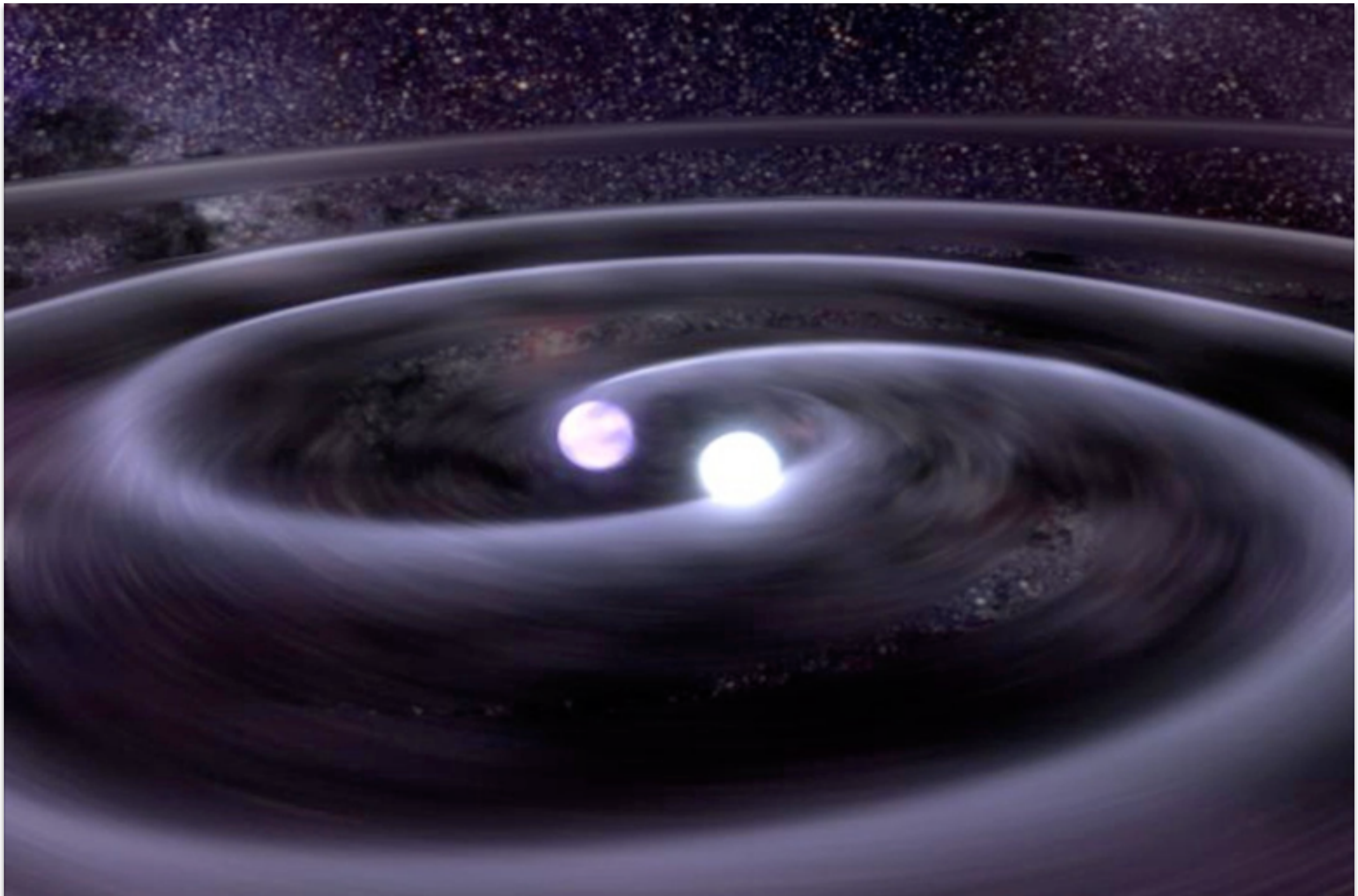
Measuring the neutron-star equation of state with gravitational-wave observations

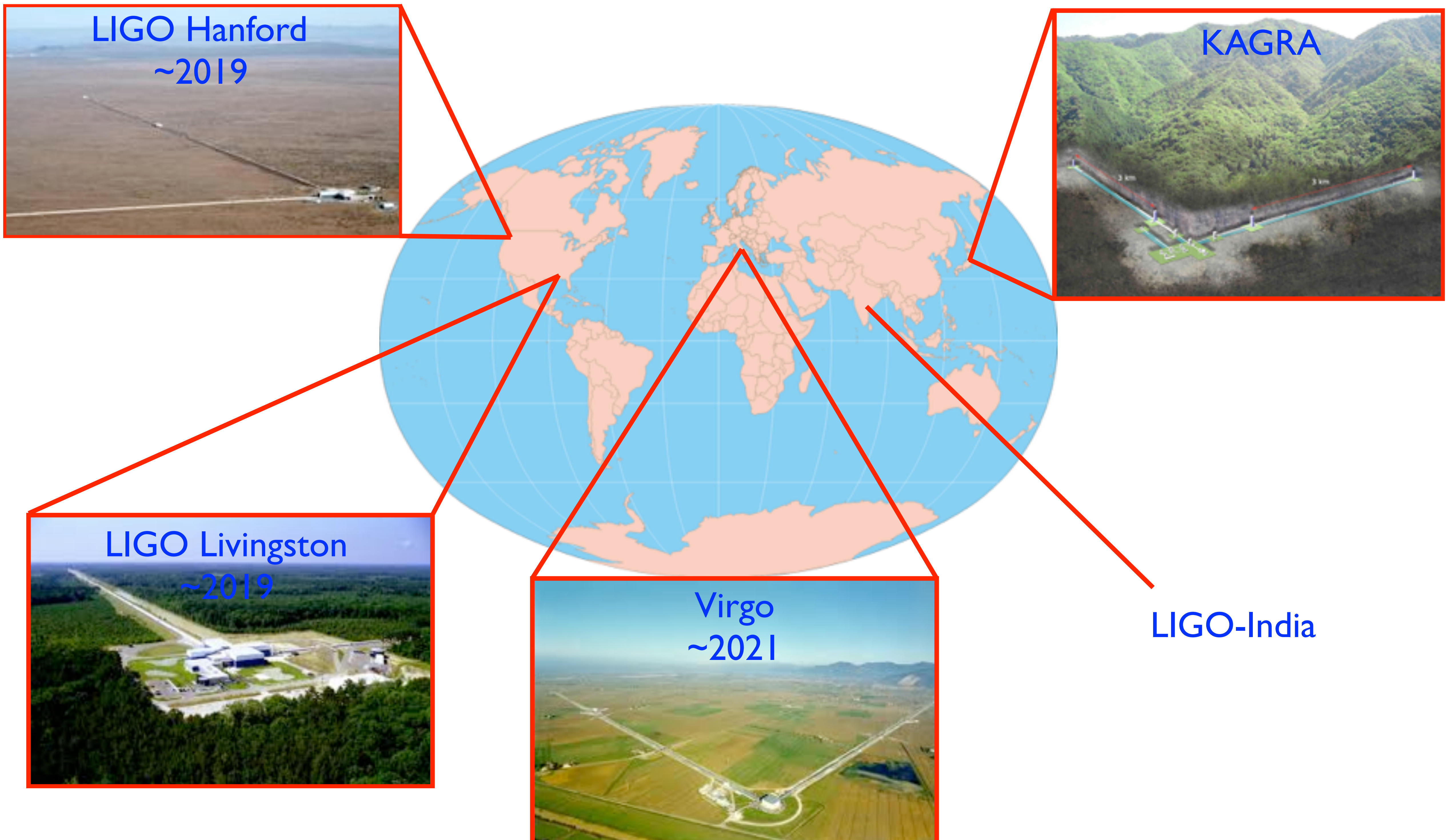


Ben Lackey
Syracuse University

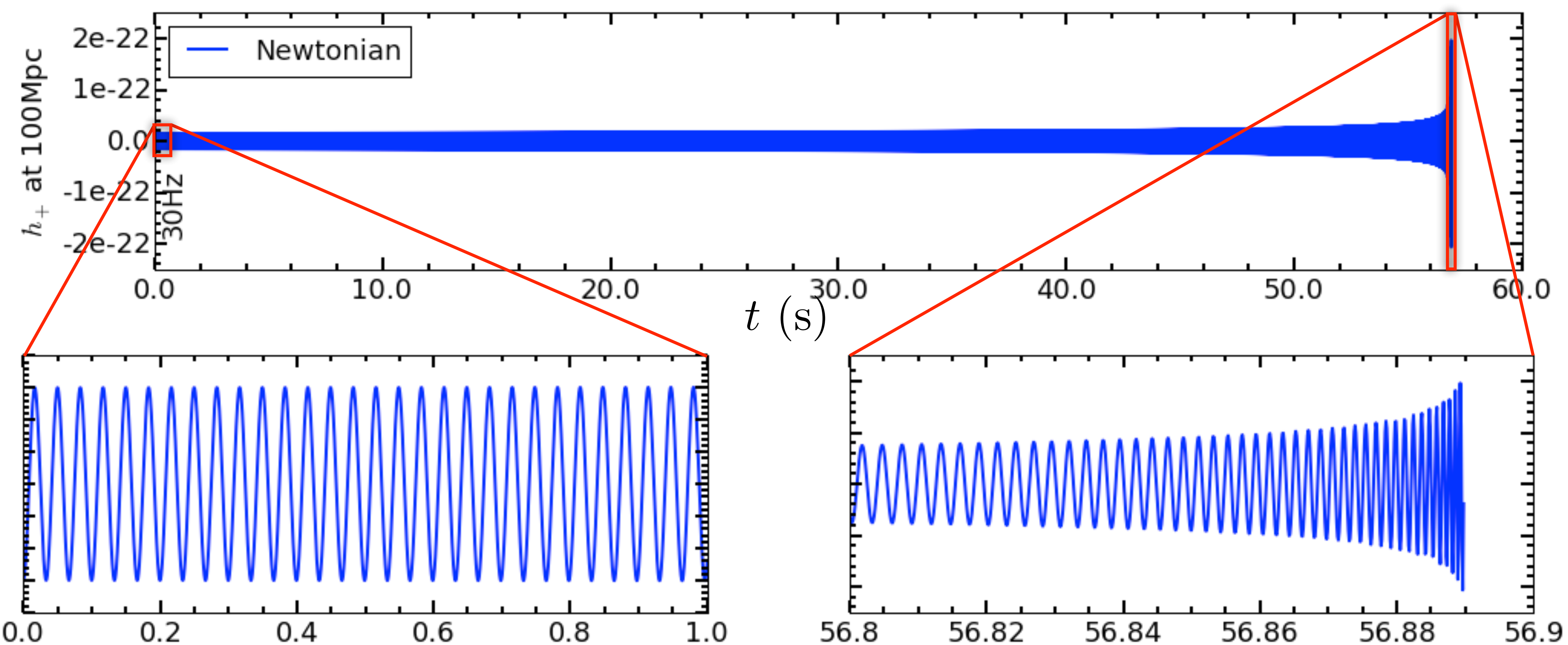
INT, Seattle, Washington, 30 June 2016

Second generation gravitational-wave detectors

- Will reach design sensitivity in the next few years
- Sensitive to gravitational-waves between ~10Hz and a few kHz



Post-Newtonian waveform without matter effects

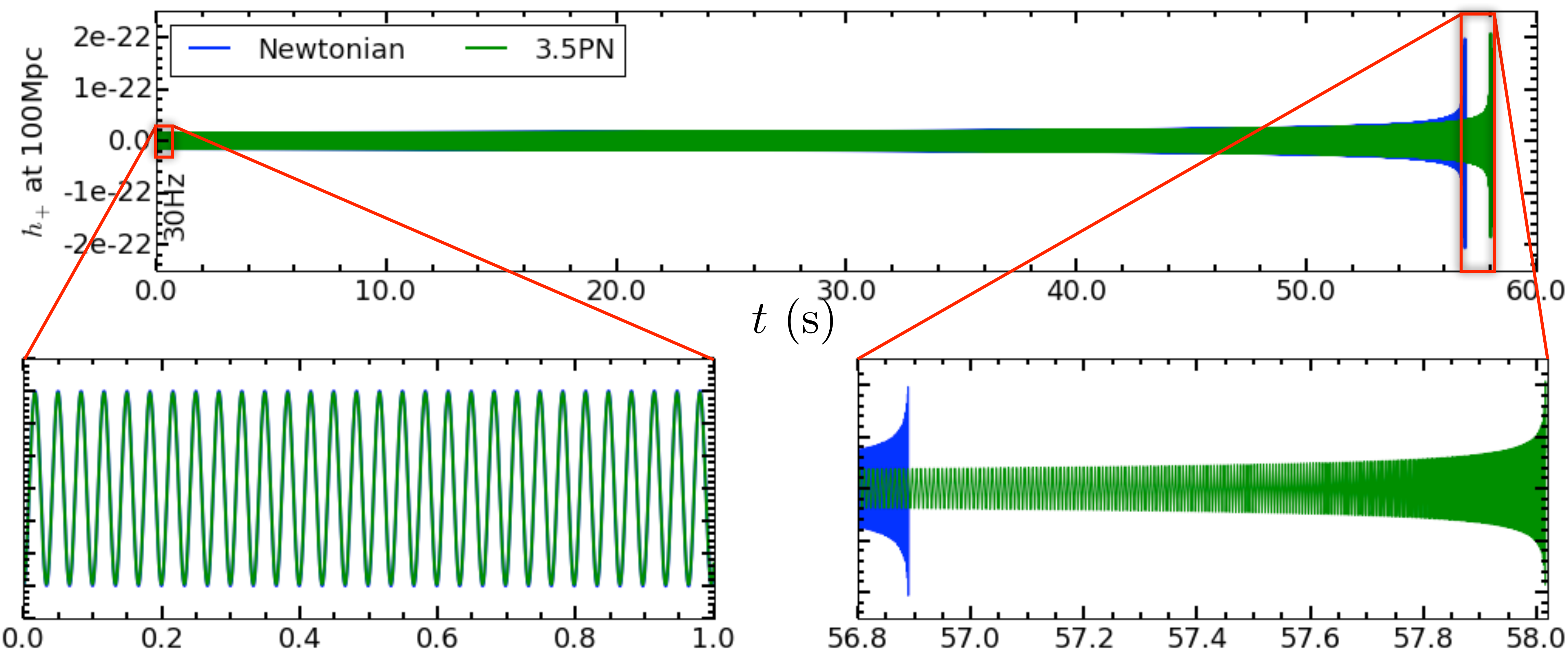


$$\mathrm{Phase}(t) = 0\mathrm{PN}(t; \mathcal{M})$$

Chirp mass:

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

Post-Newtonian waveform without matter effects



$$\text{Phase}(t) = 0\text{PN}(t; \mathcal{M}) \left[1 + \underbrace{1\text{PN}(t; \eta)}_{(v/c)^2} + \cdots + \underbrace{3.5\text{PN}(t; \eta)}_{(v/c)^7}\right]$$

Chirp mass:

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

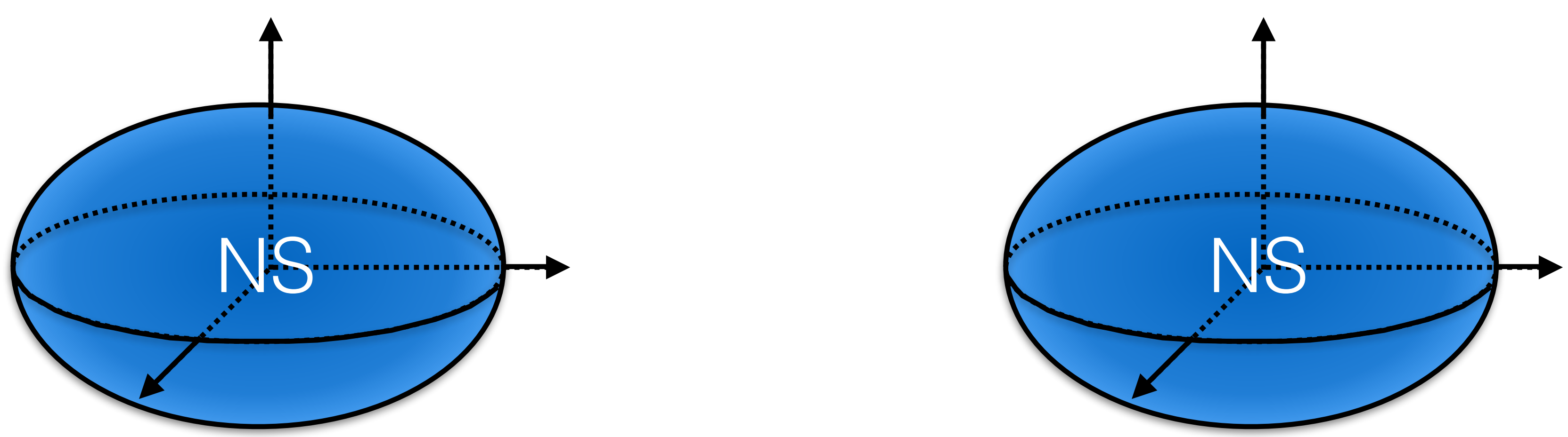
Symmetric mass ratio:

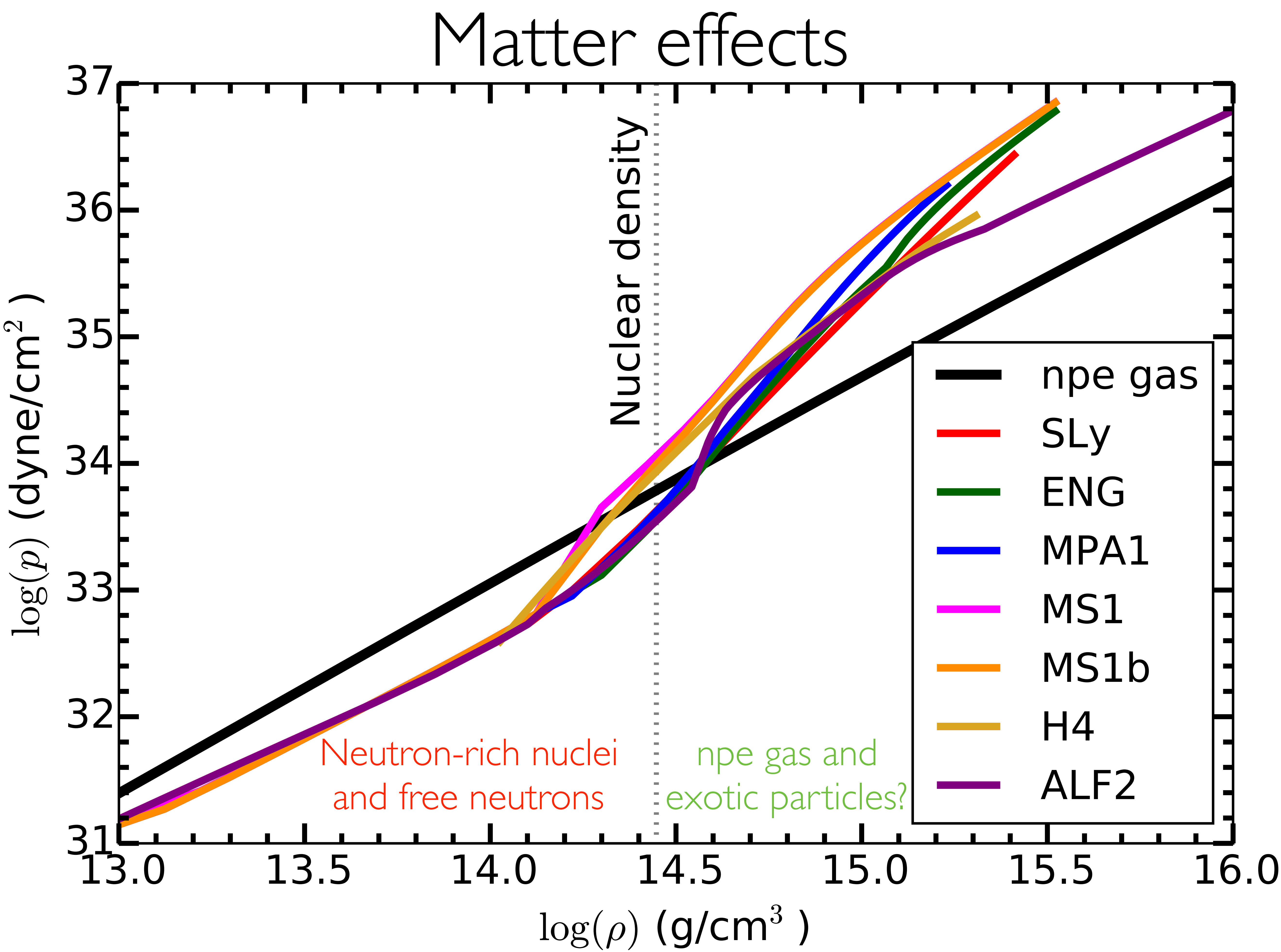
$$\eta = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

