Baryon Spectroscopy: Data Consistency and Model Discrimination

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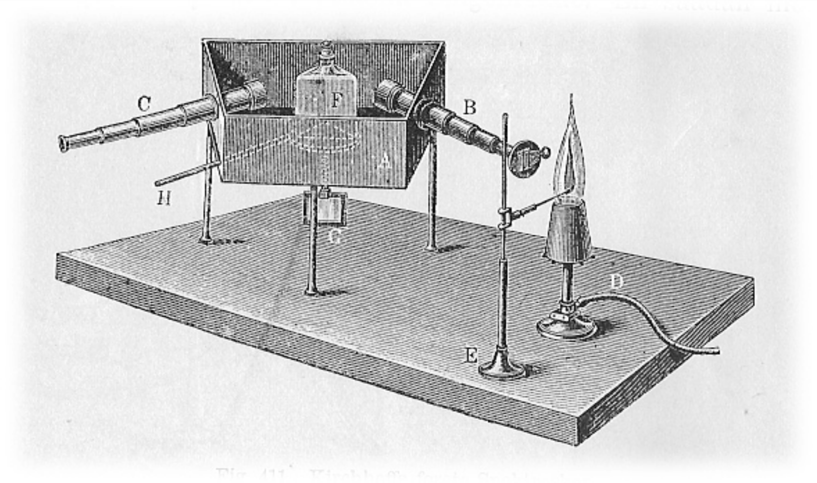
Introduction

Why Spectroscopy?



A spectrum reveals the underlying nature of the physical system.

Baryon Summary Table

Figure 1: Particle Data Group listing 2014 [1]

p	$1/2^+$	****	$\Delta(1232)$	$3/2^+$	****	Σ^+	$1/2^+$	****	Ξ^0	$1/2^+$	****	Λ_c^+	$1/2^+$	****
n	$1/2^+$	****	$\Delta(1600)$	$3/2^+$	***	Σ^0	$1/2^+$	****	Ξ^-	$1/2^+$	****	$\Lambda_c(2595)^+$	$1/2^-$	***
$N(1440)$	$1/2^+$	****	$\Delta(1620)$	$1/2^-$	****	Σ^-	$1/2^+$	****	$\Xi(1530)$	$3/2^+$	****	$\Lambda_c(2625)^+$	$3/2^-$	***
$N(1520)$	$3/2^-$	****	$\Delta(1700)$	$3/2^-$	****	$\Sigma(1385)$	$3/2^+$	****	$\Xi(1620)$		*	$\Lambda_c(2765)^+$		*
$N(1535)$	$1/2^-$	****	$\Delta(1750)$	$1/2^+$	*	$\Sigma(1480)$		*	$\Xi(1690)$		***	$\Lambda_c(2880)^+$	$5/2^+$	***
$N(1650)$	$1/2^-$	****	$\Delta(1900)$	$1/2^-$	**	$\Sigma(1560)$		**	$\Xi(1820)$	$3/2^-$	***	$\Lambda_c(2940)^+$		***
$N(1675)$	$5/2^-$	****	$\Delta(1905)$	$5/2^+$	****	$\Sigma(1580)$	$3/2^-$	*	$\Xi(1950)$		***	$\Sigma_c(2455)$	$1/2^+$	****
$N(1680)$	$5/2^+$	****	$\Delta(1910)$	$1/2^+$	****	$\Sigma(1620)$	$1/2^-$	*	$\Xi(2030)$	$\geq \frac{5}{2}?$	***	$\Sigma_c(2520)$	$3/2^+$	***
$N(1685)$		*	$\Delta(1920)$	$3/2^+$	***	$\Sigma(1660)$	$1/2^+$	***	$\Xi(2120)$		*	$\Sigma_c(2800)$		***
$N(1700)$	$3/2^-$	***	$\Delta(1930)$	$5/2^-$	***	$\Sigma(1670)$	$3/2^-$	****	$\Xi(2250)$		**	Ξ_c^+	$1/2^+$	***
$N(1710)$	$1/2^+$	***	$\Delta(1940)$	$3/2^-$	**	$\Sigma(1690)$		**	$\Xi(2370)$		**	Ξ_c^0	$1/2^+$	***
$N(1720)$	$3/2^+$	****	$\Delta(1950)$	$7/2^+$	****	$\Sigma(1730)$	$3/2^+$	*	$\Xi(2500)$		*	$\Xi_c'^+$	$1/2^+$	***
$N(1860)$	$5/2^+$	**	$\Delta(2000)$	$5/2^+$	**	$\Sigma(1750)$	$1/2^-$	***				$\Xi_c'^0$	$1/2^+$	***
$N(1875)$	$3/2^-$	***	$\Delta(2150)$	$1/2^-$	*	$\Sigma(1770)$	$1/2^+$	*	Ω^-	$3/2^+$	****	$\Xi_c(2645)$	$3/2^+$	***
$N(1880)$	$1/2^+$	**	$\Delta(2200)$	$7/2^-$	*	$\Sigma(1775)$	$5/2^-$	****	$\Omega(2250)^-$		***	$\Xi_c(2790)$	$1/2^-$	***
$N(1895)$	$1/2^-$	**	$\Delta(2300)$	$9/2^+$	**	$\Sigma(1840)$	$3/2^+$	*	$\Omega(2380)^-$		**	$\Xi_c(2815)$	$3/2^-$	***
$N(1900)$	$3/2^+$	***	$\Delta(2350)$	$5/2^-$	*	$\Sigma(1880)$	$1/2^+$	**	$\Omega(2470)^-$		**	$\Xi_c(2930)$		*
$N(1990)$	$7/2^+$	**	$\Delta(2390)$	$7/2^+$	*	$\Sigma(1900)$	$1/2^-$	*				$\Xi_c(2980)$		***
$N(2000)$	$5/2^+$	**	$\Delta(2400)$	$9/2^-$	**	$\Sigma(1915)$	$5/2^+$	****				$\Xi_c(3055)$		**
$N(2040)$	$3/2^+$	*	$\Delta(2420)$	$11/2^+$	****	$\Sigma(1940)$	$3/2^+$	*				$\Xi_c(3080)$		***
$N(2060)$	$5/2^-$	**	$\Delta(2750)$	$13/2^-$	**	$\Sigma(1940)$	$3/2^-$	***				$\Xi_c(3123)$		*
$N(2100)$	$1/2^+$	*	$\Delta(2950)$	$15/2^+$	**	$\Sigma(2000)$	$1/2^-$	*				Ω_c^0	$1/2^+$	***
$N(2120)$	$3/2^-$	**				$\Sigma(2030)$	$7/2^+$	****				$\Omega_c(2770)^0$	$3/2^+$	***
$N(2190)$	$7/2^-$	****	Λ	$1/2^+$	****	$\Sigma(2070)$	$5/2^+$	*						
$N(2220)$	$9/2^+$	****	$\Lambda(1405)$	$1/2^-$	****	$\Sigma(2080)$	$3/2^+$	**				Ξ_{cc}^+		*
$N(2250)$	$9/2^-$	****	$\Lambda(1520)$	$3/2^-$	****	$\Sigma(2100)$	$7/2^-$	*						
$N(2300)$	$1/2^+$	**	$\Lambda(1600)$	$1/2^+$	***	$\Sigma(2250)$		***				Λ_b^0	$1/2^+$	***
$N(2570)$	$5/2^-$	**	$\Lambda(1670)$	$1/2^-$	****	$\Sigma(2455)$		**				$\Lambda_b(5912)^0$	$1/2^-$	***
$N(2600)$	$11/2^-$	***	$\Lambda(1690)$	$3/2^-$	****	$\Sigma(2620)$		**				$\Lambda_b(5920)^0$	$3/2^-$	***
$N(2700)$	$13/2^+$	**	$\Lambda(1710)$	$1/2^+$	*	$\Sigma(3000)$		*				Σ_b	$1/2^+$	***
			$\Lambda(1800)$	$1/2^-$	***	$\Sigma(3170)$		*				Σ_b^*	$3/2^+$	***

