



**“r-modes”**  
or  
**Astrophysics meets Architecture**  
or  
**Designing the perfect window**

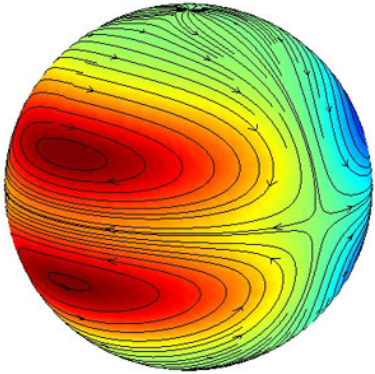
Seattle July 2016



UNIVERSITY OF  
**Southampton**

**Nils Andersson**  
na@maths.soton.ac.uk

# the r-modes



The r-modes belong to a large class of “inertial” modes (dominated by the Coriolis force).

May be driven unstable by the emission of gravitational waves at all rates of rotation (in inviscid stars).

Actual instability window depends on uncertain core physics.

The  $l=m=2$  r-mode grows (due to current multipole radiation) on a timescale

$$t_{\text{gw}} \approx 50 M_{1.4}^{-1} R_{10}^{-4} P_{-3}^6 \text{ s}$$

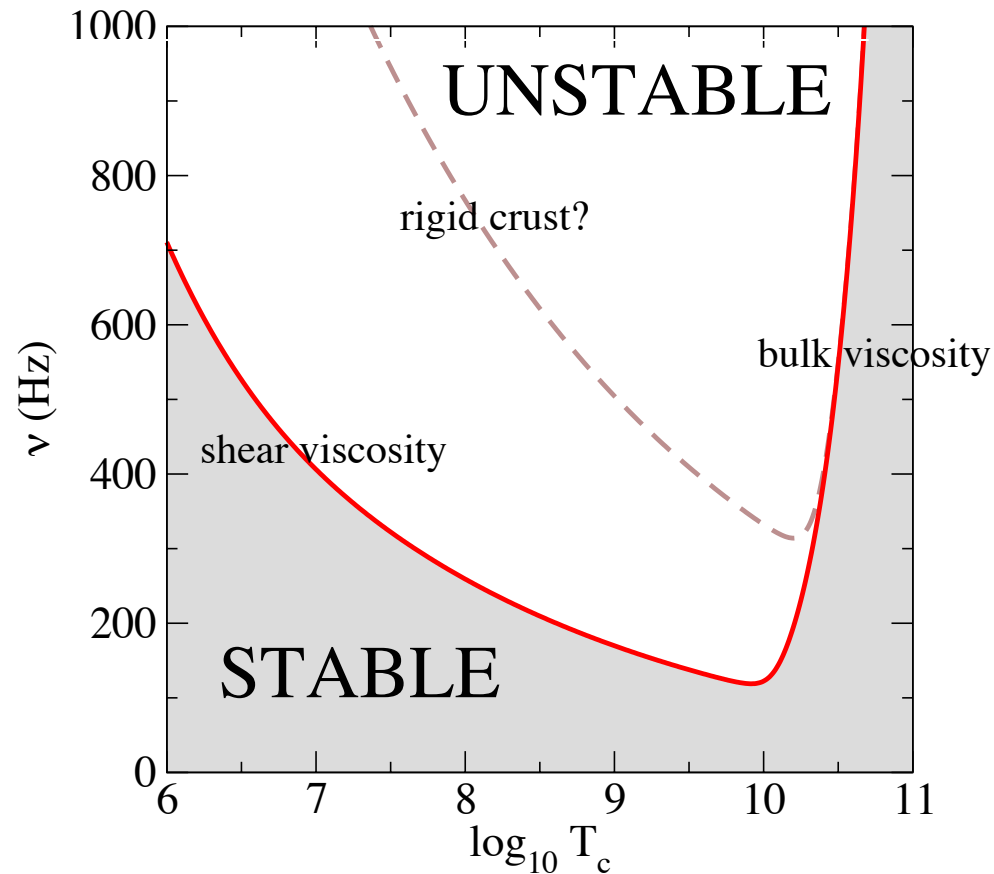
Viscosity may stabilise the star. At low temperature, shear viscosity is expected to dominate. For nn scattering we have

$$t_{\text{sv}} \approx 7 \times 10^7 M_{1.4}^{-5/4} R_{10}^{23/4} T_9^2 \text{ s}$$

Bulk viscosity is important at high temperatures (requires density perturbation which arises at second order in  $\Omega$ )

$$t_{\text{bv}} \approx 3 \times 10^{11} M_{1.4} R_{10}^{-1} P_{-3}^2 T_9^{-6} \text{ s}$$

# conservative window



In principle, we should not observe any “usual” pulsar inside the instability region. Use this to “rule out” theoretical models?

