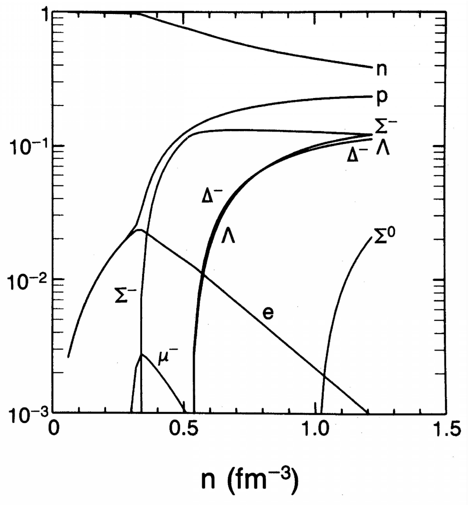
Formation of $\Delta(1232)$ in Neutron Stars

Bao-Jun Cai
Texas A&M University-Commerce

B.J. Cai, F.J. Fattoyev, B.A. Li, and W.G. Newton
Phys. Rev. C **92**, 015802 (2015) [arXiv:1501.01680]

The Phases of Dense Matter/INT-16-2b, 7/11-8/12, 2016

formation of $\Delta(1232)$ in neutron stars (Glendenning, 1985)



$$e^- + n \rightarrow \Sigma^- + \nu_e$$

$$\Delta^- + p \rightarrow n + n$$

$$\rho^{crit}_{\Sigma^-} \approx 2\rho_0$$

$$\rho^{crit}_{\Delta^-} \approx 3\rho_0$$

$$\rho^{crit}_{\Sigma^-} < \rho^{crit}_{\Delta^-}$$

$$L \equiv 3\rho_0 \partial E_{sym}(\rho)/\partial\rho|_{\rho_0} \approx 90\,\mathrm{MeV}$$
$$J_0 \equiv 27\rho_0^3 \partial^3 E_0(\rho)/\partial\rho^3|_{\rho_0} \approx -460\,\mathrm{MeV}$$

generally, people think that $\Delta(1232)$ played little role in neutron stars density isomers

π condensation Neutron Stars

cooling pattern and efficiency

star quakes and pulsar glitches

...

$\Delta(1232)$

. . .

π, K, $\cdots$, Heavy Ion Collisions

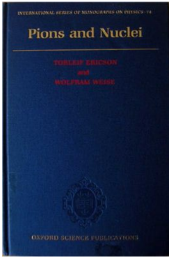
unresolved issue in NS (Shapiro et al. , 1983)

NN $\rightarrow$ NΔ dynamics, π^-/π^+ ratio, $\cdots$

8.11 Unresolved Issues: The Δ Resonance

There are a number of nagging problems, requiring difficult calculations, that remain to be resolved before the equation of state can be understood even at densities around $2\rho_{\rm nuc}$. Some of these (e.g., many-body computational schemes) have been mentioned already. While it would be inappropriate in this book to go into detail on all these topics, we do at least wish to summarize some of the problems, and indicate the expected sign of the various effects on the equation of state.

One of the unresolved issues involves the Δ resonance, an excited state of the nucleon with mass 1236 MeV, and quantum numbers $t = \frac{3}{2}$, $J = \frac{3}{2}$. Pion exchange between two nucleons can produce virtual intermediate states which are NN, NΔ, or $\Delta\Delta$. As pointed out by Green and Haapakoski (1974), such attractive pion exchange processes will be suppressed in a dense nuclear medium due to modifications of the intermediate state energy as well as the Pauli exclusion principle (i.e., many of the states are occupied). Accordingly, some of the attractive



$\rho_\Delta^{\rm crit}$ — why it is large — simple consideration

$$|\mathbf{k}| = k_{\rm F} = (3\pi^2\rho/2)^{1/3}$$

$$\sqrt{s} = 2E_{\rm single}^{\rm N,free} = 2(k_{\rm F}^2 + m_{\rm N}^2)^{1/2}$$

$$m_\Delta^{\rm max} = 2(k_{\rm F}^2 + m_{\rm N}^2)^{1/2} - m_{\rm N}$$

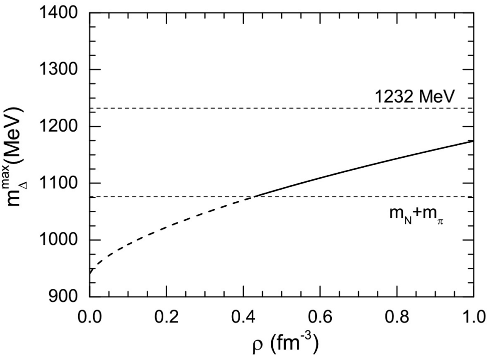
$$\Delta \rightarrow {\rm N} + \pi$$

$$\downarrow$$

$$m_\Delta^{\rm min} = m_{\rm N} + m_\pi \approx 1076\,{\rm MeV}$$

$$m_\Delta^{\rm max} = m_\Delta^{\rm min} = 1076\,{\rm Mev} \longrightarrow \rho_\Delta^{\rm crit} \approx 3\rho_0$$

$$m_\Delta^{\rm max} = m_\Delta^0 = 1232\,{\rm MeV} \longrightarrow \rho_\Delta^{\rm crit} > 1.0\,{\rm fm}^{-3}$$



medium effects/mass dependence of $\Delta(1232)$ are thus important for determining $\rho_\Delta^{\rm crit}$