Baryon Spectrum (LQCD)

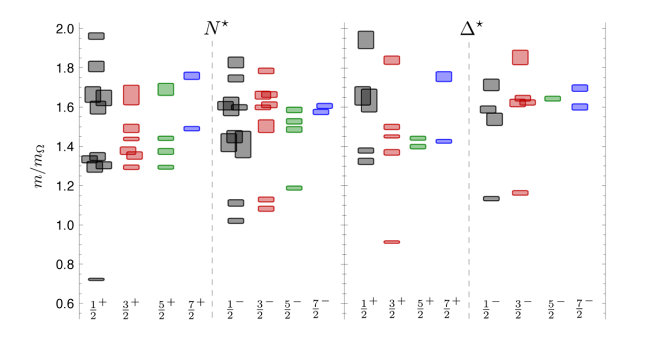


Figure 2: Lattice QCD calculation of baryon spectrum. From [2]

- Both lattice- and quark model calculations predict more states than observed

Resonance Hunting

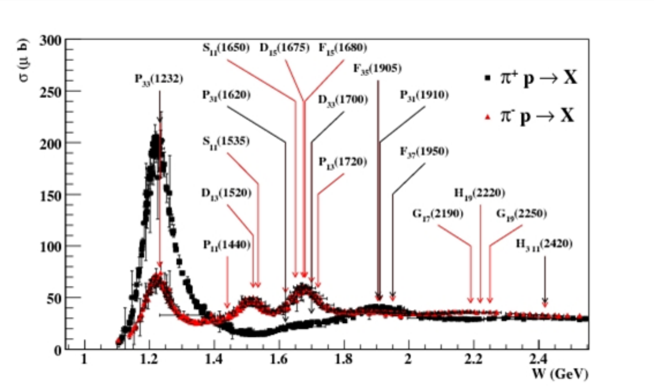


Figure 3: Most resonance information is from partial wave analysis (PWA) of πN scattering

Resonance decays to other channels

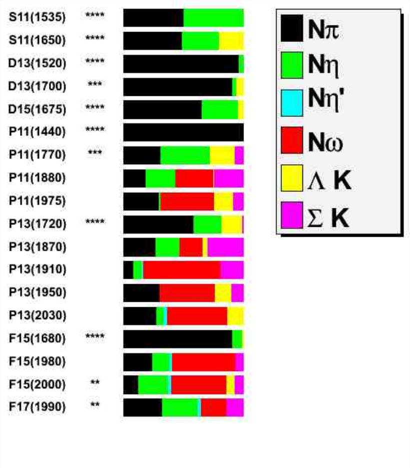


Figure 4: Some resonances predicted to decay into strange channels [3].

