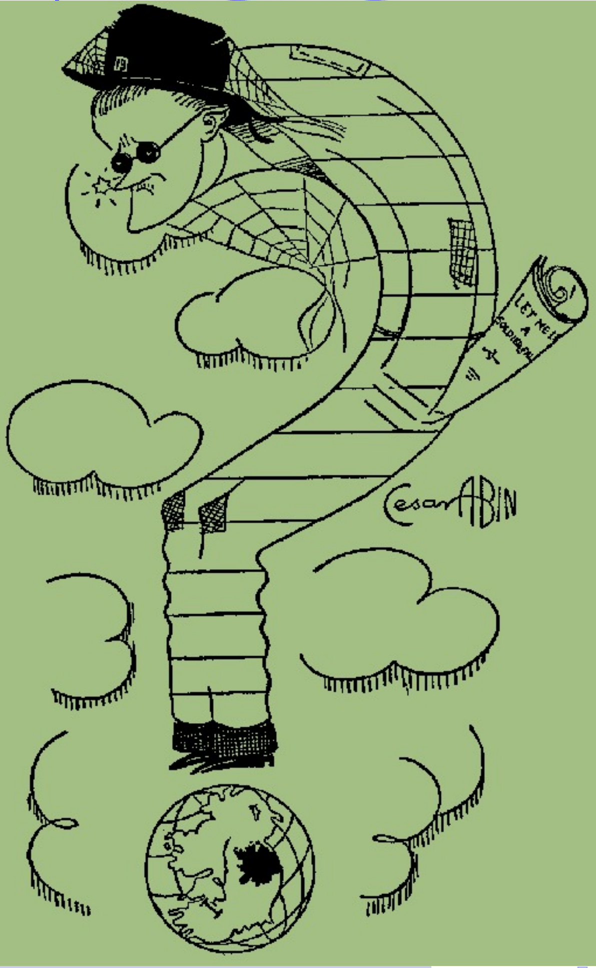


Happy
BLOOMSDAY
June 16

Five lines of text and ten pages of notes.

JJ: U, 1

A Pedestrian's Perspective of Applying Bayesian Statistics in Effective Field Theories

H. W. Grießhammer

Institute for Nuclear Studies
The George Washington University, DC, USA

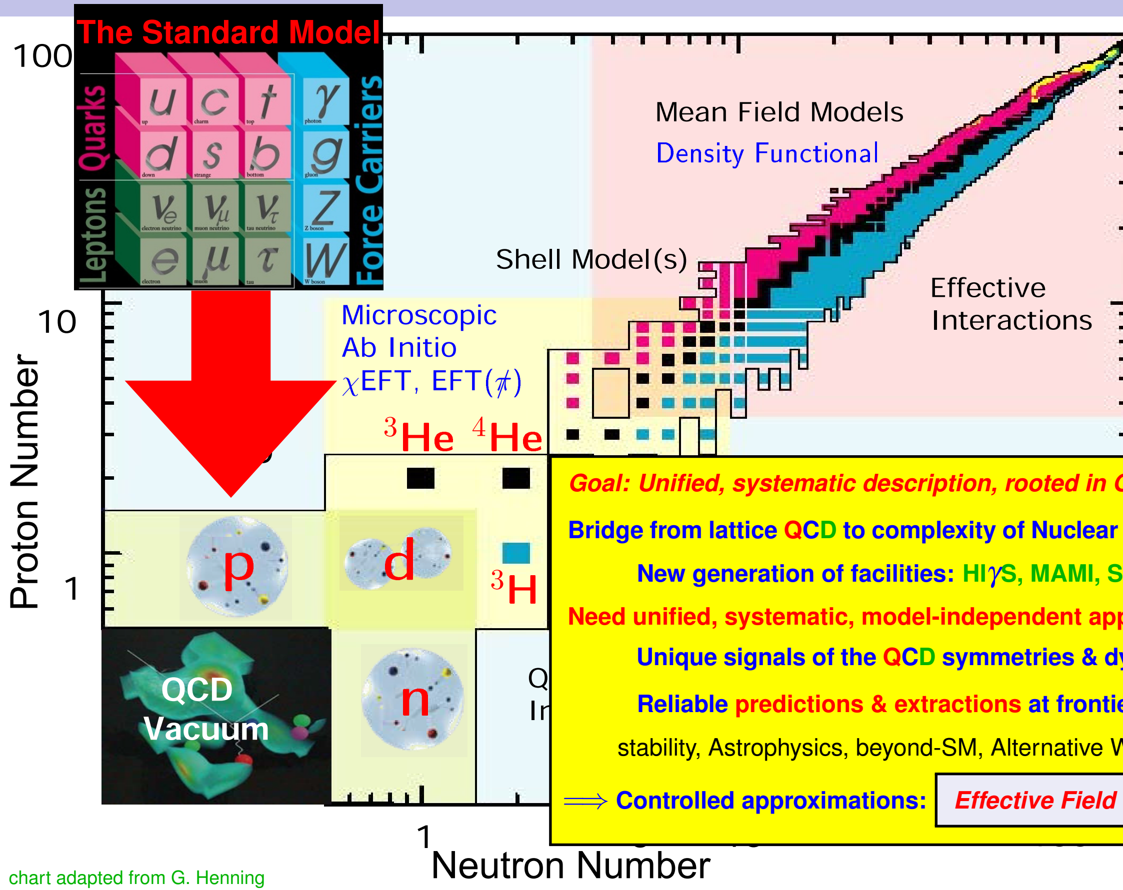
1. Error-Bars for Nuclear Physics!
2. Compton Scattering $\gamma X \to \gamma X$ Explores Dynamics
3. Some Bayesian Questions & Opportunities
4. Concluding Statements and Disclaimer

How to bridge between QCD and Nuclear Physics?

BUQEYE COLLABORATION Phys. Rev. **C92** (2015) 024005 [1506.01343] etc.

Polarisabilities & Bayes in χEFT for lattice-QCD: hg/JMcG/DRP Europ. Phys. J. **A52** (2016) 139

1. Error-Bars for Nuclear Physics!



Goal: Unified, systematic description, rooted in QCD.

Bridge from lattice QCD to complexity of Nuclear Physics:

New generation of facilities: HIγS, MAMI, SNS,. . .

Need unified, systematic, model-independent approach:

Unique signals of the QCD symmetries & dynamics;

Reliable predictions & extractions at frontiers:

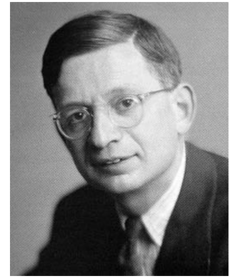
stability, Astrophysics, beyond-SM, Alternative Worlds,. . .

$\Longrightarrow$ **Controlled approximations:** ***Effective Field Theories***

chart adapted from G. Henning

Model: Capture *some* aspects with lots of data – no "fail" but "tuned". Cargo Cult mode.

The Trouble With Nuclear Physics



In fact the trouble in the recent past has been a surfeit of different *models* [of the nucleus], each of them successful in explaining the behavior of nuclei *in some situations*, and each in *apparent contradiction with other successful models* or with our ideas about nuclear forces. Rudolph E. Peierls: "The Atomic Nucleus", *Scientific American* **200 (1959)**, no. 1, p. 75; emphasis added

Theory: Comprehensive, prescriptive, predictive, may fail. Explain-All-To-Some-Degree mode.

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Weinberg's "Folk Theorem": Throw In the Kitchen Sink

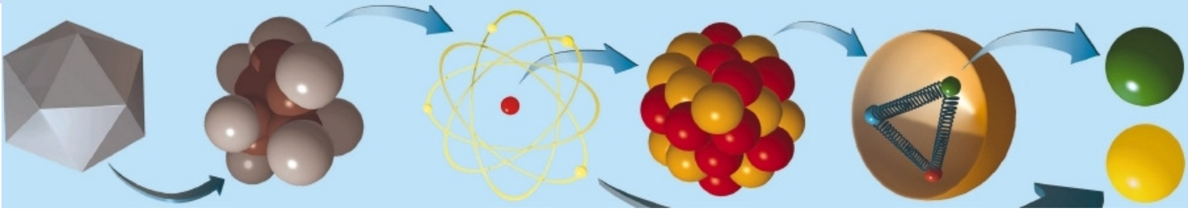
As long as you let it be the most general possible Lagrangian consistent with the symmetries of the theory, you're simply writing down the most general theory you could possibly write down.

Original: Weinberg: Physica 96A (1979) 327 – here 1997 version

Problems: What interacts, what symmetries?
Infinitely many terms. $\Longrightarrow$ Impossible to calculate.

