Statistical Analysis of Nucleon-Nucleon interactions

INT Program

Bayesian Methods in Nuclear Physics

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June 22, 2016
Seattle, WA

LLNL-PRES-695520
This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract DE-AC52-07NA27344. Lawrence Livermore National Security, LLC

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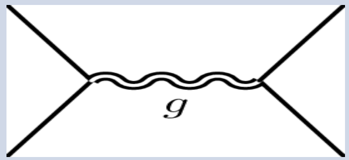
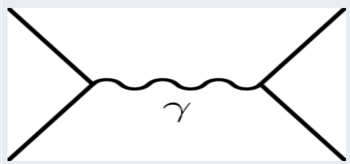
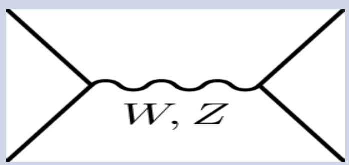
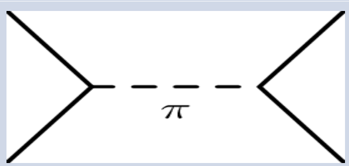
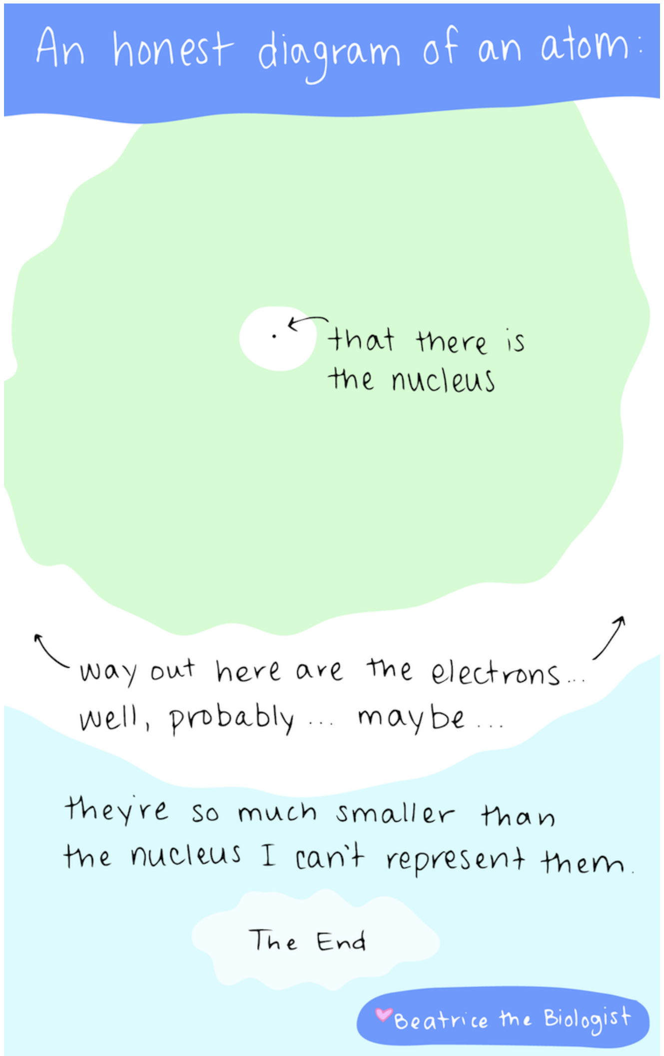
Four fundamental interactions

		Intensity	Range	Exchange
Gravitational		6×10^{-39}	Infinite	Gravitons?
Electromagnetic		1/137	Infinite	photons
Weak		10^{-6}	10^{-8} m	W^+, W^-, Z
Strong		1	10^{-15} m	gluons, π

Strong interaction has the largest intensity
but a very short range.

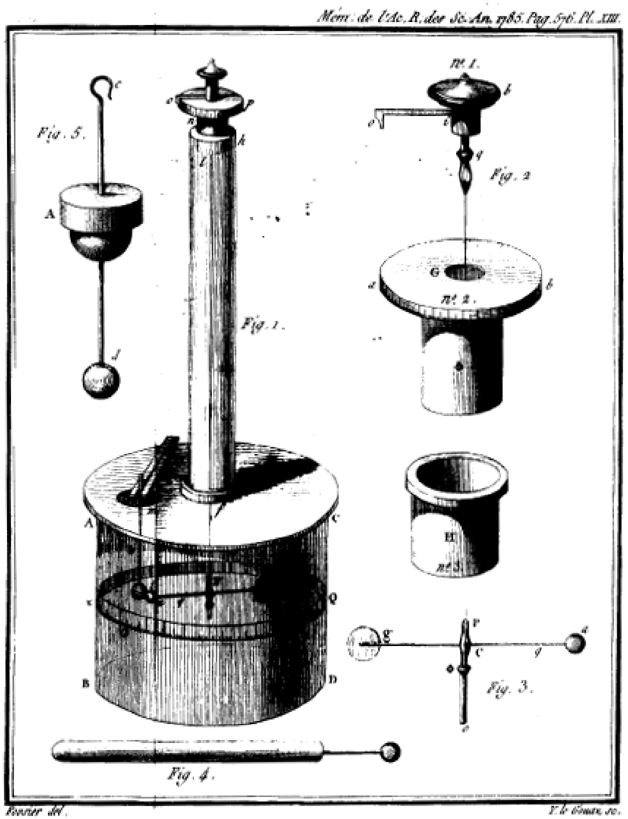
Scales



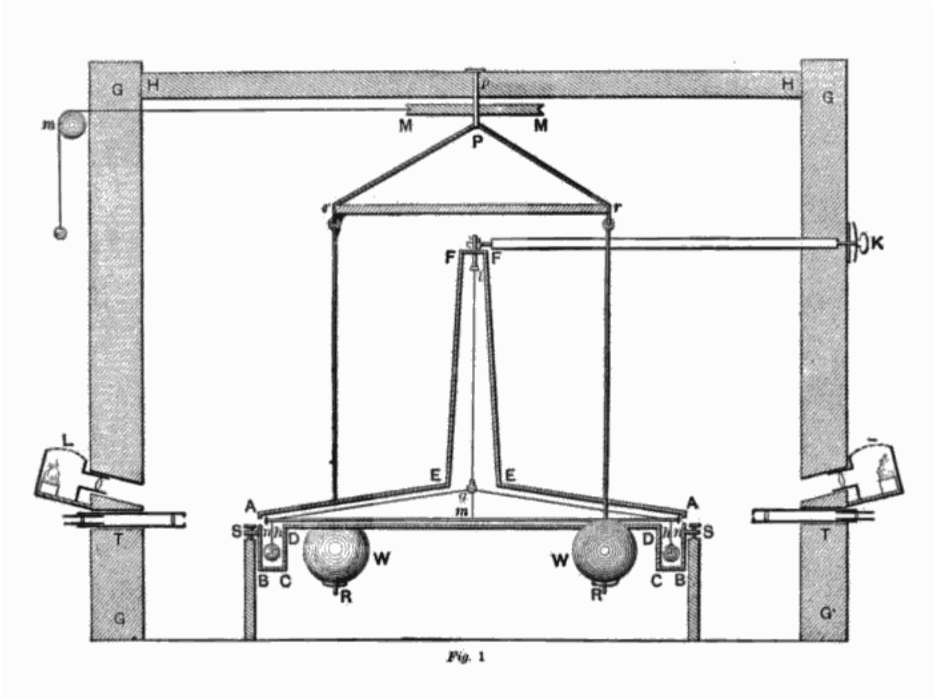
- Atomic scale
  - 1 Angstrom = 10^{-10} m
  - Bohr radius = 0.529 Å
  - Phosphorus atom ~ 1 Å
- Nuclear scale
  - 1 Fermi = 10^{-15} m
  - Proton radius ~ 0.85 fm
  - Inter-nucleon distance ~ 2 fm
  - Gold nucleus ~ 8.45 fm

How to determine the interactions?

