INT Program INT-16-2a

Bayesian Methods in Nuclear Physics

June 24, 2016

Bayes this!

Identifying and quantifying theoretical uncertainties.

Doron Gazit

Racah Institute of Physics

Hebrew University of Jerusalem

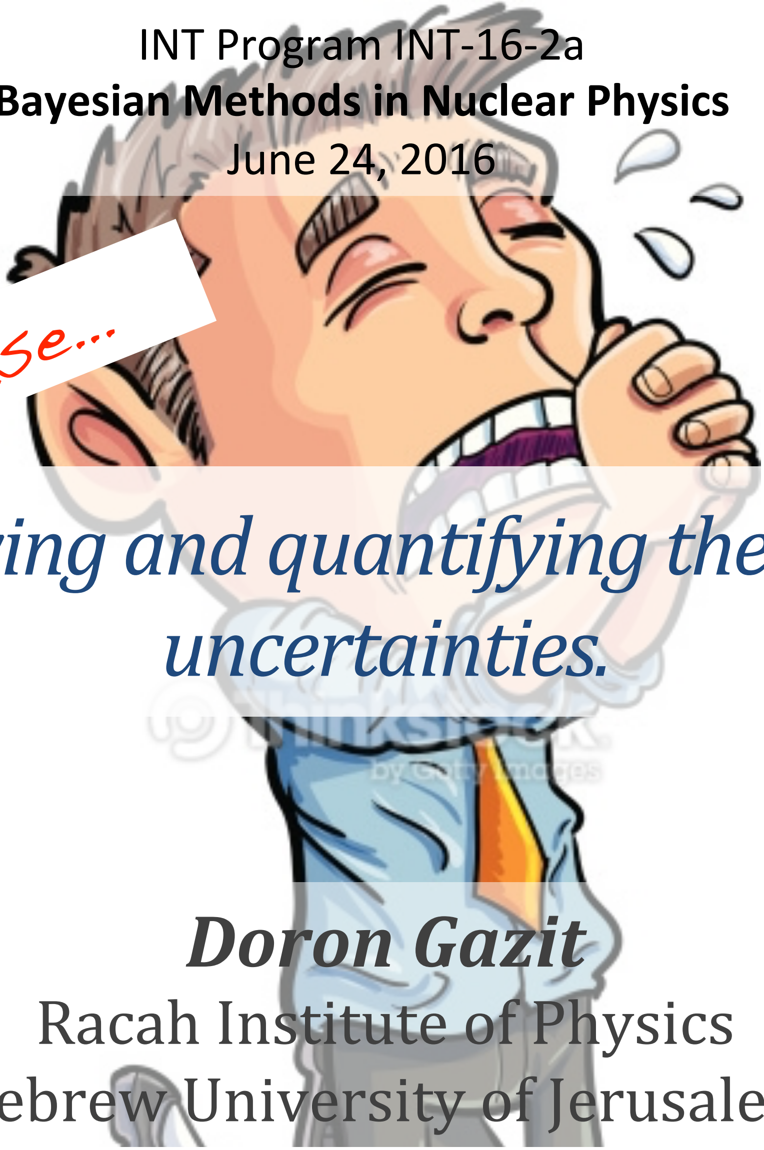
האוניברסיטה העברית בירושלים

THE HEBREW UNIVERSITY OF JERUSALEM

Outline

- In the talk I will present a few recent problems where theoretical uncertainty assessment has become a main part of the challenge:
  - ***Nuclear physics of light nuclei***: tale of two effective field theory descriptions of light nuclei
  - ***Nuclear fusion rates***: predicting the proton-proton fusion rate in the Sun.
  - ***Atomic physics of in extreme density and temperature***: the solar abundance problem.
- In all these problems, my feeling is that I've used the "chi-by-eye" version of theoretical uncertainty assessment:
  - Is there a better, more systematic, approach?

INT Program INT-16-2a
Bayesian Methods in Nuclear Physics
June 24, 2016



Bayes this, please...

Identifying and quantifying theoretical uncertainties.

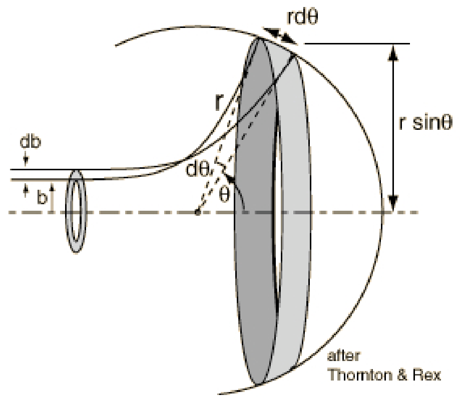
Doron Gazit
Racah Institute of Physics
Hebrew University of Jerusalem

האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

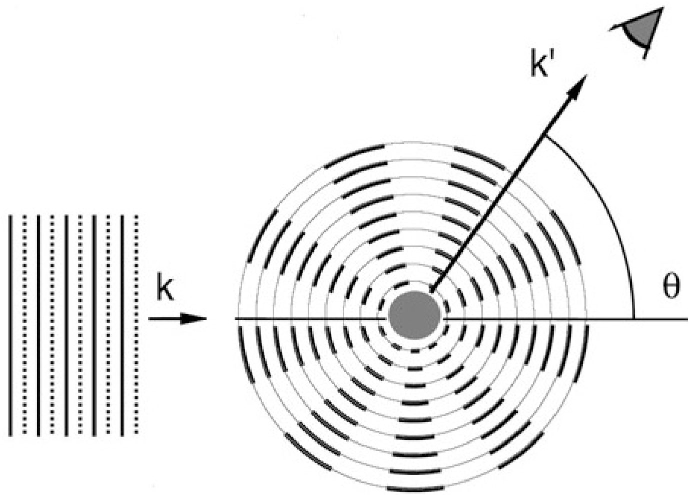
CORRELATIONS IN NUCLEAR PHYSICS AND FINE TUNING.

Strong interactions at low-energies:

- Interaction is assessed using the “cross-section”:



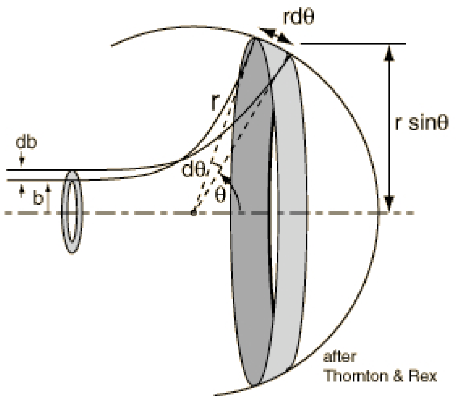
classical



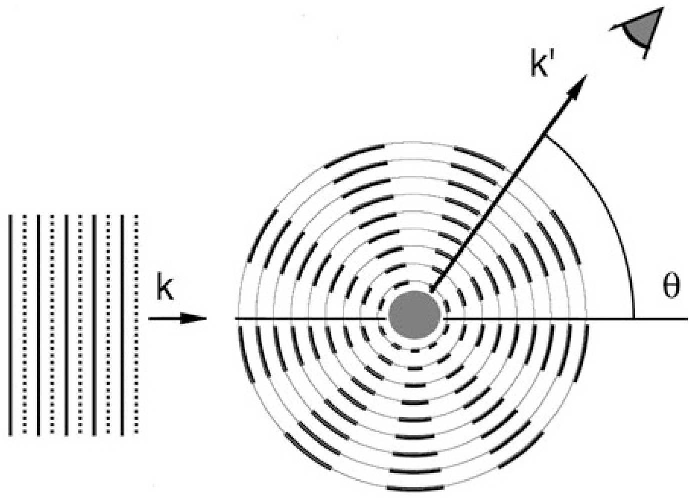
quantum

Strong interactions at low-energies:

- Interaction is assessed using the "cross-section":



classical



quantum

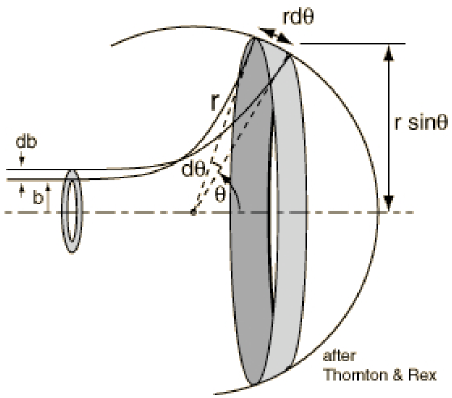
Defining "strong interaction" at low energies:

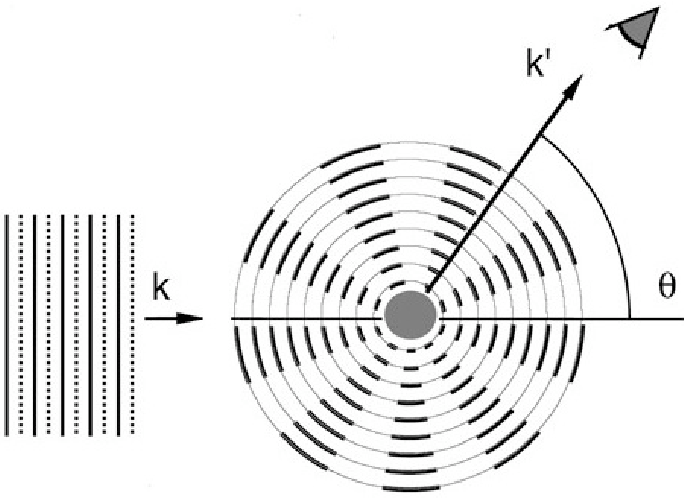
$$\sigma = 4\pi a^2 \rightarrow \infty$$

a – "scattering length" – extent of the wave function (layman term)

Strong interactions at low-energies:

- Interaction is assessed using the “cross-section”:





classical

quantum

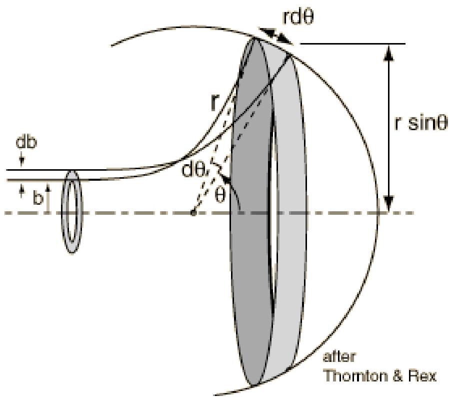
Defining “strong interaction” at low energies:

Infinity in physics: $\sigma = 4\pi a^2 >> 4\pi \left(\begin{array}{c} all\, other \\ length\, scales \end{array} \right)^2$

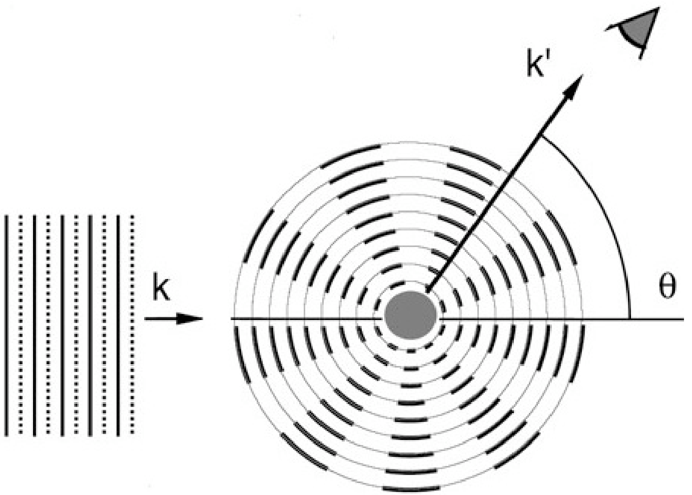
a – “scattering length” – extent of the wave function (layman term)

Strong interactions at low-energies:

- Interaction is assessed using the "cross-section":



classical



quantum

r – "effective range of the interaction"

Infinity in physics: $\sigma = 4\pi a^2 >> 4\pi r^2$

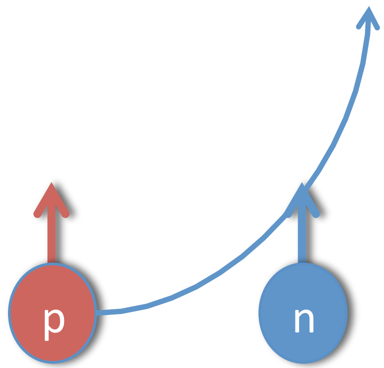
a – "scattering length" – extent of the wave function (layman term)

Nuclear physics at very low energies is strongly interacting!

$$\sigma = 4\pi a^2 >> 4\pi r^2$$

$$a \approx 10\, fm >> r \approx 1\, fm$$

Nuclear physics at very low energies is strongly interacting!

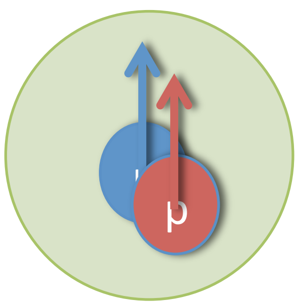


$$\sigma = 4\pi a^2 >> 4\pi r^2$$

$$a \approx 10\, fm >> r \approx 1\, fm$$

1. Nucleon-nucleon scattering experiments,
2. deuteron (bound state of neutron and proton) "vanishing" binding energy – from dimensional analysis in the presence of one dominant length scale.

$$E = -\frac{\hbar^2}{2ma^2} \to 0$$



>>

1. "Size" of the nucleus.

Nuclear physics at very low energies is strongly interacting!

$$\sigma = 4\pi a^2 >> 4\pi r^2$$

$$a \approx 10\, fm >> r \approx 1\, fm$$

1. Nucleon-nucleon scattering experiments,
2. deuteron (bound state of neutron and proton) "vanishing" binding energy – from dimensional analysis in the presence of one dominant length scale.

$$E = -\frac{\hbar^2}{\ } \rightarrow 0$$

>>

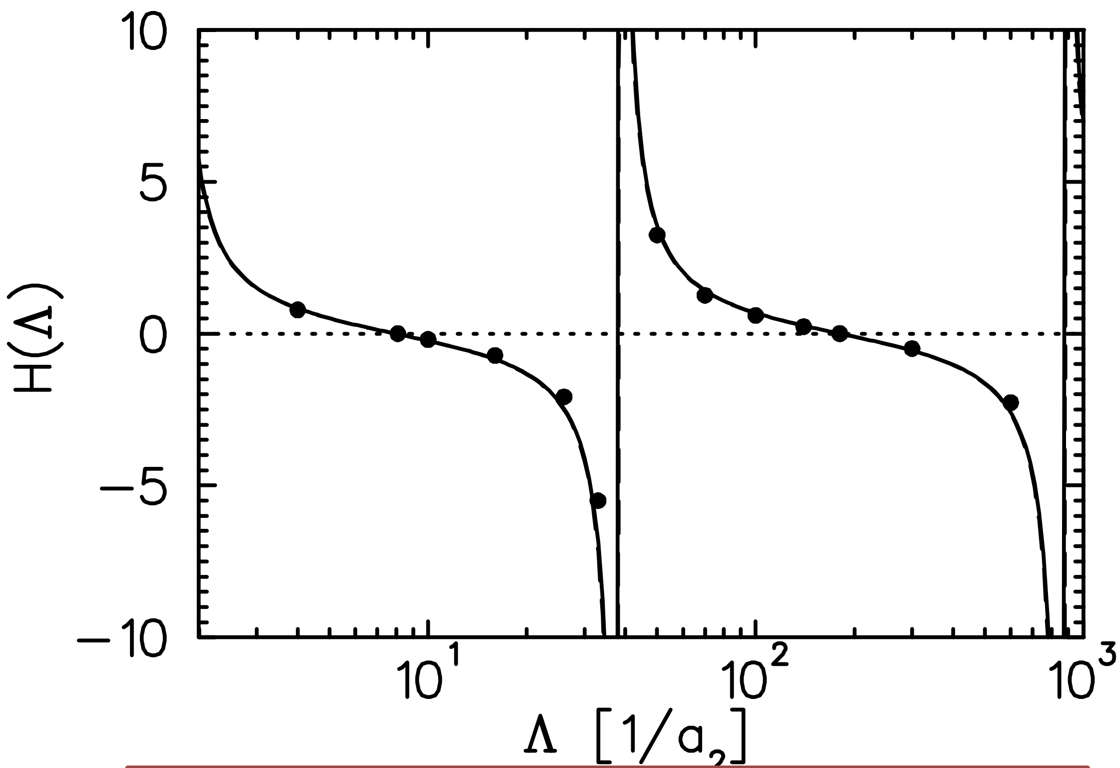
1. "Size" of the nucleus.

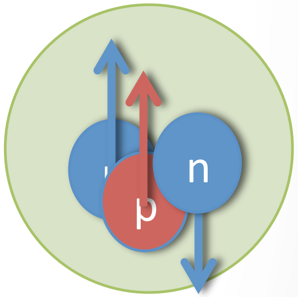
"Pionless" effective field theory of nuclear physics:

1. Leading order – r/a→0; Next to leading order – r/a linear corrections
2. The EFT is applicable at low energies, in which only nucleons are valid degrees of freedom. Most general interaction consistent with power counting.
3. If Lagrangian consistent with QCD symmetries: a QCD prediction.
4. Renormalization group invariance: given the EFT Lagrangian, low energy observables are insensitive to details at high energies: introduce a cutoff Λ, and verify that observables are independent of Λ.

$\not{\pi}$EFT @ LO: 3 particles ground state

Bedaque, Hammer, van-Kolck: (1999)
What is the binding energy of triton?



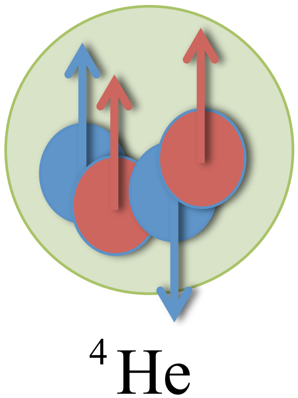


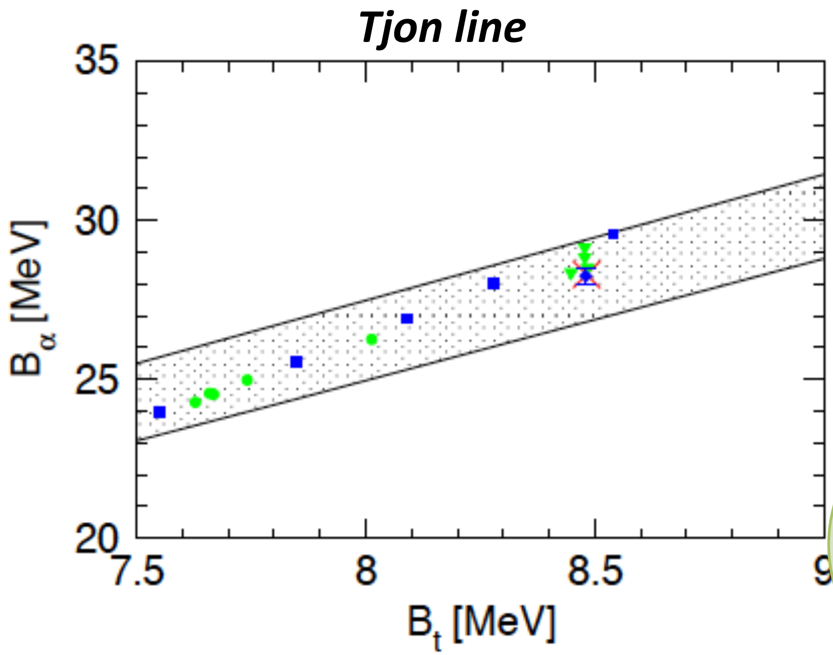
^{3}H

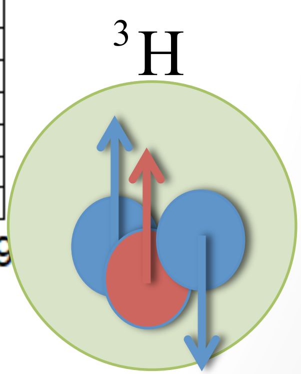
triton B.E. at LO has strong cutoff dependence → add 3-body contact at LO

$\not\pi$EFT – and Correlations in light nuclei

- No 4 body parameter at LO.
- The binding energy of the 4-body ground state should be correlated to 3-body ground state.
- Phenomenological "Tjon" correlation (1975) originates in Pionless EFT!







Platter (2006), Platter, Hammer, Meissner (2005), Kirscher, Griesshammer, Hofmann (2007)

Nuclear physics at very low energies is strongly interacting!

$$r \approx 1\, fm$$

Finite range of the interaction indicates the existence of a massive particle that intermediates the strong force: The pion!

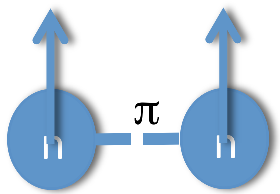
$$m_\pi \approx 135 \mathrm{MeV/c^2} \sim \frac{1}{r}\frac{\hbar c}{c^2}$$

Nuclear physics at very low energies is strongly interacting!

$$r \approx 1\, fm$$

Finite range of the interaction indicates the existence of a massive particle that intermediates the strong force: The pion!

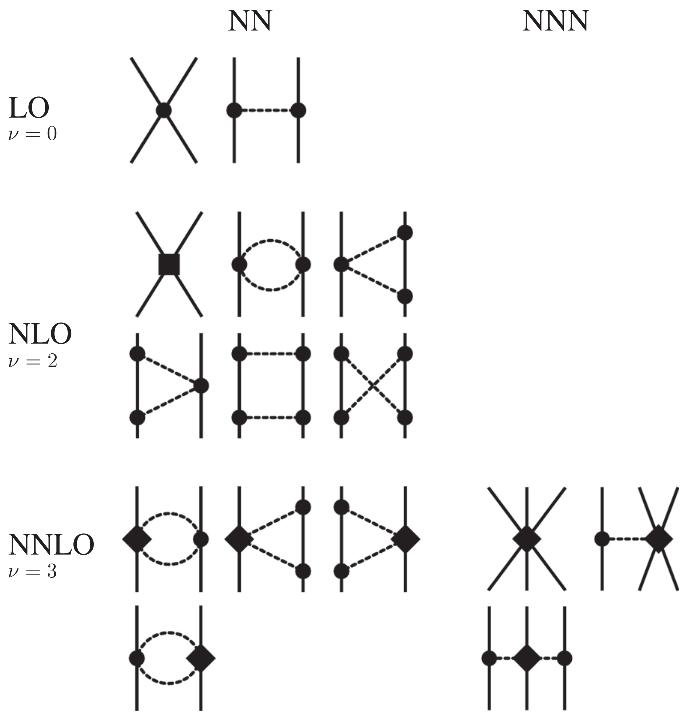
$$m_\pi \approx 135\mathrm{MeV/c^2} \sim \frac{1}{r}\frac{\hbar c}{c^2}$$



"Chiral" effective field theory of nuclear physics:

1. Expansion about zero momentum and zero pion mass of a theory of nucleons interacting via contacts and pion exchanges.
2. The EFT is applicable at energies up to few hundered MeV/c.
3. The Lagrangian is intimately related to QCD fundamental symmetries: a QCD prediction!
4. The common approach is NOT renormalization group invariant.

χEFT potential



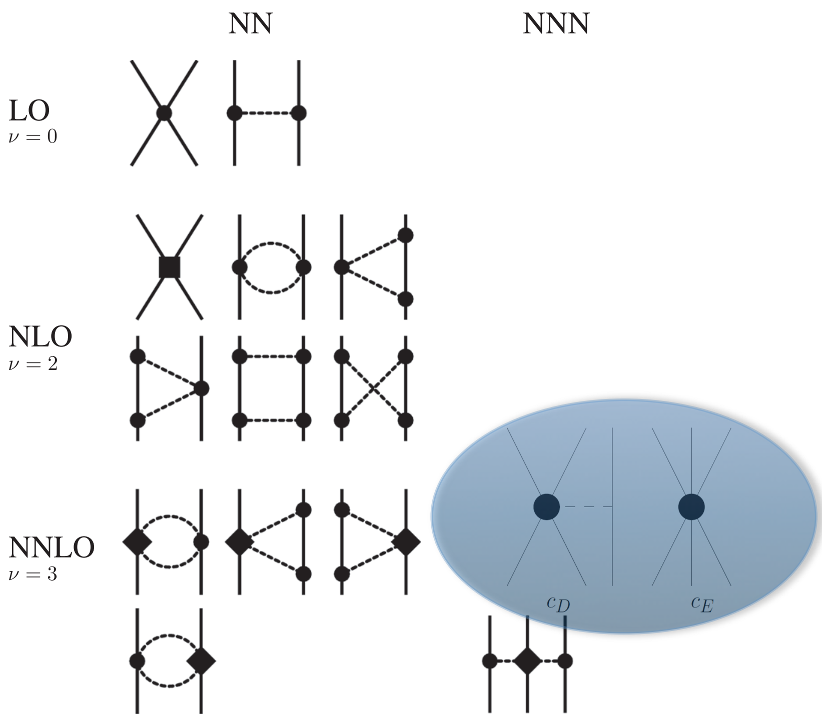
28 total parameters @ NNLO,
Fitted to hundreds of
data points!