Meson Photoproduction

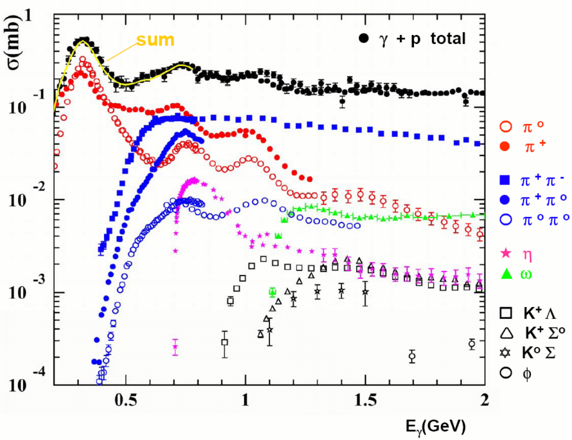


Figure 5: Comparison of photoproduction channels

Kaon Photoproduction

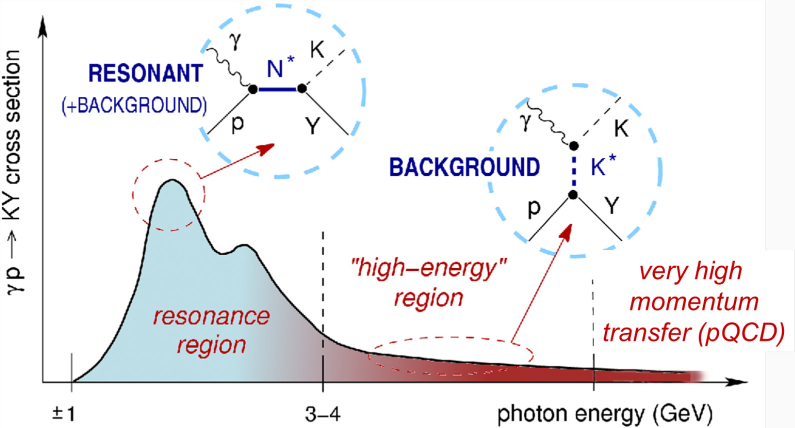


Figure 6: Energy dependence of cross section

Kaon Photoproduction

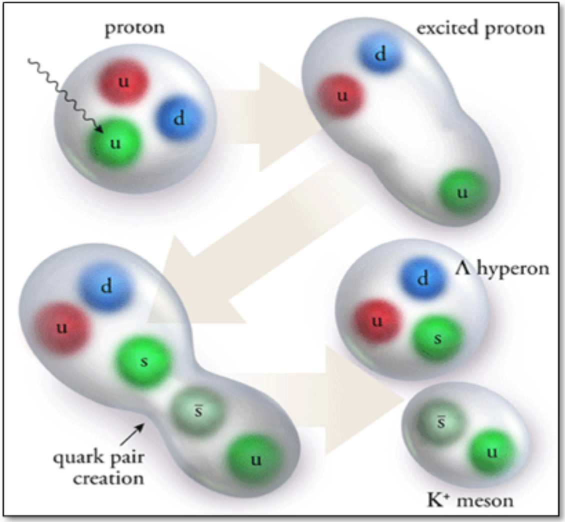


Figure 7: Possible production scenario

$\vec{\gamma}p \to K\Lambda$ Kinematics

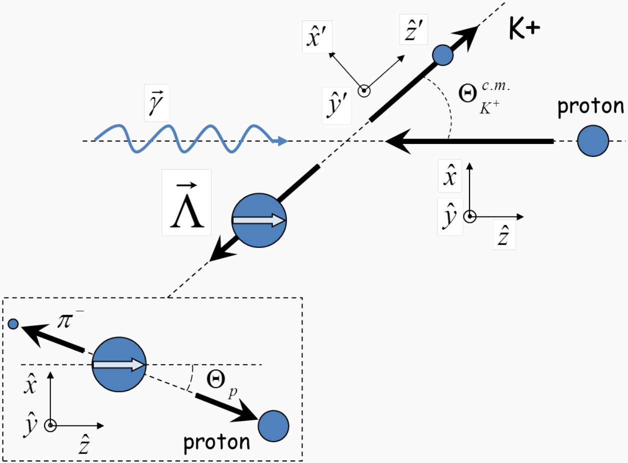


Figure 8: Taken from [4]. Kinematic variables are W (hadronic mass) and $\theta_{\text{c.m.}}$ (scattering angle).

The transversity basis

Transversity amplitudes b_j ($j = 1, 2, 3, 4$): quantization axis perpendicular to reaction plane and the linear photon polarizations J_x and J_y

$$\begin{aligned} b_1 &= {}_y\langle +|J_y|+\rangle_y, \\ b_2 &= {}_y\langle -|J_y|-\rangle_y, \\ b_3 &= {}_y\langle +|J_x|-\rangle_y, \\ b_4 &= {}_y\langle -|J_x|+\rangle_y. \end{aligned}$$

Normalized transversity amplitudes (NTA) a_j ($j = 1, 2, 3, 4$)

$$a_j \equiv \frac{b_j}{\sqrt{|b_1|^2 + |b_2|^2 + |b_3|^2 + |b_4|^2}},$$

The a_j are functions of W (hadronic mass) and $\theta_{\mathrm{c.m.}}$ (scattering angle)