(b) Way Out: What You See Is What You Get

Weinberg: "folk lore theorem"



To probes with wavelength λ, object of size R appears — point-like for $\lambda \gg R$, blurry for $\lambda \gtrsim R$, composed for $\lambda \lesssim R$.

• **Example Radiation Multipoles**: $P_{El} \xrightarrow{\lambda \gg R} \sum_{\text{ang. mom. } l} a_l \left(\frac{\text{size } R}{\text{wavelength } \lambda} \right)^{2l}$ e.g. atoms: $\frac{R \sim 1\text{Å}}{\lambda \sim 5000\text{Å}}$.

Converges if **Separation of Scales** $Q = \frac{\text{target size } R}{\text{resolution } \lambda} < 1$ **&** a_l **of natural size** $\Longrightarrow$ **error-estimate, space for improvement**

(b) Way Out: What You See Is What You Get

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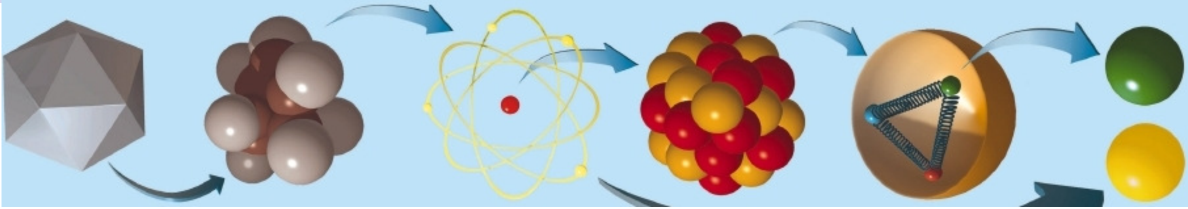

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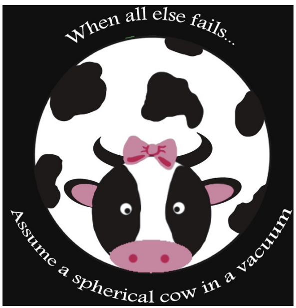


Tenet: **Short-distance physics does not have to be right for a good calculation, because a low-energy process cannot probe details of the high-energy structure.**

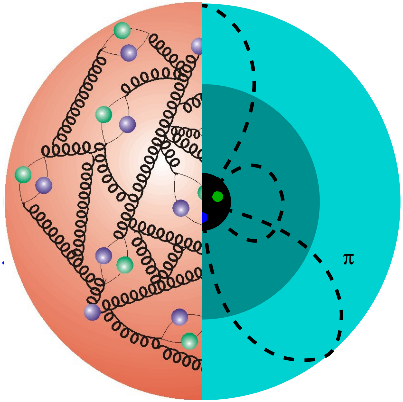
$\Longrightarrow$ ***Effective Field Theories***

Identify those degrees of freedom and symmetries which are *appropriate* to resolve the *relevant* Physics at the scale of interest.

Systematic approximation of real world with **estimate of theoretical uncertainties.**

(c) Chiral Effective Field Theory of Nuclear Physics

At low energies, quarks & gluons rearrange into new, *effective* low-energy degrees of freedom: Nucleons, Pions, $\Delta(1232)$.



$$\mathcal{L}_{\chi\text{EFT}} = (D_\mu \pi^a)(D^\mu \pi^a) - m_\pi^2\, \pi^a \pi^a + \dots$$

$$+ N^\dagger [\mathrm{i}\, D_0 + \frac{\vec{D}^2}{2M} + \frac{g_A}{2 f_\pi}\, \vec{\sigma}\cdot\vec{D}\pi + \dots] N + C_0 \left(N^\dagger N\right)^2 + H_0 \left(N^\dagger N\right)^3 + \dots$$

Correct long-range $+$ symmetries: Chiral SSB, gauge, iso-spin,...

$\Longrightarrow$ *Write most general Interaction Lagrangean permitted.*

Short-range: ignorance into *minimal* parameter-set at given order.

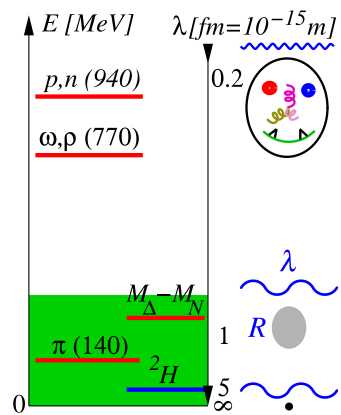
Coefficients from experiment or QCD or...

"The Power Counting":

Systematic ordering in $Q = \dfrac{\text{typ. momentum } \sim m_\pi}{\text{breakdown scale } \sim 1\text{ GeV}} \approx \dfrac{1}{5\dots7}$.

Controlled approximation: model-independent, error-estimate.

Space for improvement.



$\Longrightarrow$ Chiral Effective Field Theory χEFT $\equiv$ low-energy QCD

(d) What Can Possibly Go Wrong??

Expand observables as $\mathcal{O} = c_0 + c_1\,Q^1 + c_2\,Q^2 + \ldots$

with $Q = \dfrac{\text{typ. momentum } p_{\text{typ.}}}{\text{breakdown scale } \bar{\Lambda}_{\text{EFT}}} < 1.$

Check assumptions:

– $p_{\text{typ.}} \nearrow \bar{\Lambda}_{\text{EFT}} \implies Q \not\ll 1$?

"EFTs carry seed of own destruction." D. R. Phillips

– No separation/jungle of scales? e.g. N^* at $2\ \text{GeV}$

– Wrong constituents/degrees of freedom?

new d.o.f. e.g. QED at $100\ \text{GeV}$ without W, Z

phase transition changes d.o.f. $N, \pi \to$ quarks, gluons

– Nature does *not* have assumed symmetry?

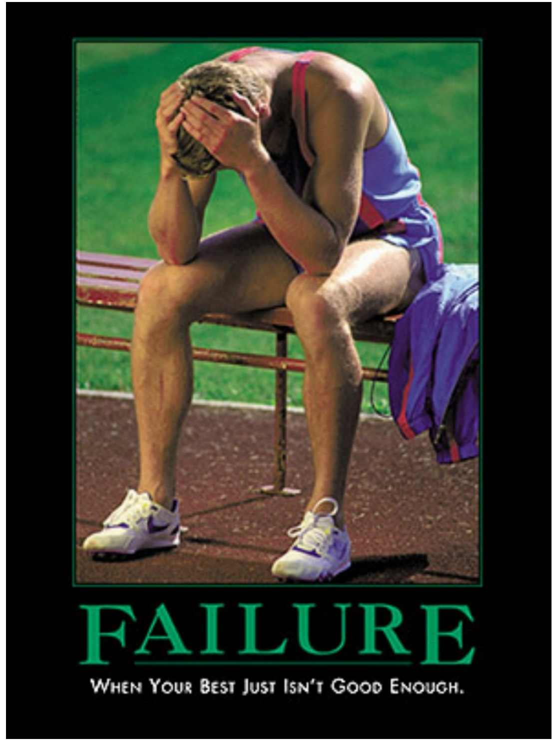
e.g. impose Parity in weak interactions

Check Quantitatively Predicted Convergence Pattern:

