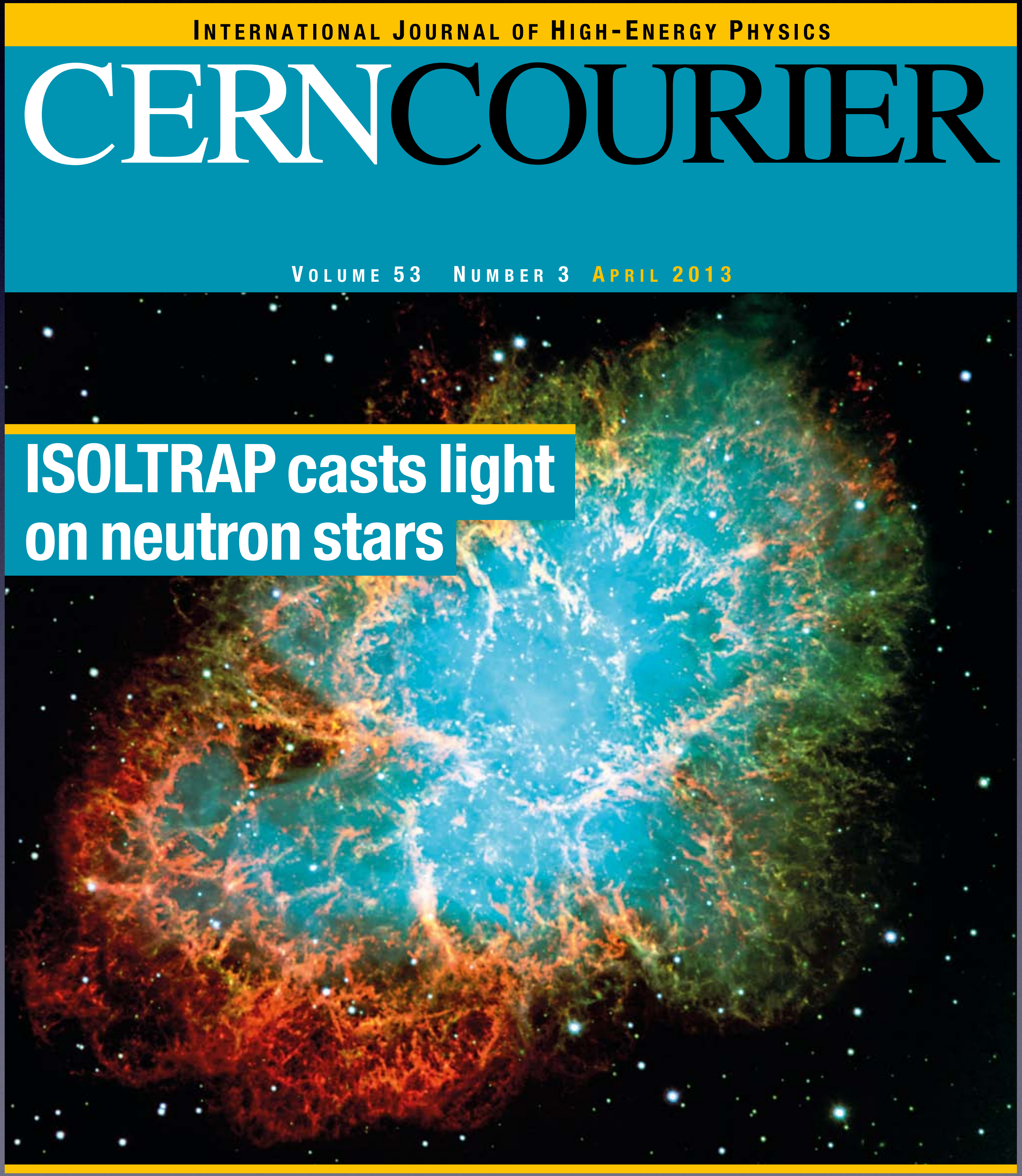
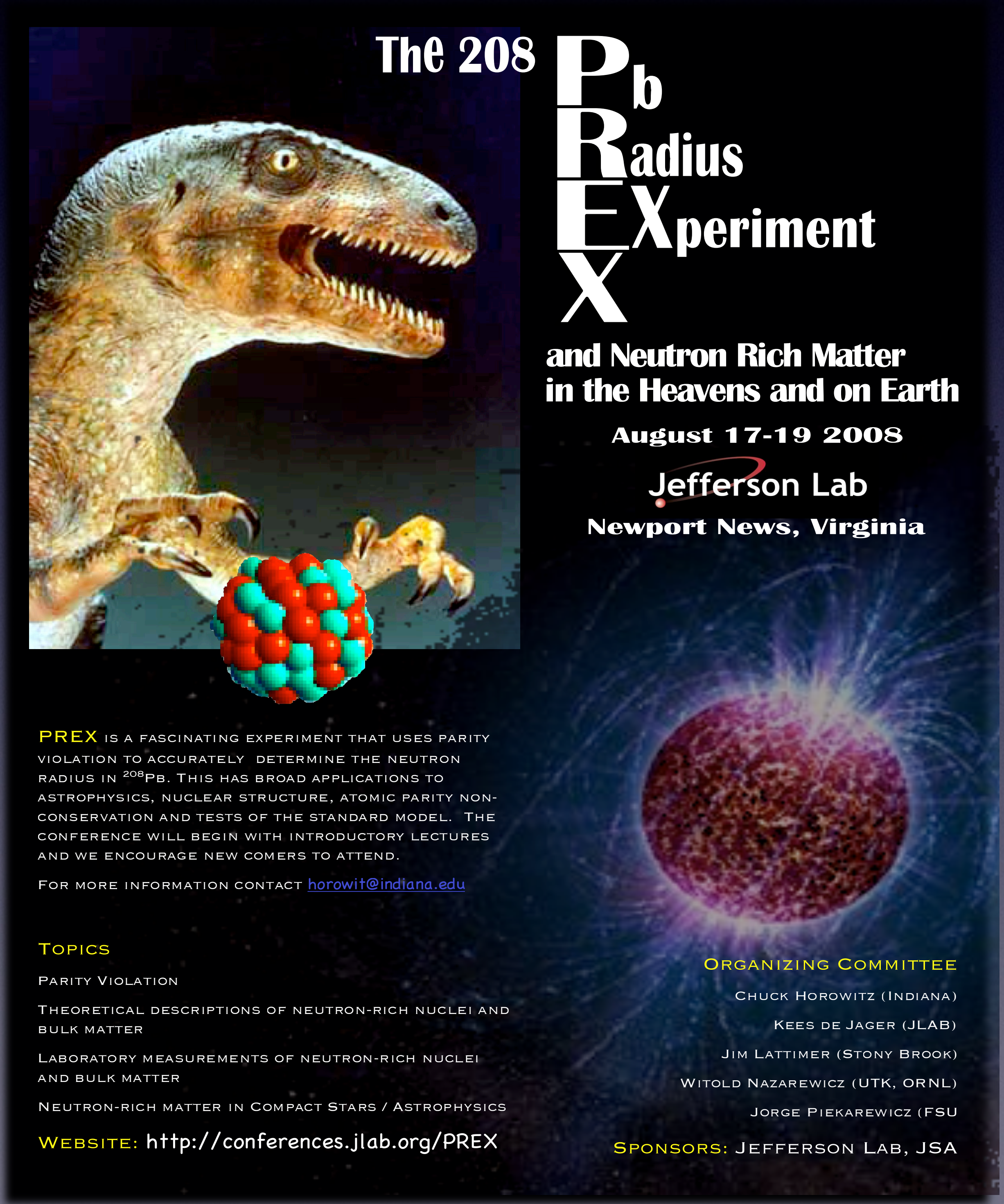
Constraining Neutron Star Properties from Laboratory Experiments

Bayesian Methods in Nuclear Physics
INT Program INT-16-2a
Jorge Piekarewicz (FSU)





My Collaborators

My FSU Collaborators

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- Brad Futch
- Jutri Taruna
- **Farrukh Fattoyev**
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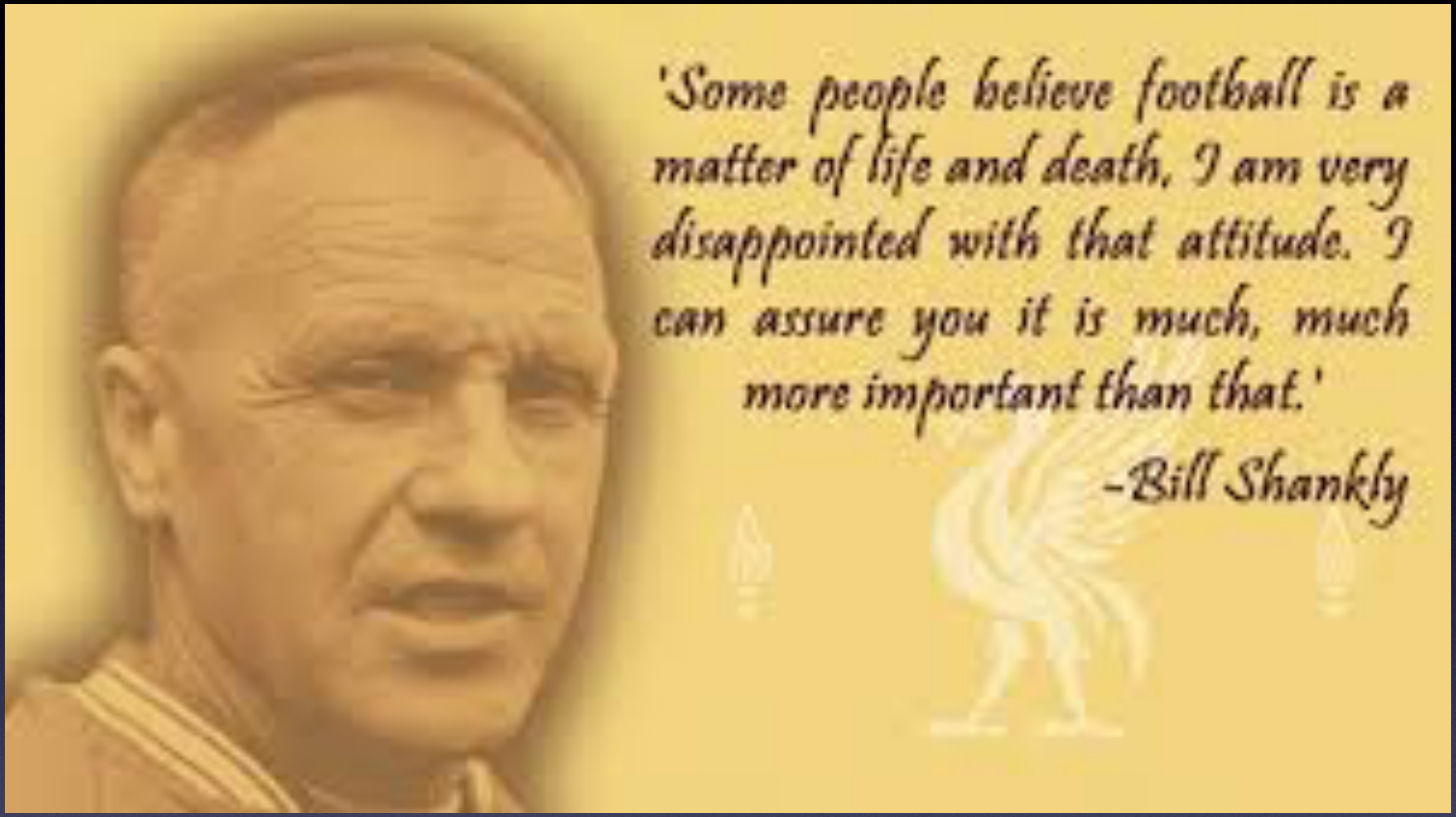
Me … between we and EURO2016



William "Bill" Shankly was a Scottish footballer and manager who is best known for his time as manager of Liverpool. Shankly came from a small Scottish mining community as one of five brothers who played football professionally.

Our Marching Orders …

For Physicists:
We would like speakers to follow these instructions:

1. *Provide a high-level (=pedagogical) introduction* to both the basic science questions you are trying to answer and an introduction to the statistical methods you are using (Bayesian or not). Please avoid jargon and remember that most attendees will not be experts in your field!

