Quark matter in compact stars: the Constant Sound Speed parameterization

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Alford, Han, Prakash, arXiv:1302.4732
Alford, Burgio, Han, Taranto, Zappalà, arXiv:1501.07902
Ranea-Sandoval, Han, Orsaria, Contrera, Weber, Alford, arXiv:1512.09183

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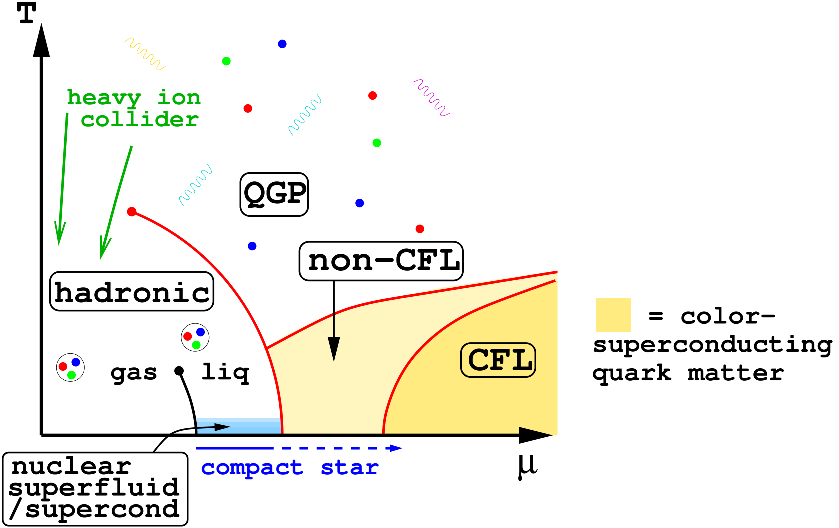
Schematic QCD phase diagram



M. Alford, K. Rajagopal, T. Schäfer, A. Schmitt, arXiv:0709.4635 (RMP review)

A. Schmitt, arXiv:1001.3294 (Springer Lecture Notes)

Signatures of quark matter in compact stars

Observable ← Microphysical properties (and neutron star structure) ← Phases of dense matter

	Property	Nuclear phase	Quark phase
mass, radius	eqn of state $\varepsilon(p)$	known up to $\sim n_{\rm sat}$	unknown; many models
spindown (spin freq, age)	bulk viscosity shear viscosity	Depends on phase:	Depends on phase:
		$n\,p\,e$	unpaired
cooling (temp, age)	heat capacity neutrino emissivity thermal cond.	$n\,p\,e,\mu$ $n\,p\,e,\Lambda,\ \Sigma^-$ n superfluid	CFL CFL-K^0 2SC
glitches (superfluid, crystal)	shear modulus vortex pinning energy	p supercond π condensate K condensate	CSL LOFF 1SC …

Constraining QM EoS by observing $M(R)$

There is lots of literature about specific models of quark matter, e.g.

- MIT Bag Model; (Alford, Braby, Paris, Reddy, `nucl-th/0411016`)
- NJL models; (Paoli, Menezes, `arXiv:1009.2906`; Bonanno, Sedrakian, `arXiv:1108.0559`)
- PNJL models (Blaschke et. al, `arXiv:1302.6275`; Orsaria et. al.; `arXiv:1212.4213`)
- hadron-quark NLσ model (Negreiros et. al., `arXiv:1006.0380`)
- 2-loop perturbation theory (Kurkela et. al., `arXiv:1006.4062`)
- MIT bag, NJL, CDM, FCM, DSM (Burgio et. al., `arXiv:1301.4060`)

We need a model-independent parameterization of the quark matter EoS:

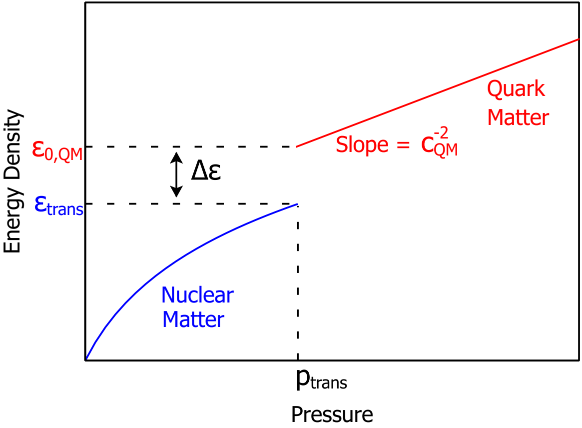
- framework for relating different models to each other
- observational constraints can be expressed in universal terms

