Nuclear Pasta Observables

Matt Caplan
Indiana University

Institute for Nuclear Theory
August 1, 2016

Neutron stars

- The crust is a crystalline lattice, while the core is uniform nuclear matter, like a nucleus. What's in between these two phases?

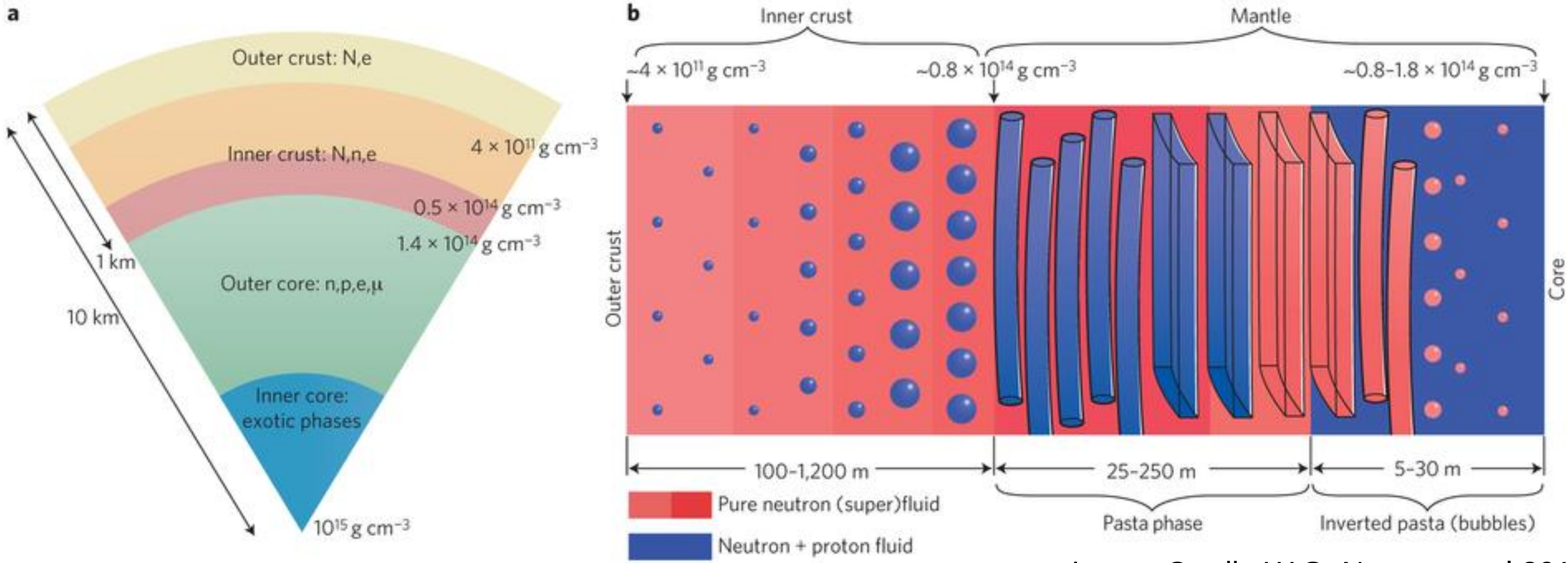


Image Credit: W.G. Newton et al 2012

Non-Spherical Nuclei

- First theoretical models of the shapes of nuclei near n_0
  - 1983: Ravenhall, Pethick, & Wilson
  - 1984: Hashimoto, H. Seki, and M. Yamada
- *Frustration*: Competition between proton-proton Coulomb repulsion and strong nuclear attraction
- Nucleons adopt non-spherical geometries near the saturation density to minimize surface energy

Shape of Nuclei in the Crust of Neutron Star 321

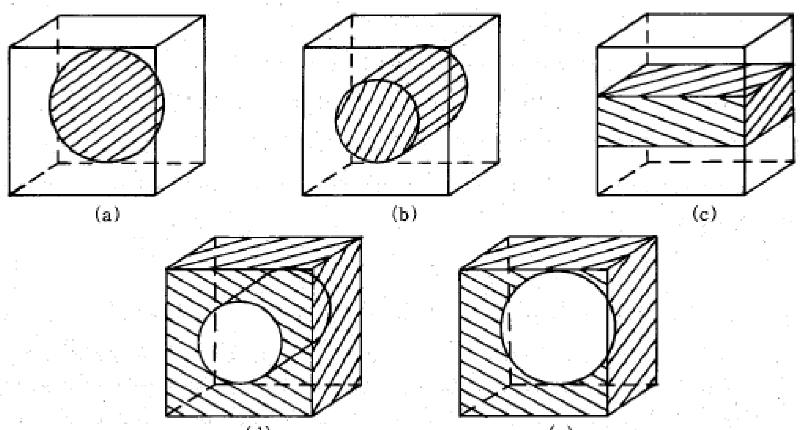
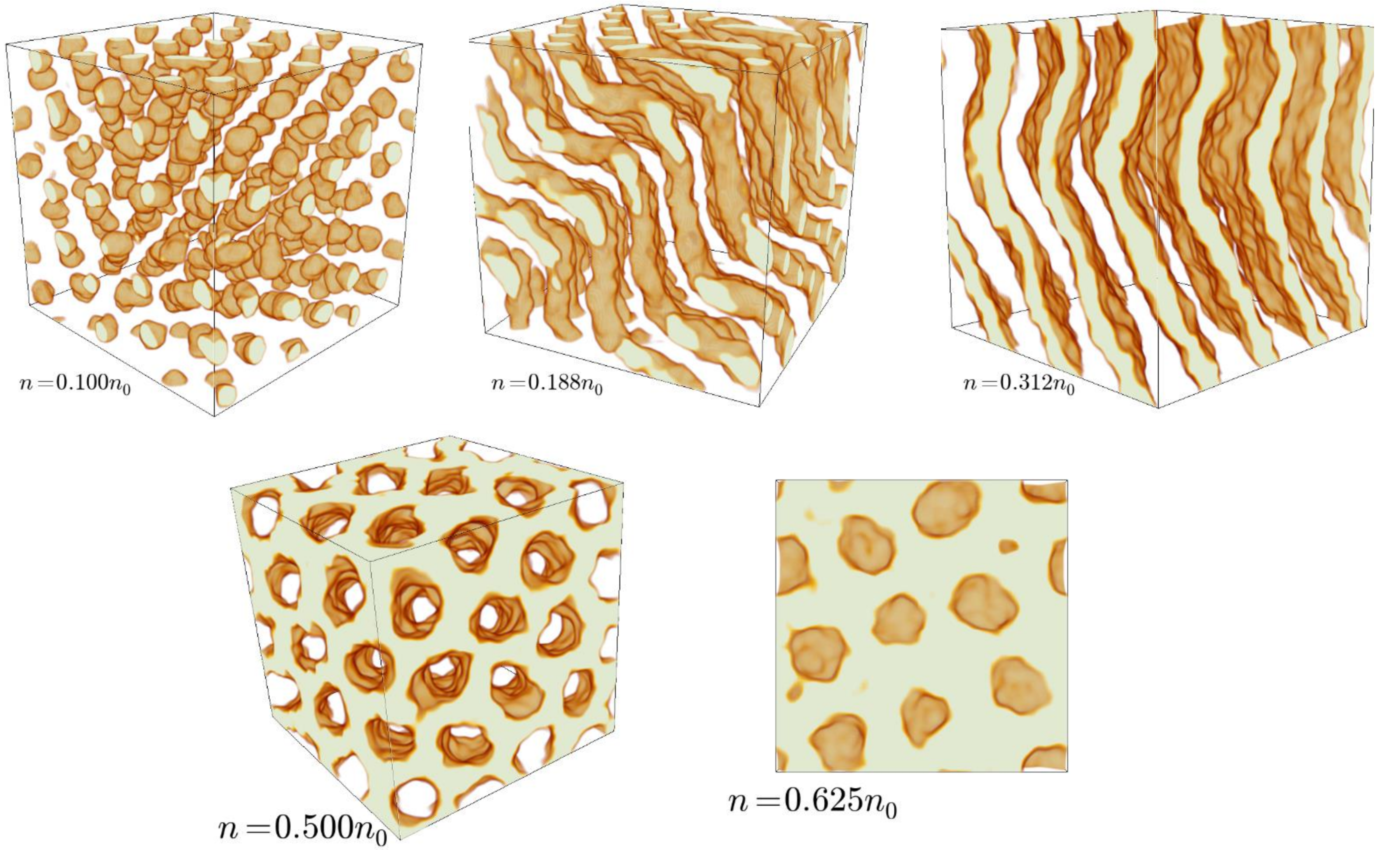


Fig. 1. Candidates for nuclear shapes. Protons are confined in the hatched regions, which we call nuclei. Then the shapes are, (a) sphere, (b) cylinder, (c) board or plank, (d) cylindrical hole and (e) spherical hole. Note that many cells of the same shape and orientation are piled up to form the whole space, and thereby the nuclei are joined to each other except for the spherical nuclei (a).

Nuclear Pasta



Shape of Nuclei in the Crust of Neutron Star

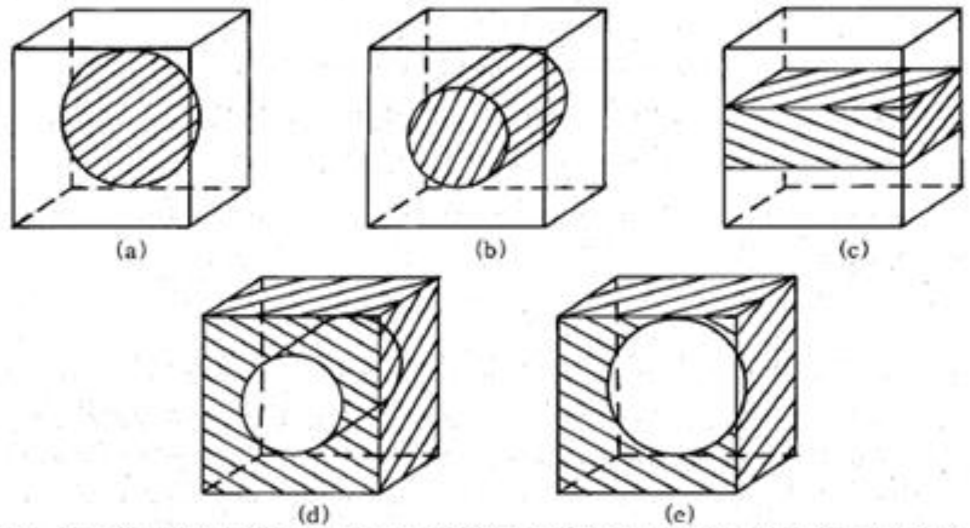


Fig. 1. Candidates for nuclear shapes. Protons are confined in the hatched regions, which we call nuclei. Then the shapes are, (a) sphere, (b) cylinder, (c) board or plank, (d) cylindrical hole and (e) spherical hole. Note that many cells of the same shape and orientation are piled up to form the whole space, and thereby the nuclei are joined to each other except for the spherical nuclei (a).

Classical Pasta Formalism

- **Classical Molecular Dynamics** with IUMD on Big Red II

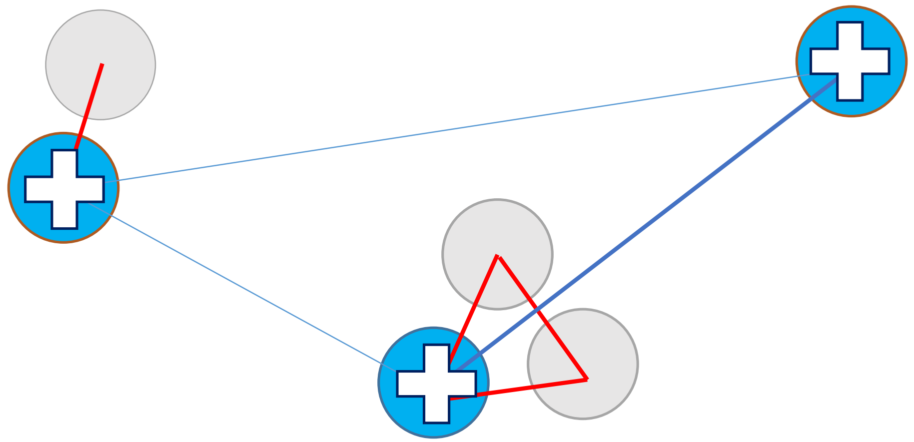
$$V_{np}(r_{ij}) = a e^{-r_{ij}^2/\Lambda} + [b - c] e^{-r_{ij}^2/2\Lambda}$$

$$V_{nn}(r_{ij}) = a e^{-r_{ij}^2/\Lambda} + [b + c] e^{-r_{ij}^2/2\Lambda}$$

$$V_{pp}(r_{ij}) = a e^{-r_{ij}^2/\Lambda} + [b + c] e^{-r_{ij}^2/2\Lambda} + \frac{\alpha}{r_{ij}} e^{-r_{ij}/\lambda}$$

a	b	c	Λ	λ
110 MeV	-26 MeV	24 MeV	1.25 fm^2	10 fm

- Short range nuclear force
- Long range Coulomb force



Nucleus	Monte-Carlo $\langle V_{tot} \rangle$ (MeV)	Experiment (MeV)
^{16}O	-7.56±0.01	-7.98
^{40}Ca	-8.75±0.03	-8.45
^{90}Zr	-9.13±0.03	-8.66
^{208}Pb	-8.2 ±0.1	-7.86

Horowitz et al. (2004) PhysRevC 69, 045804

Classical Pasta Formalism

- **Classical Molecular Dynam…** IUM… Red II

…	Λ	λ
…	$1.25\,\mathrm{fm}^2$	$10\,\mathrm{fm}$

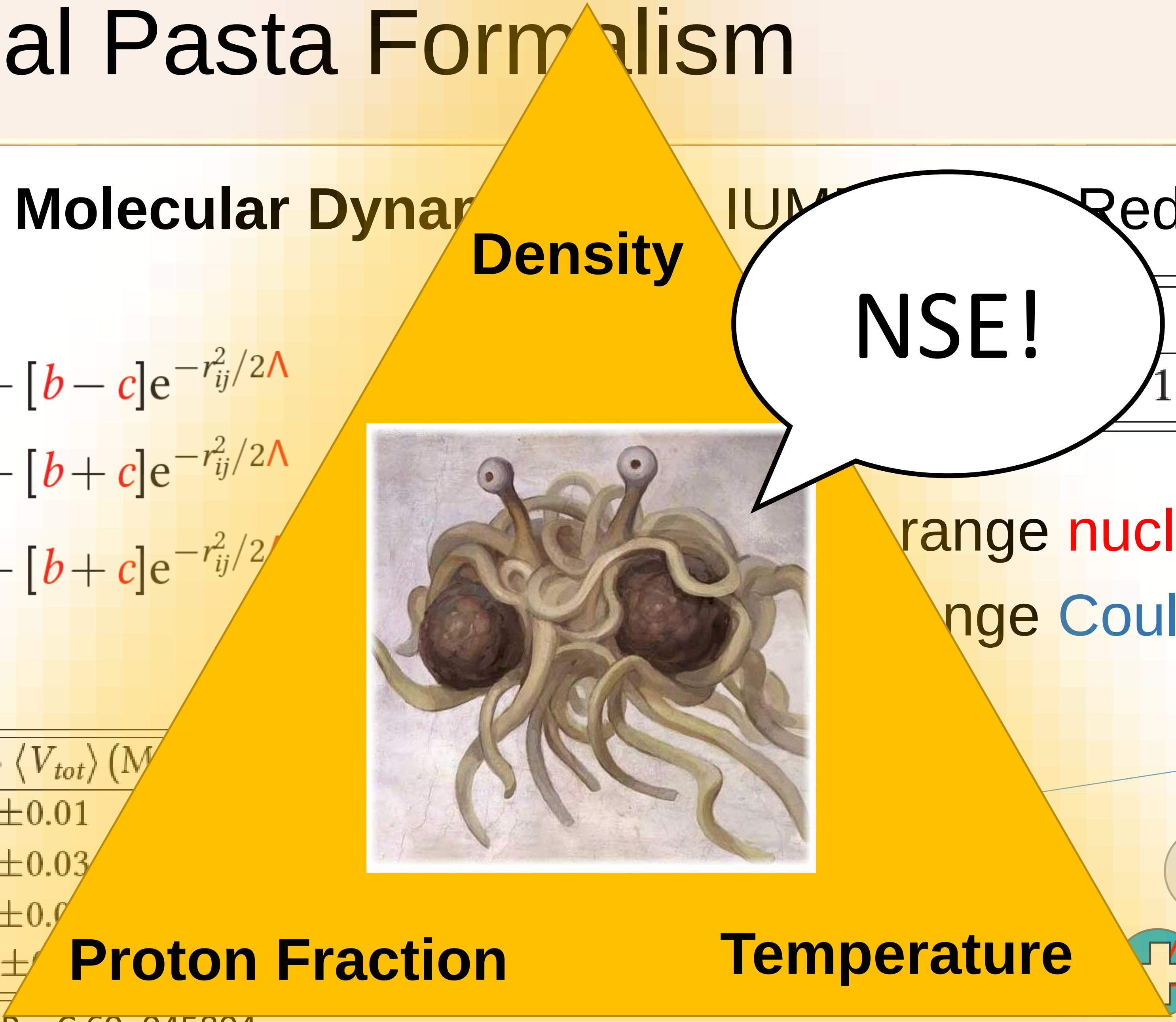
$$V_{np}(r_{ij}) = a e^{-r_{ij}^2/\Lambda} + [b - c] e^{-r_{ij}^2/2\Lambda}$$

$$V_{nn}(r_{ij}) = a e^{-r_{ij}^2/\Lambda} + [b + c] e^{-r_{ij}^2/2\Lambda}$$

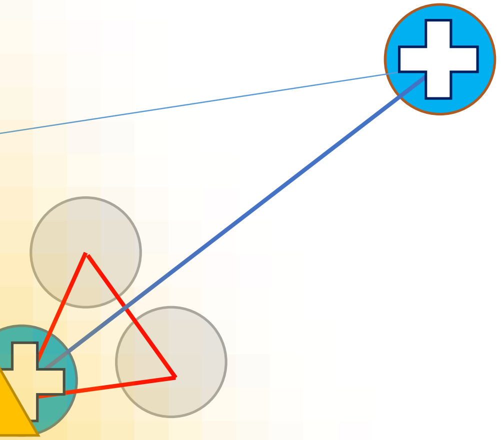
$$V_{pp}(r_{ij}) = a e^{-r_{ij}^2/\Lambda} + [b + c] e^{-r_{ij}^2/2 \ldots}$$

…range nuclear force

…nge Coulomb force



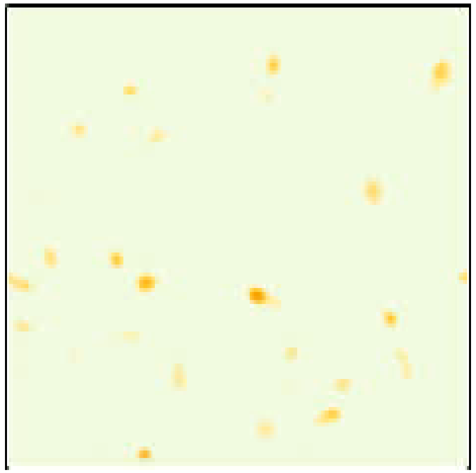
NSE!



