ESTIMATING AND CHECKING TRUNCATION ERRORS IN EFFECTIVE FIELD THEORY

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for the BUQEYE collaboration
(Bayesian Uncertainty Quantification: Errors for Your EFT)

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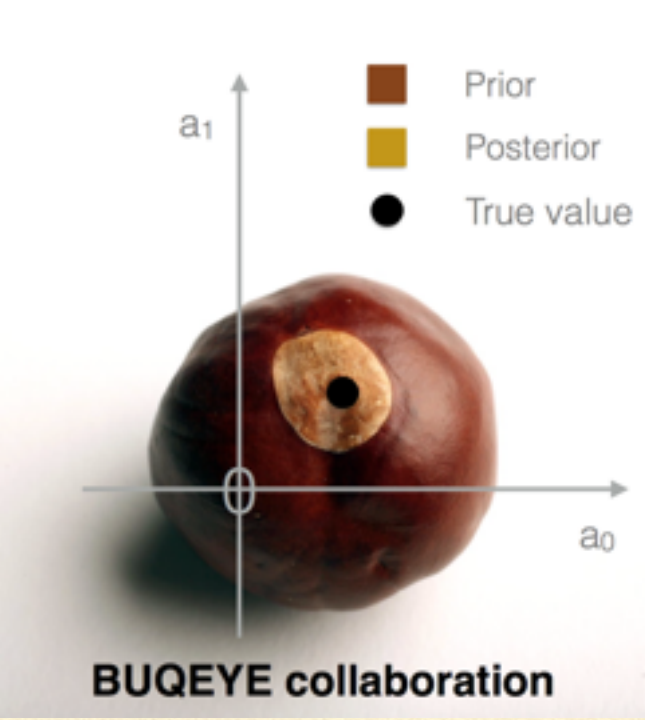

Research supported by the US DOE and NSF

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- Simpler theory that reproduces results of full theory at long distances
- Short-distance details irrelevant for long-distance (low-momentum) physics, e.g. multipole expansion
- Expansion in ratio of physical scales: p/Λ_b
- Symmetries of underlying theory limit possibilities: all possible terms up to a given order present in EFT
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Control over unknown short-distance dynamics
$\Rightarrow$error grows as first omitted term in expansion

A GENERIC EFT

$$g(x) = \sum_{i=0}^{k} \mathcal{A}_i(x) x^i$$

$$x = \frac{p}{\Lambda_b}$$

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- $f_i(x,\mu)$ is a calculable function, that encodes IR physics at order i
- c_i is a low-energy constant (LEC): encodes UV physics at order i. Must be fit to data
- Complications: multiple light scales, multiple functions at a given order, skipped orders, ….

TOY MODEL

$$y(x) = \sum_{i=0}^{k} c_i x^i + \delta(x) + \epsilon; \quad \delta(x) = \sum_{i=k+1}^{k_{\max}} c_i x^i$$

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- One light scale, non-analytic pieces absent

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- **Parameter estimation for LECs:** given data on y over a range of x how best to determine the $\{c_i\}$?
  - Fit range: manage trade-off between more data (decreased statistical error) and importance of $N^{k+1}LO$ (increased truncation error)
  - Overfitting (too many terms given data) and underfitting (too few terms to describe it)

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- Help others who must deal with the same issues

BAYESIAN TOOLS

Thomas Bayes (1701?-1761)

http://www.bayesian-inference.com

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Posterior Normalization

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Likelihood → $\mathrm{pr}(\mathrm{data}|x,I)$

Prior → $\mathrm{pr}(x|I)$

Posterior → $\mathrm{pr}(x|\mathrm{data},I)$

Normalization → $\mathrm{pr}(\mathrm{data}|I)$

Marginalization: $$\mathrm{pr}(x|\mathrm{data},I) = \int dy\, \mathrm{pr}(x,y|\mathrm{data},I)$$

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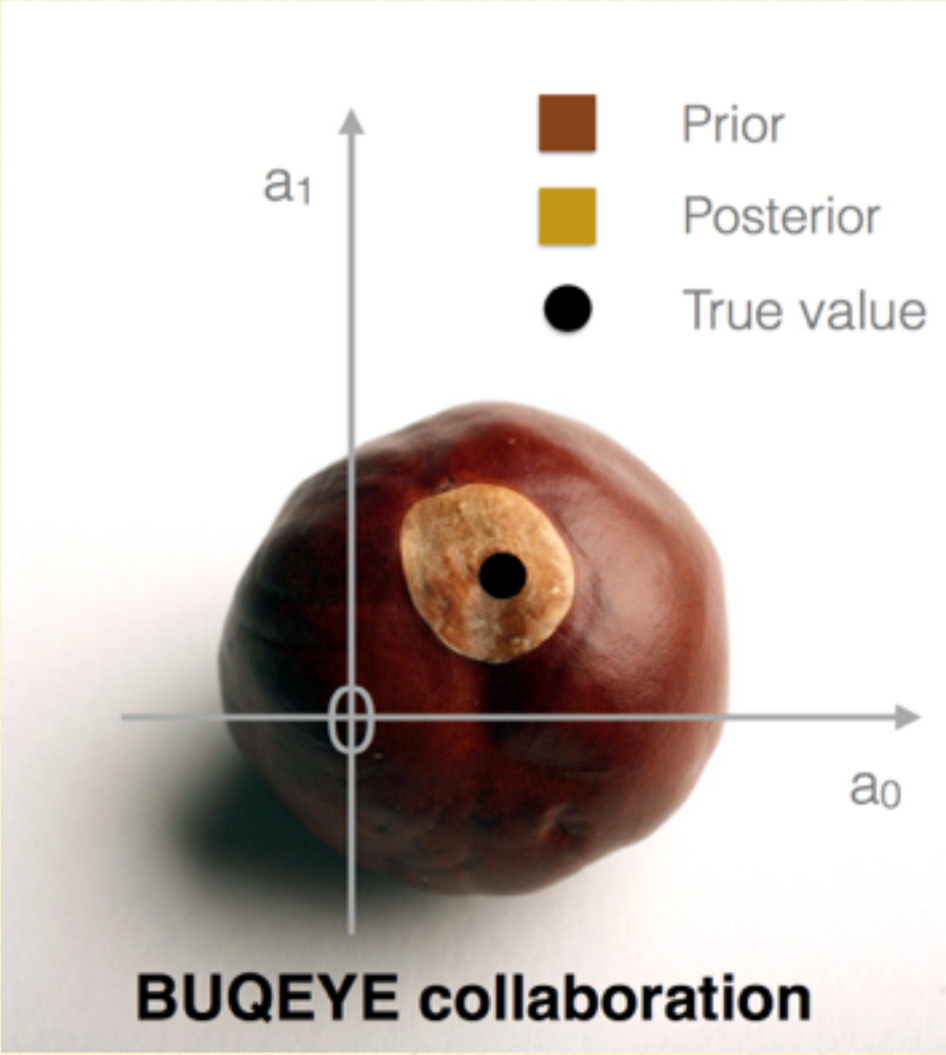
Allows us to integrate out "nuisance" (e.g. higher-order) parameters

OUTLINE

- Introduction: EFT and Bayes
- Truncation errors in χEFT
- Checking the residuals: error plots
- Evidence ratio
- Summary



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R. J. Furnstahl, N. Klco, D. R. Phillips and S. Wesolowski, Phys. Rev. C **92**, 024005 (2015)

S. Wesolowski, N. Klco, R. J. Furnstahl, D. R. Phillips and A. Thapaliya, J. Phys. G. **43**, 074001 (2016)

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- One possibility: $c_3=\max\{c_0,c_1,c_2\}$

Epelbaum, Krebs, Meissner (2014)
cf. McGovern, Griesshammer, Phillips (2013); many others.