chemical and β- equilibrium in npe$\mu\Delta$ matter

reactions:

$$\mathrm{n} \to \mathrm{p} + \mathrm{e} + \overline{\nu}_\mathrm{e}, \quad \mathrm{p} + \mathrm{e} \to \mathrm{n} + \nu_\mathrm{e}$$
$$\mathrm{p} + \mu \to \mathrm{n} + \nu_\mu, \quad \mathrm{n} \to \mathrm{p} + \mu + \overline{\nu}_\mu, \quad \mathrm{e} \to \mu + \nu_\mathrm{e} + \overline{\nu}_\mu$$
$$\Delta^{++} + \mathrm{n} \leftrightarrow \mathrm{p} + \mathrm{p}$$
$$\Delta^{+} + \mathrm{n} \leftrightarrow \mathrm{n} + \mathrm{p}$$
$$\Delta^{0} + \mathrm{p} \leftrightarrow \mathrm{p} + \mathrm{n}$$
$$\Delta^{-} + \mathrm{p} \leftrightarrow \mathrm{n} + \mathrm{n}$$

conditions:

$$\begin{aligned}
\mu_\mathrm{e} &= \mu_\mathrm{n} - \mu_\mathrm{p} \\
\mu_\mu &= \mu_\mathrm{n} - \mu_\mathrm{p} \\
\mu_{\Delta^{++}} &= 2\mu_\mathrm{p} - \mu_\mathrm{n} \\
\mu_{\Delta^{+}} &= \mu_\mathrm{p} \\
\mu_{\Delta^{0}} &= \mu_\mathrm{n} \\
\mu_{\Delta^{-}} &= 2\mu_\mathrm{n} - \mu_\mathrm{p}
\end{aligned}$$

ingredients on determining the $\rho_{\Delta}^{\text{crit}}$

for instance, for $\Delta^- + \mathrm{p} \to \mathrm{n} + \mathrm{n}$, $\mu_{\Delta^-} = 2\mu_{\mathrm{n}} - \mu_{\mathrm{p}} = \mu_{\mathrm{n}} + \mu_{\mathrm{n}} - \mu_{\mathrm{p}}$,

$$\frac{k_{\mathrm{F}}^{\mathrm{n},2}}{2m_{\mathrm{dirac}}^*} = \underbrace{m_{\Delta^-} - m_{\mathrm{N}}}_{\text{mass dependence}} - \underbrace{\underbrace{4E_{\mathrm{sym}}(\rho)\delta}_{\text{EOS dependence}} + \underbrace{\Sigma_{\mathrm{S}}^{\Delta} - \Sigma_{\mathrm{S}}^{\mathrm{N}}}_{\text{scalar self-energy}} + \underbrace{\Sigma_{\mathrm{V}}^{\Delta^-} - \Sigma_{\mathrm{V}}^{\mathrm{n}}}_{\text{vector self-energy}}}_{\text{in-medium effects}}$$

with $m_{\mathrm{dirac}}^* \equiv m_{\mathrm{dirac,N}}^*$,

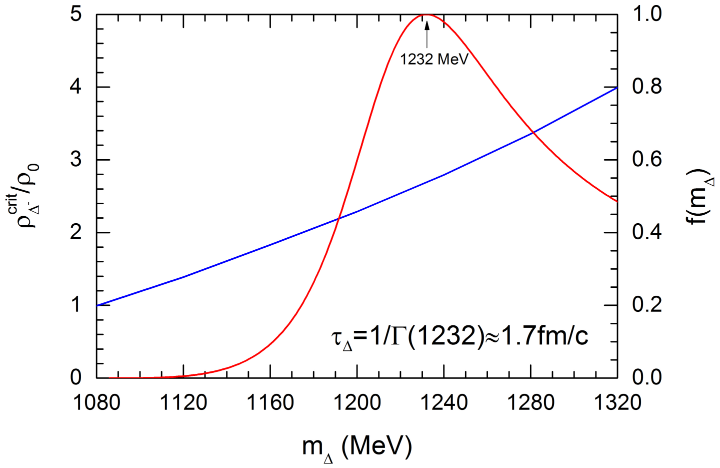
$$k_{\mathrm{F}}^{\mathrm{n}} = \left[3\pi^2(1+\delta)\rho/2\right]^{1/3}$$

$$m_{\mathrm{dirac}}^*(\mathrm{baryon}) = m_{\mathrm{baryon}} + \Sigma_{\mathrm{S}}$$

$$\mu_{\mathrm{baryon}} = \sqrt{k_{\mathrm{F}}^2 + m_{\mathrm{dirac}}^{*2}(\mathrm{baryon})} + \Sigma_{\mathrm{V}}$$