Nucleus	Monte-Carlo $\langle V_{tot} \rangle$ (M…
^{16}O	-7.56±0.01
^{40}Ca	-8.75±0.03
^{90}Zr	-9.13±0.0…
^{208}Pb	-8.2 ±…

Horowitz et al. (2004) PhysRevC 69, 045804

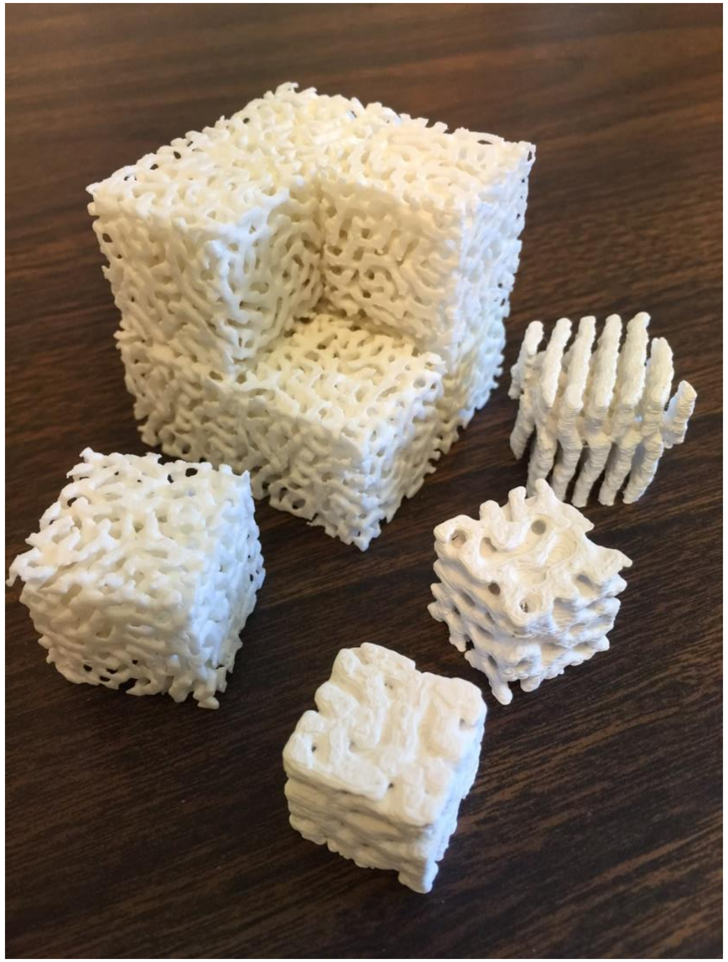
Gold Nucleus
For Scale



$$n = 0.1200 \mathrm{fm}^{-3}$$

Pass around the 3D printed cubes

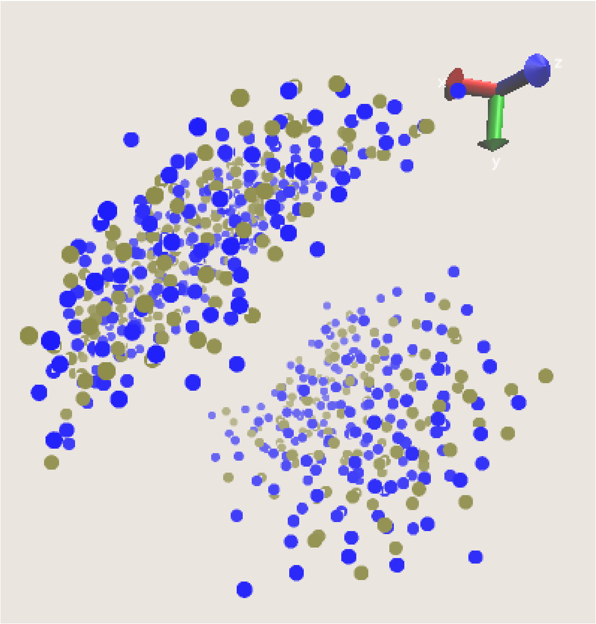
- I always forget so I made a slide to remind myself that I have them.

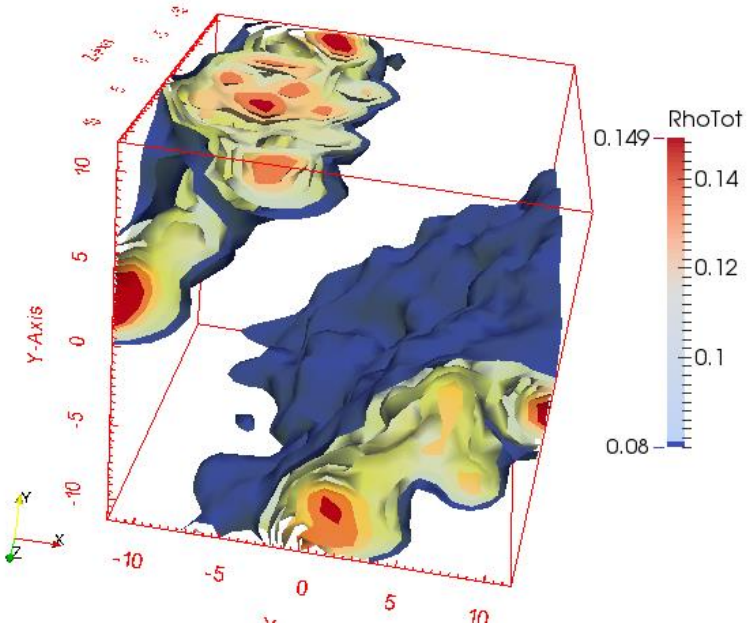


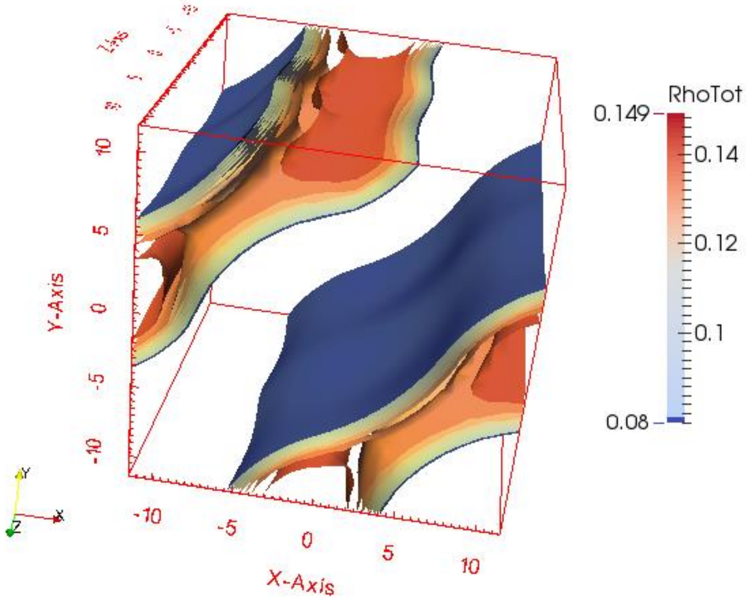
Classical and Quantum MD

- We can use the classical pasta to initiate the quantum codes
- Classical structures remain stable when evolved via HF

Classical Points → **Folded with Gaussian** → **Equilibrated Wavefunctions**

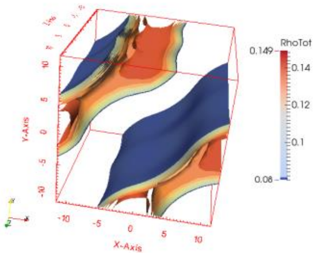






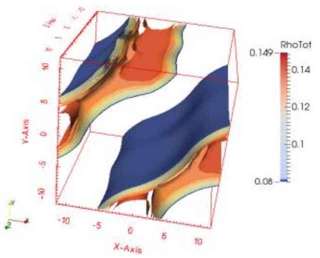
Classical and Quantum MD

800 nucleons
24 fm



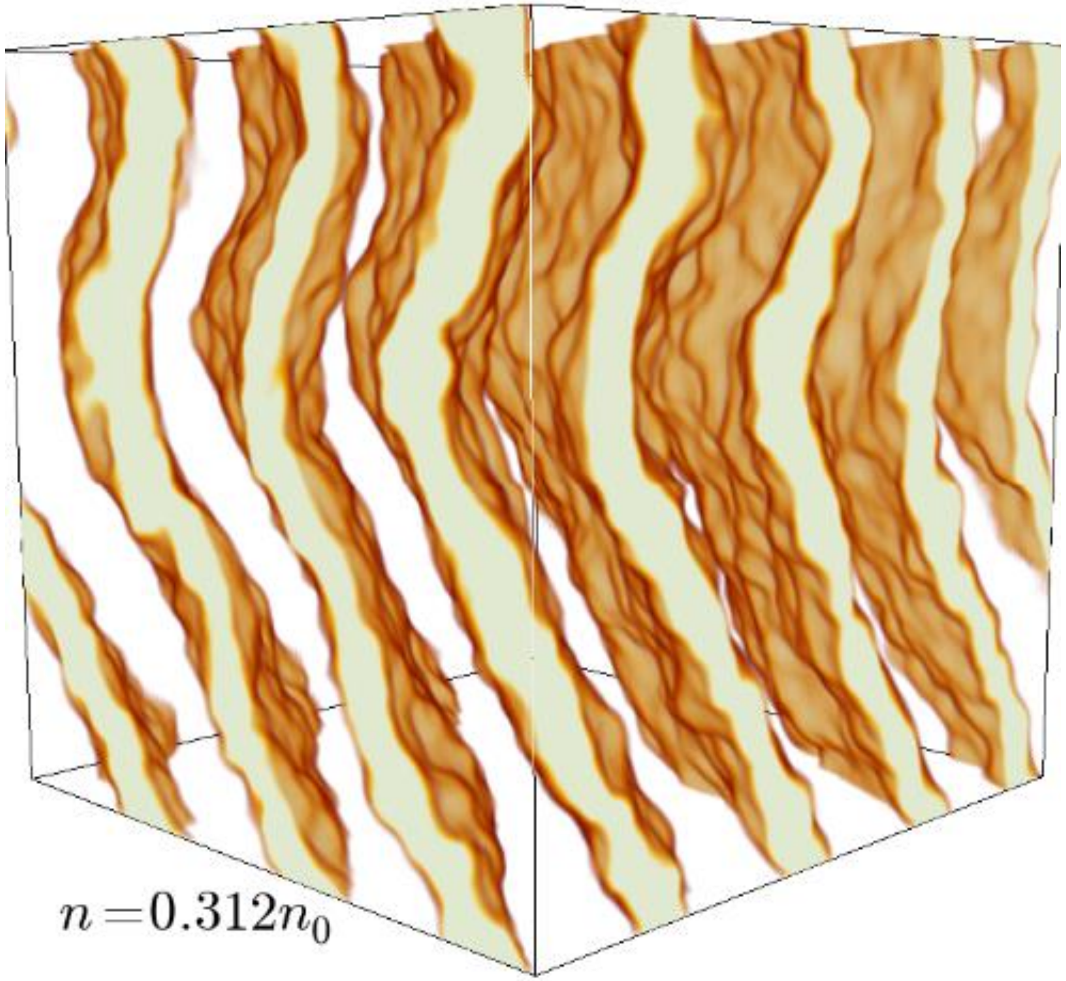
Classical and Quantum MD

800 nucleons
24 fm



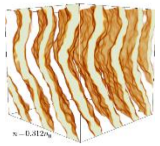
51,200 nucleons

100 fm

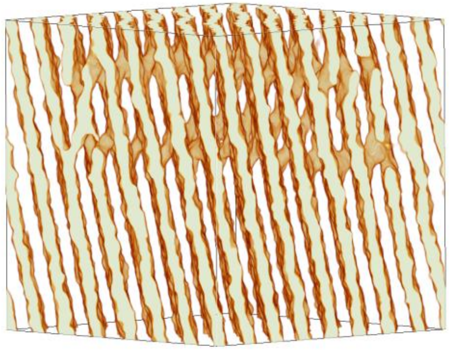


Molecular Dynamics

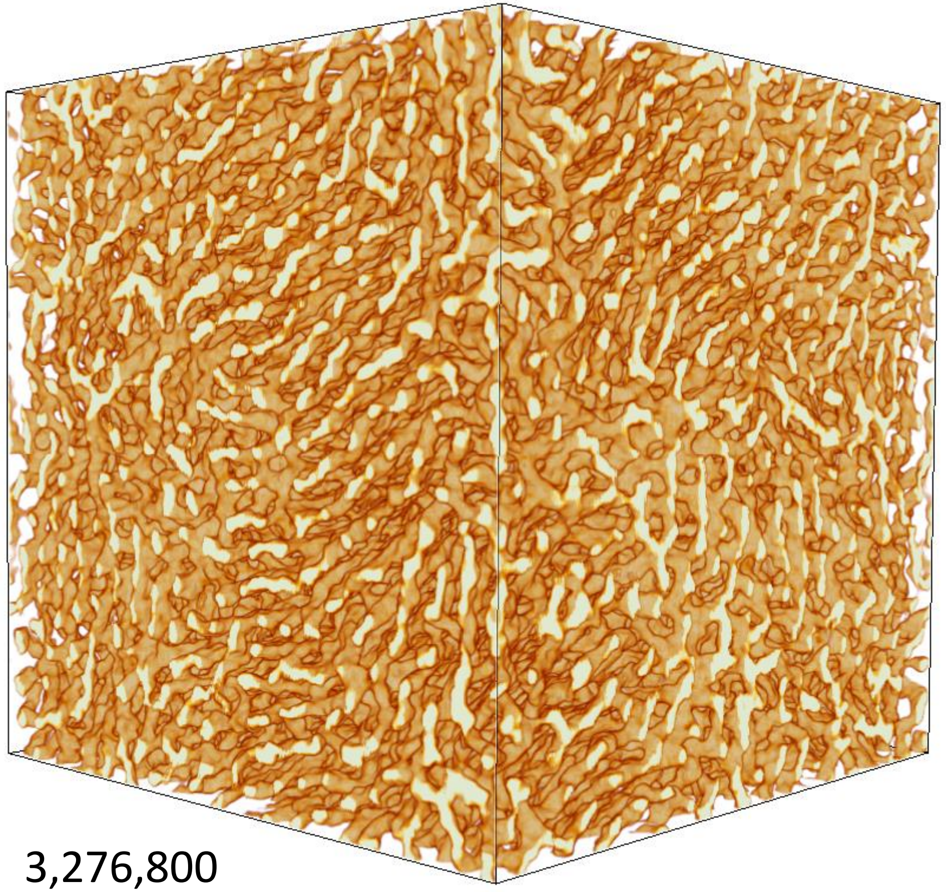
- We have evolved simulations of 409,600 nucleons, 819,200 nucleons, 1,638,400 nucleons, and 3,276,800 nucleons