Matter effects

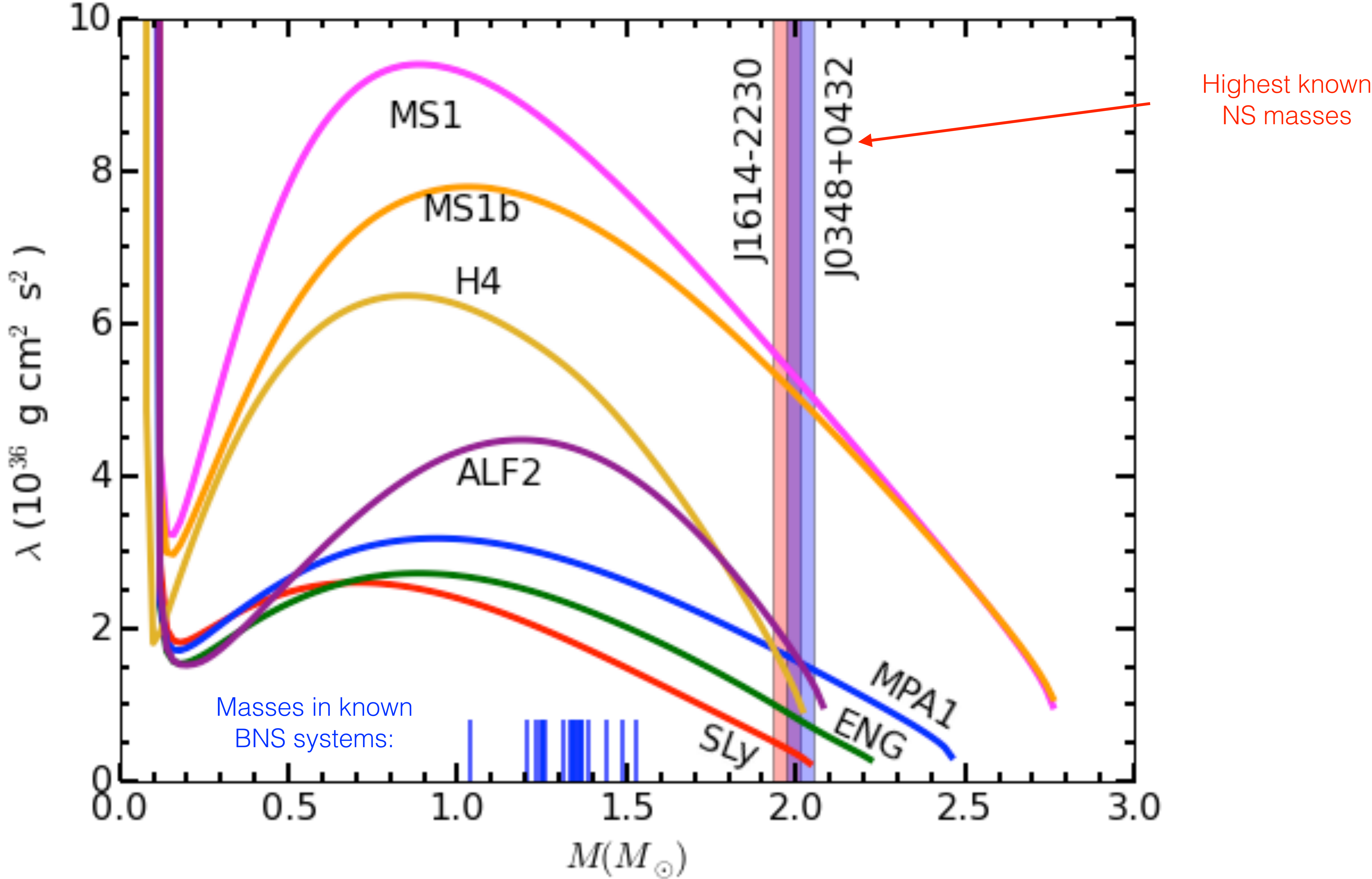
- Tidal field created by companion induces a quadrupole moment in the NS
    - Tidal field: $\mathcal{E}_{ij}$
    - Quadrupole moment: Q_{ij}
- Amount of deformation depends on stiffness of EOS via the tidal deformability λ:

$$Q_{ij} = -\lambda(\mathrm{EOS}, M)\mathcal{E}_{ij}$$




Matter effects
Nuclear density
npe gas
SLy
ENG
MPA1
MS1
MS1b
H4
ALF2
Neutron-rich nuclei
and free neutrons
npe gas and
exotic particles?
$\log(p)$ (dyne/cm^2)
$\log(\rho)$ (g/cm^3)
31
32
33
34
35
36
37
13.0
13.5
14.0
14.5
15.0
15.5
16.0

Matter effects



Matter effects

- Tidal effects first appear at same order as 5PN point-particle terms
- Leading term $\tilde{\Lambda}$ is a linear combination of the tidal deformabilities of each object

$$\text{Phase}(t) = 0\text{PN}(t;\mathcal{M})\left[1 + \underbrace{1\text{PN}(t;\eta)}_{(v/c)^2} + \cdots + \underbrace{3.5\text{PN}(t;\eta)}_{(v/c)^7} + \underbrace{5\text{PN}(t;\tilde{\Lambda})}_{(v/c)^{10}}\right]$$

$$\tilde{\Lambda} = \frac{8}{13}\left[(1+7\eta-31\eta^2)(\Lambda_1+\Lambda_2) + \sqrt{1-4\eta}(1+9\eta-11\eta^2)(\Lambda_1-\Lambda_2)\right]$$

Matter effects

- Tidal effects first appear at same order as 5PN point-particle terms
- Leading term $\tilde{\Lambda}$ is a linear combination of the tidal deformabilities of each object
- Effect of remainder term $\delta\tilde{\Lambda}$ is ~10-100 times smaller

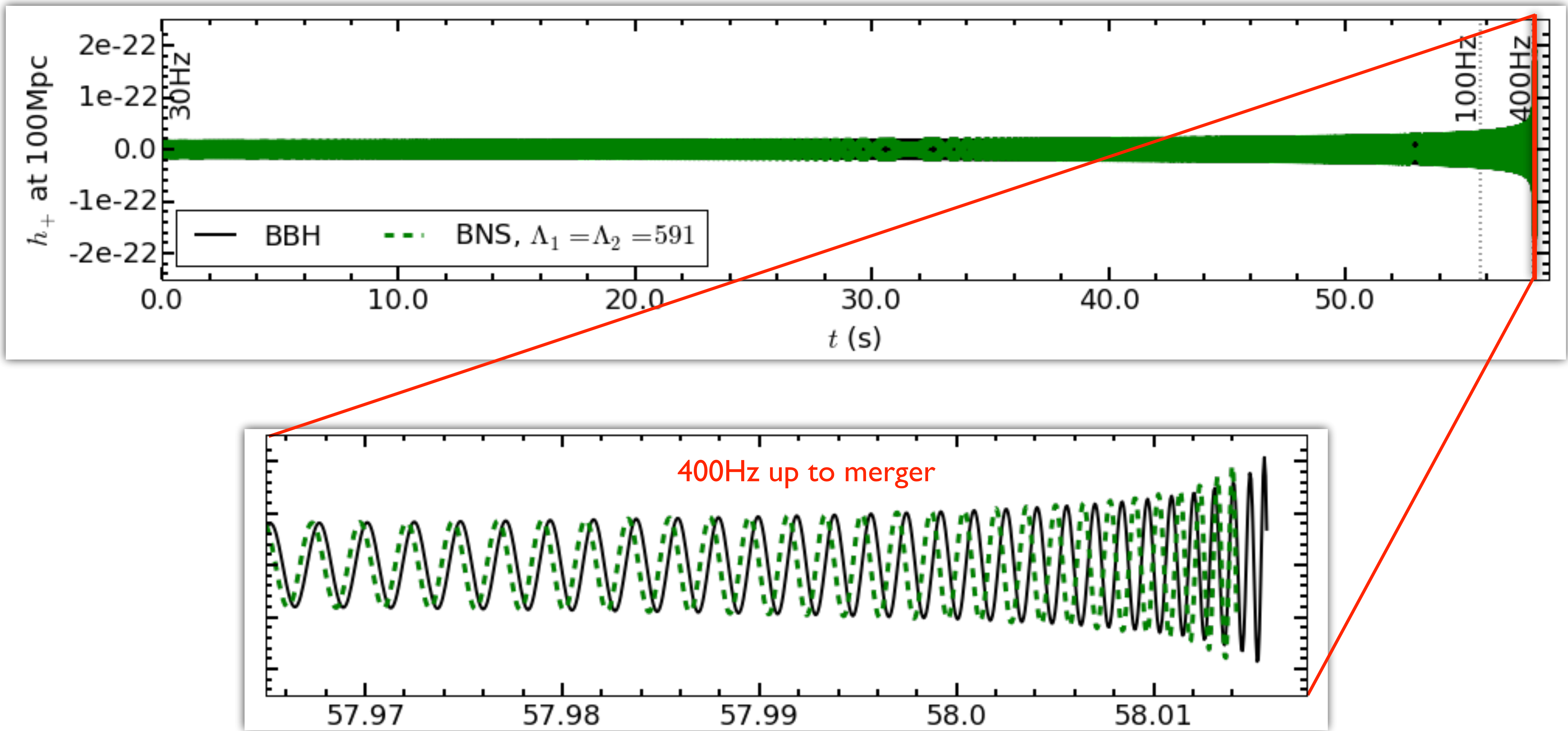
$$\mathrm{Phase}(t) = \mathrm{0PN}(t;\mathcal{M})\left[1 + \underbrace{\mathrm{1PN}(t;\eta)}_{(v/c)^2} + \cdots + \underbrace{\mathrm{3.5PN}(t;\eta)}_{(v/c)^7} + \underbrace{\mathrm{5PN}(t;\tilde{\Lambda})}_{(v/c)^{10}} + \underbrace{\mathrm{6PN}(t;\tilde{\Lambda},\delta\tilde{\Lambda})}_{(v/c)^{12}}\right]$$

$$\tilde{\Lambda} = \frac{8}{13}\left[(1+7\eta-31\eta^2)(\Lambda_1+\Lambda_2) + \sqrt{1-4\eta}(1+9\eta-11\eta^2)(\Lambda_1-\Lambda_2)\right]$$

$$\delta\tilde{\Lambda} = \frac{1}{2}\left[\sqrt{1-4\eta}\left(1-\frac{13272}{1319}\eta+\frac{8944}{1319}\eta^2\right)(\Lambda_1+\Lambda_2) + \left(1-\frac{15910}{1319}\eta+\frac{32850}{1319}\eta^2+\frac{3380}{1319}\eta^3\right)(\Lambda_1-\Lambda_2)\right]$$

Matter effects

- Both NSs contribute to tidal effect
- Leads to phase shift of 5–15 radians



Parameter estimation

- Can estimate the parameters $\vec{\theta}$ of each inspiral from the data ***d*** with Bayes' theorem:

Posterior, Prior, Likelihood

$$p(\vec{\theta}|d) = \frac{p(\vec{\theta})p(d|\vec{\theta})}{p(d)}$$

Evidence

- Time series of stationary, Gaussian noise ***n*** has the distribution

$$p_n[n(t)] \propto e^{-(n,n)/2} \qquad (a,b) = 4\mathrm{Re}\int_0^\infty \frac{\tilde{a}(f)\tilde{b}(f)}{S_n(f)}\,df$$

- (data from detector ***d***) = (noise ***n***) + (model of GW signal $m(\vec{\theta})$)

$$p(d|\vec{\theta}) \propto e^{-(d-m,d-m)/2}$$

Parameter estimation

- Can estimate the parameters $\vec{\theta}$ of each inspiral from the data ***d*** with Bayes' theorem:

Posterior, Prior, Likelihood

$$p(\vec{\theta}|d) = \frac{p(\vec{\theta})p(d|\vec{\theta})}{p(d)}$$

Evidence

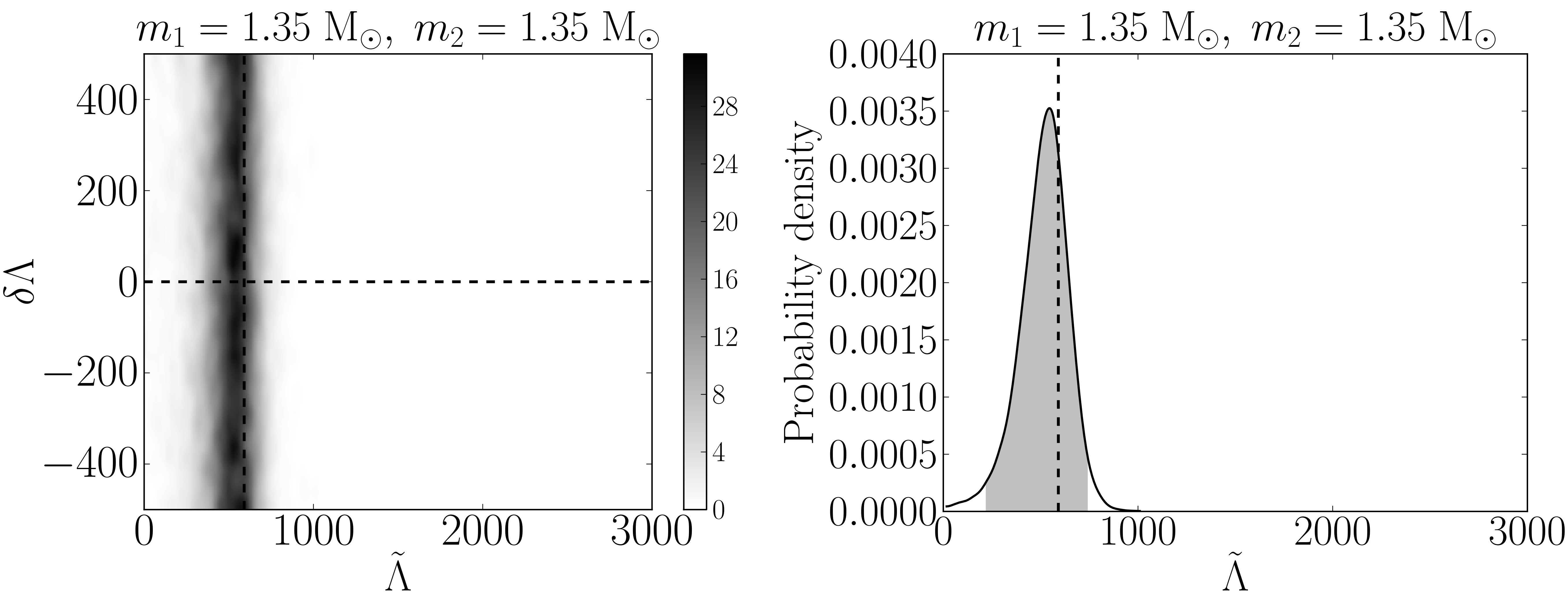
- Can sample the posterior with Markov chain Monte Carlo (MCMC), then marginalize over nuisance parameters

$$p(\mathcal{M}, \eta, \tilde{\Lambda}|d_n) = \int p(\vec{\theta}|d_n) d\vec{\theta}_{\rm nuisance}$$

We only care about masses and tidal parameters here

Parameter estimation

- Result of MCMC simulation for system with SNR=30
- Bayesian parameter estimation for aLIGO-aVirgo network:

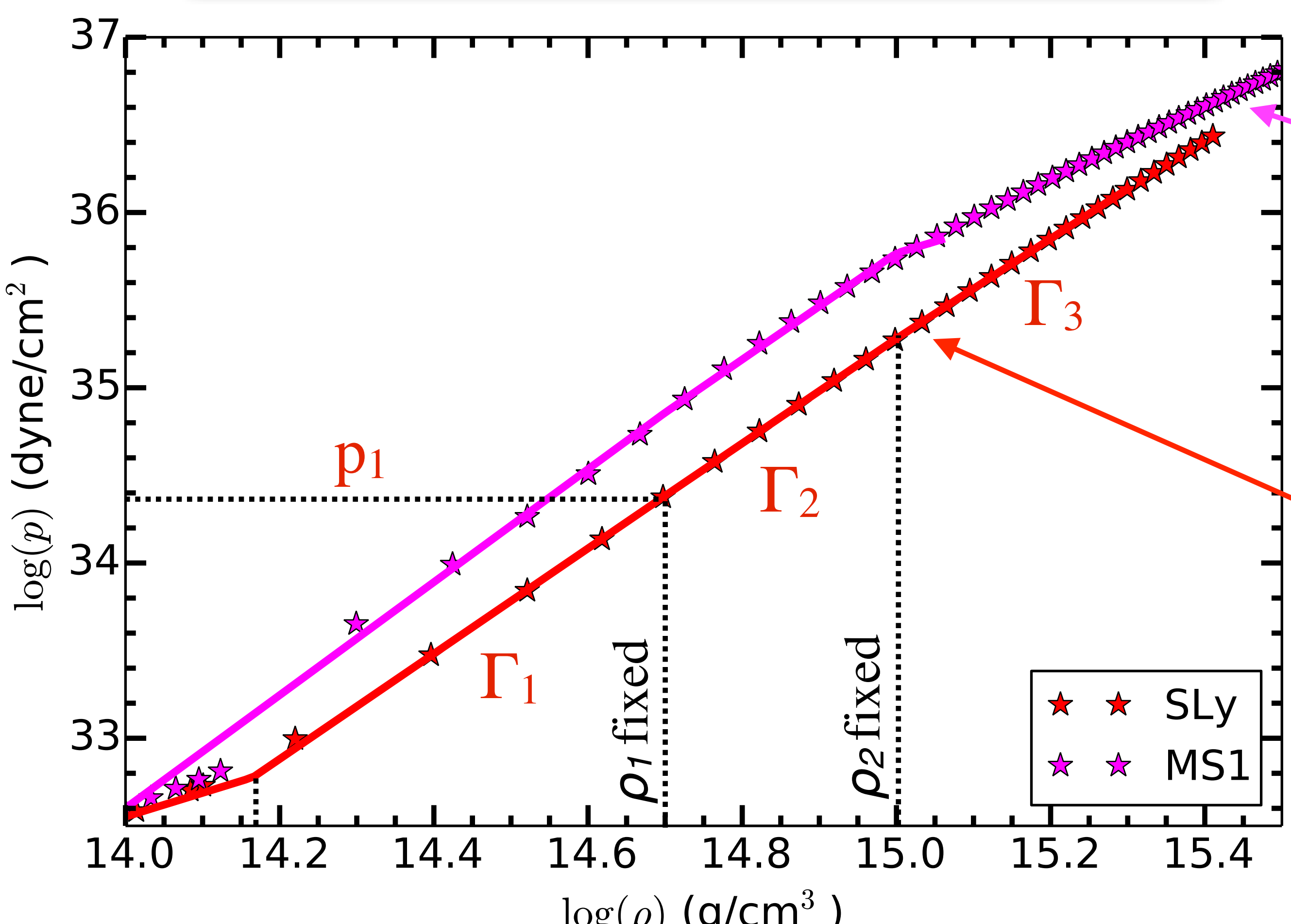


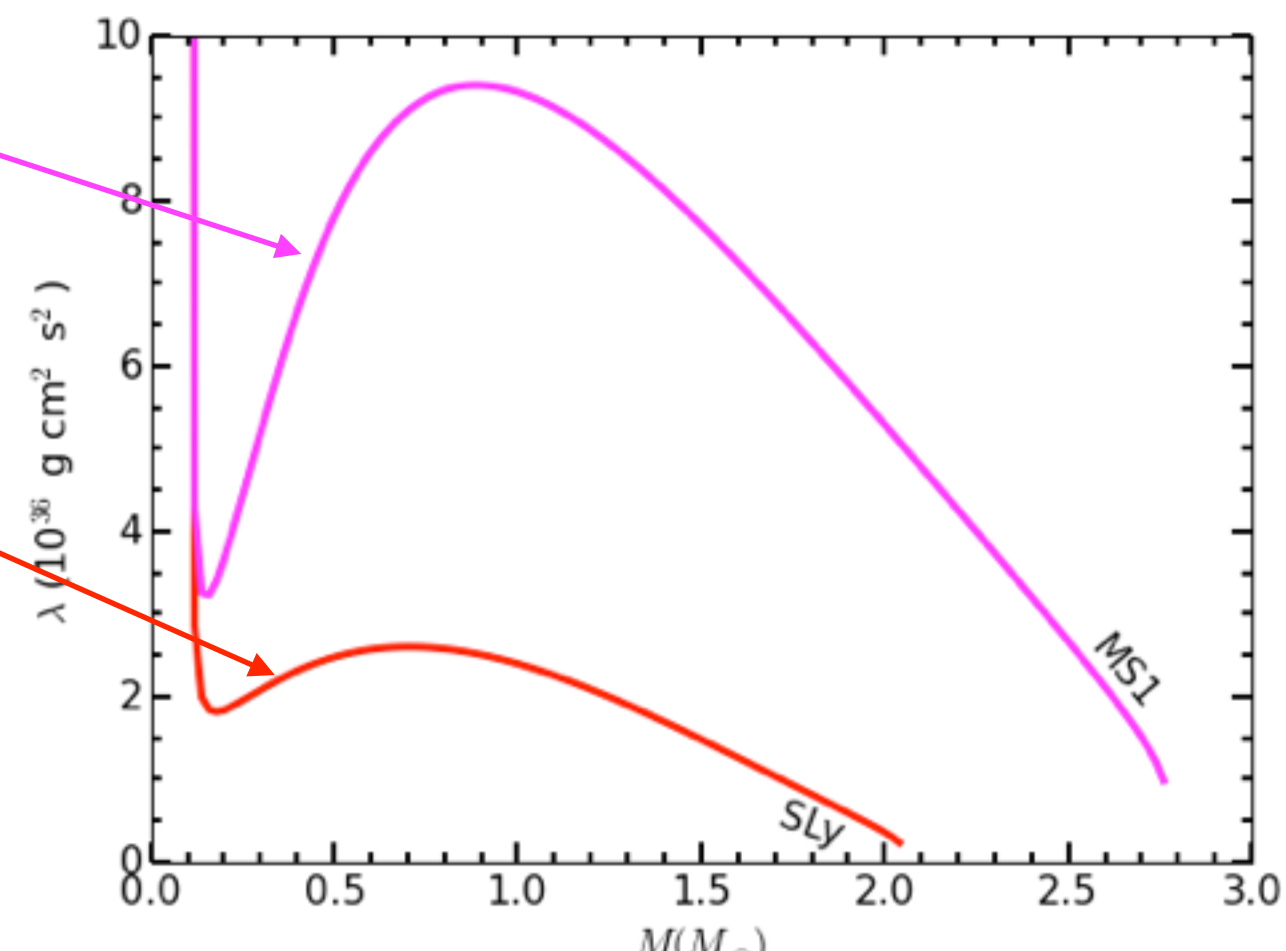
L. Wade et al. PRD 89, 103012 (2014)