“Easy” for infinite range



- Coulomb (1785)
  - “Premier mémoire sur l'electrcité et le magnetisme”

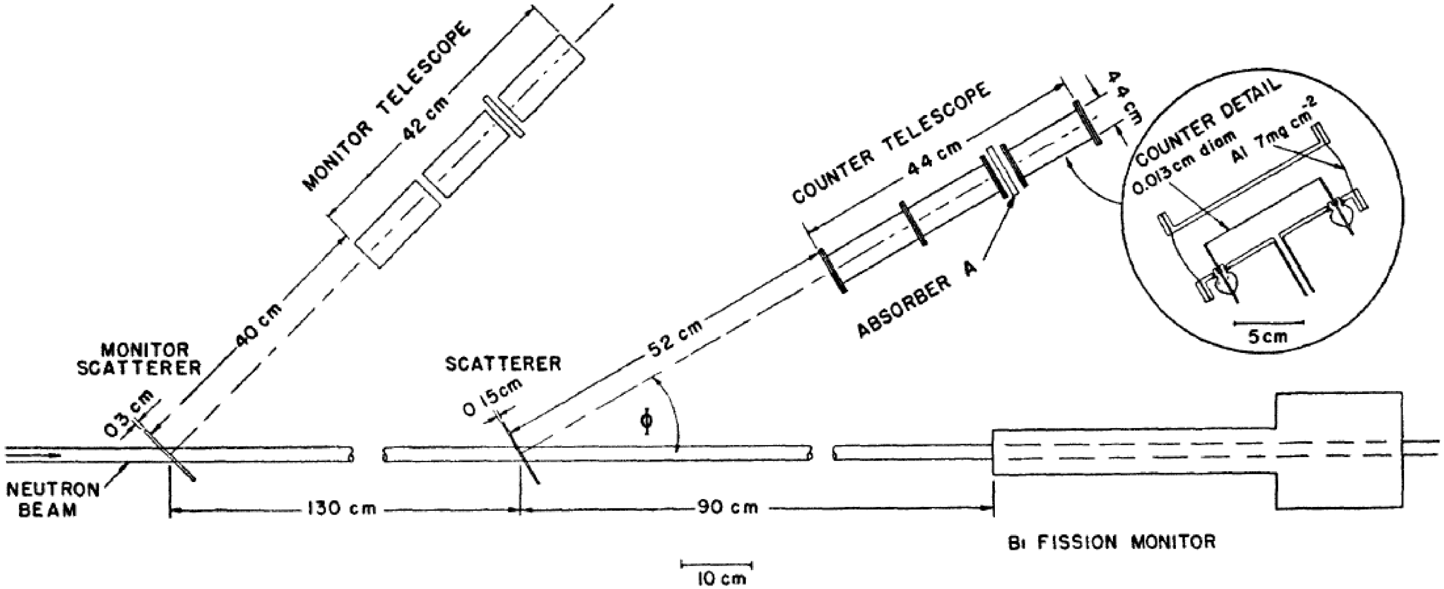


- Cavendish (1798)
  - “Experiment to determine the density of the earth”
- Newton (1687)
  - “ Philosophiæ Naturalis Principia Mathematica”

How to determine the interactions?

“Not so easy” for the short range

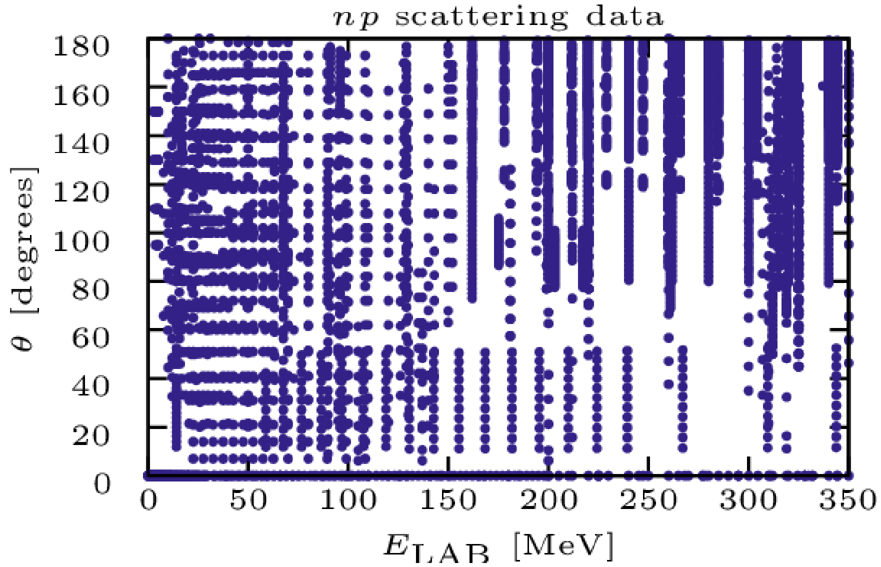
- Scattering experiments
  - Over 20 different observables for energy and angle
- Phenomenological potentials
  - Least squares fit $$\chi^2 = \sum_i \frac{(E_i - T_i)^2}{\sigma_i^2}$$

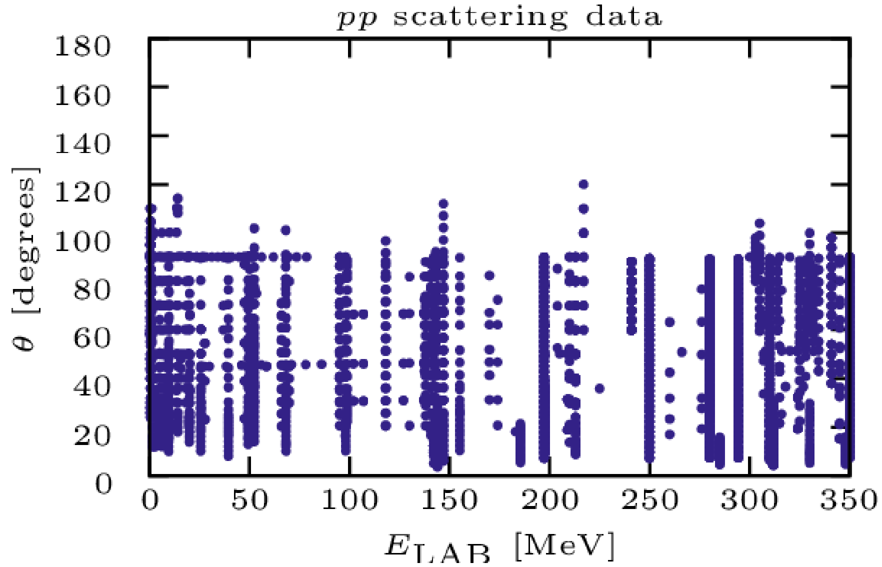


- Quantum chromodynamics (QCD)

Scattering experiments

- Study of the interaction between nucleons for over 60 years
- More than 7800 scattering data since the 1950's
- Several phenomenological models



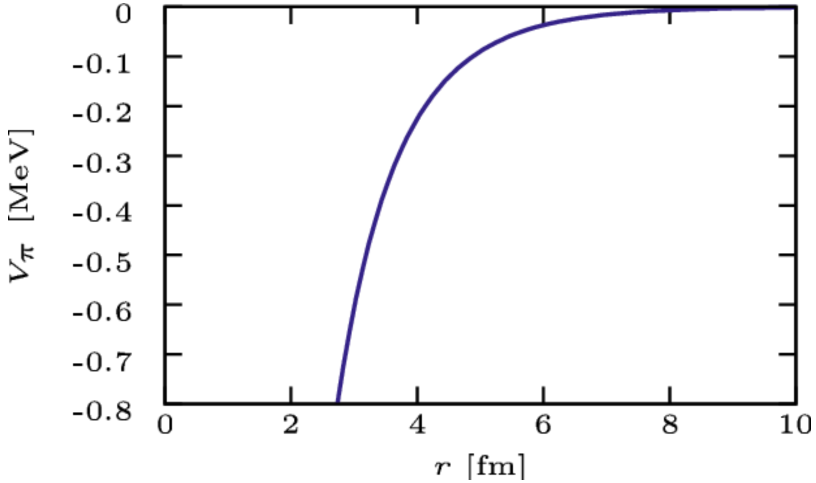


The distribution of the data is relevant

Yukawa potential (1935)

- Exchange of a scalar field with mass
- pion-nucleon coupling constant
- Good description for large distance

$$V_\pi(r) = -f_\pi^2 \frac{e^{-m_\pi r}}{r}$$



Is there a signal for charge dependence?

Phenomenological potentials

- One big family of models
- Hamada-Johnston, Yale, Paris, Bonn, Nijmegen, Reid, Argonne, Granada, …
- $\chi^2/N \sim 1$ in 1993
- One pion exchange for long range part
- ~ 40 parameters for short and intermediate range
- Different results in nuclear structure calculations

Statistical and Systematic error estimates are recent or missing.