PROBABILITY FOR EFT COEFFICIENTS

Furnstahl, Klco, DP, Wesolowski, PRC, 2015

after Cacciari and Houdeau, JHEP, 2011

- General EFT series for observable to order k: $X = X_0 \sum_{i=0}^{k} c_i x^i$
- Compute conditional probability distribution: pr(c_{k+1}|c_0,…,c_k,I)
- I=information about χEFT, e.g. naturalness
- "Prior A": $\mathrm{pr}(c_n|\bar{c}) = \frac{1}{2\bar{c}}\theta(\bar{c} - |c_n|);\ \mathrm{pr}(\bar{c}) = -\frac{1}{2\ln(\epsilon)\bar{c}}\theta\left(\frac{1}{\epsilon} - \bar{c}\right)\theta(\bar{c} - \epsilon)$
- Uniformly distributed coefficients up to maximum, maximum distributed uniformly in its logarithm. $\epsilon \to 0+$ at end
- Prior expectations will guide result, but they are not be all and end all
- Maximum of coefficients informed by known coefficients

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- This is generic, but the integrals are simple in the case of "Prior A"

$$\mathrm{pr}(\bar{c}|c_0, c_1, \ldots, c_k) \propto \begin{cases} 0 & \text{if } \bar{c} < \max\{c_0, \ldots, c_k\} \\ 1/\bar{c}^{k+2} & \text{if } \bar{c} > \max\{c_0, \ldots, c_k\} \end{cases}$$

$$\mathrm{pr}(c_{k+1}|c_0, c_1, \ldots, c_k) \propto \begin{cases} 1 & \text{if } c_{k+1} < c_{\max} \\ \left(\frac{c_{\max}}{c_{k+1}}\right)^{k+2} & \text{if } c_{k+1} > c_{\max} \end{cases}$$

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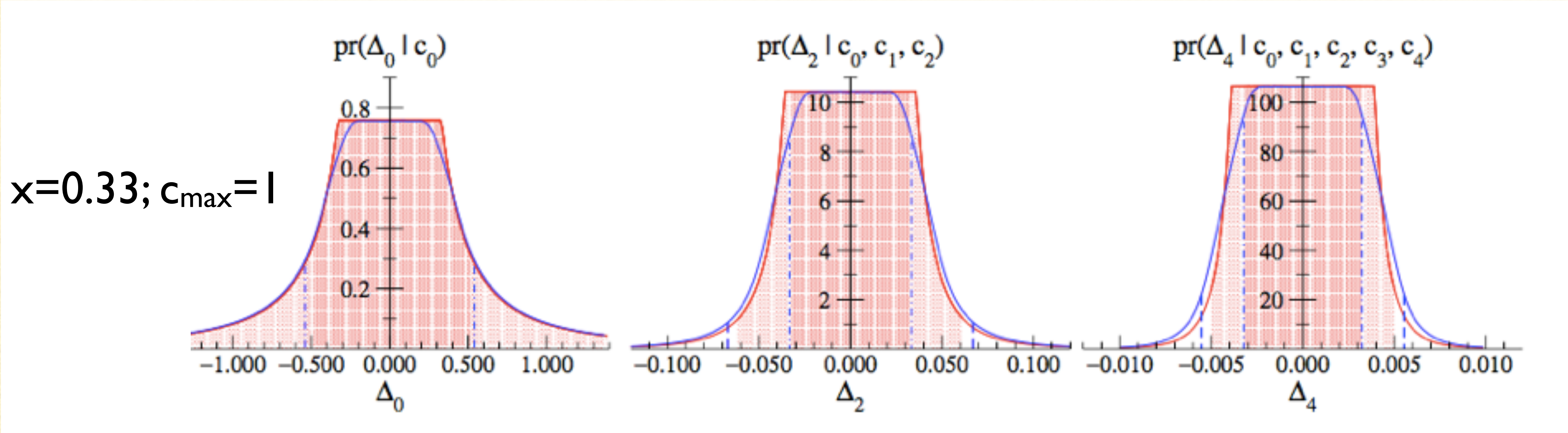
- $pr(\Delta_k) \propto X_0\, x^{k+1}\, pr(c_{k+1})$
- 68%, 95% DOB intervals from integration of probability distribution



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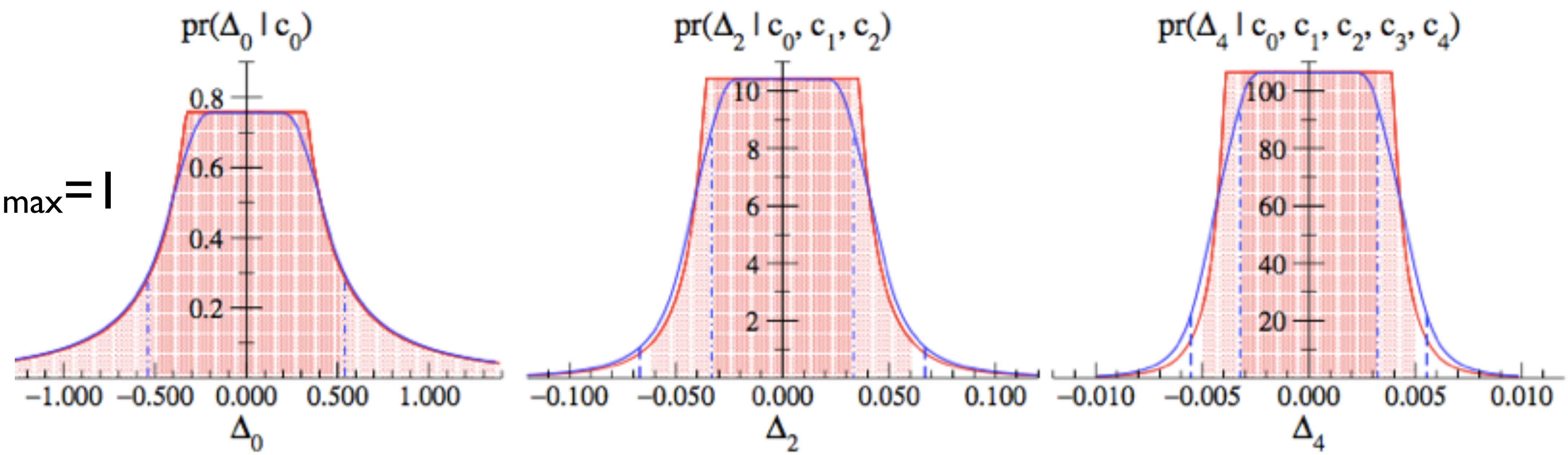

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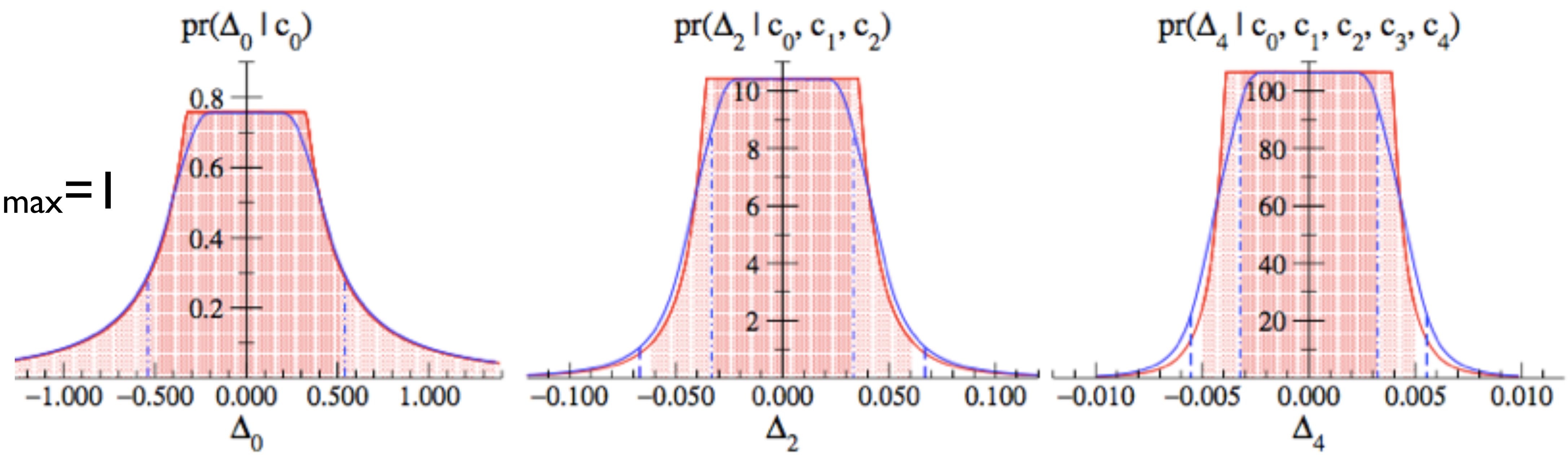

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- $[-c_{max} X_0\, x^{k+1}, c_{max} X_0\, x^{k+1}]$ is a $\dfrac{k+1}{k+2} * 100\%$ DoB interval

I DON'T LIKE THAT PRIOR!

- Modify Set A to restrict cbar to a finite range, e.g. $A_{[0.25,4]}$
- Set B: give cbar a log-normal prior: $\mathrm{pr}(\bar{c}) = \frac{1}{\sqrt{2\pi}\bar{c}\sigma} e^{-(\log \bar{c})^2/2\sigma^2}$
- Set C: $\mathrm{pr}(c_n|\bar{c}) = \frac{1}{\sqrt{2\pi}\bar{c}} e^{-c_n^2/2\bar{c}^2}$; $\mathrm{pr}(\bar{c}) \propto \frac{1}{\bar{c}}\theta(\bar{c} - \bar{c}_<)\theta(\bar{c}_> - \bar{c})$
- Same formulas as before can be invoked. Now numerical.