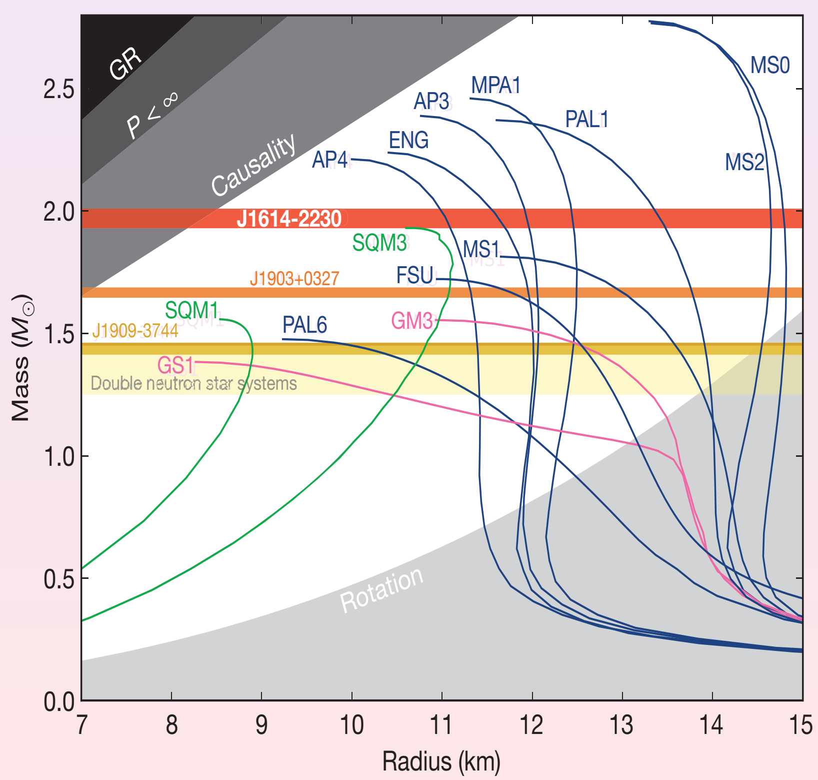
2. To help get your Bayesian statistical colleagues better engaged, be sure to describe features of the data, models, and possibly other information that is being used. Also explain the goals of your investigation: *Are you focused on estimating key parameters? Making predictions? Choosing between competing models? Seeking understanding?* Also, special features of your investigation will be important to present (e.g. computationally demanding models, data quality).

Neutron Stars: Unique Cosmic Laboratories

- Neutron stars are the remnants of massive stellar explosions (CCSN)
  - Bound by gravity — NOT by the strong force
  - Catalyst for the formation of exotic state of matter
  - Satisfy the Tolman-Oppenheimer-Volkoff equation ($v_{esc}/c \sim 1/2$)

- Only Physics that the TOV equation is sensitive to: Equation of State
  - EOS must span about 11 orders of magnitude in baryon density

- Increase from 0.7 → 2 Msun transfers ownership to Nuclear Physics!
- Predictions on stellar radii differ by several kilometers!



$$\frac{dM}{dr} = 4\pi r^2 \mathcal{E}(r)$$

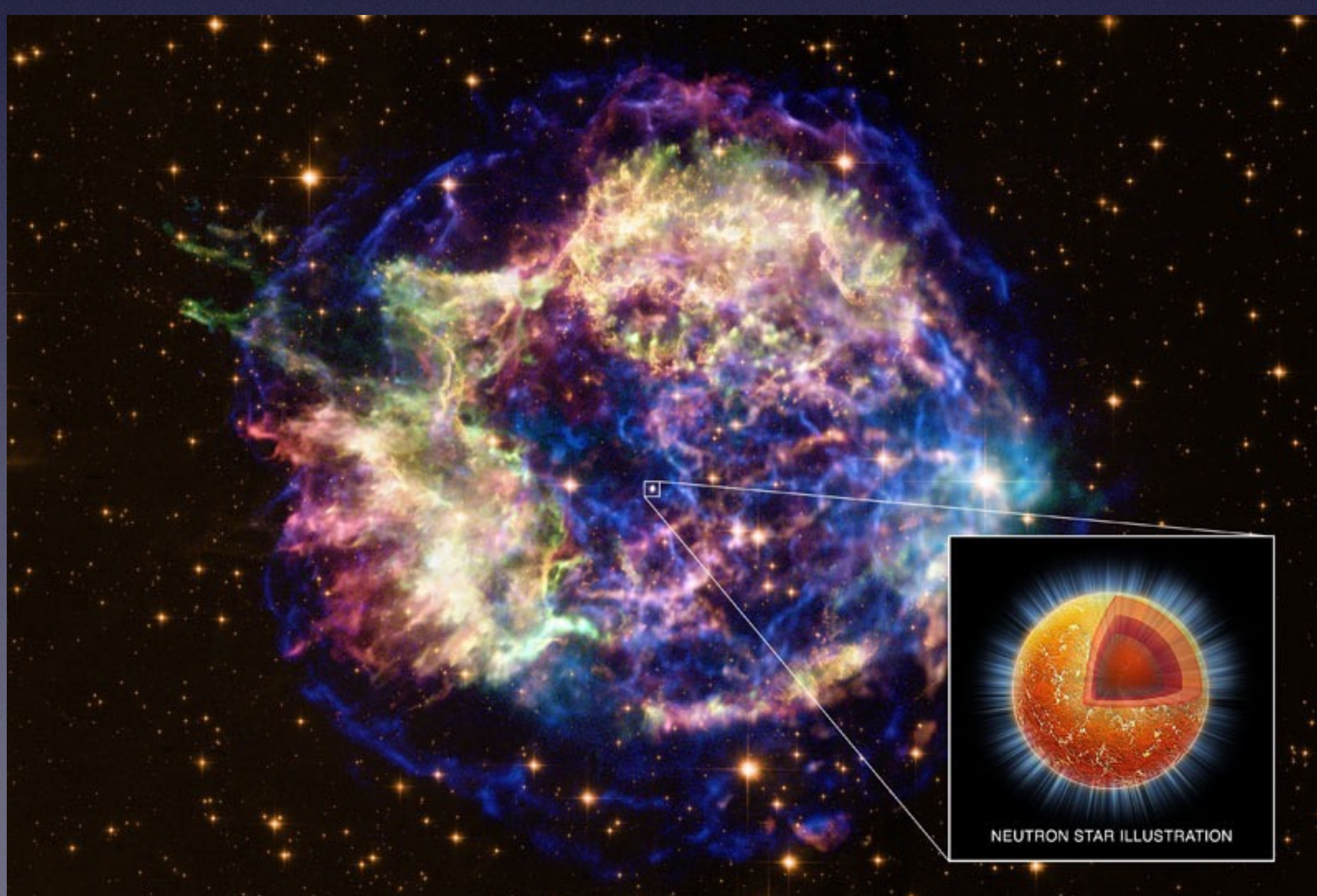
$$\frac{dP}{dr} = -G\frac{\mathcal{E}(r)M(r)}{r^2}\left[1+\frac{P(r)}{\mathcal{E}(r)}\right]\left[1+\frac{4\pi r^3 P(r)}{M(r)}\right]\left[1-\frac{2GM(r)}{r}\right]^{-1}$$

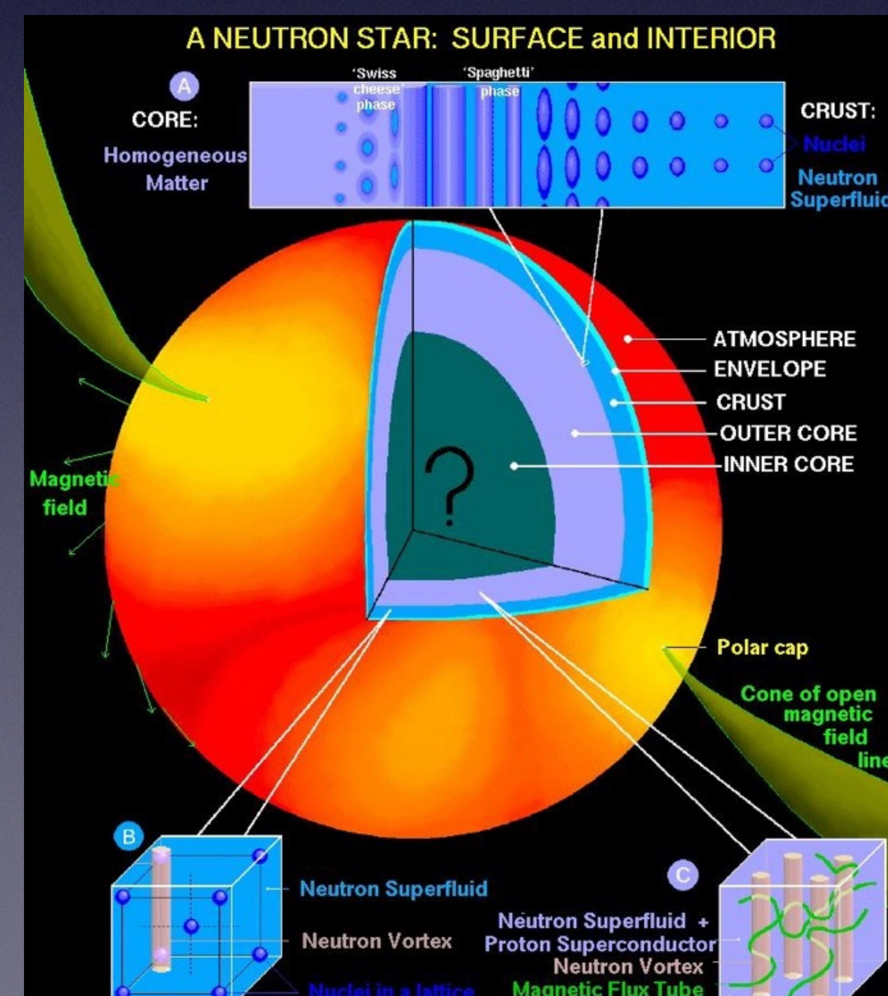
Need an EOS: $P = P(\mathcal{E})$ relation

Nuclear Physics Critical

The Anatomy of a Neutron Star

- Atmosphere (10 cm): Shapes Thermal Radiation ($L=4\pi\sigma R^2T^4$)
- Envelope (100 m): Huge Temperature Gradient (10^8K $\leftrightarrow$ 10^6K)
- Outer Crust (400 m): Coulomb Crystal (Exotic neutron-rich nuclei)
- Inner Crust (1 km): Coulomb Frustration ("Nuclear Pasta")
- Outer Core (10 km): Uniform Neutron-Rich Matter (n,p,e,μ)
- Inner Core (?): Exotic Matter (Hyperons, condensates, quark matter)