51,200



409,600

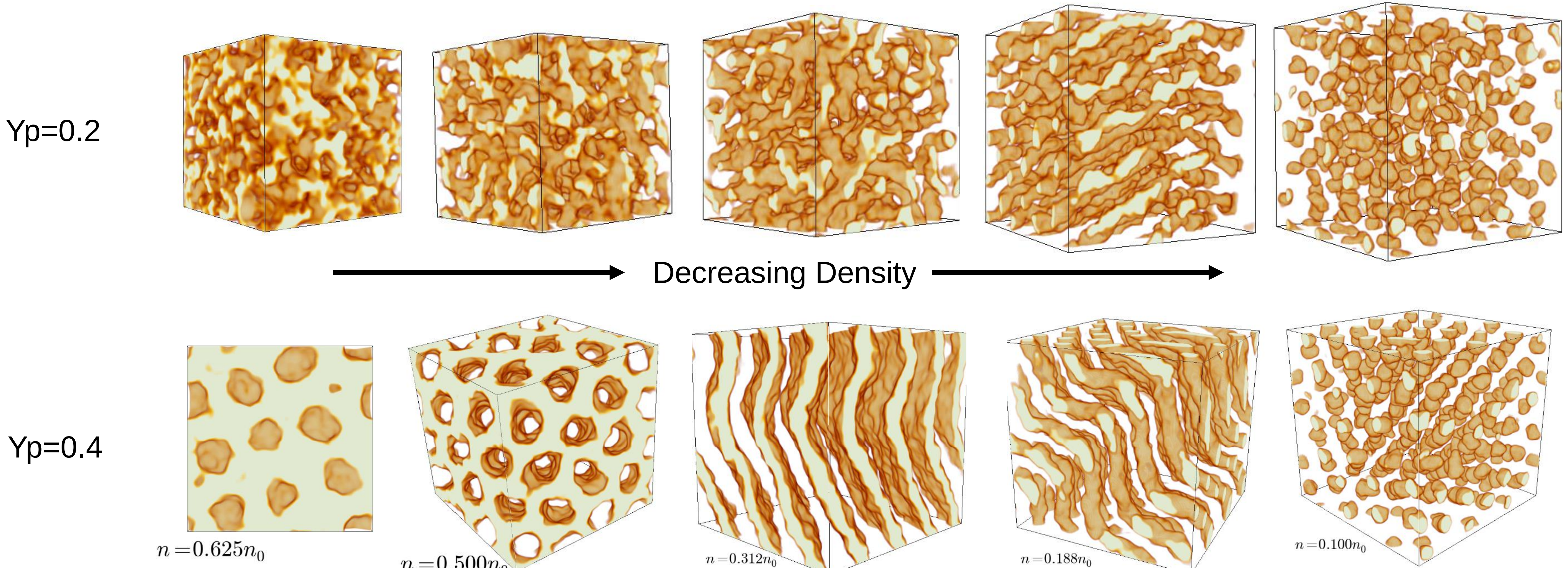


3,276,800

Phase Diagrams

Phase Diagrams

- Simulate pasta with constant temperature and proton fraction
- Observe phase transitions as a function of density



“Thermodynamic” Curvature

- Use curvature as a thermodynamic quantity
- Discontinuities in curvature indicate phase changes

V	Volume
$A = \int_{\partial K} dA$	Surface Area
$B = \int_{\partial K} (\kappa_1 + \kappa_2)\,/4\pi\, dA$	Mean Breadth
$\chi = \int_{\partial K} (\kappa_1 \cdot \kappa_2)\,/4\pi\, dA$	Euler Characteristic

0

$$\int_M K\, dA + \int_{\partial M} k_g\, ds = 2\pi\chi(M)$$

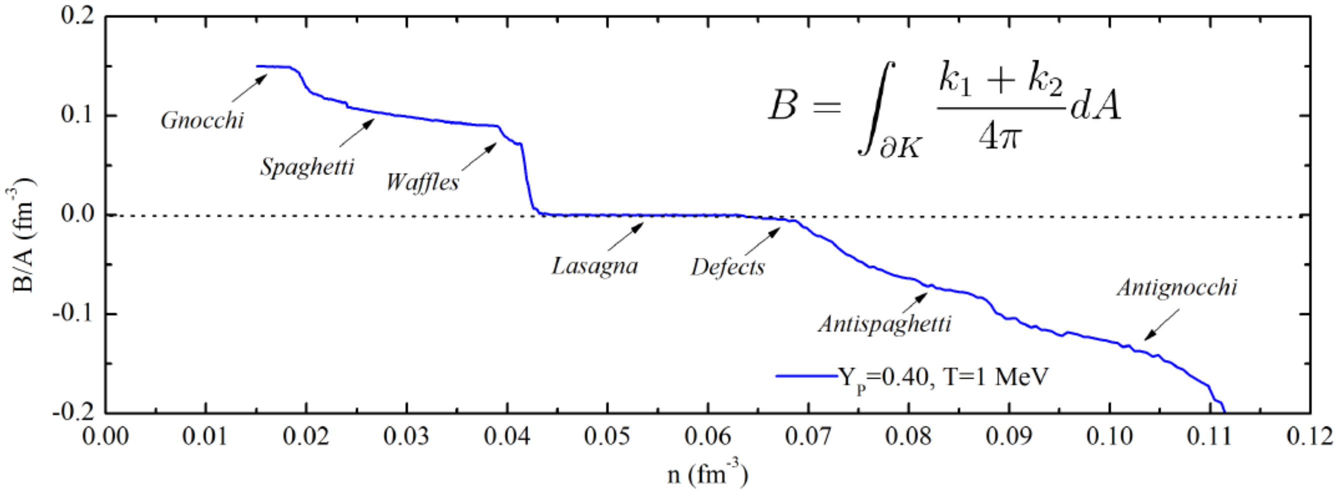
$$\chi(M) = 2 - 2g$$

Sphere		2
Torus (Product of two circles)		0
Double torus		−2
Triple torus		−4

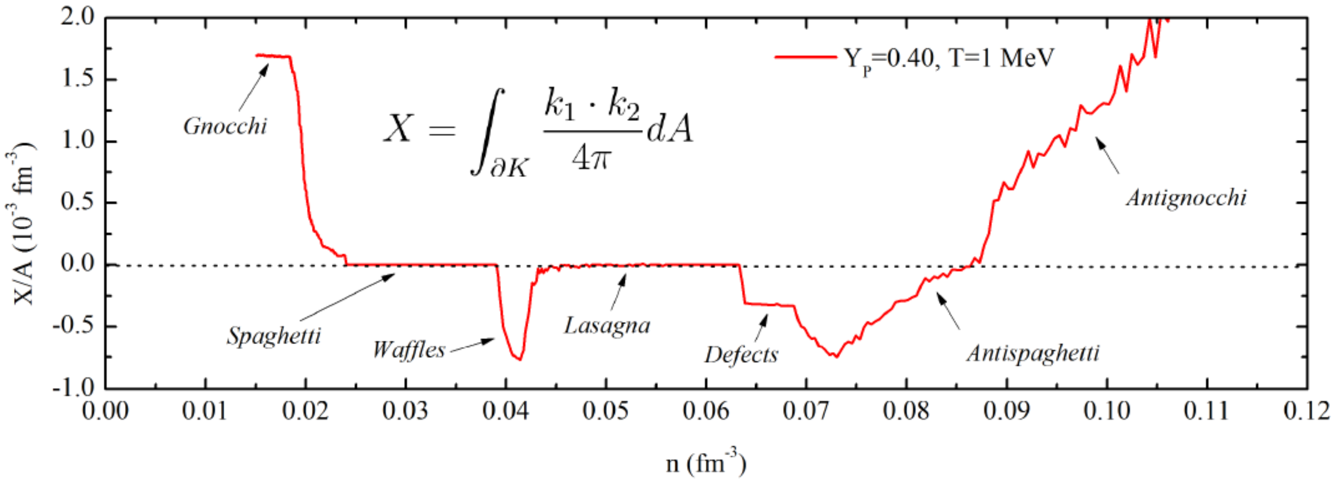
- Pieces + Cavities - Holes

“Thermodynamic” Curvature

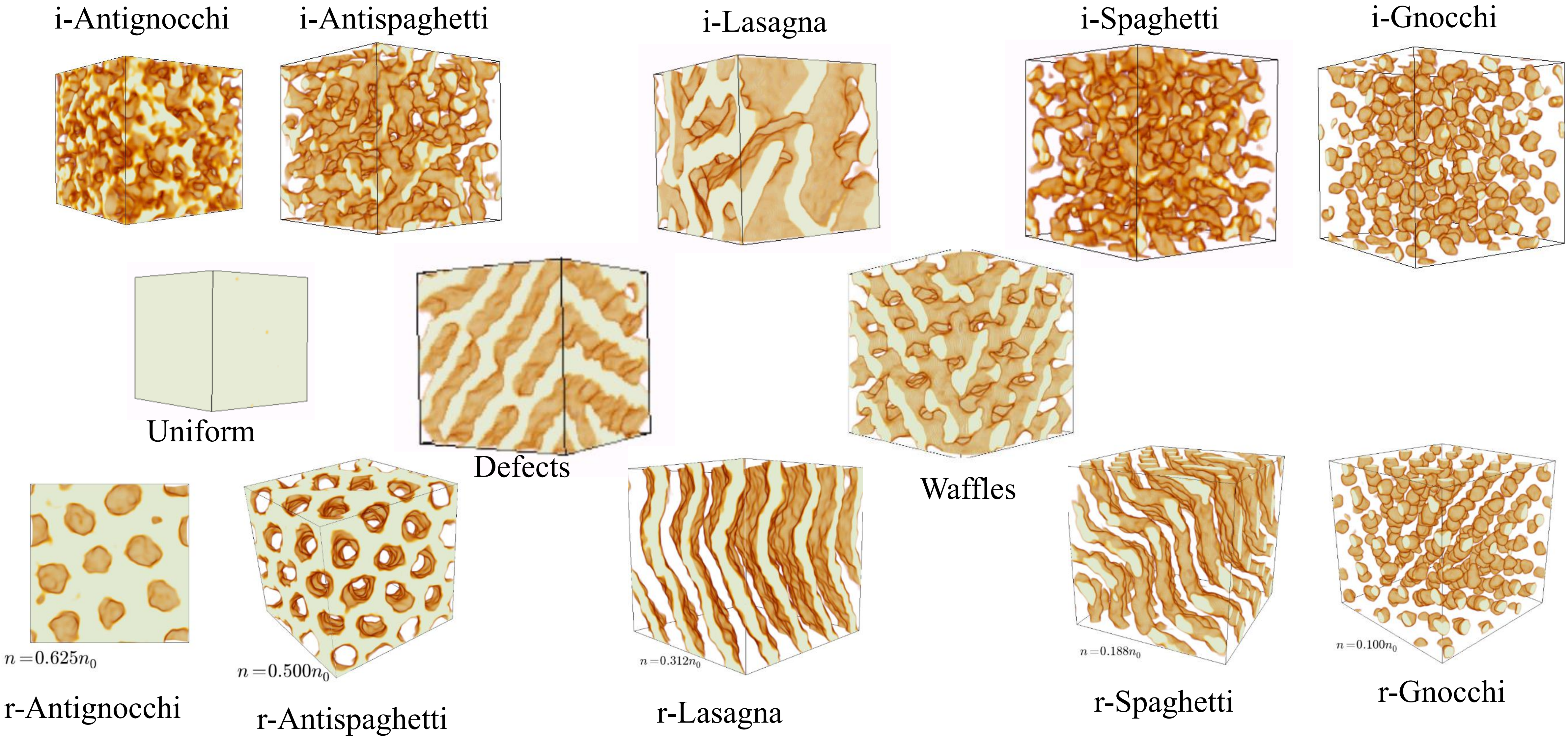
- Use curvature as a thermodynamic quantity
- Discontinuities in curvature indicate phase changes

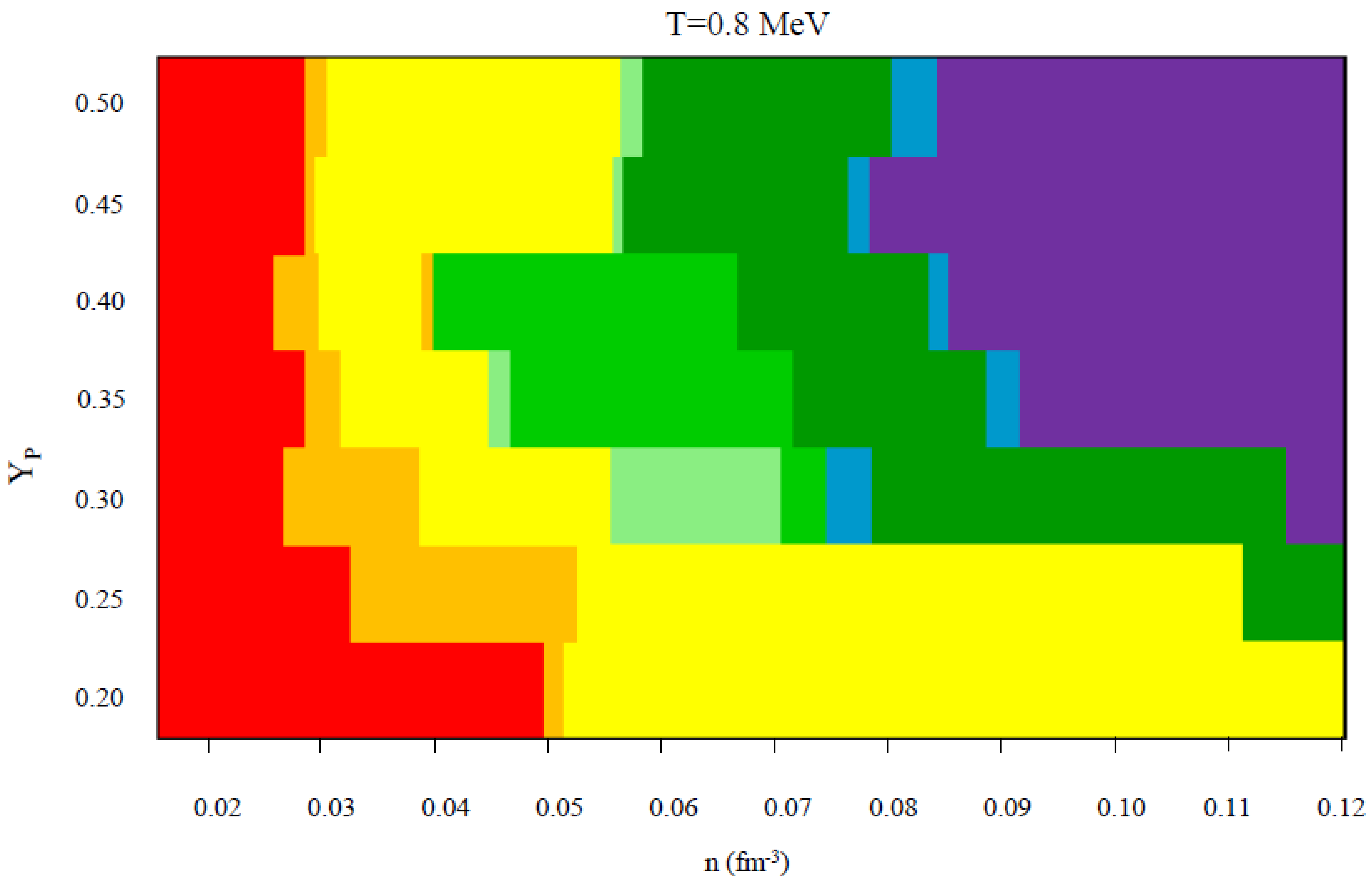


V	Volume
$A = \int_{\partial K} dA$	Surface Area
$B = \int_{\partial K} (\kappa_1 + \kappa_2) / 4\pi \, dA$	Mean Breadth
$\chi = \int_{\partial K} (\kappa_1 \cdot \kappa_2) / 4\pi \, dA$	Euler Characteristic

