# Empirical equation of state and X-rays observations of neutron stars

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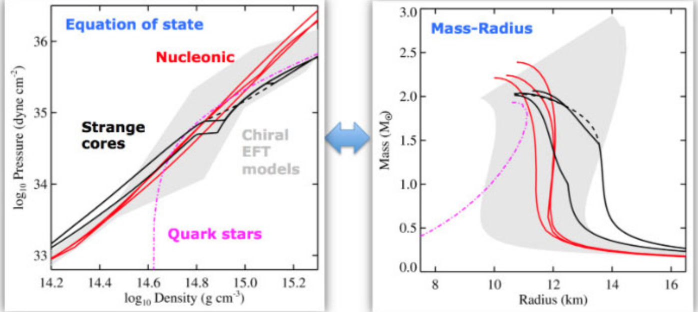
# Link between EoS and M-R relation :

T.O.V equations (Tolmann-Oppenheimer-Volkov) :

$$\frac{dm(r)}{dr} = 4\pi r^2 \epsilon(r), \tag{1}$$

$$\frac{dP(r)}{dr} = -\frac{G\epsilon(r)m(r)}{r^2}\left(1+\frac{P(r)}{\epsilon(r)c^2}\right)\left(1+\frac{4\pi P(r)r^3}{m(r)c^2}\right)\times\left(1-\frac{r_{sh}}{r}\frac{m(r)}{M}\right)^{-1}. \tag{2}$$

Equation of state : $P(\epsilon) \leftrightarrow$ mass-radius relation.



Anna Watts et al, 2014

# Constraining nuclear equation of state from thermal radiation of neutron stars :

- Devellopement of pulsar X-ray astronomy → strong limits on the equation of state at high density (Özel et al 2010 , Steiner et al 2010).
- Tool : Thermal radiation from the surface of neutron stars.
- Devellopement of atmosphere models (Heinke et al 2006).
- Applying Bayesian analysis for several qLMXBs (Guillot et al 2013, Lattimer & Steiner 2014, Heinke et al 2014, Özel et al 2015, Bogdanov 2016).

# Nuclear equation of state (EoS) :

Various kinds of EoS :

- Purely nucleonic → no phase transition, **smooth EoS** with $n, y_e, T$.
- Phase transition :
  - hadronic matter : onset of hyperons, pion condensate ...
  - pure quark stars ?
  - hybrid stars ?

Motivation for choosing **pure nucleonic matter** :

- smooth EoS
- extrapolation of nuclear physics knowledge (and uncertainties) towards high densities and low $Y_p$.
- Define M-R boundaries for smooth EoS.

# The EoS model based on empirical parameters :

## Definition of the empirical parameters :

$$\epsilon(n,\delta) = \epsilon_{IS} + \delta^2 \epsilon_{IV} \tag{3}$$

$$\epsilon_{IS} = E_{sat} + \frac{1}{2} K_{sat} x^2 + \frac{1}{3!} Q_{sat} x^3 + \frac{1}{4!} Z_{sat} x^4 + o(x^5) \tag{4}$$

$$\epsilon_{IV} = E_{sym} + L_{sym} x + \frac{1}{2} K_{sym} x^2 + \frac{1}{3!} Q_{sym} x^3 + \frac{1}{4!} Z_{sym} x^4 + o(x^5) \tag{5}$$

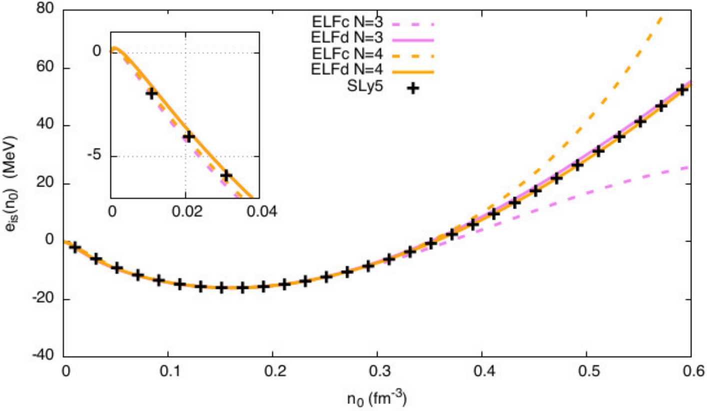
## Taylor expansion around $n_0$ for the energy density :

$$\epsilon(n,\delta) = t(n,\delta) + \sum_{\alpha \geq 0}^{N} v_\alpha(\delta) \frac{x(n)^\alpha}{\alpha!} u_\alpha^N(x), \quad \delta = \frac{(n_n - n_p)}{n}, \quad x = \frac{(n - n_0)}{3n_0}$$

$$v_\alpha(\delta) = v_{\alpha,IS} + v_{\alpha,IV} \delta^2$$

(Margueron, Casali, Gulminelli, in preparation)