CSS: a fairly generic QM EoS

Model-independent parameterization with

- Sharp 1st-order transition
- Constant [density-indp] Speed of Sound (CSS)

$$\varepsilon(p) = \varepsilon_{\rm trans} + \Delta\varepsilon + c_{\rm QM}^{-2}(p - p_{\rm trans})$$



QM EoS params:

$p_{\rm trans}/\varepsilon_{\rm trans}$

$\Delta\varepsilon/\varepsilon_{\rm trans}$

$c^2_{\rm QM}$

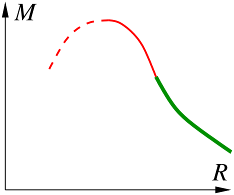
Zdunik, Haensel, arXiv:1211.1231; Alford, Han, Prakash, arXiv:1302.4732

Hybrid star $M(R)$

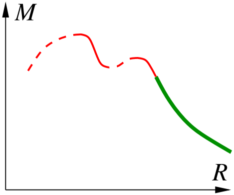
Hybrid star branch in $M(R)$ relation has 4 typical forms

$\Delta\varepsilon < \Delta\varepsilon_{\mathrm{crit}}$
small energy density jump at phase transition

"Connected"

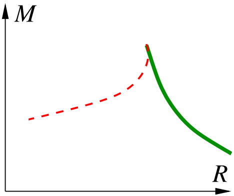


"Both"

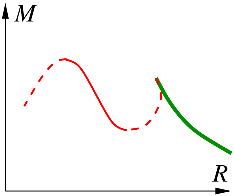


$\Delta\varepsilon > \Delta\varepsilon_{\mathrm{crit}}$
large energy density jump at phase transition

"Absent"

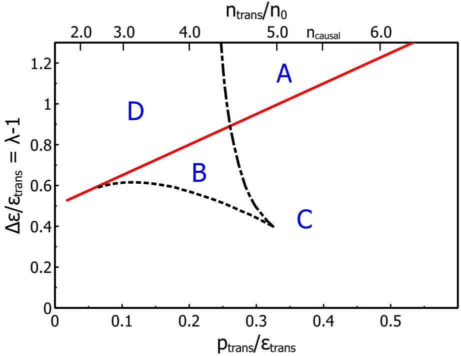


"Disconnected"

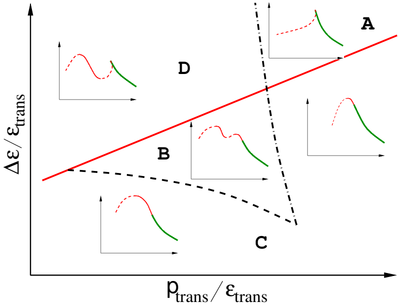


CSS "Phase diagram" of hybrid star $M(R)$

Soft NM + CSS($c^2_{\mathrm{QM}} = 1$)



Schematic



Above the red line ($\Delta\varepsilon > \Delta\varepsilon_{\mathrm{crit}}$), connected branch disappears

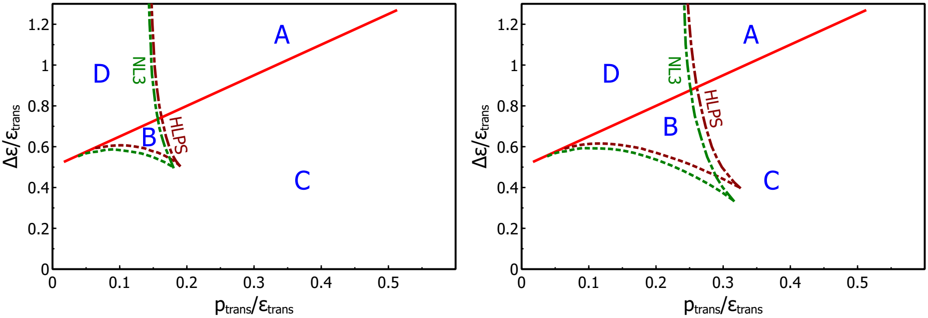
$$\frac{\Delta\varepsilon_{\mathrm{crit}}}{\varepsilon_{\mathrm{trans}}} = \frac{1}{2} + \frac{3}{2}\frac{p_{\mathrm{trans}}}{\varepsilon_{\mathrm{trans}}}$$