– Convergence?: ***Coefficients of Natural Size?***

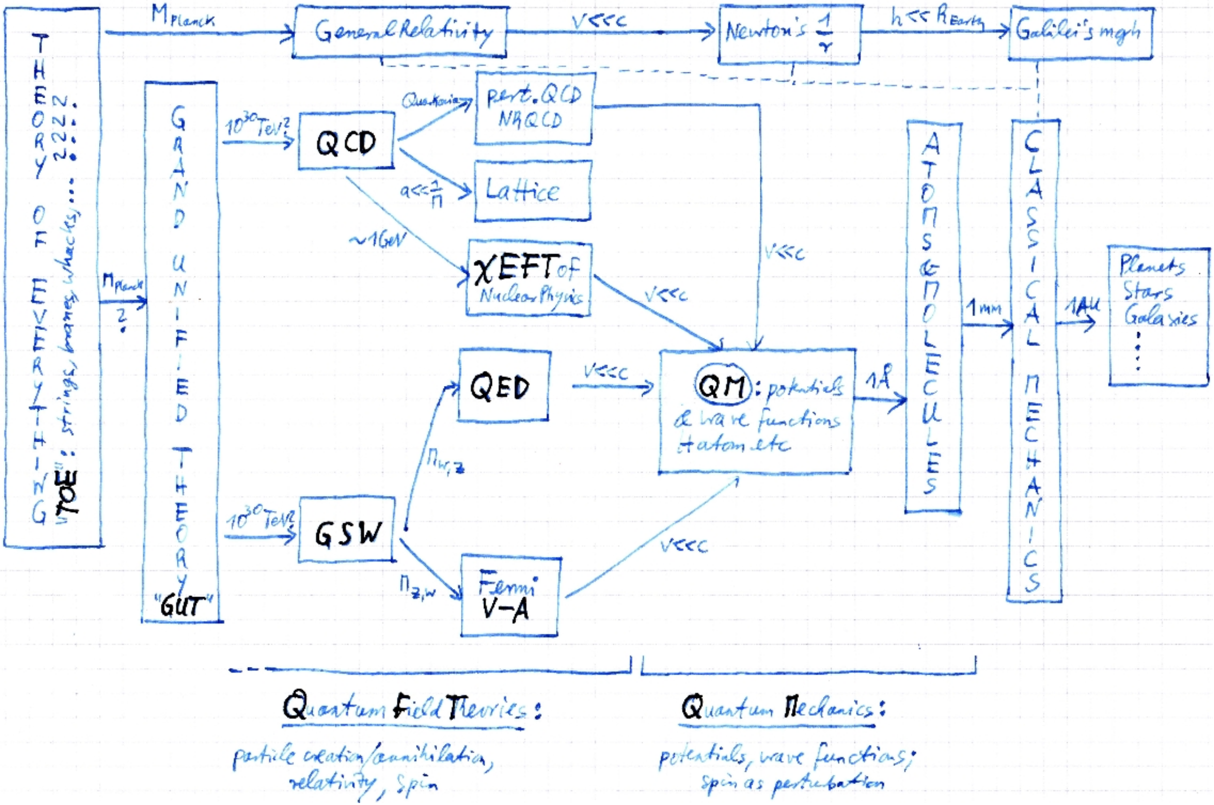
– Order by order smaller corrections.

– Order by order less cut-off/RScheme dependence.



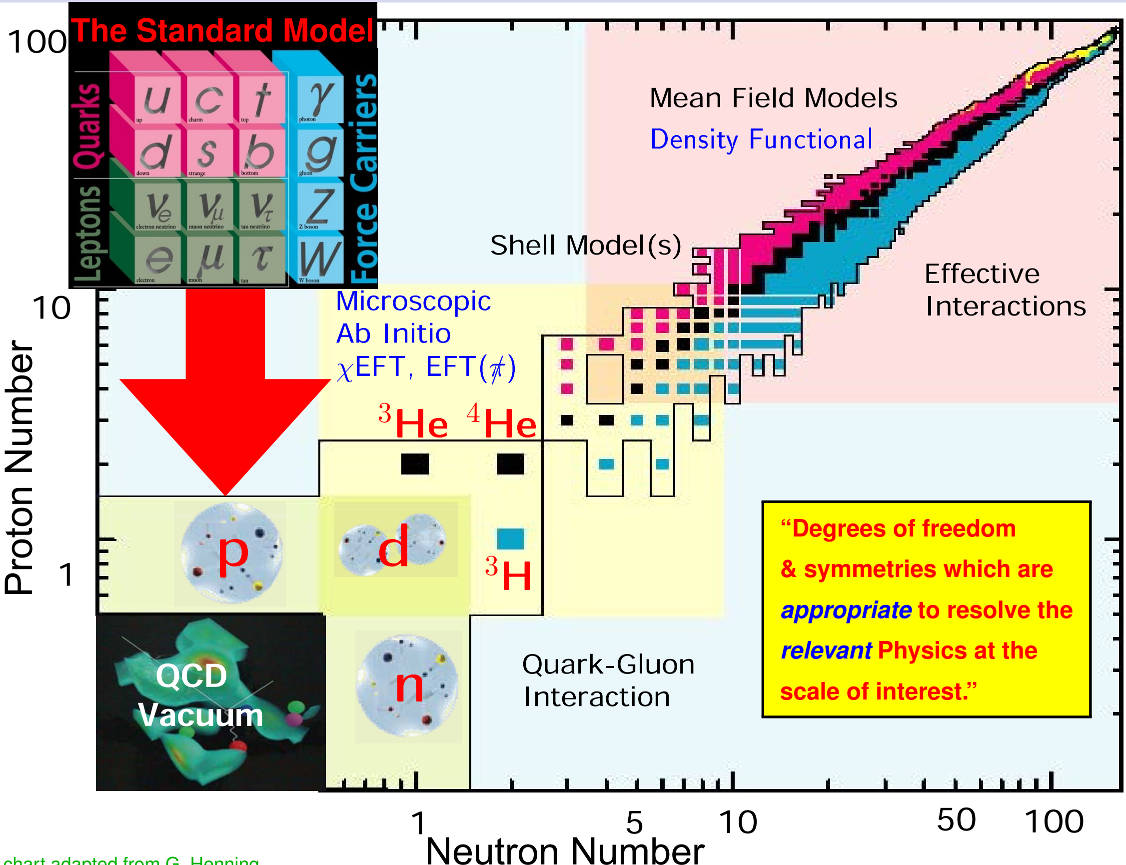
Falsifiability: Convergence to Nature tests assumptions. – *After* theoretical uncertainties determined.

(e) The Onion We Call Nature: The World Is Effective



All Physics Theories applicable only in a *limited energy range* (except in-effective TOE. . .).

(f) Nuclear Physics as a Series of Effective Field Theories



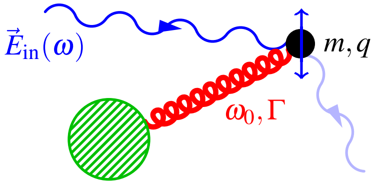
"Degrees of freedom & symmetries which are *appropriate* to resolve the *relevant* Physics at the scale of interest."

chart adapted from G. Henning

2. Compton Scattering $\gamma X \to \gamma X$ Explores Dynamics

(a) Polarisabilities: Stiffness of Charged Constituents in El.- Mag. Fields

Example: induced electric dipole radiation from harmonically bound charge, damping Γ Lorentz/Drude 1900/1905

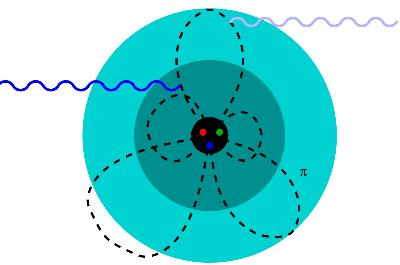


$$\vec{d}_{\rm ind}(\omega) = \underbrace{\frac{q^2}{m}\,\frac{1}{\omega_0^2 - \omega^2 - \mathrm{i}\Gamma\omega}}_{=:\,4\pi\,\alpha_{E1}(\omega)}\,\vec{E}_{\rm in}(\omega)$$

$$\mathcal{L}_{\rm pol} = 2\pi\Big[\underbrace{\alpha_{E1}(\omega)\,\vec{E}^2 + \beta_{M1}(\omega)\,\vec{B}^2}_{} + \dots\Big]$$

electric, magnetic scalar dipole
"displaced volume" $[10^{-3}\ \mathrm{fm}^3]$

$\Longrightarrow$ **Clean, perturbative probe of $\Delta(1232)$ properties, nucleon spin-constituents, χiral symmetry of pion-cloud & its breaking (proton-neutron difference).**



– fundamental hadron property $\Longrightarrow$ **link to emergent lattice-QCD results**

Alexandru/Lee/. . . 2005-, NPLQCD 2006-, LHPC 2007-, Leinweber/. . . 2013

(b) There Is Money In It...

2015 LRP: ***Great progress has been made in determining the electric and magnetic polarizabilities. Within the next few years, data are expected from [HIγS] that will allow accurate extraction of proton-neutron differences and spin polarizabilities....***

2015 QCD White Paper: "*Synergistic Blend of Theory and Experiment*"

Lattice QCD: relate to fundamental interactions

$\longrightarrow$ *polarQCD* (Alexandru/Lee) 2005-; *NPLQCD* 2006-; *LHPC* (Engelhardt) 2007-; Leinweber/... (Adelaide) 2013

Experiment: Significant investments; data taken/scheduled/approved:

HIγS (DOE): a central goal; > 3000 hrs committed at $60 - 100$ MeV

proton doubly & beam pol. (E-06-09/10) deuteron beam pol. (E-18-09, running)

^{3}He unpol & doubly pol. (E-07-10, E-08-16) ^{4}He unpol ^{6}Li unpol. (E-15-11, first!)

A2 @ MAMI (DFG: 5-year SFB): running, data cooking and planned

proton $100 - 400$ MeV: beam & target pol.

deuteron, ^{3}He, ^{4}He unpol., beam & target pol.

MAXlab: data cooking deuteron $100 - 160$ MeV: unpol.

