The Roles of Nuclear Physics and the Maximum Mass in Constraining the Neutron Star Radius

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Outline

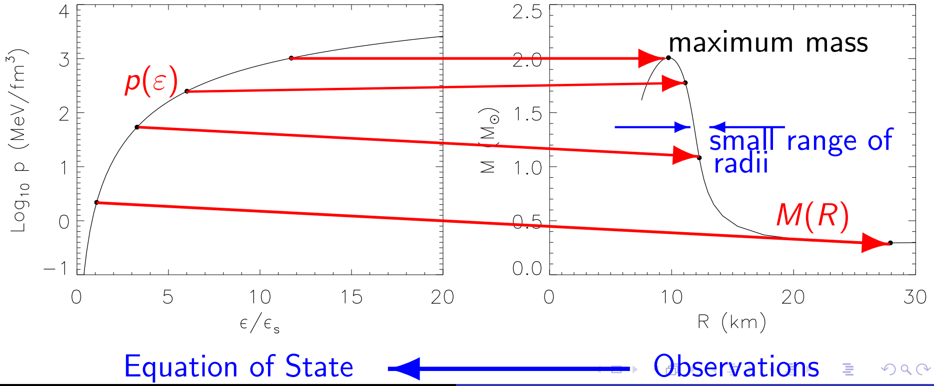
- The Dense Matter Equation of State and Neutron Star Structure
  - General Causality, Maximum Mass and GR Limits
  - Neutron Matter and the Nuclear Symmetry Energy
  - Theoretical and Experimental Constraints on the Symmetry Energy
- Extrapolating to High Densities with Piecewise Polytropes
- Constraints on Neutron Star Radii
- Universal Relations
- Observational Constraints on Radii
  - Photospheric Radius Expansion Bursts
  - Thermal Emission from Quiescent Binary Sources
  - Effects of Systematic Uncertainties
- Further Observations of Masses and Radii

Neutron Star Structure

Tolman-Oppenheimer-Volkov equations

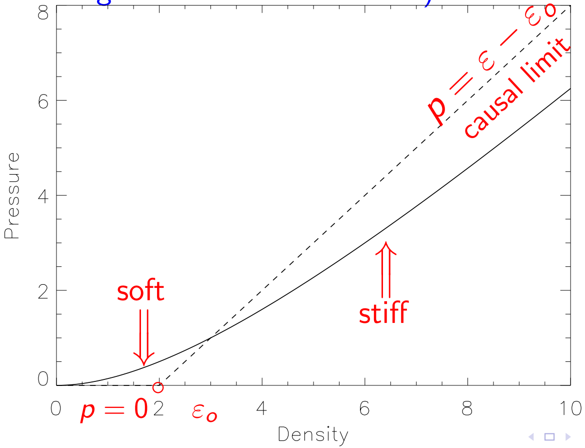
$$\frac{dp}{dr} = -\frac{G}{c^4}\frac{(mc^2 + 4\pi pr^3)(\varepsilon + p)}{r(r - 2Gm/c^2)}$$

$$\frac{dm}{dr} = 4\pi\frac{\varepsilon}{c^2}r^2$$



Extremal Properties of Neutron Stars

- The most compact and massive configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff" (Koranda, Stergioulas & Friedman 1997).



ε_o is the only EOS parameter

The TOV solutions scale with ε_o

$w = \varepsilon/\varepsilon_o$
$y = p/\varepsilon_o$
$x = r\sqrt{G\varepsilon_o}/c^2$
$z = m\sqrt{G^3\varepsilon_o}/c^2$

Causality + GR Limits and the Maximum Mass

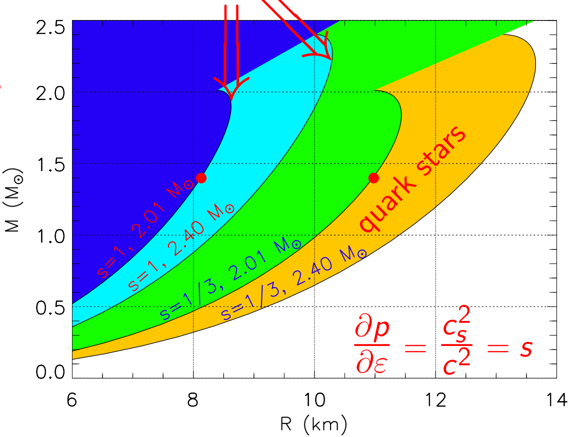
A lower limit to the maximum mass sets a lower limit to the radius for a given mass.

Similarly, a precision upper limit to R sets an upper limit to the maximum mass.

$R_{1.4} > 8.15 M_\odot$ if $M_{max} \geq 2.01 M_\odot$.

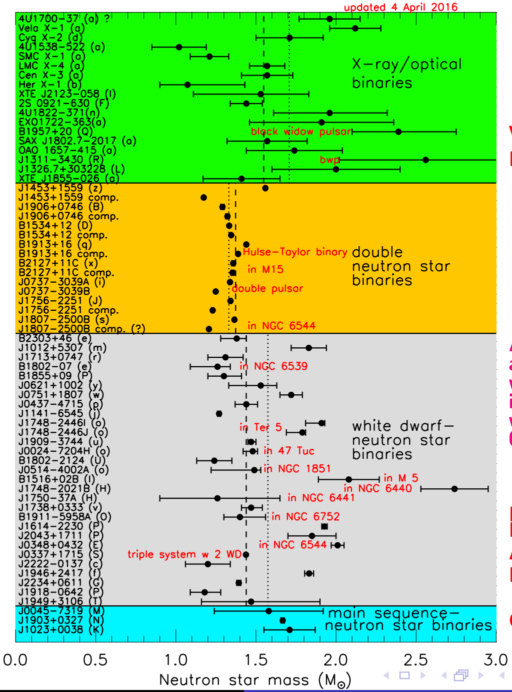
$M_{max} < 2.4 M_\odot$ if $R < 10.3$ km.

