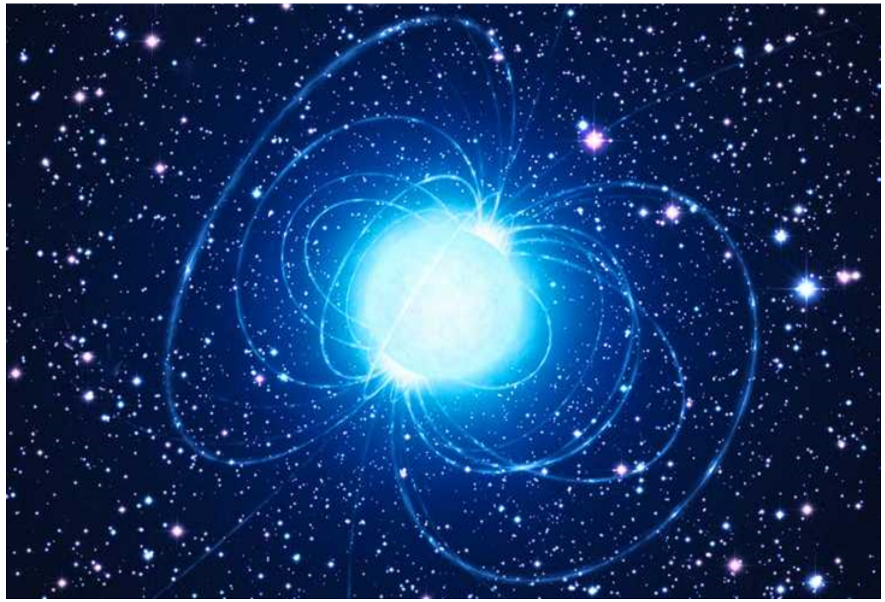
Ilaria Caiazzo
University of British Columbia



Modelling Pulsar Glitches with different Equations of State

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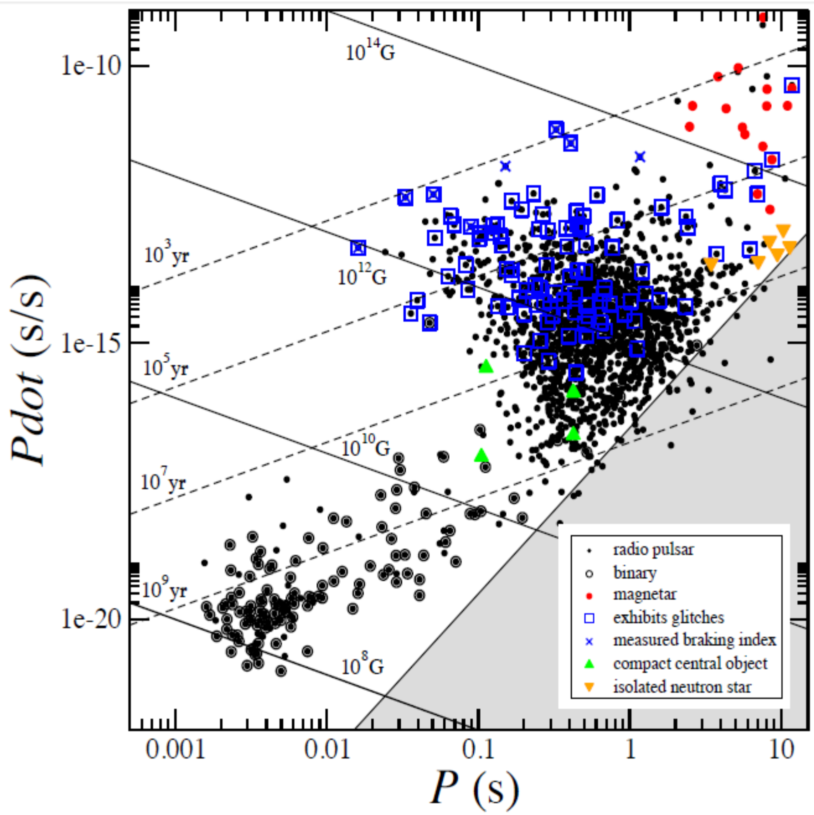
comp
star

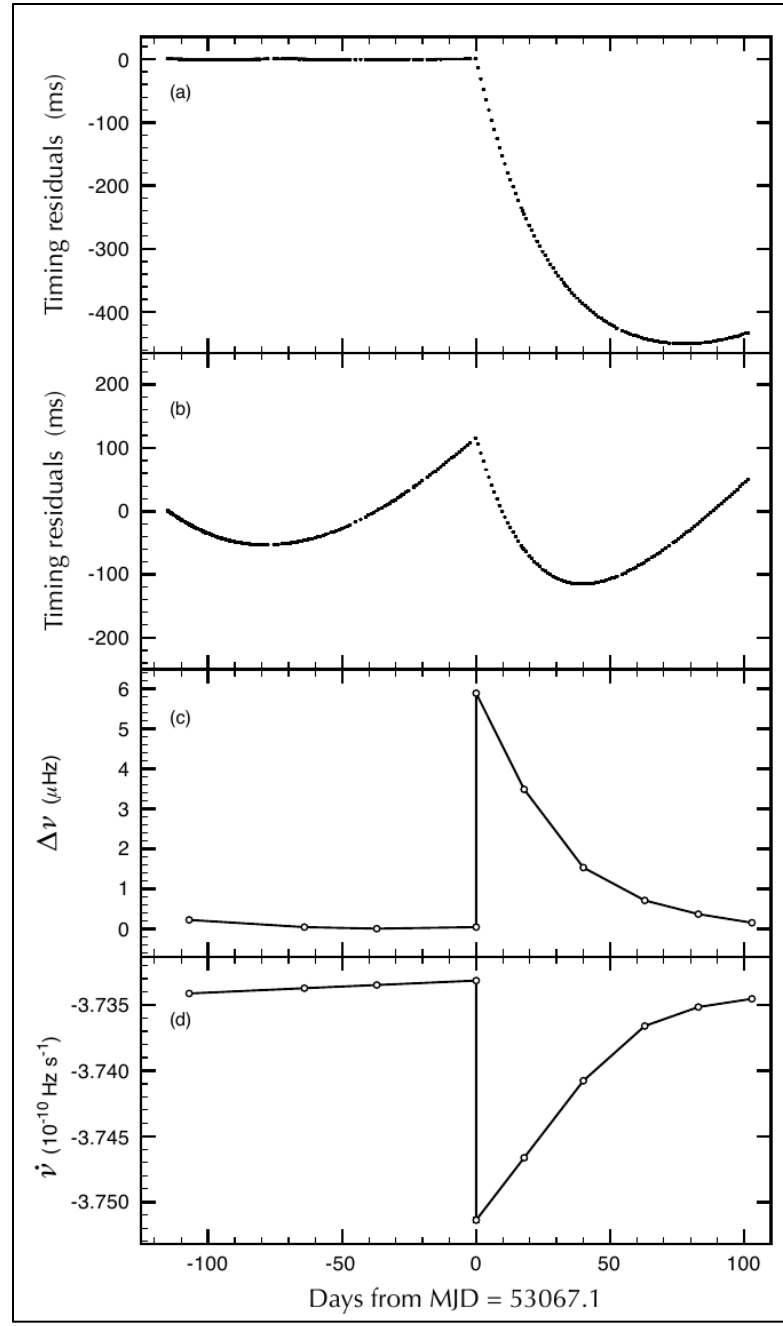
Pulsar Glitches

- Sudden jumps in the spin frequencies:

$$\frac{\Delta\nu}{\nu} \sim 10^{-11} - 10^{-5}$$

- *Giant* or *Vela-like* glitches:
- $\frac{\Delta\nu}{\nu} \sim 10^{-6} - 10^{-5}$, $\left|\frac{\Delta\nu'}{\nu'}\right| \sim 10^{-4} - 1$
  - ➢ rough periodicity.
- Slow post glitch relaxation

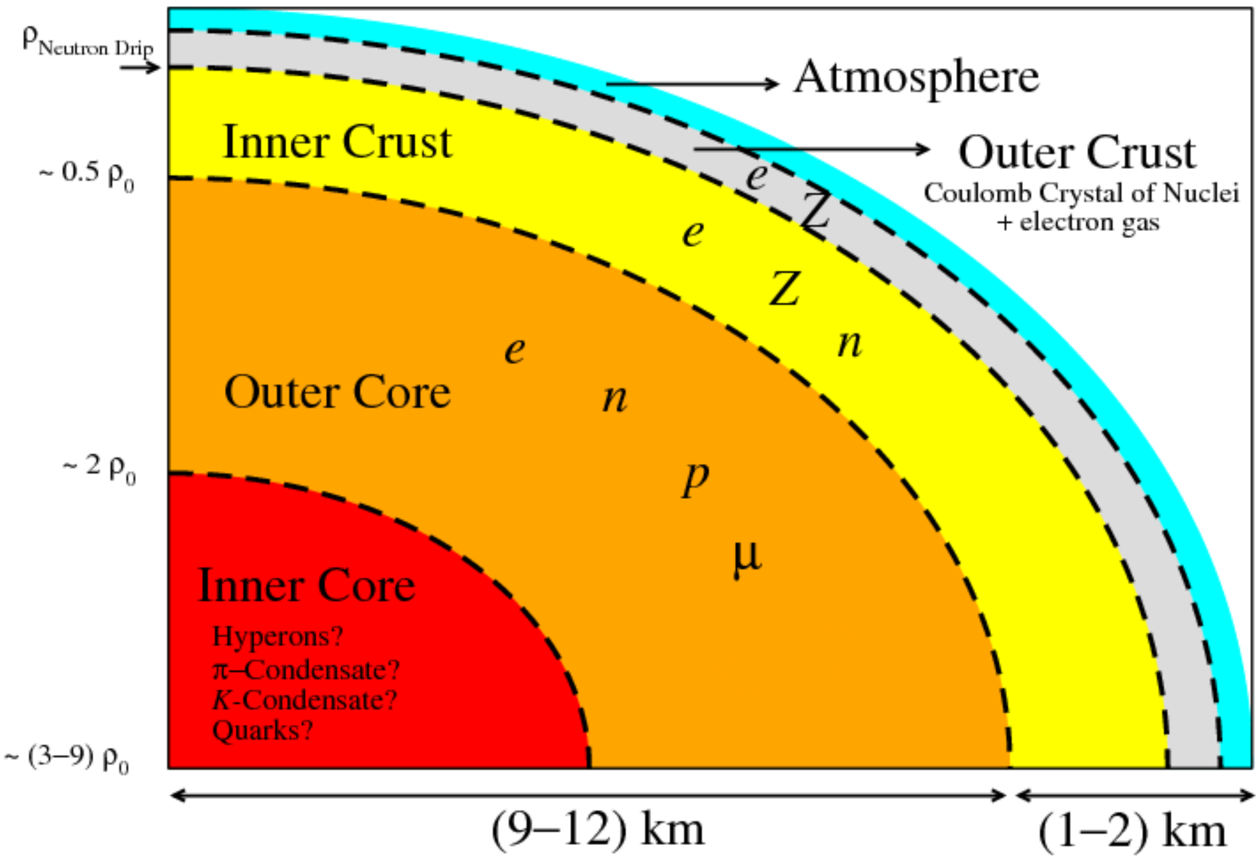




Espinoza et al.