Chiral EFT: data consistency, binding effects, analysis, extraction

Goal: Unified framework with reliable error bars for proton, deuteron, ^{3}He (elastic & inelastic) into $\Delta(1232)$ region.

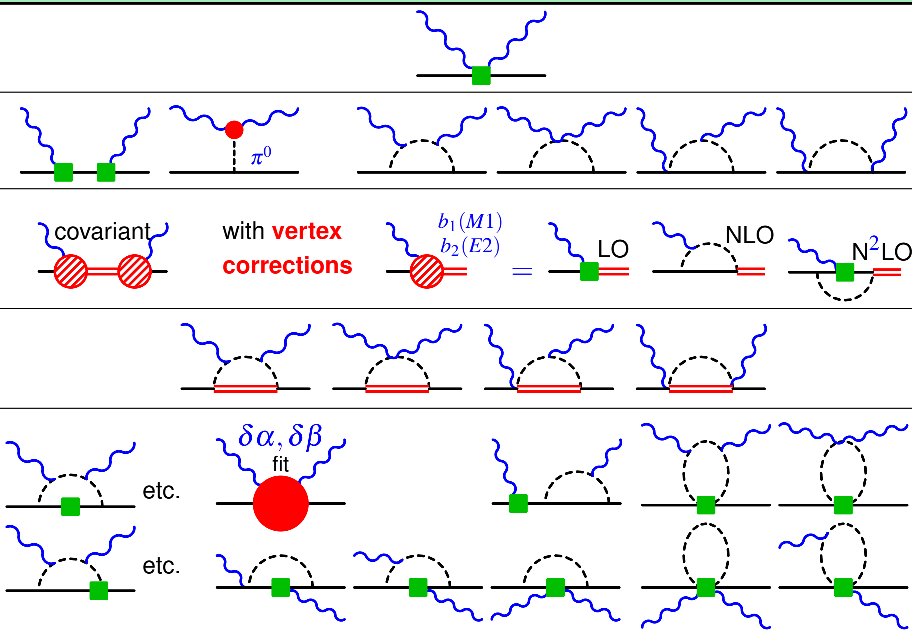
(c) All 1N Contributions to N^{4}LO

Unified Amplitude: gauge & RG invariant set of all contributions which are

in low régime $\omega \lesssim m_\pi$ at least N^{4}LO $(e^2\delta^4)$: accuracy $\delta^5 \lesssim 2\%$;

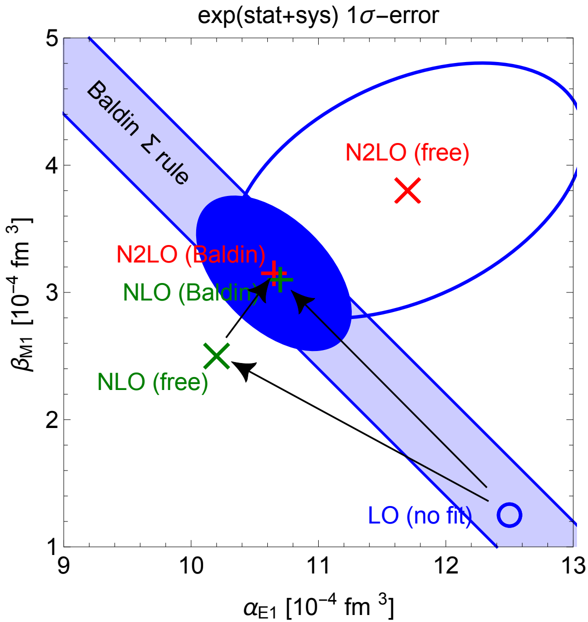
or in high régime $\omega \sim M_\Delta - M_N$ at least NLO $(e^2\delta^0)$: accuracy $\delta^2 \lesssim 20\%$.

	$\omega \lesssim m_\pi$	$\sim M_\Delta - M_N \approx 300$ MeV
Row 1	$e^2\delta^0$ LO	$e^2\delta^0 \searrow$ NLO
Row 2	$e^2\delta^2$ N^{2}LO	$e^2\delta^1$ N^{2}LO
Row 3	$e^2\delta^3$ N^{3}LO	$e^2\delta^{-1} \nearrow$ LO
Row 4	$e^2\delta^3$ N^{3}LO	$e^2\delta^1$ N^{2}LO
Row 5	$e^2\delta^4$ N^{4}LO	$e^2\delta^2$ N^{3}LO



Unknowns: short-distance $\delta\alpha, \delta\beta \Longleftrightarrow$ static α_{E1}, β_{M1}

(d) Fit Discussion: Parameters and Uncertainties



Residual Theoretical Uncertainty

McGovern/Phillips/hg: EPJ**A49** 12 (2013); many before

Observable/Series

$\mathcal{O} = c_0 + c_1 \delta^1 + c_2 \delta^2 +$ **unknown** $\times \delta^3$

Convergence pattern of $\alpha_{E1} - \beta_{M1}$ by

most conservative/worst-case of:

(1) $\delta \approx \frac{2}{5}$ of NLO→N²LO;

(2) $\delta^2 \approx \frac{1}{6}$ of LO→NLO;

(3) $\delta^2 \approx \frac{1}{6}$ of LO→N²LO. ⟸

Fit Stability: floating norms within exp. sys. errors; vary dataset, b_1, vertex dressing,...

(e) (Dis)Agreement Significant Only When All Error Sources Explored

Editorial PRA 83 (2011) 040001

physical effects not included in the calculation from the beginning, such as electron correlation and relativistic corrections. It is of course never possible to state precisely what the error is without in fact doing a larger calculation and obtaining the higher accuracy. However, the same is true for the uncertainties in experimental data. The aim is to estimate the uncertainty, not to state the exact amount of the error or provide a rigorous bound.

There are many cases where it is indeed not practical to give a meaningful error estimate for a theoretical calculation; for example, in scattering processes involving complex systems. The comparison with experiment itself provides a test of our theoretical understanding. However, there is a broad class of papers where estimates of theoretical uncertainties can and should be made. Papers presenting the results of theoretical calculations are expected to include uncertainty estimates for the calculations whenever practicable, and especially under the following circumstances:

1. If the authors claim high accuracy, or improvements on the accuracy of previous work.
2. If the primary motivation for the paper is to make comparisons with present or future high precision experimental measurements.
3. If the primary motivation is to provide interpolations or extrapolations of known experimental measurements.

These guidelines have been used on a case-by-case basis for the past two years. Authors have adapted well to this, resulting in papers of greater interest and significance for our readers.

The Editors

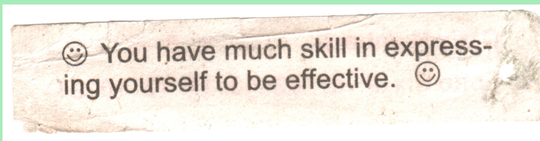
$$\alpha_{E1}^{p} = 10.65 \pm 0.35_{\text{stat}} \pm 0.2_{\Sigma} \pm 0.3_{\text{theory}}$$

Non-Theory Errors: Numerical $\Longrightarrow$ better computers.

Statistical/parameter $\Longrightarrow$ better data.

Theoretical uncertainty: Truncation of Physics

EFT claim: systematic in $Q = \dfrac{\textit{typ. low scale } p_{\textit{typ}}}{\textit{typ. high scale } \overline{\Lambda}_{\textit{EFT}}}$



Scientific Method: Quantitative results with corridor of theoretical uncertainties for *falsifiable predictions*.

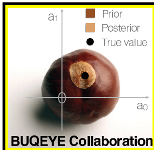
Need procedure which is established, economical, reproducible: room to argue about "error on the error".

"Double-Blind" Theory Errors: Assess with pretense of no/very limited data.

(f) Fit Discussion: What Does "Conservative" Error Mean?

Observable/Series $\mathcal{O} = \delta^n \left(c_0 + c_1 \delta^1 + c_2 \delta^2 + \textbf{unknown} \times \delta^3 \right) \implies$

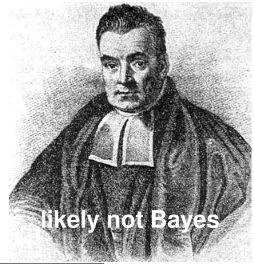
Estimate next term *"most conservatively"* as $|\textbf{unknown } c_3| \lesssim \max\{|c_0|;|c_1|;|c_2|\}$.



No infinite sampling pool; data fixed; more data changes confidence.

