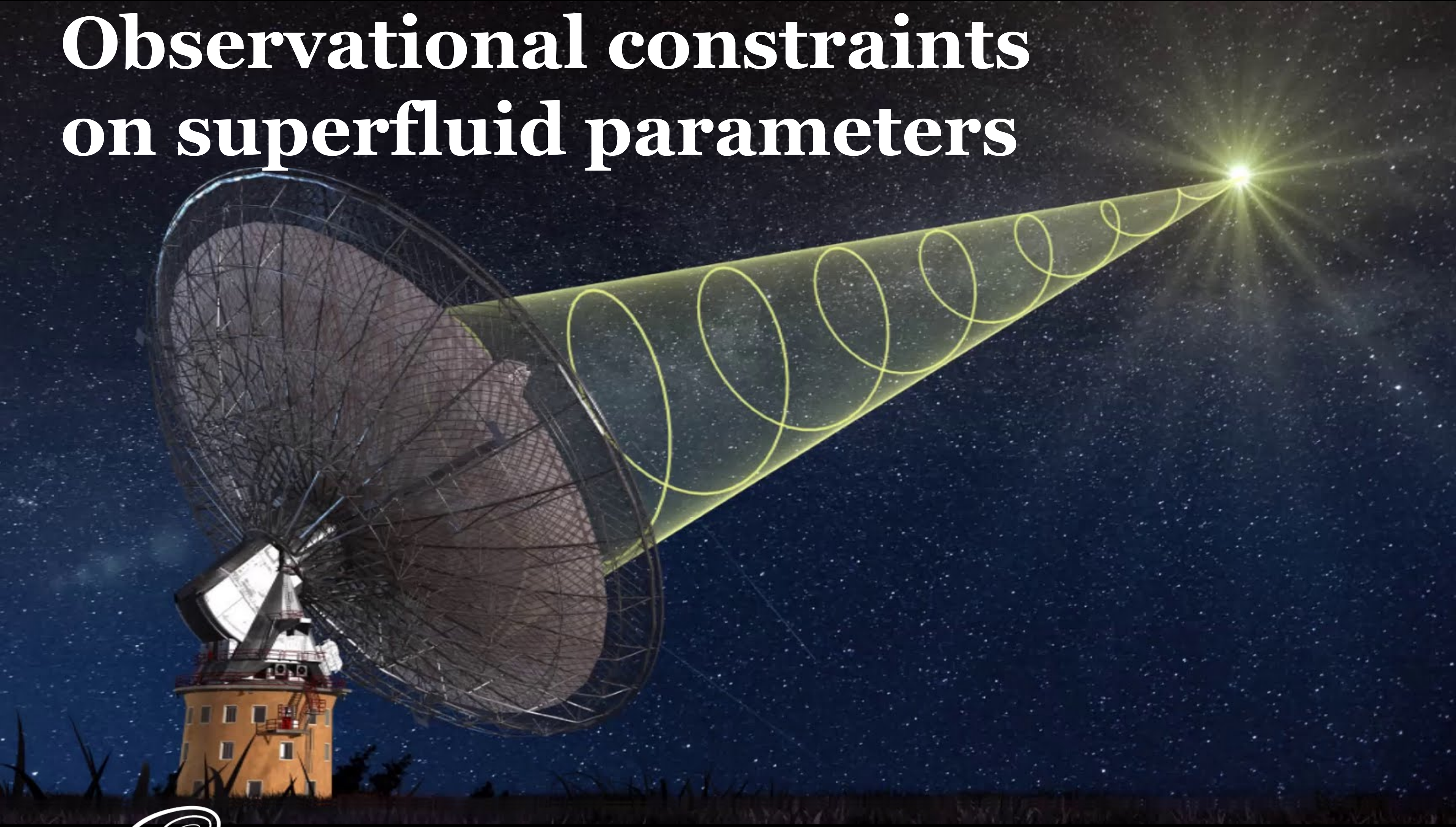
# Observational constraints on superfluid parameters



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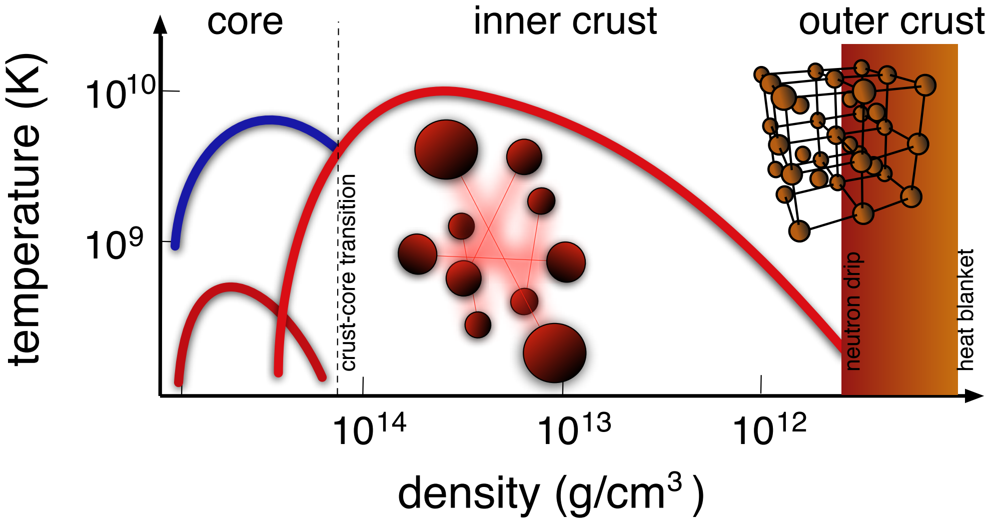
# scope

Mature neutron stars are "cold" ($10^8$K<< $T_{Fermi}$=$10^{12}$K) so they **should be** either solid or superfluid.