PHYSICAL REVIEW X **6,** 011019 (2016)

Uncertainty Analysis and Order-by-Order Optimization of Chiral Nuclear Interactions

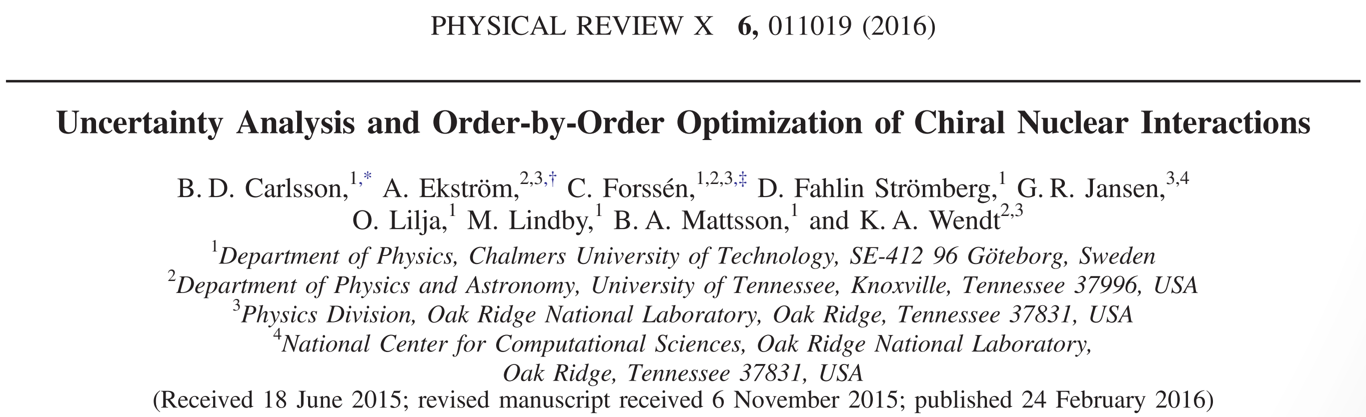
B. D. Carlsson,[1,*] A. Ekström,[2,3,†] C. Forssén,[1,2,3,‡] D. Fahlin Strömberg,[1] G. R. Jansen,[3,4]
O. Lilja,[1] M. Lindby,[1] B. A. Mattsson,[1] and K. A. Wendt[2,3]
[1]*Department of Physics, Chalmers University of Technology, SE-412 96 Göteborg, Sweden*
[2]*Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA*
[3]*Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA*
[4]*National Center for Computational Sciences, Oak Ridge National Laboratory,*
Oak Ridge, Tennessee 37831, USA
(Received 18 June 2015; revised manuscript received 6 November 2015; published 24 February 2016)

χEFT potential



28 total parameters @ NNLO, Fitted to hundreds of data points!

Power counting leads to 3-body forces at Next-to-next-to-leading order.



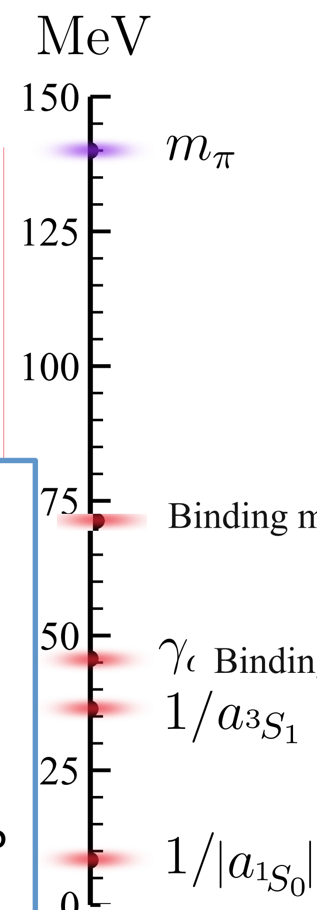
Two QCD EFTs

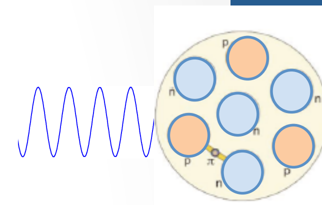
At very low energies both EFTs are applicable!
Can we learn something from that?

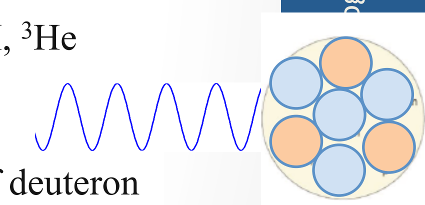
1. EFTs have inherent inaccuracy, as they include neglected orders.
2. However, often higher orders include new parameters, that need to be fitted to observables.

EFT Uncertainty Challenge 1:

1. How does one choose the observable to fit?
2. Are there right/wrong choices?

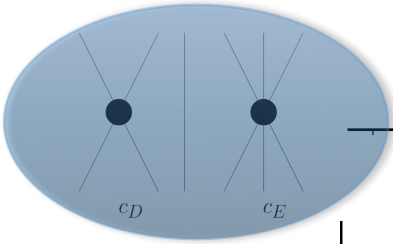


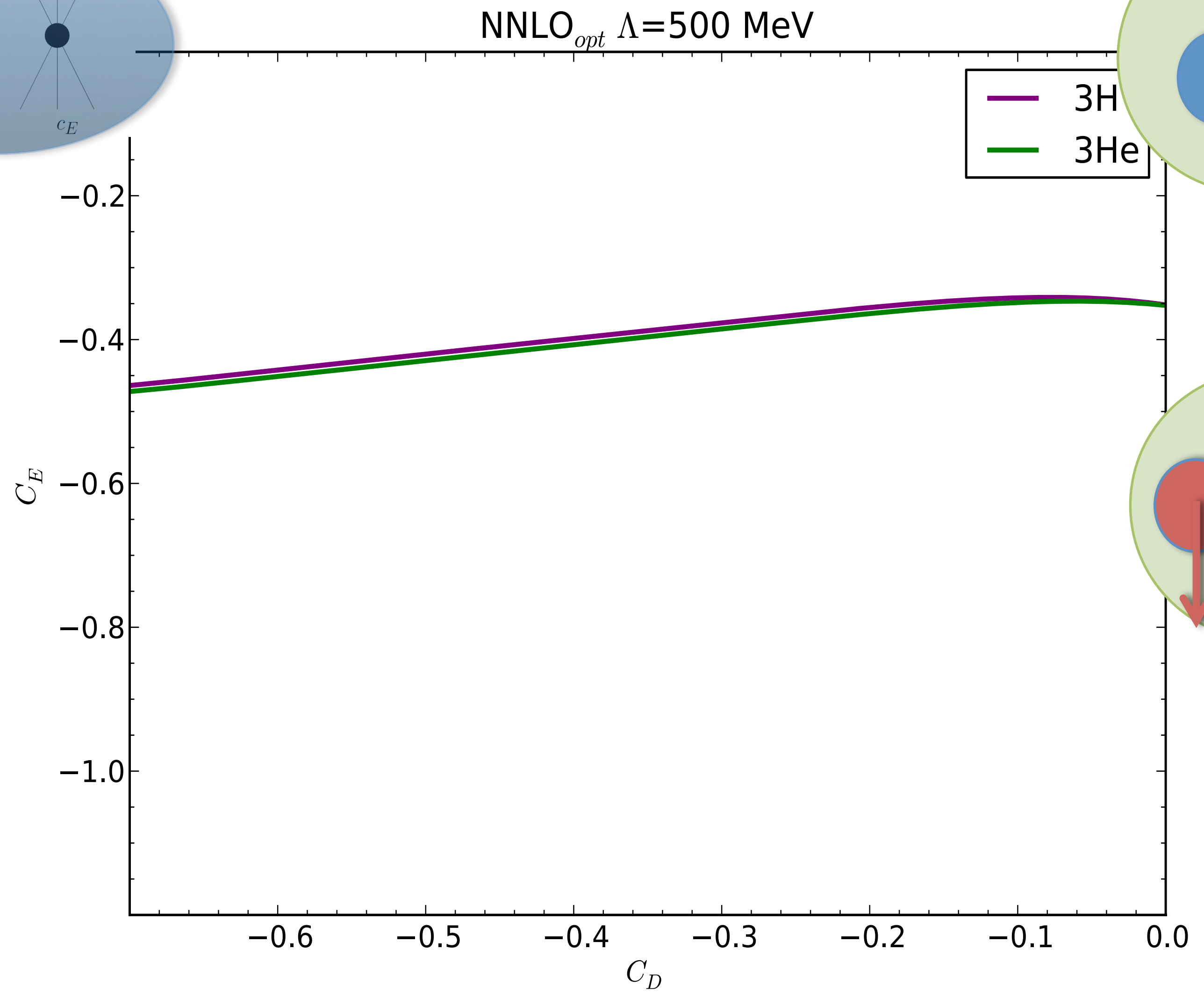


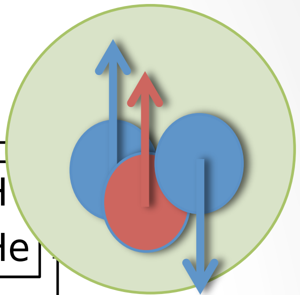


- $a_{^3S_1} \approx 5.4$ fm, $a_{^1S_0} \approx -23.7$ fm $\gg 1/m_\pi \approx 1.4$ fm
- effective ranges (1.8 fm, 2.7 fm) are natural

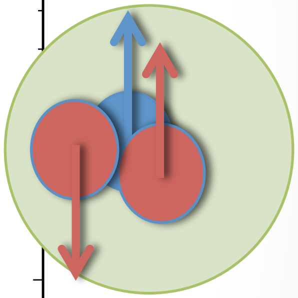
χEFT – three body problem in the (c_D, c_E) plane.







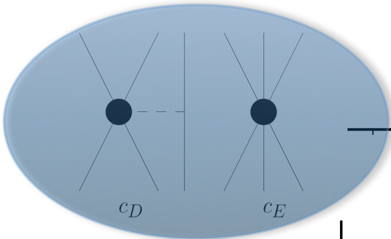
^{3}H

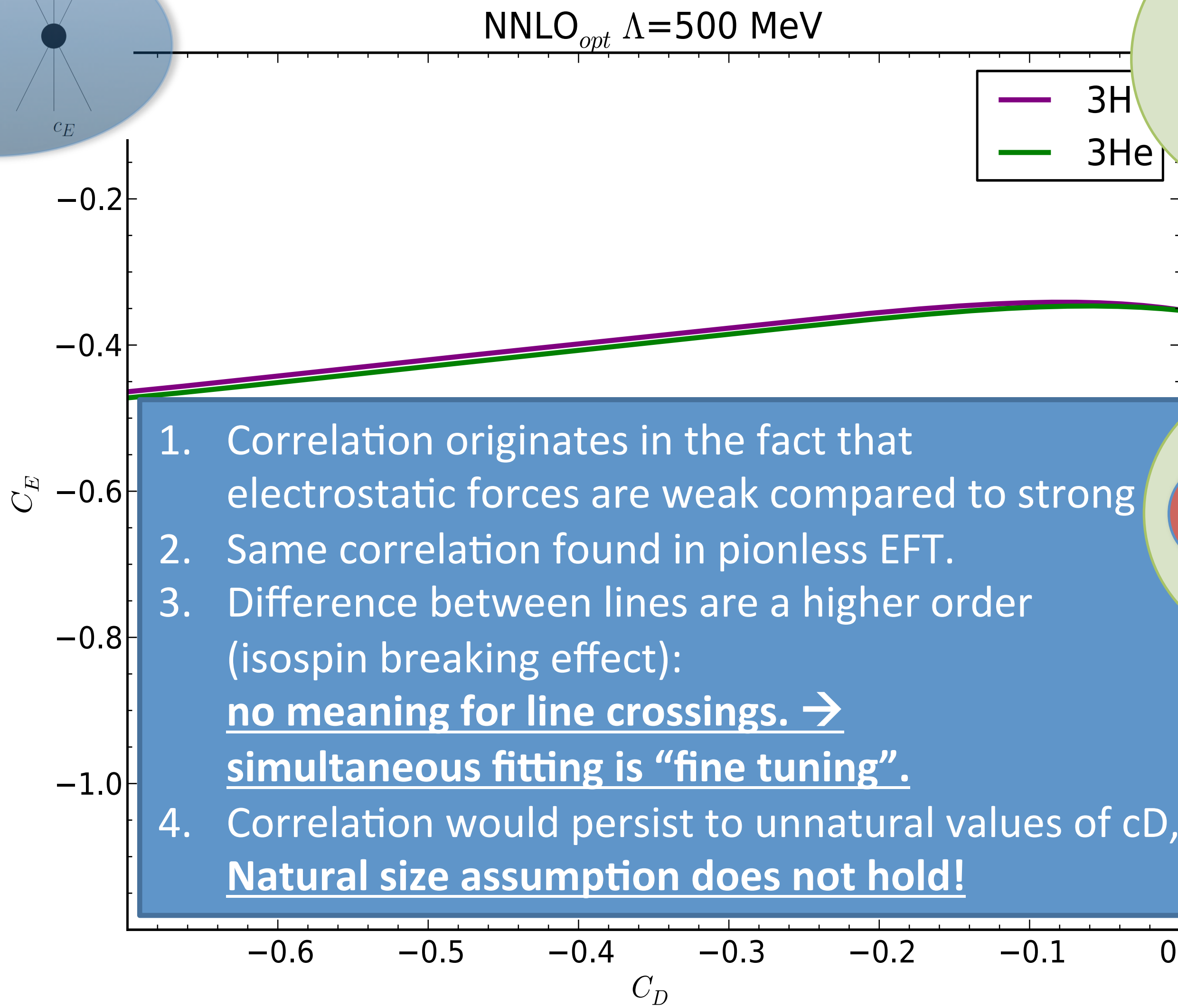


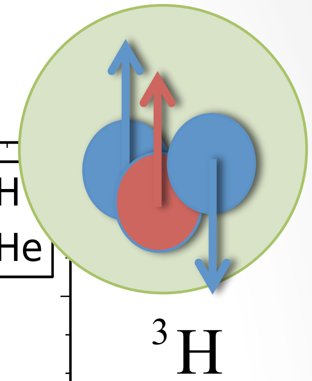
^{3}He

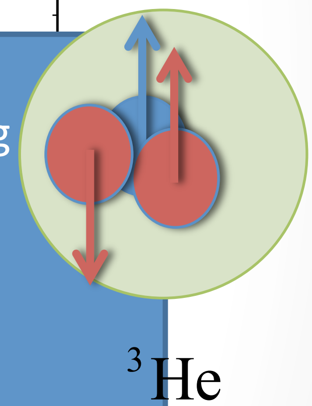
Lupu, Barnea, DG, arXiv: arXiv: 1508.05654

χEFT – three body problem in the (c_D,c_E) plane.





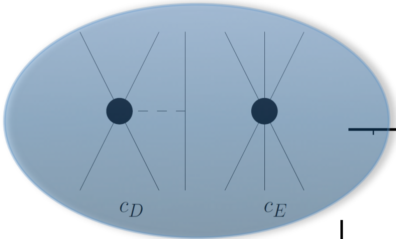


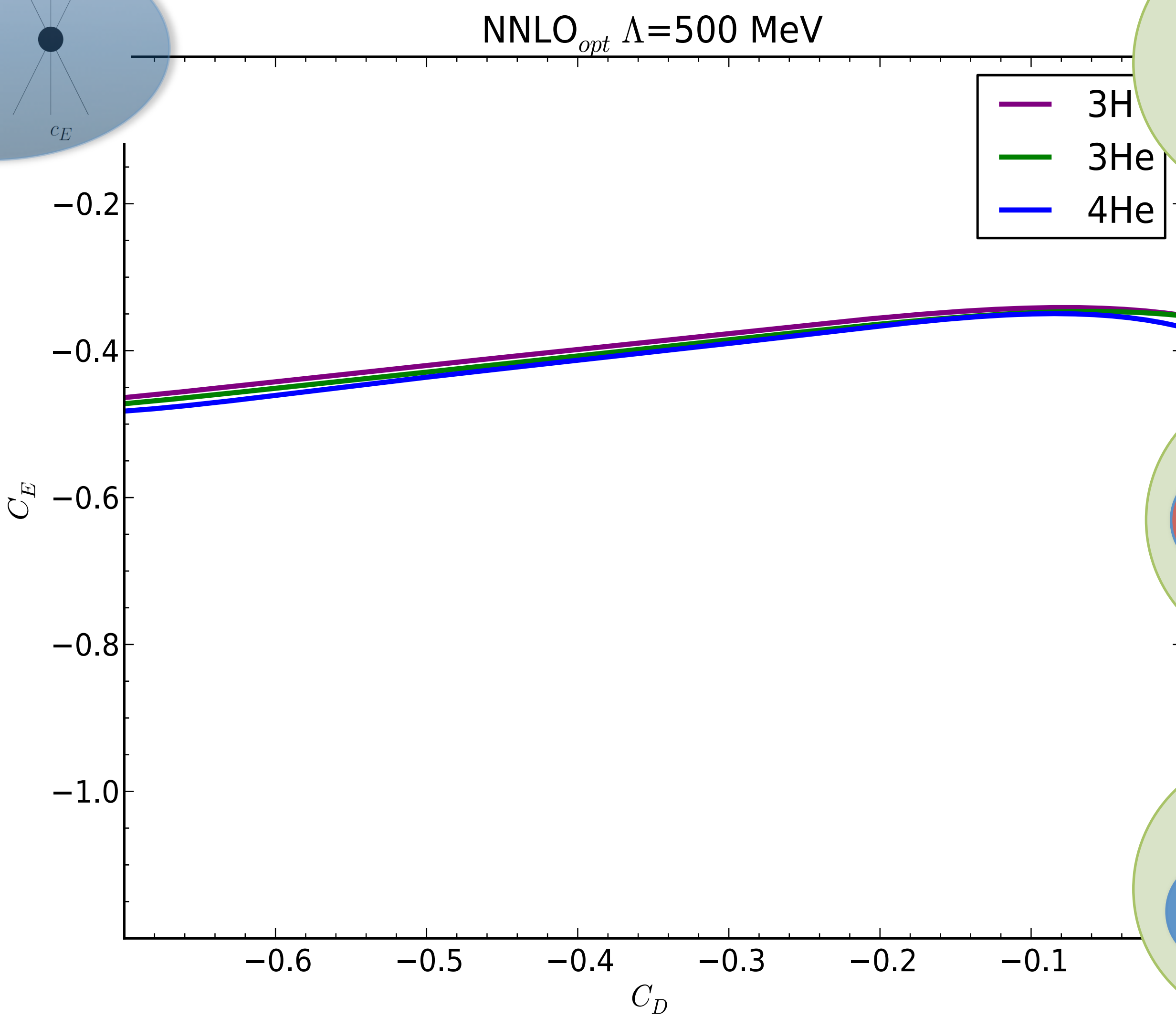


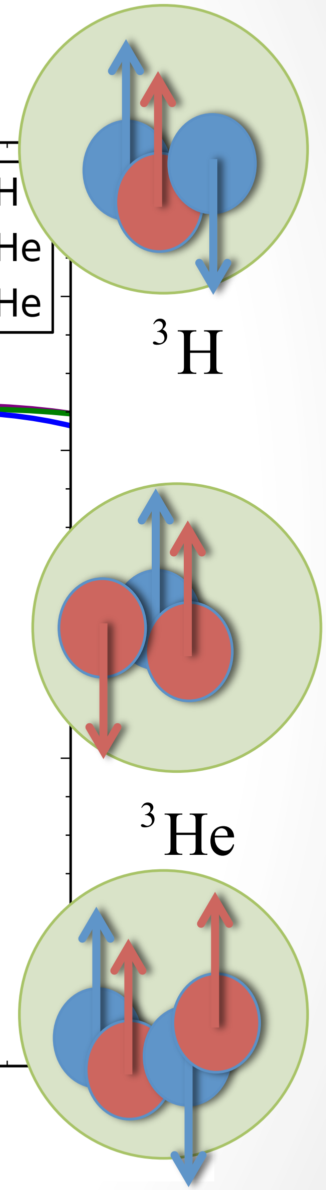
1. Correlation originates in the fact that electrostatic forces are weak compared to strong
2. Same correlation found in pionless EFT.
3. Difference between lines are a higher order (isospin breaking effect): **no meaning for line crossings. → simultaneous fitting is "fine tuning".**
4. Correlation would persist to unnatural values of cD, cE: **Natural size assumption does not hold!**