Sources of uncertainty

- Numerical (Implementation)
  - Inexact solution method
  - Inherent to any numerical calculation
- Systematic (Model dependence)
  - Any model makes assumptions
  - Different representations for the NN interaction
- Statistical (Fitting bias)
  - Statistical fluctuations in any measurement
  - Uncertainty in data ➡ Uncertainty in parameters

Assuming independence among them

$$(\Delta F)^2 = (\Delta F^{\text{num}})^2 + (\Delta F^{\text{sys}})^2 + (\Delta F^{\text{stat}})^2$$

Anatomy of phenomenological models
fitted to the Granada database

Short and Intermediate range

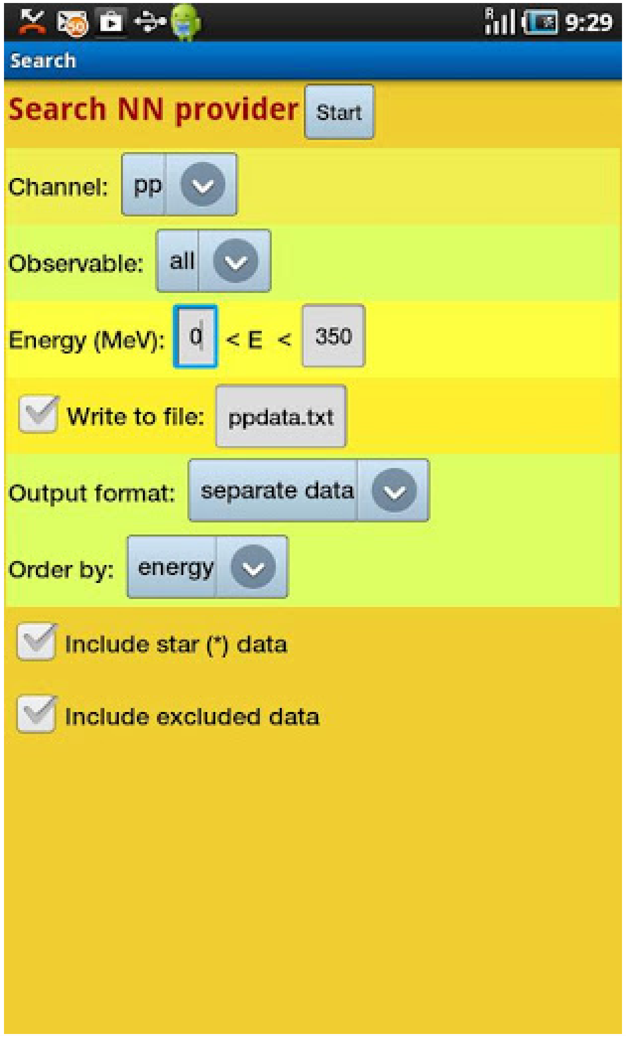
- Delta Shells
  - Coarse grained
  - Simplified calculations
  - High momentum components
- Sum of Gaussian functions
  - Smooth and soft
  - Nuclear structure calculations
  - Not as fast

Long range

- Electromagnetic contributions
  - Small but crucial
- One pion exchange
  - Proper analytic behavior
- Optional
  - Two pion exchange
  - Δ degree of freedom
    - Born approximation

Six different phenomenological models

Granada database



- NN scattering data from 1950 to 2013
  - http://nn-online.org/
  - http://gwdac.phys.gwu.edu/
  - NN Provider for Android
    - Google play store

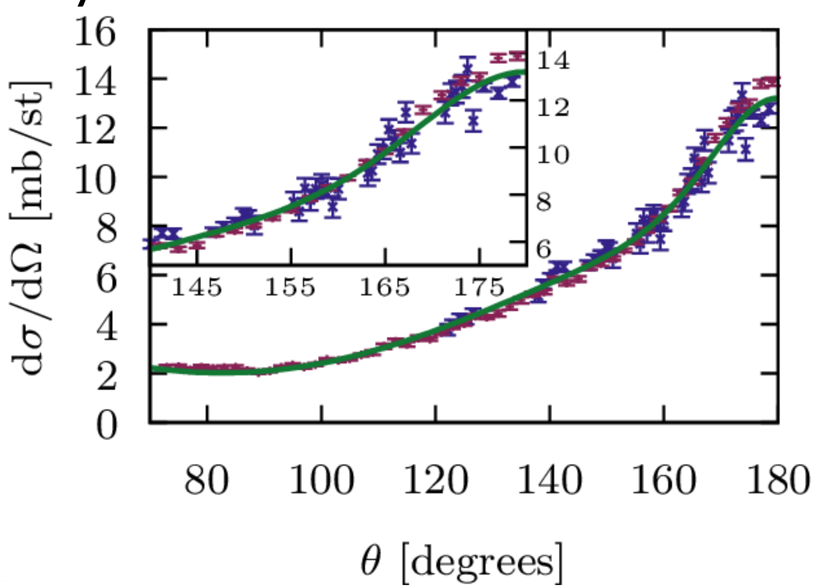
[Amaro, RNP, Ruiz-Arriola]

  - http://www.ugr.es/~amaro/nndatabase/
- 2868 pp and 4991 np data

Fitting NN scattering observables

Selection of data

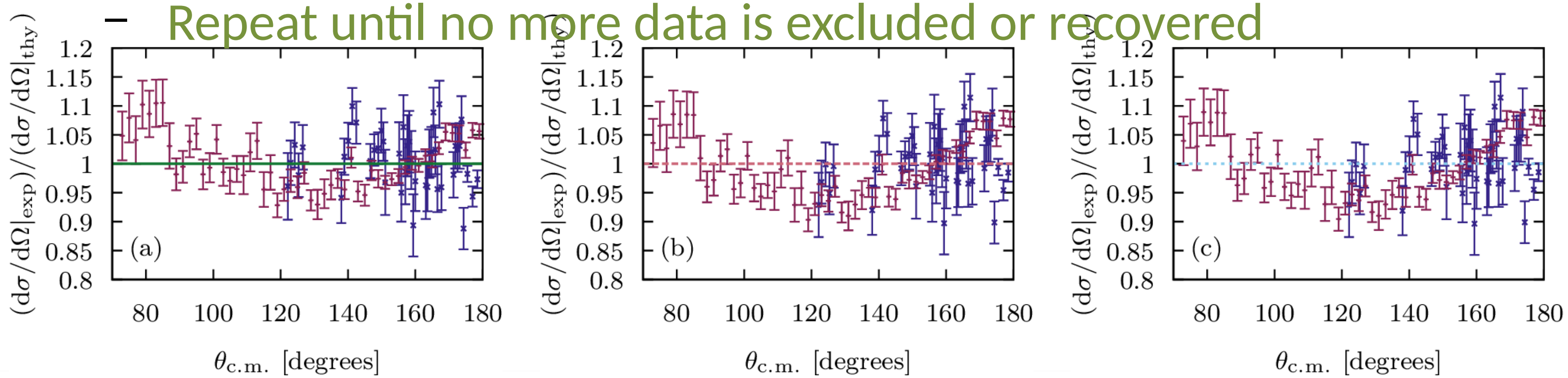
- Direct fits to all data NEVER give χ^2/d.o.f. ≈ 1
  - Restrictive model ? ➡ Improve model
  - Mutually incompatible data ➡ Reject incompatible data
- np $d\sigma/d\Omega$ at 162 MeV
- Statistical and systematic errors may be over or underestimated
- 3σ criterion
  - Fit all data (χ^2/d.o.f. > 1)
  - Remove sets with improbably high or low χ^2
  - Refit parameters



Fitting NN scattering observables

Recovering data

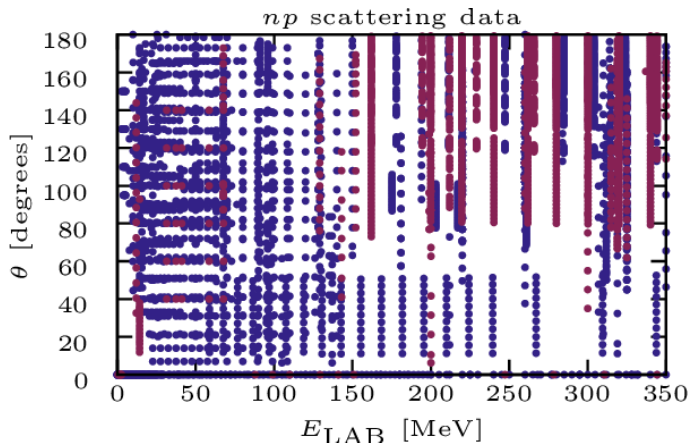
- Mutually incompatible data
  - Which experiment is correct?
  - Is any of the two correct?
  - Maximization of experimental consensus
- Exclude data sets inconsistent with the rest of the database
  - Fit to all data (χ^2/d.o.f. > 1)
  - Remove data sets with improbably high or low χ^2 (3σ criterion)
  - Refit parameters
  - Re-apply 3σ criterion to all data
  - Repeat until no more data is excluded or recovered