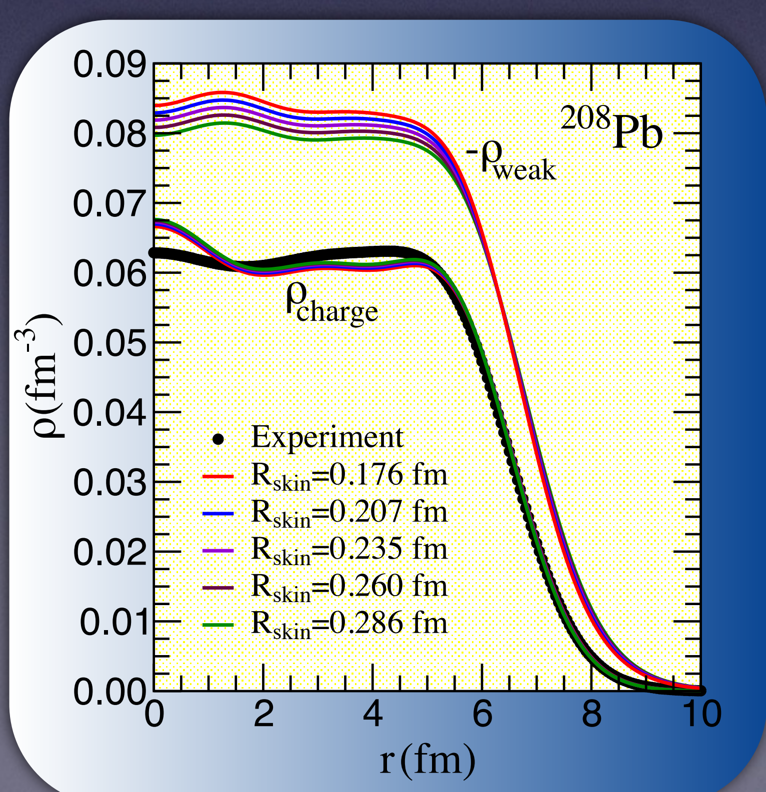


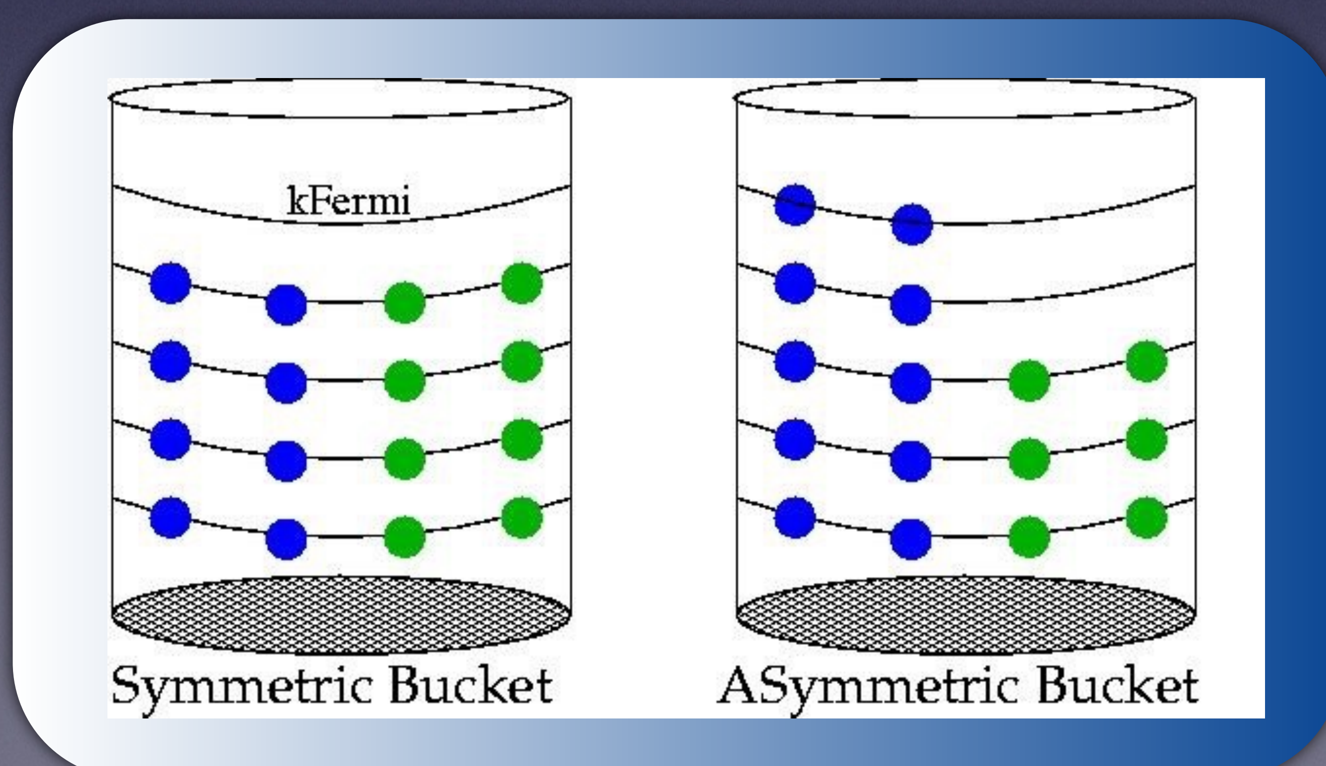
The Equation of State of Neutron-Rich Matter

- The EOS of asymmetric matter: α=(N-Z)/A; $x=(\rho-\rho_0)/3\rho_0$; T=0
  - $\rho_0 \simeq 0.15$ fm-3 — saturation density ↔ nuclear density

$$\mathcal{E}(\rho,\alpha) \simeq \mathcal{E}_0(\rho) + \alpha^2 \mathcal{S}(\rho) \simeq \left(\epsilon_0 + \frac{1}{2}K_0 x^2\right) + \left(J + Lx + \frac{1}{2}K_{\rm sym}x^2\right)\alpha^2$$

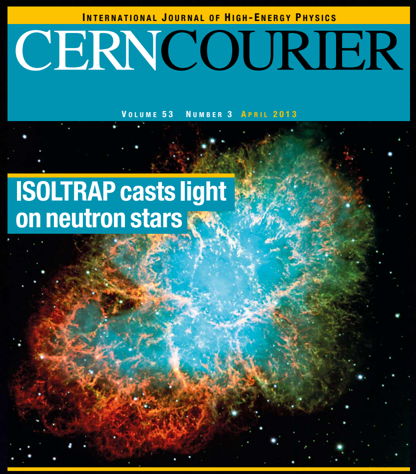
- Symmetric nuclear matter saturates:
  - $\varepsilon_0 \simeq$ -16 MeV — binding energy per nucleon ↔ nuclear masses
  - $K_0 \simeq$ 230 MeV — nuclear incompressibility ↔ nuclear "breathing" mode
- Density dependence of symmetry energy poorly constrained:
  - J $\simeq$ 30 MeV — symmetry energy ↔ masses of neutron-rich nuclei
  - L $\simeq$? — symmetry slope ↔ neutron skin (R_n-R_p) of heavy nuclei ?





The Composition of the Outer Crust

High sensitivity to nuclear masses

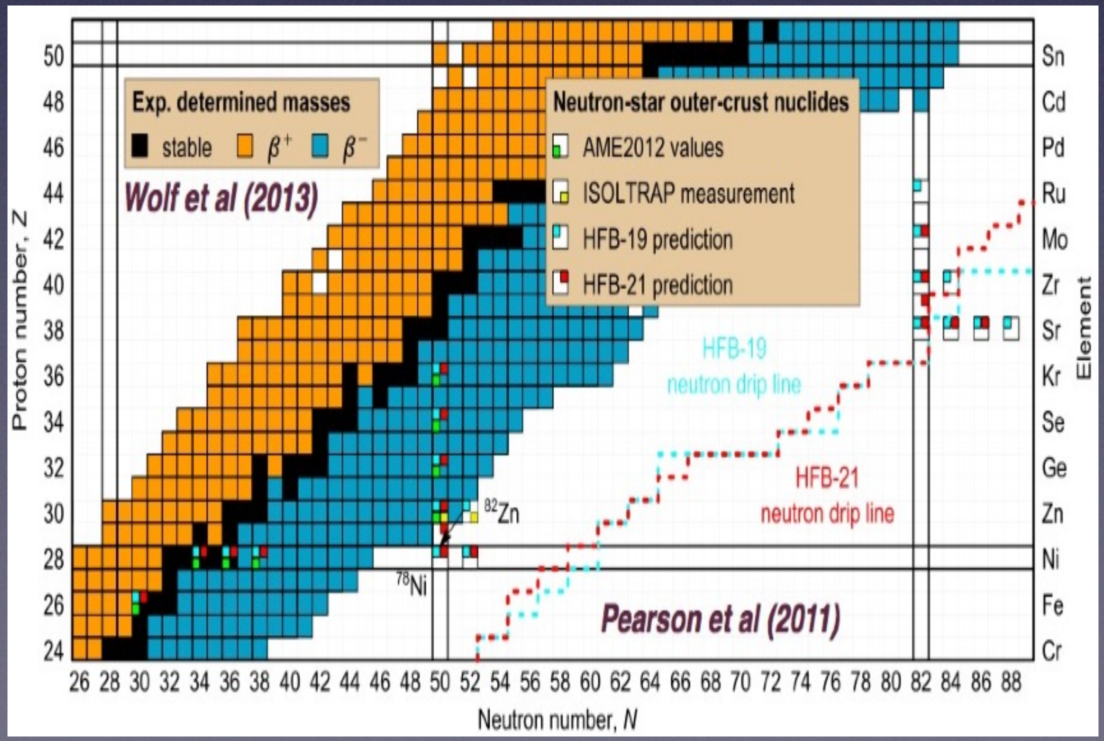


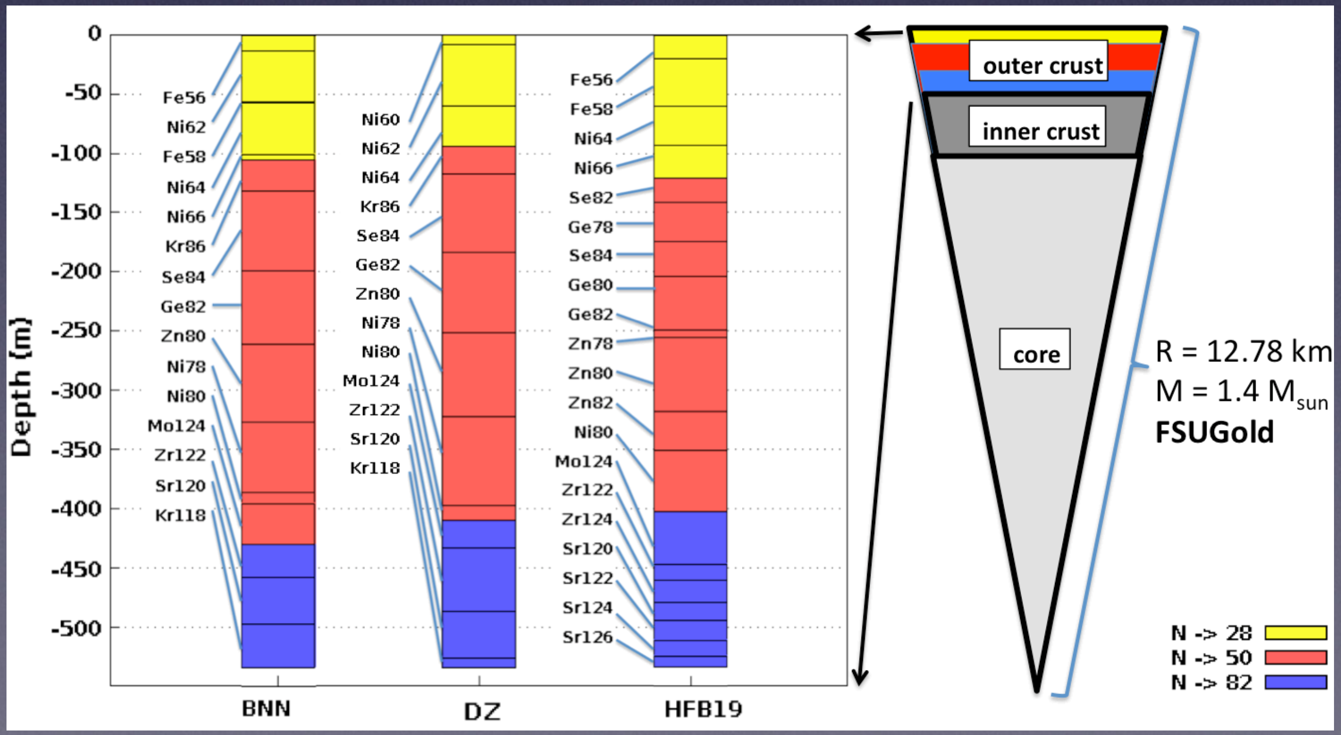
- System unstable to cluster formation
  - BCC lattice of neutron-rich nuclei imbedded in e-gas

- Composition emerges from relatively simple dynamics
  - Subtle composition between electronic and symmetry energy

$$E/A_{\rm tot} = M(N,Z)/A + \frac{3}{4} Y_e^{4/3} {\rm k_F} + {\rm lattice}$$

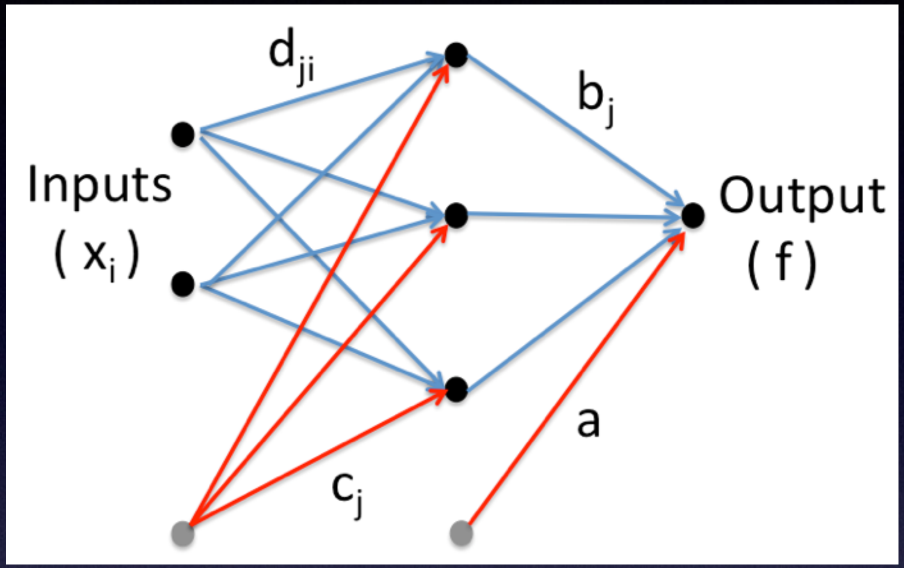
- Precision mass measurements of exotic nuclei is essential
  - Both for neutron-star crusts and r-process nucleosynthesis





Feed-Forward Neural Networks

- Neural networks are non-linear functions that map n inputs to m outputs
- Feed-forward neural networks are "universal approximators"



$x = (N, Z) \mapsto M_{\rm exp}(N, Z) = t$

$$p(x,t|\omega) = \exp\left(-\frac{1}{2}\sum_{i=1}^{N}\left(\frac{t_i - f(x_i,\omega)}{\Delta t_i}\right)^2\right)$$

$$f(x,\omega) = a + \sum_{j=1}^{H} b_j \tanh\left(c_j + \sum_{i=1}^{I} d_{ji} x_i\right)$$

Bayesian Neural Networks: $p(\omega|x,t) = \dfrac{p(x,t|\omega)p(\omega)}{p(x,t)}$

- Treat the training of the network as Bayesian inference problem
- Use Bayes' theorem to infer the posterior probability that a neural network model describes a given set of data (e.g., masses, radii, …)
- Furnishes an estimate of the model uncertainty …
- Selection of the prior, hyper-prior, hyper-parameters often feels like voodoo art!

