Bayesian Analysis for ${}^{7}_{4}\mathrm{Be} + \mathrm{p} \to {}^{8}_{5}\mathrm{B} + \gamma$ Based on Effective Field Theory

Xilin Zhang
University of Washington

In collaboration with K. Nollett (San Diego State U.) and D. Phillips (Ohio U.)

INT Program INT-16-2a, "Bayesian Methods in Nuclear Physics", June, 2016

Outline

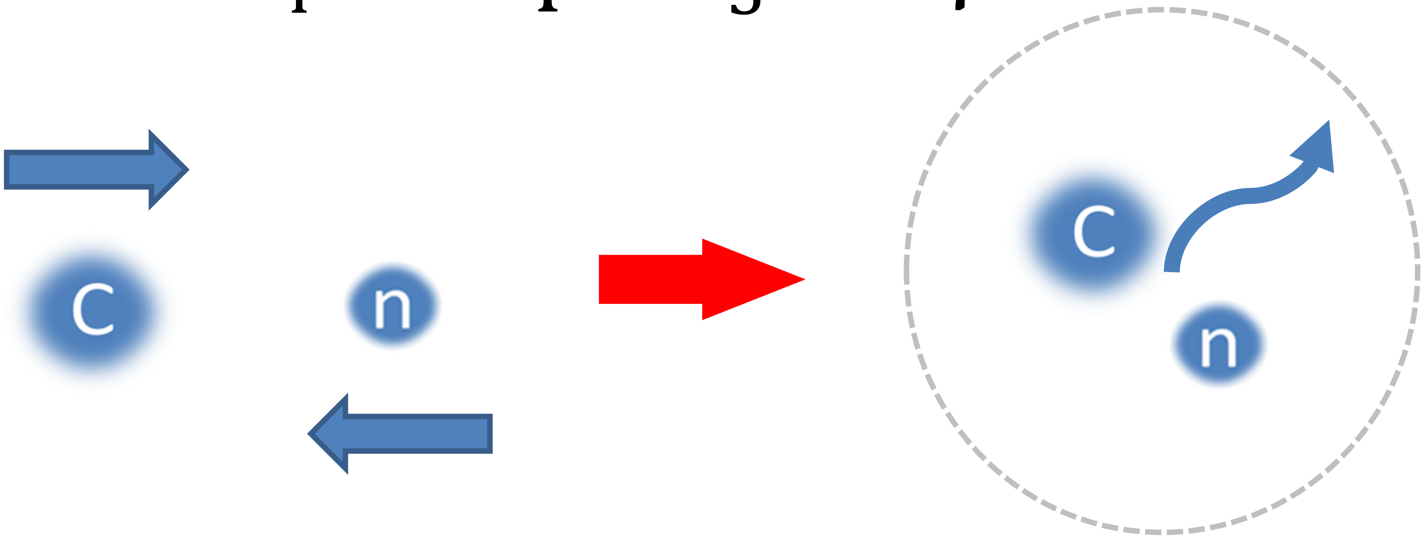
- Motivation
- Be7 capture in EFT: next-to-leading order (NLO)
- Bayesian analysis
- Questions

Radiative Capture Reaction

$$^{7}_{4}\mathrm{Be} + \mathrm{p} \rightarrow {}^{8}_{5}\mathrm{B} + \gamma$$

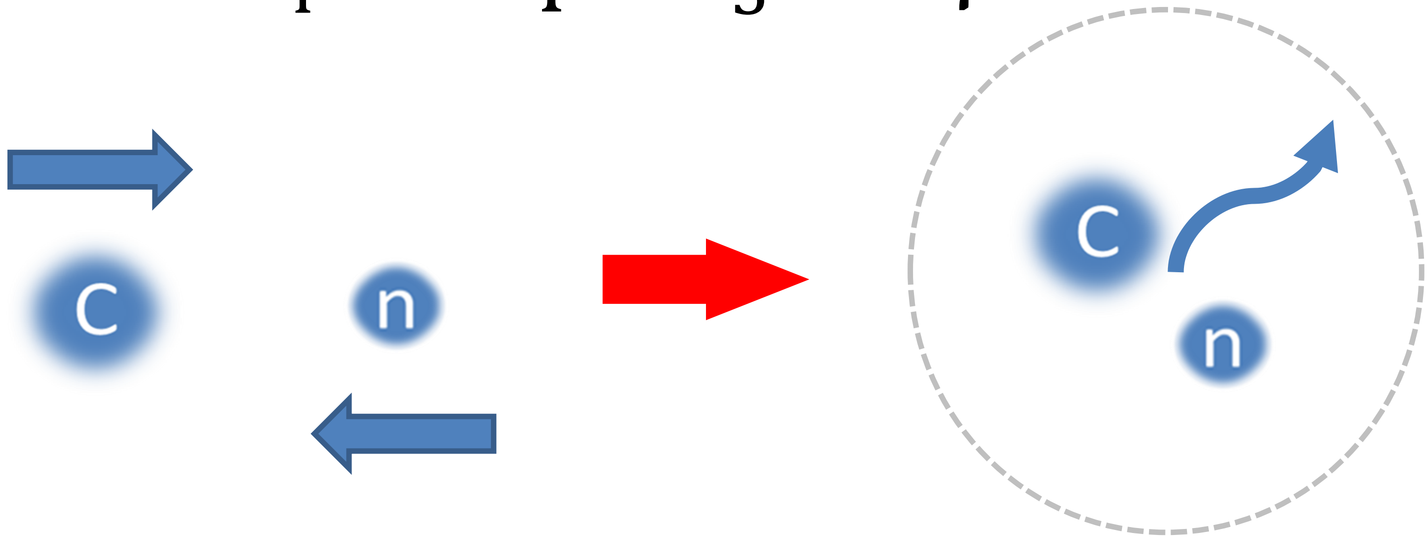
Radiative Capture Reaction

$$^{7}_{4}\mathrm{Be} + \mathrm{p} \rightarrow\, ^{8}_{5}\mathrm{B} + \gamma$$



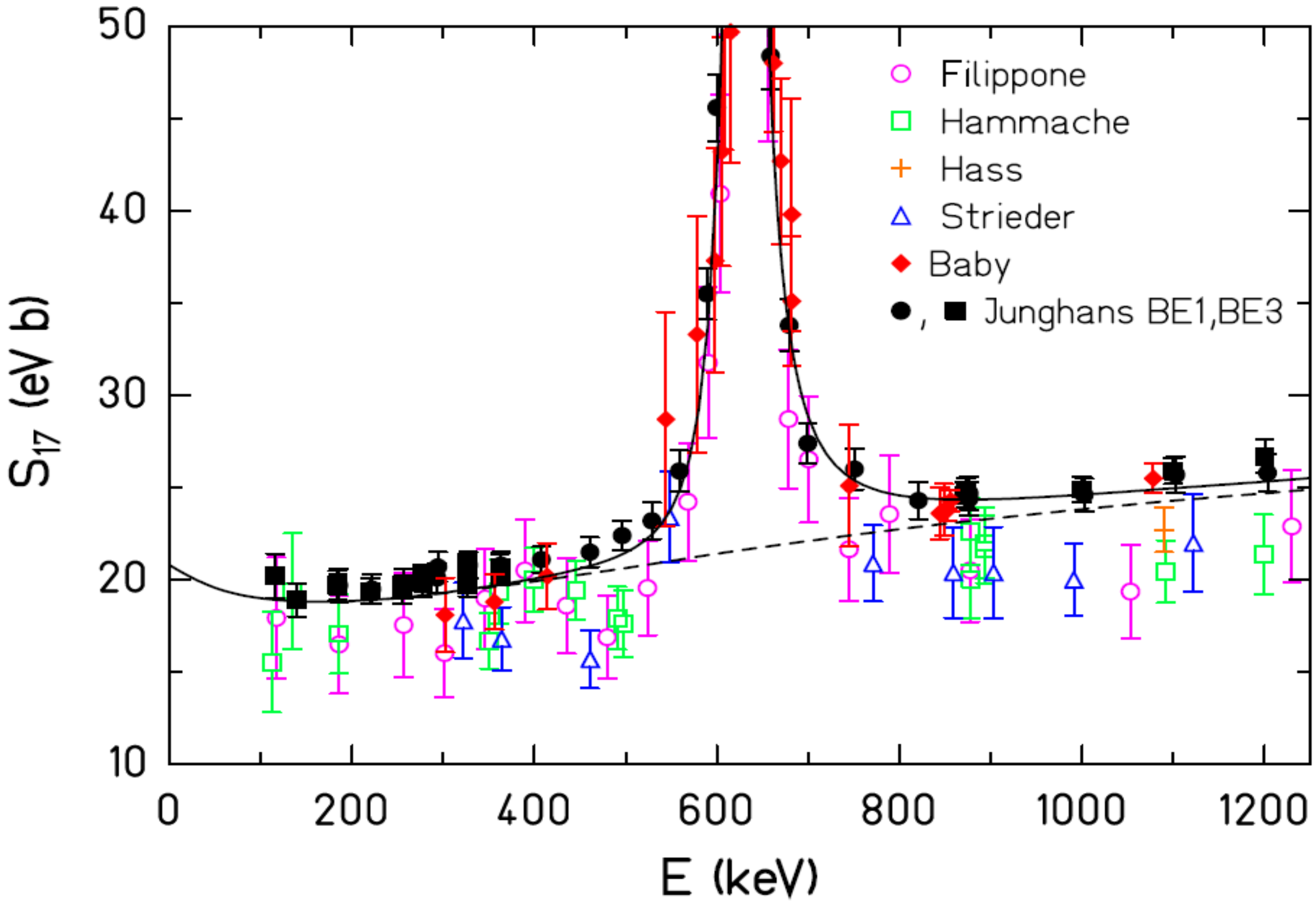
Radiative Capture Reaction

$$^{7}_{4}\mathrm{Be} + \mathrm{p} \rightarrow {}^{8}_{5}\mathrm{B} + \gamma$$

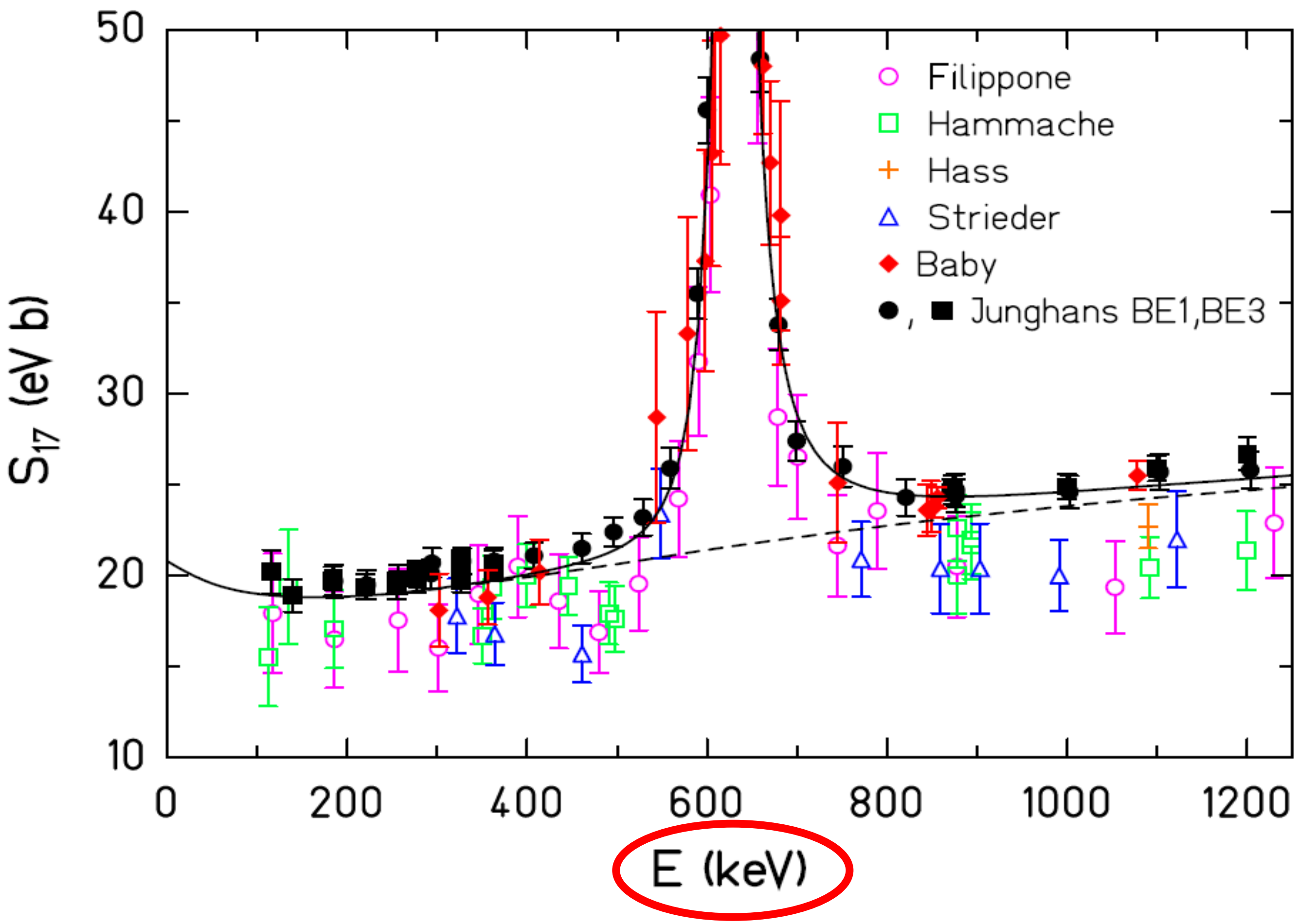


- Kinetic energy (E) between core (C) and nucleon(n)
- Photon takes away all the energy: Q value + E
- Particles carry spin (2 channels→ 2 sets of parameters)
- Electromagnetic dipole radiation (charge separation), and governed by strong interaction

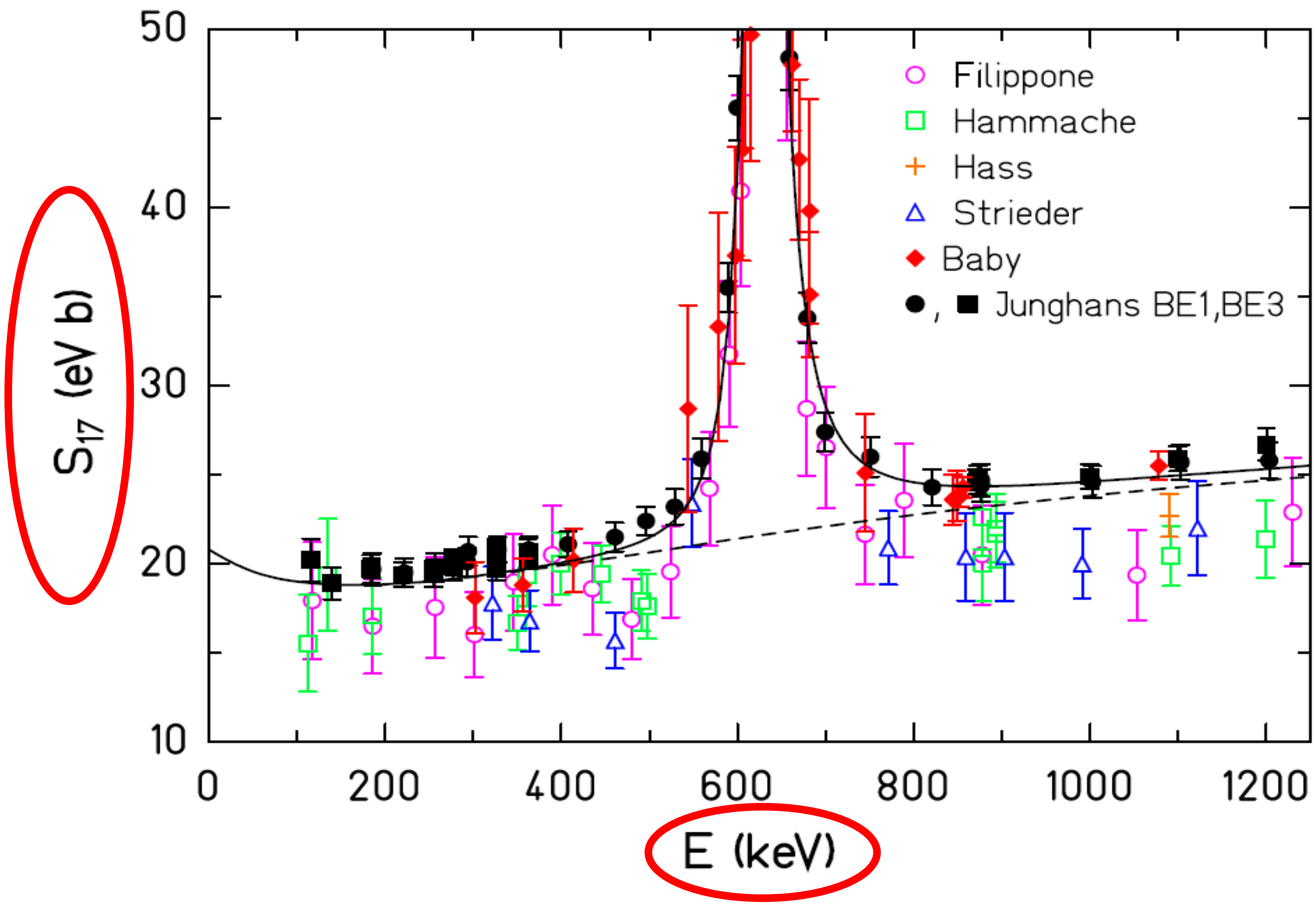
Radiative Capture Reaction



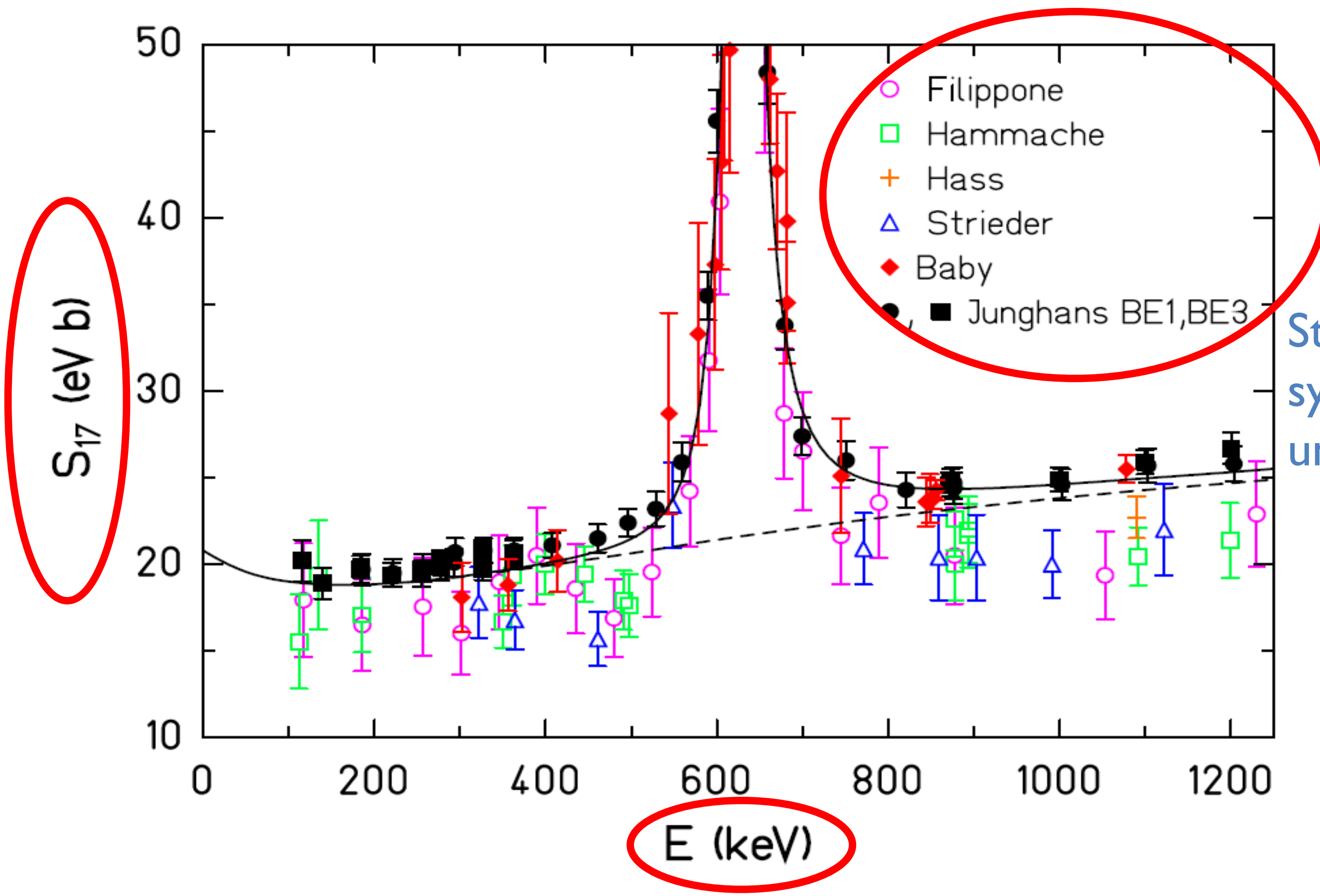
Radiative Capture Reaction



Radiative Capture Reaction

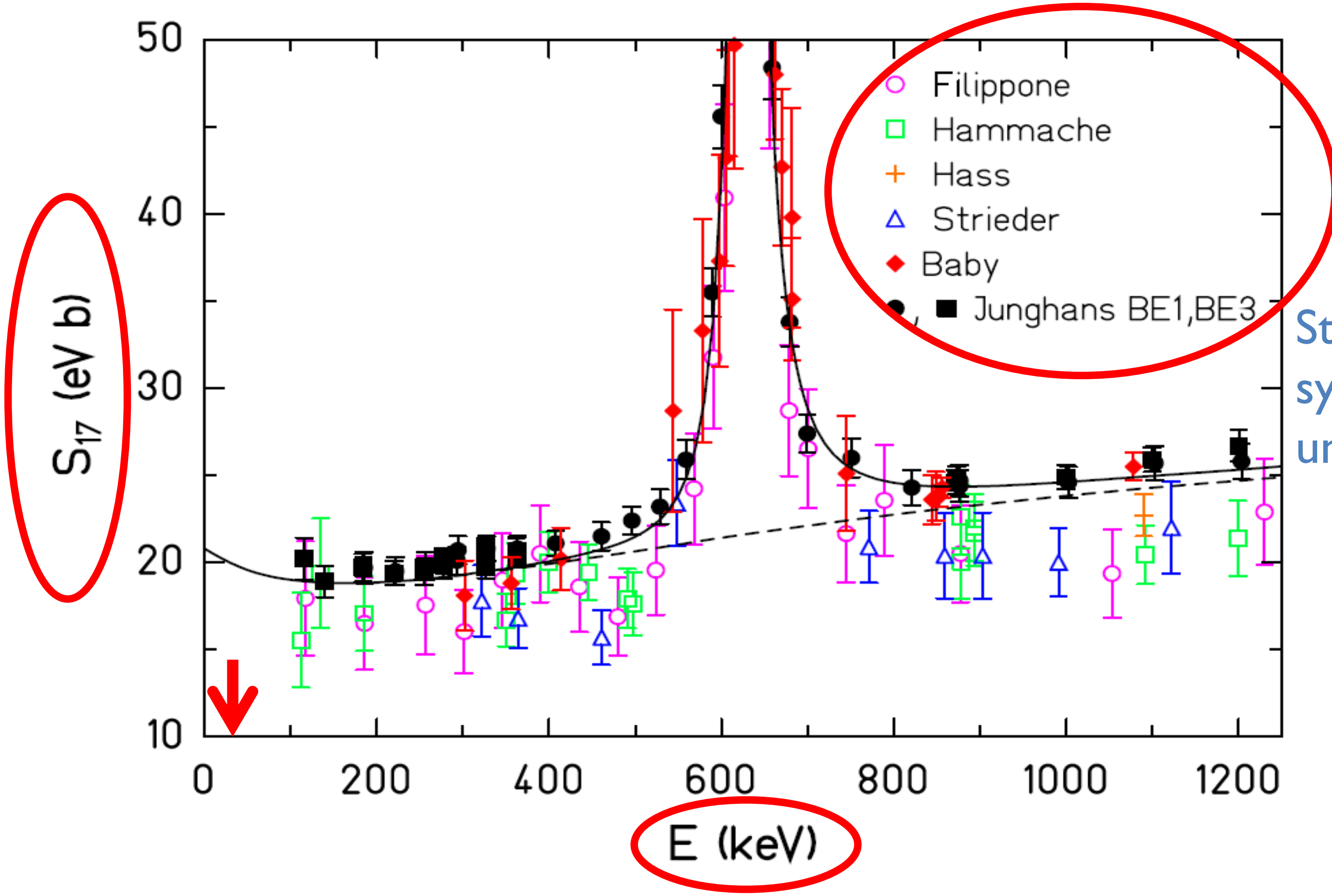


Radiative Capture Reaction



Statistical & systematical uncertainties

Radiative Capture Reaction



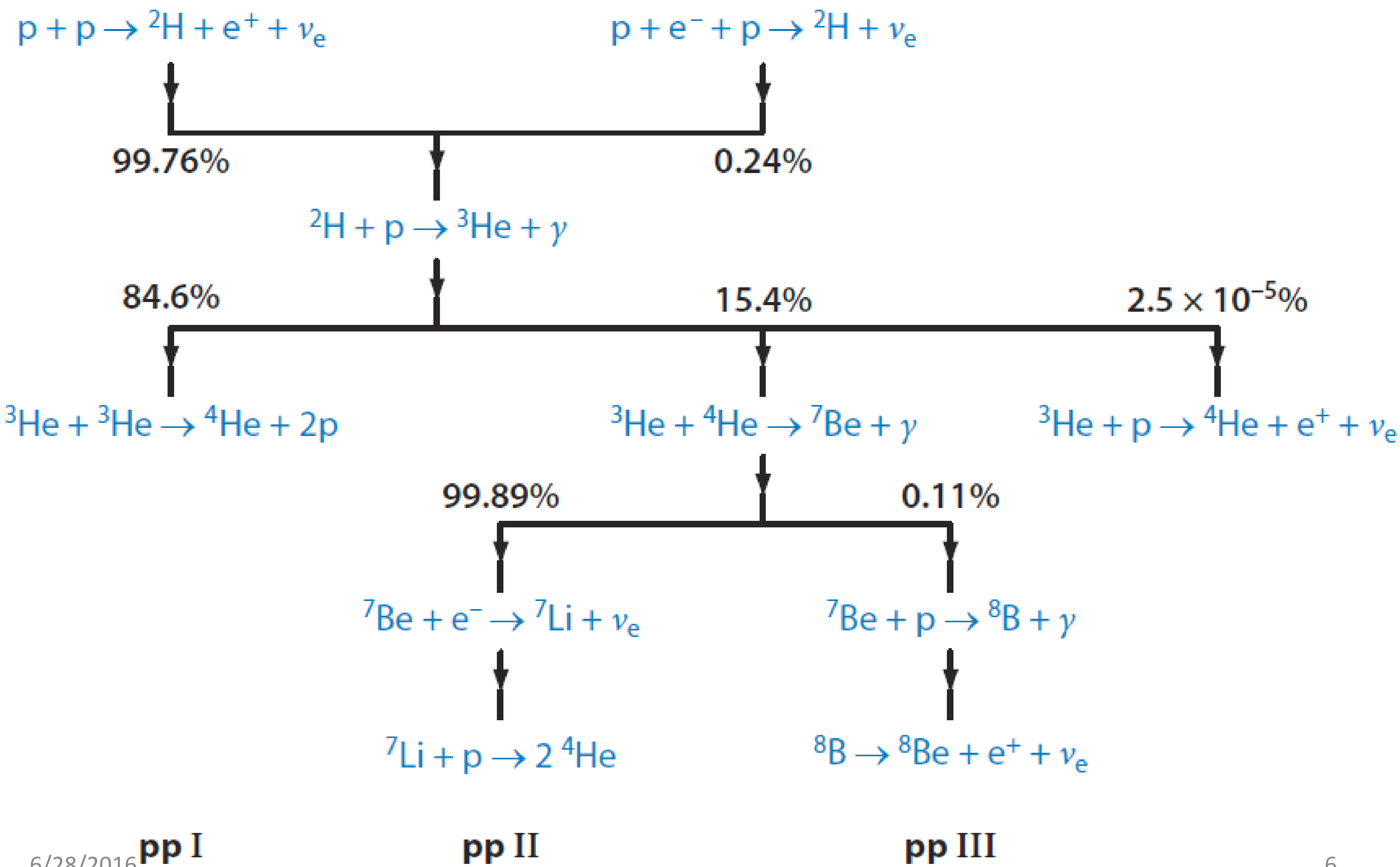
Statistical & systematical uncertainties

Goal is to infer the S factor and its uncertainty at near-zero energies based on theory