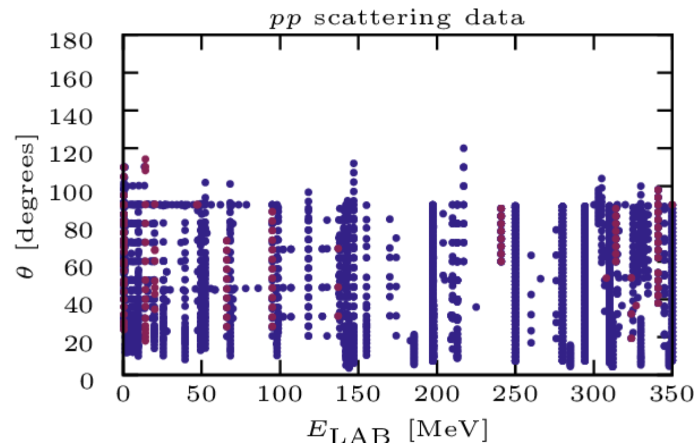


Fitting NN scattering observables

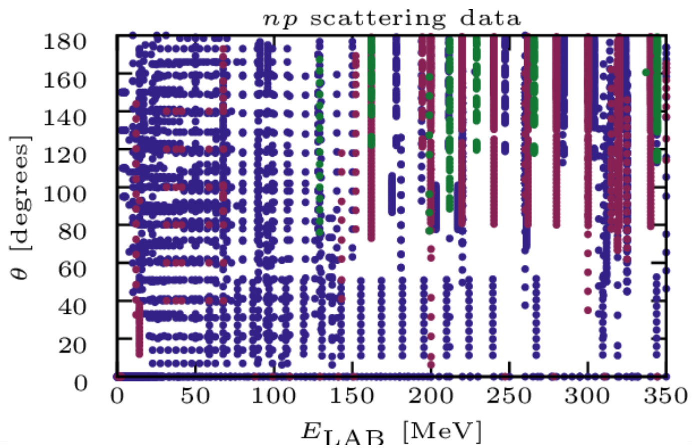
Recovering data

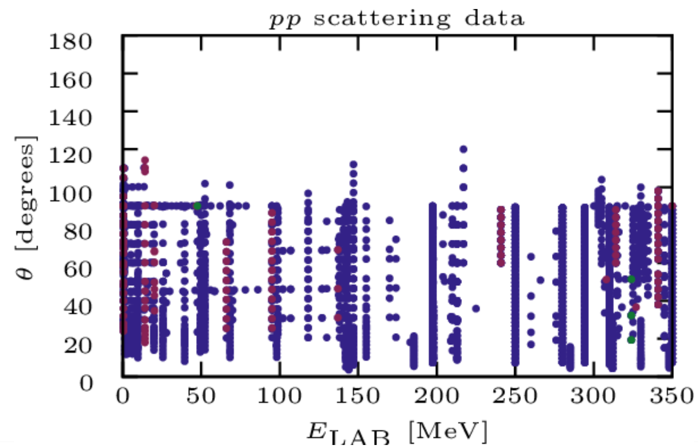
Usual Nijmegen 3σ criterion (1677 rejected data)





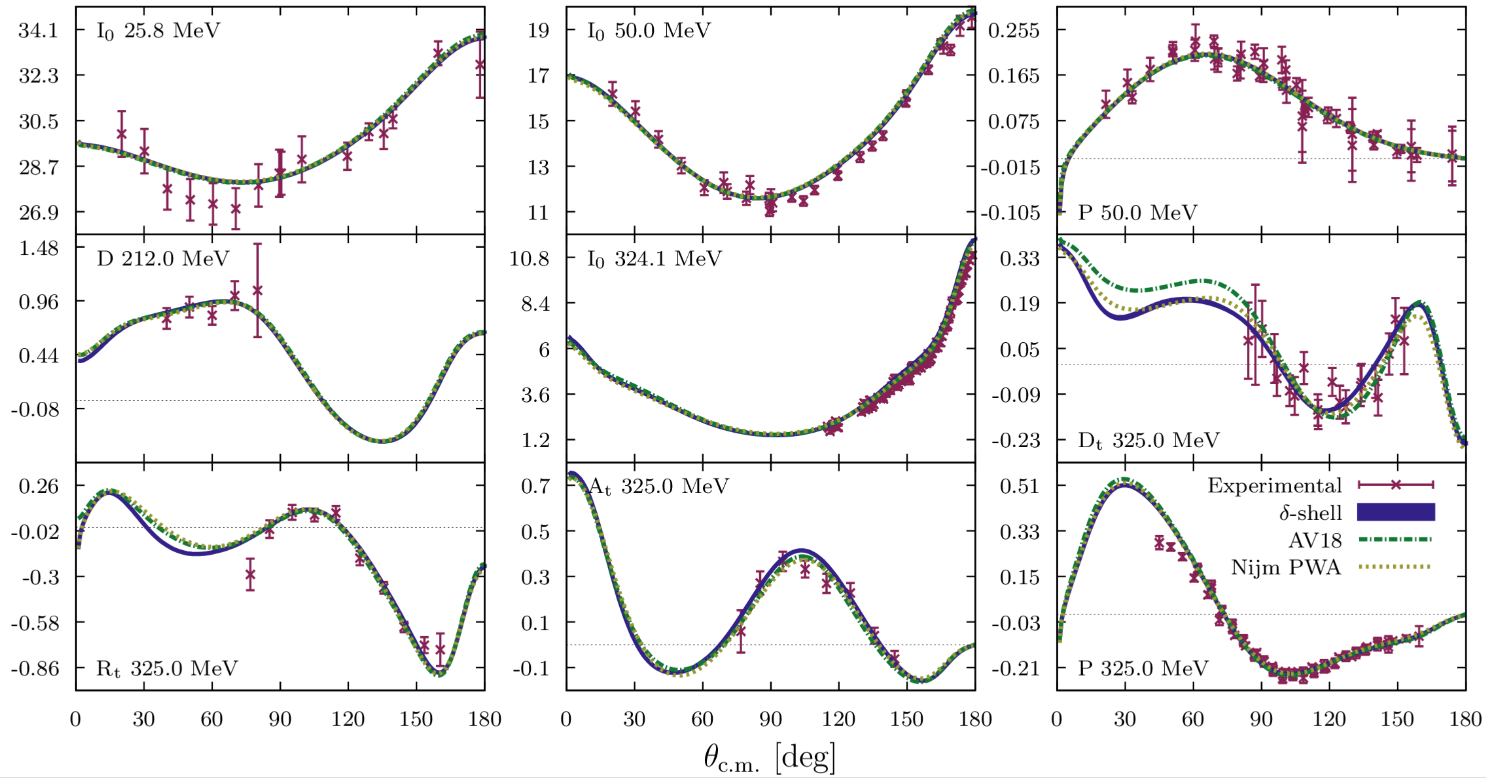
300 recovered data with Granada procedure (consistent database)





Fitting NN scattering observables

- Comparing with other models and experimental data



χ^2/d.o.f. = 1.06 with $N = 2747|_{pp} + 3691|_{np}$

[RNP, Amaro & Ruiz-Arriola. Phys.Rev.C88 (2013) 024002]

Fitting NN scattering observables

- Different models fitted to the *same* database

Potential	T_{LAB}	N_{Data}	$N_{parameters}$	χ^2/d.o.f.
DS - OPE	350	6713	46	1.05
DS - χTPE	350	6712	33	1.08
DS - ΔBorn	350	6719	31	1.06
Gauss - OPE	350	6712	42	1.07
Gauss - χTPE	350	6712	31	1.09
Gauss - ΔBorn	350	6712	30	1.14

[RNP, Amaro & Ruiz Arriola. ArXiv:1410.8097v3]

Predictions are different
Source of *systematic* uncertainties

Fitting NN scattering observables

Testing the normality of residuals

- Experiments by counting events → Poissonian statistics

- Large number of events → Normal statistics
- Crucial assumption

$$R_i = \frac{O_i^{\exp} - O_i^{theor}\left(p_1, p_2, \ldots p_P\right)}{\Delta O_i^{\exp}}$$

  follows the standard normal distribution

- χ^2/d.o.f. = 1 ± (2/d.o.f.)$^{1/2}$

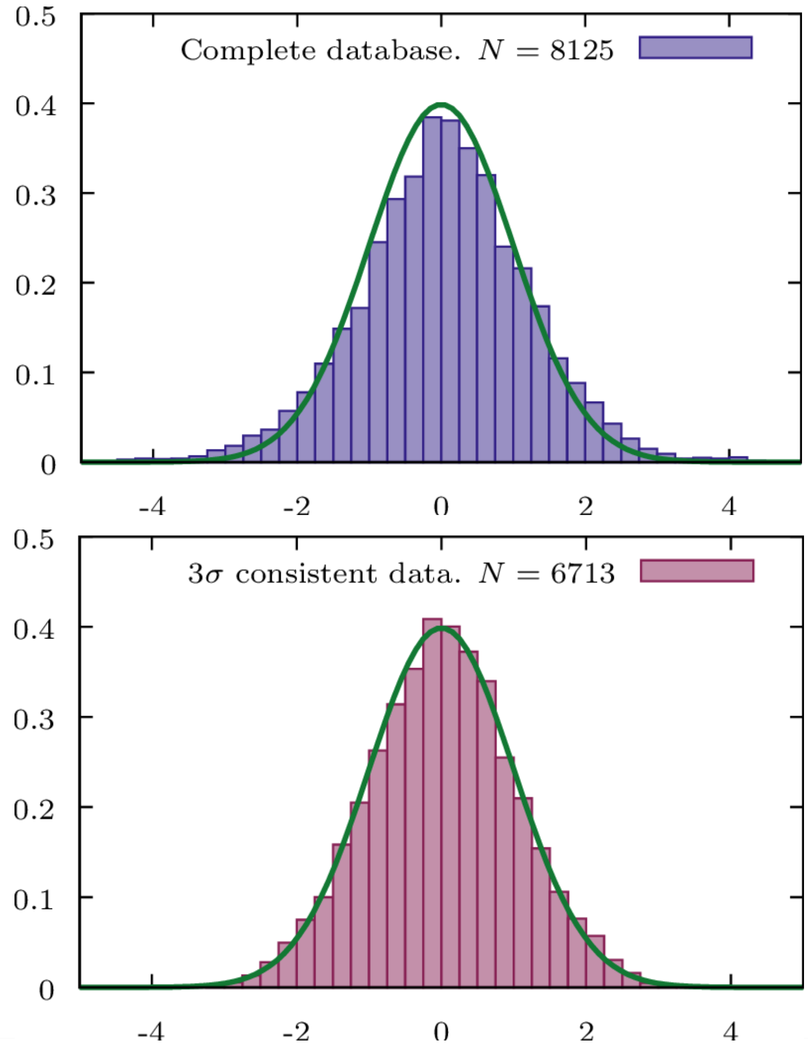
- Can be different from *N*(0,1), but it has to be known

Can only be checked *a posteriori*

Fitting NN scattering observables

Testing the normality of residuals

- Empirical distribution P_{emp}
- Normal distribution N(0,1)
- Finite size fluctuations
- Discrepancies between P_{emp} and $N(0,1)$
- How large is too large?
- Normality tests
  - Quantifying discrepancies
  - Test statistic T
  - Critical values



Fitting NN scattering observables

Tail Sensitive test

- Quantitative test with a graphical representation

  Aldor-Noiman et al. The American Statistician, 67(4):249–260, 2013.