Best constraint from young systems: The X-ray pulsar J0537-6910, currently spinning at 16 ms, would have been born with a period in the range 6-9 ms.

# LMXBs

Accreting neutron stars in LMXBs rotate well below the break-up limit; some kind of speed-limit may be enforced.

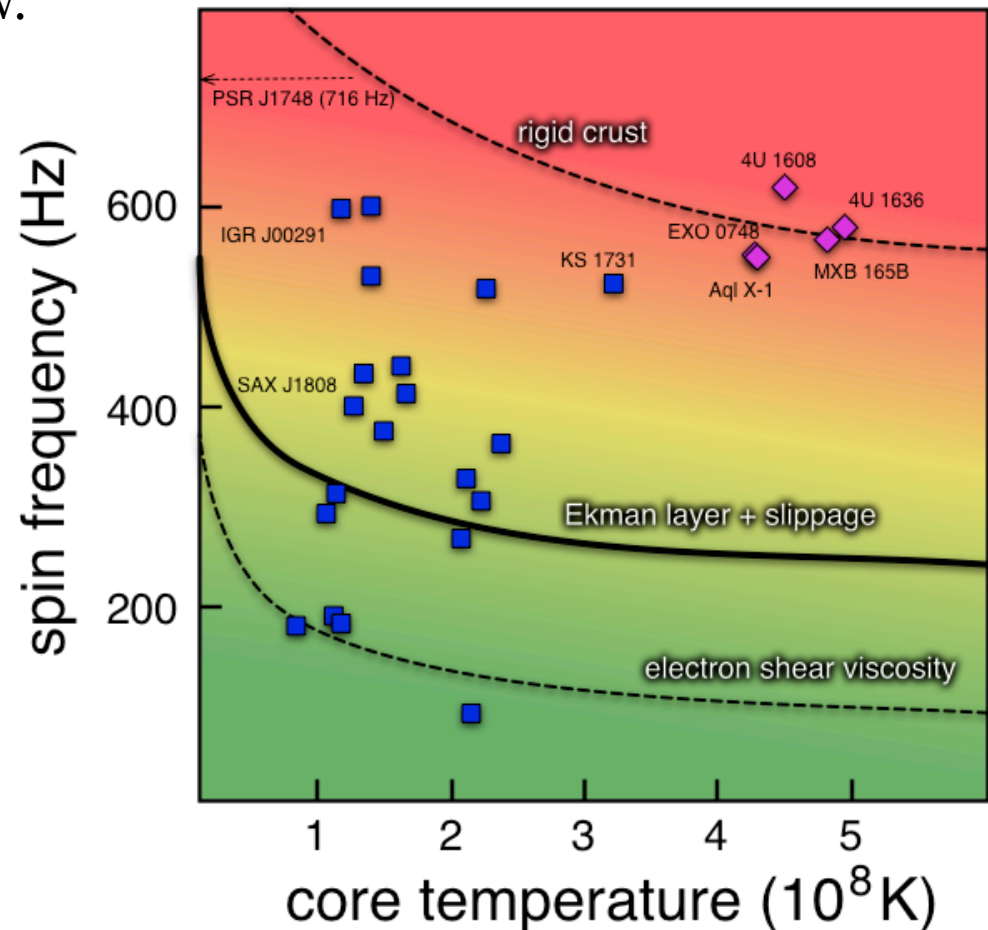
The r-modes could provide an explanation. Many systems lie inside the “conservative” instability window.

**Rigid crust** with viscous (Ekman) boundary layer would lead to sufficient damping...

...but the crust is more like jelly, so the effect is reduced (“slippage”).