$\vec{\gamma}p \to K\Lambda$ Reaction Amplitudes

Type	Observable	Transversity representation	Helicity representation
S	σ	$\|a_1\|^2 + \|a_2\|^2 + \|a_3\|^2 + \|a_4\|^2$	$\|h_1\|^2 + \|h_2\|^2 + \|h_3\|^2 + \|h_4\|^2$
	Σ	$\|a_1\|^2 + \|a_2\|^2 - \|a_3\|^2 - \|a_4\|^2$	$2\Re(h_1 h_4^* - h_2 h_3^*)$
	P	$\|a_1\|^2 - \|a_2\|^2 + \|a_3\|^2 - \|a_4\|^2$	$2\Im(h_1 h_3^* + h_2 h_4^*)$
	T	$\|a_1\|^2 - \|a_2\|^2 - \|a_3\|^2 + \|a_4\|^2$	$2\Im(h_1 h_3^* + h_2 h_4^*)$
BT	E	$2\Re(a_1 a_3^* + a_2 a_4^*)$	$\|h_1\|^2 - \|h_2\|^2 + \|h_3\|^2 - \|h_4\|^2$
	F	$2\Im(a_1 a_3^* - a_2 a_4^*)$	$2\Re(h_1 h_2^* + h_3 h_4^*)$
	G	$2\Im(a_1 a_3^* + a_2 a_4^*)$	$-2\Im(h_1 h_4^* + h_2 h_3^*)$
	H	$-2\Re(a_1 a_3^* - a_2 a_4^*)$	$-2\Im(h_1 h_3^* - h_2 h_4^*)$
BR	C_x	$-2\Im(a_1 a_4^* - a_2 a_3^*)$	$2\Re(h_1 h_3^* + h_2 h_4^*)$
	C_z	$2\Re(a_1 a_4^* + a_2 a_3^*)$	$\|h_1\|^2 + \|h_2\|^2 - \|h_3\|^2 - \|h_4\|^2$
	O_x	$2\Re(a_1 a_4^* - a_2 a_3^*)$	$-2\Im(h_1 h_2^* - h_3 h_4^*)$
	O_z	$2\Im(a_1 a_4^* + a_2 a_3^*)$	$2\Im(h_1 h_4^* - h_2 h_3^*)$
TR	T_x	$2\Re(a_1 a_2^* - a_3 a_4^*)$	$-2\Re(h_1 h_4^* + h_2 h_3^*)$
	T_z	$2\Im(a_1 a_2^* - a_3 a_4^*)$	$-2\Re(h_1 h_2^* - h_3 h_4^*)$
	L_x	$-2\Im(a_1 a_2^* + a_3 a_4^*)$	$2\Re(h_1 h_3^* - h_2 h_4^*)$
	L_z	$2\Re(a_1 a_2^* + a_3 a_4^*)$	$\|h_1\|^2 - \|h_2\|^2 - \|h_3\|^2 + \|h_4\|^2$

$\vec{\gamma} p \rightarrow K\Lambda$ Polarized Cross Section

$$
\begin{aligned}
\sigma_{Total} = \sigma_0\{ & 1 - P_L^\gamma P_T^T P_y^R \sin(\phi)\cos(2\phi) + \Sigma(-P_L^\gamma \cos(2\phi) + P_T^T P_y^R \sin(\phi)) \\
& + T(P_T^T \sin(\phi) - P_L^\gamma P_y^R \cos(2\phi)) + P(P_y^R - P_L^\gamma P_T^T \sin(\phi)\cos(2\phi)) \\
& + E(-P_C^\gamma P_L^T + P_L^\gamma P_T^T P_y^R \cos(\phi)\sin(2\phi)) + F(P_C^\gamma P_T^T \cos(\phi) + P_L^\gamma P_L^T P_y^R \sin(2\phi)) \\
& - G(P_L^\gamma P_L^T \sin(2\phi) + P_C^\gamma P_T^T P_y^R \cos(\phi)) - H(P_L^\gamma P_T^T \cos(\phi)\sin(2\phi) - P_C^\gamma P_L^T P_y^R) \\
& - C_x(P_C^\gamma P_x^R - P_L^\gamma P_T^T P_z^R \sin(\phi)\sin(2\phi)) - C_z(P_C^\gamma P_z^R + P_L^\gamma P_T^T P_x^R \sin(\phi)\sin(2\phi)) \\
& - O_x(P_L^\gamma P_x^R \sin(2\phi) + P_C^\gamma P_T^T P_z^R \sin(\phi)) - O_z(P_L^\gamma P_z^R \sin(2\phi) - P_C^\gamma P_T^T P_x^R \sin(\phi)) \\
& + L_x(P_L^T P_x^R + P_L^\gamma P_T^T P_z^R \cos(\phi)\cos(2\phi)) + L_z(P_L^T P_z^R - P_L^\gamma P_T^T P_x^R \cos(\phi)\cos(2\phi)) \\
& + T_x(P_T^T P_x^R \cos(\phi) - P_L^\gamma P_L^T P_z^R \cos(2\phi)) + T_z(P_T^T P_z^R \cos(\phi) + P_L^\gamma P_L^T P_x^R \cos(2\phi))\}
\end{aligned}
$$

Figure 10: Cross section as a function of beam ($P^\gamma_{C,L}$), target ($P^T_{L,T}$) and recoil ($P^R_{x,y,z}$) polarization

Baryon Spectroscopy in Practice

Usual process:

Issues:

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- Progress by fitting observables (cross sections, asymmetries) for several channels

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- How accurate do measurements require to be?

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- Different model frameworks (i.e. different theory groups) and different model content (choice of resonances, etc.)
- Amplitude analysis ideally

Issues:

- How accurate do measurements require to be?
- How do we deal with measurements from different experiments?

Model Discrimination

Distinguishing Objects

- Resolve two objects
- Actual angular “distance”
- Instrumental resolution (aperture limit)
- Rayleigh Criterion: 1st diffraction minimum of object 1 $\leq$ distance to centre of object 2

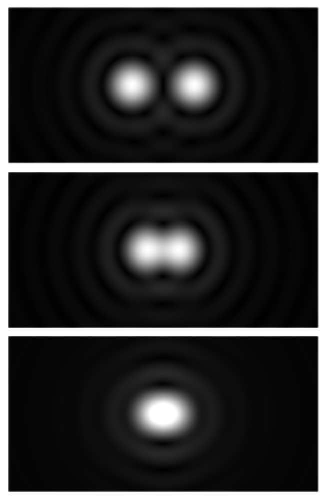


Figure 11: Airy disk near Rayleigh Criterion.

