Small bits of cold dense matter

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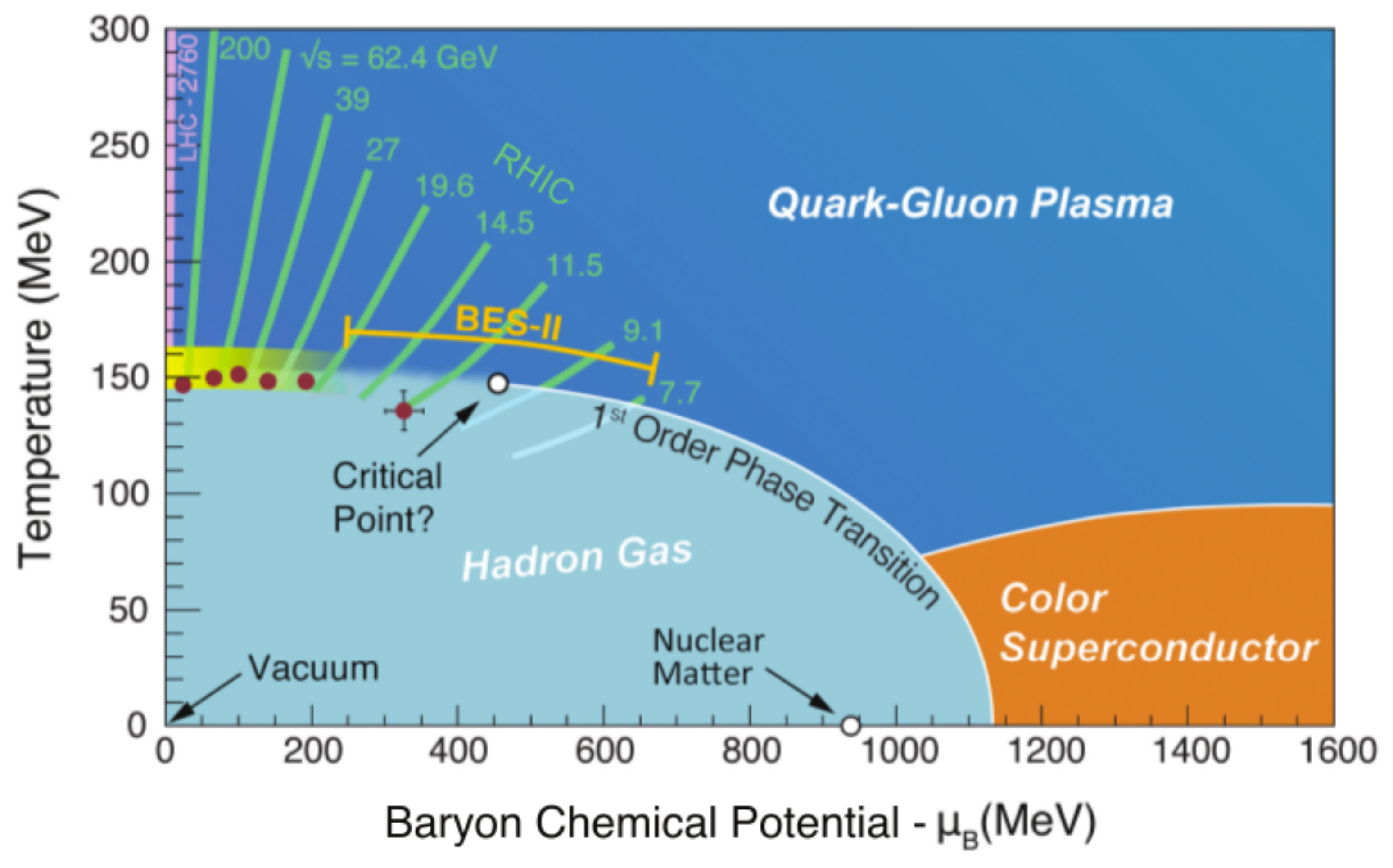
- QCD EOS (T~0 at high densities) and neutron stars
- Dense fermionic matter and the sign problem
- Degrees of Freedom at different densties
- Exploiting Boundary conditions for small particle number
- Selected results for N=3, 4, 6, 8, 10, 14
- Comparison to free quarks
- Outlook