**Saturation amplitude** due to mode-coupling is too large to allow evolution far into instability region.

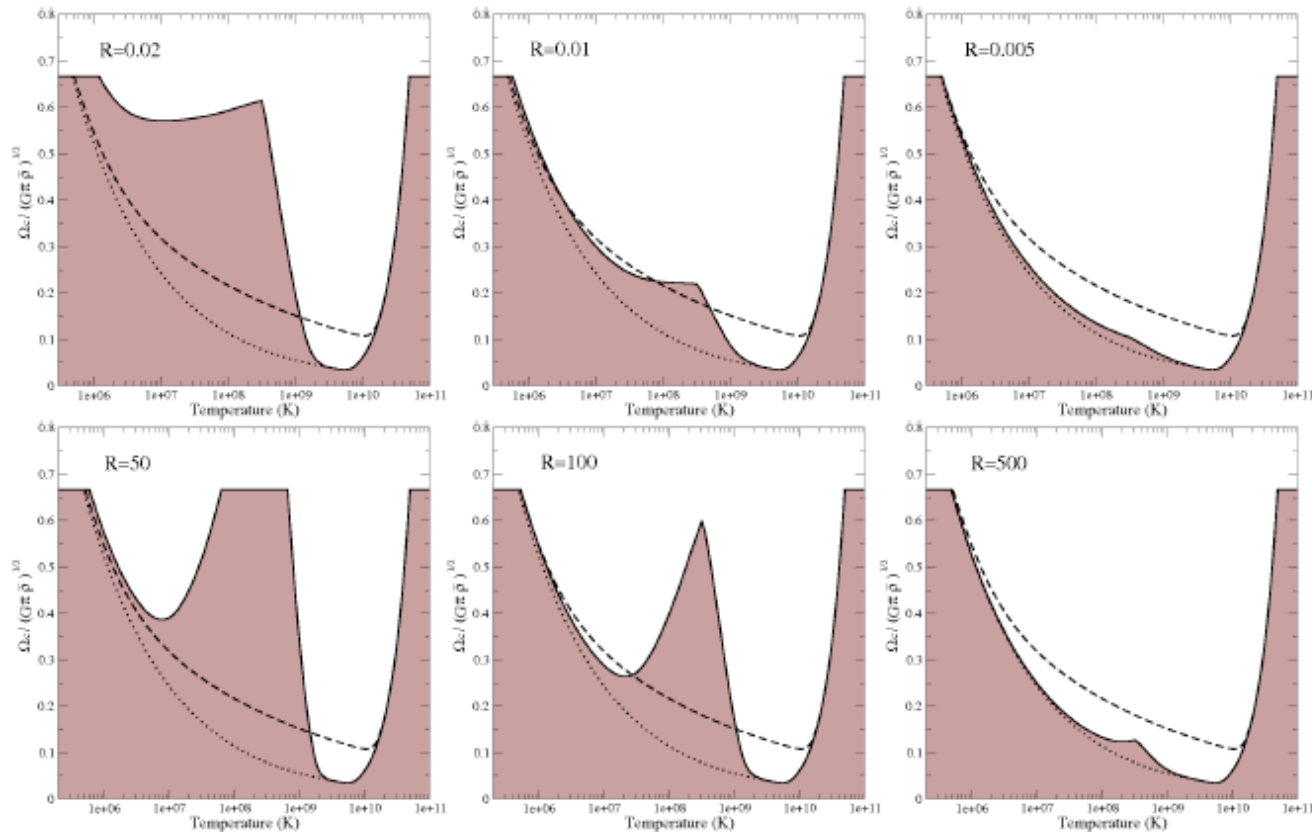
Note: No evidence in the data for “clustering” near an instability threshold.



# variable windows

Vortex mediated mutual friction is an important mechanism in superfluid neutron star dynamics, but has little impact on the r-modes for “expected” parameters.

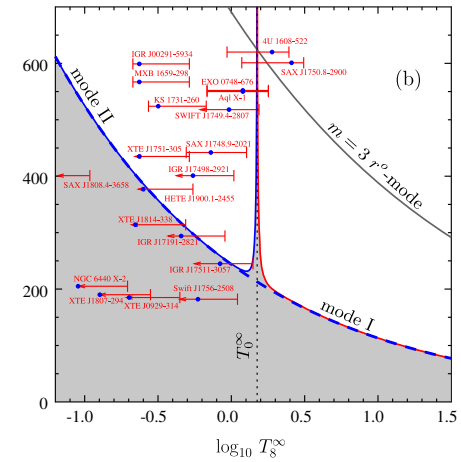
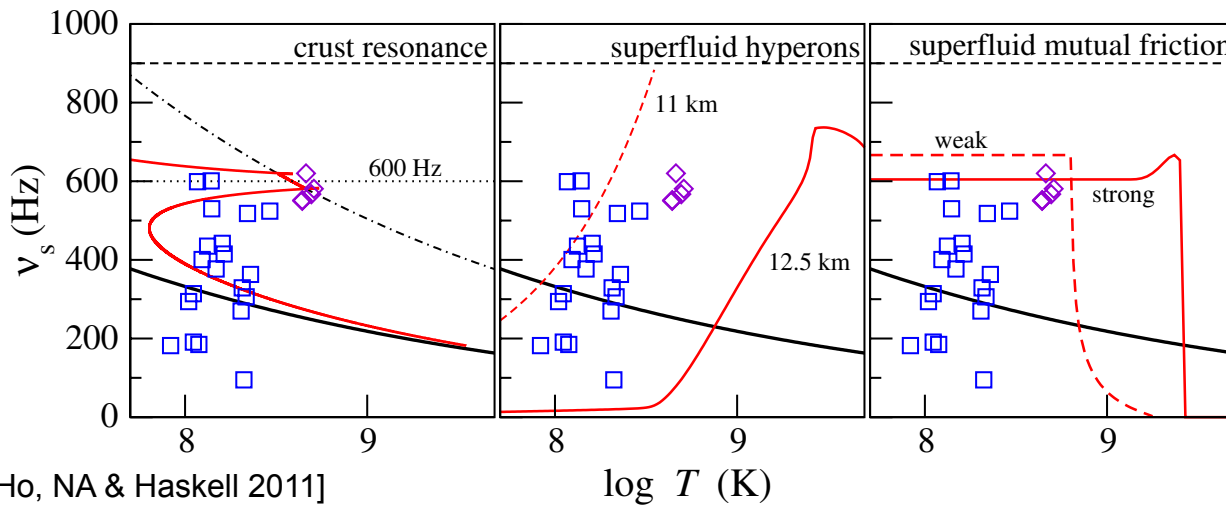
Would need to be stronger by a factor of about 100 to resolve the problem.