**Crust** – superfluid neutrons (singlet) coexist with nuclear lattice

**Outer core** – superfluid neutrons (triplet) coexist with superconducting protons

**Inner core** – possible exotic phases, like colour superconducting quarks

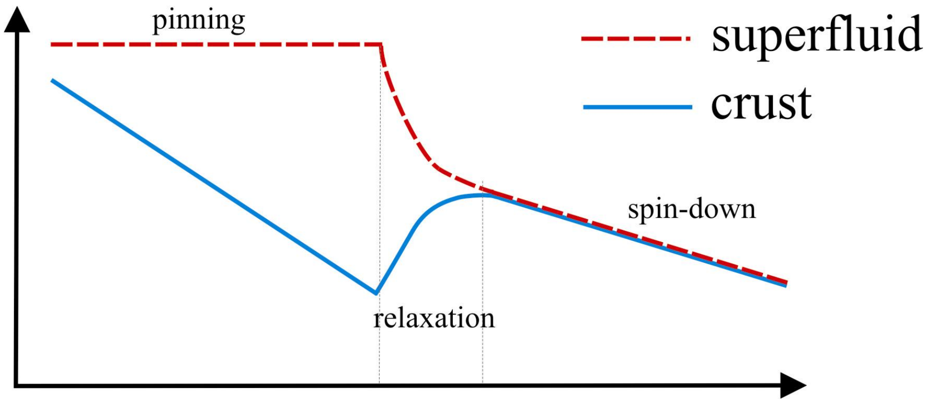
Anticipated since 1950's; nuclear physics calculations indicate "BCS-like" pairing gaps for neutrons and protons.

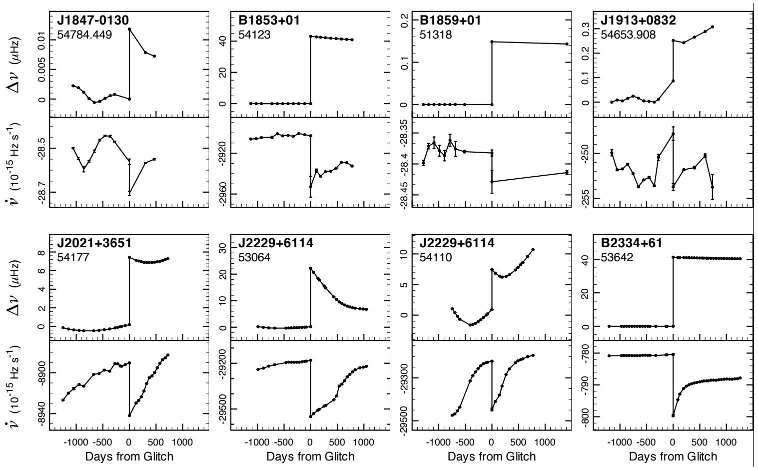
Evidence from cooling (the curious case of Cas A) and timing variability (pulsar glitches).

**Can we use observations to constrain theory?**

# glitches

“Standard” model for glitches involves transfer of angular momentum from an internal superfluid component (rotating via vortices) to the star’s crust.



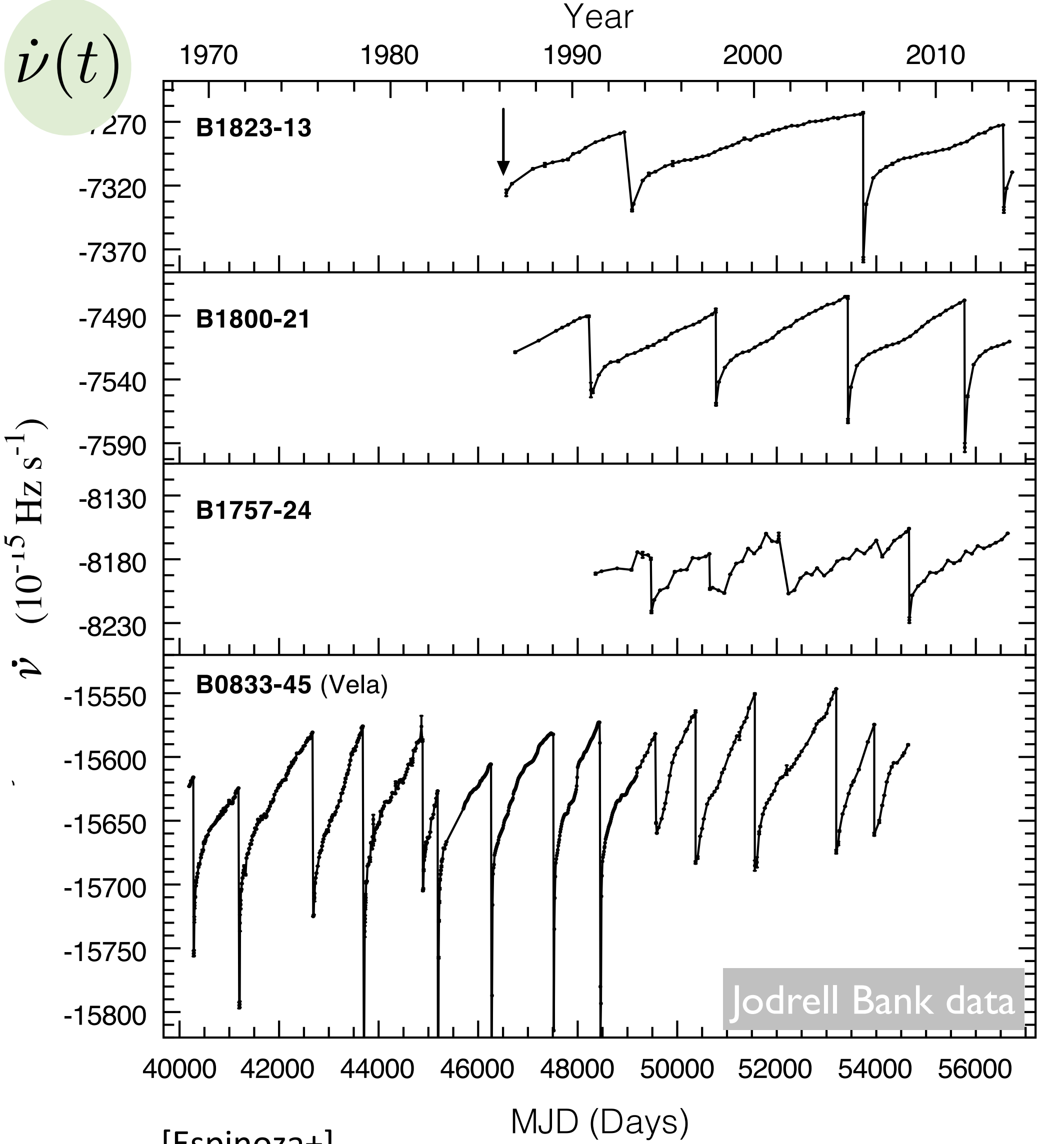


1. the crust slows down due to magnetic braking
2. the superfluid can only spin down if vortices move outwards
3. if the vortices are pinned (to the crust), the superfluid lags behind
4. at some critical level, a large number of vortices are released. As a result the crust is spun up.