Distinguishing Objects

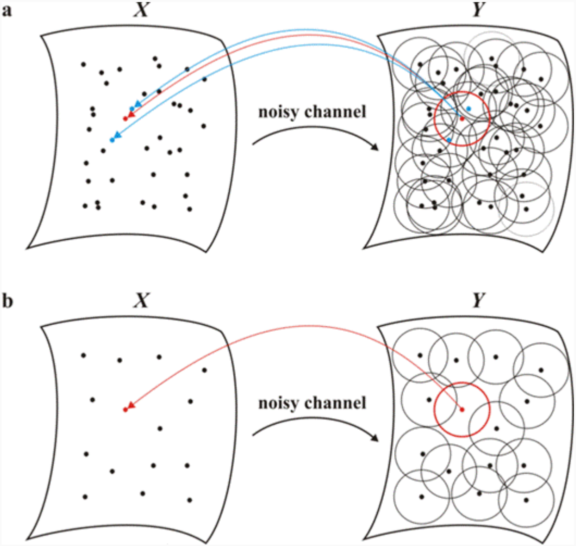
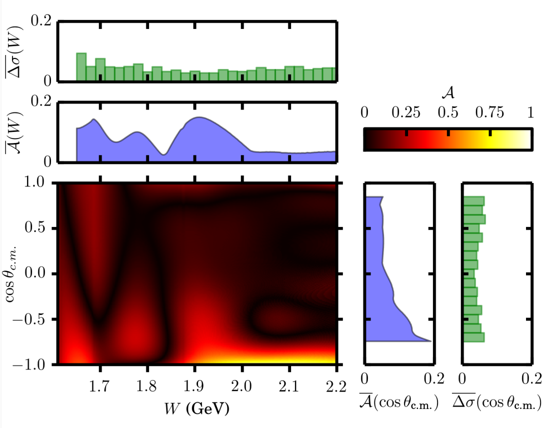


Figure 12: Mapping between Amplitudes (X) and Observables (Y).

Model Discrimination from Cross Sections

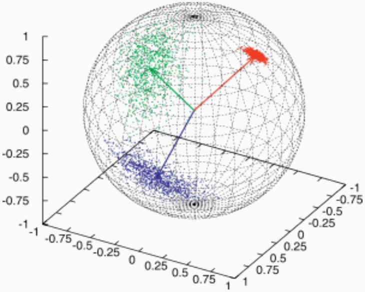


$$\mathcal{A}[A,B] = \left| \frac{\frac{d\sigma}{d\Omega}(A) - \frac{d\sigma}{d\Omega}(B)}{\frac{d\sigma}{d\Omega}(A) + \frac{d\sigma}{d\Omega}(B)} \right|$$

- Measure for difference between the c.s. predictions
- Example: BnGa2014-02 vs. RPR-2011 predictions for $\gamma p \to K^+\Lambda$
- Experimental resolution: $\Delta\sigma = \left(\Delta\frac{d\sigma}{d\Omega}\right) / \frac{d\sigma}{d\Omega}$
- $\overline{\mathcal{A}}(\text{th}) \approx \overline{\Delta\sigma}(\text{expt})$
- ArXiv: [5]

Model discrimination: distance in amplitude space

Measure to discriminate between two models for $p(\gamma, K^+)\Lambda$ in amplitude space?



- $4D$-vector representation for NTA

$$\vec{\mathcal{M}}_1(s,t) = (a_1 \;\; a_2 \;\; a_3 \;\; a_4)^T$$

vectors on a 3-sphere in $\mathbb{C}^4$
(unit 7-sphere in $\mathbb{R}^8$)

- Distance between two models

$$\mathcal{D}\left[\vec{\mathcal{M}}_1, \vec{\mathcal{M}}_2\right] = \arccos \operatorname{Re}\left(\vec{\mathcal{M}}_1^\dagger \cdot \vec{\mathcal{M}}_2\right)$$

- Dependence on arbitrary phase: $\vec{\mathcal{M}}_2(\alpha_4' = 0)$ and vary α_4 in $\vec{\mathcal{M}}_1(\alpha_4 = 0)$ such that $\mathcal{D}\left[\vec{\mathcal{M}}_1, \vec{\mathcal{M}}_2\right]$ is minimized

Example Comparison

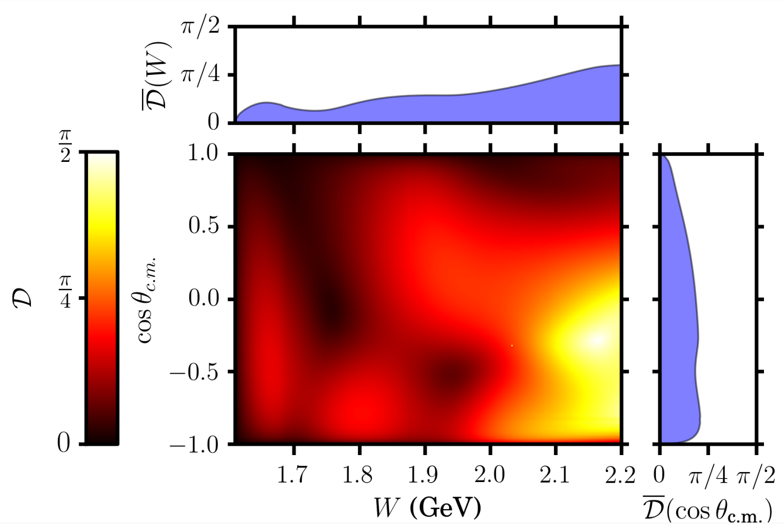
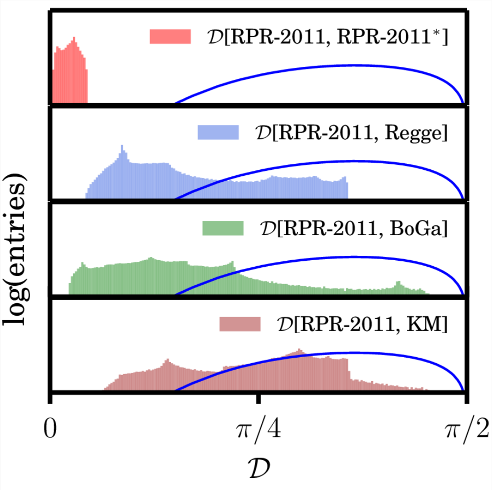