$M - R$ curves for minimally compact EOS



$$\frac{\partial p}{\partial \varepsilon} = \frac{c_s^2}{c^2} = s$$

If quark matter exists in the interior, the minimum radii are substantially larger.



vanKerkwijk 2010

Romani et al. 2012

Although simple average mass of w.d. companions is 0.23 $M_\odot$ larger, weighted average is 0.04 $M_\odot$ smaller

Demorest et al. 2010

Fonseca et al. 2016

Antoniadis et al. 2013

Barr et al. 2016

Champion et al. 2008

Mass-Radius Diagram and Theoretical Constraints

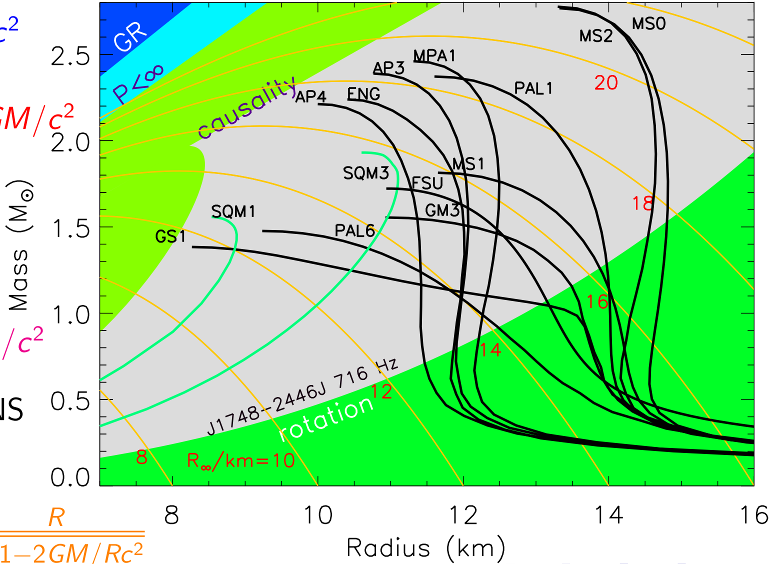
GR:
$R > 2GM/c^2$

$P < \infty$:
$R > (9/4)GM/c^2$

causality:
$R \gtrsim 2.9GM/c^2$

— normal NS
— SQS

— $R_\infty = \frac{R}{\sqrt{1-2GM/Rc^2}}$



Neutron Star Radii and Nuclear Symmetry Energy

- Radii are highly correlated with the neutron star matter pressure around $n_s - 2n_s \simeq (0.16 - 0.32)$ fm^{-3}. (Lattimer & Prakash 2001)
- Neutron star matter is nearly purely neutrons, $x \sim 0.04$.
- Nuclear symmetry energy

$$S(n) \equiv E(n, x = 0) - E(n, 1/2)$$

$$E(n,x) \simeq E(n,1/2) + S_2(n)(1-2x)^2 + S_4(n)(1-2x)^4 \ldots$$

$$S(n) \simeq S_2(n) \simeq S_v + \frac{L}{3n_s}(n - n_s) + \frac{K_{sym}}{18}\left(\frac{n - n_s}{n_s}\right)^2 \ldots$$

- $S_v \sim 32$ MeV; $L \sim 50$ MeV from nuclear systematics.
- Neutron matter energy and pressure at n_s:

$$E(n_s, 0) \simeq S_v + E(n_s, 1/2) = S_v - B \sim 16 \text{ MeV}$$

$$p(n_s, 0) = \left(n^2 \frac{\partial E(n,0)}{\partial n}\right)_{n_s} \simeq \frac{L n_s}{3} \sim 2.5 \text{ MeV fm}^{-3}$$

Nuclear Experimental Constraints

The liquid droplet model is a useful frame of reference. Its symmetry parameters S_v and S_s are related to S_v and L:

$$\frac{S_s}{S_v} \simeq \frac{aL}{r_o S_v}\left[1 + \frac{L}{6S_v} - \frac{K_{sym}}{12L} + \dots\right].$$

- Symmetry contribution to the binding energy:

$$E_{sym} \simeq S_v A I^2 \left[1 + \frac{S_s}{S_v A^{1/3}}\right]^{-1}.$$

- Giant Dipole Resonance (dipole polarizability)

$$\alpha_D \simeq \frac{AR^2}{20 S_v}\left(1 + \frac{5}{3}\frac{S_s}{S_v A^{1/3}}\right).$$

- Neutron Skin Thickness

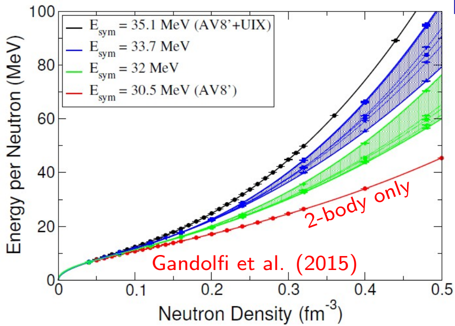
$$r_{np} \simeq \sqrt{\frac{3}{5}}\frac{2 r_o I}{3}\frac{S_s}{S_v}\left(1 + \frac{S_s}{S_v A^{1/3}}\right)^{-1}\left(1 + \frac{10}{3}\frac{S_s}{S_v A^{1/3}}\right).$$

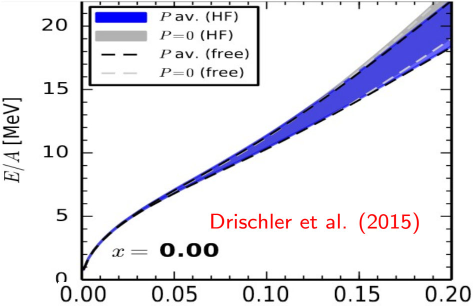
Theoretical Neutron Matter Calculations

Nuclei provide information for matter up to n_s.

Theoretical studies, beginning from fitting low-energy neutron scattering data and few-body calculations of light nuclei, can probe higher densities.