Superfluidity

- Long post-glitches relaxation times → Observative proof for the presence of superfluidity inside NSs
- Pairing gap in cold nuclear matter ~ 1-2 MeV → Protons and neutrons are expected to be **superfluid** inside NS ($T \lesssim 1$ keV)



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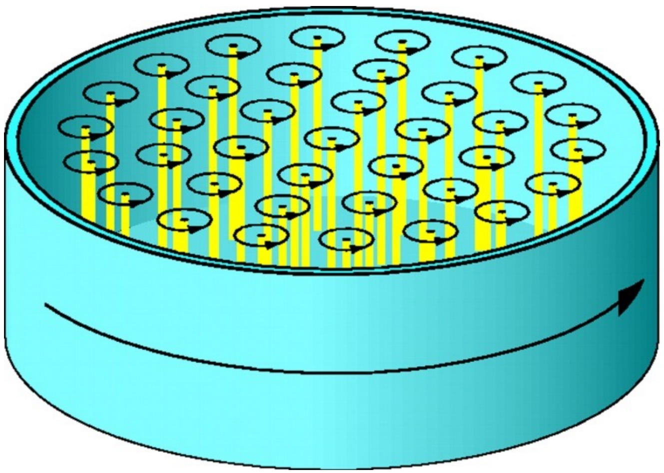
- $\nabla \times \boldsymbol{v}_n = 0$ → a superfluid can rotate only by forming an array of quantized **vortices** each with vorticity $\kappa = \frac{h}{2m_n}$

$$\oint \boldsymbol{p}_n \cdot d\boldsymbol{l} = \frac{hN(x,t)}{2}$$



Lounasmaa and Thuneberg

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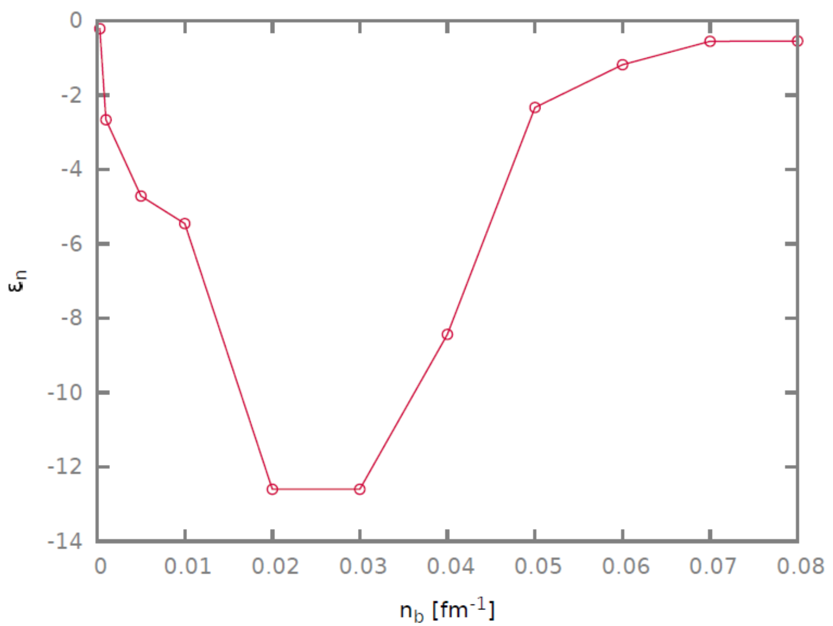
$$\boldsymbol{p}_n = m_n[\boldsymbol{v}_n + \epsilon_n(\boldsymbol{v}_p - \boldsymbol{v}_n)]$$



Chamel 2012

Superfluidity

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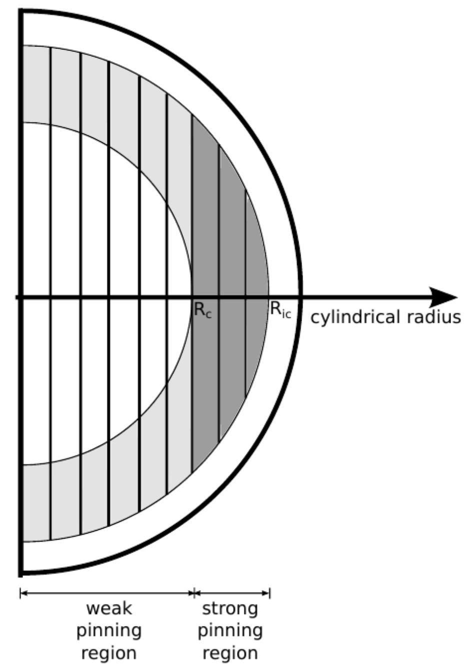
- **Entrainment:** a non dissipative drag arises when a superfluid is in contact with another fluid.

$$\boldsymbol{p}_n = m_n[\boldsymbol{v}_n + \epsilon_n(\boldsymbol{v}_p - \boldsymbol{v}_n)]$$

- **Crustal pinning:** due to the interaction between a vortex line and a nucleus there is a preferred position for the vortex → **interstitial** or **nuclear** pinning

  The totality of the interactions between a vortex line and nuclei in the crustal lattice results in a potential well in which the vortex line is pinned



Seveso S.

The two components model for glitches

'Normal' component: charged particles (core) and ion lattice (crust)

Superfluid component: neutrons (core) and dripped neutrons (crust)

- The normal component slows down (external electromagnetic torque)
- The neutron superfluid can slow down only if it expels a vortex
- Vortices are pinned → a lag builds up among the two components
- When a **critical lag** is reached, vortices are set free. The stored angular momentum is transferred to the crust, causing a glitch.

The Snowplow Model

Pierre Pizzochero

Stefano Seveso

Marco Antonelli

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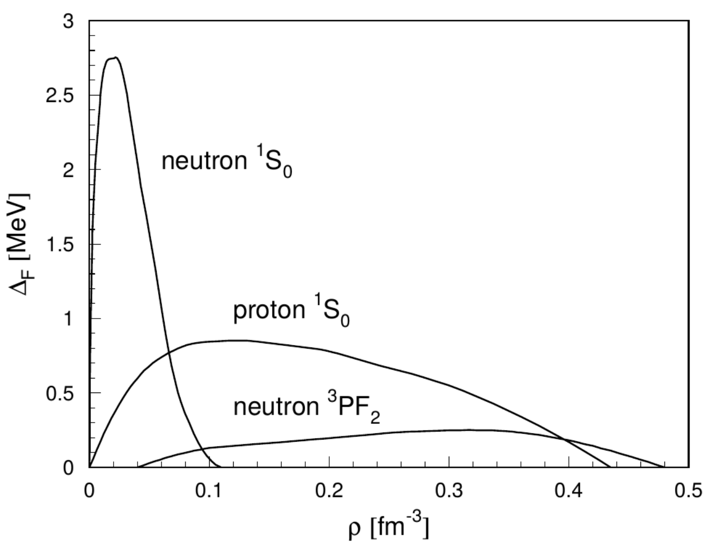
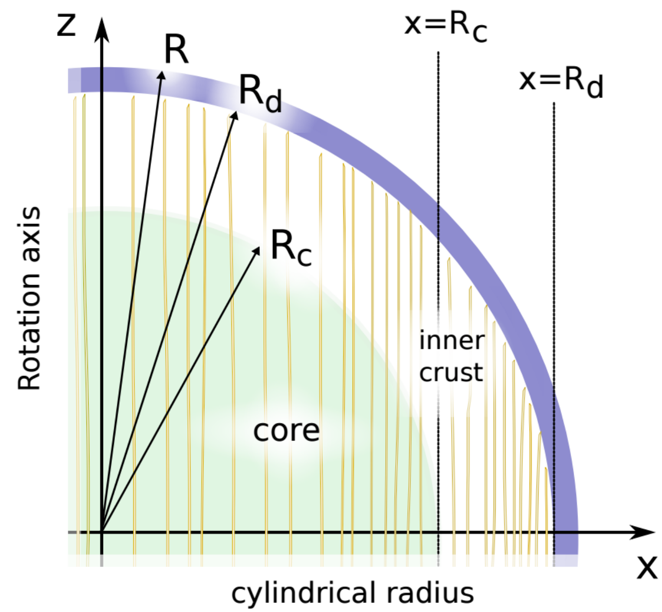
The Snowplow Model

Assumptions:

- Vortex lines in the crust and the core are connected
- Vortex lines are straight
- No Hyperons, no quarks



Lombardo and Schulze



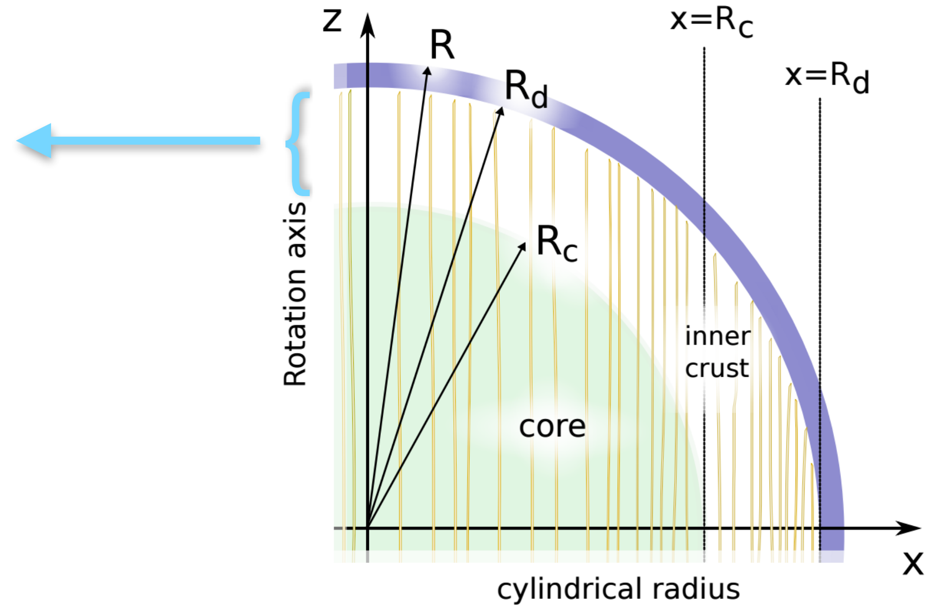
Antonelli et al.

The Snowplow Model

TOTAL FORCE ON A VORTEX LINE

Pinning force: $F_{pin}(x)$

depends on how much of the vortex is immersed in the crust and then on the cylindrical radius



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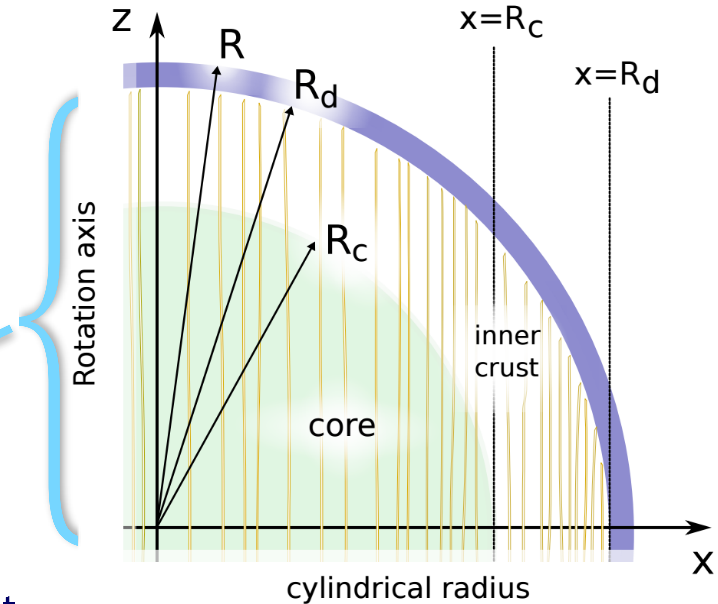


Antonelli et al.

Magnus force:

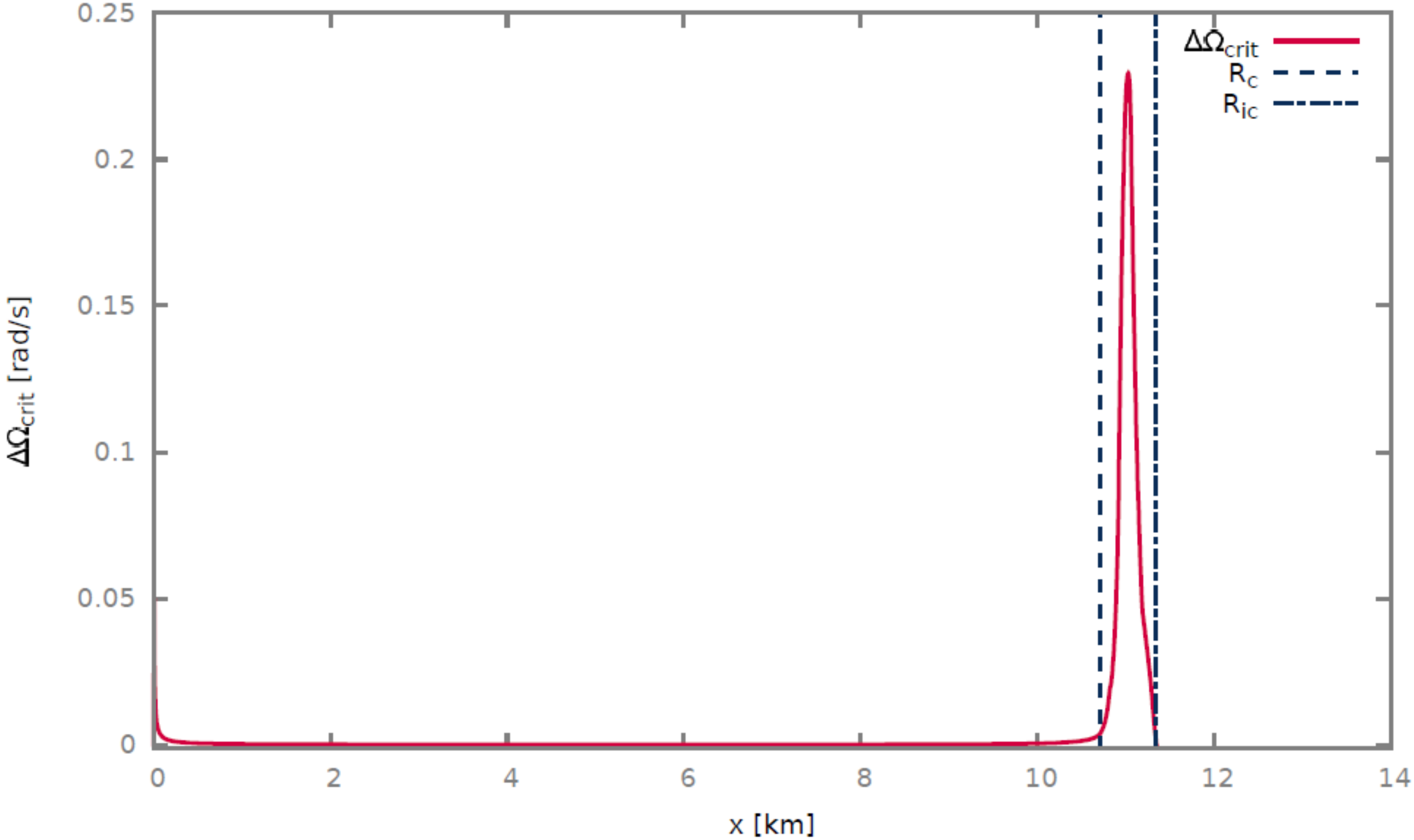
the vortex line is forced to rotate along with the crust

→ different angular velocity with respect to the fluid in which it's immersed

a hydrodynamical lift proportional to the lag arises

$$F_M(x,t) \propto \Delta\Omega(x,t)$$

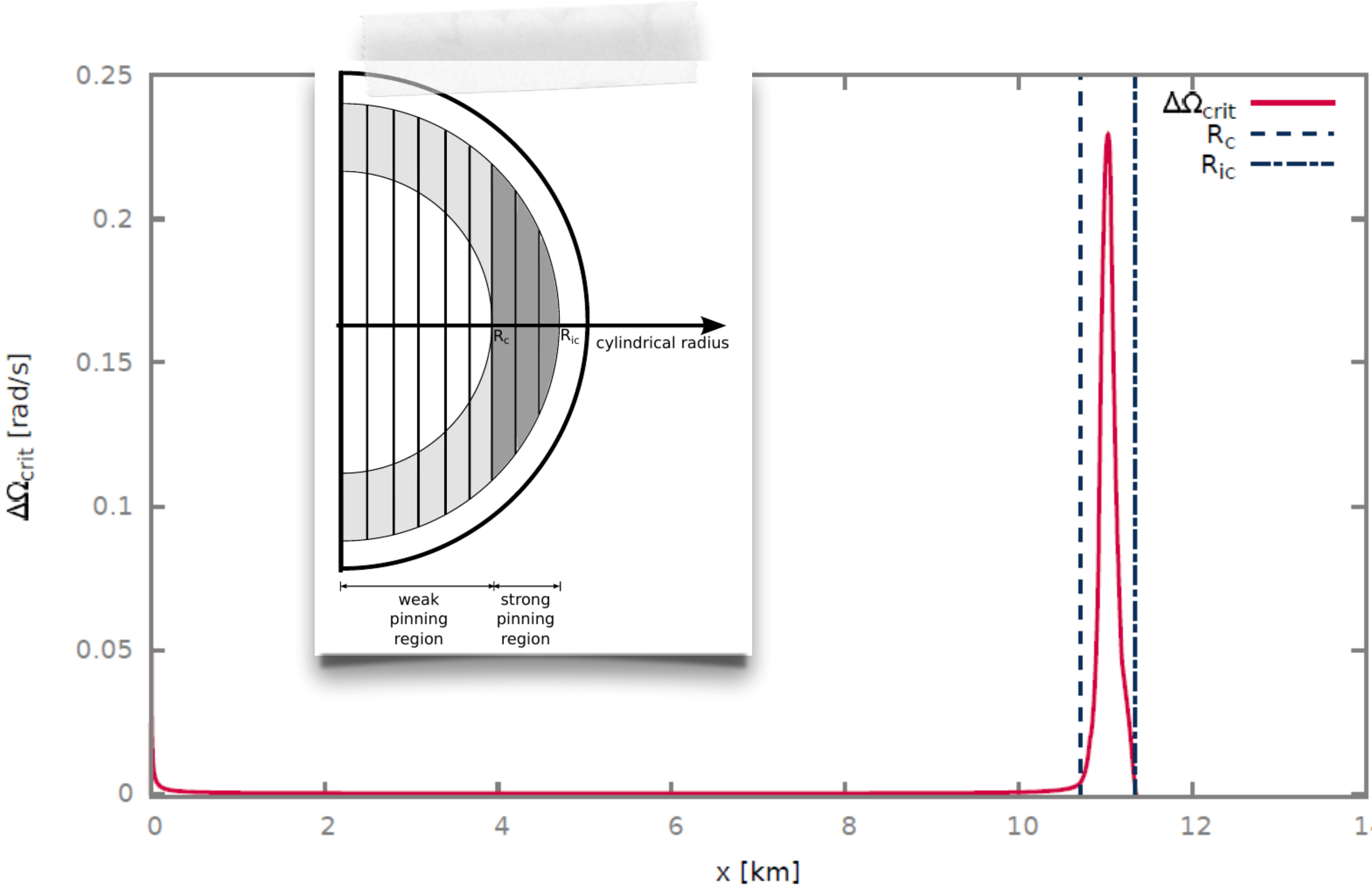
The Snowplow Model



$$F_M(x) = F_{pin}(x)$$ → **CRITICAL LAG**

The vortices within that cylindrical radius are depinned by the magnus force

The Snowplow Model



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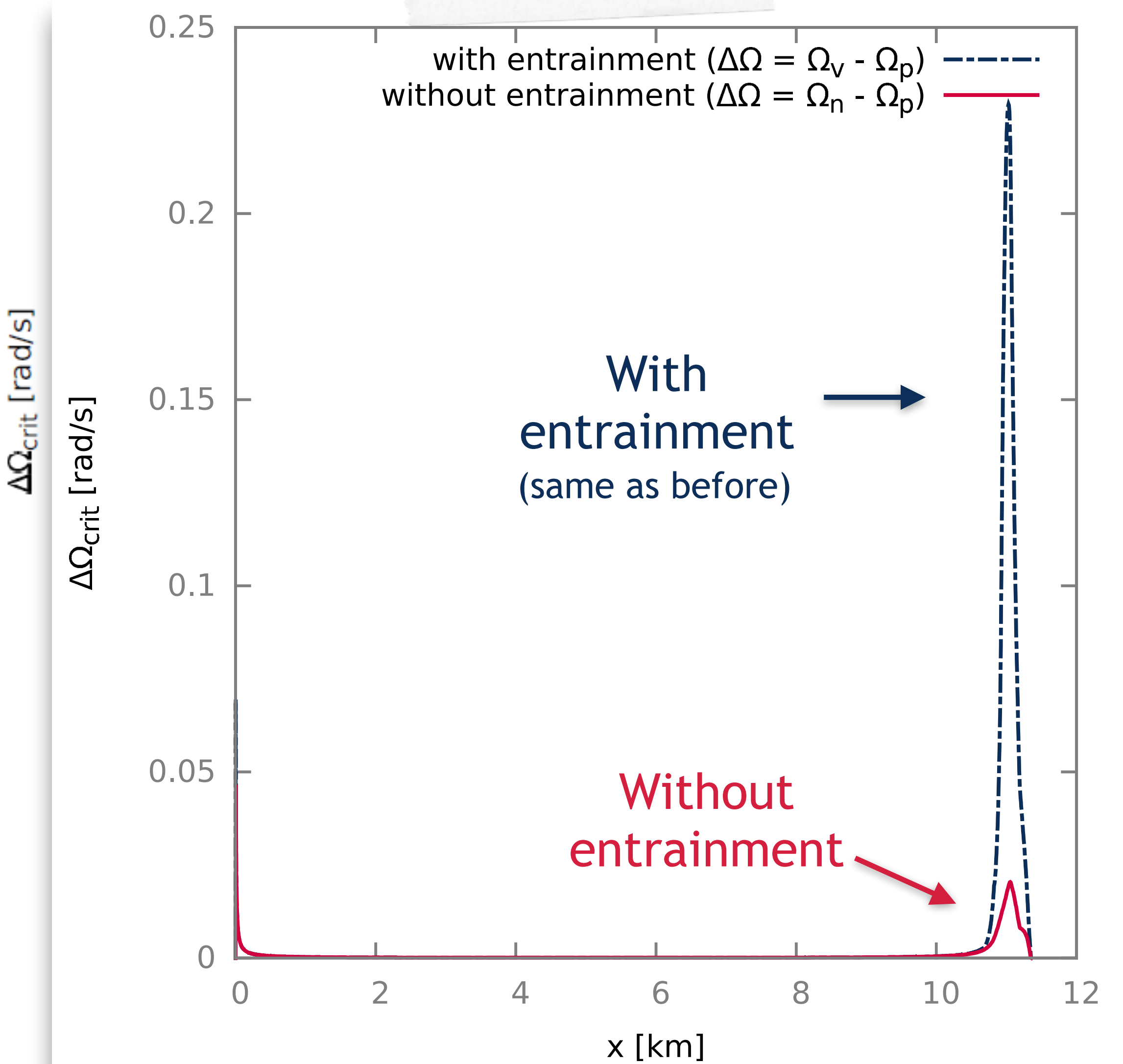
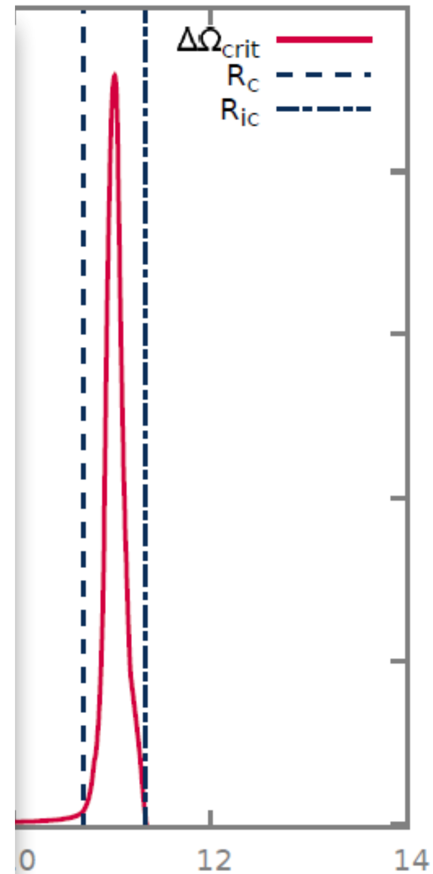




ICAL LAG

The vortices within that cylindrical radius
are depinned by the magnus force

The Snowplow Model

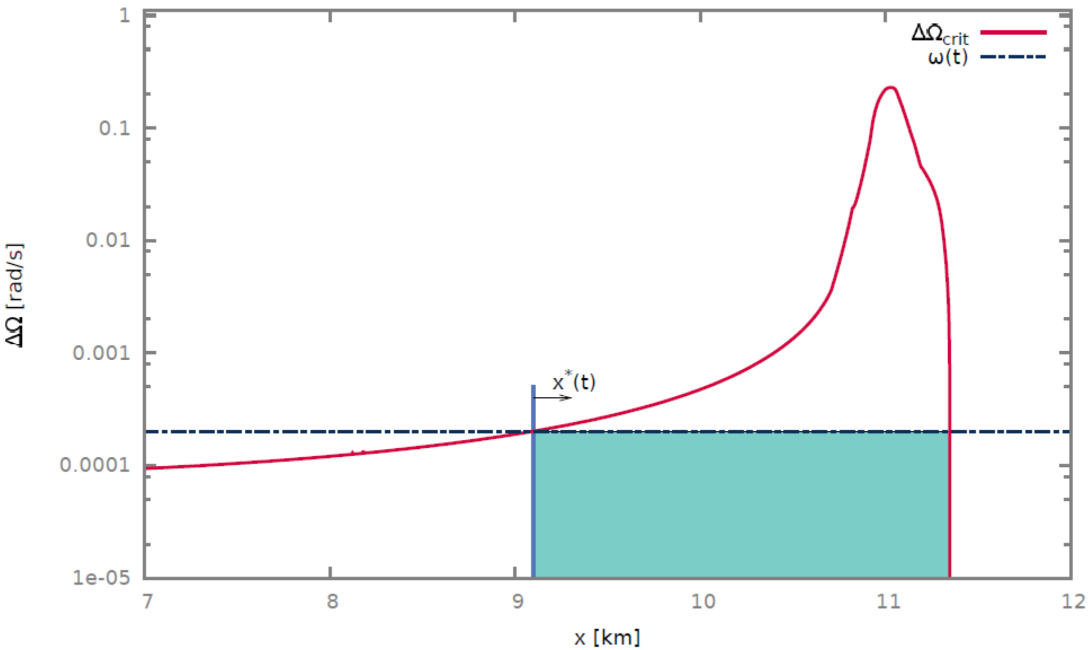
Critical lag first reached in the core → Formation of a **VORTEX SHEET** which moves towards the outer crust as the lag increases