[Espinoza+]

**No quantitative models** explain the range of observed behaviour…

$\dot{\nu}(t)$



[Espinoza+]

For systems that exhibit regular (large) glitches, like Vela, the data is “consistent” with a vortex “unpinning” model with a critical lag as trigger.

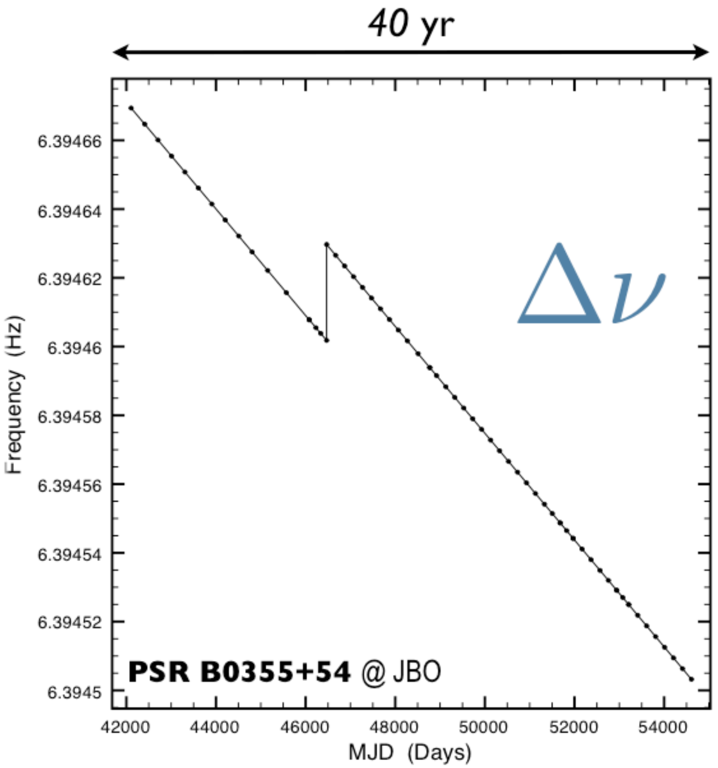
Suggest unpinning of vortices at relative rotation;

$$\Delta\Omega/\Omega_p \approx 5\times10^{-4}$$

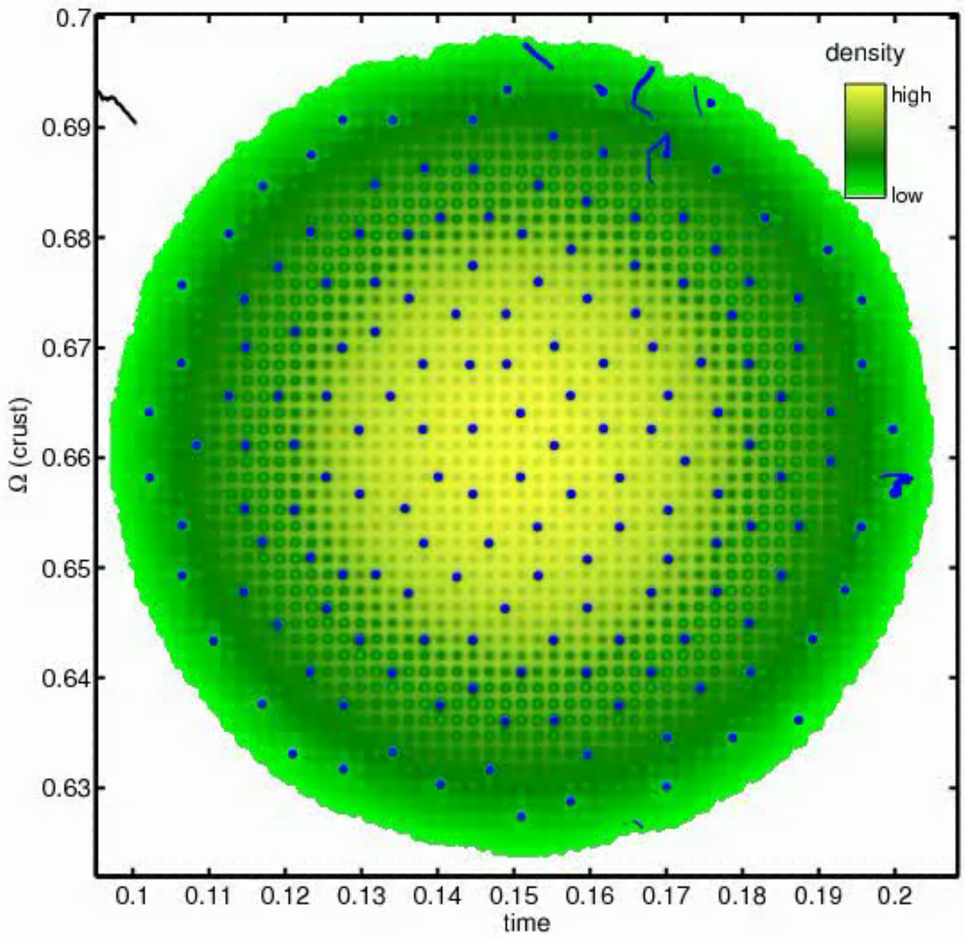
Still far from detailed picture;

What triggers the glitches in the first place?

How is the angular momentum transferred?



[Espinoza+]



[Haskell+]

Vortex simulations (Gross-Pitaevskii), suggest vortices move in “avalanches”. Would explain why glitches come in a distribution of sizes…

… but the dissipation mechanism in the model is ad hoc and there are questions about scaling the results to neutron stars (from fluid element to 10km).

# two-fluid model

Need hydrodynamics: Phenomenological model inspired by classic two-fluid model for superfluid Helium (atoms and the excitations, e.g. phonons).