(Seidov, 1971; Schaeffer, Zdunik, Haensel, 1983; Lindblom, gr-qc/9802072)

Disconnected branch exists in regions D and B.

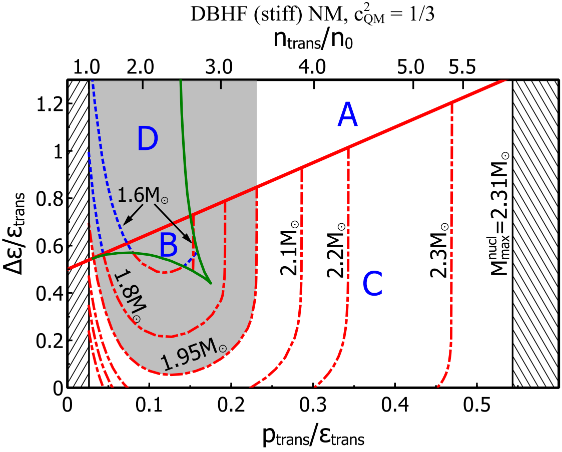
Sensitivity to NM EoS and $c^2_{\rm QM}$

$c^2_{\rm QM} = 1/3$ $\qquad$ $c^2_{\rm QM} = 1$



- NM EoS (HLPS=soft, NL3=hard) does not make much difference.
- Higher $c^2_{\rm QM}$ favors disconnected branch.

Constraints on QM EoS from $M_{\rm max}$



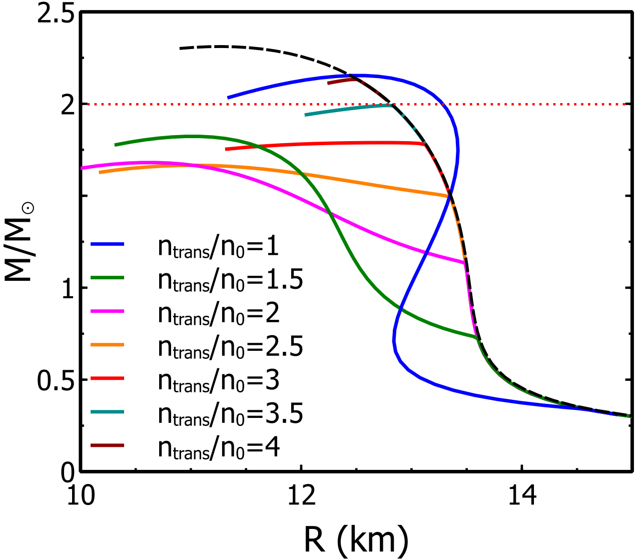
- Increasing $\Delta\varepsilon$ reduces $M_{\rm max}$
- Increasing $p_{\rm trans}$ at first reduces then increases $M_{\rm max}$

$2\,M_\odot$ observation allows two scenarios:

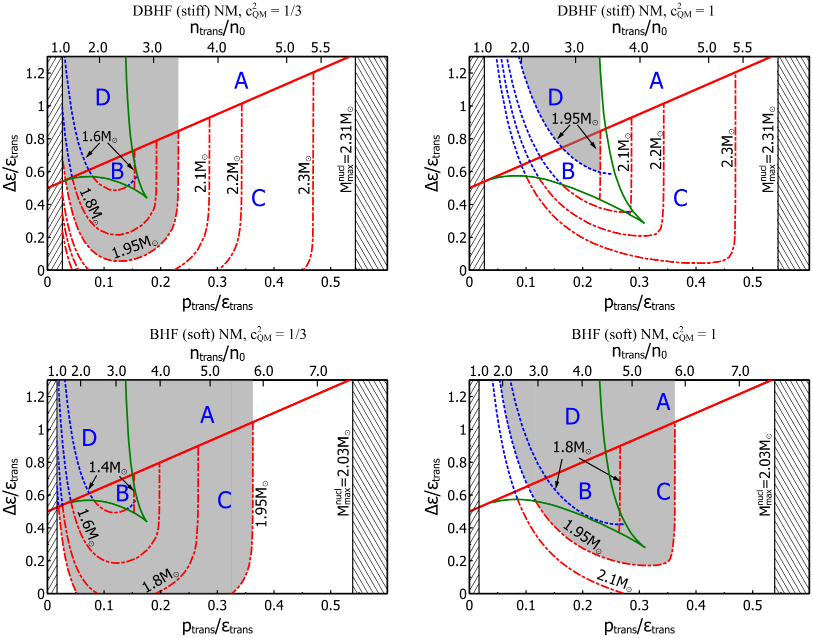
- high $p_{\rm trans}$: very small connected branch
- low $p_{\rm trans}$: modest $\Delta\varepsilon$, no disconnected branch.

Low p_{trans} and high p_{trans} windows

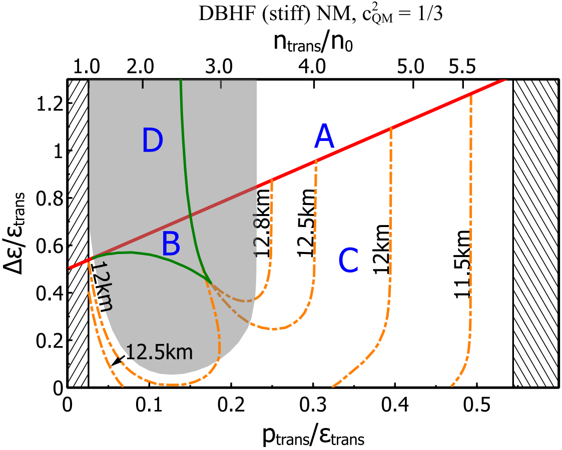
DBHF (stiff) NM, $c^2_{QM} = 1/3$, $\Delta\varepsilon/\varepsilon_{trans} = 0.4$



Constraints on QM EoS from $M_{\max}$



Radius of heaviest star $R_{\mathrm{max}M}$

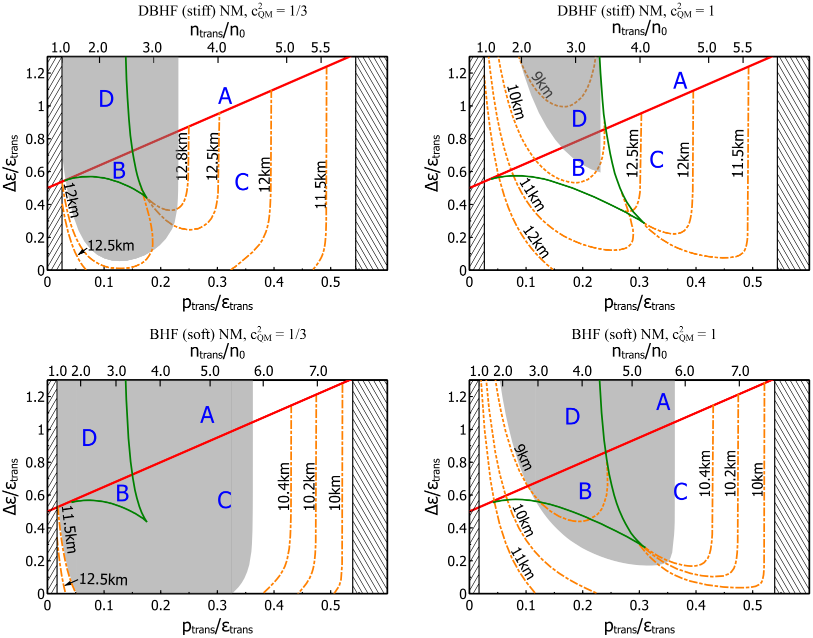


Heaviest star is typically the smallest, so lower limit on $R_{\mathrm{max}M}$ is the minimum radius of compact stars.