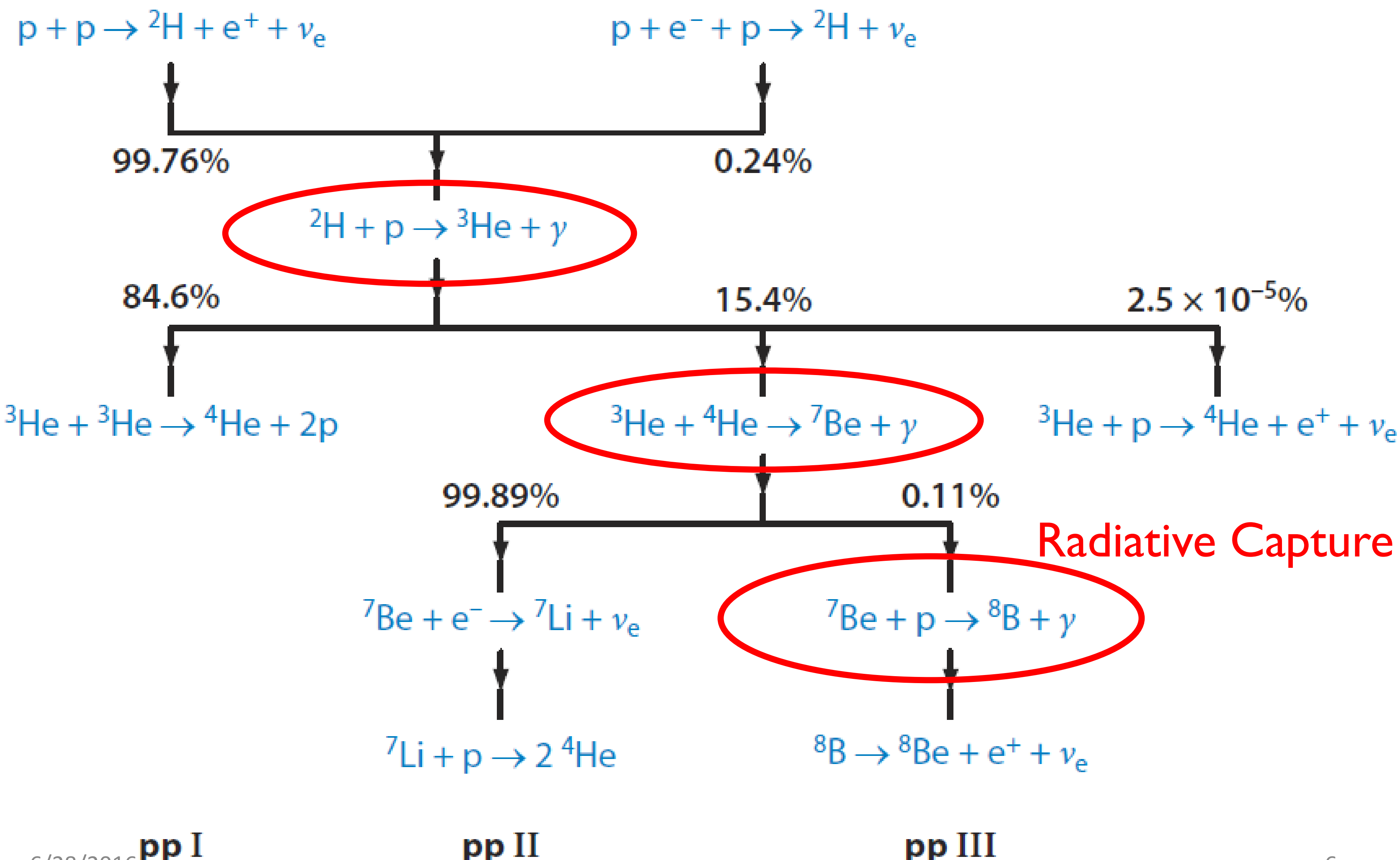
Motivations

W.C. Haxton, R.G. Hamish Robertson, and Aldo M. Serenelli, Annu.Rev. Astron. Astrophys. 51, 21 (2013)

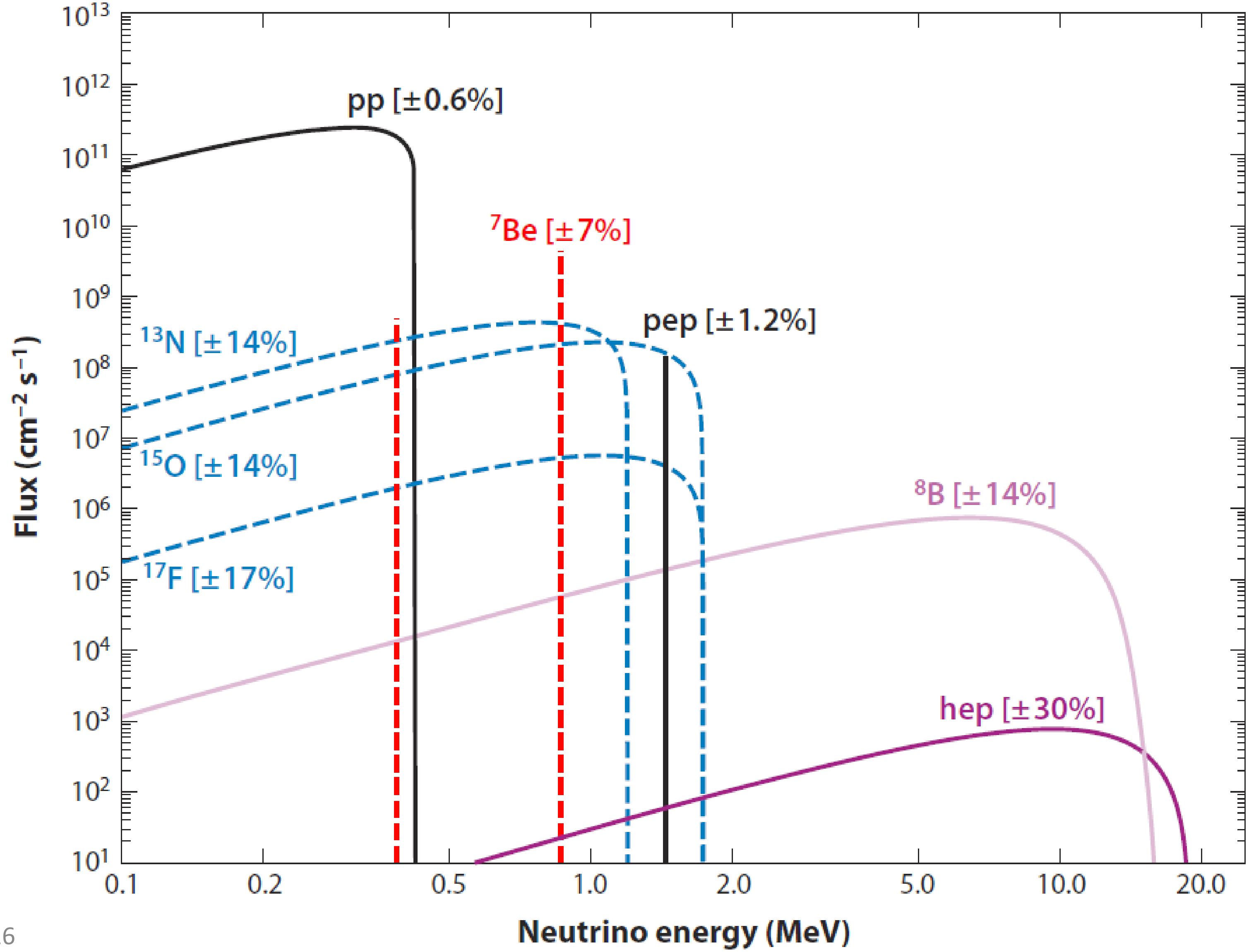
Solar neutrino generation



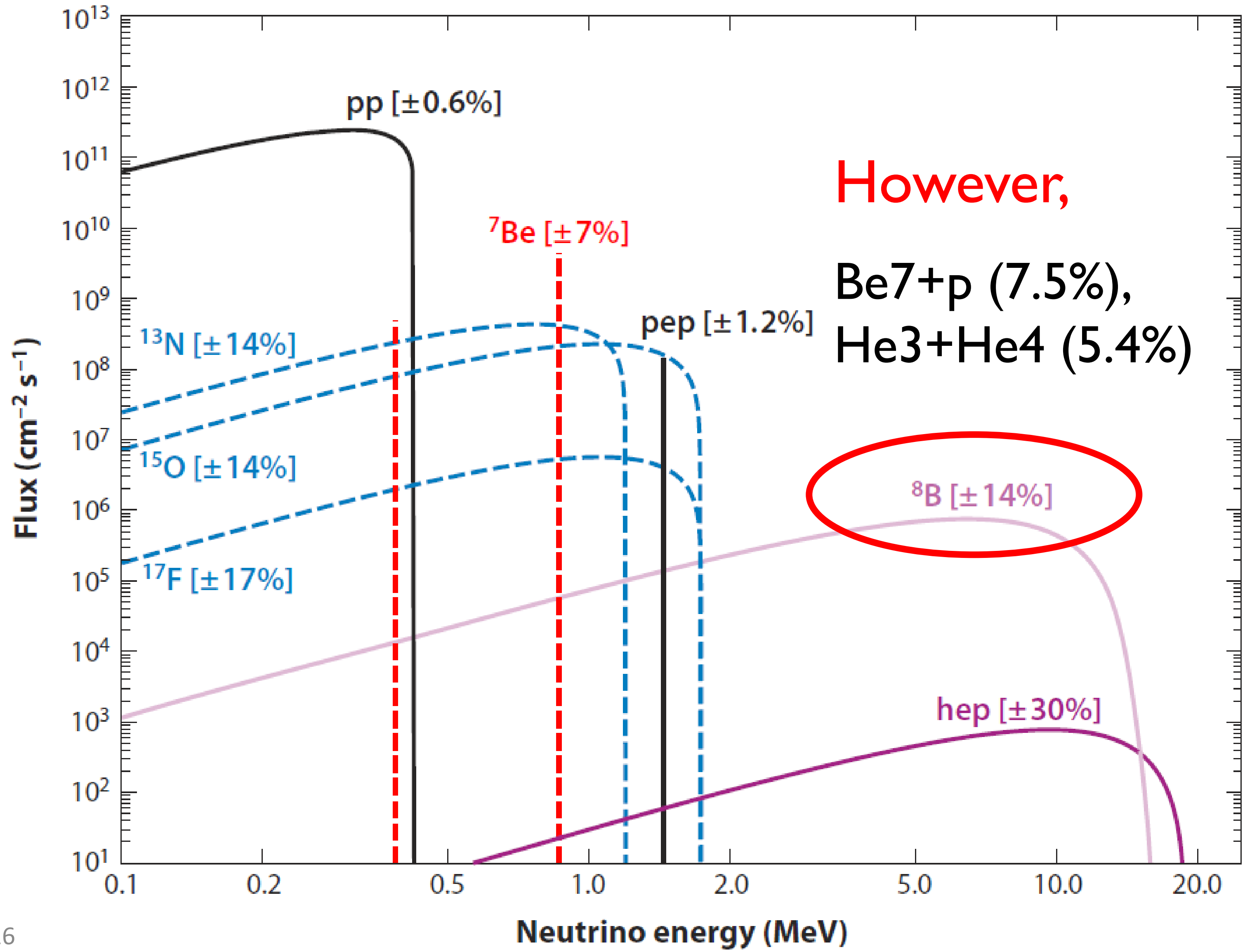
Solar neutrino generation



Solar neutrino generation



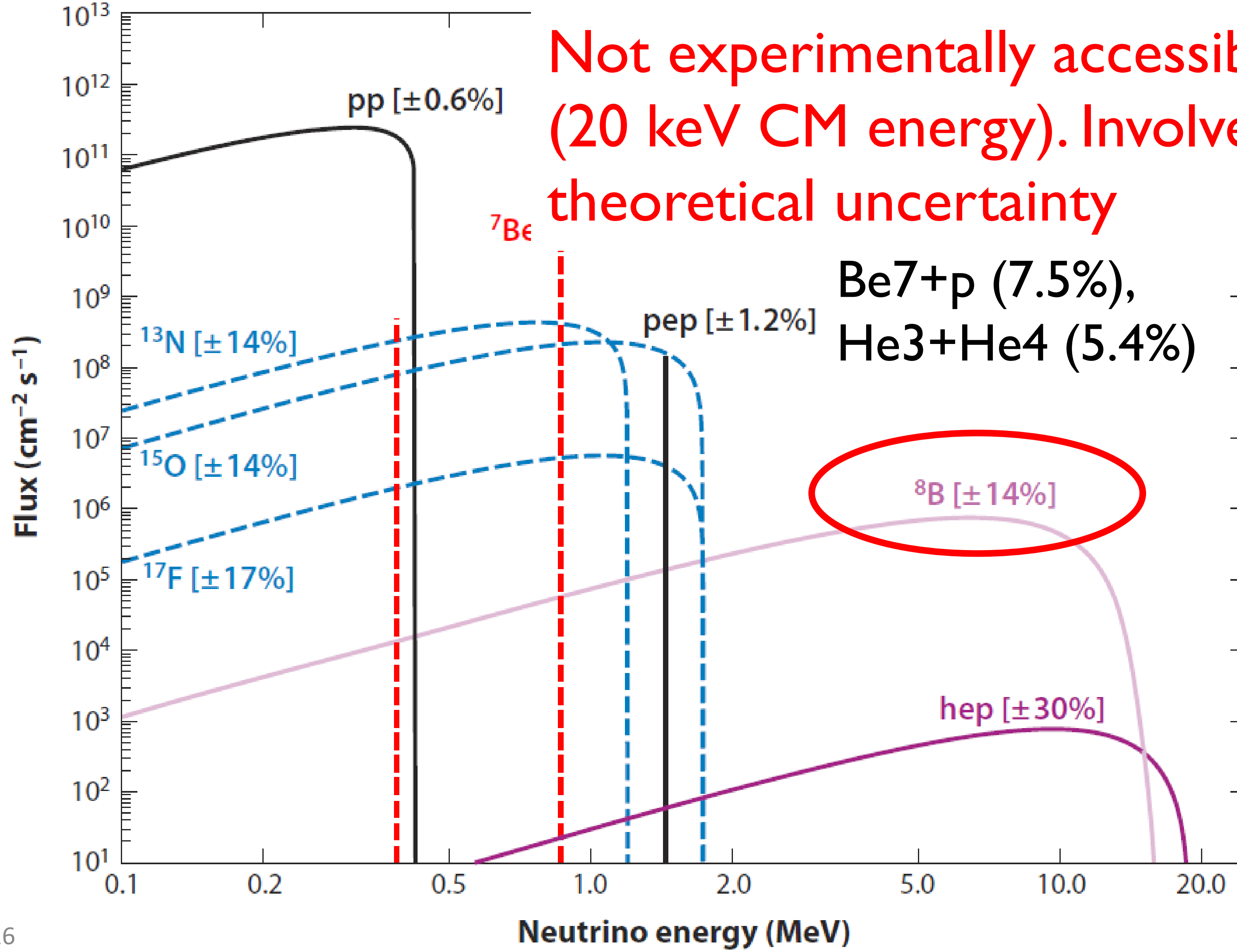
Solar neutrino generation



However,

Be7+p (7.5%),
He3+He4 (5.4%)

Solar neutrino generation



Not experimentally accessible (20 keV CM energy). Involve theoretical uncertainty

Be7+p (7.5%), He3+He4 (5.4%)

The capture reaction cross sections impact solar neutrino oscillation experiments, and solar modeling.

Solar abundance problem

Solar abundance problem

Table 1 Standard solar model characteristics are compared to helioseismic values, as determined by Basu & Antia (1997, 2004)

Property[a]	GS98-SFII	AGSS09-SFII	Solar
$(Z/X)_S$	0.0229	0.0178	–
Z_S	0.0170	0.0134	–
Y_S	0.2429	0.2319	0.2485 ± 0.0035
$R_{CZ}/R_\odot$	0.7124	0.7231	0.713 ± 0.001
$\langle \delta c/c \rangle$	0.0009	0.0037	0.0
Z_C	0.0200	0.0159	–
Y_C	0.6333	0.6222	–
Z_{ini}	0.0187	0.0149	–
Y_{ini}	0.2724	0.2620	–

Based on surface properties from 1-D convection zone simulation

Based on surface properties from 3-D convection zone simulation

Solar abundance problem

Table 1 Standard solar model characteristics are compared to helioseismic values, as determined by Basu & Antia (1997, 2004)

Property[a]	GS98-SFII	AGSS09-SFII	Solar
$(Z/X)_S$	0.0229	0.0178	–
Z_S	0.0170	0.0134	–
Y_S	0.2429	0.2319	0.2485 ± 0.0035
$R_{CZ}/R_\odot$	0.7124	0.7231	0.713 ± 0.001
$\langle \delta c/c \rangle$	0.0009	0.0037	0.0
Z_C	0.0200	0.0159	–
Y_C	0.6333	0.6222	–
Z_{ini}	0.0187	0.0149	–
Y_{ini}	0.2724	0.2620	–

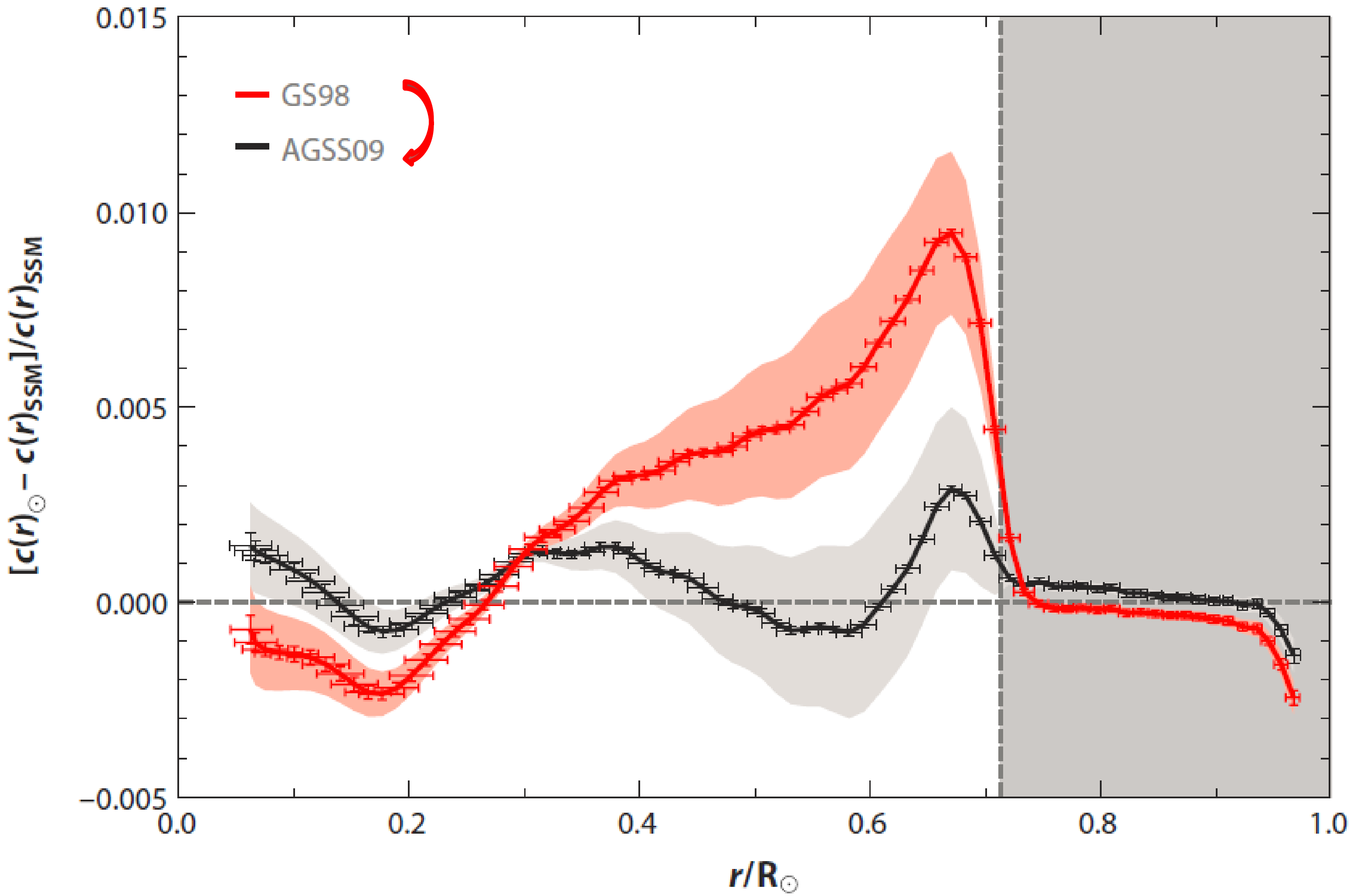
Based on surface properties from 1-D convection zone simulation

High metallicity
High core T
Large neutrino flux

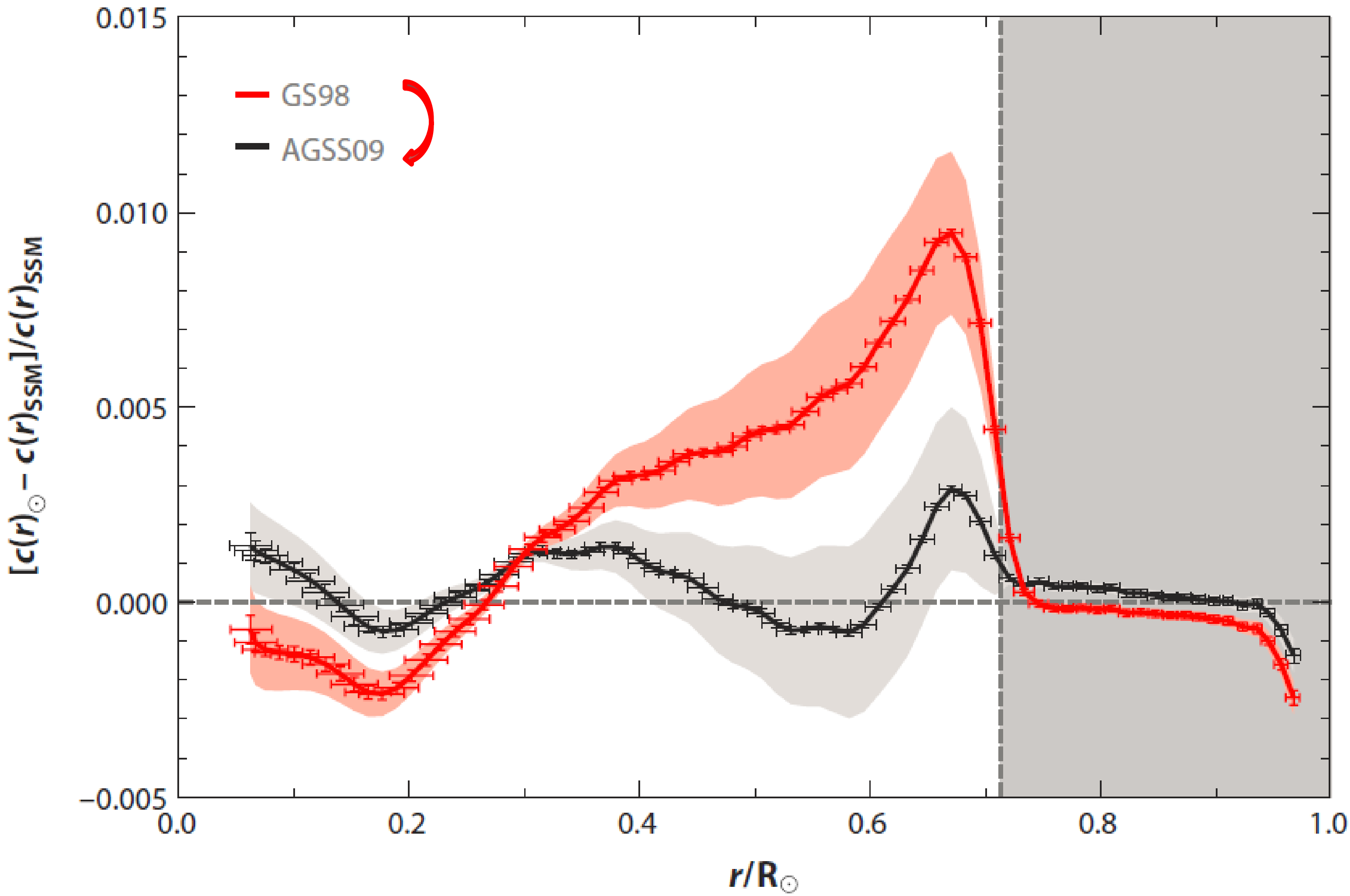
Based on surface properties from 3-D convection zone simulation

Low metallicity
Low core T
Small neutrino flux

Solar abundance problem: Helioseismology



Solar abundance problem: Helioseismology



The revised SSM does NOT agree with Helioseismology measurements

Solar abundance problem: Neutrinos

Table 2 Standard solar model (SSM) neutrino fluxes from the GS98-SFII and AGSS09-SFII SSMs, with associated uncertainties (averaging over asymmetric uncertainties)[a]

ν flux	E_ν^{max} (MeV)	GS98-SFII	AGSS09-SFII	Solar	Units
$p + p \to {}^2H + e^+ + \nu$	0.42	$5.98(1 \pm 0.006)$	$6.03(1 \pm 0.006)$	$6.05(1^{+0.003}_{-0.011})$	10^{10} cm^{-2} s^{-1}
$p + e^- + p \to {}^2H + \nu$	1.44	$1.44(1 \pm 0.012)$	$1.47(1 \pm 0.012)$	$1.46(1^{+0.010}_{-0.014})$	10^8 cm^{-2} s^{-1}
${}^7Be + e^- \to {}^7Li + \nu$	0.86 (90%)	$5.00(1 \pm 0.07)$	$4.56(1 \pm 0.07)$	$4.82(1^{+0.05}_{-0.04})$	10^9 cm^{-2} s^{-1}
	0.38 (10%)				
${}^8B \to {}^8Be + e^+ + \nu$	~15	$5.58(1 \pm 0.14)$	$4.59(1 \pm 0.14)$	$5.00(1 \pm 0.03)$	10^6 cm^{-2} s^{-1}
${}^3He+p \to {}^4He+e^++\nu$	18.77	$8.04(1 \pm 0.30)$	$8.31(1 \pm 0.30)$	—	10^3 cm^{-2} s^{-1}
${}^{13}N \to {}^{13}C + e^+ + \nu$	1.20	$2.96(1 \pm 0.14)$	$2.17(1 \pm 0.14)$	≤ 6.7	10^8 cm^{-2} s^{-1}
${}^{15}O \to {}^{15}N + e^+ + \nu$	1.73	$2.23(1 \pm 0.15)$	$1.56(1 \pm 0.15)$	≤ 3.2	10^8 cm^{-2} s^{-1}
${}^{17}F \to {}^{17}0 + e^+ + \nu$	1.74	$5.52(1 \pm 0.17)$	$3.40(1 \pm 0.16)$	≤ 59	10^6 cm^{-2} s^{-1}
χ^2 / P^{agr}		3.5/90%	3.4/90%		

Solar abundance problem: Neutrinos

Table 2 Standard solar model (SSM) neutrino fluxes from the GS98-SFII and AGSS09-SFII SSMs, with associated uncertainties (averaging over asymmetric uncertainties)[a]

ν flux	E_ν^{max} (MeV)	GS98-SFII	AGSS09-SFII	Solar	Units
$p + p \rightarrow {}^2H + e^+ + \nu$	0.42	$5.98(1 \pm 0.006)$	$6.03(1 \pm 0.006)$	$6.05(1^{+0.003}_{-0.011})$	10^{10} cm^{-2} s^{-1}
$p + e^- + p \rightarrow {}^2H + \nu$	1.44	$1.44(1 \pm 0.012)$	$1.47(1 \pm 0.012)$	$1.46(1^{+0.010}_{-0.014})$	10^8 cm^{-2} s^{-1}
${}^7Be + e^- \rightarrow {}^7Li + \nu$	0.86 (90%)	$5.00(1 \pm 0.07)$	$4.56(1 \pm 0.07)$	$4.82(1^{+0.05}_{-0.04})$	10^9 cm^{-2} s^{-1}
	0.38 (10%)				
${}^8B \rightarrow {}^8Be + e^+ + \nu$	~15	$5.58(1 \pm 0.14)$	$4.59(1 \pm 0.14)$	$5.00(1 \pm 0.03)$	10^6 cm^{-2} s^{-1}
${}^3He + p \rightarrow {}^4He + e^+ + \nu$	18.77	$8.04(1 \pm 0.30)$	$8.31(1 \pm 0.30)$	—	10^3 cm^{-2} s^{-1}
${}^{13}N \rightarrow {}^{13}C + e^+ + \nu$	1.20	$2.96(1 \pm 0.14)$	$2.17(1 \pm 0.14)$	≤ 6.7	10^8 cm^{-2} s^{-1}
${}^{15}O \rightarrow {}^{15}N + e^+ + \nu$	1.73	$2.23(1 \pm 0.15)$	$1.56(1 \pm 0.15)$	≤ 3.2	10^8 cm^{-2} s^{-1}
${}^{17}F \rightarrow {}^{17}O + e^+ + \nu$	1.74	$5.52(1 \pm 0.17)$	$3.40(1 \pm 0.16)$	≤ 59	10^6 cm^{-2} s^{-1}
χ^2 / P^{agr}		3.5/90%	3.4/90%		

Solar abundance problem: Neutrinos

Table 2 Standard solar model (SSM) neutrino fluxes from the GS98-SFII and AGSS09-SFII SSMs, with associated uncertainties (averaging over asymmetric uncertainties)[a]

