Probing Neutron Star Interiors with Gravitational Waves

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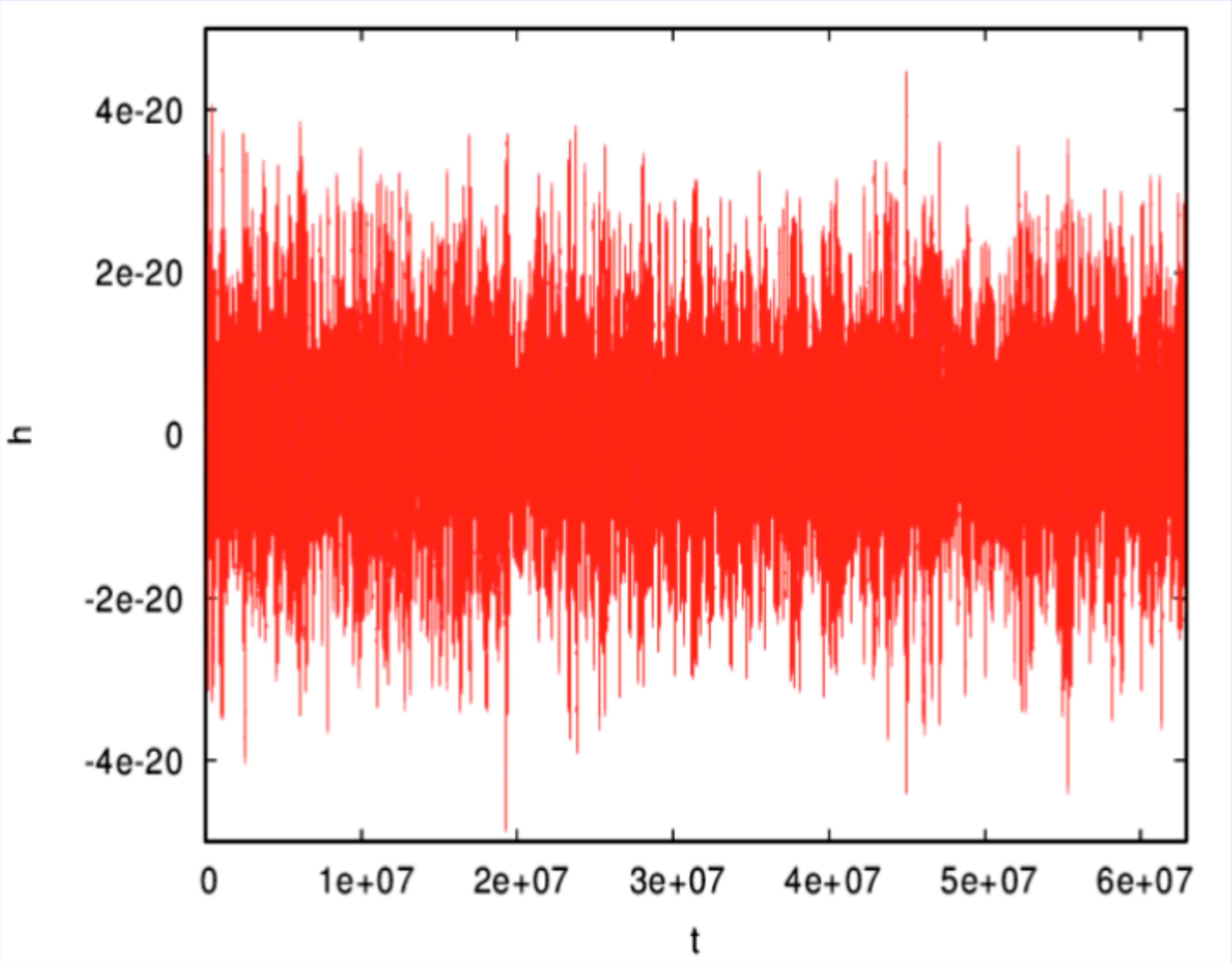
Montana State University

with Kent Yagi, Antoine Klein, Neil Cornish, and Nico Yunes

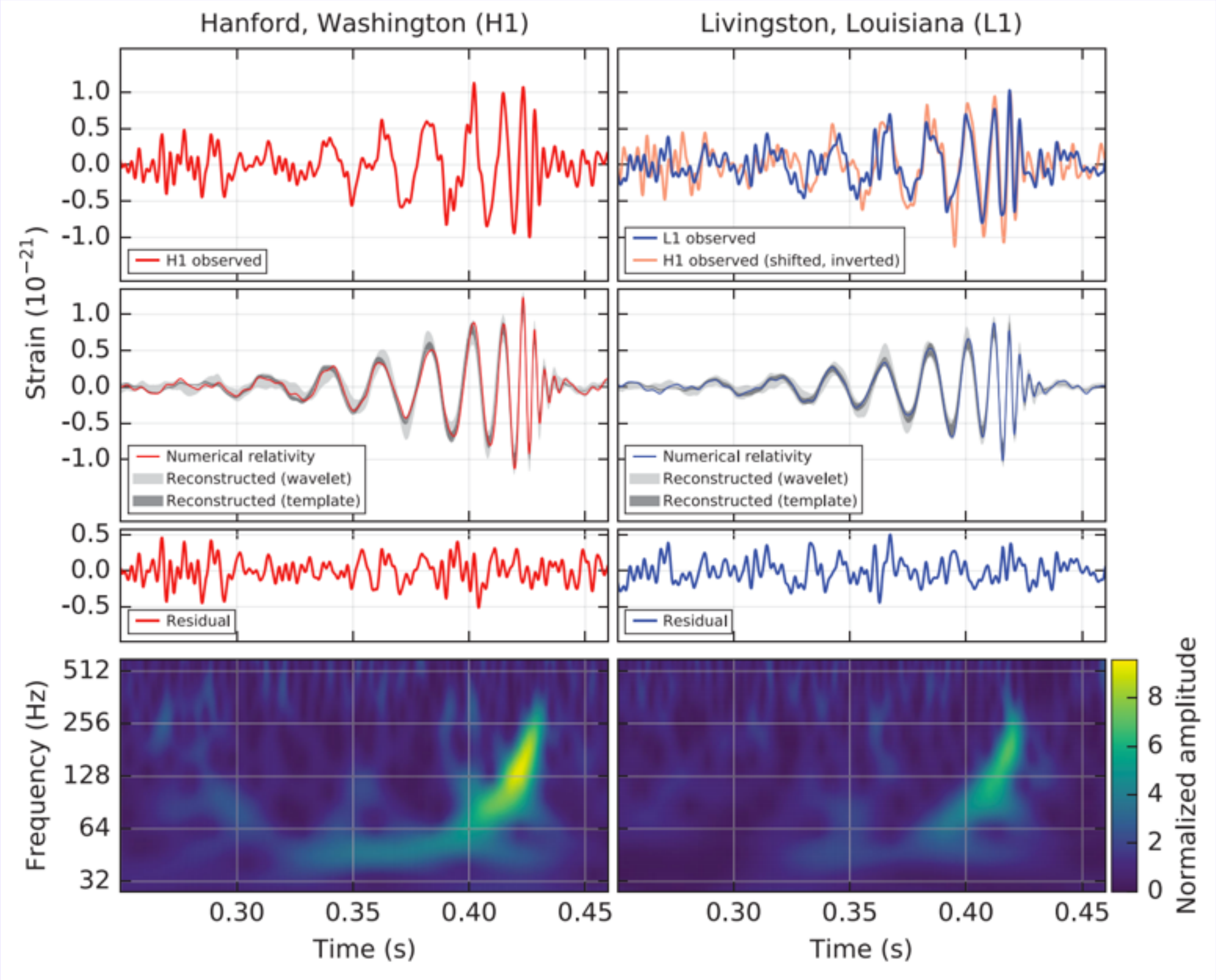
INT, 2016

Can you spot the Gravitational Wave?



Cornish

Exciting Times for Gravitational Wave Astrophysics!



