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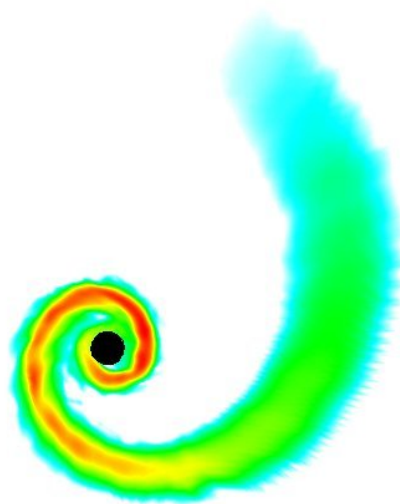
Constraining the neutron star equation of state with gravitational wave observations

IUCAA



Sukanta Bose

Supported in part by NSF grant PHY-1506497 & IUSSTF(India).
Based on LIGO-DCC-G1601432

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Outline

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- Advanced LIGO detectors and first direct GW detections
  (on behalf of LIGO Scientific and Virgo Collaborations)
- Expected neutron star (NS) merger rates in advanced detectors
- Imprint of NS EoS on gravitational-wave signals
- How accurately can gravitational wave (GW) observations constrain the NS equation of state (EoS)?
- Effects of NS-NS populations, NS mass distributions on EOS constraints
- Post-merger waveforms, electro-magnetic counterparts, etc.

The gravitational-wave signal "GW150914", extracted from noise

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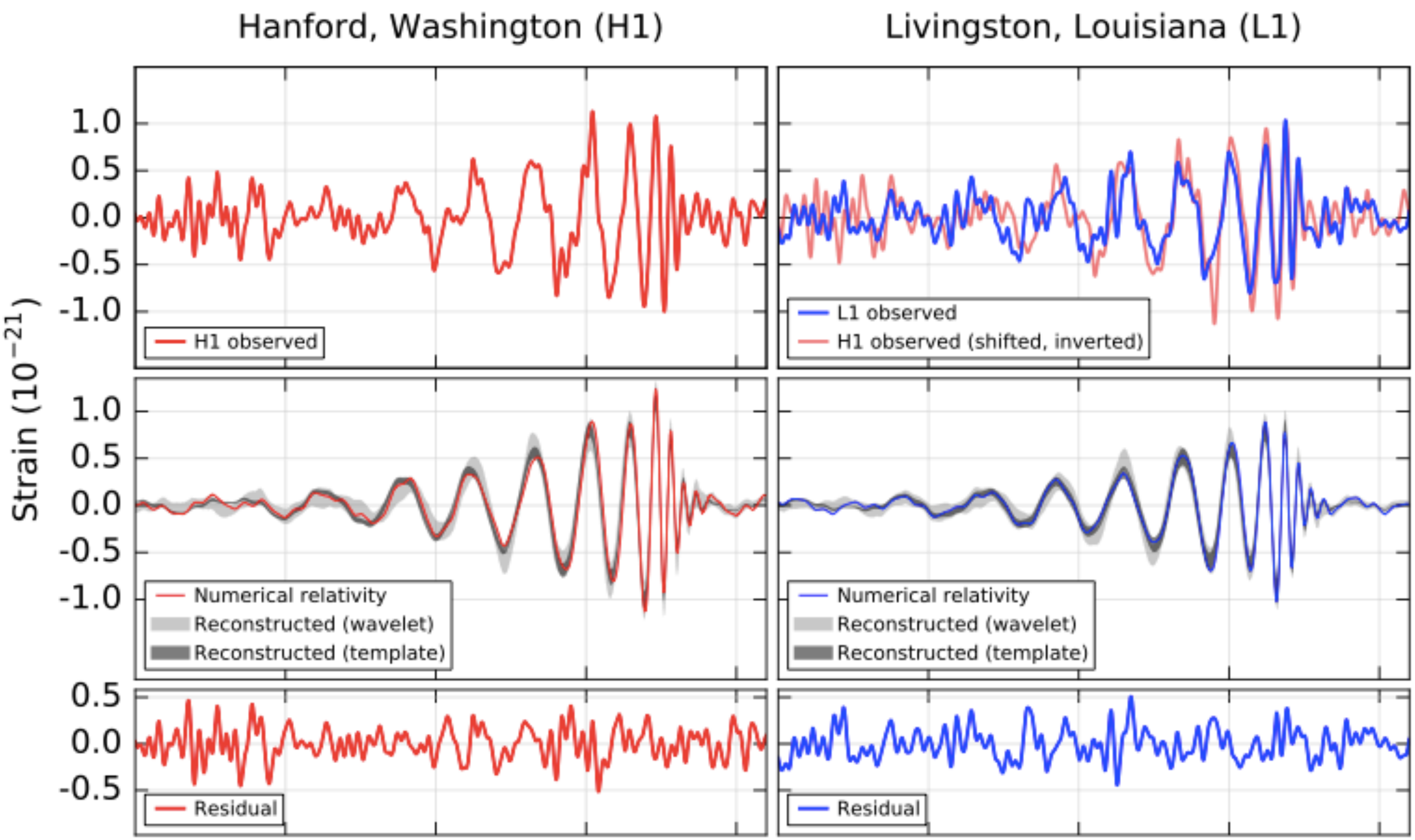

Abbott et al. (LVC), Observation of Gravitational Waves from a Binary Black Hole Merger," Phys. Rev. Lett. 116, 061102 (2016).

The first direct detection of a gravitational-wave event: GW150914

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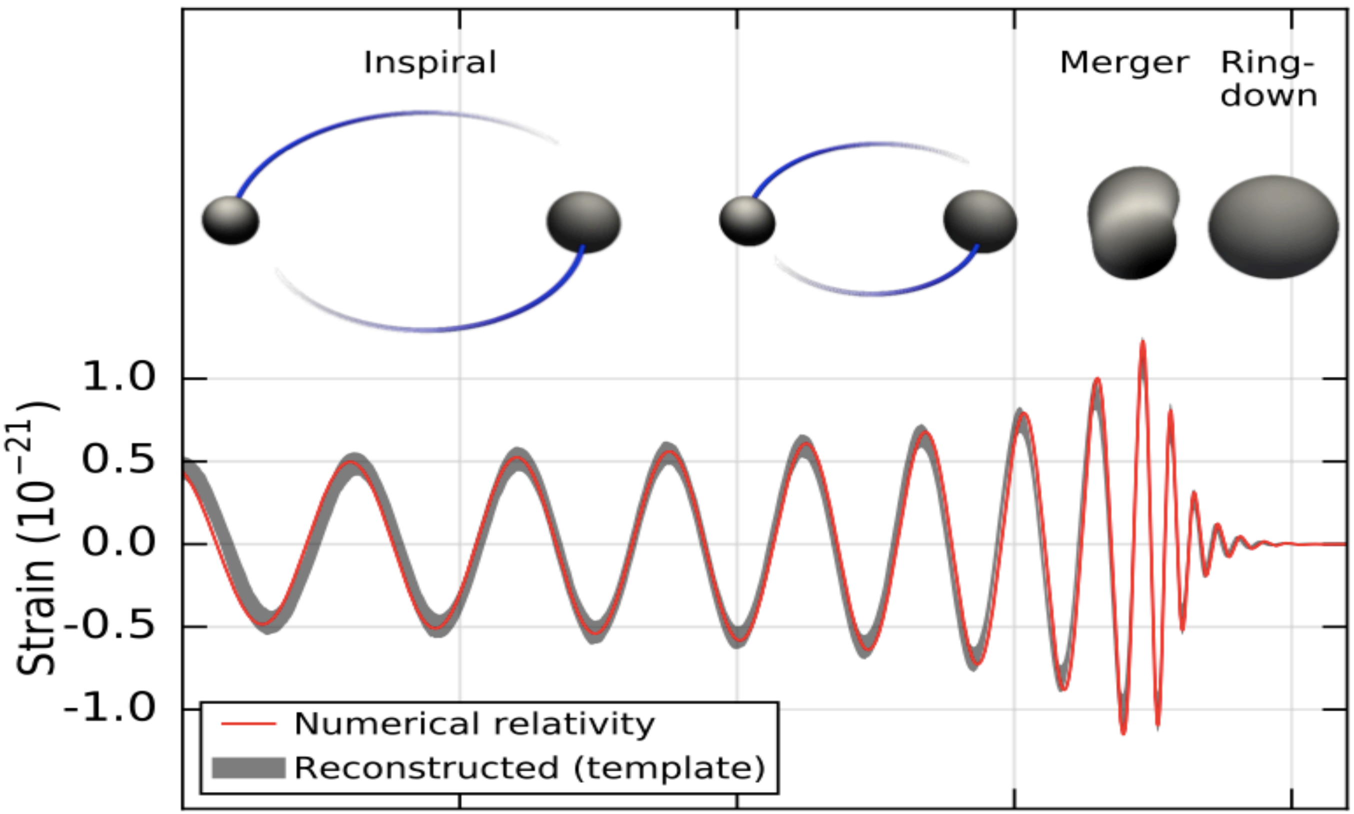
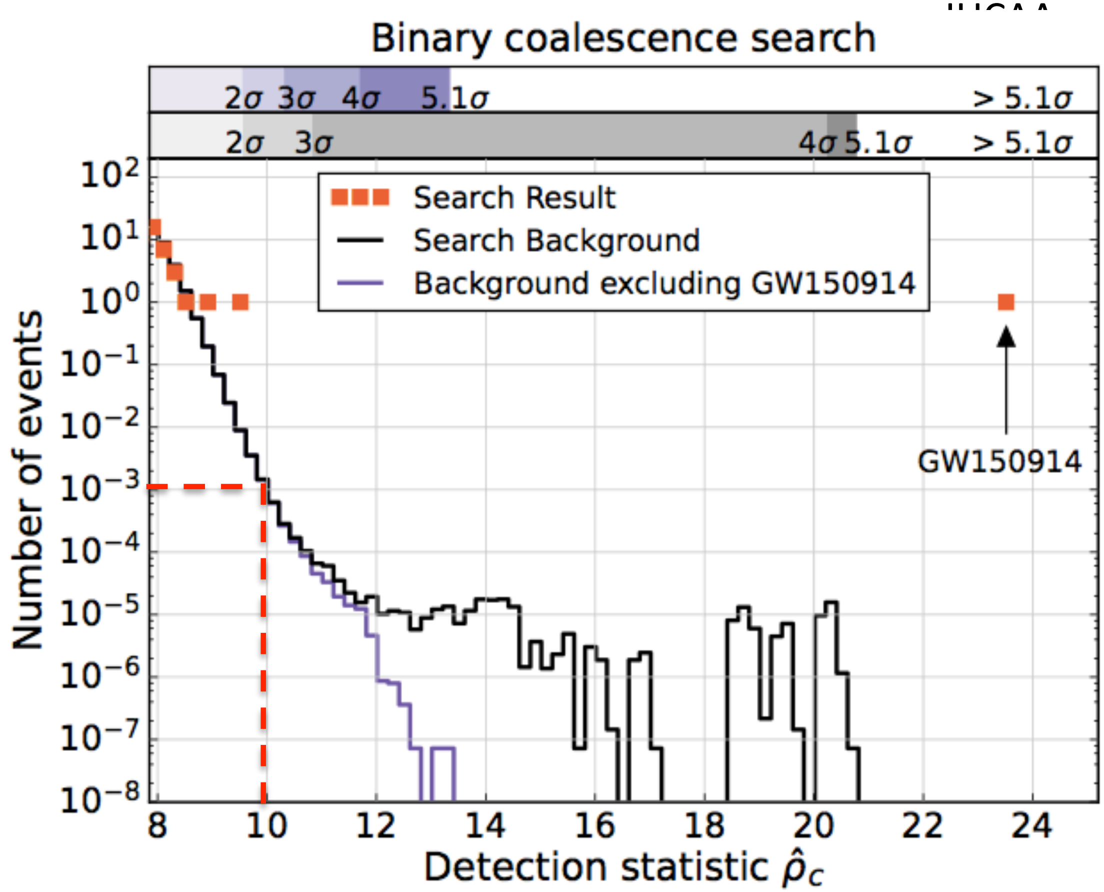


Detection paper: B. Abbott et al., PRL 116, 061102 (2016).

Significance of GW150914

This search concluded that a noise event mimicking GW150914 would be extremely rare – less than once in about 200,000 years.

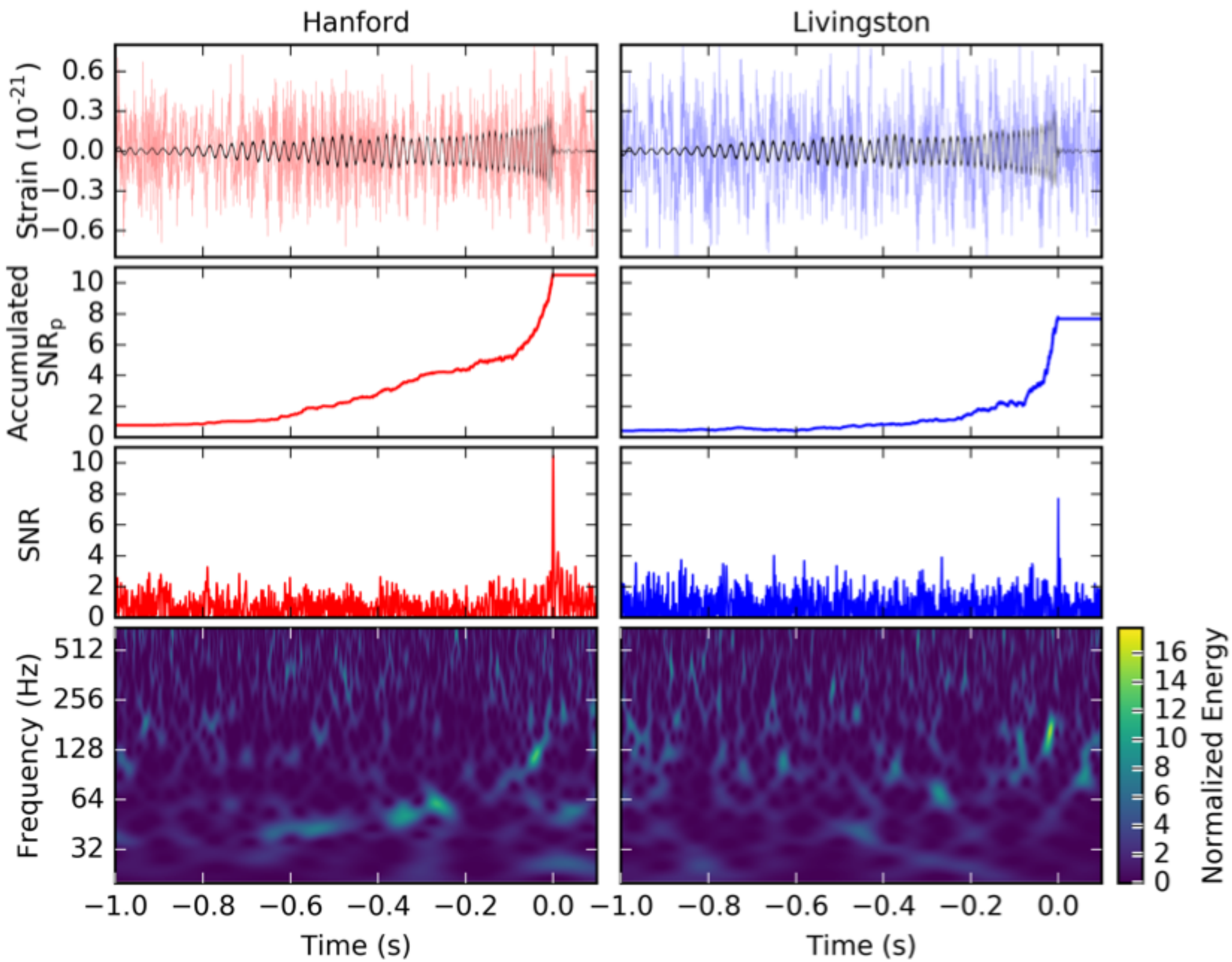
This is a value that has a detection significance of more than 5 'sigma' , i.e., ~1 false alarm in 3.5 million trials.



Results from our binary coalescence search showing how extremely rare it would be for noise alone to produce an event like GW150914.

Abbott et al. (LVC), Observation of Gravitational Waves from a Binary Black Hole Merger," Phys. Rev. Lett. 116, 061102 (2016).

GW151226: A second detection!



Abbott et al., Phys. Rev. Lett., 116, 241103 (2016)

GW151226, at over 5-sigma

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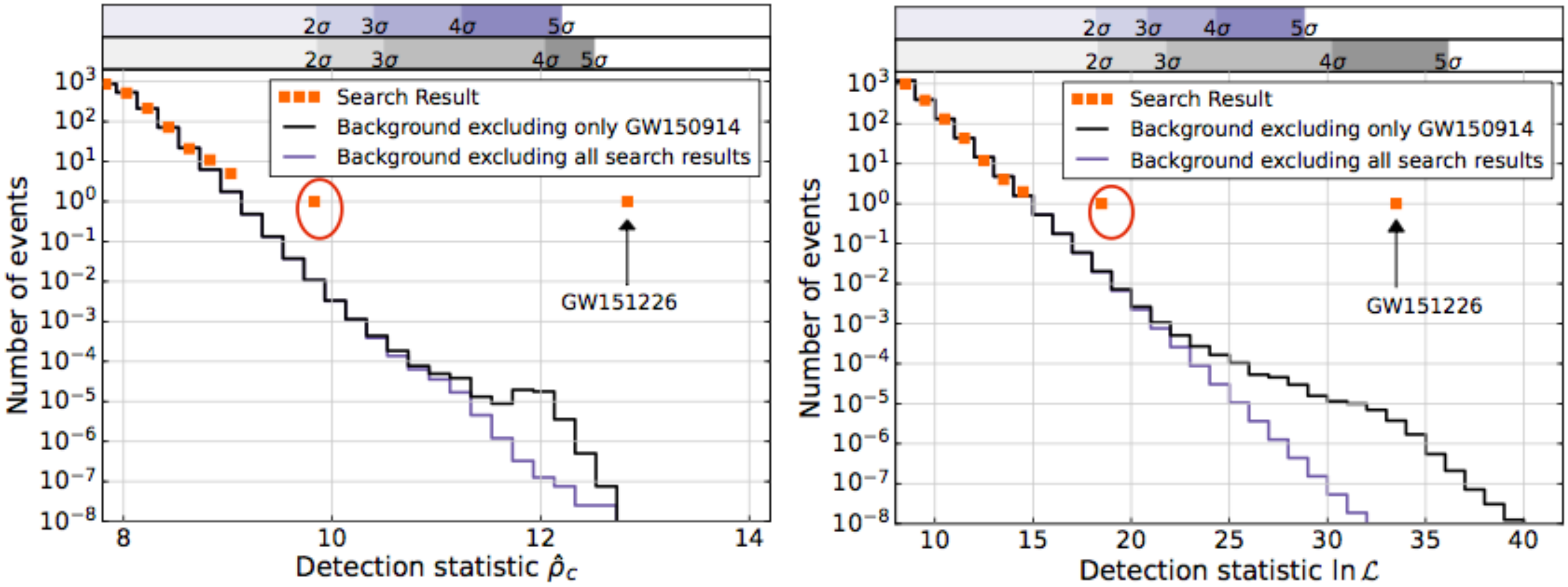

Abbott et al., Phys. Rev. Lett., 116, 241103 (2016)

GW151226 source parameters

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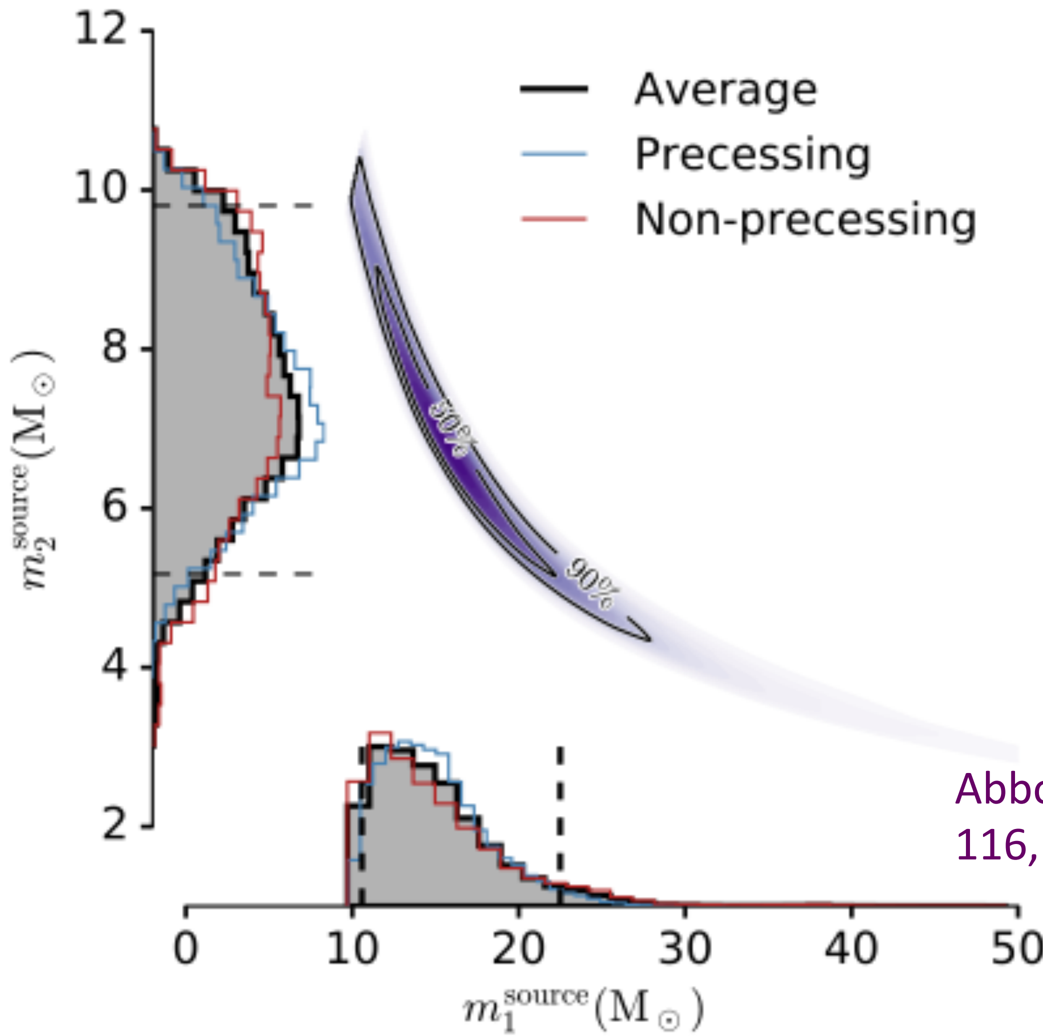
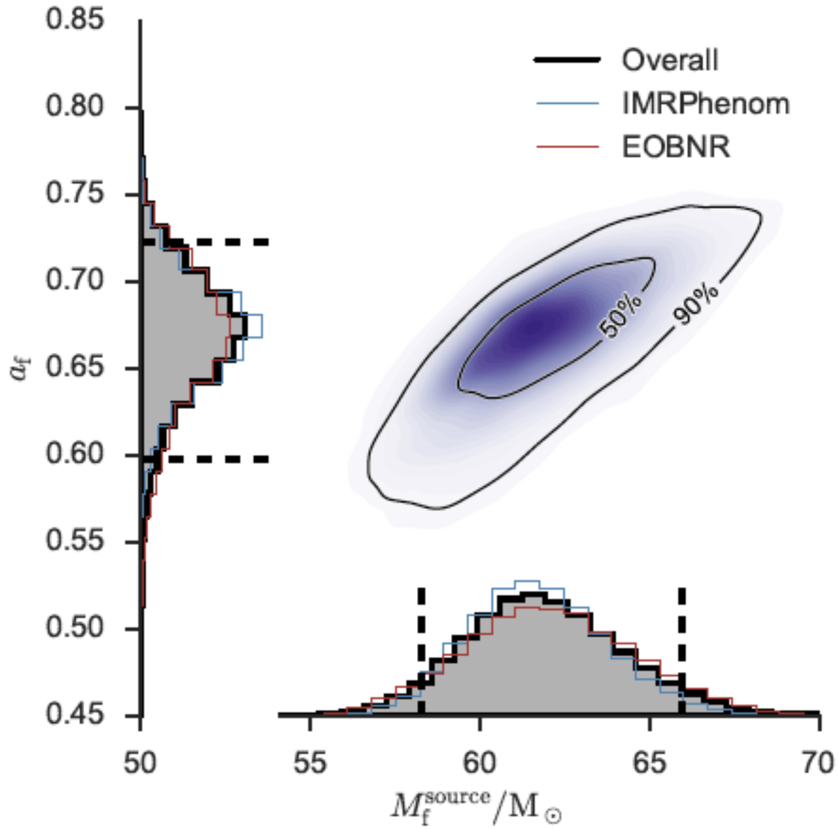


Abbott et al., Phys. Rev. Lett., 116, 241103 (2016)

Characterizing the final BH in GW150914



LVC, arXiv:1602.03840; A. Ghosh et al. (2016).

IndIGO researchers collaborated with LSC members in using fitting formulae from GR numerical simulations that relate masses and spins of component BHs to the final remnant. [Slide courtesy: K. G. Arun]

Did they emit anything visible?

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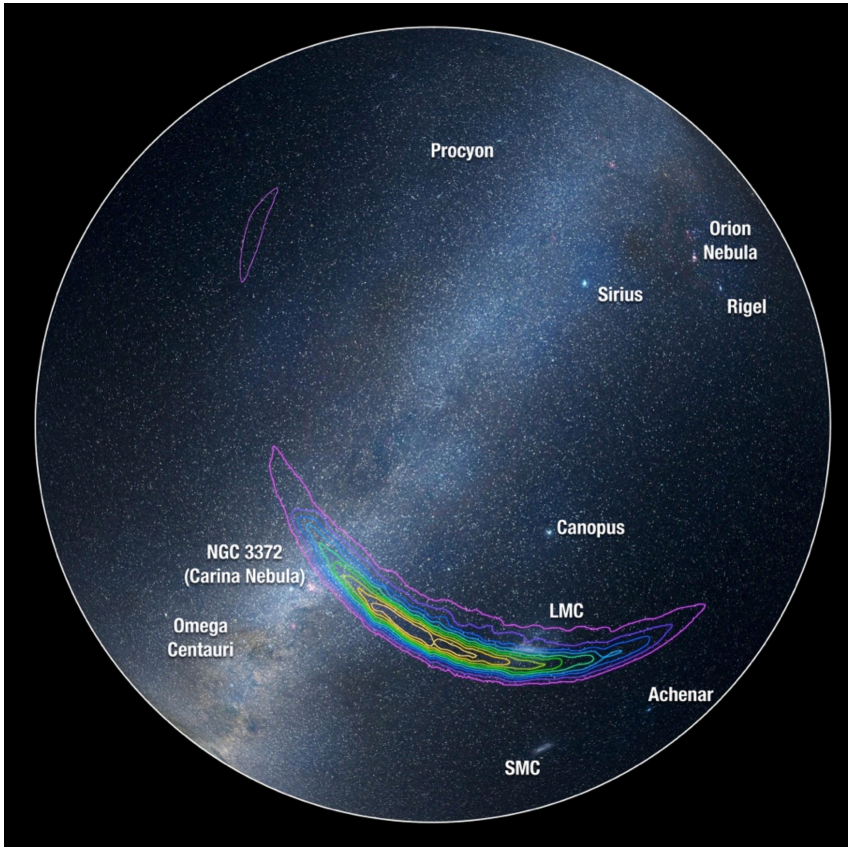
Source Localization and *electromagnetic counterpart* follow up of GW150914:

- Javed Rana, Varun Bhalerao, & Akshat Singhal followed-up a few tiles of this large sky-patch for a possible optical counterpart. We also formulated a scanning method.
- No EM counterpart found till date.