$\implies$ **Call upon the Reverend Bayes!**

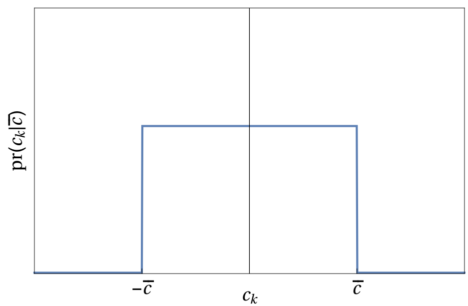
see e.g. **BUQEYE collaboration** Furnstahl/Phillips/... 1506.01343



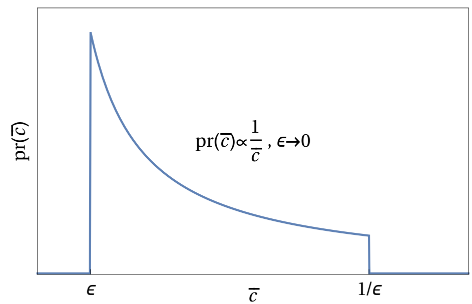
Bayes makes you specify your premises/assumptions about series.

Priors: leading-omitted term dominates ($\delta \ll 1$); putative distributions of *all* c_k's and of largest value $\bar{c}$ in series.

"Least informed/informative": All values c_k equally likely, given upper bound $\bar{c}$ of series.



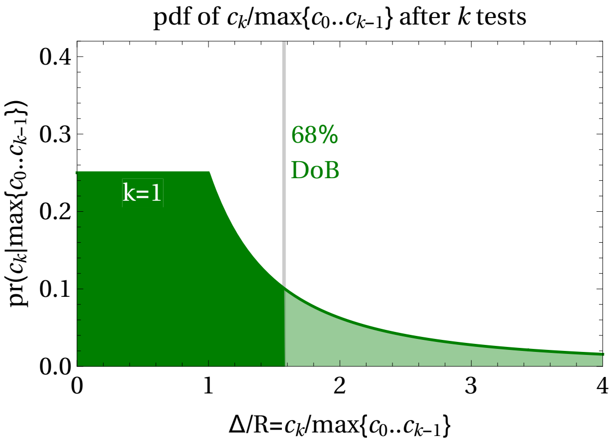
"Any upper bound": $\ln$-uniform prior sets no bias on scale of $\bar{c}$.



Information: Convergence LO$\to$NLO$\to$N^{2}LO gives probable "largest number" $R = \delta^k \max\{|c_0| \ldots |c_{k-1}|\}$.

Result: **Posterior** $\equiv$ ***Degree of Belief (DoB)*** that next term $c_k \delta^k$ differs from order-k central value by Δ.

$$\mathrm{pr}(\Delta|\max.\ R, \text{order } k) \propto \int_0^\infty \mathrm{d}\bar{c}\ \mathrm{pr}(\bar{c})\ \mathrm{pr}(c_k = \frac{\Delta}{\delta^k}|\bar{c}) \prod_n^{k-1} \mathrm{pr}(c_n|\bar{c}) \to \frac{k}{k+1}\frac{1}{2R}\begin{cases} 1 & |\Delta| \leq R \\ \left(\frac{R}{|\Delta|}\right)^{k+1} & |\Delta| > R \end{cases}$$



order	DOB in $\pm R$	σ	$\Delta(95\%)$
LO	50%	$1.6\,R$	$11R = 7\sigma$
Gauß	68.27%	$1.0\,R$	2.0σ

Quantifying One's Beliefs

Information: Convergence LO⟶NLO⟶N^{2}LO gives **probable "largest number"** $R = \delta^k \max\{|c_0| \ldots |c_{k-1}|\}$.

Result: **Posterior** $\equiv$ ***Degree of Belief (DoB)*** that next term $c_k \delta^k$ differs from order-k central value by Δ.

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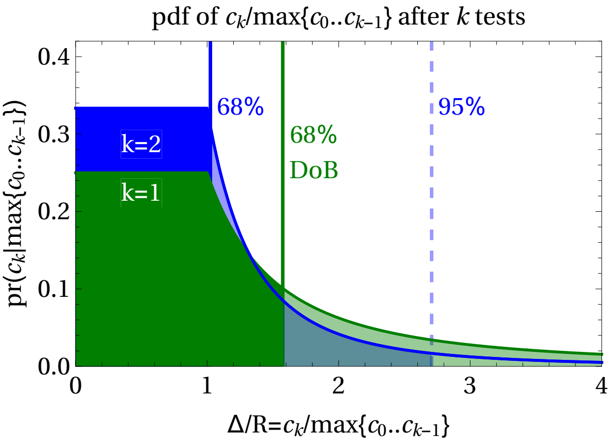


order	DOB in $\pm R$	σ	$\Delta(95\%)$
LO	50%	$1.6\,R$	$11R = 7\sigma$
NLO	66.7%	$1.0\,R$	$2.7R = 2.6\sigma$
Gauß	68.27%	$1.0\,R$	2.0σ

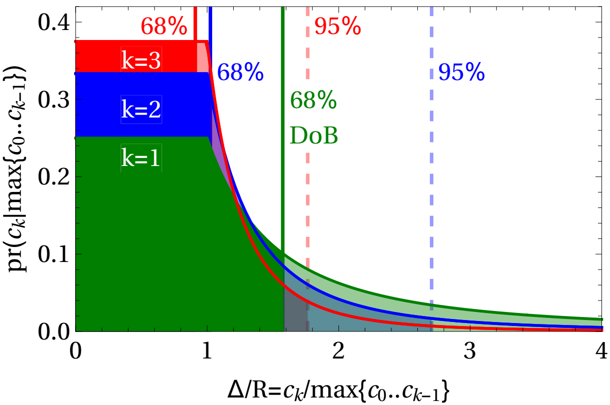
Quantifying One's Beliefs

Information: Convergence LO→NLO→N^{2}LO gives probable "largest number" $R = \delta^k \max\{|c_0| \dots |c_{k-1}|\}$.

Result: Posterior $\equiv$ ***Degree of Belief (DoB)*** that next term $c_k \delta^k$ differs from order-k central value by Δ.

$$\mathrm{pr}(\Delta|\max.\ R, \text{order } k) \propto \int_0^\infty \mathrm{d}\bar{c}\ \mathrm{pr}(\bar{c})\ \mathrm{pr}(c_k = \frac{\Delta}{\delta^k}|\bar{c}) \prod_n^{k-1} \mathrm{pr}(c_n|\bar{c}) \to \frac{k}{k+1}\frac{1}{2R}\begin{cases} 1 & |\Delta| \leq R \\ \left(\frac{R}{|\Delta|}\right)^{k+1} & |\Delta| > R \end{cases}$$

pdf of $c_k/\max\{c_0..c_{k-1}\}$ after k tests



order	DOB in $\pm R$	σ	$\Delta(95\%)$
LO	50%	$1.6\,R$	$11R = 7\sigma$
NLO	66.7%	$1.0\,R$	$2.7R = 2.6\sigma$
N^{2}LO	75%	$0.9\,R$	$1.8R = 1.9\sigma$
k	$\frac{k}{k+1}\%$		
Gauß	68.27%	$1.0\,R$	2.0σ

For "high enough" order, largest number R limits $\gtrsim 68\%$ **degree-of-belief interval.**

Varying priors: When $k \geq 2$ orders known, DoBs with different assumptions about $\bar{c}$, c_n vary by $\lesssim \pm 20\%$.

Posterior pdf *not Gauß'ian:* Plateau & power-law tail. – Do not add in quadrature in convolution!

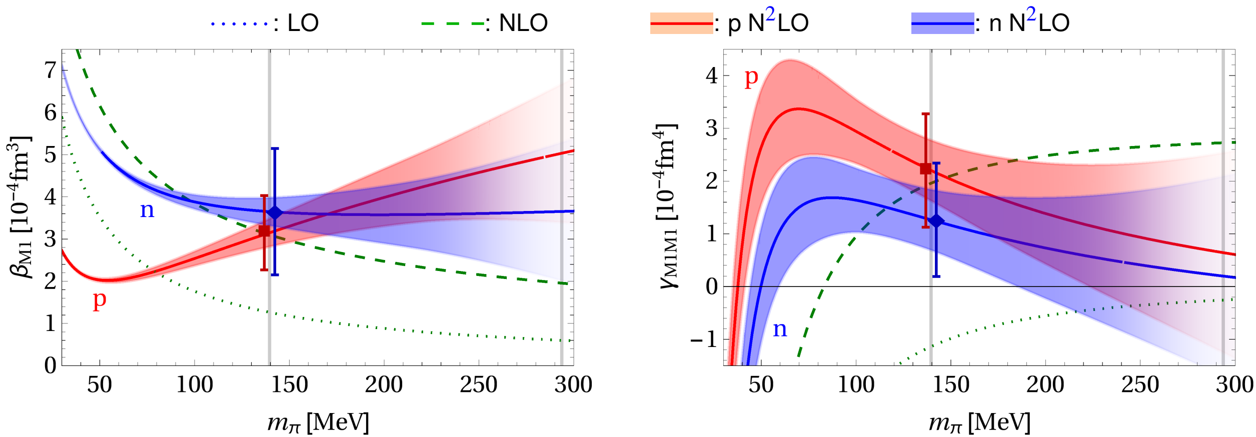
$\Longrightarrow$ ***Interpretation of all theory uncertainties, with these priors; "$A \pm \sigma$": 68% DoB interval*** $[A - \sigma; A + \sigma]$.