# Ability of the EoS to mimic known EoS : example of SLy5



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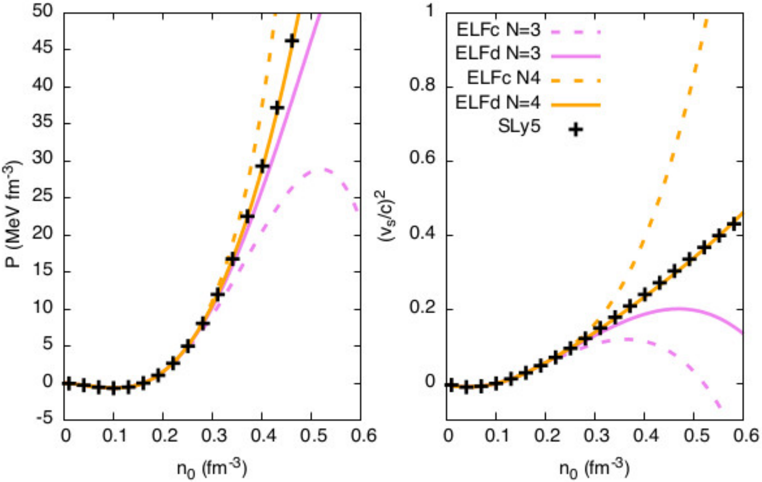

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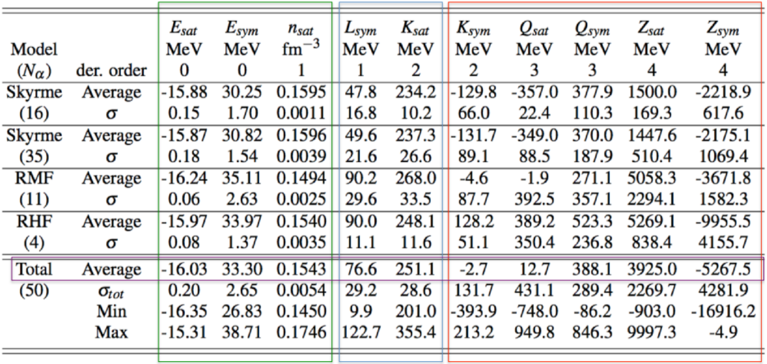
# Our present knowledge of the empirical parameters :

Empirical parameters for various effective approaches :