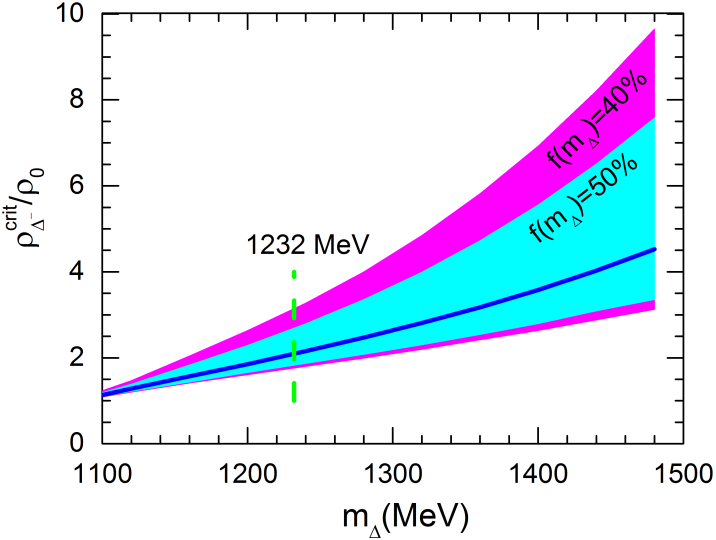
$$\mu_{\mathrm{n}} - \mu_{\mathrm{p}} \approx 4E_{\mathrm{sym}}(\rho)\delta, \quad E_{\mathrm{sym}}(\rho) \equiv 2^{-1}\partial^2 E(\rho,\delta)/\partial\delta^2|_{\delta=0}$$

large uncertainties come especially from poor knowledge on the self-energy difference nonlinear Walecka Lagrangian including $\Delta(1232)$ [Cai *et al.*, PRC **92**, 015802 (2015)]

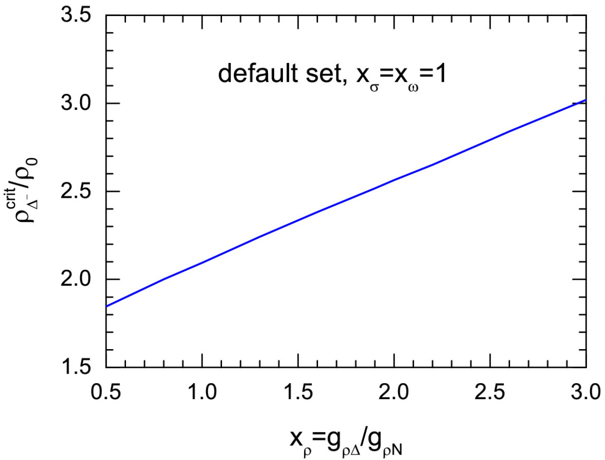


$$f(m_\Delta) = \frac{1}{4}\frac{\Gamma^2(m_\Delta)}{(m_\Delta - m^0_\Delta)^2 + \Gamma^2(m_\Delta)/4}, \quad \Gamma(m_\Delta) = \frac{0.47q^3}{m^2_\pi + 0.6q^2}, \quad q = \left[\left(\frac{m^2_\Delta - m^2_N + m^2_\pi}{2m_\Delta}\right)^2 - m^2_\pi\right]^{1/2}$$

$$f(m_\Delta) = \frac{1}{4}\frac{\Gamma^2(m_\Delta)}{(m_\Delta - m_\Delta^0)^2 + \Gamma^2(m_\Delta)/4}$$



$\rho^{\rm crit}_{\Delta^-}$ remains underdetermined — $x_\rho = g_{\rho\Delta}/g_{\rho \rm N}$ completely unknown



$$\frac{(k_{\rm F}^{\rm n})^2}{2m^*_{\rm dirac}} \approx \Phi_\Delta + g_{\sigma\rm N}(1-x_\sigma)\overline{\sigma} - g_{\omega\rm N}(1-x_\omega)\overline{\omega}_0 - 6(1-x_\rho)E^{\rm pot}_{\rm sym}(\rho)\delta - 4E^{\rm kin}_{\rm sym}(\rho)\delta$$

$\rho^{\rm crit}_{\Delta}$ for four charge states ($x_i = g_{i\Delta}/g_{i\rm N}$)

the critical conditions are

$$\rho^{\rm crit}_{\Delta-} : \frac{(k_{\rm F}^{\rm n})^2}{2m^*_{\rm dirac}} \approx \Phi_\Delta + g_{\sigma\rm N}(1-x_\sigma)\overline{\sigma} - g_{\omega\rm N}(1-x_\omega)\overline{\omega}_0 - 6(1-x_\rho)E^{\rm pot}_{\rm sym}(\rho)\delta - 4E^{\rm kin}_{\rm sym}(\rho)\delta$$

$$\rho^{\rm crit}_{\Delta^0} : \frac{(k_{\rm F}^{\rm n})^2}{2m^*_{\rm dirac}} \approx \Phi_\Delta + g_{\sigma\rm N}(1-x_\sigma)\overline{\sigma} - g_{\omega\rm N}(1-x_\omega)\overline{\omega}_0 - 2(1-x_\rho)E^{\rm pot}_{\rm sym}(\rho)\delta$$

$$\rho^{\rm crit}_{\Delta+} : \frac{(k_{\rm F}^{\rm p})^2}{2m^*_{\rm dirac}} \approx \Phi_\Delta + g_{\sigma\rm N}(1-x_\sigma)\overline{\sigma} - g_{\omega\rm N}(1-x_\omega)\overline{\omega}_0 + 2(1-x_\rho)E^{\rm pot}_{\rm sym}(\rho)\delta$$

$$\rho^{\rm crit}_{\Delta++} : \frac{(k_{\rm F}^{\rm p})^2}{2m^*_{\rm dirac}} \approx \Phi_\Delta + g_{\sigma\rm N}(1-x_\sigma)\overline{\sigma} - g_{\omega\rm N}(1-x_\omega)\overline{\omega}_0 + 6(1-x_\rho)E^{\rm pot}_{\rm sym}(\rho)\delta + 4E^{\rm kin}_{\rm sym}(\rho)\delta$$

