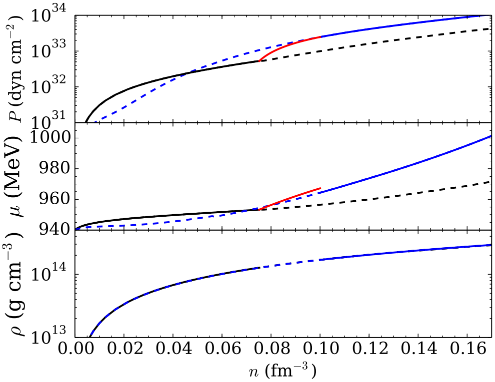

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Matched EOS

For $n_1 < n < n_2$:

- assume a form for $P(n)$.
- $\mu(n) = \mu(n_1) + \int_{n_1}^{n} \frac{\mathrm{d}P(n)}{n}$
- $\rho(n) = n\mu(n) - P(n)$

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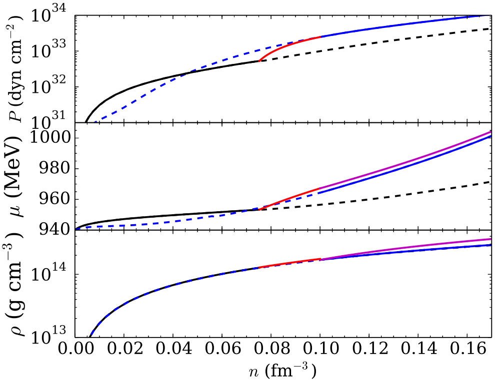

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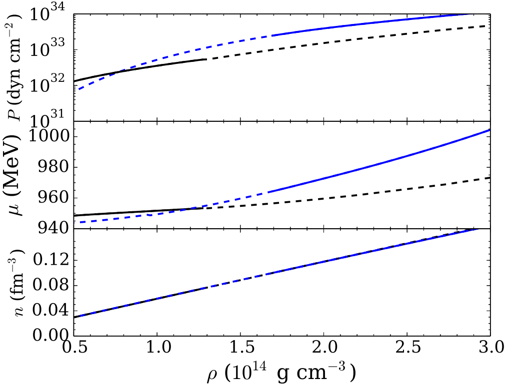
- but in general jump in μ at n_2
- for $n > n_2$ $\mu(n) \rightarrow \mu(n) + (\mu(n_2) - \mu_{\rm co}(n_2))$
- for $n > n_2$, $\rho(n) \rightarrow \rho(n) + (n - n_2)(\mu(n_2) - \mu_{\rm co}(n_2))$.

But $P(\rho)$ enters the TOV equation $\rightarrow M - R$ relation affected.

1. Thermodynamically consistent ‘gluing’

Example: as a function of ρ



Matched EOS

For $\rho_1 < \rho < \rho_2$:

- crust: SLy4 for $\rho \leq \rho_1 = 1.27 \times 10^{14}$ g cm^{-3},
- core: NL3 for $\rho \geq \rho_2 = 1.69 \times 10^{14}$ g cm^{-3}

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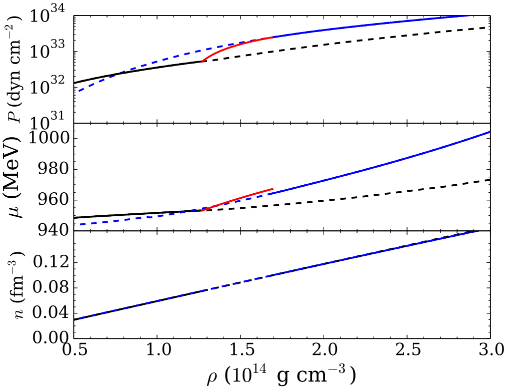
Example: as a function of ρ



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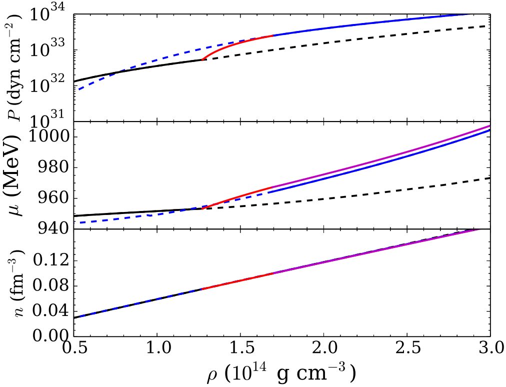
Matched EOS

For $\rho_1 < \rho < \rho_2$:

- assume a form for $P(\rho)$.
- $n(\rho) = n_1 \exp\left(\int_{\rho_1}^{\rho} \frac{\mathrm{d}\rho}{P(\rho)+\rho}\right)$.

1. Thermodynamically consistent ‘gluing’

Example: as a function of ρ



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For $\rho_1 < \rho < \rho_2$:

- assume a form for $P(\rho)$.
- $n(\rho) = n_1 \exp\left(\int_{\rho_1}^{\rho} \frac{\mathrm{d}\rho}{P(\rho)+\rho}\right)$.
- but in general jump in μ at ρ_2
- for $\rho > \rho_2$, $n(\rho) = n_{\mathrm{co}}(\rho) n(\rho_2)/n_{\mathrm{co}}(\rho_2)$.

$P(\rho)$ and $M-R$ relation unaffected but the microscopic approach given by $n(\rho)$ modified.