[1. J. Rana, A. Singhal, B. Gadre, V. Bhalerao, S. Bose, arXiv:1603.01689 [astro-ph.IM];
2. Kasliwal,…,V. Bhalerao, J. Rana, A. Singhal,…, et al., arXiv:1602.08764 [astro-ph.IM];
3. Abbott et al. (LVC), "Localization and broadband follow-up of the gravitational-wave transient GW150914," arXiv: 1602.08492 [astro-ph.HE].]



[Sky at the time of the event. View is from the South Atlantic Ocean, North at the top, with the Sun rising and the Milky Way diagonally from NW to SE.]

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Sky localization error regions

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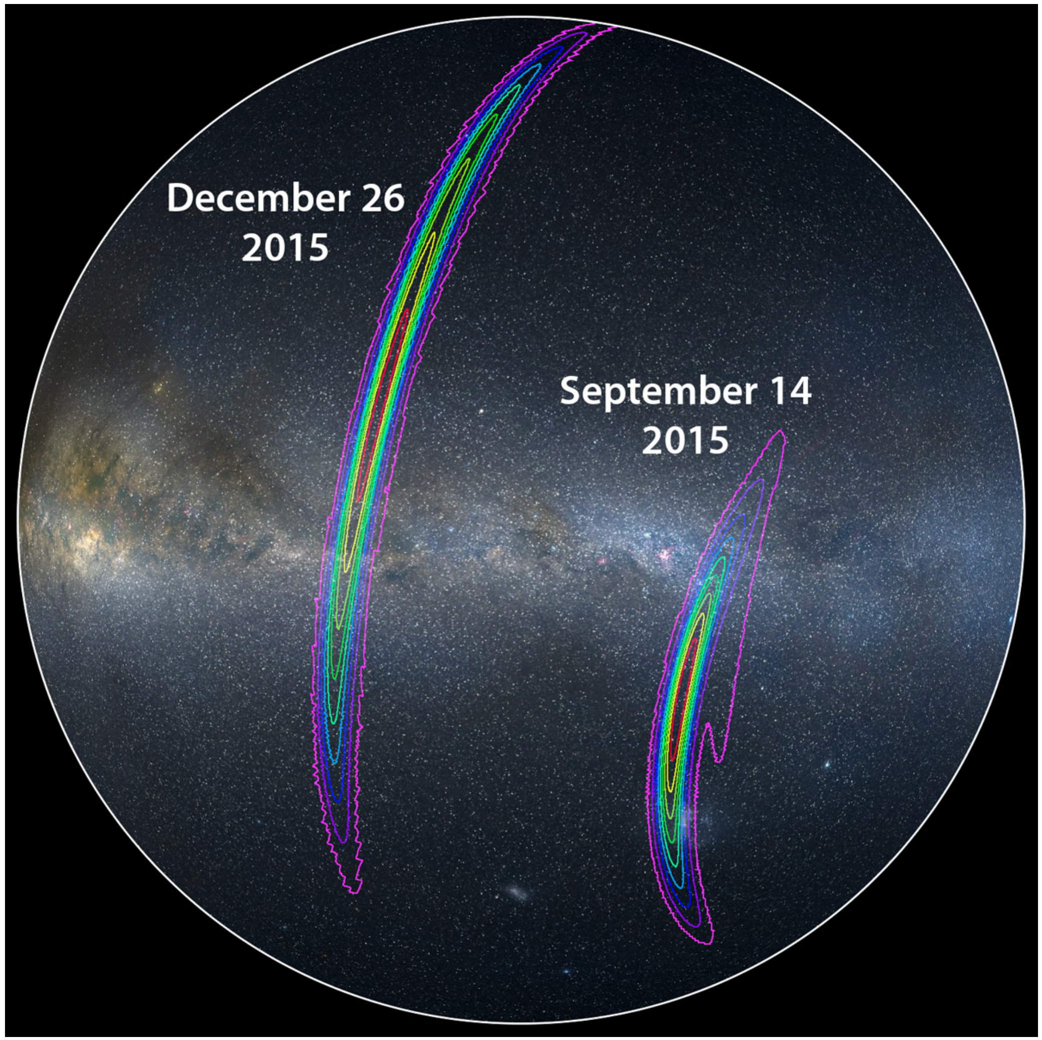
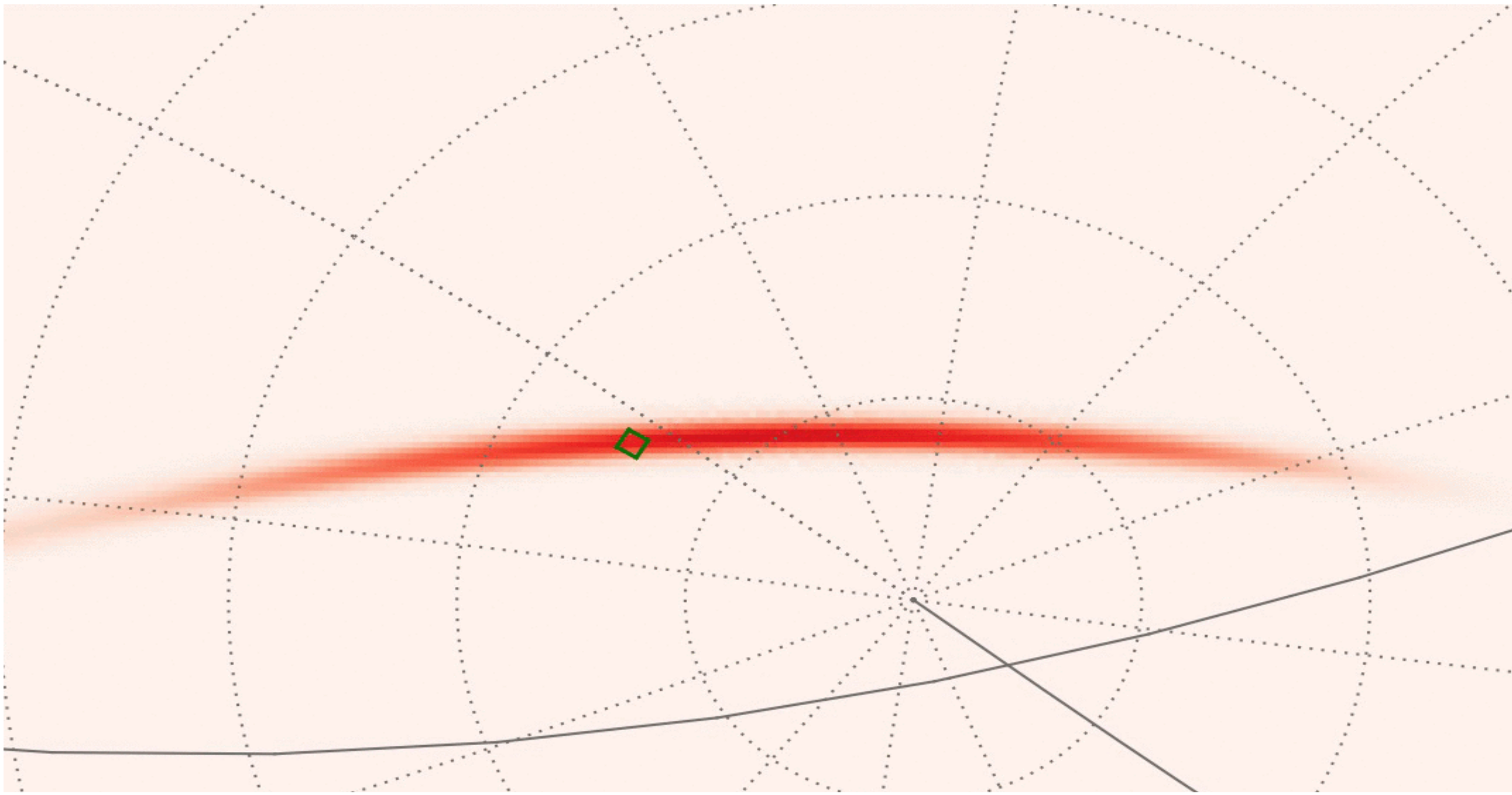


[Courtesy: Singer et al.]

Scheduling telescope observations

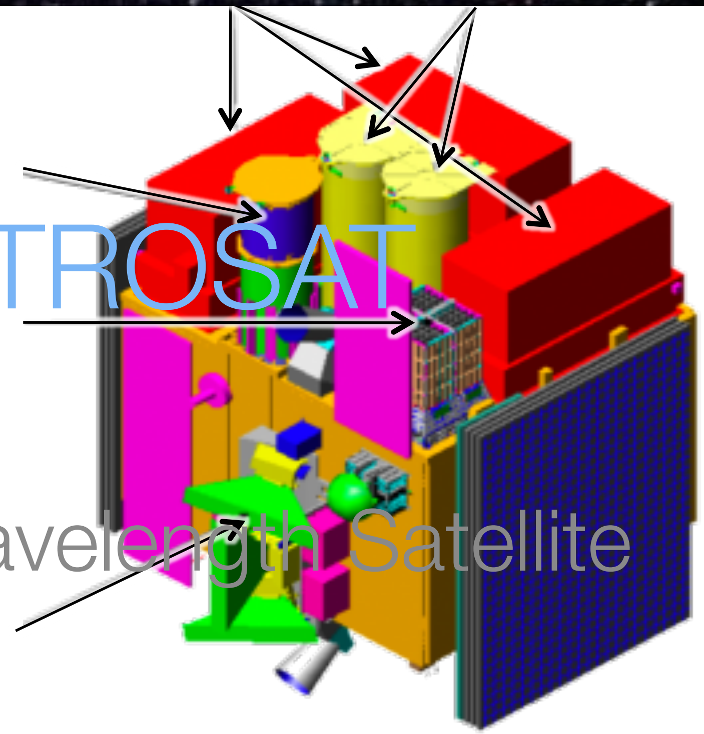


J. Rana, A. Singhal, B. Gadre,
V. Bhalerao, S. Bose, arXiv:1603.01689

SXT

ASTROSAT

CZTI



A Multi-Wavelength Satellite

SSM

LIGO-India gets Govt. of India's "in principle" approval

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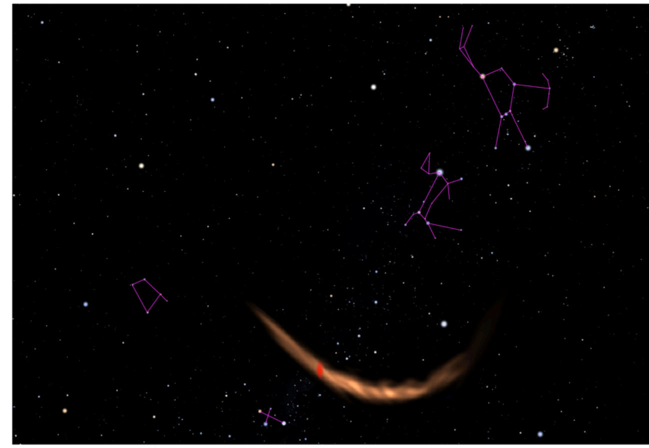
SCI-TECH » SCIENCE

NEW DELHI, February 17, 2016

Updated: February 18, 2016 01:49 IST

Union Cabinet clears LIGO-India gravitational wave observatory



TOPICS

World

India

applied science

cosmology

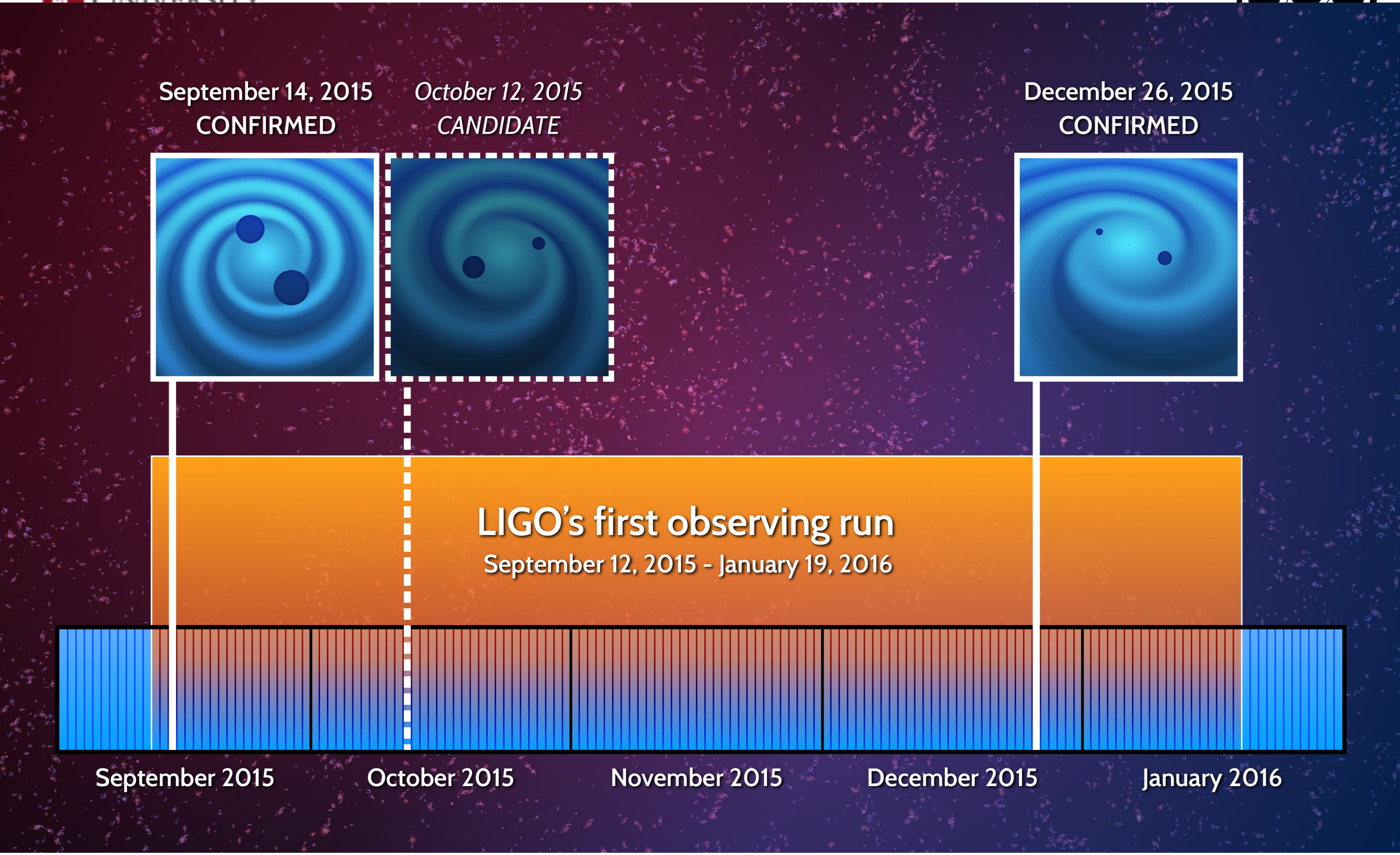
physics

science and

The project is piloted by the Department of Atomic Energy and Department of Science and Technology.

Days after an international team of scientists, including several from India, formally announced that it had detected gravitational waves from deep space, the Union Cabinet, chaired by Prime Minister Narendra Modi, said it had, "in principle," approved a proposal to have a gravitational wave detector in India.

Those connected with the project said it was an important development and marked the government formally acknowledging it but a final decision regarding the money, and how it would be spent, was still some time away. Current estimates suggest the project would cost at least Rs. 1,200 crore. As *The Hindu*



Abbott et al., arXiv:1606.04856 [gr-qc] (2016)

A broadband detector & its noise budget

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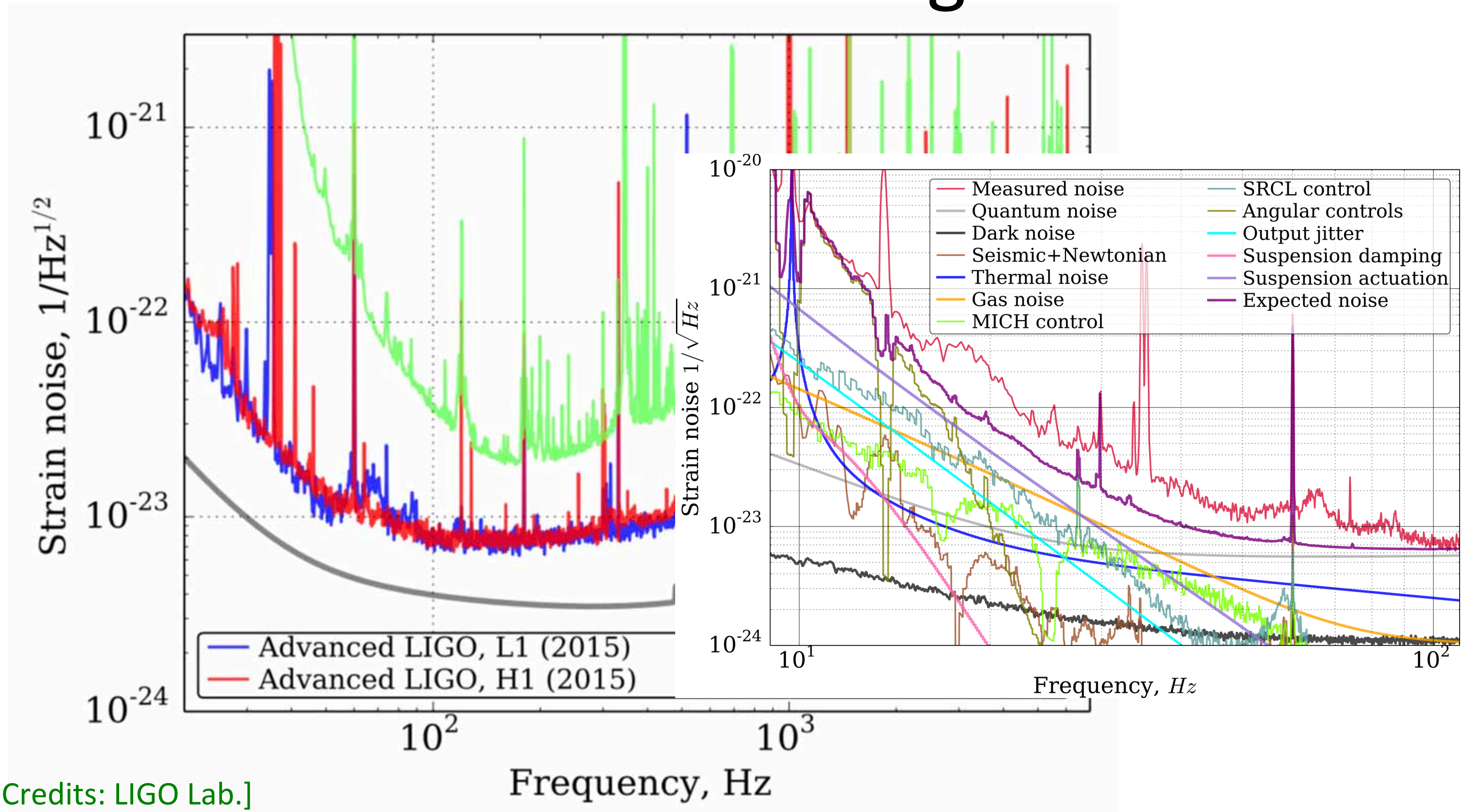

[Credits: LIGO Lab.]

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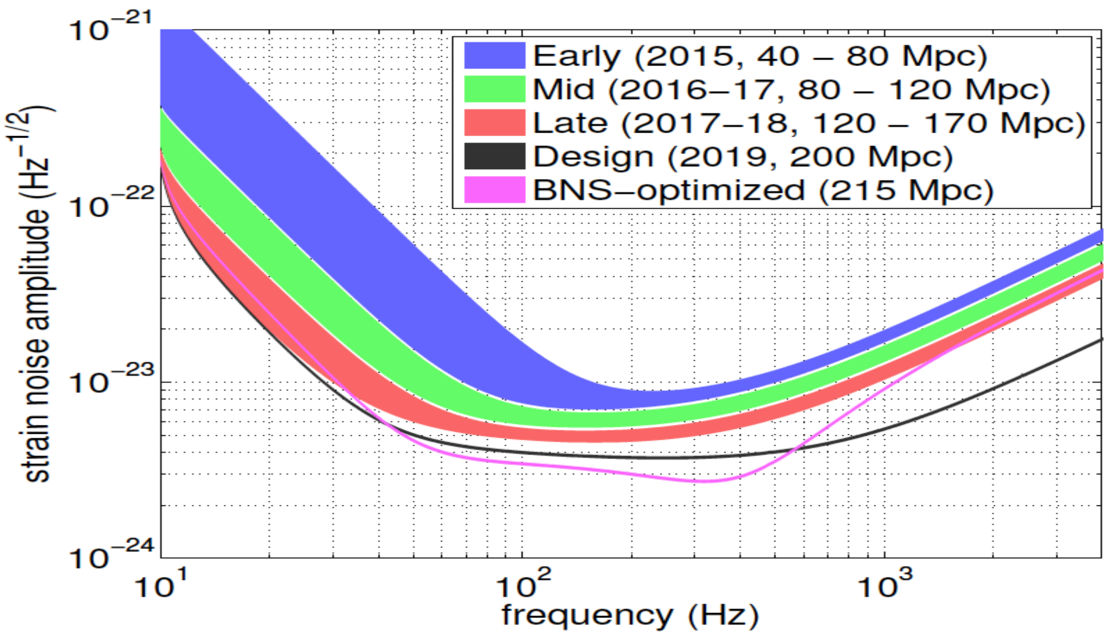
Multiple observations: Projected improvement in GWOs

IUCAA



[J. Aasi et al., LIGO Scientific and Virgo Collaborations., arXiv:1304.0670 [gr-qc] (2013).]

Epoch	Estimated Run Duration	$E_{GW} = 10^{-2} M_{\odot} c^2$ Burst Range (Mpc) LIGO	Virgo	BNS Range (Mpc) LIGO	Virgo	Number of BNS Detections	% BNS Localized within 5 deg^2	20 deg^2
2015	3 months	40 – 60	–	40 – 80	–	0.0004 – 3	–	–
2016–17	6 months	60 – 75	20 – 40	80 – 120	20 – 60	0.006 – 20	2	5 – 12
2017–18	9 months	75 – 90	40 – 50	120 – 170	60 – 85	0.04 – 100	1 – 2	10 – 12
2019+	(per year)	105	40 – 80	200	65 – 130	0.2 – 200	3 – 8	8 – 28
2022+ (India)	(per year)	105	80	200	130	0.4 – 400	17	48

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The GW signal

The GW strain in a detector depends at least on 9 parameters :

$$h(t) = h_+(t)F^+ + h_\times(t)F^\times$$
$$= \frac{A(t)}{D}\cos\left(2\varphi(t) - \vartheta_0\right) = \frac{A(t)}{D}\mathrm{Re}\left[e^{i(2\varphi(t) - \vartheta_0)}\right] ,$$

where the *effective* distance is, $D \equiv \dfrac{r}{\sqrt{F^{+2}\left(1+\cos^2\iota\right)^2 + 4F^{\times 2}\cos^2\iota}}$,

and the *effective* initial phase is, $\vartheta_0 \equiv \arctan\left[\dfrac{2F^\times \cos\iota}{F^+\left(1+\cos^2\iota\right)}\right] + 2\varphi_0.$

In the Fourier domain:

$$\tilde{h}(f) = \left|\tilde{h}(f)\right| e^{i\psi(f)},$$

where

$$\left|\tilde{h}(f)\right| = \frac{2c}{D}\left(\frac{5G\mu}{96c^3}\right)^{1/2}\left(\frac{GM}{\pi^2 c^3}\right)^{1/3} f^{-7/6}.$$

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Chirp mass & its significance

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Chirp mass:

$$M_{\text{chirp}} = \eta^{3/5} M$$

The leading order phase:

$$\psi(f) = 2\pi f t_{\text{ISCO}} + \frac{3}{128}\left(\frac{\pi G M_{\text{chirp}} f}{c^3}\right)^{-5/3} - \phi_{\text{ISCO}},$$

Duration of signal in detector band:

$$\Delta t_{\text{chirp}} = \frac{5}{128}\left(\frac{\pi G M_{\text{chirp}} f_{\text{seismic}}}{c^3}\right)^{-5/3}.$$

Higher order corrections (including spin)

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$$h(f) \equiv C f^{-7/6} \exp\{-i[\Psi(f) - \pi/4]\}$$

$$\chi \equiv (1+\delta)\chi_1/2 + (1-\delta)\chi_2/2,$$

$$\beta = 113\chi/12,$$

$$\sigma_0 = \left(-\frac{12769(4\eta-81)}{16(76\eta-113)^2}\right)\chi^2,$$

$$\gamma_0 = \left(\frac{565\left(17136\eta^2+135856\eta-146597\right)}{2268(76\eta-113)}\right)\chi,$$

$$\begin{aligned}\Psi(f) = 2\pi f t_0 + \phi_0 + \frac{3}{128\eta v_f^5}\Bigg\{1 &+ v_f^2\left[\frac{55\eta}{9} + \frac{3715}{756}\right] \\ &+ v_f^3[4\beta - 16\pi] \\ &+ v_f^4\left[\frac{3085\eta^2}{72} + \frac{27145\eta}{504} + \frac{15293365}{508032} - 10\sigma_0\right] \\ &+ v_f^5\left[\frac{38645\pi}{756} - \frac{65\pi\eta}{9} - \gamma_0\right](3\ln(v_f)+1) \\ &+ v_f^6\Bigg[-\frac{6848\gamma_E}{21} - \frac{127825\eta^3}{1296} + \frac{76055\eta^2}{1728} \\ &\quad + \left(\frac{2255\pi^2}{12} - \frac{15737765635}{3048192}\right)\eta - \frac{640\pi^2}{3} \\ &\quad + \frac{11583231236531}{4694215680} - \frac{6848\ln(4v_f)}{21}\Bigg] \\ &+ v_f^7\left[-\frac{74045\pi\eta^2}{756} + \frac{378515\pi\eta}{1512} + \frac{77096675\pi}{254016}\right]\Bigg\},\end{aligned}$$

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How can GWs constrain the NS EOS?

IUCAA

- How stiff or soft the NS EOS is determines how much the NS will flex (with quadrupole moment Q_{ij}) in an external tidal field, E_{ij} .