ν flux	E_ν^{max} (MeV)	GS98-SFII	AGSS09-SFII	Solar	Units
$p + p \rightarrow {}^2H + e^+ + \nu$	0.42	$5.98(1 \pm 0.006)$	$6.03(1 \pm 0.006)$	$6.05(1^{+0.003}_{-0.011})$	10^{10} cm^{-2} s^{-1}
$p + e^- + p \rightarrow {}^2H + \nu$	1.44	$1.44(1 \pm 0.012)$	$1.47(1 \pm 0.012)$	$1.46(1^{+0.010}_{-0.014})$	10^8 cm^{-2} s^{-1}
${}^7Be + e^- \rightarrow {}^7Li + \nu$	0.86 (90%)	$5.00(1 \pm 0.07)$	$4.56(1 \pm 0.07)$	$4.82(1^{+0.05}_{-0.04})$	10^9 cm^{-2} s^{-1}
	0.38 (10%)				
${}^8B \rightarrow {}^8Be + e^+ + \nu$	~15	$5.58(1 \pm 0.14)$	$4.59(1 \pm 0.14)$	$5.00(1 \pm 0.03)$	10^6 cm^{-2} s^{-1}
${}^3He + p \rightarrow {}^4He + e^+ + \nu$	18.77	$8.04(1 \pm 0.30)$	$8.31(1 \pm 0.30)$	—	10^3 cm^{-2} s^{-1}
${}^{13}N \rightarrow {}^{13}C + e^+ + \nu$	1.20	$2.96(1 \pm 0.14)$	$2.17(1 \pm 0.14)$	≤ 6.7	10^8 cm^{-2} s^{-1}
${}^{15}O \rightarrow {}^{15}N + e^+ + \nu$	1.73	$2.23(1 \pm 0.15)$	$1.56(1 \pm 0.15)$	≤ 3.2	10^8 cm^{-2} s^{-1}
${}^{17}F \rightarrow {}^{17}0 + e^+ + \nu$	1.74	$5.52(1 \pm 0.17)$	$3.40(1 \pm 0.16)$	≤ 59	10^6 cm^{-2} s^{-1}
χ^2 / P^{agr}		3.5/90%	3.4/90%		

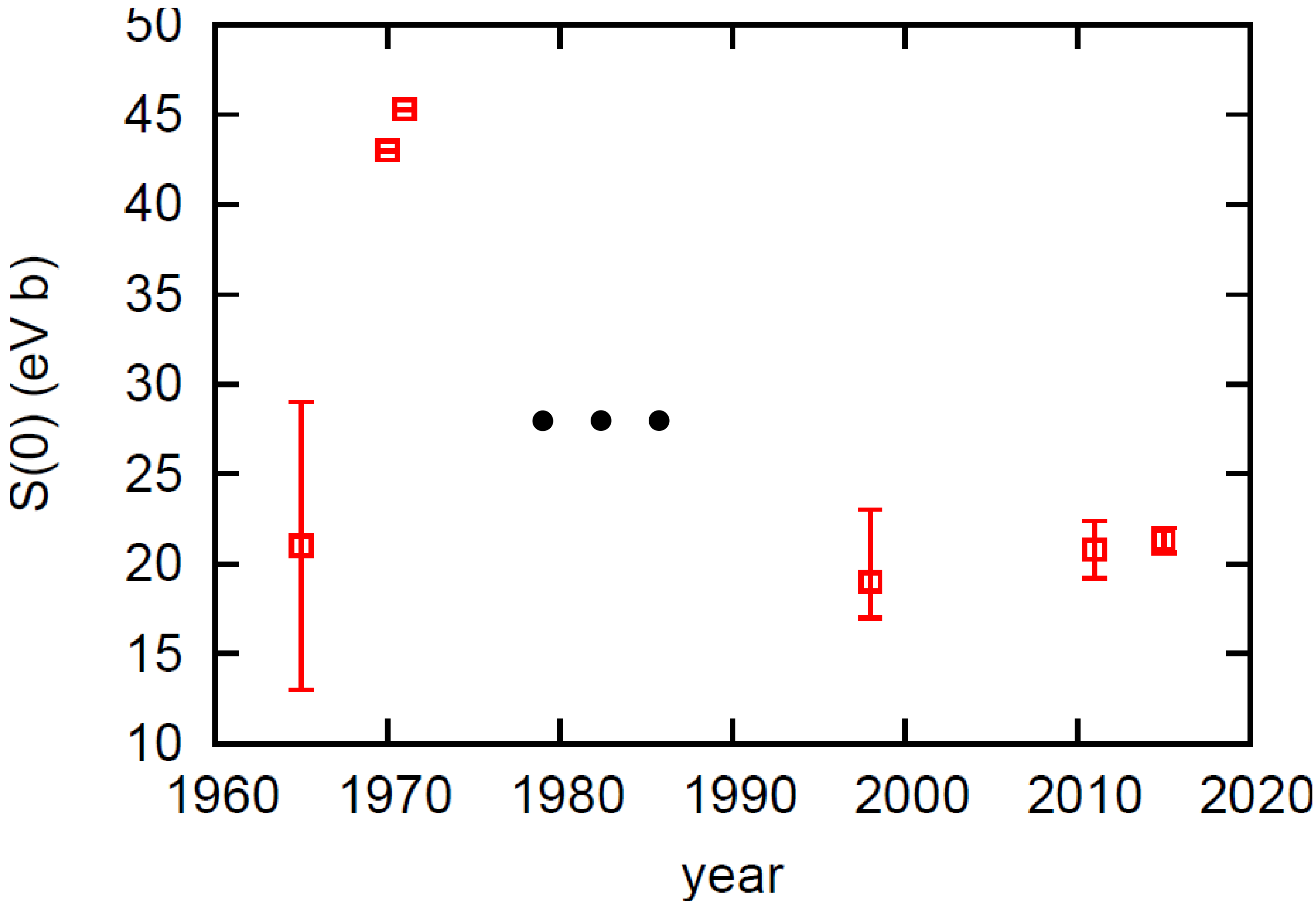
Two models could be differentiated IF the theoretical errors and those of solar neutrino experiments on 8B neutrino flux can be reduced.

EFT at N2LO

A simple picture due to **scale separation**; systematic expansion (Lagrangian); uncertainty estimate

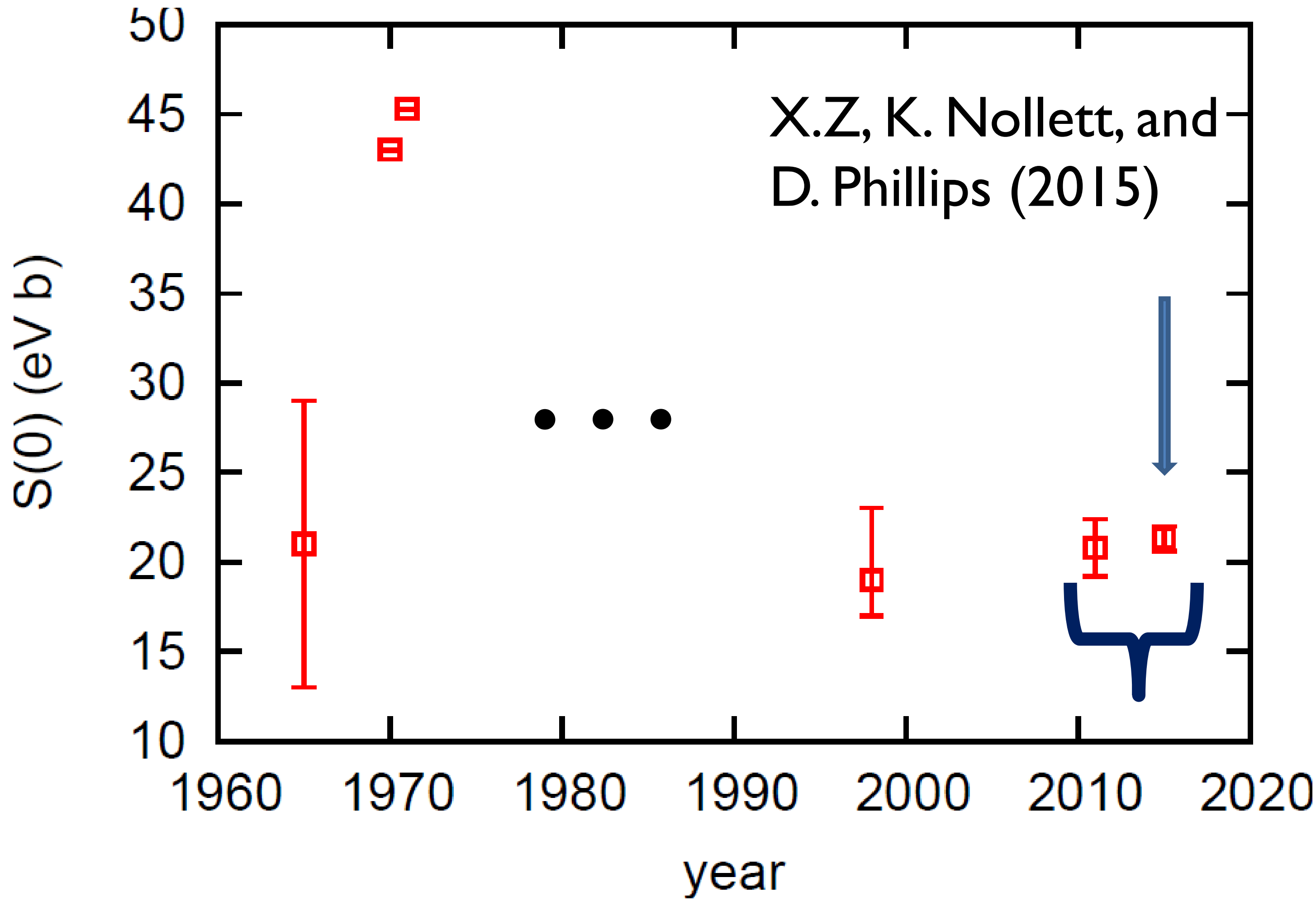
X.Z., K. Nollett and D. Phillips, PRC 89, 051602 (2014)
PLB 751, 535(2015); EPJ Web Conf. 113, 06001 (2016).

Then and now



Tombrello(1965), Aurdal(1970),
Rev.Mod.Phys.(1998), **Rev.Mod.Phys(2011)**

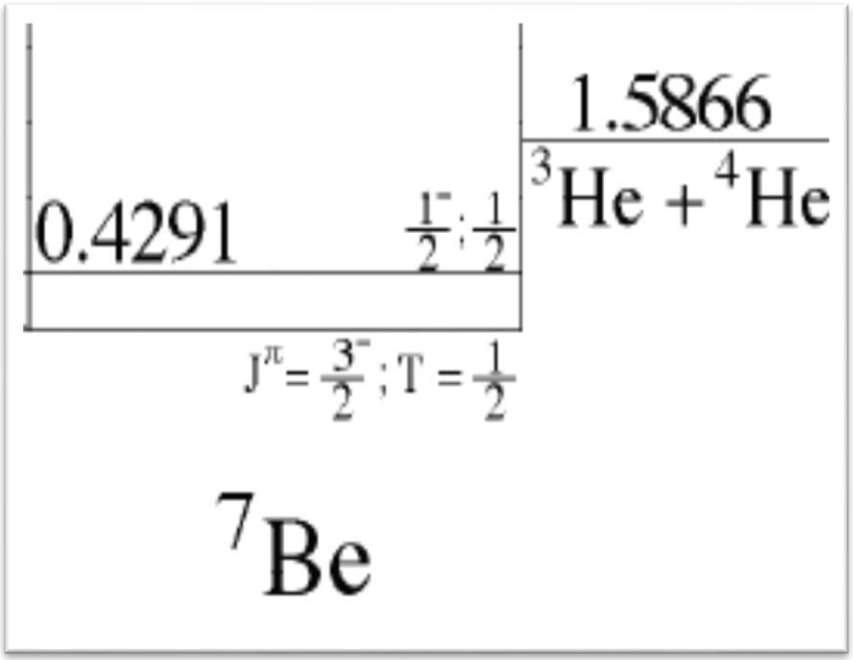
Then and now



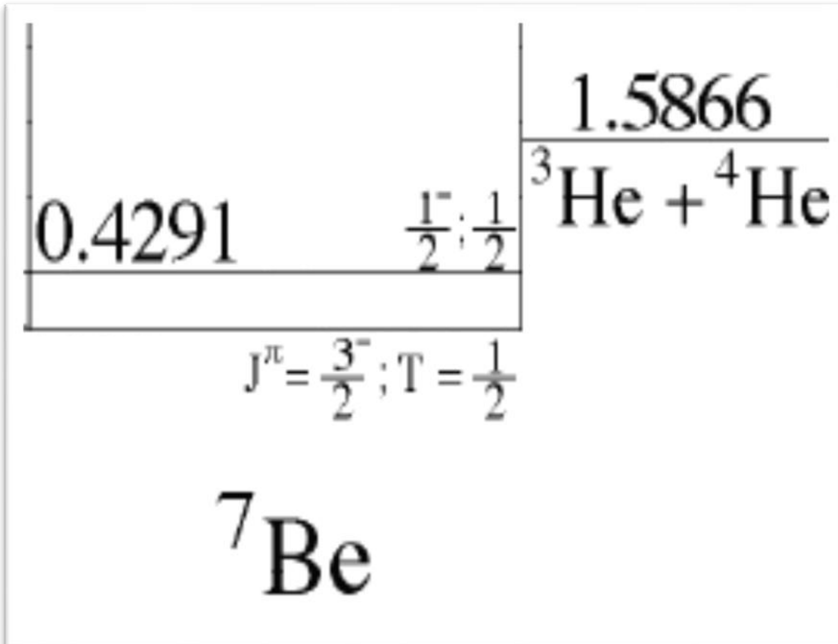
Tombrello(1965), Aurdal(1970), Rev.Mod.Phys.(1998), **Rev.Mod.Phys(2011)**

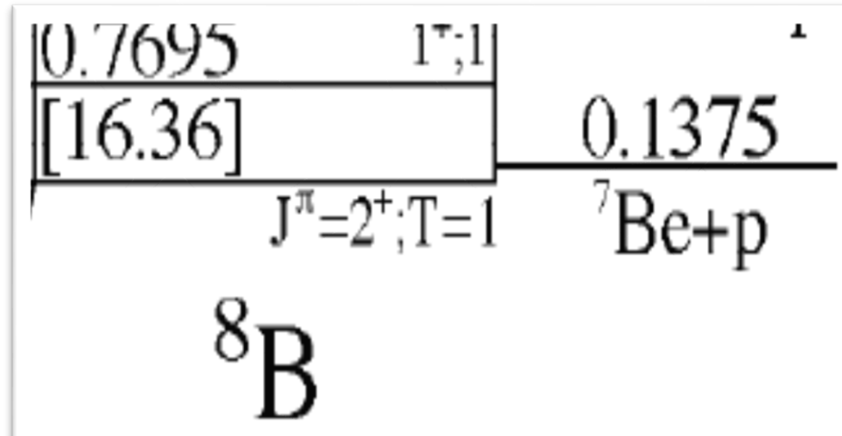
Based on the same data

Scale separation: spectrum

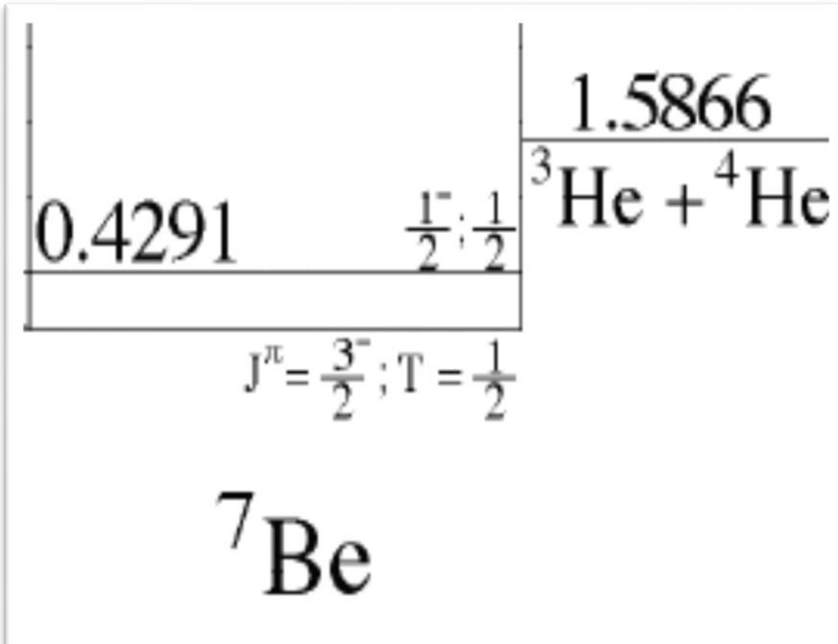


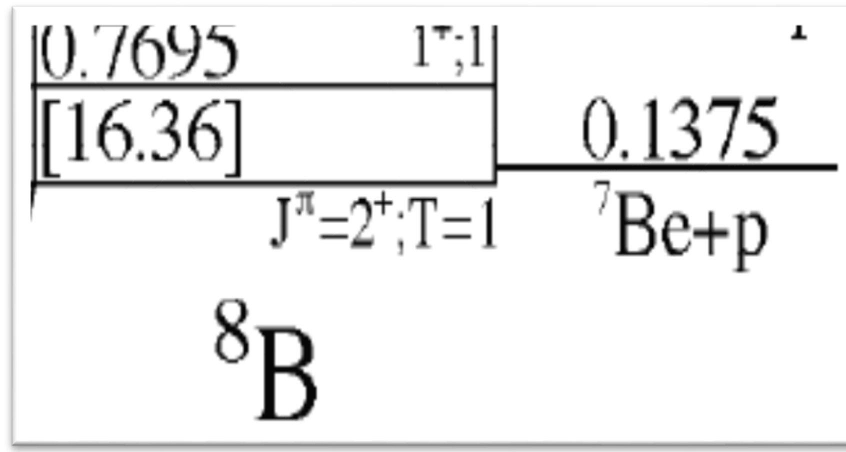
Scale separation: spectrum





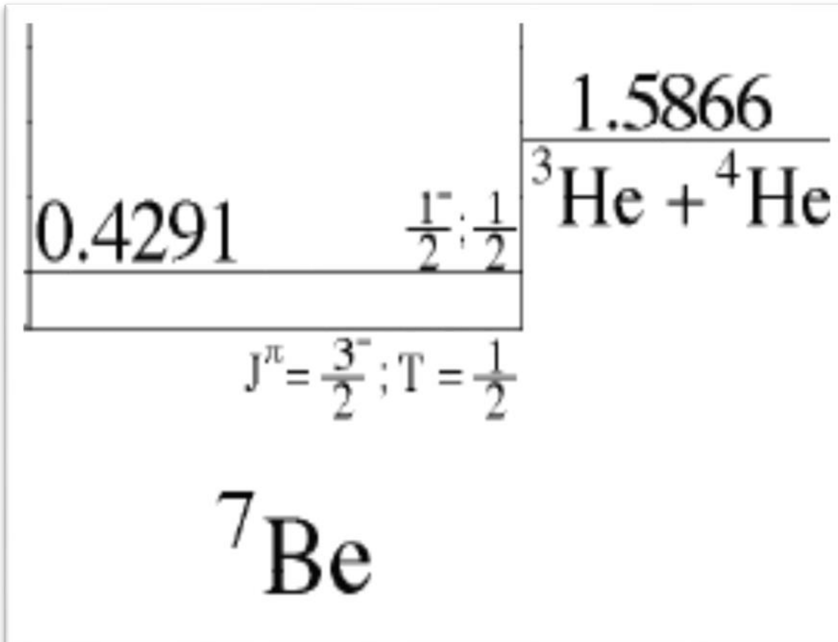
Scale separation: spectrum

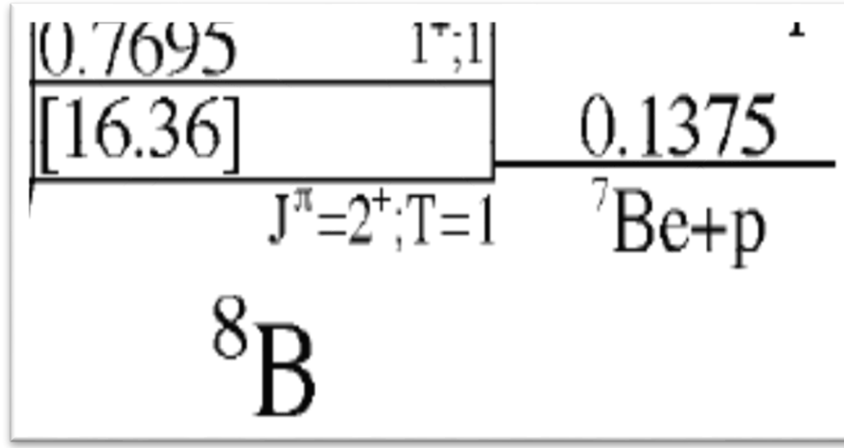




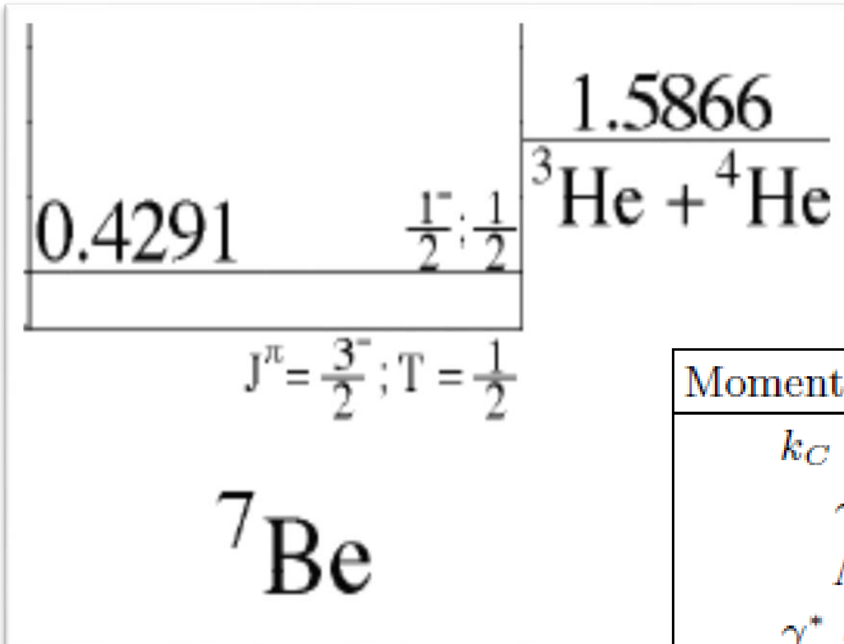
- B8: a **shallow** bound state in terms of proton+Be7
- Proton-Be7 s-wave has **large** scattering lengths
- Length scale ~ 1/(momentum scale)

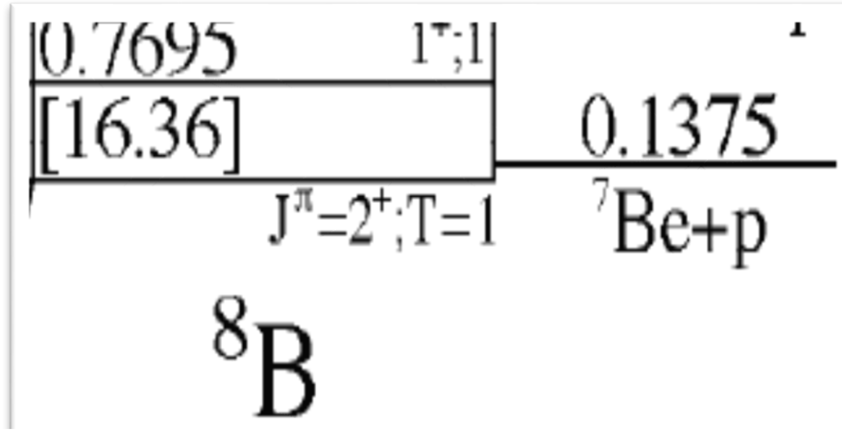
Scale separation: spectrum





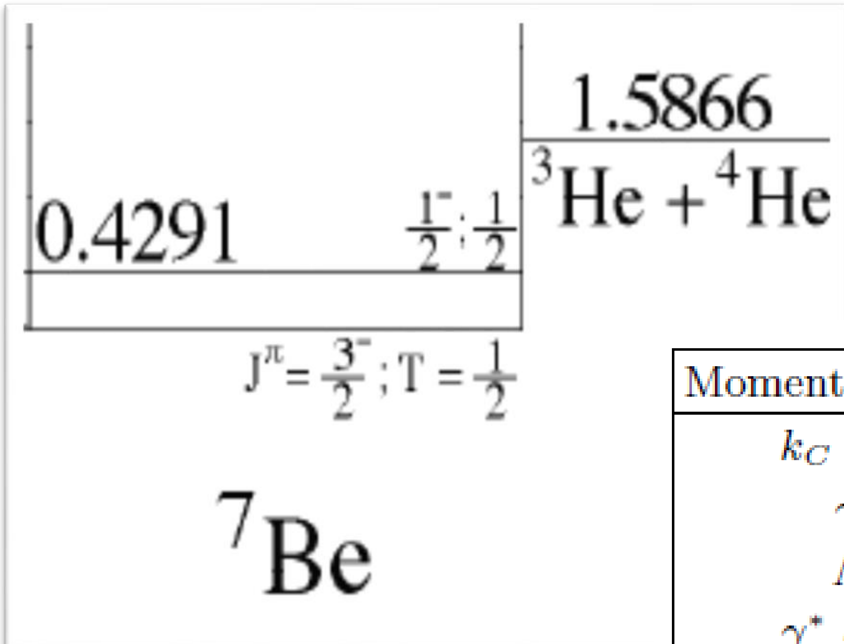
Scale separation: spectrum

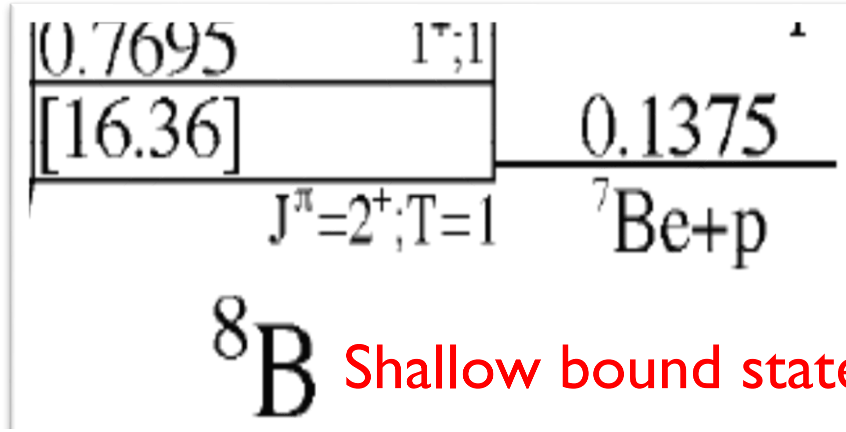




Momentum scale	Definition	Value
$k_C \sim \gamma$	$Q_c Q_n \alpha_{EM} M_{\mathrm{R}}$	24.02 MeV
γ	$\sqrt{2M_R B_{^8\mathrm{B}}}$	15.04 MeV
Λ	$\sqrt{2M'_R B_{^7\mathrm{Be}}}$	70 MeV
$\gamma^* \sim \gamma$	$\sqrt{2M_R(B_{^8\mathrm{B}} + E^*)}$	30.53 MeV
$\gamma_\Delta \sim \gamma$	$\sqrt{2M_R E^*}$	26.57 MeV
$a_{^3S_1}, a_{^5S_2} \sim 1/\gamma$	scattering lengths	Varies
$r_0 \sim 1/\Lambda$	$l = 0$ effective ranges	Varies
$a_1 \sim \gamma^{-2}\Lambda^{-1}$	scattering volume	1054.1 fm^3
$r_1 \sim \Lambda$	$l = 1$ effective "range"	-0.34 fm^{-1}