Measuring the EOS directly

- The tidal deformability is calculated from the EOS
- This can be inverted to find EOS parameters from observations of the tidal parameters and masses

$$p(\rho) = \begin{cases} K_1 \rho^{\Gamma_1}, & \rho_0 < \rho < \rho_1 \\ K_2 \rho^{\Gamma_2}, & \rho_1 < \rho < \rho_2 \\ K_3 \rho^{\Gamma_3}, & \rho > \rho_2 \end{cases} \longleftrightarrow \begin{aligned} \lambda_1 &= \lambda_1[p(\rho), m_1] \\ \lambda_2 &= \lambda_2[p(\rho), m_2] \end{aligned}$$





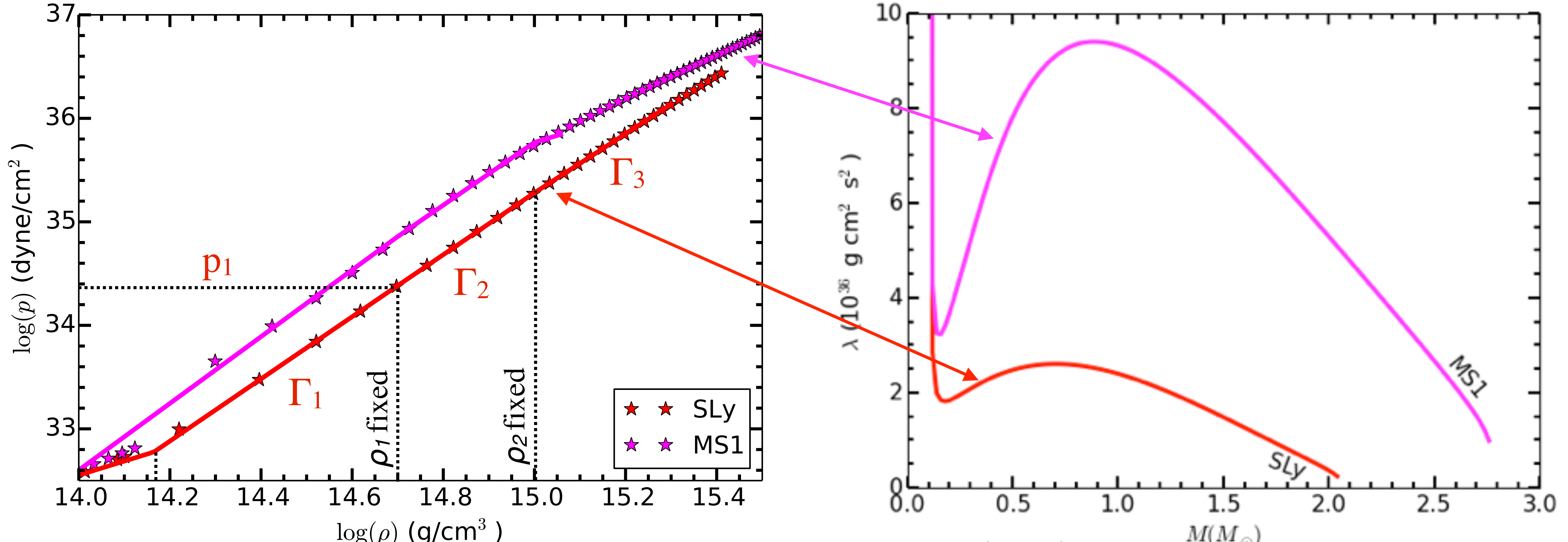
B. Lackey, L. Wade. PRD 91, 043002 (2015)

Measuring the EOS directly

- Use Bayes' theorem again to estimate masses and EOS parameters:

$$\underbrace{p(\vec{x}|d_1 \ldots d_N)}_{\text{Posterior}} = \frac{\overbrace{p(\vec{x})}^{\text{Prior}}\overbrace{p(d_1 \ldots d_N|\vec{x})}^{\text{Likelihood}}}{\underbrace{p(d_1 \ldots d_N)}_{\text{Evidence}}}$$

$$\vec{x} = \{\log(p_1), \Gamma_1, \Gamma_2, \Gamma_3, \mathcal{M}_1, \eta_1, \ldots, \mathcal{M}_N, \eta_N\}$$



B. Lackey, L. Wade. PRD 91, 043002 (2015)

Measuring the EOS directly

- Use Bayes' theorem again to estimate masses and EOS parameters:

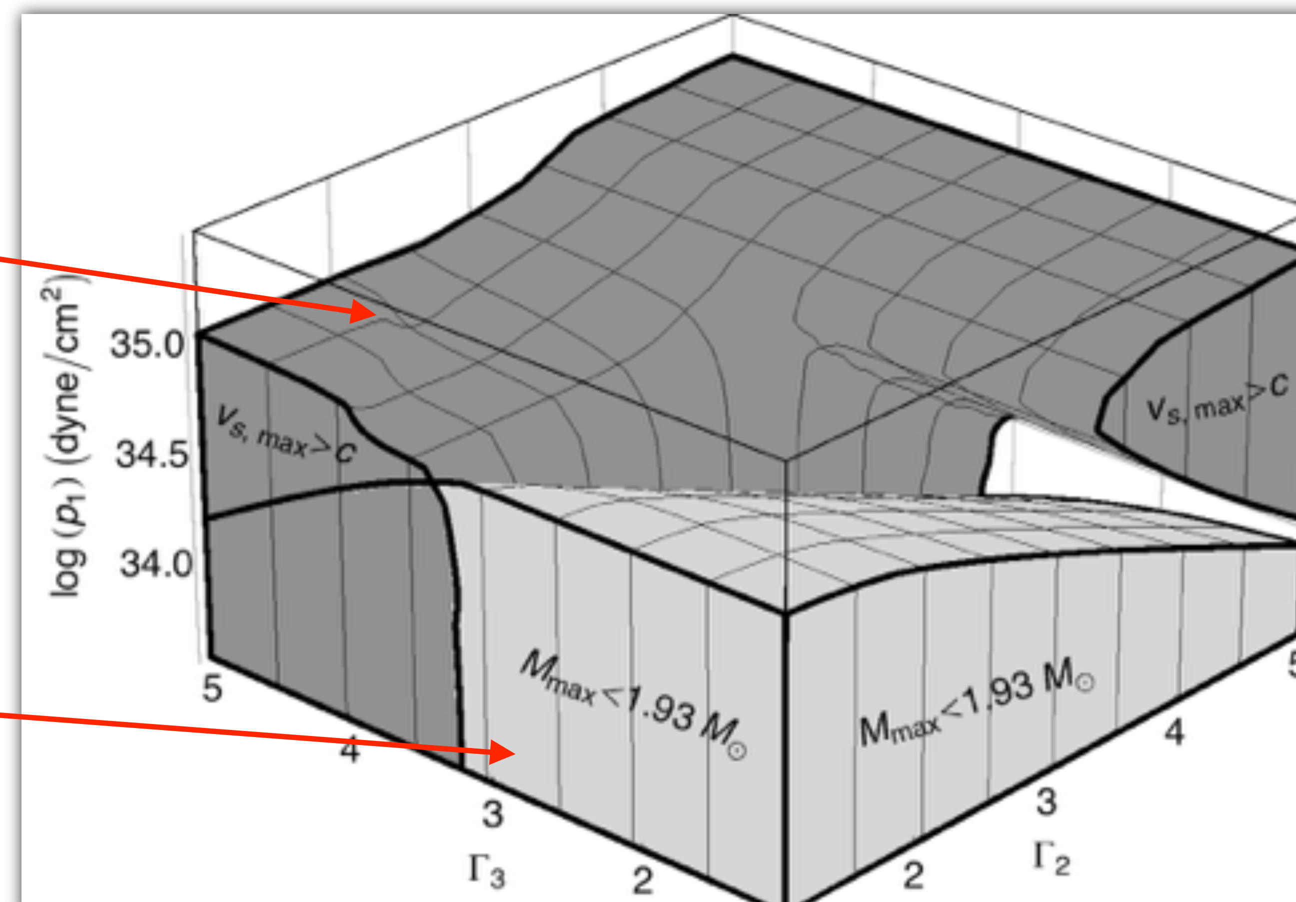
Posterior, Prior, Likelihood, Evidence

$$p(\vec{x}|d_1 \ldots d_N) = \frac{p(\vec{x})p(d_1 \ldots d_N|\vec{x})}{p(d_1 \ldots d_N)}$$

- **Causality**: The speed of sound must be less than the speed of light

$$v_s = \sqrt{dp/d\epsilon} < c$$

- **Maximum mass**: The EOS must allow for masses that are greater than observed NSs (at least $1.93 M_\odot$)



B. Lackey, L. Wade. PRD 91, 043002 (2015)

Measuring the EOS directly

- Use Bayes' theorem again to estimate masses and EOS parameters:

Posterior — Prior — Likelihood — Evidence

$$p(\vec{x}|d_1 \dots d_N) = \frac{p(\vec{x})p(d_1 \dots d_N|\vec{x})}{p(d_1 \dots d_N)}$$

- Likelihood is the product of the marginalized distributions for each event

Marginalized posterior for single event

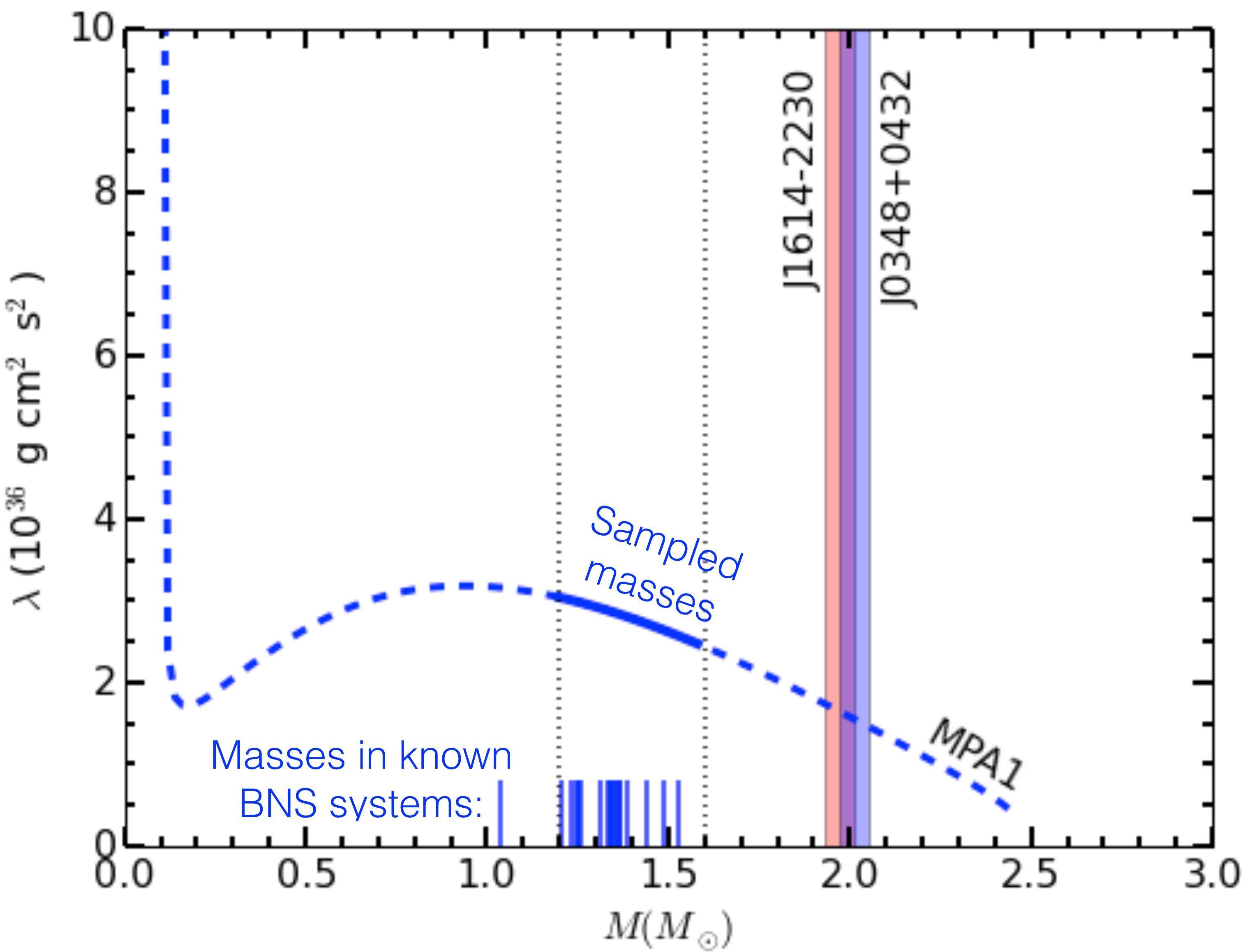
$$p(d_1, \dots, d_N|\vec{x}) = \prod_{n=1}^{N} p(\mathcal{M}_n, \eta_n, \tilde{\Lambda}_n|d_n)|_{\tilde{\Lambda}_n = \tilde{\Lambda}(\mathcal{M}_n, \eta_n, \mathrm{EOS})}$$

- The 4 EOS parameters are found from an MCMC simulation of the 4+2N parameters by marginalizing over the 2N mass parameters

B. Lackey, L. Wade. PRD 91, 043002 (2015)

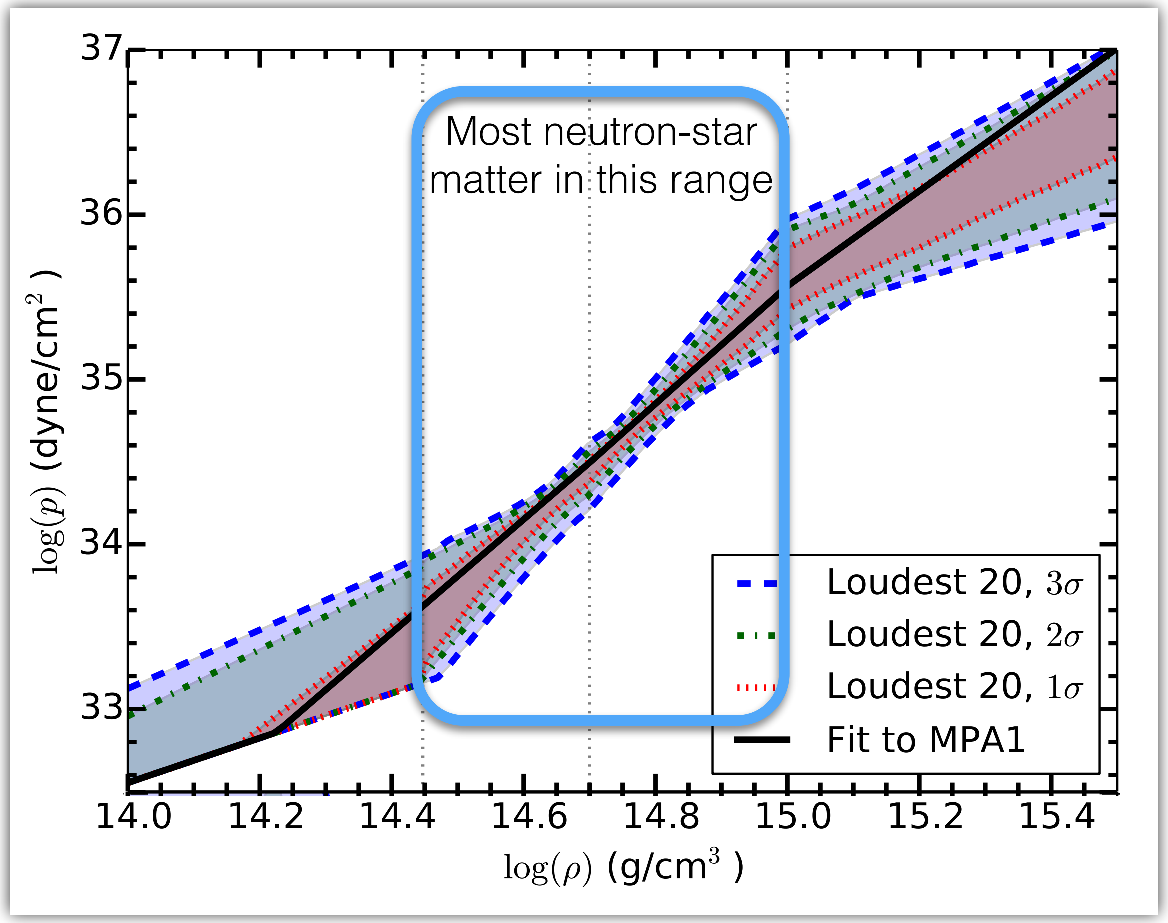
Measuring the EOS directly

- We simulated a year of events for the aLIGO-aVirgo network
  - Each detector had ~40 BNS events/year with SNR>8
  - Sampled the masses uniformly in the range $1.2M_\odot$–$1.6M_\odot$
  - Assumed MPA1 as the true EOS



B. Lackey, L. Wade. PRD 91, 043002 (2015)

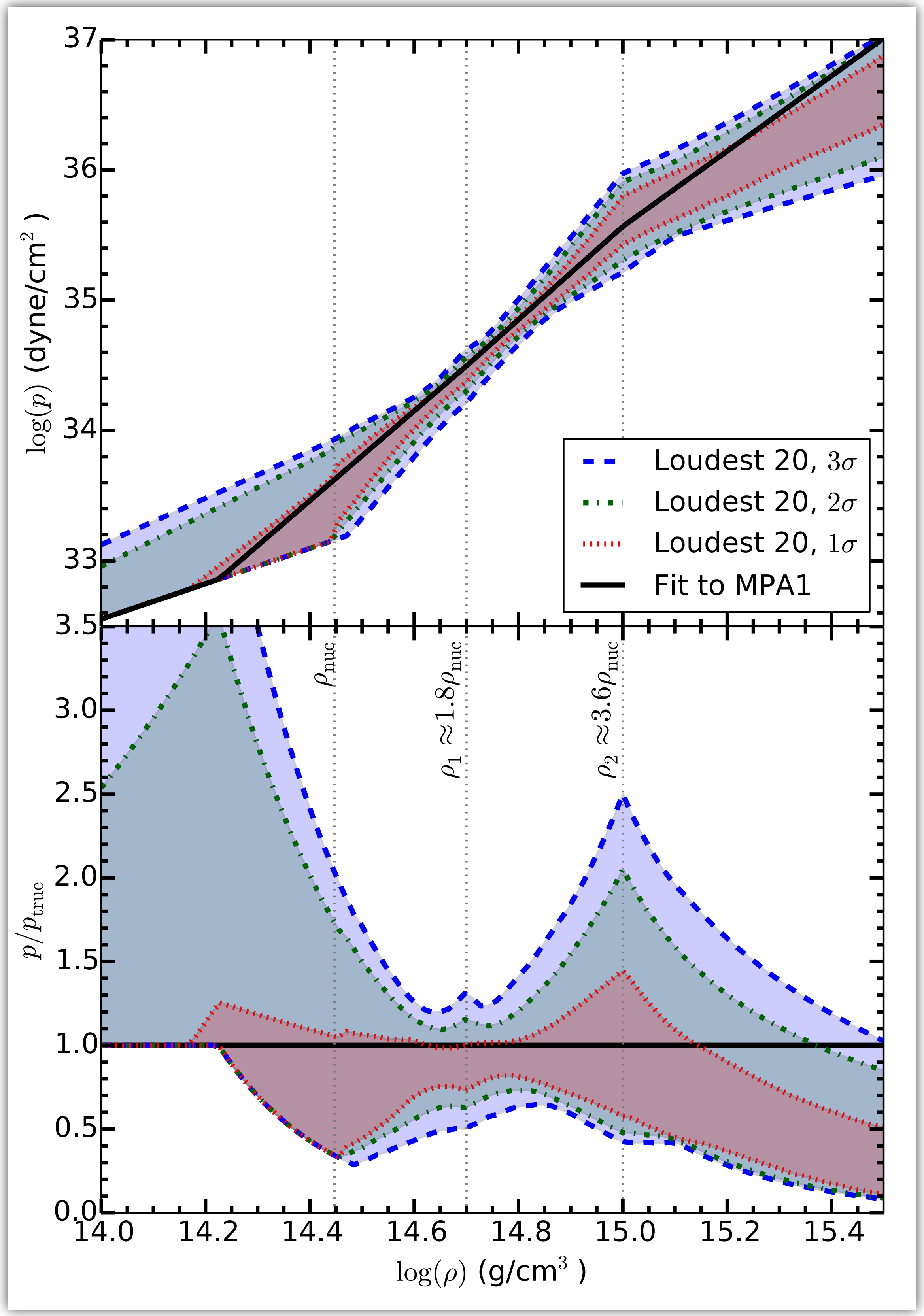
Measuring the EOS directly



- 68% credible region
- 95% credible region
- 99.7% credible region

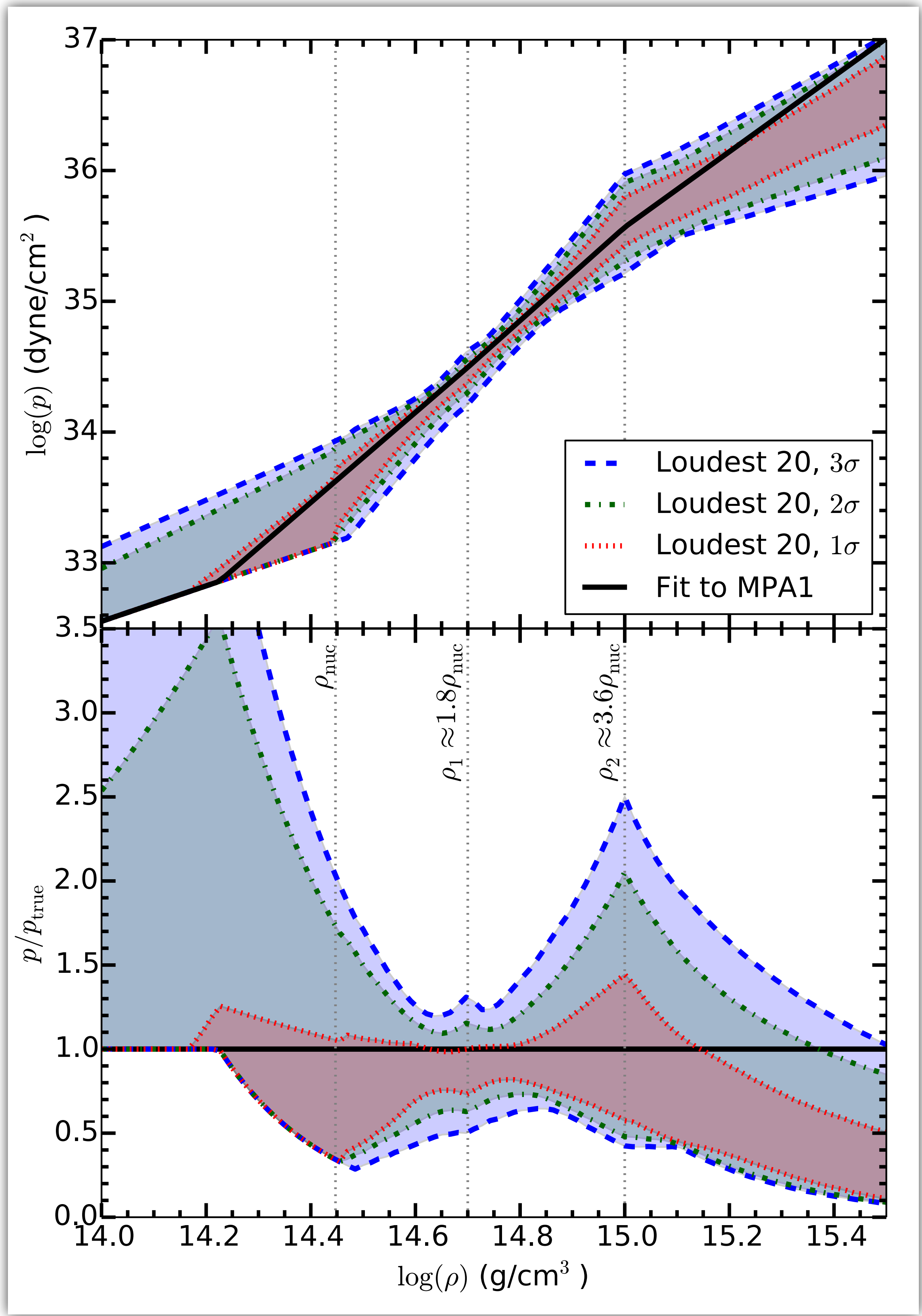
B. Lackey, L. Wade. PRD 91, 043002 (2015)