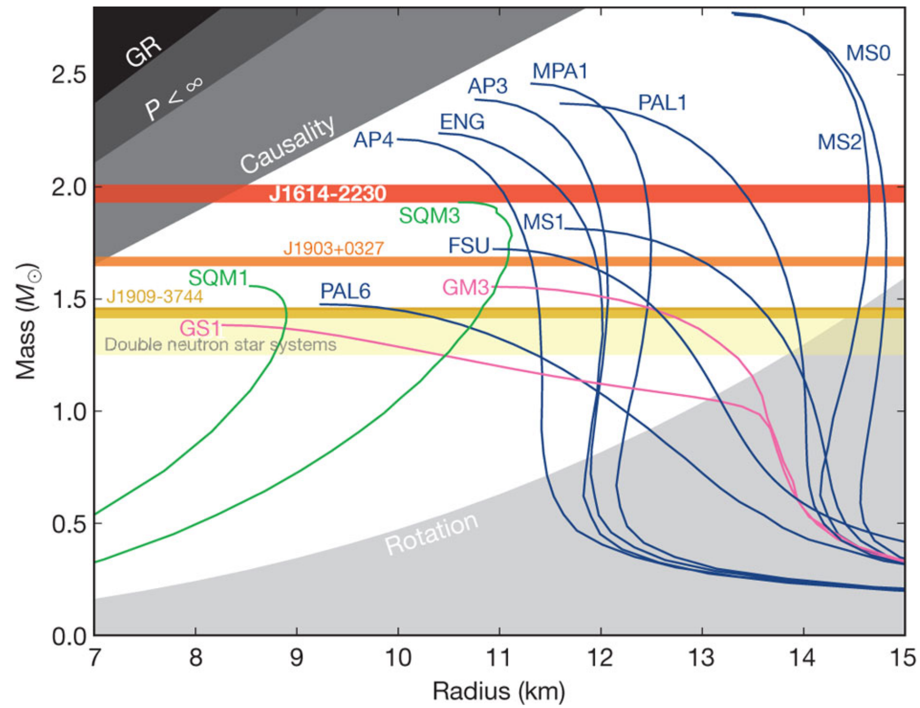
The EOS parameter to be measured is λ, where $Q_{ij} = -\lambda E_{ij}$, and

$$\frac{\lambda}{M^5} = \frac{2}{3} k_2 \left(\frac{R}{M} \right)^5 \approx 10^2 - 10^5.$$

k_2 is the "second Love number".

It is bigger for stiffer EOS.

- The flexing of a neutron star affects the GW emitted by it.



[Credits: J. Lattimer.]

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NS tide in waveforms

Total phase = Point-particle phase + Tidal phase-correction.

Point-particle phase has non-spinning and spinning (aligned or anti-aligned) terms up to 3.5pN. We add test-particle non-spinning corrections from 4pN to 7.5pN, in part, to bridge the gap that otherwise exists between 3.5pN terms and the terms where tidal corrections come in (5pN…).

Tidal phase-correction is:

$$\psi_{\text{tidal}} = \sum_{i=1}^{2} \frac{3\lambda_i}{128\eta M^5}\left[-\frac{24}{\chi_i}\left(1+\frac{11\eta}{\chi_i}\right)\left(\frac{v}{c}\right)^5 - \frac{5}{28\chi_i}\left(3179-919\chi_i-2286\chi_i^2+260\chi_i^3\right)\left(\frac{v}{c}\right)^7\right],$$

[Vines, Flanagan, Hinderer, arXiv:1101.1673v1.; Damour, Nagar, Villain, PRD85, 123007 (2012).

where $v=(M\omega)^{1/3}$, $\chi_i = m_i/M$ and "i" is binary component index.

$M = m_1 + m_2$ and $\eta = m_1 m_2 / M^2$.

New: We add the aforementioned point particle terms to improve upon waveforms used in the past

Effect of EOS on GW signals

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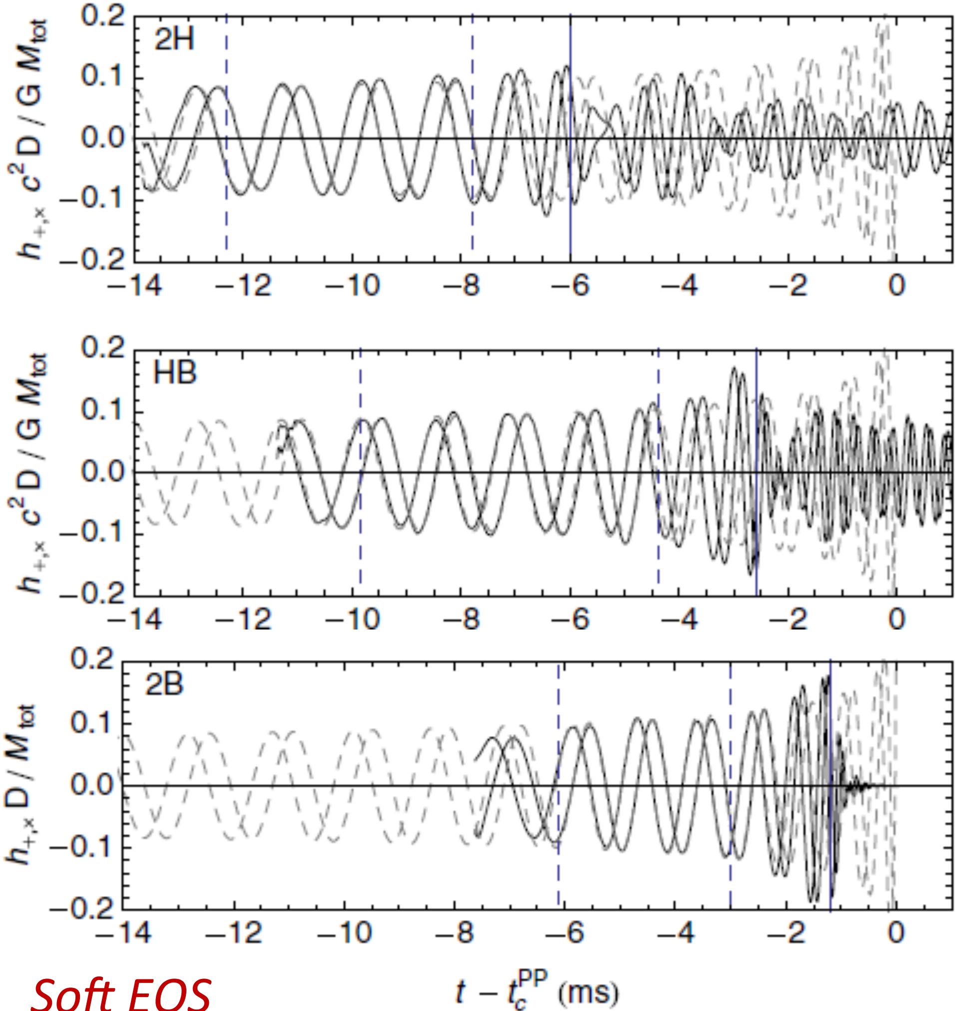
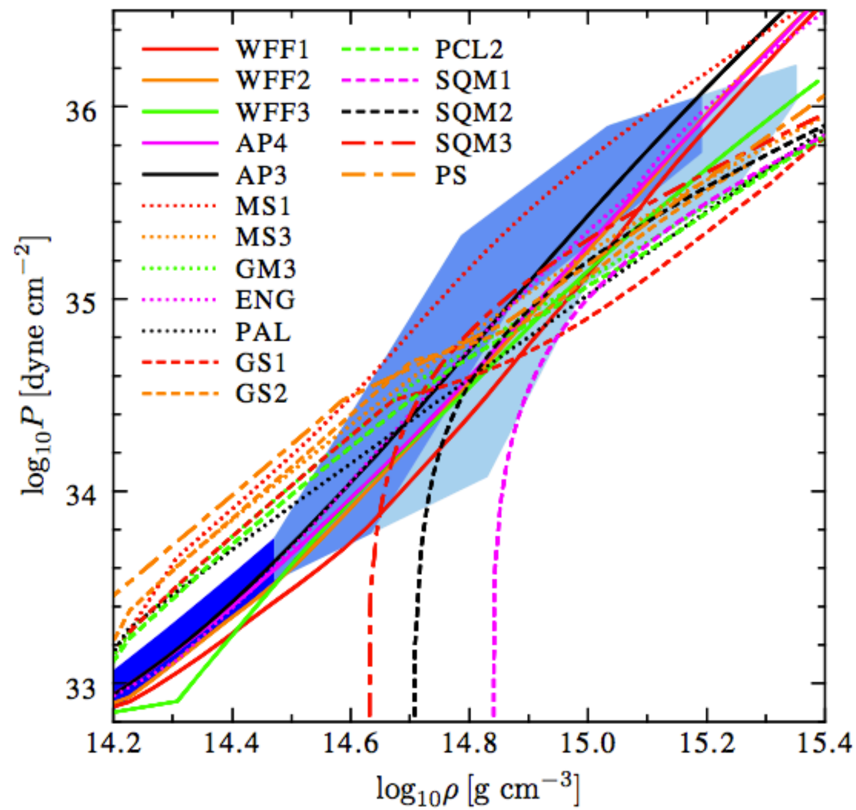
Stiff EOS





Soft EOS

[Read et al. Phys.Rev. D79 (2009) 124033;
Hebeler, Schwenk, Eur.Phys.J. A50 (2014) 11]

Chirp mass well determined but not individual masses

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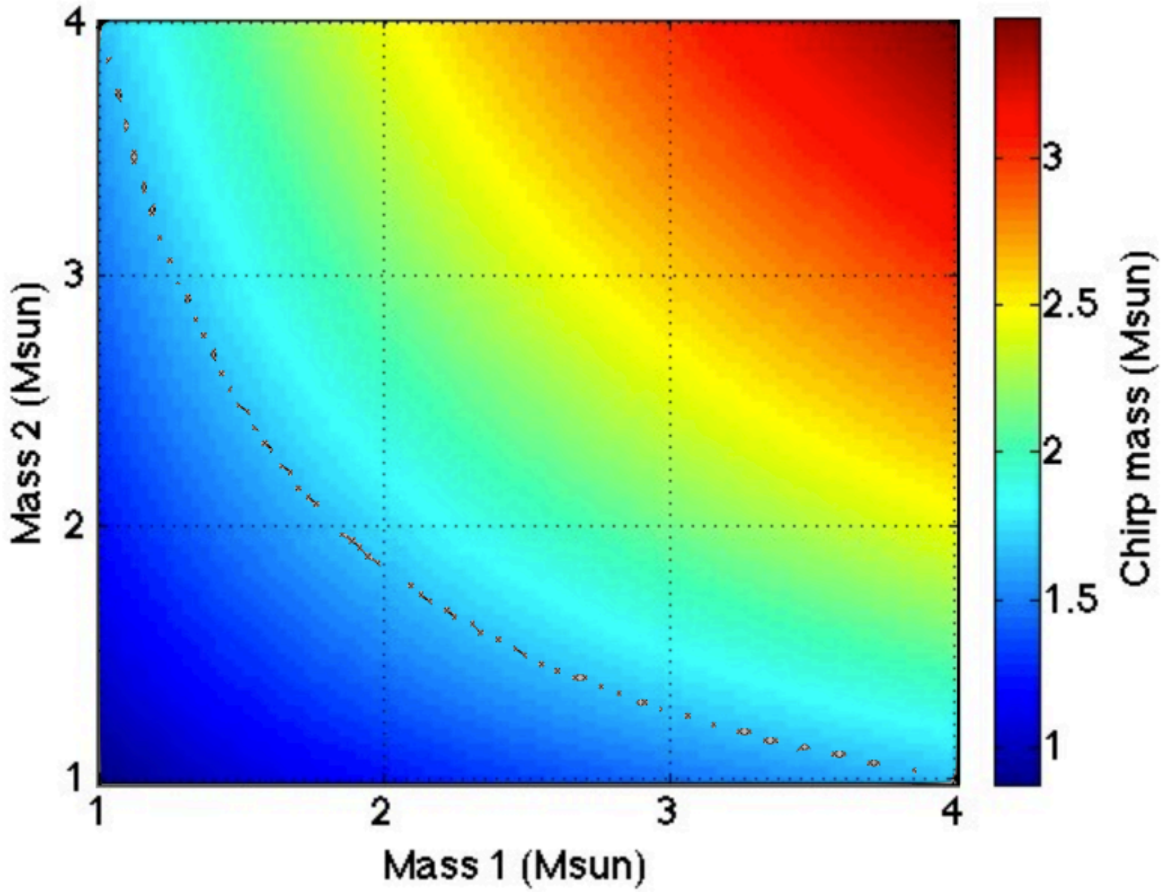
* If M_{BH}= 4Msun and
M_{NS}= [1, 2]$Msun$, then:

M_c= [1.67, 2.43] $Msun$

Total mass, symmetrized mass-ratio & chirp mass:

$$M = m_1 + m_2, \quad \eta = \frac{m_1 m_2}{M^2} \ ,$$

$$M_c = \eta^{3/5} M \ .$$



BNS mass errors from GWs

System	$\Delta\mathcal{M}_c/\mathcal{M}_c$		Δq	
Credible Level	68%	95%	68%	95%
$1\,M_\odot - 1\,M_\odot$	0.00497%	0.0104%	0.123	0.197
$1.4\,M_\odot - 1.4\,M_\odot$	0.00883%	0.0188%	0.132	0.212
$1\,M_\odot - 2.5\,M_\odot$	0.0176%	0.0355%	0.0138	0.028
$2.5\,M_\odot - 2.5\,M_\odot$	0.0246%	0.0522%	0.149	0.239

Errors are for SNR = 20, spinless systems in aLIGO-AdV.

Rodriguez et al. ApJ 784, 114 (2014).

BNS mass errors from GWs

IUCAA

	HLV			
System	$\Delta M_1/M_1$		$\Delta M_2/M_2$	
Credible Level	68%	95%	68%	95%
$1\,M_\odot - 1\,M_\odot$	7.17%	11.9%	6.39%	10.3%
$1.4\,M_\odot - 1.4\,M_\odot$	7.77%	13%	6.87%	11.1%
$1\,M_\odot - 2.5\,M_\odot$	1.86%	3.74%	1.59%	3.23%
$2.5\,M_\odot - 2.5\,M_\odot$	9.02%	15%	7.82%	12.6%

Errors are for SNR = 20, spinless systems in aLIGO-AdV.

Rodriguez et al. ApJ 784, 114 (2014).

Double neutron star systems: Populations

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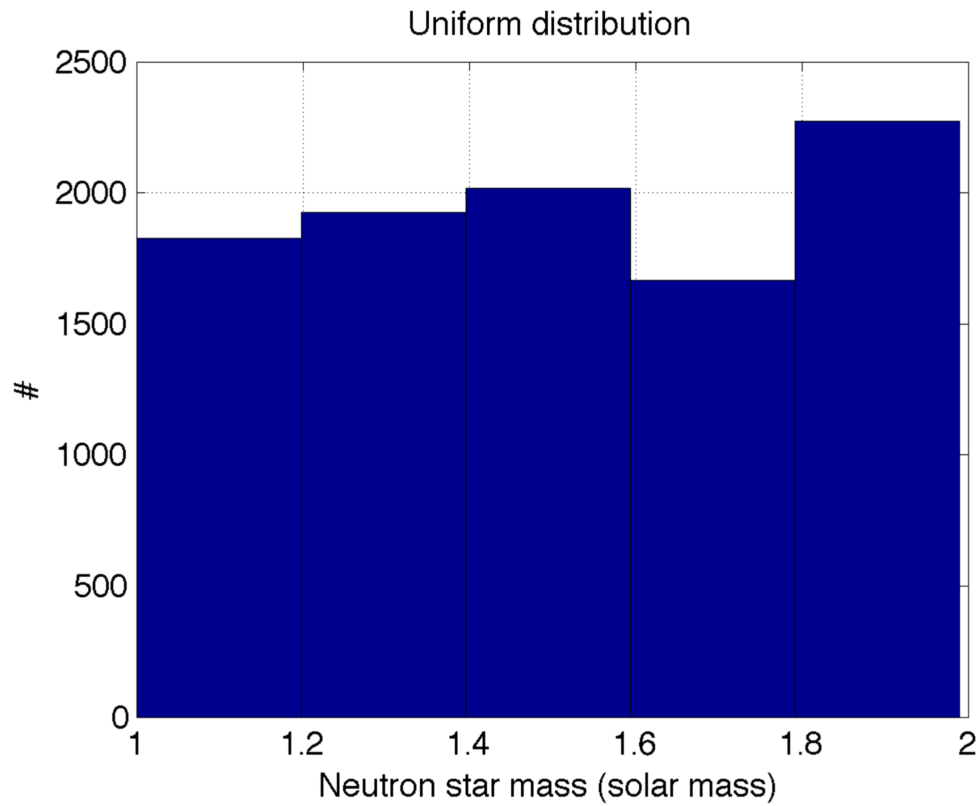
- Uniform distribution of NS masses, limited to the range [1, 2] M_sun.

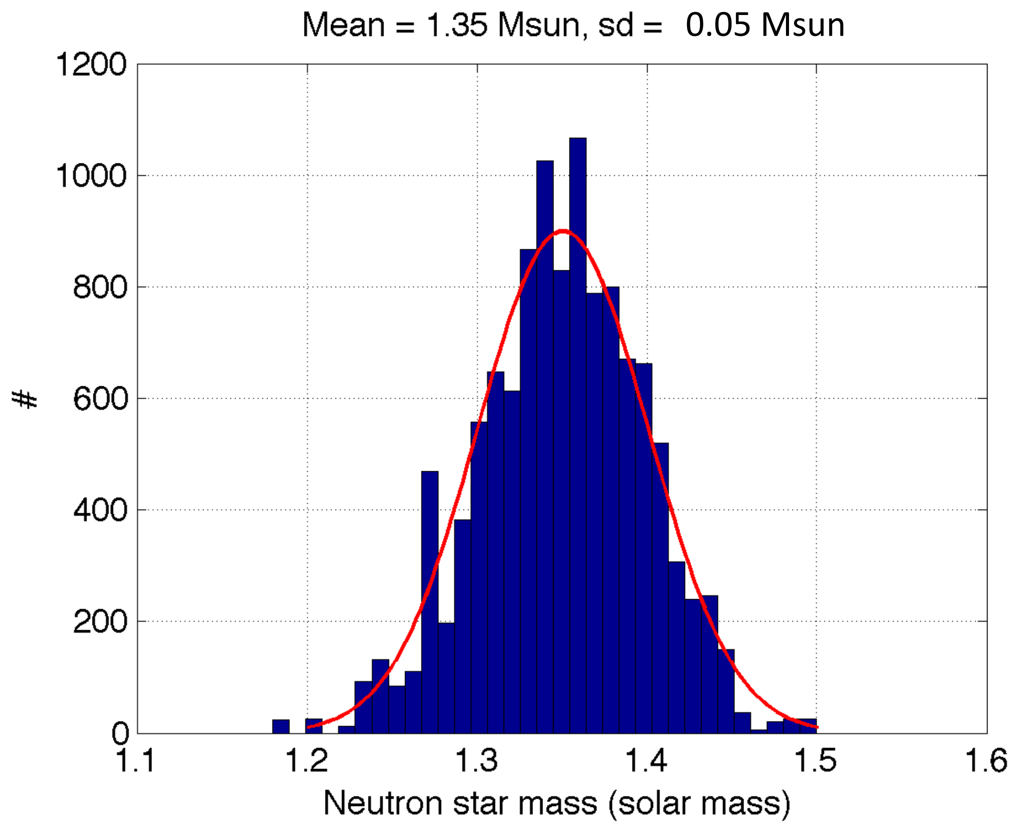


Double neutron star systems

Populations

IUCAA

- Gaussian distribution of NS masses with mean = 1.35Msun, sd = 0.05Msun:

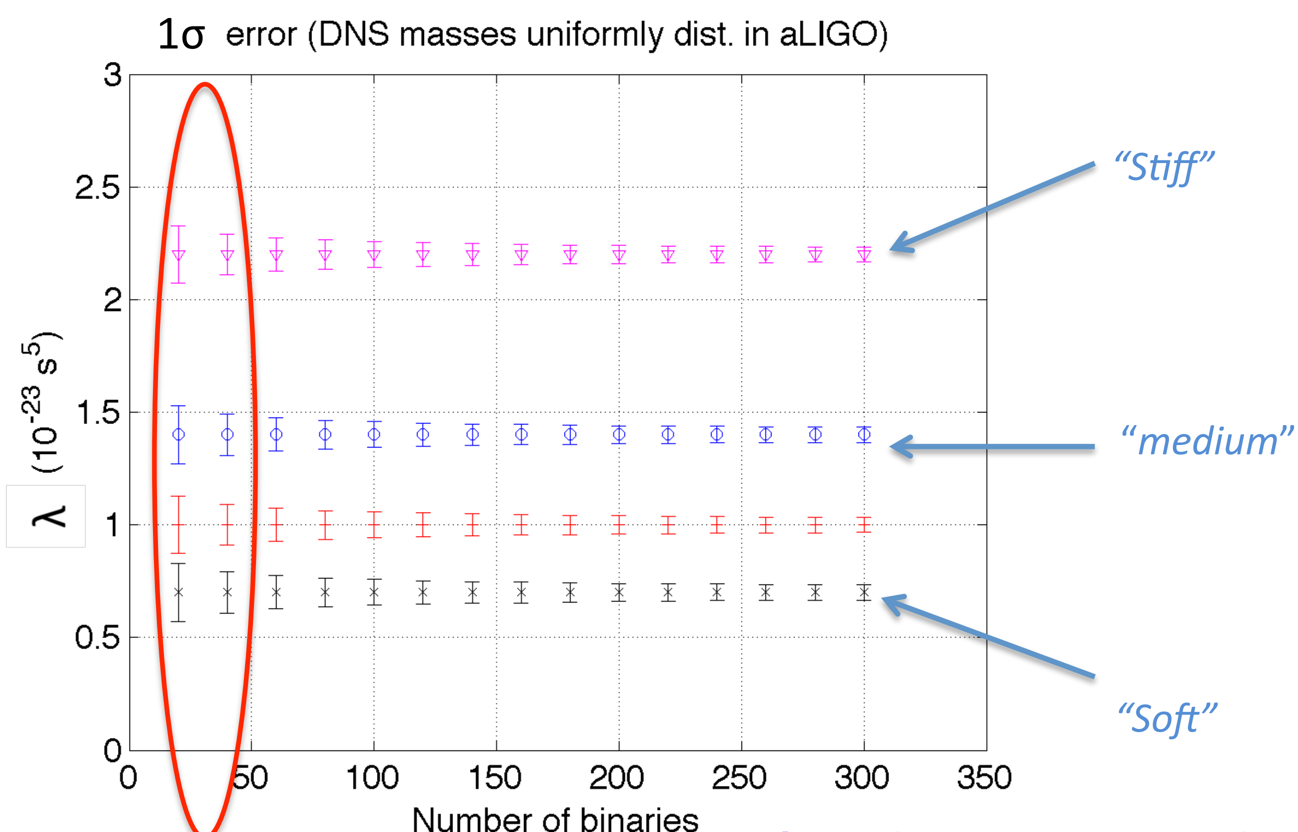


Error in determining λ

Double neutron star systems

IUCAA

- ***Uniform*** distribution of NS masses, limited to the range [1, 2] M_sun.



[SB et al. in preparation; see also: Agathos et al., arXiv:1503.05405 (2015).]

Two past studies

Double Neutron Stars

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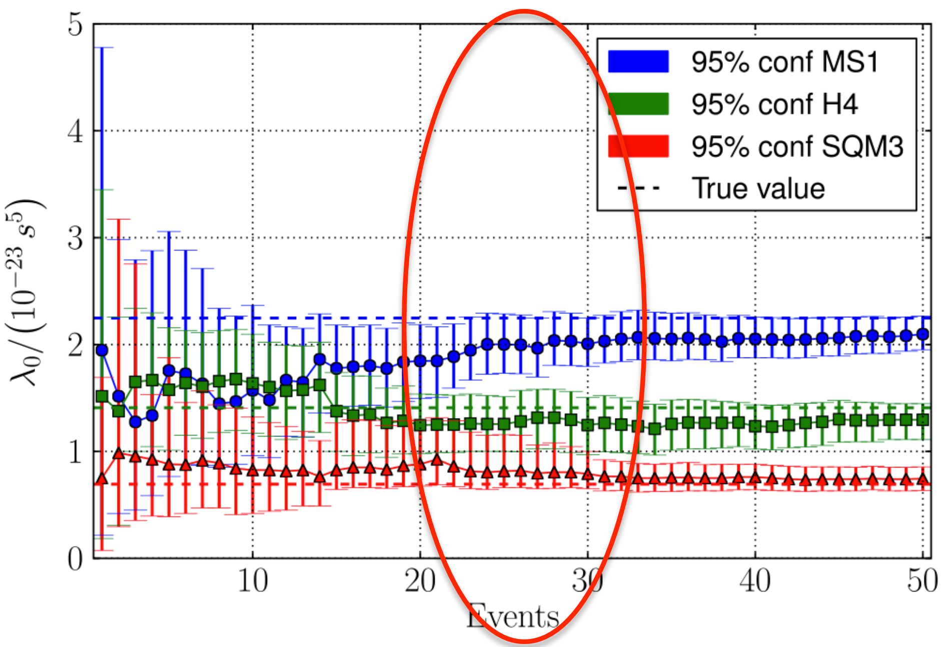
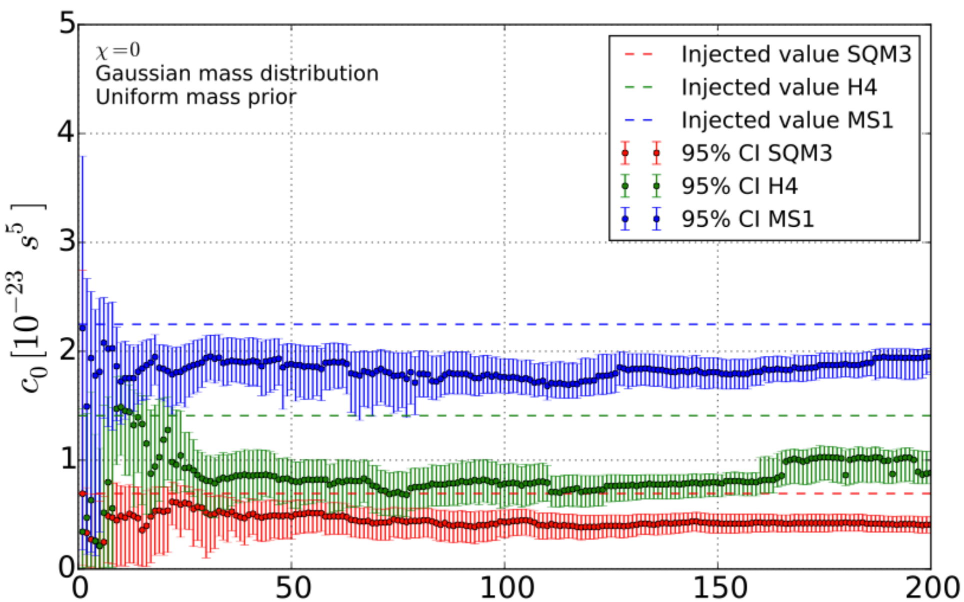
- Uniform distribution of NS masses, limited to the range [1, 2] M_sun:
- Gaussian distribution of NS masses with mean = 1.35Msun, sd = 0.05Msun:



[del Pozzo et al., PRL 111, 071101 (2013).]