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LIGO

From now on…

- Gravitational Waves and Data Analysis
- Neutron Star Compact Binaries
- Model Selection
- Results

Gravitational Waves

Definition: Wave-like perturbation of the gravitational field

$$\Box h_{\mu\nu} = T_{\mu\nu}$$

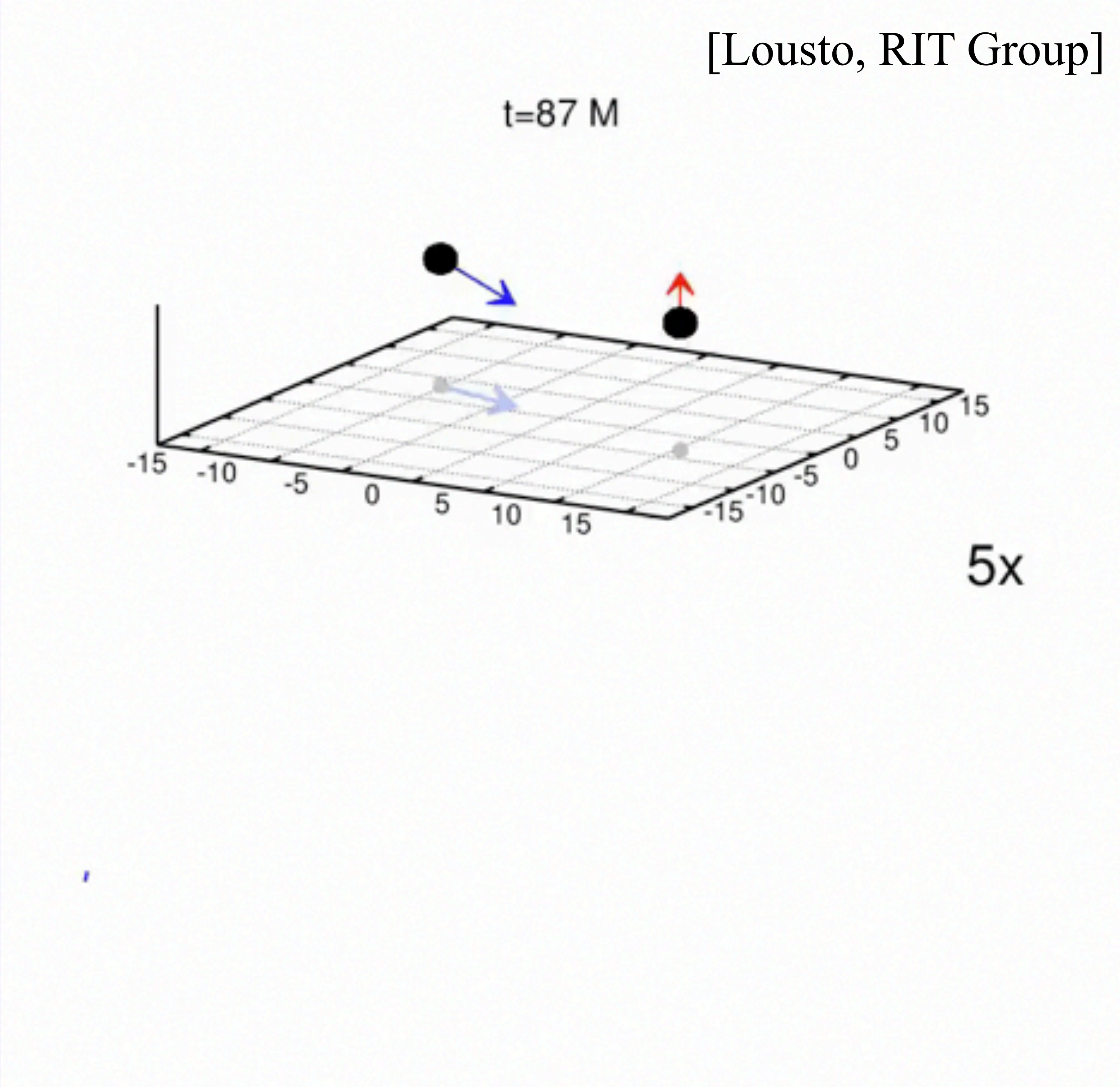
Generation: Accelerating masses (changing quadrupole and higher multipole moments)

$$h_{ij} \sim \frac{1}{R}\frac{d^2 Q_{ij}}{dt^2}$$

Amplitude: Small

$$h \sim \frac{G}{c^4}\frac{mu^2}{R} \sim 10^{-22}$$

Propagation: Light speed, weakly interacting



Spectrum: Kepler 3rd Law: $f \sim \sqrt{\frac{m}{r_{12}^3}} \sim \frac{1}{m}$, $E_{rad} \sim \% m$

Example: for GW150914, $E_{\rm GW} \sim 3M_\odot \sim 10^3 E_{SN} \sim 0.6 E_{\rm GRB}$

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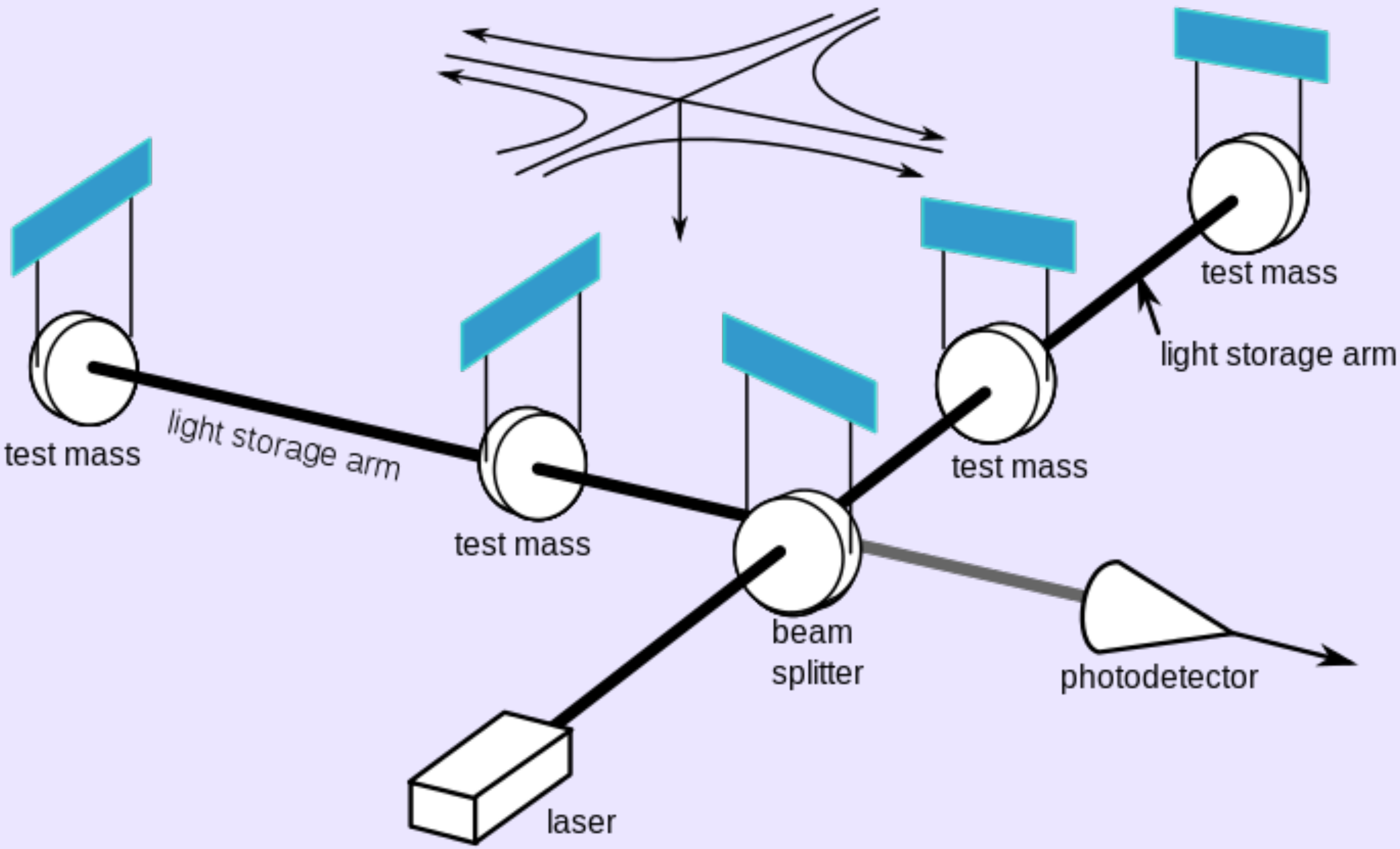
Gravitational Wave Detectors



$$h = \frac{\Delta L}{L} \Rightarrow \Delta L = 10^{-19} m$$

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Gravitational Wave Data Analysis

$$d = R[h(\vec{x})] + n$$

data — detector response — noise

Fit the data with a theoretical model for the GW signal

1) Get data $\qquad d$

2) Select a model $\qquad h'(\vec{x})$

3) Calculate the residual $\qquad r = d - R[h'(\vec{x})]$

4) Is this just noise? $\qquad p(r) = p(n) = p(d|h'(\vec{x}))$

Likelihood

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Bayesian Probability Theory

Degree of belief interpretation of probability

Initial Understanding + New Observations = Updated Understanding

$p(\vec{x})$ $p(d|\vec{x})$ $p(\vec{x}|d)$

Prior + Likelihood = Posterior

Bayes' Theorem

$$p(\vec{x}|d, M) = \frac{p(\vec{x}|M)p(d|\vec{x}, M)}{p(d|M)}$$

Evidence

$$p(d|M) = \int p(\vec{x}|M)p(d|\vec{x}, M)d\vec{x}$$

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Bayesian Model Selection

Probability of model M $$p(M|d) \sim p(M)p(d|M)$$

Odds ratio
$$\mathcal{O}_{ij} = \frac{p(M_i|d)}{p(M_j|d)}$$
$$= \frac{p(M_i)p(d|M_i)}{p(M_j)p(d|M_j)}$$

Bayes Factor

(the evidence from before)