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- You don't like these? Pick your own and follow the rules…
- First omitted term approximation

EKM'S NN SCATTERING ANALYSIS

Epelbaum, Krebs, Meissner, PRC, 2015

χEFT: $\mathscr{L}$(N,π)→V$^{(k)}$→δ

$$\sigma_{np}(E_{\rm lab}) = \sigma_{\rm LO} \sum_{n=0}^{k} c_n(p_{\rm rel}) \left(\frac{p_{\rm rel}}{\Lambda_b}\right)^n$$

$$x = \frac{p_{\rm rel}}{\Lambda_b}$$

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- Here: R=0.9 fm data

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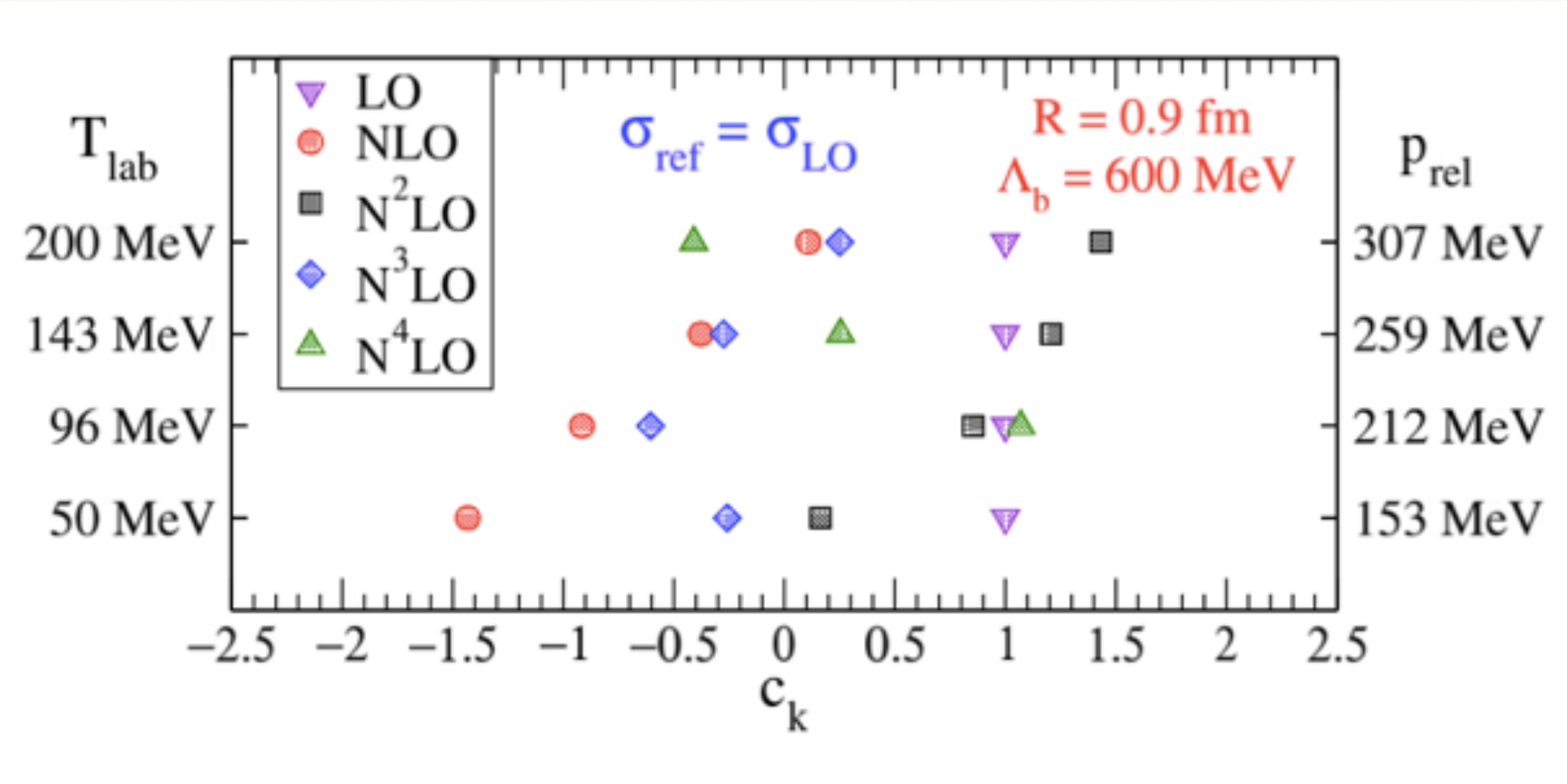

EKM'S NN SCATTERING ANALYSIS

Epelbaum, Krebs, Meissner, PRC, 2015

- NN cross section at T_{lab}=50, 96, 143, 200 MeV
- Potential regulated by local function, parameterized by R
- EKM identify Λ_b=600 MeV for smaller R values
- Here: R=0.9 fm data
- Results at LO, NLO, N^{2}LO, N^{3}LO, N^{4}LO (k=0, 2, 3, 4, 5)
- One outlier. Fitting procedure?

χEFT: $\mathscr{L}(N,\pi) \to V^{(k)} \to \delta$

$$\sigma_{np}(E_{\rm lab}) = \sigma_{\rm LO} \sum_{n=0}^{k} c_n(p_{\rm rel}) \left(\frac{p_{\rm rel}}{\Lambda_b}\right)^n$$