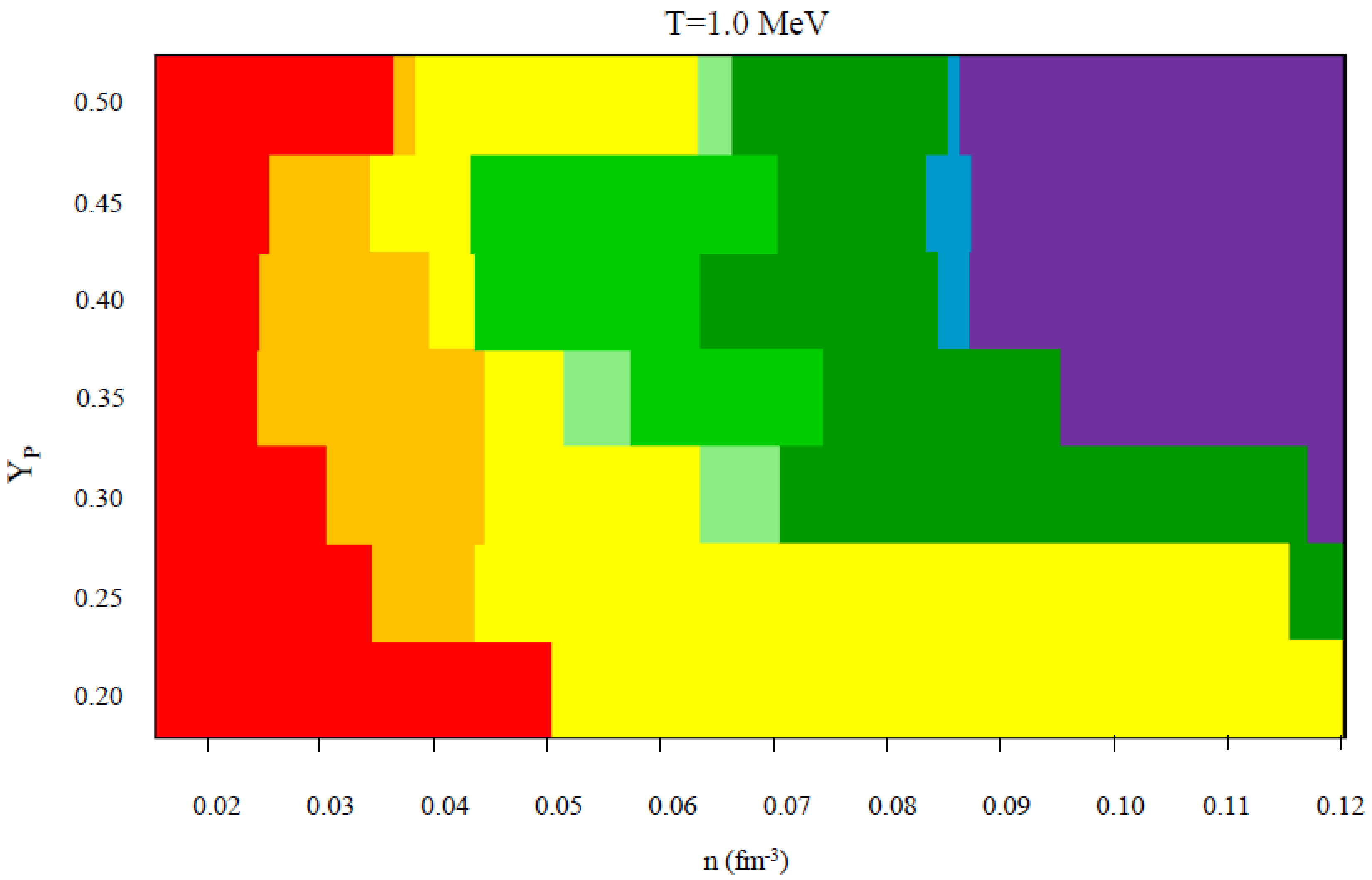


Phases

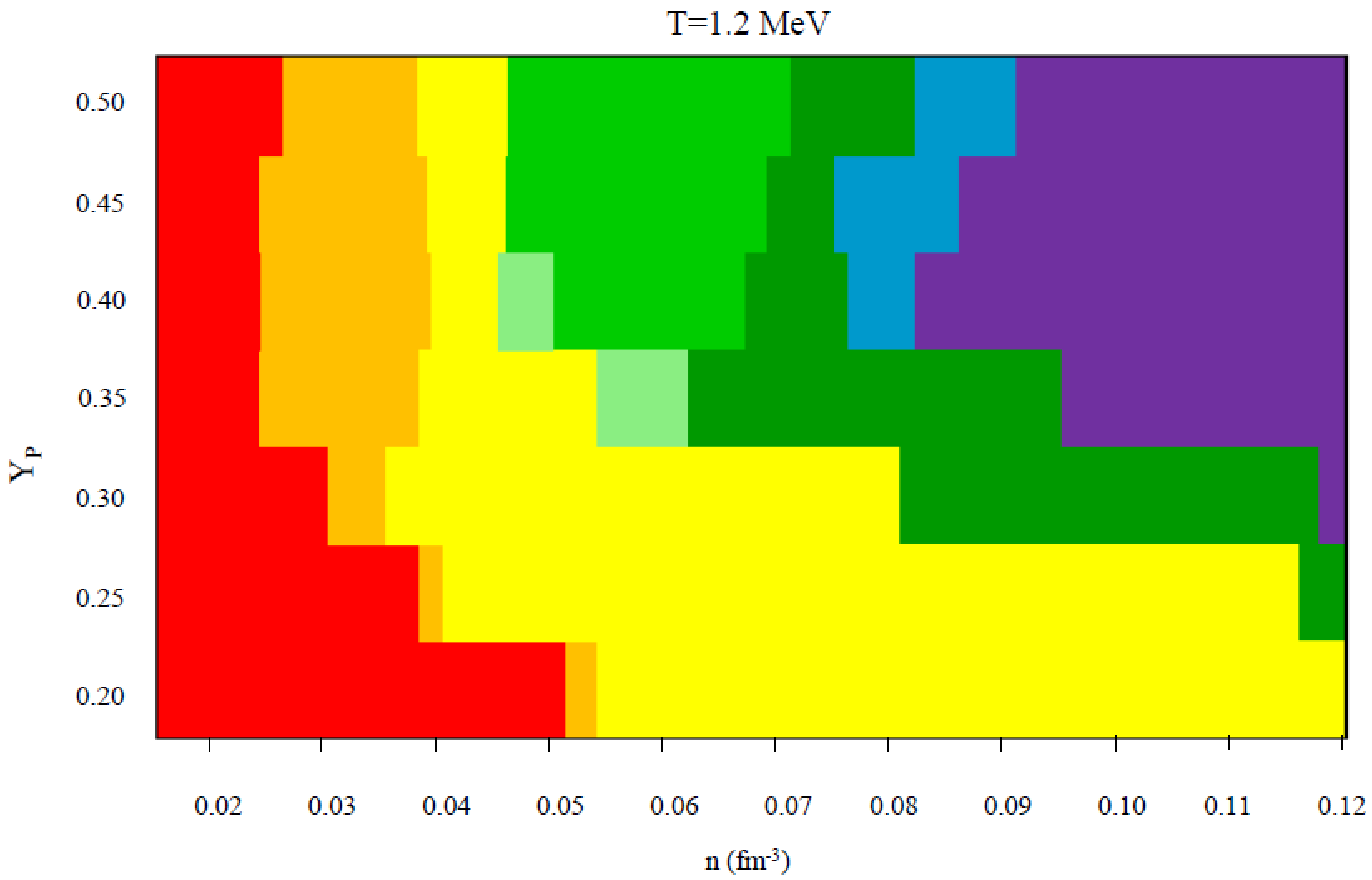

i-Antignocchi
i-Antispaghetti
i-Lasagna
i-Spaghetti
i-Gnocchi
Uniform
Defects
Waffles
$n = 0.625n_0$
r-Antignocchi
$n = 0.500n_0$
r-Antispaghetti
$n = 0.312n_0$
r-Lasagna
$n = 0.188n_0$
r-Spaghetti
$n = 0.100n_0$
r-Gnocchi



	$B < 0$	$B \sim 0$	$B > 0$
$\chi > 0$	sph b		sph
$\chi \sim 0$	rod-1 b	slab	rod-1
$\chi < 0$	rod-2 b	rod-3	rod-2



	$B < 0$	$B \sim 0$	$B > 0$
$\chi > 0$	sph b		sph
$\chi \sim 0$	rod-1 b	slab	rod-1
$\chi < 0$	rod-2 b	rod-3	rod-2

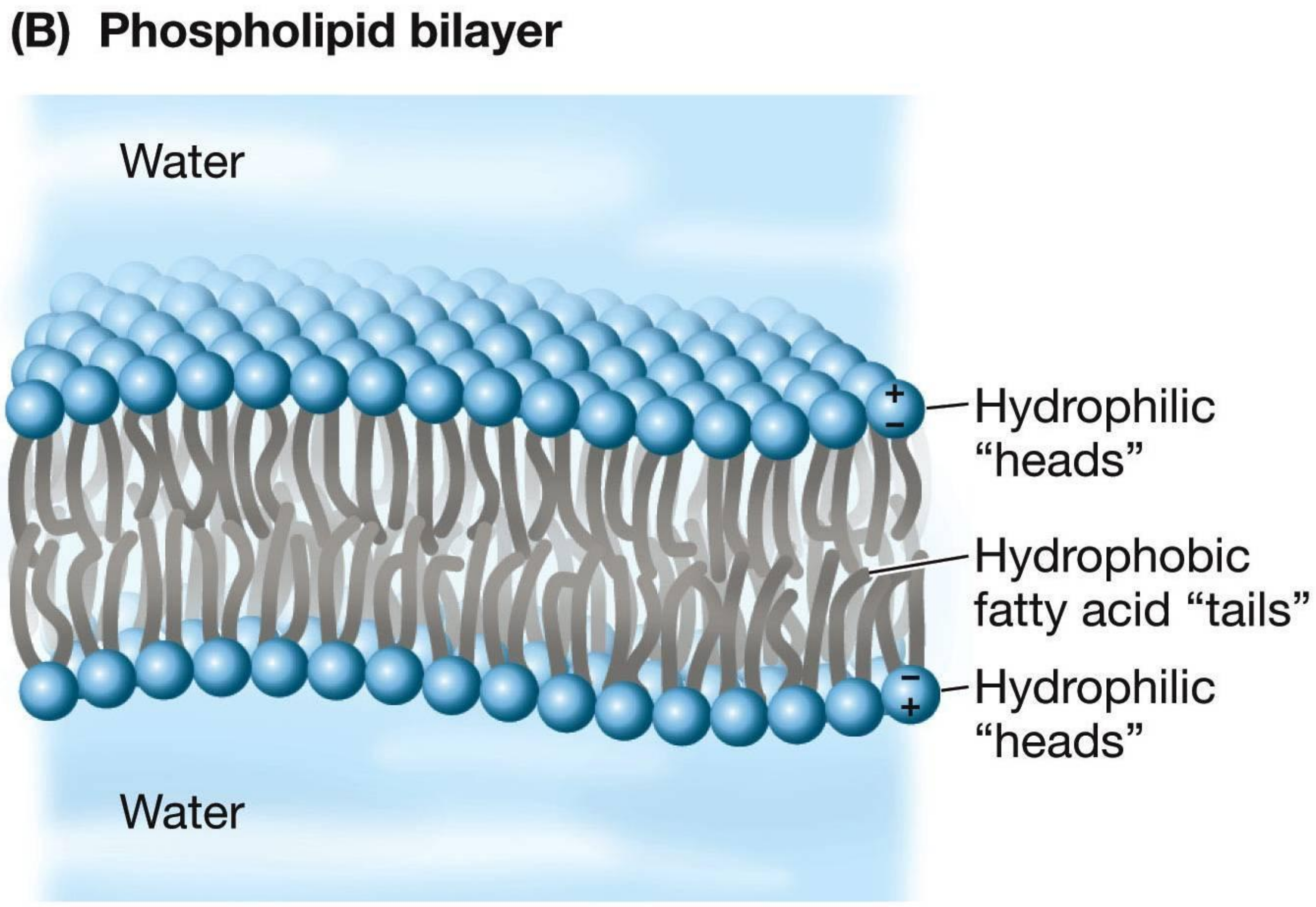


	$B < 0$	$B \sim 0$	$B > 0$
$\chi > 0$	sph b		sph
$\chi \sim 0$	rod-1 b	slab	rod-1
$\chi < 0$	rod-2 b	rod-3	rod-2

Soft Matter

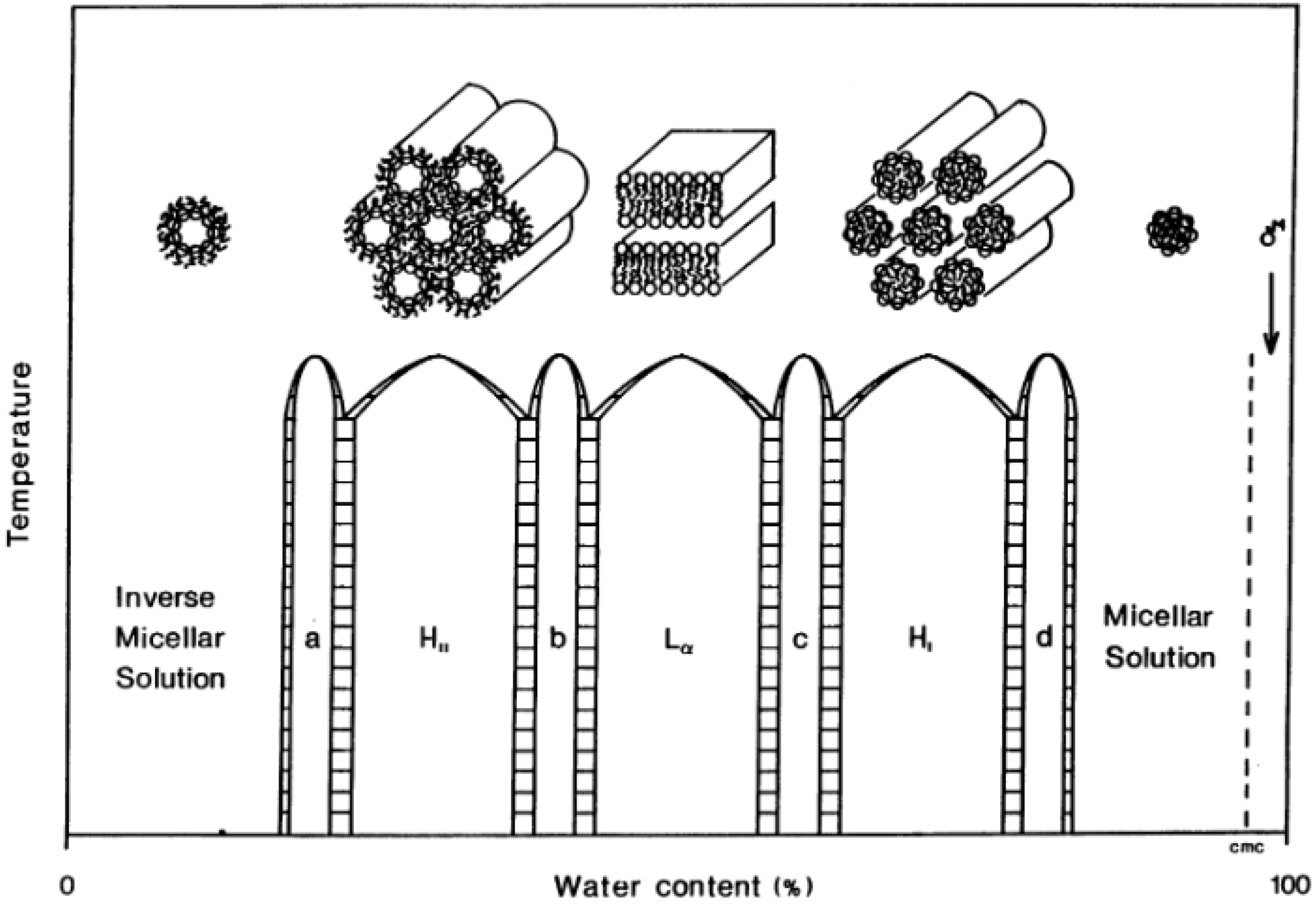
Self Assembly

- Well studied in phospholipids: hydrophilic heads and hydrophobic tails self assemble in an aqueous solution



LIFE 8e, Figure 3.20 (Part 2)

Self Assembly



Seddon, BBA 1031, 1–69 (1990)