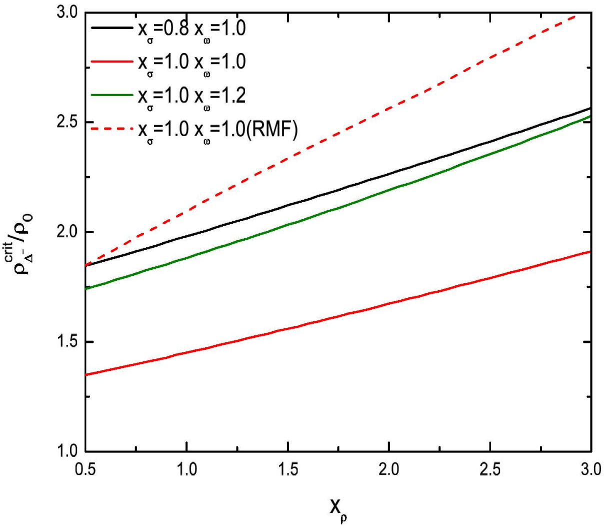
with $\Phi_\Delta \equiv m_\Delta - m_{\rm N}$ (mass difference) and

$$E^{\rm pot}_{\rm sym}(\rho) = 2^{-1}\rho g^2_{\rho\rm N}(m^2_\rho + \Lambda_{\rm V} g^2_{\rho\rm N} g^2_{\omega\rm N}\overline{\omega}^2_0)^{-1}, \quad E^{\rm kin}_{\rm sym}(\rho) = 6^{-1}k^2_{\rm F}(k^2_{\rm F} + m^{*,2}_{\rm dirac})^{-1/2}$$

thus

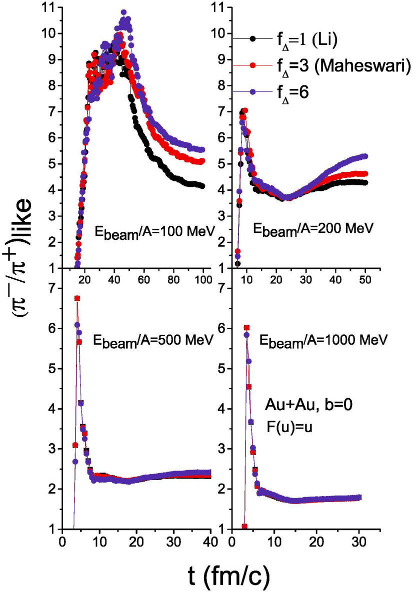
$$\boxed{\rho^{\rm crit}_{\Delta-} < \rho^{\rm crit}_{\Delta^0} < \rho^{\rm crit}_{\Delta+} < \rho^{\rm crit}_{\Delta++}, \text{ for } x_\sigma = x_\omega = x_\rho = 1 \text{ (universal baryon-meson coupling)}}$$

confirmed by RHF calculations

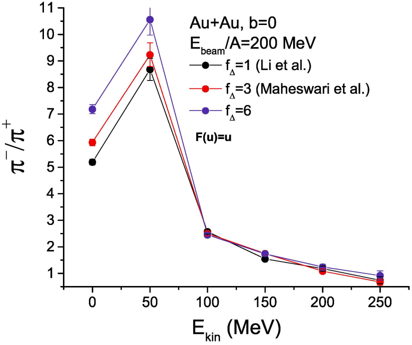


Zhu *et al.*, 2016 (arXiv:1607.04007)

effects of x_ρ on the π^-/π^+ ratio



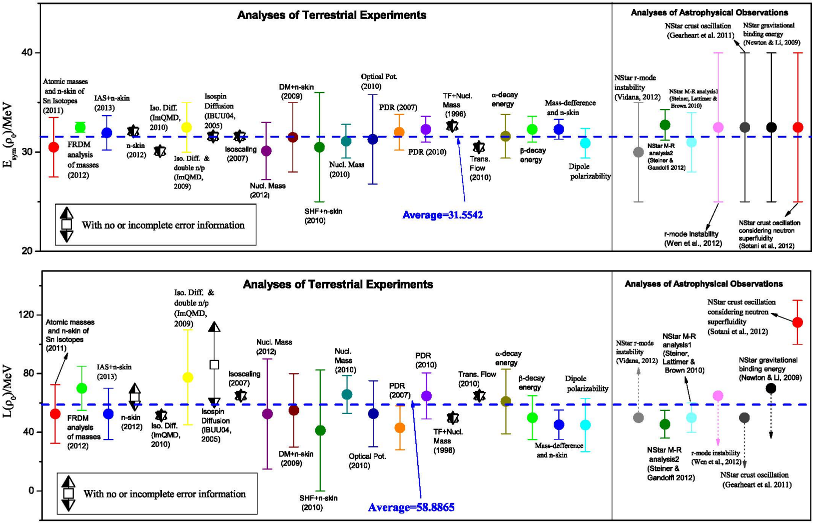
$f_\Delta \leftrightarrow 3x_\rho$



At deeply subthreshold beam energies, the effect of the Δ symmetry potential on the total π^-/π^+ ratio is significant. At $E_{\rm beam}/A = 200$ MeV, for example, the total π^-/π^+ ratio increases by about 25% when the f_Δ is increased from 1 to 6. As one expects, the final $(\pi^-/\pi^+)_{\rm like}$ ratio is higher with a larger f_Δ value, which amplifies the difference in potentials for Δ^- and Δ^{++}. The beam energy dependence