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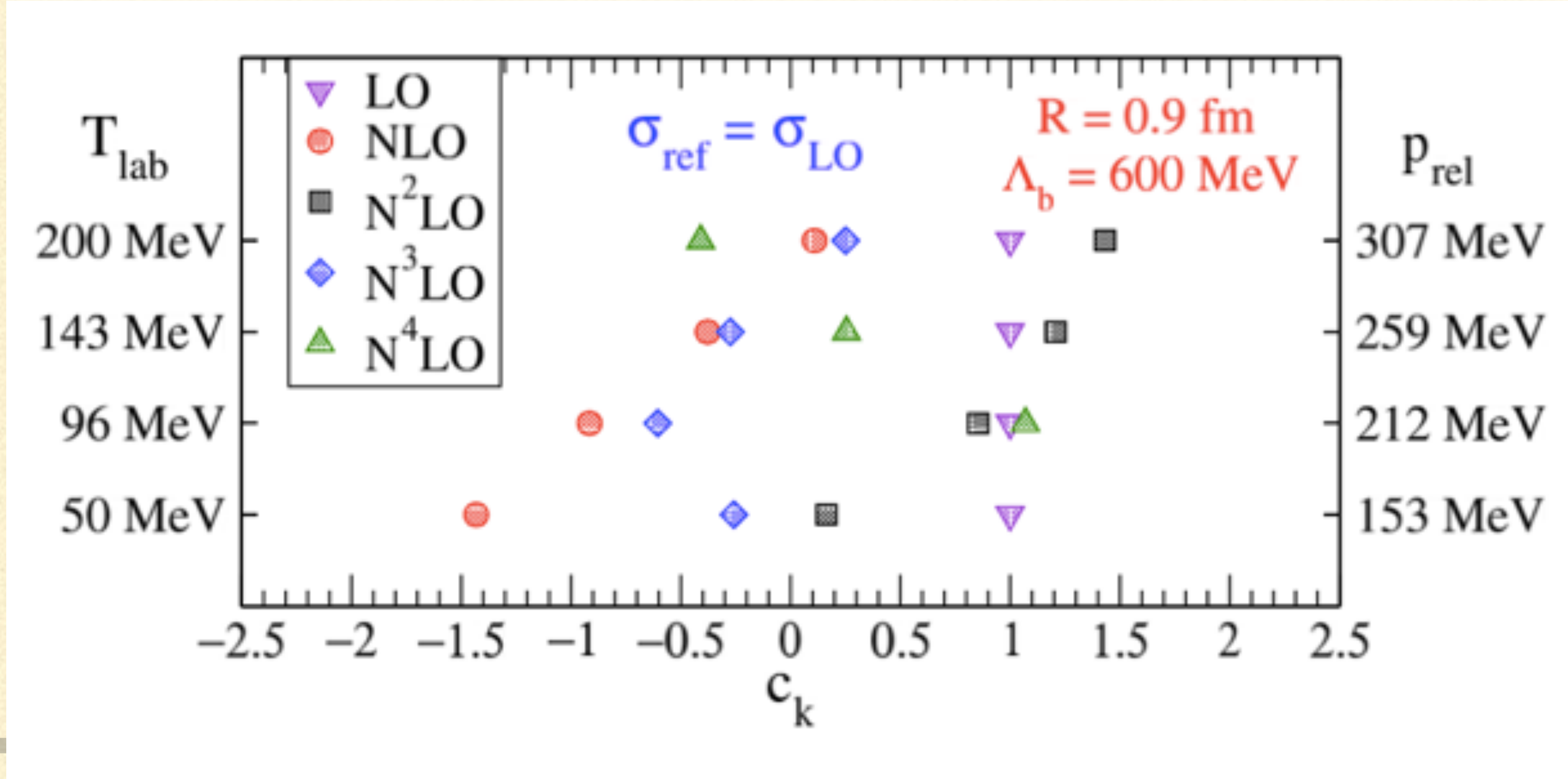
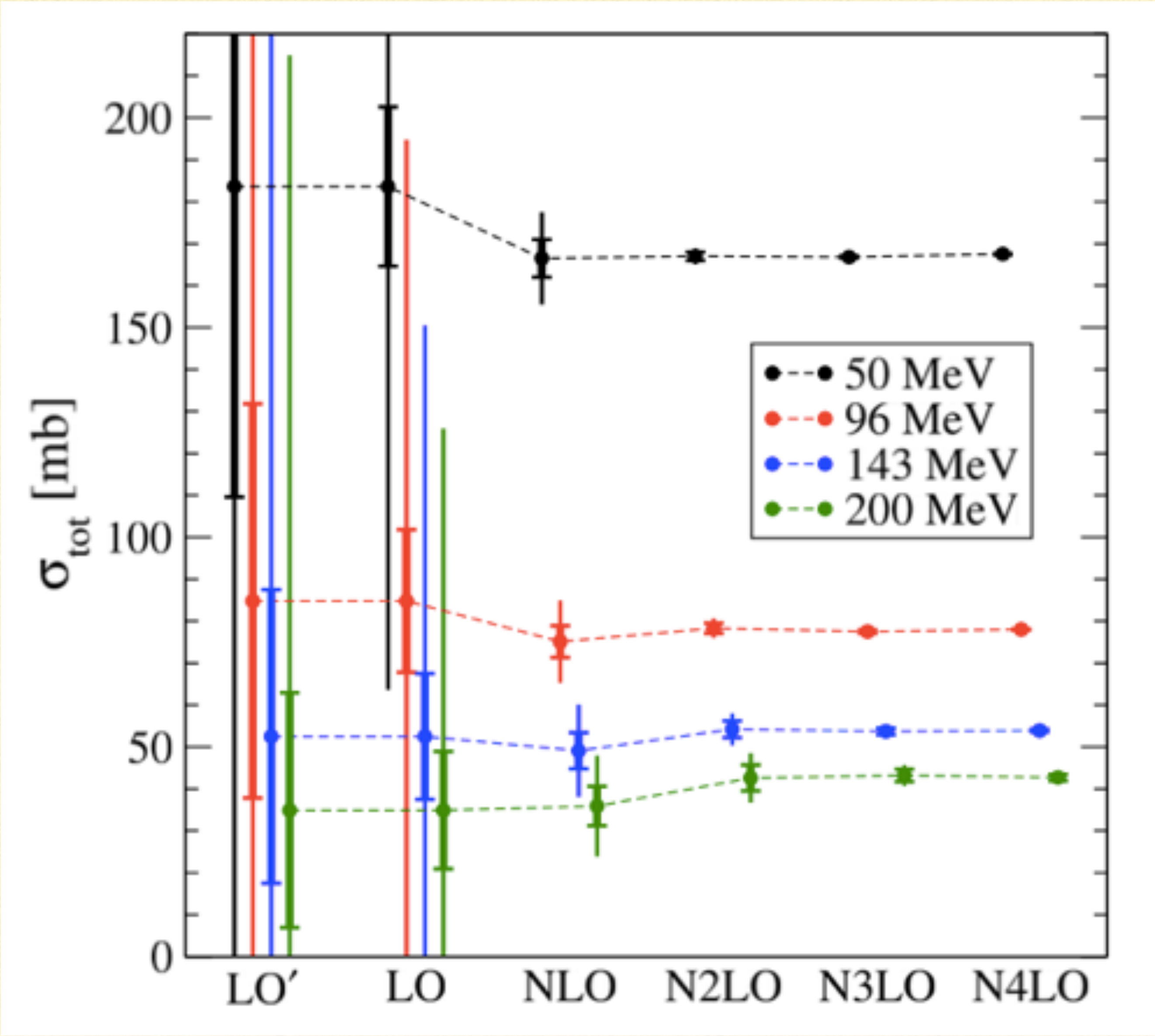
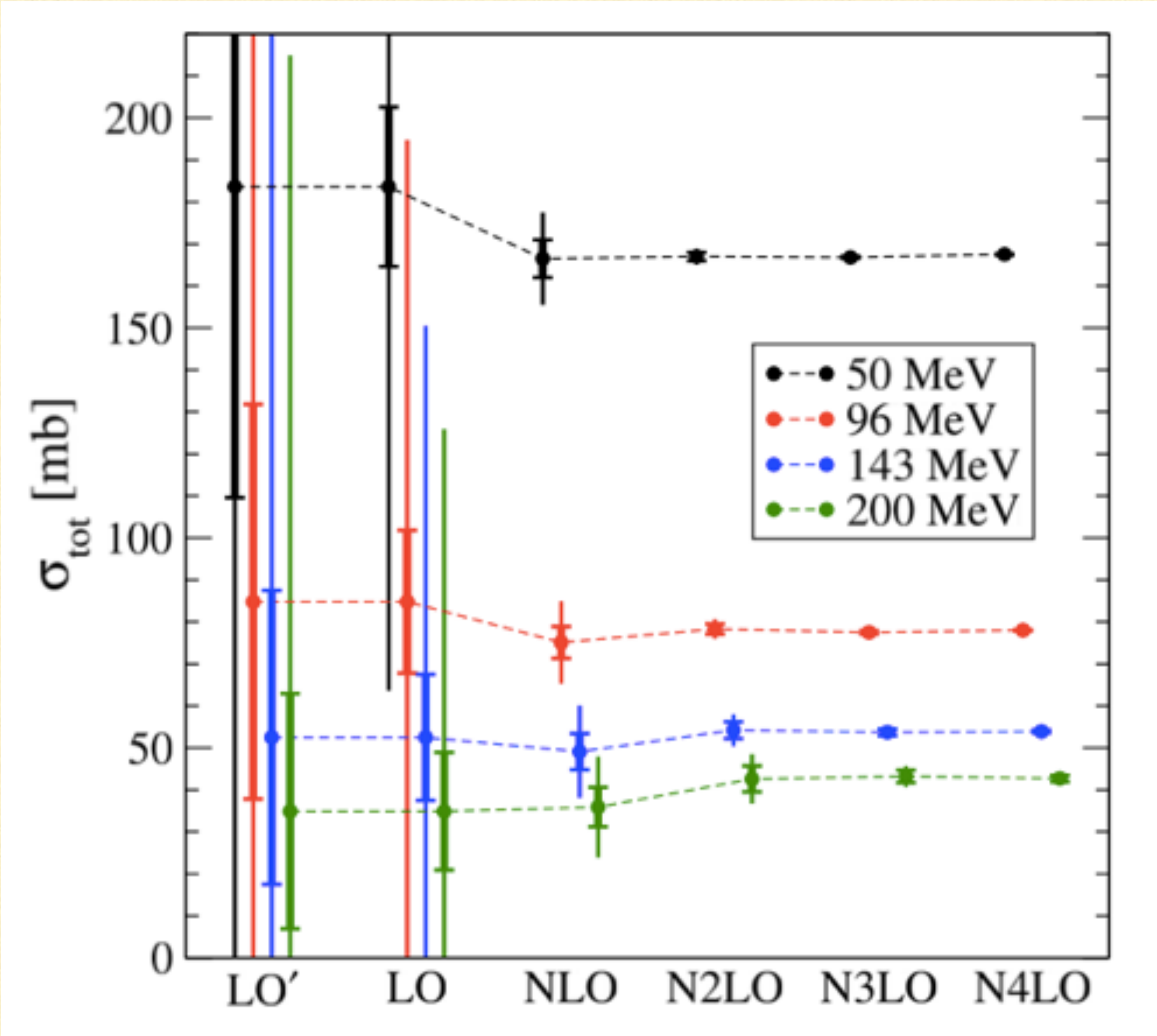