Select the training set
Validate on the excluded set
Predict …often involves extrapolations!

DFT meets BNN

PHYSICAL REVIEW C **93**, 014311 (2016)

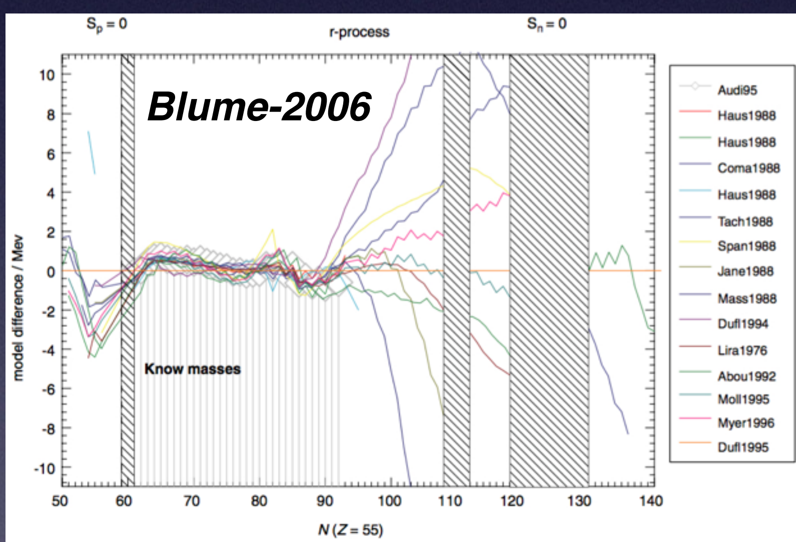
Nuclear mass predictions for the crustal composition of neutron stars: A Bayesian neural network approach

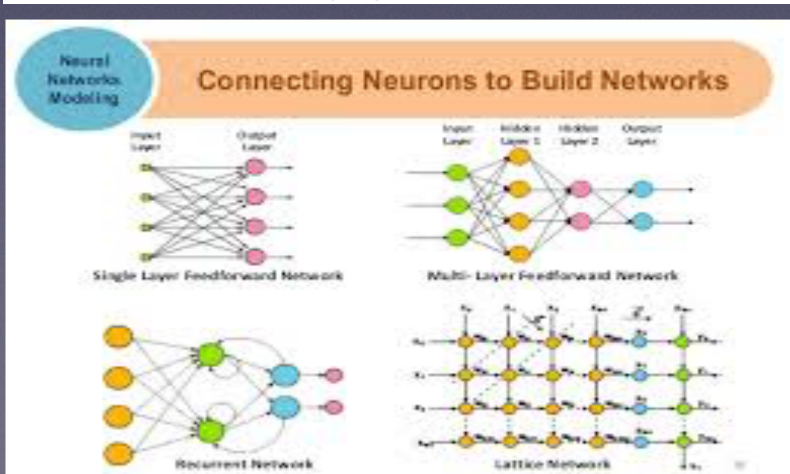
R. Utama,* J. Piekarewicz,† and H. B. Prosper‡
Department of Physics, Florida State University, Tallahassee, Florida 32306, USA

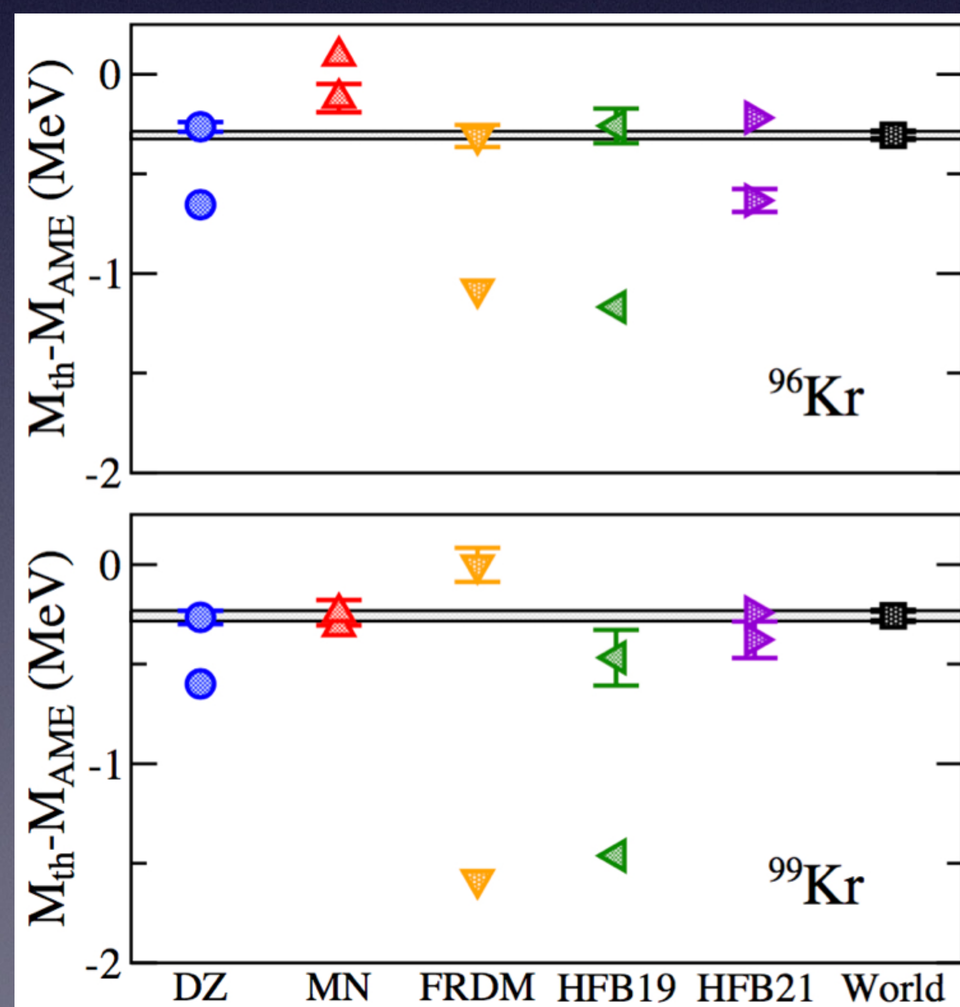
- Use DFT to predict nuclear masses
- Train BNN by focusing on residuals

$$M(N,Z) = M_{DFT}(N,Z) + \delta M_{BNN}(N,Z)$$

- Systematic scattering greatly reduced
- Predictions supplemented by theoretical errors







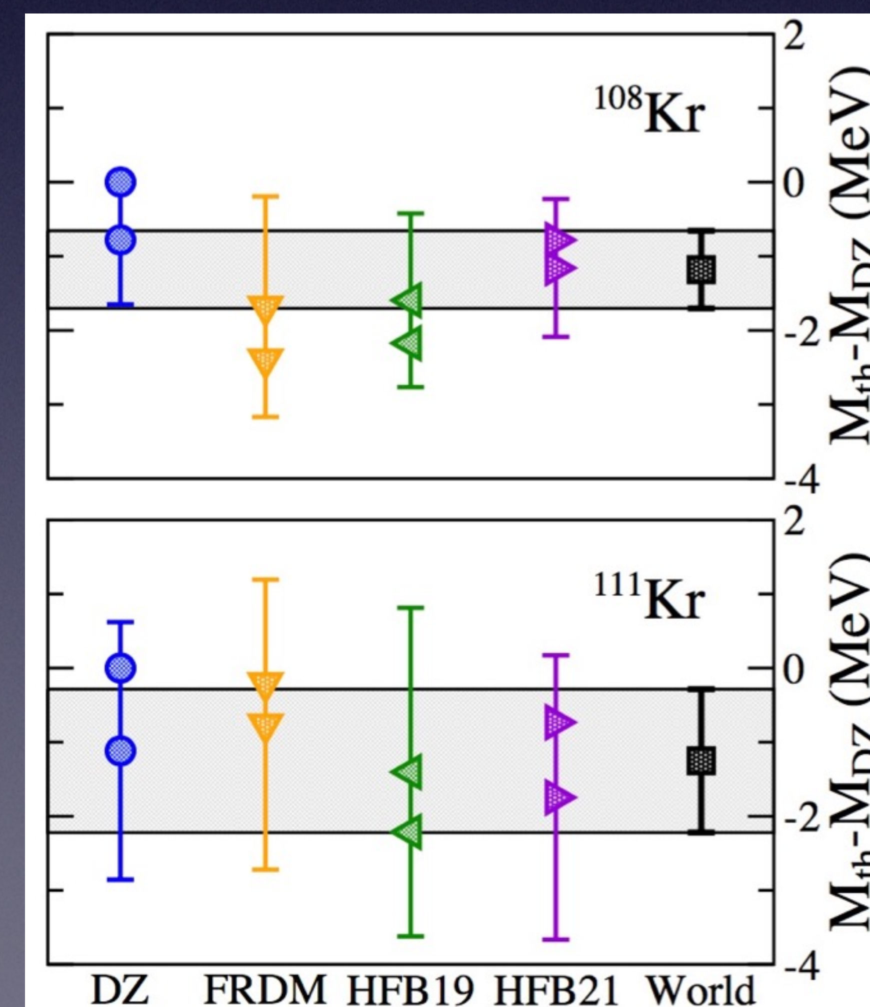


Image Reconstructions meets BNN

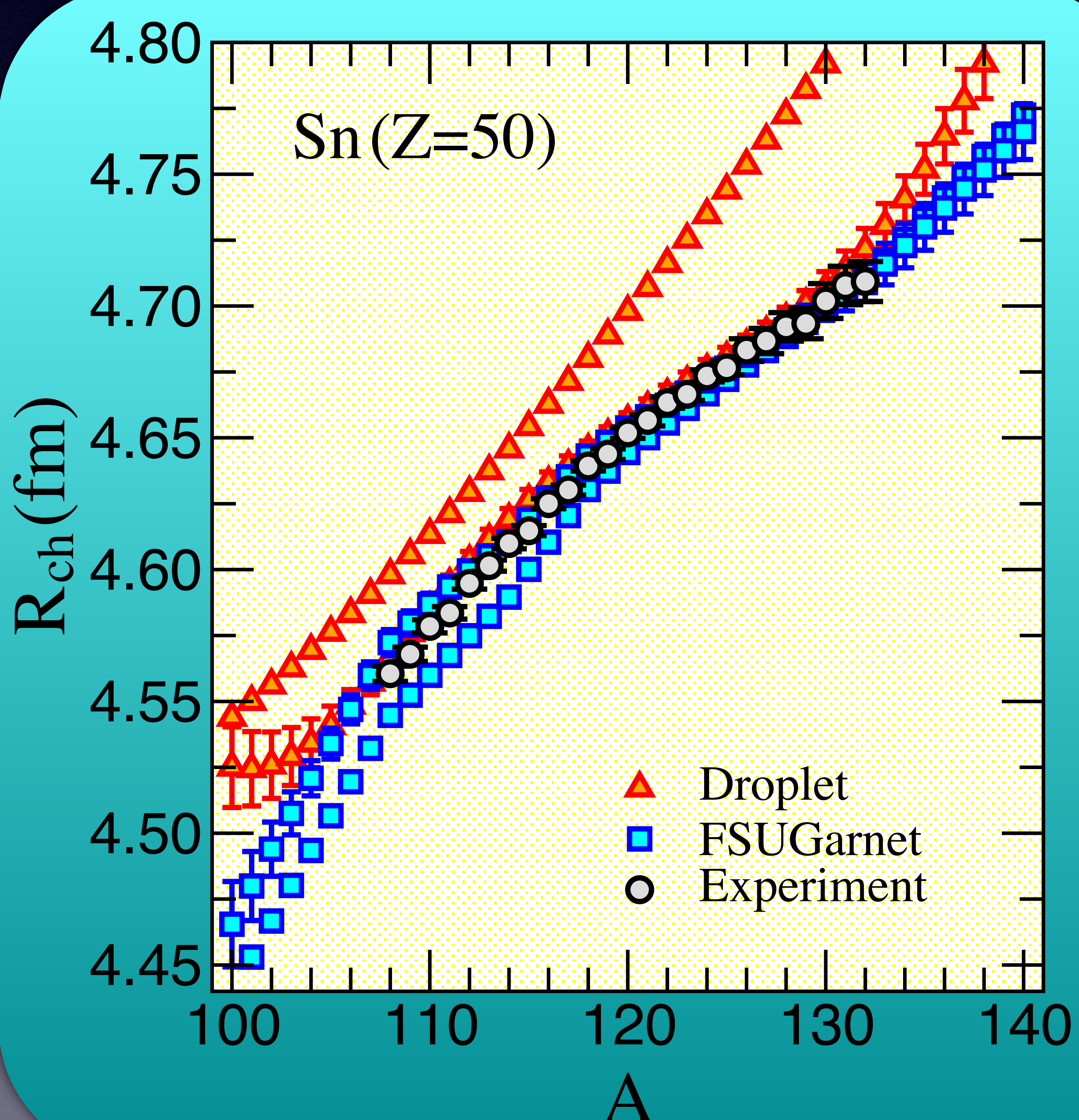
- Nature provides precise image of the world
- Models (DFT) aim to reproduce such image
- Image reconstruction (BNN) provides fine tuning