[Agathos et al., arXiv:1503.05405 (2015).]

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Two past studies
Double Neutron Stars

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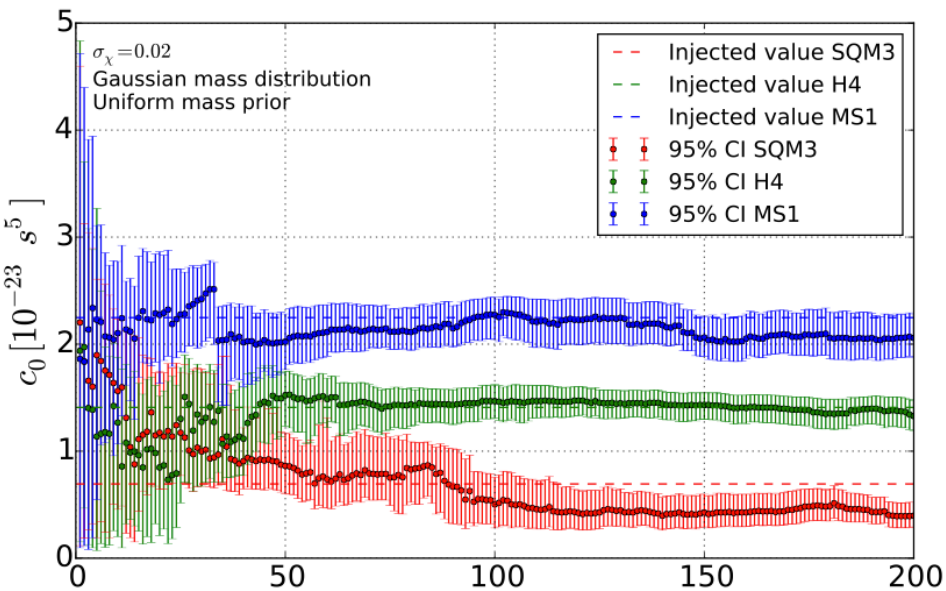
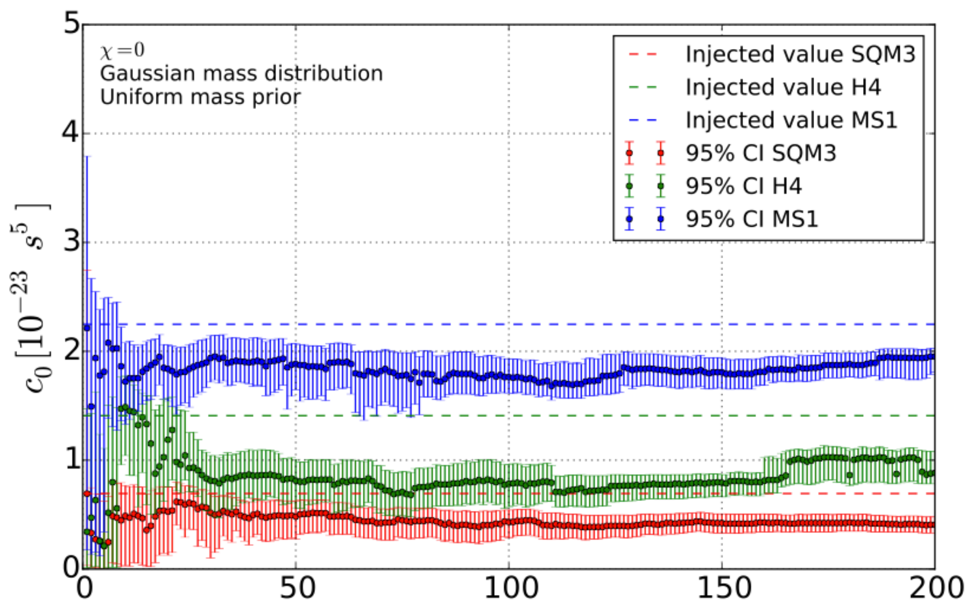
- Spin's effect: Gaussian distribution of NS spins: mean = 0, sd = 0.02.



- Gaussian distribution of NS masses with mean = 1.35Msun, sd = 0.05Msun:



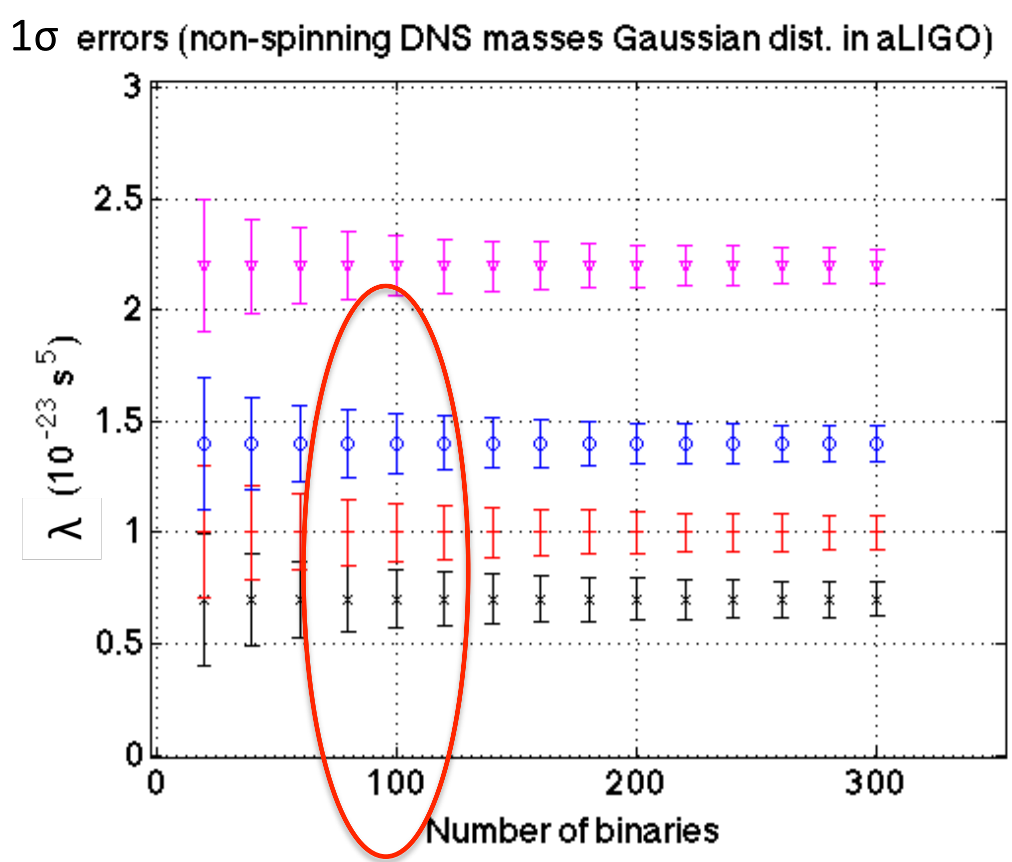
Spin appears to improve accuracy for small lambda, and reduce systematic errors.

[Agathos et al., arXiv:1503.05405 (2015).]

Error in determining λ
Double neutron star systems

IUCAA

- ***Gaussian*** distribution of NS masses, with mean = 1.35Msun, sd = 0.05Msun.



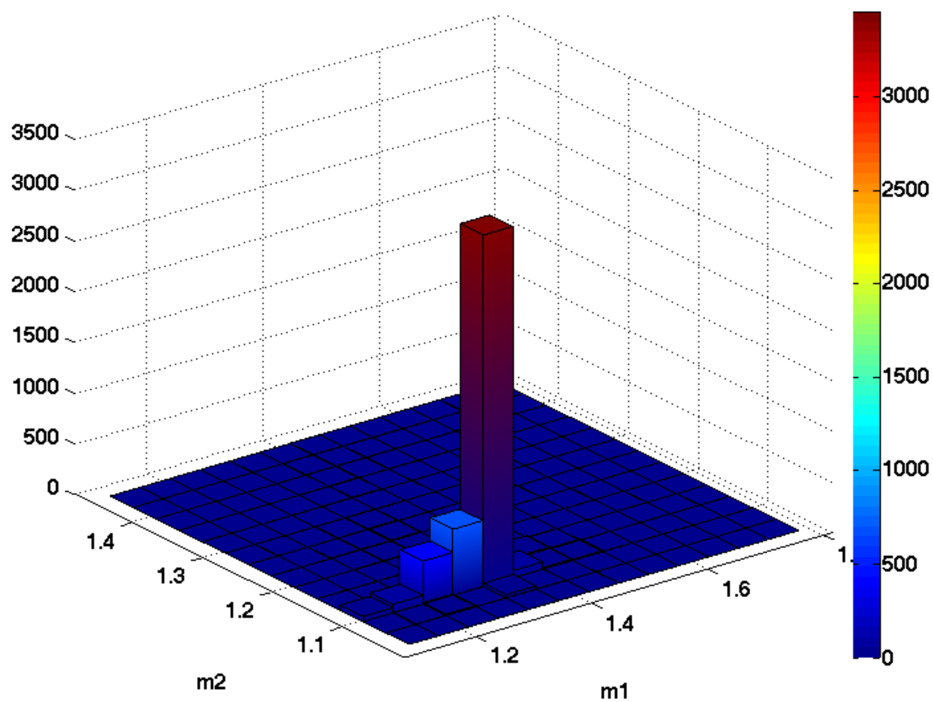
[SB et al. in preparation]

Double neutron star systems

Populations

IUCAA

- Solar distribution of DNS mass pairs:

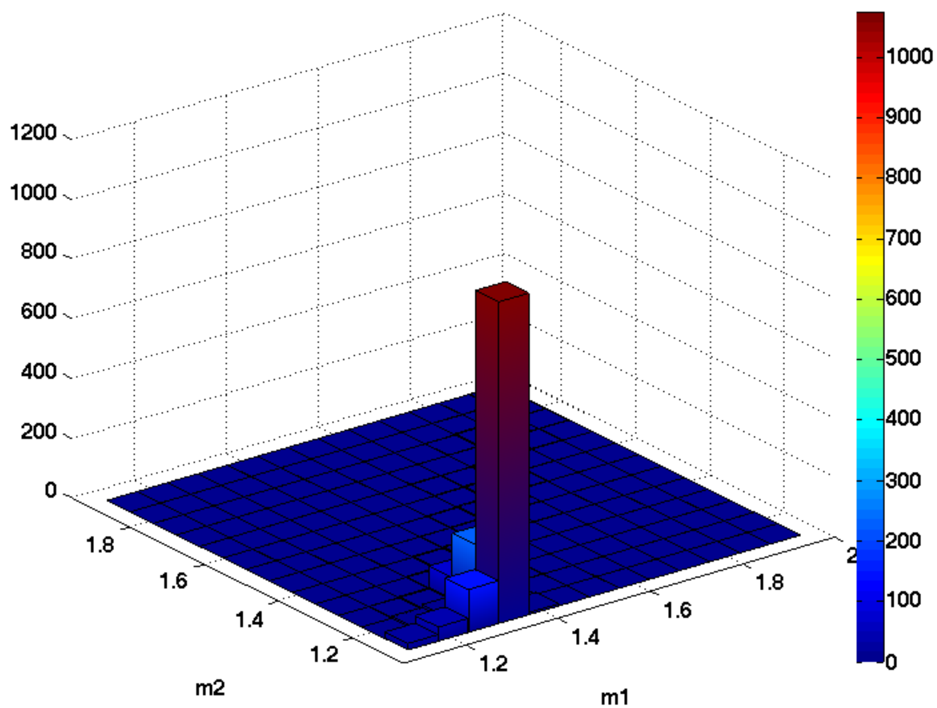


[SB et al. in preparation]

Double neutron star systems
Populations

IUCAA

- Sub-solar distribution of DNS mass pairs:



[SB et al. in preparation]

Error in determining λ

For four NS population models

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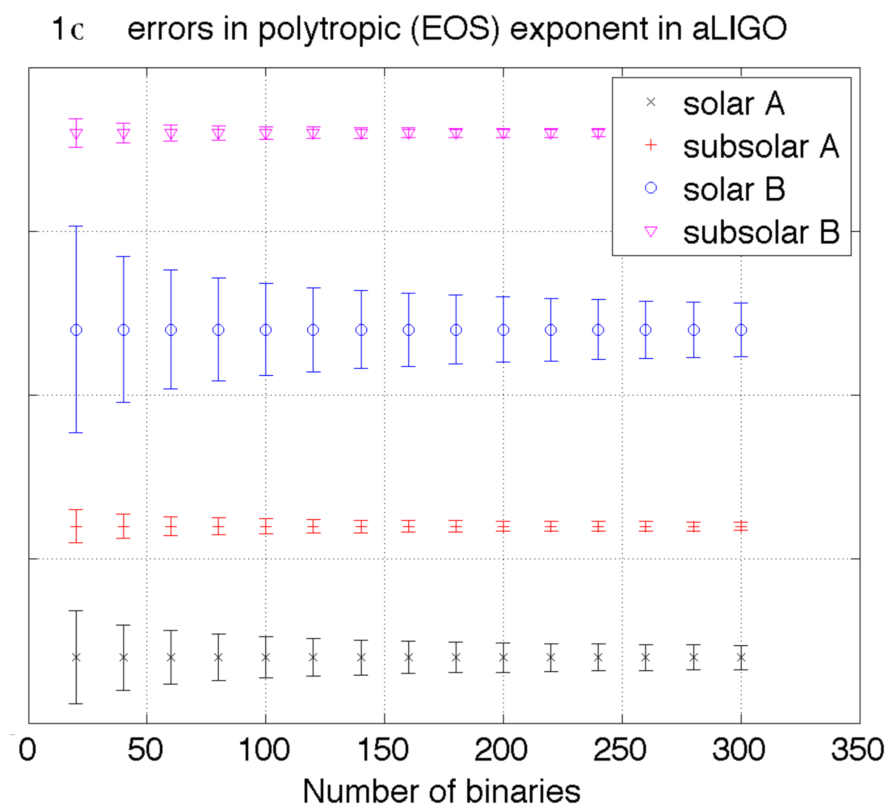
- Four different distributions of NS masses:

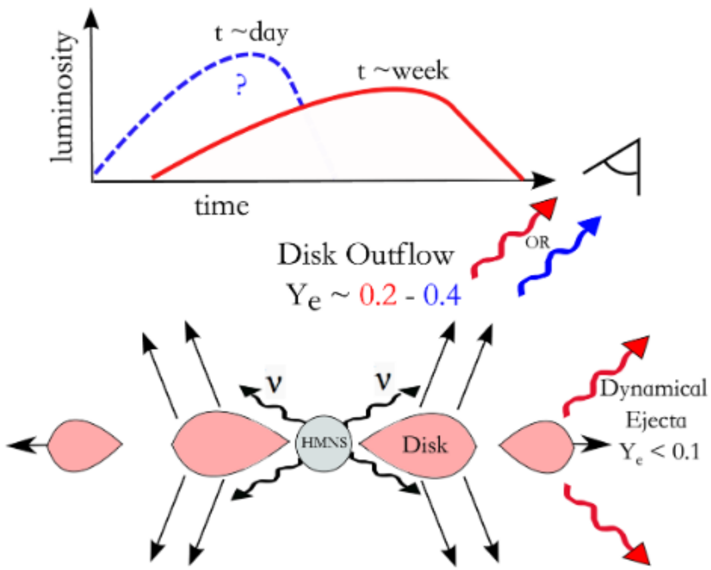
Four different Population Synthesis models

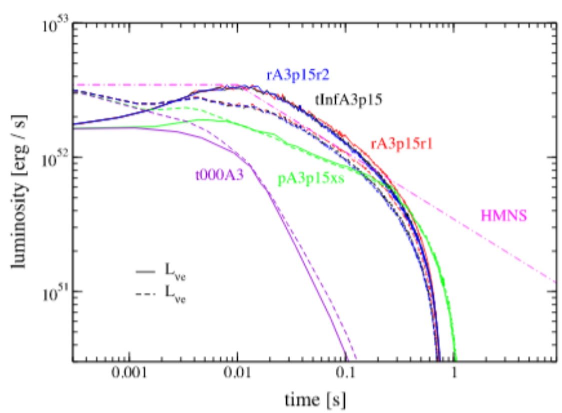


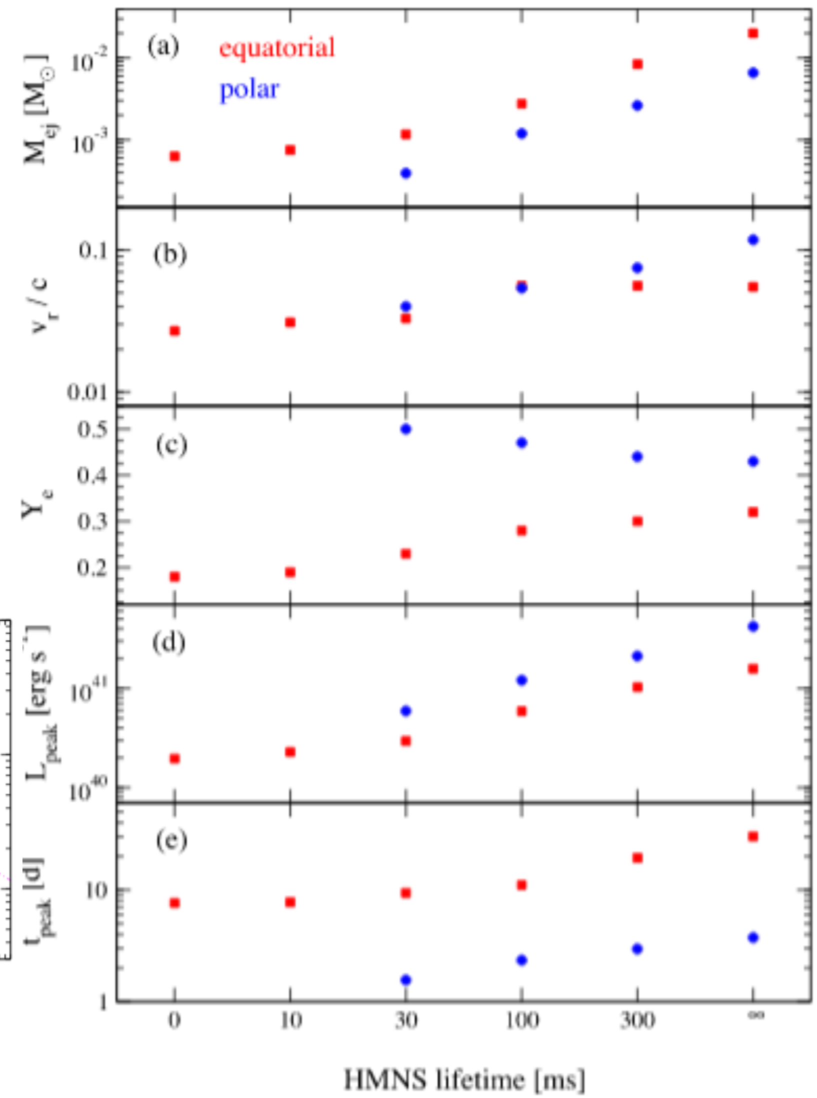
[SB et al. in preparation]

Kilonovae can probe nature of BNS remnants?

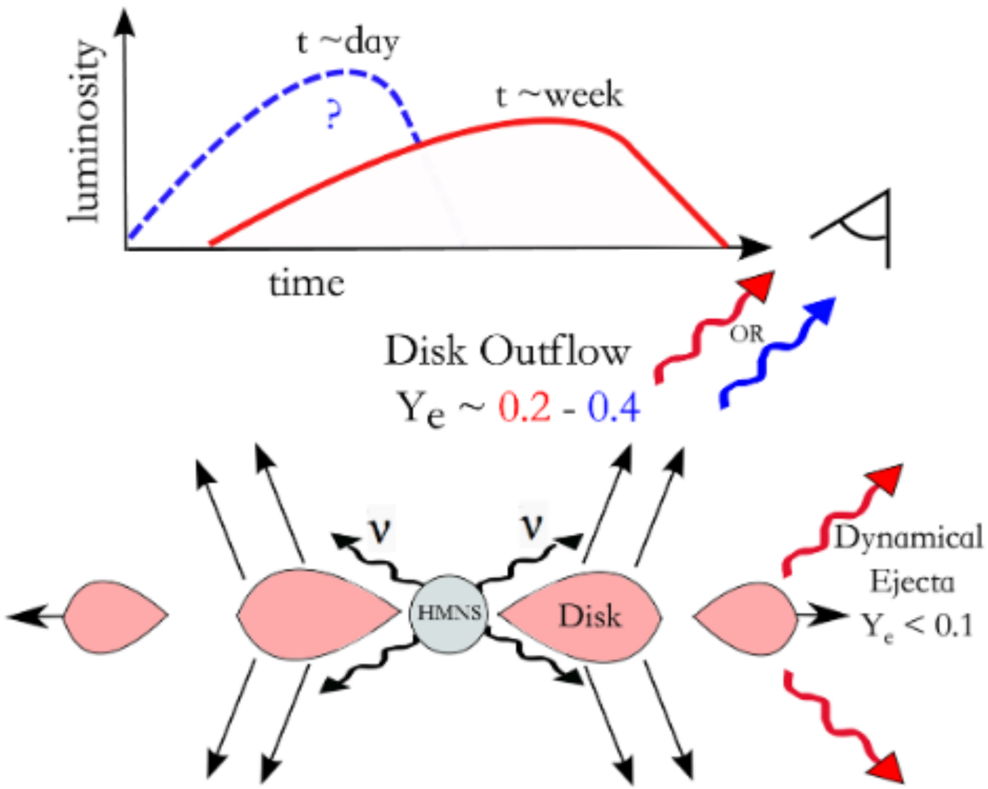
IUCAA







Kilonovae can probe nature of BNS remnants?



Metzger et al. MNRAS (2014)]

Post-merger signal and NS EoS

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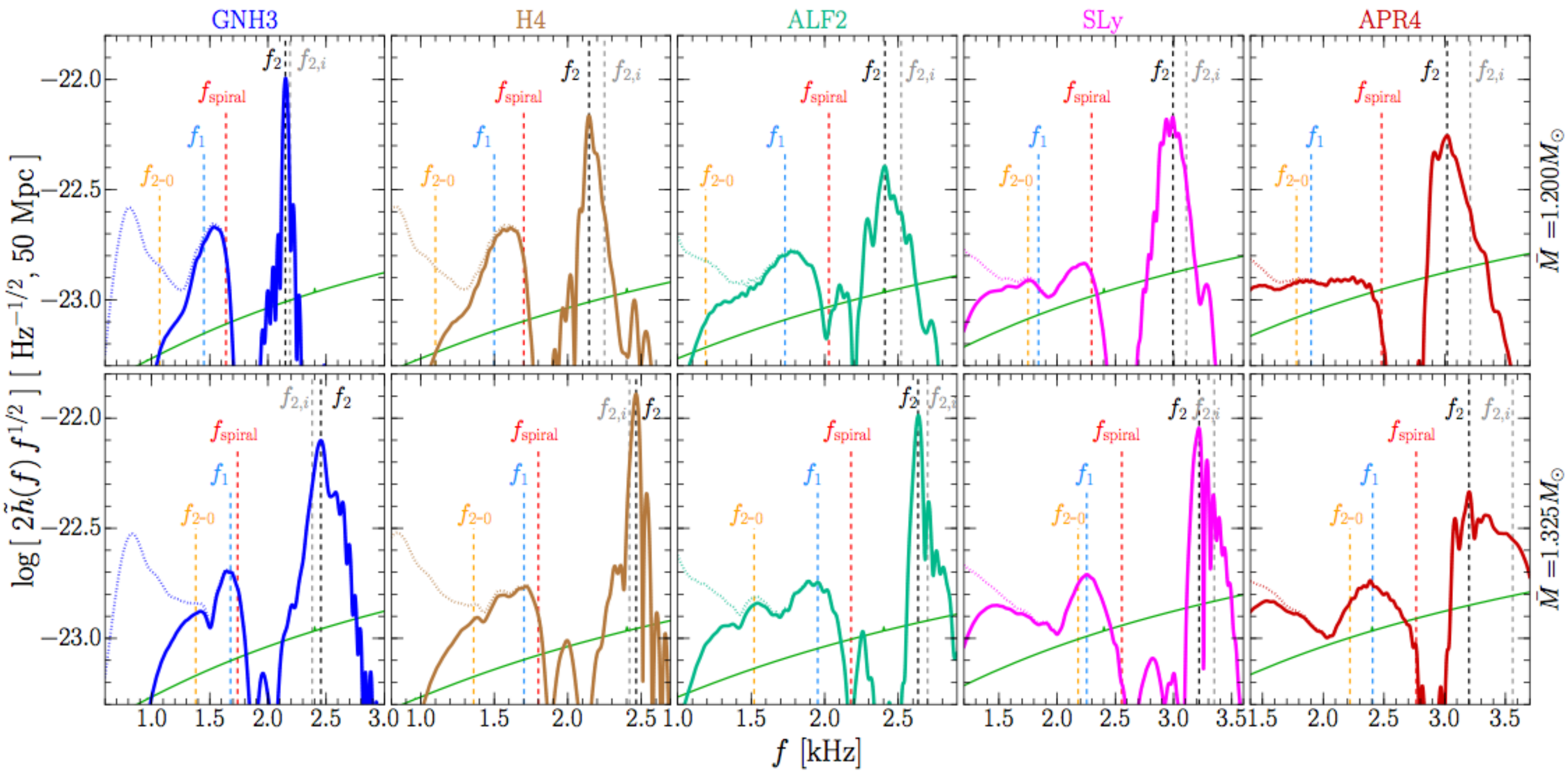

[Rezzolla & Takami, 2016]

Modeling the postmerger waveforms

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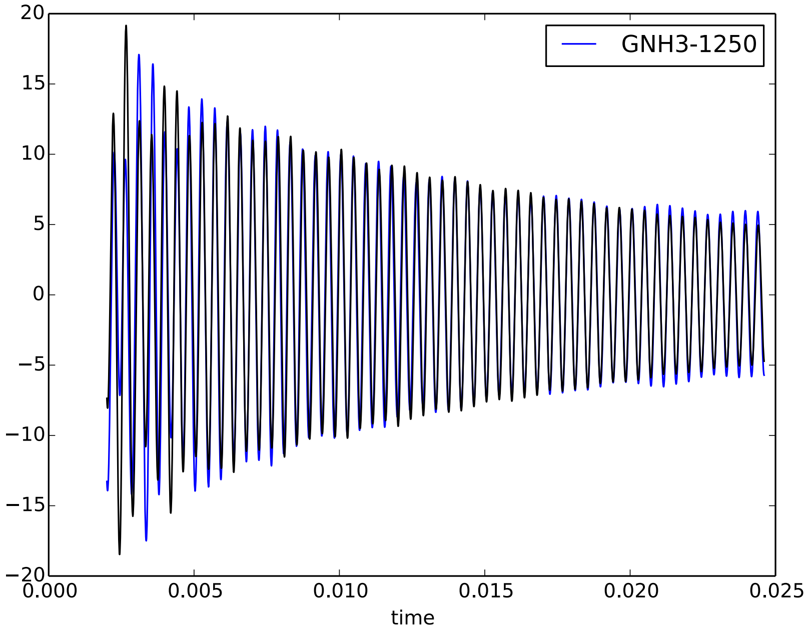