- Quantile-Quantile plot
  - Theoretical quantiles

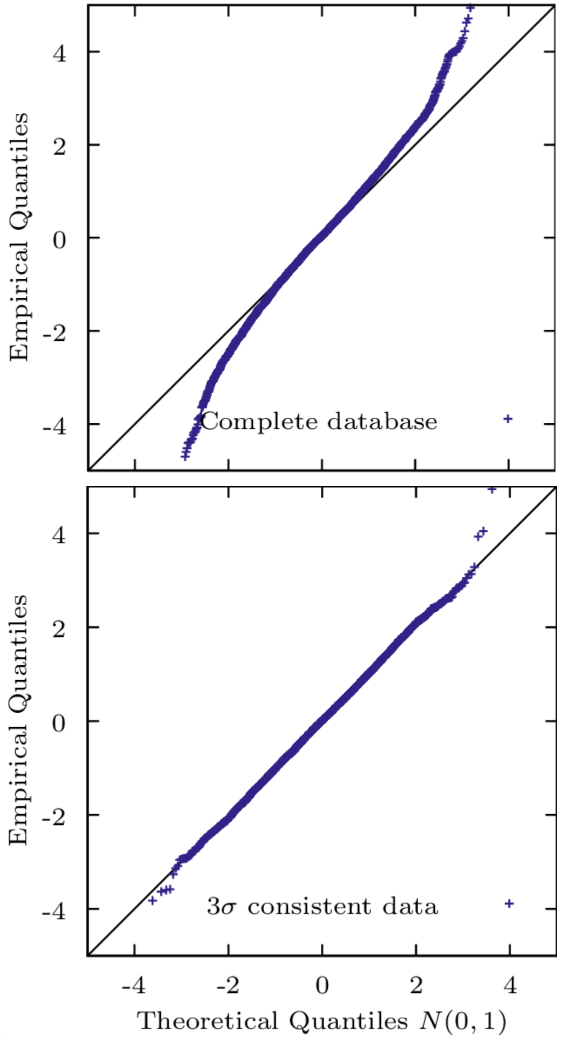
$$\frac{i}{N+1} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x_i^{th}} e^{-\frac{t^2}{2}} dt$$

  - Empirical Quantiles

$$x_1^{emp} < x_2^{emp} < \ldots < x_N^{emp}$$

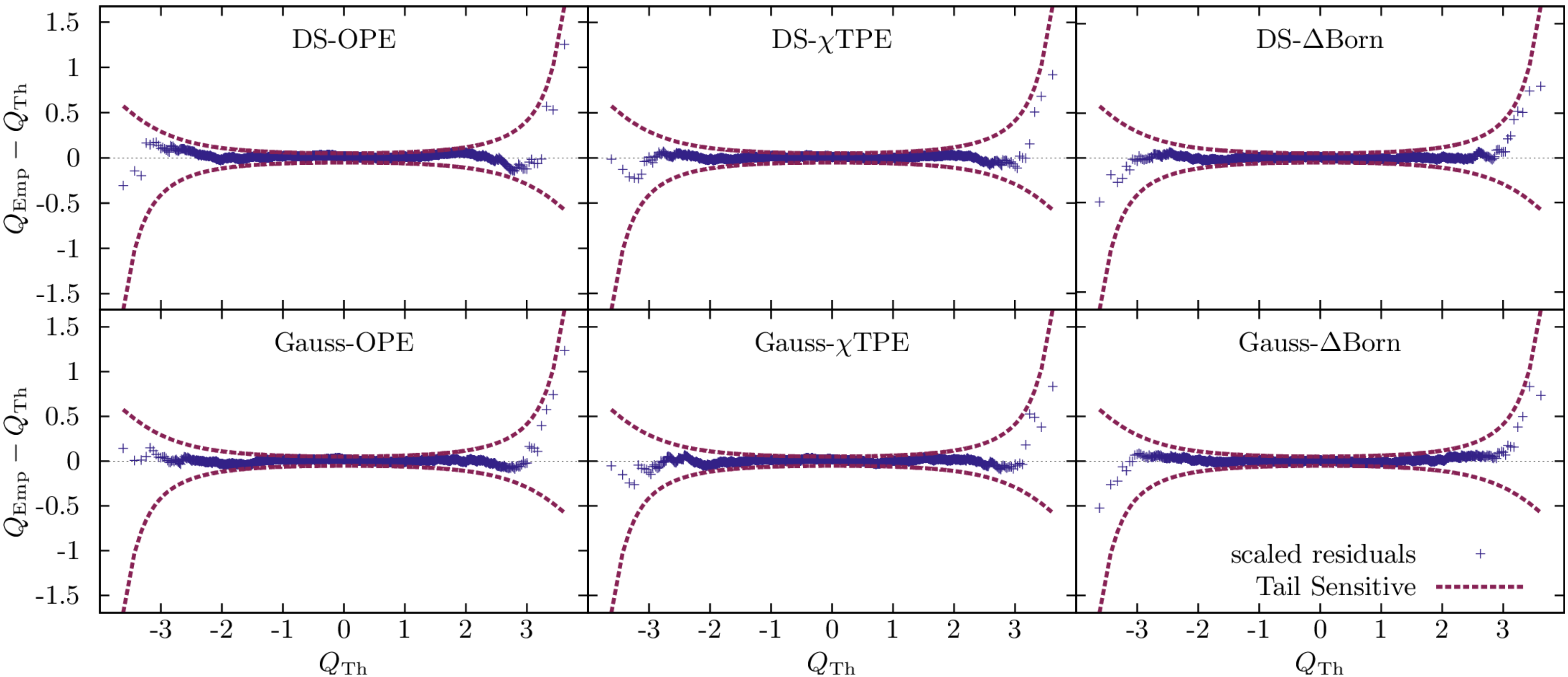
  - Mapping (x_i^{th}, x_i^{emp})
  - $\mathrm{Lim}_{N\to\infty} (x_i^{emp} - x_i^{th}) = 0$
  - Confidence bands
    - Recipe and tables available at

      J. Phys. G: Nucl. Part. Phys. 42 (2015) 034013



Fitting NN scattering observables

Testing the normality of residuals



Six statistically equivalent representations of the NN interaction
Their discrepancies won't come from the data