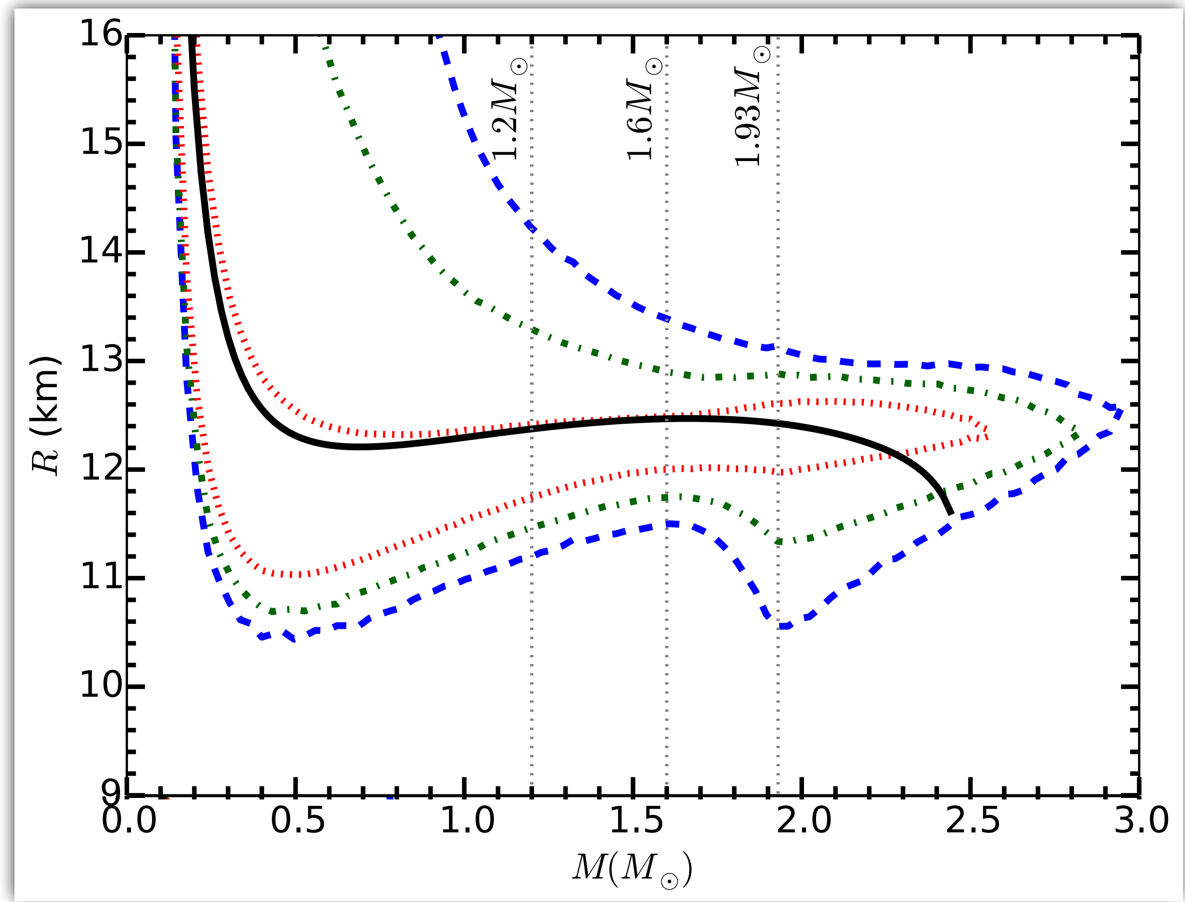
Measuring the EOS directly



B. Lackey, L. Wade. PRD 91, 043002 (2015)

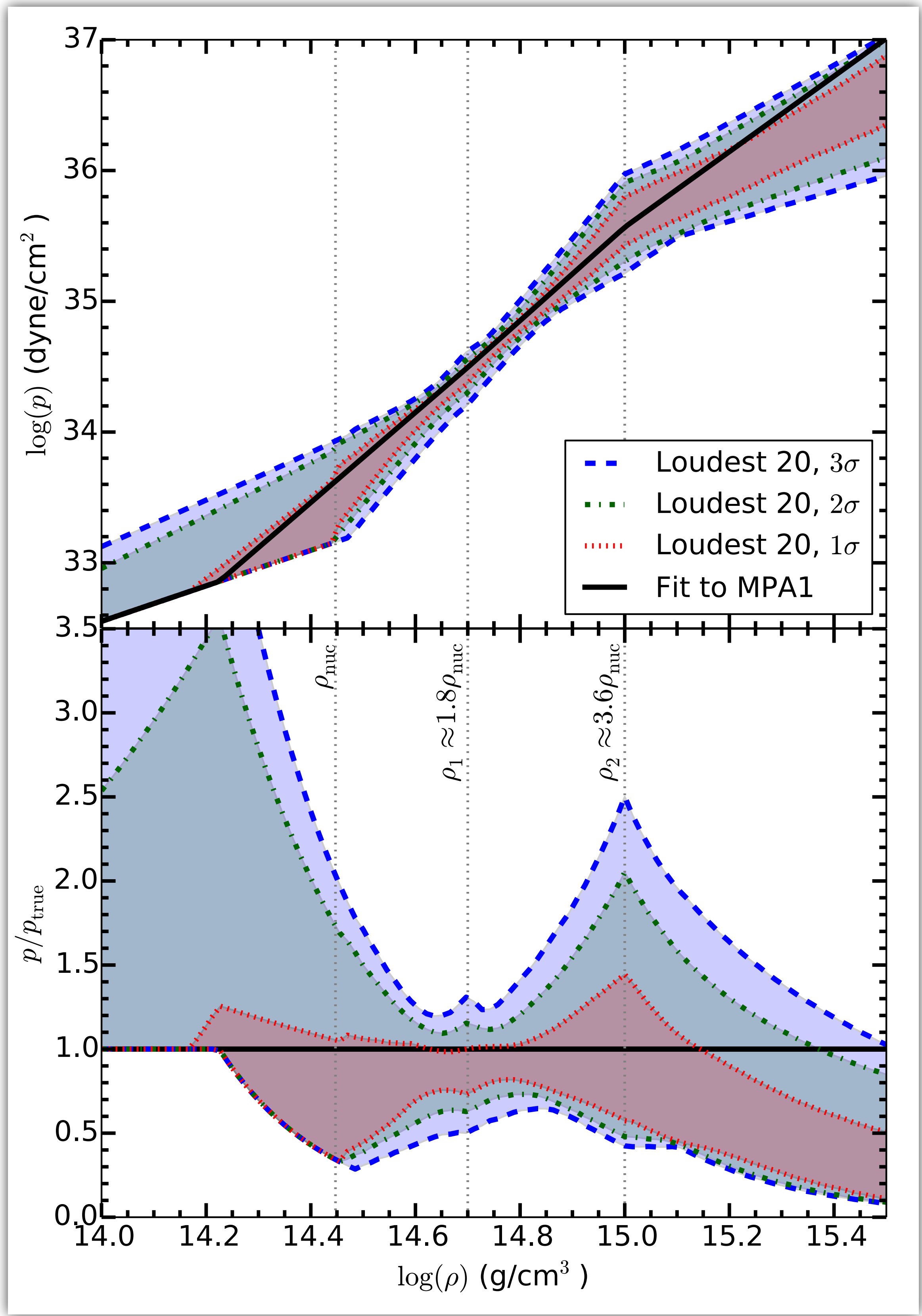
Measuring the EOS directly

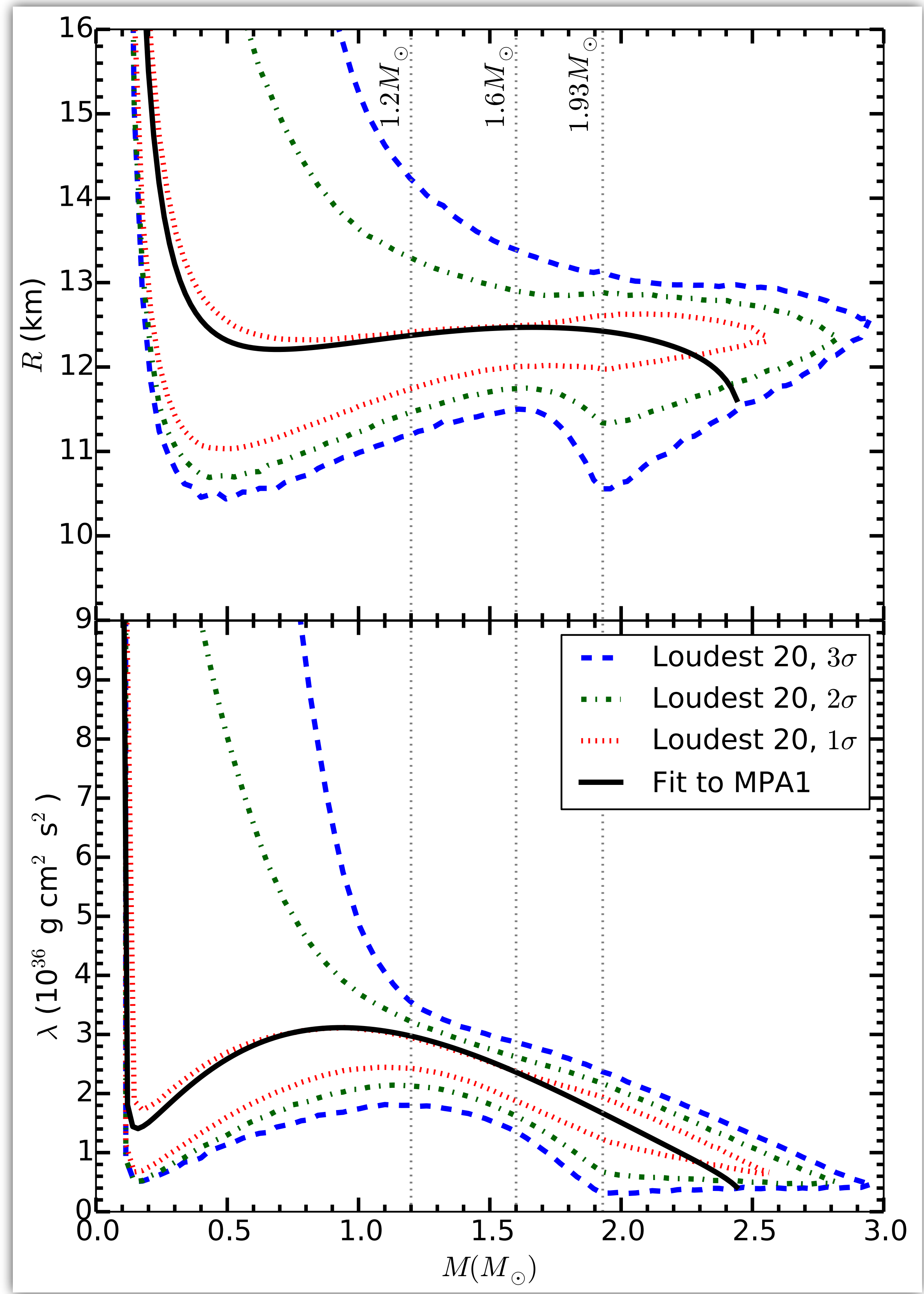




B. Lackey, L. Wade. PRD 91, 043002 (2015)

Measuring the EOS directly

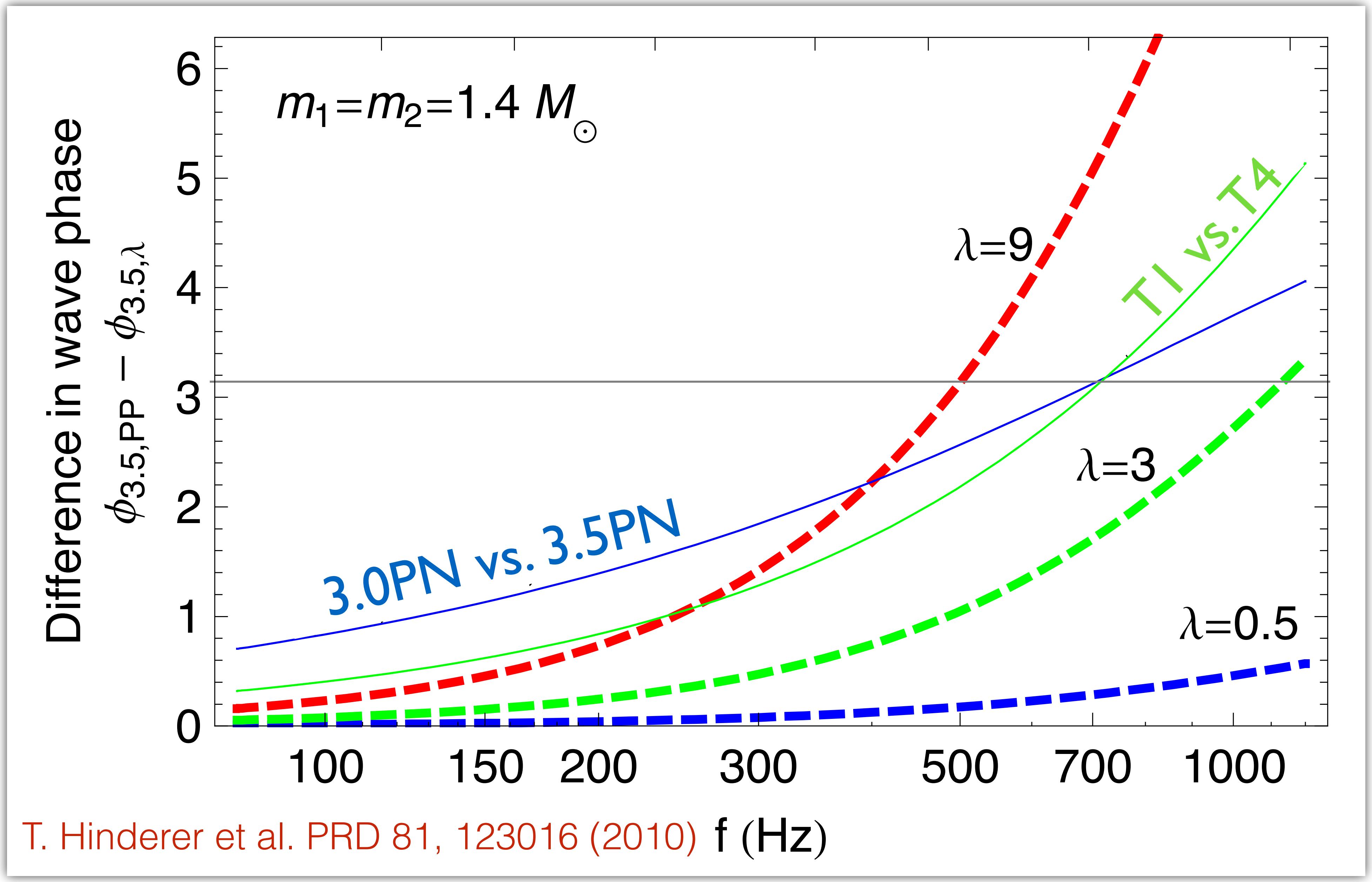




B. Lackey, L. Wade. PRD 91, 043002 (2015)

Is the post-Newtonian waveform accurate enough?

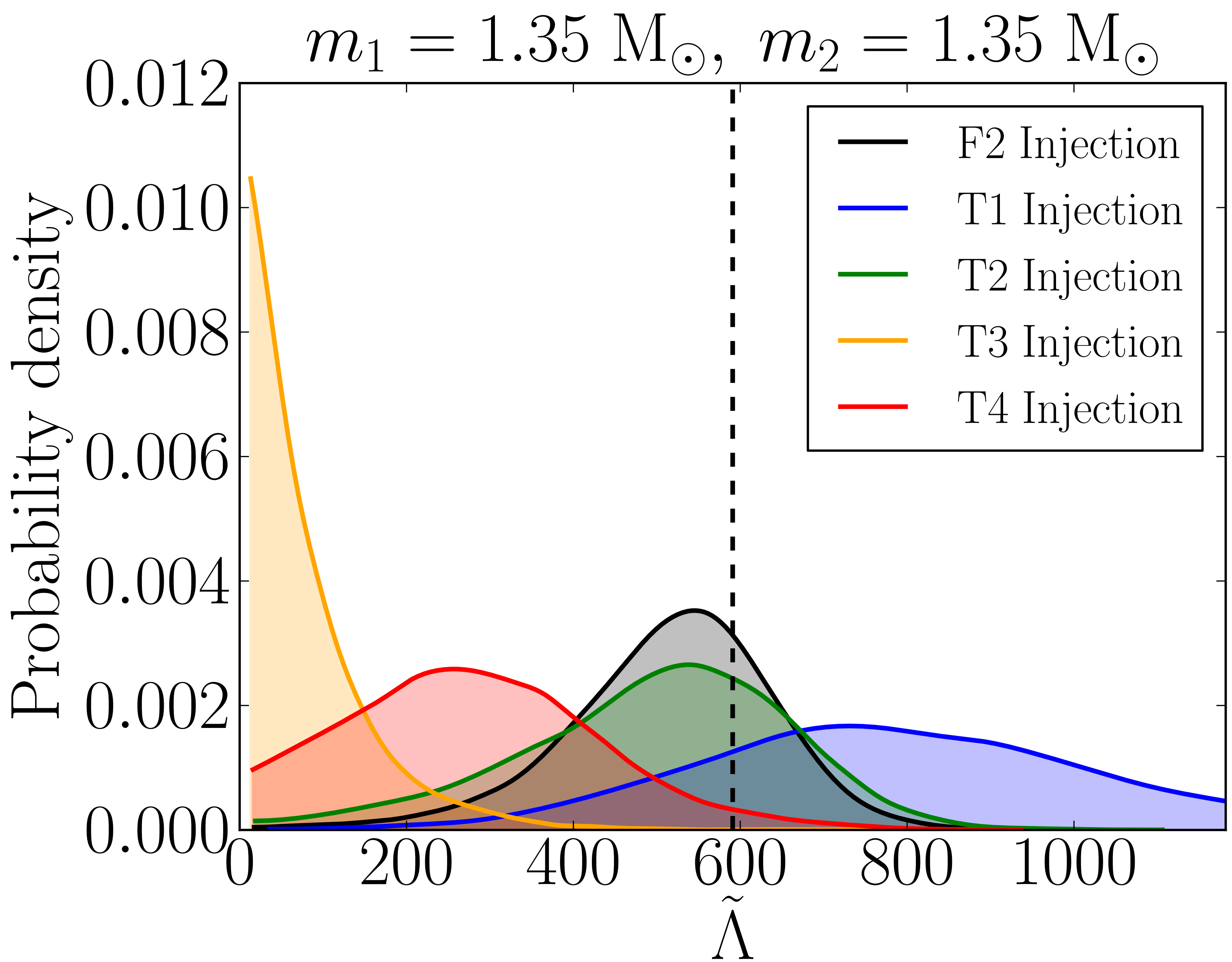
- Phase difference between 3PN and 3.5PN: ~1GW cycle
- At a given post-Newtonian order, there are many ways to calculate the phase
  - Approximants: Taylor**T1**, …, Taylor**T4**, Taylor**F2**
  - Phase difference between approximants: ~1GW cycle