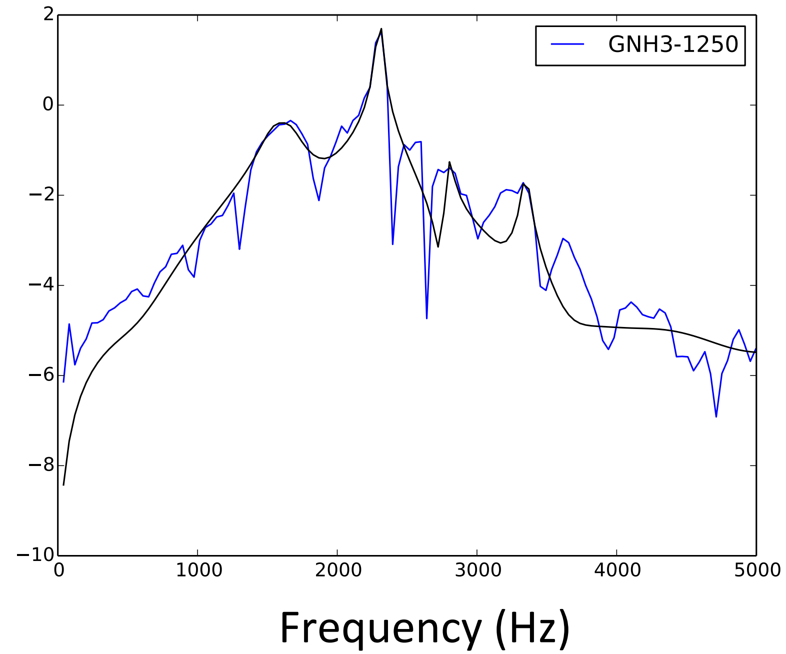

[K. Chakravarti, SB et al. (2016)]

Modeling the postmerger waveforms

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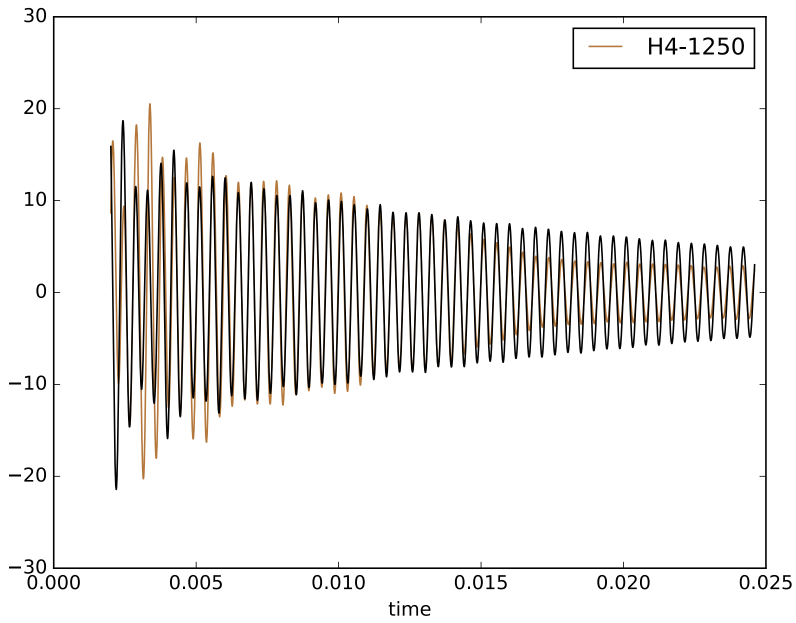

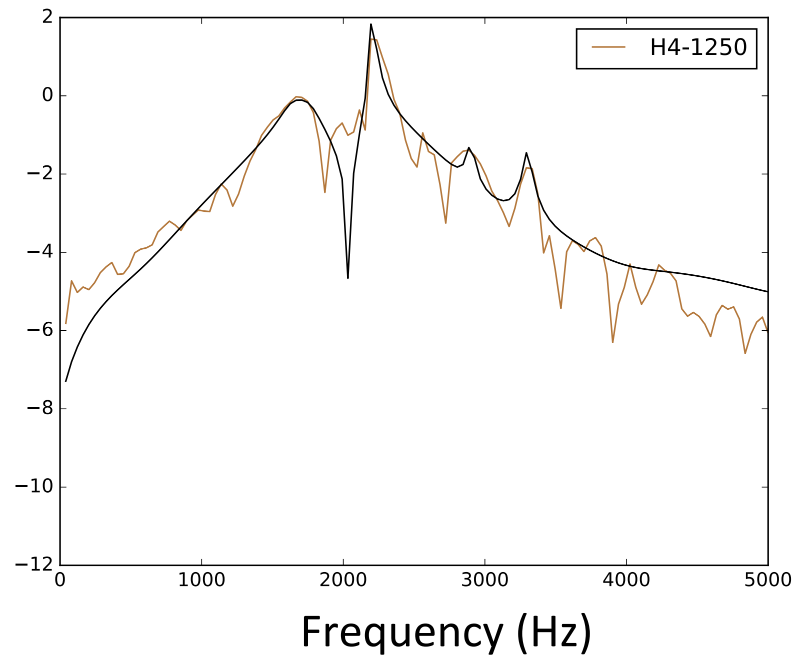

[K. Chakravarti, SB et al. (2016)]

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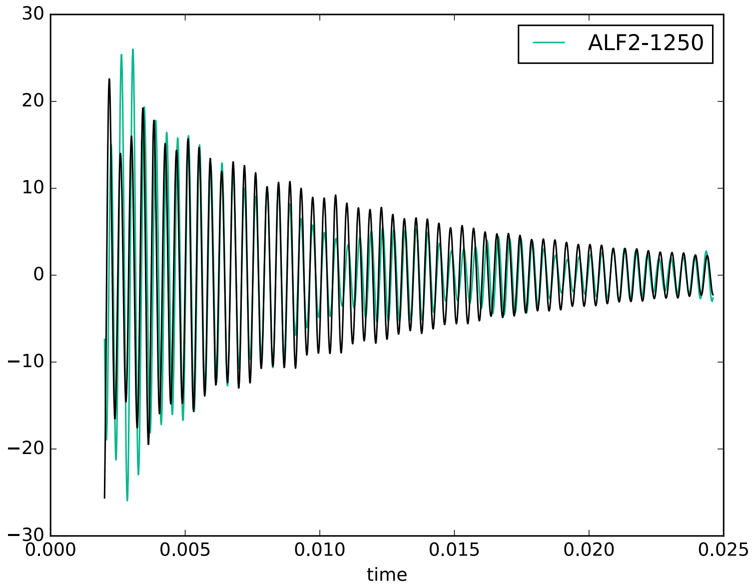
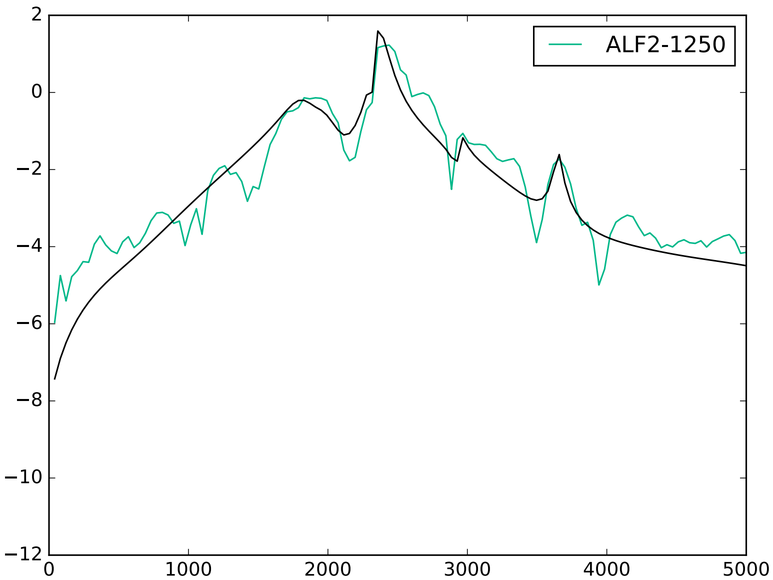
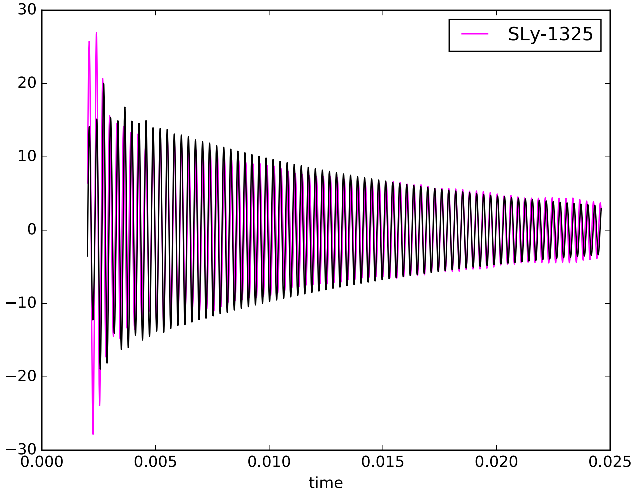
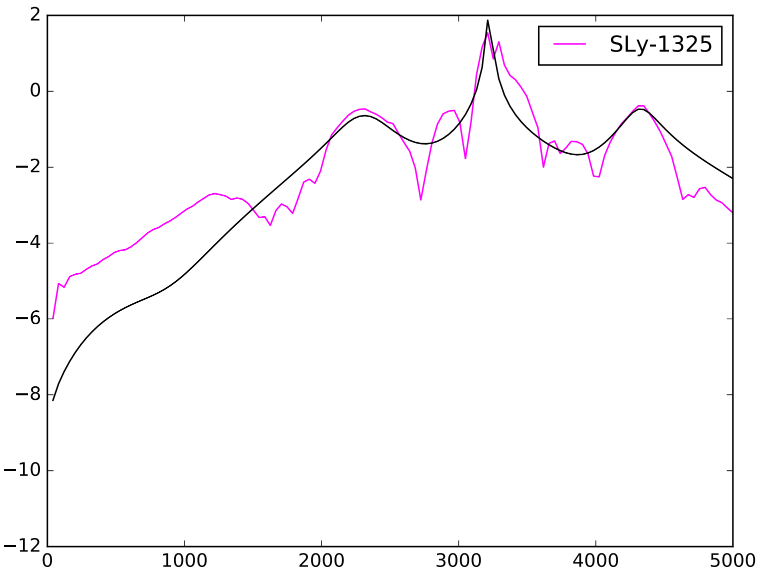





SLy-1325
time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EoS from postmerger signals

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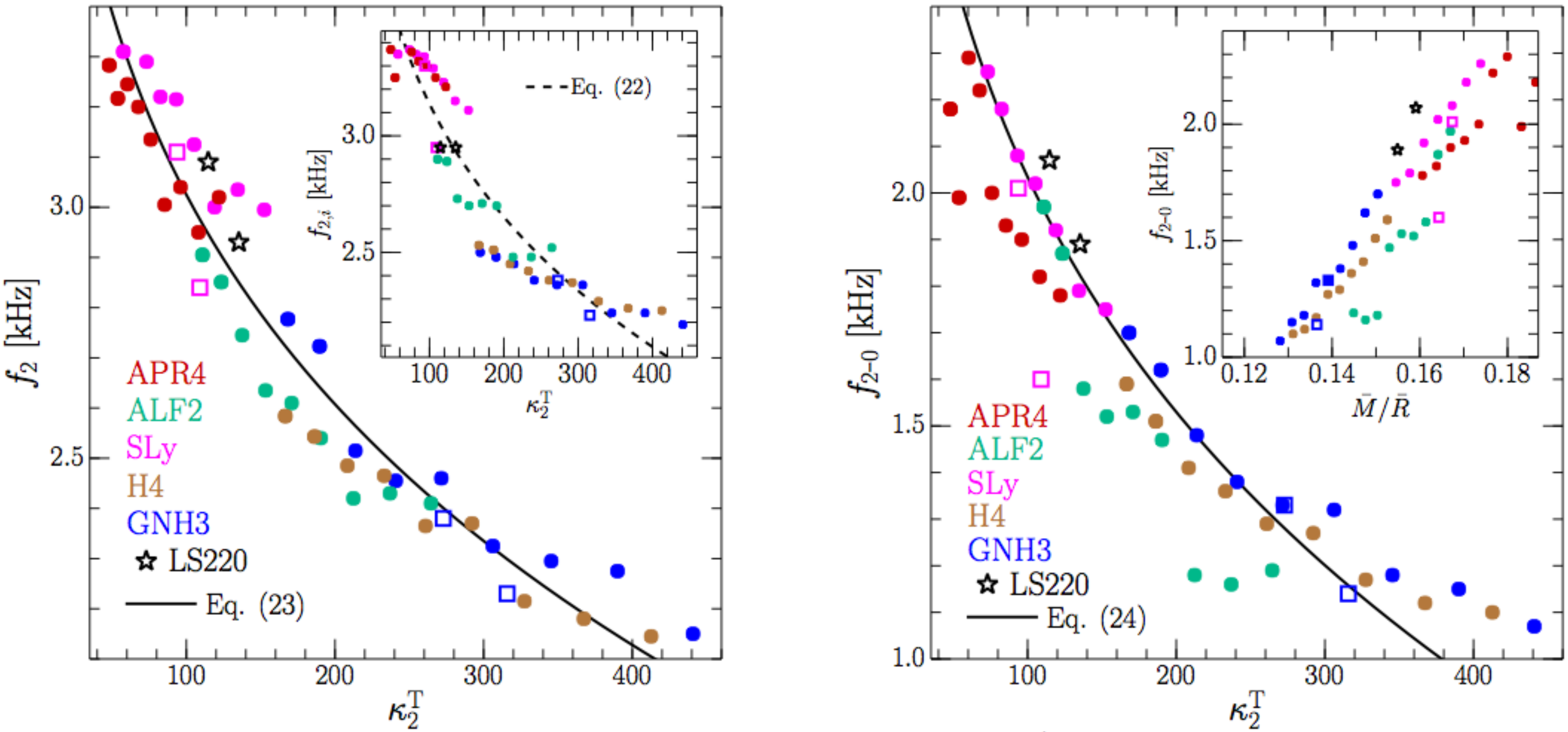

$$f_2 \approx 5.832 - 1.118 \left(\kappa_2^T\right)^{1/5} \quad \text{kHz}$$

Rezolla, Takami, arxiv (2016).

EoS from postmerger signals

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$$f_2 \approx 5.832 - 1.118 \left(\kappa_2^T\right)^{1/5} \quad \text{kHz}$$

$$\kappa_2^T \equiv 2\left[q\left(\frac{X_A}{C_A}\right)^5 k_2^A + \frac{1}{q}\left(\frac{X_B}{C_B}\right)^5 k_2^B\right], \qquad (11)$$

where A and B refer to the primary and secondary stars in the binary

$$q \equiv \frac{M_B}{M_A} \leq 1, \qquad X_{A,B} \equiv \frac{M_{A,B}}{M_A + M_B}, \qquad (12)$$

and C is the compaction.

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"New" physics & astronomy

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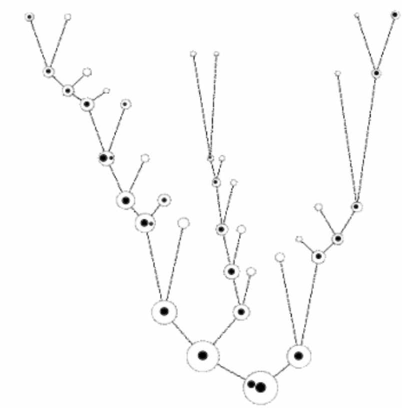
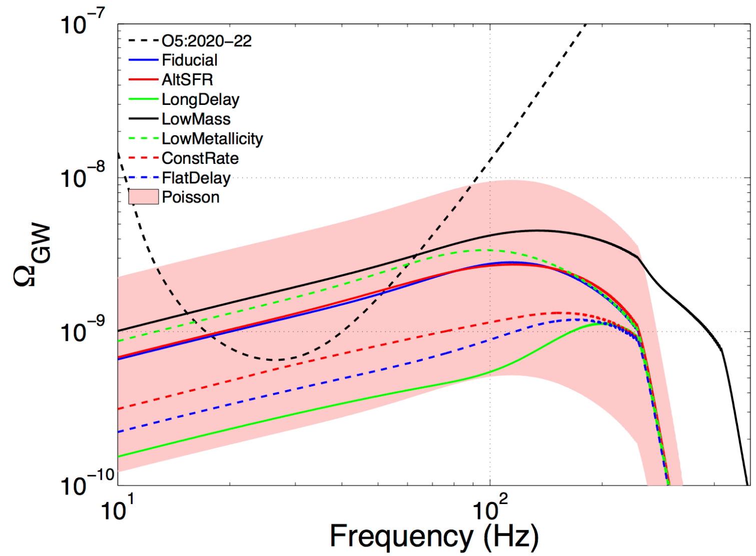
- Binary black holes:
  - How can massive stellar black holes form & then *tango*? Do super-massive black holes form hierarchically, seeded by less massive stellar / intermediate mass black holes?
  - Can their mergers produce electro-magnetic counterparts? See, e.g.,

    A. Loeb, "Electromagnetic Counterparts to Black Hole Mergers Detected by LIGO," ApJ Letters (2016).
  - Cosmography: BBH are *standard sirens* that can provide distance measurements; with host galaxy identifications, one can obtain redshifts. Key to alternative measurements of the Hubble parameter & the dark energy EOS.
  - Can BBH mergers be so many that they actually form a confusion noise / stochastic background in our data? .
  - Observe quasinormal modes and orbital precession.



Hierarchical SMBH formation. [Credits: M. Volonteri.]



Abbott et al. (LVC), Phys. Rev. Lett. 116, 131102 (2016)

"New" physics & astronomy

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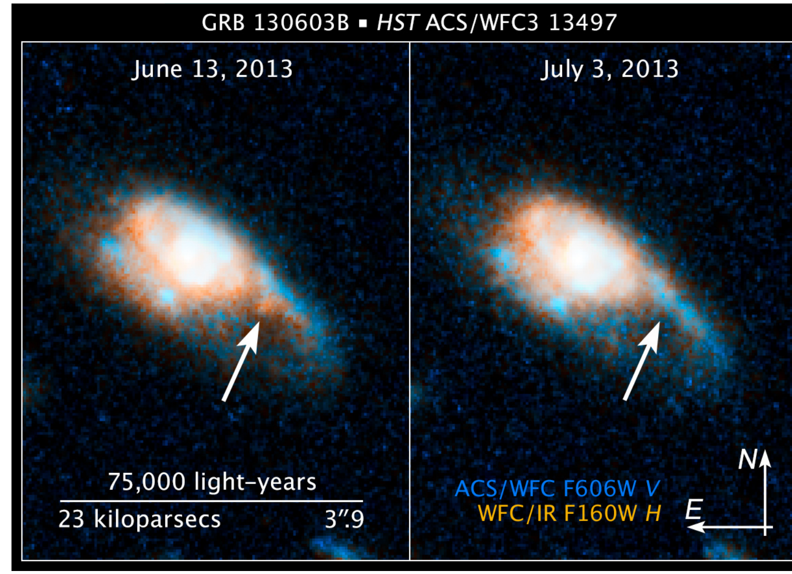
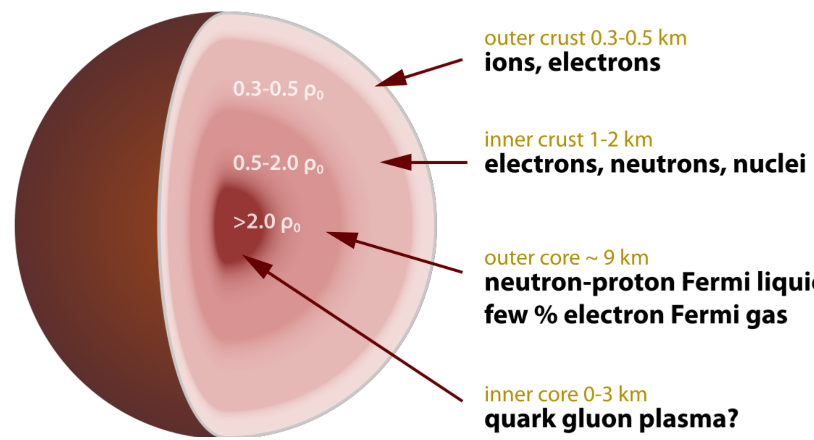
Tanvir et al., Nature (2013).



- Involve neutron stars:
  - Do kilonovae/macronovae and short GRBs share a common origin?
  - What is the equation of state of a neutron star?
  - What are the host galaxies of binary NS systems like?
  - Can GWs help track how BNS formation rate evolved with redshift?



[Credits: Wikipedia.]

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"New" physics & astronomy

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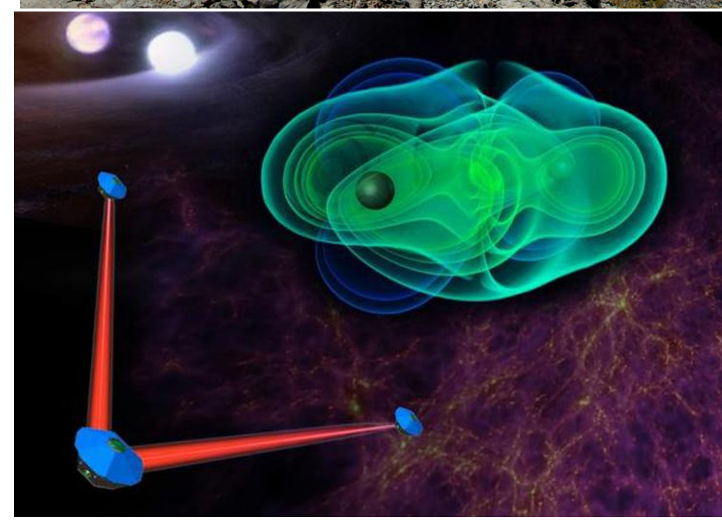
- Future missions / surveys / experiments:
  - LSST: Rate of EM-GW coincidences & the SGRB beaming angle
  - SKA: NS mergers with radio counterparts can inform us about the circum-merger environments.
  - GW missions (proposed): Combine information about the same system from multiple missions, e.g., Einstein's Telescope, eLISA, DECIGO, etc.
  - ASTROSAT: Is following up GW triggers (beginning with GW151226).
- More exotic inquiries:
  - Are (stellar or heavier) primordial black holes a significant component of dark matter?

LSST [Credits: LSST Project Office.]



eLISA [Credits: eLISA consortium.]

In conclusion

- Indian scientists collaborated with their LSC colleagues to detect and characterize GW150914 and GW151226.
- Searching for an EM counterpart was difficult owing to a large sky-error region. KAGRA, LIGO-India & Virgo can help by reducing that error region.
- The LIGO-India project is moving ahead.
- Coincident GW-EM observations hold out promise for interesting new discoveries, including those of unimagined phenomena

Non-linear coupling in neutron star mergers

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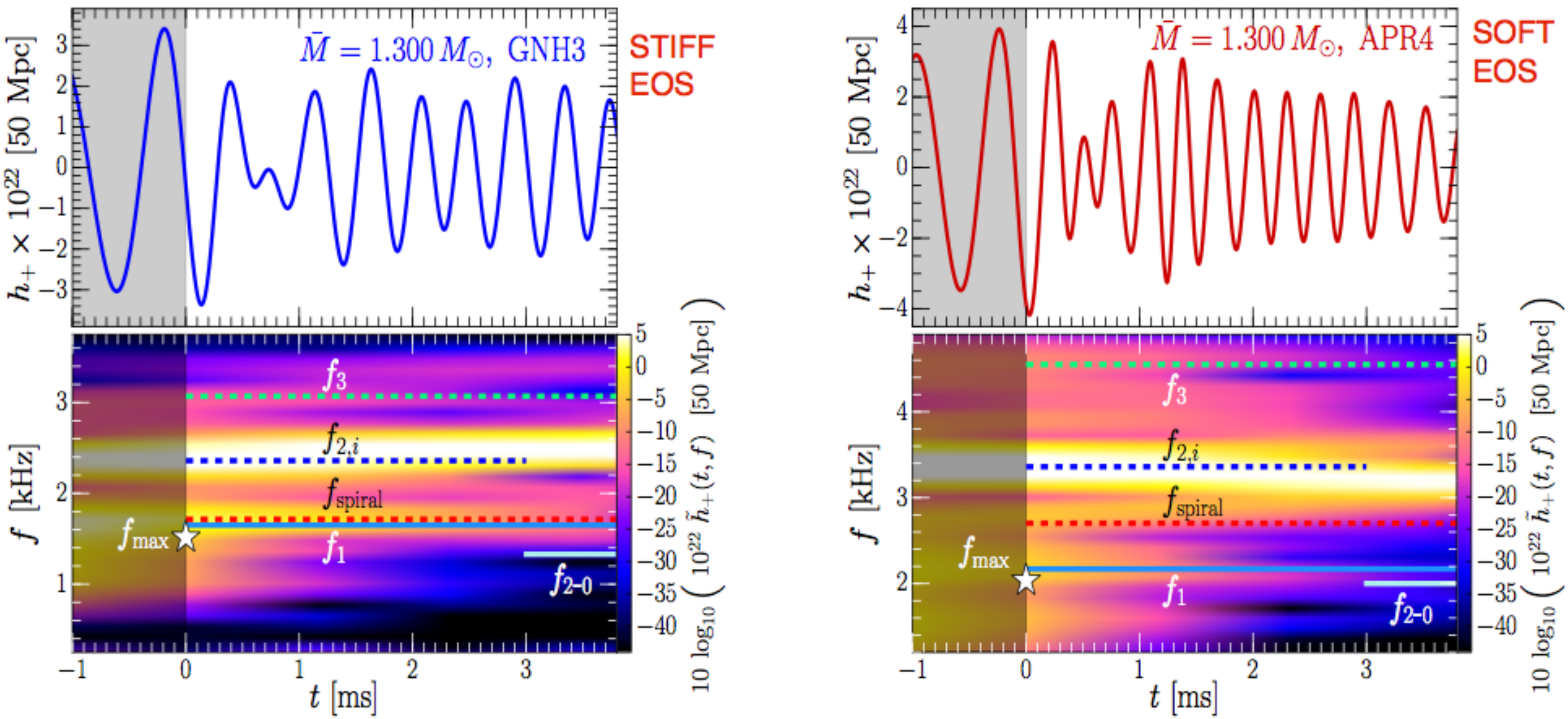

[Rezzolla & Takami, 2016]

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Find “cheaper” coordinates

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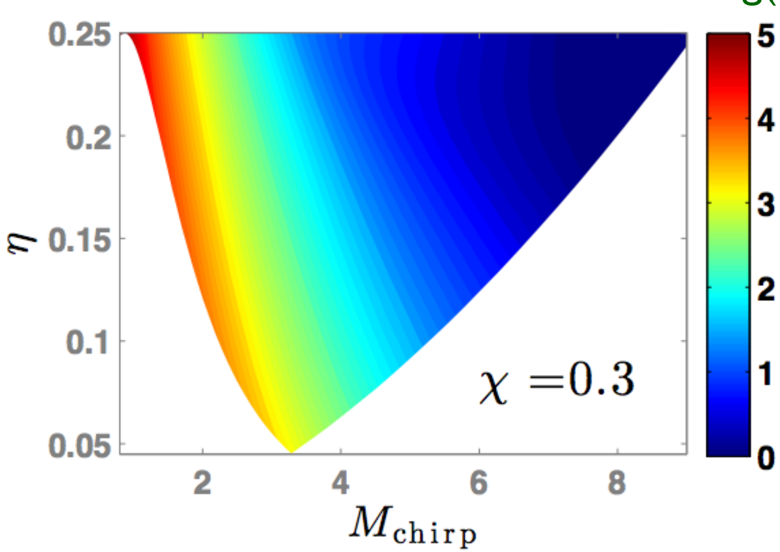
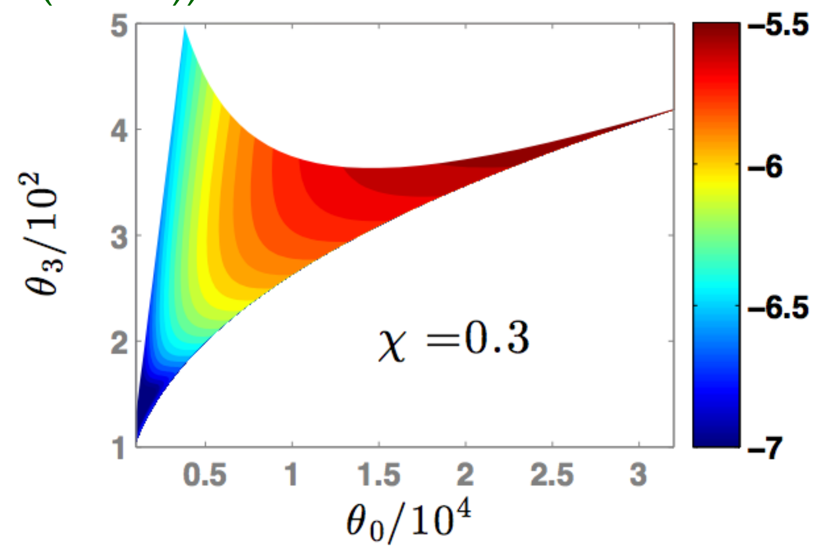
- Possible to find coordinates (chirp times) in which the metric is slowly varying (still, not constant).