Scale separation: spectrum



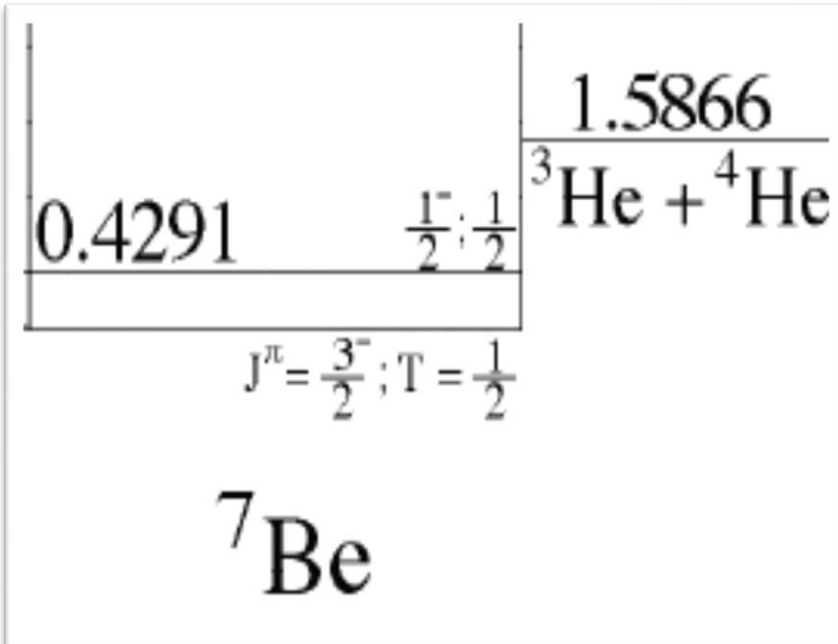


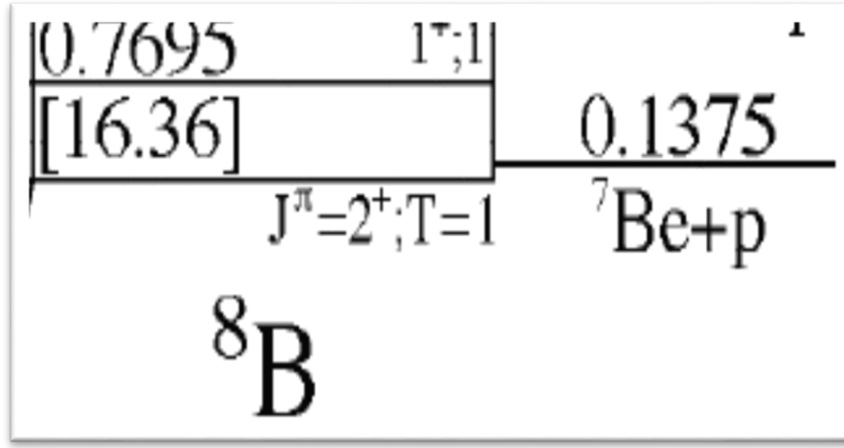
Momentum scale	Definition	Value
$k_C \sim \gamma$	$Q_c Q_n \alpha_{EM} M_{\mathrm{R}}$	24.02 MeV
γ	$\sqrt{2M_R B_{^8\mathrm{B}}}$	15.04 MeV
Λ	$\sqrt{2M'_R B_{^7\mathrm{Be}}}$	70 MeV
$\gamma^* \sim \gamma$	$\sqrt{2M_R(B_{^8\mathrm{B}} + E^*)}$	30.53 MeV
$\gamma_\Delta \sim \gamma$	$\sqrt{2M_R E^*}$	26.57 MeV
$a_{^3S_1}, a_{^5S_2} \sim 1/\gamma$	scattering lengths	Varies
$r_0 \sim 1/\Lambda$	$l=0$ effective ranges	Varies
$a_1 \sim \gamma^{-2}\Lambda^{-1}$	scattering volume	1054.1 fm^3
$r_1 \sim \Lambda$	$l=1$ effective "range"	-0.34 fm^{-1}

$$\frac{\gamma}{\Lambda}, \frac{k}{\Lambda} \approx 0.2$$

$$\eta = \frac{k_C}{k} \sim 1$$

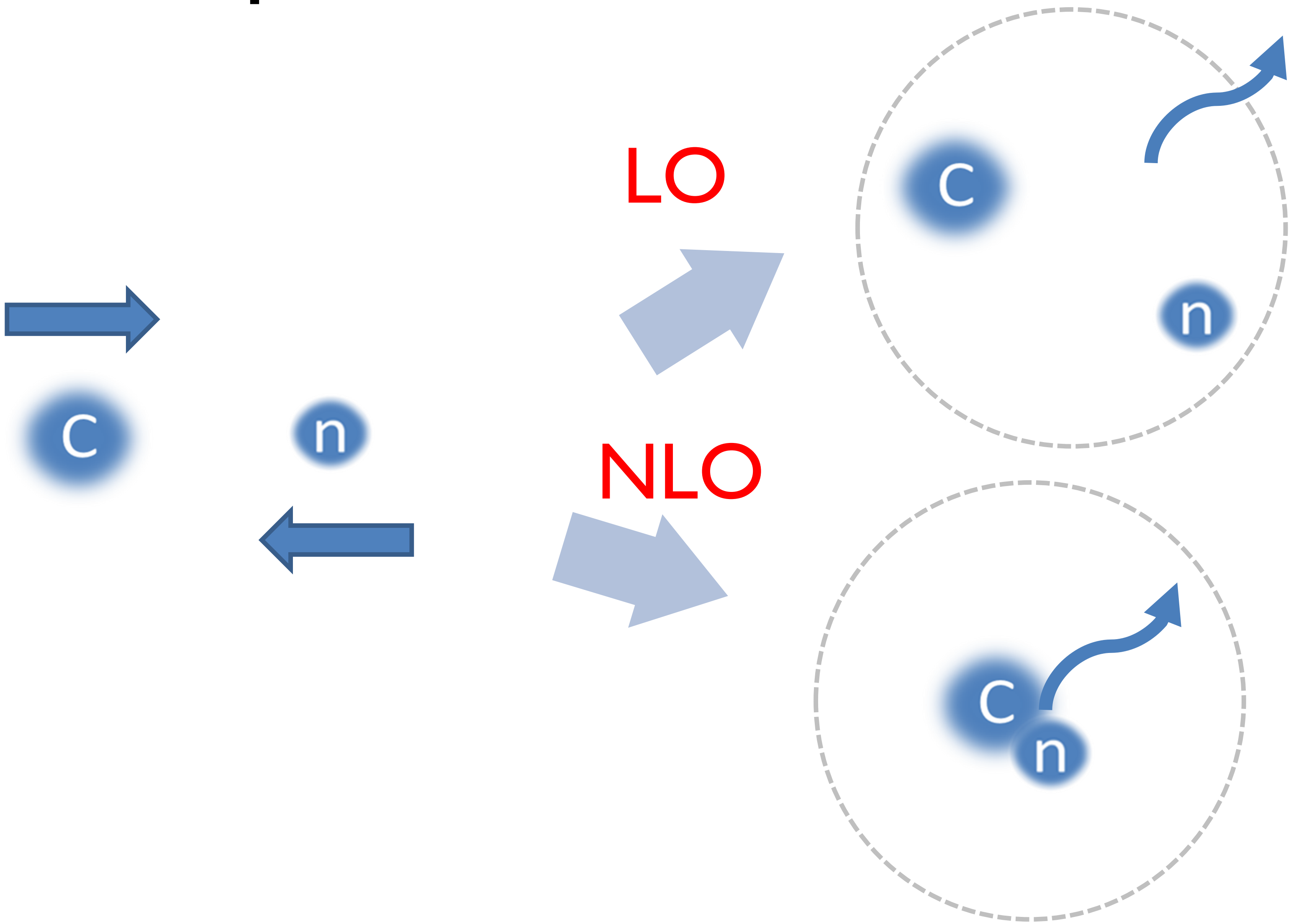
Scale separation: spectrum



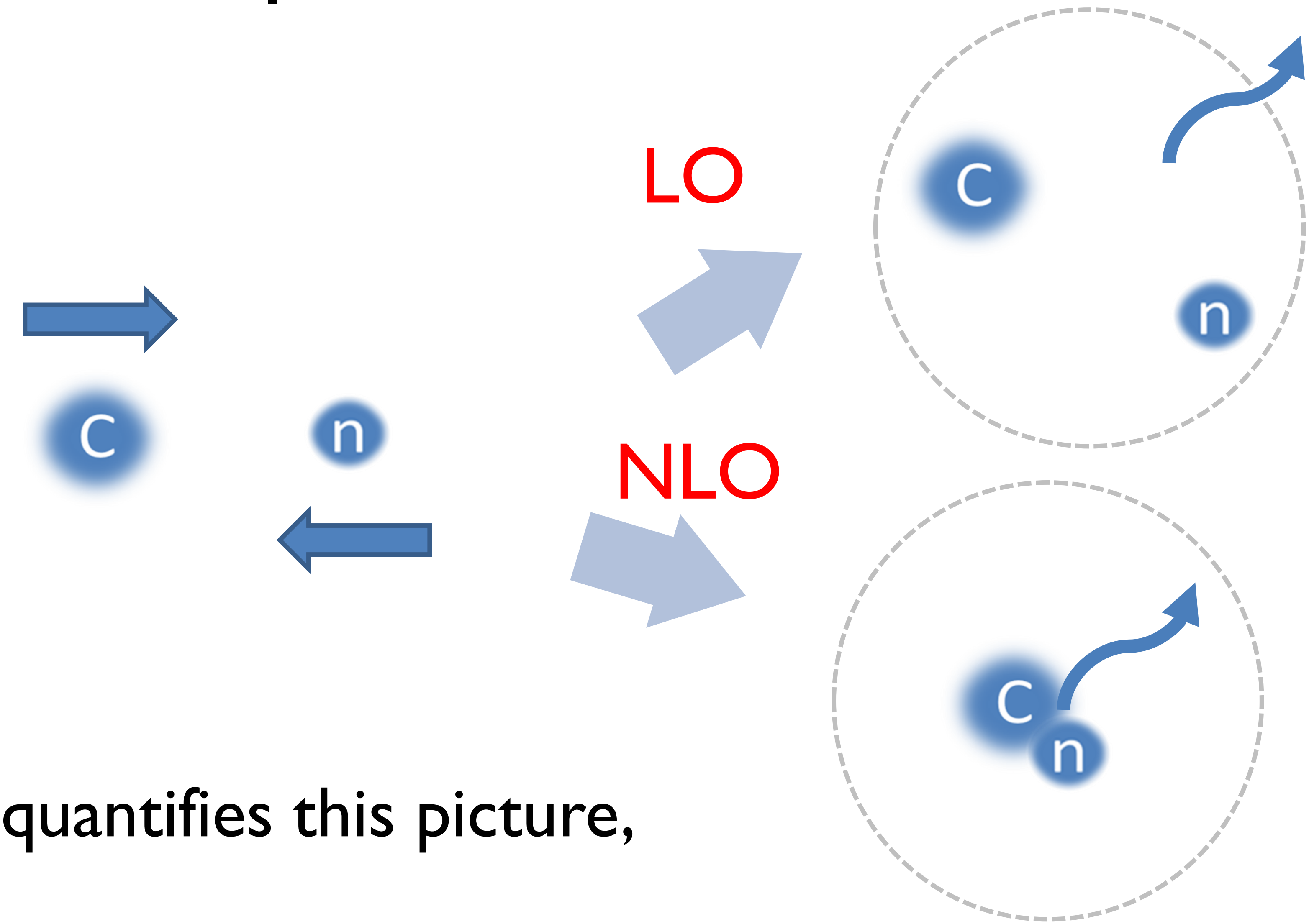


Be and proton total spin can be 1 or 2, giving two independent reaction "channels"→ two sets of parameters

Scale separation: reaction

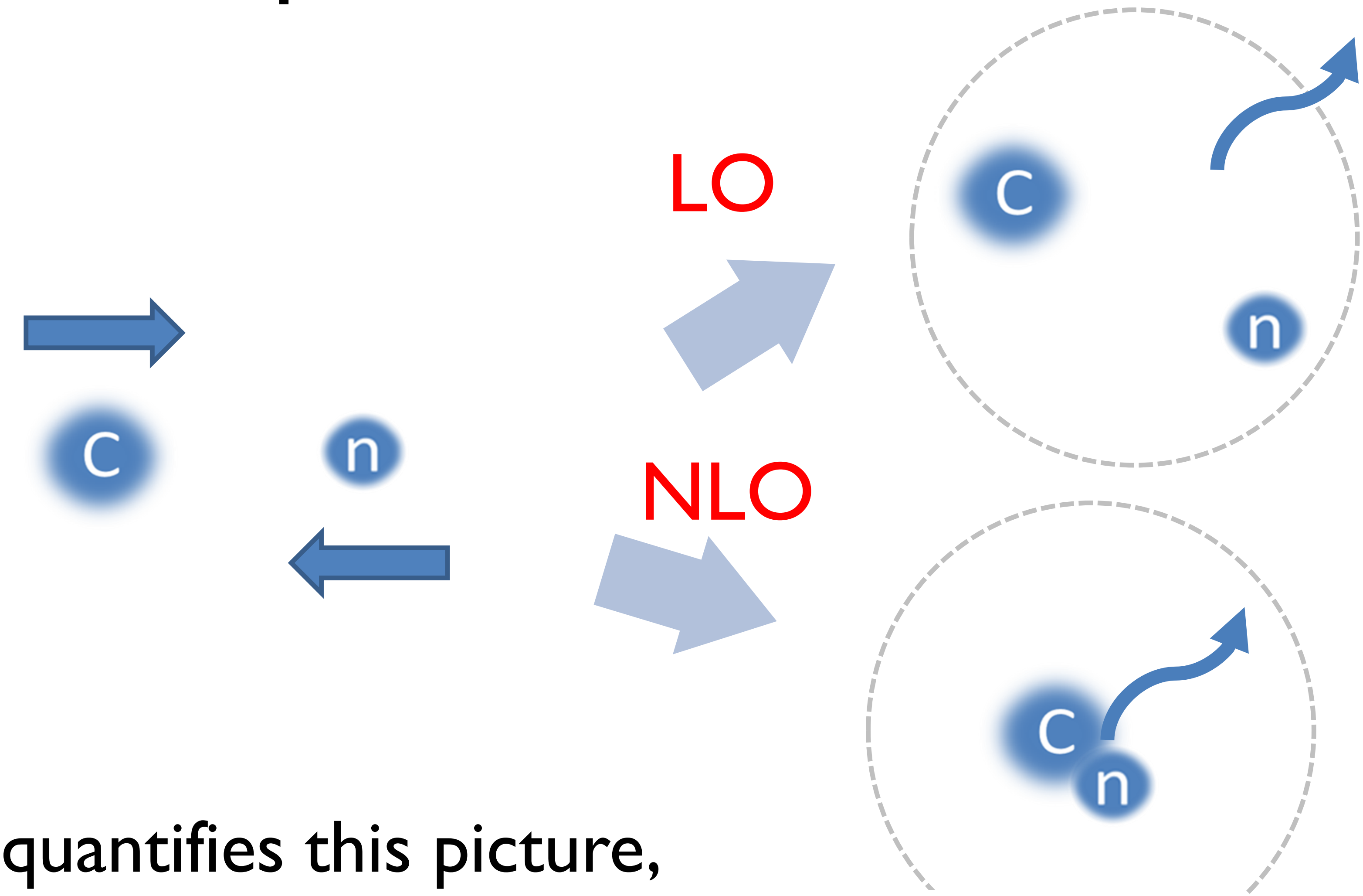


Scale separation: reaction



EFT quantifies this picture,

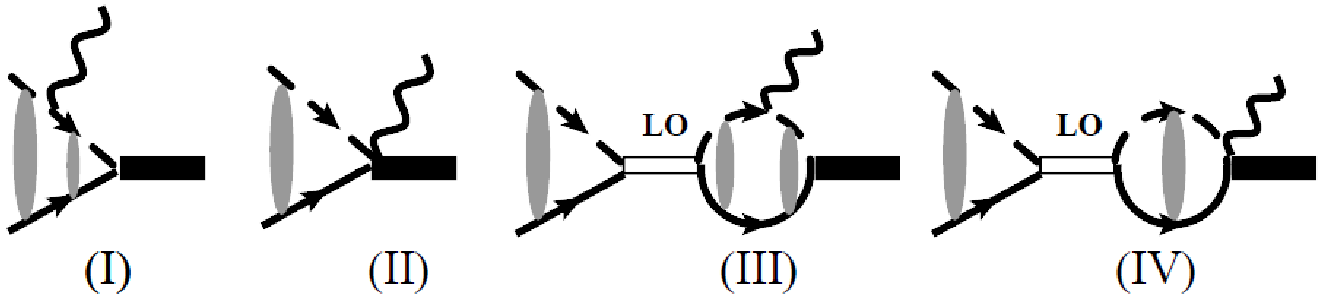
Scale separation: reaction



EFT quantifies this picture,

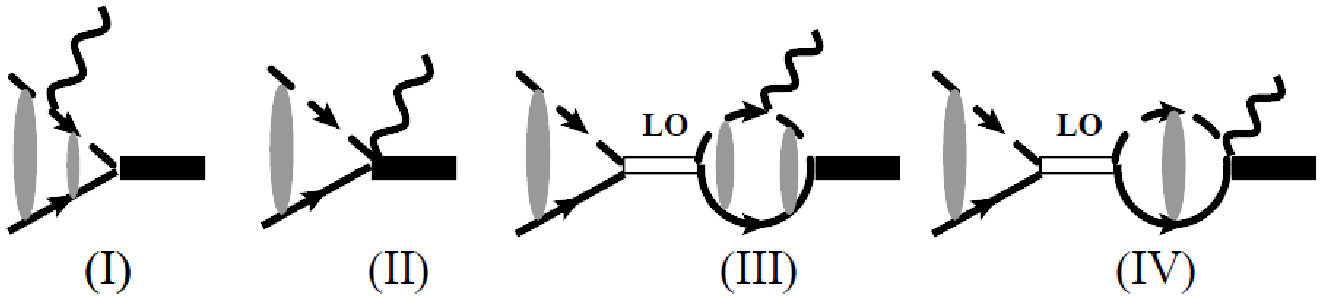
by expanding S-matrix in terms of $Q_{\text{low}}/\Lambda \approx 0.2$.

EFT: N2LO

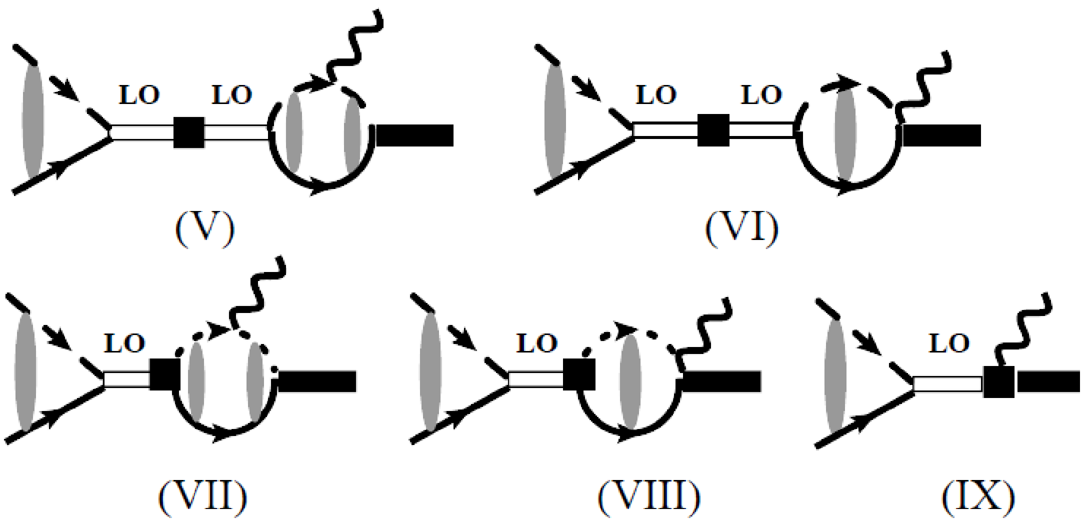


LO: 4 parameters including $C_{(^3P_2)}, C_{(^5P_2)}, a_{(^3S_1)}, a_{(^5S_2)}$

EFT: N2LO

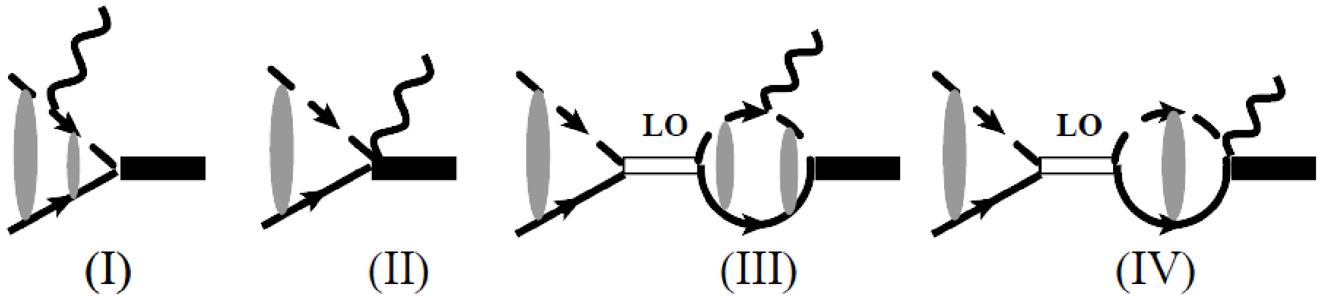


LO: 4 parameters including $C_{(^3P_2)}, C_{(^5P_2)}, a_{(^3S_1)}, a_{(^5S_2)}$

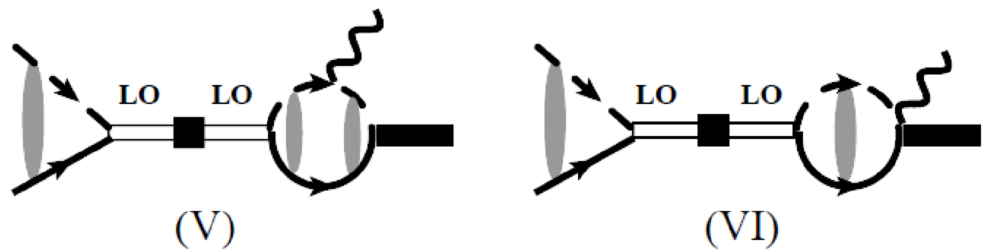


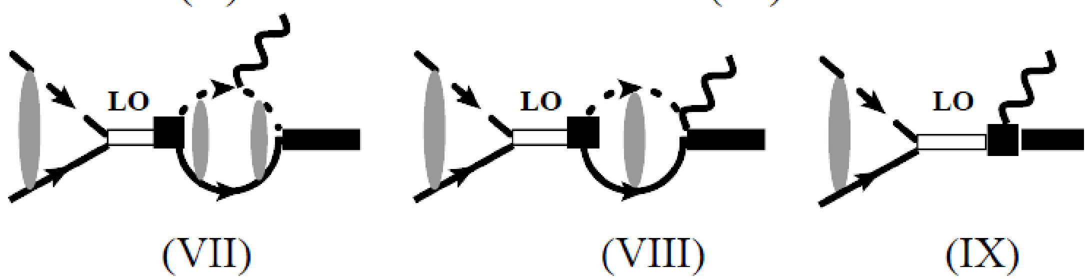
NLO: another 5 parameters including $r_{(^3S_1)}, r_{(^5S_2)}, \varepsilon_1, L_{E1}, L_{E2}$

EFT: N2LO



LO: 4 parameters including $C_{(^3P_2)}, C_{(^5P_2)}, a_{(^3S_1)}, a_{(^5S_2)}$

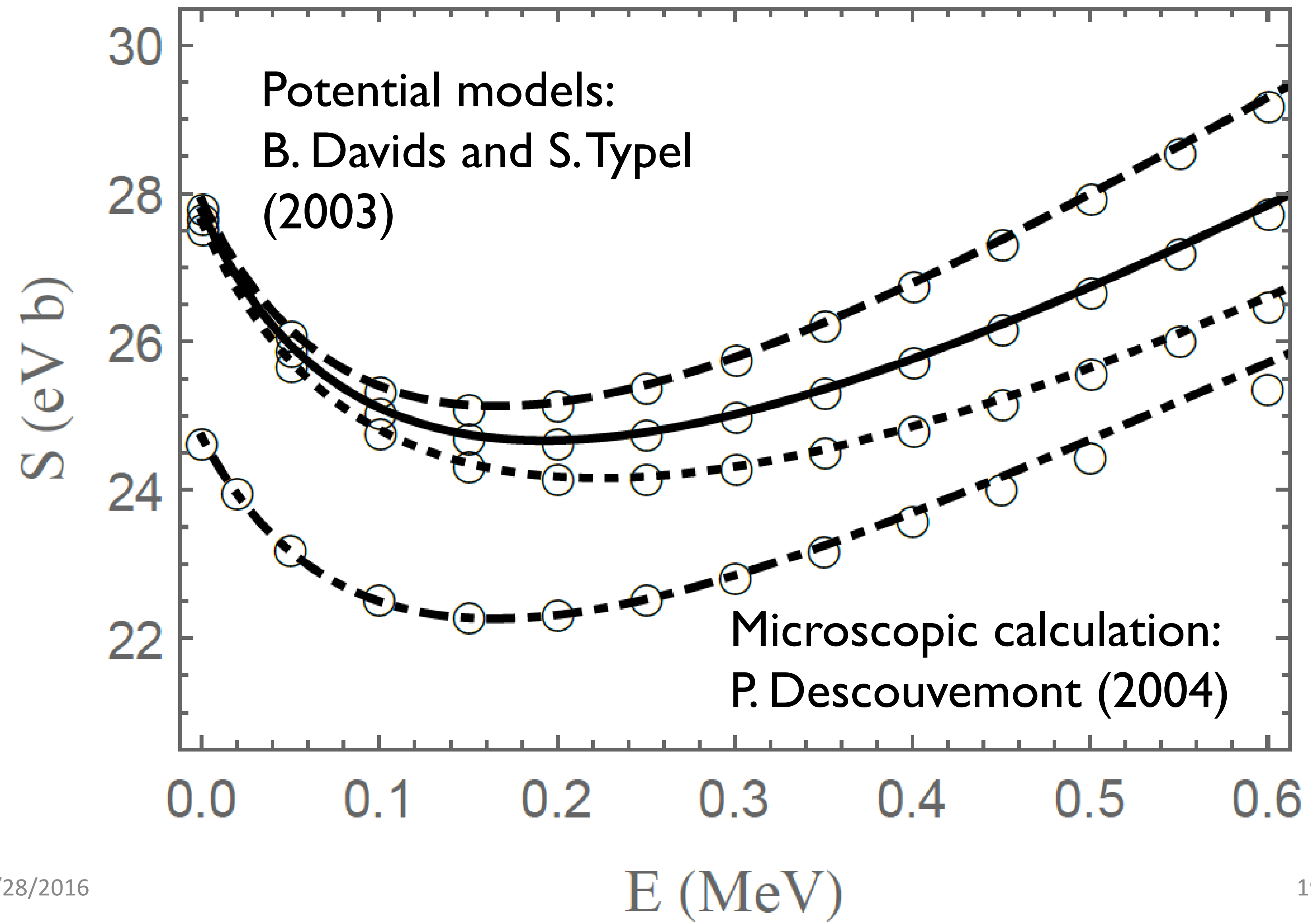




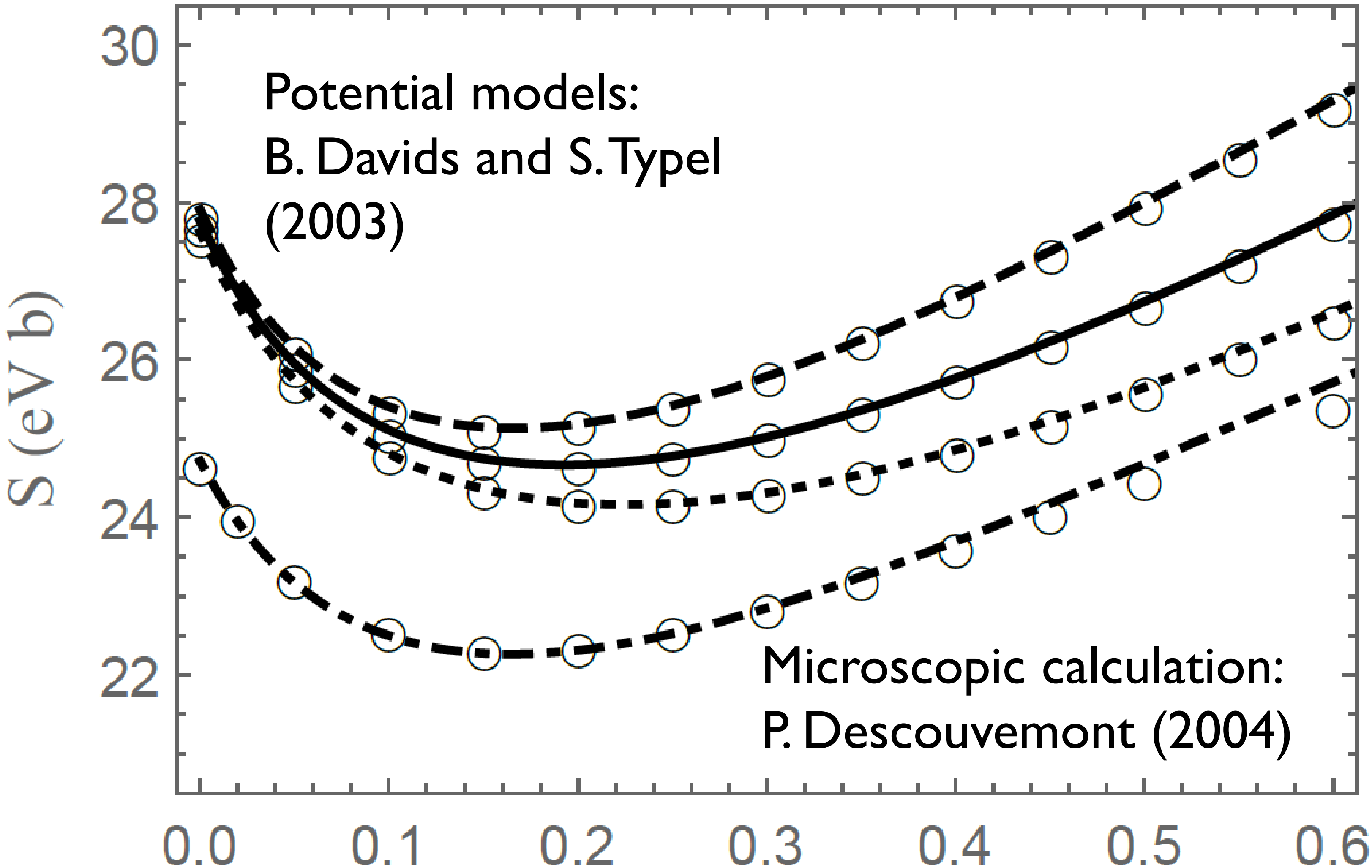
Core excitation

NLO: another 5 parameters including $r_{(^3S_1)}, r_{(^5S_2)}, \varepsilon_1, L_{E1}, L_{E2}$

Model independence



Model independence



EFT reproduces other models

Bayesian Analysis

Bayesian Analysis

Bayesian Analysis

$$\Pr\left(\vec{g},\{\xi_i\}\middle|D;T\right)\propto \Pr\left(D\middle|\vec{g},\{\xi_i\};T\right)\times \Pr\left(\vec{g},\{\xi_i\}\middle|T\right)$$

Bayesian Analysis

$$\Pr\left(\vec{g},\{\xi_i\}\middle|D;T\right)\propto \Pr\left(D\middle|\vec{g},\{\xi_i\};T\right)\times \Pr\left(\vec{g},\{\xi_i\}\middle|T\right)$$