fixed

Explore in
small interval

Explore in
large interval

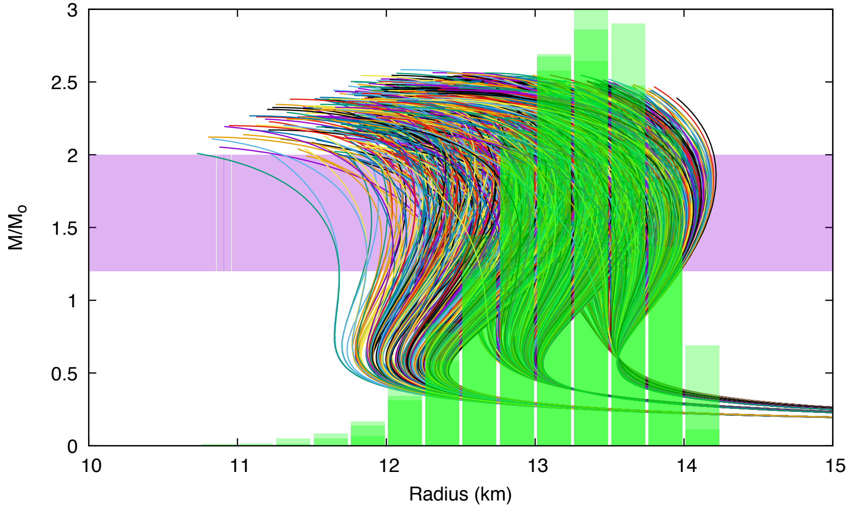


| Model ($N_\alpha$) | der. order | $E_{sat}$ MeV 0 | $E_{sym}$ MeV 0 | $n_{sat}$ fm$^{-3}$ 1 | $L_{sym}$ MeV 1 | $K_{sat}$ MeV 2 | $K_{sym}$ MeV 2 | $Q_{sat}$ MeV 3 | $Q_{sym}$ MeV 3 | $Z_{sat}$ MeV 4 | $Z_{sym}$ MeV 4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Skyrme | Average | -15.88 | 30.25 | 0.1595 | 47.8 | 234.2 | -129.8 | -357.0 | 377.9 | 1500.0 | -2218.9 |
| (16) | $\sigma$ | 0.15 | 1.70 | 0.0011 | 16.8 | 10.2 | 66.0 | 22.4 | 110.3 | 169.3 | 617.6 |
| Skyrme | Average | -15.87 | 30.82 | 0.1596 | 49.6 | 237.3 | -131.7 | -349.0 | 370.0 | 1447.6 | -2175.1 |
| (35) | $\sigma$ | 0.18 | 1.54 | 0.0039 | 21.6 | 26.6 | 89.1 | 88.5 | 187.9 | 510.4 | 1069.4 |
| RMF | Average | -16.24 | 35.11 | 0.1494 | 90.2 | 268.0 | -4.6 | -1.9 | 271.1 | 5058.3 | -3671.8 |
| (11) | $\sigma$ | 0.06 | 2.63 | 0.0025 | 29.6 | 33.5 | 87.7 | 392.5 | 357.1 | 2294.1 | 1582.3 |
| RHF | Average | -15.97 | 33.97 | 0.1540 | 90.0 | 248.1 | 128.2 | 389.2 | 523.3 | 5269.1 | -9955.5 |
| (4) | $\sigma$ | 0.08 | 1.37 | 0.0035 | 11.1 | 11.6 | 51.1 | 350.4 | 236.8 | 838.4 | 4155.7 |
| Total | Average | -16.03 | 33.30 | 0.1543 | 76.6 | 251.1 | -2.7 | 12.7 | 388.1 | 3925.0 | -5267.5 |
| (50) | $\sigma_{tot}$ | 0.20 | 2.65 | 0.0054 | 29.2 | 28.6 | 131.7 | 431.1 | 289.4 | 2269.7 | 4281.9 |
| | Min | -16.35 | 26.83 | 0.1450 | 9.9 | 201.0 | -393.9 | -748.0 | -86.2 | -903.0 | -16916.2 |
| | Max | -15.31 | 38.71 | 0.1746 | 122.7 | 355.4 | 213.2 | 949.8 | 846.3 | 9997.3 | -4.9 |

(Margueron, Casali, Gulminelli, in preparation)

# Dependence of M-R on the empirical parameters :

$+$ constrains : $0 < v_s^2 < c^2$ and $\epsilon_{IV}(n) > 0$ for $n < 4n_{sat}$



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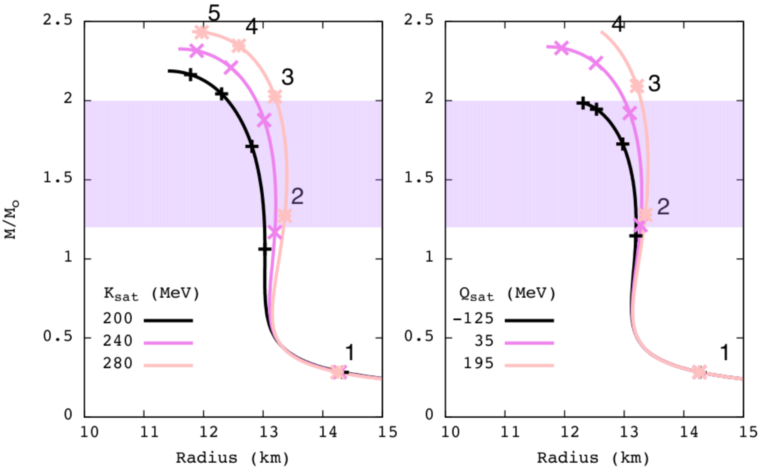