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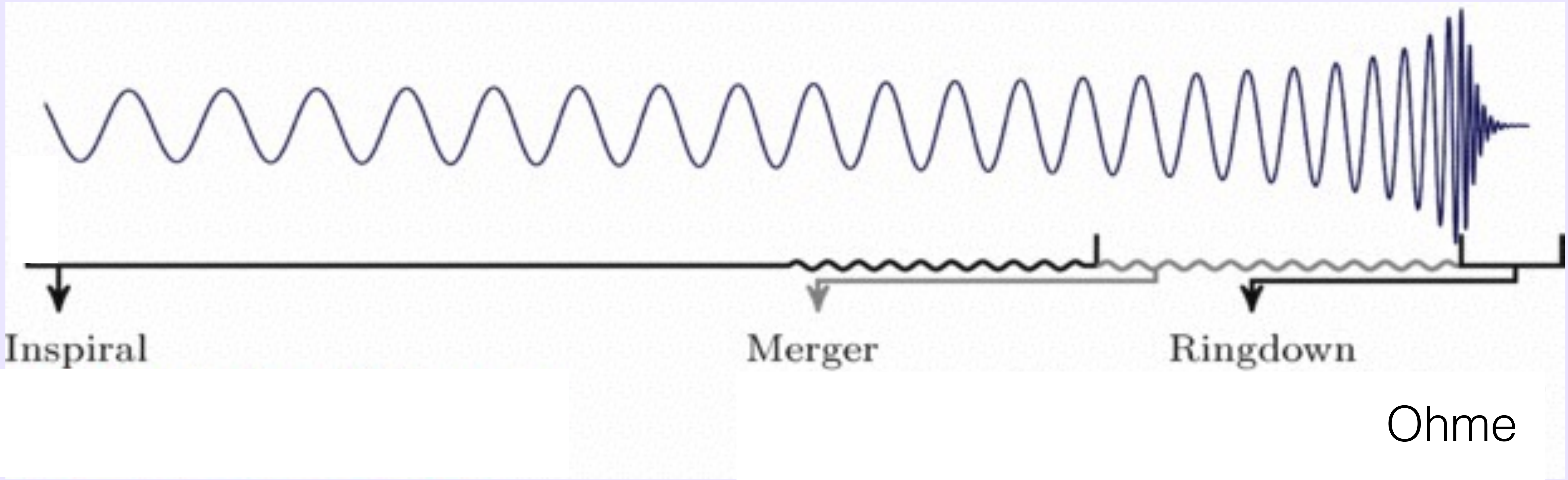
Quasicircular Compact Binary Inspirals



NS/NS:	[20 mins, 10,000 cycles, (10-a few k)Hz]	~400km
BH/NS:	[(3-7) mins, (1,000-5,000) cycles, (10-merger)Hz]	~(500-700)km
BH/BH:	[secs - mins, (100-700) cycles, (10-merger)Hz]	~(600-1000)km

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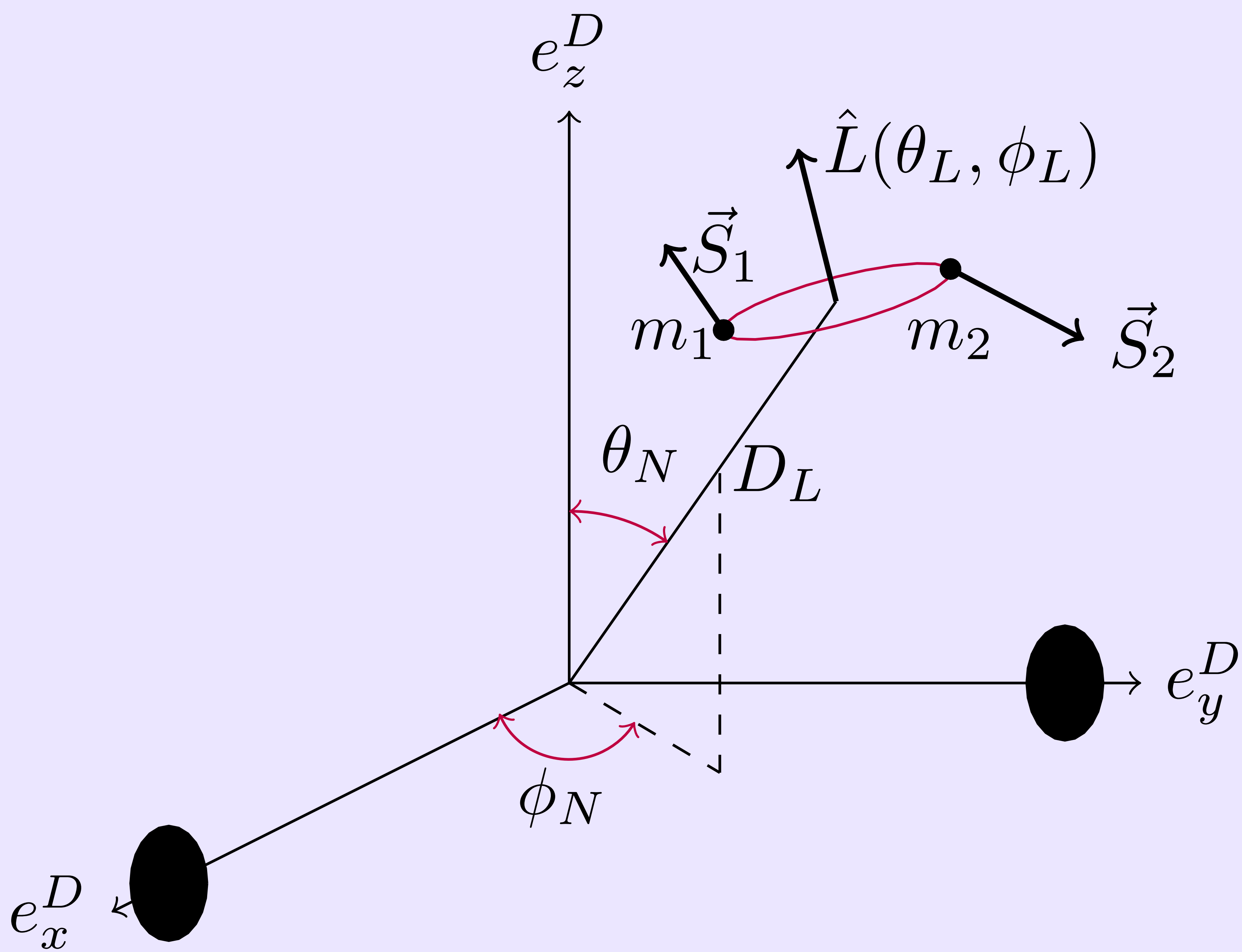
Compact Binary System



$$\vec{x} = (m_1, m_2, \vec{S}_1, \vec{S}_2, D_L, \theta_N, \phi_N, \theta_L, \phi_L, t_c, \phi_c)$$

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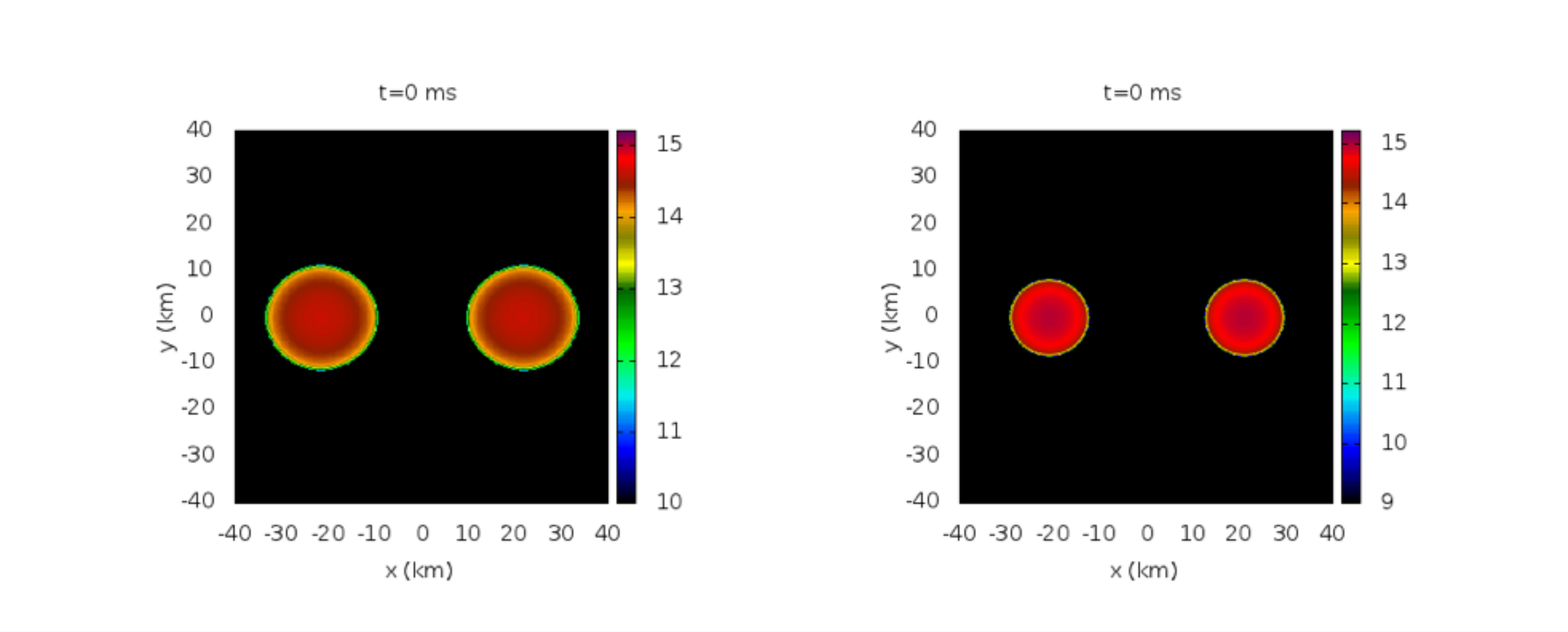
Coalescing Neutron Stars and Nuclear Physics