Figure 13: Distance measure in amplitude space for BnGa versus RPR-2011

Model discrimination: distance in amplitude space

Reaction: $\gamma p \to K^+\Lambda$



- Blue line: random samples in NTA amplitude space
- $\mathcal{D}$[RPR-2011,RPR2011*]: Resolution required to hunt a resonance ($D_{13}(1900)$)
- $\mathcal{D}$[RPR-2011,Regge]: Resolution required to determine “the” background
- $\mathcal{D}$[RPR-2011,KM]: Resolution required to discriminate between RPR-2011 and Kaon-MAID

Extract $r_3 e^{i\delta_3^4}$ at ($W = 1.8$ GeV, $\theta_{c.m.} = -0.1$) from data

1. Bootstrap: M sets of data $\{A_i^j \pm \delta A_i^j, i = 1, ..., N\}, j = 1, ..., M$
2. χ^2 fit to extract amplitudes for each set of synthetic data
3. Histogram solutions in amplitude space

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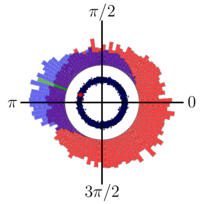
Red: accuracy = 0.1; Blue: accuracy = 0.01



(i) "mathematically complete set": $\{A_i^{\mathrm{exp}}\}_1 = \{\frac{d\sigma}{d\Omega}, \Sigma, T, P, C_x, O_x, E, F\}$

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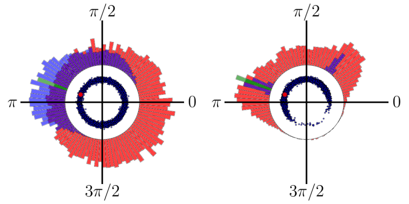

(i) "mathematically complete set": $\{A_i^{\rm exp}\}_1 = \{\frac{d\sigma}{d\Omega}, \Sigma, T, P, C_x, O_x, E, F\}$
(ii) $\{A_i^{\rm exp}\}_2 = \{A_i^{\rm exp}\}_1 + \{C_z, O_z, G\}$

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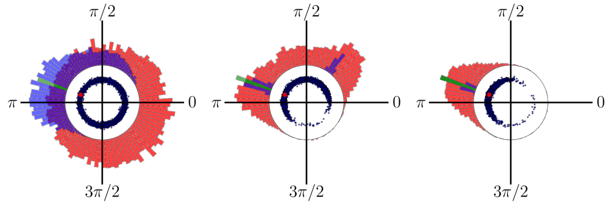

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(iii) $\{A_i^{\exp}\}_3 = \{A_i^{\exp}\}_2 + \{H\}$

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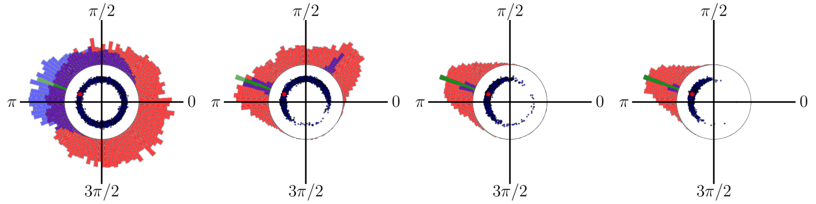
Red: accuracy = 0.1; Blue: accuracy = 0.01



(i) "mathematically complete set": $\{A_i^{\mathrm{exp}}\}_1 = \{\frac{d\sigma}{d\Omega}, \Sigma, T, P, C_x, O_x, E, F\}$

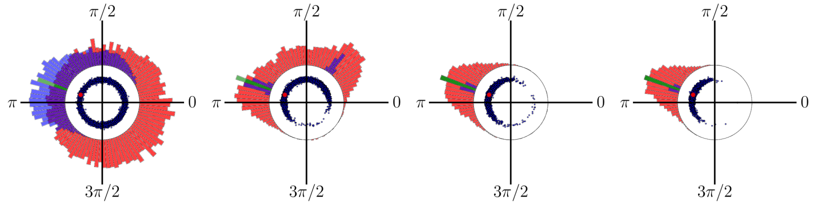
(ii) $\{A_i^{\mathrm{exp}}\}_2 = \{A_i^{\mathrm{exp}}\}_1 + \{C_z, O_z, G\}$

(iii) $\{A_i^{\mathrm{exp}}\}_3 = \{A_i^{\mathrm{exp}}\}_2 + \{H\}$

(iv) $\{A_i^{\mathrm{exp}}\}_4 = \{A_i^{\mathrm{exp}}\}_3 + \{T_x, T_z, L_x, L_z\}$

Extract $r_3 e^{i\delta_3^4}$ at $(W = 1.8\ \text{GeV}, \theta_{c.m.} = -0.1)$ from data