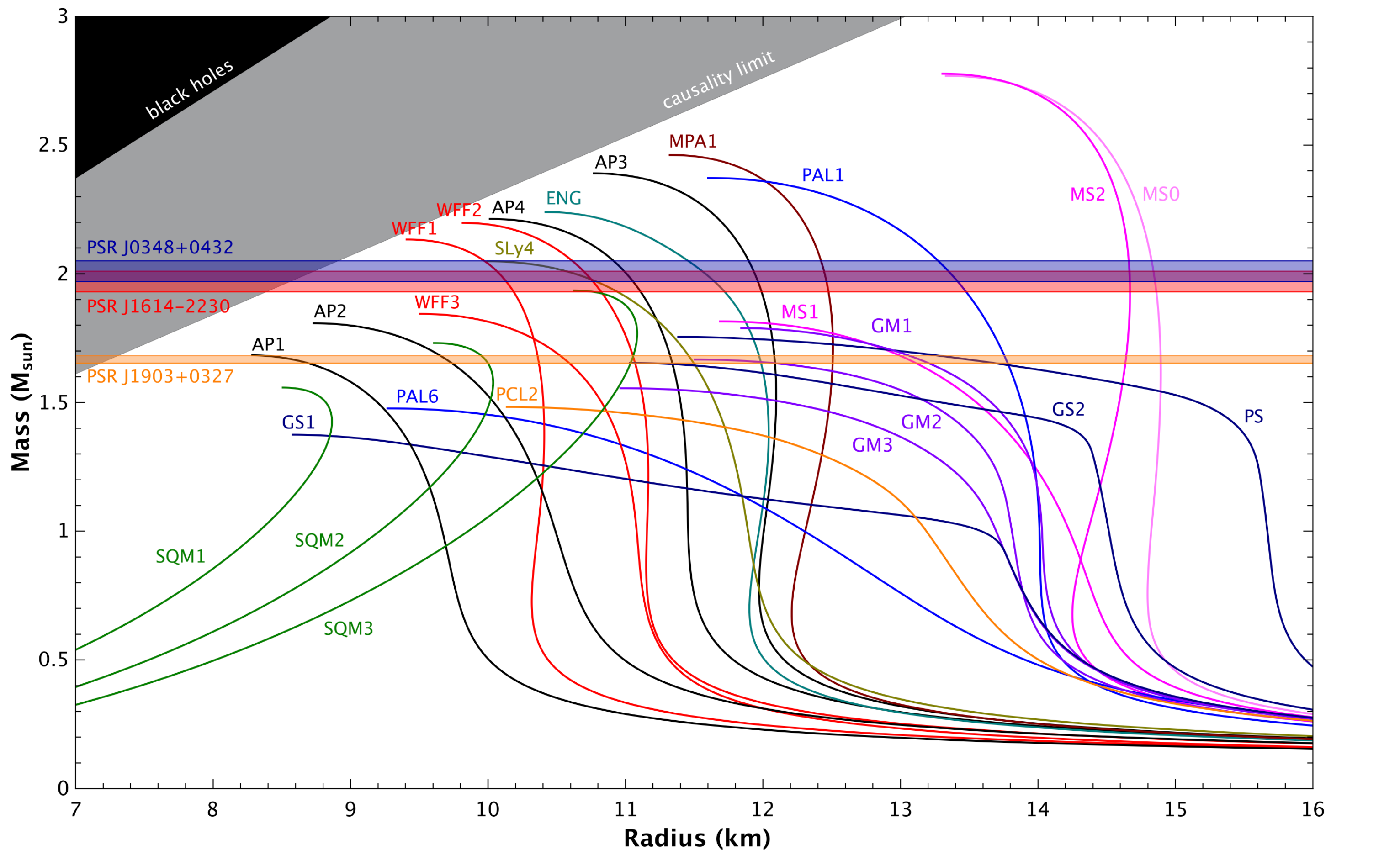
from USQCD.org



Impressive lattice QCD at **μ** = 0, and exploratory studies at **μ** > 0

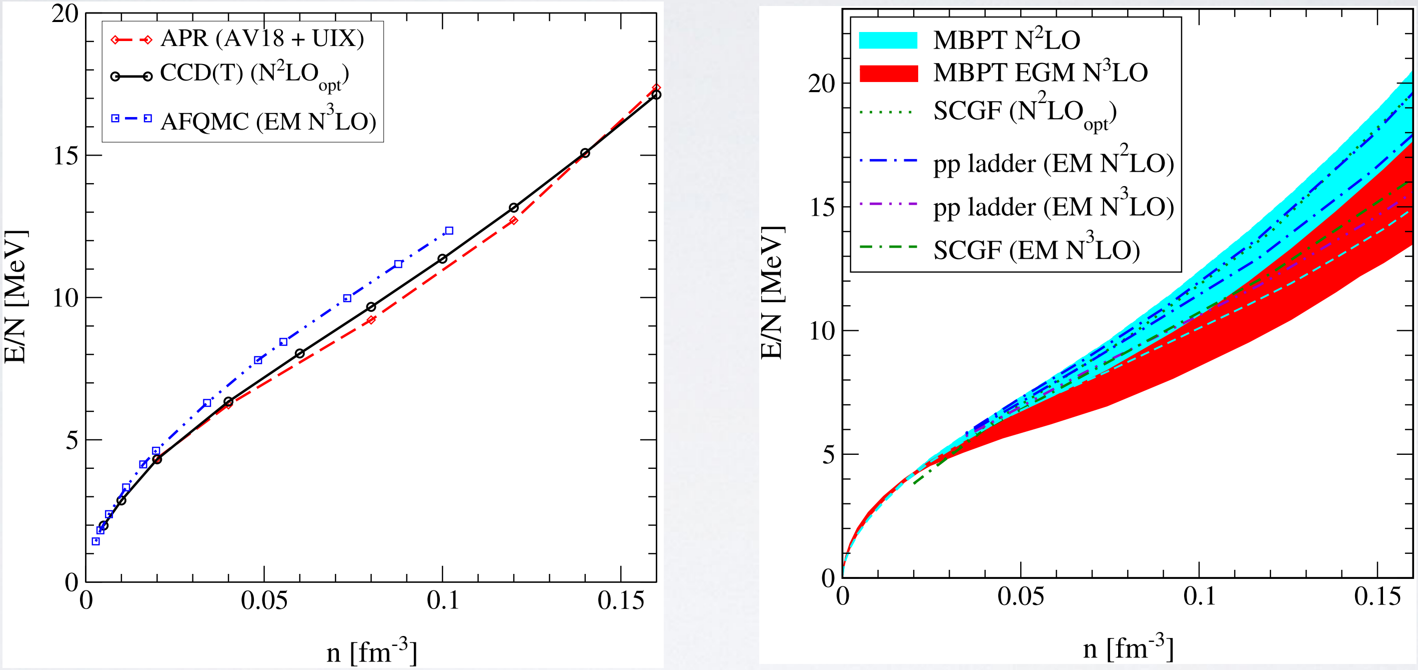
Little work at T ~ 0 for large **μ**

Neutron Star Mass Radius Relations



Selection of Mass /Radius relations before 2-solar mass neutron star observations.

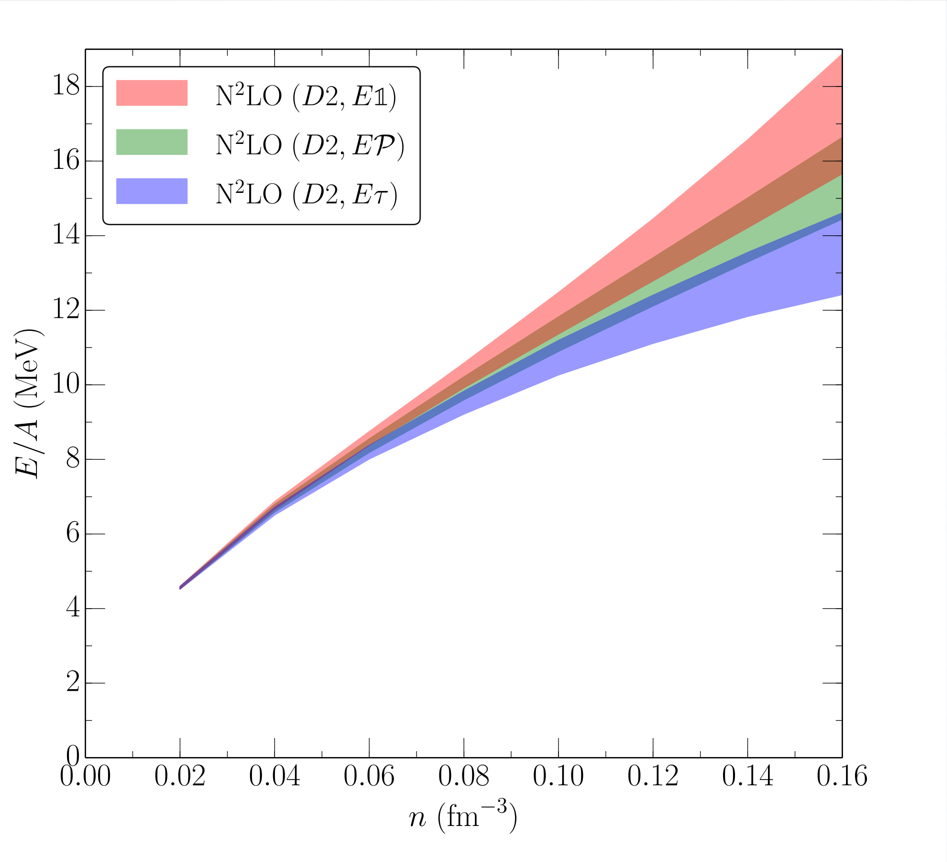
Low to Moderate Densities:
Nucleons (neutrons) primary degrees of freedom



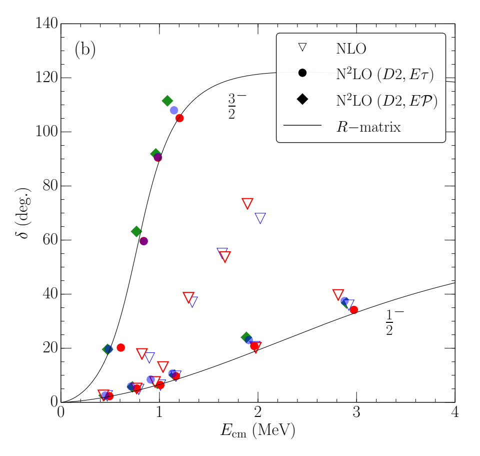
Gandolfi, Gezerlis, Carlson; Ann. Rev. Nucl. Part. Sci. 65, 303 (2015)