Li, PRC **92**, 034603 (2015)

review of the status of the L and $E_{\rm sym}(\rho_0)$

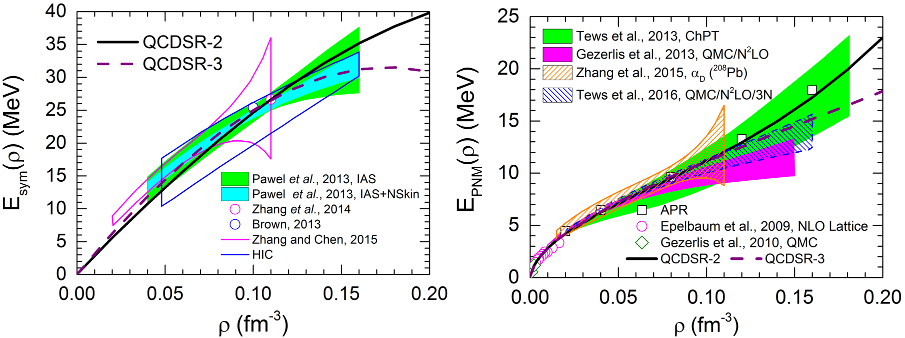


$E_{\rm sym}(\rho_0) \approx 31.6 \pm 2.7\,{\rm MeV}, \quad L \approx 59 \pm 16\,{\rm MeV}$

Li and Han, PLB **727**, 276 (2013)

L is found to be smaller than early RMF predictions

$$E_{\rm PNM}(0.10\,{\rm fm}^{-3}) \approx E_0(\rho_0)+\frac{1}{2}K_0\chi^2+E_{\rm sym}(\rho_0)+L\chi \approx 18\,{\rm MeV}-\frac{1}{8}L,\quad \chi=\frac{\rho-\rho_0}{3\rho_0}\approx-\frac{1}{8}$$

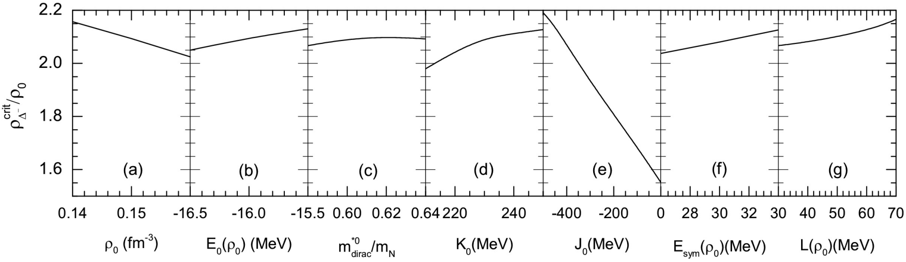


$\Pi^{\rm phen}_{\alpha\beta}(q^2)=\Pi^{\rm QCD}_{\alpha\beta}(q^2)$ (QCD sum rules) : self-energy $(\Sigma)\longleftrightarrow$ EOS $(E(\rho,\delta))$

$$\Pi^{\alpha\beta}_{\rm QCD}(x)\sim\langle j^\alpha(x)j^\beta(0)\rangle=\sum_n C_n^{\alpha\beta}\mathcal{O}_n(x),\quad \rho^{\rm phen}(s)\sim\phi\delta(s-M^2)-\frac{1}{4\pi^2}s\langle\bar{q}q\rangle_{\rm vac}\theta(s-s_0)+\cdots$$

Cai and Chen, 2016

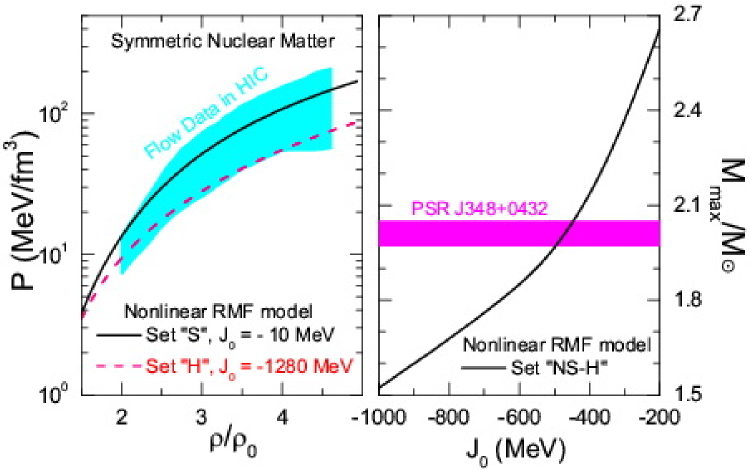
dependence of $\rho^{crit}_{\Delta^-}$ on the EOS of ANM



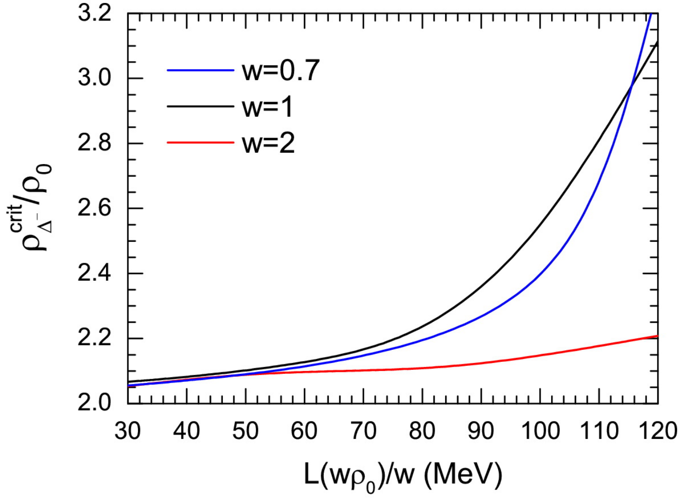
default set:

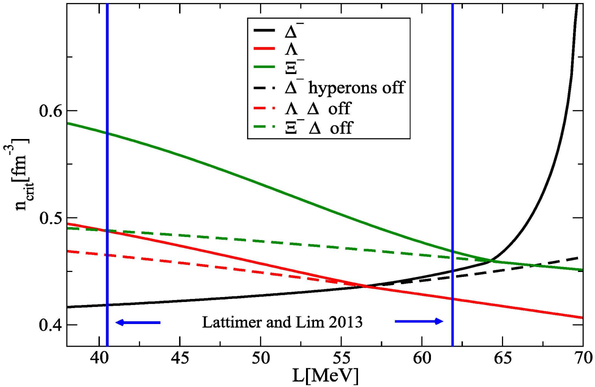
$$\rho_0 = 0.149\,\mathrm{fm}^{-3}$$
$$E_0(\rho_0) = -16.09\,\mathrm{MeV}$$
$$m^{*0}_{\mathrm{dirac}}(\rho_0)/m_{\mathrm{N}} = 0.64$$
$$K_0(\rho_0) = 230\,\mathrm{MeV}$$
$$J_0(\rho_0) = -415\,\mathrm{MeV}$$
$$E_{\mathrm{sym}}(\rho_0) = 31.17\,\mathrm{MeV}$$
$$L(\rho_0) = 48.64\,\mathrm{MeV}$$

J_0 reflects the high density behavior of the EOS



Cai and Chen, 2014 (arXiv:1402.4242)

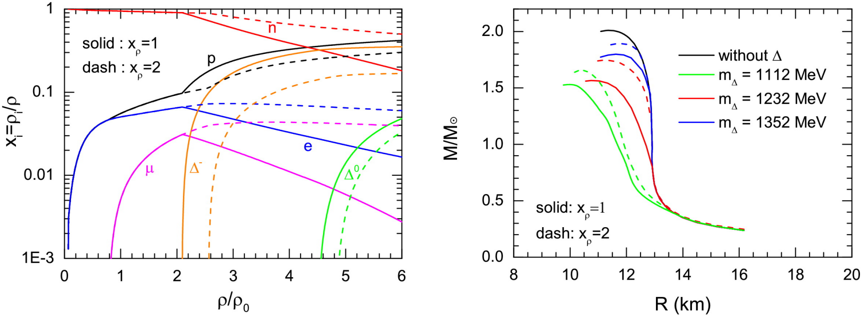

3.2
3.0
2.8
2.6
2.4
2.2
2.0
$\rho^{crit}_{\Delta^-}/\rho_0$
w=0.7
w=1
w=2
30
40
50
60
70
80
90
100
110
120
$L(w\rho_0)/w$ (MeV)



We discuss the formation of Δ isobars in neutron star matter. We show that their threshold density strictly correlates with the density derivative of the symmetry energy of nuclear matter, the L parameter. By restricting L to the range of values indicated by recent experimental and theoretical analysis, i.e. 40 MeV $\lesssim L \lesssim$ 62 MeV, we find that Δ isobars appear at a density of the order of 2÷3 times nuclear matter saturation density, i.e. the same range for the appearance of hyperons. The range of values of the couplings of the Δs with the mesons is restricted by the analysis of the data obtained from photoabsorption, electron and pion scattering on nuclei. If the potential of the Δ in nuclear matter is close to the one indicated by the experimental data then the equation of state becomes soft enough that a "Δ puzzle" exists, similar to the "hyperon puzzle" widely discussed in the literature.

Drago *et al.*, PRC **90**, 065809 (2014)

NS including degrees of freedom of $\Delta(1232)$ (Δ puzzle)



- modified fractions of the lighter particles e and μ will after possible kaon condensation
- boost of the proton fraction may have impact on cooling processes in neutron stars

- formation densities of $\Delta(1232)$ remain underdetermined:
  - in-medium mass m_Δ
  - $E_{\rm sym}^{\rm kin}$ and $E_{\rm sym}^{\rm pot}$ separately
  - skewness of the SNM J_0
  - completely unknown ρ-Δ coupling $g_{\rho\Delta}$

  $\rho_{\Delta^-}^{\rm crit}$ can be very small ($\approx \rho_0$)

- composition/structure of neutron stars
- reducing mass/radius of neutron stars (Δ puzzle)

thanks for your attention

equation of state (energy per nucleon) for asymmetric nuclear matter is,

$$E(\rho,\delta) \approx \underbrace{E(\rho,0)}_{\equiv E_0(\rho)} + \boxed{\underbrace{E_{\text{sym}}(\rho)}_{\text{symmetry energy}} \quad \delta^2} + E_{\text{sym},4}(\rho)\delta^4 + \mathcal{O}(\delta^6)$$