MS1 AP4

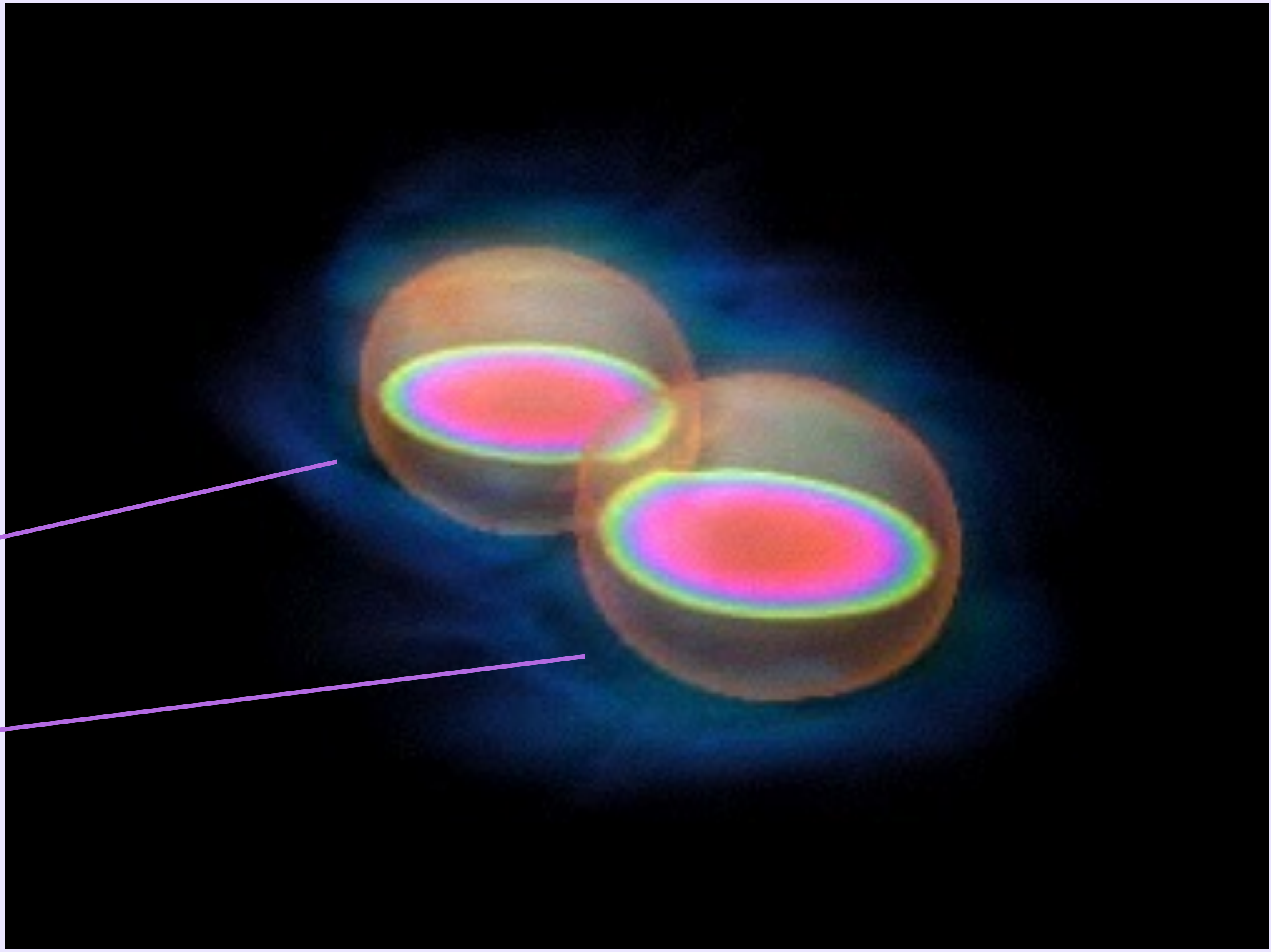


Hotokezaka et al.

Neutron Star Inspirals

Tidal deformability

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$



Calder

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Tidal Deformability

$$\vec{x} = (m_1, m_2, \vec{S}_1, \vec{S}_2, D_L, \theta_N, \phi_N, \theta_L, \phi_L, t_c, \phi_c)$$

$$+\{\lambda_i(m_i, \mathrm{EoS}), Q_i(m_i, \mathrm{EoS})\}$$

=19 parameters

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Tidal Deformability

We can measure the tidal deformability*
with a few bright sources

Read et al. (2009)
Del Pozzo et al. (2013)
Wade et al. (2014)
Agathos et al. (2015)
Lackey and Wade (2015)

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Equation of State

EoS	Method/Model	Composition
AP4	variational	n, p, e, μ
GCR	variational	n
SV	SHF	n, p, e, μ
SGI, SkI4	SHF	n, p, e, μ
DBHF$^{(2)}$(A)	BHF	n, p, e, μ
MPa	BHF	n, p, e, μ
G4, GA-FSU2.1	RMF	n, p, e, μ
SGI-YBZ6-S$\Lambda\Lambda$3, SkI4-YBZ6-S$\Lambda\Lambda$3	SHF	n, p, e, μ, H
NlY5KK*	BHF	n, p, e, μ, H
MPaH	BHF	n, p, e, μ, H
H4	RMF	n, p, e, μ, H
SGI178	SHF	n, p, e, μ, K
SV222	SHF	n, p, e, μ, K
GA-FSU2.1-180	RMF	n, p, e, μ, K
ALF4, ALF5	variational	n, p, e, μ, π, Q
GCR-ALF	variational	n, Q
SQM3	MIT bag	Q (u, d, s)

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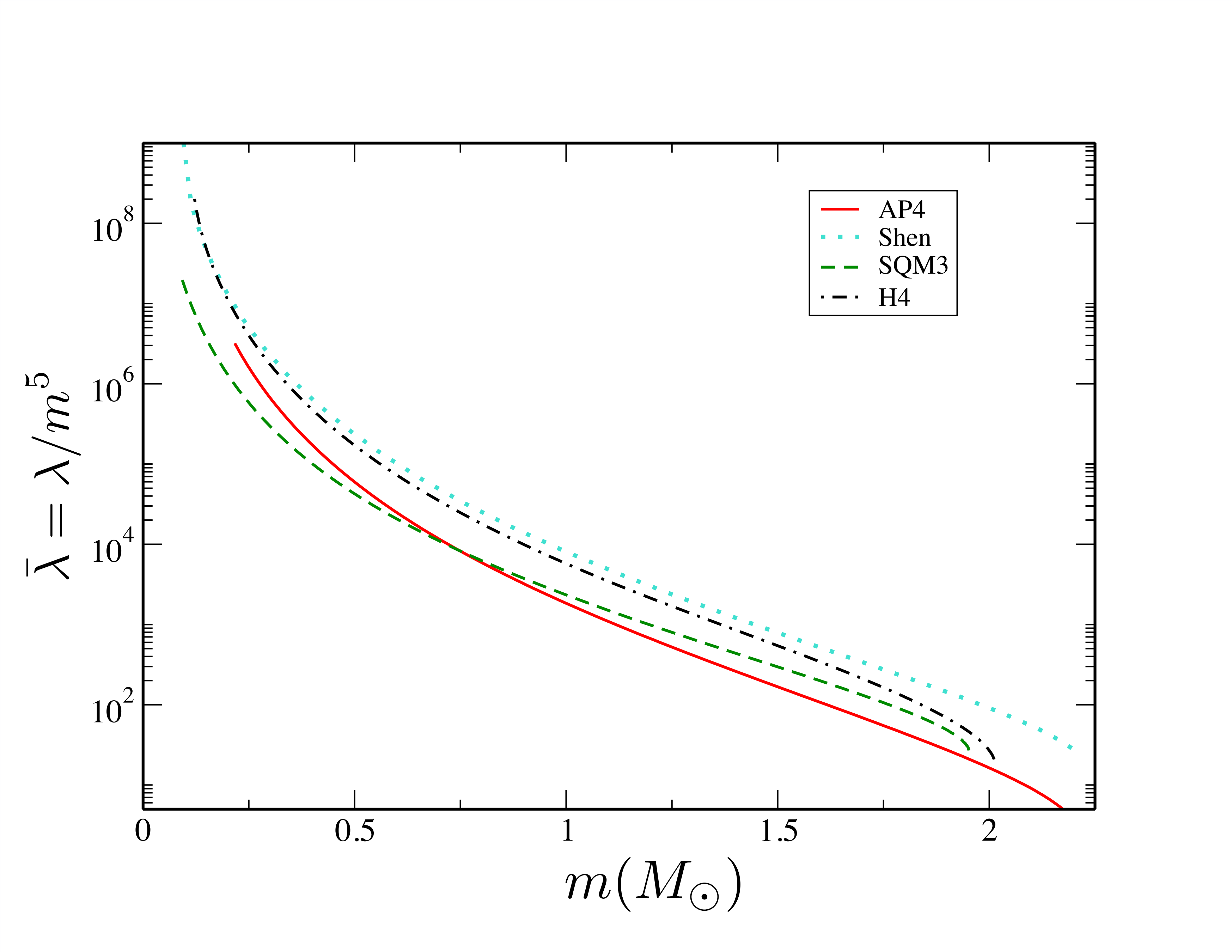