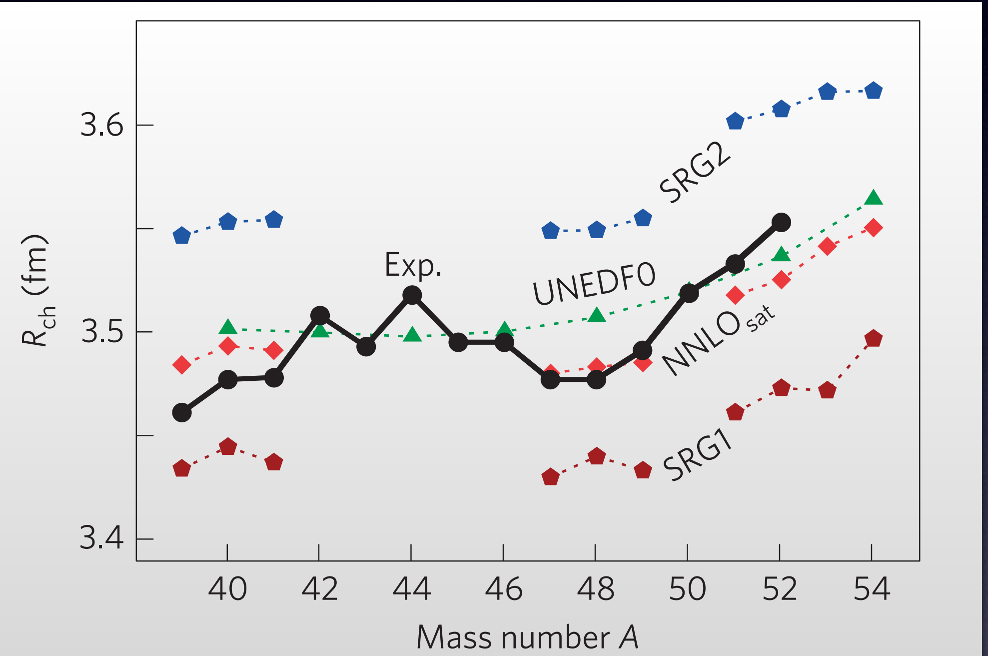
DFT meets BNN: Nuclear Charge Radii

ARTICLES
PUBLISHED ONLINE: 8 FEBRUARY 2016 | DOI: 10.1038/NPHYS3645
nature physics

Unexpectedly large charge radii of neutron-rich calcium isotopes

R.F. Garcia Ruiz et al





$$R_{\rm ch}(N,Z) = R_{\rm ch}^{DFT}(N,Z) + \delta R_{\rm ch}^{BNN}(N,Z)$$

Charge Radii along the Isotopic Chain in Sn: Textbook example of DFT+BNN refinement!

DFT meets BNN: Nuclear Charge Radii

ARTICLES
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Unexpectedly large charge radii of neutron-rich calcium isotopes

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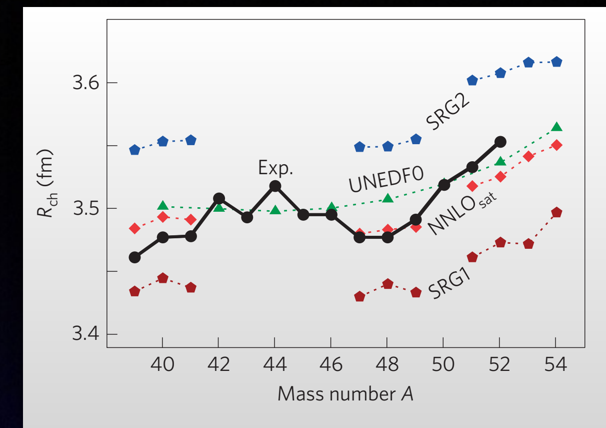

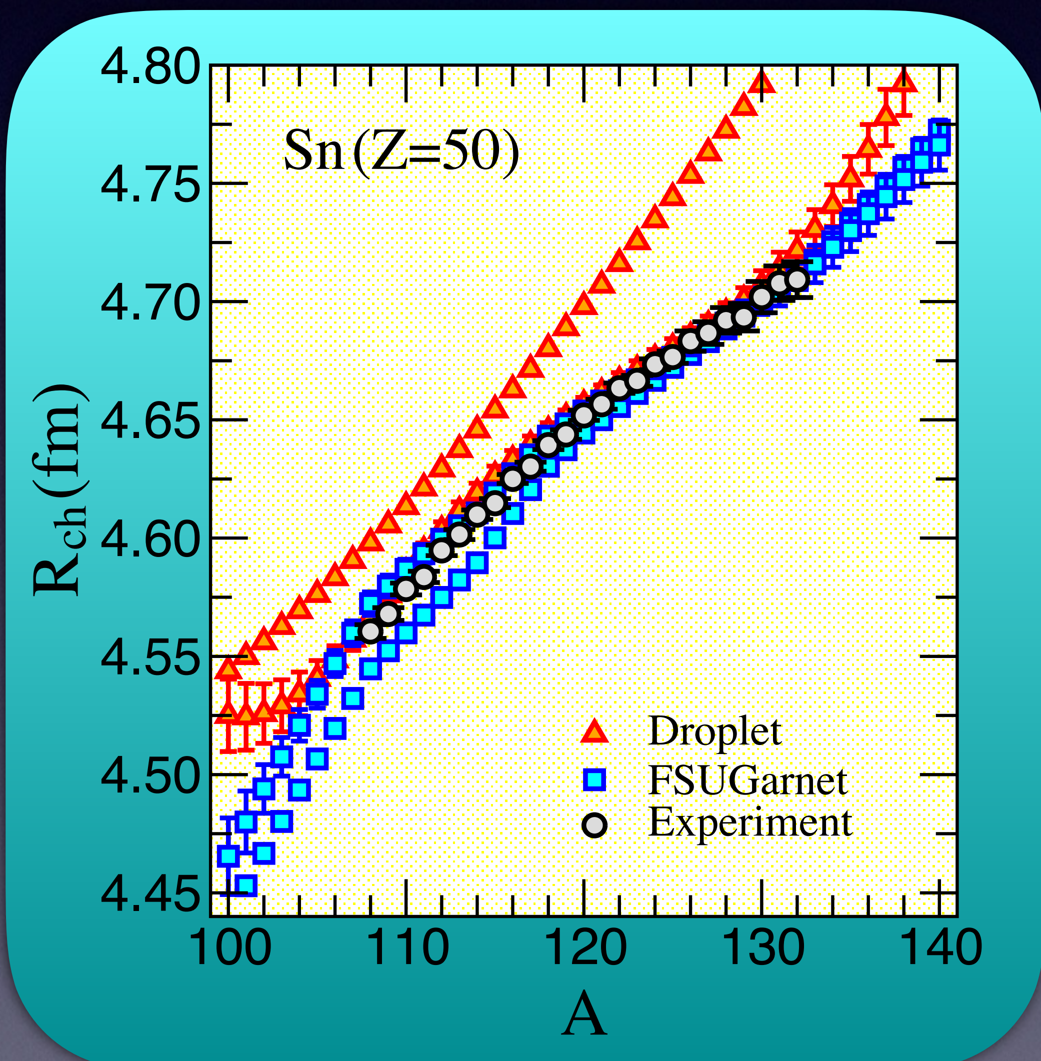

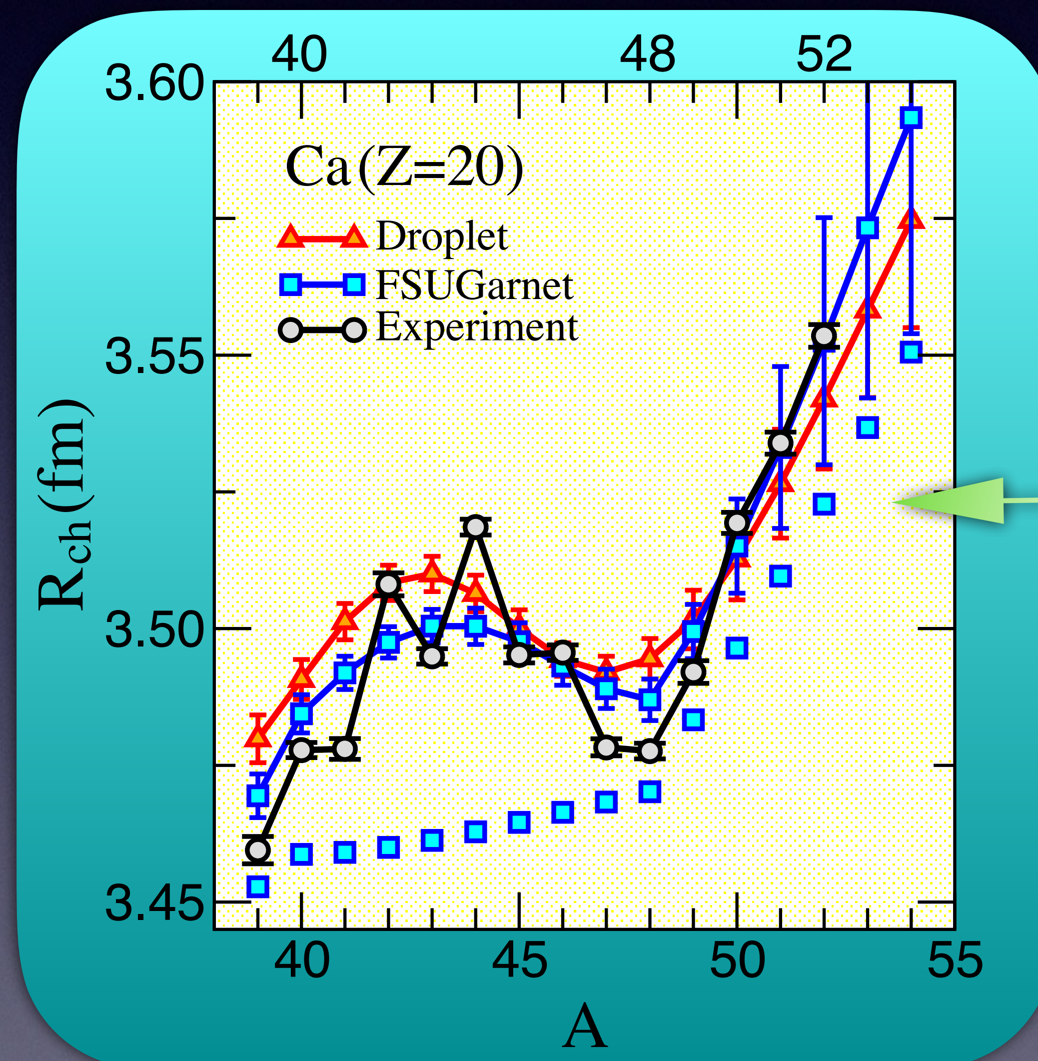

Even after a bit of cheating …

- Sn: Textbook example of DFT+BNN refinement!
- Ca: It is a riddle, wrapped in a mystery, inside an enigma!