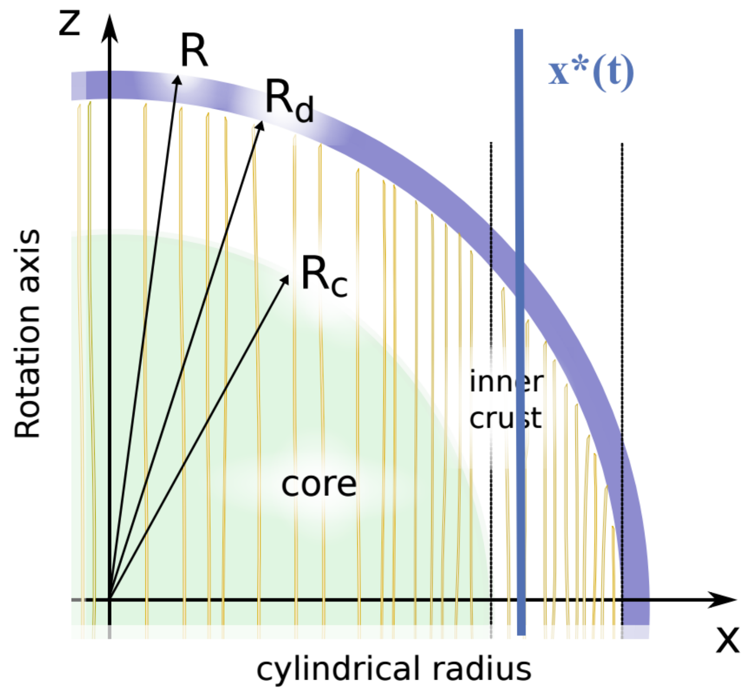
lag between the two components as the crust slows down:

$$\omega(t) = |\dot{\Omega}_c| t$$

position of the vortex sheet:

$$x^*(t)$$





Highlighted region: angular momentum that can be released to the crust if a glitch is triggered at a time *t* after the previous glitch

$$\Delta L_{gl}(t)$$

The Snowplow Model

If ΔL_{gl} is transferred to the crust, the subsequent jump in angular velocity will be:

$$\Delta\Omega_{gl}(t) = \frac{\Delta L_{gl}(t)}{(1-Q)I_{tot} + QY_{gl}I_{tot}}$$

Where:

- Y_{gl} is the fraction of vorticity coupled to the crust in the timescale of a glitch
- Q is the fraction in moment of inertia of the superfluid component:

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OBSERVED

FROM EOS + M

FREE PARAMETERS

Where:

- Y_{gl} is the fraction of vorticity coupled to the crust in the timescale of a glitch
- Q is the fraction in moment of inertia of the superfluid component:

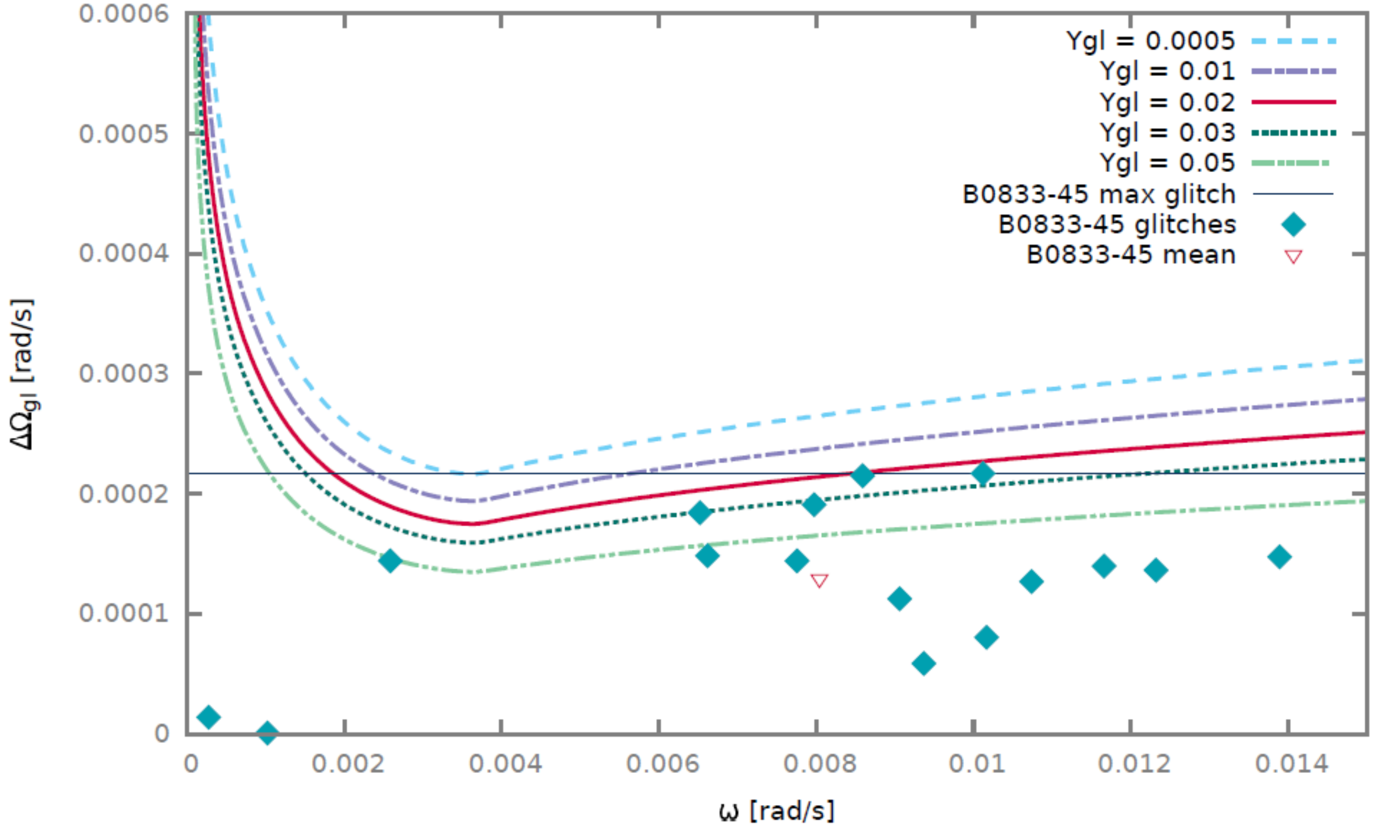
Estimating masses with the snowplow model

$$\Delta\Omega_{gl}(t) = \Delta\Omega_{gl}(\omega)$$

$$\omega = |\dot{\Omega}_p|t$$

We have obtained a profile that depends on two parameters:

$$\mathrm{M}\ ,\ Y_{gl}$$

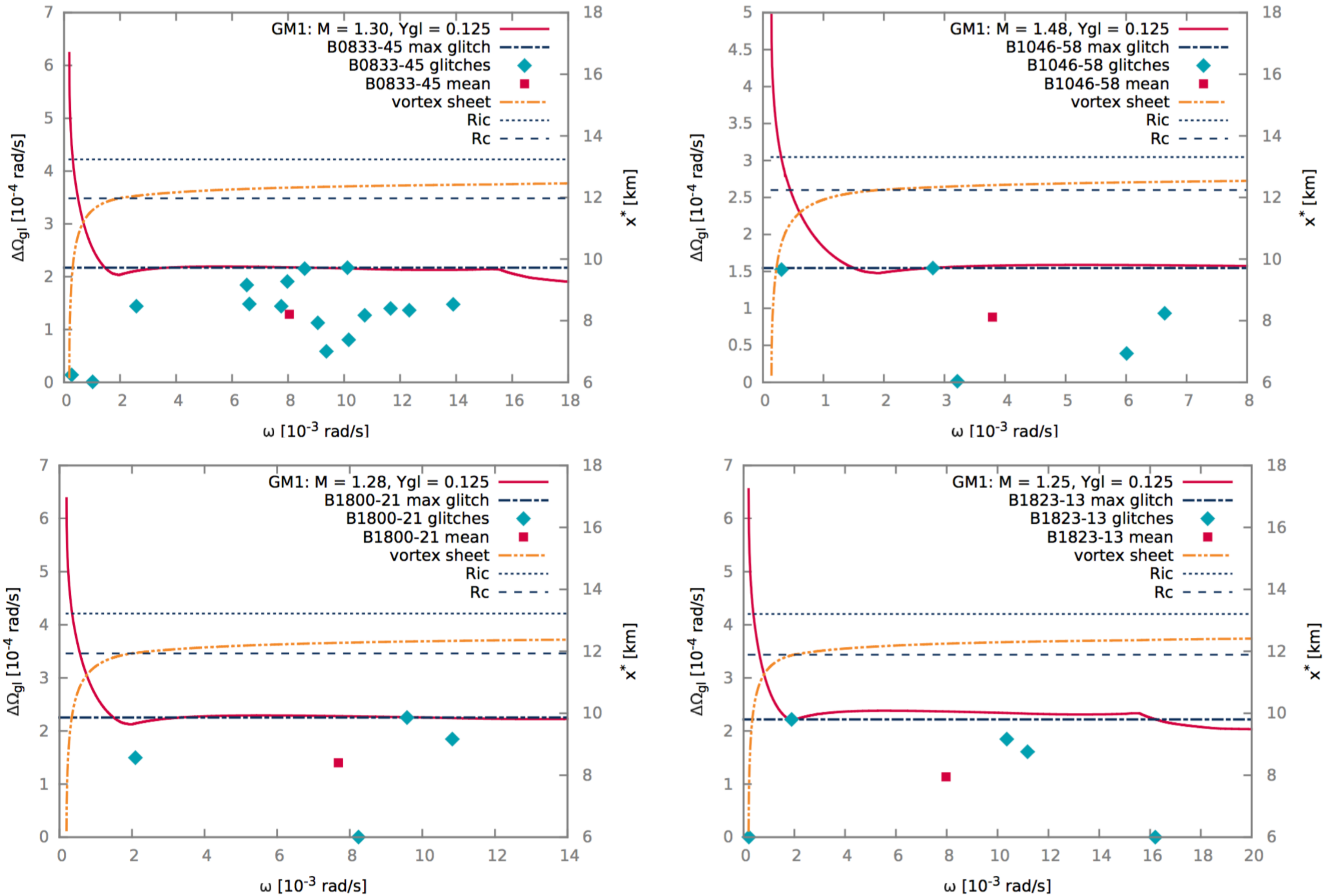


Procedure:

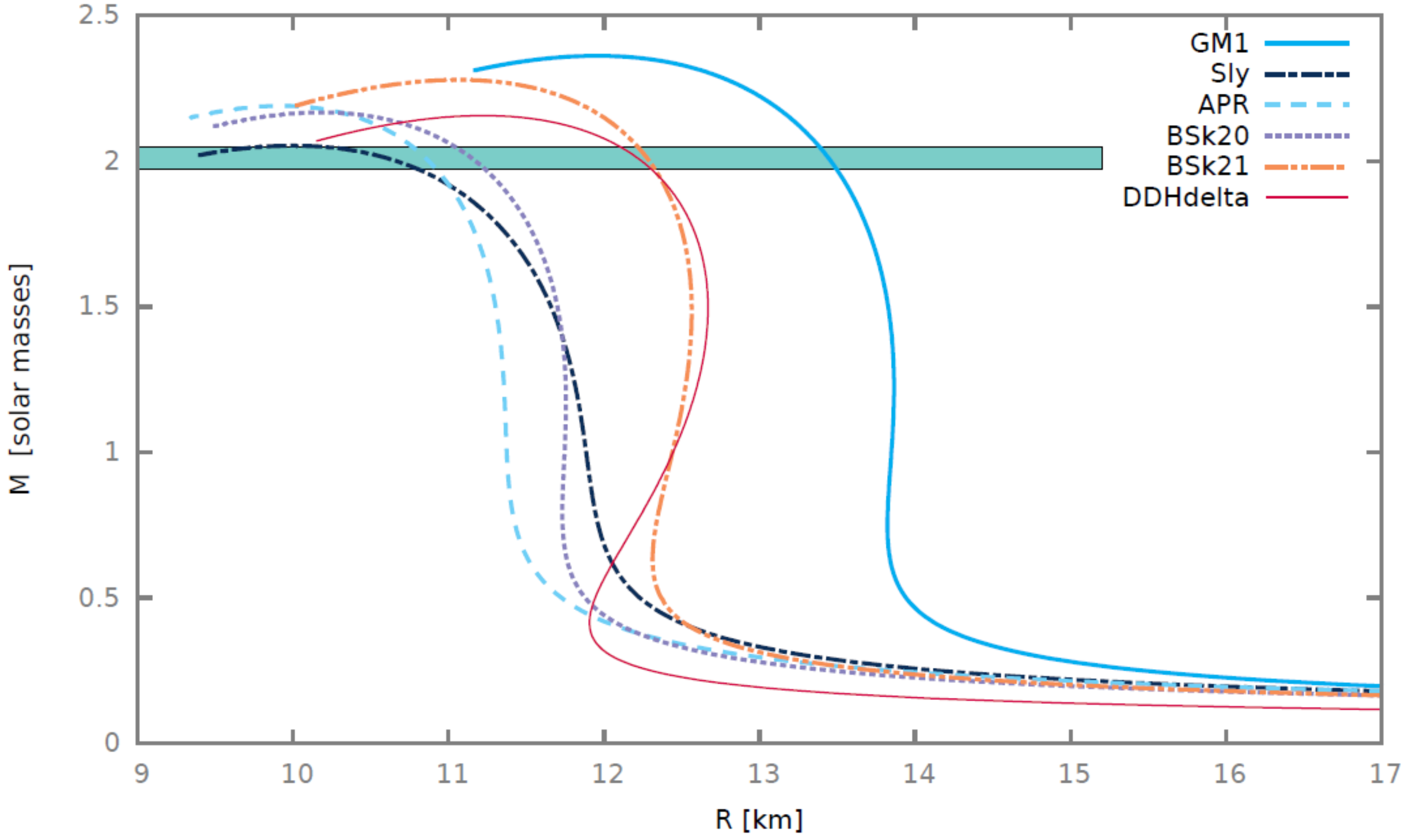
- Set M_{Vela} to a certain value (pulsar B0833-45)
- Find Y_{gl}
- Employ the same Y_{gl} to find the other pulsars' masses.

N.B. those estimates are upper limits for the masses

Fitting the masses



EoS employed



Six equations of state:

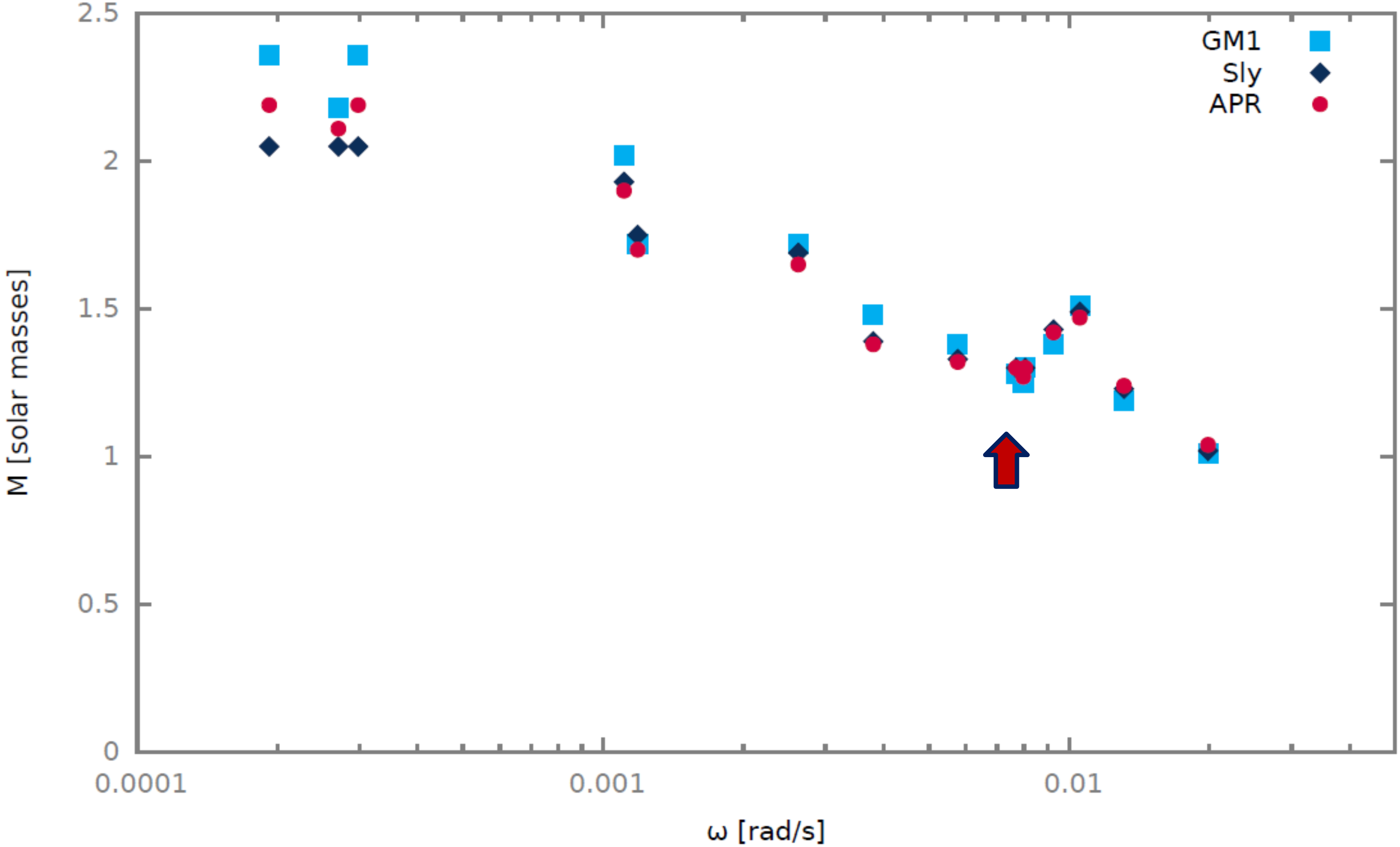
GM1, Sly, APR, DDHdelta, BSk20 e BSk21

EoS employed

Name	EoS in the core	Composition in the core	EoS in the crust	Composition in the crust
APR [1]	Akmal et al. 1998	Zuo et al. 2004 AV_{18}	Douchin and Haensel 2001	Consistent
APR [2]	Akmal et al. 1998	Zuo et al. 2004 AV_{18} + TBF	Douchin and Haensel 2001	Consistent
Sly	Douchin and Haensel 2001	Consistent	Douchin and Haensel 2001	Consistent
BSk20	Fantina et al. 2013	Consistent	Fantina et al. 2013	Consistent
BSk21	Fantina et al. 2013	Consistent	Fantina et al. 2013	Consistent
DDHdelta [1]	Gaitanos et al. 2004	Consistent	Grill et al. 2014	Negele and Vautherin 1973
DDHdelta [2]	Gaitanos et al. 2004	Consistent	Grill et al. 2014	Baldo et al. 2008
DDHdelta [3]	Gaitanos et al. 2004	Consistent	Grill et al. 2014	Douchin and Haensel 2001
GM1	Glendenning and Moszkowski 1991	Consistent	Douchin and Haensel 2001	Consistent

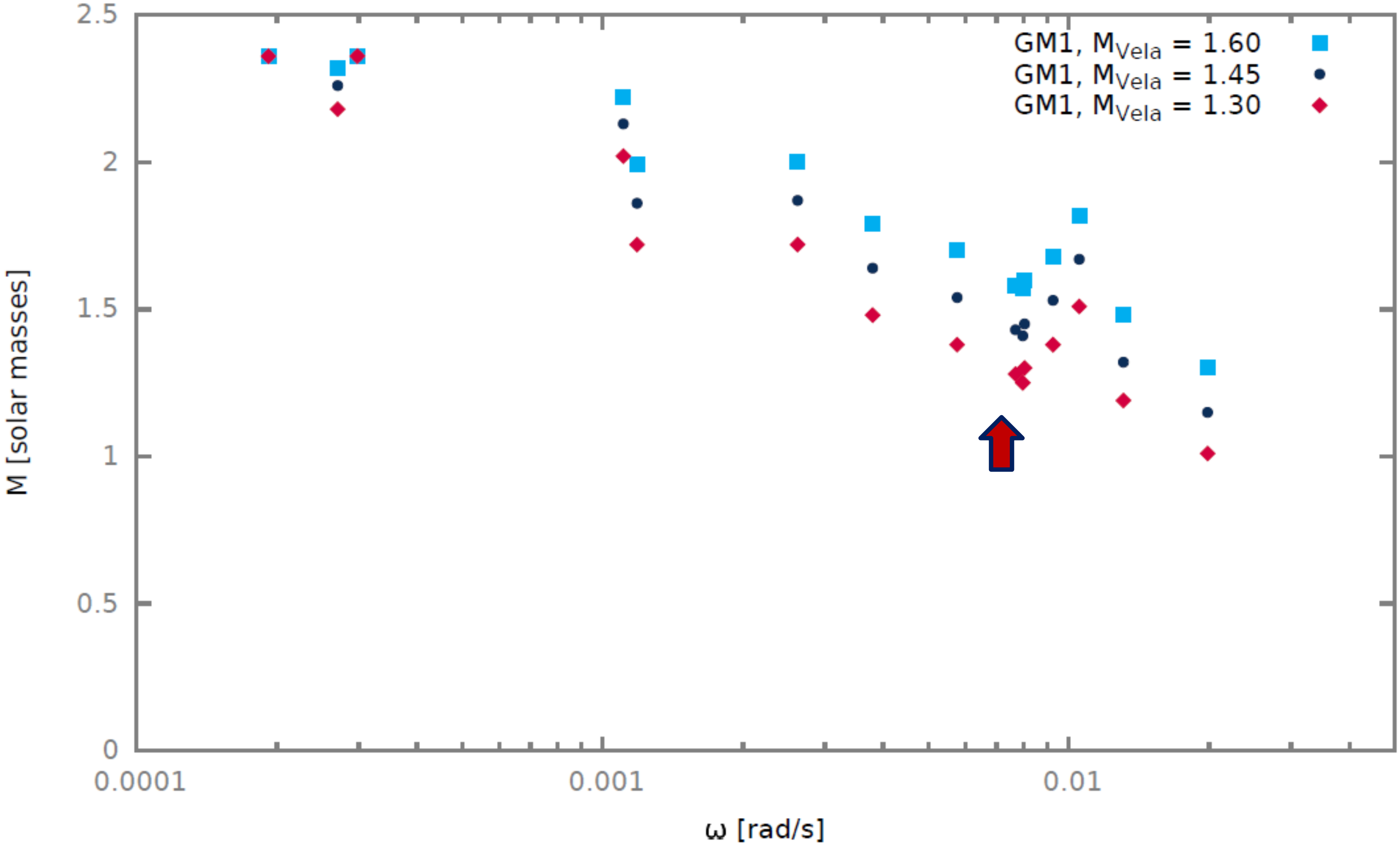
Mass estimates for 3 EoSs

$M_{Vela} = 1.3\ M_{\odot}$, with 3 equations of state: GM1, Sly and APR



Mass estimate for 3 values of M_{Vela}

GM1 for 3 values of M_{Vela} : 1.3, 1.45 e 1.6 $M_{\odot}$.