Lupu, Barnea, DG, arXiv: arXiv: 1508.05654

χEFT – a Tjon line representation

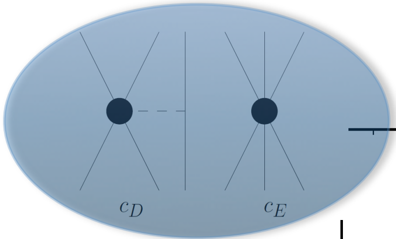


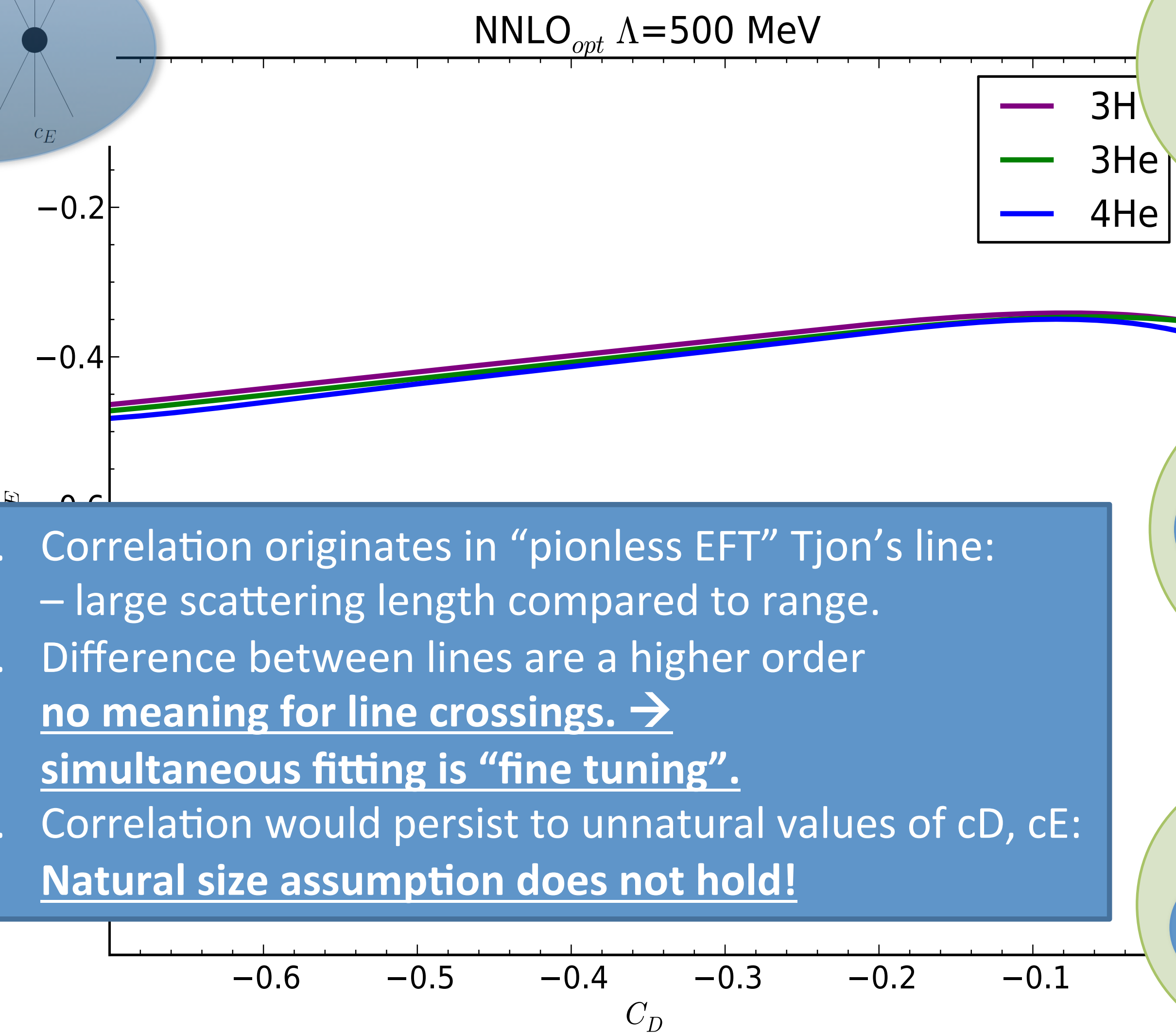


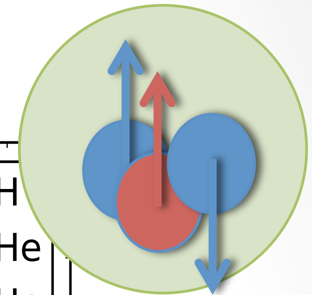


Lupu, Barnea, DG, arXiv: arXiv: 1508.05654

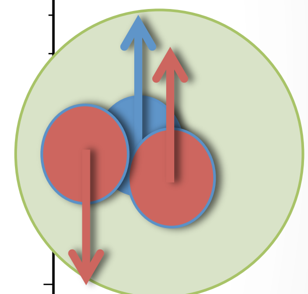
χEFT – a Tjon line representation



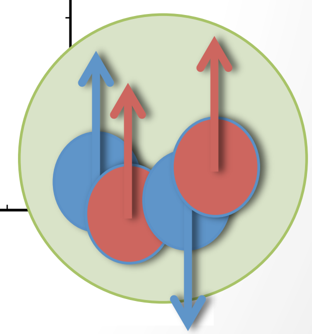




^{3}H



^{3}He

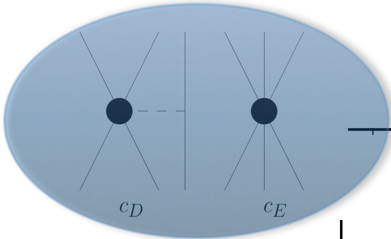


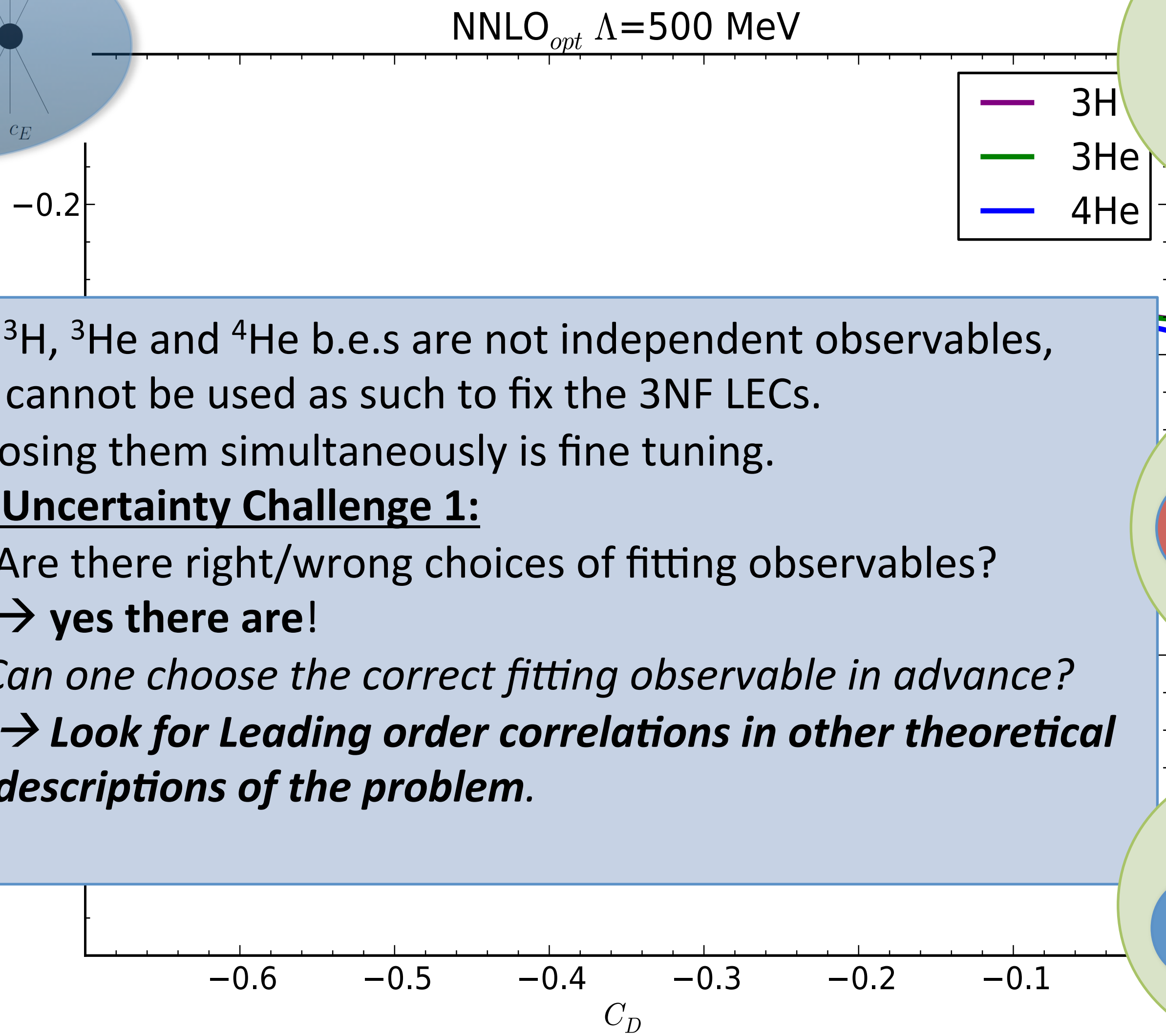
^{4}He

1. Correlation originates in “pionless EFT” Tjon’s line:
   – large scattering length compared to range.
2. Difference between lines are a higher order
   <u>no meaning for line crossings. →</u>
   <u>simultaneous fitting is “fine tuning”.</u>
3. Correlation would persist to unnatural values of cD, cE:
   <u>Natural size assumption does not hold!</u>

Lupu, Barnea, DG, arXiv: arXiv: 1508.05654

χEFT – a Tjon line representation



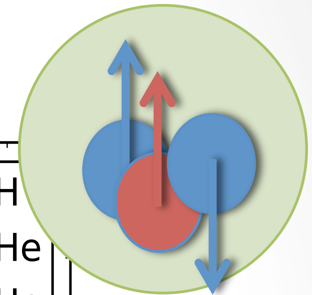


i.e., ^{3}H, ^{3}He and ^{4}He b.e.s are not independent observables, and cannot be used as such to fix the 3NF LECs.
Choosing them simultaneously is fine tuning.

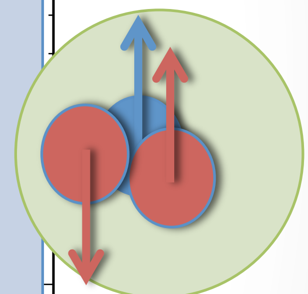
EFT Uncertainty Challenge 1:

1. Are there right/wrong choices of fitting observables?
   → **yes there are**!
2. *Can one choose the correct fitting observable in advance?*
   → ***Look for Leading order correlations in other theoretical descriptions of the problem.***

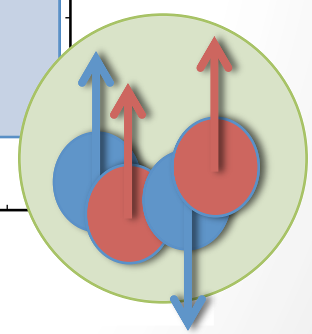
Lupu, Barnea, DG, arXiv: arXiv: 1508.05654



^{3}H



^{3}He

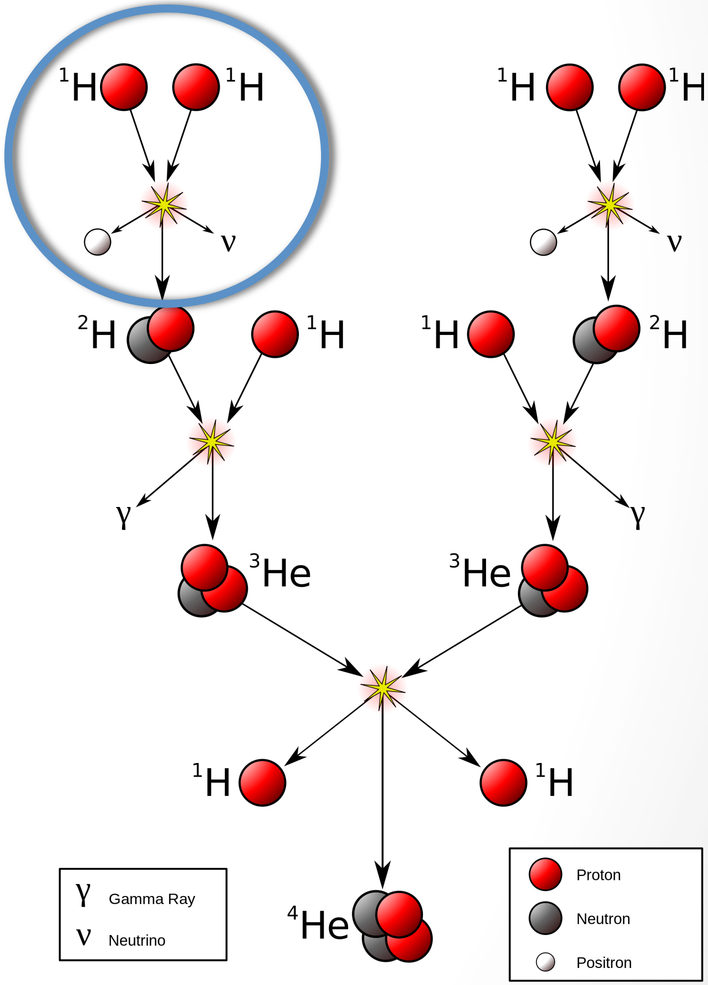


^{4}He

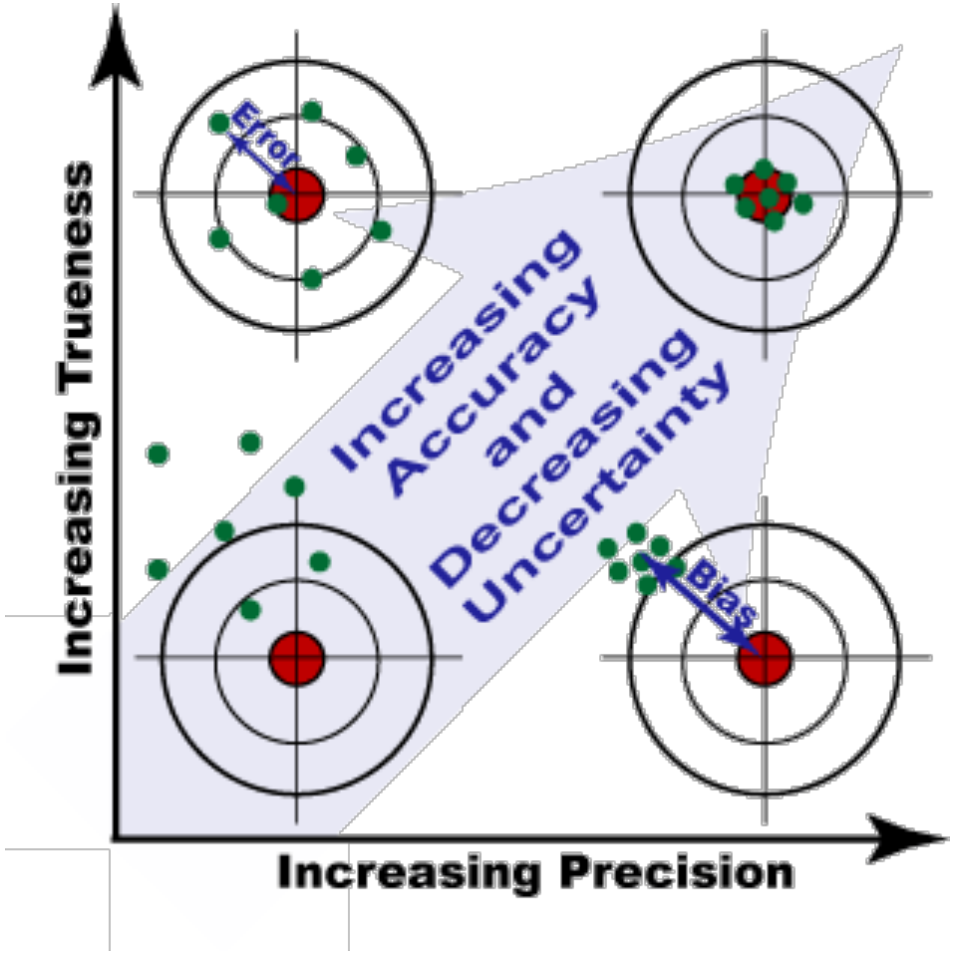
NUCLEAR FUSION REACTION RATES IN THE SUN: ASSESSING THE ACCURACY OF NUCLEAR PHYSICS PREDICTIONS

Proton-proton fusion in the Sun cannot be measured terrestrially.

Reliable theoretical prediction needed.

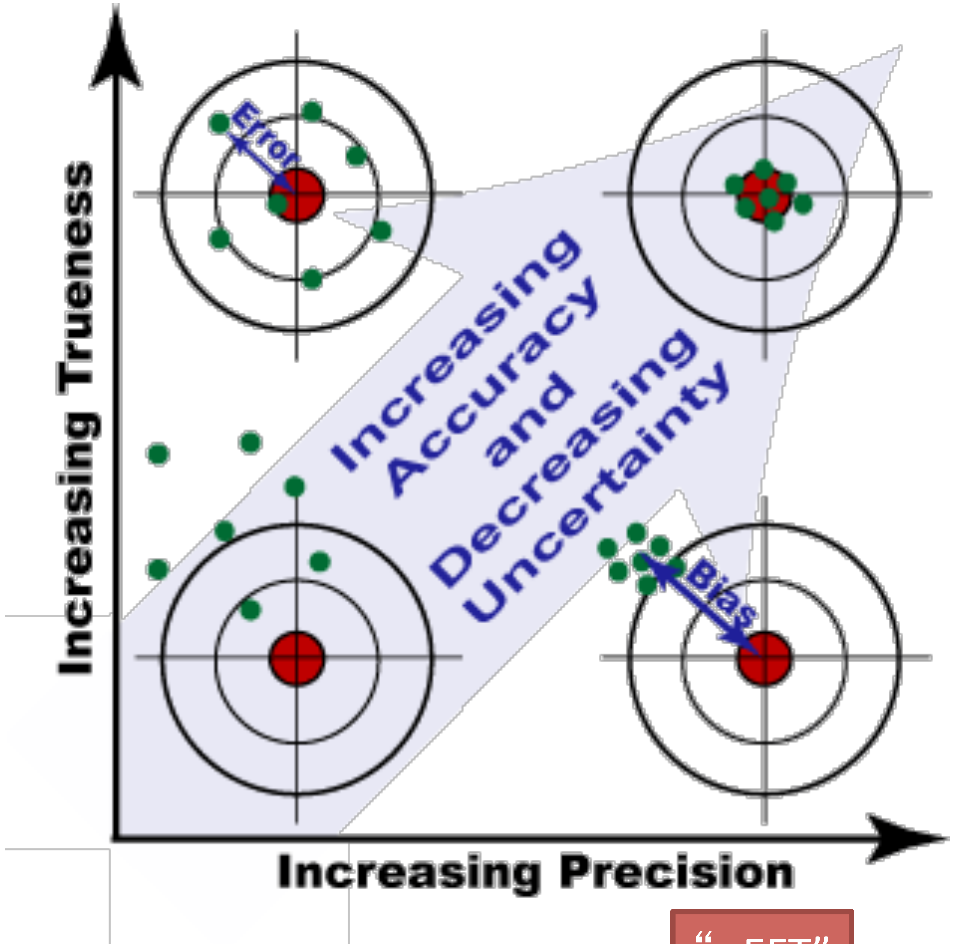


Precision, Uncertainty, and predictions



Precision, Uncertainty, and predictions

Widely believed:



"χEFT"

Weak proton-proton fusion in the Sun – theory standards

SFII – Adelberger et al., Rev. Mod. Phys. 83, 195 (2011)

$$4.01(1 \pm 0.009) \times 10^{-25}\ \mathrm{MeV\,b} \quad \text{potential models},$$
$$4.01(1 \pm 0.009) \times 10^{-25}\ \mathrm{MeV\,b} \quad \text{EFT}^*,$$
$$3.99(1 \pm 0.030) \times 10^{-25}\ \mathrm{MeV\,b} \quad \text{pionless EFT}.$$

SFII recommended value (2011): $S_{11}(0) = 4.01(1 \pm 0.009) \times 10^{-25}\ \mathrm{MeV\,b}.$

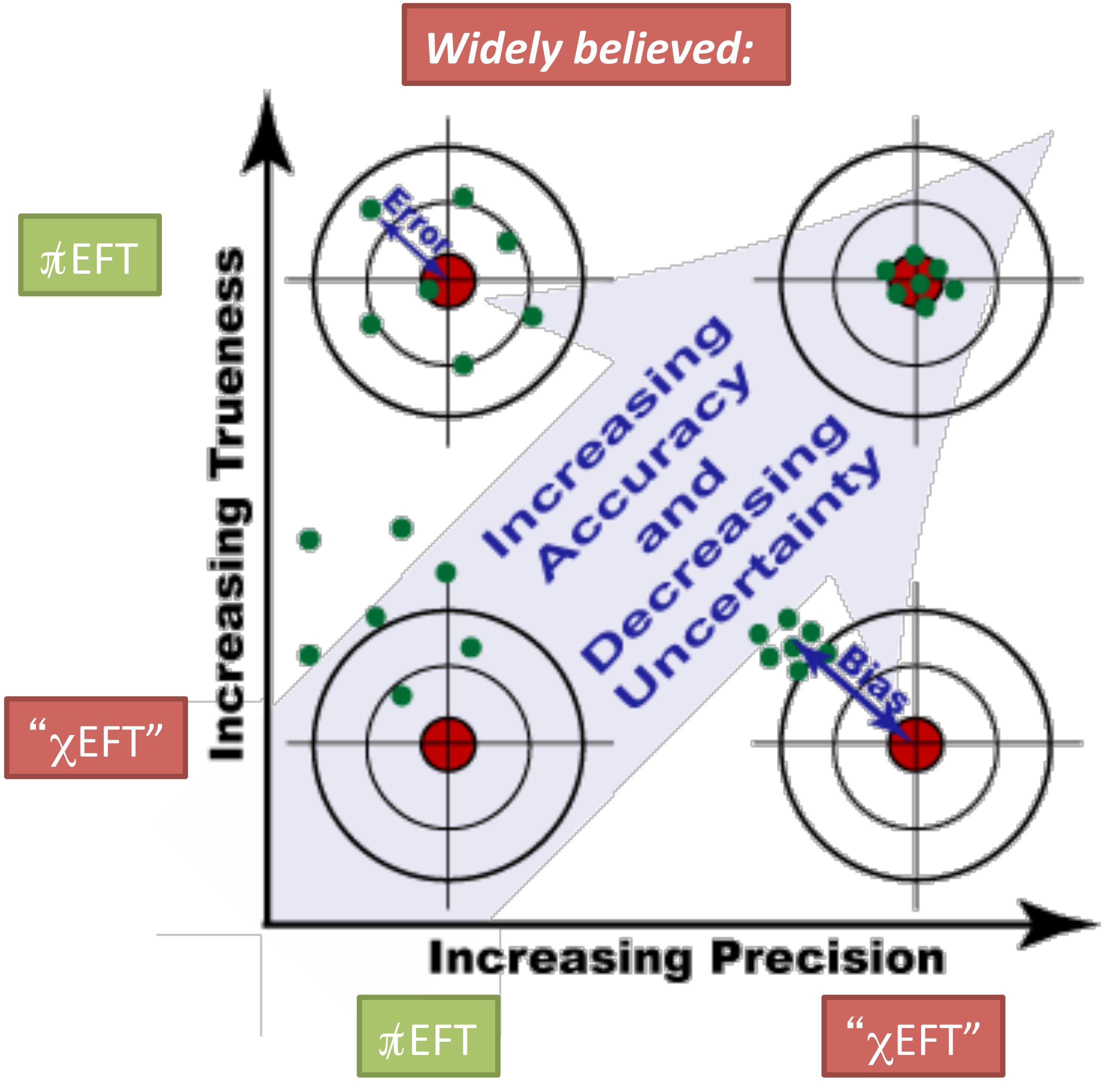
"χEFT" calculation by Marcucci et al., Phys. Rev. Lett. (2013):
Use consistent ^3H decay-rate to constrain consistently axial MEC (DG, Quaglioni, Navratil, PRL 2009), and predict pp-fusion rate.

$$S(0) = (4.030 \pm 0.006) \times 10^{-23}\ \mathrm{MeV\ fm}^2$$

Including: p-wave contribution (+0.5%), full EM (-0.25-(-0.75)%), difference between 500 and 600 MeV cutoff and potential models.

Recently Archaya et al (1603.01593) *χEFT:* $S(0) = (4.081^{+0.024}_{-0.032}) \times 10^{-23}\ \mathrm{MeV\ fm}^2$

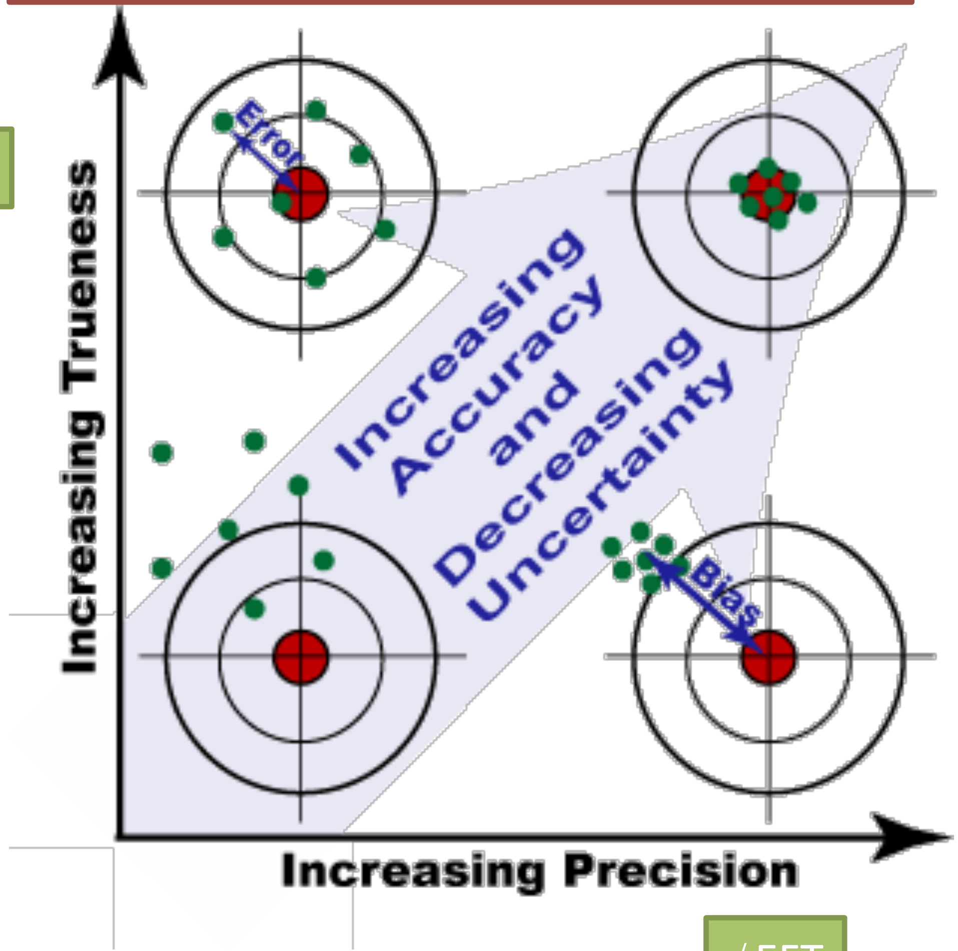
Precision, Uncertainty, and predictions



Precision, Uncertainty, and predictions

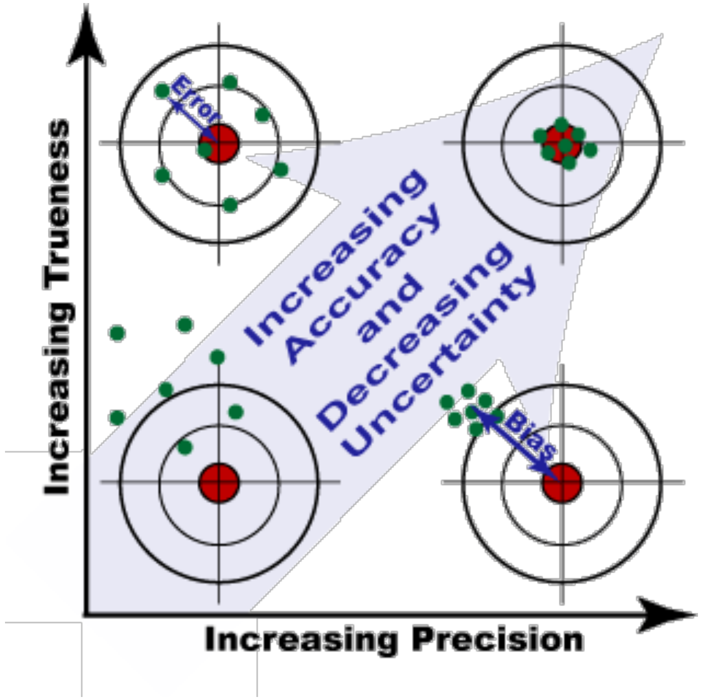
Can we reach precision physics with $\not\pi$EFT?

$\not\pi$EFT



$\not\pi$EFT

Precision, Uncertainty, and predictions



Role of $\not\pi$EFT:
Coherent and sysytematic (theoretical) uncertainty quantification.
Big question: is precision physics a possible frontier of $\not\pi$EFT?

We revisit the pp-fusion problem within pionless EFT, fixing the unknown LEC using triton decay.

Advantages of $\not\pi$EFT for proton-proton fusion:

1. Small number of parameters.
2. Two NLO $\not\pi$ EFT arrangements.
3. A “cheat-sheet” in the electromagnetic sector.
4. Cutoff independence up to infinity.