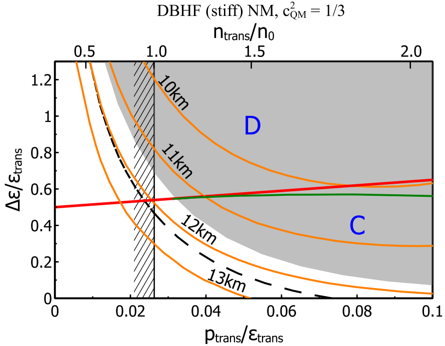
High p_{trans}: very short connected hybrid branch, radius like that of heaviest hadronic star.

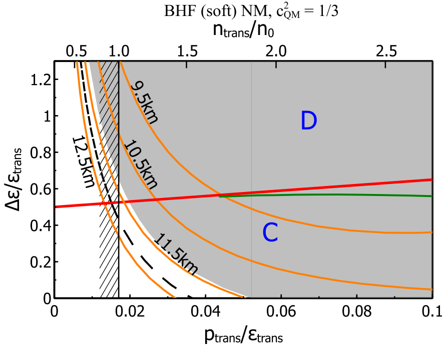
Low p_{trans}: need to zoom in.

Constraints on QM EoS from $R_{\max M}$



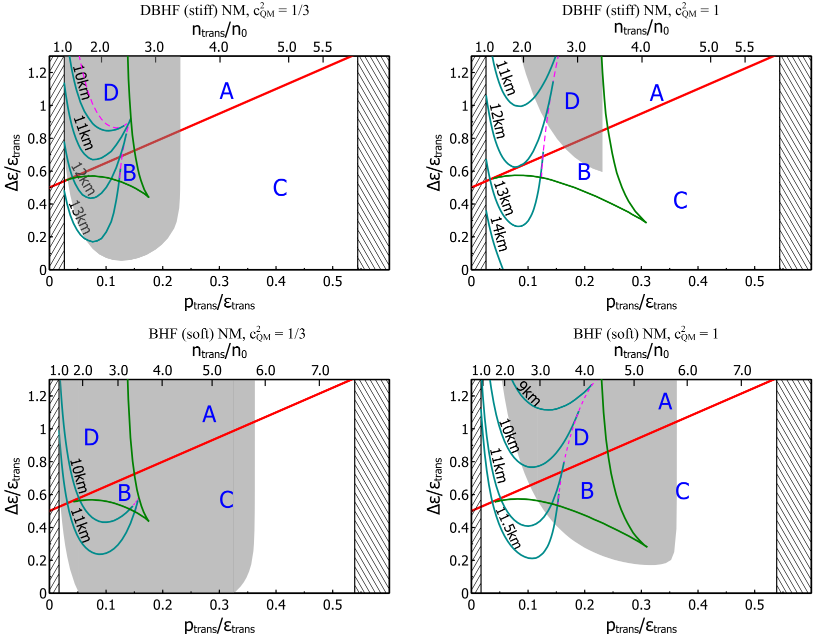
Focus on low p_{trans} and $c^2_{\mathrm{QM}} = 1/3$



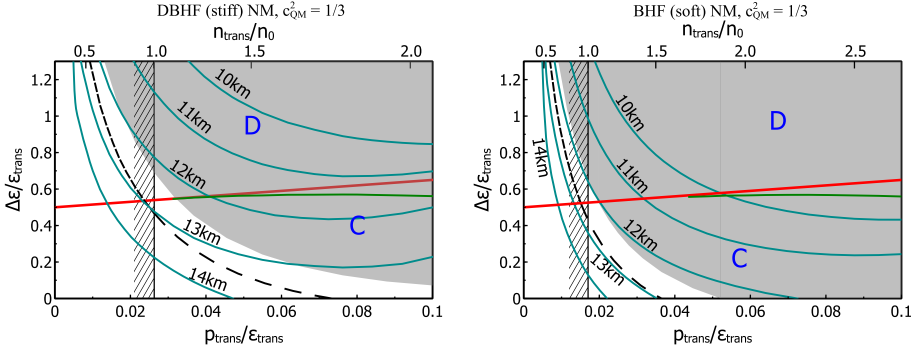


- $R_{\mathrm{max}M}$ contours closely follow mass contours
- $M_{\mathrm{max}} > 1.95\, M_\odot$ requires $R > 11.25\,\mathrm{km}$
- dashed line is $M_{\mathrm{max}} = 2.1\, M_\odot$, requires $R > 12.1\,\mathrm{km}$
- Observation of a smaller star $\Rightarrow$ high transition pressure or $c^2_{\mathrm{QM}} > 1/3$