The 'maximum obtainable mass' for Vela

- Maximum obtainable mass for Vela ($Y_{gl} = 0$) for each EoS:

EoS	$M_{V\,\max}$ ($Y_{gl} = 0$) with x_p as defined in tab 4.2 [$M_\odot$]	with $x_p = 0.05$ in the core [$M_\odot$]	with $x_p = 0.05$ in the crust [$M_\odot$]	with $x_p = 0.05$ in the whole star [$M_\odot$]
APR [1]	1.37	1.64	1.26	1.47
APR [2]	1.19	1.64	1.11	1.47
Sly	1.44	1.64	1.31	1.47
BSk20	1.29	1.39	1.30	1.41
BSk21	1.17	1.28	1.18	1.30
DDHdelta [1]	0.97	0.83	0.89	0.75
DDHdelta [2]	1.00	0.85	0.89	0.75
DDHdelta [3]	1.07	0.92	0.89	0.75
GM1	1.70	1.93	1.47	1.63

Vela's cooling (Kaminker et al.) → Realistic interval: 1.4 e 1.65 $M_\odot$

Conclusions

- Strong correlation between M and the averaged critical lag ω of the 15 pulsars.
- Unified description of glitches: low-mass neutron stars correspond to the strong glitchers and more massive stars to the weaker ones.
- Results don't change much for different EoSs.
- If we change M_{Vela} , the mass estimates shift of the same value, leaving the trend unchanged.
- The 'maximum obtainable mass' for Vela, on the contrary, is heavily affected by microphysical inputs.

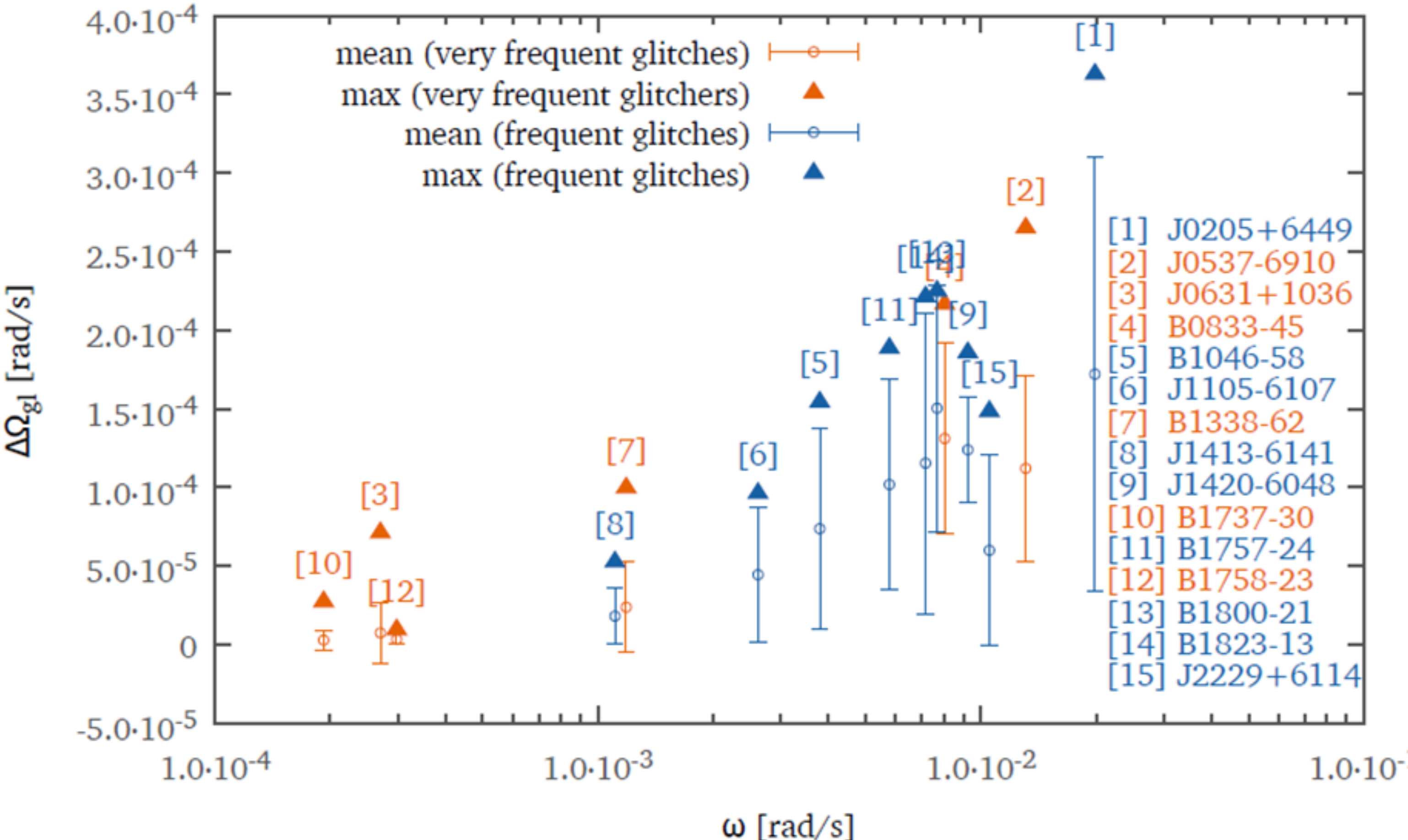
Perspectives

- Extend the model trying to include bended vortices? Turbulence? Phase transitions?
- Parallel with hydrodynamical simulations.
- More observations.

Thank you!

15 “glitchers”

15 pulsars that have been seen glitching at least 5 times



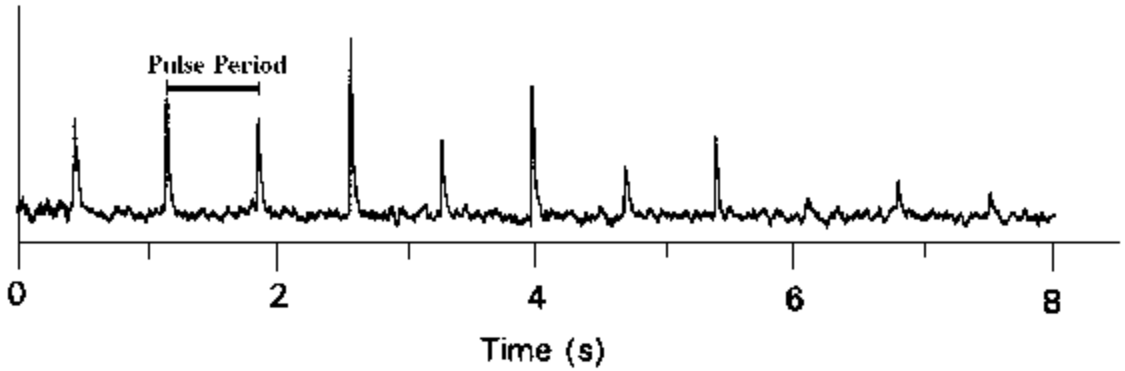
Seveso S.

- Frequent: 5-7
- Very frequent: > 12

$$\omega = \langle \, |\dot{\Omega}_p| t_{gl} \, \rangle$$

Pulsar Timing

TOAs :
Times **O**f **A**rrival
of pulses at the
observatory



Manchester, R.N. and *Taylor, J.H.*, *Pulsars, Freeman, 1977*

How to measure the period:

- Observe the TOAs.

Pulsar Timing

TOAs :
Times Of Arrival
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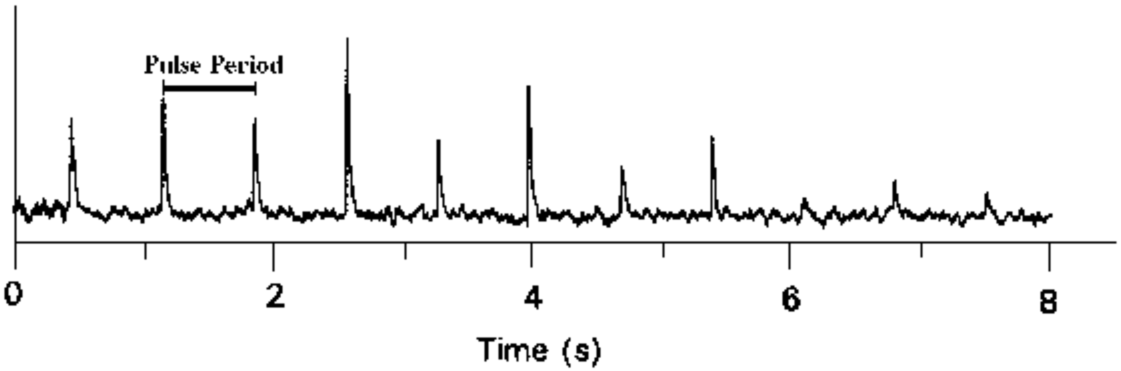

Manchester, R.N. and *Taylor, J.H.*, *Pulsars, Freeman, 1977*

How to measure the period:

- Observe the TOAs.
- Insert them in a simple spin down model:

$$N = \nu_0(t - t_0) + \frac{1}{2}\,\dot{\nu}_0(t - t_0)^2 + \frac{1}{6}\,\ddot{\nu}_0(t - t_0)^3 + \;\ldots$$

Pulsar Timing

TOAs :
Times Of Arrival
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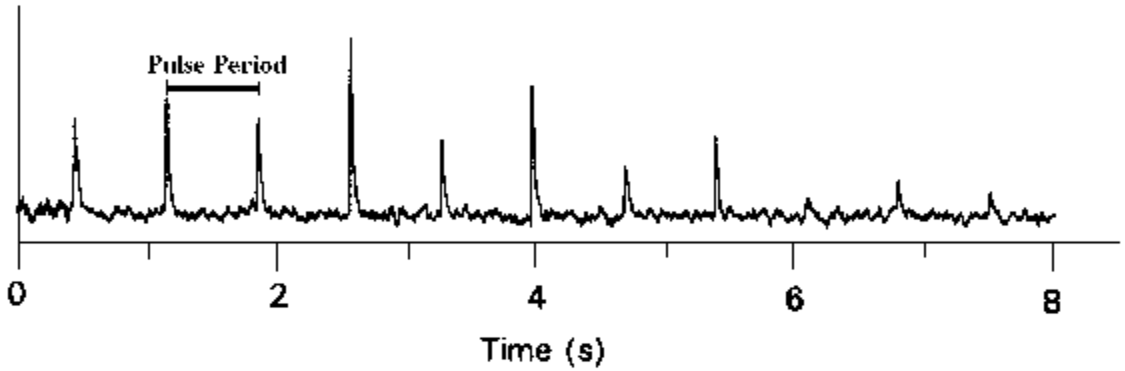

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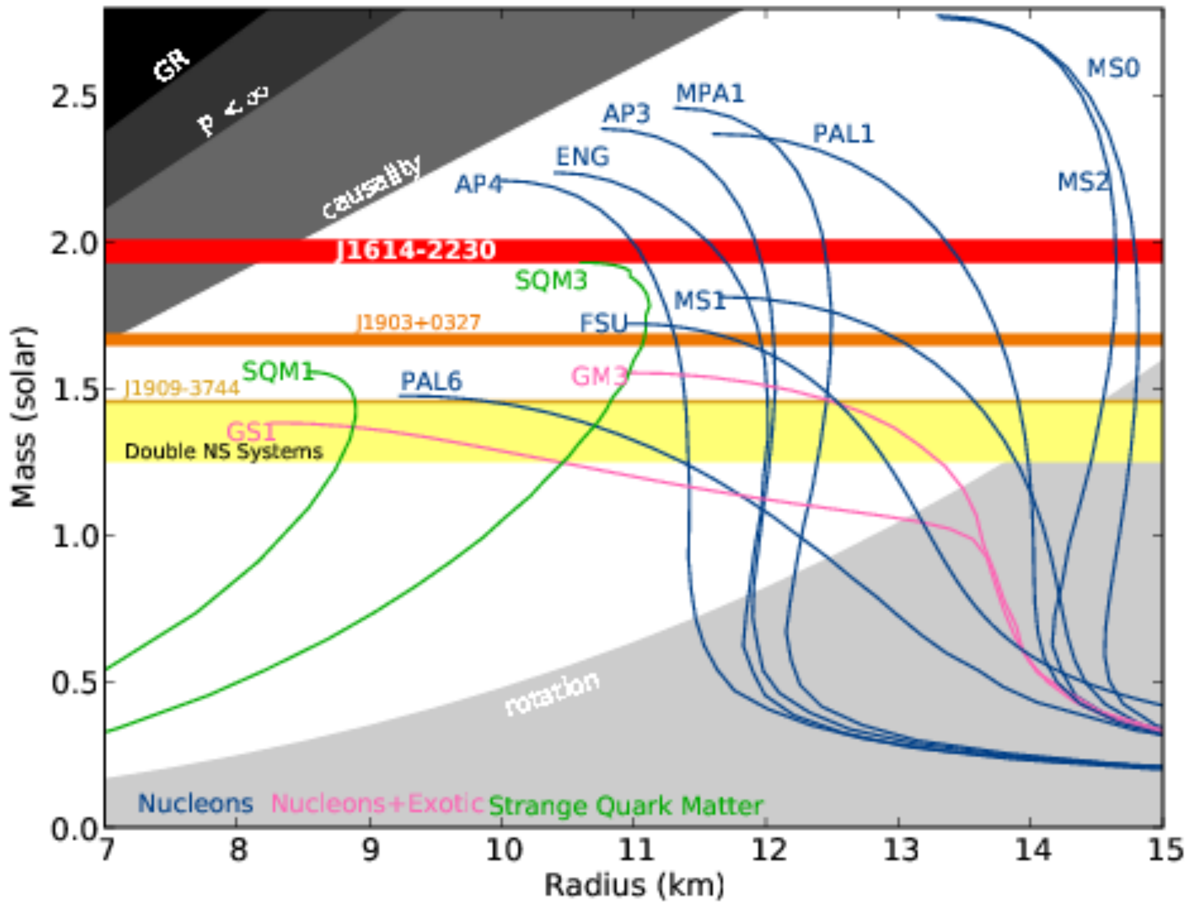
$$N = \nu_0(t - t_0) + \frac{1}{2}\,\dot{\nu}_0(t - t_0)^2 + \frac{1}{6}\,\ddot{\nu}_0(t - t_0)^3 + \ \ldots$$