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ALF4, ALF5	variational	n, p, e, μ, π, Q
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SQM3	MIT bag	Q (u, d, s)

Can GWs Distinguish NS Binaries with Different Internal Composition ?

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Model Selection

We need to calculate the evidence and the odds ratio

$$\mathcal{O}_{ij} = \frac{p(M_i)}{p(M_j)} \frac{p(d|M_i)}{p(d|M_j)}$$

How much we believe in each model <u>before</u> acquiring the data. Based on our previous experience, observational evidence, and theoretical understanding of the Universe.

Which of two competing model fits the data at hand better.

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Bayes Factor

$$\mathrm{BF} = \frac{p(d|M_i)}{p(d|M_j)}$$

When is the BF 'large enough'?

BF	Interpretation
<1	Negative
<3	Barely worth mentioning
<10	Strong
<100	Very Strong
>100	Desicive

Jeffreys scale of BF interpretation

The Evidence (or the ratio)

- Laplace Approximation
- Schwarz-Bayes Information Criterion
- Reversible Jump MCMC
- Thermodynamic Integration
- Nested Sampling
- Savage-Dickey Density Ratio

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Model Parameters

$$\vec{x} = (m_1, m_2, \vec{S}_1, \vec{S}_2, D_L, \theta_N, \phi_N, \theta_L, \phi_L, t_c, \phi_c)$$

$$+\{\mathrm{EoS}\}$$

15 *continuous* parameters, and 1 *discrete*

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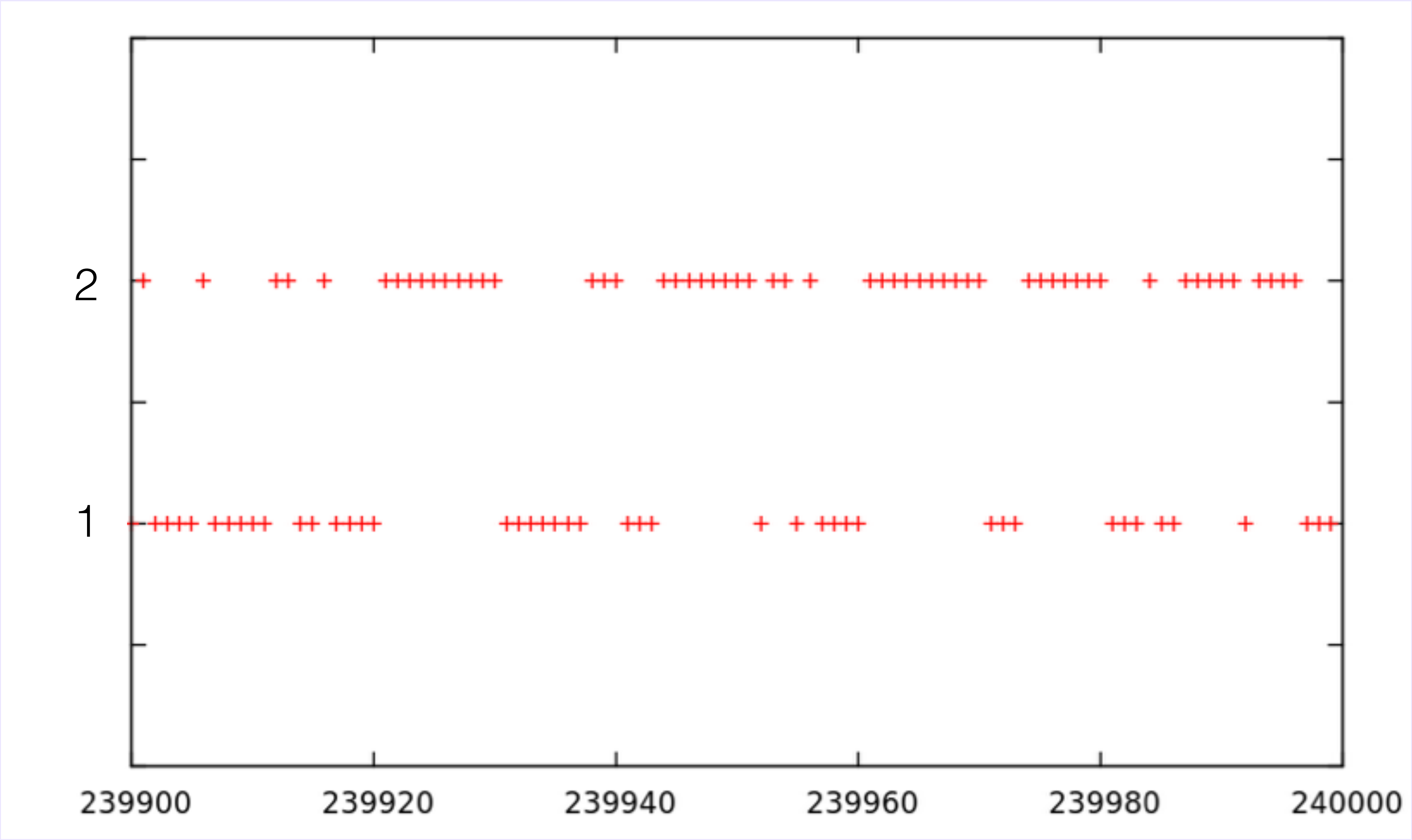
Reversible Jump Markov Chain Monte Carlo



$$\text{Bayes Factor} = \frac{\#\text{ of iterations in model 1}}{\#\text{ of iterations in model 2}}$$

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Errors (with RJMCMC)

$$\text{Bayes Factor} = \frac{\#\text{ of iterations in model 1}}{\#\text{ of iterations in model 2}}$$

For well-mixed chains

$$\text{Var(BF)} = \text{BF}^2 \left(\frac{N_1 - N_{12}}{N_1 N_{12}} + \frac{N_2 - N_{21}}{N_2 N_{21}} \right)$$

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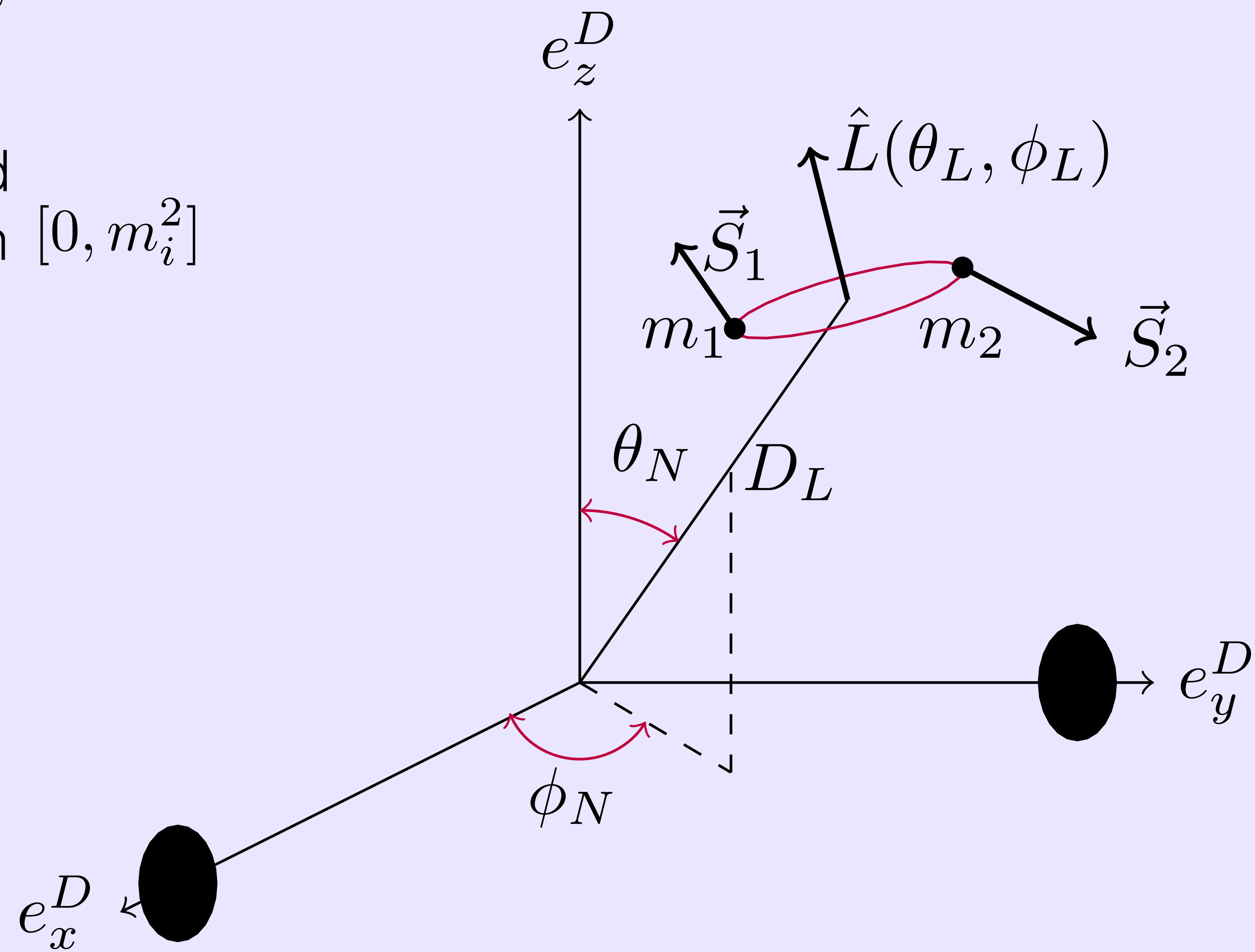
Prior