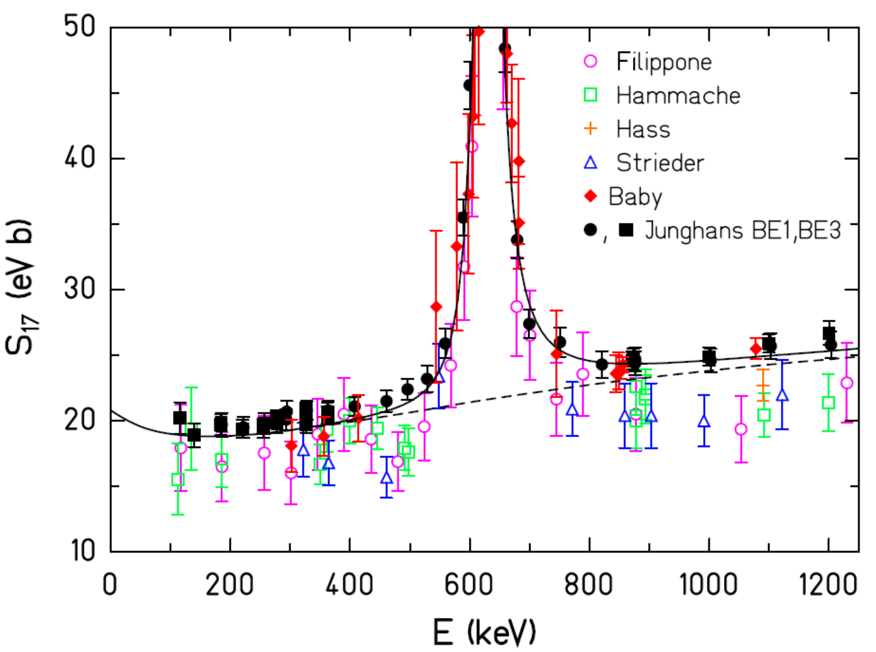
↑

Data. Here only En<0.5 MeV direct capture data are used, including Junghans, Filippone, Hammache, Baby (**32** in total)

Bayesian Analysis

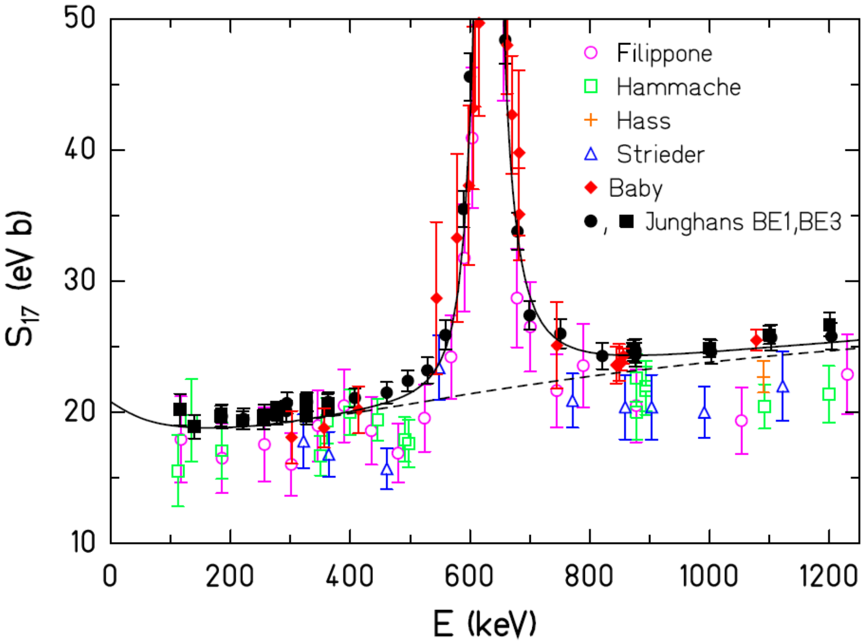
$$\Pr\left(\vec{g},\{\xi_i\}\middle|D;T\right)\propto\Pr\left(D\middle|\vec{g},\{\xi_i\};T\right)\times\Pr\left(\vec{g},\{\xi_i\}\middle|T\right)$$

Data. Here only En<0.5 MeV direct capture data are used, including Junghans, Filippone, Hammache, Baby (**32** in total)



Bayesian Analysis

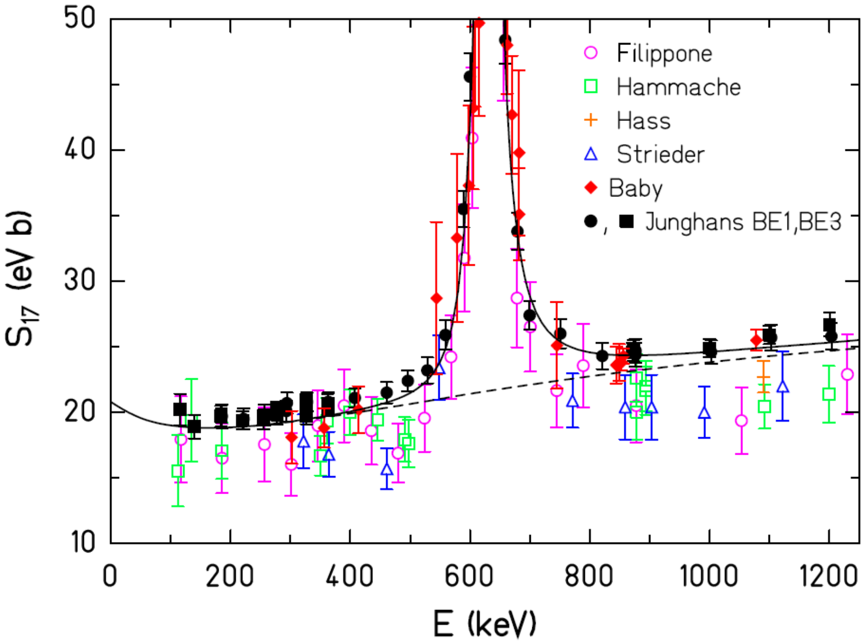
$$\Pr\left(\vec{g},\{\xi_i\}\middle|D;T\right)\propto \Pr\left(D\middle|\vec{g},\{\xi_i\};T\right)\times \Pr\left(\vec{g},\{\xi_i\}\middle|T\right)$$



Bayesian Analysis

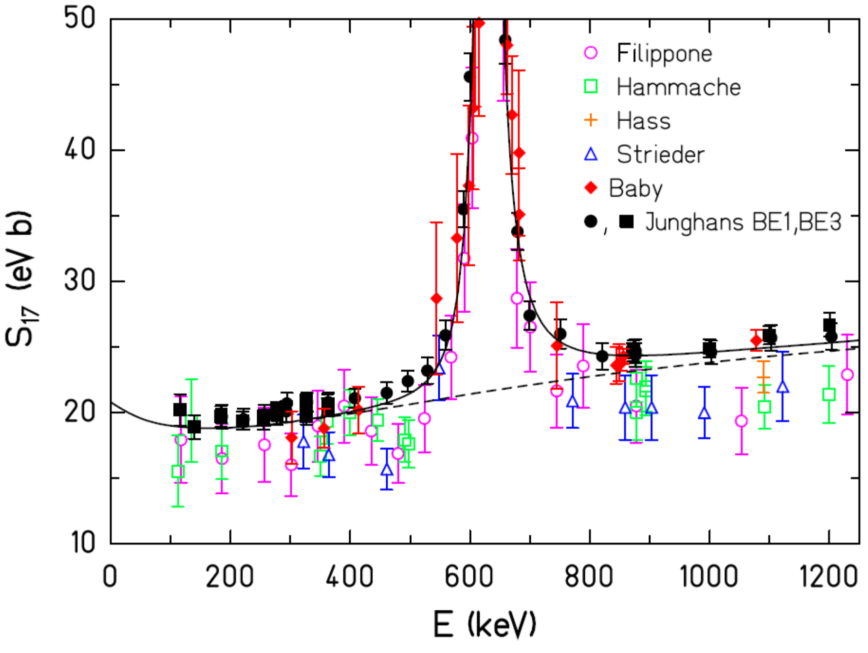
$$\Pr\left(\vec{g},\{\xi_i\}\middle|D;T\right) \propto \Pr\left(D\middle|\vec{g},\{\xi_i\};T\right) \times \Pr\left(\vec{g},\{\xi_i\}\middle|T\right)$$

Theory, here S factor



Bayesian Analysis

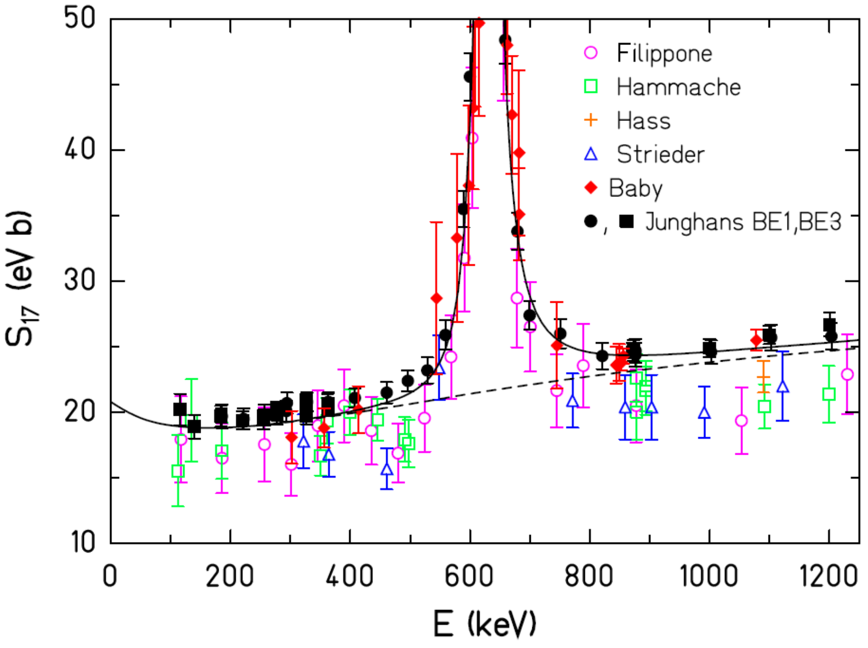
$$\Pr\left(\vec{g},\{\xi_i\}\middle|D;T\right) \propto \Pr\left(D\middle|\vec{g},\{\xi_i\};T\right) \times \Pr\left(\vec{g},\{\xi_i\}\middle|T\right)$$



Bayesian Analysis

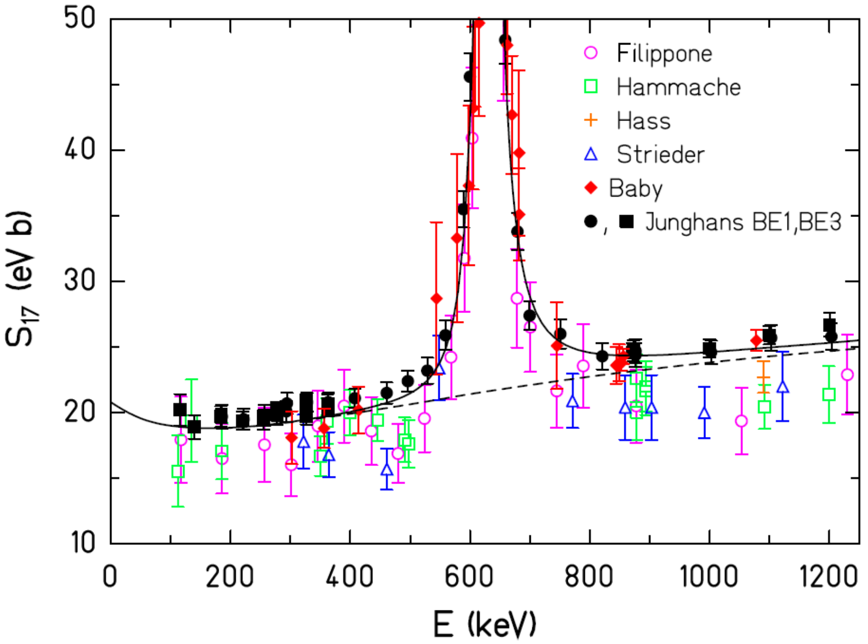
$$\Pr\left(\vec{g},\{\xi_i\}\middle|D;T\right) \propto \Pr\left(D\middle|\vec{g},\{\xi_i\};T\right) \times \Pr\left(\vec{g},\{\xi_i\}\middle|T\right)$$

EFT parameters



Bayesian Analysis

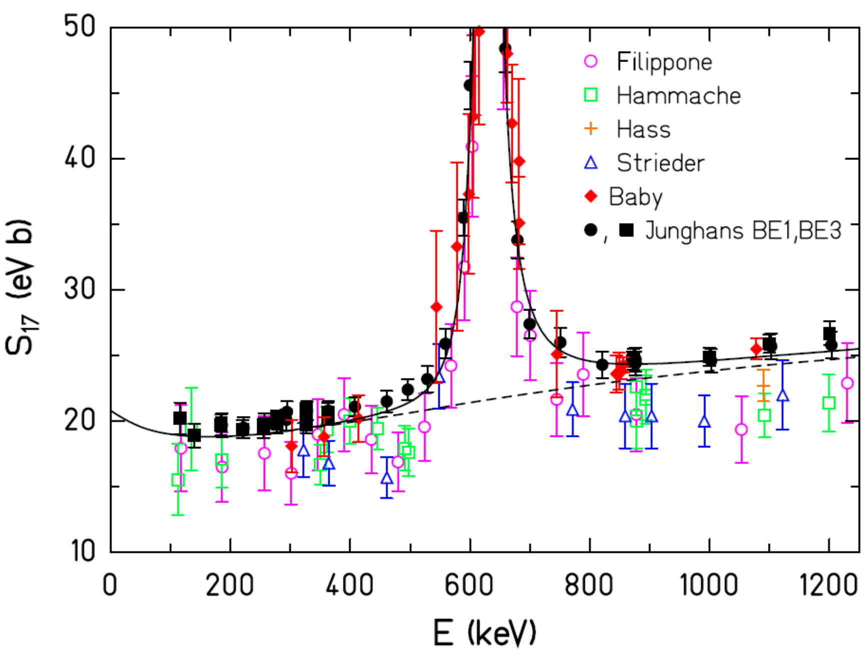
$$\Pr\left(\vec{g},\{\xi_i\}\middle|D;T\right)\propto\Pr\left(D\middle|\vec{g},\{\xi_i\};T\right)\times\Pr\left(\vec{g},\{\xi_i\}\middle|T\right)$$



Bayesian Analysis

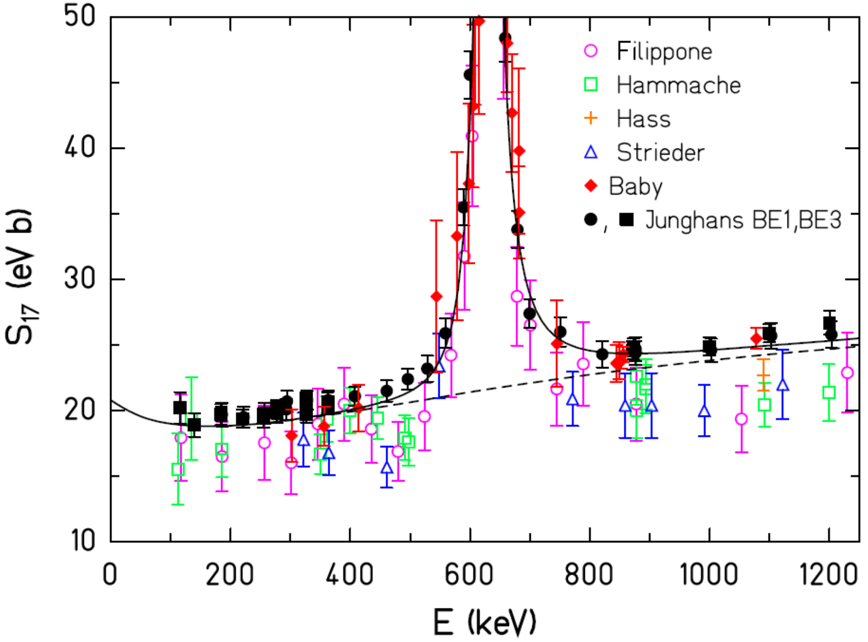
$$\Pr\left(\vec{g},\{\xi_i\}\middle|D;T\right)\propto \Pr\left(D\middle|\vec{g},\{\xi_i\};T\right)\times \Pr\left(\vec{g},\{\xi_i\}\middle|T\right)$$

Systematic error variables



Bayesian Analysis

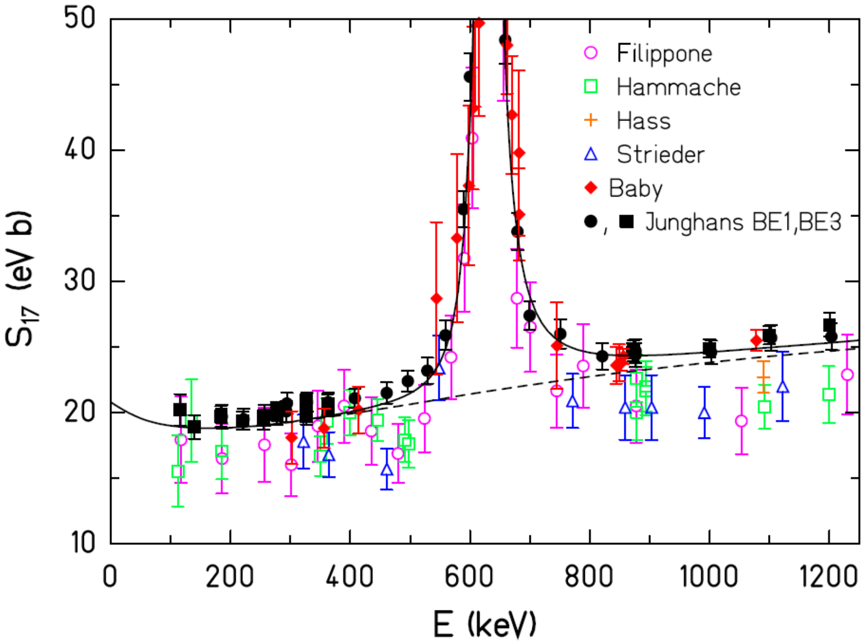
$$\Pr\left(\vec{g},\{\xi_i\}\middle|D;T\right)\propto \Pr\left(D\middle|\vec{g},\{\xi_i\};T\right)\times \Pr\left(\vec{g},\{\xi_i\}\middle|T\right)$$



Bayesian Analysis

$$\Pr\left(\vec{g},\{\xi_i\}\middle|D;T\right) \propto \Pr\left(D\middle|\vec{g},\{\xi_i\};T\right) \times \Pr\left(\vec{g},\{\xi_i\}\middle|T\right)$$

$$\Pr\left(D\middle|\vec{g},\{\xi_i\};T\right) \propto Exp\left(-\frac{\chi^2}{2}\right);\ \chi^2 = \sum_{i=1}^{\#data} \frac{\left[S\left(\vec{g};E_i\right)\left(1-\xi_i\right)-D_i\right]^2}{\sigma_i^2}$$



Bayesian Analysis

$$\Pr\left(\vec{g},\{\xi_i\}\middle|D;T\right)\propto \Pr\left(D\middle|\vec{g},\{\xi_i\};T\right)\times \Pr\left(\vec{g},\{\xi_i\}\middle|T\right)$$

$$\Pr\left(D\middle|\vec{g},\{\xi_i\};T\right)\propto Exp\left(-\frac{\chi^2}{2}\right);\ \chi^2=\sum_{i=1}^{\#data}\frac{\left[S\left(\vec{g};E_i\right)\left(1-\xi_i\right)-D_i\right]^2}{\sigma_i^2}$$

Bayesian Analysis

$$\Pr\left(\vec{g},\{\xi_i\}\middle|D;T\right)\propto \Pr\left(D\middle|\vec{g},\{\xi_i\};T\right)\times \Pr\left(\vec{g},\{\xi_i\}\middle|T\right)$$

$$\Pr\left(D\middle|\vec{g},\{\xi_i\};T\right)\propto Exp\left(-\frac{\chi^2}{2}\right);\ \chi^2=\sum_{i=1}^{\#data}\frac{\left[S\left(\vec{g};E_i\right)\left(1-\xi_i\right)-D_i\right]^2}{\sigma_i^2}$$

$$\Pr\left(\vec{g},\{\xi_i\}\middle|T\right)\propto Exp\left(-\sum_j^{\#sys-err}\frac{\xi_j^{\ 2}}{2\sigma_{\xi_j}^2}\right)\times Exp\left(-\sum_l^{\#para}\frac{\left(g_l-g_l^0\right)^2}{2\sigma_{g_l}^2}\right)$$ or flat dis.

Bayesian Analysis

$$\Pr\left(\vec{g},\{\xi_i\}\middle|D;T\right)\propto \Pr\left(D\middle|\vec{g},\{\xi_i\};T\right)\times \Pr\left(\vec{g},\{\xi_i\}\middle|T\right)$$

$$\Pr\left(D\middle|\vec{g},\{\xi_i\};T\right)\propto Exp\left(-\frac{\chi^2}{2}\right);\ \chi^2=\sum_{i=1}^{\#data}\frac{\left[S\left(\vec{g};E_i\right)\left(1-\xi_i\right)-D_i\right]^2}{\sigma_i^2}$$

$$\Pr\left(\vec{g},\{\xi_i\}\middle|T\right)\propto Exp\left(-\sum_j^{\#sys-err}\frac{\xi_j^{\ 2}}{2\sigma_{\xi_j}^2}\right)\times Exp\left(-\sum_l^{\#para}\frac{\left(g_l-g_l^0\right)^2}{2\sigma_{g_l}^2}\right)$$ or flat dis.

$$\Pr\left(\vec{g}\middle|D;T\right)=\int\prod_i d\xi_i\ \Pr\left(\vec{g},\{\xi_i\}\middle|D;T\right)$$

Bayesian Analysis

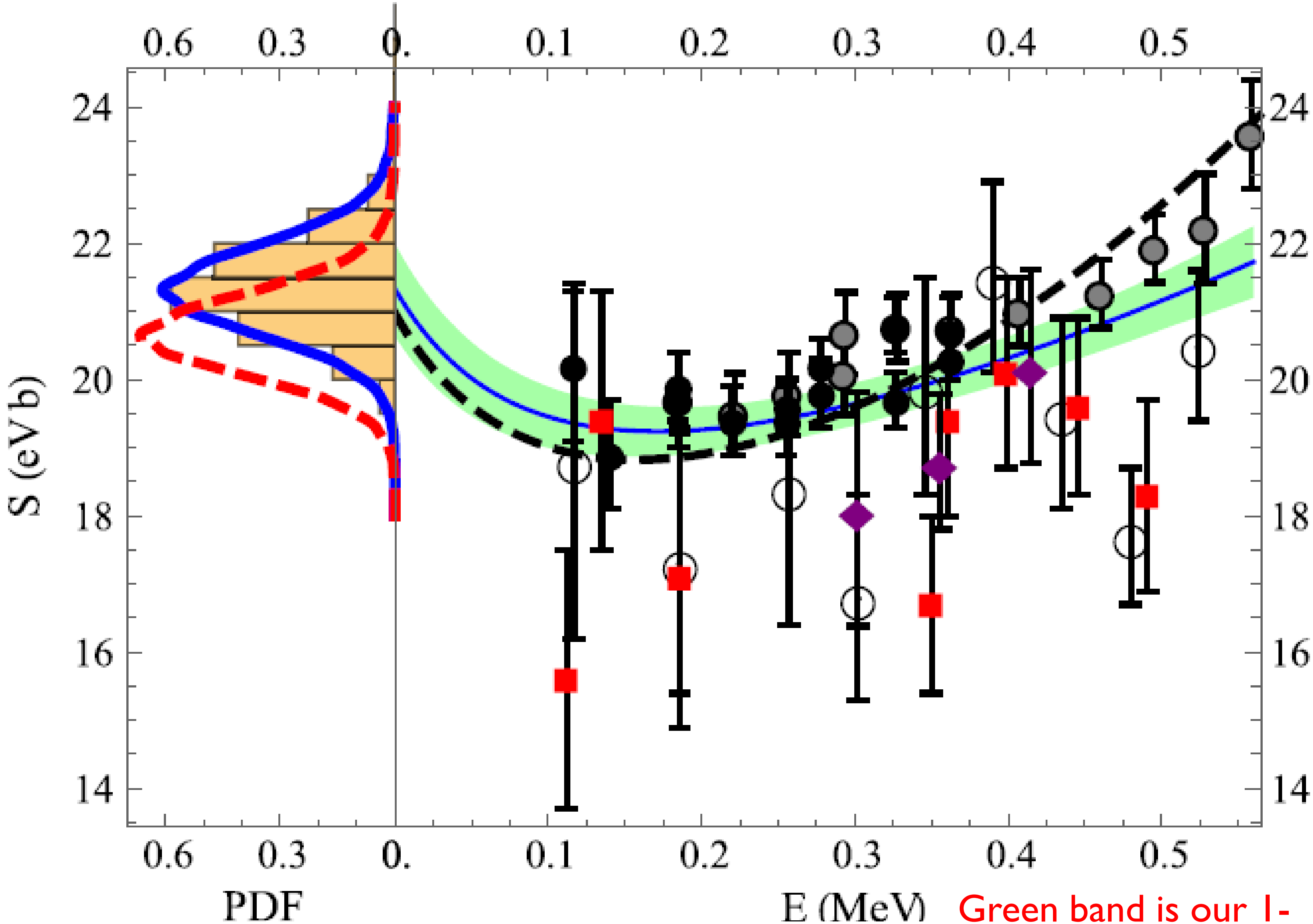
$$\Pr\left(\vec{g},\{\xi_i\}\middle|D;T\right)\propto \Pr\left(D\middle|\vec{g},\{\xi_i\};T\right)\times \Pr\left(\vec{g},\{\xi_i\}\middle|T\right)$$

$$\Pr\left(D\middle|\vec{g},\{\xi_i\};T\right)\propto Exp\left(-\frac{\chi^2}{2}\right);\ \chi^2=\sum_{i=1}^{\#data}\frac{\left[S\left(\vec{g};E_i\right)\left(1-\xi_i\right)-D_i\right]^2}{\sigma_i^2}$$

$$\Pr\left(\vec{g},\{\xi_i\}\middle|T\right)\propto Exp\left(-\sum_{j}^{\#sys-err}\frac{\xi_j^{\ 2}}{2\sigma_{\xi_j}^2}\right)\times Exp\left(-\sum_{l}^{\#para}\frac{\left(g_l-g_l^0\right)^2}{2\sigma_{g_l}^2}\right)\text{ or flat dis.}$$

$$\Pr\left(\vec{g}\middle|D;T\right)=\int\prod_i d\xi_i \Pr\left(\vec{g},\{\xi_i\}\middle|D;T\right)$$

Monte-Carlo Markov-Chain ➔ ensemble of parameters according to the parameter distributions



Junghans BE1 and BE3 (filled circle), Filippone (open circle), Baby (filled diamond), Hammache (filled box)

Green band is our 1-standard deviation error band: 3% error

	S (eV b)	S'/S (MeV^{-1})	S''/S (MeV^{-2})
Median	21.33 [20.67]	-1.82 [-1.34]	31.96 [22.30]
$+\sigma$	0.66 [0.60]	0.12 [0.12]	0.33 [0.34]
$-\sigma$	0.69 [0.63]	0.12 [0.12]	0.37 [0.38]

S(0 keV) [S(20 keV)]

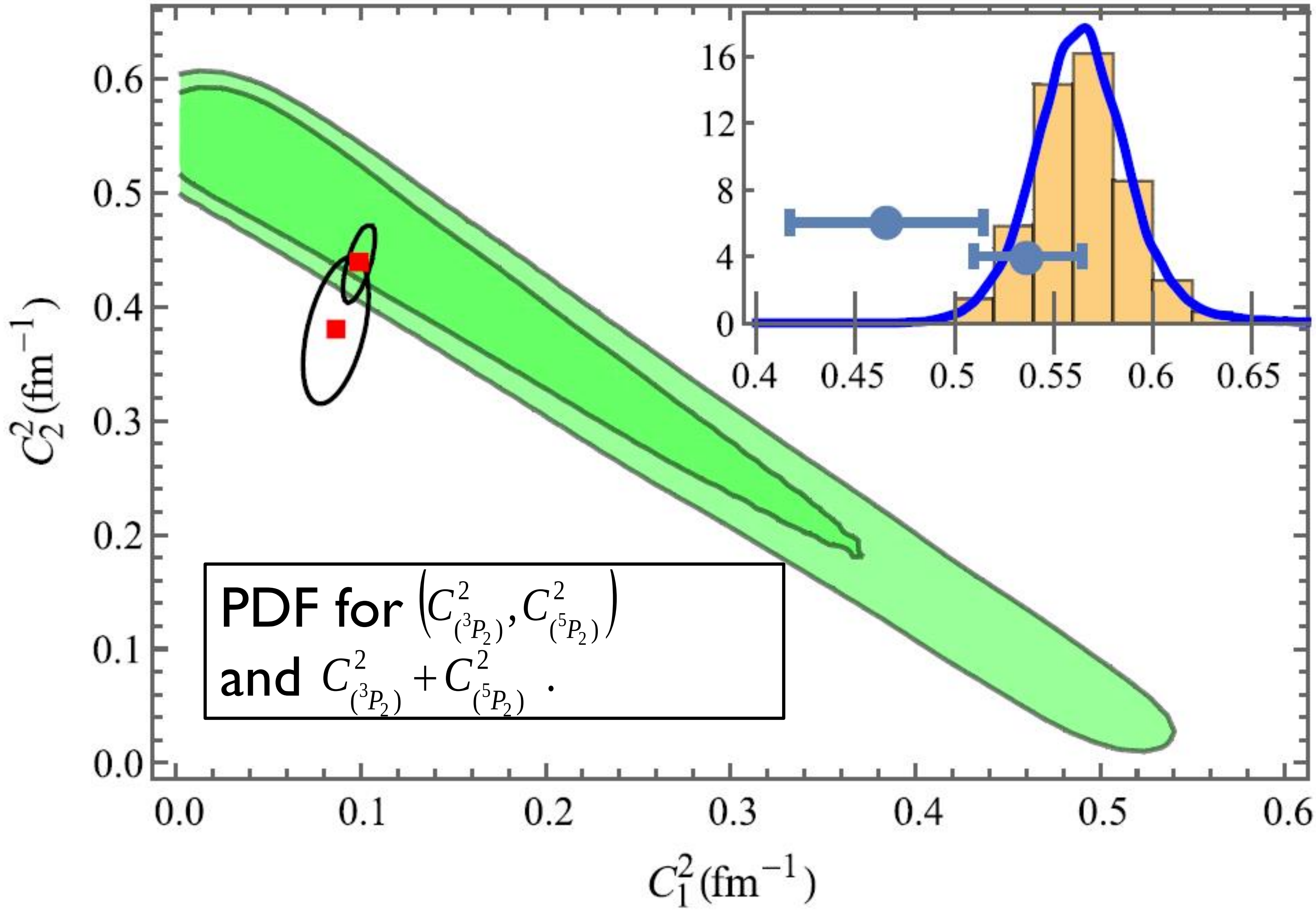
E. G. Adelberger, et.al., Rev. Mod. Phys. 83, 195 (2011)
recommend: $S(0) = 20.8 \pm 0.7$ (expt) ± 1.4 (theor) eV b