Observable $\mathcal{O} = c_0(m_\pi) + c_1(m_\pi)\delta^1 + c_2(m_\pi)\delta^2 +$ **unknown** $\times\, \delta^3$.

χ**EFT: explicit** m_π**-dependence, parameters fixed at** m_π^{phys}.

Propagating Uncertainties: Bayesian order-by-order as before, now at each m_π.

Some new terms linear in m_π. $\Longrightarrow$ **Conservatively expand in** $\delta(m_\pi) = 0.4 \times \frac{m_\pi}{m_\pi^{\text{phys}}}$, **fade as** $m_\pi \nearrow \frac{m_\pi^{\text{phys}}}{0.4}$.



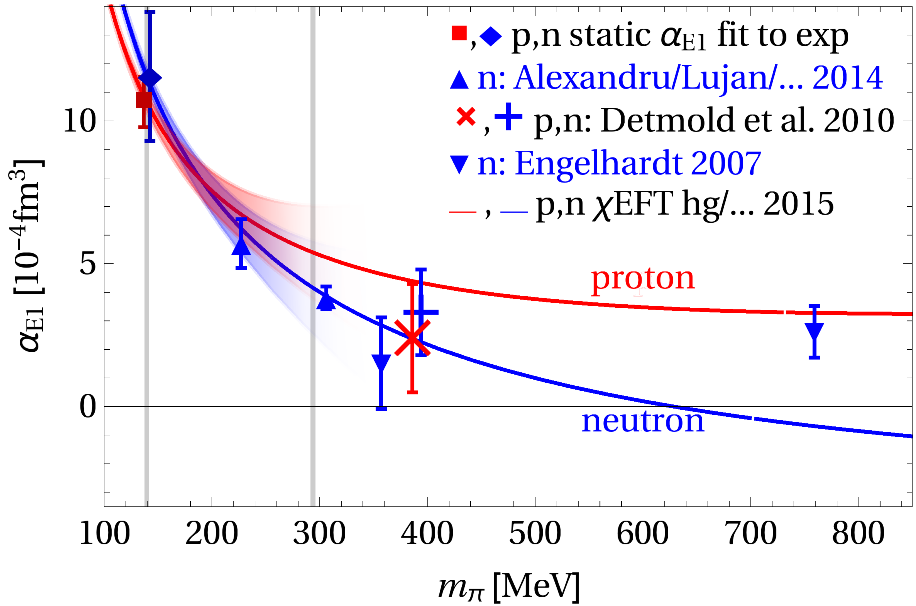
At physical $m_\pi = 140$ MeV: **paramagnetic** $\Delta(1232)$ **fine-tuned against diamagnetic NLO** π**N loops.**

Only physical point has no substantial isospin splitting.

(h) Electric Polarisabilities: This Is Not A Fit

Criteria: $m_\pi \ll \Lambda_\chi \approx 800\ \text{MeV}$, **extrapolated to infinite volume, fully dynamical (except for charging sea).**

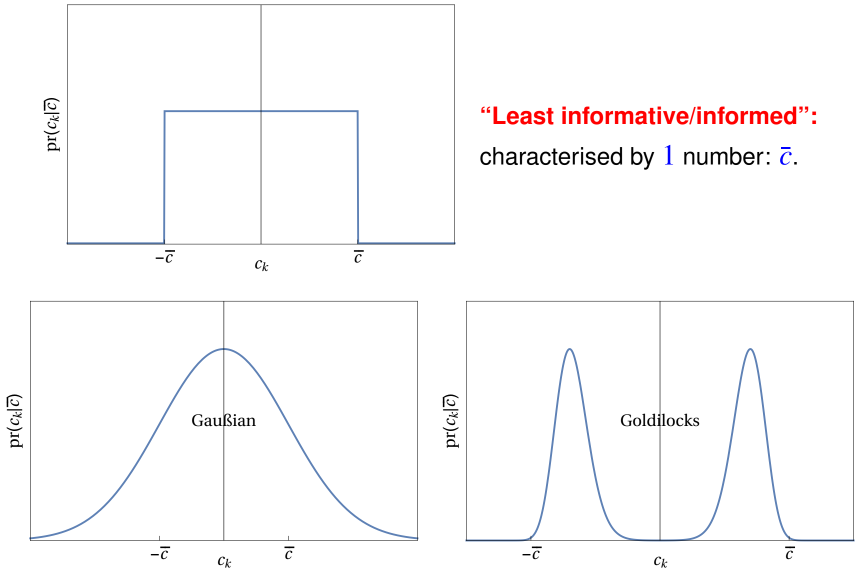
Lattice computations use χEFT for infinite-volume and partial-quenching: Detmold/Tiburzi/Walker-Loud 2006.



Eventually, use χEFT's functional form & uncertainties to extrapolate lattice to physical point.

(a) Prior Choice: What is “Natural Size”? (SCOTUS: I Know It When I see It.)

Observable/Series $\mathcal{O} = c_0 + c_1\delta^1 + c_2\delta^2 +$ **unknown** $\times\, \delta^3$ with ***“naturally-sized coefficients”*** c_i.



“Least informative/informed”:

characterised by 1 number: $\bar{c}$.

More informed choices: more complicated structures, more thought, more parameters: $\bar{c}$, typ. size, spread,. . .

BUQEYE (Wesolowski/Klco/. . .): When $k \geq 2$ orders known, DoBs with different assumptions about $\bar{c}$, c_n vary by $\lesssim \pm 20\%$ for some “reasonable priors”.

(b) More Bayes Comments and Questions

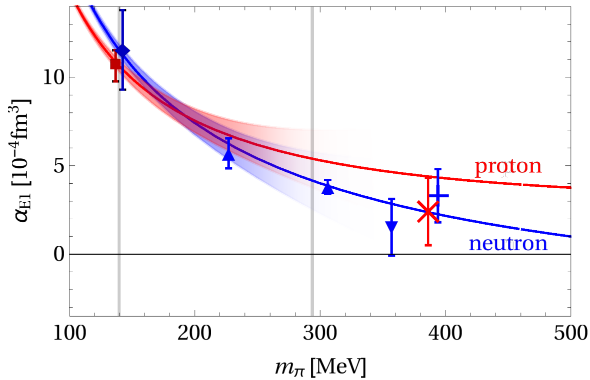
Observable/Series $\mathcal{O} = c_0 + c_1\delta^1 + c_2\delta^2 +$ **unknown** $\times\, \delta^3$ with ***"naturally-sized coefficients"*** c_i.

– **This is Tiny Data:** Usually only few orders known; symmetries may force some to exact zero: not counted in k.

– **Achilles Heel:** δ as input, but e.g. thermal triton capture $\sigma(nd \to t\gamma, \text{EFT}(\not\pi)) = [0.485 + 0.011 + 0.007]\text{mb}$.

Is split of terms in series into $c_i \times \delta^i$ artificial, or just convenient?

– $\delta(k) = \dfrac{\text{typ. momenta } k, \gamma, m_\pi}{\text{breakdown scale } \bar{\Lambda}_{\text{EFT}}}$ depends on k, not on process.



$\Longrightarrow$ Infer $\delta(k)$ from host of processes?

Determine breakdown scale $\bar{\Lambda}_{\text{EFT}}$ of theory?

How to account for correlations in k, m_π?

How to implement physical knowledge?

(monotonic function, $\alpha_{E1} < 0$ forbidden,...)

Fit/Extrapolation: How to reflect that $\delta(k)$ changes?

Can one extrapolate "by persistence" to where EFT does not converge (well)?

How to test Theory against Data to identify correct symmetries, degrees of freedom,...?

The Three Big Lies of Nuclear Physics

Nuclear Power is Safe.

They have Weapons of Mass Destruction.

My Power-Counting is Systematic.