RESULTS

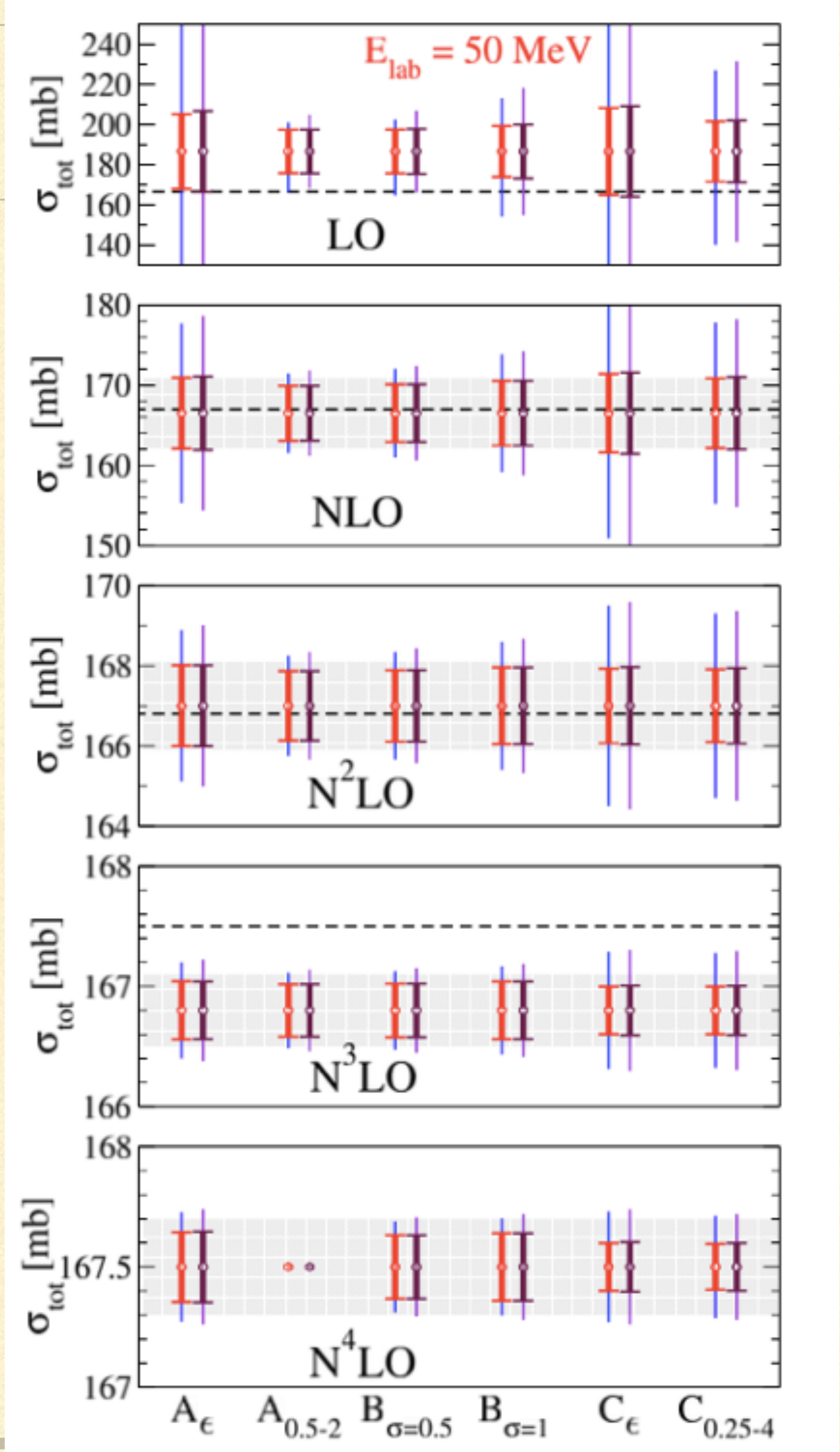


200
150
100
50
0
σ_{tot} [mb]
50 MeV
96 MeV
143 MeV
200 MeV
LO′
LO
NLO
N2LO
N3LO
N4LO

RESULTS

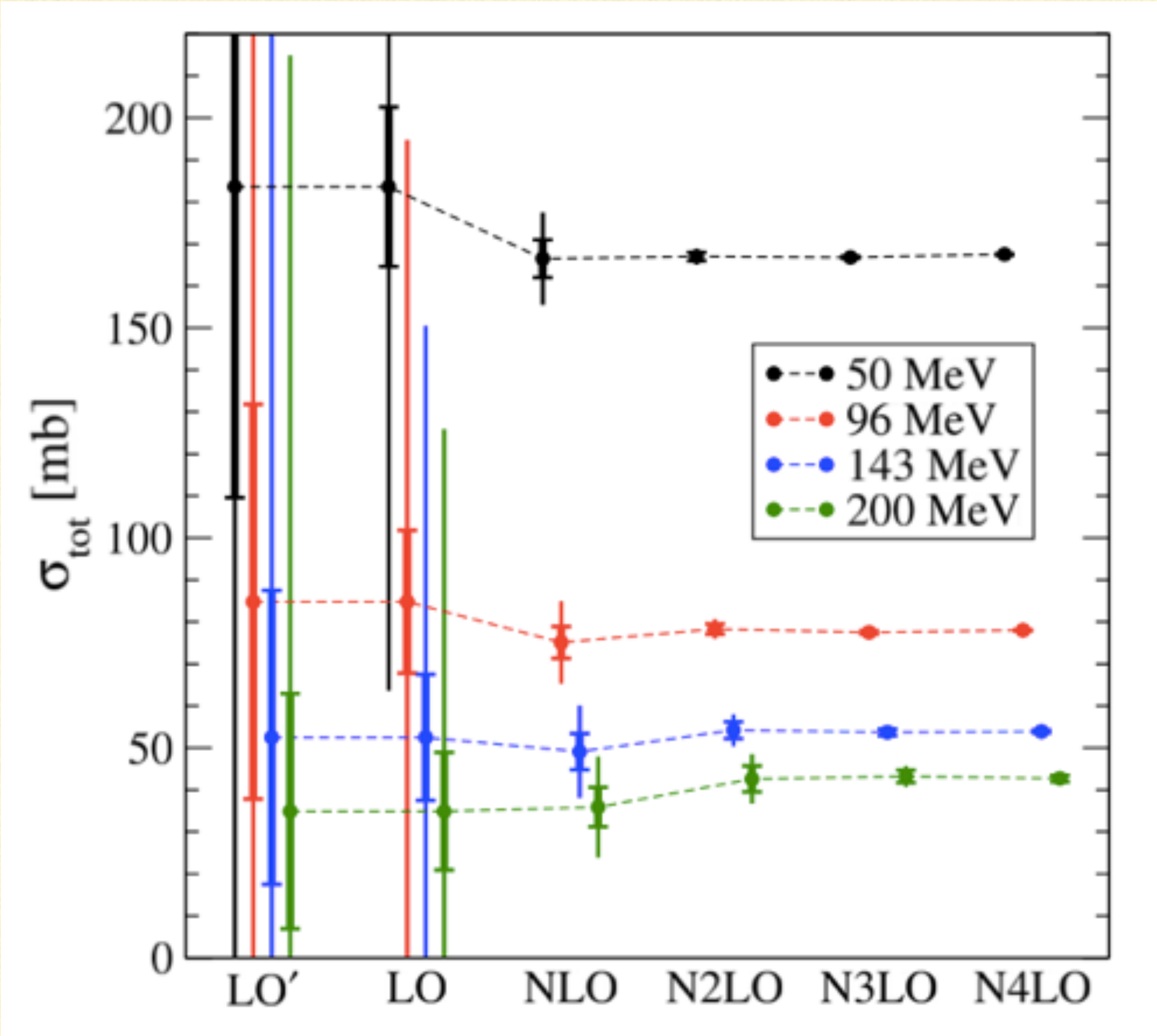


σtot [mb]
0
50
100
150
200
LO′
LO
NLO
N2LO
N3LO
N4LO
50 MeV
96 MeV
143 MeV
200 MeV