- electrons/muons in the core are coupled (electromagnetically) to the protons on very short timescales
- vortices and fluxtubes are sufficiently dense that smooth-averaging can be performed

$$\partial_t n_{\mathrm{x}} + \nabla_i (n_{\mathrm{x}} v_{\mathrm{x}}^i) = 0$$

$$(\partial_t + v_{\mathrm{x}}^j \nabla_j) p_i^{\mathrm{x}} + \nabla_i (\Phi + \tilde{\mu}_{\mathrm{x}}) + \varepsilon_{\mathrm{x}} w_j^{\mathrm{yx}} \nabla_i v_{\mathrm{x}}^j = f_i^{\mathrm{x}}$$

where the relative velocity is $w_i^{\mathrm{yx}} = v_i^{\mathrm{y}} - v_i^{\mathrm{x}}$ and the momenta are given by

$$p_i^{\mathrm{x}} = v_i^{\mathrm{x}} + \varepsilon_{\mathrm{x}} w_i^{\mathrm{yx}}$$

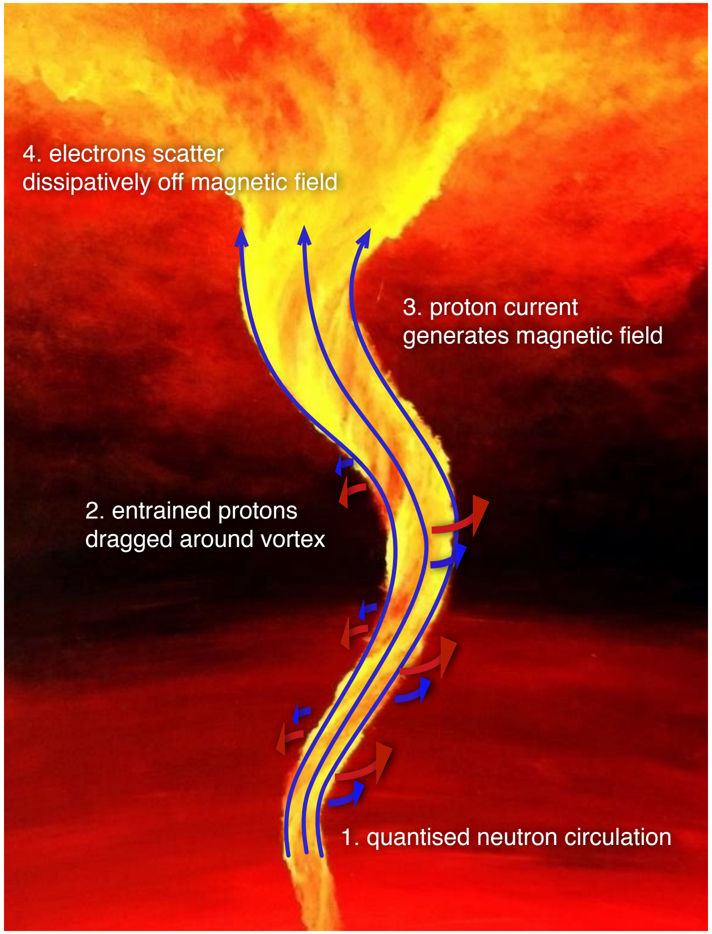
This encodes the **entrainment effect**, due to which the velocity of each fluid does not have to be parallel to its momentum.

Can be thought of in terms of an “effective mass”; $\rho_{\mathrm{p}} \varepsilon_{\mathrm{p}} = n_{\mathrm{p}} (m_{\mathrm{p}} - m_{\mathrm{p}}^*)$

# mutual friction

Compared to the Navier-Stokes equations, a multi-fluid system may have many additional dissipation channels (largely unexplored!).

In a superfluid, the presence of vortices leads to “mutual friction”.



Standard form (for a straight vortex array);

$$f_i^{\mathrm{mf}} = \frac{R}{1+R^2}\varepsilon_{ijk}\widehat{\omega}_{\mathrm{n}}^j \varepsilon^{klm}\omega_l^{\mathrm{n}} w_m^{\mathrm{np}} + \frac{R^2}{1+R^2}\varepsilon_{ijk}\omega_{\mathrm{n}}^j w_{\mathrm{np}}^k$$

where

$$\omega_{\mathrm{n}}^i = \varepsilon^{ijk}\nabla_j p_k^{\mathrm{n}}$$

– electron scattering off vortices leads to $R<<1$

– vortex/fluxtube interaction may lead to a stronger effect (velocity dependent)

# relaxation

Usual form for mutual friction leads to a model that predicts that the system evolves according to

$$\begin{matrix} n_\mathrm{n}\partial_t p_i^\mathrm{n} + \ldots = f_i \\ n_\mathrm{p}\partial_t p_i^\mathrm{p} + \ldots = -f_i \end{matrix} \quad \rightarrow \quad \frac{m_\mathrm{p}^*}{m_\mathrm{p}}\partial_t w_i^\mathrm{np} + \ldots \approx -\frac{B\kappa n_\mathrm{v}}{x_\mathrm{p}} w_i^\mathrm{np}$$

following a glitch event. This corresponds to a typical coupling timescale

$$t_d \approx 10P(s)\left(\frac{m_\mathrm{p}^*}{m_\mathrm{p}-m_\mathrm{p}^*}\right)^2\left(\frac{x_\mathrm{p}}{0.05}\right)^{-1/6}\left(\frac{\rho}{10^{14}\ \mathrm{g/cm^3}}\right)^{-1/6}$$

Much faster than the observed relaxation time in, for example, the Vela pulsar (weeks/months), so glitches may not be associated with the core…

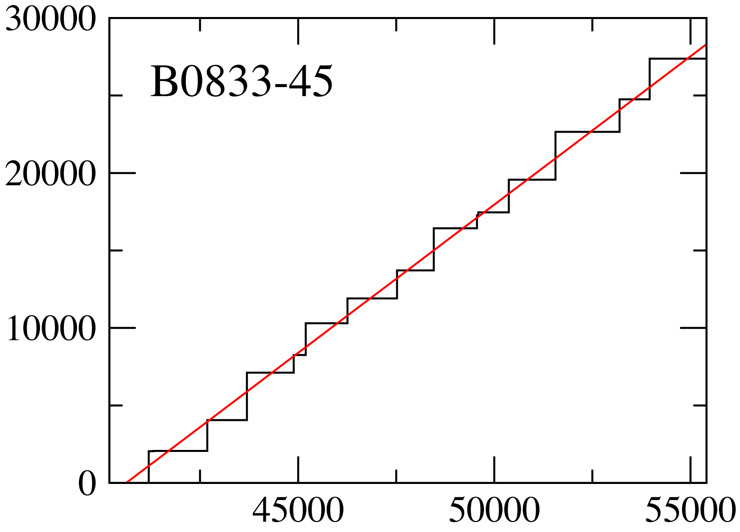
The standard view is that glitches are a manifestation of the (singlet) superfluid that permeates the crust. The interaction with the crust nuclei is expected to provide the required vortex pinning.