Shape of Nuclei in the Crust of Neutron Star

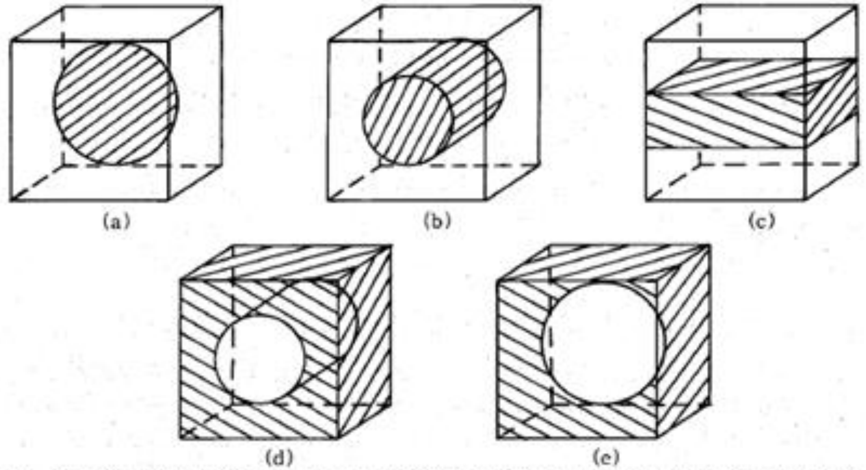
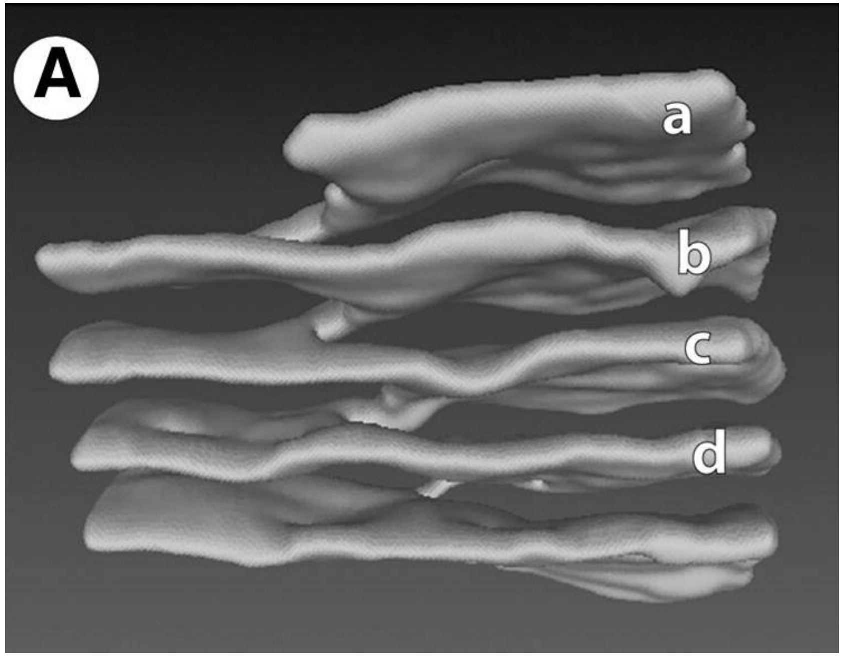


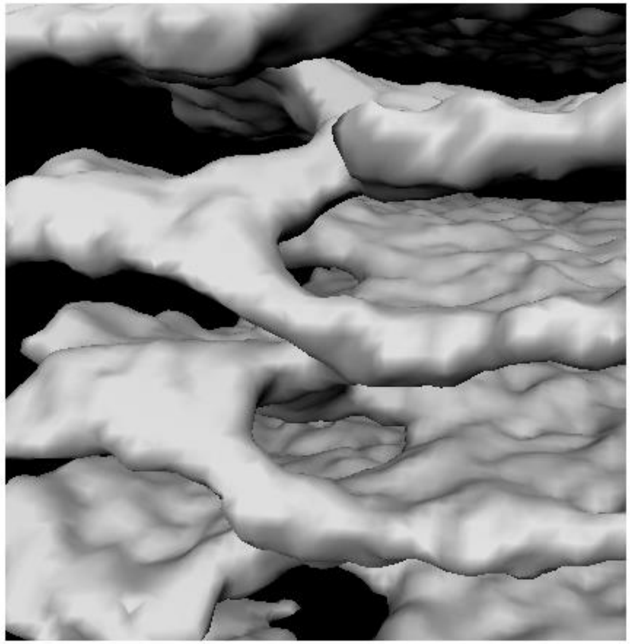
Fig. 1. Candidates for nuclear shapes. Protons are confined in the hatched regions, which we call nuclei. Then the shapes are, (a) sphere, (b) cylinder, (c) board or plank, (d) cylindrical hole and (e) spherical hole. Note that many cells of the same shape and orientation are piled up to form the whole space, and thereby the nuclei are joined to each other except for the spherical nuclei (a).

Self Assembly

- Left: Electron microscopy of helicoids in mice endoplasmic reticulum



Terasaki et al, Cell 154.2 (2013)



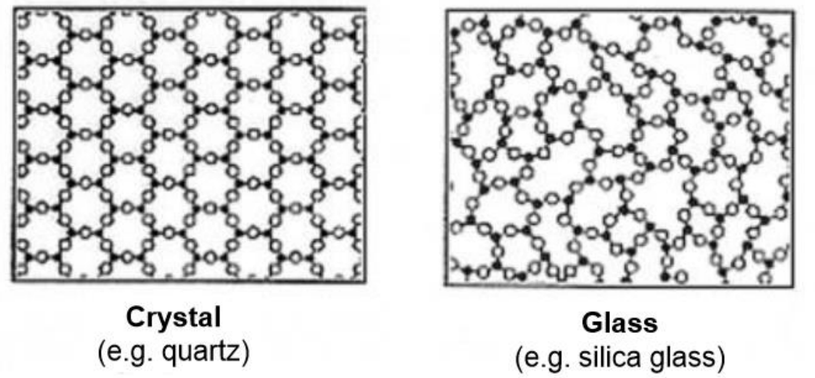
Horowitz et al, PRL.114.031102 (2015)

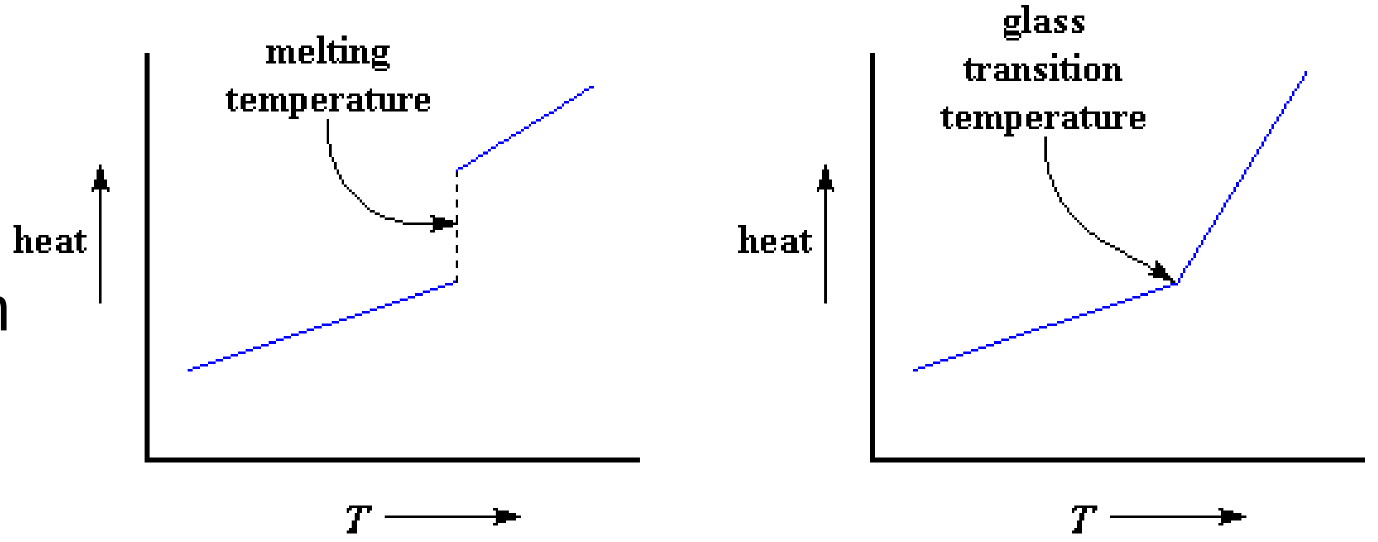
- Right: Defects in nuclear pasta MD simulations

Parking Garage Structures in astrophysics and biophysics (arXiv:1509.00410)

Caloric Curve

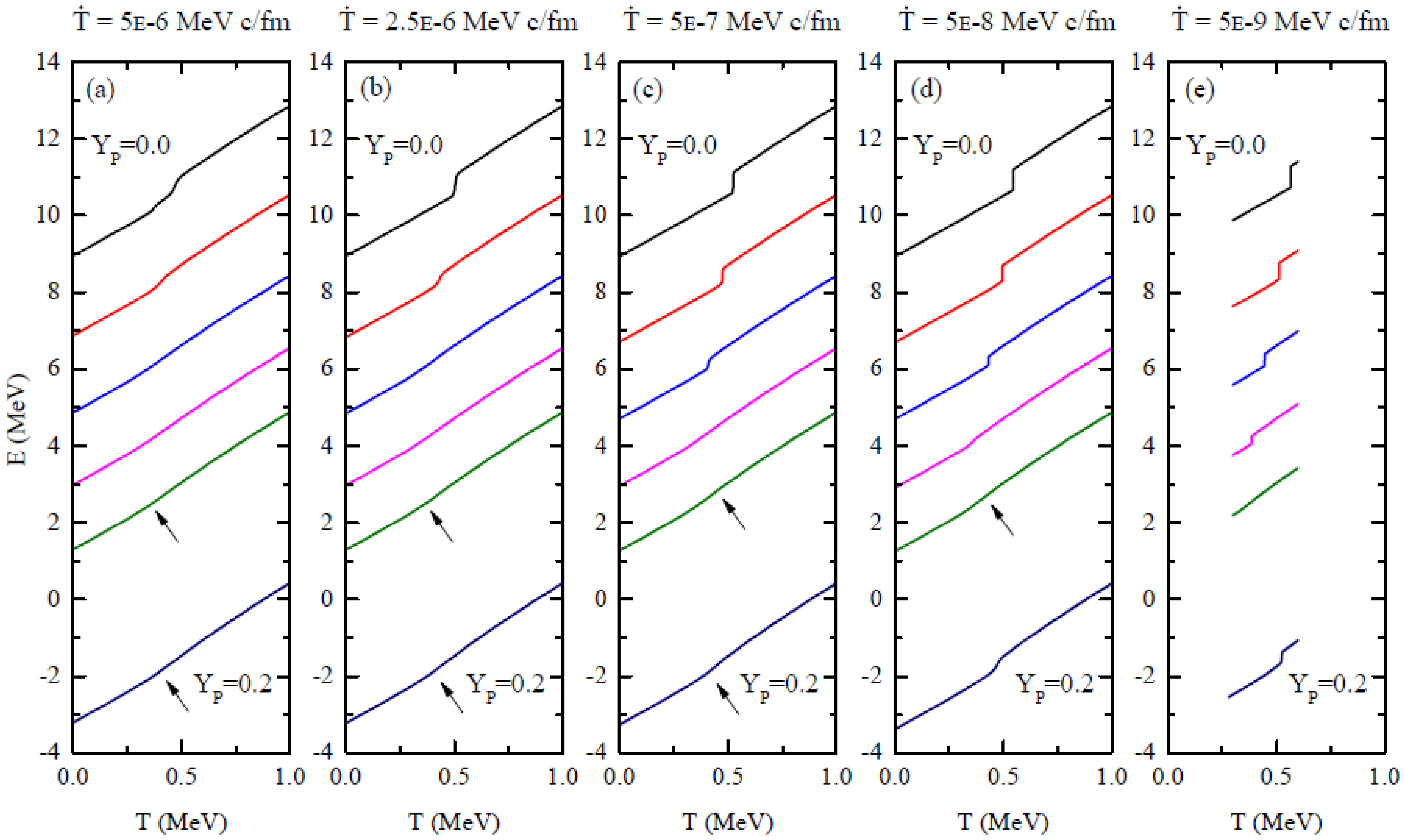
- Glass transition – impure substance forms an amorphous solid when quenched
- Solid:
  - Long Range Order
  - Nondiffusive
  - First order phase transition
- Glass:
  - Short range order
  - Low diffusion
  - Second order phase transition





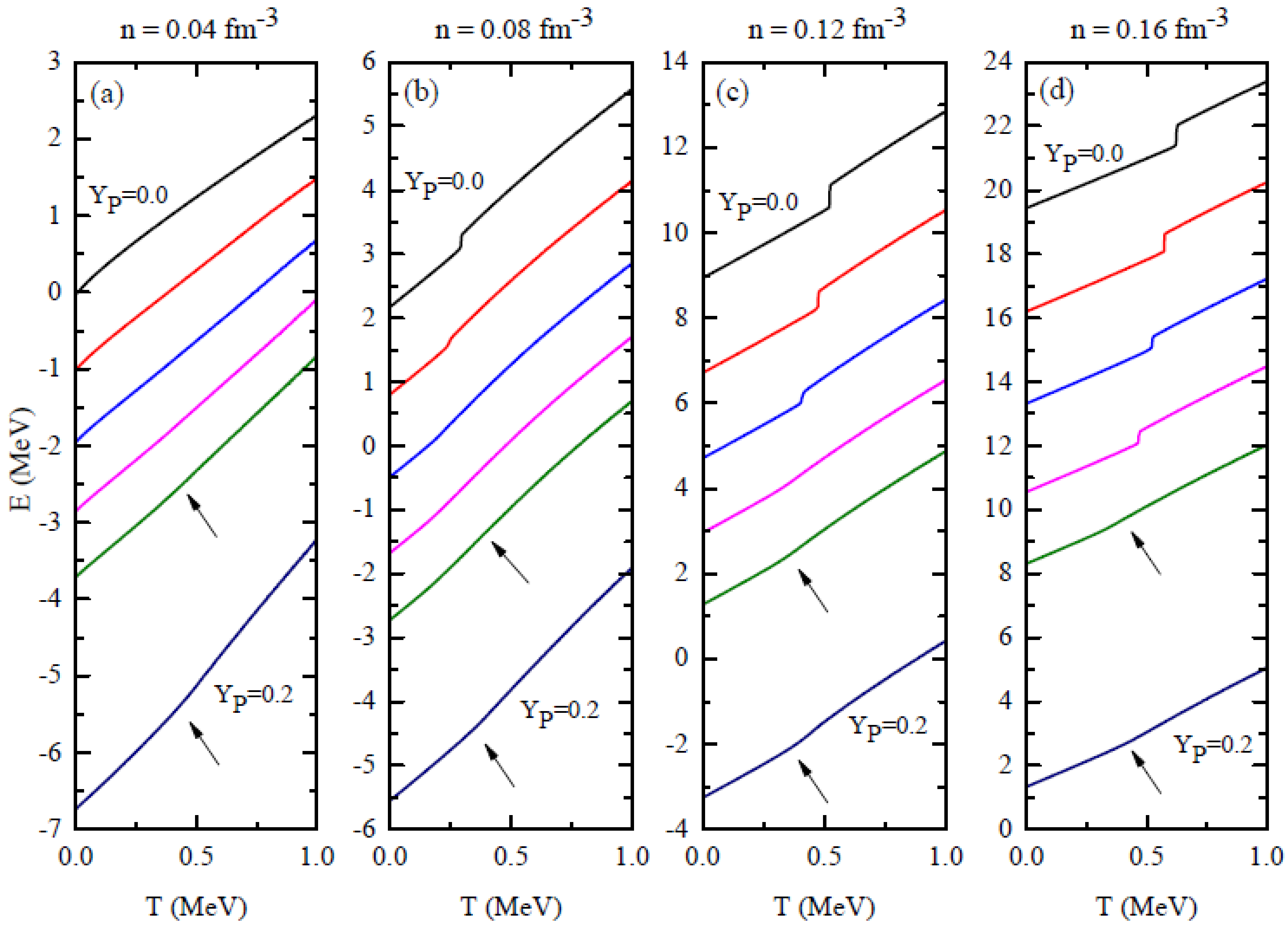
Quench Rate

- Cooling quickly and cooling slowly
- Proton fractions: Y_P = 0.0 – 0.2
- Density: n = 0.12 fm^{-3}



Caplan et al (In Prep)

Glassy Systems

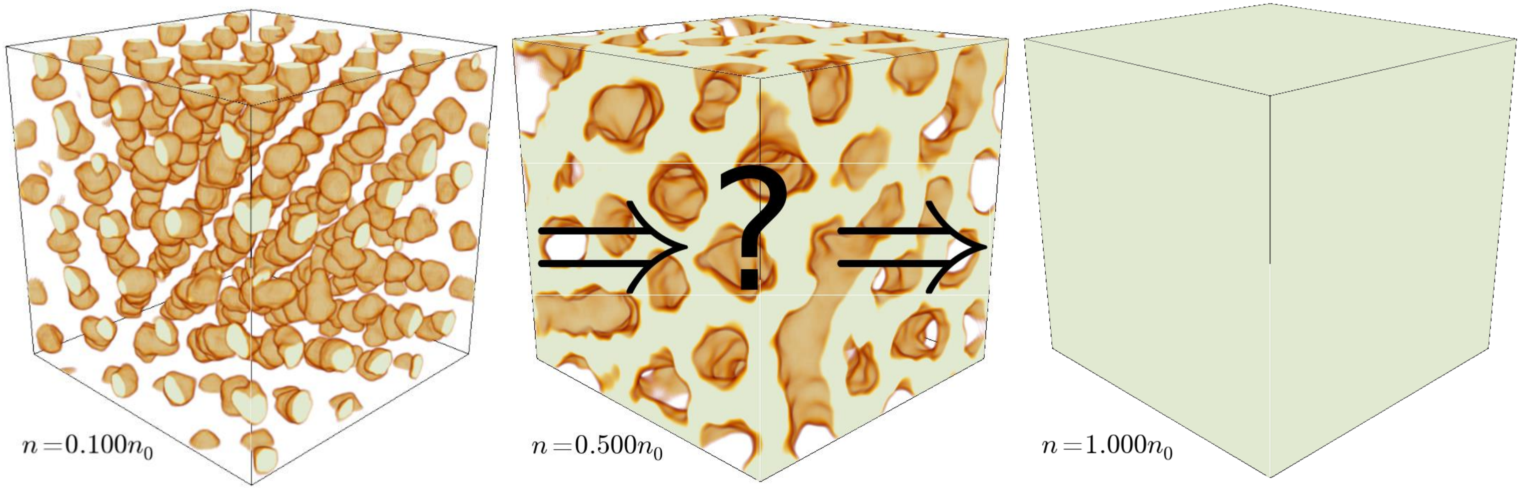


Caplan et al (In Prep)