Constraints on QM EoS from $R_{1.4\,M_\odot}$



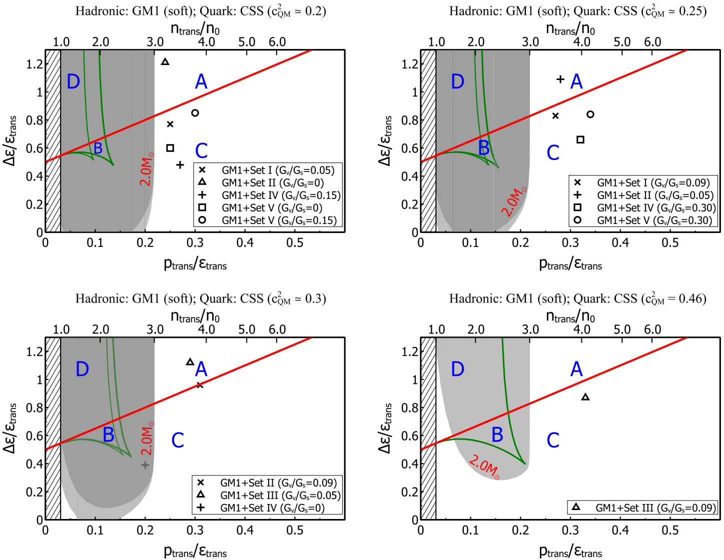
Low transition pressure and $R_{1.4\,\mathrm{M}_\odot}$



- $R_{1.4\,\mathrm{M}_\odot}$ contours roughly follow mass contours
- $M_{\mathrm{max}} > 1.95\,M_\odot$ requires $R_{1.4\,\mathrm{M}_\odot} > 12\,\mathrm{km}$ $(n_{\mathrm{trans}} \approx n_0)$, rising with n_{trans}.
- dashed line is $M_{\mathrm{max}} = 2.1\,M_\odot$, requires $R_{1.4\,\mathrm{M}_\odot} > 12.7\,\mathrm{km}$
- Observation of a smaller $1.4\,\mathrm{M}_\odot$ star $\Rightarrow$ $c^2_{\mathrm{QM}} > 1/3$.
- If p_{trans} is high then no hybrid stars have mass $1.4\,\mathrm{M}_\odot$

compare Lattimer `arXiv:1305.3510`: $R > 11\,\mathrm{km}$.

NJL models in CSS space



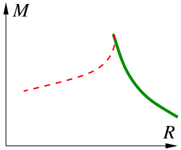
Summary of CSS

- CSS (Constant Speed of Sound) is a generic parameterization of the EoS close to a sharp first-order transition to quark matter.
- Any specific model of quark matter with such a transition corresponds to particular values of the CSS parameters
  $(p_{\rm trans}/\varepsilon_{\rm trans}, \quad \Delta\varepsilon/\varepsilon_{\rm trans}, \quad c^2_{\rm QM})$.
  Its predictions for hybrid star branches then follow from the generic CSS phase diagram.
- Every observation, e.g. observing a $2M_\odot$ neutron star, $\Rightarrow$ constraint on CSS parameters.
  E.g., for soft NM we need $c^2_{\rm QM} \gtrsim 1/3$
  (But note that $c^2_{\rm QM} = 1/3 - \mathcal{O}(\alpha_s)$ in pert QCD).
- More measurements of M and R would strengthen the constraints.
- Models of quark matter tend to have $c^2_{\rm QM} \sim 1/3$ and high transition pressure $\Rightarrow$ very short hybrid branch.