# the crust is not enough

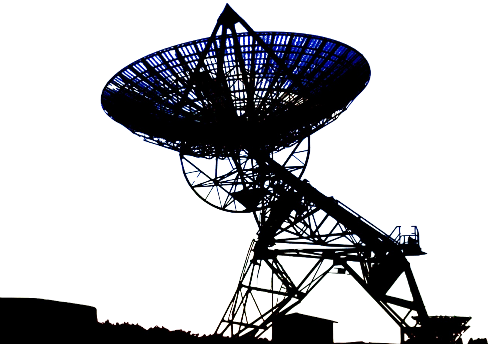
For systems that glitch regularly, one can estimate the moment of inertia of the superfluid component.

Need to involve up to 2% of the total moment of inertia.

The **crust superfluid** would be sufficient to explain the observations; as long as we do not worry about entrainment.



However, the large effective neutron mass in the crust (due to Bragg scattering of neutrons by the nuclear lattice) lowers the effective superfluid moment of inertia by a factor of 5 or so. This is problematic.



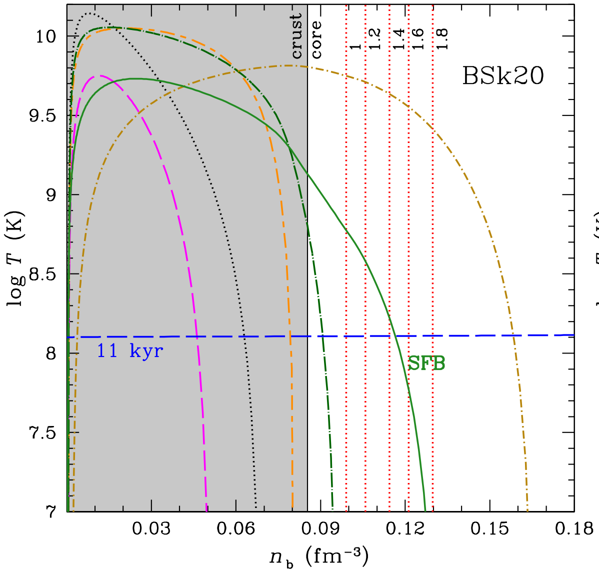
1. A fraction of the **core superfluid** could be involved, but why would the glitches be “the same size”?
2. The (singlet) pairing gap could lead to a smaller superfluid region, just large enough to explain the observations.
3. Lack of “precision”: Need more accurate “parameters”.

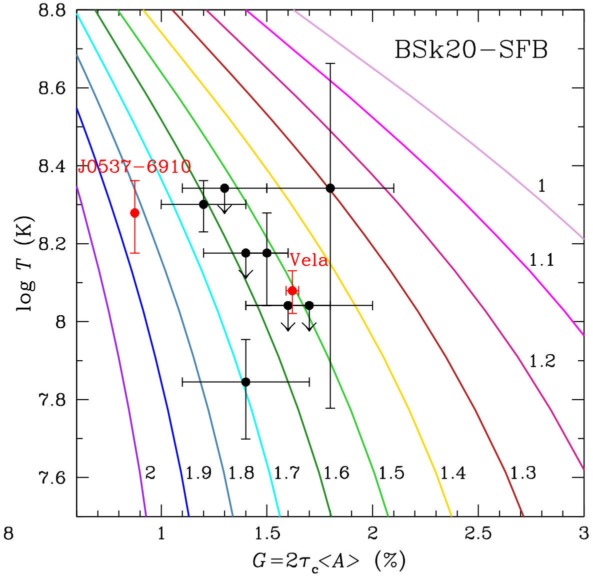
# mind the gap

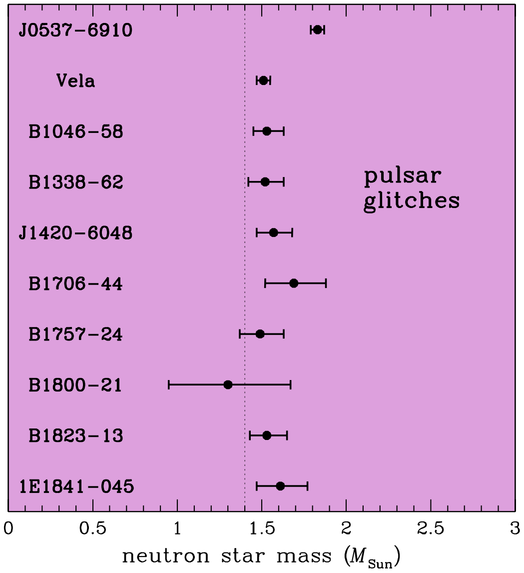
Possible resolution: Involve only the singlet superfluid in the crust + outer region of the core.

The data can then be turned into a constraint on the superfluid pairing gap (provided one has some idea of the star’s temperature, and assuming that the angular momentum reservoir is exhausted in each glitch event).

Interestingly, most available gap models **fail** this test.