	S (eV b)	S'/S (MeV^{-1})	S''/S (MeV^{-2})
Median	21.33 [20.67]	−1.82 [−1.34]	31.96 [22.30]
$+\sigma$	0.66 [0.60]	0.12 [0.12]	0.33 [0.34]
$-\sigma$	0.69 [0.63]	0.12 [0.12]	0.37 [0.38]

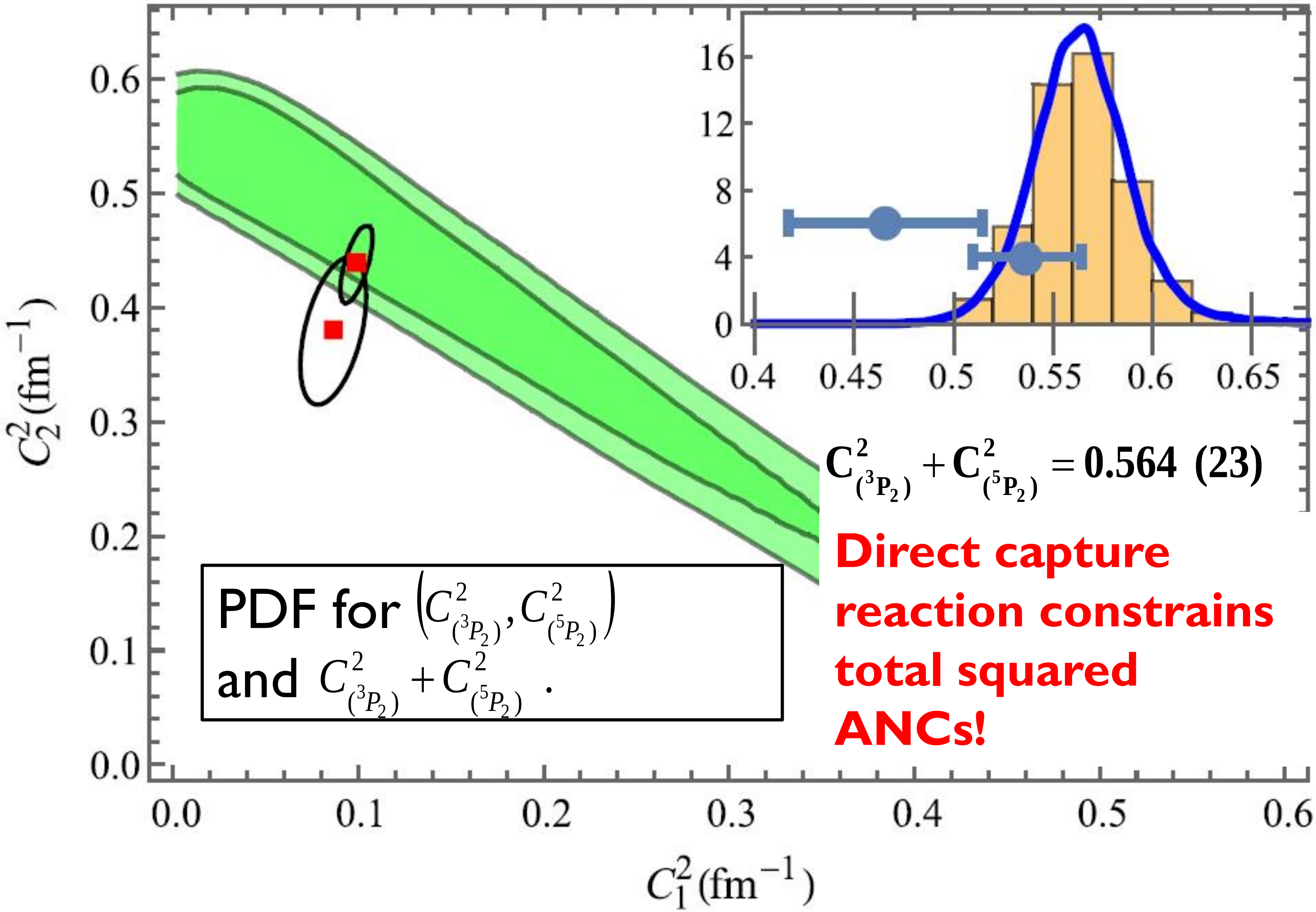
S(0 keV) [S(20 keV)]

E. G. Adelberger, et.al., Rev. Mod. Phys. 83, 195 (2011)
recommend: $S(0) = 20.8 \pm 0.7$ (expt) ± 1.4 (theor) eV b

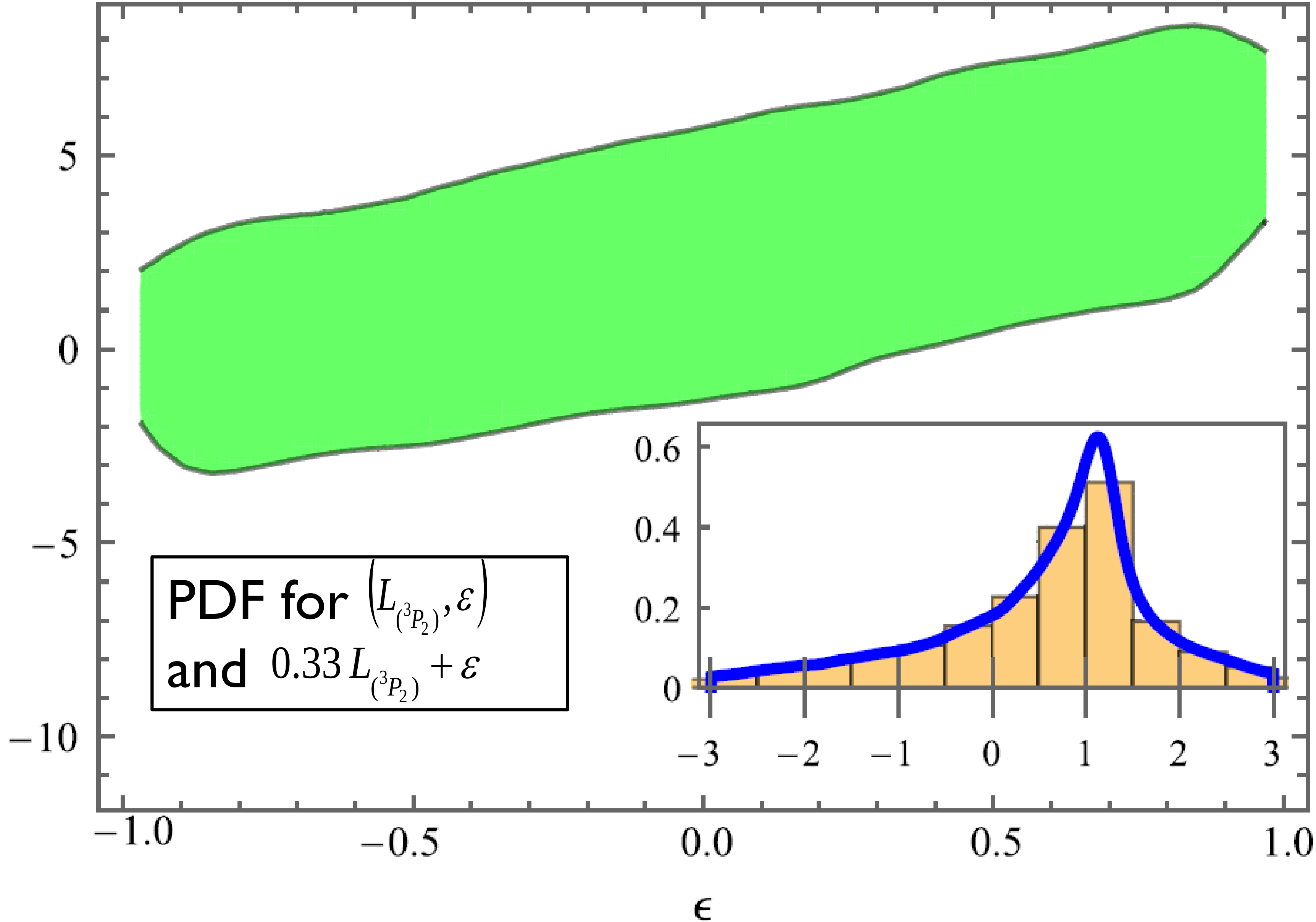
We reduce the error by more than 50%!



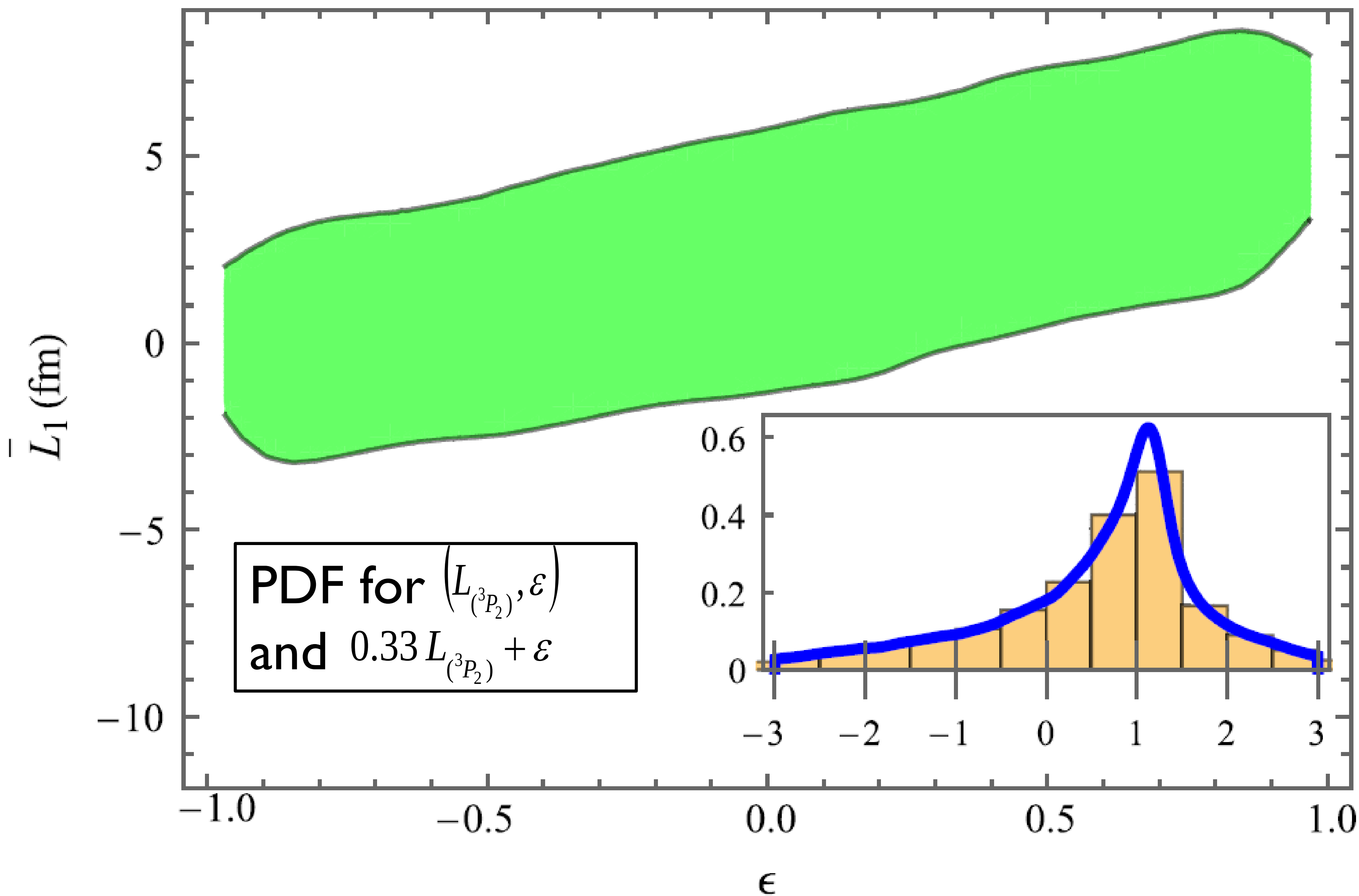
Tabacaru *et.al.*, measurements by transfer reaction (large eclipse)
Nollett *et.al.*, *ab initio* calculation (small eclipse)



Tabacaru *et.al.*, measurements by transfer reaction (large eclipse)
Nollett *et.al.*, *ab initio* calculation (small eclipse)

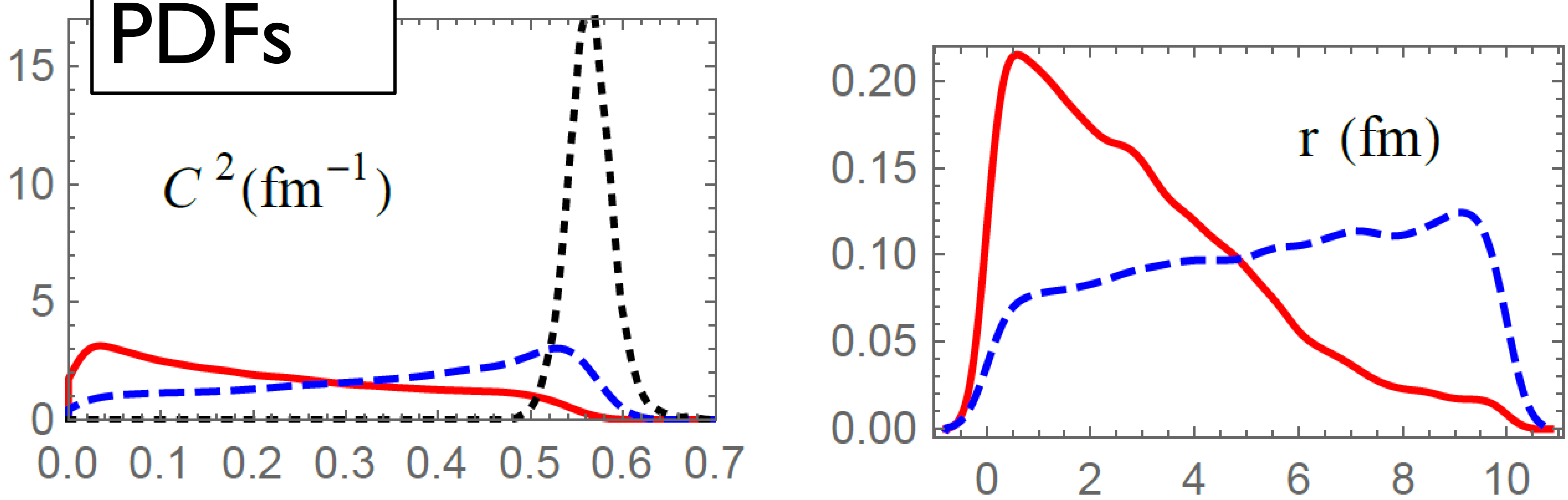


5
0
−5
−10
$\bar{L}_1$ (fm)
−1.0
−0.5
0.0
0.5
1.0
ϵ
0.6
0.4
0.2
0
−3
−2
−1
0
1
2
3
PDF for $\left(L_{(^3P_2)}, \varepsilon\right)$
and $0.33\,L_{(^3P_2)} + \varepsilon$



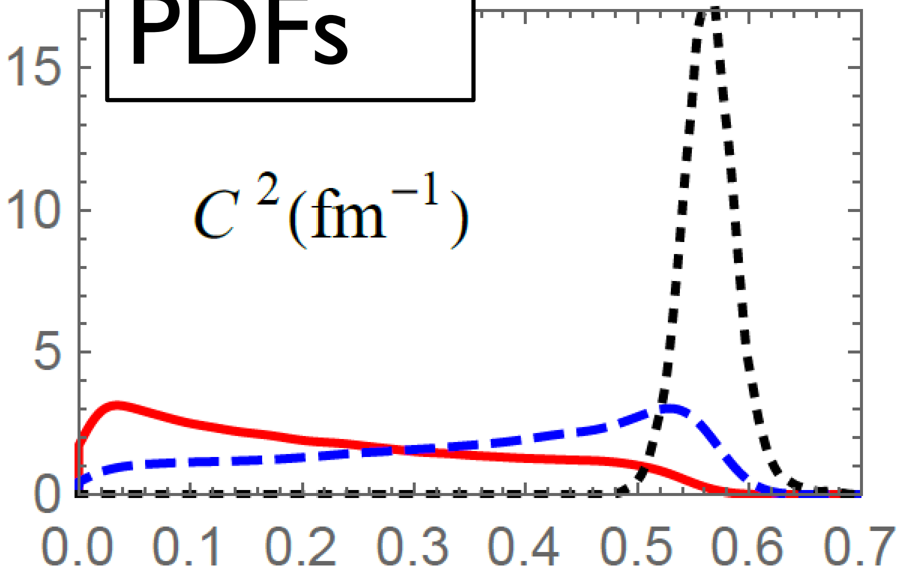
Core excitation and short range term
not distinguished by low energy data

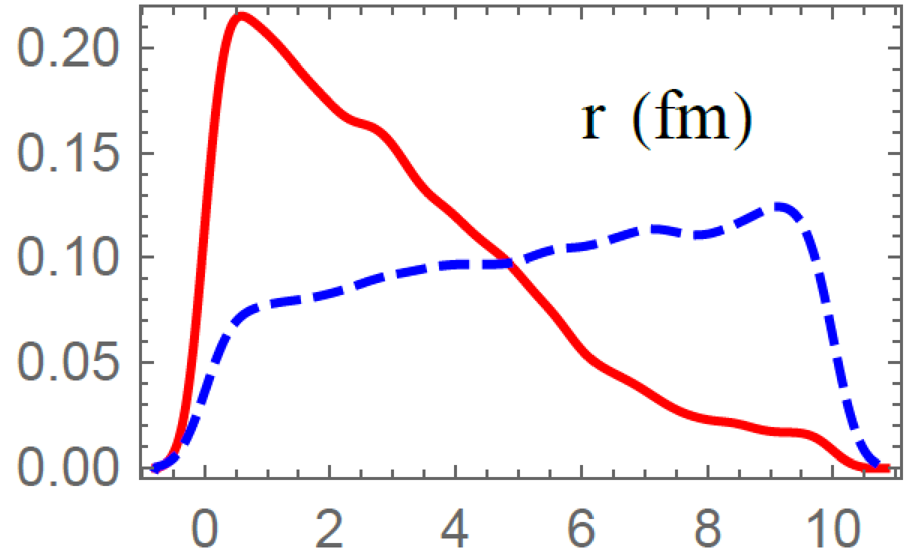
PDFs



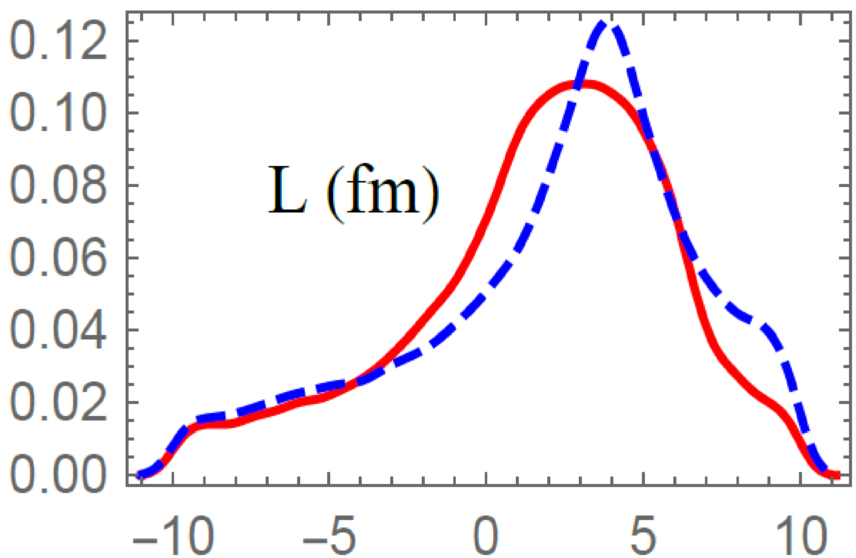
Red for S=1, Blue for S=2.

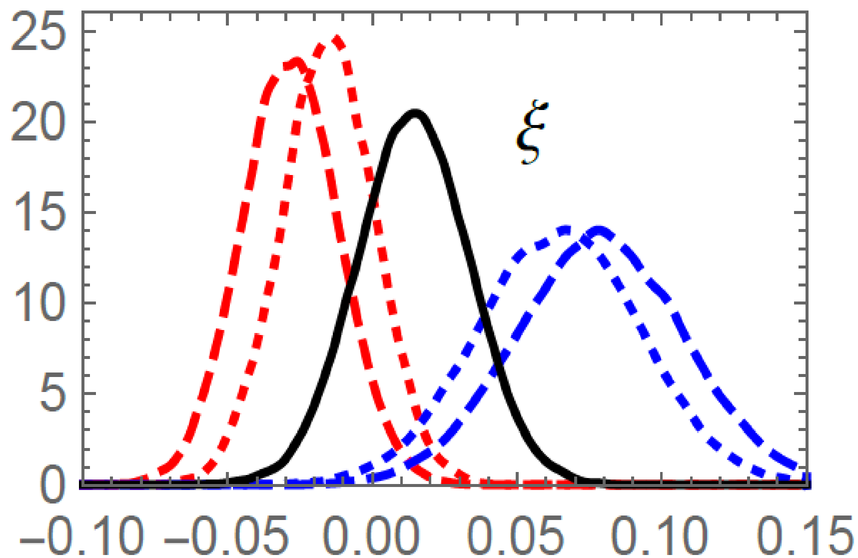
PDFs



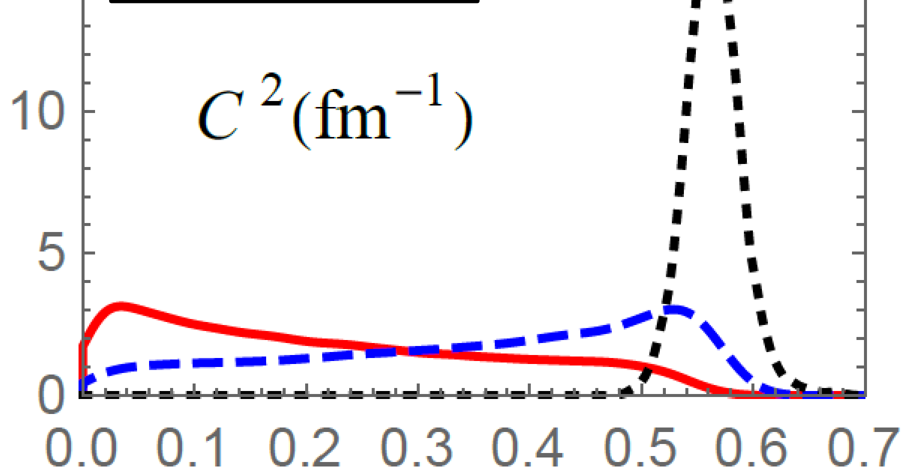


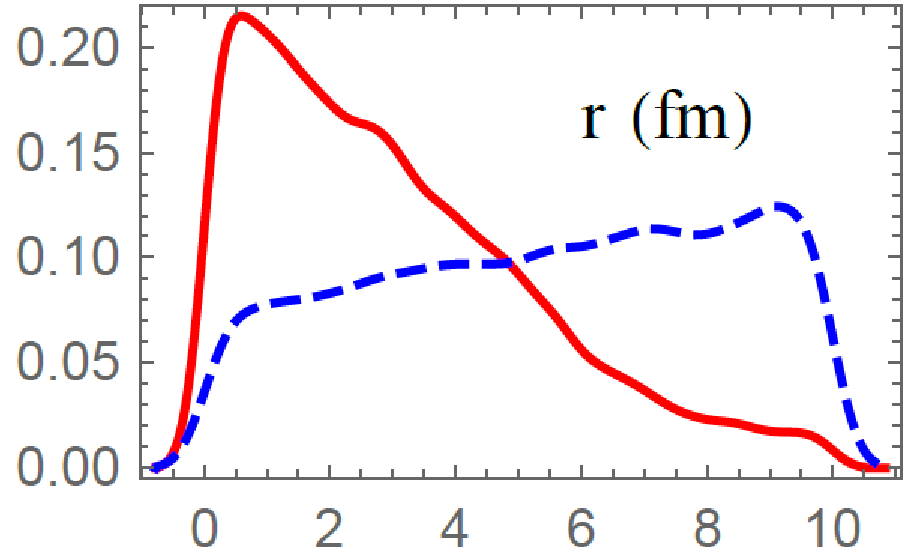
Red for S=1, Blue for S=2.



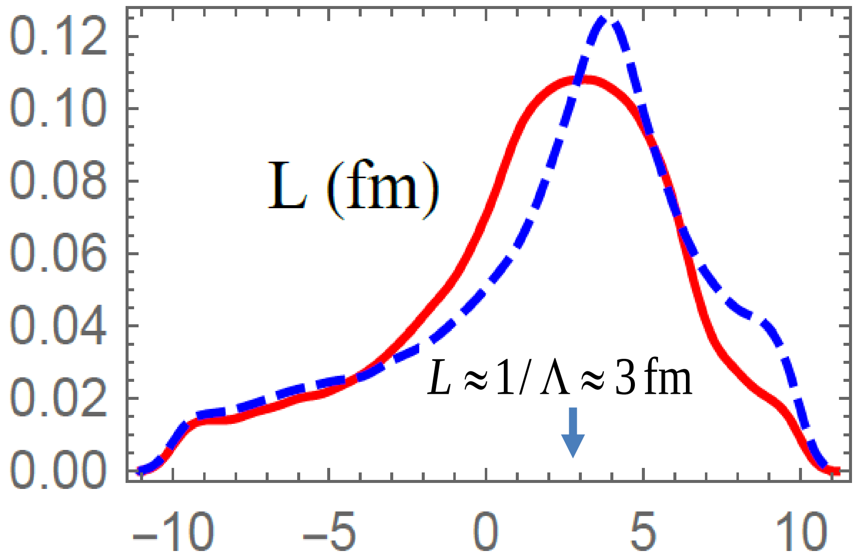


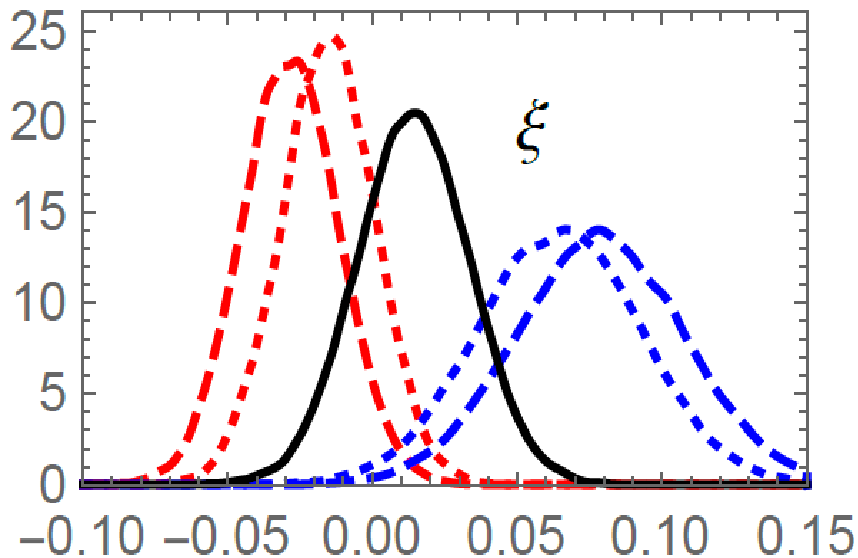
PDFs



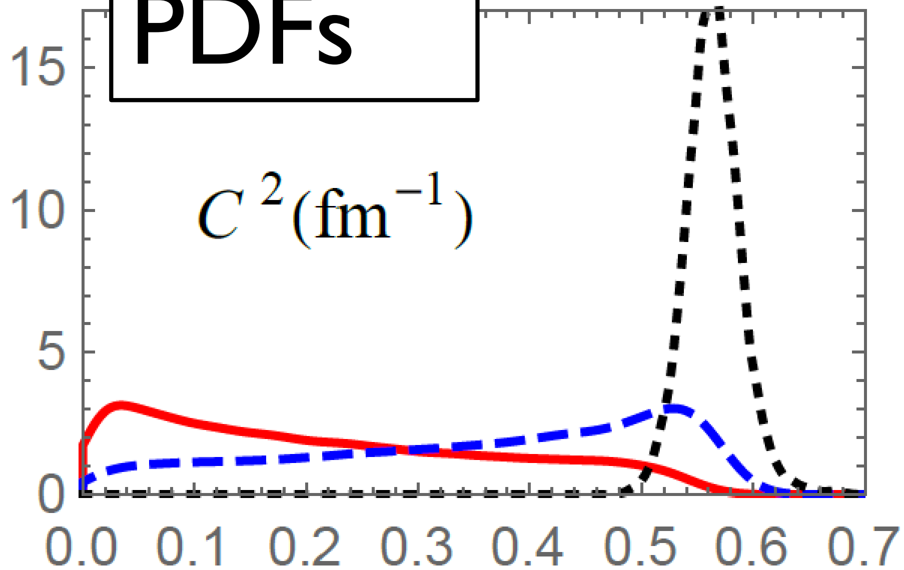


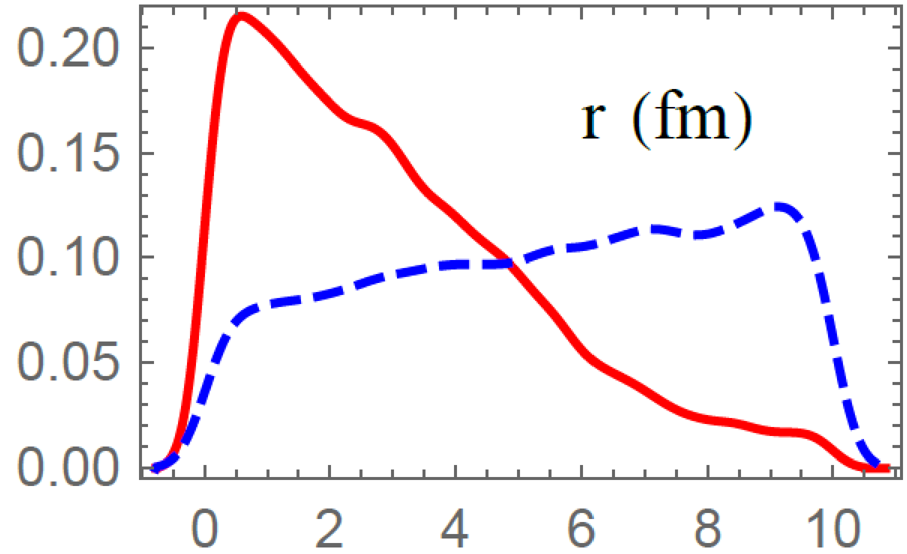
Red for S=1, Blue for S=2.