Bayes' Theorem: Application to Model Building

PHYSICAL REVIEW C **90**, 044305 (2014)

Building relativistic mean field models for finite nuclei and neutron stars

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$$P(M|D) = \frac{P(D|M)P(M)}{P(D)}$$

Posterior ← $P(M|D)$; Prior ← $P(M)$; Likelihood ← $P(D|M)$; Marginal Likelihood ← $P(D)$

- QCD is the fundamental theory of the strong interactions!
  - M: A theoretical MODEL with parameters and biases
  - D: A collection of experimental and observational DATA

- The Prior *P(M)*: An insightful transformation in DFT

$$(g_{\rm s}, g_{\rm v}, g_{\rho}, \kappa, \lambda, \Lambda_{\rm v}) \Longleftrightarrow (\rho_0, \epsilon_0, M^*, K, J, L)$$

- The Likelihood $P(D|M) \simeq exp(-\chi^2/2)$

$$\chi^2(D,M) = \sum_{n=1}^{N} \frac{\left(O_n^{(\rm th)}(M) - O_n^{(\rm exp)}(D)\right)^2}{\Delta O_n^2}$$

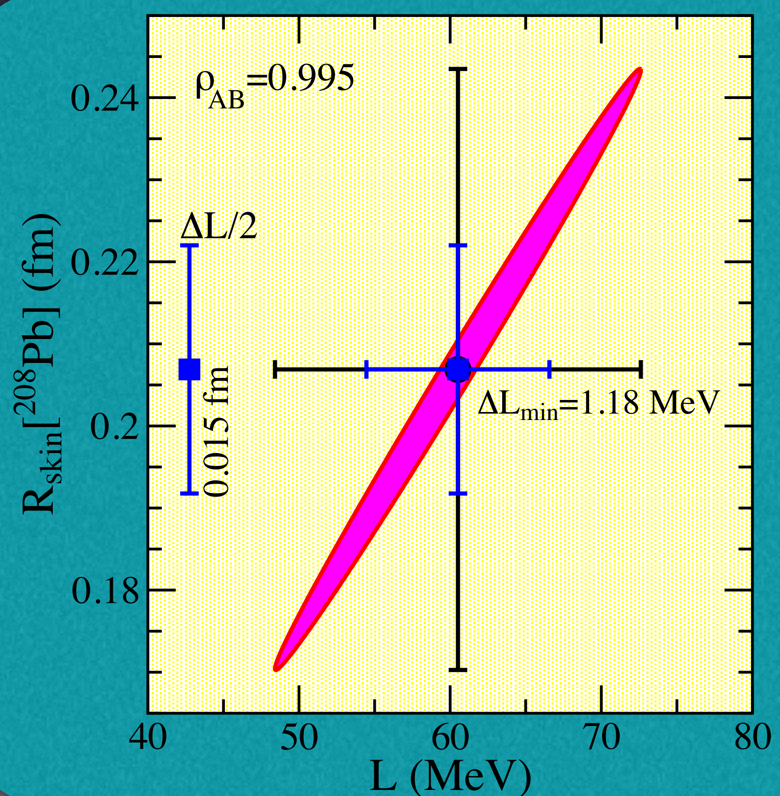
- The Marginal Likelihood; overall normalization factor

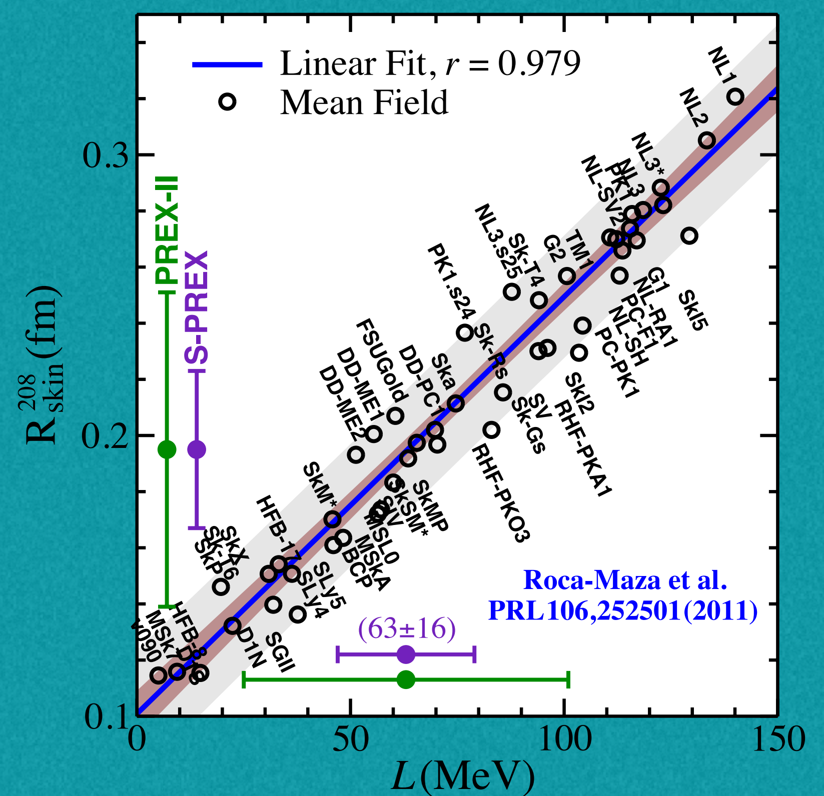
Searching for L: The Strategy

- Establish a powerful physical argument connecting L to R_{skin}
  - Where do the extra 44 neutrons in ^{208}Pb go? Competition between surface tension and the *difference* $S(\rho_0)$-$S(\rho_{surf}) \simeq$ L.
    The larger the value of L, the thicker the neutron skin of ^{208}Pb

- Ensure that "your" accurately-calibrated DFT supports the correlation
  - Statistical Uncertainty: Theoretical error bars and correlation coefficients
  - What precision in R_{skin} is required to constrain L to the desire accuracy?

- Ensure that "all" accurately-calibrated DFT support the correlation
  - Systematic Uncertainty: As with all systematic errors, much harder to quantify
    (… "all models are equal but some models are more equal than others")



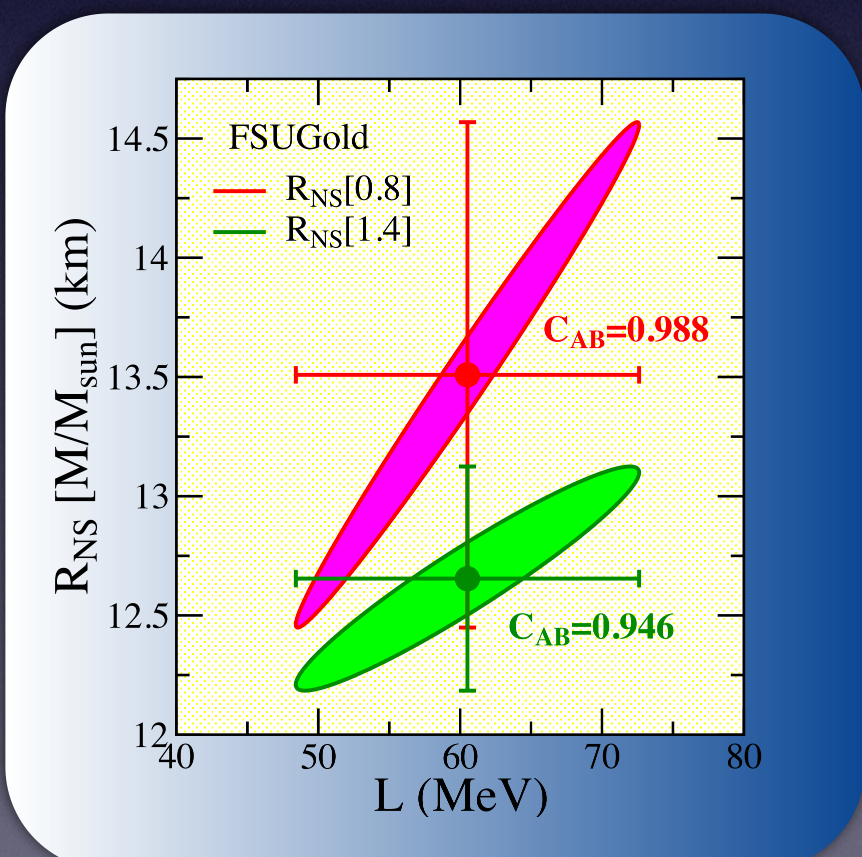


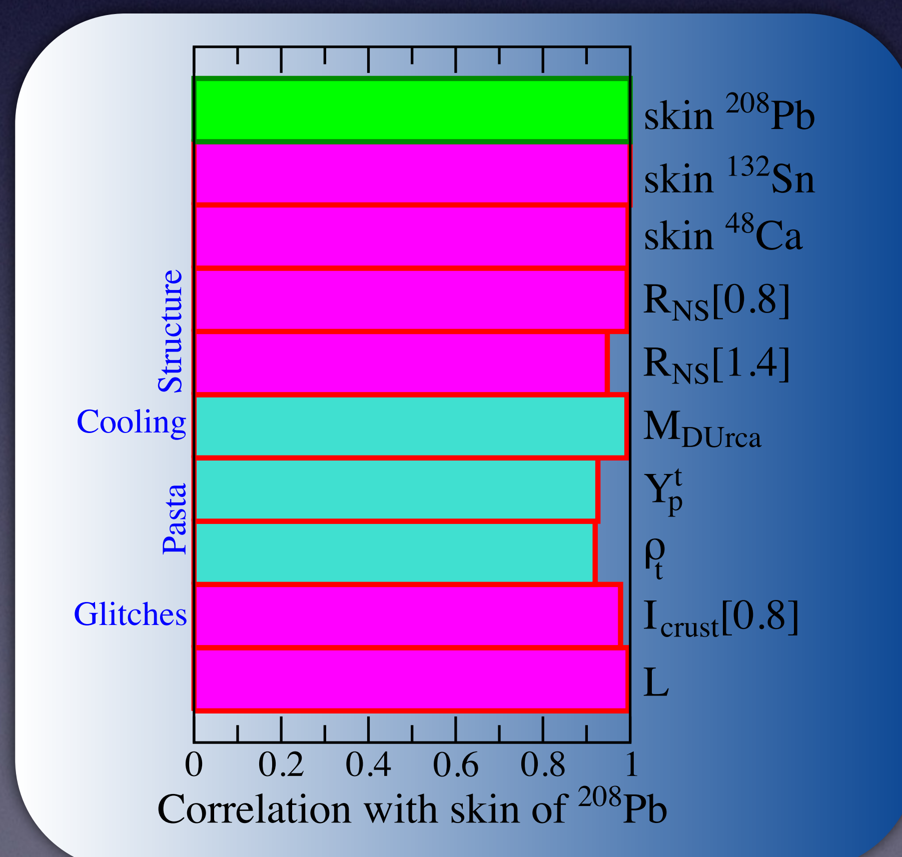
New era in Nuclear Theory where predictability will be typical and uncertainty quantification will be demanded …

Heaven and Earth

The enormous reach of the neutron skin

- Neutron-star radii are sensitive to the EOS near $2\rho_0$
  - Neutron star masses sensitive to EOS at much higher density
- Neutron skin correlated to a host of neutron-star properties
  - Stellar radii, proton fraction, enhanced cooling, moment of inertia