Reasonable agreement with different NN and 3N interactions
Uncertainty still has significant impact on mass-radius relation

Neutron Matter EOS w/ chiral 2N and 3N interactions at N2LO

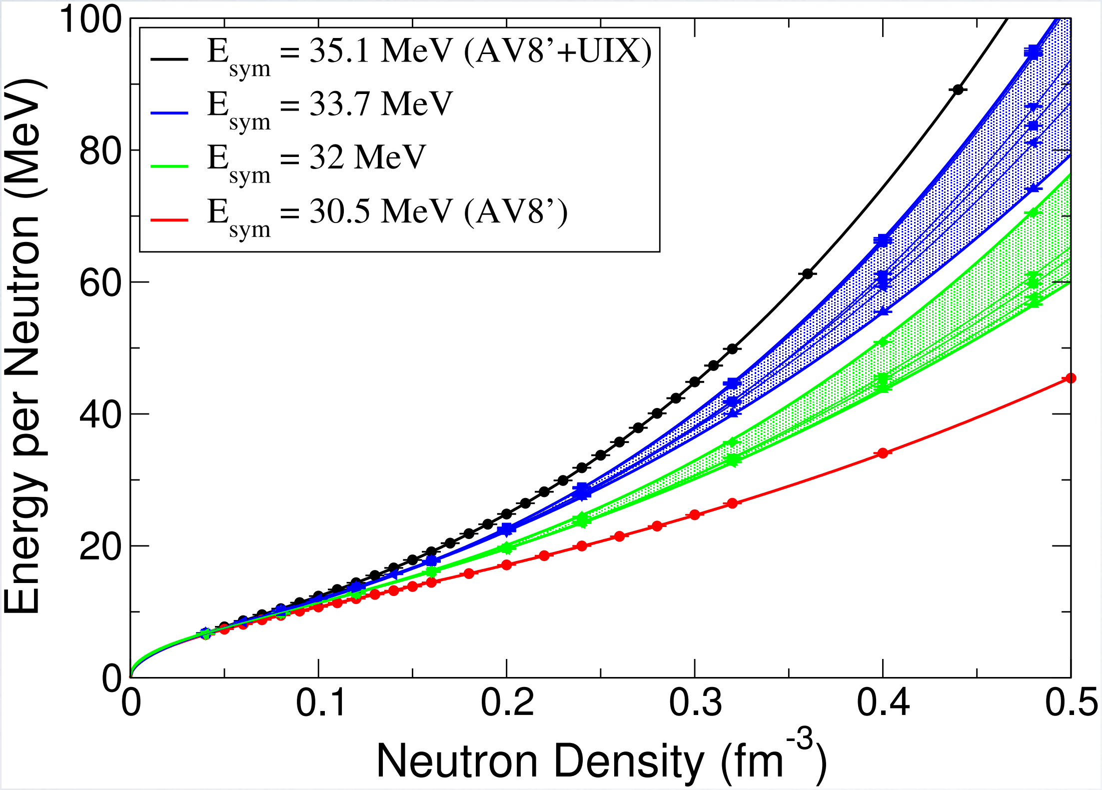


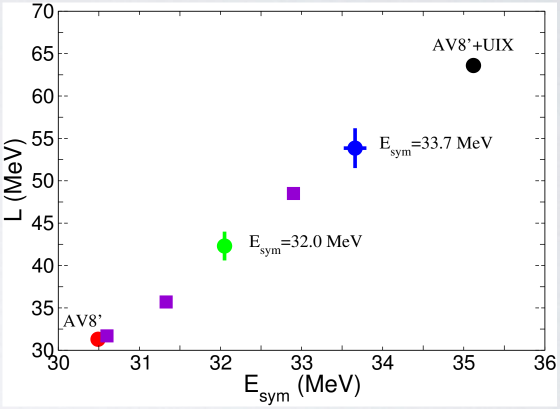
Lynn, et al., PRL 2016



- TNI fit to A=4 binding, n-alpha scattering
- Significant uncertainties from regulators, Fierz rearrangements, …
- Would like to reduce these uncertainties

Neutron Star Mass/Radius
w/ AV18 NN and UIX, Ill, and other TNI

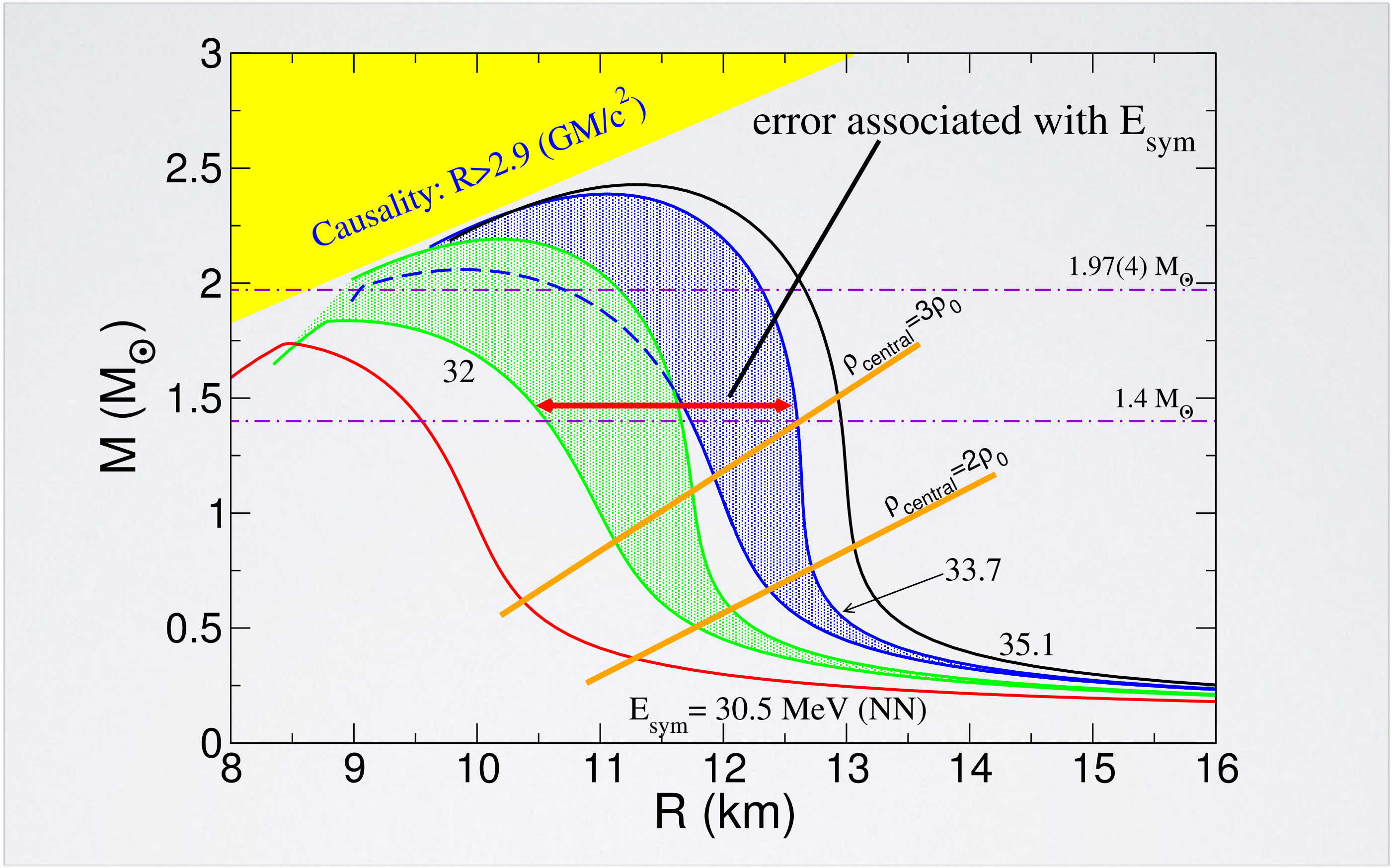




from: Gandolfi, Carlson, Reddy, arXiv:1101.1921; PRC 2013

Low energy nuclear experiments primarily sensitive to $\boldsymbol{\rho} \lesssim \boldsymbol{\rho}_0/2$