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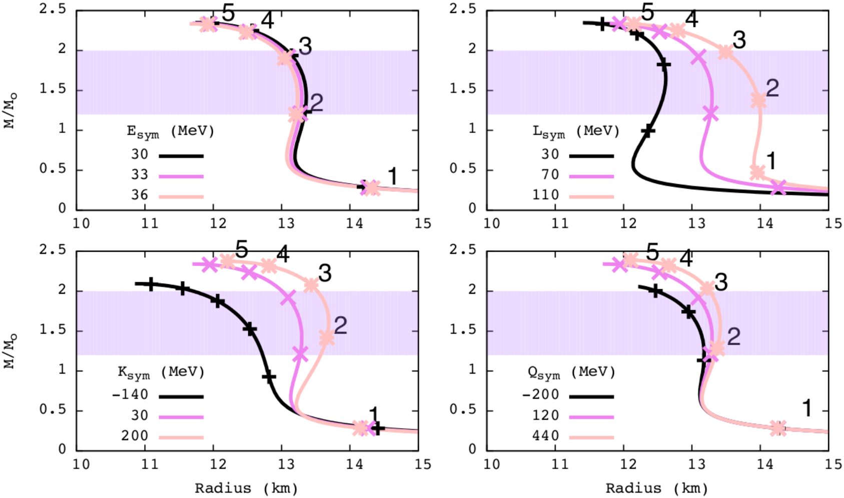
# Dependence of M-R on the empirical parameters :



(Margueron, Casali, Gulminelli, in preparation)

# a M-R simulator, function of $L_{sym}$ and $K_{sym}$ :

Polynomial expression of the radius :

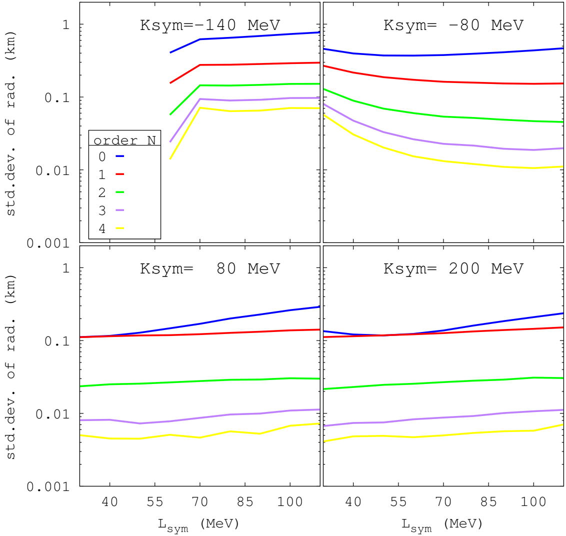
$$R(M, L_{sym}, K_{sym}) = \sum_{i=0}^{N} a_i(L_{sym}, K_{sym}) M^i \tag{6}$$

where :

$$a_i(L_{sym}, K_{sym}) = \sum_{k=0}^{2} a_{ik} L_{sym}^{k} K_{sym}^{2-k} \tag{7}$$

N=0 → Guillot et al. 2013

# a M-R simulator, function of $L_{sym}$ and $K_{sym}$ :



# Observations of thermal emission from NS :

black body : (cf. talk by Andrew Steiner)

$$F \propto T^4 (R_\infty / D)^2$$

(Rutledge et al. 1999)

- 6 low mass X-ray transcients : almost pure thermal components, low magnetic fields, constant flux, atmosphere composition is purely H, in globular clusters (well constrained distances).
- a single EoS → simultaneous analysis.

| Sources | Obs. | distance (kpc) | nH ($10^{22} cm^{-2}$) | atm. | pile-up $\alpha$ | Teff (K) |
|---|---|---|---|---|---|---|
| M13 | Chandra | $7.1 \pm 0.4$ | $[0;1]$ | H | $[0;1]$ | $[5;6.5]$ |
| M28 | Chandra | $5.5 \pm 0.3$ | $[0;1]$ | H | $[0;1]$ | $[5;6.5]$ |
| M30 | Chandra | $9 \pm 0.4$ | $[0;1]$ | H | $[0;1]$ | $[5;6.5]$ |
| NGC6304 | Chandra | $6.22 \pm 0.26$ | $[0;1]$ | H | $[0;1]$ | $[5;6.5]$ |
| NGC6397 | Chandra | $2.39 \pm 0.13$ | $[0;1]$ | H | $[0;1]$ | $[5;6.5]$ |
| $\omega$ Cen | Chandra | $4.59 \pm 0.08$ | $[0;1]$ | H | $[0;1]$ | $[5;6.5]$ |

# Modeling the spectra with Xspec :

## Spectrum model used :

- "pile-up" (Davis 2001, Bogdanov 2016), "phabs" absorbtion and "nsatmos" for the atmosphere (Heinke et al. 2006).
- $E < 2$ keV : only thermal component and no power-law