- Auxiliary Field Diffusion Quantum Monte Carlo (Gandolfi & Carlson)
- Chiral Lagrangian Expansion (Drischler, Hebeler & Schwenk)





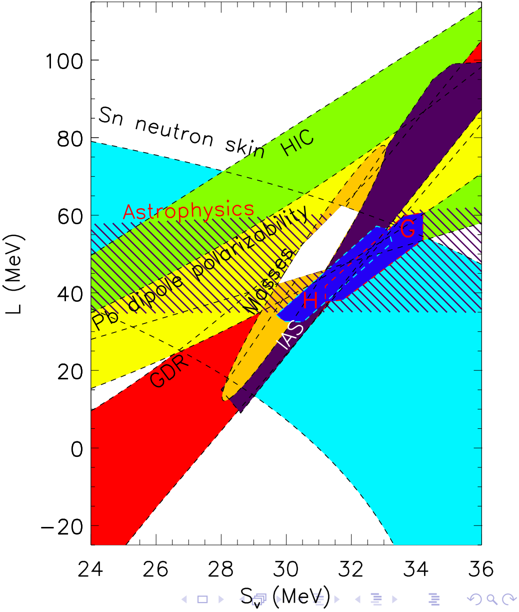
Theoretical and Experimental Constraints

H Chiral Lagrangian

G: Quantum Monte Carlo

$S_v - L$ constraints from Hebeler et al. (2012)

Neutron matter constraints are compatible with experimental constraints.

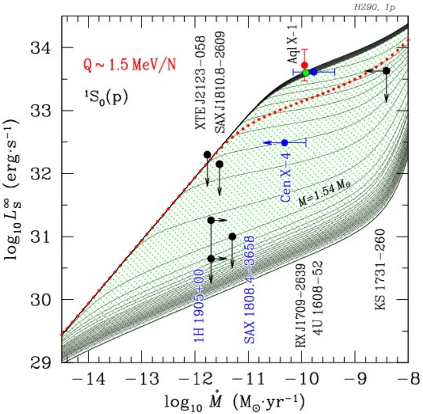


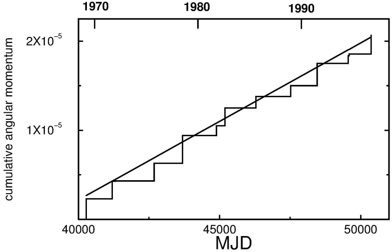
Neutron Star Crusts

The evidence is overwhelming that neutron stars have crusts.

- Neutron star cooling, both long term (ages up to millions of years) and transient (days to years), supports the existence of $\sim 0.5 - 1$ km thick crusts with masses $\sim 0.02 - 0.05 M_\odot$.
- Pulsar glitches are best explained by n $^1S_0$ superfluidity, largely confined to the crust, $\Delta I/I \sim 0.01 - 0.05$.

The crust EOS, dominated by relativistic degenerate electrons, is very well understood.





Piecewise Polytropes

Crust EOS is known: $n < n_0 = 0.4n_s$.

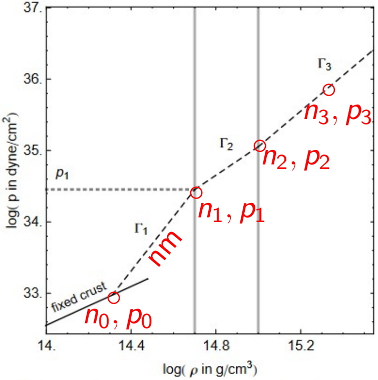
Read, Lackey, Owen & Friedman (2009) found high-density EOS can be modeled as piecewise polytropes with 3 segments.

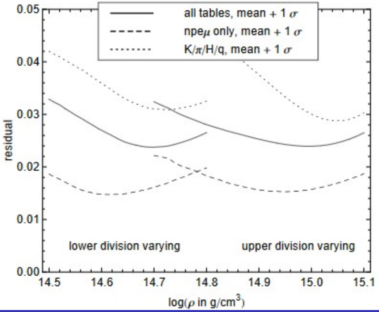
They found universal break points ($n_1 \simeq 1.85n_s, n_2 \simeq 3.7n_s$) optimized fits to the entire family of modeled EOSs.

For $n_0 < n < n_1$, assume neutron matter EOS. Arbitrarily choose $n_3 = 7.4n_s$.

For a given p_1 (or Γ_1):
$0 < \Gamma_2 < \Gamma_{2c}$ or $p_1 < p_2 < p_{2c}$.
$0 < \Gamma_3 < \Gamma_{3c}$ or $p_2 < p_3 < p_{3c}$.

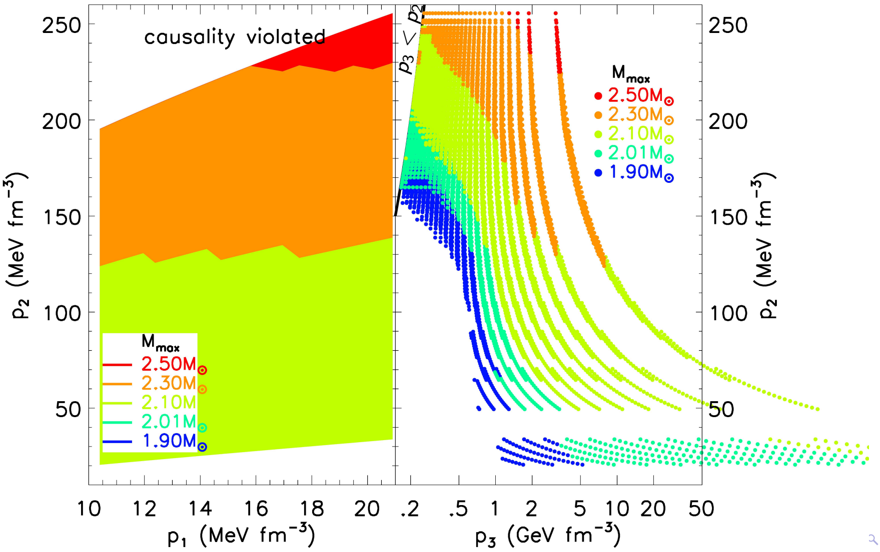
Minimum values of p_2, p_3 set by M_{max}; maximum values set by causality.