T. Hinderer et al. PRD 81, 123016 (2010)

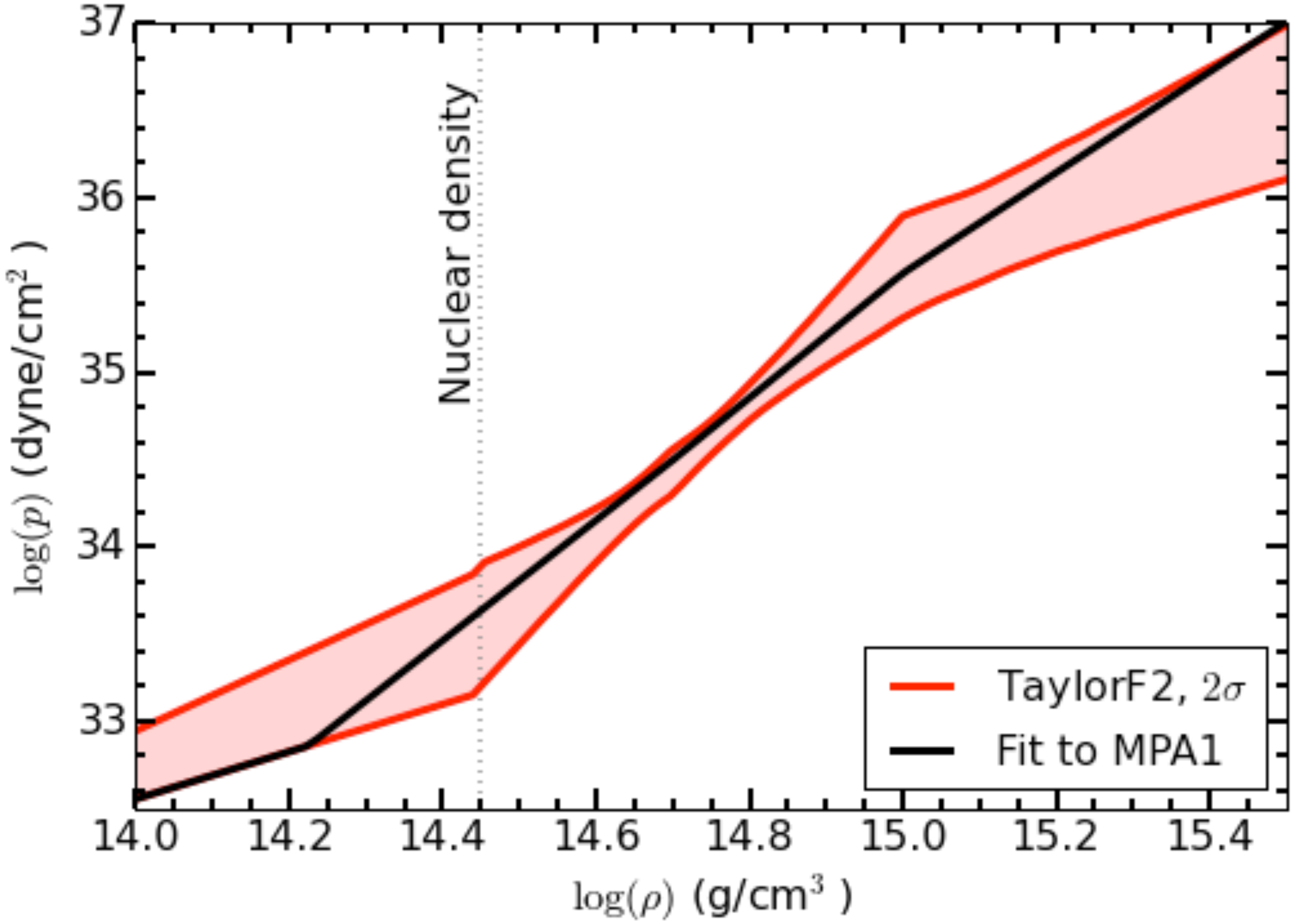
Is the post-Newtonian waveform accurate enough? **No**

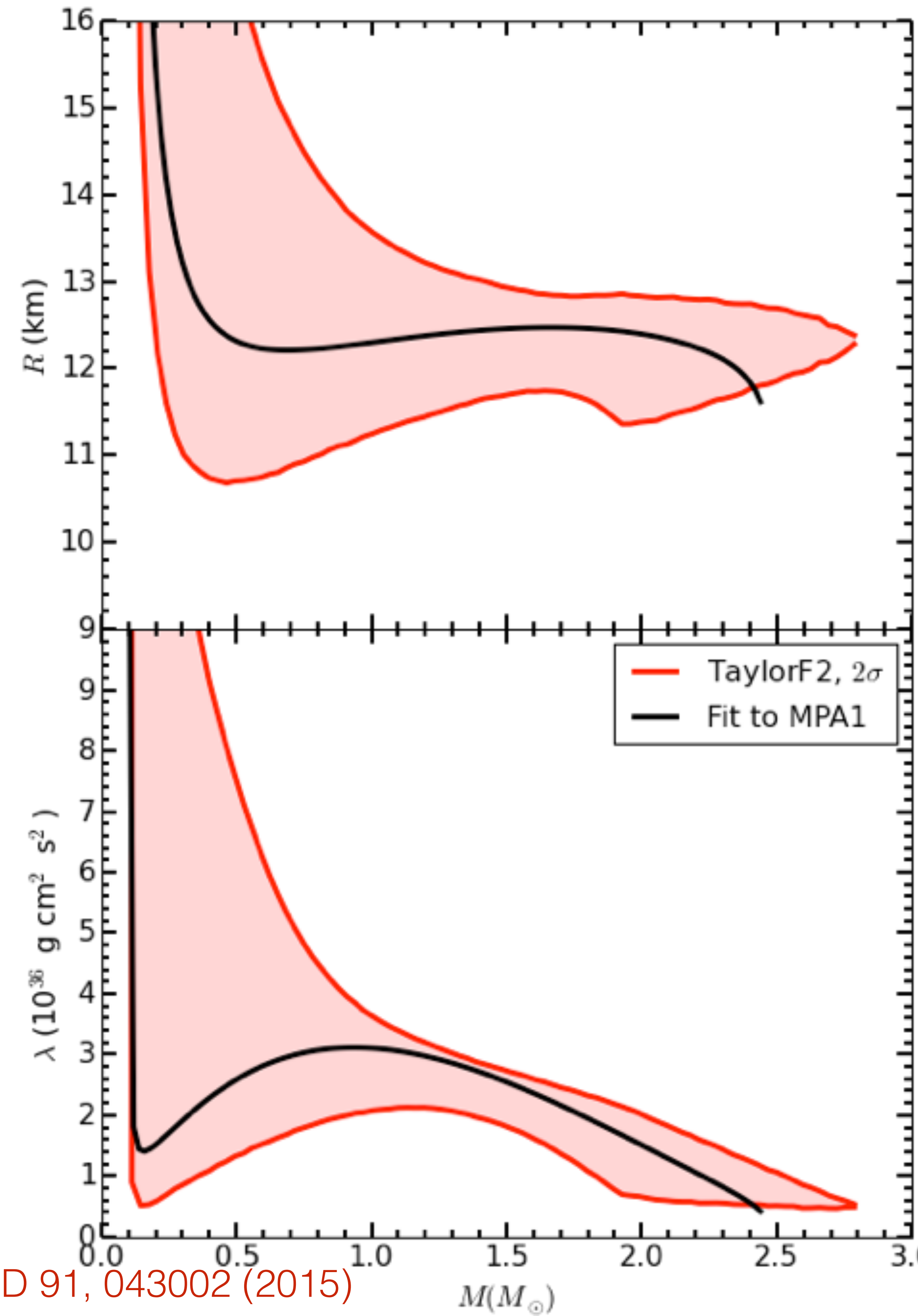
- Model used for constructing waveform template has dramatic effect on recovered tidal parameter
- Used TaylorF2 as waveform template