go a step further, every energy term can be expanded at $\rho = \rho_0$,

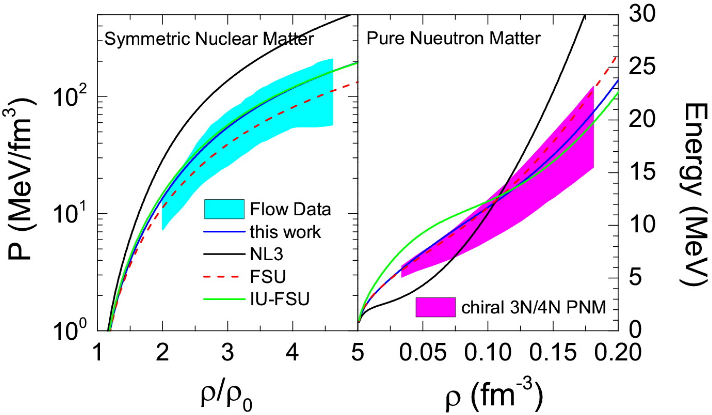
$$E_0(\rho) \approx E_0(\rho_0) + \frac{K_0}{2!}\chi^2 + \frac{J_0}{3!}\chi^3 + \frac{I_0}{4!}\chi^4 + \mathcal{O}(\chi^5), \quad \chi = \frac{\rho-\rho_0}{3\rho_0}$$

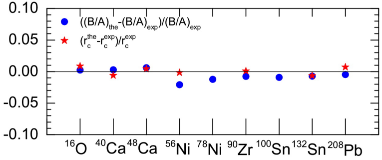
$$E_{\text{sym}}(\rho) \equiv \frac{1}{2}\frac{\partial^2 E(\rho,\delta)}{\partial\delta^2}\bigg|_{\delta=0}$$

$$\approx E_{\text{sym}}(\rho_0) + L\chi + \frac{K_{\text{sym}}}{2!}\chi^2 + \frac{J_{\text{sym}}}{3!}\chi^3 + \frac{I_{\text{sym}}}{4!}\chi^4 + \mathcal{O}(\chi^5)$$

$$E_{\text{sym},4}(\rho) \equiv \frac{1}{24}\frac{\partial^4 E(\rho,\delta)}{\partial\delta^4}\bigg|_{\delta=0}$$

$$\approx E_{\text{sym},4}(\rho_0) + L_{\text{sym},4}\chi + \frac{K_{\text{sym},4}}{2!}\chi^2 + \frac{J_{\text{sym},4}}{3!}\chi^3 + \frac{I_{\text{sym},4}}{4!}\chi^4 + \mathcal{O}(\chi^5)$$

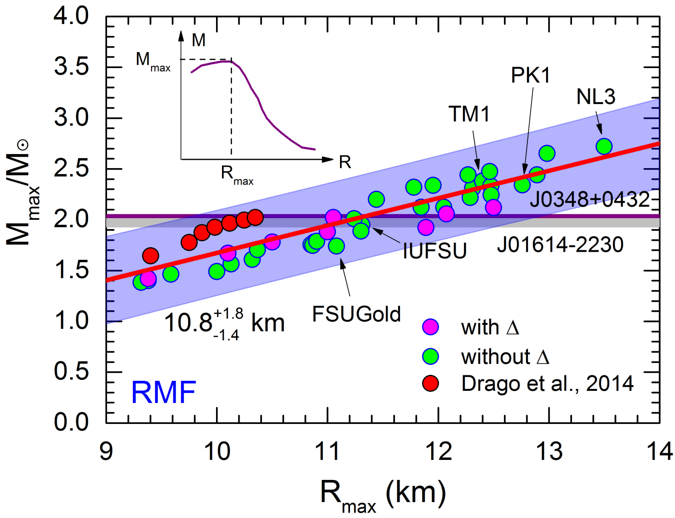


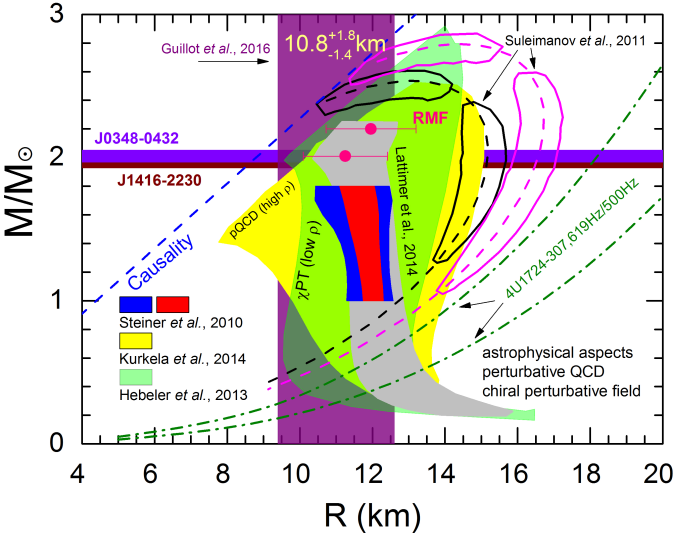


⟵ relative deviation less than 2%

$$M_{\max}^{\rm NS} \approx 2.01 M_\odot$$

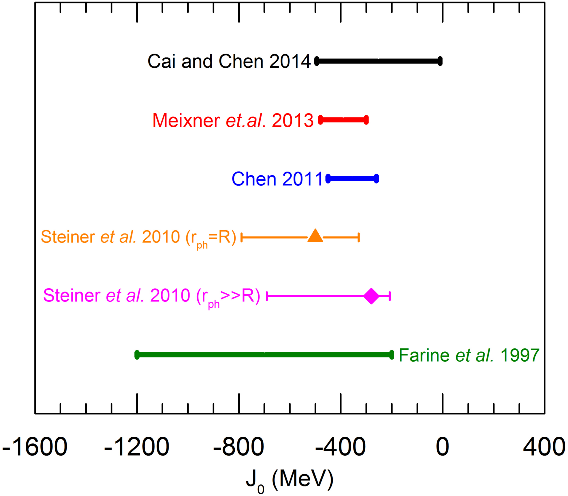
Cai, Chen, and Jiang, 2014


4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
M
M_max
R
R_max
PK1
NL3
TM1
J0348+0432
IUFSU
J01614-2230
FSUGold
10.8 +1.8 -1.4 km
with Δ
without Δ
Drago et al., 2014
RMF
9
10
11
12
13
14
R_max (km)
M_max/M⊙