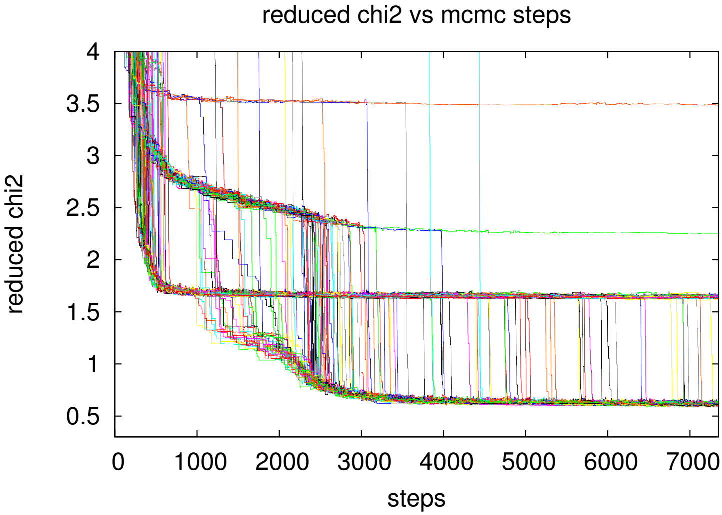
## Parameters which are allowed to vary :

- pile-up alpha
- hydrogen collumn density on the line of site (nH)
- distance to the stars (dkpc)
- the surface effective temperature (Teff)
- the mass of the stars.
- the nuclear parameters $K_{sym}$ and $L_{sym}$ :

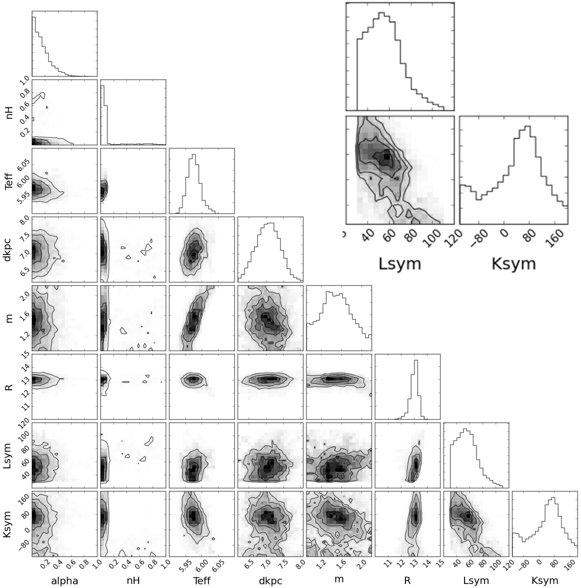
$$\text{EoS}(K_{sym},L_{sym},\text{Mass}) \rightarrow \text{Radius}$$

# Reproducing the data :

MCMC stretch move algorithm with 200 walkers (Mackey et al. 2013)

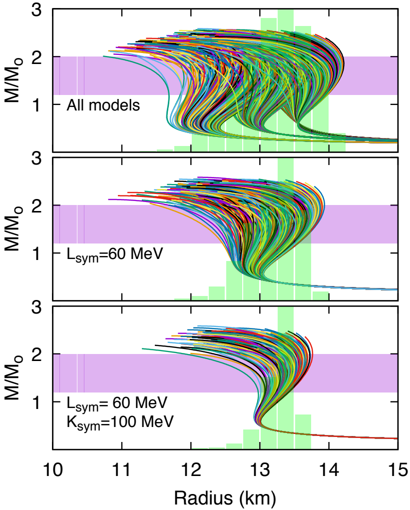


# Preliminary results (M13 parameters) :



# Conclusion :

- Present knowledge of nuclear physics + smooth EoS → $R \simeq 11.5 - 14$ km and $M < 2.5 M_\odot$
- Main source of uncertainties : $L_{sym}$ and $K_{sym}$.
- if better knowledge on $L_{sym}$ and $K_{sym}$ : reduce uncertainty $\simeq 1$ km
- measurement of R out of these boundaries → non-smooth EoS (induced by phase transition)



# Outlooks :

- Finalizing this study.
- Extension of present EoS to include quark matter.
- Further analysis of occurence of a phase transition from X-ray data (Bayes factor).
- Waiting for *Lsym* constrains from PREX.
- Confronting our results with NICER or NS mergers data.

# Thank you for attention !