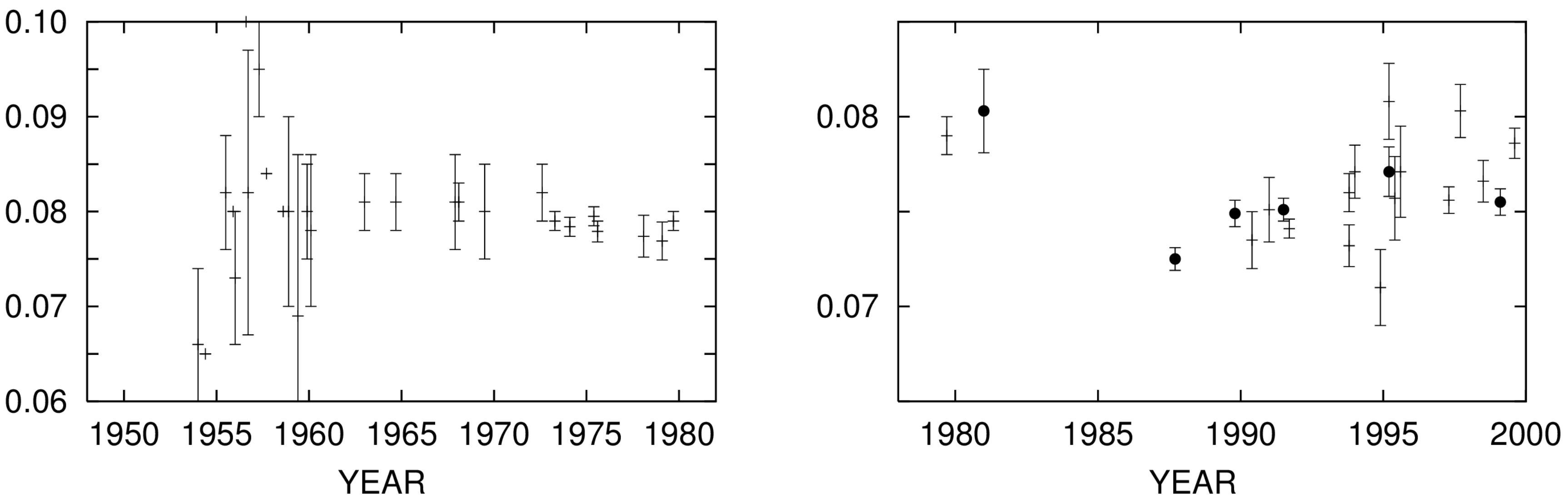
Determining f_π

- Bethe in 1940 from Deuteron properties
  - $f_\pi^{\,2} = 0.077 – 0.080$

$$V_\pi(r) = -f_\pi^2 \frac{e^{-m_\pi r}}{r}$$

- Different processes (NN, πN)
- Different values and precision

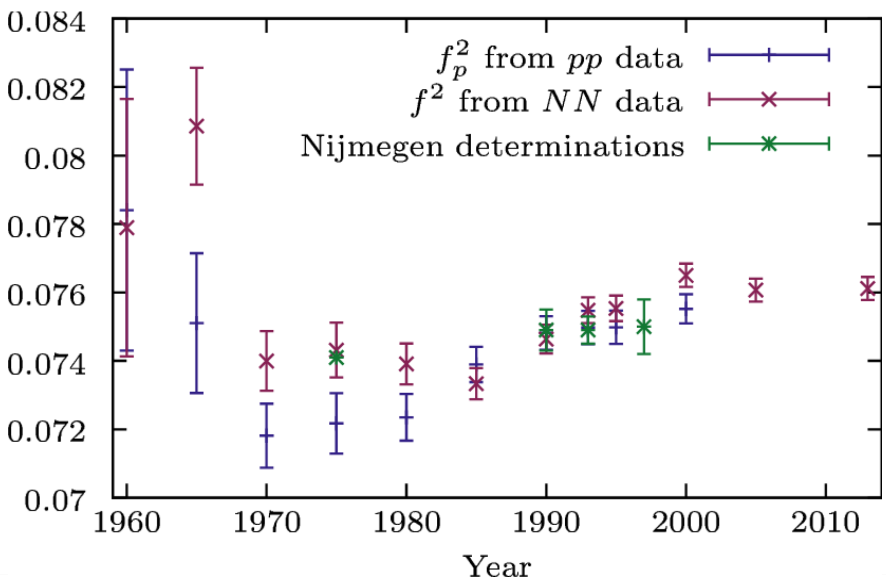
PION-NUCLEON COUPLING CONSTANT UNTIL 1980 PION-NUCLEON COUPLING CONSTANT AFTER 1980



M.E. Sainio arXiv:hep-ph/9912337 (1999)

Determining f_π

- In 1997 the Nijmegen group recommends
  - charge independent f^2=0.075
    "The present accuracies in the determination of the various coupling constants are such,that with a little improvement in the data and in the analyses these charge-independence breaking effects could be checked"



PiN Newslett. 13 (1997) 96

Determining f_π

- Looking for signals of charge dependence

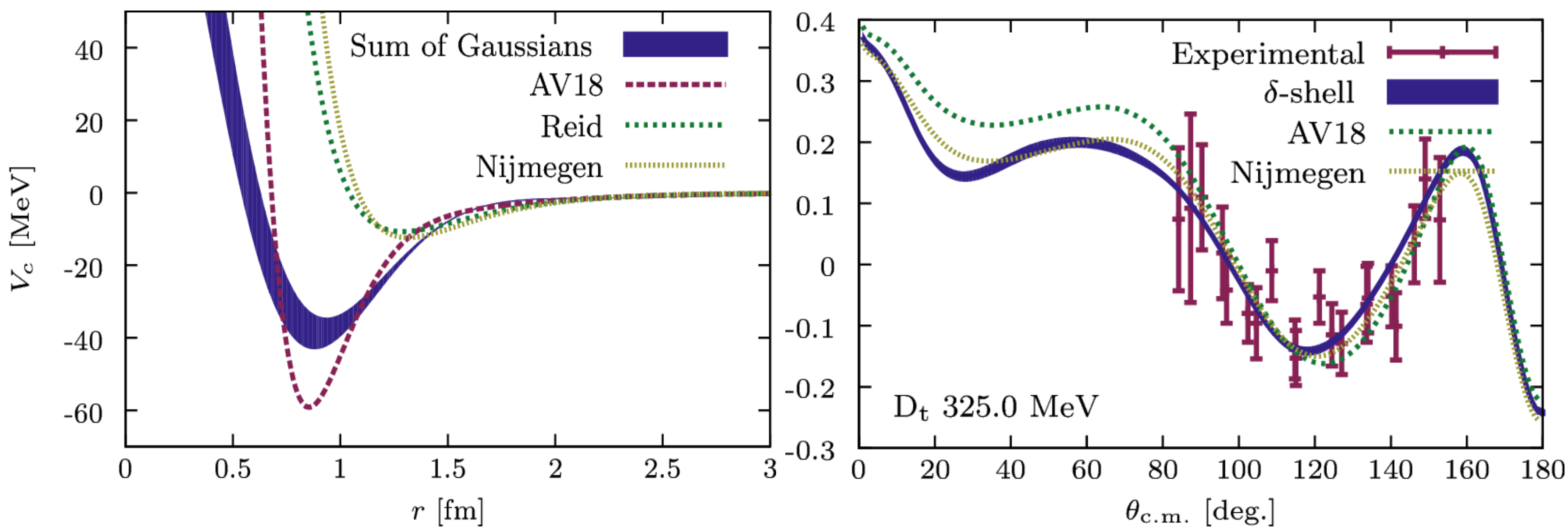
f_p^2	f_0^2	f_c^2	$N_{parameters}$	χ^2/d.o.f.
0.075	idem	idem	46	1.051
0.0761(3)	idem	idem	46+1	1.051
0.0759(4)	0.079(1)	0.0763(6)	46+3	1.043
0.0758(4)	0.080(2)	0.0765(6)	46+3+9	1.036

[RNP, Amaro & Ruiz Arriola. ArXiv:1606.00592v1]

f_0^2 is incompatible with f_p^2 and f_c^2 at the 1σ level

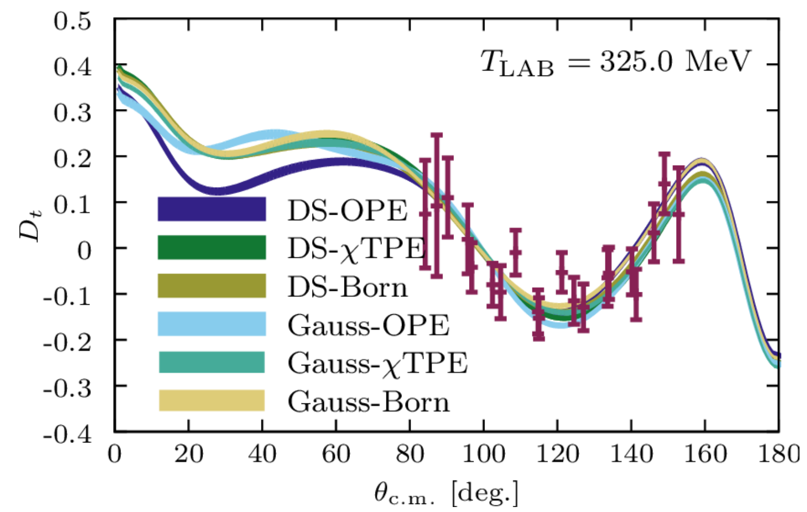
Systematic vs. statistical uncertainties

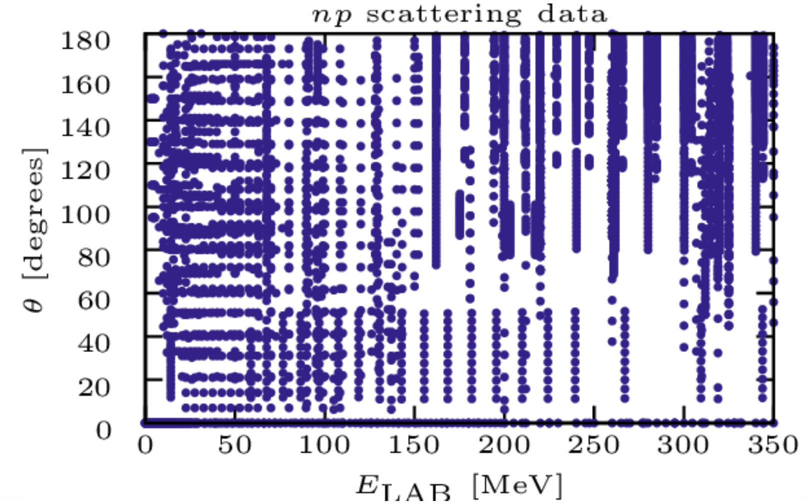
- Same data
- Different representations
- Different predictions
- Who dominates the uncertainty?