m_1 m_2	Uniform in $[0.1, 3.2] M_\odot$
$\vec{S}_2$ $\vec{S}_1$	Uniform in direction and magnitude in $[0, m_i^2]$
D_L	Uniform in volume
θ_N ϕ_N	Uniform in the sky
θ_L ϕ_L	Uniform in direction



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Likelihood: the Noise Model

$$p(d|h) = p(d - R[h]) = p(n)$$

Correlated *Gaussian* noise

$$p(n_1 ... n_N) = \frac{1}{\sqrt{\det(2\pi C)}} e^{-\frac{1}{2} n_i C^{-1}_{ij} n_j}$$

Stationary noise

$$C_{f_i f_j} \sim \delta_{ij} S(f_i)$$

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Likelihood:the Noise Model

Easier to evaluate

$$n_i C_{ij}^{-1} n_j = (n|n) \sim \int \frac{\tilde{n}(f)\tilde{n}^*(f)}{S(f)} df$$

Our noise model

$$p(d|\vec{x}) \sim e^{-\frac{(d-h(\vec{x})|d-h(\vec{x}))}{2}}$$

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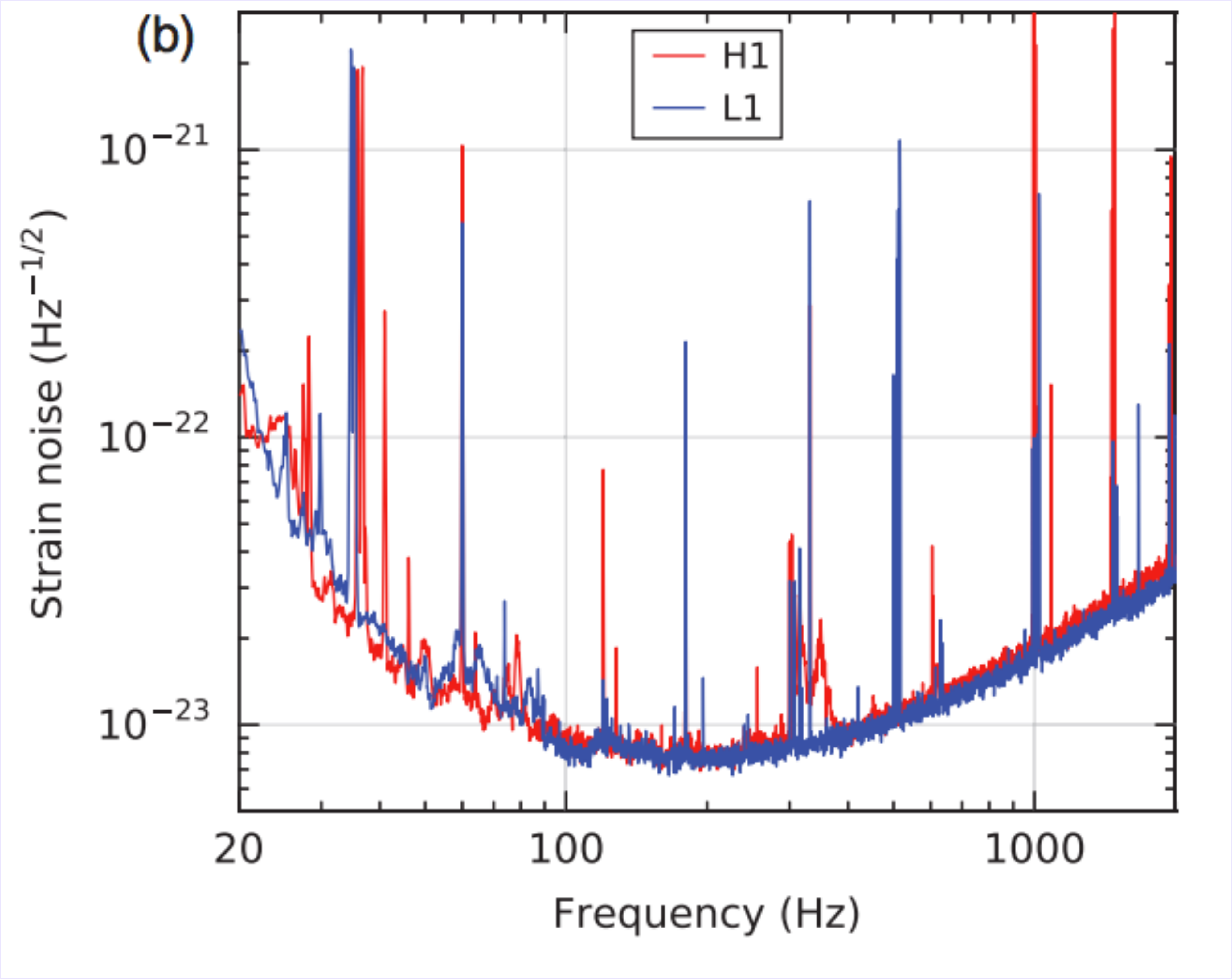
Noise

related to $S(f)$



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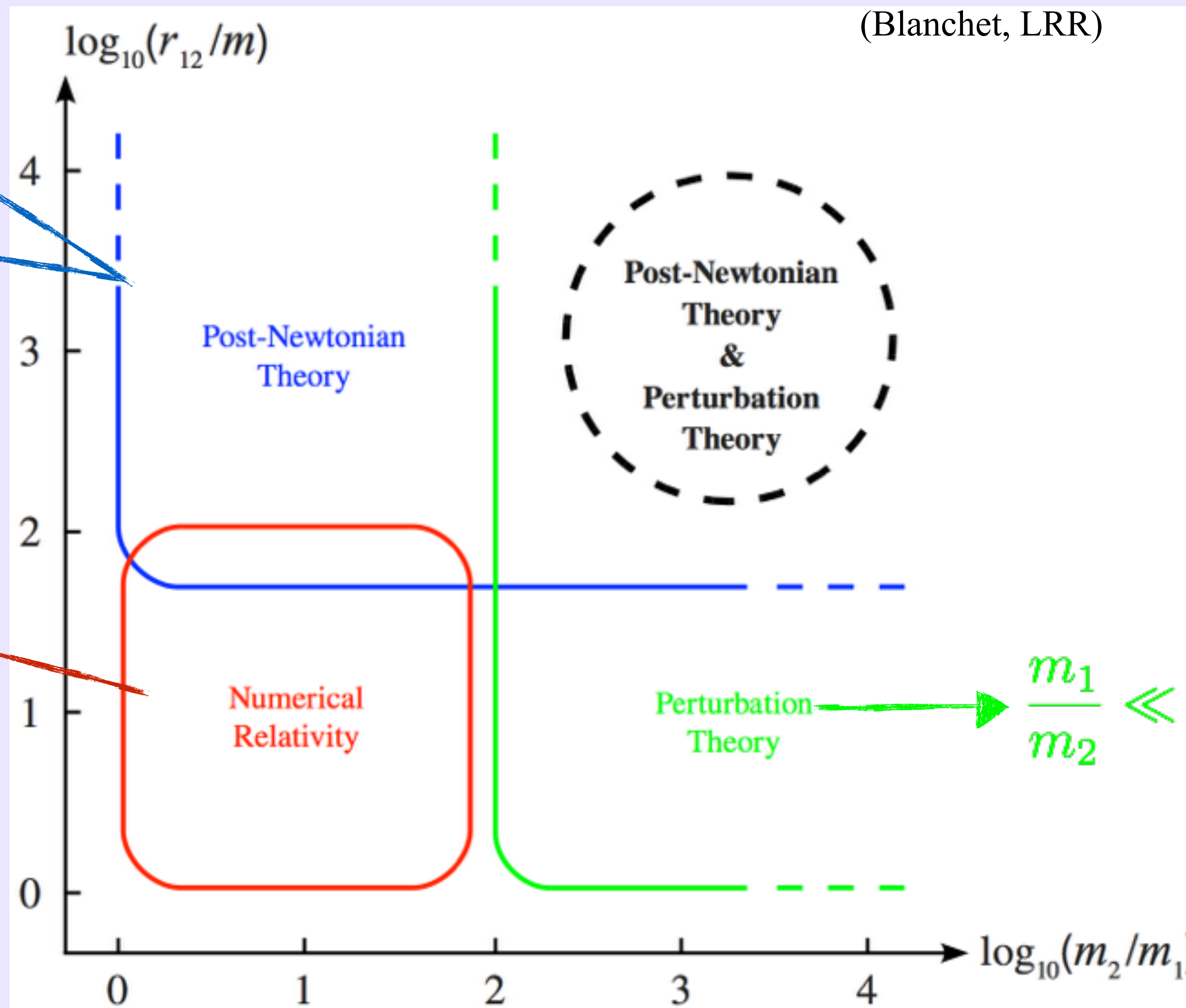
Building Models