Mass/Radius relationships for different symmetry energies



How to further reduce uncertainty at 1-4 times saturation density?
Where is the phase transition ?

Fermion Sign Problem

Exponential decay in signal to noise for quantum fermi systems
Ubiquitous: electrons, cold atoms, helium, cold atoms, NP, LQCD

Decay proportional to Bose minus Fermi Energy
QCD : A (M_N - (3/2) M)
Nucleons: A x Fermi Energy

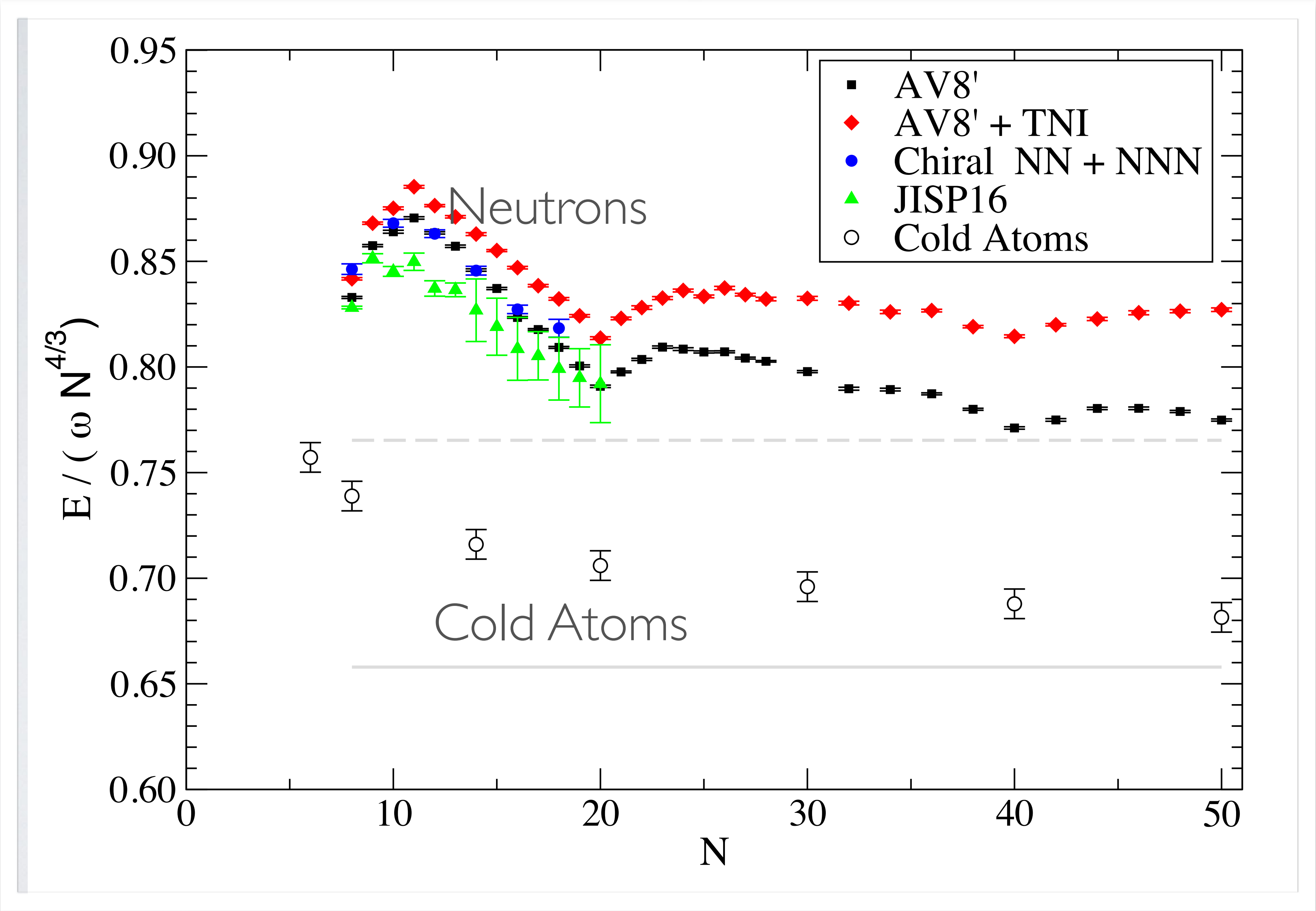
No general solution - exponentially difficult for large A

Try small A - make direct comparisons
to lattice at moderate to high densities
not necessary to go through S-matrix

Advantages: small boxes give large gaps, high excitation energies
can probe different N, boundary conditions, quantum numbers,…

Can we calibrate nuclear interactions?
Can we extrapolate to matter?
Can we begin to identify the phase transition?

Small quantum systems can identify important degrees of freedom



Gandolfi, Gezerlis, Carlson; Ann. Rev. Nucl. Part. Sci. 65, 303 (2015)

Small Bits of Neutron Matter (or neutron star matter)

QMC: Variational, GFMC, and AFDMC

Trial Wave function (s-wave BCS form) :

$$\begin{aligned}\Psi &= \prod_{i<j} [\sum_k F^k(r_{ij}) O^k(ij)]\ \Phi \\ \Phi &= \mathcal{A}\ \prod [\phi(r_{ij})(\uparrow_i \downarrow_j - \uparrow_j \uparrow_i)]\end{aligned}$$