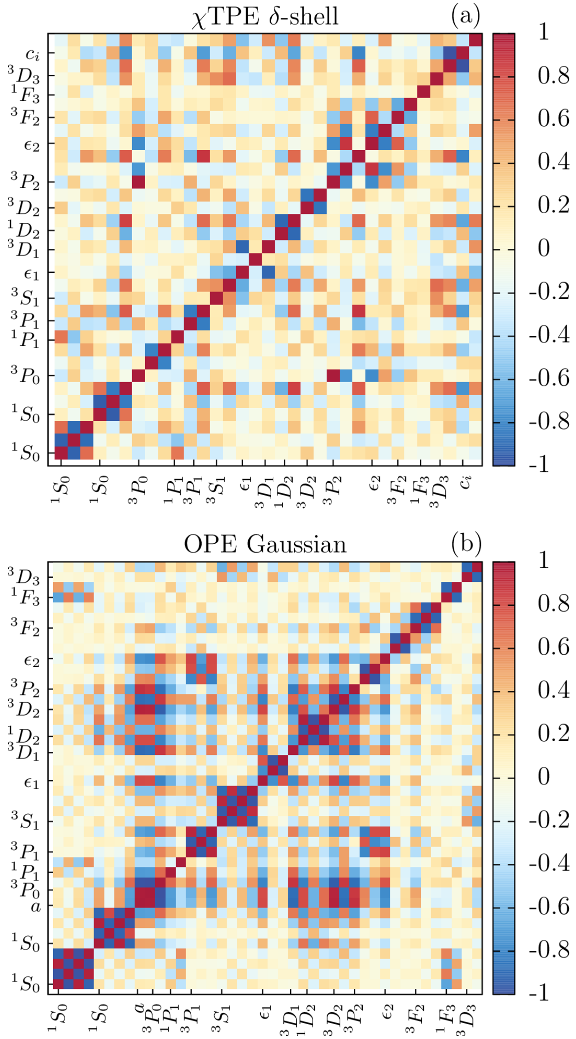
NN Systematic Uncertainty

- Data is unevenly distributed on the $(T_{\text{LAB}},\theta_{\text{c.m.}})$
- Same description in probed regions
- Incompatible predictions in unexplored areas
- A uniform experimental exploration is necessary but unlikely



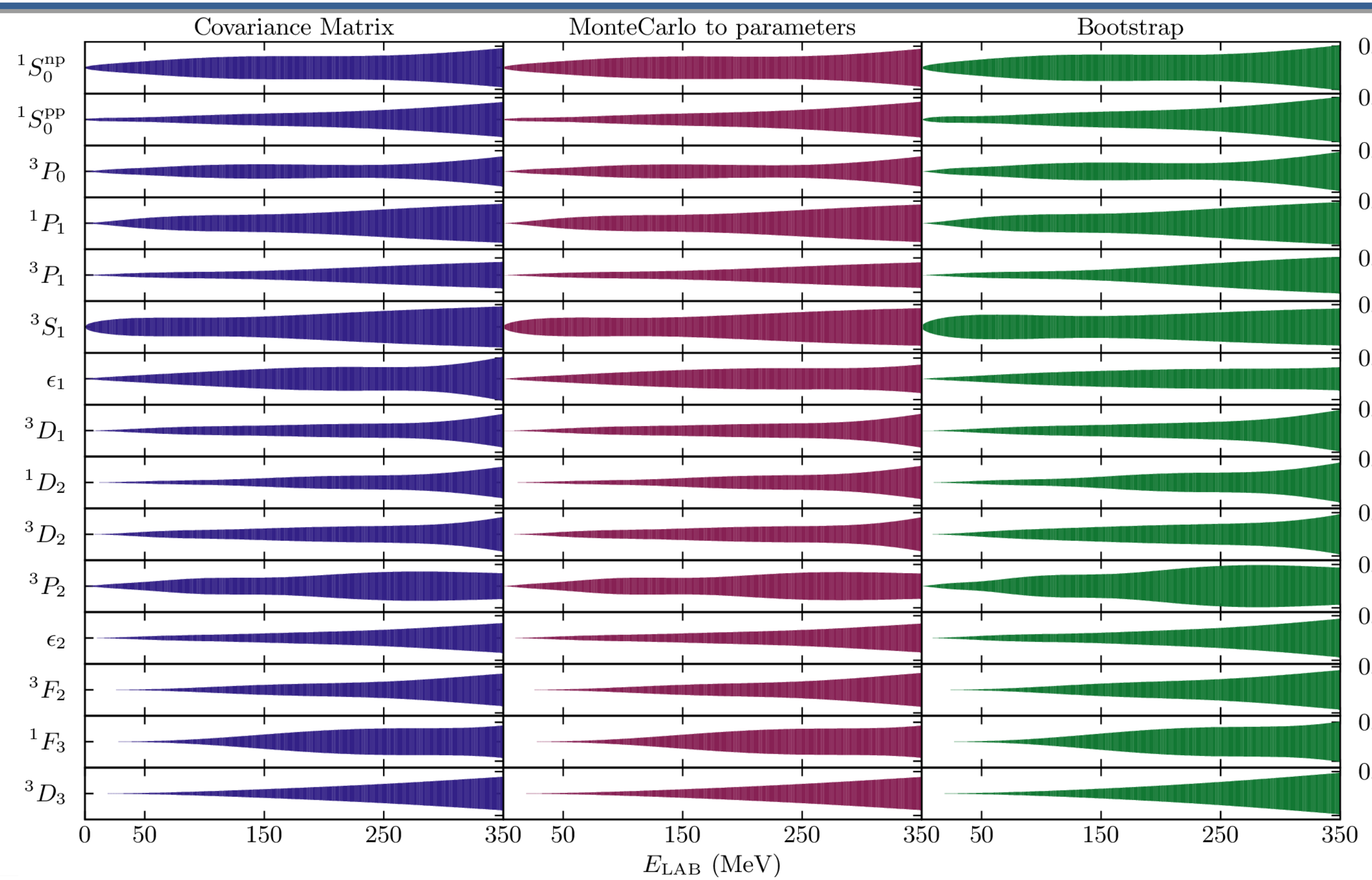


Reproducing NN uncertainties from NN data



- Propagation with covariance matrix
  - Requires to calculate derivatives
- Monte-Carlo family of potentials
- Bootstrap the data
  - Simulate data ~ $N(O_i,\sigma_i)$
  - Refit parameters
- Replicate parameters correlations
  - Simulate parameters
  - Faster, but real distribution may differ

Reproducing NN uncertainties from NN data

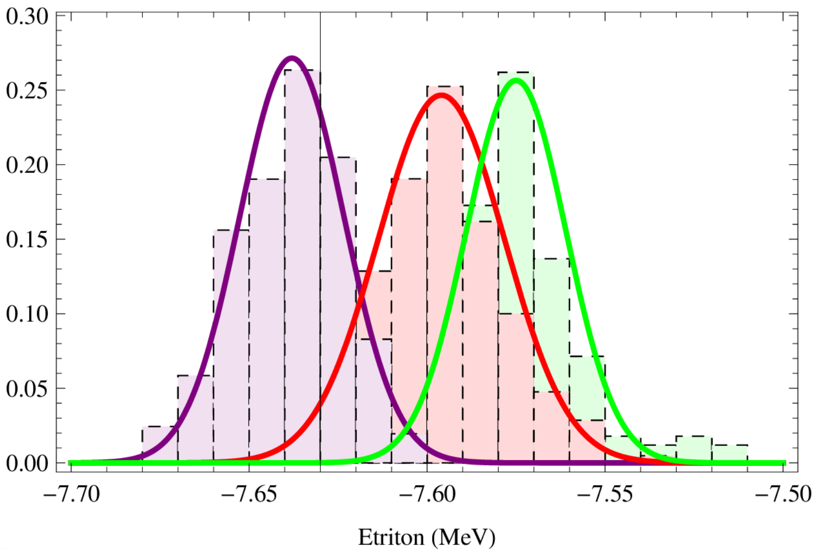


Triton Binding Energy
Hyperspherical Adiabatic Expansion Method

- Monte-Carlo simulation of N =250 potentials
- Error estimates in nuclear structure calculations
- ΔB_t^{stat} = 15(1) KeV, ΔB_t^{num} = 1 KeV

[RNP, Garrido, Amaro & Ruiz-Arriola. Phys.Rev.C99 (2014) 047001]

- N ~ 30 gives a fairly good estimate
- Reduction of target accuracy is possible
- ΔB_t^{sys} is even larger