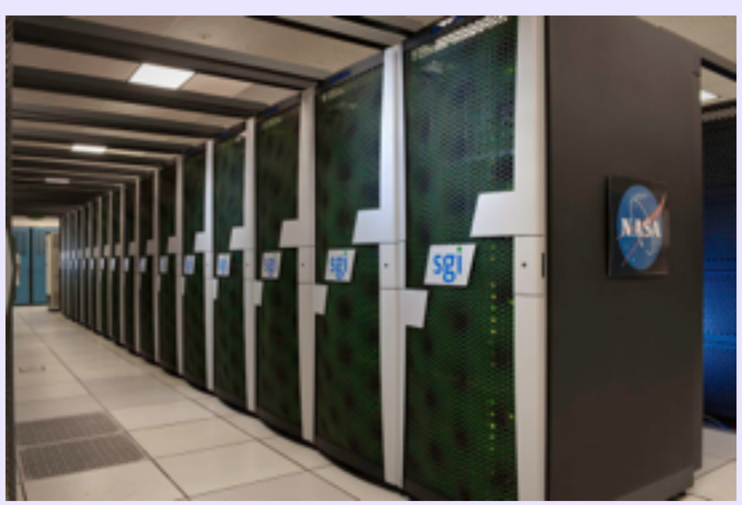
$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\frac{G m_{\text{tot}}}{c^2 r_{12}} \ll 1$$

$$\frac{v}{c} \ll 1$$



$$\frac{m_1}{m_2} \ll 1$$



Models: Inspiral GW

$$\vec{x} = (m_1, m_2, \vec{S}_1, \vec{S}_2, D_L, \theta_N, \phi_N, \theta_L, \phi_L, t_c, \phi_c) + \{\mathrm{EoS}\}$$

- GW described by $\vec{x}$, $(m_1, m_2) \leq M_{\max}(\mathrm{EoS})$
- No GW, otherwise

Reasonably fast to evaluate

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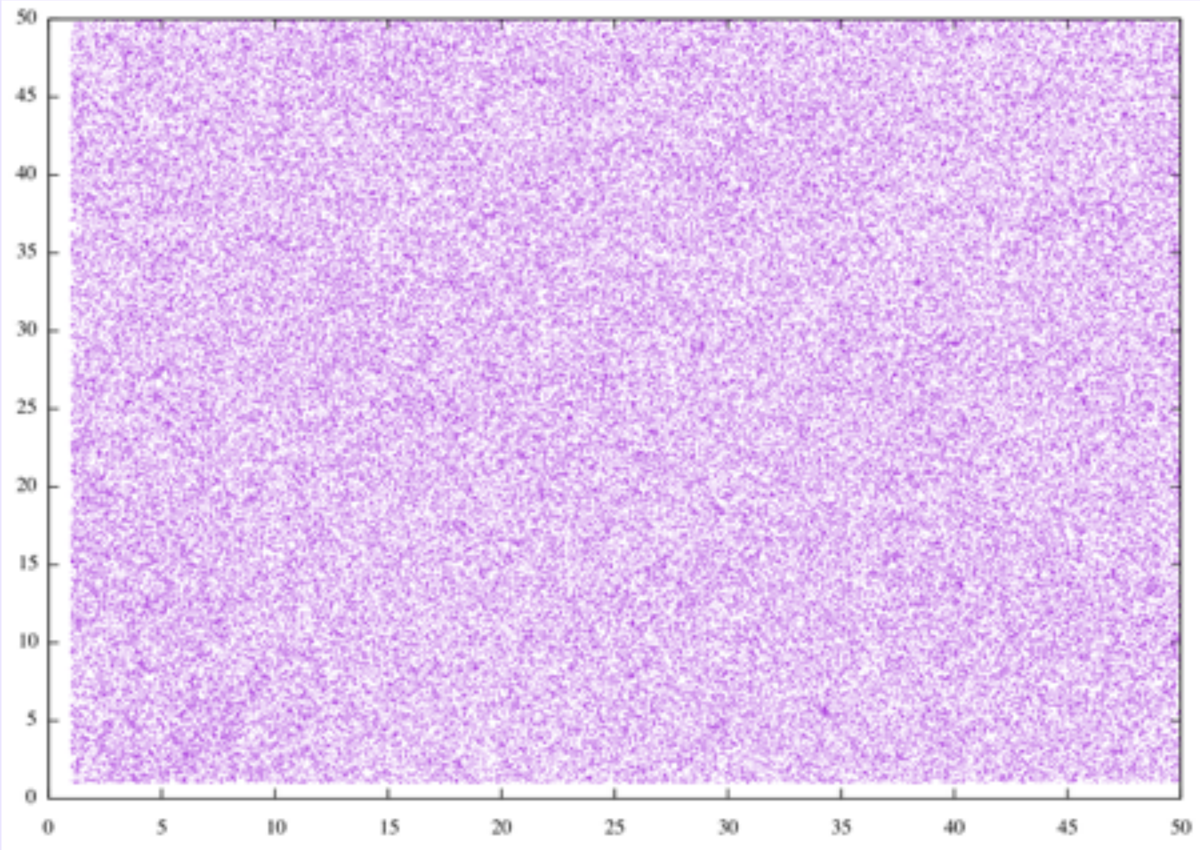
Proposal Distribution

Prior

Provides access to the entire prior volume

(essential to pass the constant likelihood test)



Jiggle

Search around the current position

$$\vec{\theta} + \vec{\epsilon}$$

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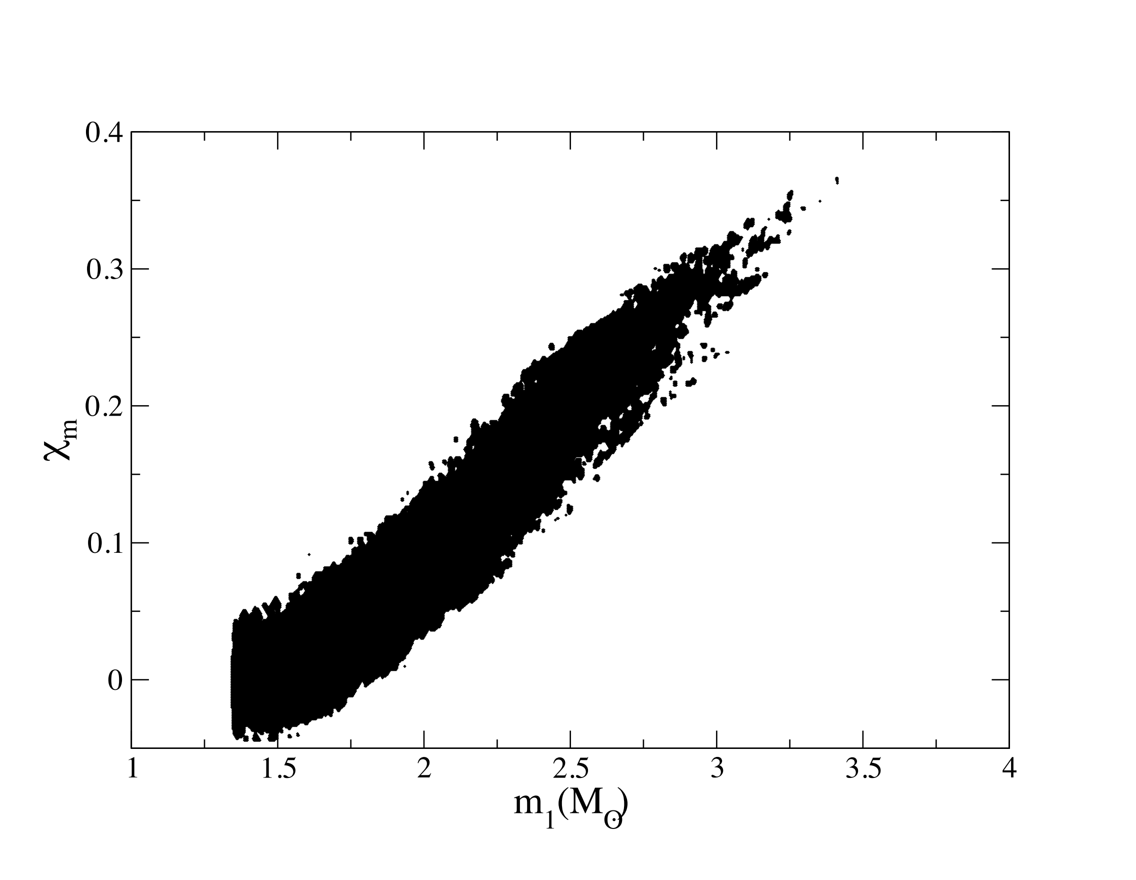
Proposal Distribution