De-Leon, Gazit, in preparation (2016)

A fully perturbative pionless EFT A=2, 3 calculation @NLO

De-Leon, Gazit, in preparation (2016)

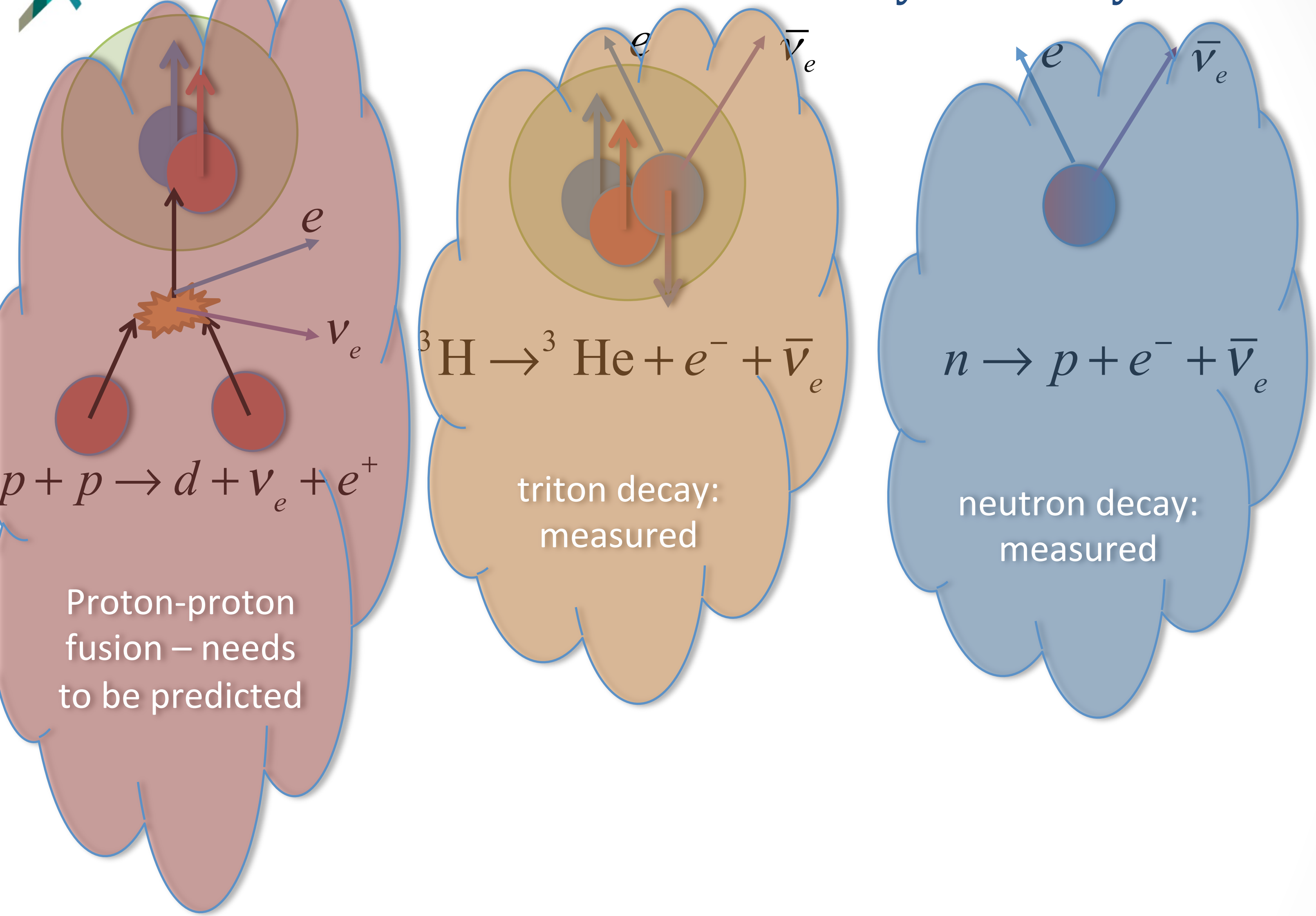
- LO Parameters:
  - nn and 2-np Scattering lengths: $^3S_1$, $^1S_0$.
  - pp scattering length.
  - Fine structure constant.
  - Three body force.
- NLO parameters:
  - 2 effective ranges.
  - Renormalizations of pp and 3NF.
  - (isospin dependent NLO 3NF
    to prevent logarithmic divergence
    in the binding energy of ^{3}He).

These parameters fixed using nuclear data

- ***Weak Interaction: LO (g_A – 1 body), NLO (L_{1A} – 2 body)***

From neutron decay

Weak observables in two and three body nuclear systems:



A fully perturbative pionless EFT A=2, 3 calculation @NLO

De-Leon, Gazit, in preparation (2016)

- LO Parameters:
  - nn and 2-np Scattering lengths: $^3S_1$, $^1S_0$.
  - pp scattering length.
  - Fine structure constant.
  - Three body force.
- NLO parameters:
  - 2 effective ranges.
  - Renormalizations of pp and 3NF.
  - (isospin dependent NLO 3NF
    to prevent logarithmic divergence
    in the binding energy of ^{3}He).

These parameters fixed using nuclear data

- ***Weak Interaction: LO (g_A – 1 body), NLO (L_{1A} – 2 body)***

From neutron decay

From triton decay!

Advantages of $\pi\!\!/$EFT for proton-proton fusion:

1. Small number of parameters.
2. Two NLO $\pi\!\!/$EFT arrangements.
3. A "cheat-sheet" in the electromagnetic sector.
4. Cutoff independence up to infinity.

De-Leon, Gazit, in preparation (2016)

The role of the deuteron tail

- Many low energy reactions depend on deuteron normalization.

$$Z_d^{-1} = i\frac{\partial}{\partial_{p_0}}\frac{1}{i\mathcal{D}_t(p_0,p)}\Big|_{p_0=\frac{\gamma_t^2}{M_N},p=0}$$

- One has a choice of rearranging the expansion:
  - rho-parameterization: $Z_d = \frac{1}{1-\gamma\rho} \approx 1 + \gamma\rho + (\gamma\rho)^2 + \ldots$
  - Z-parameterization: $Z_d = \frac{1}{1-\gamma\rho} \approx 1 + (Z_d - 1) + 0 + \ldots$

LO NLO N^{2}LO+…

Both are valid rearrangements!
Z-parameterization has quicker convergence, especially for observables sensitive to the deuteron tail.

Phillips, Rupak, Savage, Phys. Lett. **B473**, 209 (2000)
Grießhammer, Nucl. Phys. **A744**, 192 (2004)

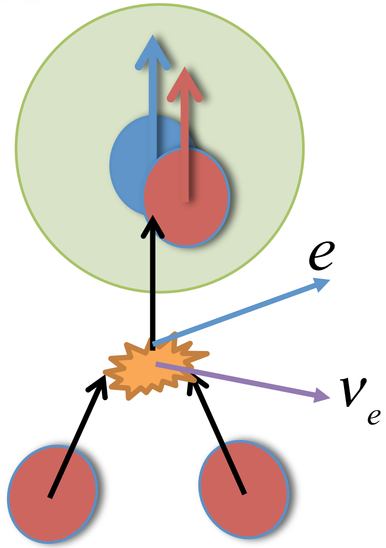
De-Leon, Gazit, in preparation (2016)

Advantages of $\pi\!\!\!/$EFT for proton-proton fusion:

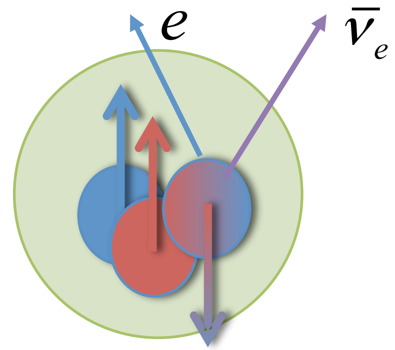
1. Small number of parameters.
2. Two NLO $\pi\!\!\!/$EFT arrangements.
3. A "cheat-sheet" in the electromagnetic sector.
4. Cutoff independence up to infinity.

De-Leon, Gazit, in preparation (2016)

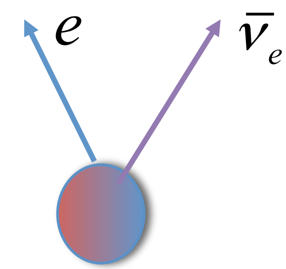
Analogy between weak and EM:



$$p+p \rightarrow d+\nu_e+e^+$$



$$^3\mathrm{H} \rightarrow {}^3\mathrm{He}+e^-+\bar{\nu}_e$$

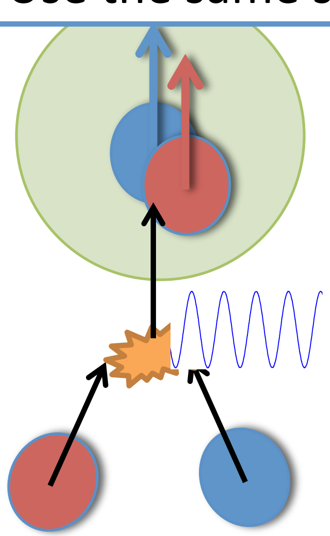


$$n \rightarrow p+e^-+\bar{\nu}_e$$

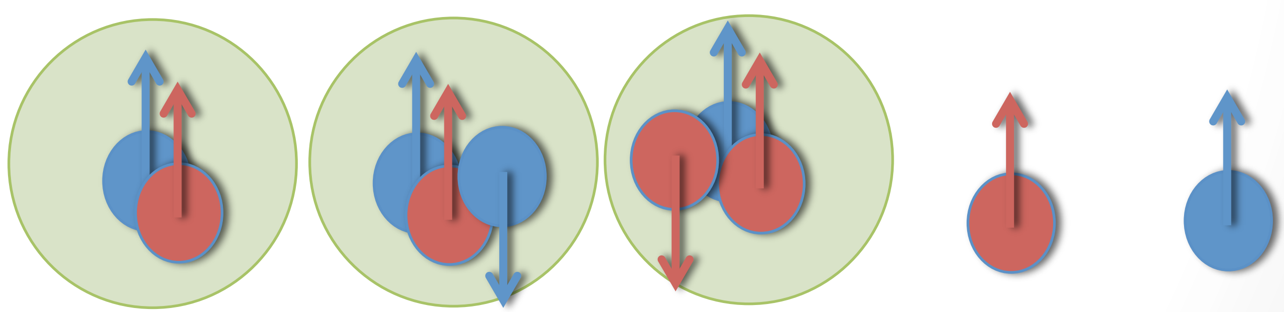
Weak observables

Use the same strategy in both cases: fix probe LECs at A=3 and predict A=2.

EM observables



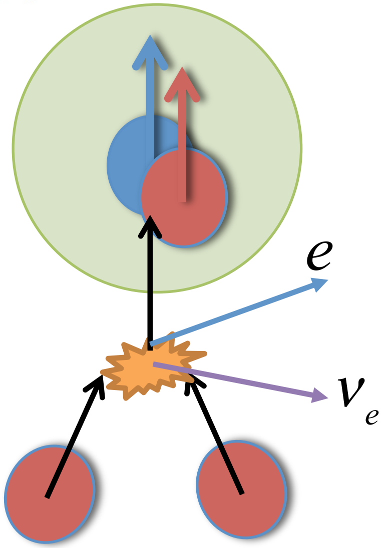
$$n+p \rightarrow d+\gamma$$



μ_d $\mu_{^3\mathrm{H}}$ $\mu_{^3\mathrm{H}}$ μ_p μ_n

De-Leon, Gazit, in preparation (2016)

Analogy between weak and EM:



$p+p\to d+\nu_e+e^+$

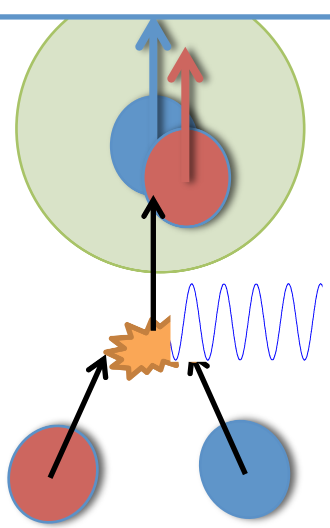
Pionless EFT reproduces low-energy electromagnetic A=2 observables when using A=3 observables as input, to a very good precision (~1%), with assessed theoretical uncertainty of about 3%.

$^3H\to{}^3He+e^-+\bar{\nu}_e$ $\quad$ $n\to p+e^-+\bar{\nu}_e$

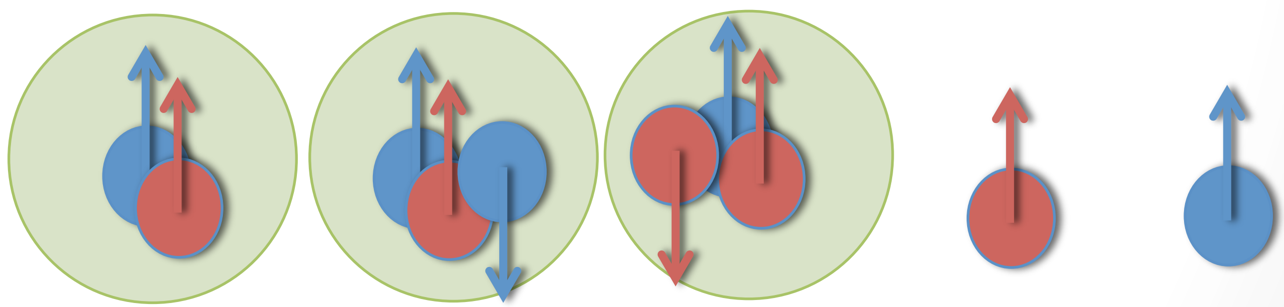
Weak observables

Use the same strategy in both cases: fix probe LECs at A=3 and predict A=2.

EM observables



$n+p\to d+\gamma$



μ_d $\quad$ $\mu_{^3H}$ $\quad$ $\mu_{^3He}$ $\quad$ μ_p $\quad$ μ_n

De-Leon, Gazit, in preparation (2016)

Advantages of $\pi\!\!\!/$EFT for proton-proton fusion:

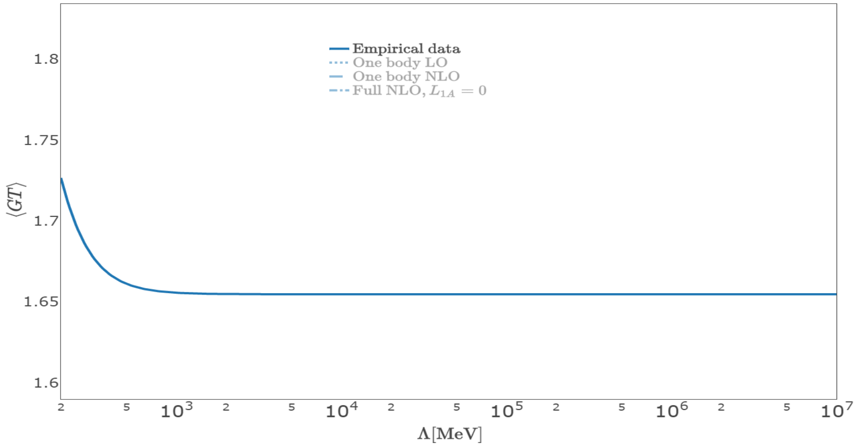
1. Small number of parameters.
2. Two NLO $\pi\!\!\!/$ EFT arrangements.
3. A "cheat-sheet" in the electromagnetic sector.
4. Cutoff independence up to infinity.

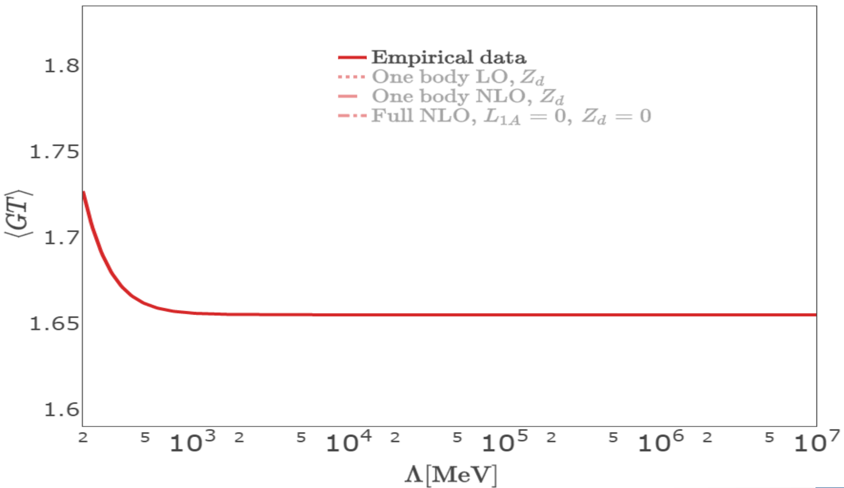
De-Leon, Gazit, in preparation (2016)

Triton decay – GT cutoff independence

Rho-parameterization **Z-parameterization**

$$ft = \frac{K}{G_F^2 V_{ud}^2 \left[\left| \left\langle {}^3\mathrm{H} \left\| \mathcal{V}_\mu^+ \right\| {}^3\mathrm{He} \right\rangle \right|^2 + \frac{f_A}{f_V} \left| \left\langle {}^3\mathrm{H} \left\| \mathcal{A}_\mu^+ \right\| {}^3\mathrm{He} \right\rangle \right|^2 \right]}$$





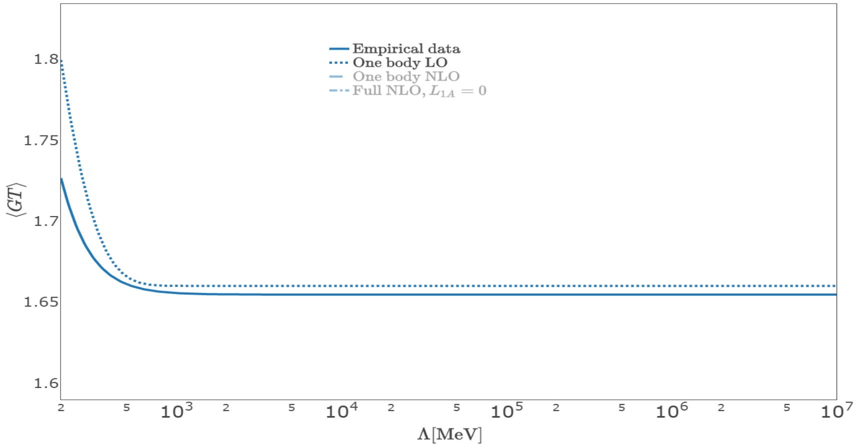
"Empirical" extraction of GT (using calculated F strength)

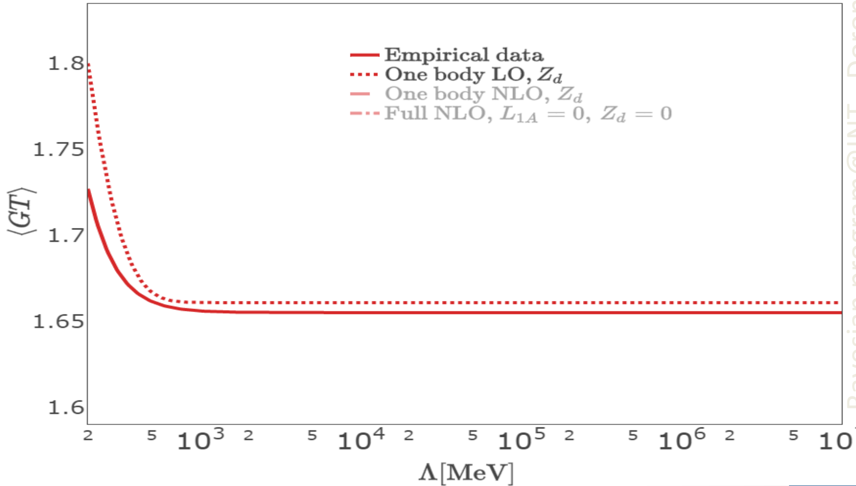
De-Leon, Gazit, in preparation (2016)

Triton decay – GT cutoff independence

Rho-parameterization **Z-parameterization**

$$ft = \frac{K}{G_F^2 V_{ud}^2 \left[\left| \left\langle {}^3\mathrm{H} \left\| \mathcal{V}_\mu^+ \right\| {}^3\mathrm{He} \right\rangle \right|^2 + \frac{f_A}{f_V} \left| \left\langle {}^3\mathrm{H} \left\| \mathcal{A}_\mu^+ \right\| {}^3\mathrm{He} \right\rangle \right|^2 \right]}$$





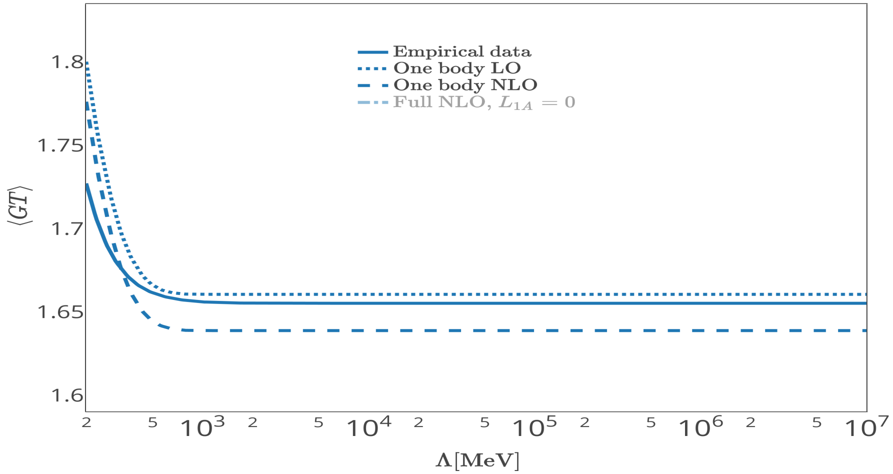
Adding the LO 1-body contribution

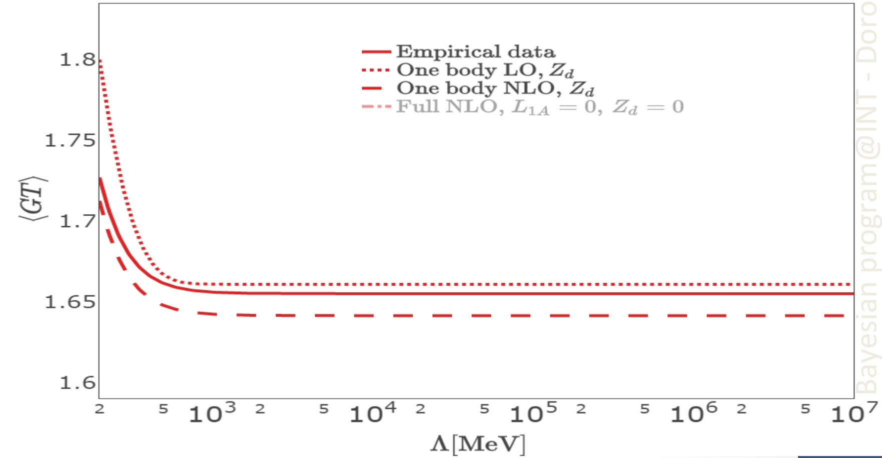
De-Leon, Gazit, in preparation (2016)

Triton decay – GT cutoff independence

Rho-parameterization **Z-parameterization**

$$ft = \frac{K}{G_F^2 V_{ud}^2 \left[\left| \left\langle {}^3\mathrm{H} \left\| \mathcal{V}_\mu^+ \right\| {}^3\mathrm{He} \right\rangle \right|^2 + \frac{f_A}{f_V} \left| \left\langle {}^3\mathrm{H} \left\| \mathcal{A}_\mu^+ \right\| {}^3\mathrm{He} \right\rangle \right|^2 \right]}$$





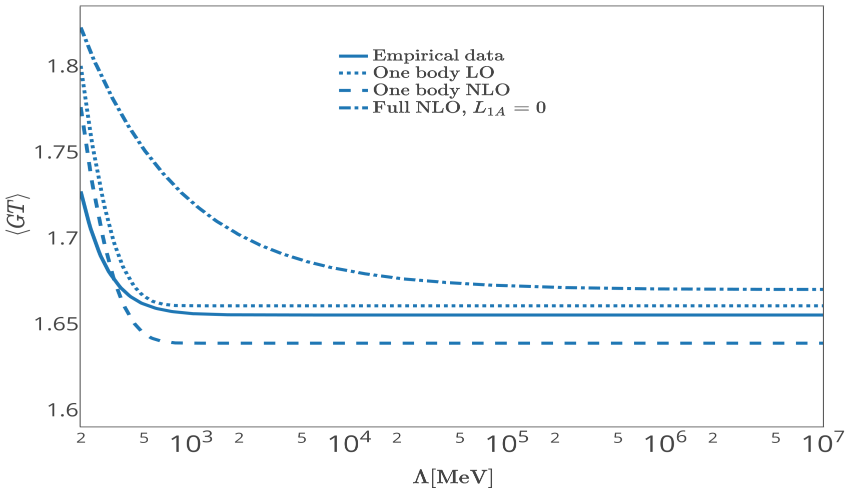
Adding the NLO 1-body contributions

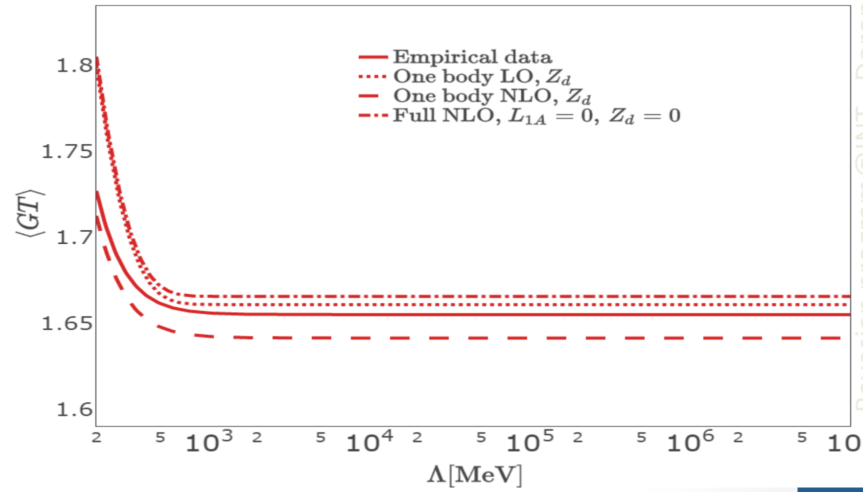
De-Leon, Gazit, in preparation (2016)