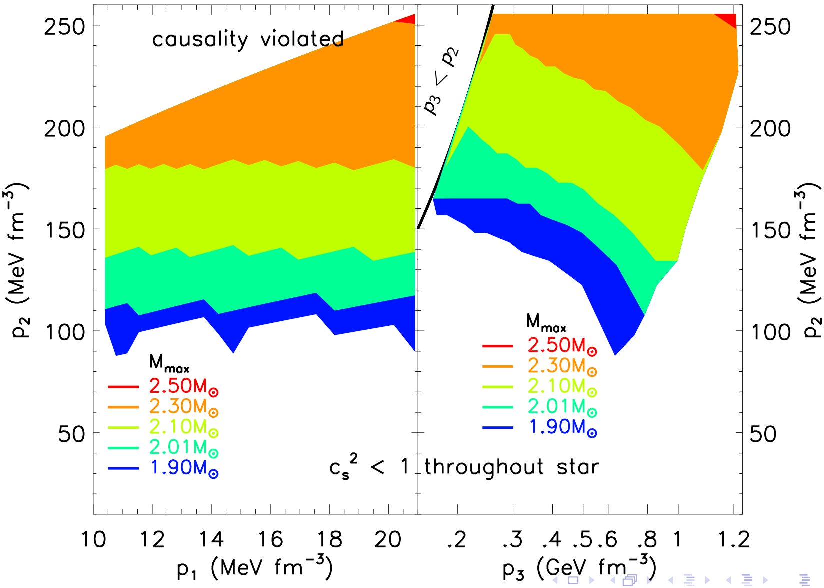


Maximum Mass and Causality Constraints

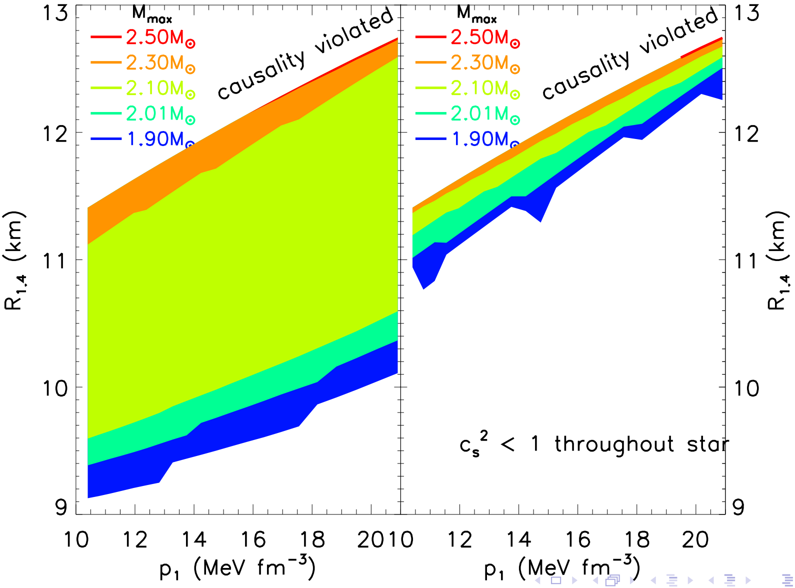
Where EOS gives $c_s > c$, force $c_s = c$.