Color shows log(det(metric))



“physical coordinates” (large variation in template density)



“chirp-time coordinates” (small variation in template density)

The spin challenge

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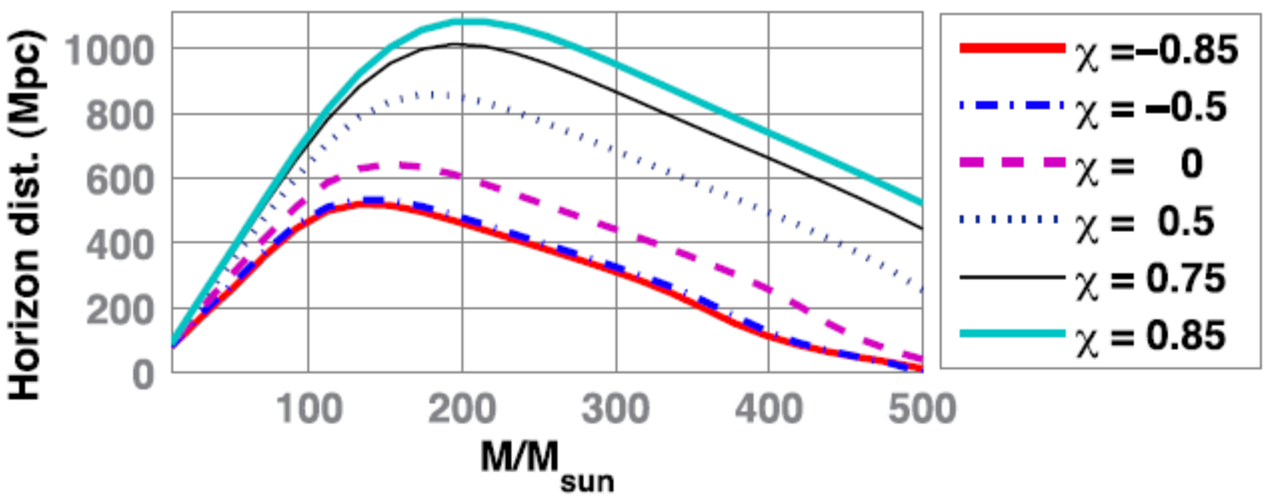

FIG. 3 (color online). Distance to optimally oriented equal-mass binaries with spin χ producing SNR 8 in Initial LIGO.

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The spin challenge: A proliferation of parameters

$$h(f) \equiv C\, f^{-7/6} \exp\{-\mathrm{i}\,[\Psi(f) - \pi/4]\}$$

The wave phase is determined by the orbital phase:

$$\begin{aligned}
\varphi(v) = \varphi_0 - \frac{1}{32 v^5 \eta}\Bigg\{ & 1 + v^2\left[\frac{55\eta}{12} + \frac{3715}{1008}\right] + v^3\left[\frac{565}{24}\left(\left(1 - \frac{76\eta}{113}\right)\boldsymbol{\chi}_s \cdot \hat{\mathbf{L}}_N + \delta \boldsymbol{\chi}_a \cdot \hat{\mathbf{L}}_N\right) - 10\pi\right] \\
& + v^4\left[(\boldsymbol{\chi}_a \cdot \hat{\mathbf{L}}_N)^2\left(150\eta - \frac{3595}{96}\right) - \frac{3595 \boldsymbol{\chi}_a \cdot \hat{\mathbf{L}}_N \boldsymbol{\chi}_s \cdot \hat{\mathbf{L}}_N \delta}{48} + \boldsymbol{\chi}_a^2\left(\frac{1165}{96} - 50\eta\right) + \frac{1165 \boldsymbol{\chi}_s \cdot \boldsymbol{\chi}_a \delta}{48}\right. \\
& \left. + (\boldsymbol{\chi}_s \cdot \hat{\mathbf{L}}_N)^2\left(-\frac{5\eta}{24} - \frac{3595}{96}\right) + \boldsymbol{\chi}_s^2\left(\frac{35\eta}{24} + \frac{1165}{96}\right) + \frac{3085\eta^2}{144} + \frac{27145\eta}{1008} + \frac{15293365}{1016064}\right] \\
& + v^5\left[\left(\boldsymbol{\chi}_a \cdot \hat{\mathbf{L}}_N\left(-\frac{35\delta\eta}{2} - \frac{732985\delta}{2016}\right) + \boldsymbol{\chi}_s \cdot \hat{\mathbf{L}}_N\left(\frac{85\eta^2}{2} + \frac{6065\eta}{18} - \frac{732985}{2016}\right) - \frac{65\pi\eta}{8} + \frac{38645\pi}{672}\right)\ln(v)\right] \\
& + v^6\left[-\frac{127825\eta^3}{5184} + \frac{76055\eta^2}{6912} + \frac{2255\pi^2\eta}{48} - \frac{15737765635\eta}{12192768} - \frac{1712\gamma_E}{21} - \frac{160\pi^2}{3} + \frac{12348611926451}{18776862720}\right. \\
& \left. - \frac{1712\ln(4v)}{21}\right] + v^7\left[-\frac{74045\pi\eta^2}{6048} + \frac{378515\pi\eta}{12096} + \frac{77096675\pi}{2032128}\right]\Bigg\},
\end{aligned}$$

$$v \equiv (m\omega)^{1/3}$$

The spin challenge: Better coordinates

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$$h(f) \equiv C\, f^{-7/6} \exp\{-i\,[\Psi(f) - \pi/4]\}$$

$$\chi \equiv (1+\delta)\chi_1/2 + (1-\delta)\chi_2/2,$$

$$\beta = 113\chi/12,$$

$$\sigma_0 = \left(-\frac{12769\,(4\eta - 81)}{16\,(76\eta - 113)^2}\right)\chi^2,$$

$$\gamma_0 = \left(\frac{565\left(17136\eta^2 + 135856\eta - 146597\right)}{2268\,(76\eta - 113)}\right)\chi,$$

$$\Psi(f) = \sum_{k=0}^{k=8}\left[\psi_k + \psi_k^{\mathrm{L}} \ln\left(\frac{f}{f_0}\right)\right]\left(\frac{f}{f_0}\right)^{\frac{k-5}{3}}, \qquad (3.12)$$

where

$$\psi_0 = \frac{3\theta_0}{5},$$

$$\psi_1 = 0,$$

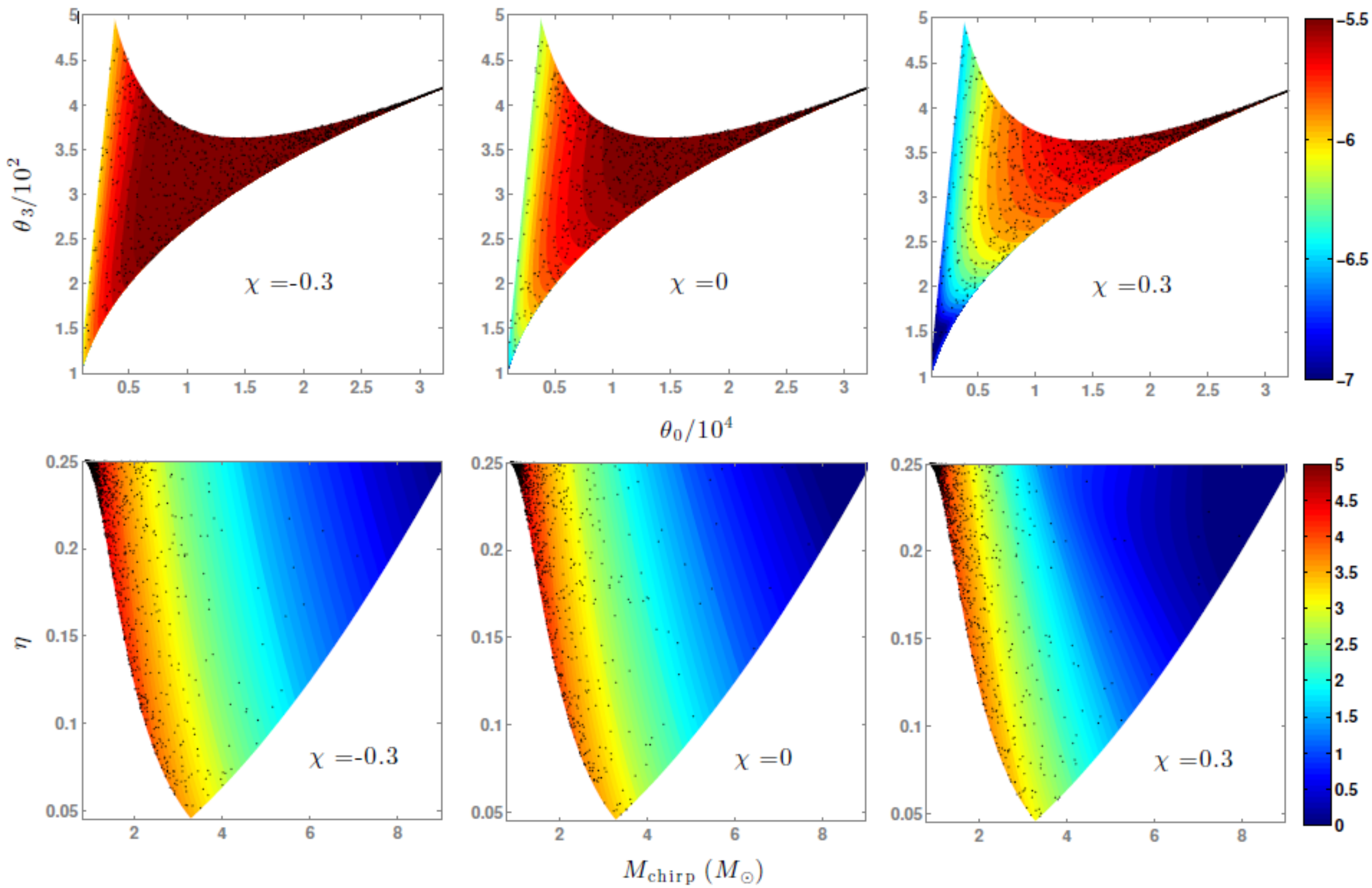
$$\psi_2 = \frac{743}{2016}\left(\frac{25}{2\pi^2}\right)^{1/3}\theta_0^{1/3}\theta_3^{2/3} + \frac{11\pi\,\theta_0}{12\,\theta_3},$$

$$\psi_3 = -\frac{3}{2}(\theta_3 - \theta_{3S}),$$

$$\psi_4 = \frac{675\,\theta_3\,\theta_{3S}^2\left(8\times 10^{2/3}\pi^{7/3}\theta_0^{2/3} - 405\sqrt[3]{10}\,\pi^{2/3}\,\theta_3^{5/3}\right)}{4\sqrt[3]{\theta_0}\left(152\sqrt[3]{10}\,\pi^{5/3}\theta_0^{2/3} - 565\,\theta_3^{5/3}\right)^2} + \frac{15293365\sqrt[3]{5}\,\theta_3^{4/3}}{10838016\times 2^{2/3}\pi^{4/3}\sqrt[3]{\theta_0}} + \frac{617\,\pi^2\,\theta_0}{384\,\theta_3^2} + \frac{5429}{5376}\left(\frac{25\pi\,\theta_0}{2\,\theta_3}\right)^{1/3},$$

$$\psi_5 = \frac{140311625\pi\theta_3^{2/3}\theta_{3S}}{180348\left(565\theta_3^{5/3} - 152\sqrt[3]{10}\pi^{5/3}\theta_0^{2/3}\right)} + \ldots$$

The spin challenge: Find better coordinates



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The source orientation & location…

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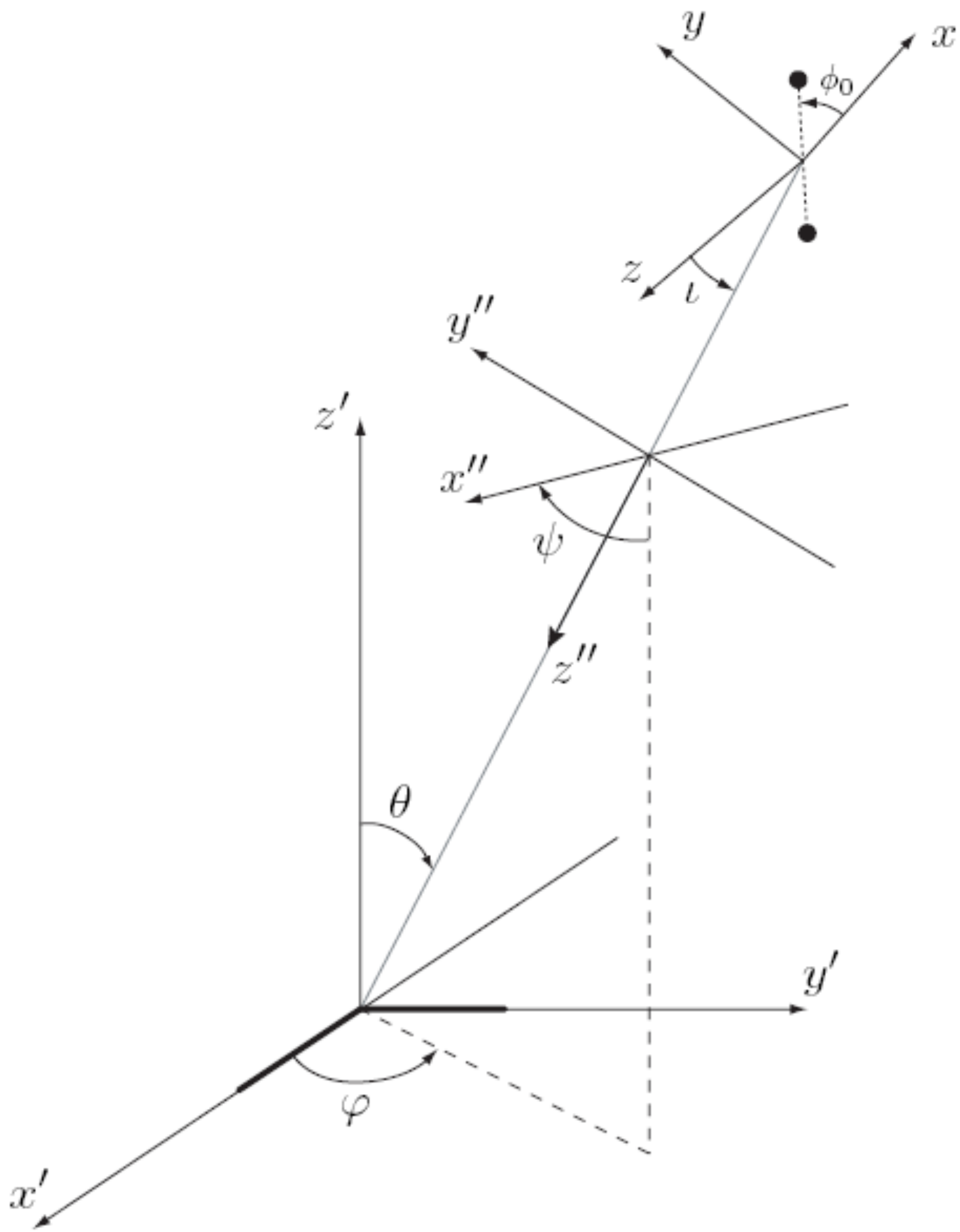

…in the detector frame.

[D. Brown, PhD Thesis '04]

The GW strain in a detector

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In the transverse, trace-free (TT) gauge, the space-space components of the wave tensor, **h**, are:

$$h_{ij}(t) = h_+(t)\ e_{ij}^+ \quad + \quad h_\times(t)\ e_{ij}^\times \ .$$

The GW strain in a detector is:

$$h = \frac{\delta L}{L} = \frac{1}{2}\mathbf{n_x}\cdot\mathbf{h}\cdot\mathbf{n_x} - \frac{1}{2}\mathbf{n_y}\cdot\mathbf{h}\cdot\mathbf{n_y}$$

[See, e.g., B. Schutz, GR, '85]

and can be re-expressed as,

$$h(t) = \mathbf{h}\cdot\mathbf{d} = h_+(t)\left[e_{ij}^+ d^{ij}(t)\right] + h_\times(t)\left[e_{ij}^\times(t) d^{ij}(t)\right]$$

$$= h_+(t)\ F^+(t;\theta,\varphi,\psi) + h_\times(t)\ F^\times(t;\theta,\varphi,\psi)\ .$$

Inspiral searches: The waveforms

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The two polarization components of a (restricted) waveform are:

$$h_+(t) = \frac{G\mu}{2c^2 r}\left(\frac{2\pi GMf}{c^3}\right)^{2/3}\left(1+\cos^2\iota\right)\cos\left(2\phi(t)-2\phi_0\right),$$

$$h_\times(t) = \frac{G\mu}{2c^2 r}\left(\frac{2\pi GMf}{c^3}\right)^{2/3}\left(2\cos\iota\right)\sin\left(2\phi(t)-2\phi_0\right),$$

Here, M = total mass,

μ = reduced mass,

f = wave-frequency = twice the orbital frequency,

ι = orbital inclination angle,

$\phi(t)$ = PN orbital phase $= \pi\int f(t)\,dt$, $2\phi_0$ = initial phase.

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Chirp mass

Chirp mass:

$$M_{\text{chirp}} = \eta^{3/5} M$$

The leading order phase:

$$\psi(f) = 2\pi f t_{\text{ISCO}} + \frac{3}{128}\left(\frac{\pi G M_{\text{chirp}} f}{c^3}\right)^{-5/3} - \phi_{\text{ISCO}} - \frac{\pi}{4},$$

Duration of signal in detector band:

$$\Delta t_{\text{chirp}} h_{\times}(t) = \frac{G\mu}{2c^2 r}\left(\frac{2\pi G M f}{c^3}\right)^{2/3} (2\cos\iota)\sin\left(2\varphi(t)\ \text{-}\ 2\varphi_0\right),$$

Here, $M =$ total mass,

$\mu =$ reduced mass,

$f =$ wave-frequency $=$ twice the orbital frequency,

$\iota =$ orbital inclination angle,

$\varphi(t) =$ PN orbital phase $= \pi \int f(t)\, dt$, $2\varphi_0 =$ initial phase,

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Signal parameters

The GW strain (2PN) in a detector depends at least on 9 parameters:

$$h(t) = h_+(t)F^+ + h_\times(t)F^\times = \frac{A(t)}{D}\cos(2\phi(t) - \vartheta_0) = \frac{A(t)}{D}\mathrm{Re}\left[e^{i(2\phi(t)-\vartheta_0)}\right],$$

where

$$A(t) = \frac{2G\mu}{c^2}(2\pi GMf(t))^{2/3},$$

the *effective* **distance is,** $D \equiv \frac{r}{\left|F^+(1+\cos^2\iota) + i2F^\times\cos\iota\right|} \propto \frac{r}{|E|},$

and the *effective* **initial phase is,** $\vartheta_0 \equiv \arctan\left[\frac{2F^\times\cos\iota}{F^+(1+\cos^2\iota)}\right] + 2\phi_0.$

Grouping the time-dependent terms together gives:

$$h(t) = a\,\mathrm{Re}\left[E(\theta,\varphi,\psi,\iota)\ H(t;m_1,m_2,t_0)\ e^{-i\vartheta_0}\right],$$ **where** $H(t) \propto f^{2/3}(t)$ **and** $a \propto \frac{1}{r}.$

Signal parameters

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The GW strain in a detector depends on 9 parameters:

$$h(t) = h_+(t)F^+ + h_\times(t)F^\times = A\,\mathrm{Re}\left[E(\theta,\varphi,\psi,\iota)\; S(t;m_1,m_2,t_0)\; e^{-i\vartheta_0}\right],$$

where $M = m_1 + m_2$, $\eta = \frac{m_1 m_2}{M^2}$, $M_c = \eta^{3/5} M$, and $A \propto \frac{1}{d_{\mathrm{eff}}(d_L)}$.

The *effective* distance is, $d_{\mathrm{eff}} \equiv \frac{d_L}{\left|F^+(1+\cos^2\iota) + i2F^\times \cos\iota\right|} = \frac{d_L}{|E|}$.

The *red* factors are amplitude-like parameters whose accuracies improve as *1/SNR*.

The *green* parameters are frequency-like factors whose accuracies improve as *1 / [SNR * (Number of wave cycles)^k]*.

$$\left(\frac{\Delta d_{\mathrm{eff}}}{d_{\mathrm{eff}}}\right)^2 = \frac{25}{36}\left(\frac{\Delta M}{M}\right)^2 + \frac{1}{4}\left(\frac{\Delta\eta}{\eta}\right)^2 + \frac{5}{6}C_{M\eta}\frac{\Delta M}{M}\frac{\Delta\eta}{\eta} - \frac{5}{3}C_{MA}\frac{\Delta M}{M}\frac{\Delta A}{A} - C_{\eta A}\frac{\Delta\eta}{\eta}\frac{\Delta A}{A} + \left(\frac{\Delta A}{A}\right)^2.$$

[P. Ajith, SB, PRD79, 084032 (2009)]

Consistency of the signal in LIGO-Hanford & LIGO-Livingston

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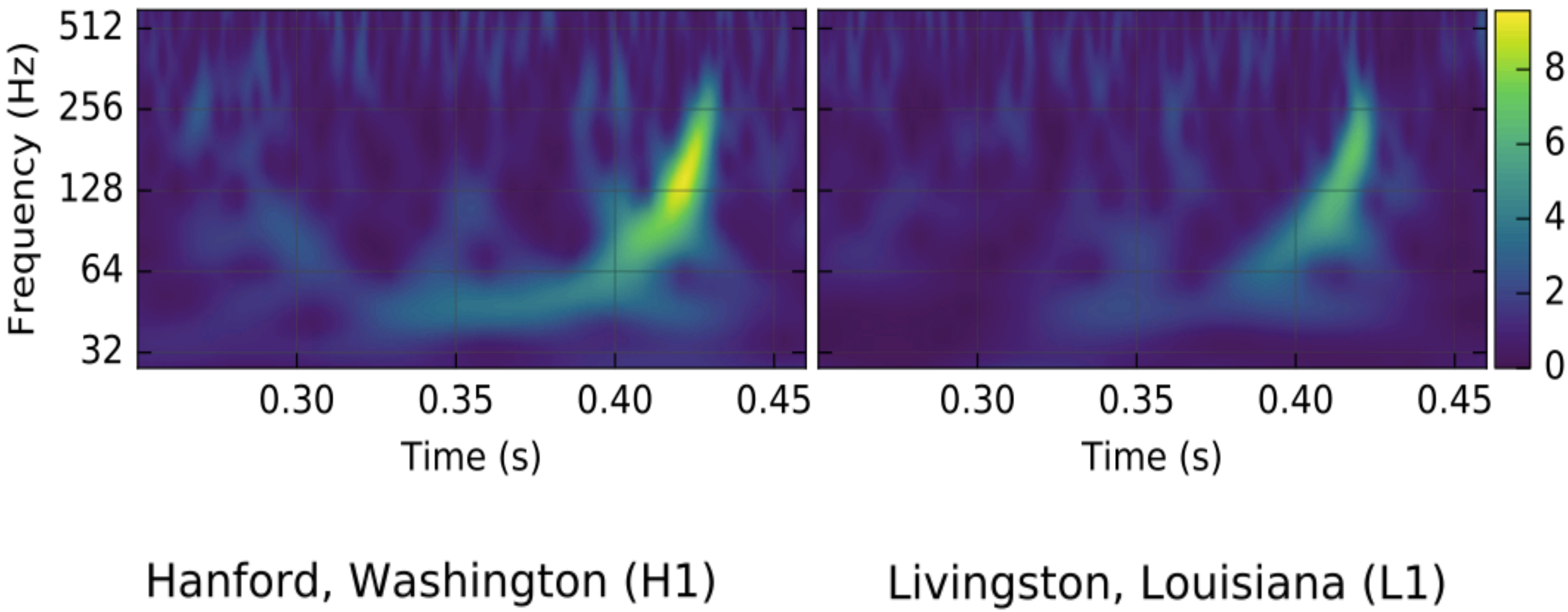
See Haris & Sumit's posters.



Interferometry with interferometers:
A. Pai, S. Dhurandhar, SB, IJMPD (2000); PRD (2001).

Abbott et al. (LVC), Observation of Gravitational Waves from a Binary Black Hole Merger," Phys. Rev. Lett. 116, 061102 (2016).

NS tide in waveforms

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Total phase = Point-particle phase + Tidal phase-correction.

Point-particle phase has non-spinning and spinning (aligned or anti-aligned) terms up to 3.5pN. We add test-particle non-spinning corrections from 4pN to 7.5pN, in part, to bridge the gap that otherwise exists between 3.5pN terms and the terms where tidal corrections come in (5pN…).

Tidal phase-correction is:

$$\psi_{\rm tidal} = \sum_{i=1}^{2} \frac{3\lambda_i}{128\eta M^5}\left[-\frac{24}{\chi_i}\left(1+\frac{11\eta}{\chi_i}\right)\left(\frac{v}{c}\right)^5 - \frac{5}{28\chi_i}\left(3179 - 919\chi_i - 2286\chi_i^2 + 260\chi_i^3\right)\left(\frac{v}{c}\right)^7 \right],$$

[Vines, Flanagan, Hinderer, arXiv:1101.1673v1.; Damour, Nagar, Villain, PRD85, 123007 (2012).

where $v = (M\omega)^{1/3}$, $\chi_i = m_i / M$ and "i" is binary component index.