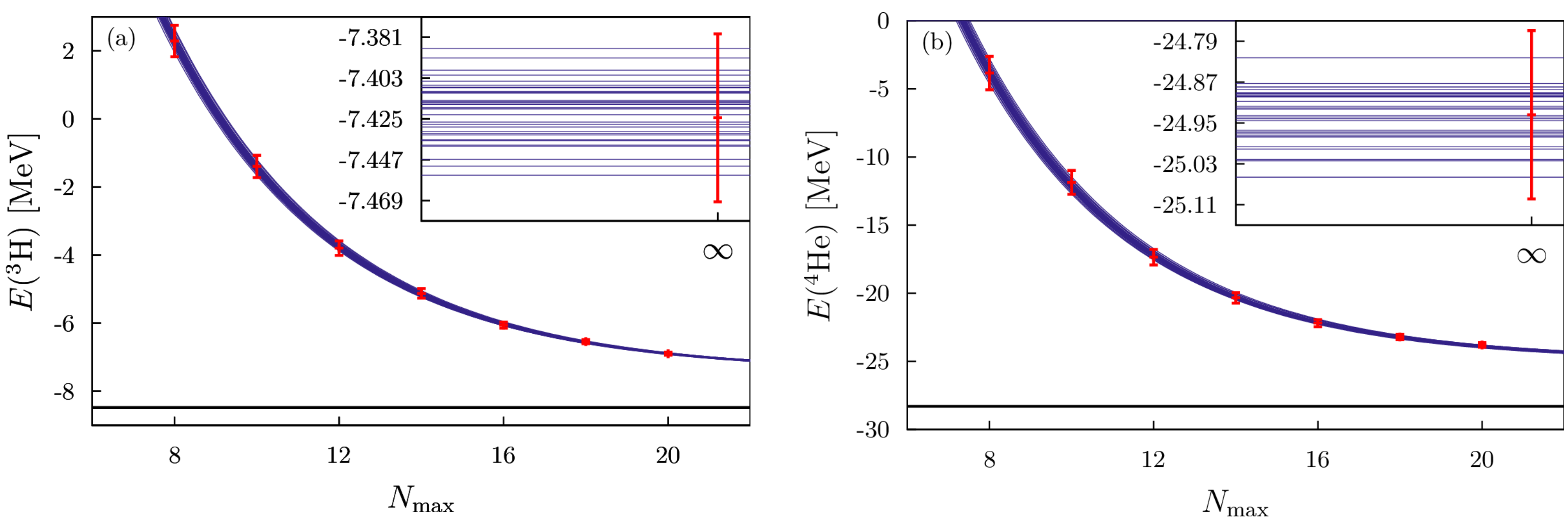


^{3}H and ^{4}He Binding Energy

No Core Full Configuration Method

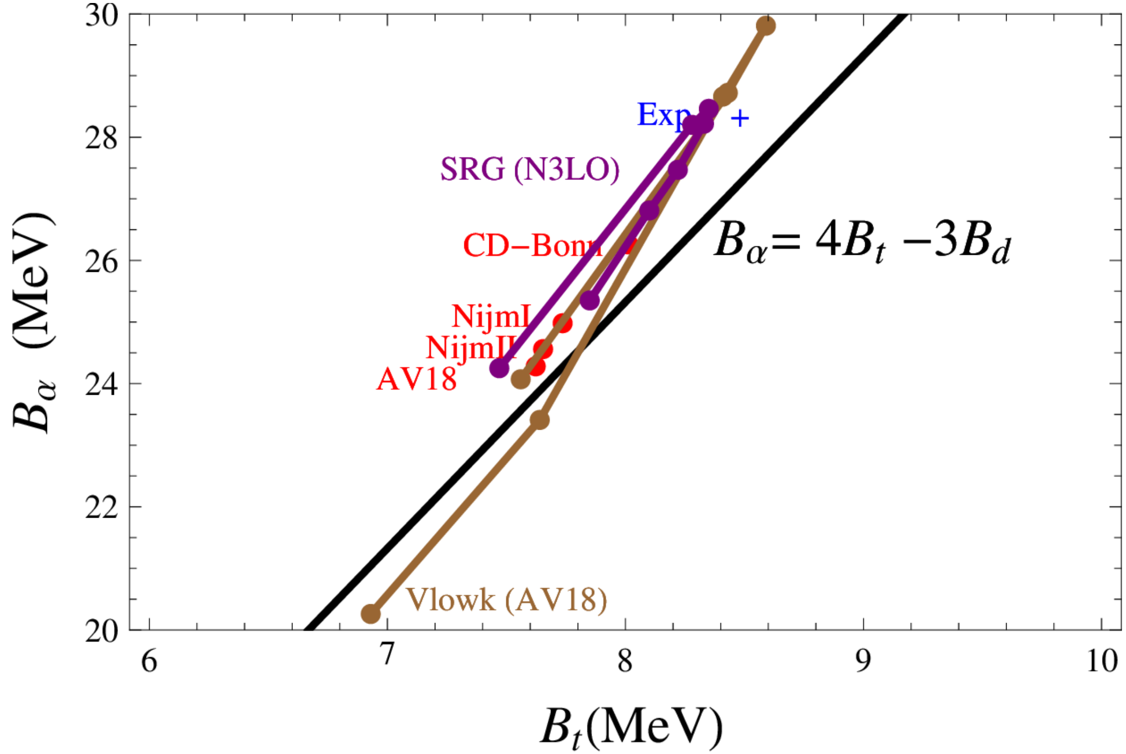
- Sum of Gaussians potential
- 33 Monte-Carlo potentials
- $\Delta(^{3}H)_{t}^{stat}$ = 15 KeV, $\Delta(^{4}He)_{t}^{stat}$ = 55 KeV

[RNP, Amaro, Ruiz-Arriola, Maris & Vary. Phys.Rev.C92 (2015) 064003]



Tjon Line correlation

- Empirical correlation between binding energy calculations



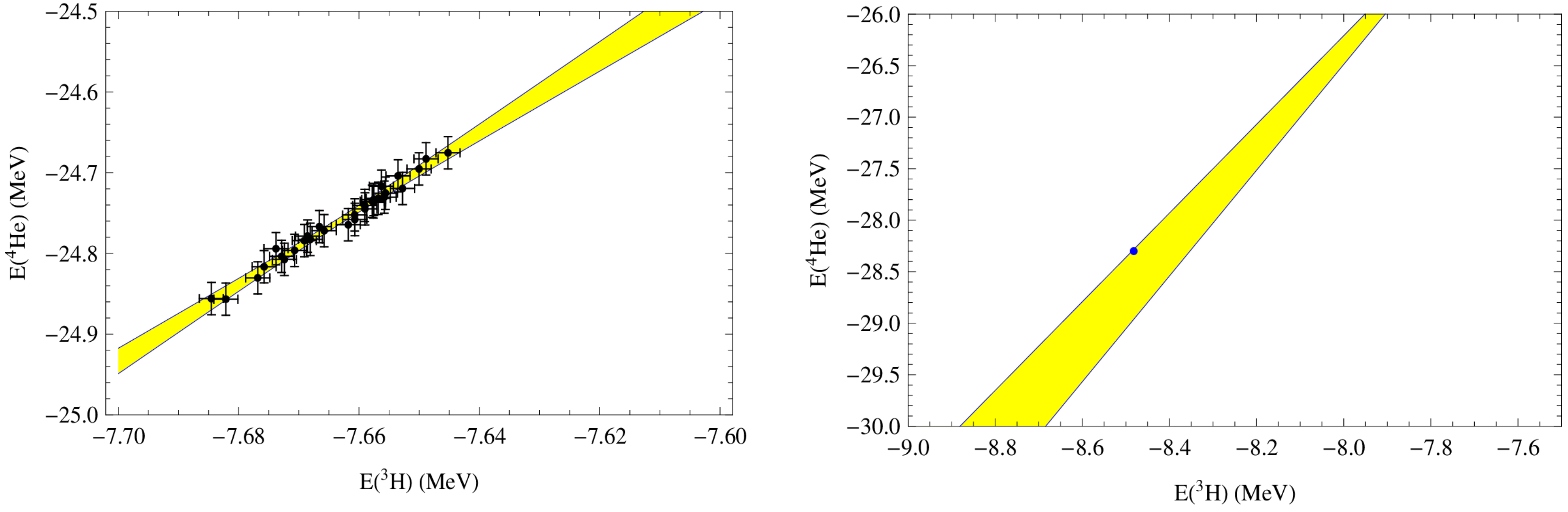
Similarity Renormalization Group: $B_\alpha = 4B_t +3B_d$

[Ruiz-Arriola, Szpiegel & Timoteo. Few Body Syst. 55 (2014) 971-975]

Tjon Line Correlation

Numerical accuracy.

- $\Delta(^3H)_t^{num}$ = 1 KeV, $\Delta(^4He)_t^{num}$ = 20 KeV



4-Body forces are masked by the numerical noise in 3 and 4 body calculations

Summary

- Nucleon Nucleon interaction
  - Over 8000 scattering data
  - Phenomenological models, least squares fit
  - Selection of data is relevant
- Statistical uncertainties
  - Normality of residuals has to be checked
  - Enough signal to determine charge dependence in f_π^2
- Systematic uncertainties
  - Dominate statistical ones
- Propagation into nuclear structure
  - Enough precision to see four body force?

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