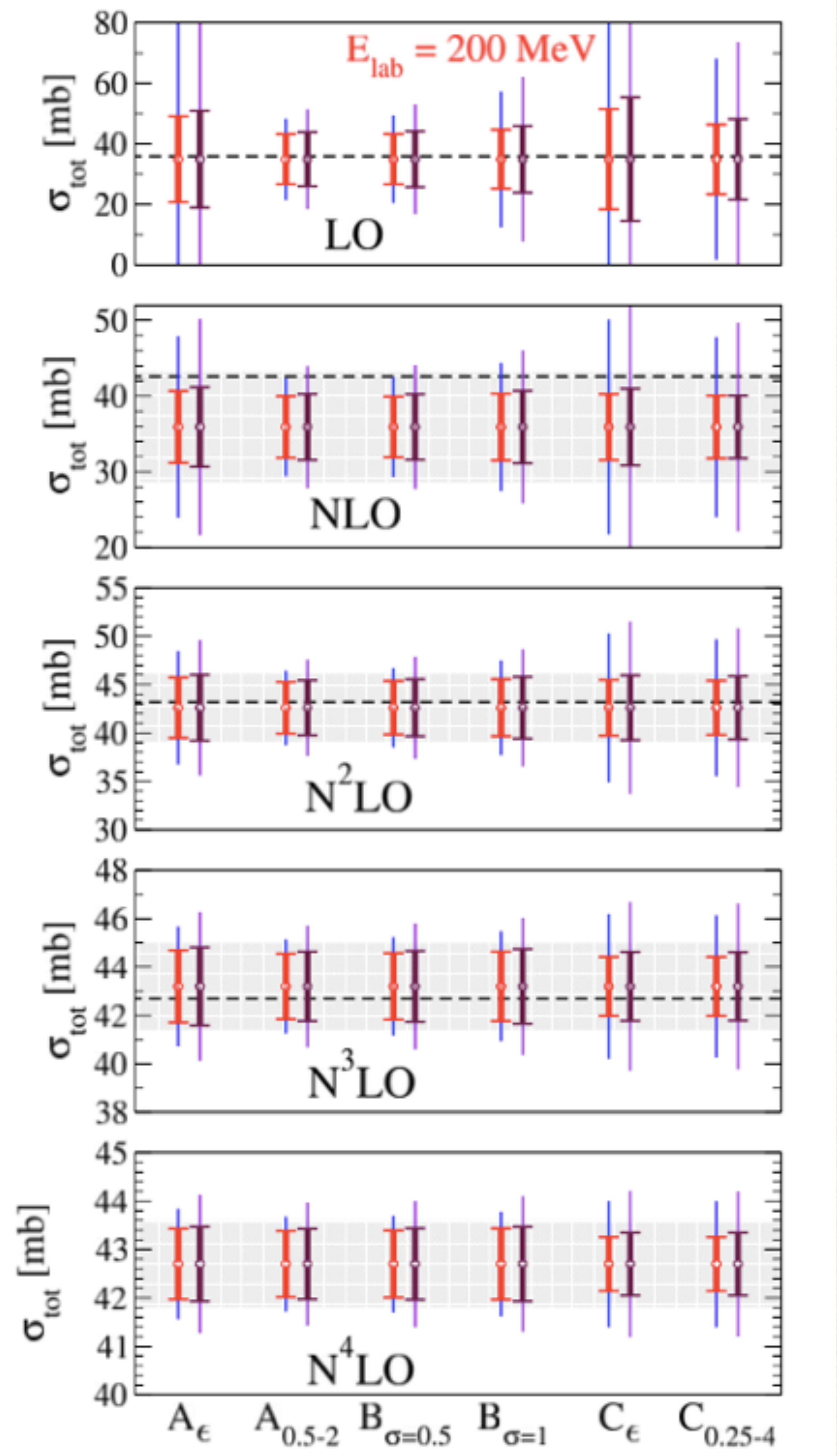


Elab = 50 MeV
σtot [mb]
LO
NLO
N2LO
N3LO
N4LO
Aϵ
A0.5-2
Bσ=0.5
Bσ=1
Cϵ
C0.25-4

RESULTS



σ_tot [mb]
0
50
100
150
200
LO′
LO
NLO
N2LO
N3LO
N4LO
50 MeV
96 MeV
143 MeV
200 MeV

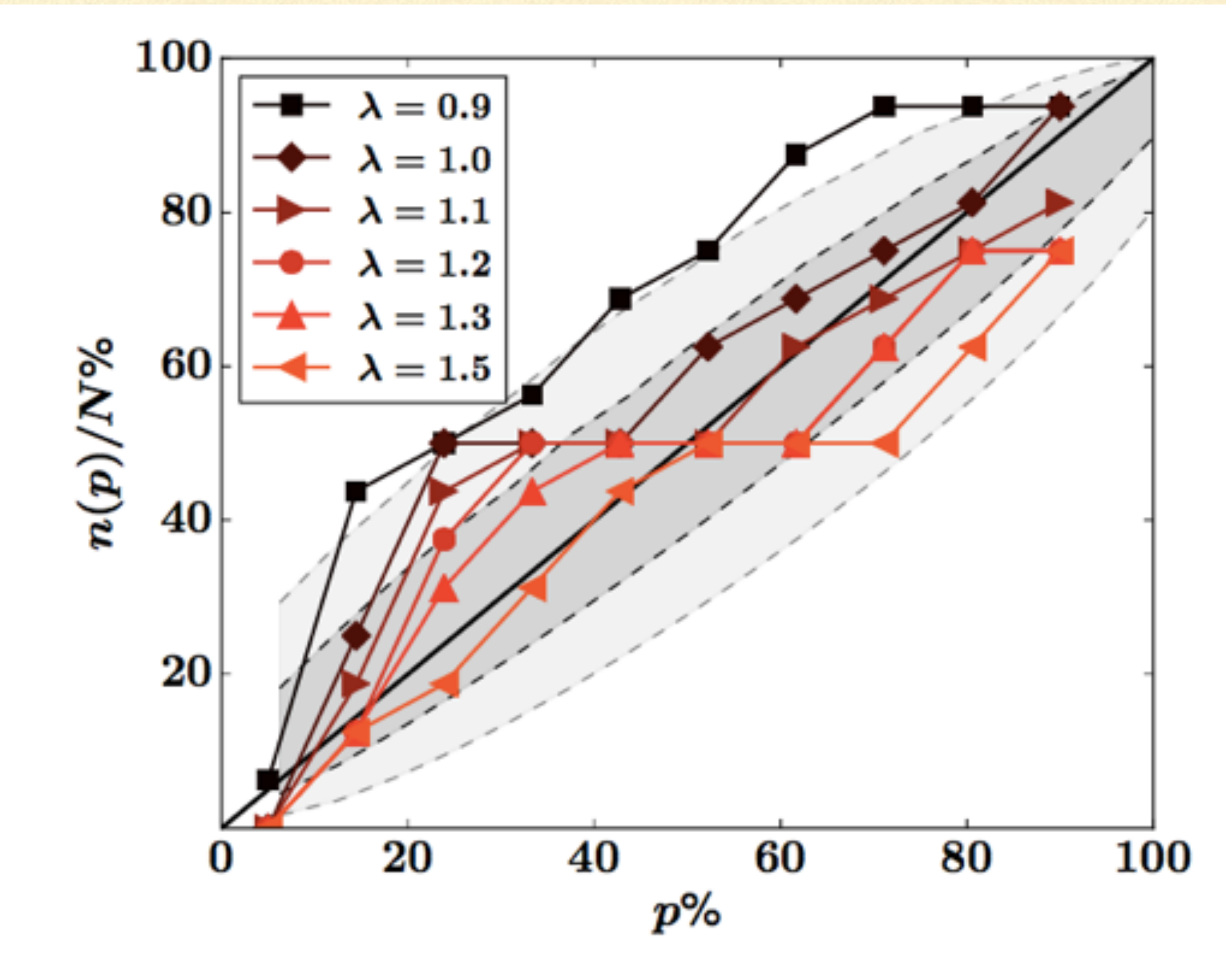


E_lab = 200 MeV
σ_tot [mb]
LO
NLO
N²LO
N³LO
N⁴LO
A_ε
A_0.5-2
B_σ=0.5
B_σ=1
C_ε
C_0.25-4

CONSISTENCY?

Furnstahl, Klco, DP, Wesolowski, PRC, 2015

after: Bagnaschi, Cacciari, Guffanti, Jenniches, 2015



- Now we consider predictions at each order, with their error bars, as data and test them to see if the procedure is consistent
- Fix a given DOB interval, compute actual success ratio and compare
- Look at this over EKM predictions at four different orders and four different energies
- Interpret in terms of rescaling of Λ_b by a factor λ

No evidence for significant rescaling of Λ_b

NORMAL NATURALNESS?

- Treat 19 coefficients as data and test for naturalness.
- Approach 1: coefficients should be normally distributed around a mean μ with a variance σ^2.