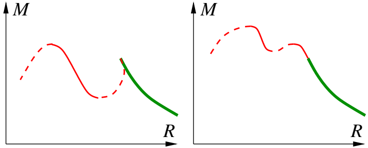
Could we identify hybrid stars via $M(R)$?

We could identify a phase transition to a high-density phase

(A) Nuclear branch ends with $dM/dR \neq 0$
occurs if $\Delta\varepsilon/\varepsilon_{\rm trans}$ is large enough



(B,D) Disconnected branch
can occur with $M_{\rm max} \gtrsim 2M_\odot$ if nuclear *and* quark matter are both stiff ($c^2_{\rm QM} \sim 1$)



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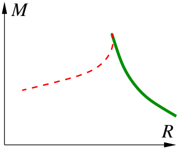

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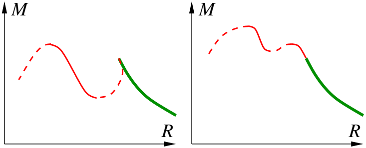


<u>We need</u>:

- better measurements of M and R
- knowledge of nuclear matter EoS

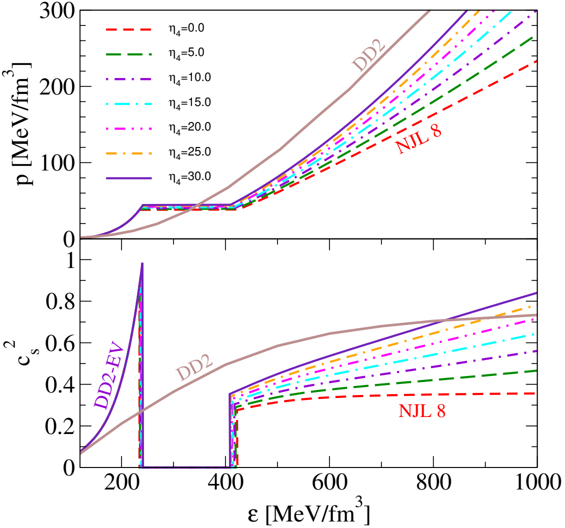
<u>We could benefit from</u>:

- theoretical constraints on parameters of QM EoS ($p_{\rm trans}/\varepsilon_{\rm trans}$, $\Delta\varepsilon/\varepsilon_{\rm trans}$, $c^2_{\rm QM}$)

Density-independent c^2_{QM} ?

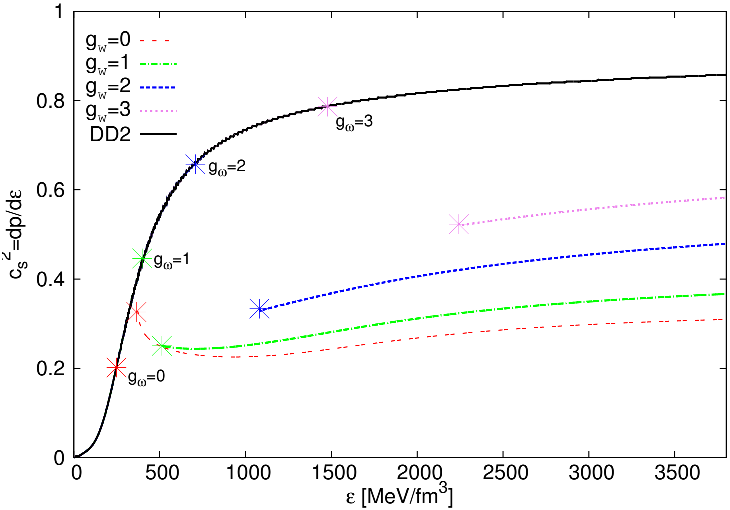
DD2-EV(nuclear) + NJL with 8 quark interactions

(Blaschke et al., `arXiv:1411.2856`)