Observables

Nuclear Pasta

- Important to many processes:
  - Thermodynamics: Late time crust cooling
  - Magnetic field decay: Electron scattering in pasta
  - Gravitational wave amplitude: Pasta elasticity and breaking strain
  - Neutrino scattering: Neutrino wavelength comparable to pasta spacing
  - R-process: Pasta is ejected in mergers

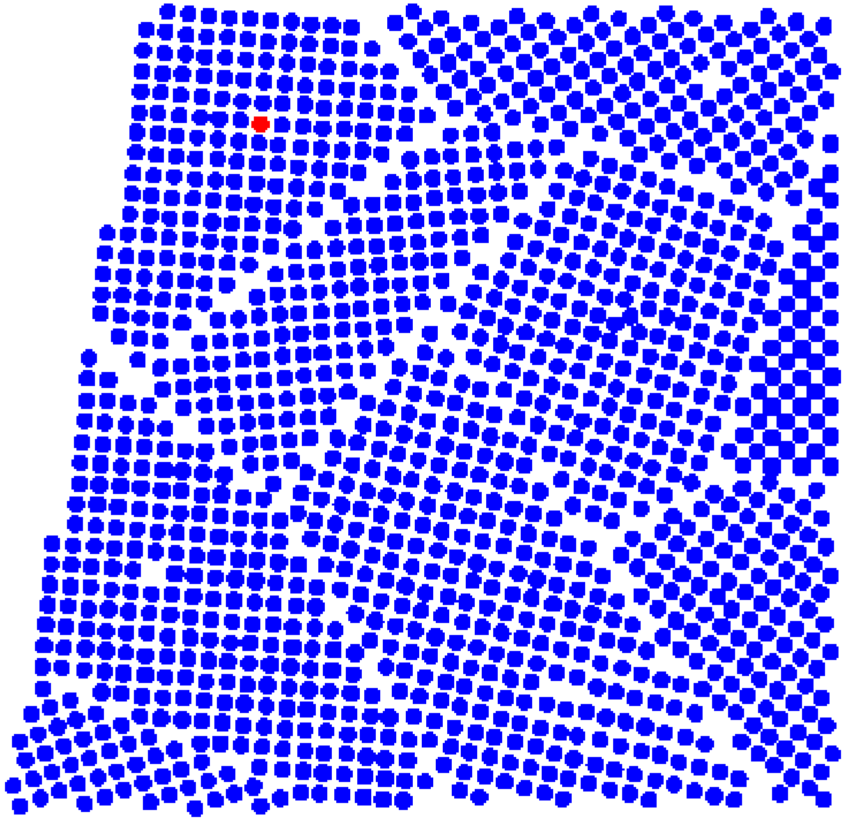


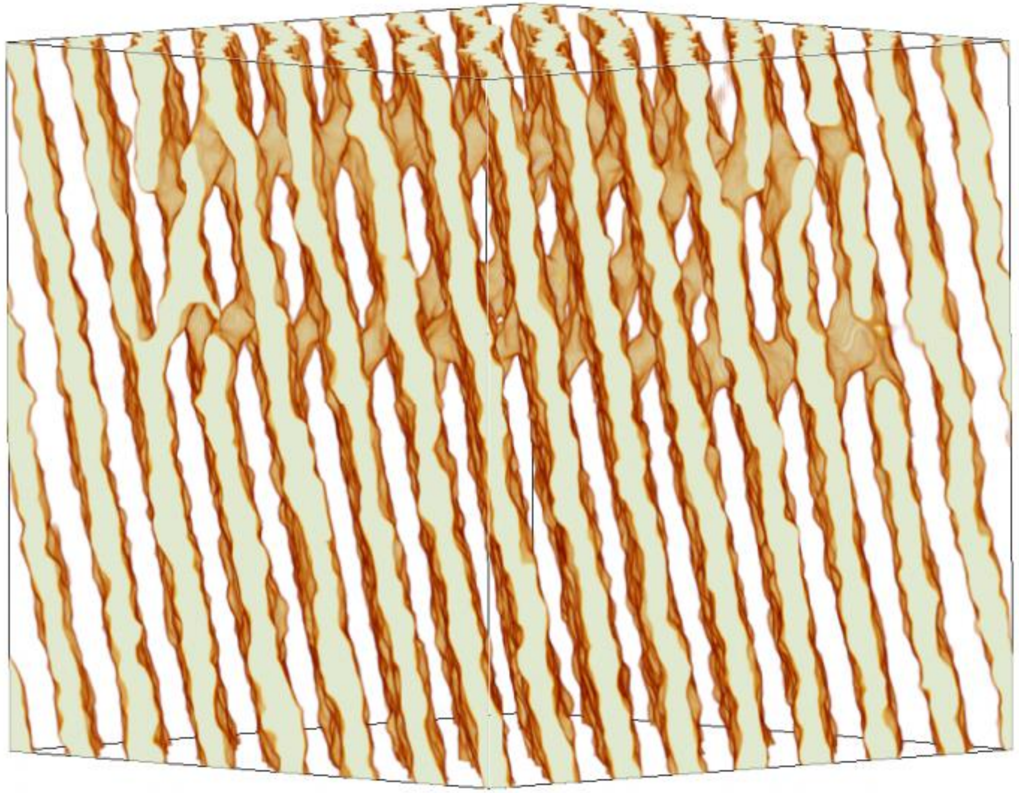
Lepton Scattering

- Supernova: Hot, proton rich
  - Neutrino scattering?
- Neutron Stars: Cold, proton poor
  - Electron scattering?

Defects

- In the same way that crystals have defects, pasta does too!
- Electrons don't scatter from *order*, they scatter from *disorder*

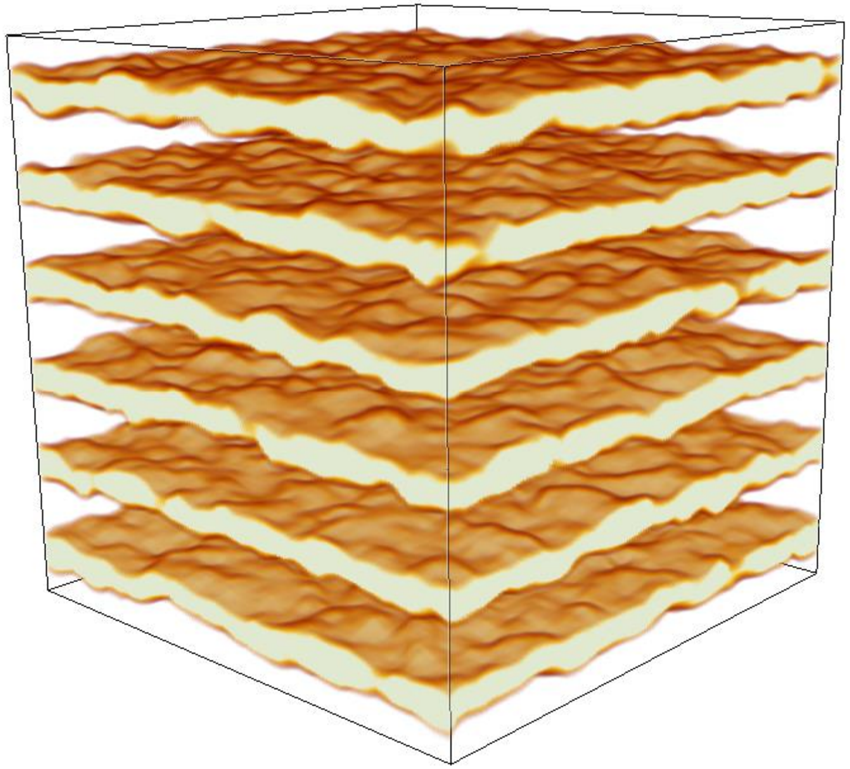




- Horowitz et al, PRL.114.031102 (2015)

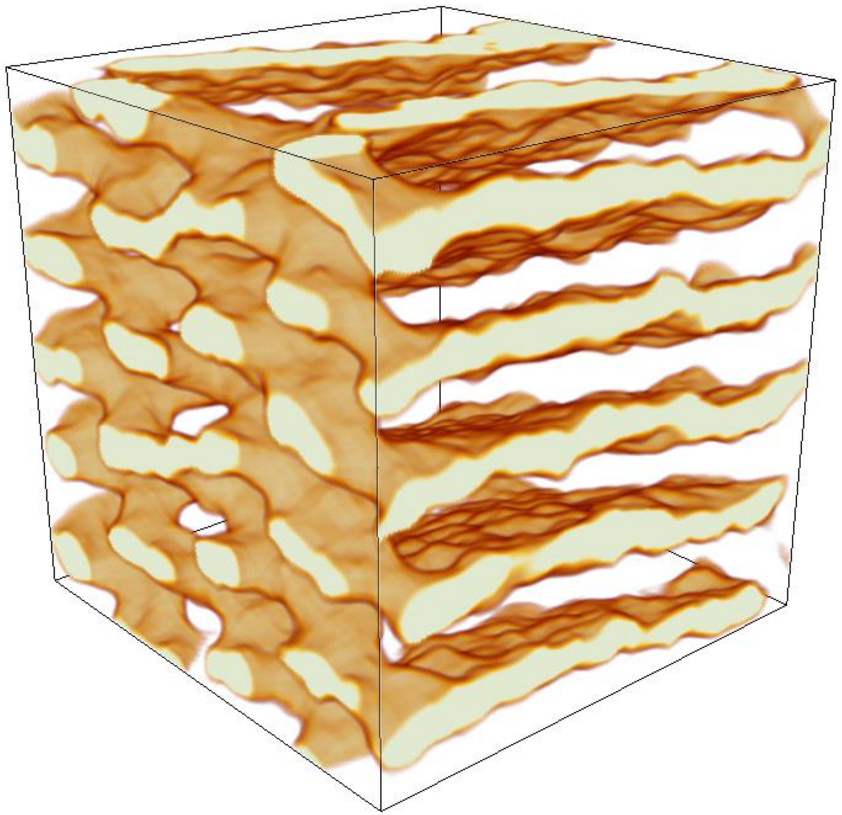
Pasta Defects

- Defects act as a site for *scattering*



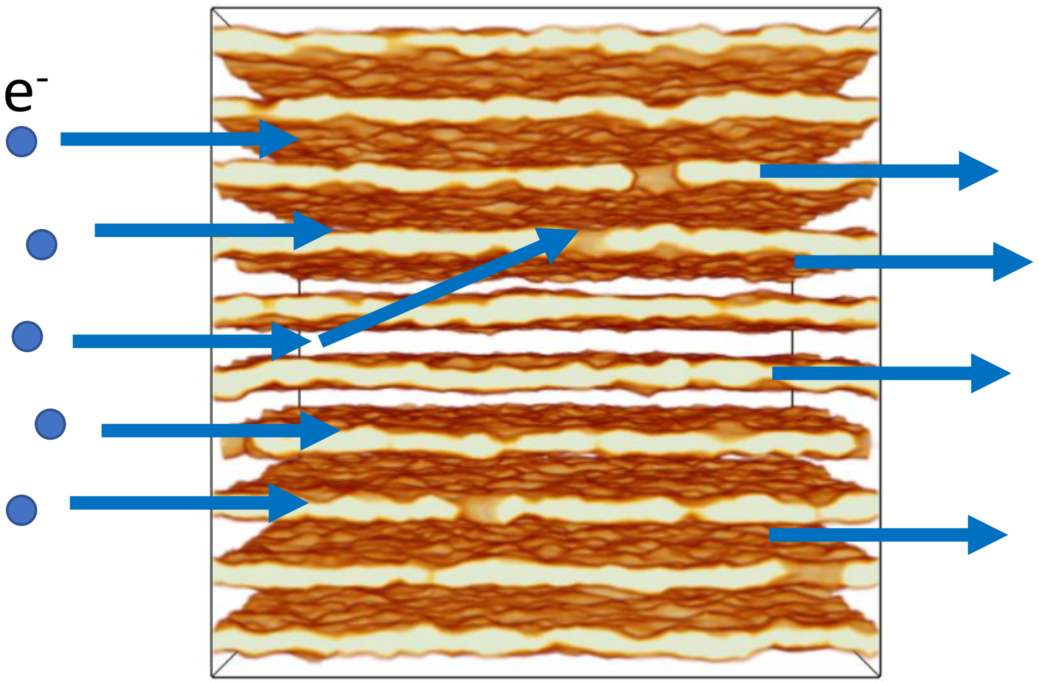
← Perfect

Defects →



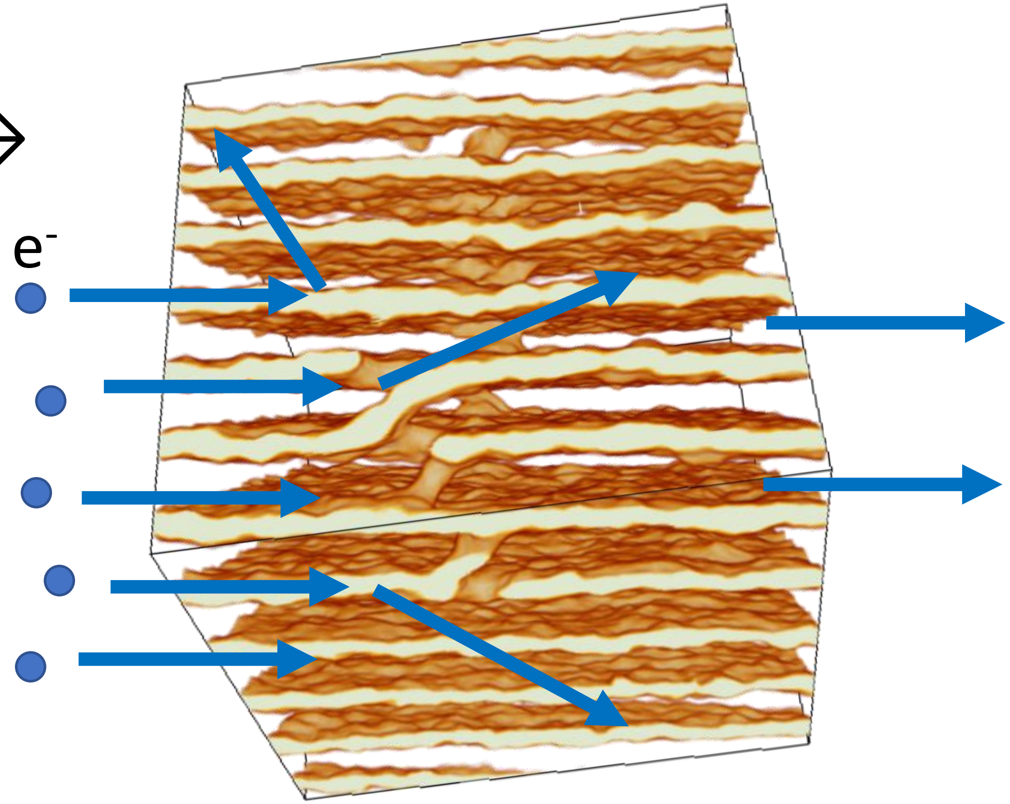
Pasta Defects

- The magnetic field decays within about 1 million years, consistent with observations (Pons et al 2013)



← Perfect

Defects →



Pons et al, Nature Physics 9, 431–434 (2013)

Lepton Scattering

- Lepton scattering from pasta influences a variety of transport coefficients:
- Shear viscosity:

$$\eta = \frac{\pi v_F^2 n_e}{20\alpha^2 \Lambda_{ep}^{\eta}},$$

- Electrical conductivity:

$$\sigma = \frac{v_F^2 k_F}{4\pi\alpha\Lambda_{ep}^{\sigma}}$$

$$\Lambda_{ep}^{\eta} = \int_0^{2k_F} \frac{dq}{q\epsilon^2(q)} \left(1 - \frac{q^2}{4k_F^2}\right)\left(1 - \frac{v_F^2 q^2}{4k_F^2}\right) S_p(q)$$

- Thermal conductivity:

$$\Lambda_{ep}^{\kappa} = \Lambda_{ep}^{\sigma} = \int_0^{2k_F} \frac{dq}{q\epsilon^2(q)} \left(1 - \frac{v_F^2 q^2}{4k_F^2}\right) S_p(q).$$