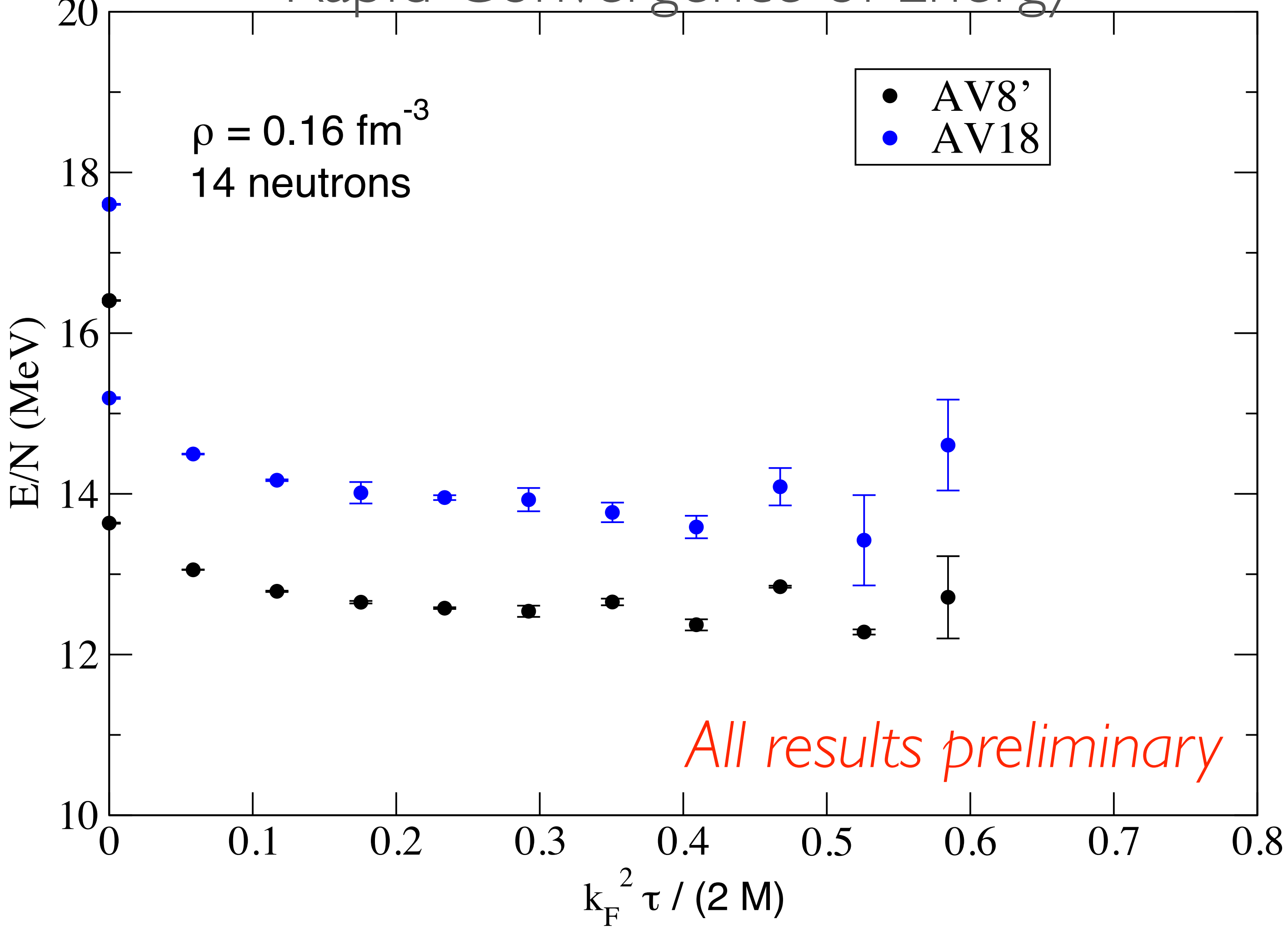
Specific choice of quantum numbers:

P=0, symmetric under all rotations

not the lowest energy state in all cases

14 Neutrons at Saturation Density w/ PBC:

Rapid Convergence of Energy



L = 4.4 fm, E_F = 55 MeV, **Δ** ~ 2 E_F ~ 100 MeV

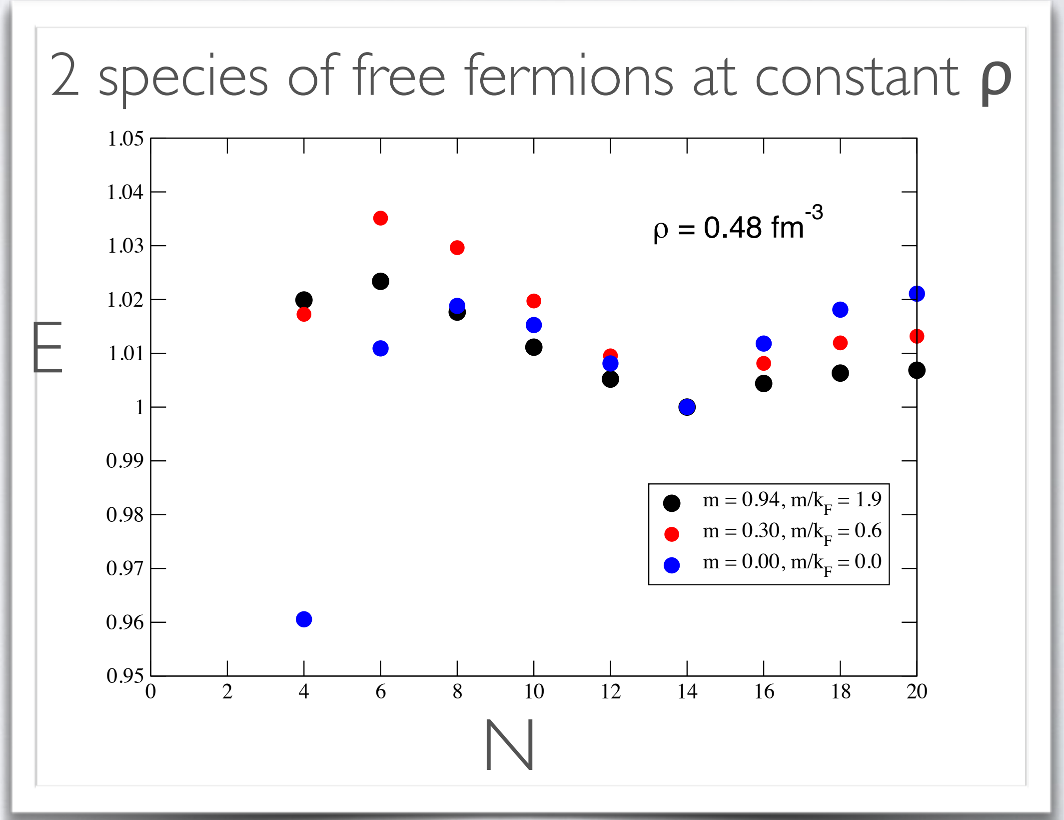
Closed shell numbers change with boundary conditions

Small system results

Do not necessarily need to connect to S-matrix,
just compare energies for specific BC, N, quantum numbers
small systems: single-particle spectra very important