Forte, Isgro, Vita, PLB, 2014

- Approach 2: see if χ^2 has size expected, assuming μ=0 and a particular σ.

$$\chi^2 = \sum_{i=1}^{N_O} \sum_{n=0}^{k} \left(\frac{|c_n^{(i)}|\lambda^n - \mu}{\sigma} \right)^2$$

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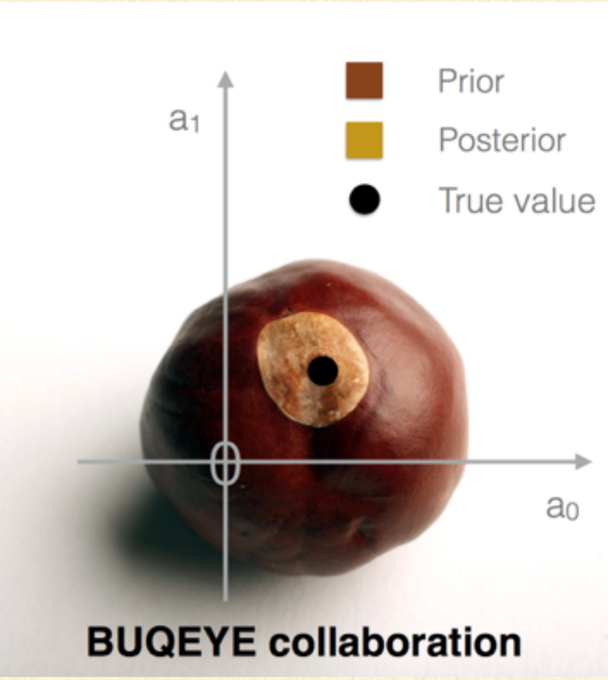
- Approach 1 (σ^2=1): $\lambda = 1.01^{+0.18}_{-0.19}$
- Approach 2 (σ^2=1): λ=1.09 gives χ^2=19. λ=1.01→1.15 consistent.

No evidence for significant rescaling of Λ_b

CAVEATS

- Naturalness of c_i's in x-expansion for NN cross section assumed. Justified for perturbative process, but justification not so clear for NN
- m_π not included in x (anticipate this is only a small effect)
- We looked at results only for one R; at larger Rs the regulator effects dominate and:
  - The distribution $\{c_n\}$ is qualitatively different;
  - Λ_b is identified as lower by EKM. Cutoff artefact, not true EFT breakdown scale
- We took EKM's LECs as given. LECs themselves have statistical errors, but we did not incorporate those in our analysis
- LECs also have truncation errors, which should be included in their quoted errors

SUMMARY: PART 1

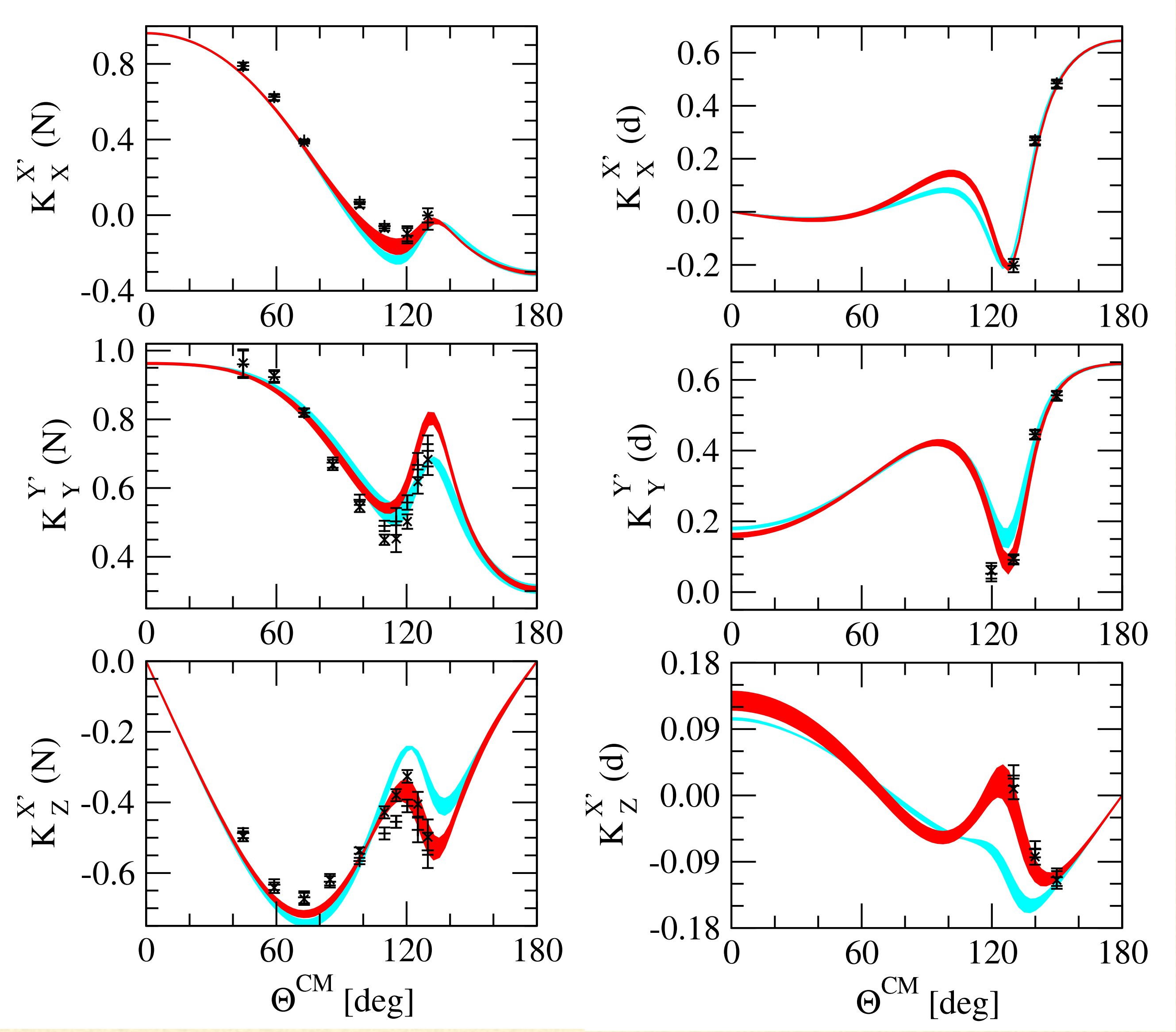


- EFTs are well set up for uncertainty quantification, since the parametric form of higher-order terms is, in principle, known.
- A Bayesian analysis of truncation error makes explicit assumptions about the pattern of EFT LECs, allowing rigorous consequences to be derived.
- "Set A_ϵ" prior justifies the standard EFT error estimation procedure; truncation errors quite stable under choice of other (reasonable) priors.
- Analysis of residuals allows us to test if the EFT is working "as advertised"

UNCERTAINTY QUANTIFICATION IN χEFT

Chiral EFT predictions for p-d spin observables with theory errors from cutoff variation

Epelbaum, Hammer, Meissner, RMP, 2009



This talk: truncation errors

Standard technique in few-nucleon χEFT calculations had been to vary cutoff in reasonable range.

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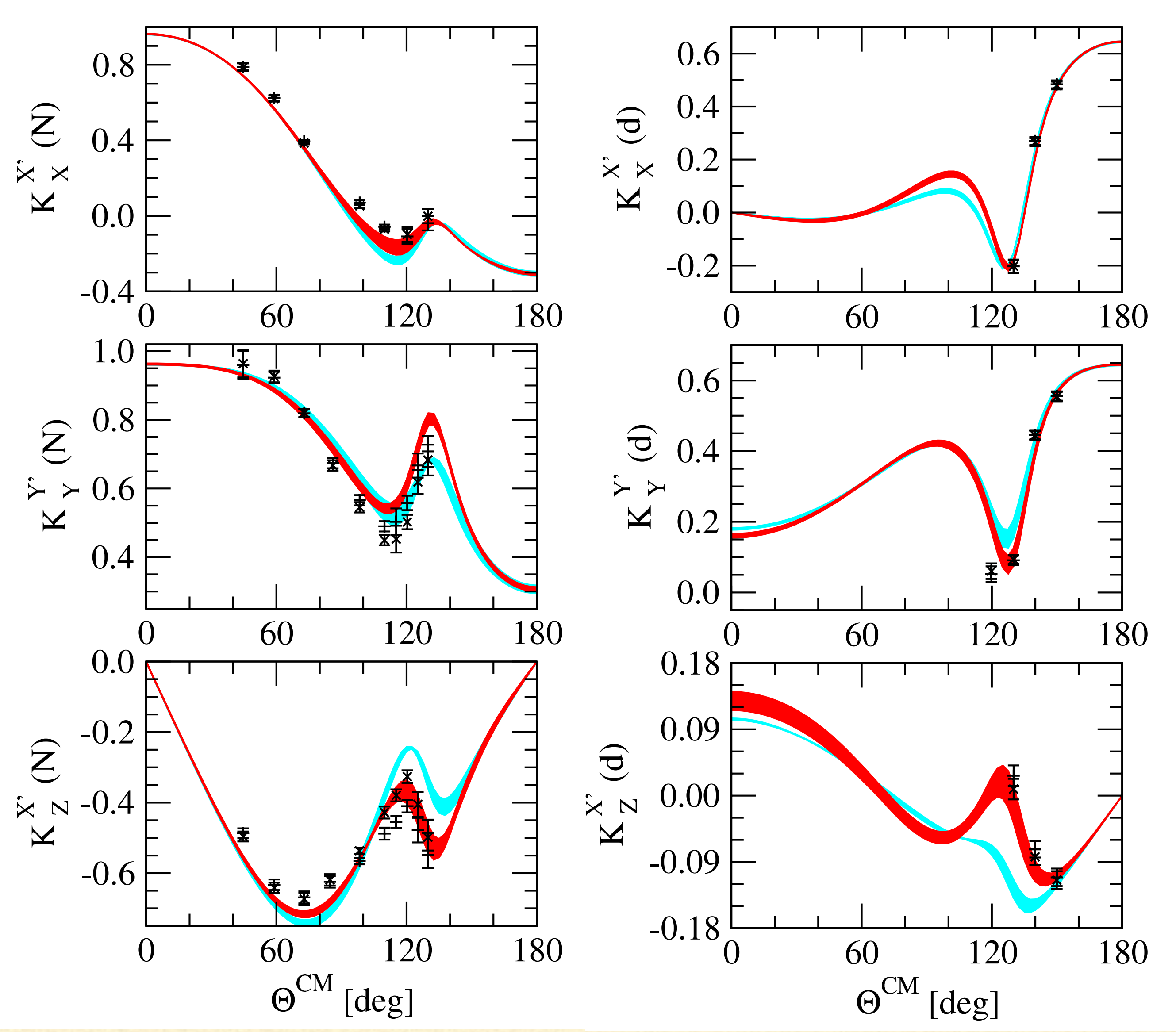