Compare bootstrap method:



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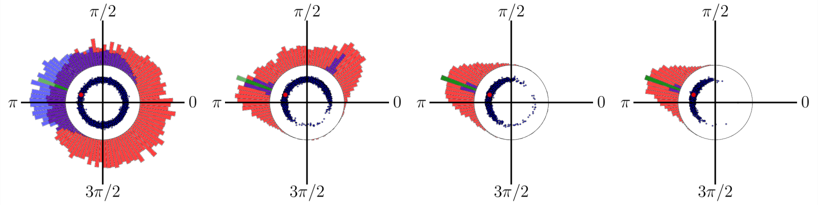
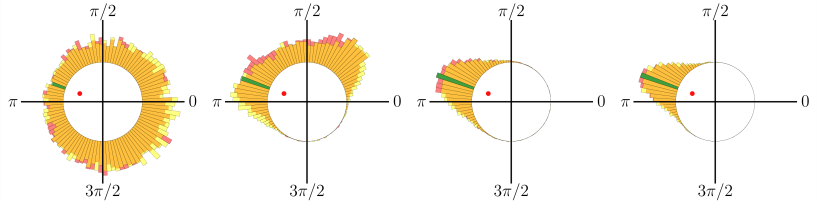
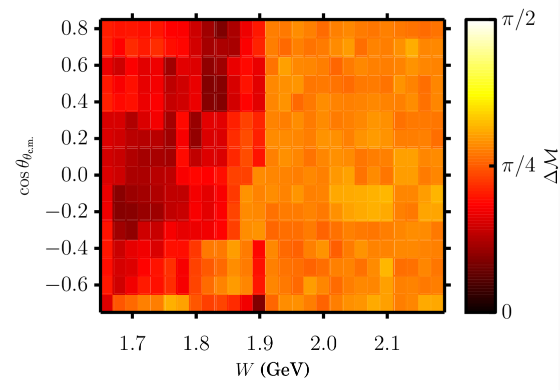


To MCMC (nested sampling):



Resolving power of $p(\gamma, K^+)\Lambda$ polarization data?



The darker the color, the better the reaction amplitudes are determined by the data

- All data in grids:
  1. $\Delta W = 20$ MeV
  2. $\Delta \cos\theta_{c.m.} = 0.1$.
- 2241 single polarization observables (Σ, P, T)
- 452 double polarization observables (beam-recoil, target-recoil, beam-target)

Model Discrimination

Key Points:

Questions:

Model Discrimination

Key Points:

- Introduced distance measure between models in amplitude space.

Questions:

Model Discrimination

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- Experimental data must lead to PDFs in amplitude space that have smaller dispersions than characteristic distances between models.

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Questions:

- How to extend this for distributions over kinematic variables?

Data Consistency

Fierz Identity Comparison: $\gamma + p \rightarrow K + \Lambda$

For $\gamma + N \rightarrow \text{p.s. meson} + \text{baryon}$

$$O_x^2 + O_z^2 + C_x^2 + C_z^2 + \mathbf{\Sigma}^2 - T^2 + P^2 = 1$$

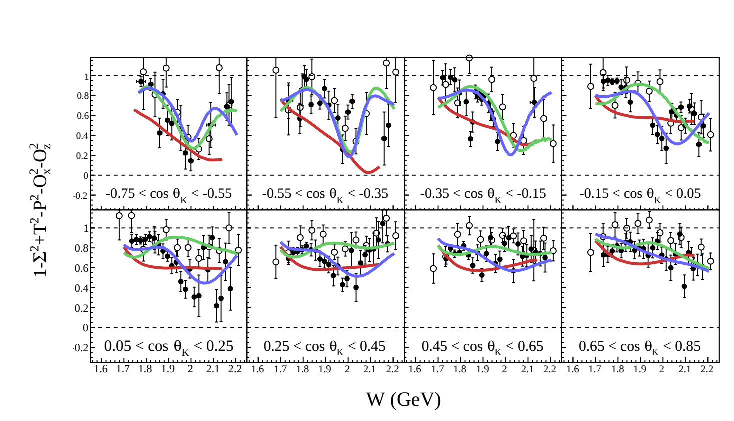

Figure 14: Open circles - $C_x^2 + C_z^2$ [4]; Filled - $1 - \mathbf{\Sigma}^2 + T^2 - P^2 - O_x^2 - O_z^2$ [6]

Data Consistency Idea

The constraints among observables, e.g.:

$$O_x^2 + O_z^2 + C_x^2 + C_z^2 + \mathbf{\Sigma}^2 - T^2 + P^2 = 1$$

stem from the constraint among amplitudes:

$$|a_1|^2 + |a_2|^2 + |a_3|^2 + |a_4|^2 = 1$$

i.e. surface of a unit 7-sphere in $\mathbb{R}^8$

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i.e. surface of a unit 7-sphere in $\mathbb{R}^8$

- Can we map PDFs in observable space to PDF in amplitude space?
- If so, can we project amplitude PDF back into a joint observable PDF?