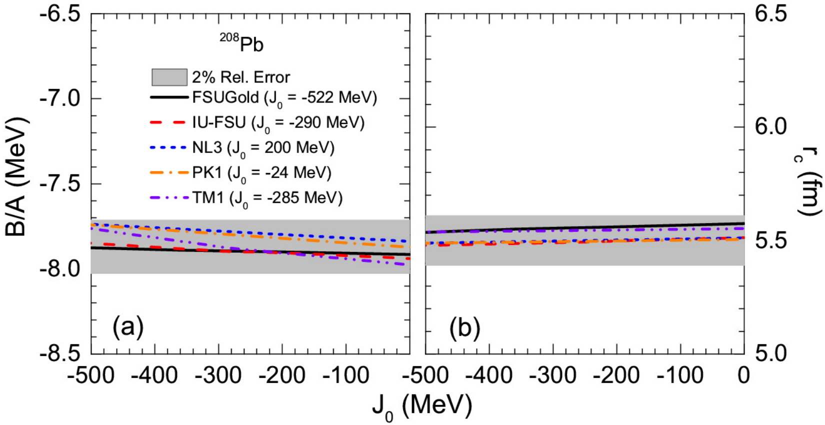
3
2
1
0
4
6
8
10
12
14
16
18
20
M/M⊙
R (km)
Guillot et al., 2016
10.8 +1.8 -1.4 km
Suleimanov et al., 2011
J0348-0432
J1416-2230
RMF
Lattimer et al., 2014
pQCD (high ρ)
χPT (low ρ)
Causality
4U1724-307 619Hz/500Hz
Steiner et al., 2010
Kurkela et al., 2014
Hebeler et al., 2013
astrophysical aspects
perturbative QCD
chiral perturbative field

uncertainties on J_0 are large



Cai and Chen, 2014 (arXiv:1402.4242)

effects of J_0 on nuclear structure quantities are small



Lagrangian:

$$\begin{aligned}\mathcal{L} =&\overline{\psi}_{\mathrm{N}}\left[\gamma_\mu(i\partial^\mu - g_{\omega\mathrm{N}}\omega^\mu - g_{\rho\mathrm{N}}\vec{\tau}_{\mathrm{N}}\cdot\vec{\rho}^{\mu}) - (m_{\mathrm{N}} - g_{\sigma\mathrm{N}}\sigma)\right]\psi_{\mathrm{N}} \\ &+ \overline{\psi}_{\Delta\nu}\left[\gamma_\mu(i\partial^\mu - g_{\omega\Delta}\omega^\mu - g_{\rho\Delta}\vec{\tau}_{\Delta}\cdot\vec{\rho}^{\mu}) - (m_{\Delta} - g_{\sigma\Delta}\sigma)\right]\psi_{\Delta}^{\nu} \\ &+ \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}b_{\sigma\mathrm{N}}m_{\mathrm{N}}(g_{\sigma\mathrm{N}}\sigma)^3 - \frac{1}{4}c_{\sigma\mathrm{N}}(g_{\sigma\mathrm{N}}\sigma)^4 \quad \text{(Boguta \textit{et al.}, 1977)} \\ &+ \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{4}c_{\omega\mathrm{N}}(g_{\omega\mathrm{N}}^2\omega_\mu\omega^\mu)^2 \\ &+ \frac{1}{2}m_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}^{\mu} - \frac{1}{4}\vec{\rho}_{\mu\nu}\cdot\vec{\rho}^{\mu\nu} \\ &+ \frac{1}{2}\left(g_{\rho\mathrm{N}}^2\vec{\rho}_\mu\cdot\vec{\rho}^{\mu}\right)\Lambda_{\mathrm{V}}g_{\omega\mathrm{N}}^2\omega_\mu\omega^\mu \quad \text{(Müller \textit{et al.}, 1996; Todd-Rutel \textit{et al.}, 2005)}\end{aligned}$$

scalar self-energy:

$$\Sigma_{\mathrm{S}}^{\mathrm{N}} = -g_{\sigma\mathrm{N}}\overline{\sigma}, \quad \Sigma_{\mathrm{S}}^{\Delta} = -g_{\sigma\Delta}\overline{\sigma}$$

vector self-energy:

$$\Sigma_{\mathrm{V}}^{\mathrm{N}} = g_{\omega\mathrm{N}}\overline{\omega}_0 + \tau_3^{\mathrm{p/n}}g_{\rho\mathrm{N}}\overline{\rho}_0^{(3)}, \quad \Sigma_{\mathrm{V}}^{\Delta} = g_{\omega\Delta}\overline{\omega}_0 + \tau_3^{i}g_{\rho\Delta}\overline{\rho}_0^{(3)}$$