- Minimize the fractional part of N

Time Residual : $\frac{N - \lfloor N \rfloor}{\nu_0}$

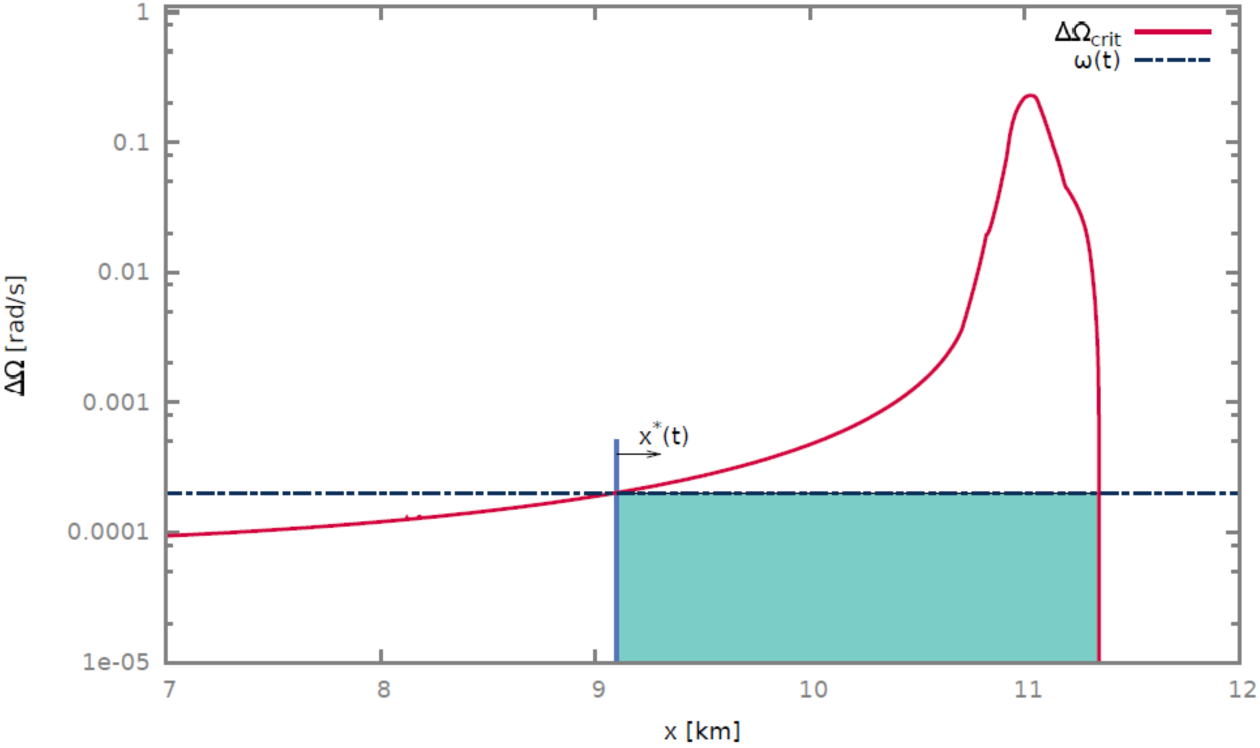
Time residuals are normally distributed around zero but …

Why studying glitches?



Demorest et al.

The Snowplow Model



Highlighted region:

$$\Delta L_{gl}(t)$$

angular momentum that can be released to the crust at a certain time t after the previous glitch.

- $$\Delta L_{gl}(t) = 2\,\pi \int_{x^*(t)}^{R_{ic}} x^3\,dx \int_0^{z(x)} \min[\omega(t), \Delta\Omega_{crit}(x)] \left(\rho(r) + \frac{P(r)}{c^2}\right) \frac{\bar{\omega}}{\Omega}(r)\, \frac{1 - x_p(r)}{1 - \epsilon_n(r)}\, e^{\lambda(r) - \phi(r)}\, dz$$

- $\omega(t) = \left|\dot{\Omega}_p\right| t$; $x_p(r)$ = mass fraction of the normal component;
- $\phi(r)$ and $\lambda(r)$ = structural functions of the TOV metric;
- $\bar{\omega} = \Omega - \omega_d$ = difference between the spin frequency of the star and the dragging frequency.

Moment of inertia for slow rotation

In general relativity, the inertial frames inside a rotating fluid are not at rest with
respect to the distant stars

- ω_d = dragging frequency of an inertial frame
- Ω = angular velocity of the star
- $\bar{\omega} = \Omega - \omega_d$

Hartle approach: (Hartle J. B., 1967, Astrophys. J. 150, 1005)

$$\frac{1}{r^4}\frac{d}{dr}\left(r^4 e^{-\phi+\lambda}\frac{d\bar{\omega}}{dr}\right) + \frac{4}{r}\left(\frac{d}{dr}e^{-\phi+\lambda}\right)\bar{\omega} = 0$$

Boundary conditions:

- regularity at $r = 0$;
- vanishing of ω_d at infinity
- $\bar{\omega}$ continuous at the stellar surface

In order to obtain the Schwarzschild solution for $r \geq R$ we have to impose:

$$e^{-\phi+\lambda} = 1$$

The Snowplow Model

If ΔL_{gl} is transferred to the crust, the subsequent jump in angular velocity will be:

$$\Delta\Omega_{gl}(t) = \frac{\Delta L_{gl}(t)}{(1-Q)I_{tot} + QY_{gl}I_{tot}}$$

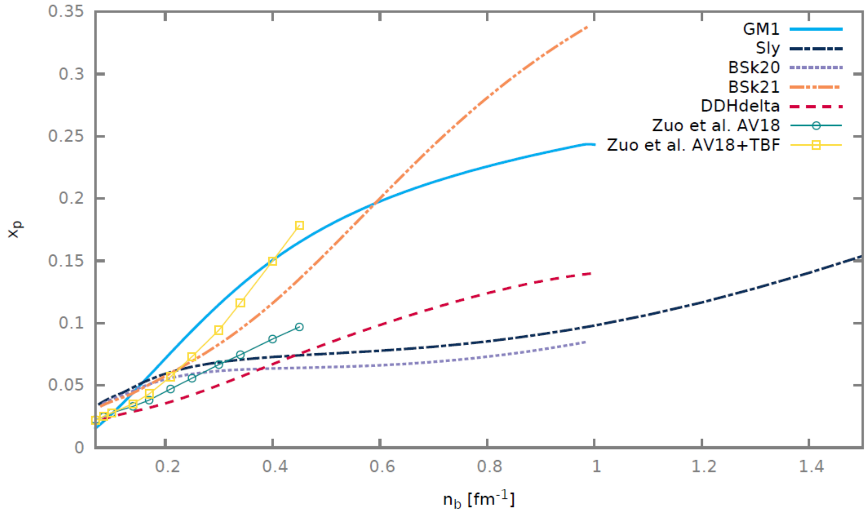
Where:

- Y_{gl} is the fraction of vorticity coupled to the crust in the timescale of a glitch
- Q is the fraction in moment of inertia of the superfluid component:

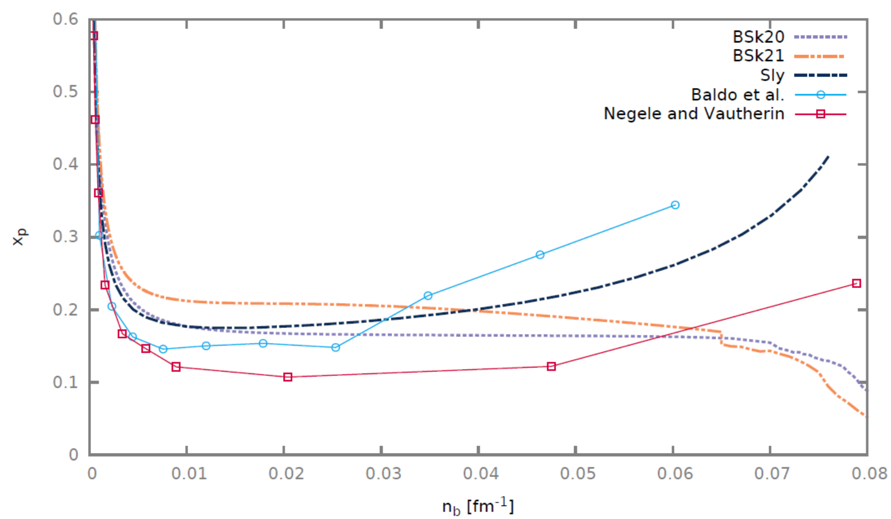
$$Q = \frac{I_v}{I_{tot}} = \frac{8\pi}{3I_{tot}} \int_0^R r^4 \left(\rho(r) + \frac{P(r)}{c^2}\right) \frac{\bar{\omega}}{\Omega}(r)\, \frac{1 - x_p(r)}{1 - \epsilon_n(r)} e^{\lambda(r)-\phi(r)}\, dr$$

Microphysical inputs

Compositions consistent with the EoSs.

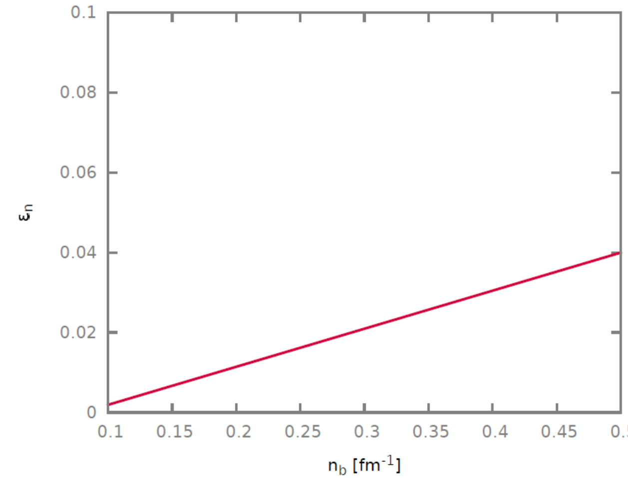


Normal fraction in the core

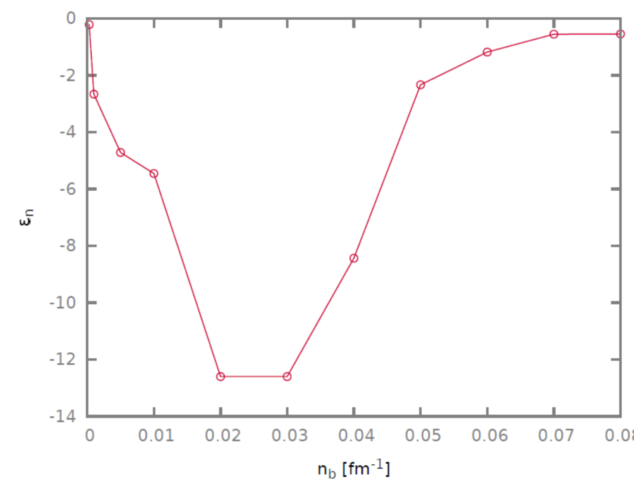


Normal fraction in the crust

Entrainment parameter ϵ_n in the core (Chamel e Haensel) and in the crust (Chamel).