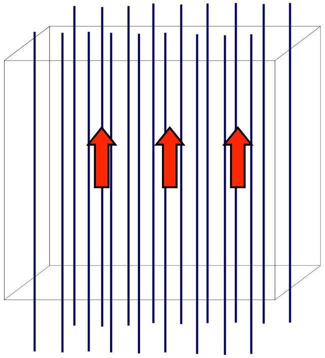


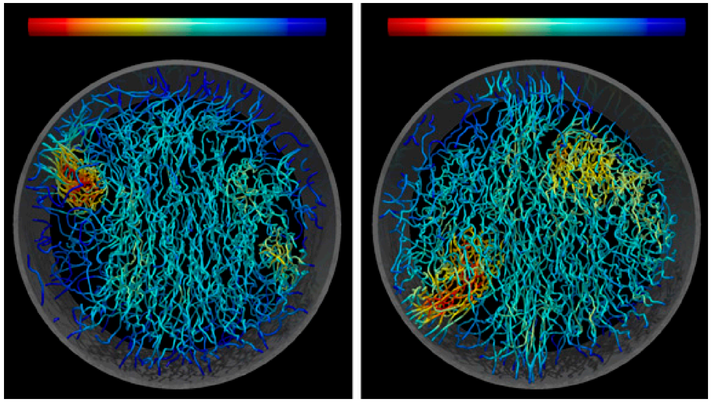
If we take the pairing gap as given, we can **infer the mass** of a glitching pulsar.

# turbulence

Unfortunately… the vortices are unlikely to form a regular array.

If there is a large scale flow along the vortex array, then short wavelength inertial modes become unstable (Glaberson-Donnely).





The system becomes turbulent (overwhelming evidence from experiments), and the mutual friction may have a different form;

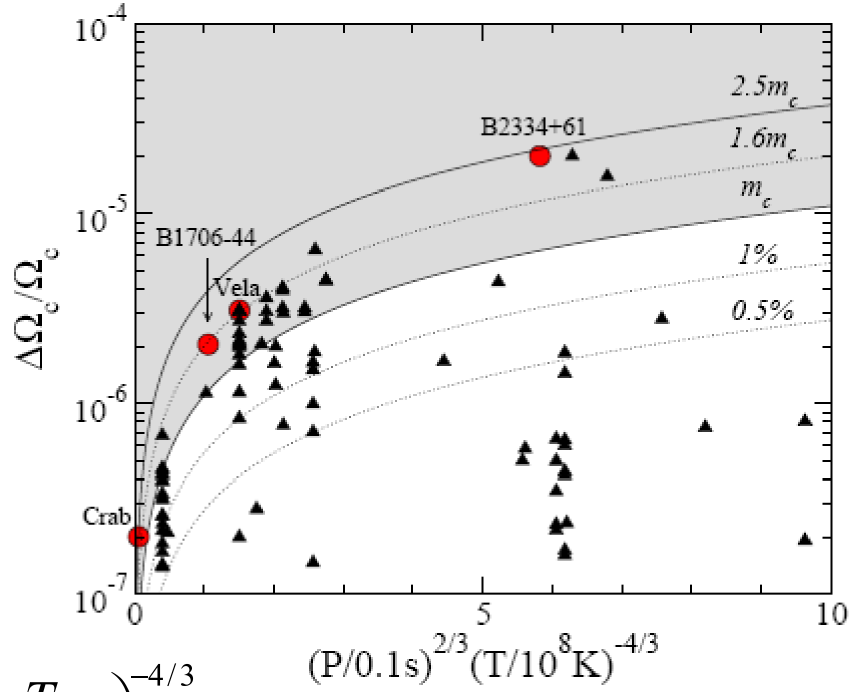
$$f_i = \frac{8\pi^2 \rho_{\mathrm{n}}}{3\kappa} \left( \frac{\chi_1}{\chi_2} \right)^2 B^3 w_{\mathrm{pn}}^2 w_i^{\mathrm{pn}}$$

Leads to non-exponential relaxation (locally)…

**Need** to understand polarised turbulence (more complicated "averaging").

# superfluid instability

Global r-mode calculation for model with mutual friction and different background rotation rates shows that short wave-length become **dynamically** unstable beyond critical rotational lag in system with **strong** coupling.



Balance mode growth and shear viscosity damping to get;

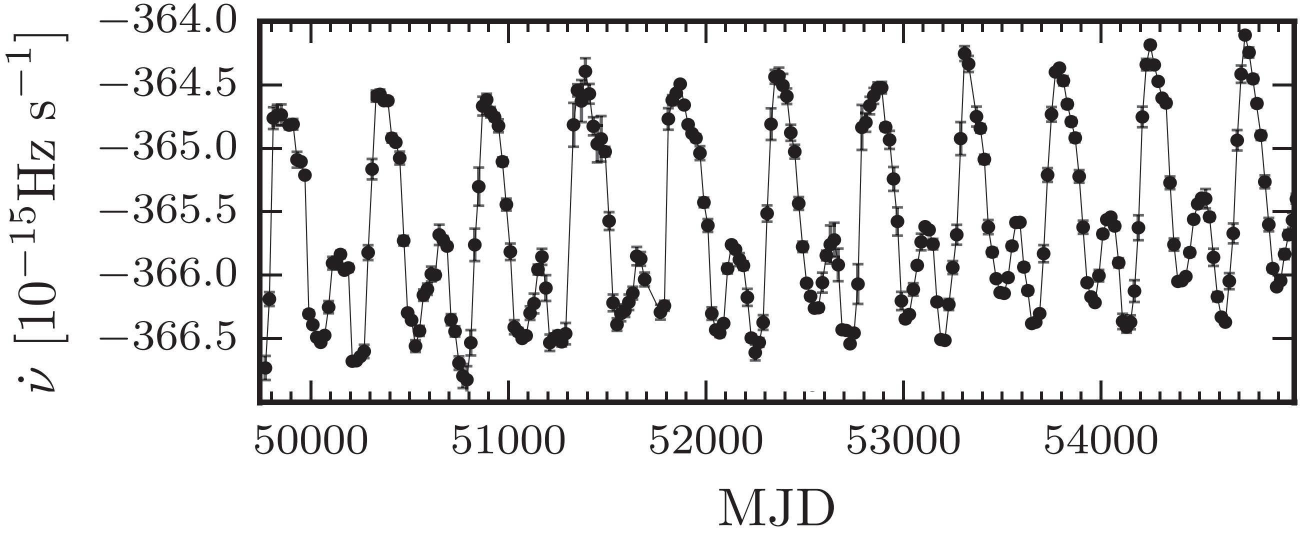
$$\left.\frac{\Omega_n-\Omega_p}{\Omega_p}\right|_{\text{critical}}\approx 6\times10^{-5}\left(\frac{P}{0.1\text{s}}\right)^{2/3}\left(\frac{T}{10^8\,\text{K}}\right)^{-4/3}$$

Plausible link to the mechanism that triggers pulsar glitches and the onset of vortex turbulence.