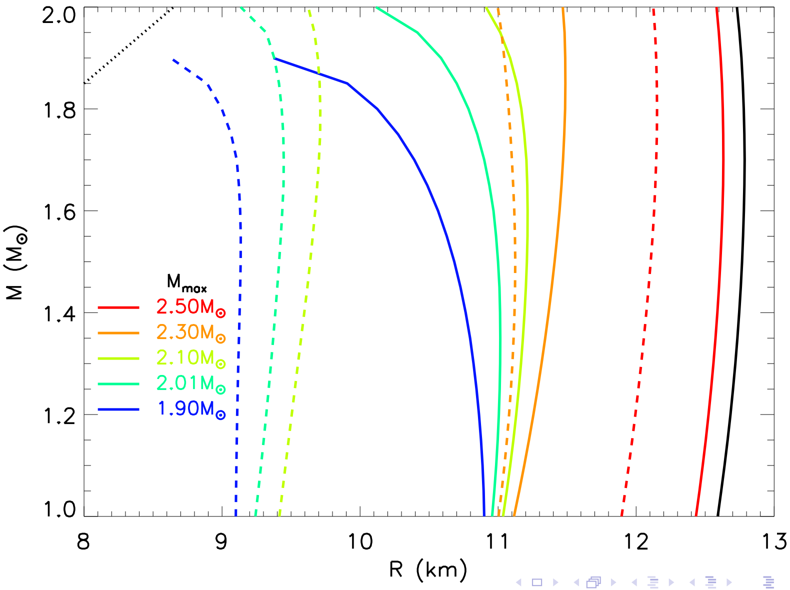
Maximum Mass and Causality Constraints



Radius - p_1 Correlation



Mass-Radius Constraints from Causality



Context

$$P(\mathcal{M}|\mathcal{D}) \propto P(\mathcal{D}|\mathcal{M})P(\mathcal{M})$$

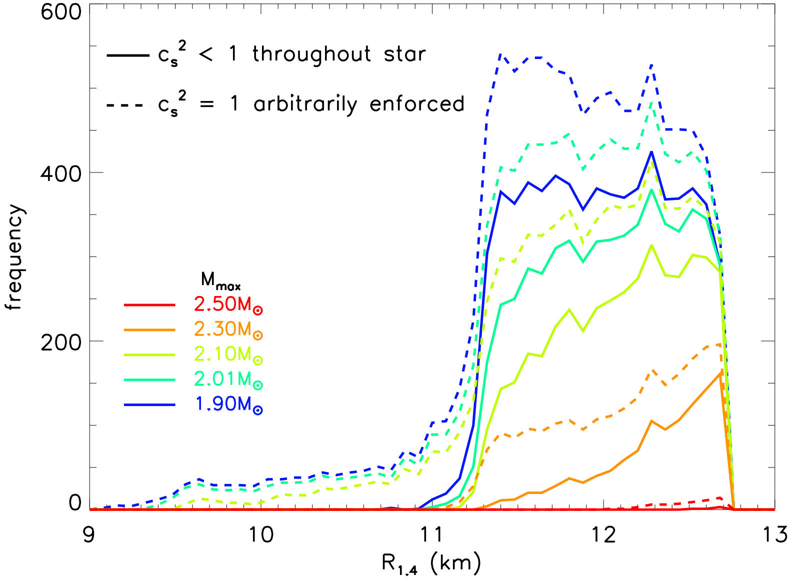
$$\mathcal{M} \to (p_1, p_2, p_3), \qquad \mathcal{D} \to M(R)$$

$$P(p_i) = \int dp_j dp_k P(\mathcal{D}|\mathcal{M})P(\mathcal{M})$$

$$P(\hat{R}|\hat{M}) = \int dp_1 dp_2 dp_3 P(\mathcal{D}|\mathcal{M})P(\mathcal{M})\delta[R(\hat{M}, p_1, p_2, p_3) - \hat{R}]$$

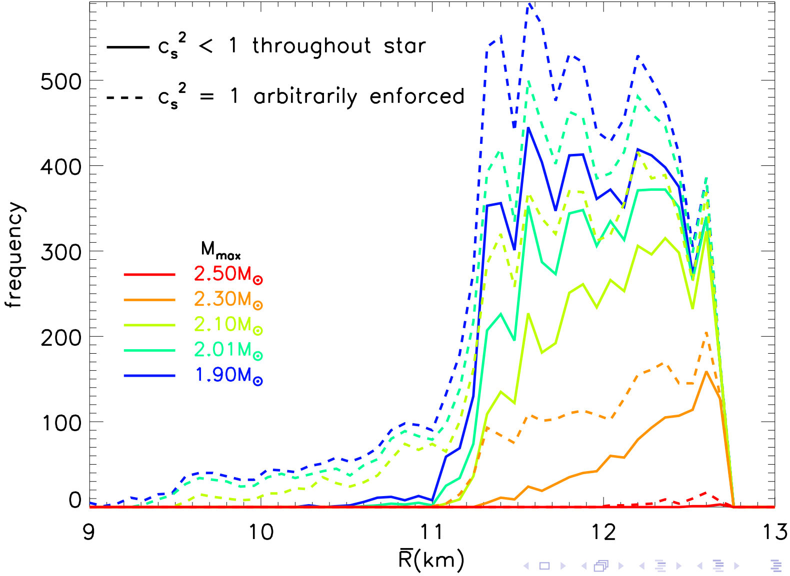
$$P(\hat{R}) = \int dM dp_1 dp_2 dp_3 P(M) P(\mathcal{D}|\mathcal{M})P(\mathcal{M})\delta[R(M, p_1, p_2, p_3) - \hat{R}]$$