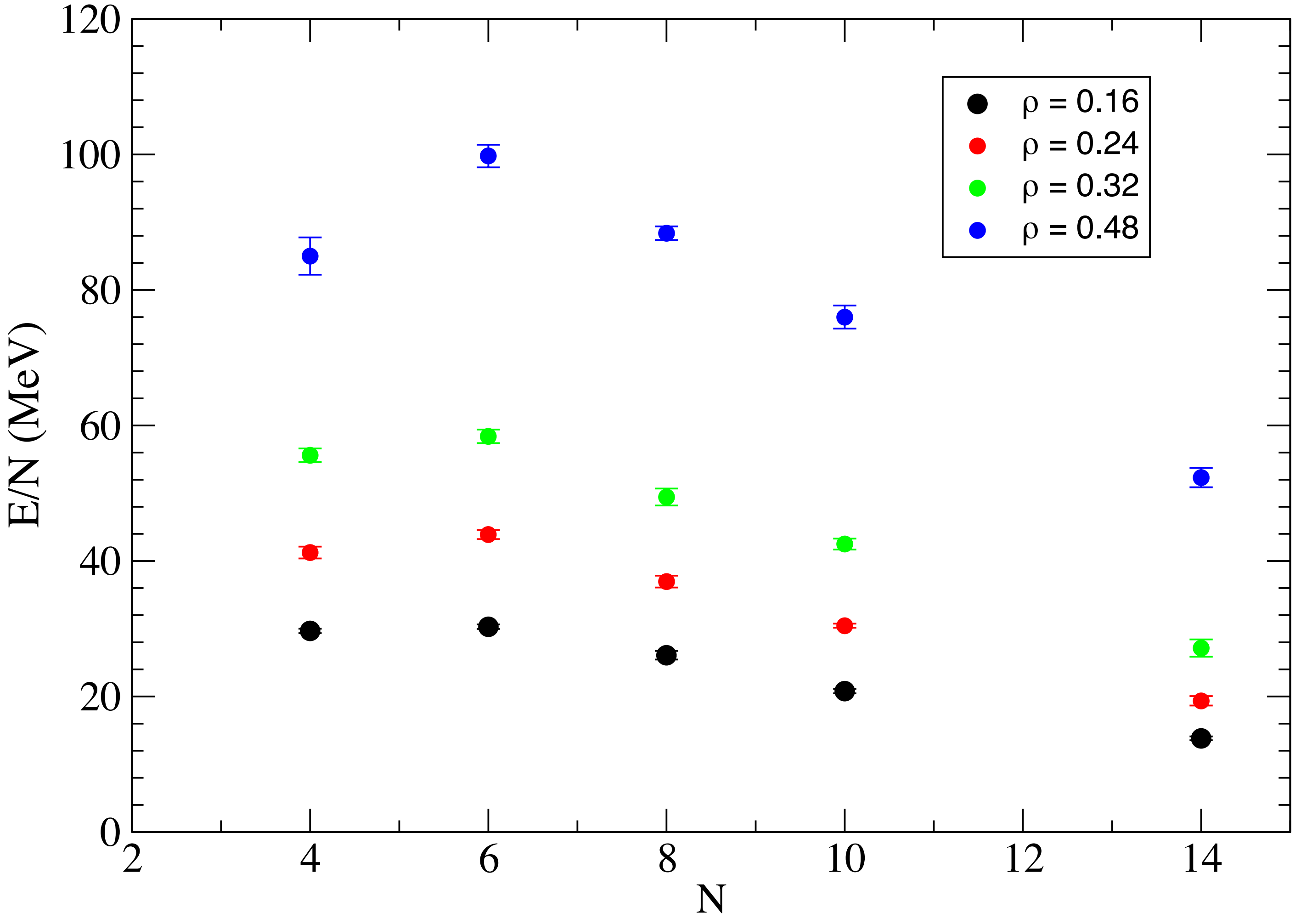
Results depend upon:
- degrees of freedom (degeneracies)
- relativistic vs. non-relativistic dispersion
- closed shells

Valuable to have results for:
- particle number
- boundary conditions
- quantum numbers
- small charges
- strangeness
- . . .

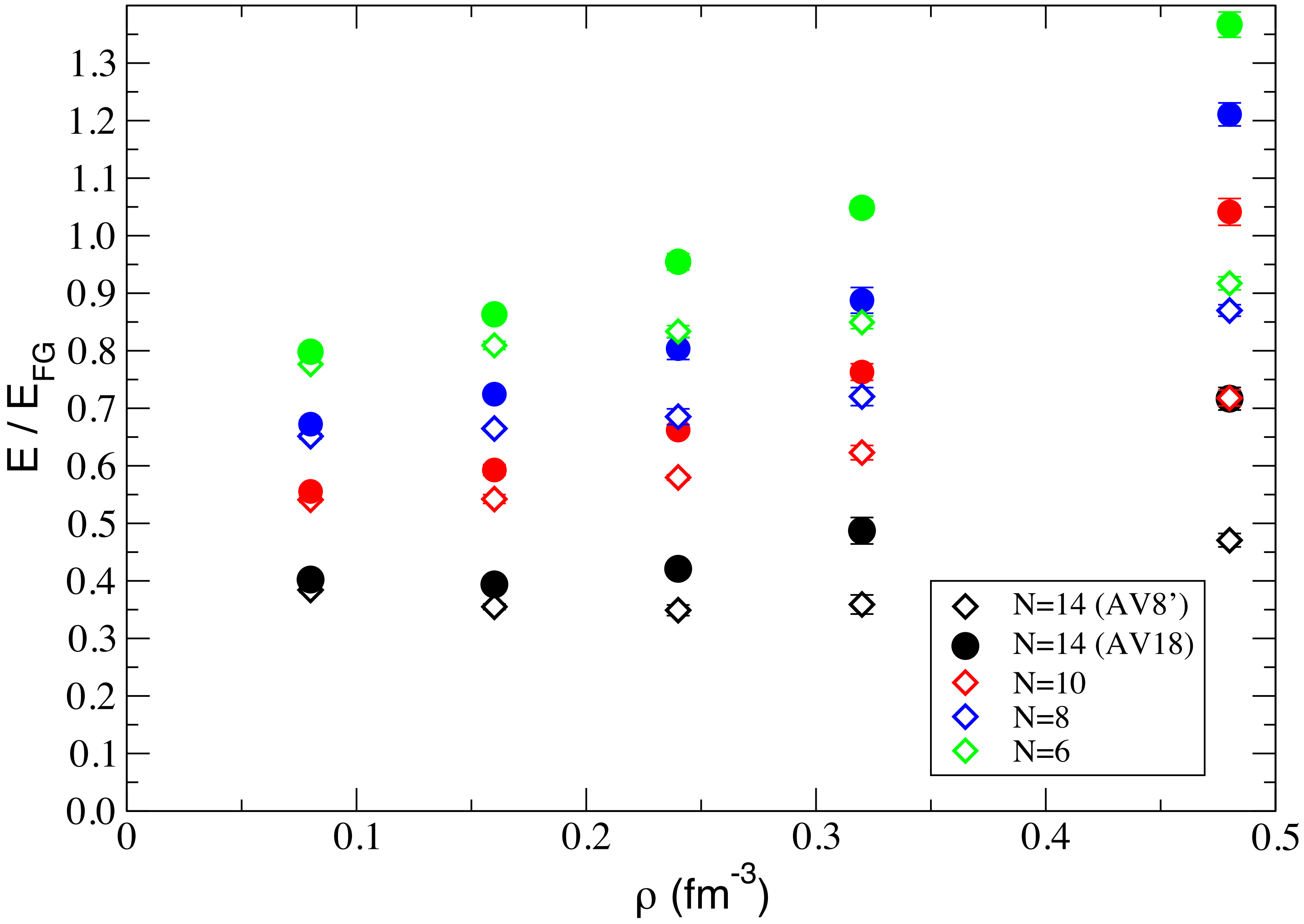


2 species of free fermions at constant ρ

Neutrons: energies versus N at constant density (AV18, PBC)