# free precession

Free precession is the most general motion of a solid body. (“Chandler wobble”)

Neutron star will precess if the crust is deformed in some way. Expect small deformations and long period precession.



[Ashton+]

Strongest observational evidence (?): 1009d (or 500d) periodicity in PSR B1828-11

Since the precession motion is a normal mode of the coupled core-crust system it depends on the interior dynamics and the presence of superfluidity.

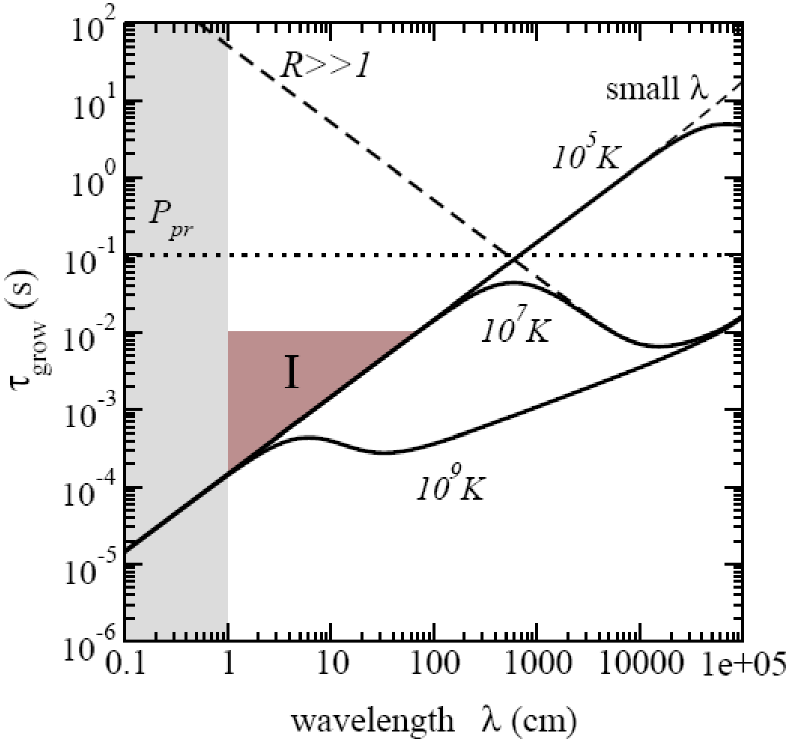
# fast precession?

Long period precession may not be possible if there is significant pinning between vortices and magnetic fluxtubes in the star's core.

Perhaps the core is not a type II superconductor, after all?

Local analysis shows that short wavelength waves may be unstable in a precessing star.

Strong coupling/fast precession motion is generically unstable.



May explain why precessing neutron stars are rare.

**Need** to consider the hydrodynamics asssociated with precession. This is a very hard problem given the range of timescales involved.

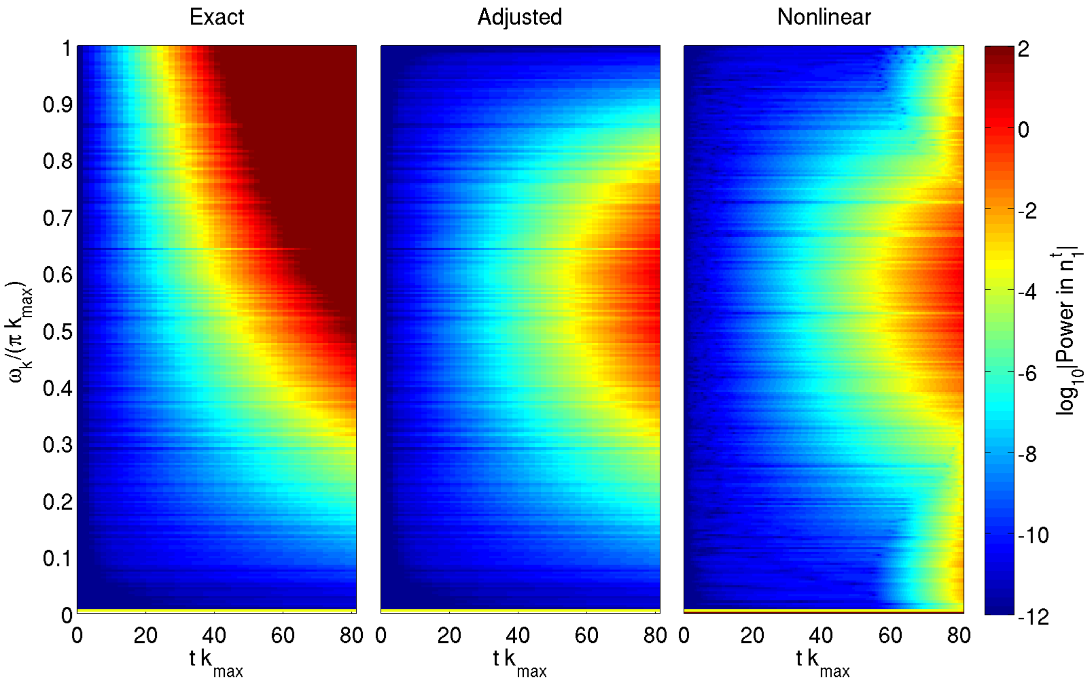
# two-stream instability

Lesson: Superfluid systems with relative flow are generically unstable.

— similar to the two-stream instability known to operate in plasmas

— analogous to the Kelvin-Helmholtz instability, although here the two fluids are **interpenetrating**

— sets in once the relative flow between the two components of the system reaches a critical level

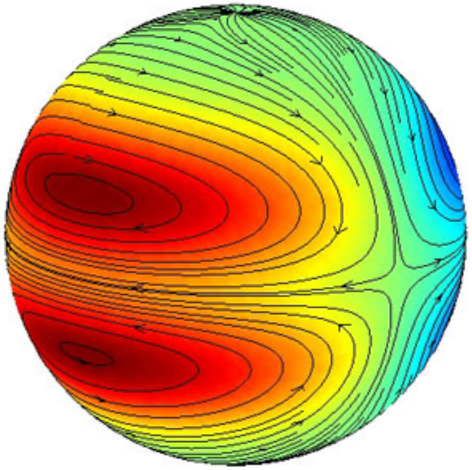


Simulations suggest the instability develops as in the linear case.