Piecewise-Polytrope $R_{M=1.4}$ Distributions

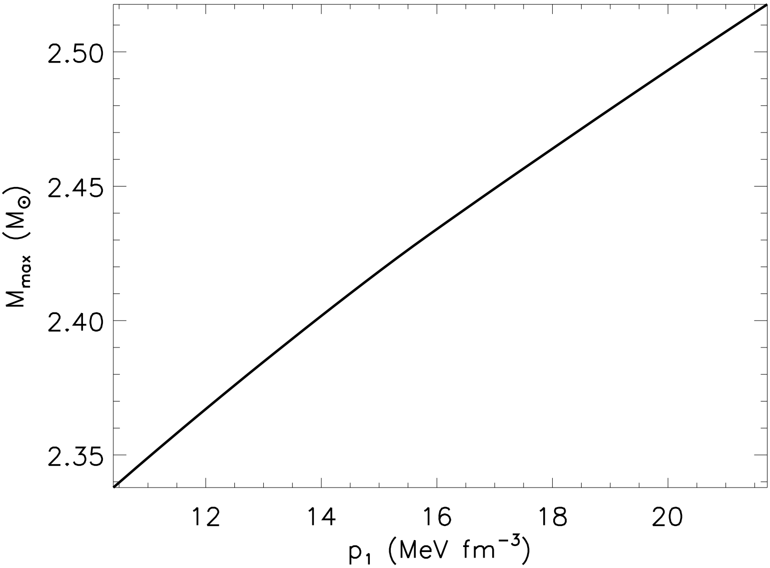


Piecewise-Polytrope Average Radius Distributions

Assumes $P(M)$ from observed pulsar-timing masses



Upper Limits to Maximum Mass



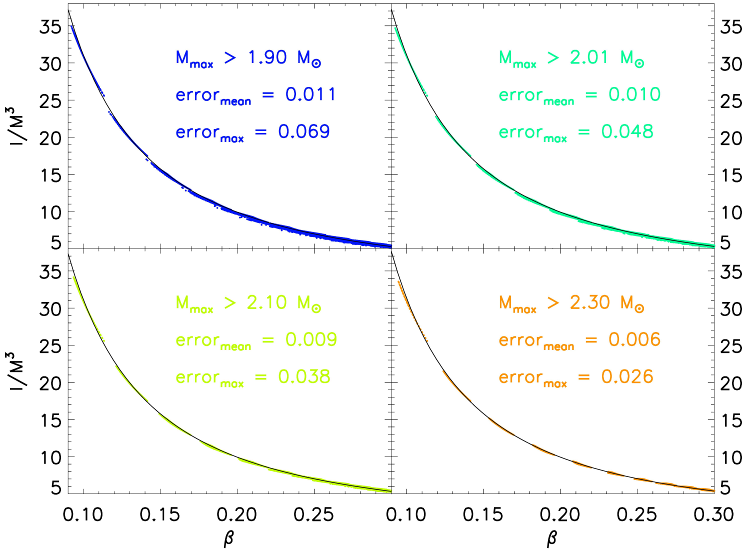
Universal Relations

With the assumptions

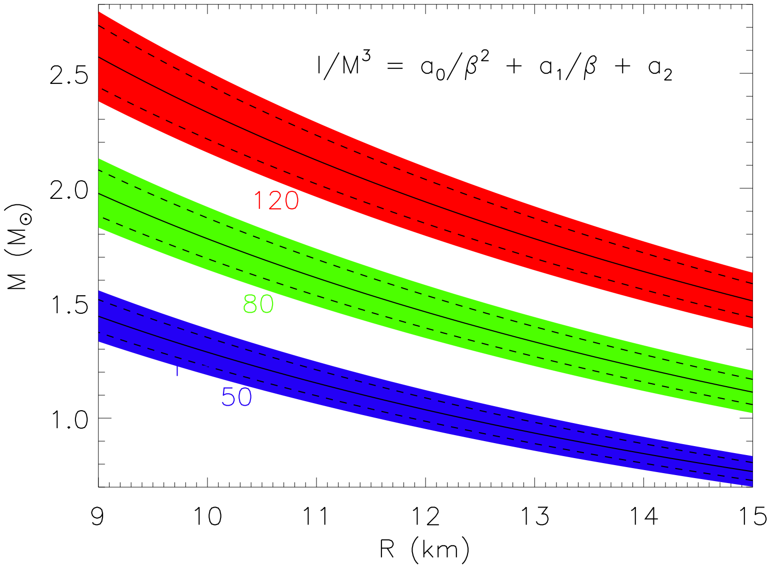
- Known crust EOS
- Bounded neutron matter EOS ($p_{min} < p_1 < p_{max}$)
- Two piecewise polytropes for $p > p_1$
- Causality is not violated
- M_{max} is limited from below

tight correlations among the compactness, moment of inertia, binding energy and tidal deformability result.

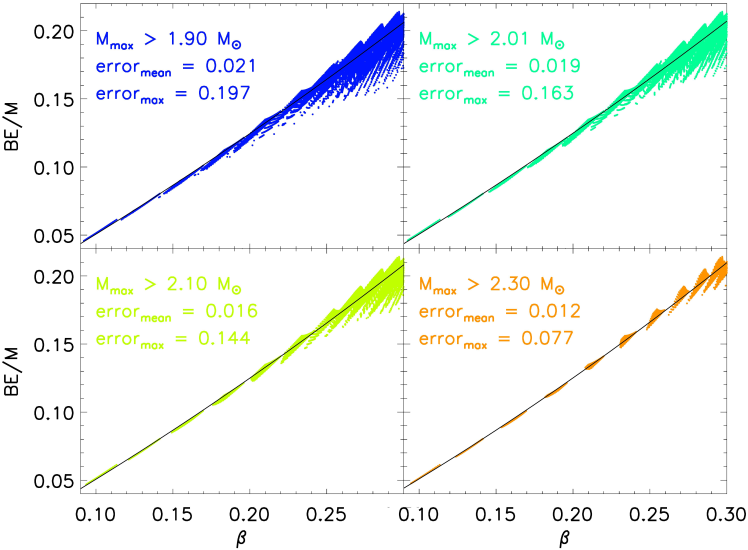
Moment of Inertia - Compactness Correlations



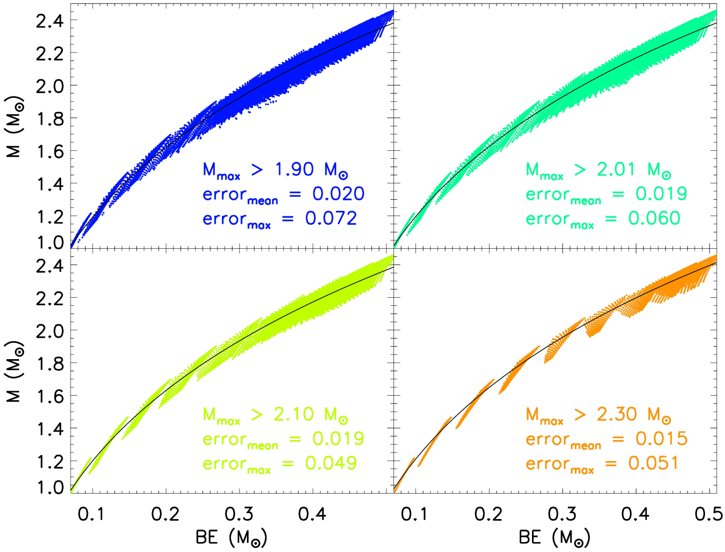
Moment of Inertia - Radius Constraints



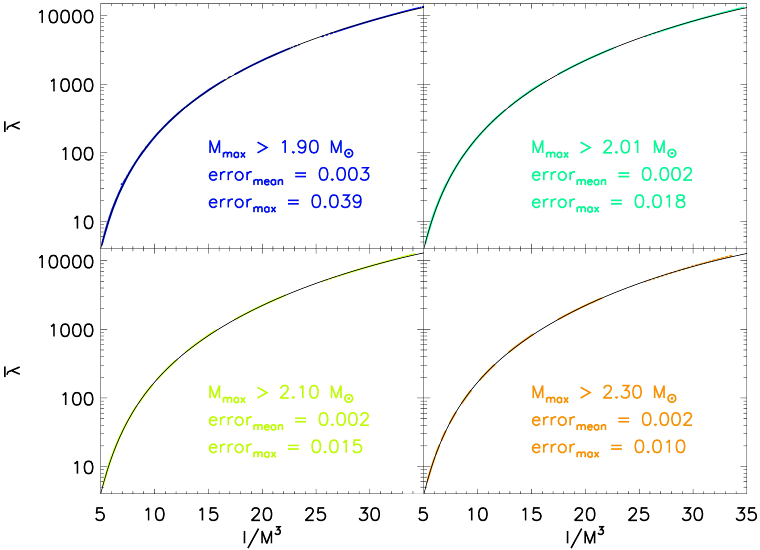
Binding Energy - Compactness Correlations