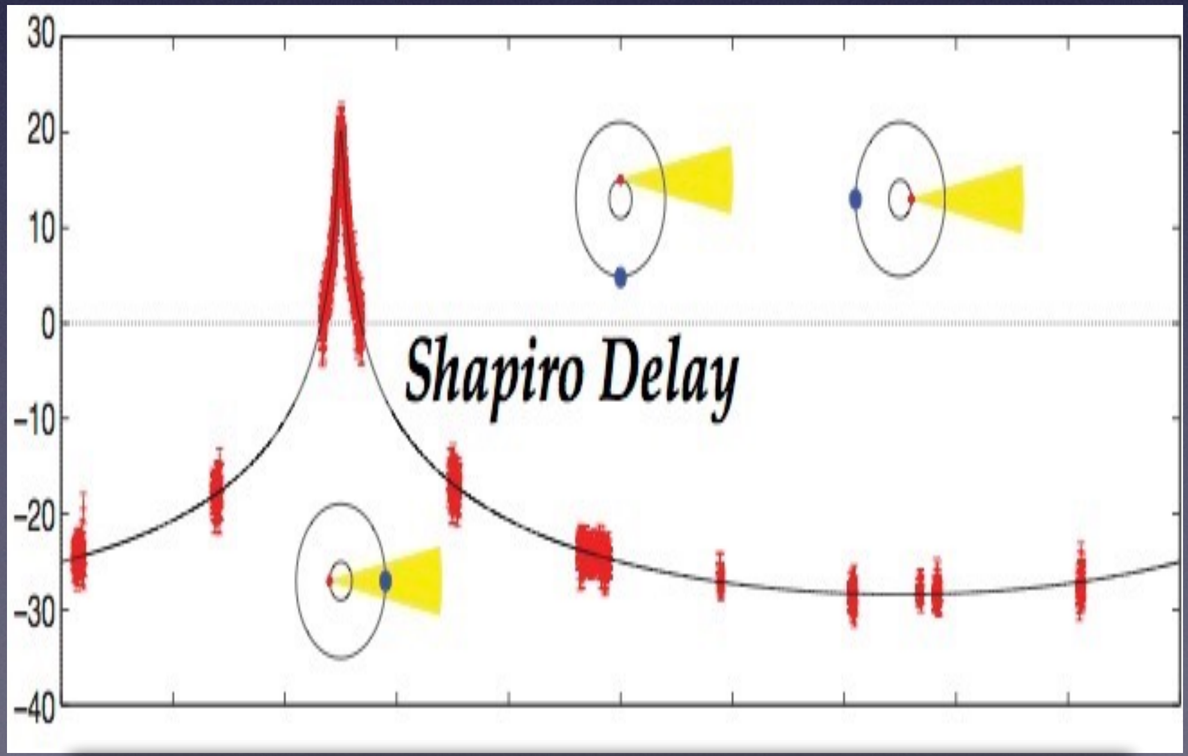


PHYSICAL REVIEW A 83, 040001 (2011)
Editorial: Uncertainty Estimates

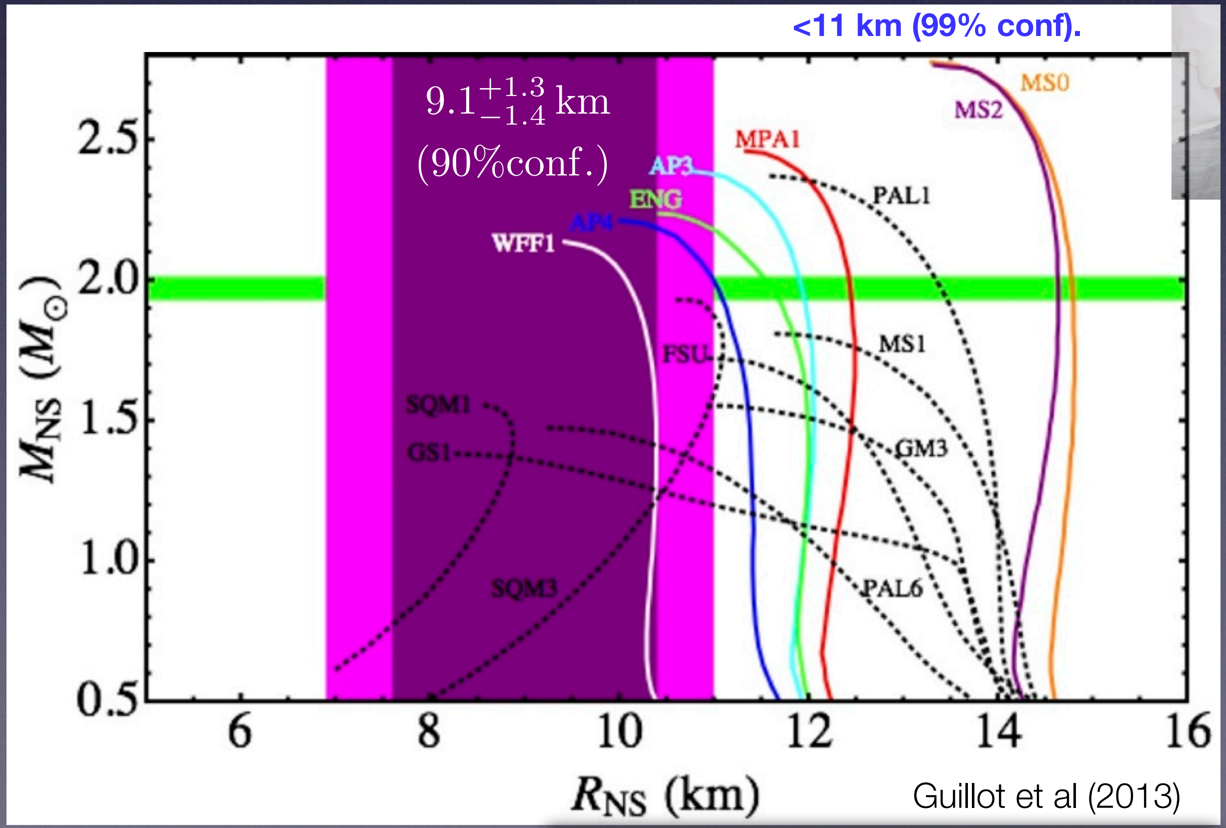
Papers presenting the results of theoretical calculations are expected to include uncertainty estimates for the calculations whenever practicable …

Addressing Future Challenges

- Same dynamical origin to neutron skin and NS radius
  - Same pressure pushes against surface tension and gravity!
  - Correlation involves quantities differing by 18 orders of magnitude!
  - NS radius may be constrained in the laboratory (PREX-II, SPREX, …)
- However, a significant tension has recently emerged!
  - Stunning observations have established the existence of massive NS
  - Recent observations has suggested that NS have small radii; How small?
  - Extremely difficult to reconcile both; perhaps evidence of a phase transition?



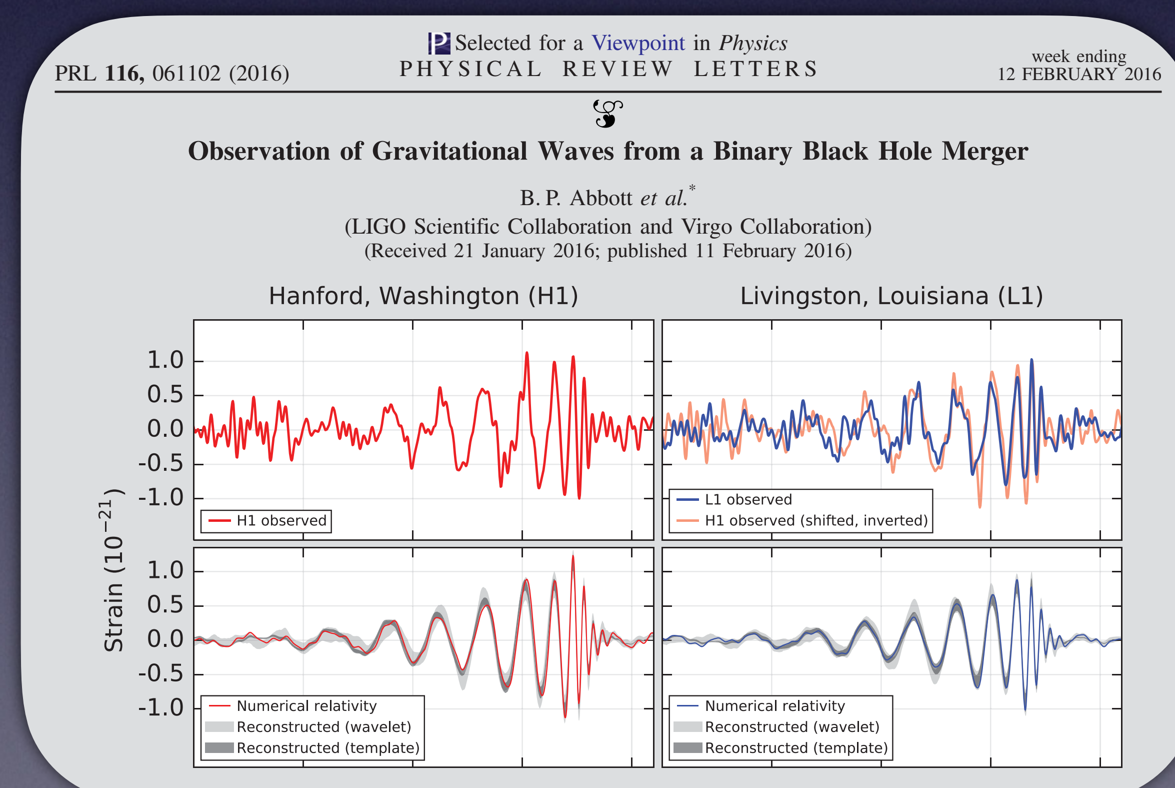
Time delay due to NS radiation dipping into gravitational well of WD!



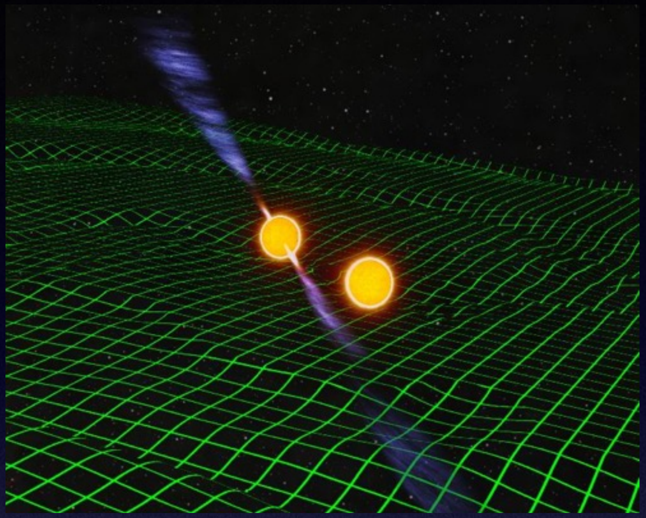
WFF1 violates causality!

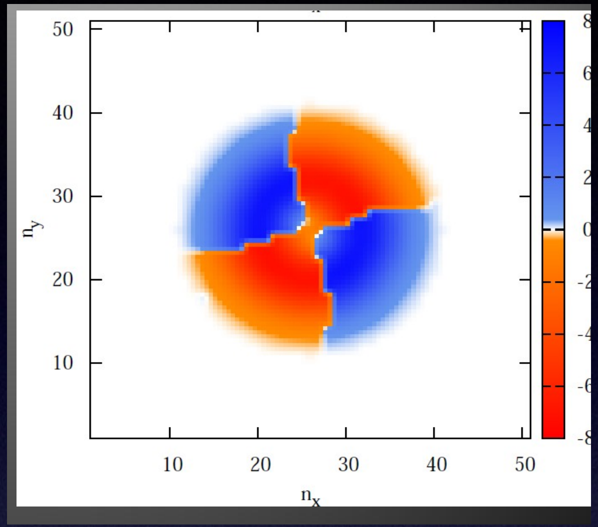
“We have detected gravitational waves. We did it”
David Reitze, February 11, 2016

- The dawn of gravitational wave astronomy
  - Initial black hole masses are 36 and 29 solar masses
  - Final black hole mass is 62 solar masses, 3 solar masses radiated in GW