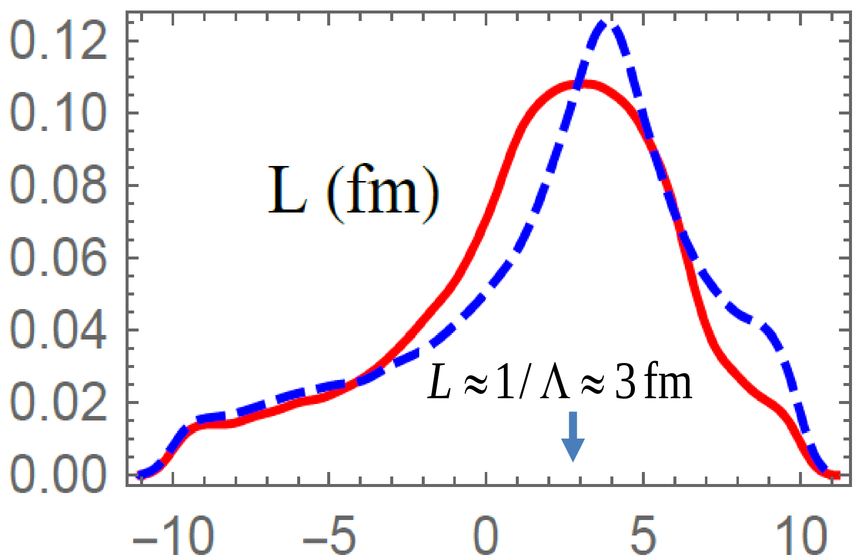


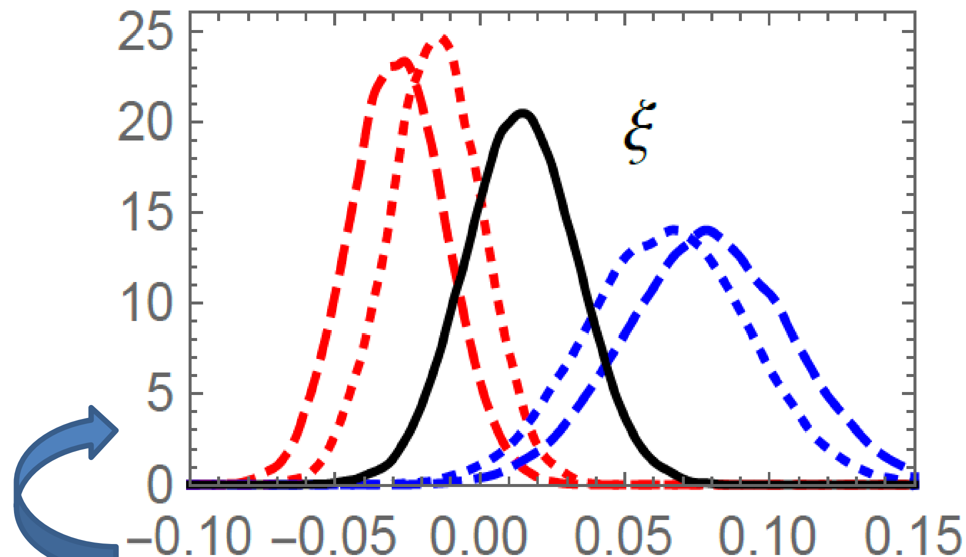
PDFs





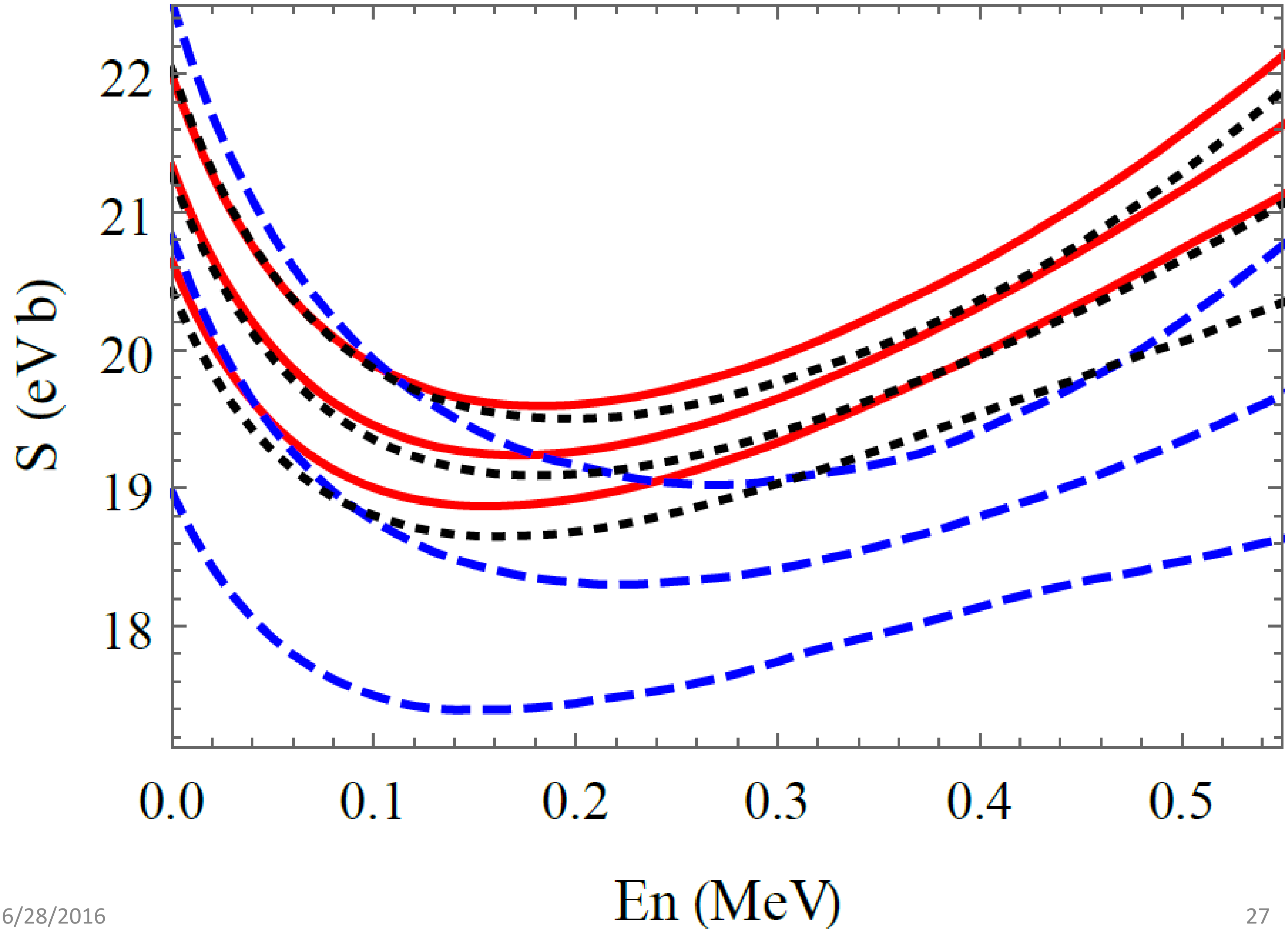
Red for S=1, Blue for S=2.



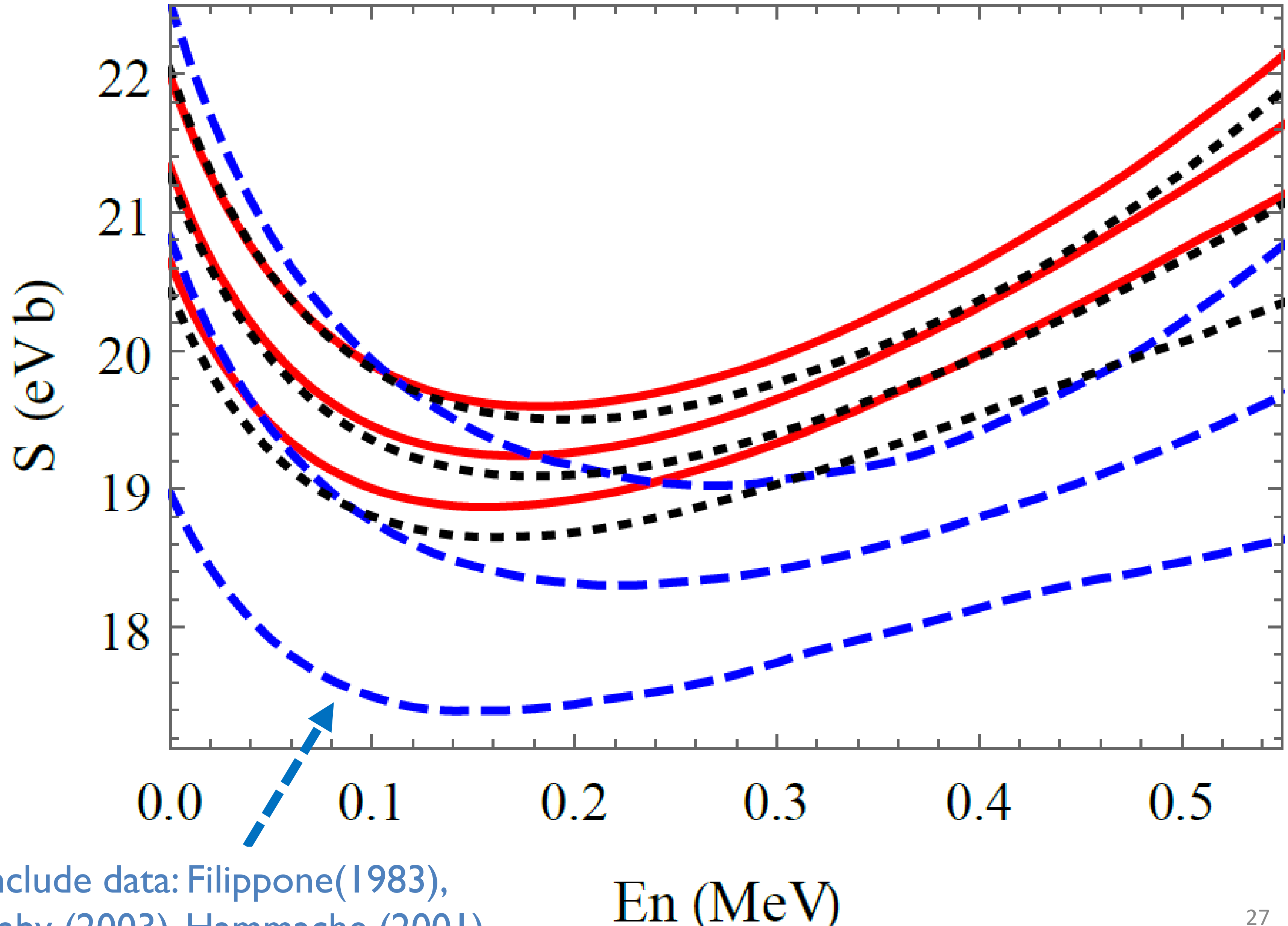


From left to right: Junghans (BE1and BE3) Baby, Hammache, Filipponne

Choice of data sets

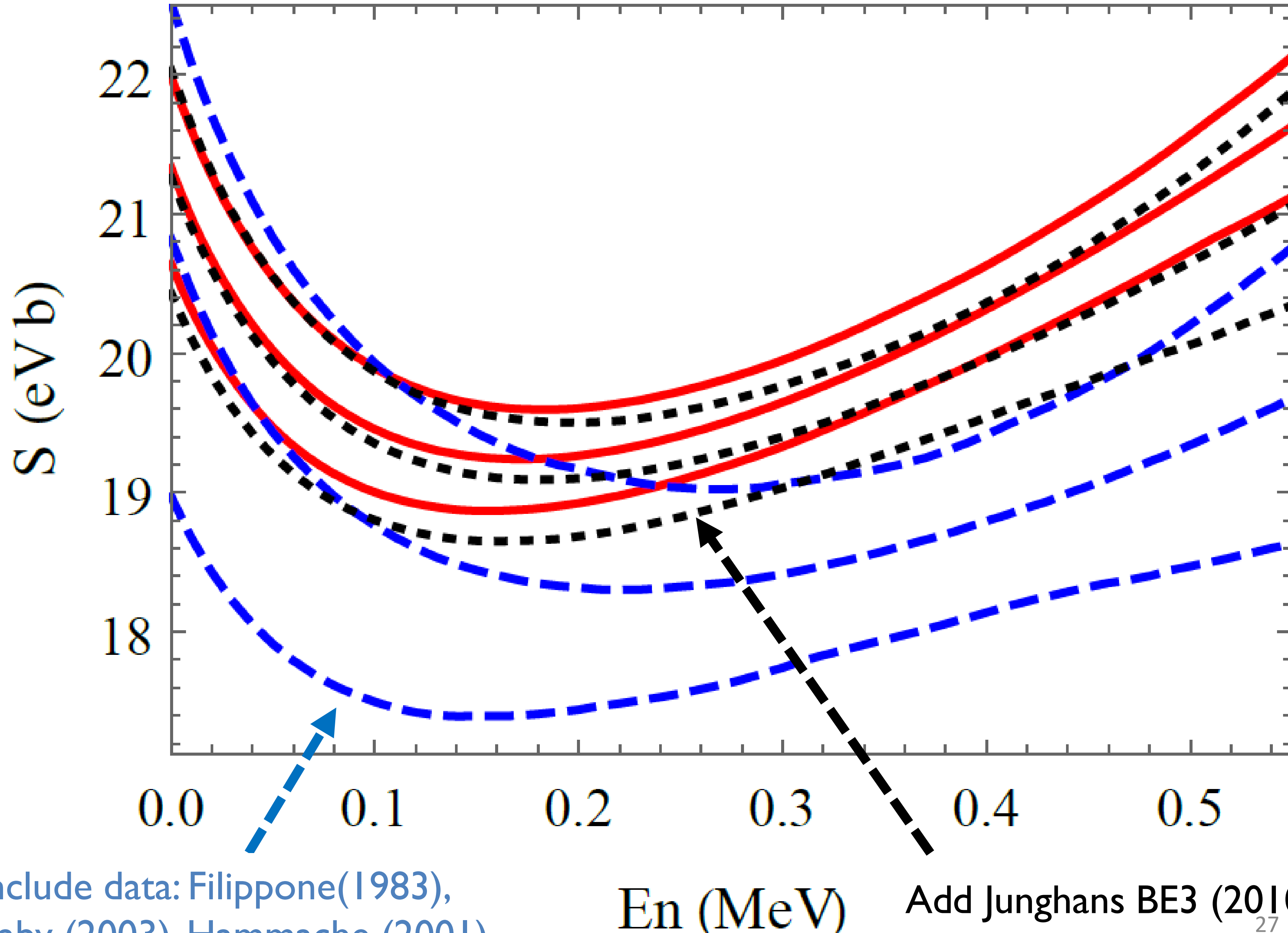


Choice of data sets



Include data: Filippone(1983), Baby (2003), Hammache (2001)

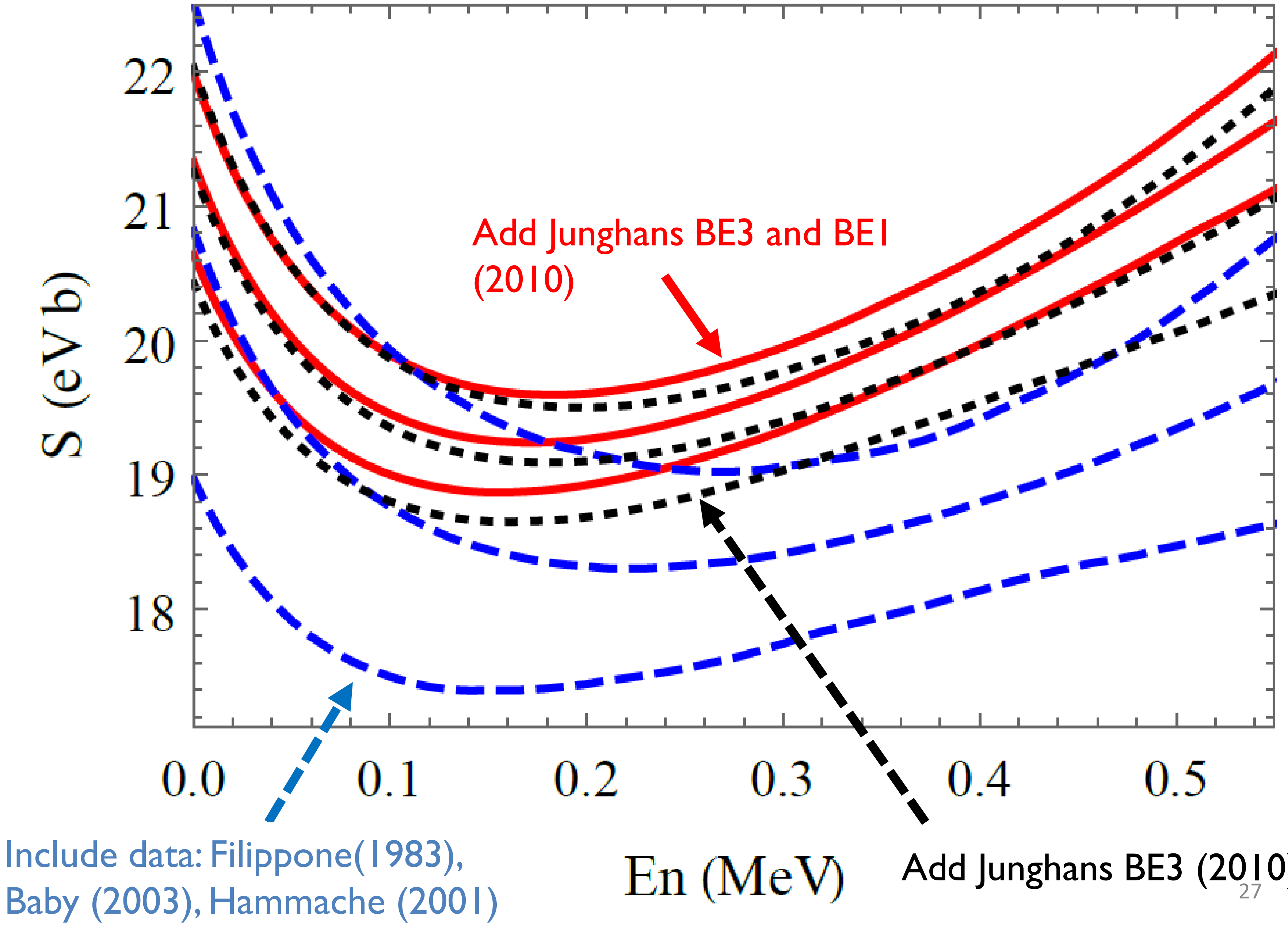
Choice of data sets



Include data: Filippone(1983), Baby (2003), Hammache (2001)

Add Junghans BE3 (2010)

Choice of data sets



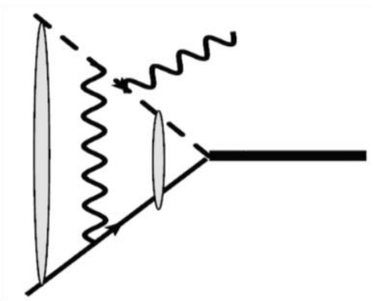
EFT N2LO corrections

EFT N2LO corrections

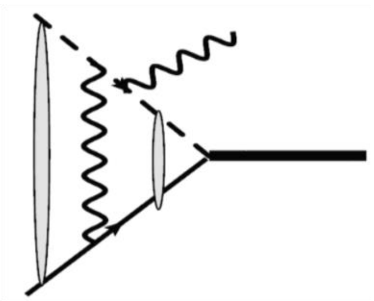
- E2, M1 contributions (S factor): < 0.01%

EFT N2LO corrections

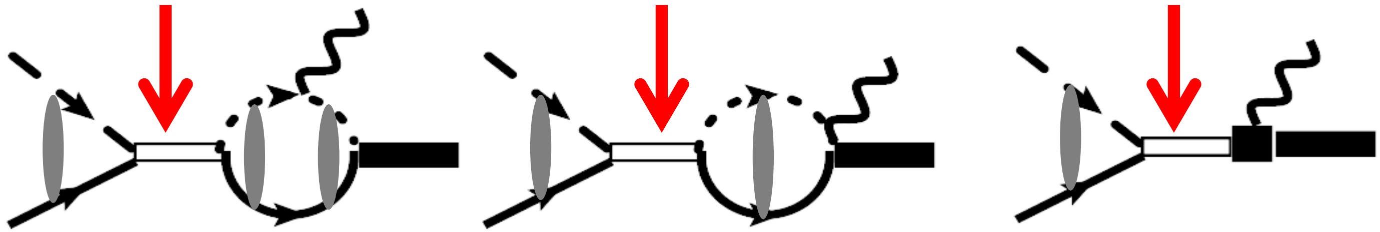
- E2, M1 contributions (S factor): < 0.01%
- Radiative corrections: ~0.1%



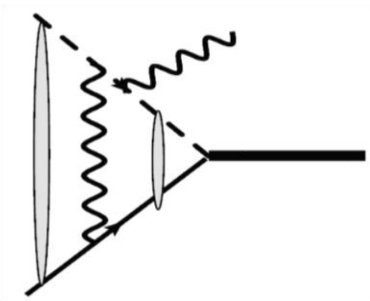
EFT N2LO corrections



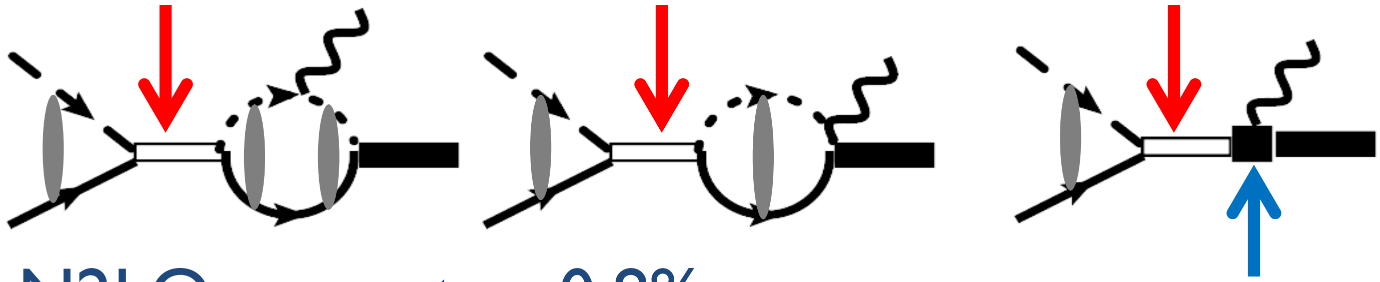
- E2, M1 contributions (S factor): < 0.01%
- Radiative corrections: ~0.1%
- EFT s-wave scattering: ~0.8%



EFT N2LO corrections

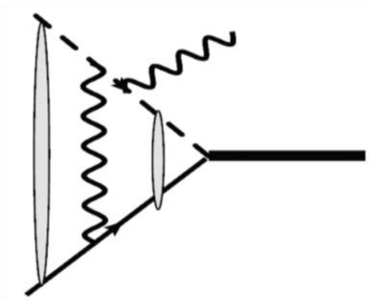


- E2, M1 contributions (S factor): < 0.01%
- Radiative corrections: ~0.1%
- EFT s-wave scattering: ~0.8%

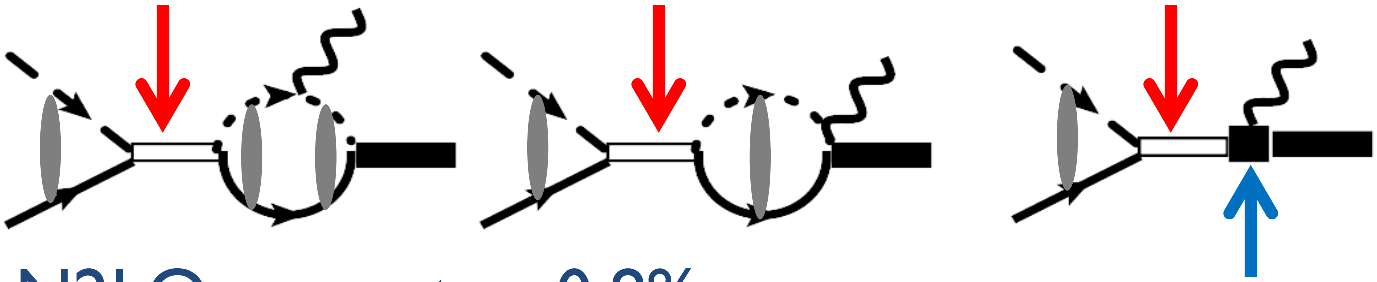


- EFT N2LO currents: ~0.8%

EFT N2LO corrections

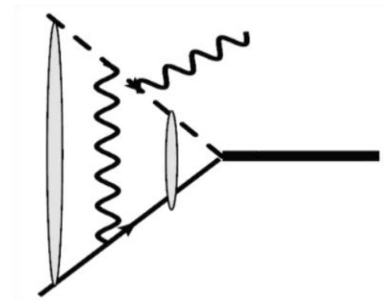


- E2, M1 contributions (S factor): < 0.01%
- Radiative corrections: ~0.1%
- EFT s-wave scattering: ~0.8%