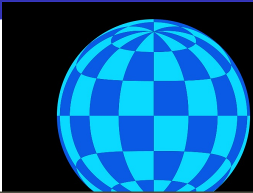
Binding Energy - Mass Correlations



Tidal Deformatibility - Moment of Inertia



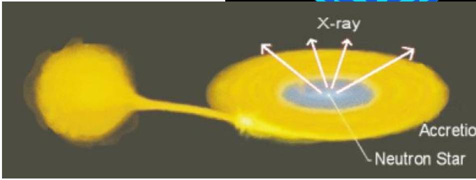
Simultaneous Mass/Radius Measurements



- Measurements of flux $F_\infty = (R_\infty/D)^2 \sigma T_{\rm eff}^4$ and color temperature $T_c \propto \lambda_{\rm max}^{-1}$ yield an apparent angular size (pseudo-BB):

  $R_\infty/D = (R/D)/\sqrt{1 - 2GM/Rc^2}$

- Observational uncertainties include distance D, interstellar absorption N_H, atmospheric composition

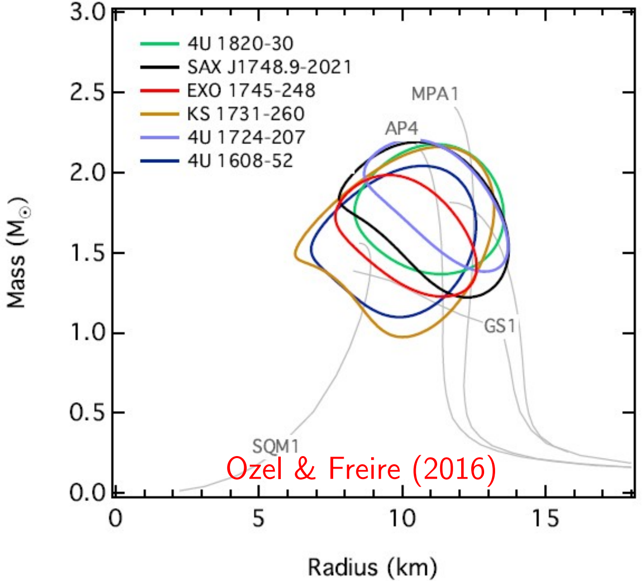


Best chances are:

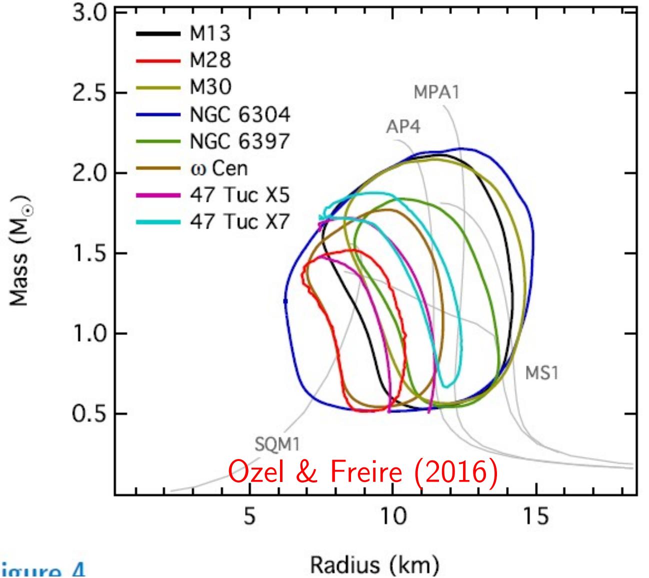
  - Isolated neutron stars with parallax (atmosphere ??)
  - Quiescent low-mass X-ray binaries (QLMXBs) in globular clusters (reliable distances, low B H-atmosperes)
  - Bursting sources (XRBs) with peak fluxes close to Eddington limit (gravity balances radiation pressure)