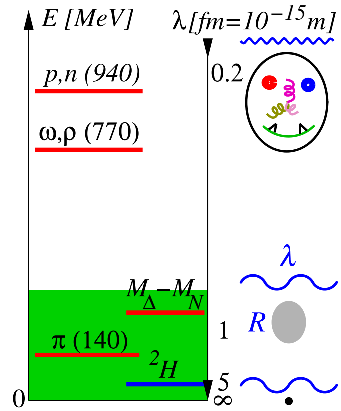
(c) Chiral Effective Field Theory of Nuclear Physics

Correct long-range + symmetries: Chiral SSB, gauge, iso-spin,…

$\Longrightarrow$ ***Write most general Interaction Lagrangean permitted.***

Short-range: ignorance into *minimal* parameter-set at given order.

Coefficients from experiment or QCD or…



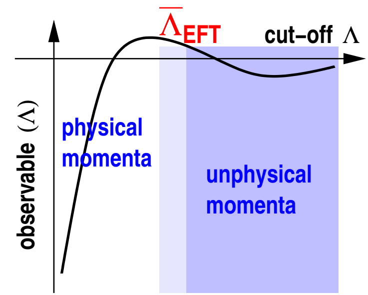
"The Power Counting":

Systematic ordering in $Q = \dfrac{\text{typ. momentum } p_{\text{typ}}}{\text{breakdown scale } \overline{\Lambda}_{\text{EFT}}} \ll 1$

Controlled approximation: model-independent, error-estimate.

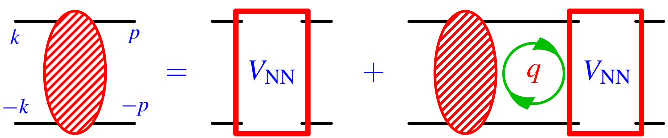
Space for improvement.

$\Longrightarrow$ **Chiral Effective Field Theory χEFT $\equiv$ low-energy QCD**



$\Longrightarrow$ saturated at $\overline{\Lambda}_{\text{EFT}} \lesssim \Lambda$.

Shallow real/virtual QCD bound states $\Longrightarrow$ Few-N non-perturbative!



$\Longrightarrow$ Analytic results rare; regularisation by **cut-off** $\Lambda \overset{!!}{\neq} \overline{\Lambda}_{\text{EFT}}$.

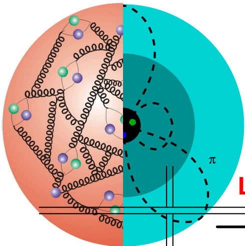
(d) Few-Nucleon Interactions in χEFT

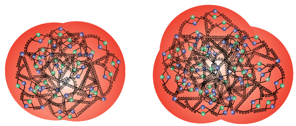
Weinberg, Ordóñez/Ray/van Kolck, Friar/Coon, Kaiser/Brockmann/Weise, Epelbaum/Glöckle/Meißner, Entem/Machleidt, Kaiser, Higa/Robilotta, Epelbaum, . . .

$\frac{\text{typ. momentum}}{\text{breakdown scale}} \ll 1$

Long-Range: correct symmetries and IR degrees of freedom: **Chiral Dynamics**

Short-Range: symmetries constrain contact-ints to simplify UV: **Minimal parameter-set**





Hierarchy: 2NF-effects $\gg$ 3NF-effects $\gg$ 4NF-effects

	LO	NLO	N^2LO	N^3LO
$2N$ ints	2 parameter	$\propto p^2$ $+7$ parameter	$+0$ parameter	etc. $\propto p^4$ $+15 = 24$ param.
$3N$ ints	—	—	D E 2 parameter	etc. parameter-free, in progress
$4N$ ints	—	—	—	etc. parameter-free

(e) NN χEFT Power Counting Comparison

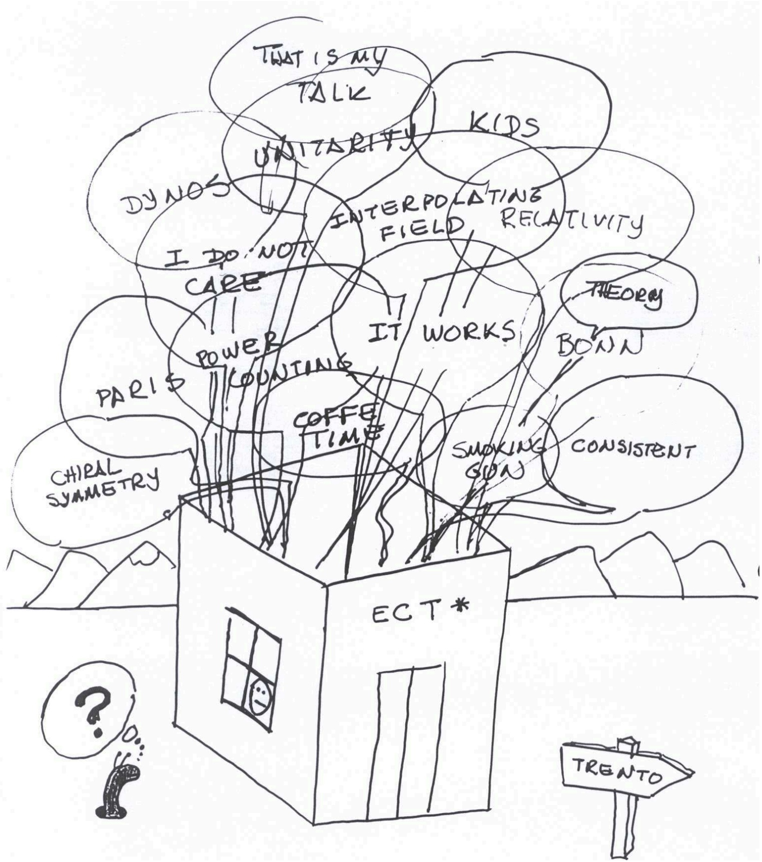
prepared for Orsay Workshop by Grießhammer 7.3.2013
based on and approved by the authors in private communications

Derived with explicit & implicit assumptions; contentious issue.
Proposed order Q^n at which counter-term enters *differs*. $\Longrightarrow$ Predict *different* accuracy, # of parameters.

<table>
<tr><th>wave</th><th>order</th><th>Yang/Long
PRC86(2012) 024001 etc.</th><th>Pavon Valderrama
PRC74 (2006) 054001 etc.</th><th>Birse
PRC74 (2006) 014003</th></tr>
<tr><td>$^1S_0$</td><td>LO</td><td colspan="3">-1</td></tr>
<tr><td></td><td>NLO</td><td colspan="3">0</td></tr>
<tr><td></td><td>N^{2}LO</td><td>1</td><td colspan="2">2</td></tr>
<tr><td>$^3S_1$</td><td>LO</td><td colspan="3">-1</td></tr>
<tr><td></td><td>NLO</td><td>1</td><td>2</td><td>$\frac{1}{2}$</td></tr>
<tr><td>$^3SD_1$</td><td>LO</td><td>1</td><td>$-\frac{1}{2}$</td><td>-1</td></tr>
<tr><td></td><td>NLO</td><td></td><td>2</td><td>$\frac{1}{2}$</td></tr>
<tr><td>$^3D_1$</td><td>LO</td><td></td><td>$-\frac{1}{2}$</td><td>-1</td></tr>
<tr><td></td><td>NLO</td><td></td><td>2</td><td>$\frac{1}{2}$</td></tr>
<tr><td>$^3P_0$ (attr. triplet)</td><td>LO</td><td colspan="2">-1</td><td>$-\frac{1}{2}$</td></tr>
<tr><td>TPE</td><td>LO</td><td>1</td><td colspan="2">2</td></tr>
<tr><td colspan="2"># of param. at Q^{-1}</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td colspan="2"># of param. at Q^0</td><td>4</td><td>6</td><td>6</td></tr>
<tr><td colspan="2"># of param. at Q^1</td><td>8</td><td>6</td><td>9</td></tr>
</table>

Weinberg: LO: 2; NLO: $+0$; N^{2}LO: $+7 = 9$ – different channels; consistency questioned Beane/...2002; Nogga/...2005

With same χ^2, proposal with least parameters *wins*: minimum information bias.