No evidence of nonlinear saturation.

**Need** to explore the role of dissipation.

# seismology



Neutron stars have a zoo of oscillation modes, more or less directly associated with the various "restoring forces" in the system.

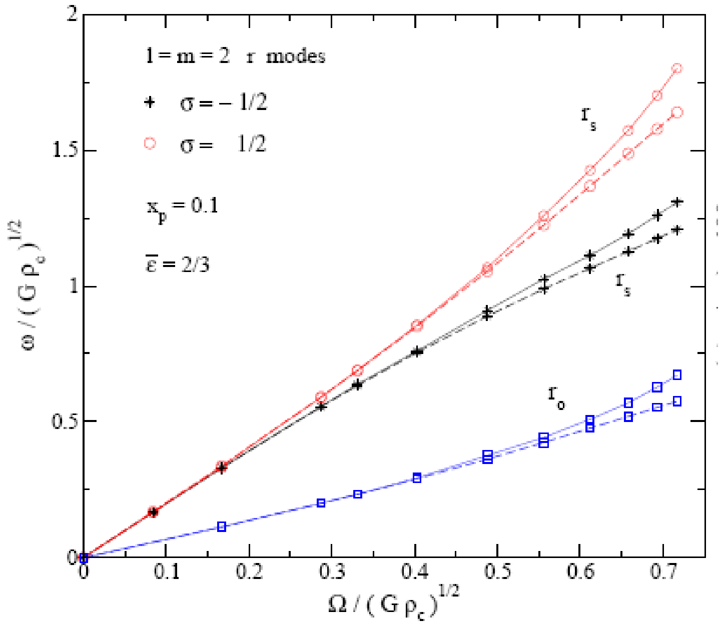
In principle, observations can be used to probe the star's interior.

Requires detailed models with as "realistic" physics as possible.

Superfluids have additional degrees of freedom (cf. second sound).

- Acoustic modes restored by pressure.
- Superfluid modes restored by deviation from chemical equilibrium.

**Need** consistent interior composition and superfluid parameters.

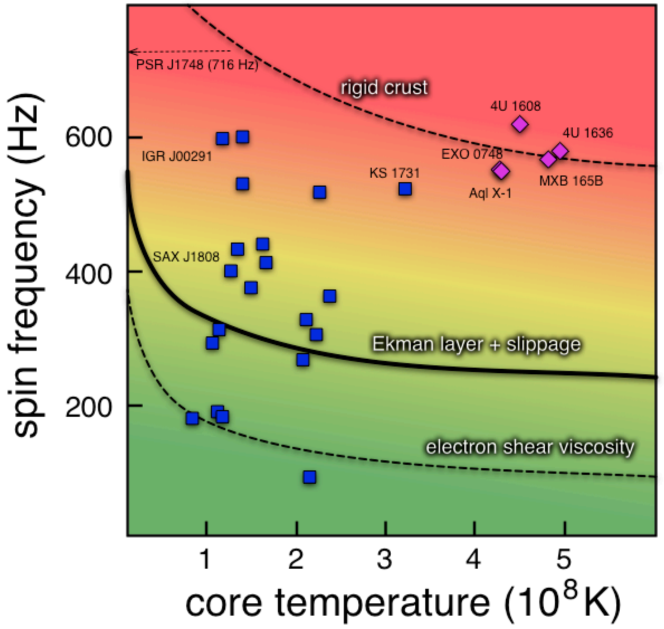


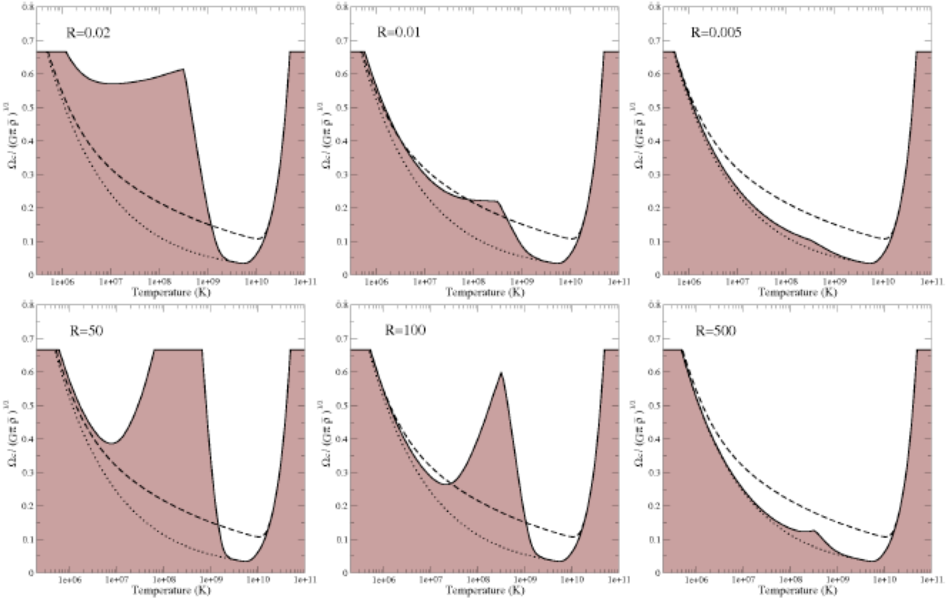
# the r-modes

Given the “best estimate” for the main r-mode damping mechanisms, many observed accreting neutron stars in LMXBs **should be** unstable.

Saturation amplitude due to mode-coupling is too large to allow evolution far into instability region.

The magnetic field may play an important role, even if it is too weak to affect the nature of the r-mode itself.





Stronger than expected mutual friction could, in principle, provide an explanation, but...

**Need** to understand the microphysics.

# final thoughts

Superfluidity impacts on both the gradual evolution (cooling/spindown/magnetic field decay) of neutron stars and their dynamics.

Strong evidence for the presence of superfluid components from pulsar glitches, and one can make interesting inferences from the data (weighing isolated stars?) but detailed modelling remains a real challenge.