Test Case: π-N Scattering

Two amplitudes, four observables:

$$\frac{d\sigma}{d\Omega} = |f|^2 + |g|^2$$
$$A = |f|^2 - |g|^2$$
$$R = -2\,\mathrm{Re}\,(fg^\star)$$
$$P = 2\,\mathrm{Im}\,(fg^\star)$$

Normalize:

$$|f|^2 + |g|^2 = 1$$

Constraint:

$$A^2 + R^2 + P^2 = 1$$

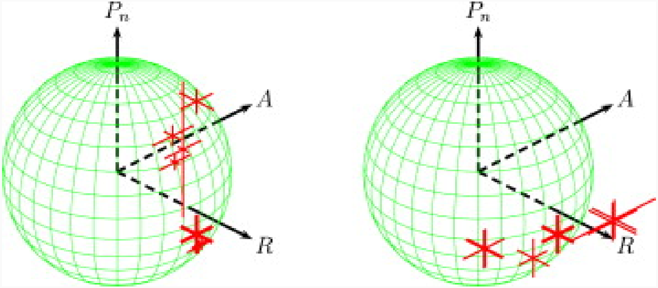


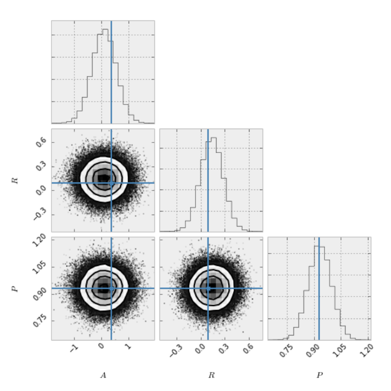
Figure 15: $\pi^- p$ (left) and $\pi^+ p$ (right) polarization observables

Test Case: π-N Scattering

- Generate "true" synthetic data
- Generate statistical uncertainty
- Sample from $\mathcal{N}(\mu, \sigma)$
- Add systematic error

Observables	A	R	P
"True" values	0.35	0.09	0.93
"Smeared"	0.10 ± 0.45	0.14 ± 0.14	0.93 ± 0.06
Systematic Error	0.04	0.06	-0.09

Test Case: π-N Scattering

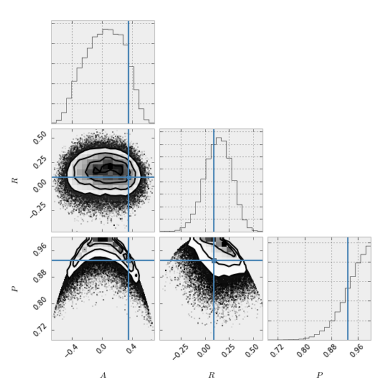


Unconstrained PDF

- Use emcee
- Sample from 3D Gaussian
- Mean and standard deviation from smeared data
- Assume uncorrelated measurements
- Corner plot with true values indicated

Observables	A	R	P
"True" values	0.35	0.09	0.93
Unconstrained MCMC	0.10 ± 0.44	0.14 ± 0.14	0.93 ± 0.06

Test Case: π-N Scattering



Constrained PDF

- Use emcee
- Sample from amplitude space
- Calculate likelihood from 3D Gaussian
- Corner plot with true values indicated

Observables	A	R	P
"True" values	0.35	0.09	0.93
Unconstrained MCMC	0.04 ± 0.25	0.14 ± 0.14	0.95 ± 0.04

Next steps

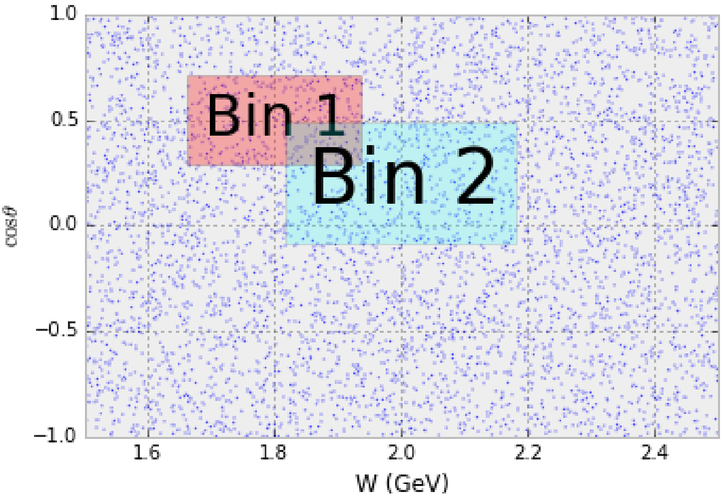
π-N Scattering Roadmap

- Generate large sample of synthetic data
- For each data set:
    - select different experimental uncertainty
    - select different systematic uncertainty
- Analyse all sets statistically
- Apply to measured data

Further work

- Apply procedure to pseudoscalar meson photoproduction
- Other reactions?

Question: How to cope with different bins?



Data Consistency

Key Points:

Questions:

Data Consistency

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- Independent polarization measurements lead to observables that are projections of the same amplitudes

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- Map observable PDFs into amplitude space PDFs and combine.

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- Inverse map of amplitude PDF to observable space

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- Needs to be extended to pseudoscalar meson photoproduction (4 amplitudes), and other reactions?

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Questions:

- Can this be used to detect inconsistent data?

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- Map observable PDFs into amplitude space PDFs and combine.
- Inverse map of amplitude PDF to observable space
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Questions:

- Can this be used to detect inconsistent data?
- How to deal with kinematic bins that partially overlap?

Conclusion

Summary

Summary

Baryon Spectroscopy

Model Discrimination

Data Consistency

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Baryon Spectroscopy

- We are still not sure of the spectrum of baryons

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- We need an analogue of a Rayleigh Criterion

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- {Work in progress}: Create joint observable PDFs

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- We are still not sure of the spectrum of baryons

Model Discrimination

- We need an analogue of a Rayleigh Criterion

Data Consistency

- {Work in progress}: Create joint observable PDFs
- Clean or process data for model inference

Collaborators

[In addition to members of the Glasgow group]

- CLAS Collaboration: Meson Photoproduction measurements
- J. Nys and J. Ryckebusch (University of Gent, Belgium): Model Discrimination

Backup Slides

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