$$F_{\rm Edd} = \frac{cGM}{\kappa D^2}\sqrt{1 - 2GM/Rc^2}$$

PRE $M - R$ Estimates



QLMXB $M - R$ Estimates



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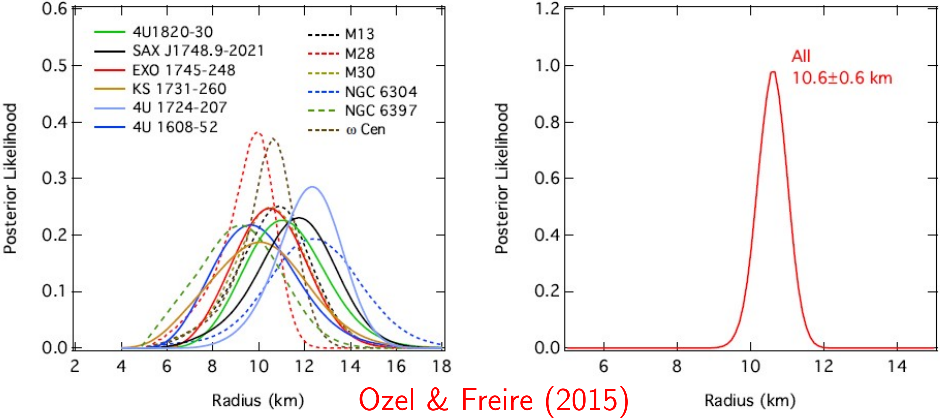
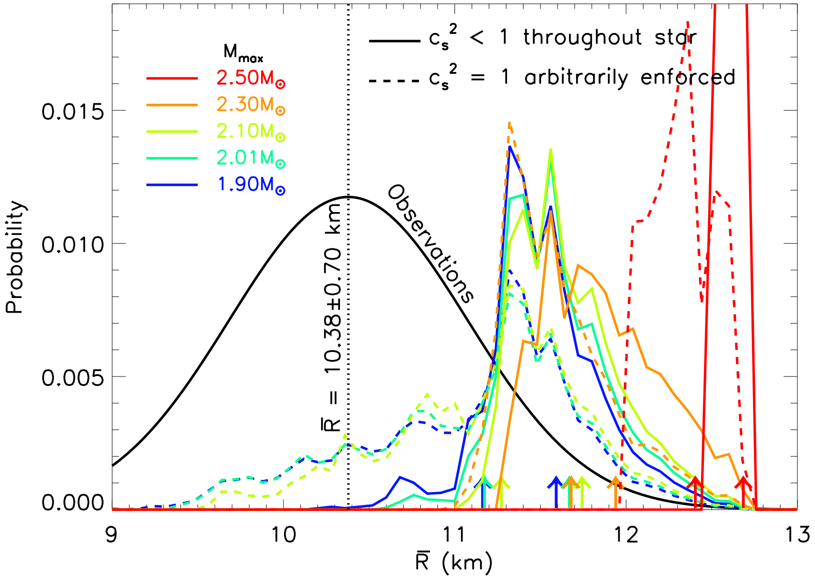
Combined R fits

Assumed $P(M)$ is that measured from pulsar timing ($\bar{M} = 1.4 M_\odot$).

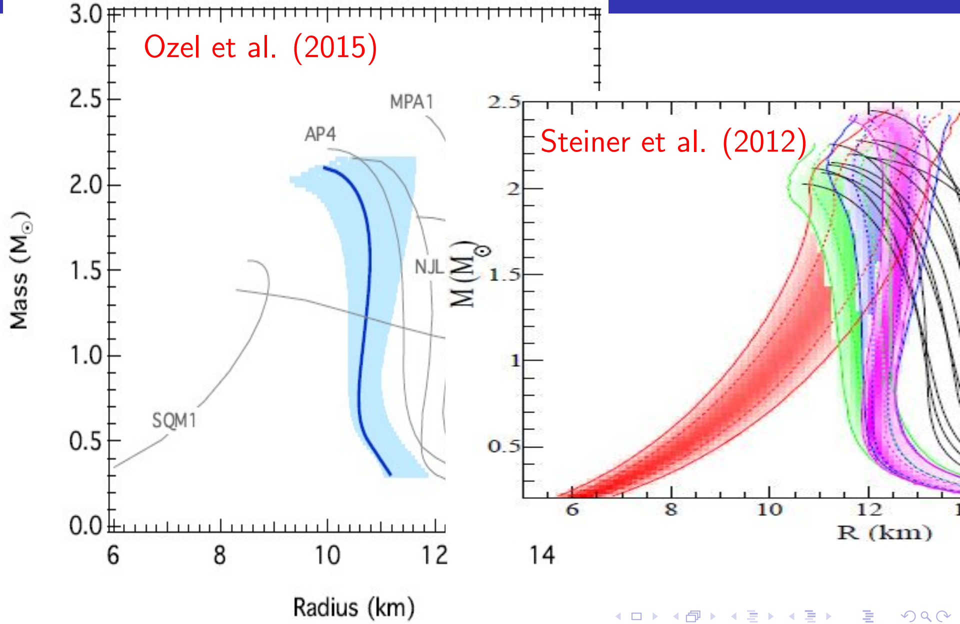


Ozel & Freire (2015)

Folding Observations with Piecewise Polytropes



Bayesian Analyses



Role of Systematic Uncertainties

Apparent tension between nuclear physics expectations and astronomical observations for the value of the mean radius.

Possible explanations:

- Non-uniform temperature distributions

$$R^2 T^4 = R_1^2 T_1^4 + R_2 T_2^4$$

$$R^2 \left(3 - \frac{h\nu_0}{kT}\right) \simeq 0$$

$$R_1^2 \left(3 - \frac{h\nu_0}{kT_1}\right) e^{-h\nu_0/kT_1} + R_2^2 \left(3 - \frac{h\nu_0}{kT_2}\right) e^{-h\nu_0/kT_2} \simeq 0$$

$$R_1 = R_2 = Ry/\sqrt{2}, \qquad x = T_2/T_1, \qquad z = T/T_1$$

$$z^4 = y^2(1+x^4)/2, \qquad z = 1 + (1 - z/x)e^{-3z/x}e^{3z}$$

For $0 < x < 1$, $z < 1$ and $y > 1$, so that

$$R_1^2 + R_2^2 = y^2 R^2 > R^2$$

- Interstellar absorption
- Atmospheric composition: In quiescent sources, He or C atmospheres can produce about 50% larger radii.
- Non-spherical geometries: In bursting sources, improper to use spherically-symmetric Eddington flux formula.
- Disc shadowing: In burst sources, leads to underprediction of $A = f_c^{-4}(R_\infty/D)^2$, overprediction of $\alpha \propto 1/\sqrt{A}$, and underprediction of $R_\infty \propto \sqrt{\alpha}$.

Conclusions

- Neutron matter calculations and nuclear experiments are consistent with each other and set reasonably tight constraints on symmetry energy behavior near the nuclear saturation density.
- These constraints, together with assumptions that neutron stars have hadronic crusts and are causal, predict neutron star radii $R_{1.4}$ in the range 12.0 ± 1.0 km.
- Astronomical observations of photospheric radius expansion X-ray bursts and quiescent sources in globular clusters suggest $R_{1.4} \sim 10.5 \pm 1$ km, unless maximum mass and EOS priors are implemented.
- Should observations require smaller or larger neutron star radii, a strong phase transition in extremely neutron-rich matter just above the nuclear saturation density is suggested. Or should GR be modified?