Triton decay – GT cutoff independence

Rho-parameterization **Z-parameterization**

$$ft = \frac{K}{G_F^2 V_{ud}^2 \left[\left| \left\langle {}^3\mathrm{H} \left\| \mathcal{V}_\mu^+ \right\| {}^3\mathrm{He} \right\rangle \right|^2 + \frac{f_A}{f_V} \left| \left\langle {}^3\mathrm{H} \left\| \mathcal{A}_\mu^+ \right\| {}^3\mathrm{He} \right\rangle \right|^2 \right]}$$





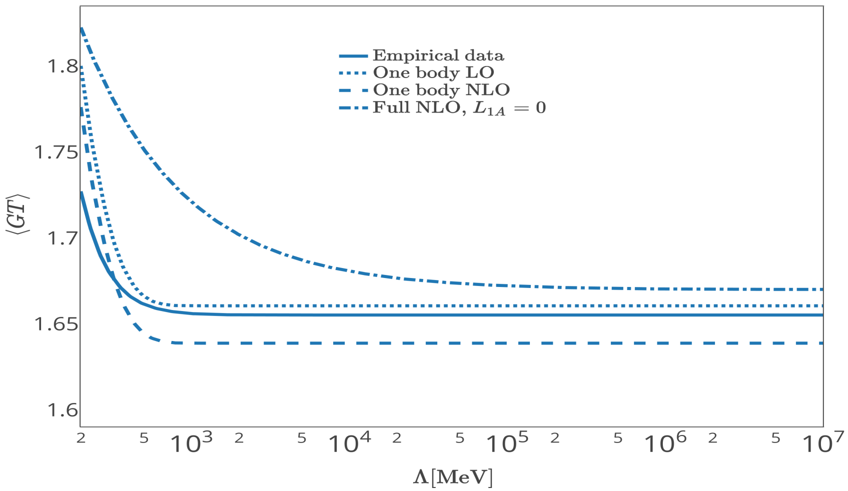
Adding all contribution, but L_{1A}

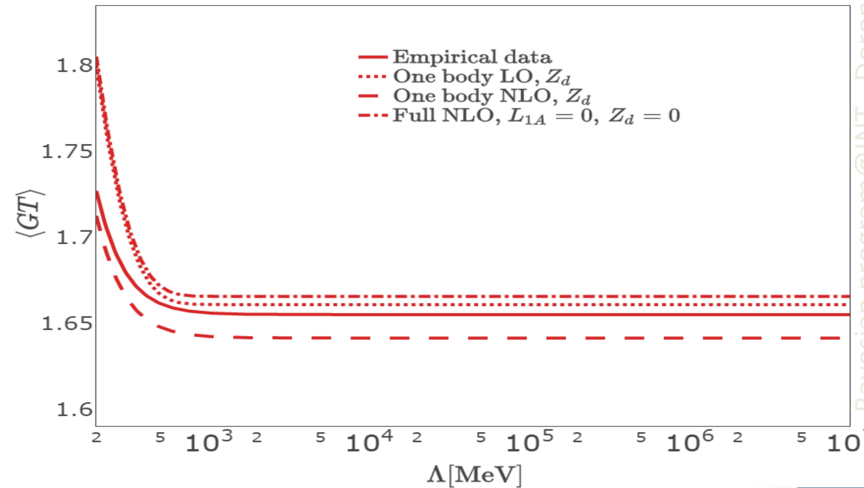
1st estimate of theoretical uncertainty:
All NLO contributions are of the same order (1-2%),
one can estimate higher order effects as the NLO contribution.

Triton decay – GT cutoff independence

Rho-parameterization **Z-parameterization**

$$ft = \frac{K}{G_F^2 V_{ud}^2 \left[\left| \left\langle {}^3\mathrm{H} \left\| \mathcal{V}_\mu^+ \right\| {}^3\mathrm{He} \right\rangle \right|^2 + \frac{f_A}{f_V} \left| \left\langle {}^3\mathrm{H} \left\| \mathcal{A}_\mu^+ \right\| {}^3\mathrm{He} \right\rangle \right|^2 \right]}$$





Adding all contributi

Translates to ±2% difference in pp fusion

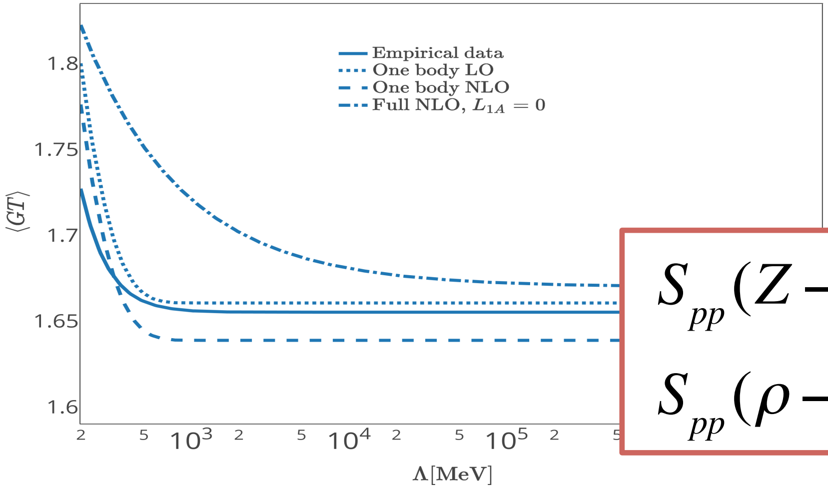
1st estimate of theoretical uncertainty:
All NLO contributions are of the same order (1-2%),
one can estimate highe

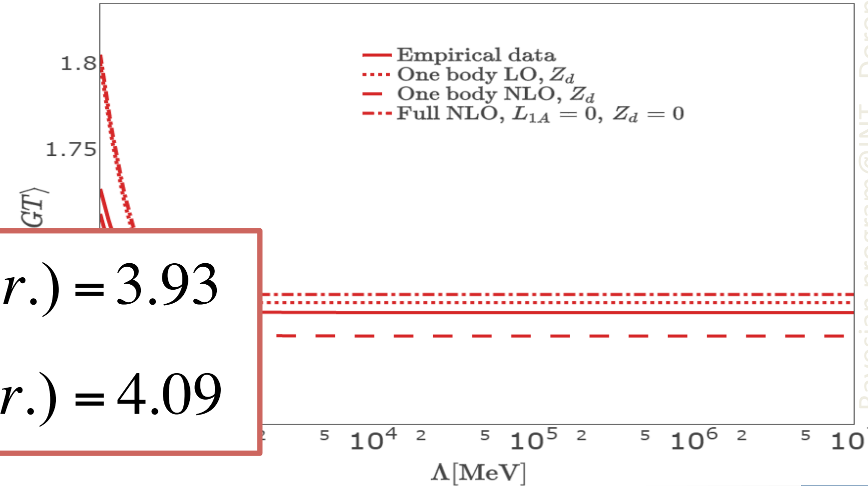
De-Leon, Gazit, in preparation (2016)

Triton decay – GT cutoff independence

Rho-parameterization **Z-parameterization**

$$ft = \frac{K}{G_F^2 V_{ud}^2 \left[\left| \left\langle {}^3\mathrm{H} \left\| \mathcal{V}_\mu^+ \right\| {}^3\mathrm{He} \right\rangle \right|^2 + \frac{f_A}{f_V} \left| \left\langle {}^3\mathrm{H} \left\| \mathcal{A}_\mu^+ \right\| {}^3\mathrm{He} \right\rangle \right|^2 \right]}$$





$$S_{pp}(Z - par.) = 3.93$$

$$S_{pp}(\rho - par.) = 4.09$$

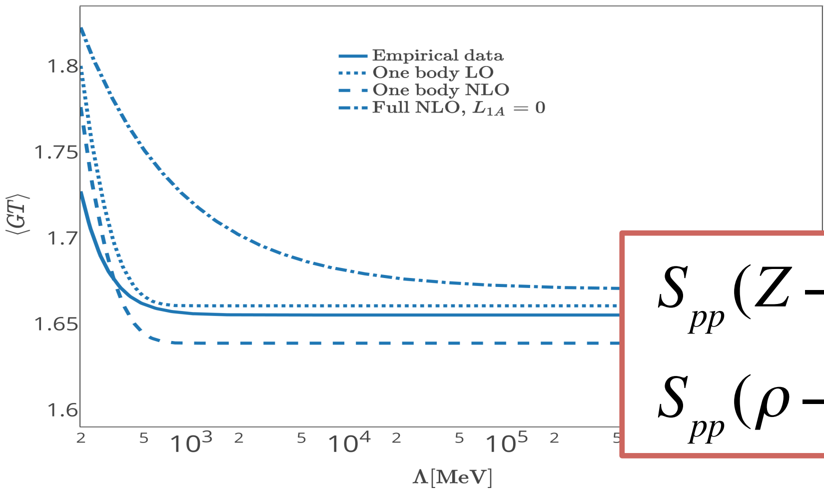
2nd estimate of theoretical uncertainty: difference between Zed and Rho Paramerizations.

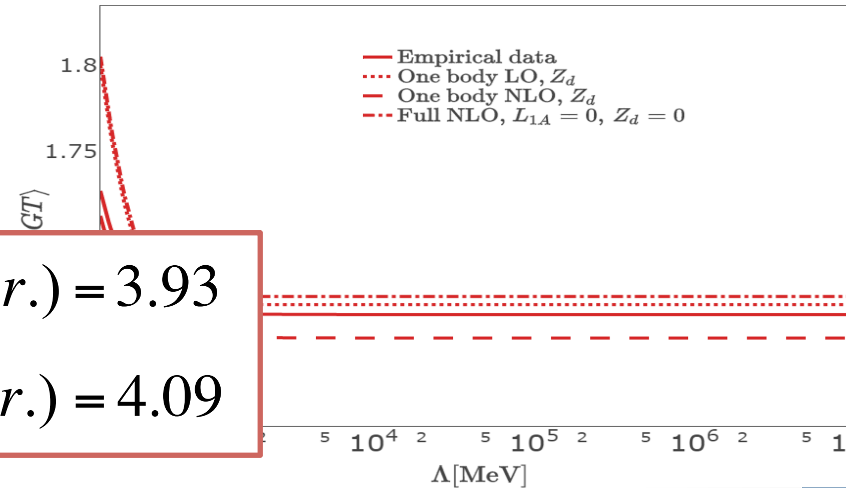
De-Leon, Gazit, in preparation (2016)

Triton decay – GT cutoff independence

Rho-parameterization **Z-parameterization**

$$ft = \frac{K}{G_F^2 V_{ud}^2 \left[\left| \left\langle {}^3\mathrm{H} \left\| \mathcal{V}_\mu^+ \right\| {}^3\mathrm{He} \right\rangle \right|^2 + \frac{f_A}{f_V} \left| \left\langle {}^3\mathrm{H} \left\| \mathcal{A}_\mu^+ \right\| {}^3\mathrm{He} \right\rangle \right|^2 \right]}$$





$$S_{pp}(Z-par.) = 3.93$$

$$S_{pp}(\rho-par.) = 4.09$$

Translates to ±2% difference in pp fusion

2nd estimate of theoretical uncertainty:
difference between Zed and Rho Paramerizations.

De-Leon, Gazit, in preparation (2016)

So… is 3% too big to be called precision physics?

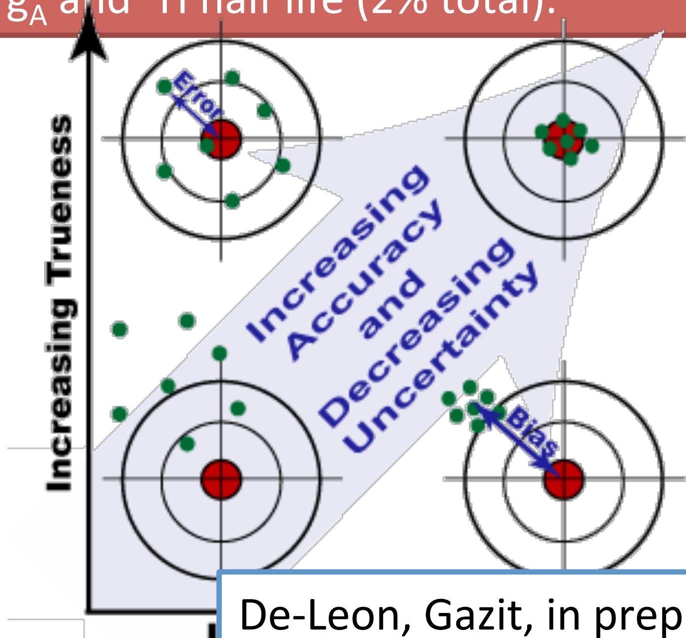
$$S_{pp}(g_A = 1.2701) = 4.01 \pm 0.08 \pm 0.07 \pm 0.04$$

$$S_{pp}(g_A = 1.276) = 4.14 \pm 0.08 \pm 0.07 \pm 0.04$$

g_A systematic uncertainty (since ~2010)

theoretical uncertainty	g_A stat. unc.	^{3}H halflife syst. unc.

i.e., theoretical uncertainty of the same order of systematic experimental error due to g_A and ^{3}H half life (2% total).



De-Leon, Gazit, in preparation (2016)

Thus,

- Pionless EFT reproduces low-energy ***electromagnetic*** observables to a very good precision (~1%), even at NLO.
- Theoretical uncertainty estimated from:
  - (Natural) Size of NLO contribution (all NLO contributions are of the same order of magnitude).
  - Difference between different arrangements of perturbative expansion.
  - Both error estimates lead to about 2% uncertainty.
- Proton-proton fusion NLO prediction and error assessment reliable!
- Uncertainty quantification challenges:

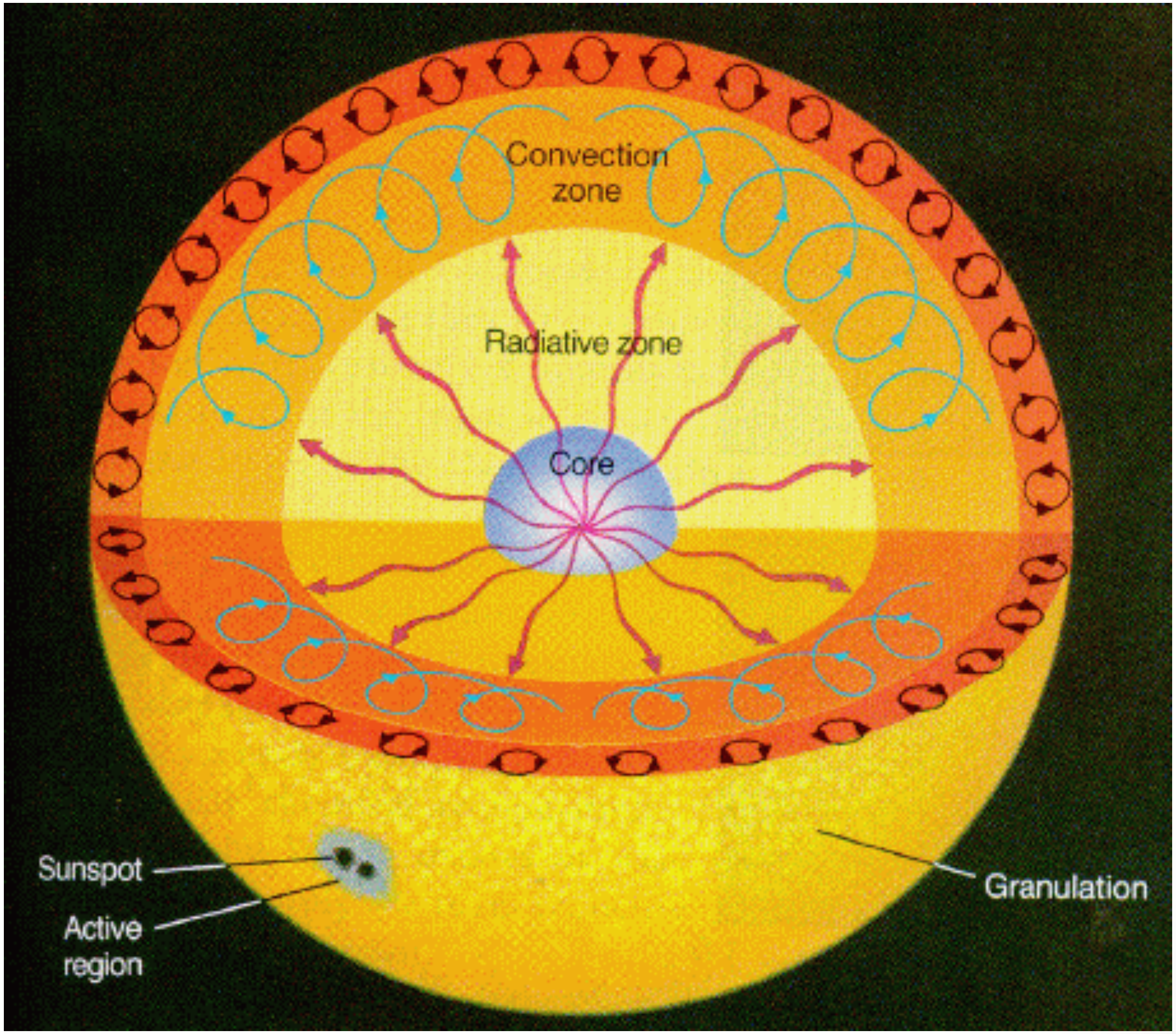
Is there a way to assign some confidence level to the theoretical uncertainty?

Is there a better way to incorporate experimental systematic uncertainties?

ATOMIC PHYSICS AT THE SOLAR INTERIOR: HOW WELL DO WE KNOW OUR SUN?

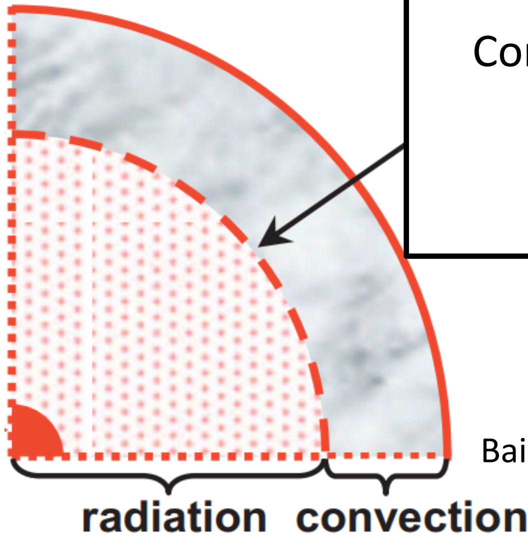
The Solar Interior

- Radiative zone: energy transport by photon diffusion.



A solar recipe

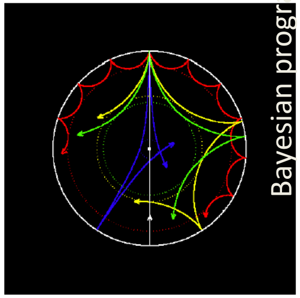
Helioseismology:
outer constraint



Neutrinos:
inner temperature

Convection zone radius: a prediction of both constraints…

Bailey, J. E., et al. 2009



differential probing of solar structure

A solar recipe

Abundances

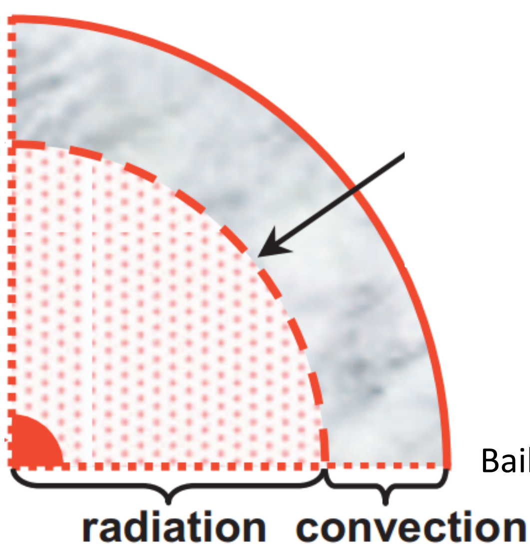
- Solar atmosphere spectra (1D\3D)
- Meteorites

Standard Solar Model (SSM)

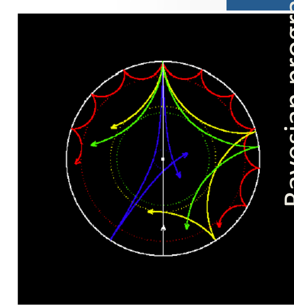
- Hydrostatic
- 1D
- **Opacities**
- Eqs. of state (EOS)
- Nuclear rates
- …..

Helioseismology: outer constraint

Neutrinos: inner temperature

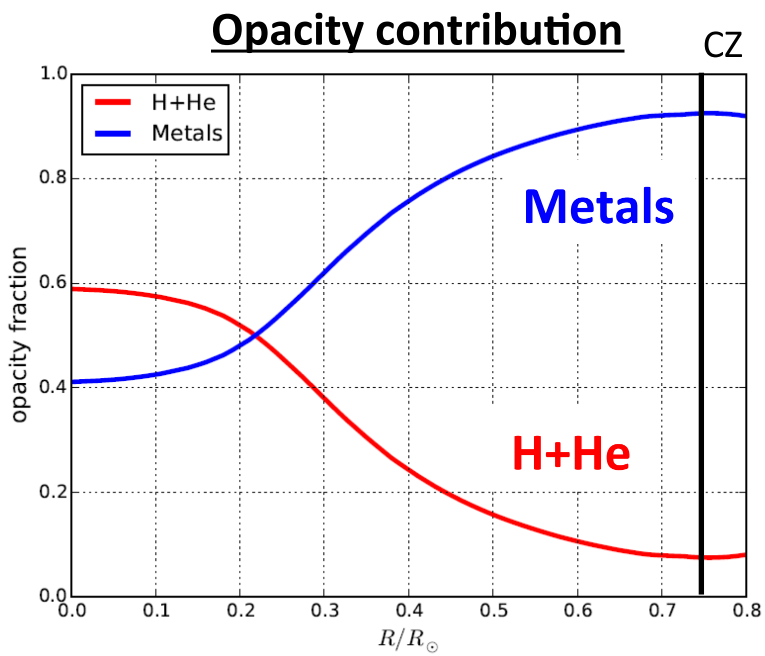


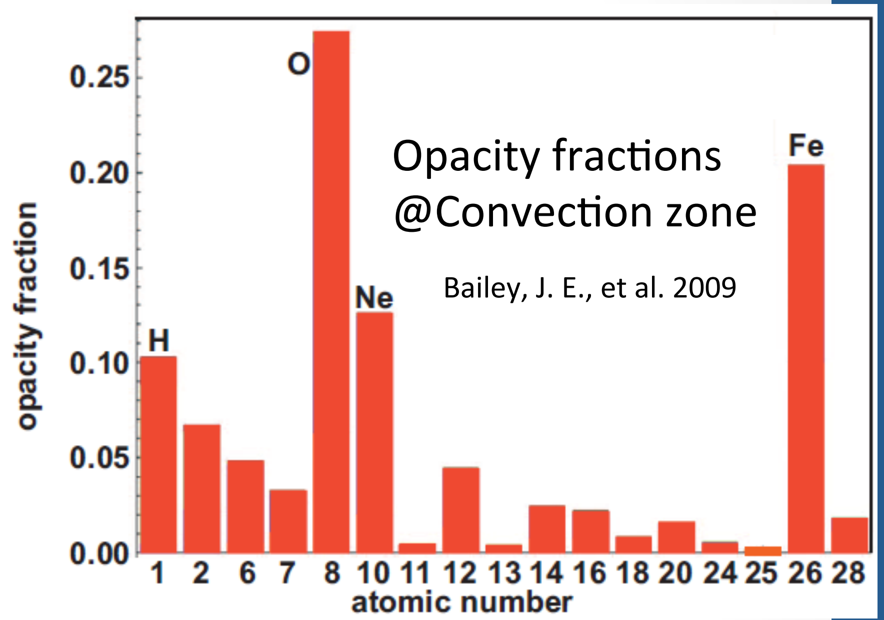
Bailey, J. E., et al. 2009



differential probing of solar structure

Metals are a major source of opacity in the Sun





- 98% H+He
- Other ~2% – "**<u>Metals</u>**"
- A **Hot-Dense** Plasma (@R_{cz}: 180 eV, 0.5 g/cc; @center: 1.5 keV, 150 g/cc)
- Pressure is not affected by these "metals".
- However "metals" have many bound electrons: contribute to opacity!