L. Wade et al. PRD 89, 103012 (2014)

Is the post-Newtonian waveform accurate enough? **No**

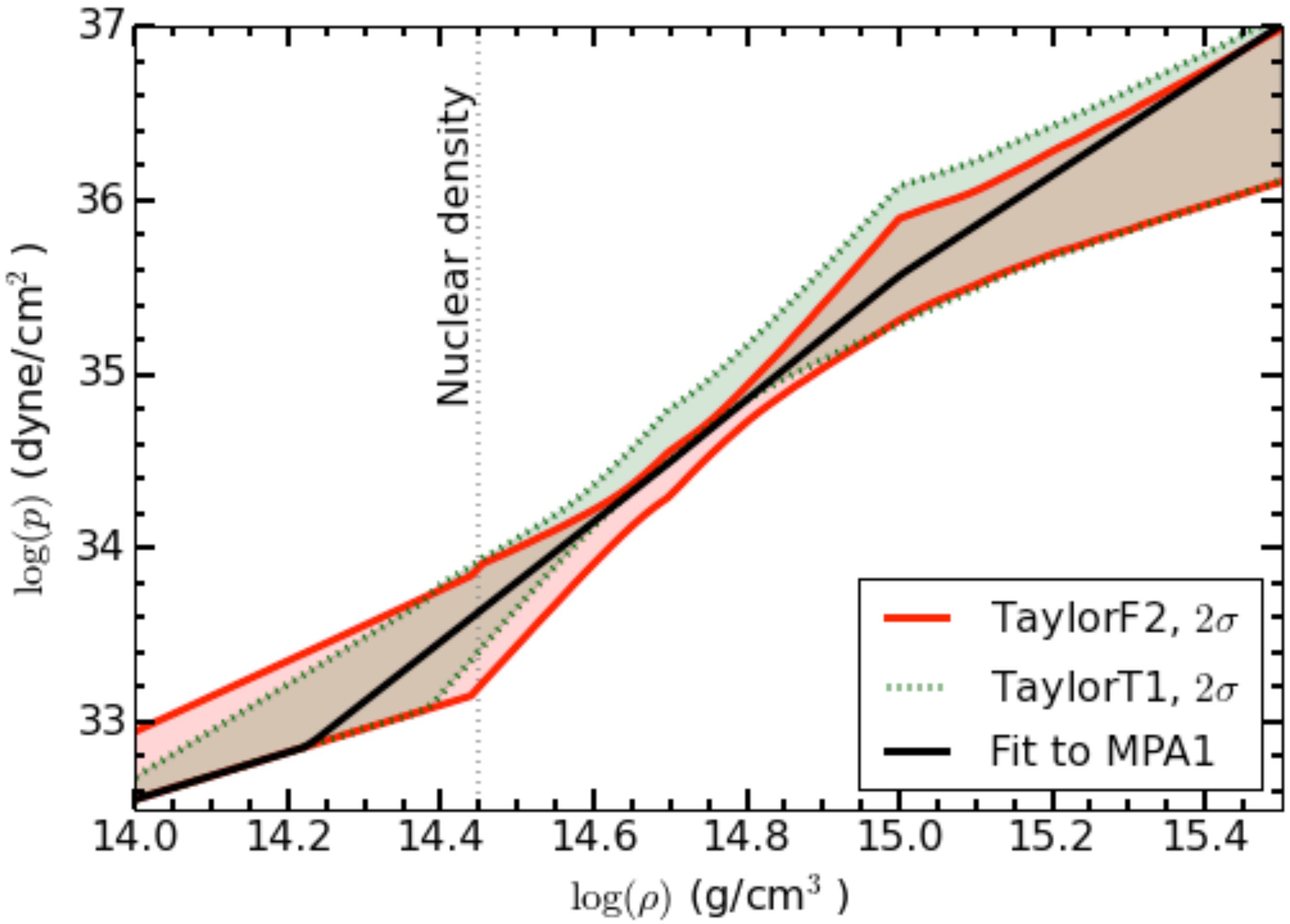




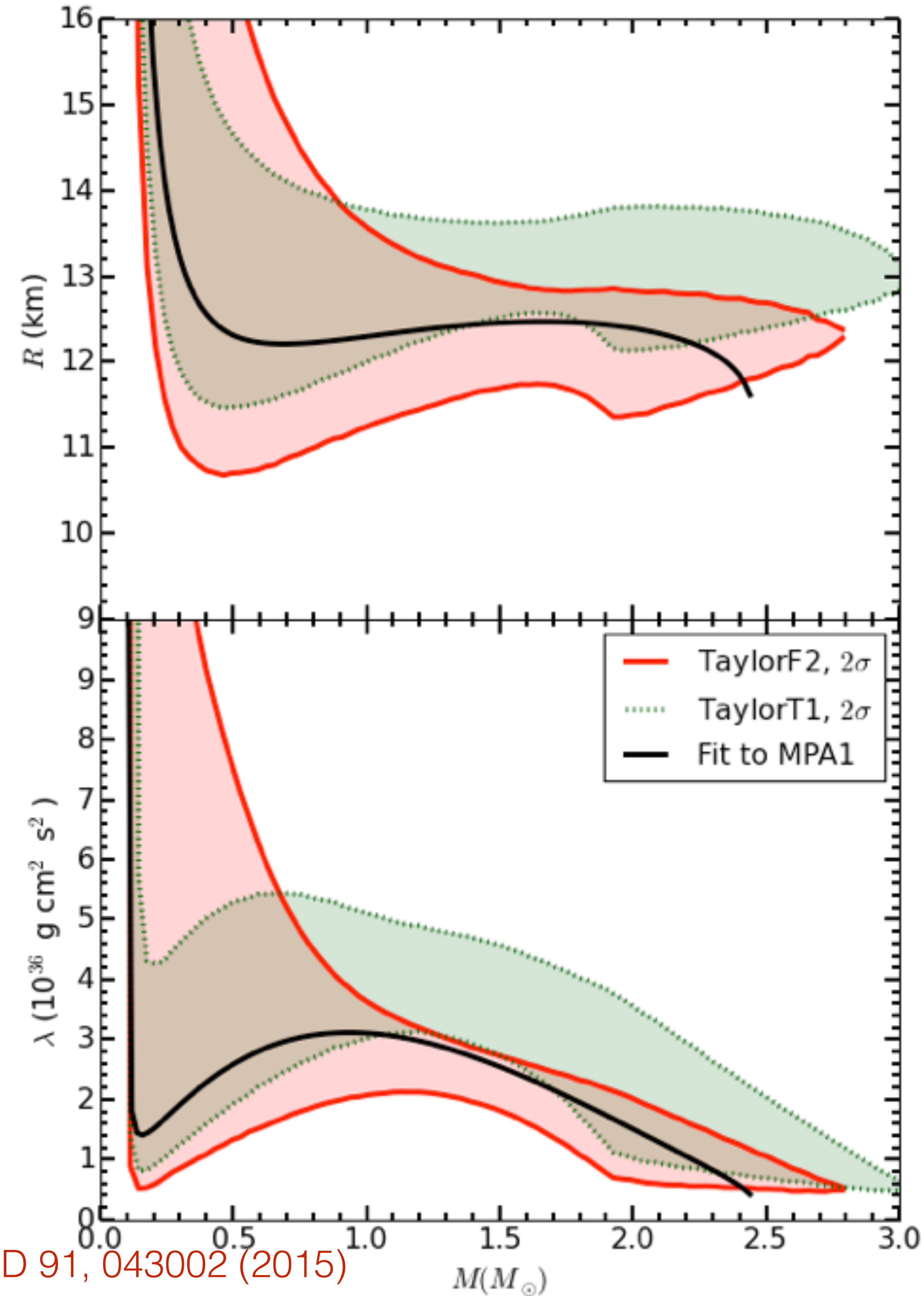
- 95% credible regions

B. Lackey, L. Wade. PRD 91, 043002 (2015)

Is the post-Newtonian waveform accurate enough? **No**

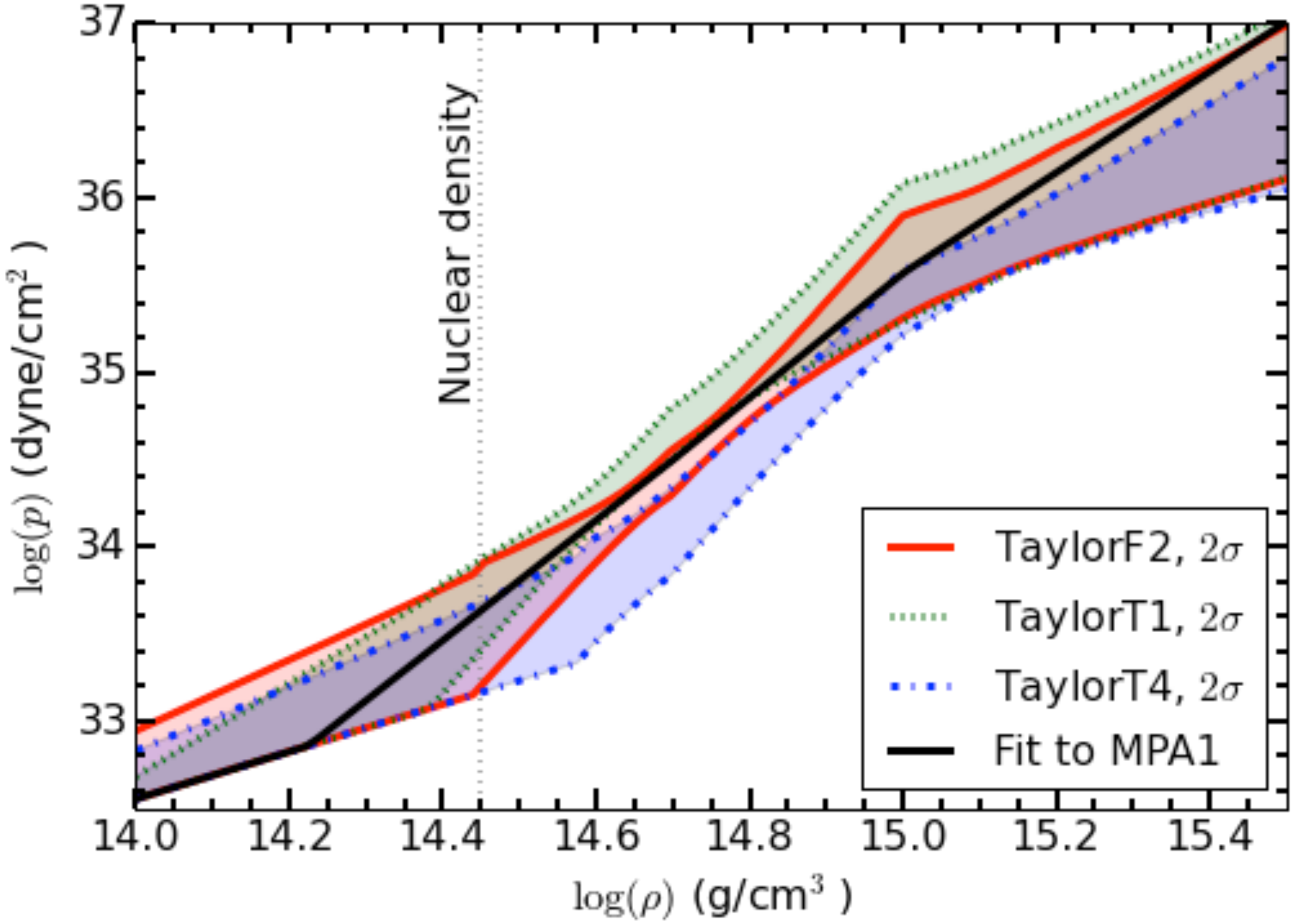


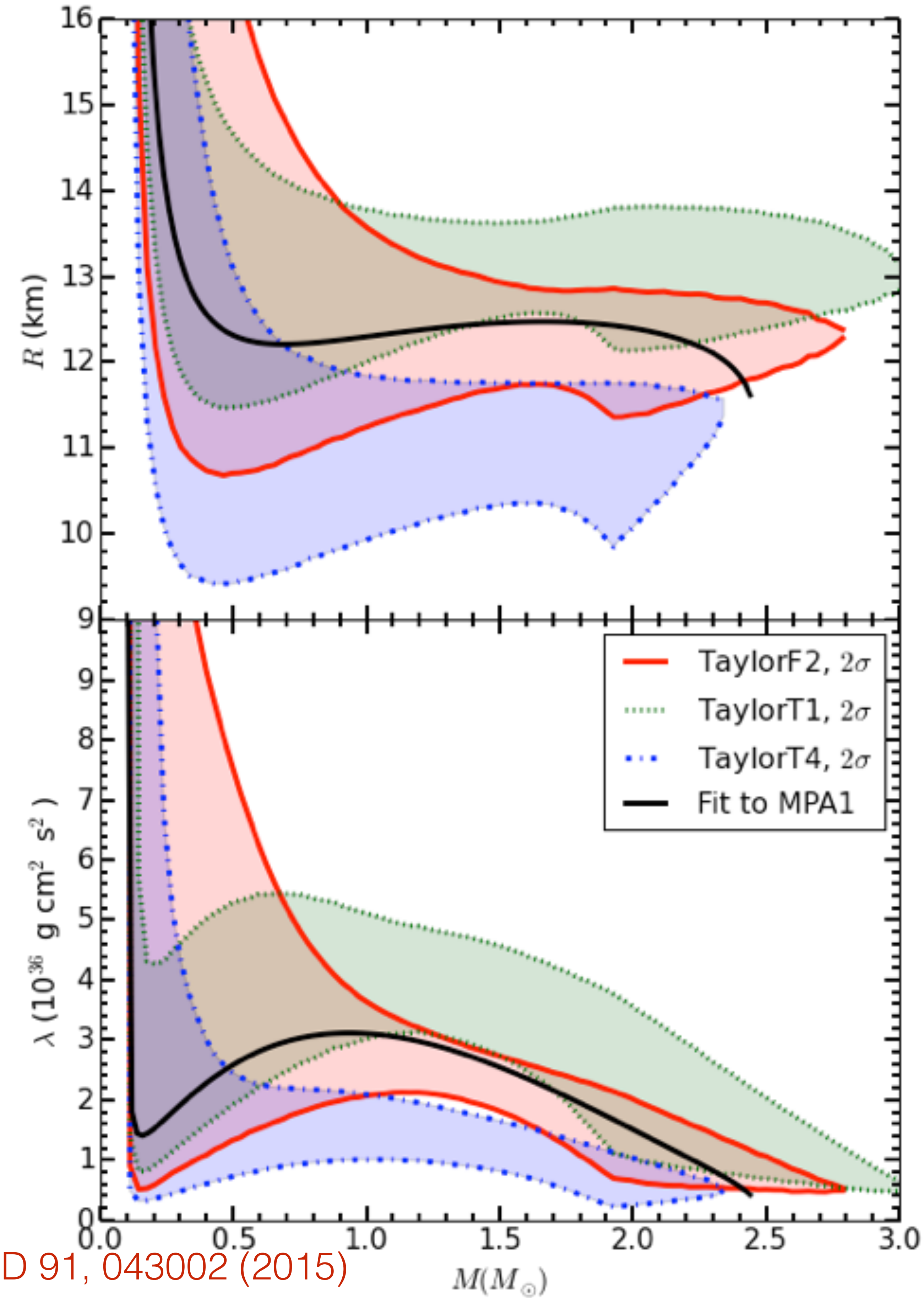
- 95% credible regions



B. Lackey, L. Wade. PRD 91, 043002 (2015)

Is the post-Newtonian waveform accurate enough? **No**





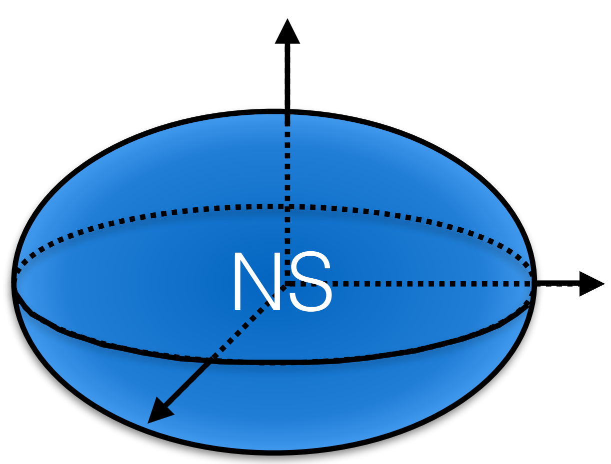
- 95% credible regions

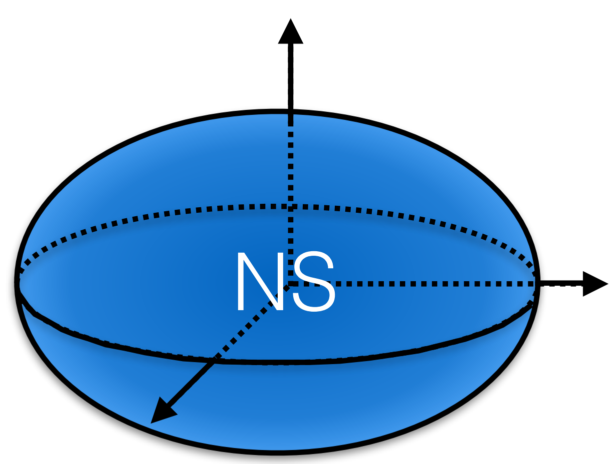
B. Lackey, L. Wade. PRD 91, 043002 (2015)

Constructing a more accurate waveform model

Additional matter effects

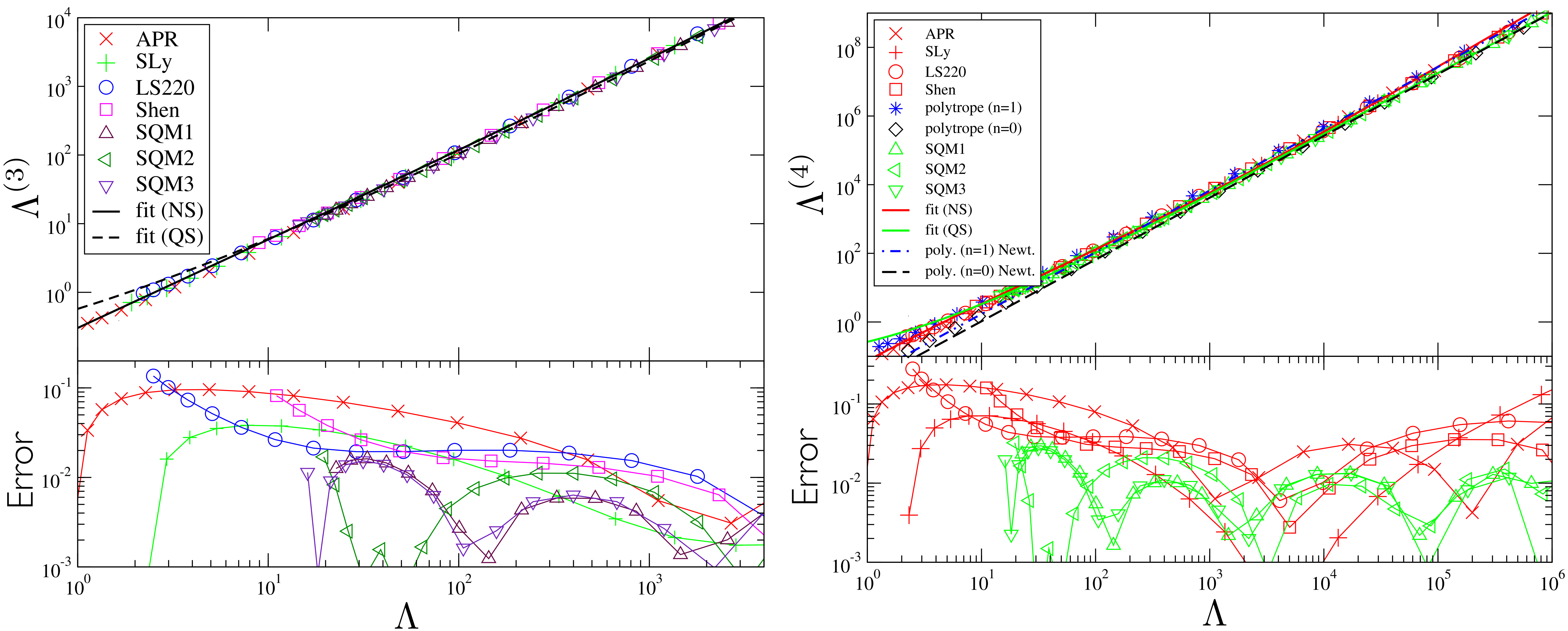
- Tidal fields $\mathcal{E}_{ij...}$ from companion star induce quadrupole Q_{ij} and higher order multipoles
  - Quadrupole: $Q_{ij} = -\Lambda(\mathrm{EOS}, m) m^5 \mathcal{E}_{ij}$ 5PN
  - Octopole: $O_{ijk} = -\Lambda^{(3)}(\mathrm{EOS}, m) m^7 \mathcal{E}_{ijk}$ 7PN
  - Hexadecapole: $H_{ijkl} = -\Lambda^{(4)}(\mathrm{EOS}, m) m^9 \mathcal{E}_{ijkl}$ 9PN





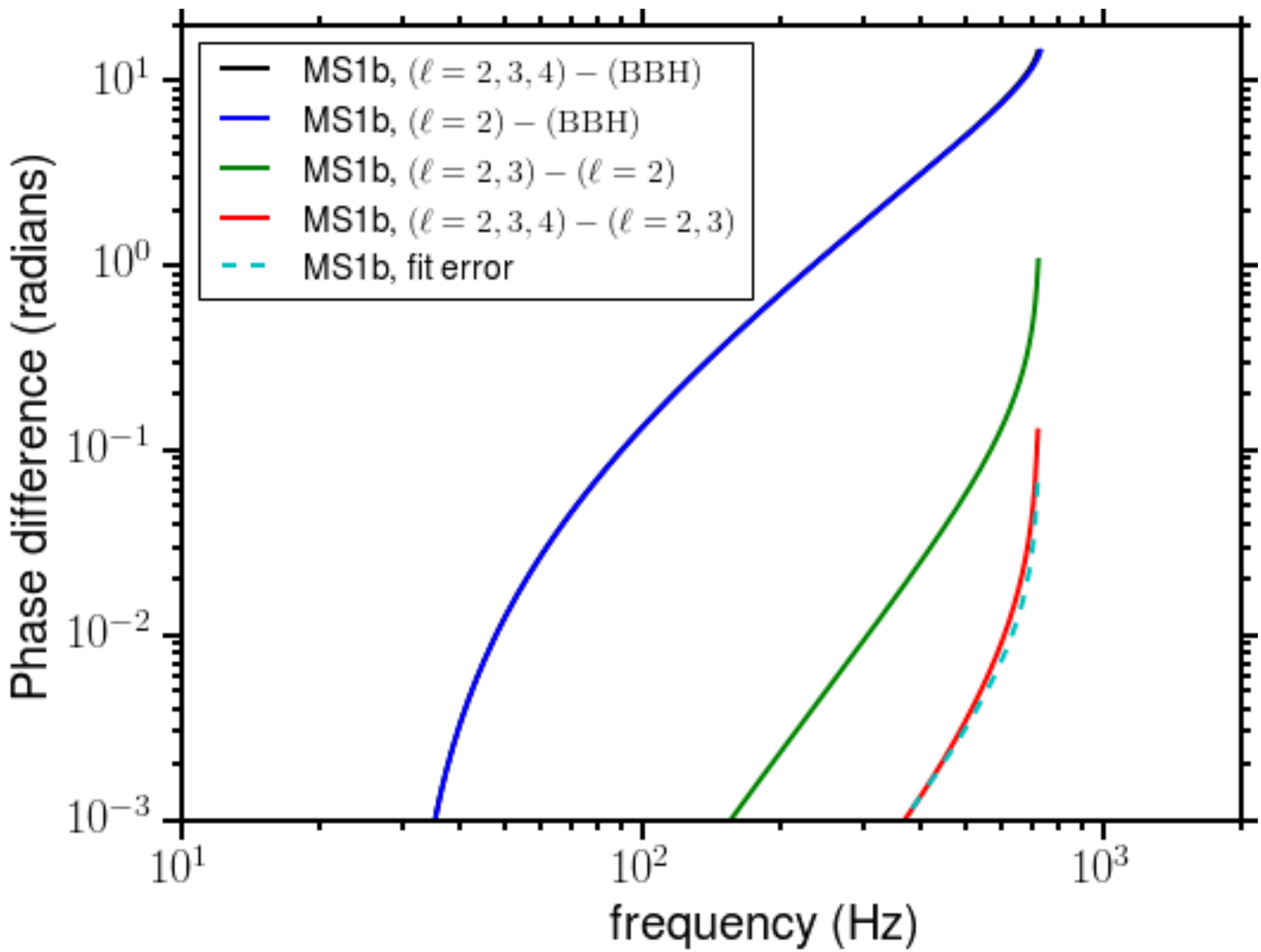
Additional matter effects

- Most NS properties are correlated in a nearly EOS-independent way
- There is effectively only one EOS-dependent parameter during inspiral



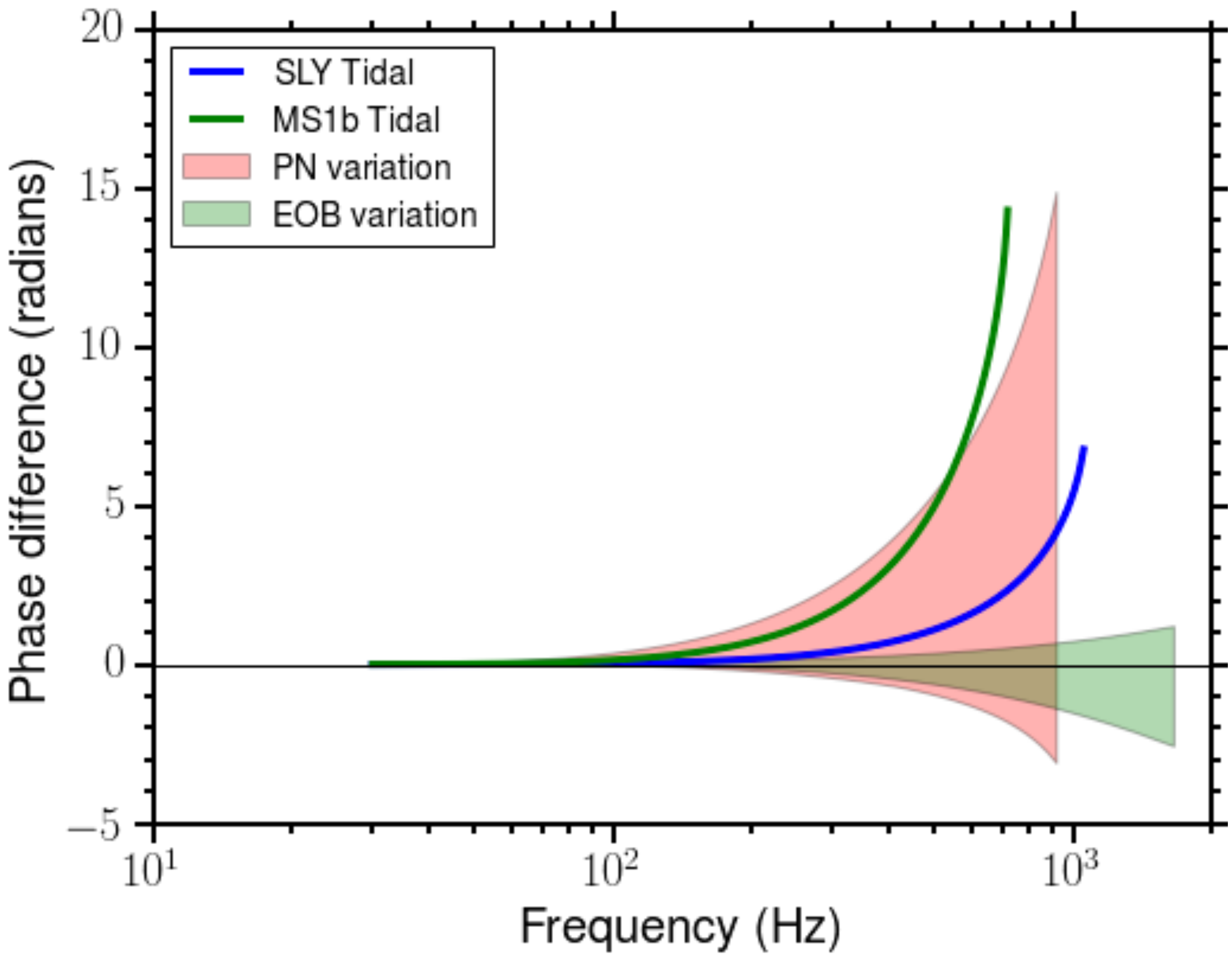
Kent Yagi. PRD 89, 043011 (2014)

Additional matter effects



More accurate BNS waveform

- Effective one body (EOB) waveforms have significantly smaller uncertainties
- Take ~10 minutes to evaluate starting at 10Hz
    - Too slow for parameter estimation



4 different PN waveform models

6 different EOB waveform models

Reduced order model

- Construct a set of orthonormal basis functions that approximate any waveform $h(t;\vec{\theta}) = A(t;\vec{\theta})e^{i\Phi(t;\vec{\theta})}$ in the parameter space

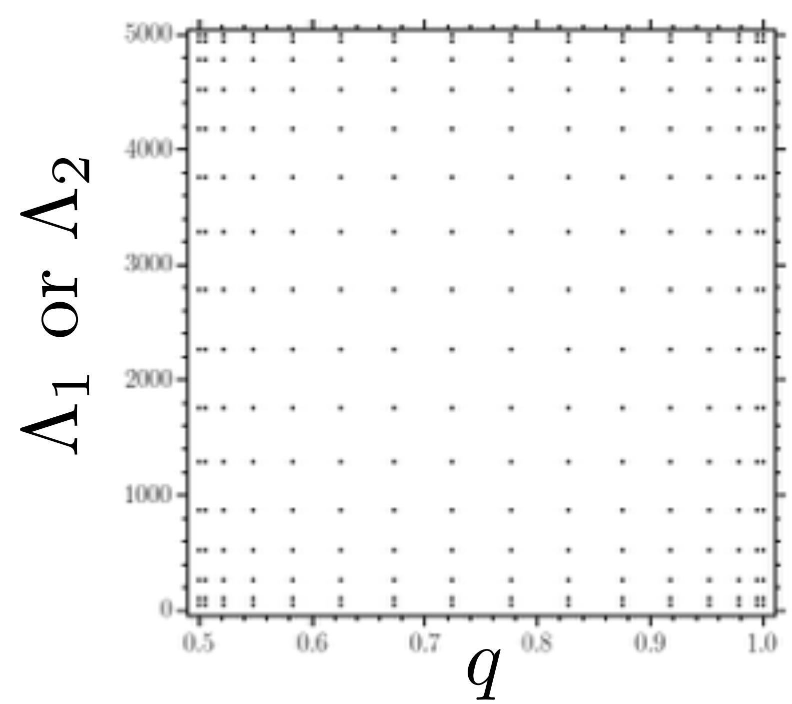
$$A(t;\vec{\theta}) \approx \sum_{i=1}^{N_A} c_i(\vec{\theta})\hat{e}_i(t) \qquad \Phi(t;\vec{\theta}) \approx \sum_{i=1}^{N_\Phi} c_i(\vec{\theta})\hat{e}_i(t)$$

Reduced order model

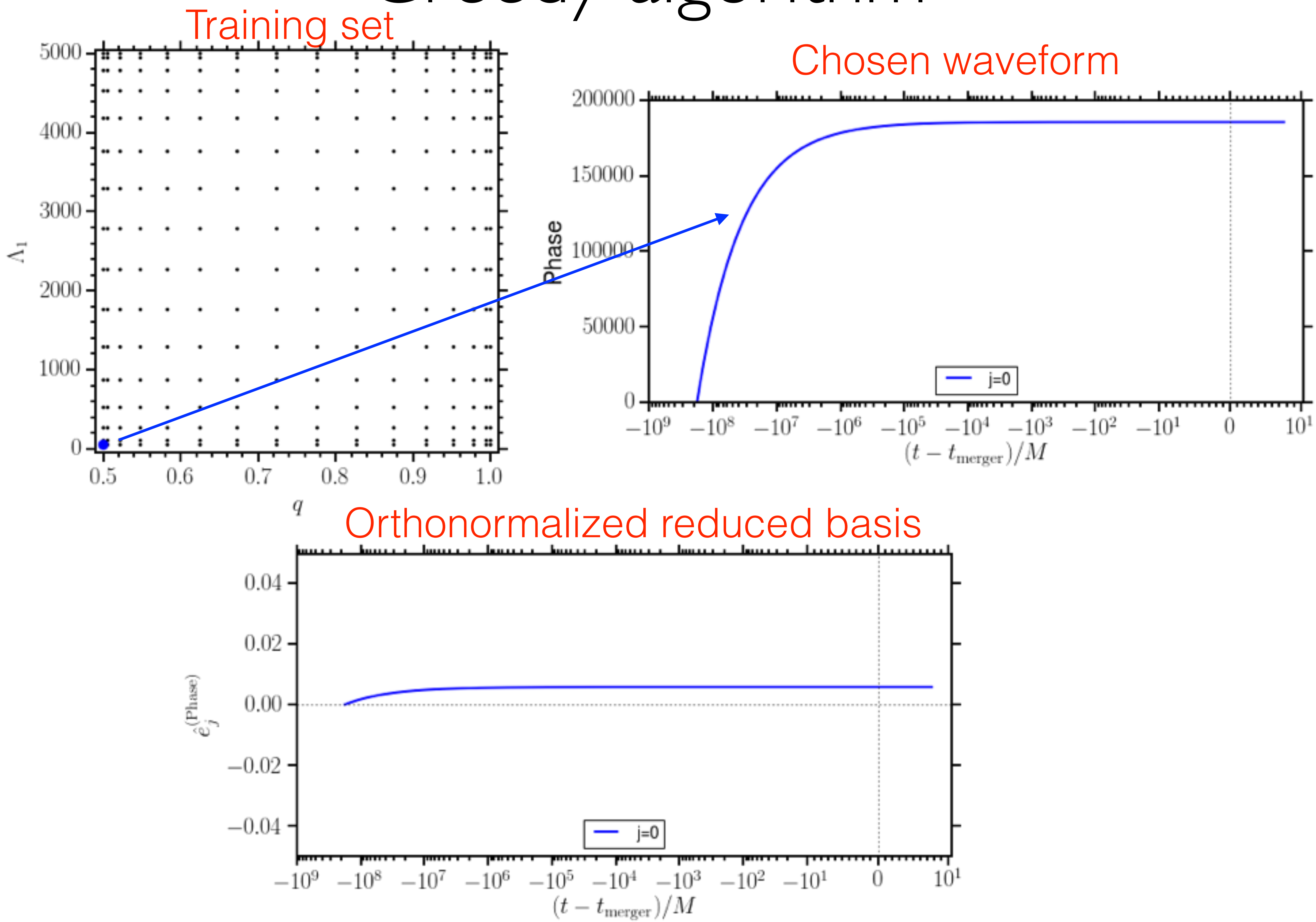
- Construct a set of orthonormal basis functions that approximate any waveform $h(t;\vec{\theta}) = A(t;\vec{\theta})e^{i\Phi(t;\vec{\theta})}$ in the parameter space

$$A(t;\vec{\theta}) \approx \sum_{i=1}^{N_A} c_i(\vec{\theta})\hat{e}_i(t) \qquad \Phi(t;\vec{\theta}) \approx \sum_{i=1}^{N_\Phi} c_i(\vec{\theta})\hat{e}_i(t)$$