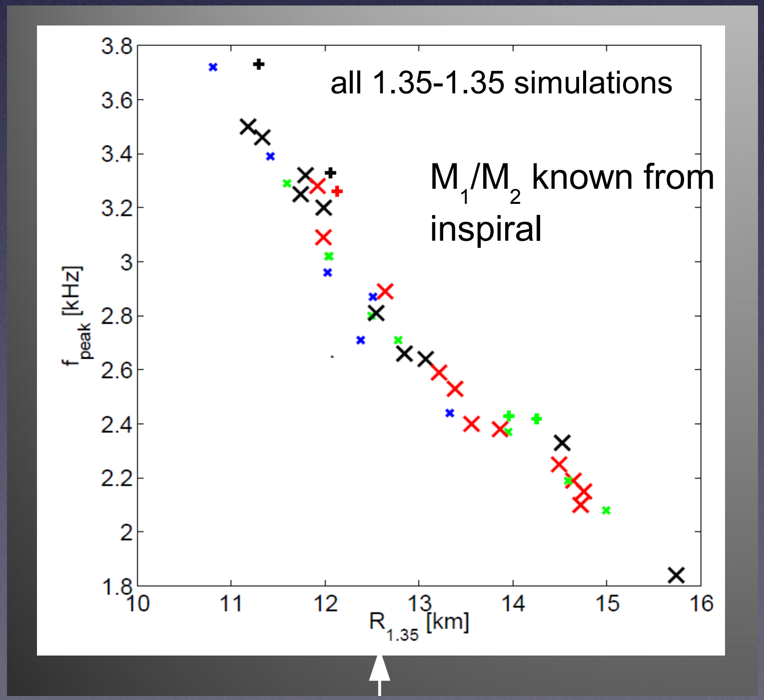


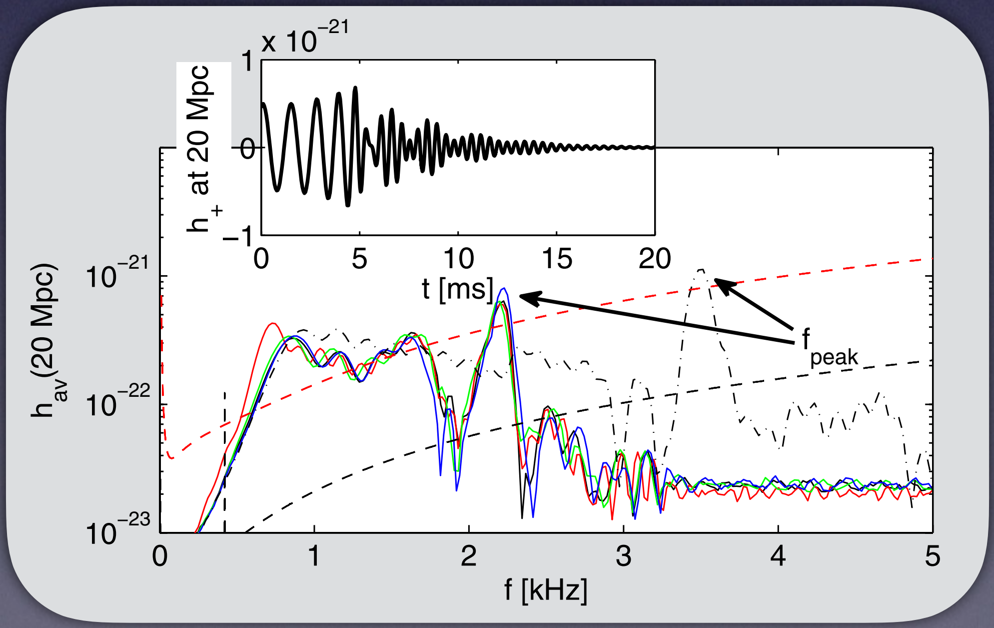
What will we learn from Neutron-Star Mergers

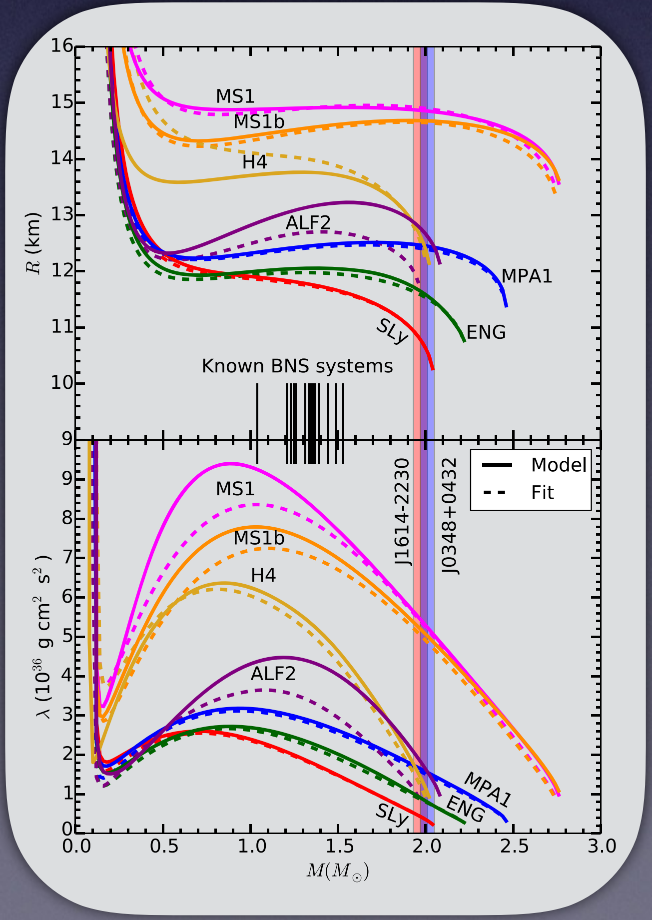




Tidal polarizability scales as R^5 …







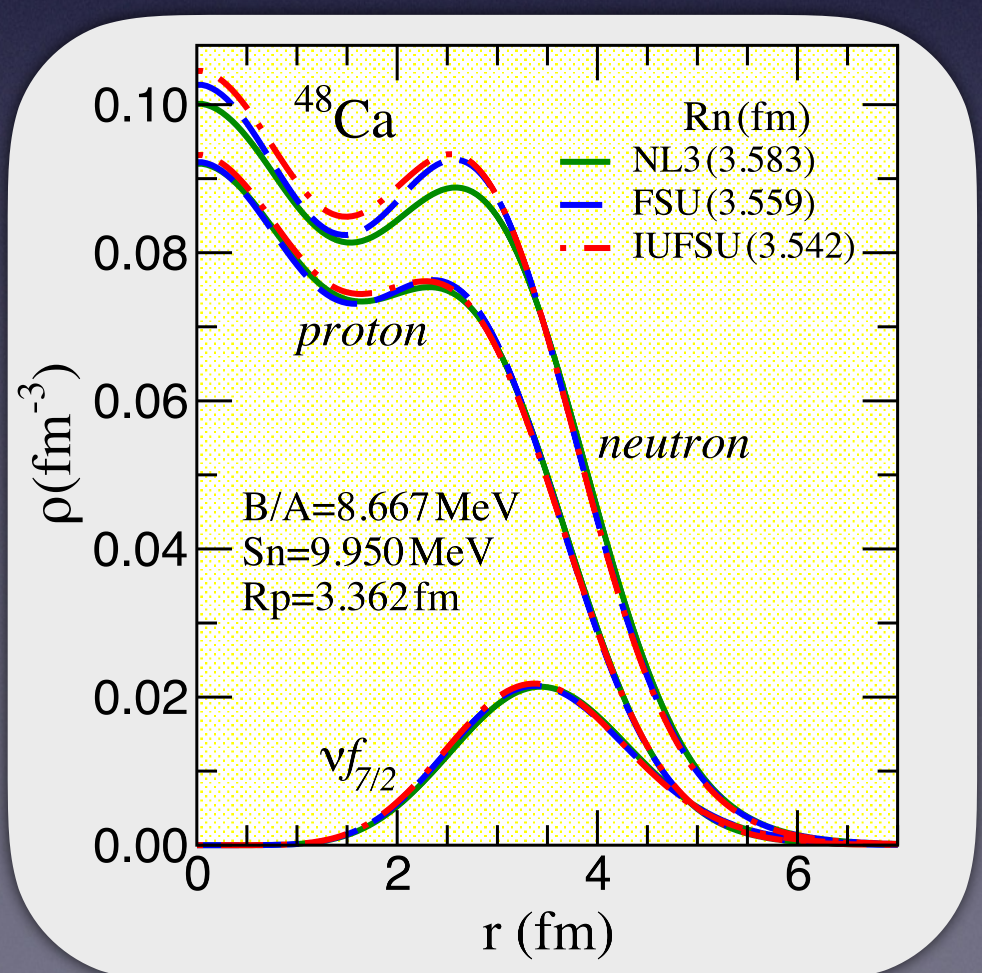
NS radius measured to better than 1km!

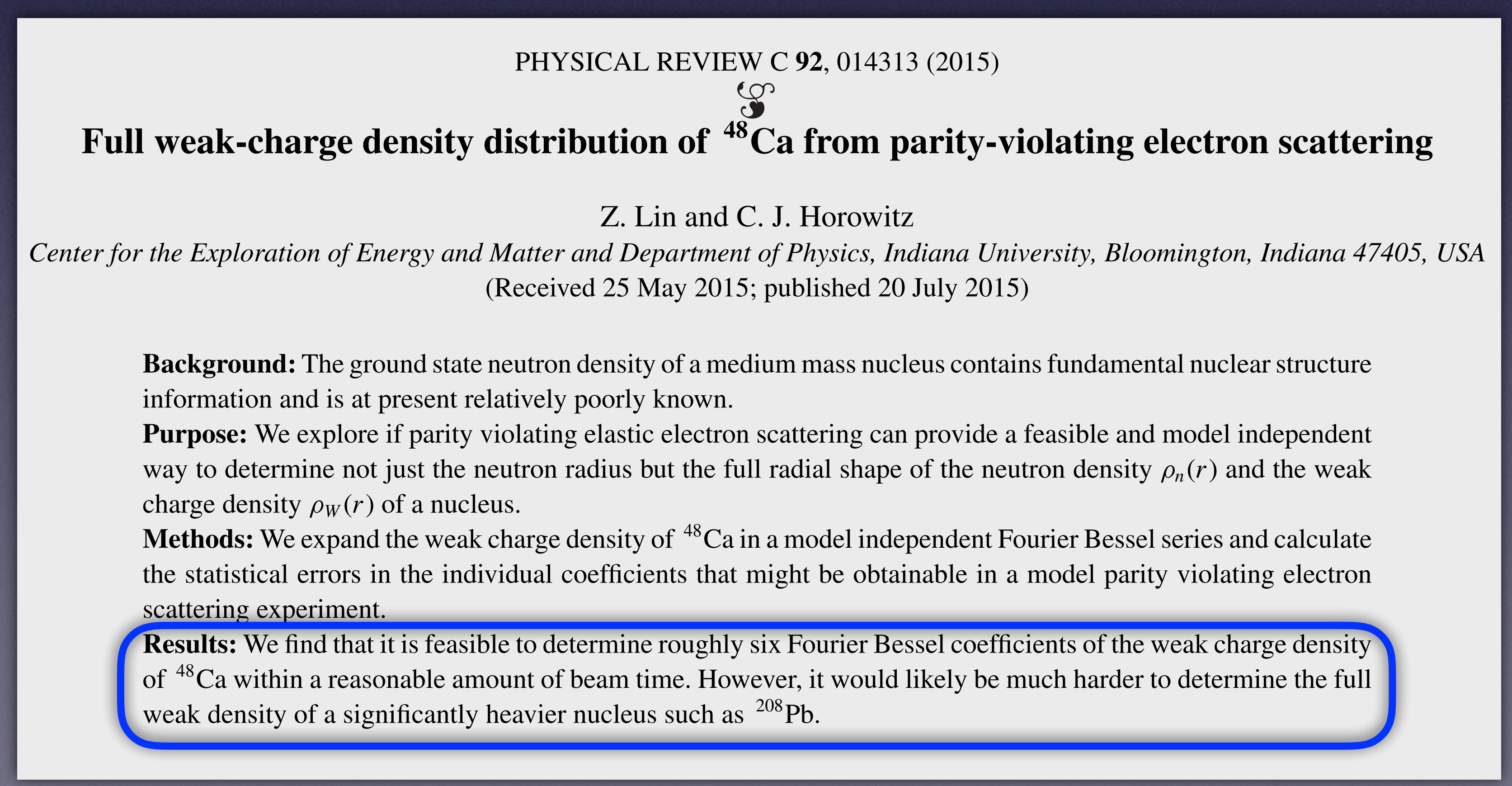
Conclusions and Open Questions

- Neutron Star Crust: DFT for “gross” features BNN for “fine tuning”
- Neutron Star Core: Bayesian inference with “robust” priors

- From the Laboratory to Nstars: Extrapolations are unavoidable!
- From Nstars to the Laboratory: Critical mass constraints have emerged!
- Nstars ⇦⇨Laboratory: Stellar radii remain poorly constrained …

- Bayesian Model Selection: Choosing between competing models?





PHYSICAL REVIEW C **92**, 014313 (2015)

Full weak-charge density distribution of ^{48}Ca from parity-violating electron scattering

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(Received 25 May 2015; published 20 July 2015)

Background: The ground state neutron density of a medium mass nucleus contains fundamental nuclear structure information and is at present relatively poorly known.
Purpose: We explore if parity violating elastic electron scattering can provide a feasible and model independent way to determine not just the neutron radius but the full radial shape of the neutron density $\rho_n(r)$ and the weak charge density $\rho_W(r)$ of a nucleus.
Methods: We expand the weak charge density of ^{48}Ca in a model independent Fourier Bessel series and calculate the statistical errors in the individual coefficients that might be obtainable in a model parity violating electron scattering experiment.
Results: We find that it is feasible to determine roughly six Fourier Bessel coefficients of the weak charge density of ^{48}Ca within a reasonable amount of beam time. However, it would likely be much harder to determine the full weak density of a significantly heavier nucleus such as ^{208}Pb.