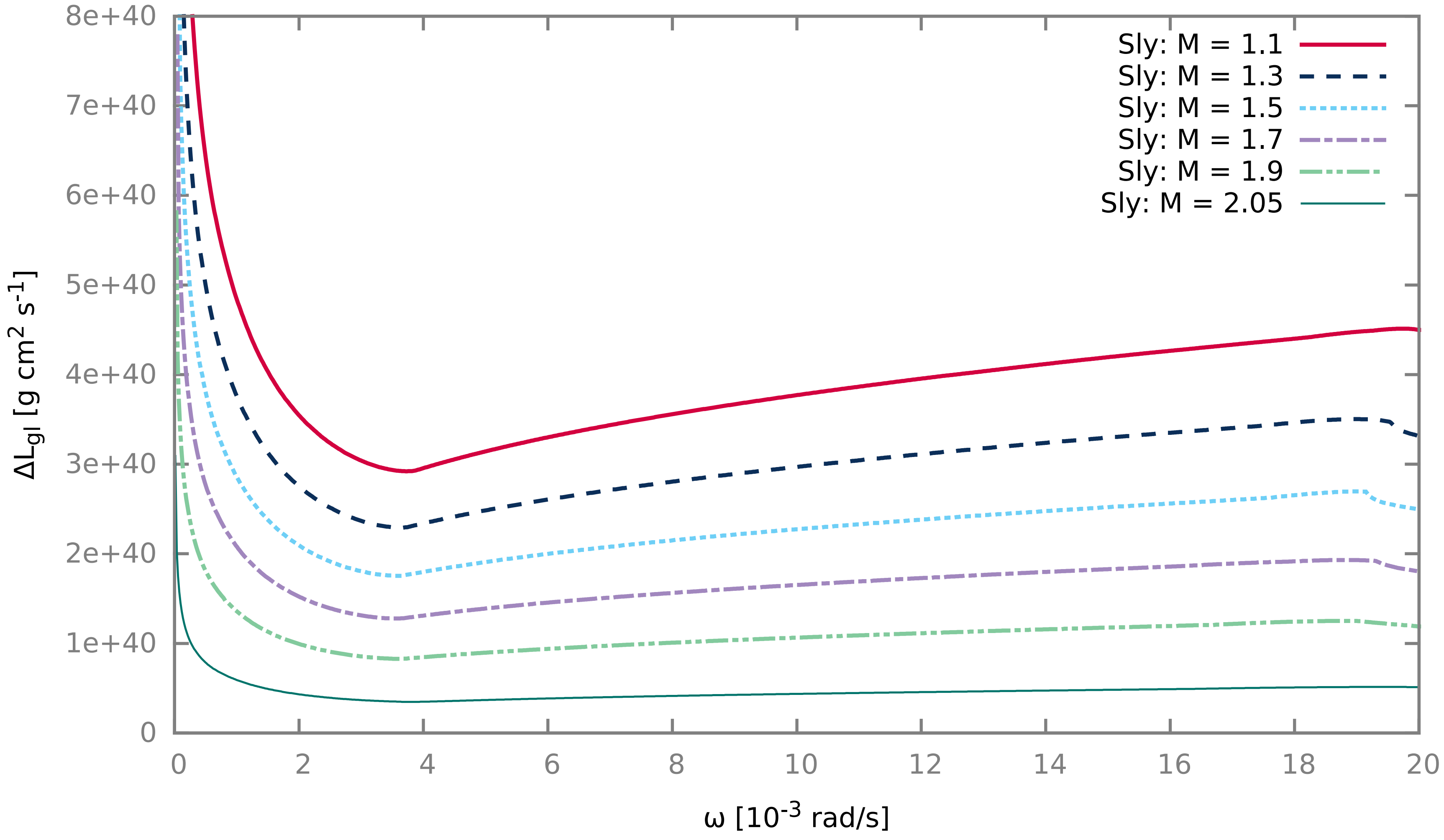


ϵ_n in the core

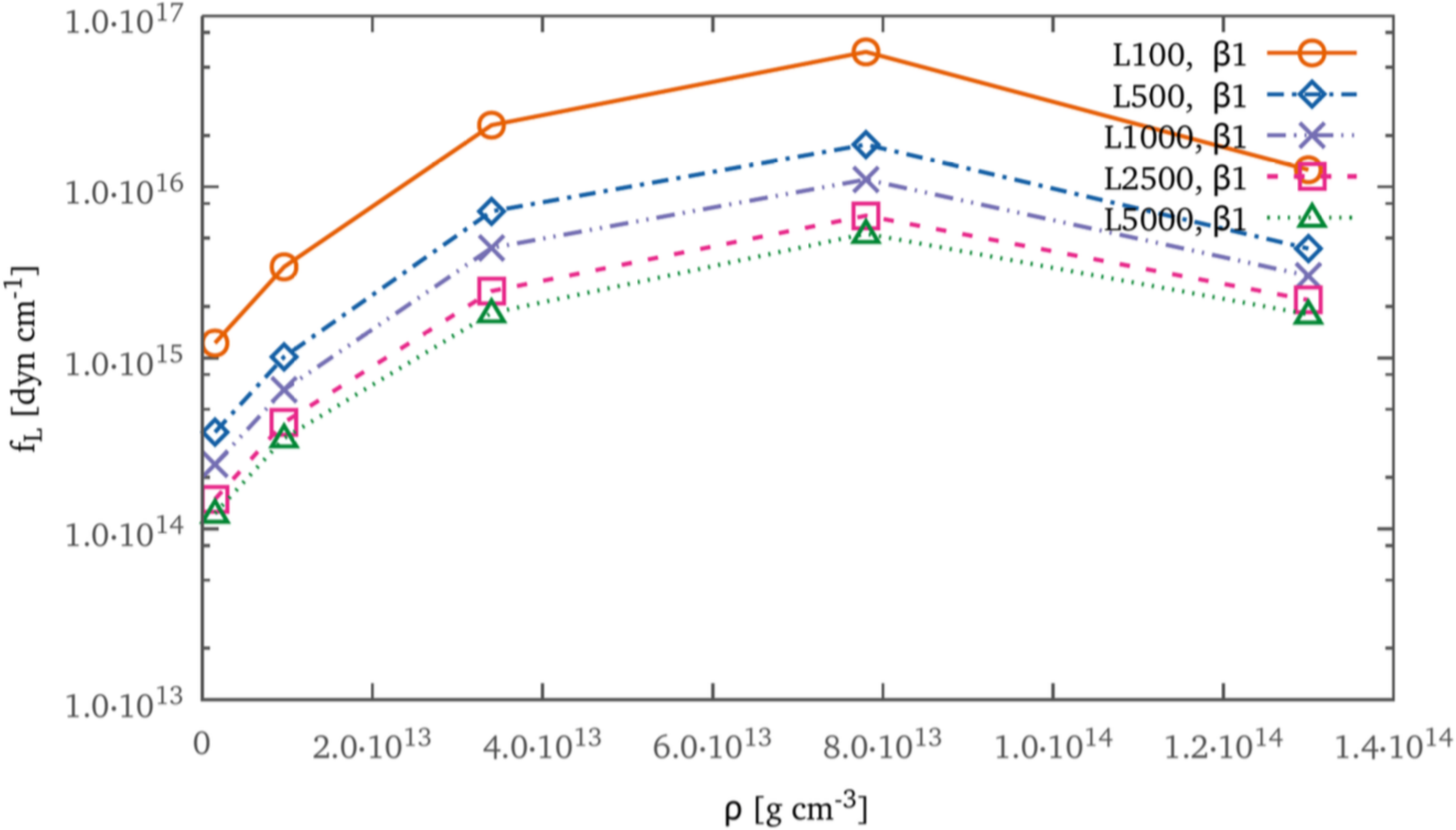


ϵ_n in the crust

Angular Momentum



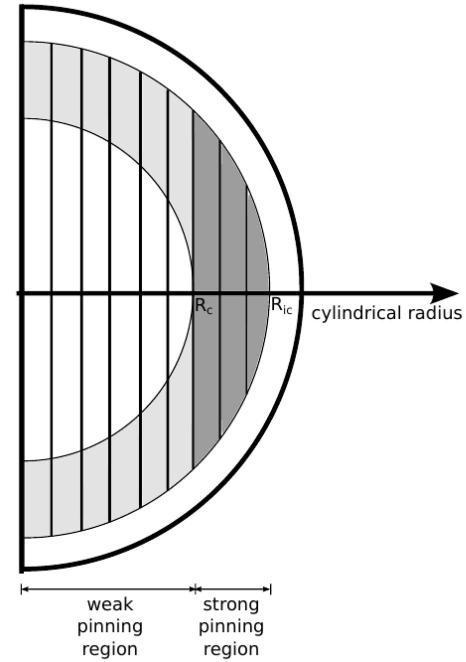
Pinning Force



Seveso et al 2014

The Snowplow Model

Pizzochero P. M., 2011, Astrophys. J. Lett. 743, L20.
Seveso S., 2015, Ph.D. thesis, Università degli Studi di Mi
Caiazzo I., 2015, Master thesis, Università degli Studi di M



Seveso S.

Two forces act on each vortex line at a cylindrical radius x:

- **The Pinning Force** (only in the crust):
  - the vortex is **blocked** in a potential well

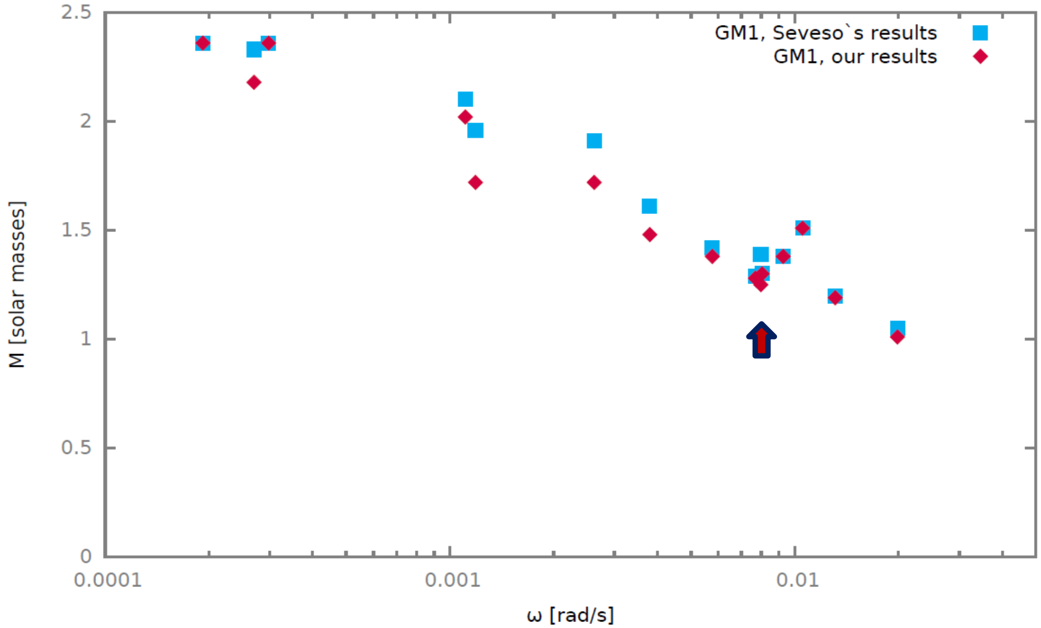
$$F_{pin}(x) = 2\int_0^{z(x)} f_{pin}[\rho(r)]\, dz$$

- **The Magnus Force:**
  - the vortex line is forced to rotate along with the crust
  - $\neq$ angular velocity respect to the superfluid
  - - a **hydrodinamical lift** $\propto$ to the lag arises

$$F_M(x) = 2\,\kappa\, x\left(\Omega_v(x) - \Omega_p\right)\int_0^{z(x)} \frac{\rho_n(r)}{1-\epsilon_n(r)}\, dz$$

- $\kappa = \frac{h}{2m_n}$; - $\Omega_v(x)$: angular velocity of the superfluid component;
- Ω_p: angular velocity of the normal component;
- $\rho_n(r)$: neutron superfluid density;
- $\epsilon_n(r)$: entrainment parameter.

Confronto con i risultati di Seveso



GM1 , $M_{Vela} = 1.3\ M_\odot$

Sly , $M_{Vela} = 1.3\ M_\odot$