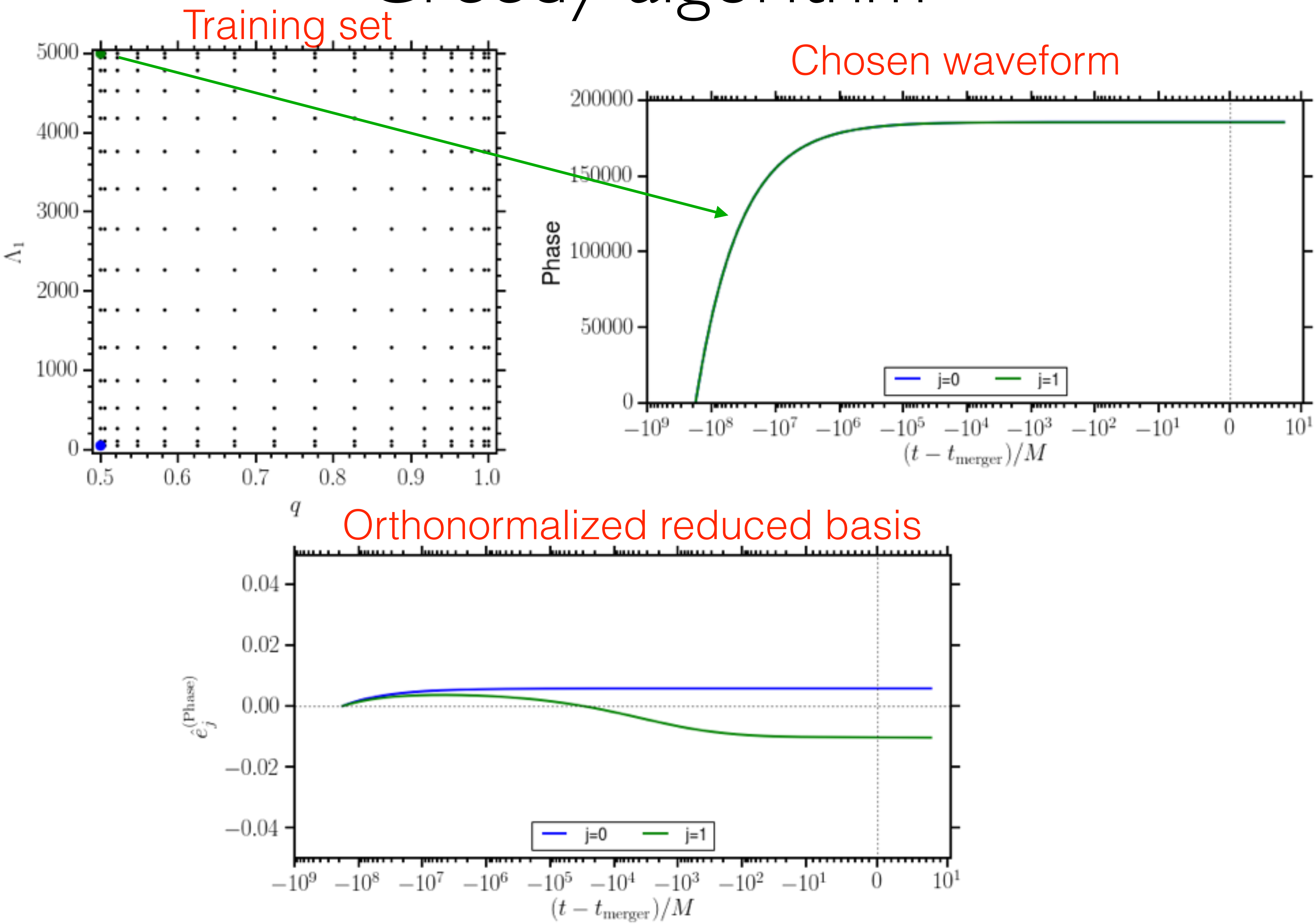
- 8-dimensional parameter space (1 mass and 3 tidal parameters for each NS)
    - Can use mass ratio q and rescale waveform with total mass M
    - Use 10% accurate fits for l=3,4 tidal parameter in terms of l=2 tidal parameter
    - Results in 3 dimensional parameter space $\vec{\theta} = \{q, \Lambda_1, \Lambda_2\}$