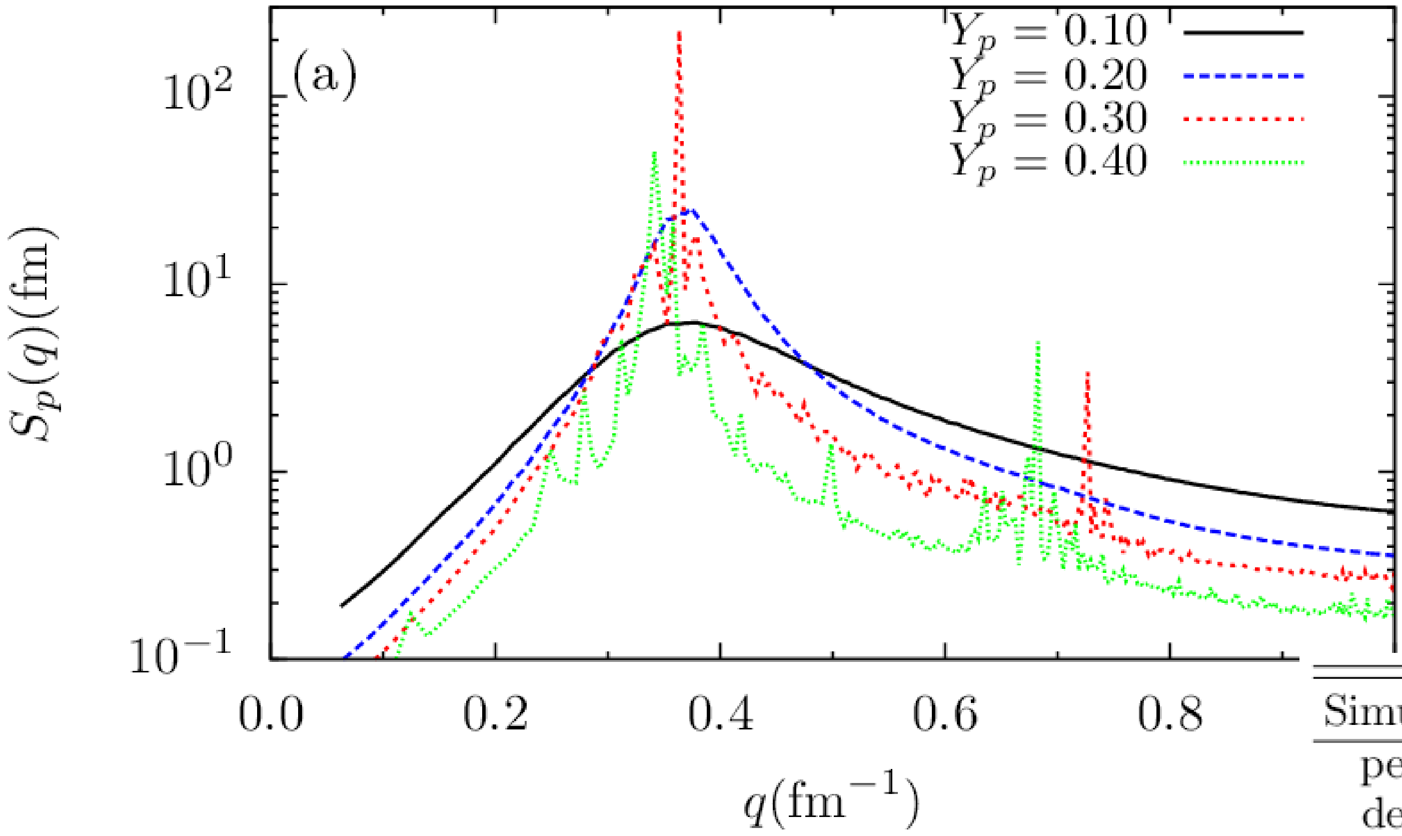
$$\kappa = \frac{\pi v_F^2 k_F k_B^2 T}{12\alpha^2 \Lambda_{ep}^{\kappa}}.$$

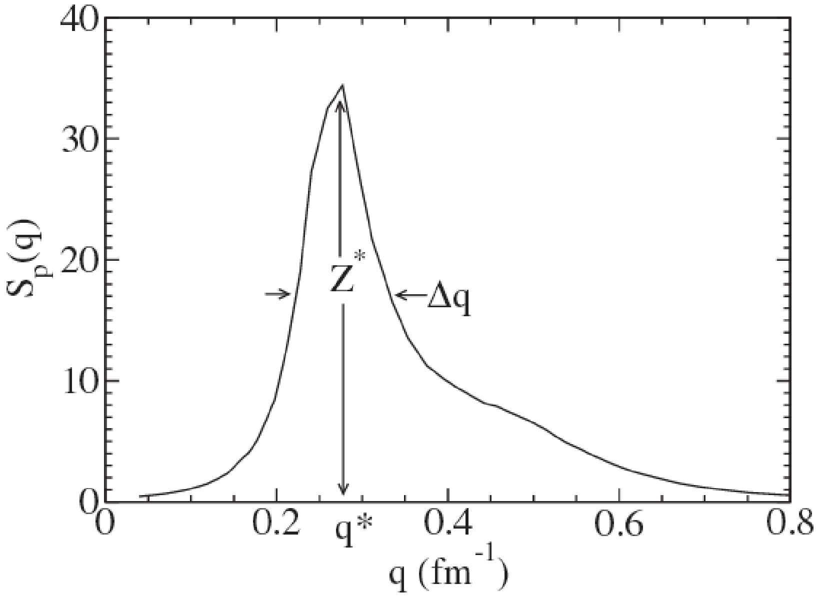
Lepton Scattering

$$S_i(\boldsymbol{q}) = \langle \rho_i^*(\boldsymbol{q},t)\rho_i(\boldsymbol{q},t)\rangle_t - \langle \rho_i^*(\boldsymbol{q},t)\rangle_t \langle \rho_i(\boldsymbol{q},t)\rangle_t$$

$$\rho_i(\boldsymbol{q},t) = N_i^{-1/2} \sum_{j=1}^{N_i} e^{i\boldsymbol{q}\cdot\boldsymbol{r}_j(t)}$$

$$\Lambda_{ep} \approx \frac{\Delta q^* Z^*}{q^*}$$





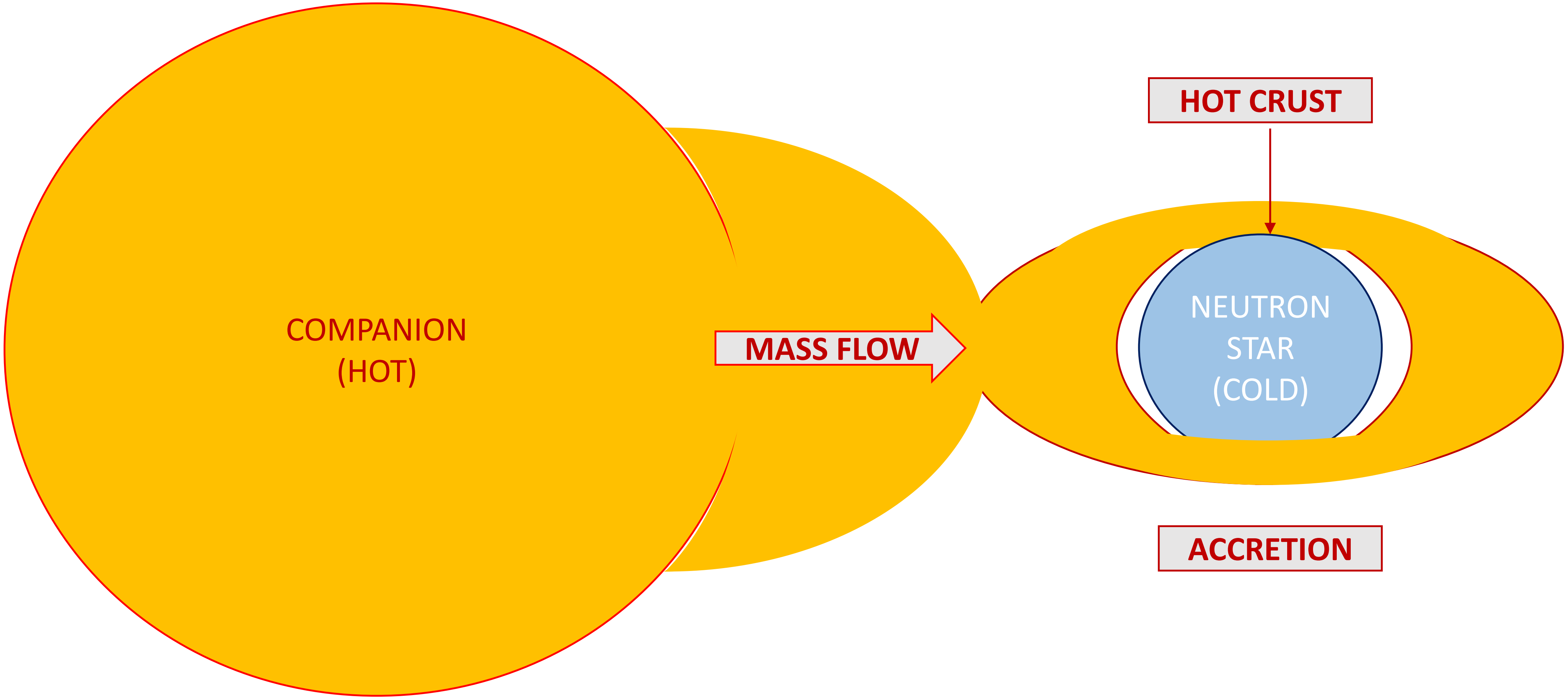
Simulation	$\bar{\eta}(\mathrm{fm}^{-3})$	$\bar{\kappa}(10^3 k_B\ \mathrm{MeV/fm})$	$\bar{Z}^*$
perfect	87.7	6.66	5.5
defects	55.5	4.15	50.2

Crust Cooling

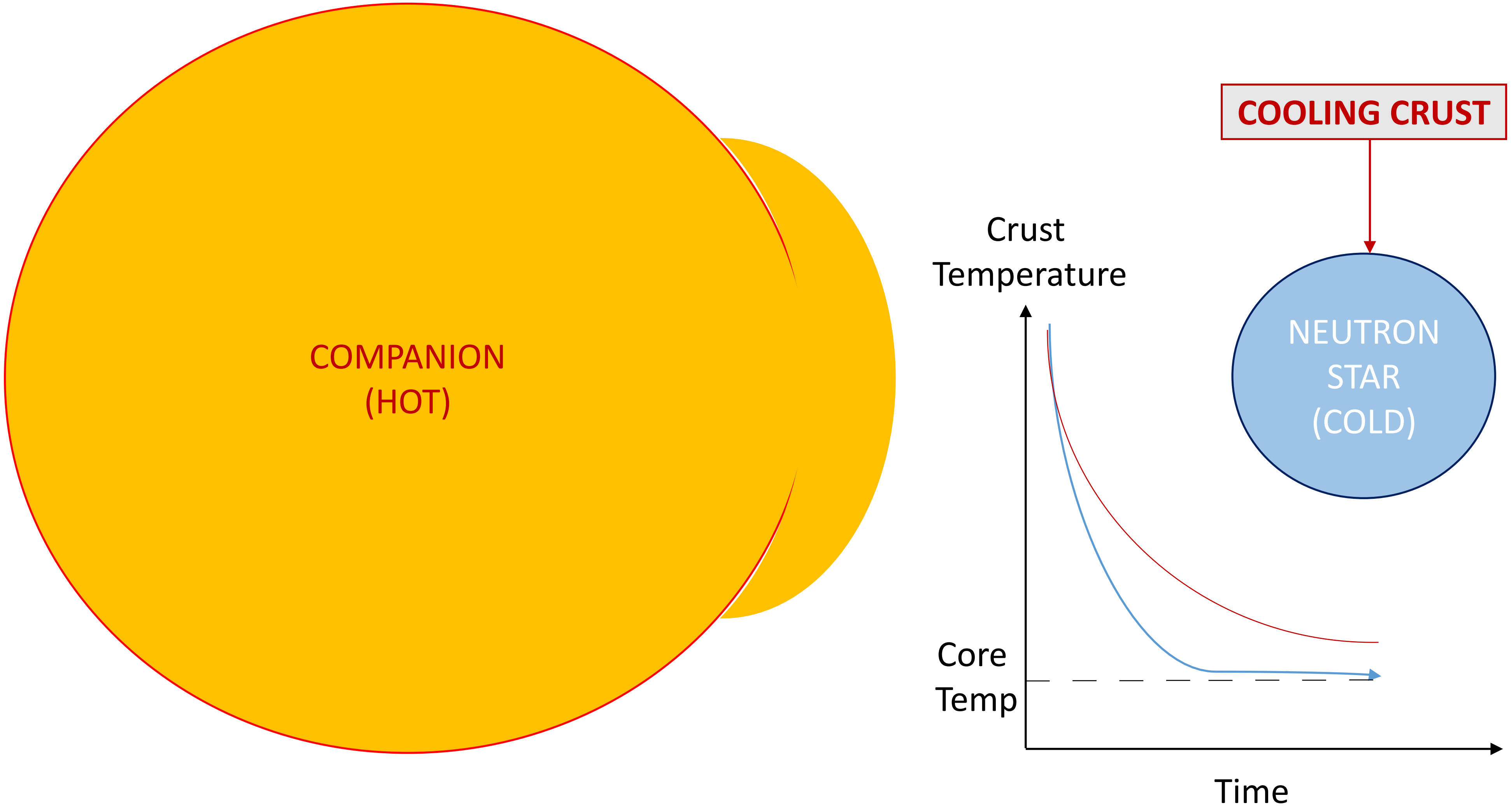
Thermal Resistivity

- So we’ve seen that pasta is electrically resistive…
- Electrons are a thermal carrier too! How does poor *thermal* conductivity effect the neutron star?

Low Mass X-Ray Binaries



Low Mass X-Ray Binaries

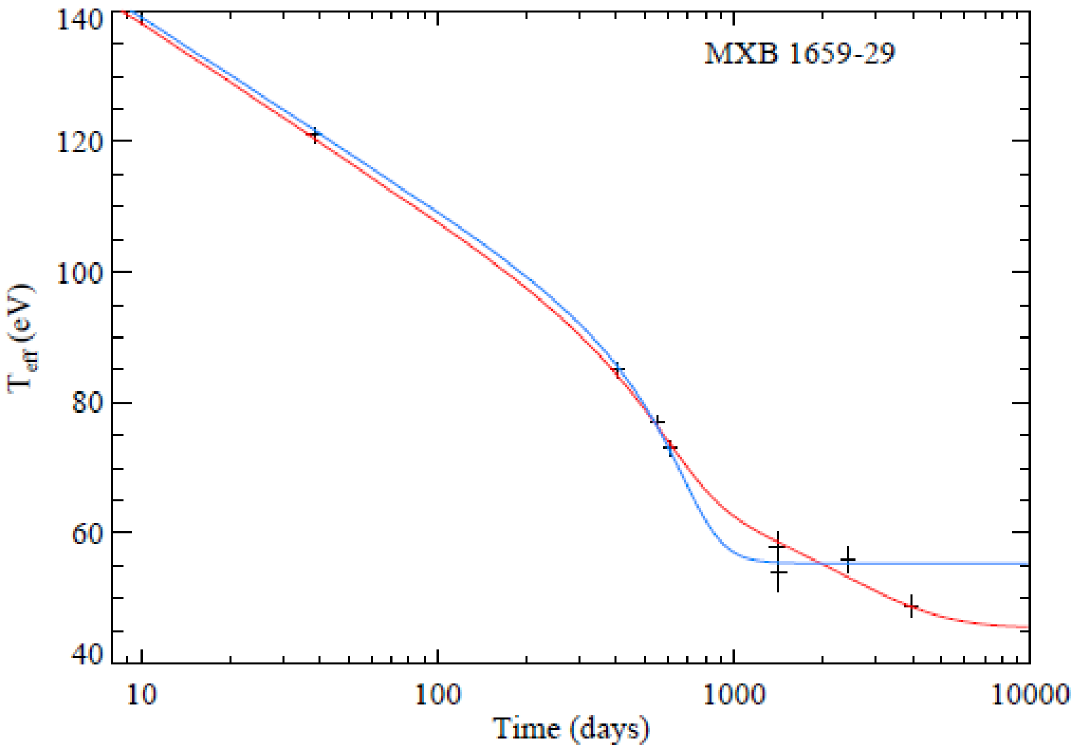


Observables – Thermal Properties

- Guess an effective impurity parameter for defects and try to fit neutron star cooling curves

$$Q_{\rm imp} \equiv n_{\rm ion}^{-1} \sum_i n_i (Z_i - \langle Z \rangle)^2$$

- Cooling curves: low mass X-ray binary MXB 1659-29
  - Blue: normal isotropic matter
    Q_{imp} = 3.5
    $T_c = 3.05 \times 10^7$ K
  - Red: impure pasta layer
    Q_{imp} = 1.5 (outer crust)
    Q_{imp} = 30 (inner crust)
    $T_c = 2 \times 10^7$ K