$M = m_1 + m_2$ and $\eta = m_1 m_2 / M^2$.

New: We add the aforementioned point particle terms to improve upon waveforms used in the past

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Details of the piecewise polytropic NS EOS

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$$\varepsilon = \rho\left[a\left(\frac{\rho}{\rho_0}\right)^{\alpha} + b\left(\frac{\rho}{\rho_0}\right)^{\beta} + m_N c^2\right], \qquad P = \rho\left[a\alpha\left(\frac{\rho}{\rho_0}\right)^{\alpha} + b\beta\left(\frac{\rho}{\rho_0}\right)^{\beta}\right],$$

m_N = nucleon mass, $\rho_0 = 0.16\ \mathrm{fm}^{-3}$ is the saturation density.

Also, $a \sim 12.71$ - 13.3 MeV, $\alpha \sim 0.48$ - 0.52,

and $b = 2 - 5$ MeV, $\beta \sim 2.1$ - 2.5.

This form was found to accurately fit QMC calculations of the EOS using nuclear Hamiltonians with realistic two- and three-body forces. [Gandolfi:2009, Gandolfi:2010b, Gandolfi:2012]

It is also consistent with recent QMC results based on chiral EFT interactions. [Wlazlowski:2014a, Gandolfi:2014, Gandolfi:2014a, Gandolfi:2015, Lynn:2015]

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Results

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$$\varepsilon = \rho\left[a\left(\frac{\rho}{\rho_0}\right)^{\alpha} + b\left(\frac{\rho}{\rho_0}\right)^{\beta} + m_N c^2\right], \qquad P = \rho\left[a\alpha\left(\frac{\rho}{\rho_0}\right)^{\alpha} + b\beta\left(\frac{\rho}{\rho_0}\right)^{\beta}\right],$$

$m_N =$ nucleon mass, $\rho_0 = 0.16\ \mathrm{fm}^{-3}$ is the saturation density.

Also, $a \sim 12.71 - 13.3$ MeV, $\alpha \sim 0.48 - 0.52$,

and $b = 2 - 5$ MeV, $\beta \sim 2.1 - 2.5$.

- If none of the four EOS parameters (a, α, b, β) are known from theory, then the observational errors (1σ) in them are larger than 100%.
- If (a, α) are known from theory, then the observational errors (1σ) in (b, β) reduce to several percent in a simulation where one assumes observations of 121 binary NS systems, all (unrealistically) at 100Mpc.

[SB et al. in preparation]

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Results

$$\varepsilon = \rho\left[a\left(\frac{\rho}{\rho_0}\right)^{\alpha} + b\left(\frac{\rho}{\rho_0}\right)^{\beta} + m_N c^2\right], \qquad P = \rho\left[a\alpha\left(\frac{\rho}{\rho_0}\right)^{\alpha} + b\beta\left(\frac{\rho}{\rho_0}\right)^{\beta}\right],$$

m_N = nucleon mass, $\rho_0 = 0.16\ \mathrm{fm}^{-3}$ is the saturation density.

Also, $a \sim 12.71$ - 13.3 MeV, $\alpha \sim 0.48$ - 0.52,

and $b = 2 - 5$ MeV, $\beta \sim 2.1$ - 2.5.

- Upcoming neutron skin experiments expect to constrain the slope of the density dependence of the symmetry energy to within an error of ΔL=41 MeV, with possible reduction to 15 MeV.

- For our EOS, L =3($a\alpha$ + $b\beta$); hence these constraints from gravitational wave observations could have an impact at the level of 5 MeV.

[SB et al. in preparation]

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Fisher information

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$s(t;\vartheta^k) = h(t;\vartheta^k) + n(t)$, where ϑ^k are signal parameters.

Or $\tilde{s}\left(f;\vartheta^k\right) = \tilde{h}\left(f;\vartheta^k\right) + \tilde{n}\left(f\right)$ in the Fourier domain.

The cross-correlation of the data with a template is:

$$C\left(\Delta\vartheta^k\right) = \left\langle s\left(\vartheta^k\right) \middle| h\left(\vartheta'^k\right)\right\rangle \propto \mathrm{Re}\left[\int_{f_1}^{f_2} \frac{\tilde{s}^*\left(f;\vartheta^k\right)\tilde{h}\left(f;\vartheta'^k\right)}{P\left(f\right)}\, df\right],$$

where $P\left(f\right)$ is the noise power-spectral density. The probability distribution of the parameter error(s) is related to that of noise, which is taken to be Gaussian:

$$p\left(n = n_0\right) \propto \exp\left[-\frac{\left\langle n_0 \middle| n_0\right\rangle}{2}\right] \propto \exp\left[-\frac{\left\langle s-h' \middle| s-h'\right\rangle}{2}\right] \propto \exp\left[-\frac{\left\langle s \middle| h'\right\rangle}{2}\right]$$

$$p\left(n = n_0\right) \propto \exp\left[-\frac{1}{2}\Gamma_{kl}\Delta\vartheta^k\Delta\vartheta^l\right], \text{ where } \Gamma_{kl} = \left\langle \frac{\partial h}{\partial\vartheta^k} \middle| \frac{\partial h}{\partial\vartheta^l}\right\rangle.$$

Fisher information matrix

For large signal-to-noise ratio, parameter error variance-covariance is:

$$\{\Delta\vartheta^k\Delta\vartheta^l\} = \left(\Gamma^{-1}\right)^{kl}.$$

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Details of the parameter estimation method

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- Let the signal parameters, θ^k, be the NS EOS parameters (a, α, b, β), with k=1-4, respectively.

- The goal is to find how much any variation in the EOS parameters changes the waveform. That information is obtained from the Fisher matrix.

$$\Gamma_{kl} = \left\langle \frac{\partial h}{\partial \vartheta^k} \middle| \frac{\partial h}{\partial \vartheta^l} \right\rangle .$$

$$\frac{\partial h}{\partial \vartheta^i} = \frac{\partial h}{\partial k_2}\frac{\partial k_2}{\partial \vartheta^i} + \sum_{p=1,2}\left[\frac{\partial h}{\partial m_p}\frac{\partial m_p}{\partial \vartheta^i} + \frac{\partial h}{\partial R_p}\frac{\partial R_p}{\partial \vartheta^i}\right],$$

where $\frac{\partial h}{\partial k_2}$ is obtained by keeping m_p and R_p fixed.

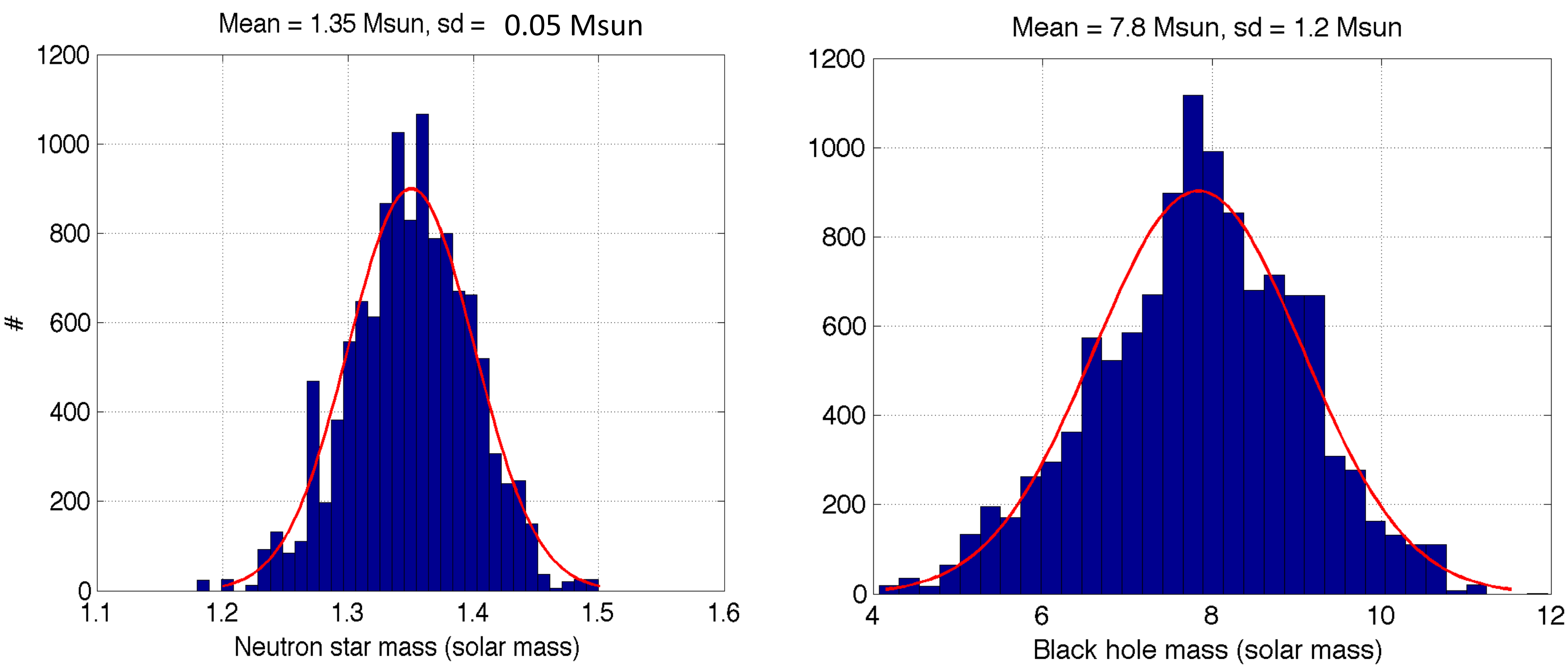
Similary for the m_p and R_p derivatives of h.

Next find parameter error variance-covariance (for large SNR):

$$\{\Delta\vartheta^k \Delta\vartheta^l\} = \left(\Gamma^{-1}\right)^{kl} .$$

NS-BH systems

- Gaussian distribution of NS and BH masses:



Where is more than half of the heavy elements produced?

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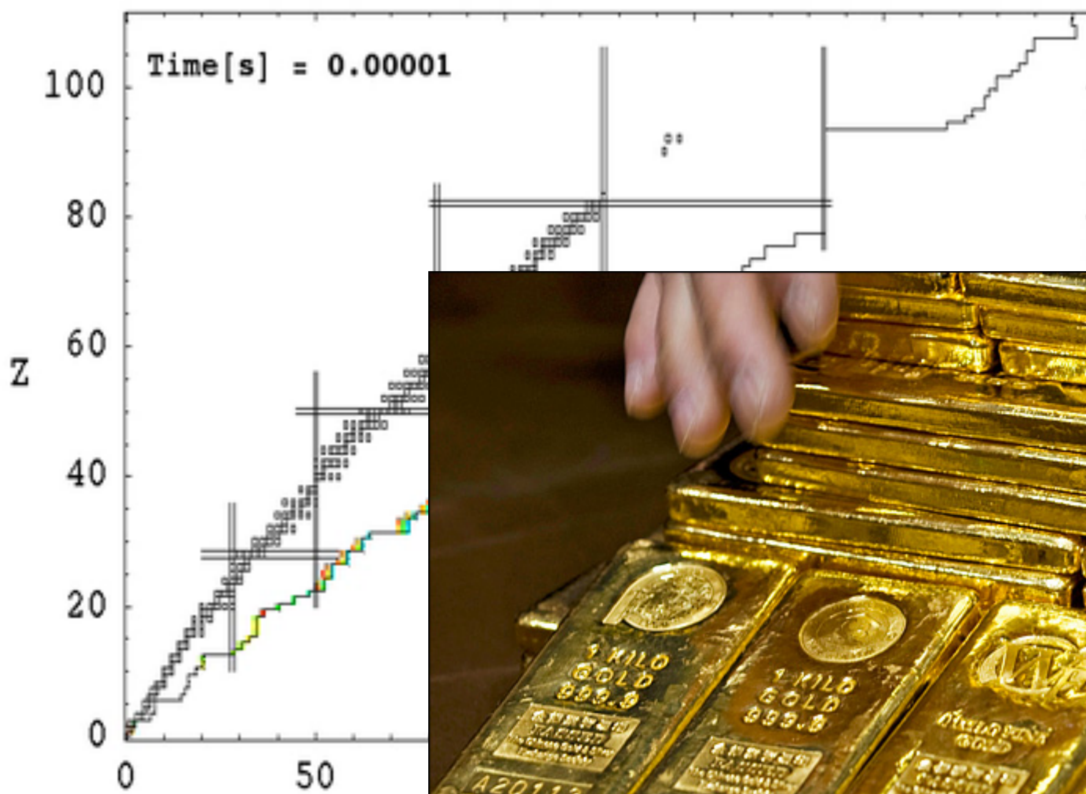

Stephane Goriely, Andreas Baus
dynamically ejected matter of n

The *spin* problem

Double neutron star systems

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- Gaussian distribution of NS masses, with mean = 1.35Msun, sd = 0.2Msun.
- Aligned spin, $J/m^2 = 0.1$.

Einstein Telescope
Double neutron star systems

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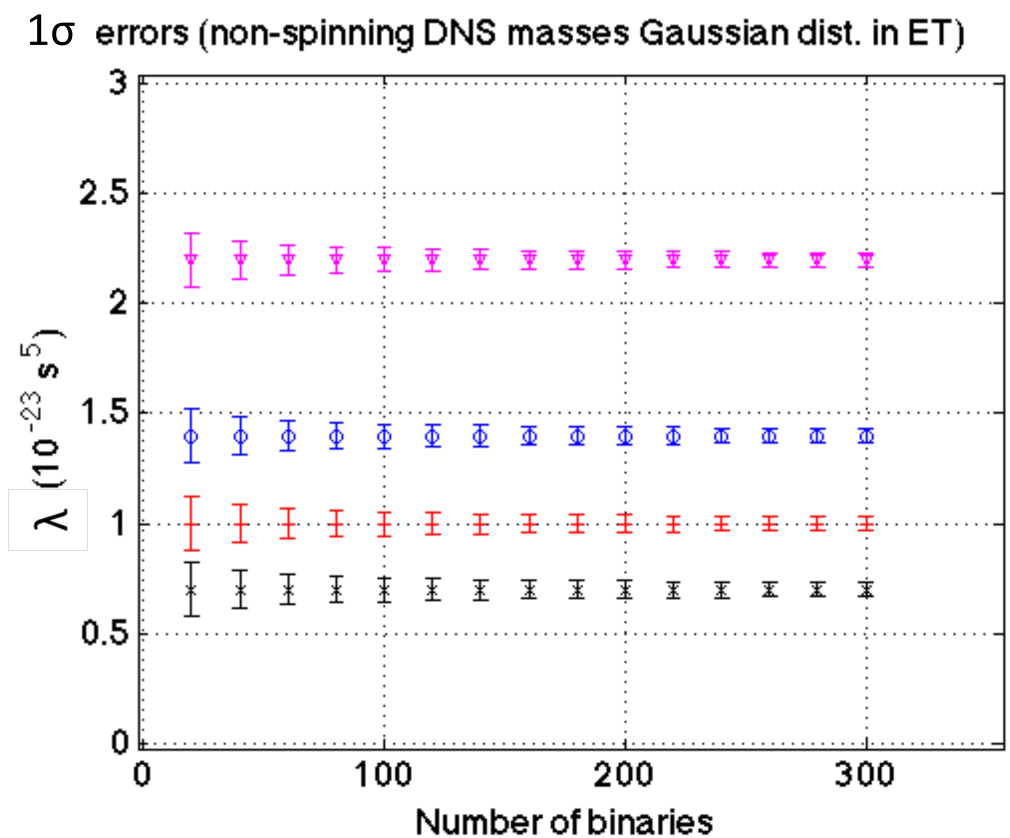
- Non-spinning DNS systems in Einstein Telescope (ET):



Populations

Double neutron star systems

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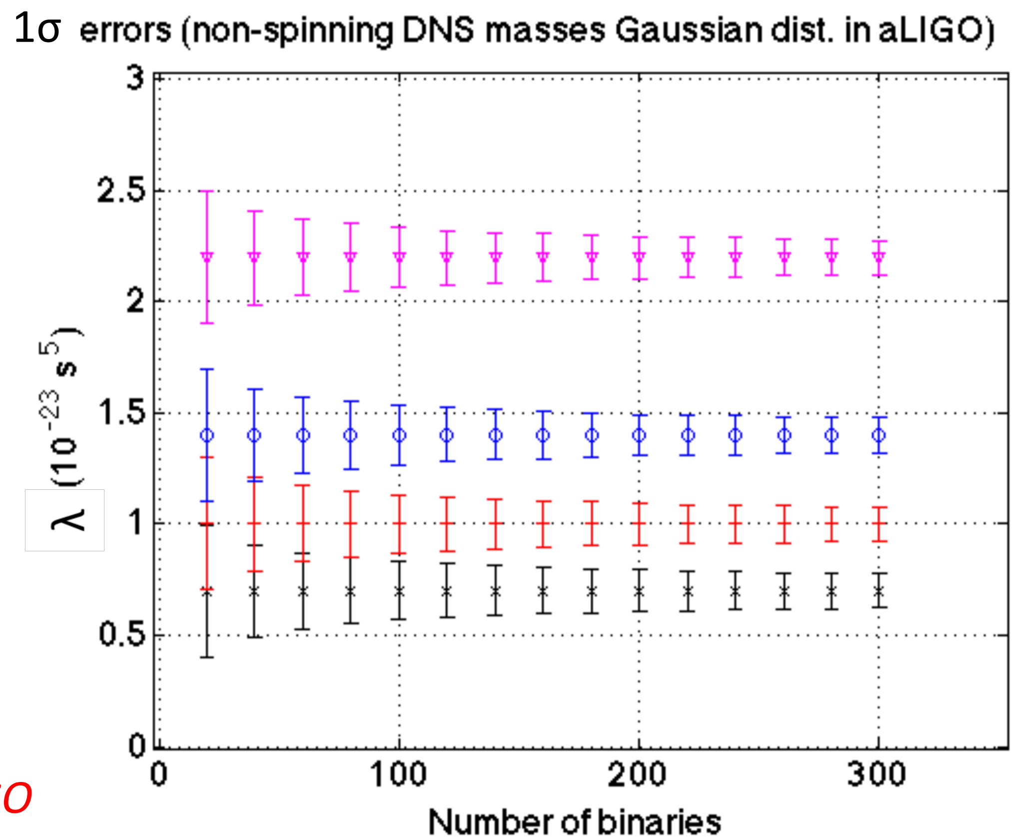
- Gaussian distribution of NS masses, with mean = 1.35Msun, sd = 0.2Msun.



Reference: aLIGO

Real nuclear matter & neutrinos

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- Need "real" NSE equation of state for composition, phase change, neutrino interaction info
- Use Lattimer-Swesty (K_0=220MeV, S_v=37MeV) EoS
  - Includes free neutrons, protons, electrons, photons, 1 species of heavy nucleus, and alpha particles
  - R=11.5km for $M_{NS}=1.4M_\odot$
- Neutrino cooling/deleptonization through leakage scheme:
  Local emission rate = linear combination of
    - Free emission rate from nucleon capture, pair annihilation, etc.
    - Diffusive emission rate, depends on optical depth

Summary of first part

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- NS mass distribution: Uniform distribution will help estimation of λ. GW observations will provide some insight into what this distribution is.
- NS spin helps or hurts λ estimation? We find that it hurts but only negligibly.
- How can waveform modeling systematics affect λ estimation? We have the tools to answer it (future work).
- In light of the prospect of LOFT, it is interesting to compare how well ET will perform.

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Take-away messages

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- Knowing certain EOS parameters from theory helps constrain, observationally, the other EOS parameters better.
- The measurement accuracy improves with the number of observed double neutron star systems.
- The effects of NS mass distribution, DNS formation scenarios, and waveform modeling on measurement accuracy can be quantified.

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Tables for NS-BH fitting factors phenomc.

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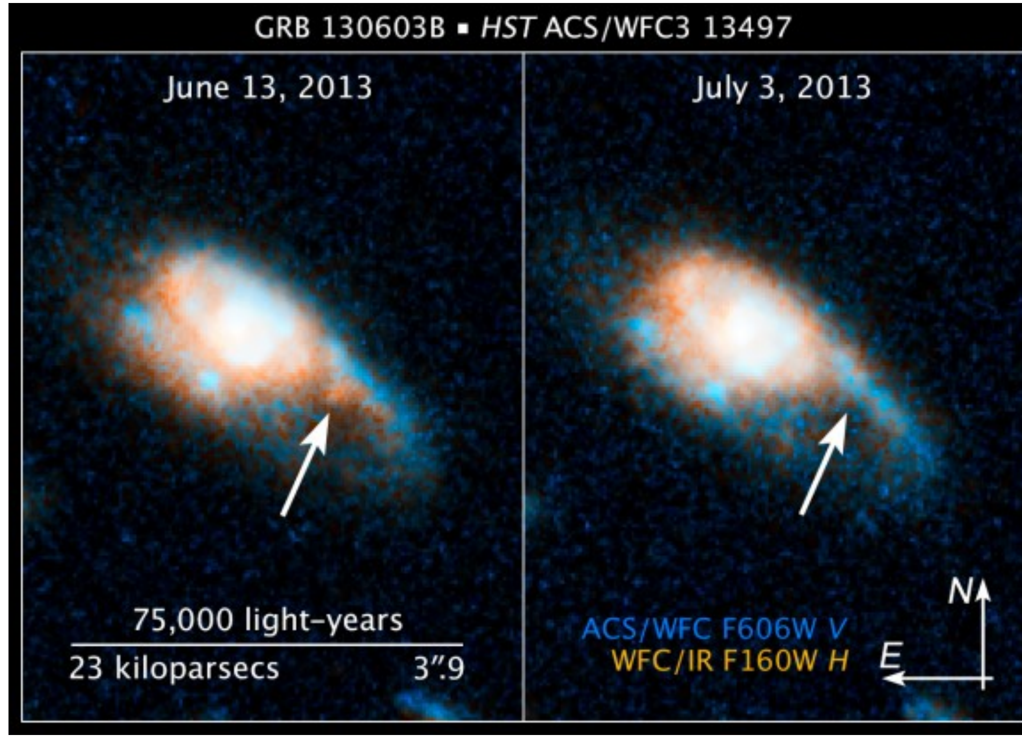
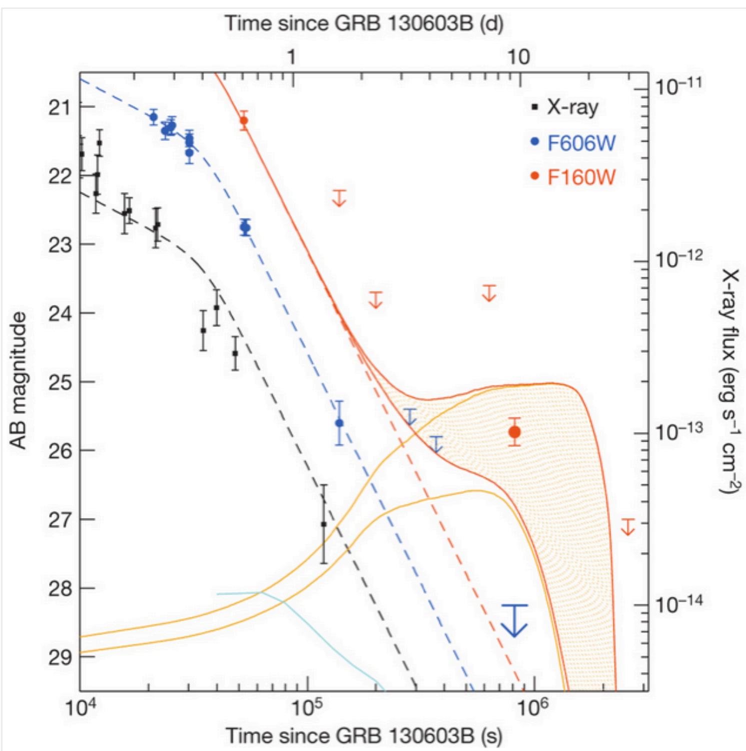
Tables for NS-BH fitting factors phenomc.

Is that a neutron star?

(Is GRB 130603B the first ever kilonova detection?)

- Late-type galaxy at z ~ 0.35
- No late-time optical counterpart (rules out Ni-decay emission as in a supernova)
- Single late-time near-infrared emission.





- N-IR emission consistent with kilonova models of CBC-NS with ejected mass ~ $10^{-2} – 10^{-1}$ Msun.

[Tanvir et al., , Nature doi:10.1038/nature12505 (2013).

What are the alternatives to GWs?

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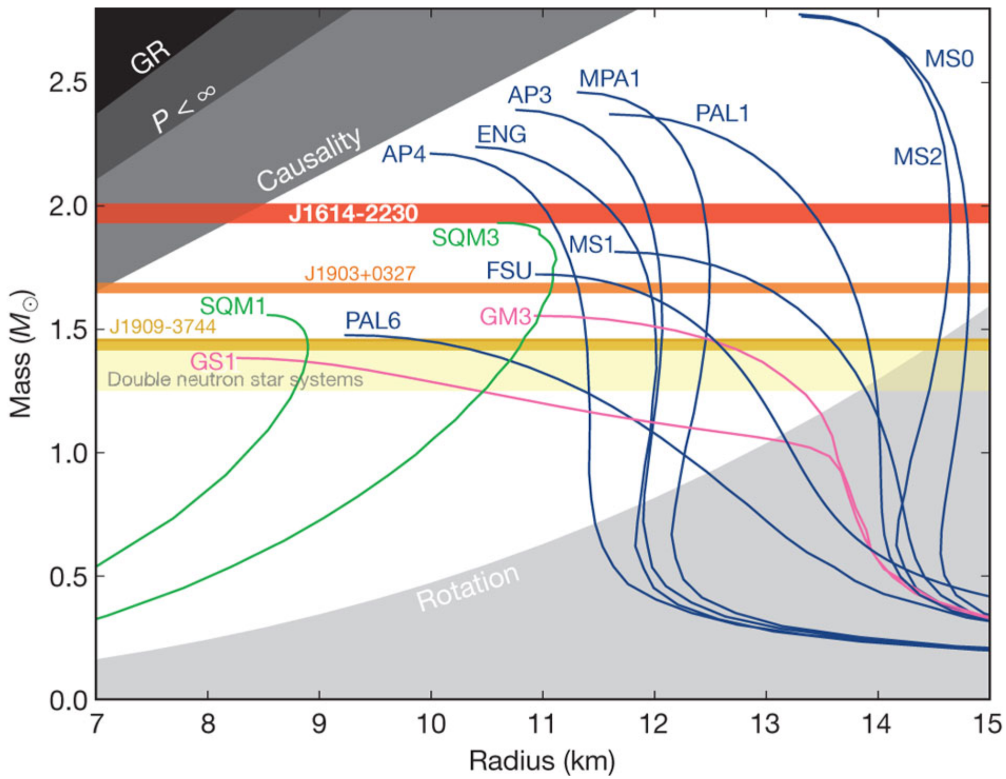

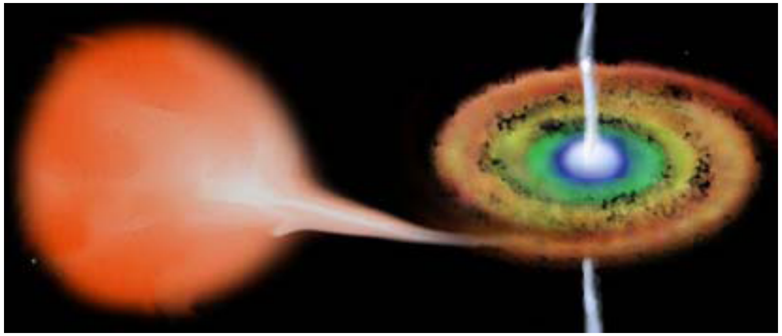

Constraints on NS EOS can be obtained from observations of *x ray* pulsars

What do X-ray observations say about NS radii?