M. Krief, A. Feigel, and D. Gazit, *"Line broadening and the solar opacity problem"*, ApJ 2016

The Rosseland Opacity

- Photon mean-free-path

$$l_v = \frac{1}{k_v} = \frac{1}{\rho \kappa_v}$$

- **Rosseland Mean**

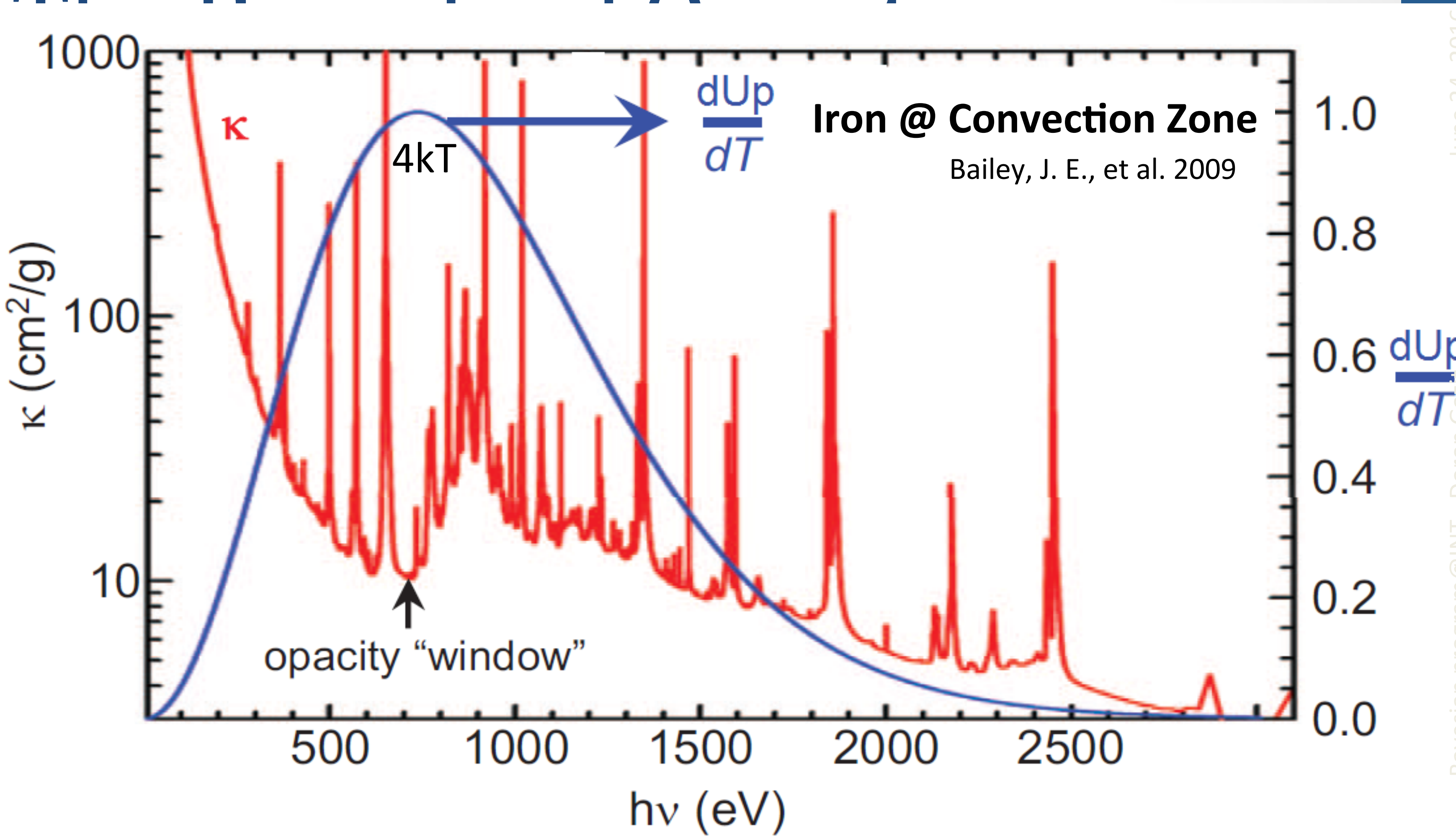
Energy Flux →

$$\mathbf{S}_v = -\frac{c l_v}{3} \boldsymbol{\nabla} U_{P,v} = -\frac{c}{3 k_v} \frac{dU_{P,v}}{dT} \boldsymbol{\nabla} T$$

↑ Planck Energy Density

$$\mathbf{S} = -\frac{c l_R}{3} \boldsymbol{\nabla} U_P$$

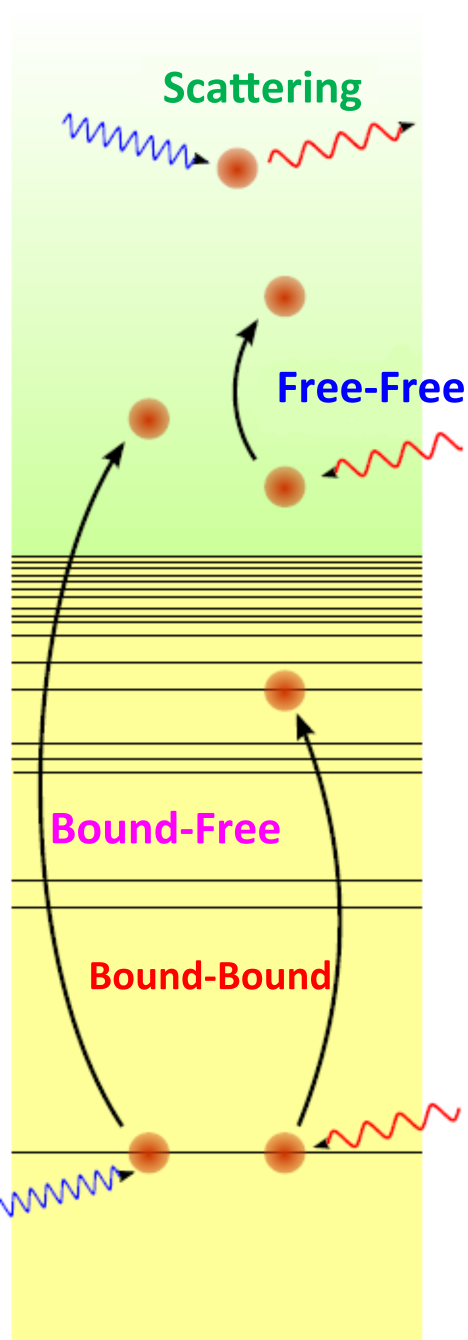
$$l_R = \frac{1}{k_R} = \frac{\int_0^\infty dv \frac{1}{k_v} \frac{dU_P}{dT}}{\int_0^\infty dv \frac{dU_P}{dT}}$$



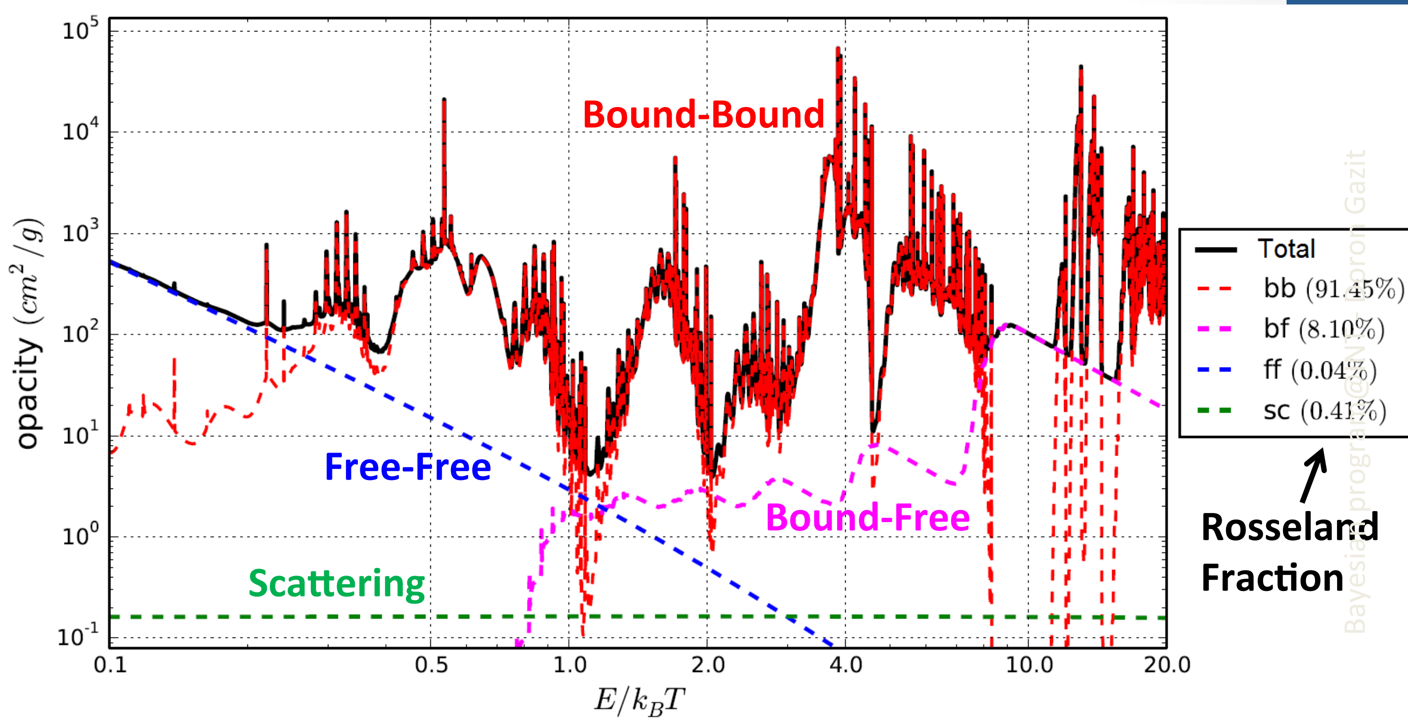
$$\mathbf{S} = -\frac{c l_R}{3} \nabla U_P$$

$$l_R = \frac{1}{k_R} = \frac{\int_0^\infty dv \frac{1}{k_v} \frac{dU_P}{dT}}{\int_0^\infty dv \frac{dU_P}{dT}}$$

Atomic Transitions



Typical opacity spectra of a mid-Z element



- Bound-Bound and Bound-Free dominate by orders of magnitude
- Existence of bound electrons strongly increases opacity

The Bound-Bound Opacity Spectra

$$\kappa_{bb}(E) = \sum_{\alpha \to \beta} \sum_{C^\alpha \to C^\beta} \sum_{\substack{a \in C^\alpha \\ b \in C^\beta}} N_a w_{ab} P_{ab}\left(E - E_{ab}\right)$$

Orbital Jumps: 10-1000

Configurations: $O(1) - O\left(10^{30}\right)$

Levels: $O(1) - O\left(10^{10}\right)$

N_a: Level Population

w_{ab}: Transition Amplitude

P_{ab}: Lineshape

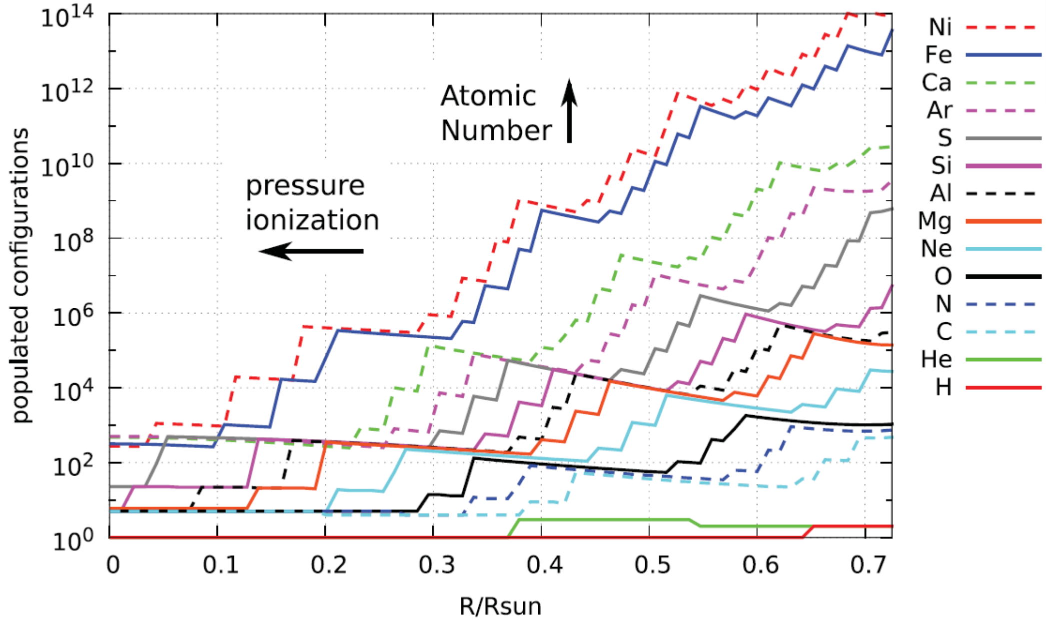
E_{ab}: Transition Energy

<u>Two major difficulties:</u>

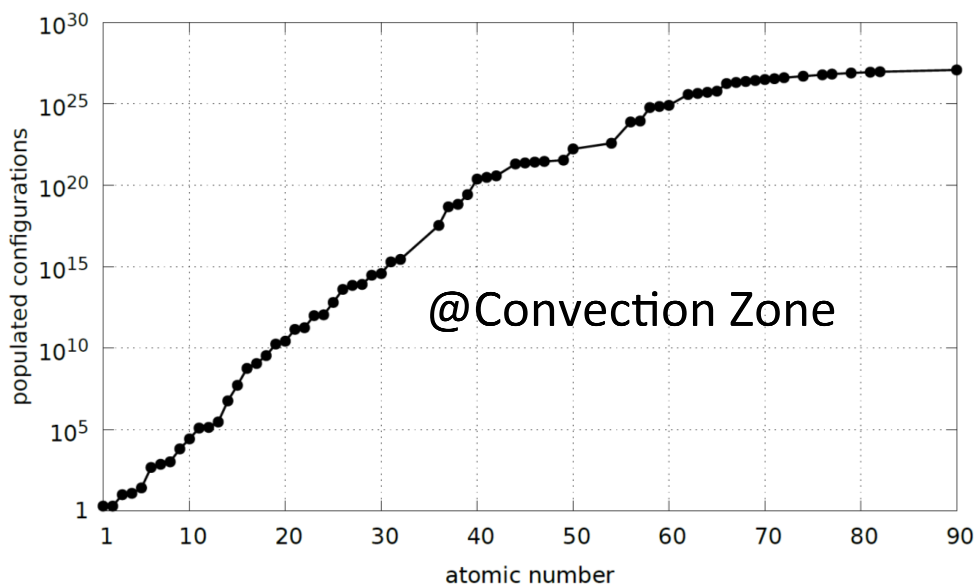
1. For mid-Z and high-Z elements - a **<u>HUGE</u>** number of lines for each pair of configurations
2. For hot plasmas - a **<u>HUGE</u>** number of atomic configurations must be included

A Huge Number of Configurations In The Solar Interior

Most abundant elements across the sun



All elements at the CZ boundary



M. Krief, A. Feigel, and D. Gazit, *"Solar opacity calculations using the super-transition-array method"* ApJ , 821:45, 2016

Unresolved Transition Arrays (UTA)

- In a hot plasma, the large number of lines between pairs of configurations often overlap and can be approximated by a single "effective" line
- Calculate only <u>the moments</u> of the effective lines

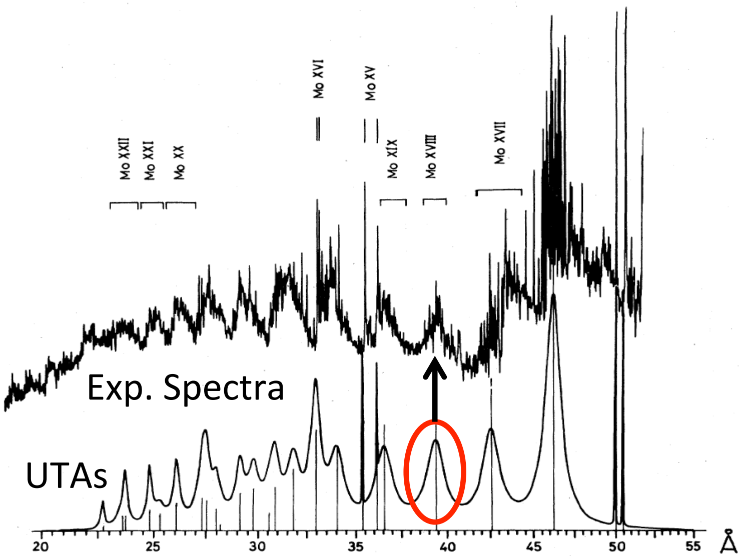
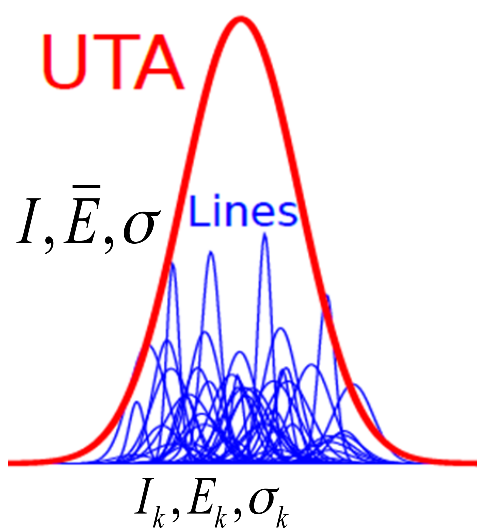
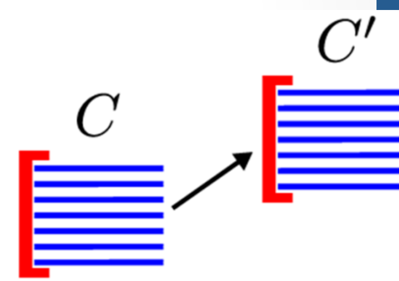


FIG. 3. Comparison between experimental (above) and theoretical (below) spectrum of Mo XV–Mo XXII.





<u>Intensity</u>

$$I = \sum_k I_k$$

<u>Transition Energy</u>

$$\bar{E} = \frac{1}{I}\sum_k I_k E_k$$

<u>Energy Variance</u>

$$\sigma^2 = \frac{1}{I}\sum_k I_k \left[\left(\bar{E} - E_k\right)^2 + \sigma_k^2\right]$$

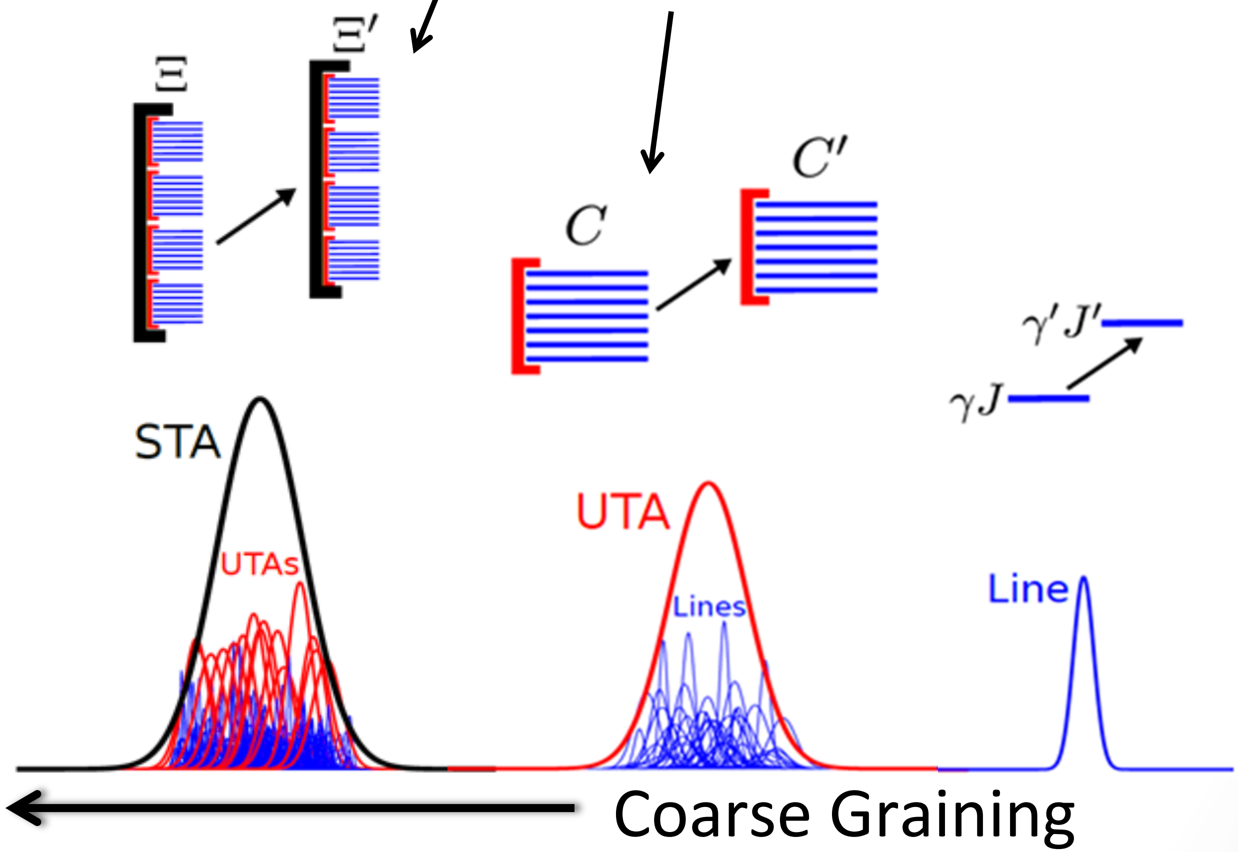
Bauche-Arnoult, C., J. Bauche, and M. Klapisch. *PRA 1979*

The Coarse-Graining Hierarchy

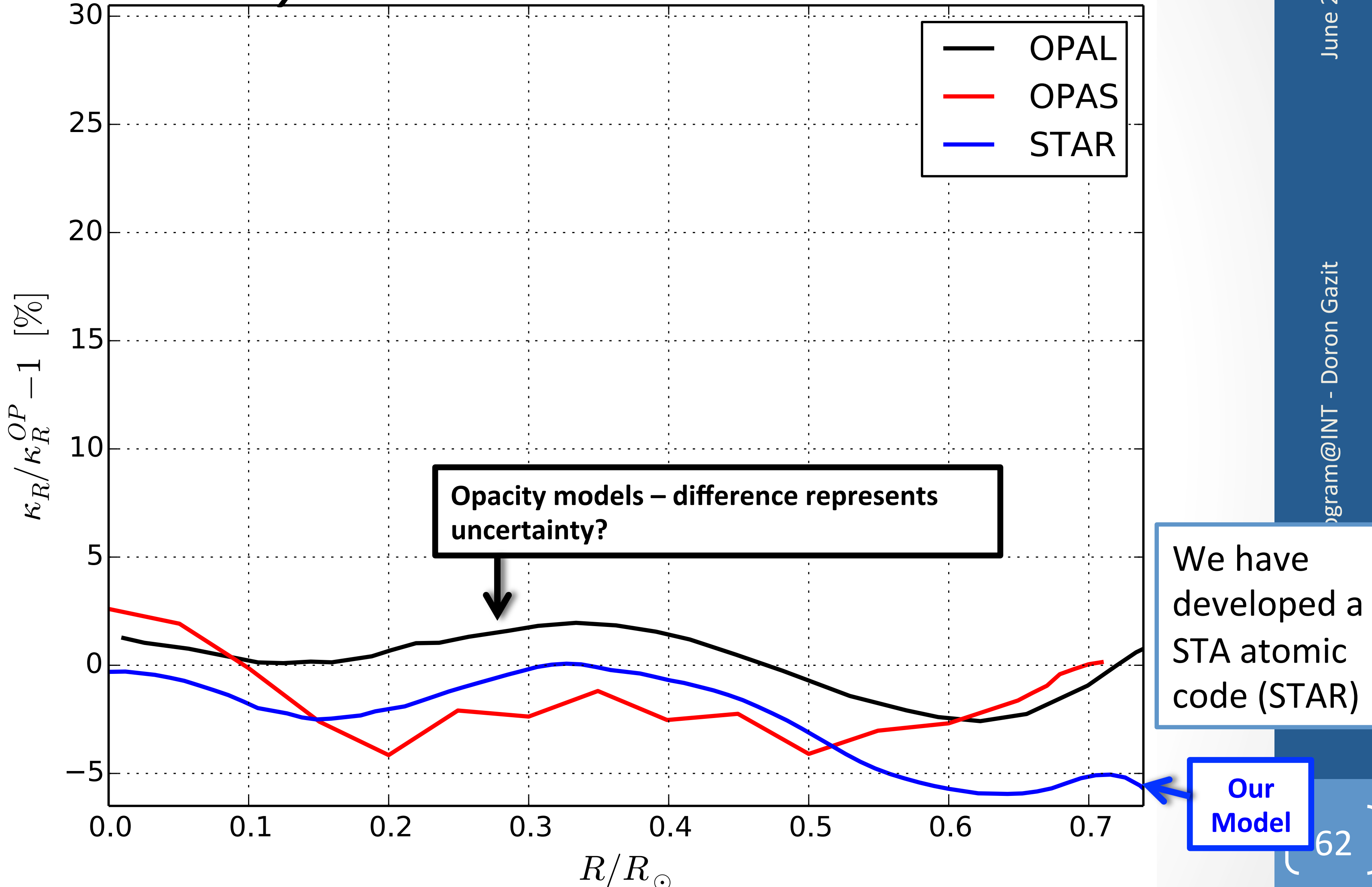
$$\kappa_{bb}(E) = \sum_{\alpha \to \beta} \sum_{C^{\alpha} \to C^{\beta}} \sum_{\substack{a \in C^{\alpha} \\ b \in C^{\beta}}} N_a w_{ab} P_{ab}\left(E - E_{ab}\right)$$

Configurations: $O(1) - O\left(10^{30}\right)$

Levels $O(1) - O\left(10^{10}\right)$



The problem is challenging – but many methods lead to the same result!