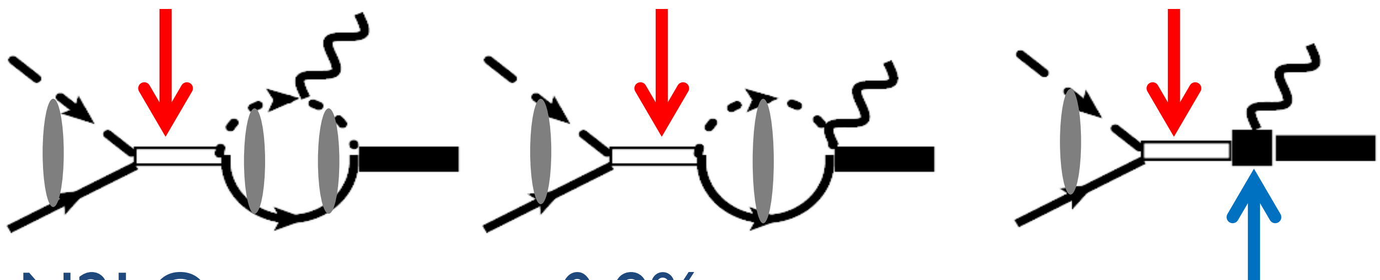


- EFT N2LO currents: ~0.8%
- Notice B8 BE=136.4(1.0) keV: ~ 0.8%

EFT N2LO corrections



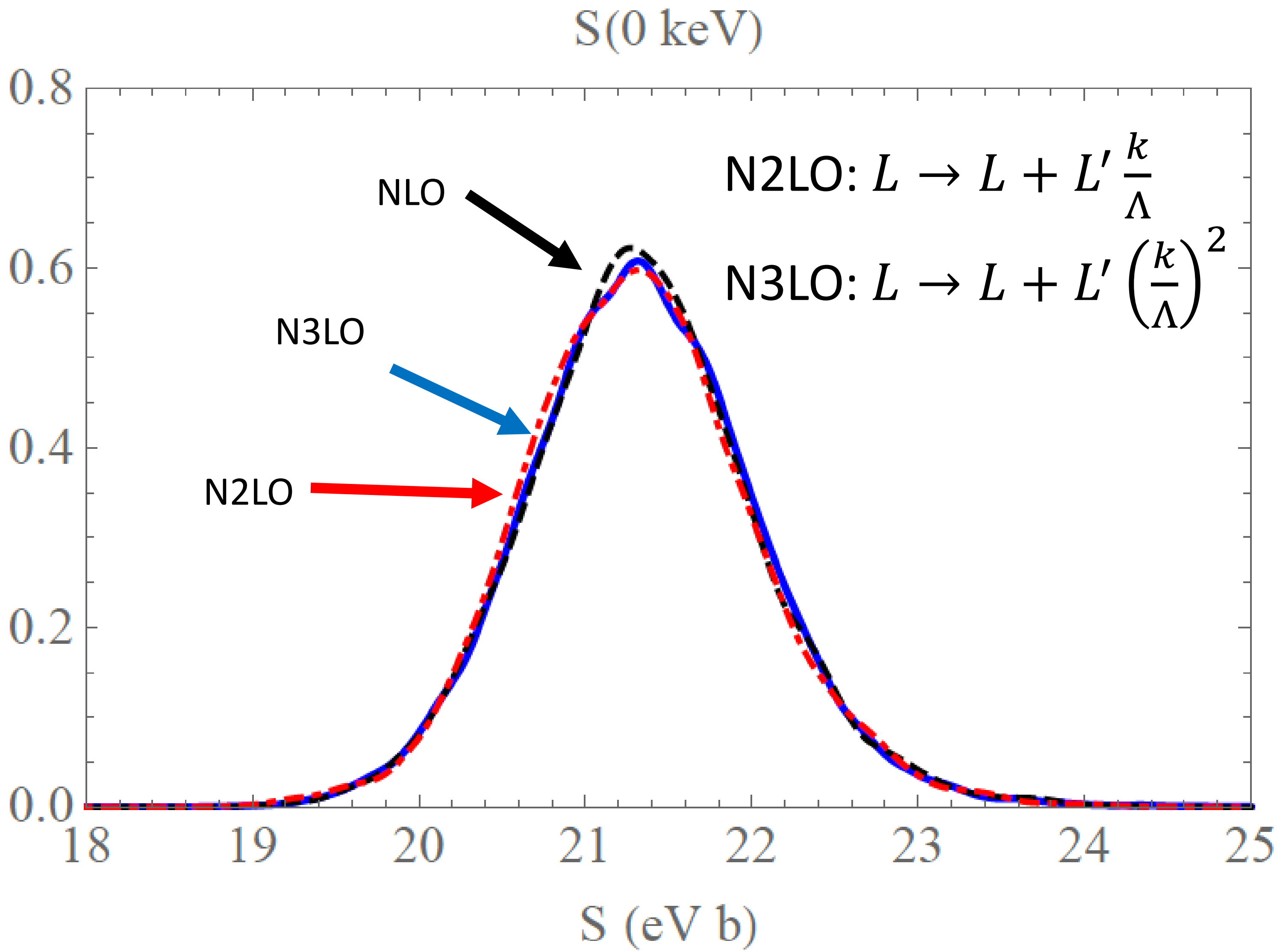
- E2, M1 contributions (S factor): < 0.01%
- Radiative corrections: ~0.1%
- EFT s-wave scattering: ~0.8%



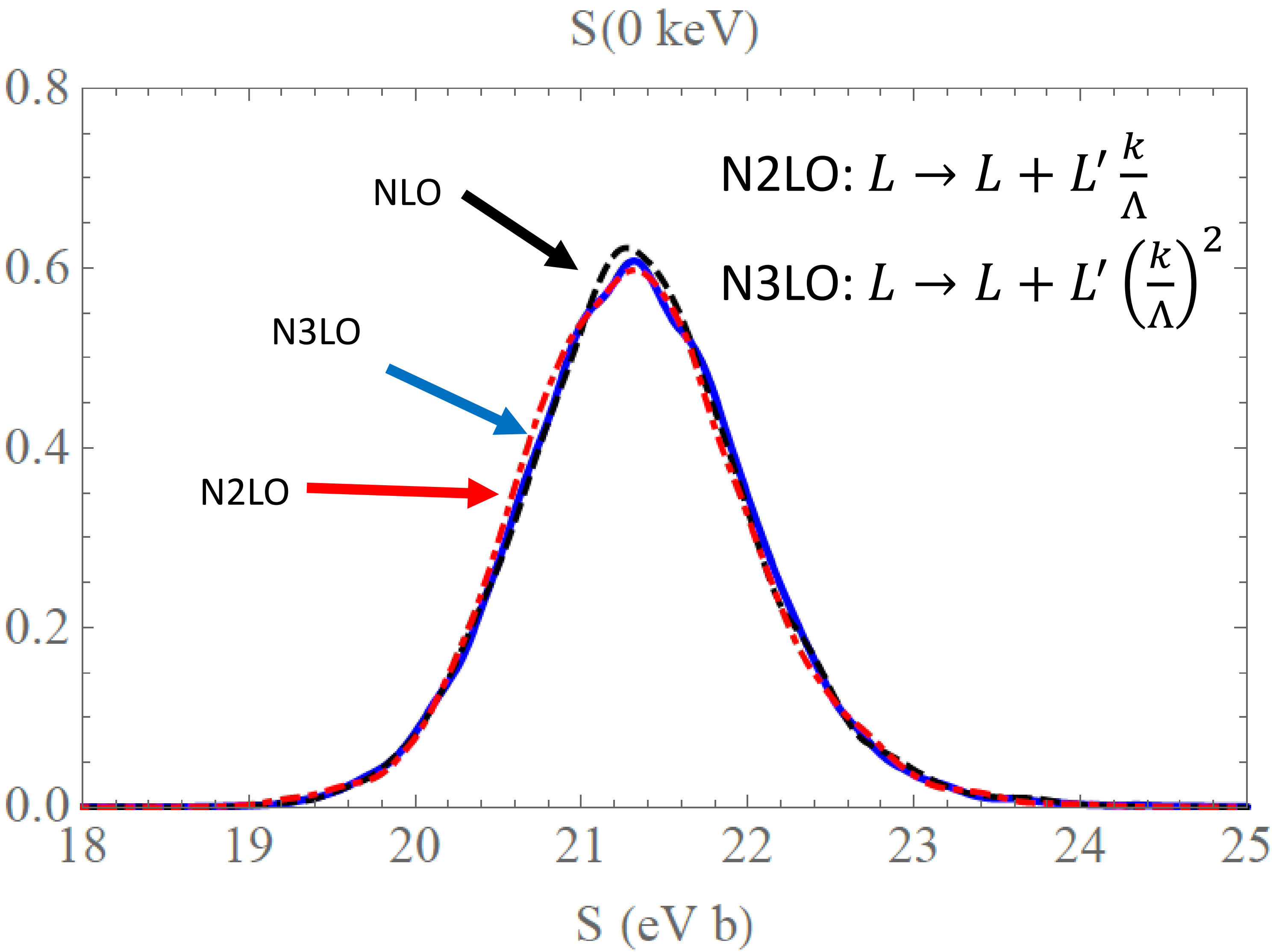
- EFT N2LO currents: ~0.8%
- Notice B8 BE=136.4(1.0) keV: ~ 0.8%

Recall EFT fitted to various potential model and RGM calculation results: deviation <~1% up to 1MeV (cm E).

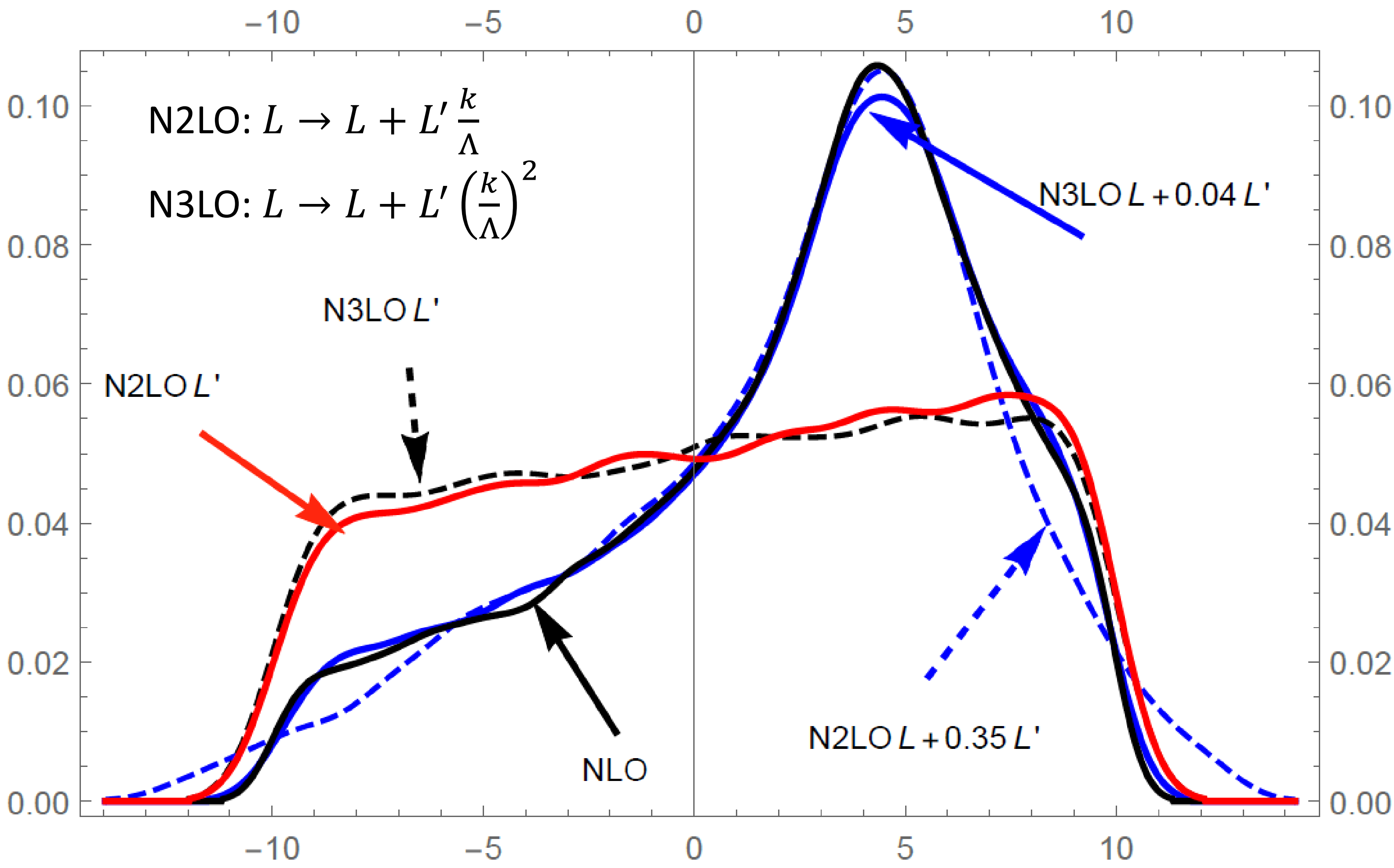
N2LO impact on Bayesian analysis



N2LO impact on Bayesian analysis



Adding N2LO shifts S(0) by << 1%.



Data couldn't give more information

A few questions

Questions

$$\Pr\left(D\middle|\vec{g},\{\xi_i\};T\right)\propto Exp\left(-\frac{\chi^2}{2}\right);\ \chi^2=\sum_{i=1}^{\#data}\frac{\left[S\left(\vec{g};E_i\right)\left(1-\xi_i\right)-D_i\right]^2}{\sigma_i^2}$$

$$\Pr\left(\vec{g},\{\xi_i\}\middle|T\right)\propto Exp\left(-\sum_{j}^{\#sys-err}\frac{\xi_j^{\ 2}}{2\sigma_{\xi_j}^2}\right)\times Exp\left(-\sum_{l}^{\#para}\frac{\left(g_l-g_l^0\right)^2}{2\sigma_{g_l}^2}\right)$$ or flat dis.

Questions

$$\Pr\left(D \middle| \vec{g}, \{\xi_i\}; T\right) \propto Exp\left(-\frac{\chi^2}{2}\right); \chi^2 = \sum_{i=1}^{\#data} \frac{\left[S(\vec{g}; E_i)(1-\xi_i) - D_i\right]^2}{\sigma_i^2}$$

$$\Pr\left(\vec{g}, \{\xi_i\} \middle| T\right) \propto Exp\left(-\sum_{j}^{\#sys-err} \frac{\xi_j^{\,2}}{2\sigma_{\xi_j}^2}\right) \times Exp\left(-\sum_{l}^{\#para} \frac{\left(g_l - g_l^0\right)^2}{2\sigma_{g_l}^2}\right) \text{ or flat dis.}$$

- How to deal with normalization floating parameter? [P.S.Baranov, A.L.L'vov *et.al.*, Physics of Particles and Nucleus, 32, 376 (2001)]

Questions

$$\Pr\left(D\middle|\vec{g},\{\xi_i\};T\right) \propto Exp\left(-\frac{\chi^2}{2}\right);\ \chi^2 = \sum_{i=1}^{\#data}\frac{\left[S\left(\vec{g};E_i\right)\left(1-\xi_i\right)-D_i\right]^2}{\sigma_i^2}$$

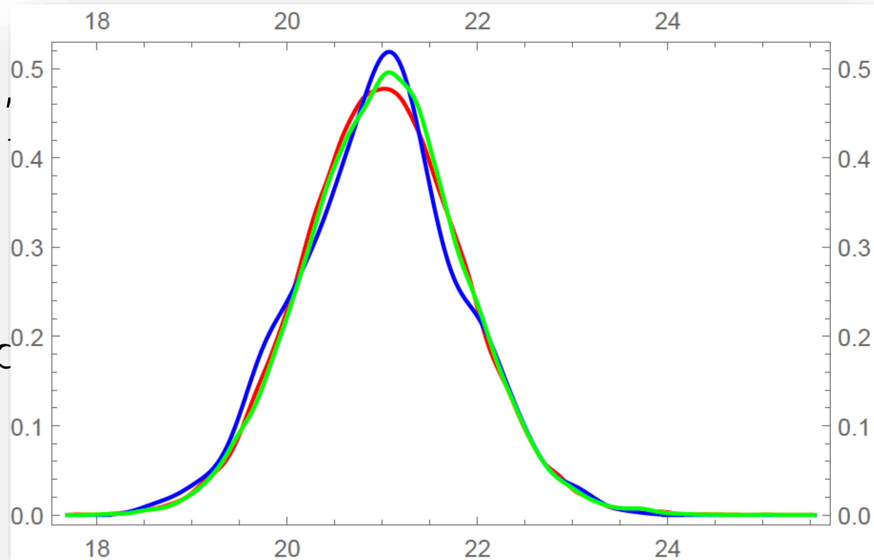
$$\Pr\left(\vec{g},\{\xi_i\}\middle|T\right) \propto Exp\left(-\sum_{j}^{\#sys-err}\frac{\xi_j^{\ 2}}{2\sigma_{\xi_j}^2}\right)\times Exp\left(-\sum_{l}^{\#para}\frac{\left(g_l-g_l^0\right)^2}{2\sigma_{g_l}^2}\right)$$ or flat dis.

- How to deal with normalization floating parameter? [P.S.Baranov, A.L.L'vov *et.al.*, Physics of Particles and Nucleus, 32, 376 (2001)]
- Assign flat priors for parameters? (red for Gaussian a0, blue and green with flat a0 prior)

Questions

$\Pr\left(D \middle| \vec{g}, \{\xi_i\}; \ldots\right.$ … $\left.\ldots E_i)(1-\xi_i)-D_i\right]^2 / \sigma_i^2$

$\Pr\left(\vec{g}, \{\xi_i\} \middle| T\right) \propto$ … $\left.\ldots l - g_l^0)^2 / 2\sigma_{g_l}^2 \right)$ or flat dis.

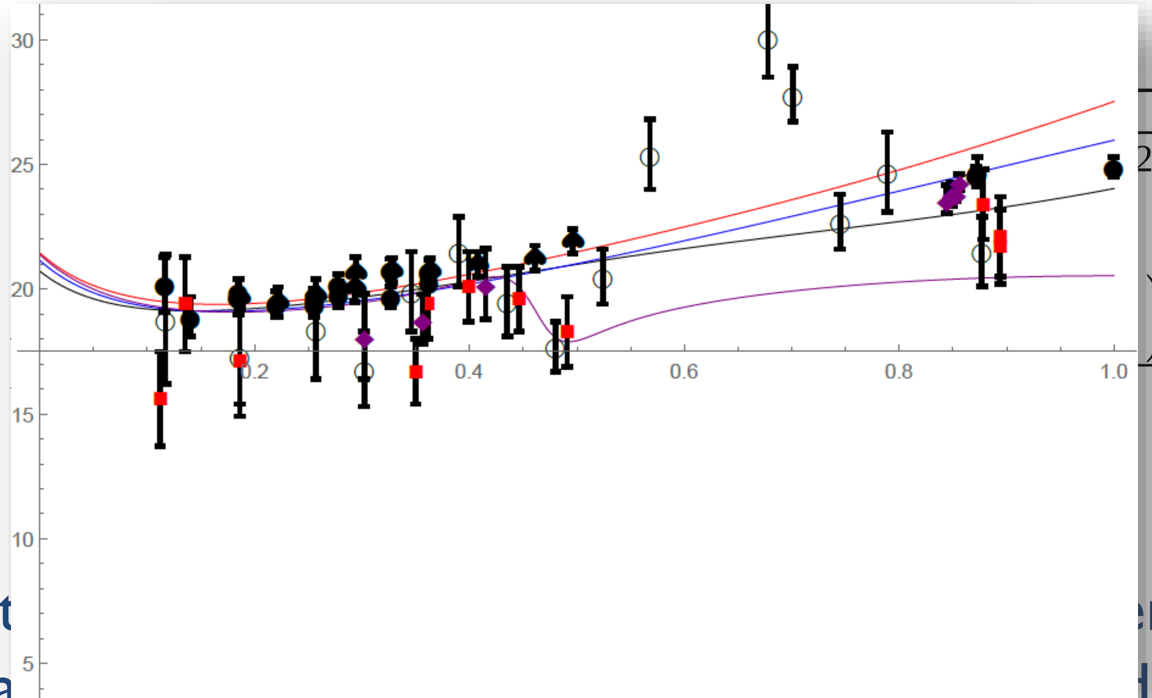


- How to deal with normalization floating parameter? [P.S.Baranov, A.L.L'vov *et.al.*, Physics of Particles and Nucleus, 32, 376 (2001)]
- Assign flat priors for parameters? (red for Gaussian a0, blue and green with flat a0 prior)

Questions

$\Pr\left(D|\vec{g},\right.$ … $\left.-\xi_i\right)-D_i\right]^2$ … 2

$\Pr\left(\vec{g},\{\xi_i\right.$ … $\left.\right)^2 \Big)$ or flat dis.



- How t … er? [P.S.Ba… … Nucleus, 32, 376 (2001)]
- Assign flat priors for parameters? (red for Gaussian a0, blue and green with flat a0 prior)
- Over fitting? Is there a best fit?

Questions

$$\Pr\left(D \middle| \vec{g},\{\xi_i\};T\right) \propto Exp\left(-\frac{\chi^2}{2}\right);\ \chi^2 = \sum_{i=1}^{\#data} \frac{\left[S\left(\vec{g};E_i\right)\left(1-\xi_i\right)-D_i\right]^2}{\sigma_i^2}$$

$$\Pr\left(\vec{g},\{\xi_i\} \middle| T\right) \propto Exp\left(-\sum_j^{\#sys-err} \frac{\xi_j^{\ 2}}{2\sigma_{\xi_j}^2}\right) \times Exp\left(-\sum_l^{\#para} \frac{\left(g_l - g_l^0\right)^2}{2\sigma_{g_l}^2}\right)$$ or flat dis.

- How to deal with normalization floating parameter? [P.S.Baranov, A.L.L'vov *et.al.*, Physics of Particles and Nucleus, 32, 376 (2001)]
- Assign flat priors for parameters? (red for Gaussian a0, blue and green with flat a0 prior)
- Over fitting? Is there a best fit?

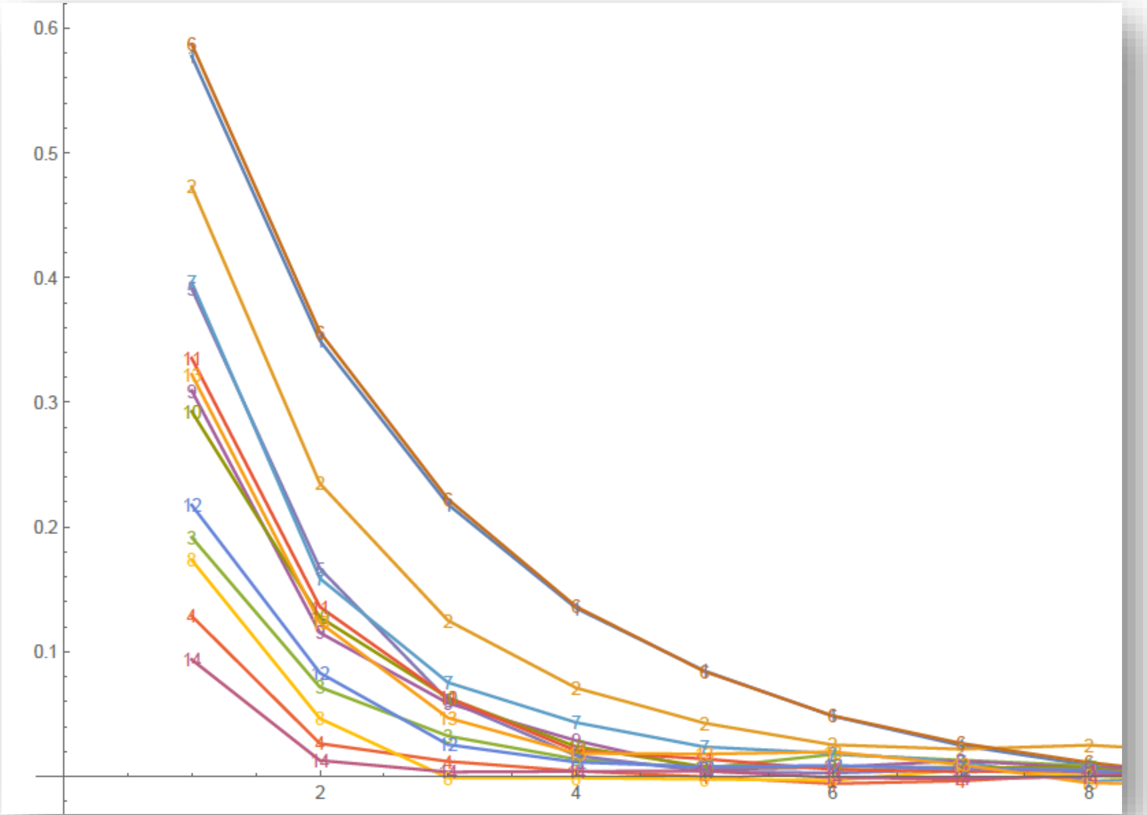
About MCMC

About MCMC

- Acceptance is 15%, good?

About MCMC

- Acceptance is 15%, good?
- How about auto-correlation length?



About MCMC

- Acceptance is 15%, good?
- How about auto-correlation length?
- Is 27000-samples enough?

About MCMC

- Acceptance is 15%, good?
- How about auto-correlation length?
- Is 27000-samples enough?
- How to estimate the error of error?

About MCMC

- Acceptance is 15%, good?
- How about auto-correlation length?
- Is 27000-samples enough?
- How to estimate the error of error?
- Comment on VEGAS algorithm due to Lepage?

Summary

- EFT works for this reaction
- Bayesian analysis is used to quantify uncertainties
- Choice of data sets, theoretical error, and choice of priors have been tested
- Questions

Back up

Solar abundance problem: Neutrinos

Solar abundance problem: Neutrinos

Extract C+N abundance

$$\frac{\phi(^{15}\mathrm{O})}{\phi(^{15}\mathrm{O})^{\mathrm{SSM}}} = \left[\frac{\phi(^{8}\mathrm{B})}{\phi(^{8}\mathrm{B})^{\mathrm{SSM}}}\right]^{0.729} x_{\mathrm{C+N}}$$
$$\times [1 \pm 0.006(\mathrm{solar}) \pm 0.027(D) \pm 0.099(\mathrm{nucl}) \pm 0.032(\theta_{12})]$$

W. C. Haxton et.al. (2013)

Solar abundance problem: Neutrinos

A-few-percent measurements

Extract C+N abundance

$$\frac{\phi(^{15}\mathrm{O})}{\phi(^{15}\mathrm{O})^{\mathrm{SSM}}} = \left[\frac{\phi(^{8}\mathrm{B})}{\phi(^{8}\mathrm{B})^{\mathrm{SSM}}}\right]^{0.729} x_{\mathrm{C+N}}$$
$$\times [1 \pm 0.006(\mathrm{solar}) \pm 0.027(D) \pm 0.099(\mathrm{nucl}) \pm 0.032(\theta_{12})]$$

W. C. Haxton et.al. (2013)

Solar abundance problem: Neutrinos

To be measured

A-few-percent measurements

Extract C+N abundance

$$\frac{\phi(^{15}\mathrm{O})}{\phi(^{15}\mathrm{O})^{\mathrm{SSM}}} = \left[\frac{\phi(^{8}\mathrm{B})}{\phi(^{8}\mathrm{B})^{\mathrm{SSM}}}\right]^{0.729} x_{\mathrm{C+N}}$$

$$\times [1 \pm 0.006(\mathrm{solar}) \pm 0.027(D) \pm 0.099(\mathrm{nucl}) \pm 0.032(\theta_{12})]$$

W. C. Haxton et.al. (2013)

Solar abundance problem: Neutrinos

To be measured

A-few-percent measurements

Extract C+N abundance

$$\frac{\phi(^{15}\mathrm{O})}{\phi(^{15}\mathrm{O})^{\mathrm{SSM}}} = \left[\frac{\phi(^{8}\mathrm{B})}{\phi(^{8}\mathrm{B})^{\mathrm{SSM}}}\right]^{0.729} x_{\mathrm{C+N}}$$
$$\times [1 \pm 0.006(\mathrm{solar}) \pm 0.027(D) \pm 0.099(\mathrm{nucl}) \pm 0.032(\theta_{12})]$$

Knowing C+N can also be used to differentiate solar models. However nuclear cross section error again needs to be reduced.

W. C. Haxton et.al. (2013)

Solar abundance problem: Neutrinos

To be measured

A-few-percent measurements

Extract C+N abundance

$$\frac{\phi(^{15}\mathrm{O})}{\phi(^{15}\mathrm{O})^{\mathrm{SSM}}} = \left[\frac{\phi(^{8}\mathrm{B})}{\phi(^{8}\mathrm{B})^{\mathrm{SSM}}}\right]^{0.729} x_{\mathrm{C+N}}$$

$$\times [1 \pm 0.006(\mathrm{solar}) \pm 0.027(D) \pm 0.099(\mathrm{nucl}) \pm 0.032(\theta_{12})]$$

Knowing C+N can also be used to differentiate solar models. However nuclear cross section error again needs to be reduced.

W. C. Haxton et.al. (2013)

Solar abundance problem: Debate!

Solar abundance problem: Debate!

A successful solar model using new solar composition data

Sunny Vagnozzi[1,2], Katherine Freese[1,3], and Thomas H. Zurbuchen[4]
[1] *The Oskar Klein Centre for Cosmoparticle Physics, Stockholm University, SE-106 91 Stockholm, Sweden*
[2] *NORDITA, KTH Royal Institute of Technology and Stockholm University, SE-106 91 Stockholm, Sweden*
[3] *Michigan Center for Theoretical Physics, Department of Physics, University of Michigan, Ann Arbor, MI 48109, USA*
[4] *Department of Climate and Space Sciences and Engineering, University of Michigan, Ann Arbor, MI 48109, USA*
sunny.vagnozzi@fysik.su.se, ktfreese@umich.edu, thomasz@umich.edu

A resolution is proposed to the "solar abundance problem", that is, the discrepancy between helioseismological observations and the predictions of solar models, computed implementing state-of-the-art photospheric abundances. We reassess the problem considering a newly determined set

Implications of solar wind measurements for solar models and composition

Aldo Serenelli,[1⋆] Pat Scott,[2] Francesco L. Villante, [3,4] Aaron C. Vincent,[5]
Martin Asplund,[6] Sarbani Basu,[7] Nicolas Grevesse,[8,9] and Carlos Peña-Garay,[10,11]

arXiv:1603.05960, 1604.05318

Solar abundance problem: Debate!

A successful solar model using new solar composition data

Sunny Vagnozzi[1,2], Katherine Freese[1,3], and Thomas H. Zurbuchen[4]
[1] *The Oskar Klein Centre for Cosmoparticle Physics, Stockholm University, SE-106 91 Stockholm, Sweden*
[2] *NORDITA, KTH Royal Institute of Technology and Stockholm University, SE-106 91 Stockholm, Sweden*
[3] *Michigan Center for Theoretical Physics, Department of Physics, University of Michigan, Ann Arbor, MI 48109, USA*
[4] *Department of Climate and Space Sciences and Engineering, University of Michigan, Ann Arbor, MI 48109, USA*
sunny.vagnozzi@fysik.su.se, ktfreese@umich.edu, thomasz@umich.edu

A resolution is proposed to the "solar abundance problem", that is, the discrepancy between helioseismological observations and the predictions of solar models, computed implementing state-of-the-art photospheric abundances. We reassess the problem considering a newly determined set

Implications of solar wind measurements for solar models and composition

Aldo Serenelli,[1⋆] Pat Scott,[2] Francesco L. Villante, [3,4] Aaron C. Vincent,[5]
Martin Asplund,[6] Sarbani Basu,[7] Nicolas Grevesse,[8,9] and Carlos Peña-Garay,[10,11]

Capture reaction study will make an impact!

arXiv:1603.05960, 1604.05318