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System	Mass (M_sun)	Radius (km)
4U 1608-248, EXO 1745-248, 4U 1820-30	1.3 – 1.95	8 - 12
KS 1731-260	<= 2.1	<= 12.5
4U 1820-30	1.58+/-0.06	9.11+/-0.4
4U 0614+09	<= 1.9	<= 15.2
the list goes on….		

X-ray observations of LMXB systems => for NS masses in the range $M \sim 1.3 – 1.95\ M_{sun}$ the NS radii are measured in the range R ~ 8-12km. Allowing for other systematics for this mass range can increase the radius upper limit to 15km.

Radius error typically of the order of > 10%

NS ellipticity from a stochastic GW background from gravitars

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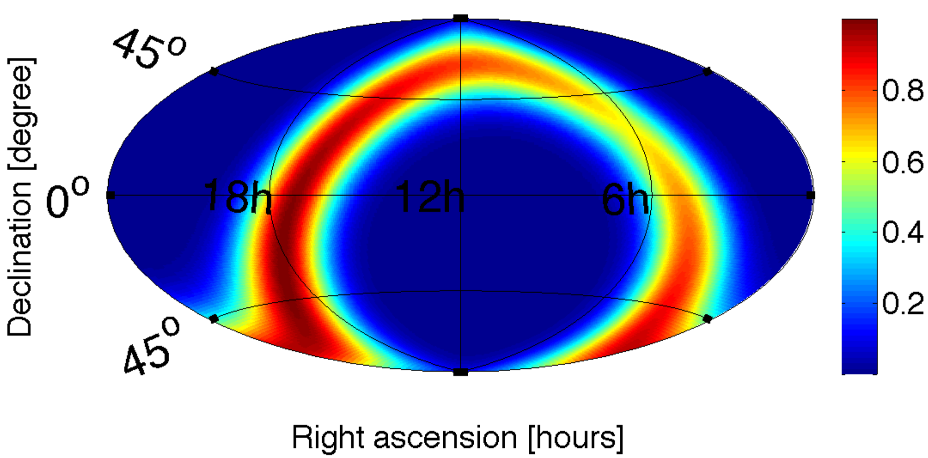

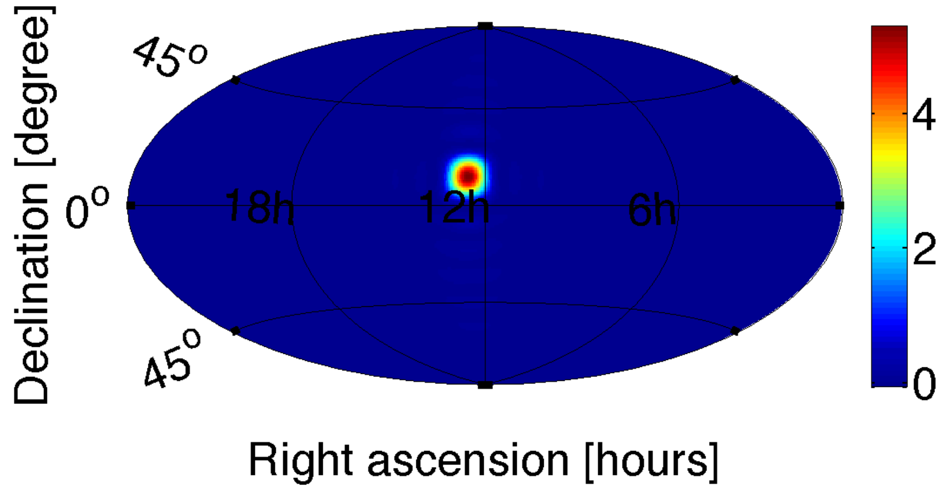

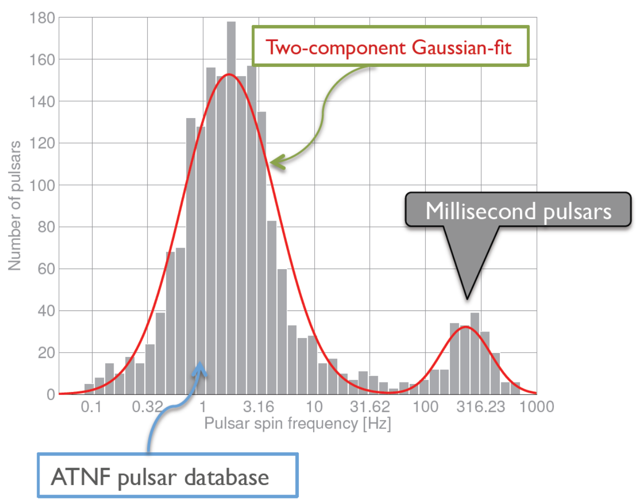

Two simulated SGWBs, one with origin confined to the Galactic disk (*left)* and another with origin in the Virgo Cluster *(right*).

About 40,000 millisecond pulsars estimated. Further the the total number of NSs in our Galaxy ≈ 10^8 for spin frequency f < 50Hz and 40,000 for f > 50 Hz.

[D. Talukder, E. Thrane, SB, T. Regimbau, PRD 89, 123008 (2014).]

NS ellipticity

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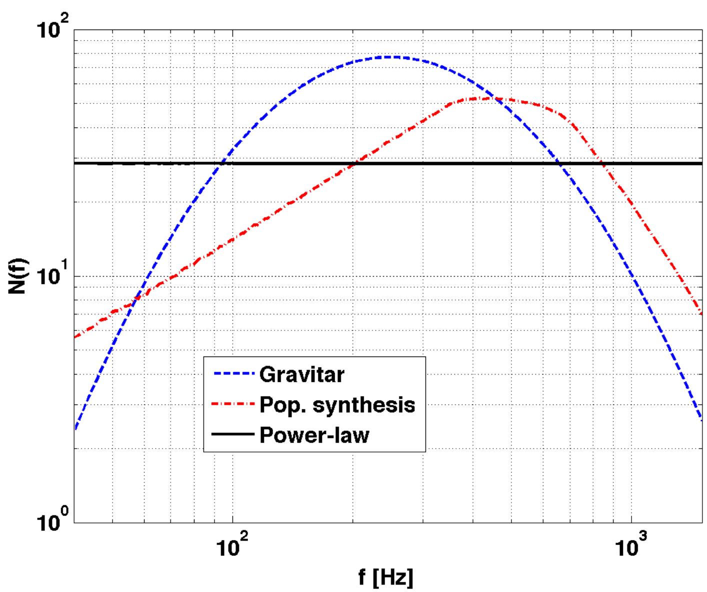

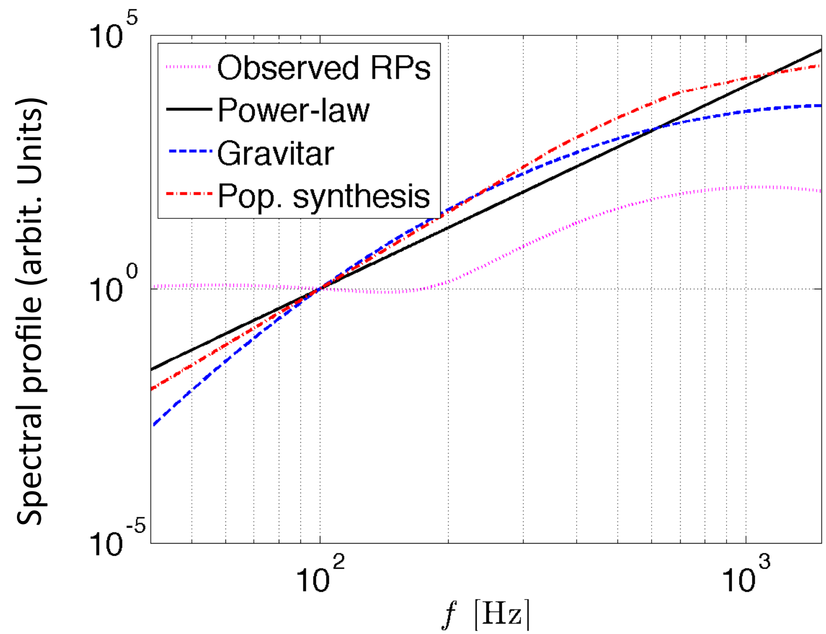

To model the spectrum, we simulate the NS spin frequency distribution assuming various population distributions, including a "power law".

We consider the population synthesis model, where the population of (millisecond) pulsars has ages uniformly distributed between 0-12 Gyr, initial magnetic fields drawn from a uniform distribution in [10^8, 10^{12}] Gauss (with no magnetic field decay), and for which the actual period is calculated assuming magnetic (dipole) braking only.

$$h(f) = 4\pi^2 \beta \frac{GI\varepsilon}{rc^4} f^2.$$

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Number of gravitars in the galaxy?

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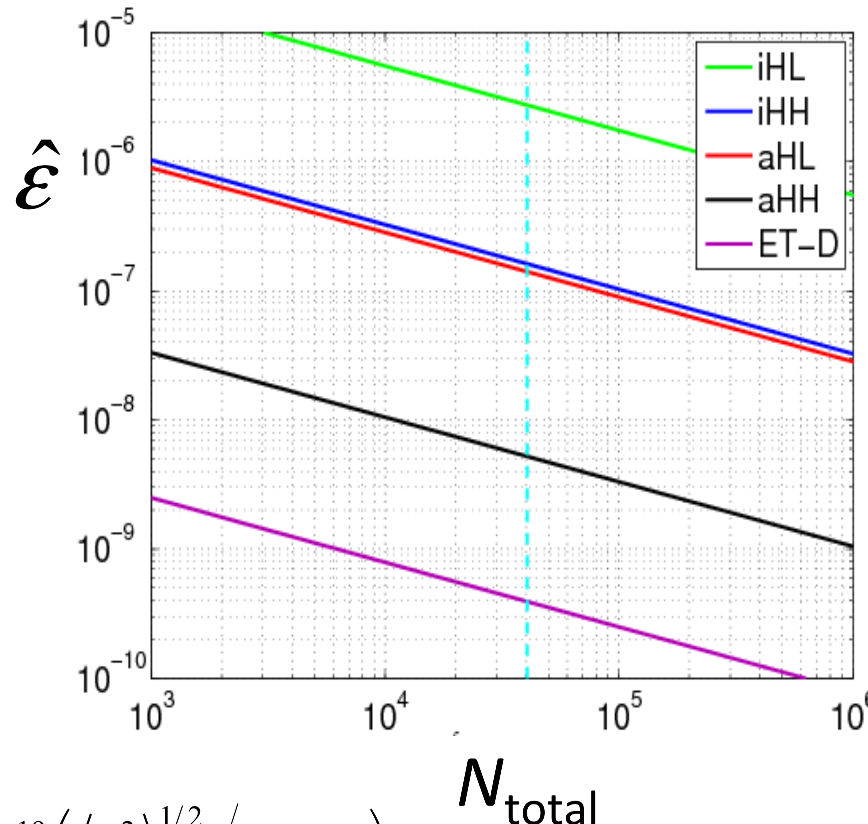
- Estimated upper limits average ellipticity ε of a neutron star in our galaxy achievable with 1 yr of integration…
- …presented as functions of the number of Galactic NSs emitting GWs with frequency in the range 600 – 1000 Hz.
- (Here, H and L denote the LIGO detectors in Hanford and Livingston with 4km long arms.)
- For the Advanced LIGO detectors in Hanford and Livingston, that upper-limit is anticipated to be 1.3×10^{-7} for N_{total} = 40,000.



$$\hat{\varepsilon}(f) \approx \frac{3\times10^{19}\left(\langle r^2\rangle^{1/2}/10\,\mathrm{kpc}\right)}{N^{1/2}(f)\left(I/1.1\times10^{45}\,\mathrm{gm}\cdot\mathrm{cm}^2\right)}\left(\frac{3H_0^2\delta f}{10\pi^2} f^{-7}\Omega(f)\right)^{1/2}.$$

Neutron star – black hole binaries

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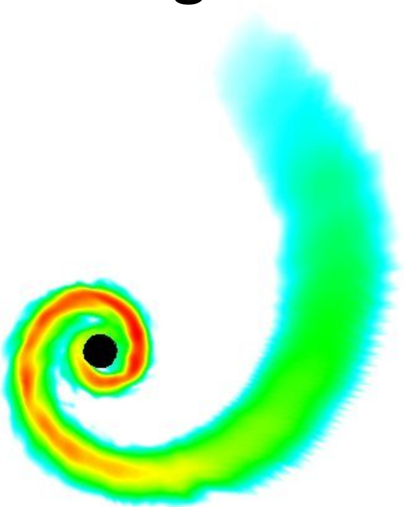
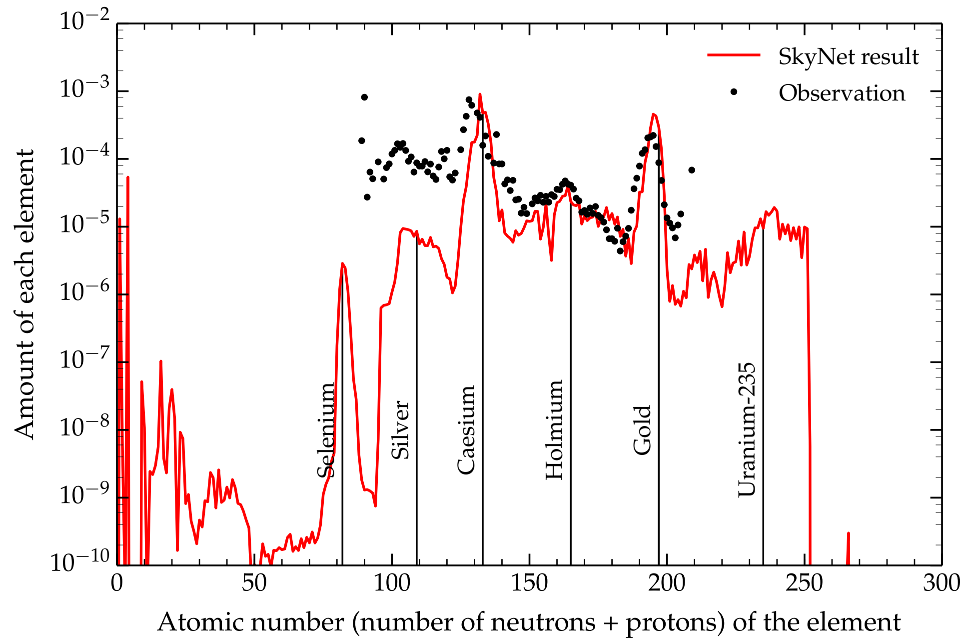
- Massive ejecta from neutron star tidal disruption
- r-process nucleosynthesis in ejecta
  - An important site for the creation of these elements?
- Kilonova signal

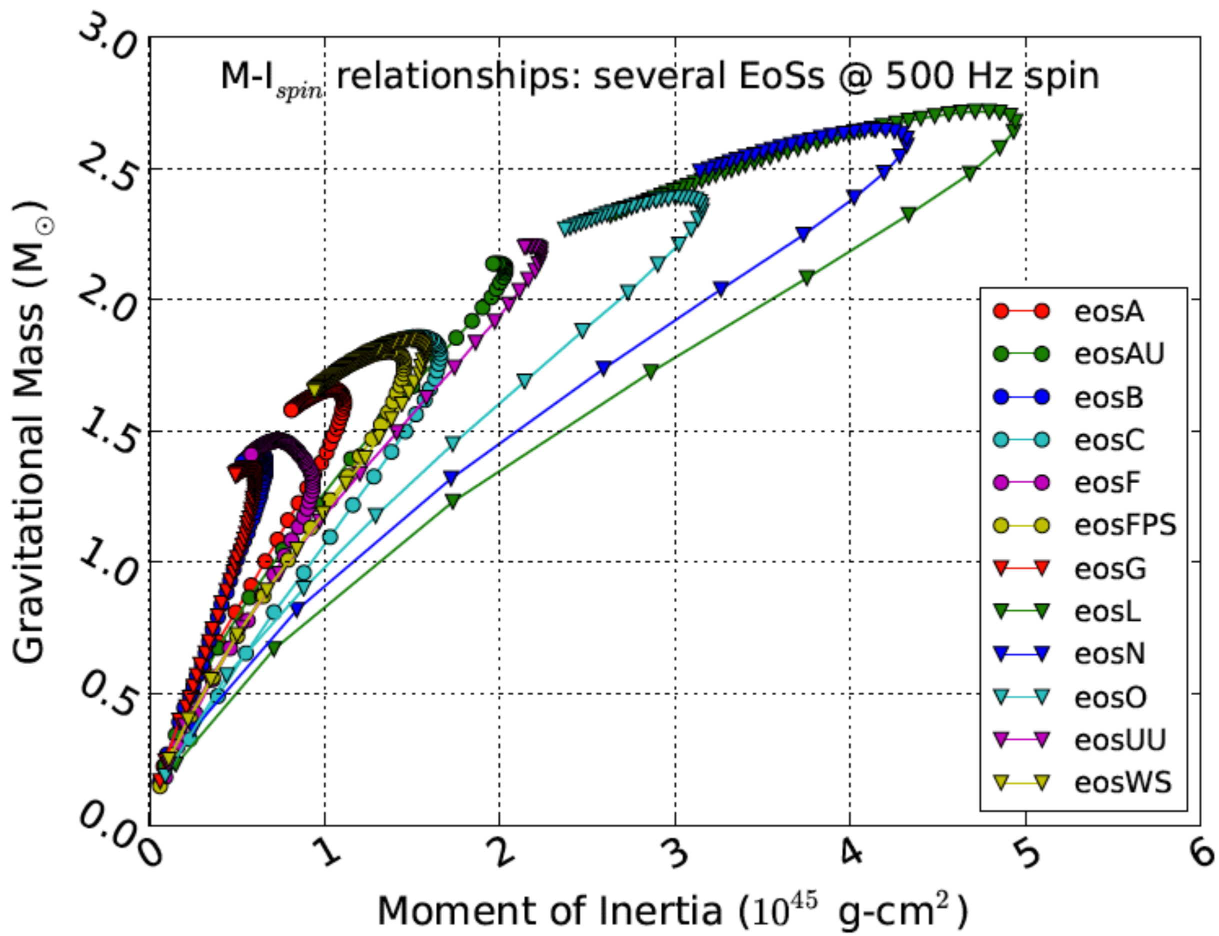


Tidal tail in a SpEC NS/BH calculation



Final abundances in outflow
From nuclear reaction network calculation
using SpEC NS/BH outflow (Lippuner *et al*, in prep)

Complementing LMXB observations with GWs



A. Mukherjee, SB, in preparation.

Complementing LMXB observations with GWs

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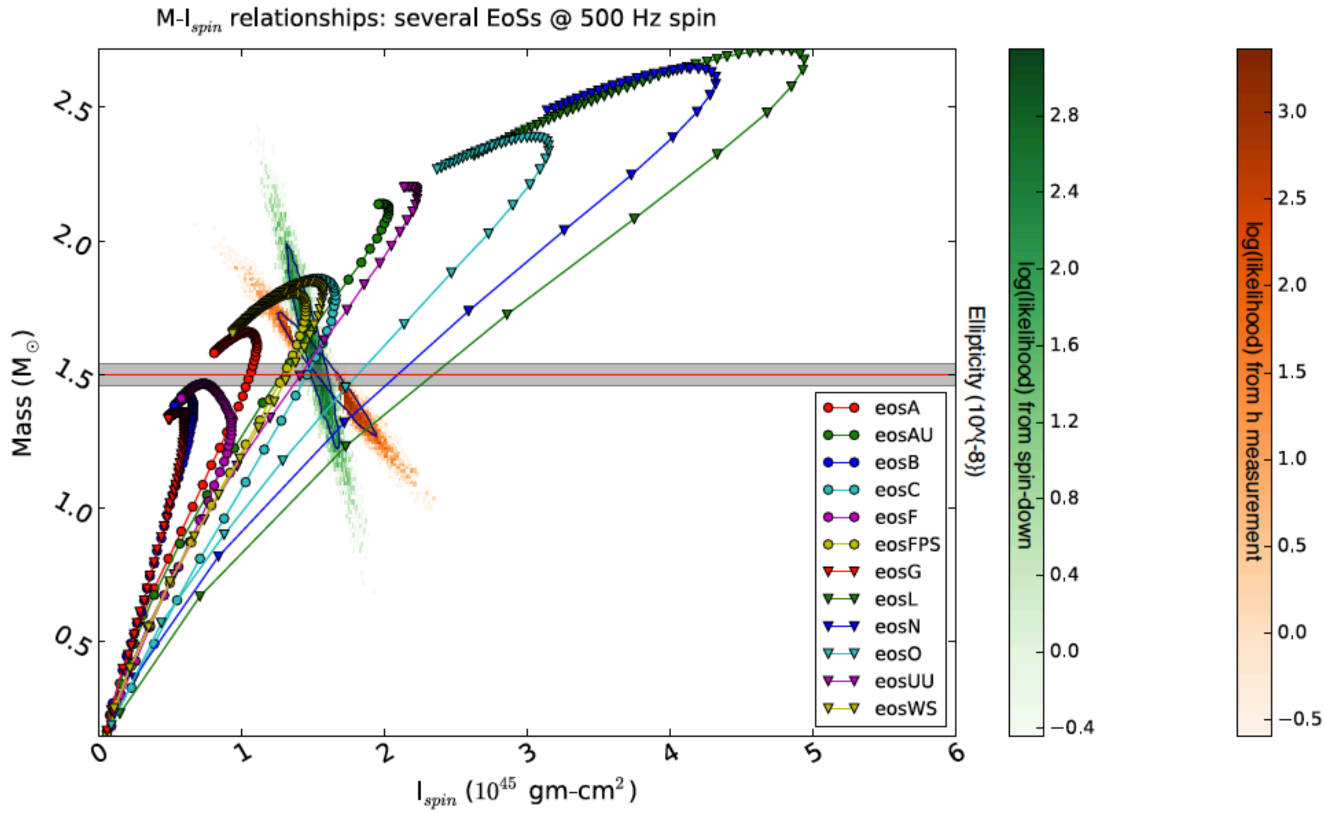

$$h(f) = 4\pi^2 \beta \frac{G[I\varepsilon]}{rc^4} f^2,$$

A. Mukherjee, SB, in preparation.

$$\dot{f} = \frac{-32G[I\varepsilon^2]}{5c^5} f^5$$

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BNS mass errors from GWs

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Table 1

Median 68% and 95% Credible Intervals for Intrinsic Parameters for Each of the Four

	HLV					
System	$\Delta\mathcal{M}_c/\mathcal{M}_c$		$\Delta M_1/M_1$		$\Delta M_2/M_2$	
Credible Level	68%	95%	68%	95%	68%	95%

Table 1

n 68% and 95% Credible Intervals for Intrinsic Parameters for Each of the Four Systems Considered

HLV									
$\mathcal{M}_c/\mathcal{M}_c$	$\Delta M_1/M_1$		$\Delta M_2/M_2$		$\Delta M_{tot}/M_{tot}$		Δq		
95%	68%	95%	68%	95%	68%	95%	68%	95%
0.0104%	7.17%	11.9%	6.39%	10.3%	0.643%	1.25%	0.123	0.197
0.0188%	7.77%	13%	6.87%	11.1%	0.746%	1.47%	0.132	0.212
0.0355%	1.86%	3.74%	1.59%	3.23%	1.48%	2.99%	0.0138	0.028
0.0522%	9.02%	15%	7.82%	12.6%	1.01%	1.94%	0.149	0.239

"New" physics & astronomy

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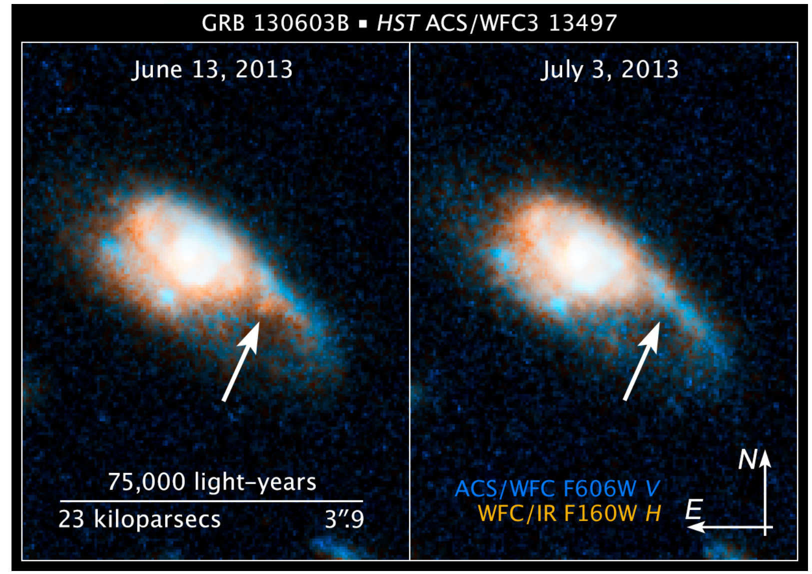
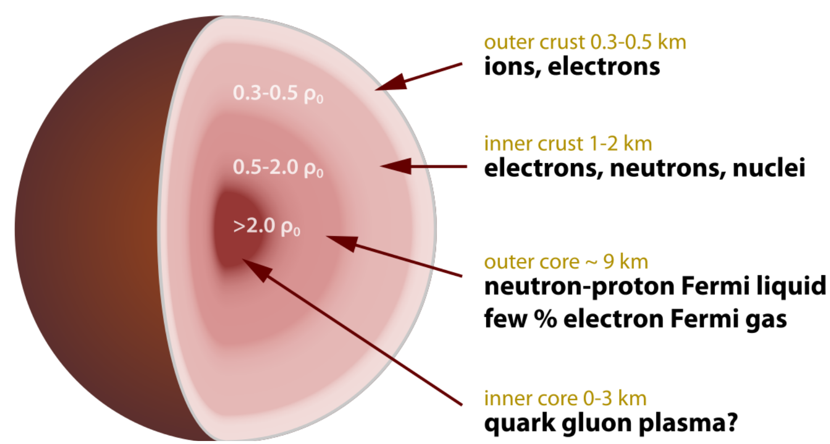
Tanvir et al., Nature (2013).



- Involve neutron stars:
  - Do kilonovae/macronovae and short GRBs share a common origin?
  - What is the equation of state of a neutron star?
  - What are the host galaxies of binary NS systems like?
  - Can GWs help track how BNS formation rate evolved with redshift?



[Credits: Wikipedia.]