# designer windows

The instability window may have a very different shape due to “resonances”;

- resonant timescale with reactions (hyperon/quark bulk viscosity)
- resonance with other modes (shear modes in crust, other inertial modes in superfluid core)



At the end of the day, the magnetic field may provide the answer...

- slippage at crust-core interface not allowed, but there is still a boundary layer due to discontinuous derivatives (how sharp is the phase transition?)
- damping due to vortex-fluxtube interactions in outer core may be very efficient and could also provide a saturation mechanism.

# XTE 1751-305

XTE 1751-305 is an accreting millisecond pulsar spinning at  $f_s = 435$  Hz.

Recent work reports evidence for coherent oscillations in RXTE data from the 2002 discovery burst at

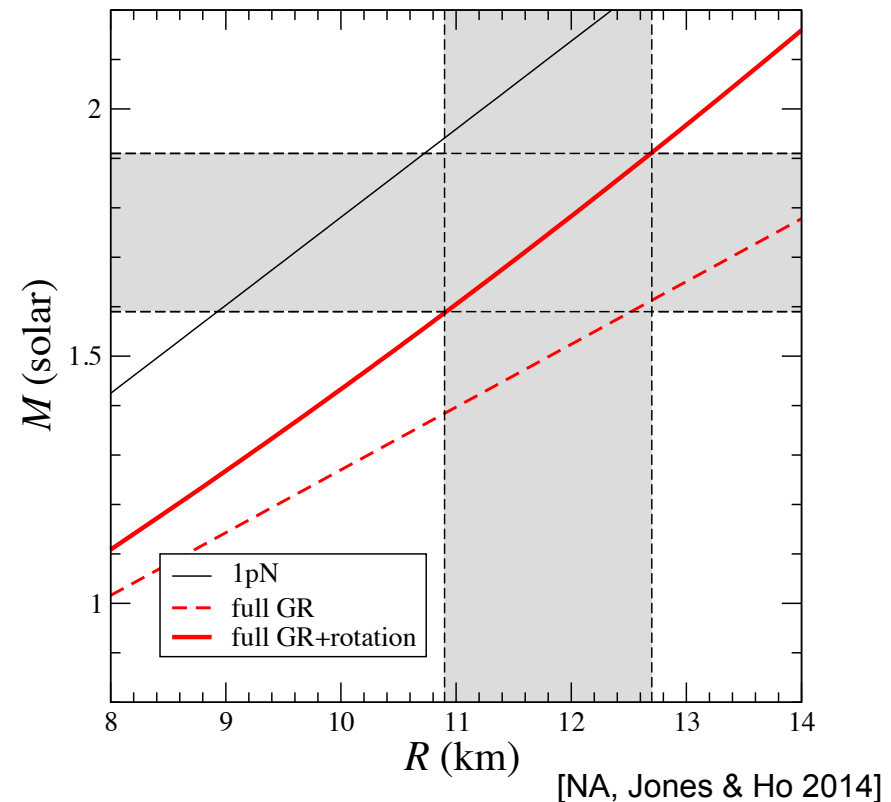
$$\mathbf{f} = \mathbf{0.5727597} \times \mathbf{f}_s$$

Provided a global oscillation modulates the light curve with the rotating frame frequency (?), the result is consistent with an r-mode once one accounts for relativistic corrections.

This would constrain the star's mass (making use of radius constraints from other X-ray burst sources).

However, the suggested amplitude is too large to be reconciled with the observed spin-evolution of the system.

There is **always** a spin-down penalty associated with r-mode excitation, even if the mode is stable.



# summary

The r-modes remains a “viable” gravitational-wave source and may be the mechanism that limits neutron star spin.

Instability window depends on core physics (composition/state of matter/transport coefficients).

Observations (in some sense) constrain theory, but...

Still have the same three questions as (nearly) 20 years ago:



1. Are the r-modes unstable in a realistic neutron star model?
2. Why does the growth of an unstable mode saturate and what is the achieved amplitude?
3. How does a star with an active instability evolve?