M. Robilotta: Impression of the Workshop on Nuclear Forces at the ECT*, Trento 1999

(f) Using Cut-Offs to Your Advantage

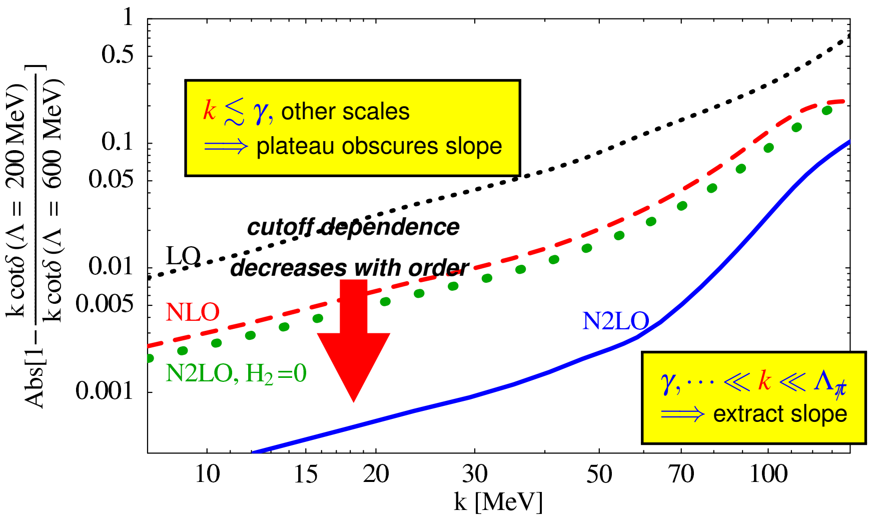
Observable $\mathcal{O}(k)$ at momentum k, order Q^n in EFT, cut-off Λ:

$$\mathcal{O}_n(k;\mu) = \underbrace{\sum_i^n \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^i \mathcal{O}_i(k, p_{\text{typ.}})}_{\text{renormalised, } \Lambda\text{-indep.}} + \underbrace{\mathcal{C}(\Lambda; k, p_{\text{typ}}, \overline{\Lambda}_{\text{EFT}}) \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^{n+1}}_{\text{residual } \Lambda\text{-dependence}}$$

residual Λ-dependence
parametrically small
$\mathcal{C}$ "of natural size"

$\Longrightarrow$ **Difference between *any two* cut-offs:** $$\frac{\mathcal{O}_n(k;\Lambda_1) - \mathcal{O}_n(k;\Lambda_2)}{\mathcal{O}_n(k;\Lambda_1)} = \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^{n+1} \times \frac{\mathcal{C}(\Lambda_1) - \mathcal{C}(\Lambda_2)}{\mathcal{C}(\Lambda_1)}$$

Ideally, *no resort to Data!* – Test consistency: Does numerics match predicted convergence pattern?



$$\left|1-\frac{k\cot\delta(\Lambda=200\ \mathrm{MeV})}{k\cot\delta(\Lambda=\infty)}\right| \sim \underbrace{\left(\frac{p_{\mathrm{typ.}}}{\Lambda_{\not\pi}}\right)^{n+1}}_{Q^{n+1}}$$

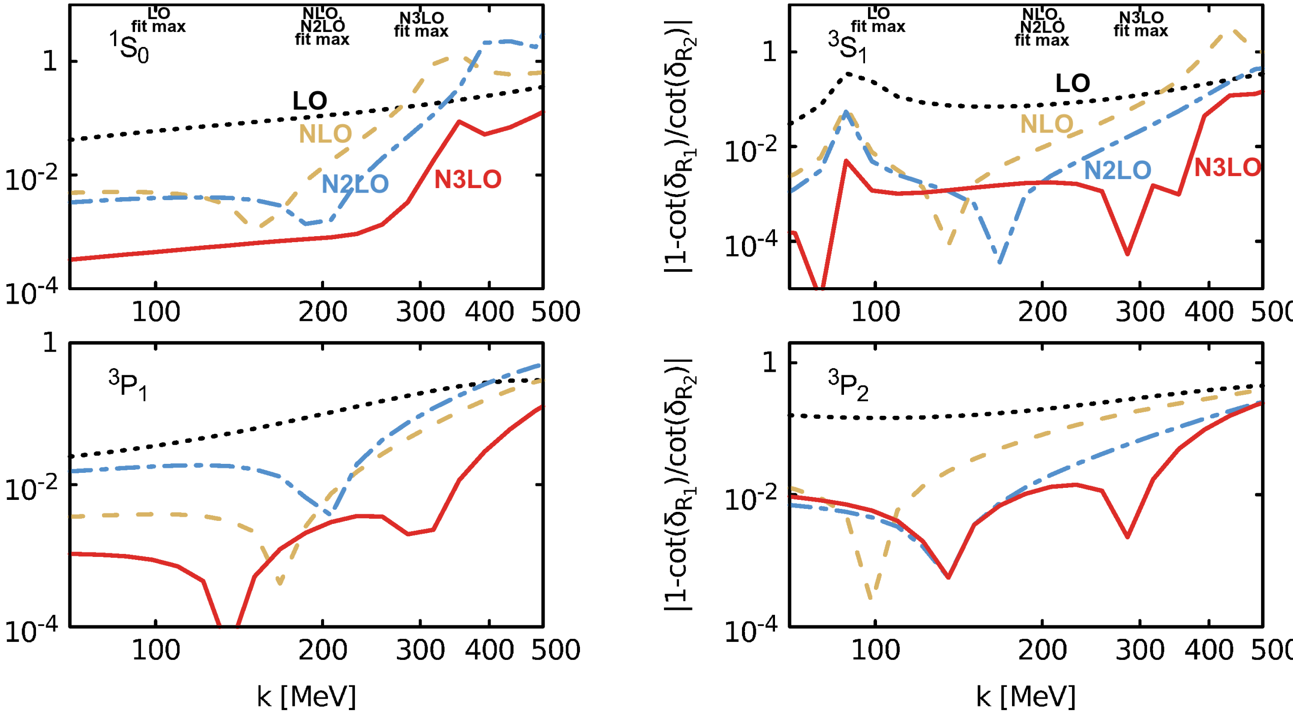
	LO	NLO	N^{2}LO	N^{2}LO without H_2
$n+1$ fitted	~ 1.9	2.9	4.8	3.1
$n+1$ predicted	2	3	4	**not renormalised**

$\Longrightarrow$ Fit to $k \in [70; 100 \dots 130]$ MeV

Slope Confirms Power Counting; Estimates $\Lambda_{\not\pi} \approx 140$ MeV; Determines Mom.-Dep. Uncertainties.

(h) Case of Interest: NN in χEFT: Fitting Parameters Obscures Slopes

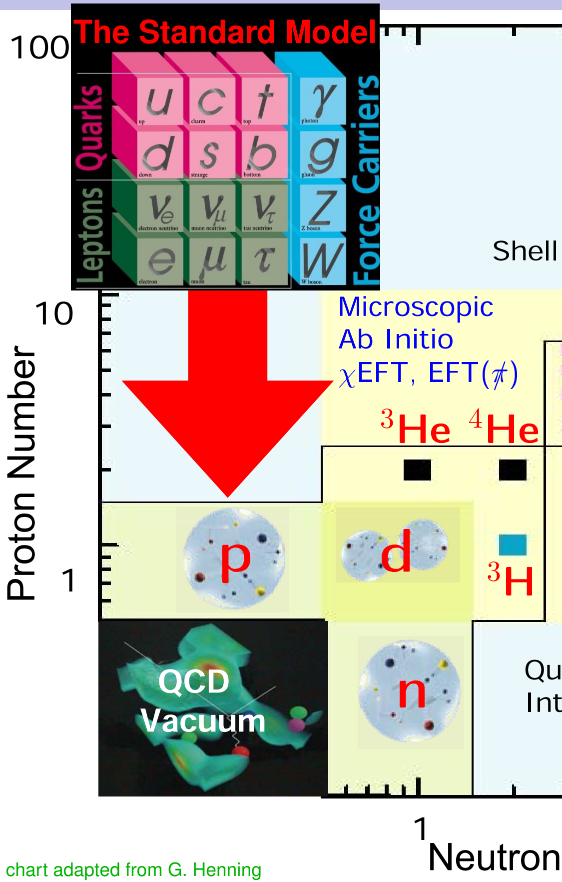
Plot stolen from **Epelbaum/Krebs/Meißner EPJA51 (2015) 5, 53**.



Inconclusive: Breakdown scale $400-500$ MeV $\Longleftrightarrow$ $\Delta(1232)$? NLO, N^{2}LO parallel? Slopes?

Coupled channels; attractive tensor? Fit- & slope-regions not clearly separated.

4. Concluding Statements and Disclaimer



Vision: ***Reliable residual theory uncertainties*:**

Reproducible & falsifiable extractions, predictions and extrapolations, also for "impossible" experiments.

Clear signals of symmetries, constituents, interactions.

Bridge to & simplify underlying theory.

Means: ***Effective Field Theories***

Model-independent, systematic, universal, simple, *minimum information bias*.

Perturbative/converging series in a small (?) parameter:

– Power Counting ≡ Importance Ordering –

Bayes comes natural and is essential!

Goal: World Domination by Uncertainty Quantification.

– *Error Bars for Physical Theories!* –

chart adapted from G. Henning

The efficient person gets the job done right. The effective person gets the right job done.