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PROBLEMS WITH CUTOFF VARIATION

- Size of error depends on how smart you are choosing regulator function;
- Depends on range of cutoffs chosen;
- Error does not necessarily decrease order-by-order;
- Only captures errors from even orders in the EFT;
- Statistical interpretation is not clear.

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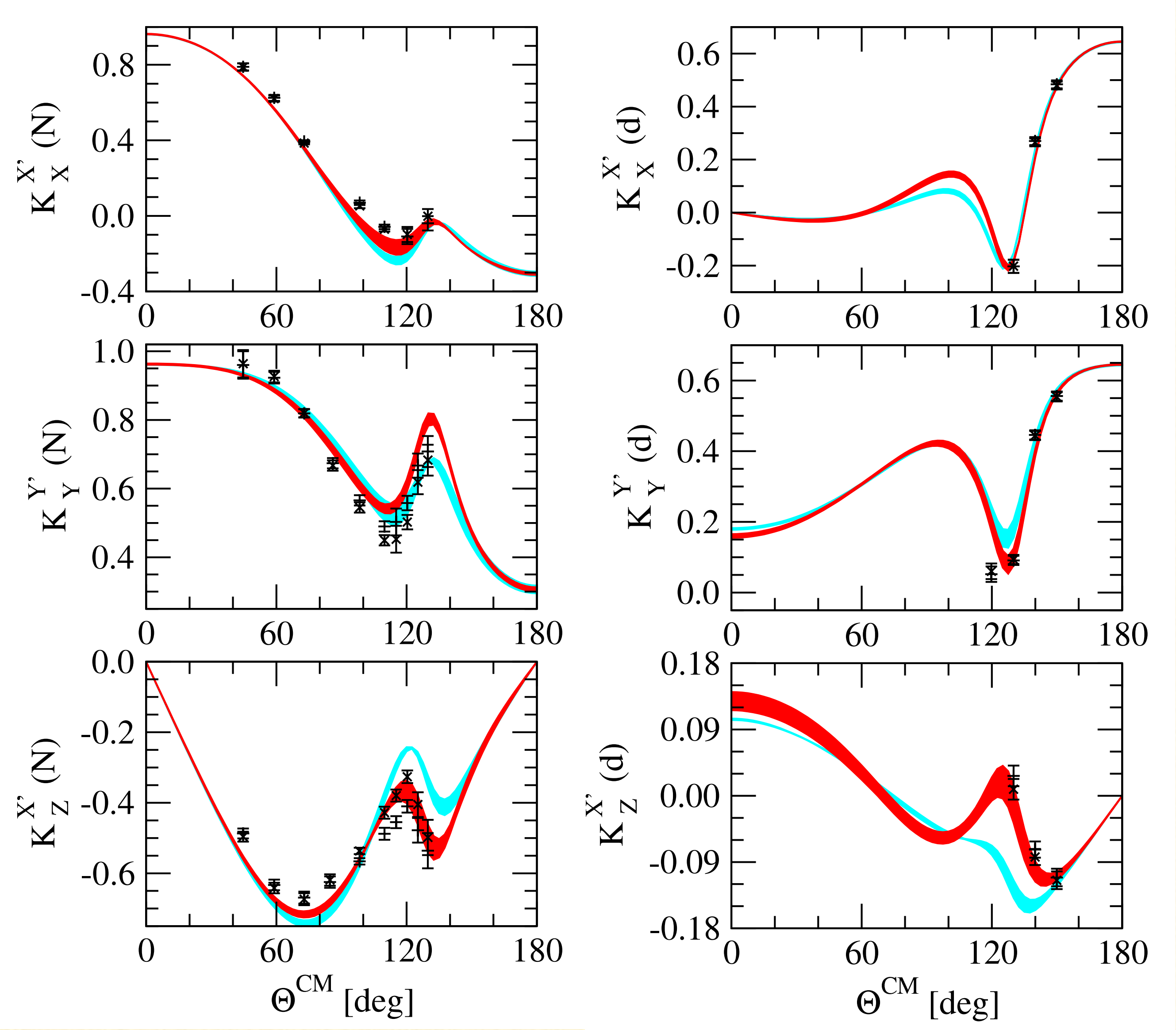

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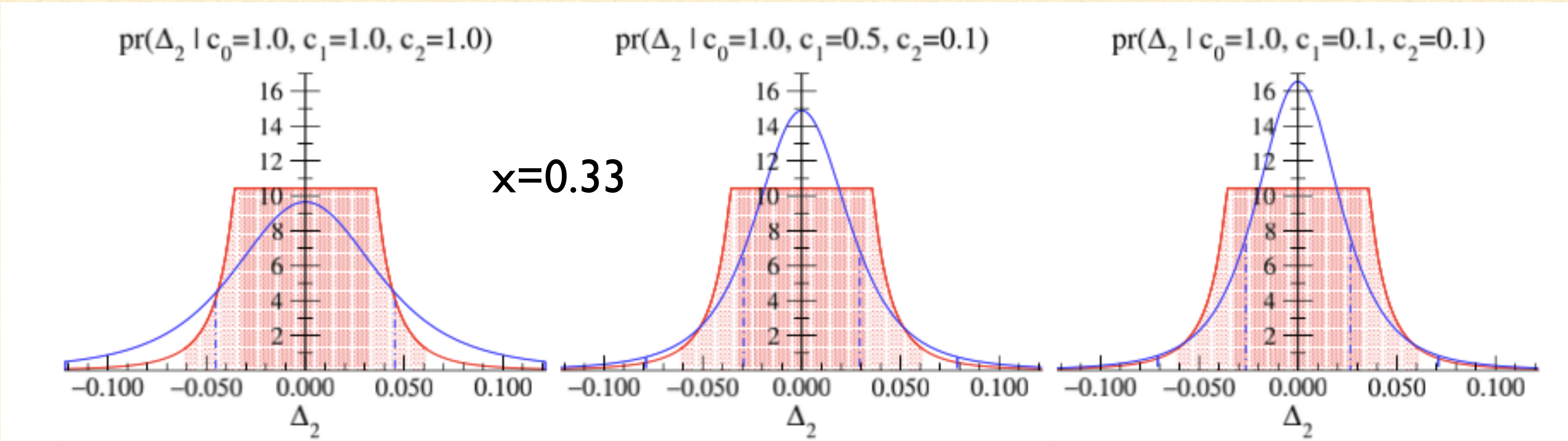
Cutoff variation is a regulator artefact which may or may not reflect full size of theory uncertainty

REPRESENTATIVE EXAMPLES



- Set C differs from Set A in that entire distribution of $\{c_n\}$ matters
- Set A_ϵ and Set C_ϵ DOB intervals closest for most uniform $\{c_n\}$
- Choice of prior matters less and less at higher orders. At and beyond k=2 different choice of priors affect 68% DOB interval by at most 10-15%
- Updating refines knowledge of coefficients: Bayesian convergence
- Bigger effect on 95% DOB interval