M. Krief, A. Feigel, and D. Gazit, "*Line broadening and the solar opacity problem*", ApJ 2016
Blancard, et. Al ApJ 745.1 (2011): 10 & Iglesias et al. ApJ 464 (1996): 943.

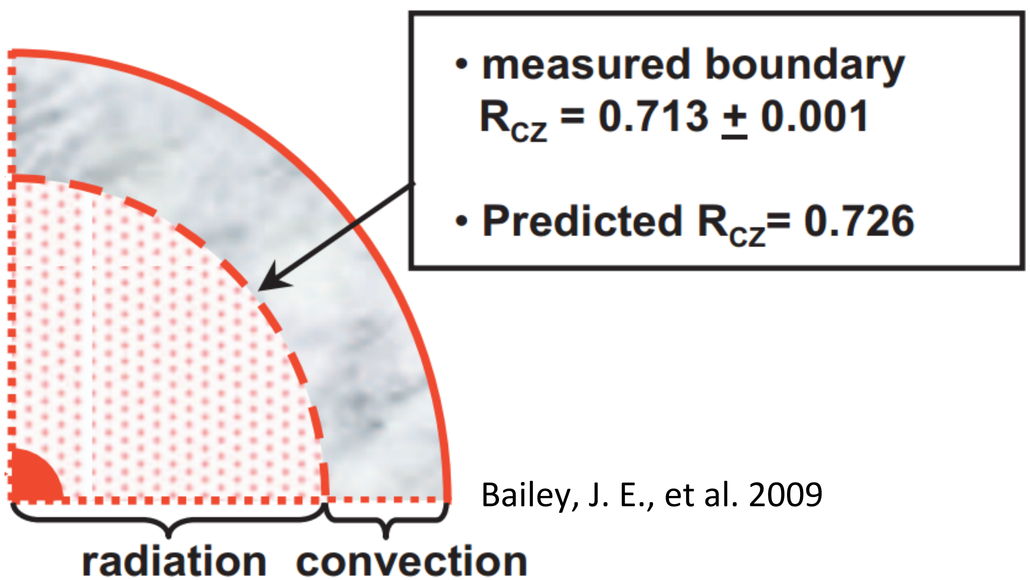
A solar recipe

Abundances

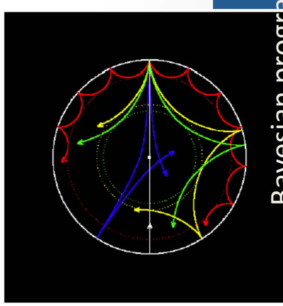
- Solar atmosphere spectra (1D\3D)
- Meteorites

→ Standard Solar Model (SSM)

- Hydrostatic
- 1D
- **Opacities**
- Eqs. of state (EOS)
- Nuclear rates
- .....



Bailey, J. E., et al. 2009



differential probing of solar structure

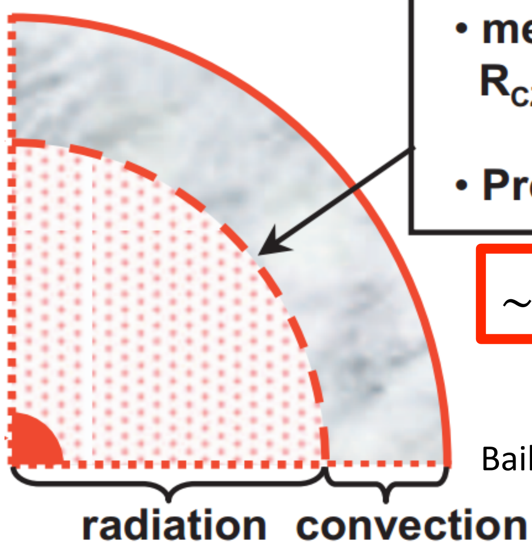
A solar recipe

Abundances

- Solar atmosphere spectra (1D\3D)
- Meteorites

Standard Solar Model (SSM)

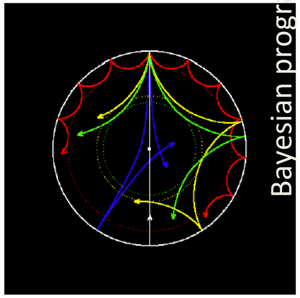
- Hydrostatic
- 1D
- **Opacities**
- Eqs. of state (EOS)
- Nuclear rates
- …..



- **measured boundary $R_{CZ} = 0.713 \pm 0.001$**
- **Predicted $R_{CZ} = 0.726$**

$\sim 10\sigma$ discrepancy!

Bailey, J. E., et al. 2009



differential probing of solar structure

Solar abundance problem

- 20%~ less metals in new abundance determination
- Metals determine most of the opacity, but not EOS
- Solar opacities are exclusively theoretical
- **Opacities are believed to be the "source" of the problem**
- Other ideas – **revised** solar models (magnetic fields, rotation, dark matter… etc.) no satisfactory model exists

1. *F.L. Villante and B. Ricci - APJ 2010*
2. *F.L. Villante* - APJ 2010
3. Bergemann, and A Serenelli. 2014

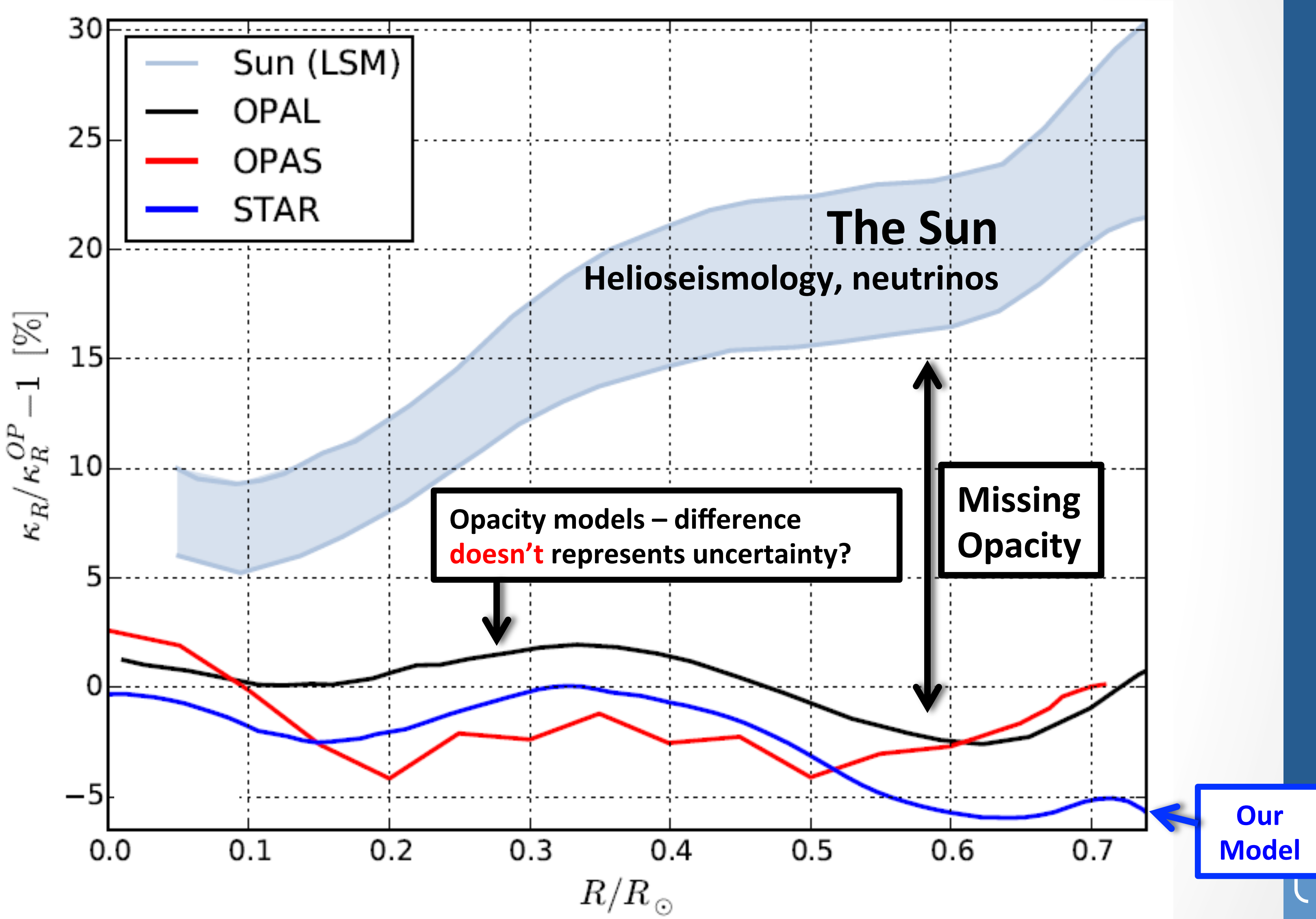
Solar ~~abundance~~ problem

opacity

- 20%~ less metals in new abundance determination
- Metals determine most of the opacity, but not EOS
- Solar opacities are exclusively theoretical
- **Opacities are believed to be the "source" of the problem**
- Other ideas – **revised** solar models (magnetic fields, rotation, dark matter… etc.) no satisfactory model exists

1. *F.L. Villante and B. Ricci - APJ 2010*
2. *F.L. Villante* - APJ 2010
3. Bergemann, and A Serenelli. 2014

Missing Opacity – a "Solar **OPACITY** Problem"



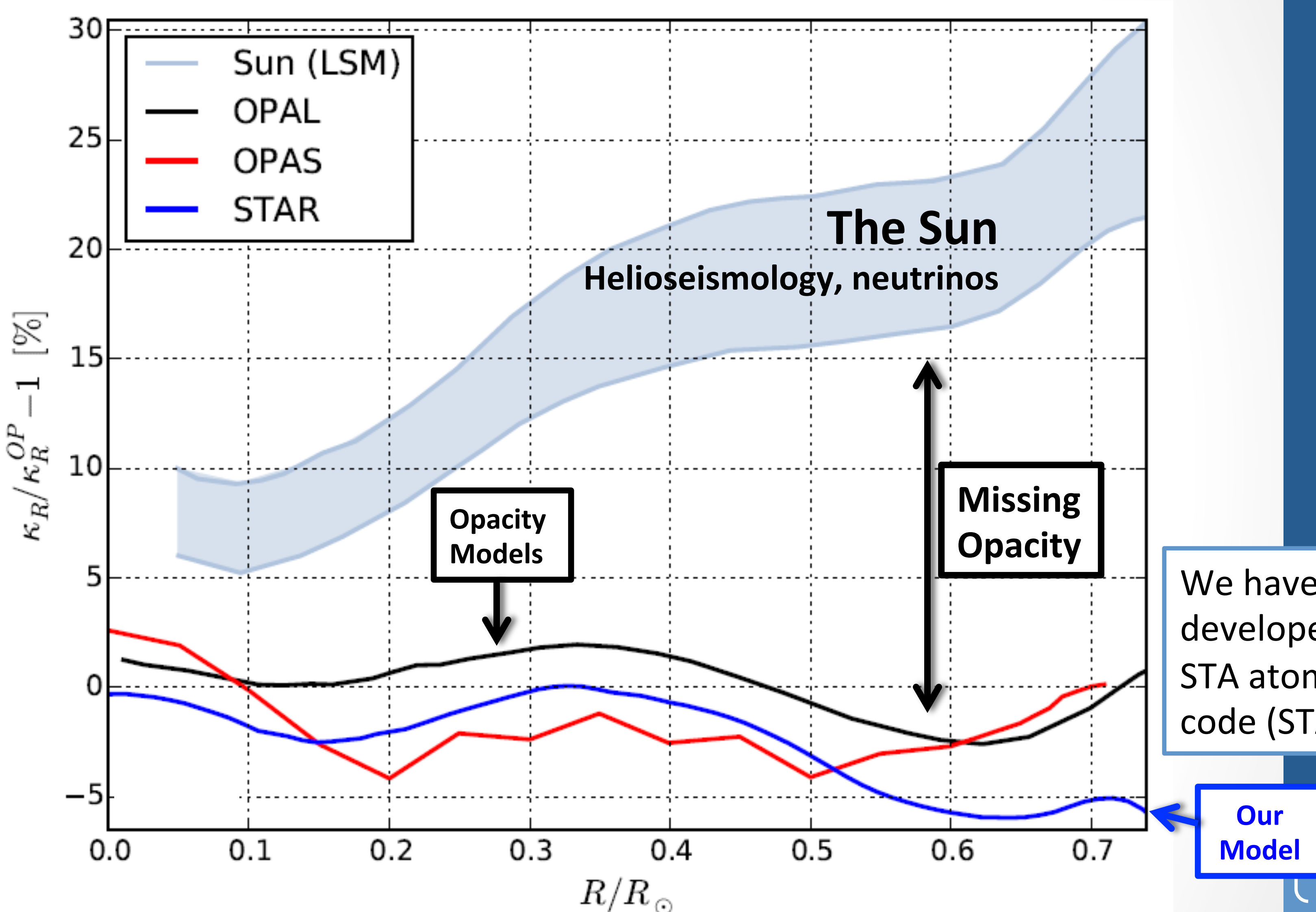
M. Krief, A. Feigel, and D. Gazit, "*Line broadening and the solar opacity problem*", ApJ 2016
Blancard, et. Al ApJ 745.1 (2011): 10 & Iglesias et al. ApJ 464 (1996): 943.

Steps Towards Solution

- Point out and check physics "beyond" current state of the art atomic models
- Alternatively, point out and quantify sources of uncertainty in atomic models and check sensitivities
- We have developed state of the art atomic models in order to investigate the source of the solar opacity problem

1. M. Krief, A. Feigel, and D. Gazit, "*Line broadening and the solar opacity problem*", ApJ 2016
2. M. Krief, A. Feigel, and D. Gazit, "*Solar opacity calculations using the super-transition-array method*" ApJ , 2016
3. M. Krief, A. Feigel "*Variance and shift of transition arrays for electric and magnetic multipole transitions*", HEDP 2015
4. M. Krief, A. Feigel "*The effect of first order superconfiguration energies on the opacity of hot dense matter*", HEDP 2015

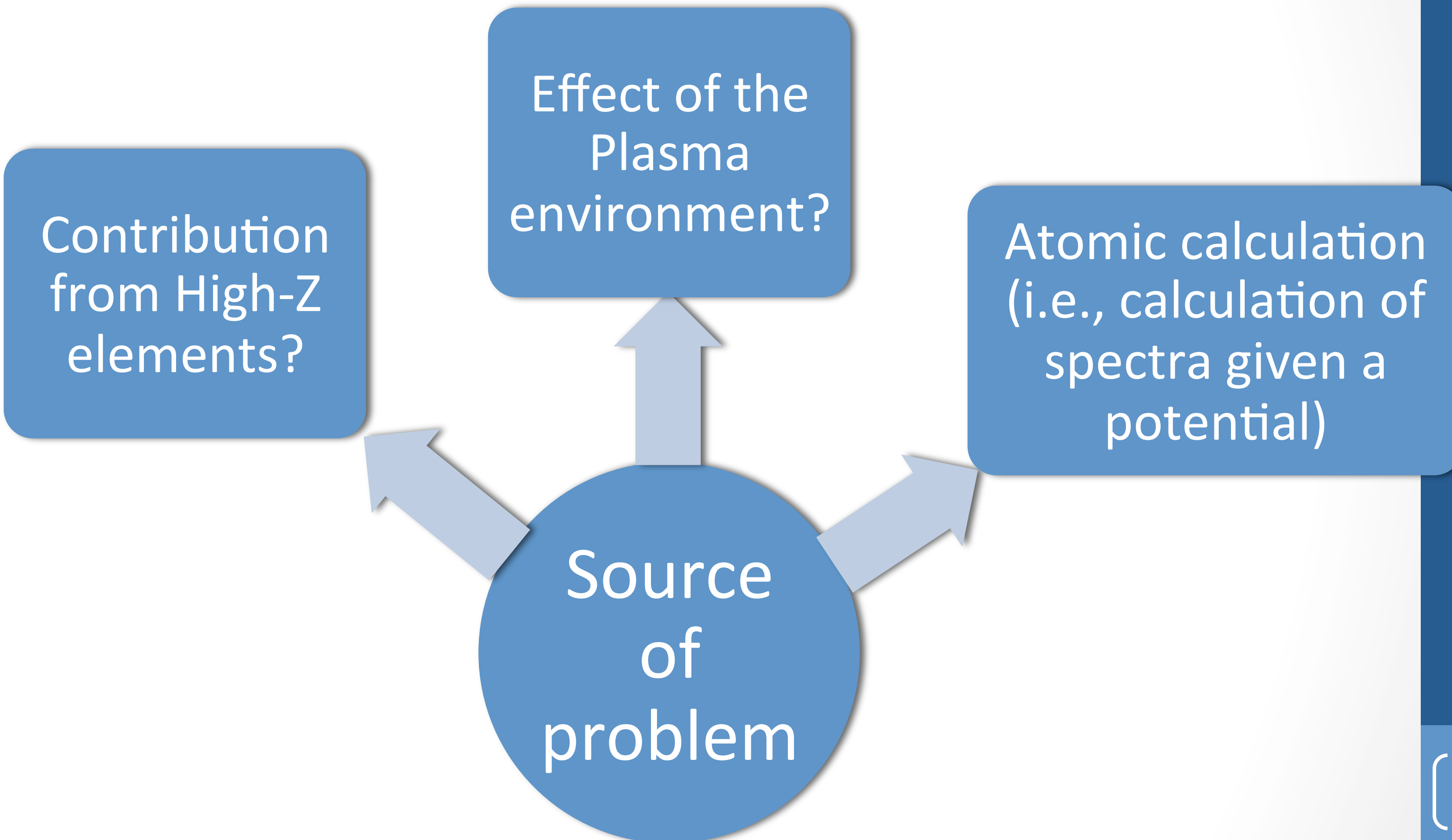
Missing Opacity – a “Solar **OPACITY** Problem”



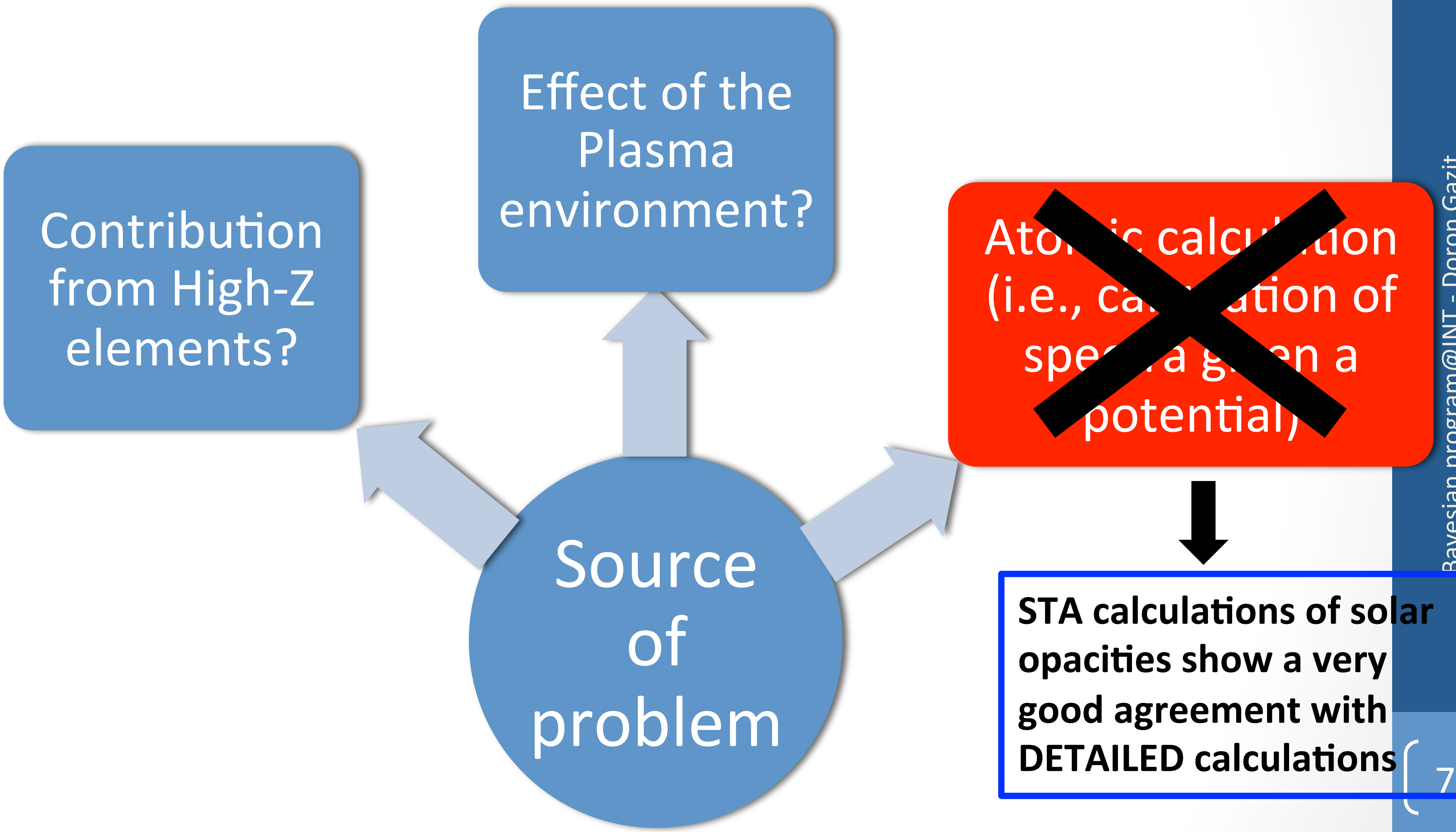
We have developed a STA atomic code (STAR)

M. Krief, A. Feigel, and D. Gazit, *“Line broadening and the solar opacity problem”*, ApJ 2016
Blancard, et. Al ApJ 745.1 (2011): 10 & Iglesias et al. ApJ 464 (1996): 943.

Possible explanations?



Possible explanations?



Possible explanations?

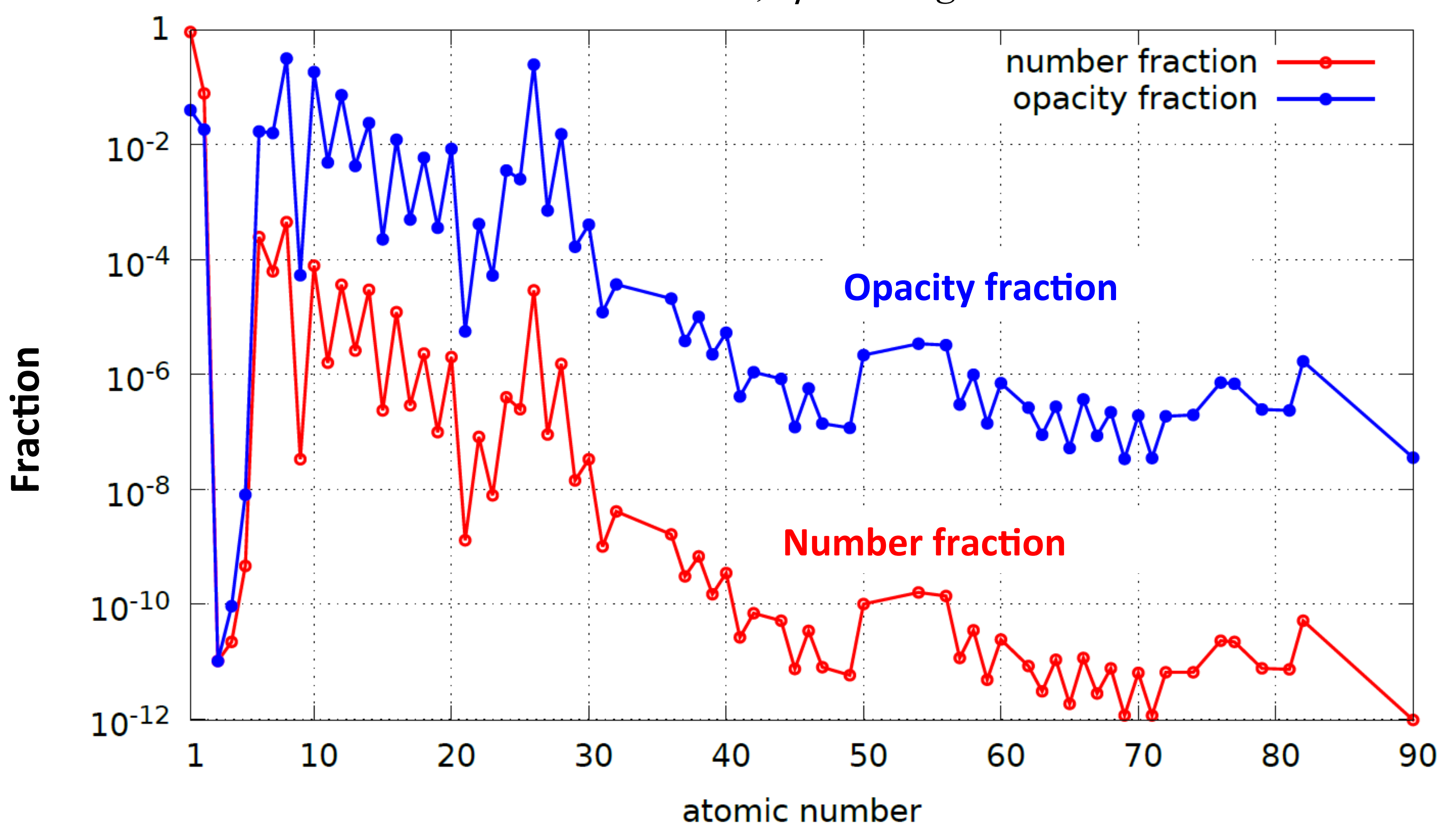
Source of problem

- Contribution from High-Z elements?
- Effect of the Plasma environment?
- ~~Atomic calculation (i.e., calculation of spectra given a potential)~~

Effect of heavy elements?