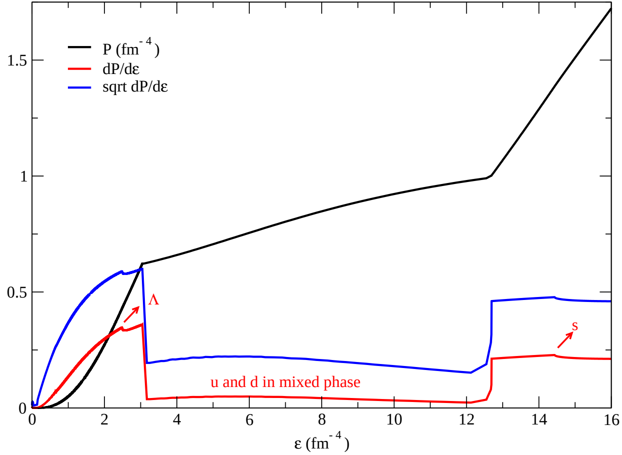
Density-independent $c^2_{\rm QM}$?

$SU(3)$ quark-meson model (quarks + Yukawa interaction via scalar and vector mesons)   (Schaffner-Bielich et al, `arXiv:1510.00180`)



Density-independent $c^2_{\rm QM}$?

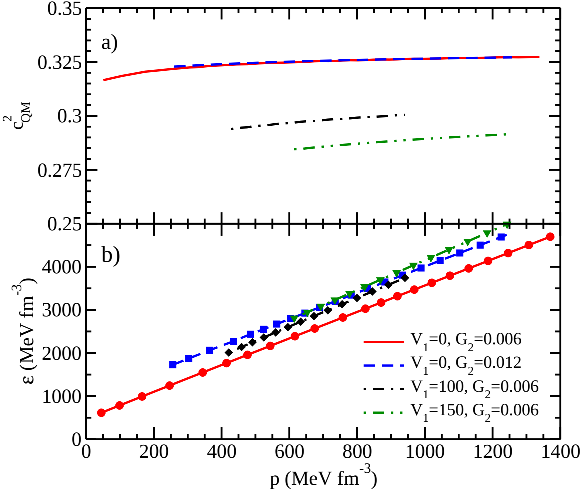
Chiral Mean Field model (quarks wuth Yukawa coupling to isoscalar and isovector mesons)   (e.g. Schramm, Dexheimer, Negreiros, arXiv:1508.04699)



Density-independent c^2_{QM} ?

Field Correlator Method

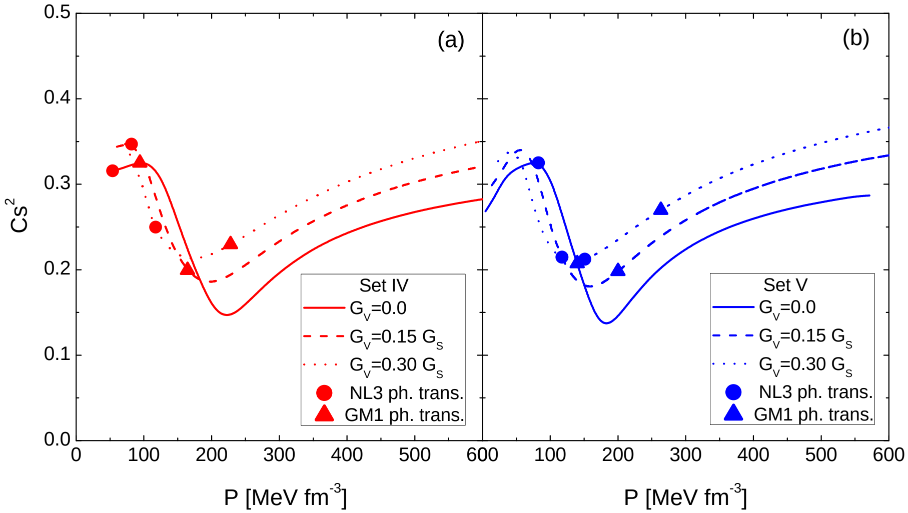
(Simonov and Trusov, hep-ph/0703228)



Density-independent $c_{\rm QM}^2$?

Local NJL model with vector repulsive interaction

(Orsaria, Rodrigues, Weber, Contrera, arXiv:1308.1657)



Density-independent $c^2_{\rm QM}$?

Non-local NJL model with vector repulsive interaction

(Orsaria, Rodrigues, Weber, Contrera, `arXiv:1308.1657`)