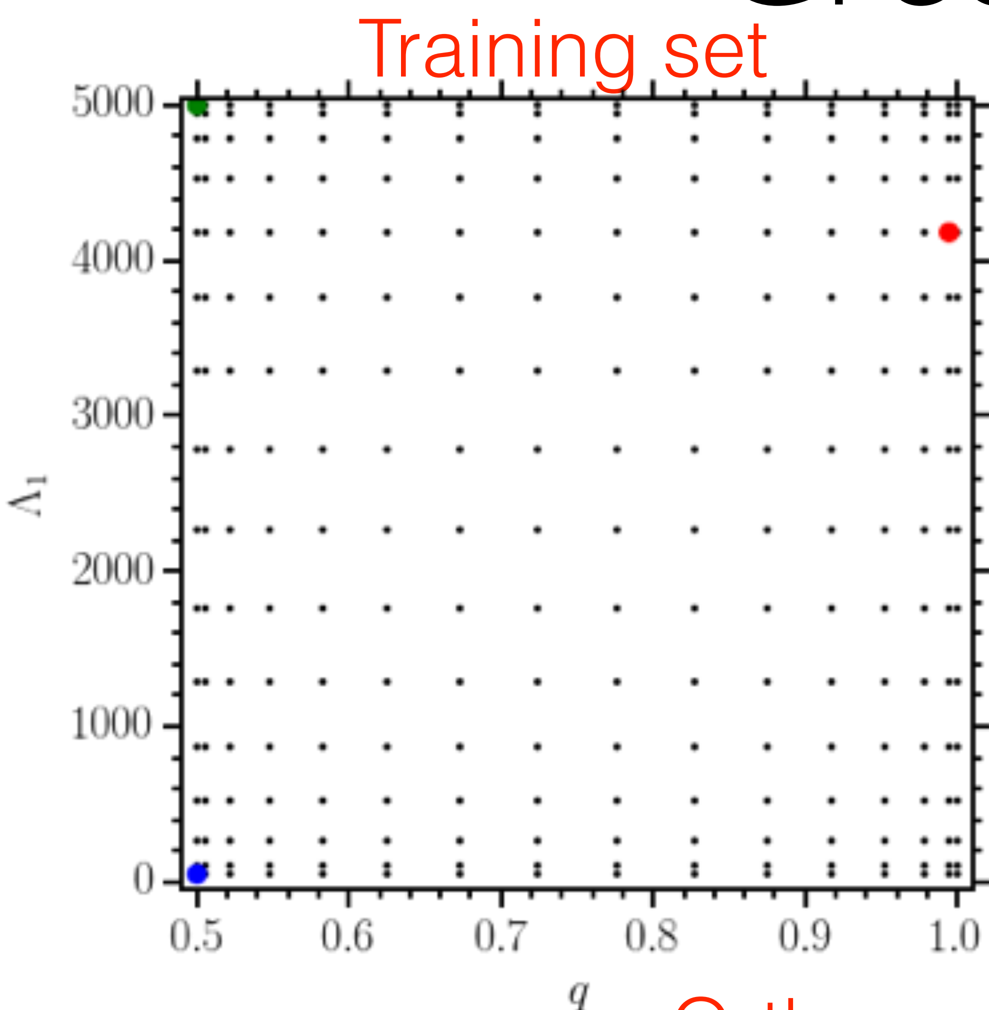
Greedy algorithm

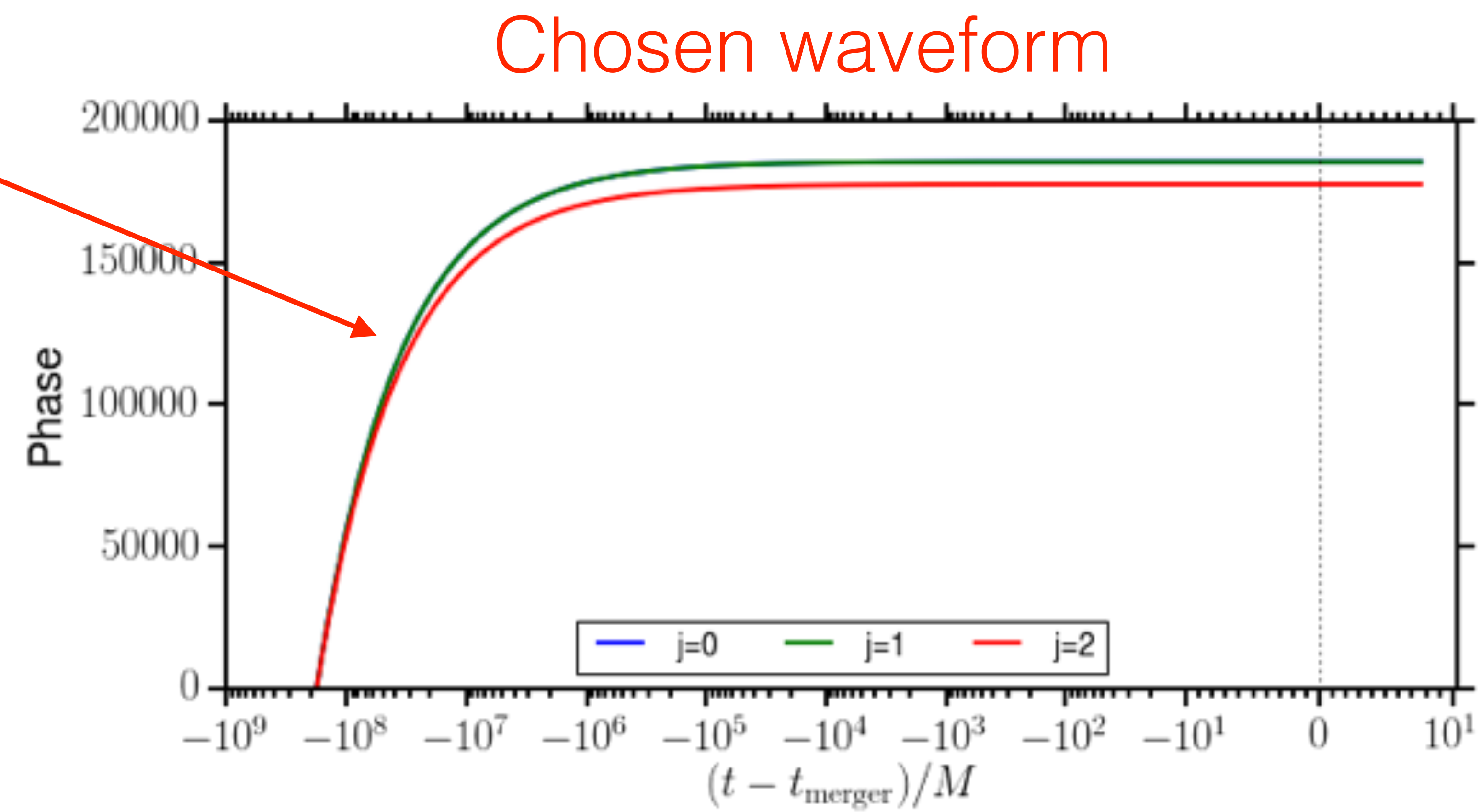


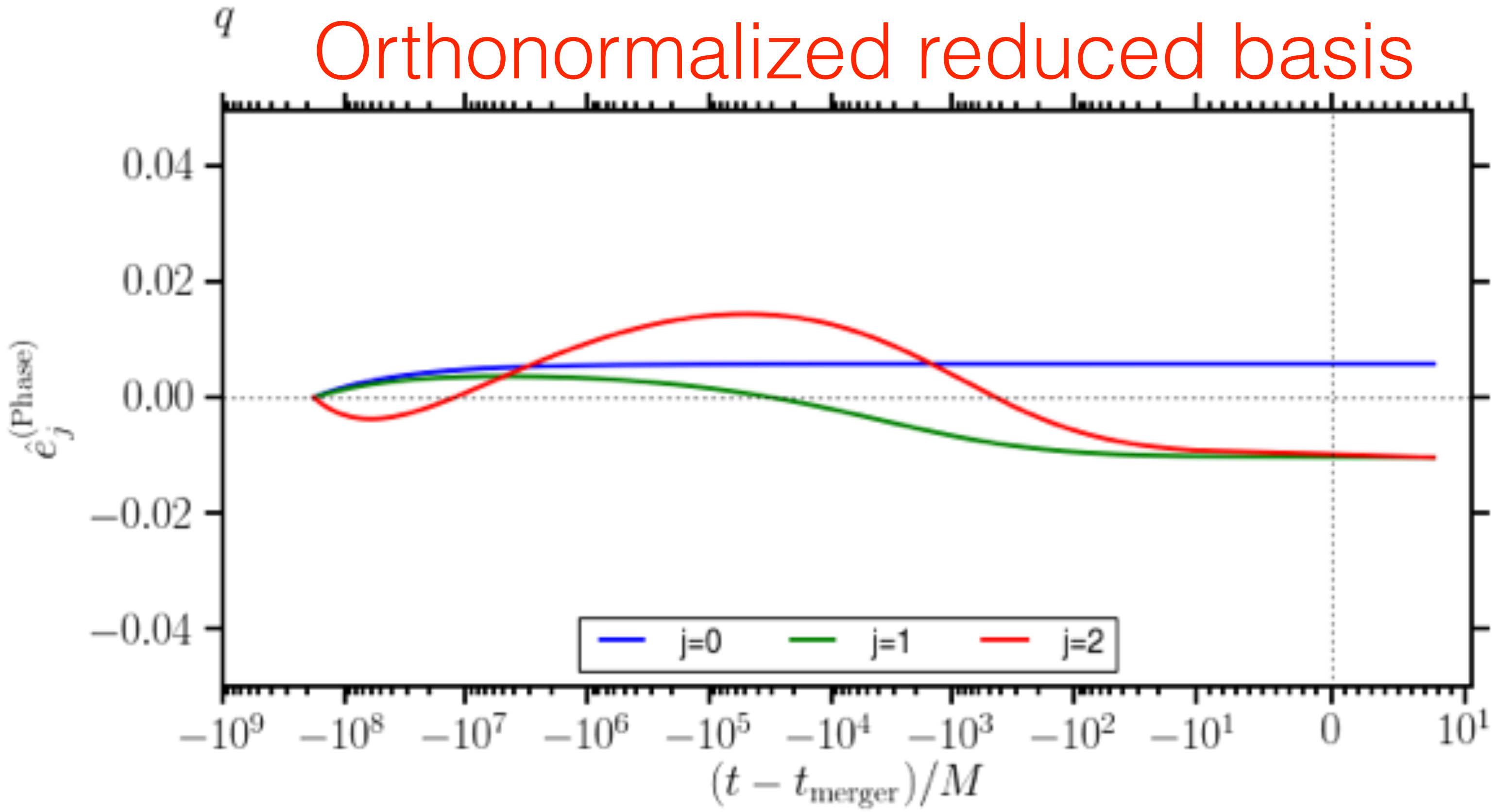
Greedy algorithm



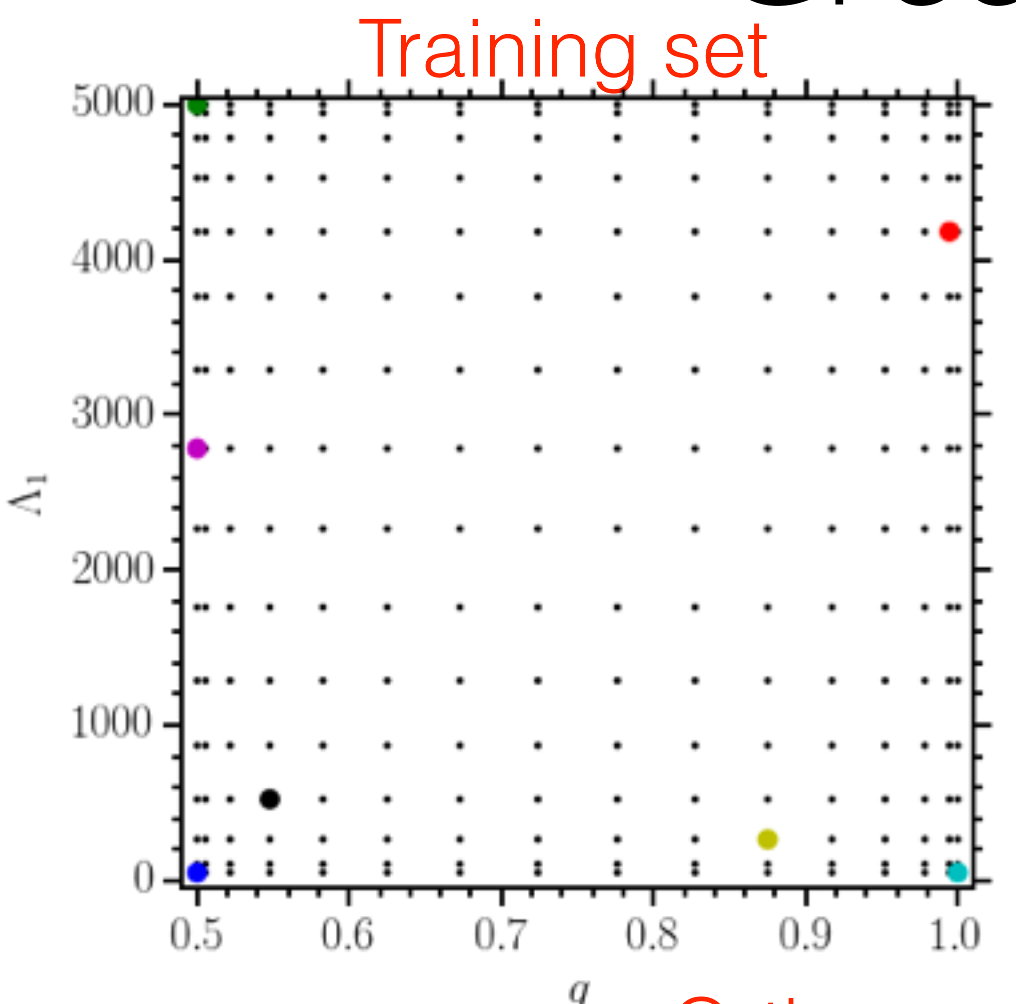
Greedy algorithm

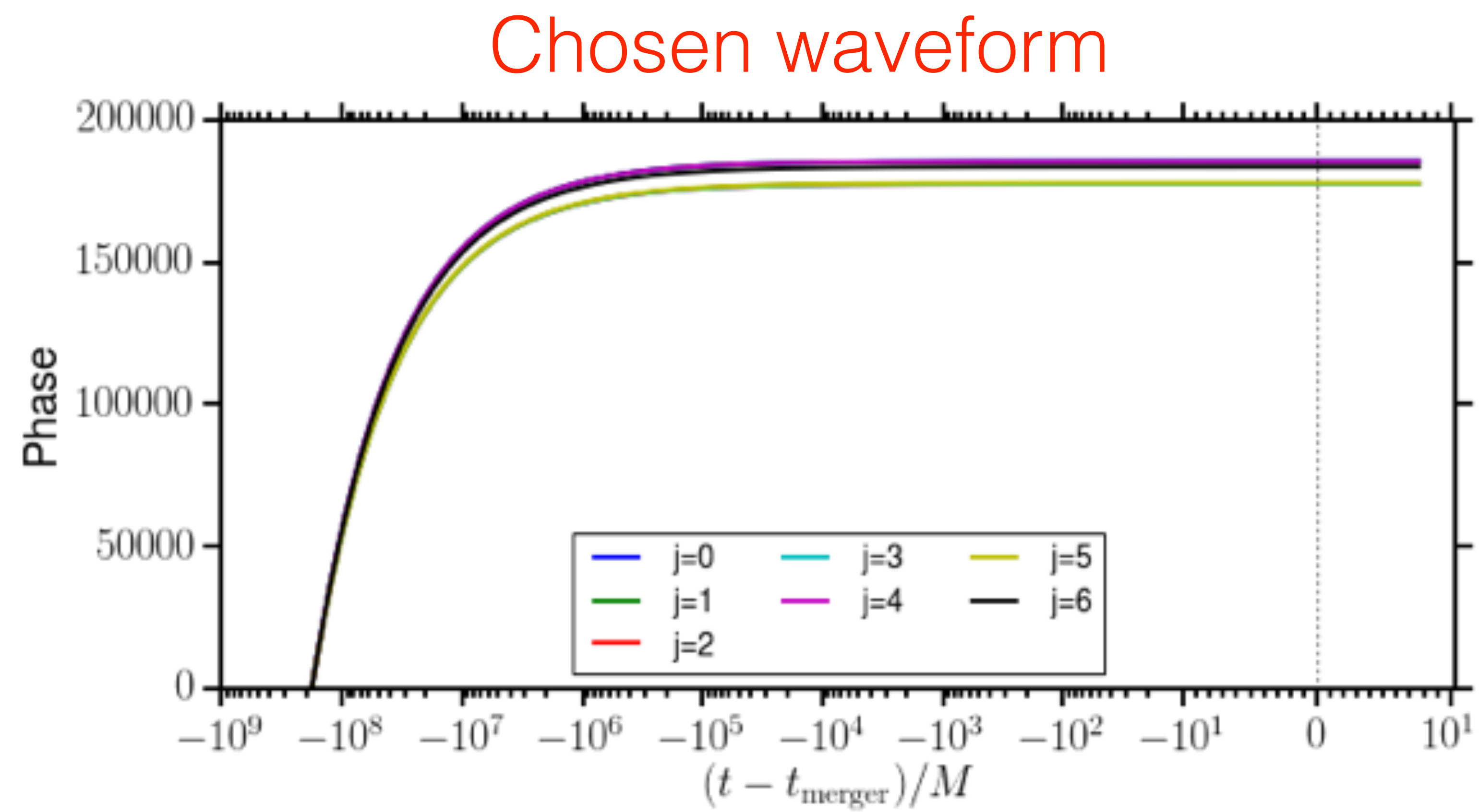


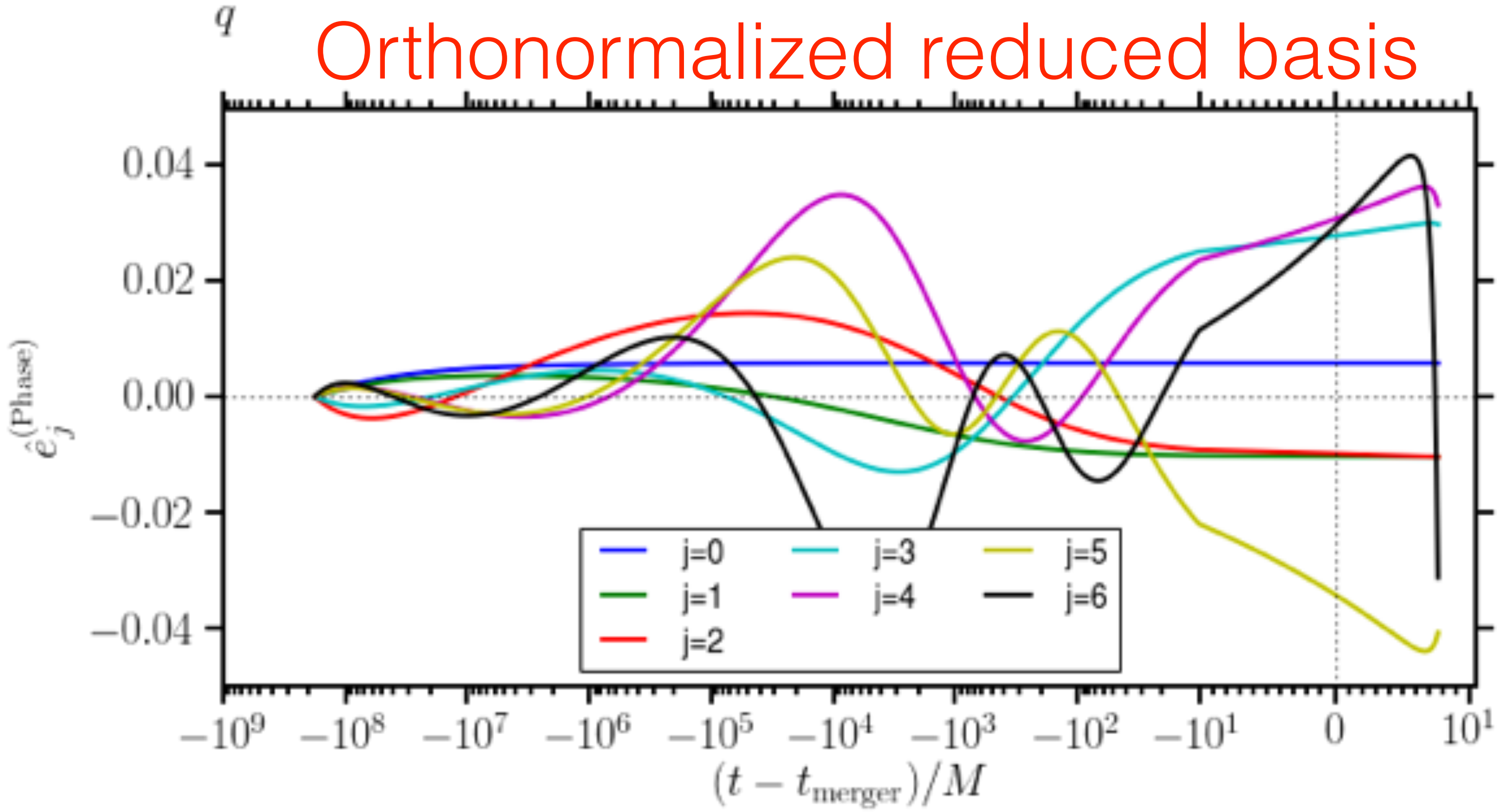




Greedy algorithm





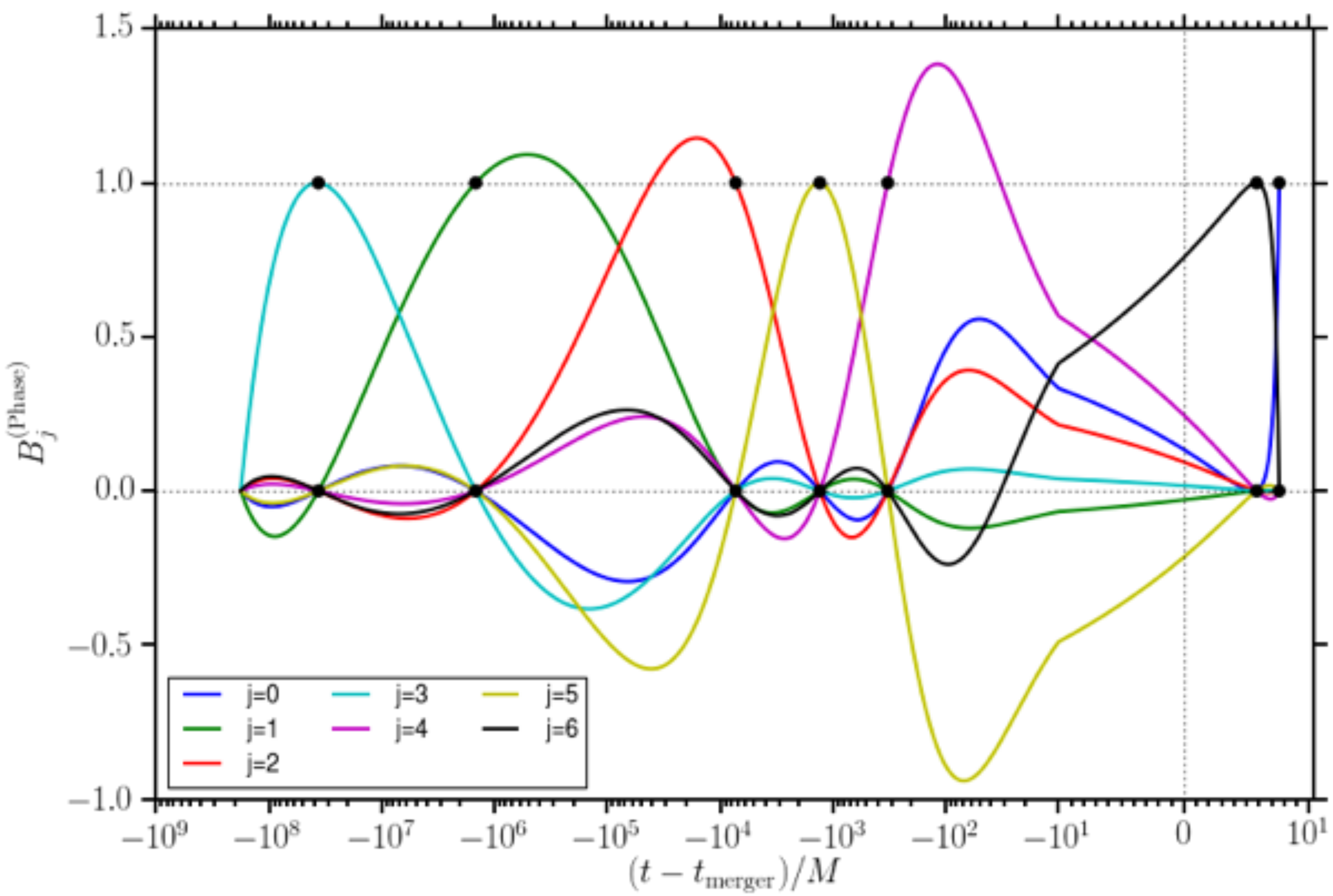


Empirical interpolation method

- Can re-express phase basis $\{e_i(t)\}$ in terms of waveform evaluated at empirical times T_j and empirical interpolants $B_j(t)$

$$\Phi(t;\vec{\theta}) \approx \sum_{j=1}^{n} \Phi(T_j;\vec{\theta}) B_j(t)$$

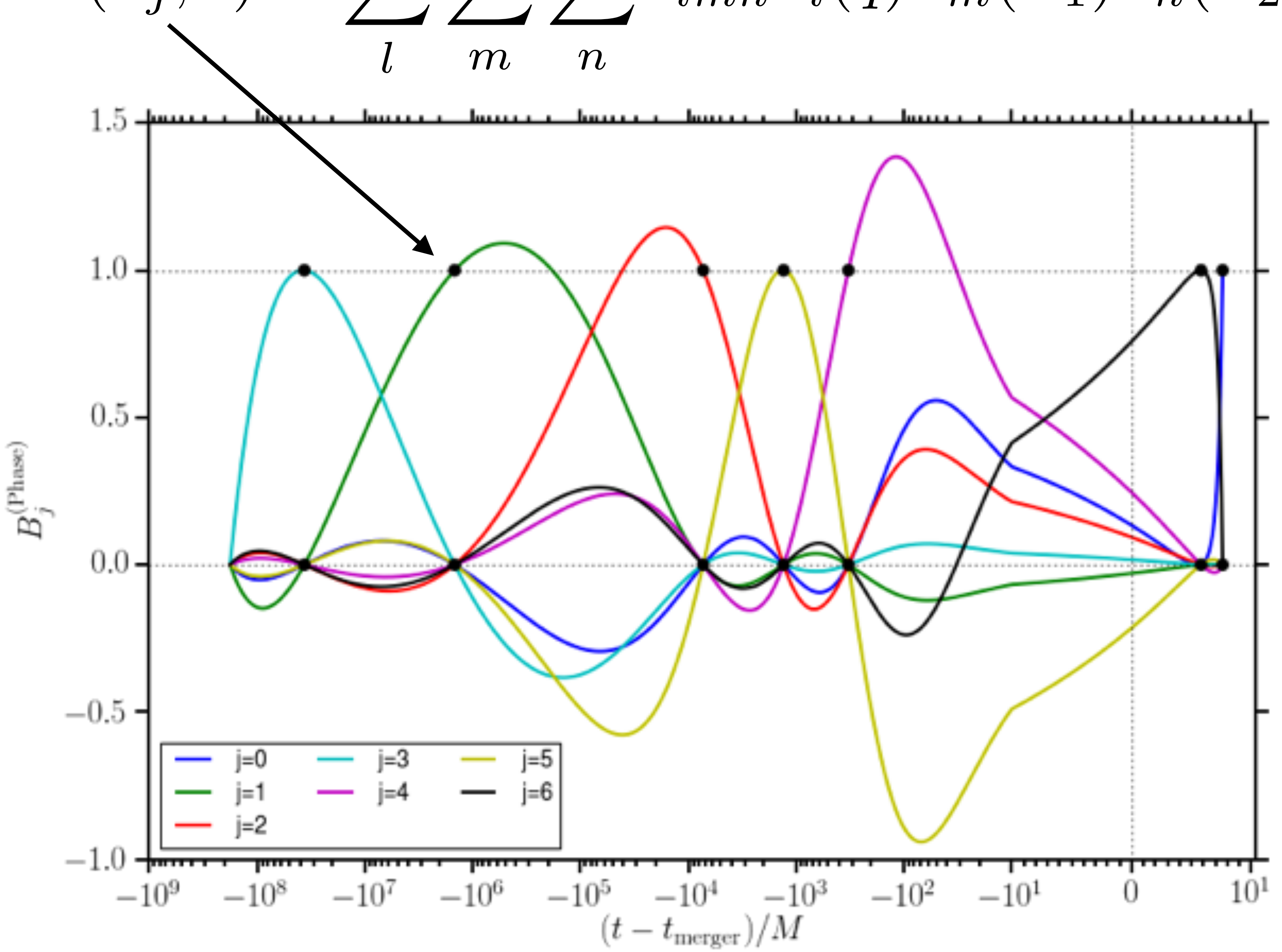
  - where $B_j(t) = \sum_{i=1}^{n} \hat{e}_i(t)(V^{-1})_{ij}$ and $V_{ji} = \hat{e}_i(T_j)$



Interpolating waveform parameters

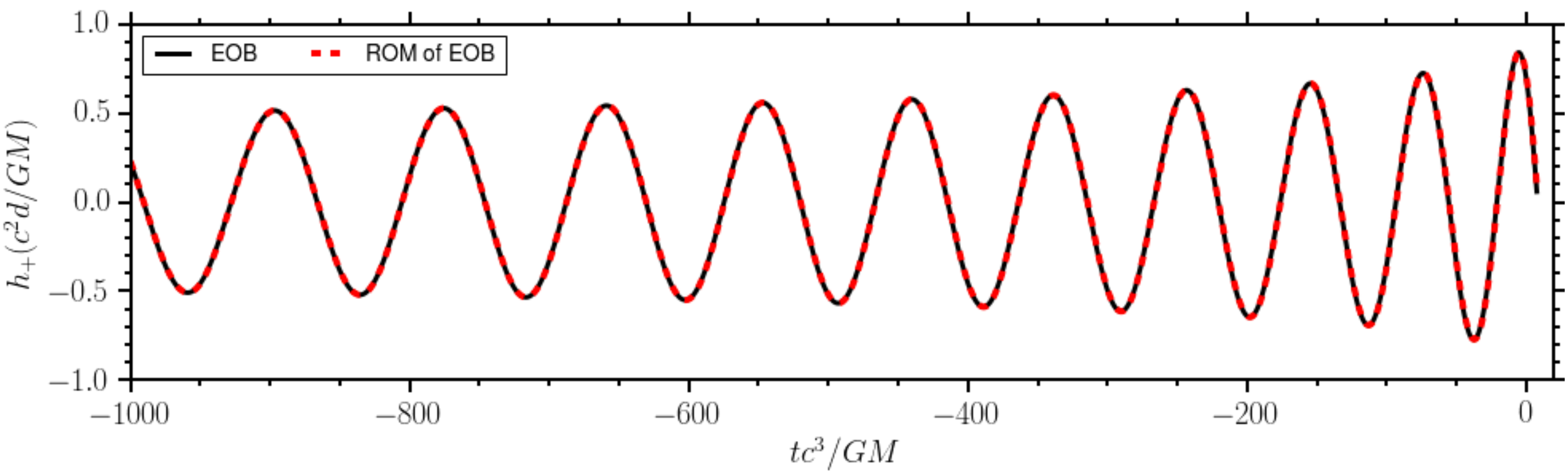
- Waveform at each empirical node T_j interpolated with Chebyshev polynomials
- Coefficients b_{lmn} calculated with Gaussian quadrature

$$\Phi(T_j;\vec{\theta}) = \sum_l \sum_m \sum_n b_{lmn} T_l(q) T_m(\Lambda_1) T_n(\Lambda_2)$$



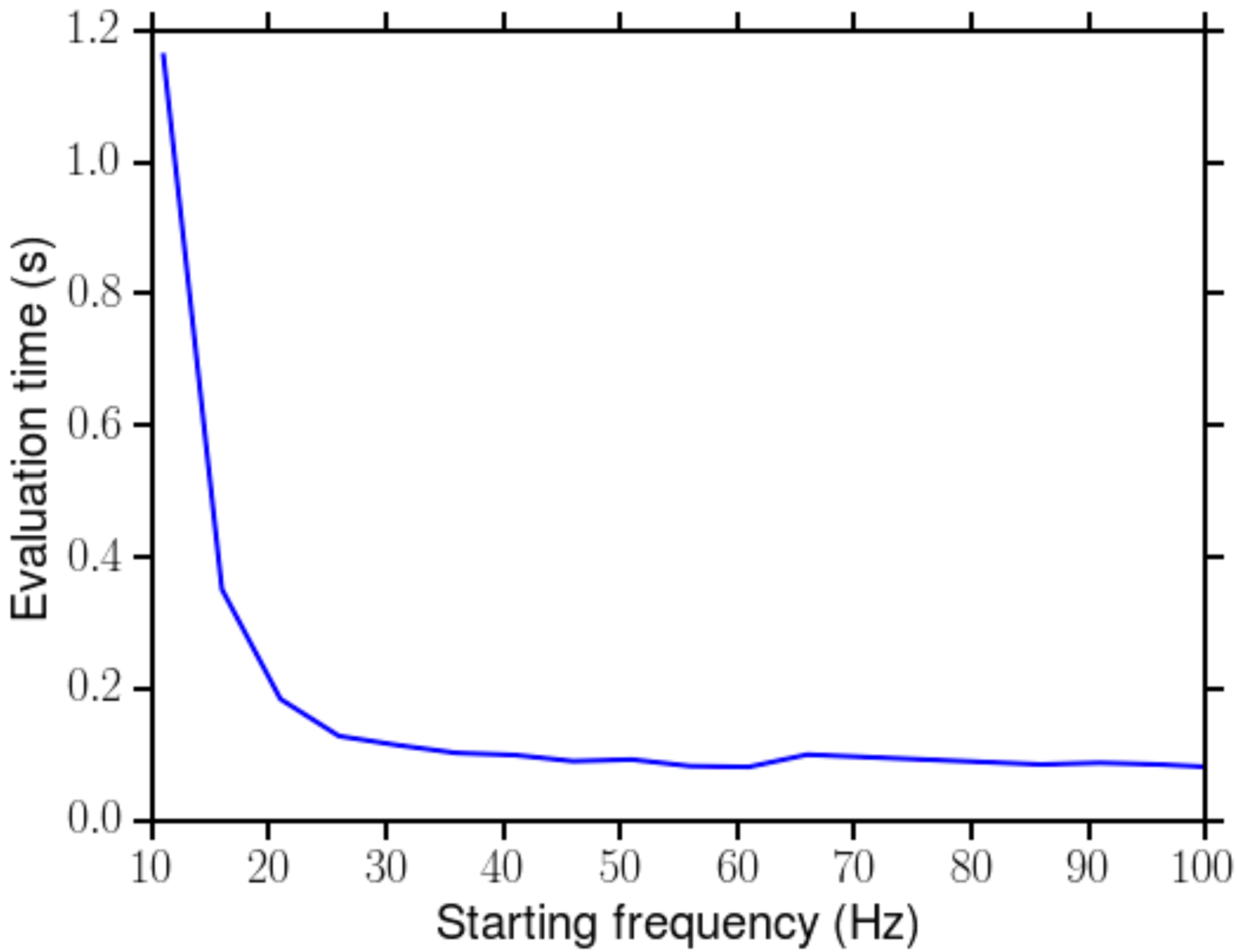
Accuracy of ROM

- Compare ROM to 10,000 waveforms randomly sampled in parameter space
    - Maximum amplitude error: ~2% (0.2% before last cycle)
    - Maximum phase error: ~0.04 radians
        - Much less than the tidal effect of ~5-10 radians



Speed of ROM

- Implementation of ROM in LAL (written in C)
  - Faster than original Matlab tidal EOB code by a factor of ~1000
  - Faster than all time-domain waveforms in LAL

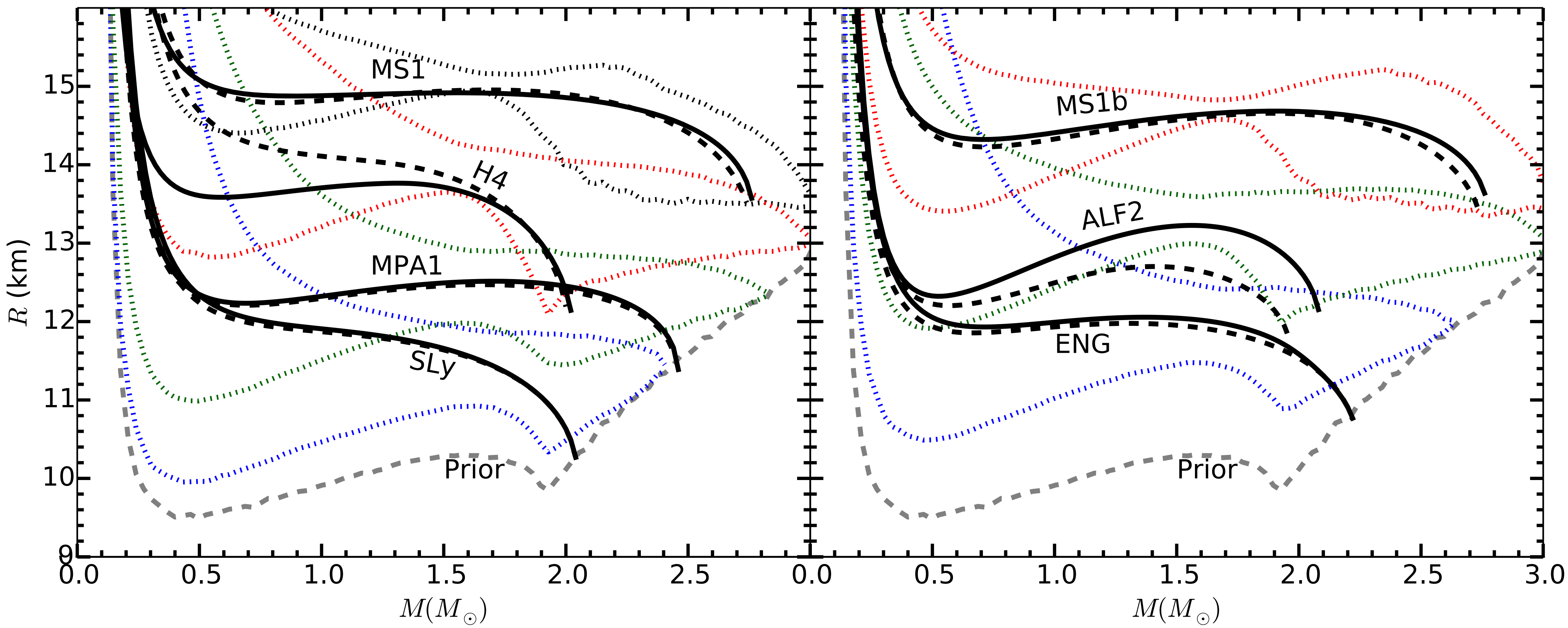


Conclusions

- EOS and NS structure can be recovered with ~1 year of BNS observations
- But, it's crucial that we have accurate and fast waveform models

Thank you

Other EOS models



- 95% credible regions

Other EOS models