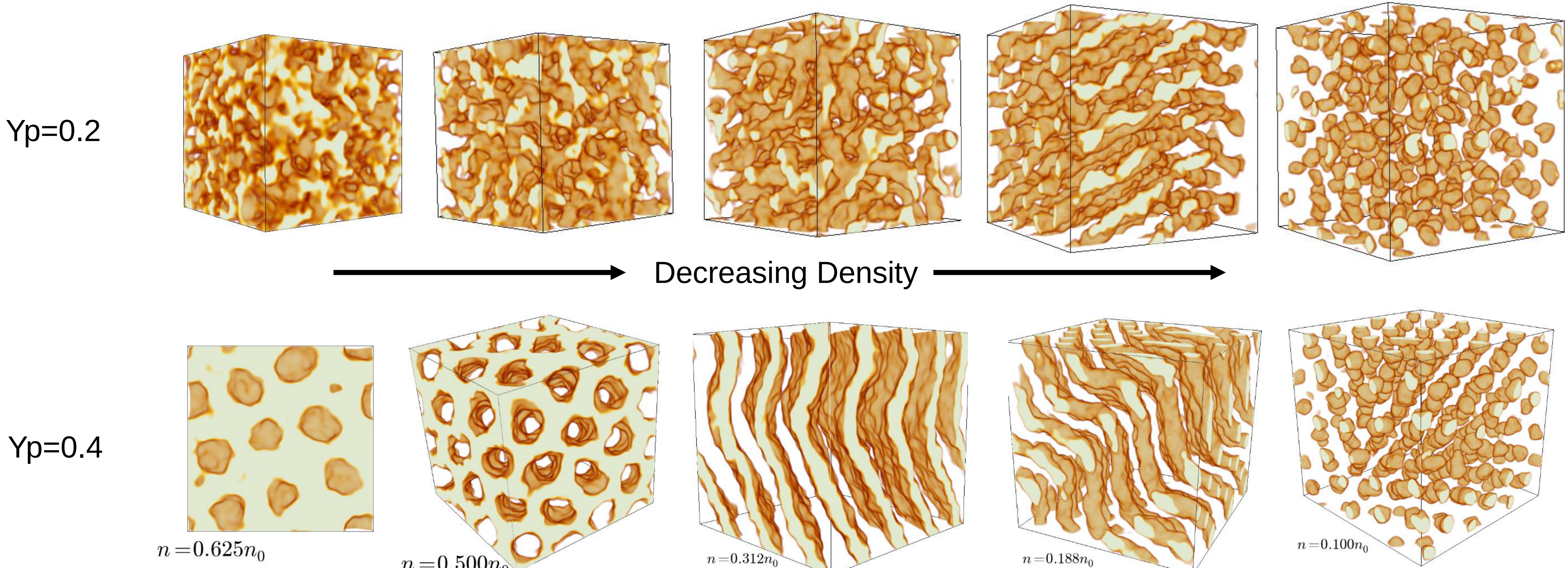


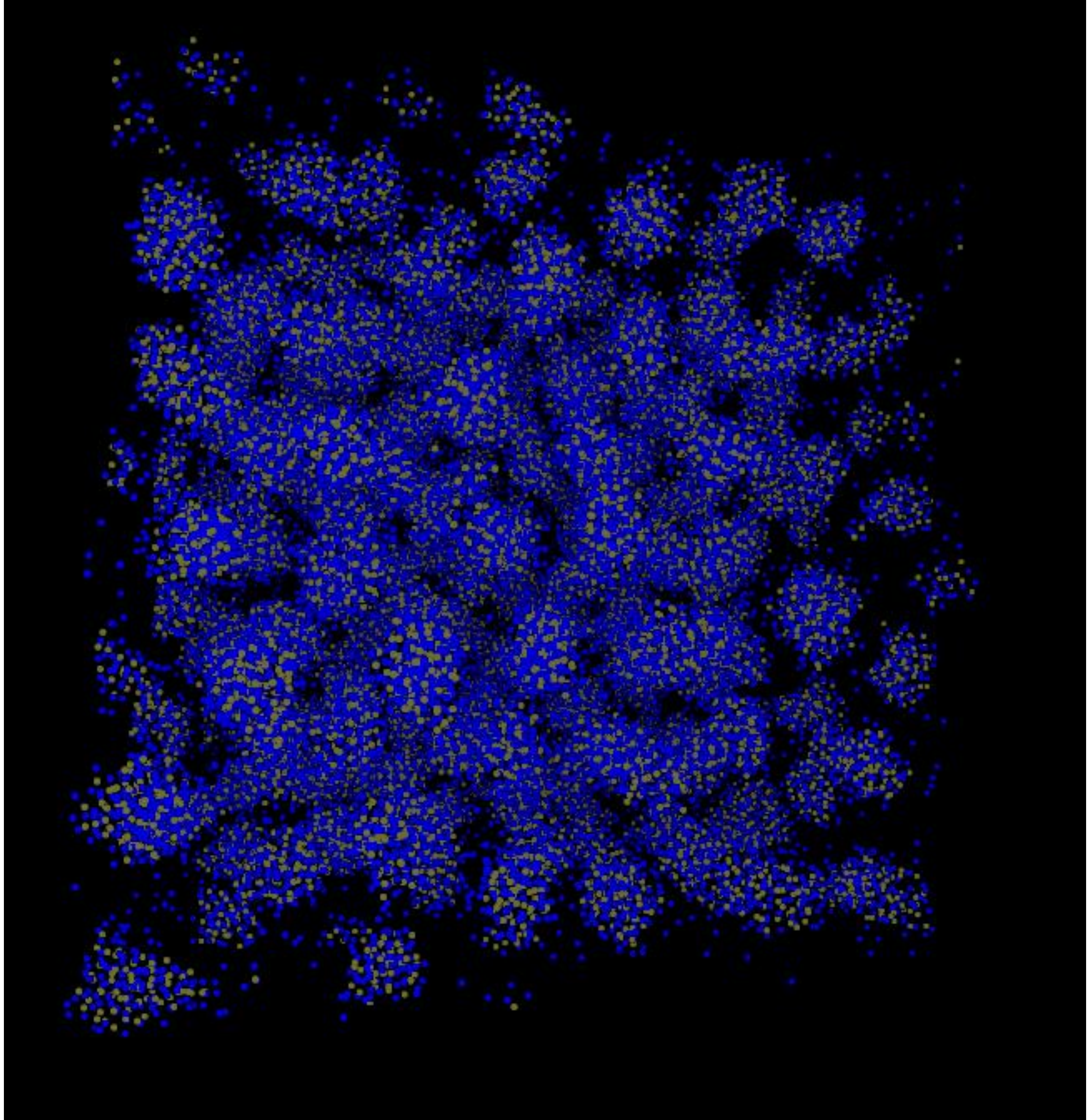
Horowitz et al (2015) PhysRevLett 114, 031102

r-process

Fragmentation

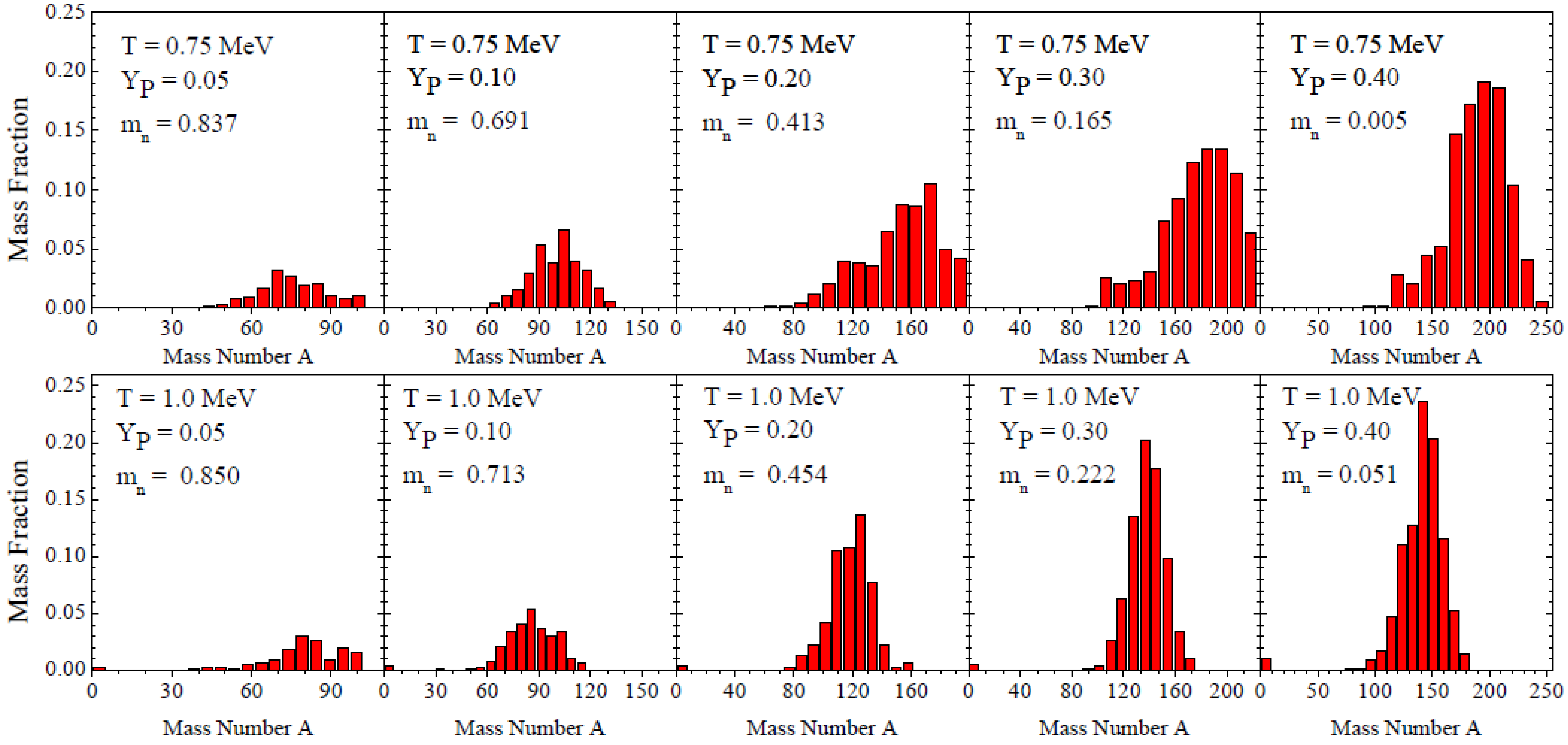
- Simulate pasta with constant temperature and proton fraction
- Observe ‘nuclei’ that form as pasta decompresses





Cluster Masses

Caplan (2015) Phys. Rev. C 91, 065802



Nuclear Statistical Equilibrium

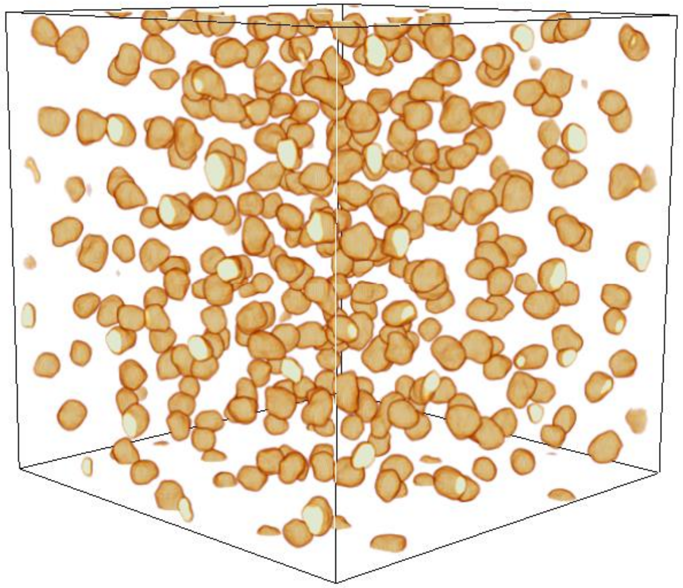
- Use a cluster algorithm to count A, Z of gnocchi
- Simulations at 1 MeV reproduce NSE for sufficiently slow evolution!

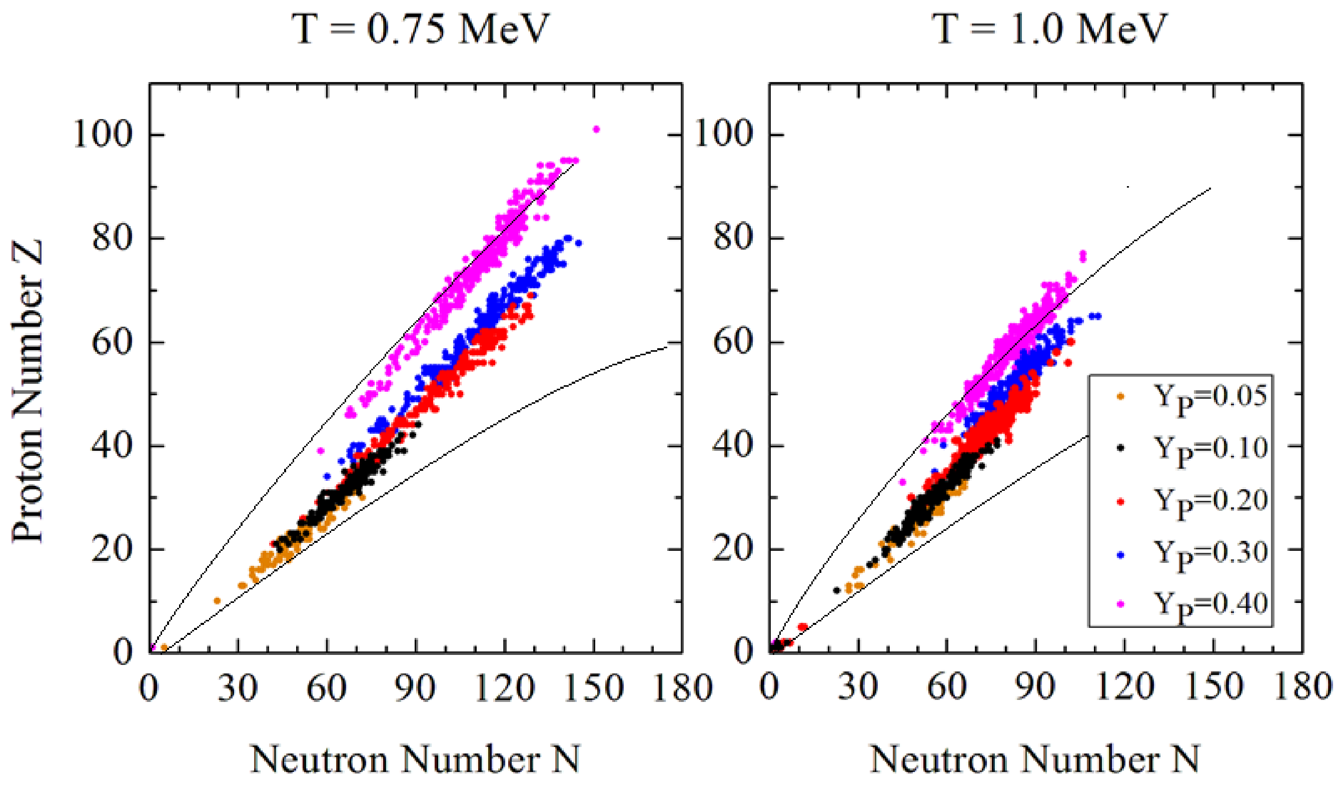
		IUMD			NSE		
Y_P	$log_{10}\dot{\xi}$	$M_{\text{free neutrons}}$	$\overline{A} \pm \sigma_A$	$\overline{Z} \pm \sigma_Z$	$M_{\text{free neutrons}}$	$\overline{A}$	$\overline{Z}$
0.1	-5	0.6538	90.12 ± 24.20	26.25 ± 7.69	Unavailable		
	-6	0.6561	94.08 ± 18.01	27.57 ± 5.69			
	-7	0.6578	99.18 ± 17.42	29.20 ± 5.57			
0.2	-5	0.3748	147.75 ± 33.88	47.31 ± 10.95	0.3335	179.3	53.80
	-6	0.3731	146.22 ± 24.96	46.69 ± 8.30			
	-7	0.3704	136.12 ± 20.32	43.29 ± 6.72			
0.3	-5	0.1359	186.47 ± 73.19	64.75 ± 24.59	0.0475	184.4	58.07
	-6	0.1377	175.13 ± 34.48	60.94 ± 12.16			
	-7	0.1367	166.74 ± 34.71	57.95 ± 12.44			
0.4	-5	0.0109	369.37 ± 426.03	149.32 ± 167.45	0.0001	194.4	77.75
	-6	0.0121	179.40 ± 29.83	72.64 ± 11.93			
	-7	0.0115	190.25 ± 29.34	76.94 ± 11.83			
	-8	0.0111	191.74 ± 24.55	77.55 ± 9.79			

Caplan (2015) Phys. Rev. C 91, 065802

Table of Nuclides

- Nuclear pasta simulations at constant proton fraction reproduce nuclear statistical equilibrium:
- Right distribution of isotopes and number of free neutrons for a given T





Caplan (2015) Phys. Rev. C 91, 065802

Summary

- Classical models give reasonable results, provided you stay in the range they are valid
- They can inform transport properties of the neutrino star crust, and be used to interpret neutron star observables.