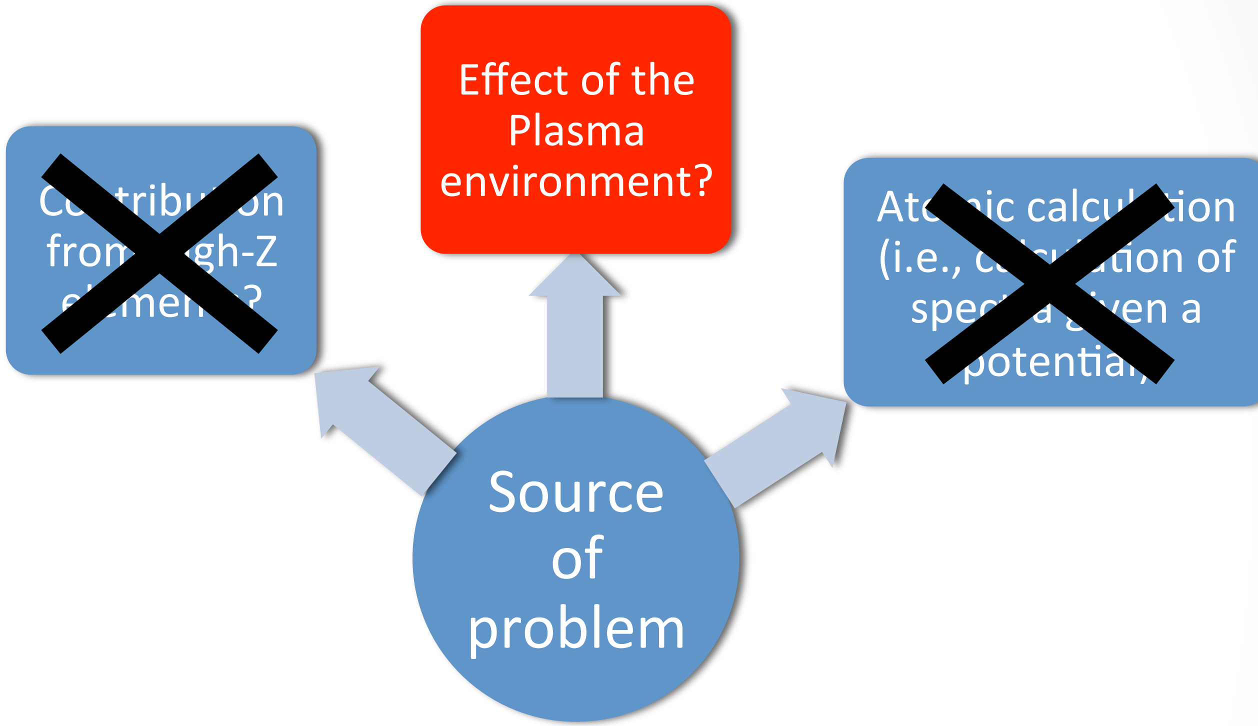
Calculation is possible only with STA

$$T = 176eV \quad , \quad \rho \approx 0.16 g/cm^3$$

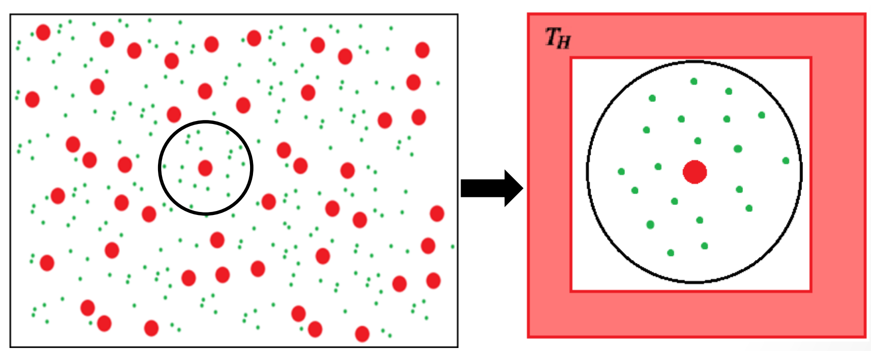


1. M. Krief, A. Feigel, and D. Gazit, *"Solar opacity calculations using the super-transition-array method"* ApJ , 821:45, 2016
2. Iglesias, C. A., Wilson, B. G., Rogers, F. J., Goldstein, W. H., Bar-Shalom, A., & Oreg, J. (1995). APJ, 445, 855-860.

Possible explanations?



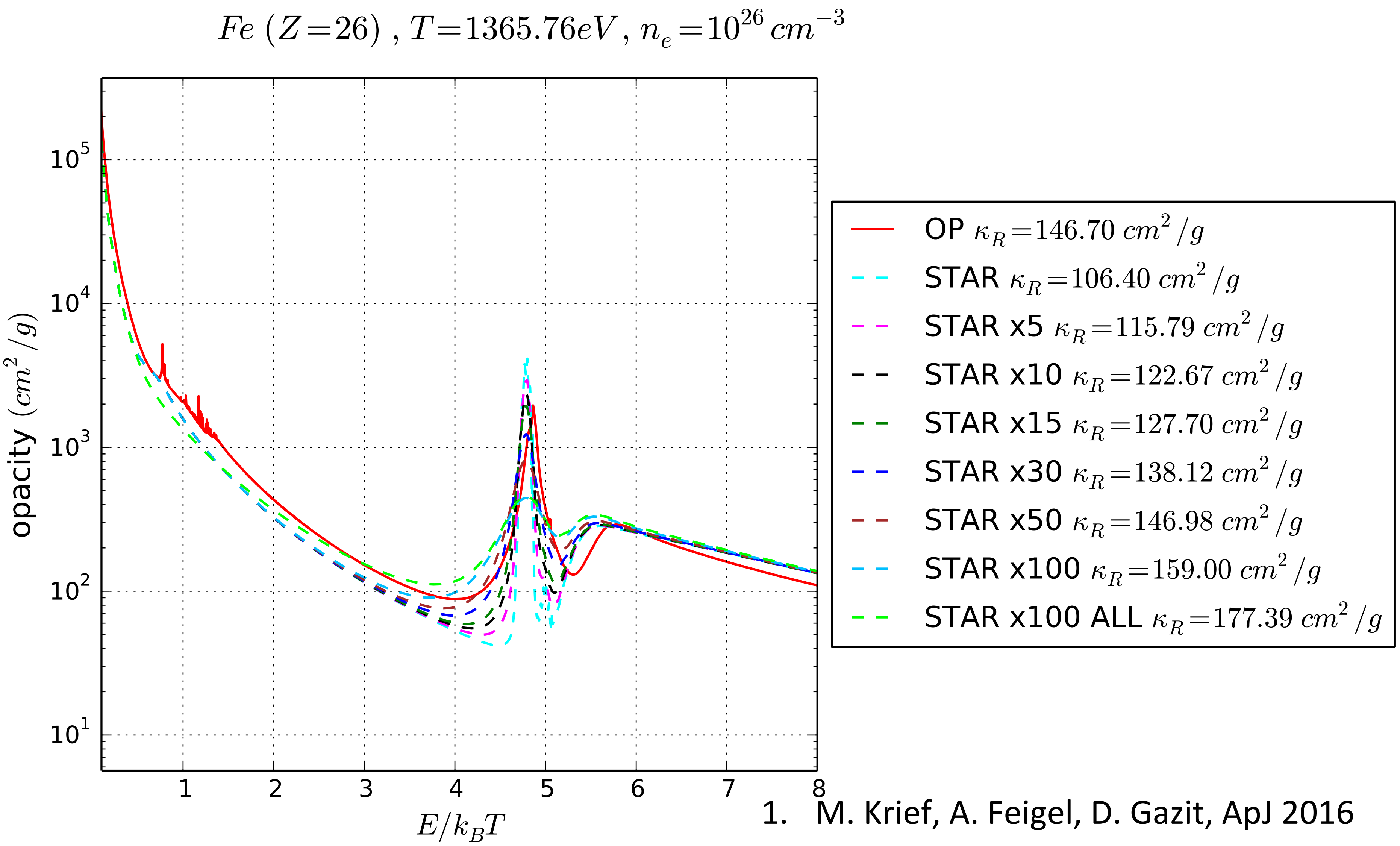
- Line Shapes
- Level populations
- Line-Shifts (screening)



Photons Escape Through Opacity "Windows"

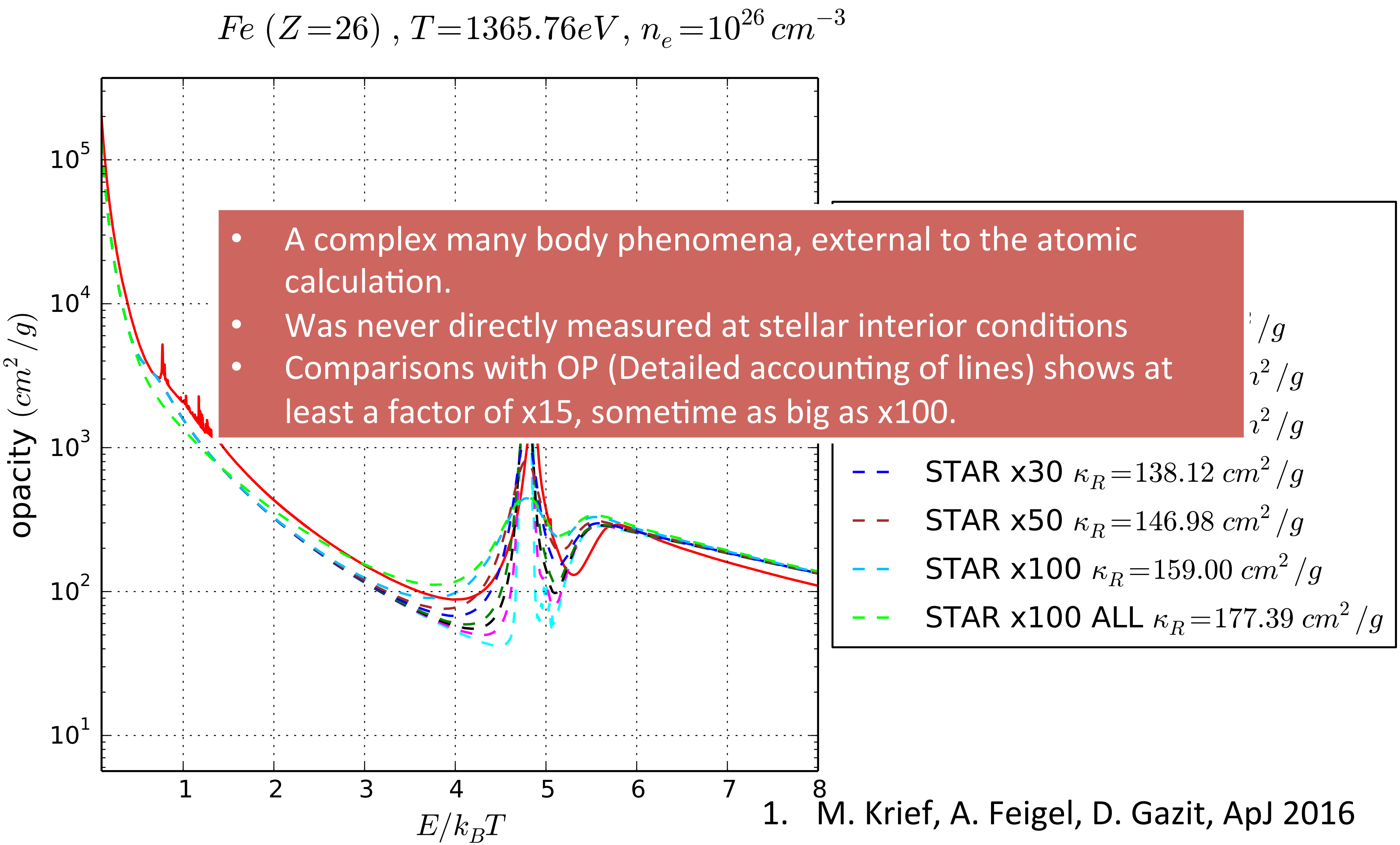
Marcel Klapisch, APIP, April 2016

Uncertainties in collisional line broadening: enormous differences between models



1. M. Krief, A. Feigel, D. Gazit, ApJ 2016

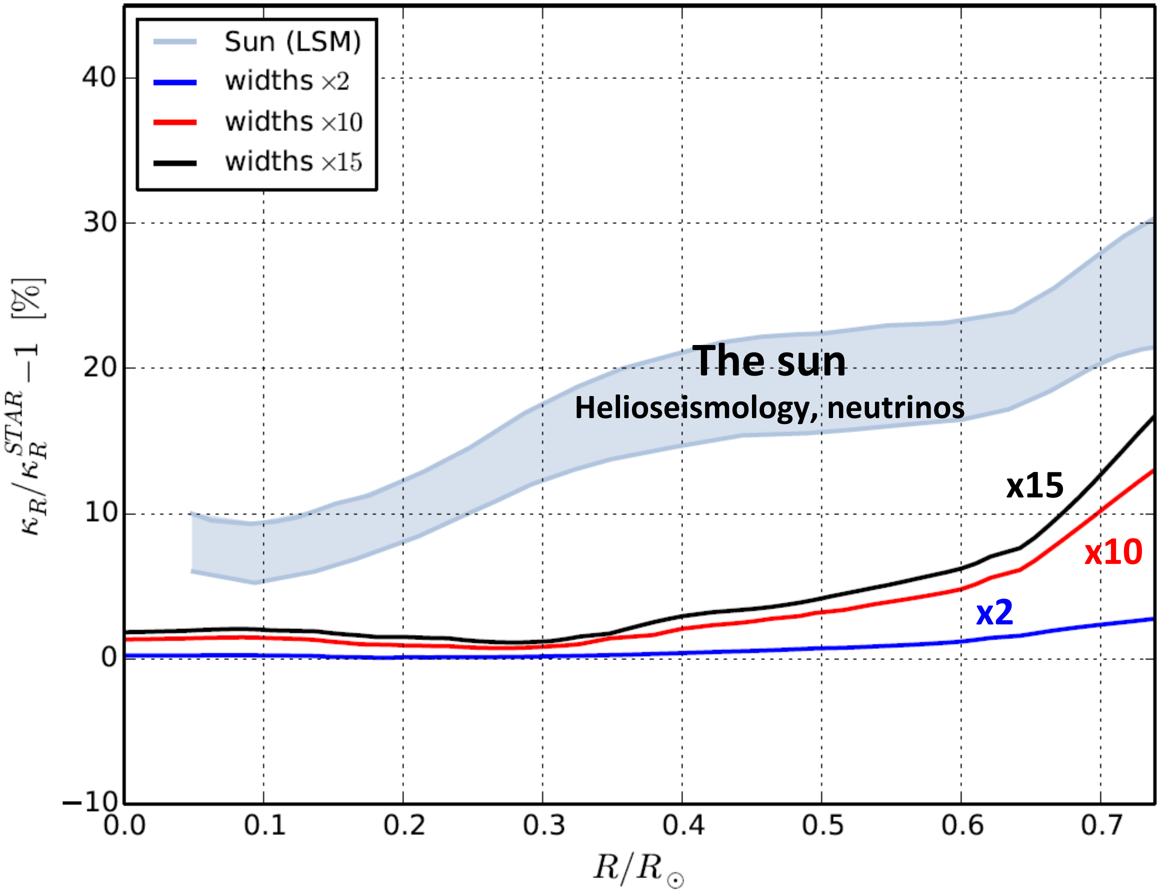
Uncertainties in collisional line broadening: enormous differences between models



- A complex many body phenomena, external to the atomic calculation.
- Was never directly measured at stellar interior conditions
- Comparisons with OP (Detailed accounting of lines) shows at least a factor of x15, sometime as big as x100.

1. M. Krief, A. Feigel, D. Gazit, ApJ 2016

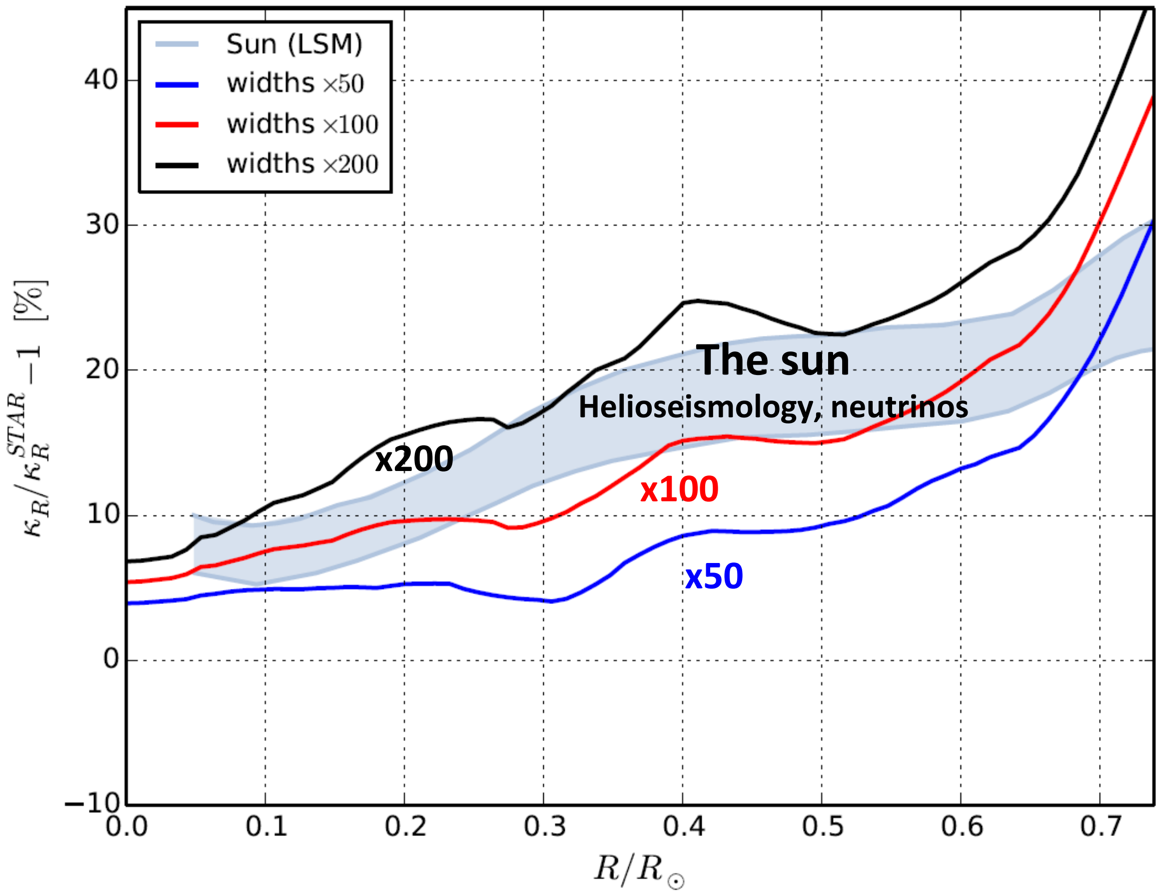
Opacity variations resulting from uncertainties in collisional line broadening



- No experimental data - <u>what is the actual uncertainty of current models</u>?

M. Krief, A. Feigel, and D. Gazit, *"Line broadening and the solar opacity problem"*, ApJ 2016

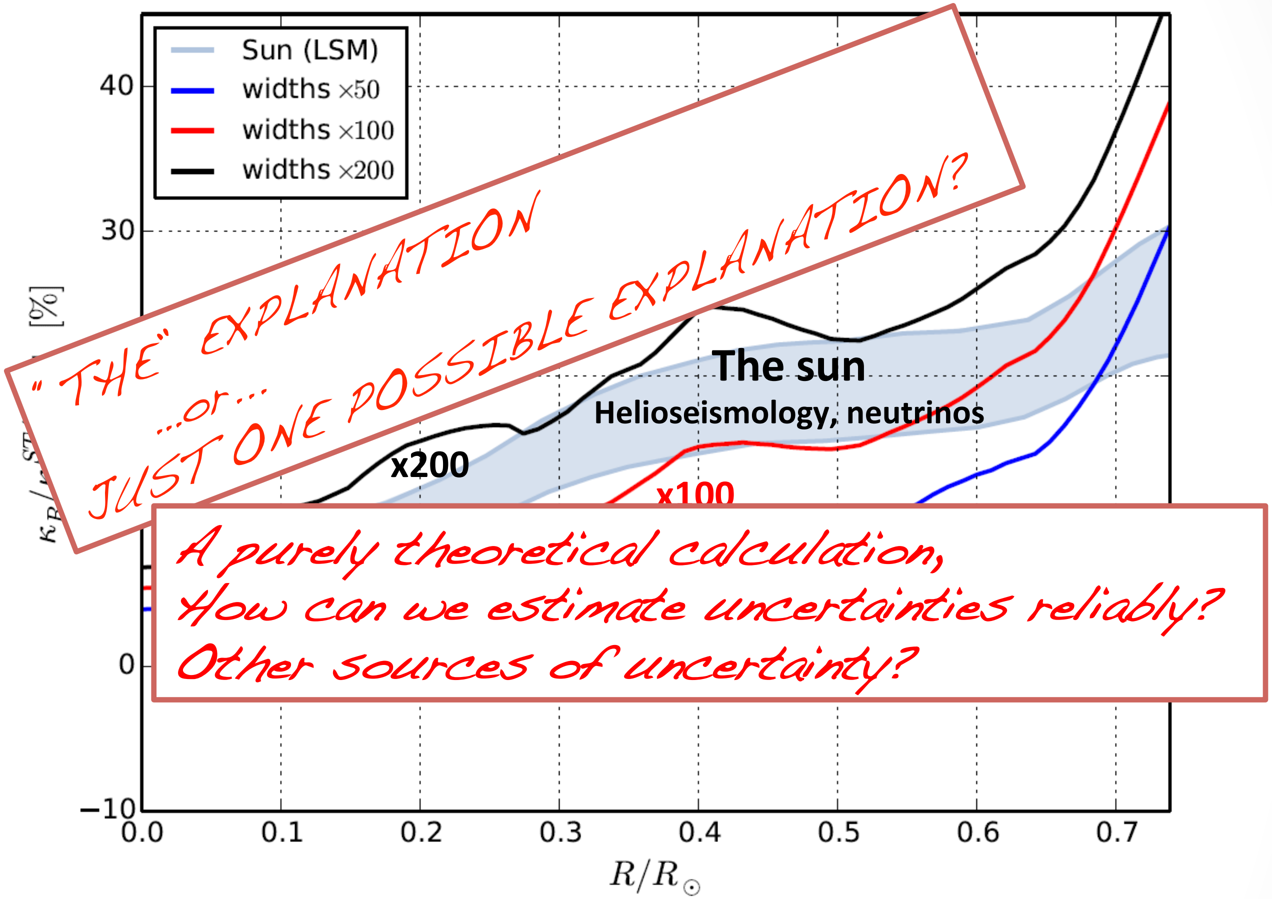
A factor of ~100 is needed to solve the problem **quantitatively** and **qualitatively**



- The uncertainty may depend on the line, atomic number, temperature and density

M. Krief, A. Feigel, and D. Gazit, *"Line broadening and the solar opacity problem"*, ApJ 2016

A factor of ~100 is needed to solve the problem **quantitatively** and **qualitatively**



- The uncertainty may depend on the line, atomic number, temperature and density

M. Krief, A. Feigel, and D. Gazit, "*Line broadening and the solar opacity problem*", ApJ 2016

A Rumsfeld type of summary ☺

https://www.youtube.com/watch?v=GiPe1OiKQuk

Summary…

- I showed a few "qualitative" approaches to "quantitatively" assess theoretical uncertainties:

"Known-knowns":

Correlations in light nuclei:
Predictions of one EFT should be reflected in other EFT: disregarding it may result in fine tuning. There are right and wrongs in choosing fitting observables.

"Known-unknowns":

Prediction of the proton-proton fusion rate:
Reliable theoretical uncertainty
1. *Natural size of sub-leading contribution.*
2. *RG invariance.*
3. *Rearranging perturbation exp*
4. *An analogue reaction where experiment exists.*
5. *(theoretical physicist's) heaven lies where large experimental uncertainty exists*

"Unknown-unknowns"

Solar opacity problem:
1. Clearly – range of models cannot be used as uncertainty quantification.
2. The fact that theory fits experiment doesn't mean it's right…

Is there a more systematic approach? If not no… please tell me! doron_gazit@me.com

The Ion-Sphere model

- The plasma is divided into spherical cells
- The density dictates the size of the “Wigner-Seitz“ cell, in which neutrality is imposed
- The surrounding plasma of each cell is considered a heat bath