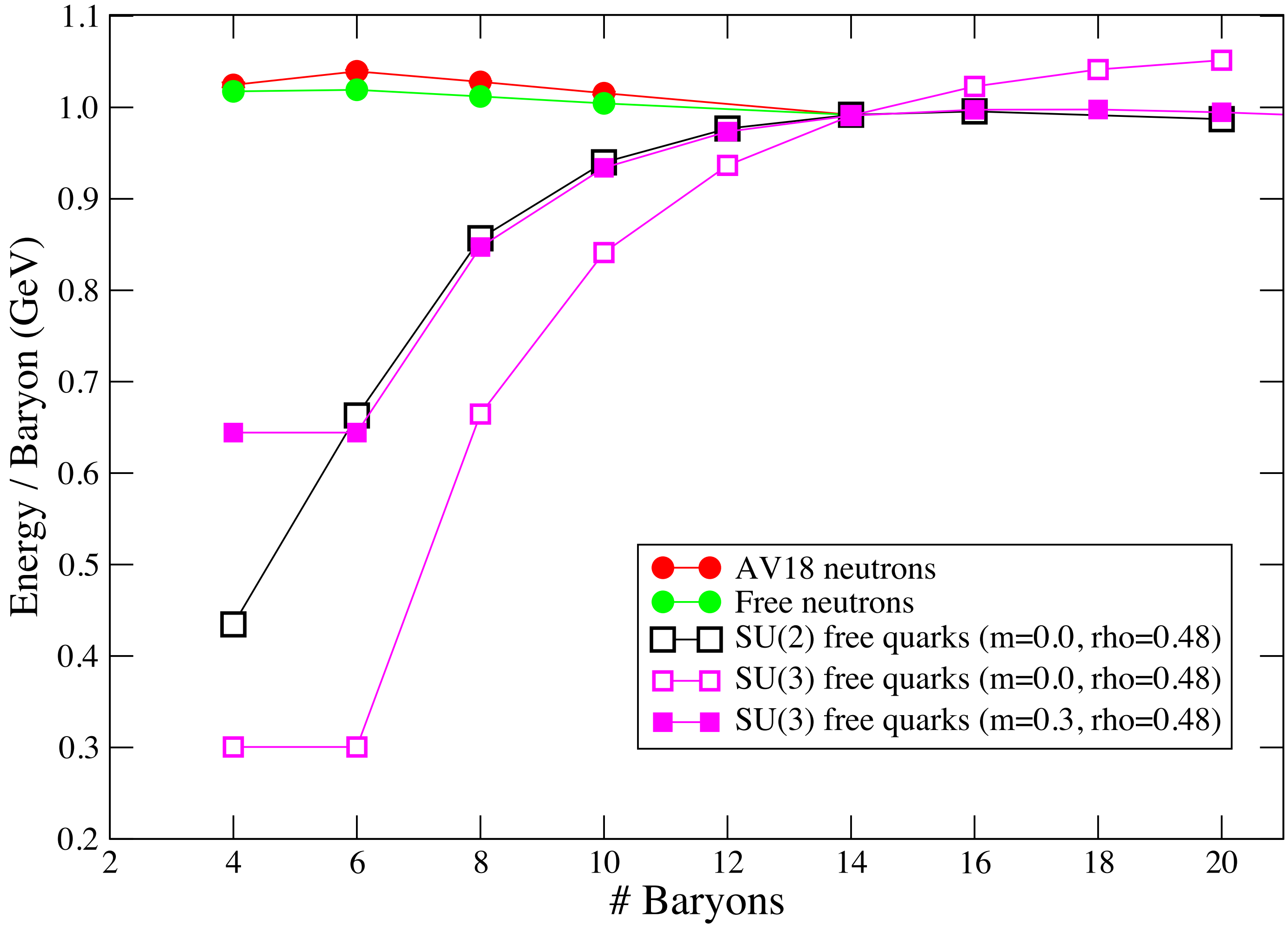
Energy versus density for different N



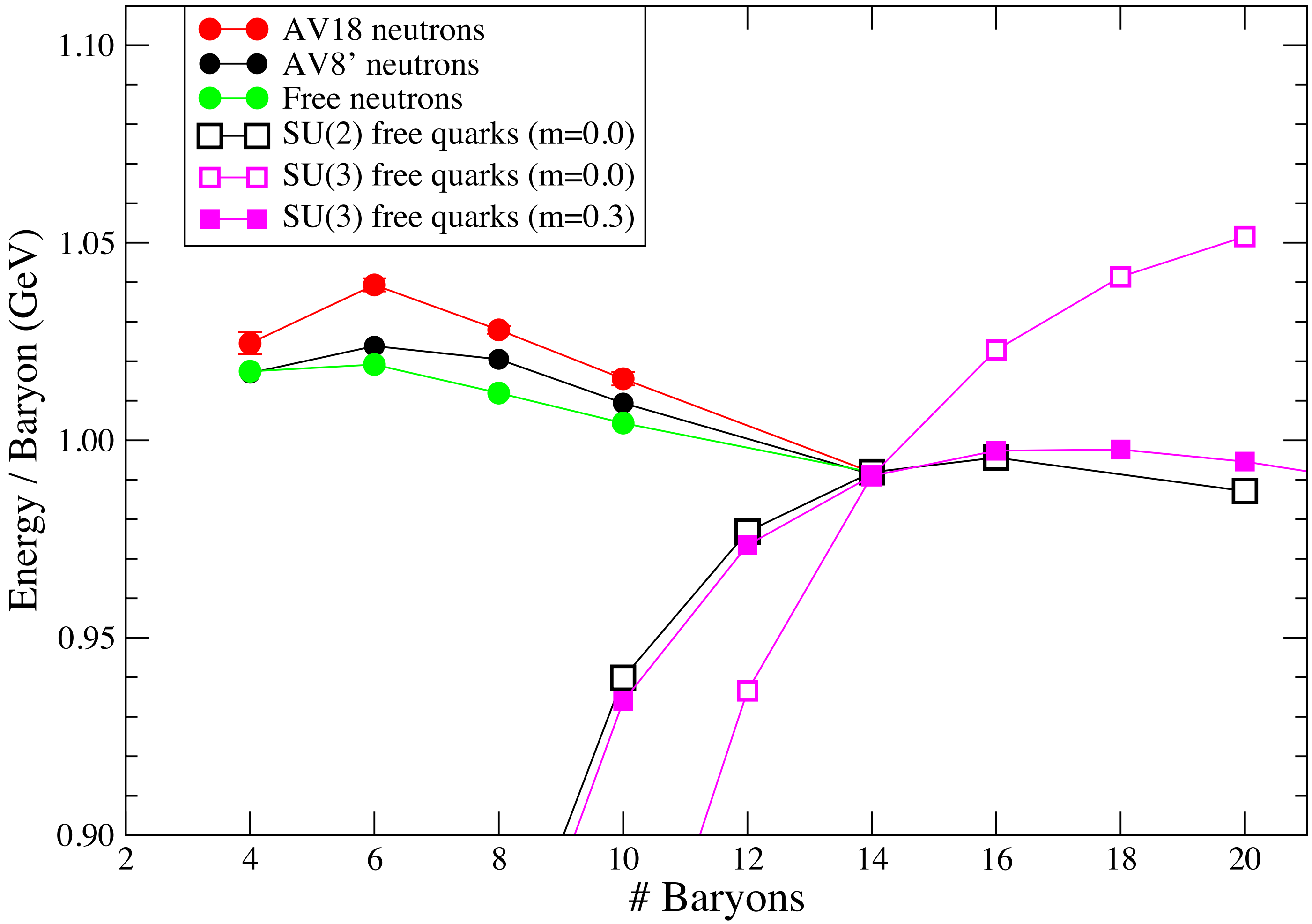
should revisit questions of sound speed with AV18 + TNI

Degrees of freedom can have a huge impact:

comparison of neutrons to free quarks



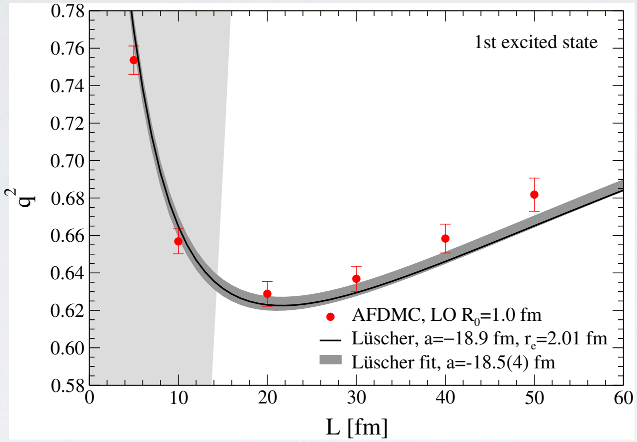
Zoom in to N ~ 10, E/A around 1 GeV



very small systems: N = 2, 3

N=2 Connections to NN scattering

1st excited state and Luscher formula



Kloss, Lynn, Tews, et. al, 2016

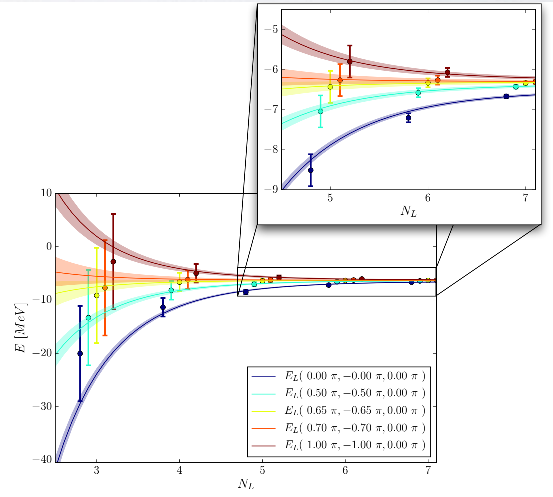
N=3

lowest k

$$\Phi \;=\; \mathcal{A}[\uparrow_1\downarrow_2\,[\exp i[k\cdot\mathbf{r_3}]\uparrow_3]$$

rho	E (MeV)
0.08	31.45(10)
0.16	41.2(2)
0.24	51.2(6)
0.32	66.0(6)
0.48	112.4(2.0)

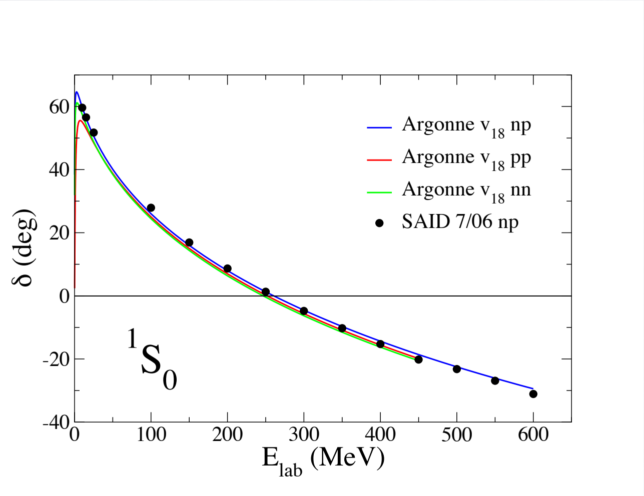
also see paper by Luu and Korber on A=3 bound states PRC 2016



N=4: low and high density

studies of possible low-energy resonances in dilute systems

N= 4 very sensitive
to nn phase shifts
at moderate/high energies

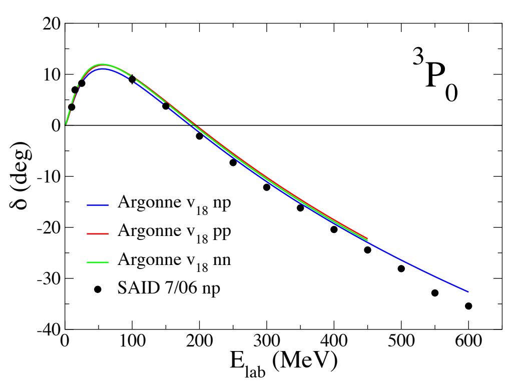


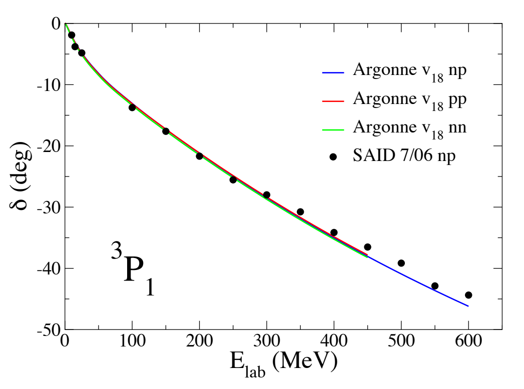
$N=4, \quad \rho = 0.16$

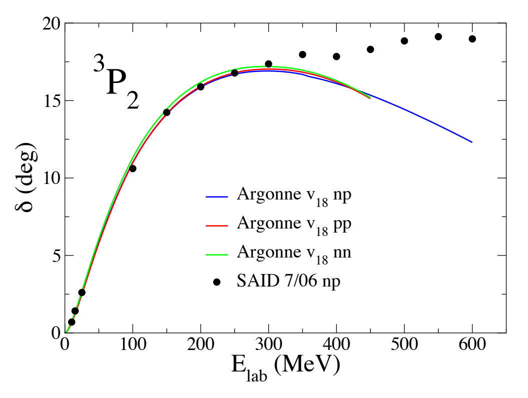
$L = 2.93$ fm

$k = (2\pi / L) = 2.14\ \mathrm{fm}^{-1}$

$E_{CM} = 190$ MeV







Other states for N = 4:

original (BCS) state:

$$\Phi \;=\; \mathcal{A}\prod[\phi(r_{ij})(\uparrow_i\downarrow_j-\uparrow_j\uparrow_i)]$$

new (p-wave) state:

$$\Phi \;=\; \mathcal{A}[\uparrow_1\downarrow_2\,[\sin[k_x\cdot\mathbf{r}_{34}]+i\sin[k_y\cdot\mathbf{r}_{34}]]\uparrow_3\uparrow_4]$$

rho	E_s (MeV)	E_p (MeV)
0.08	71.0(.5)	65.0(.5)
0.16	117(2)	92.0(1.2)

very large energy differences; still exploring other states

Small bits of cold dense matter

- *Important to constrain nuclear EOS from 1-4* $\boldsymbol{\rho}_0$
- *Small systems can provide a wealth of information*
  - more tractable: sign problem, large gaps
  - can constrain EOS (2N, 3N, … interactions)
  - start to identify region of phase transition by examining impact of different degrees of freedom
  - exploit boundary conditions, N, density
  - direct comparison of nuclear models and LQCD
- *Many other aspects to explore:*
  - explicit pions
  - evolution with pion (quark) mass
  - strangeness in dense matter
  - protons in neutron matter
  - different boundary conditions
  - models of high-density QCD
  - response functions, weak transitions, …