Fisher

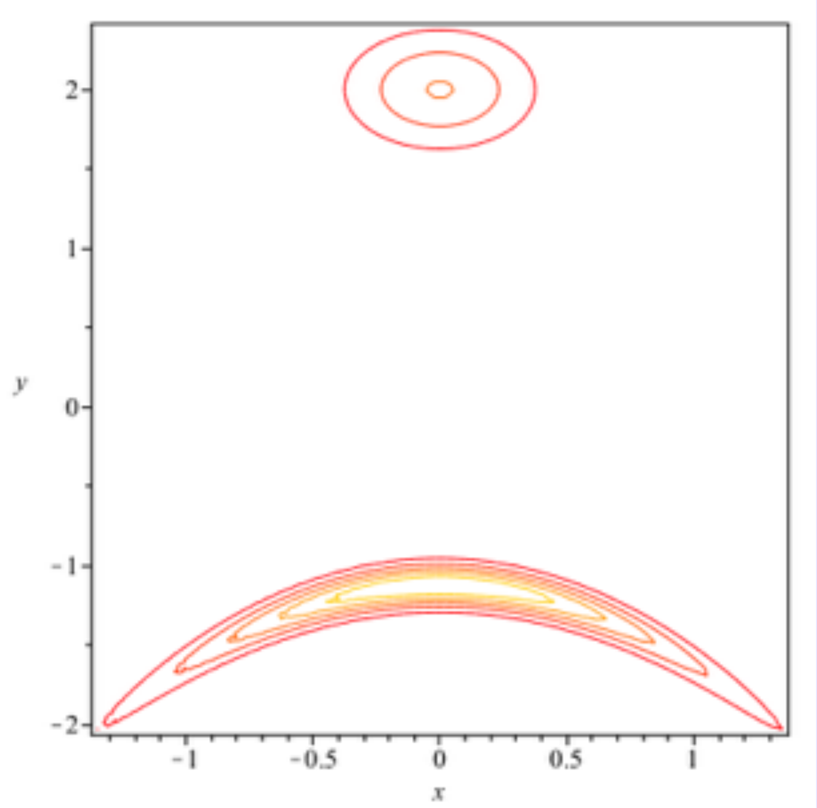
Jump along the eigendirections (scaled by the eigenvalues) of the *Fisher* Information Matrix

$$F_{ij} = (h_{,i}|h_{,j})$$



Langevin

jump along the likelihood gradient



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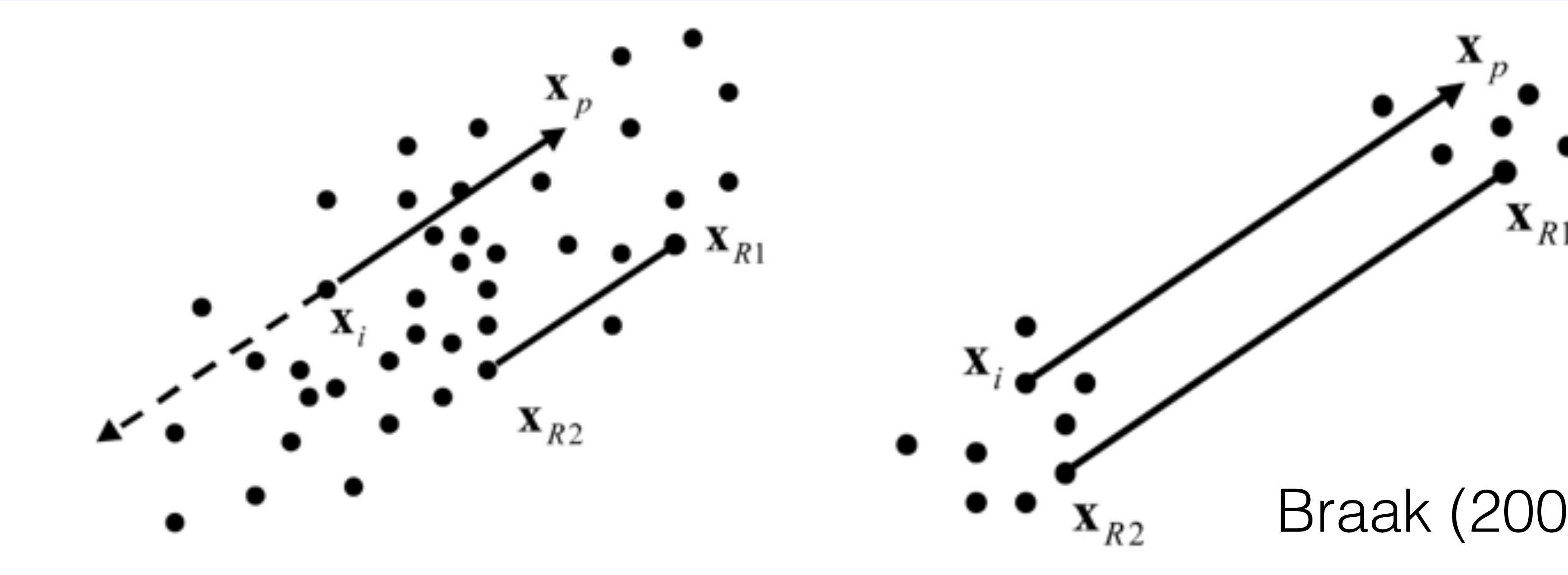
Proposal Distribution

Differential Evolution

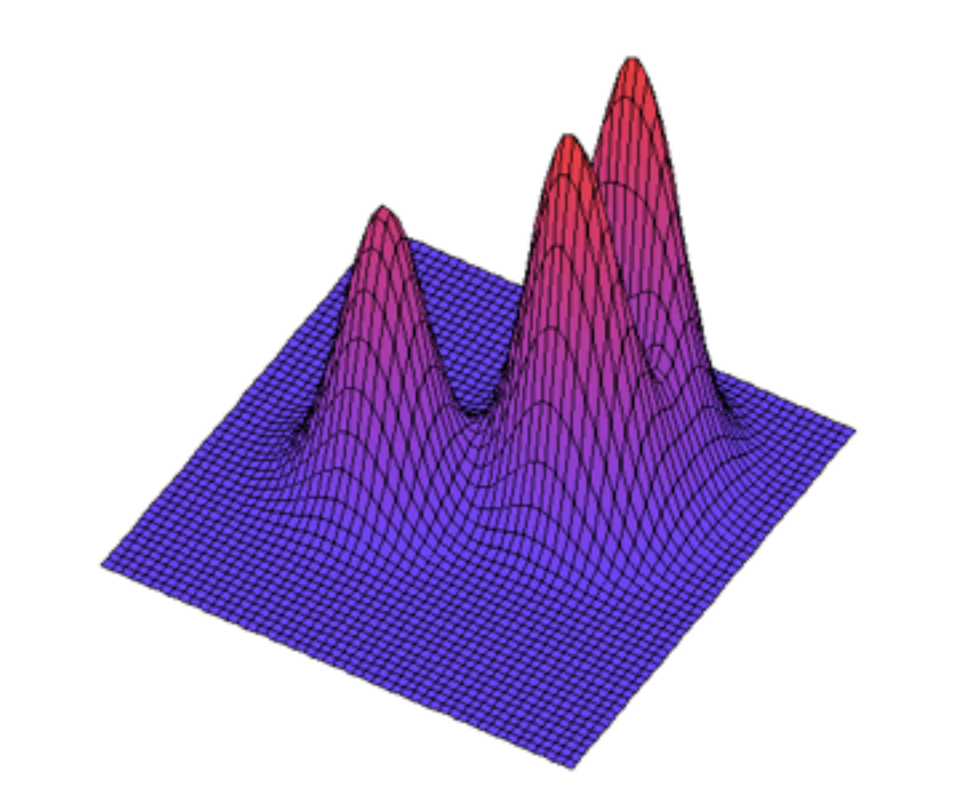
(technically it is not memoryless)



Braak (2005)

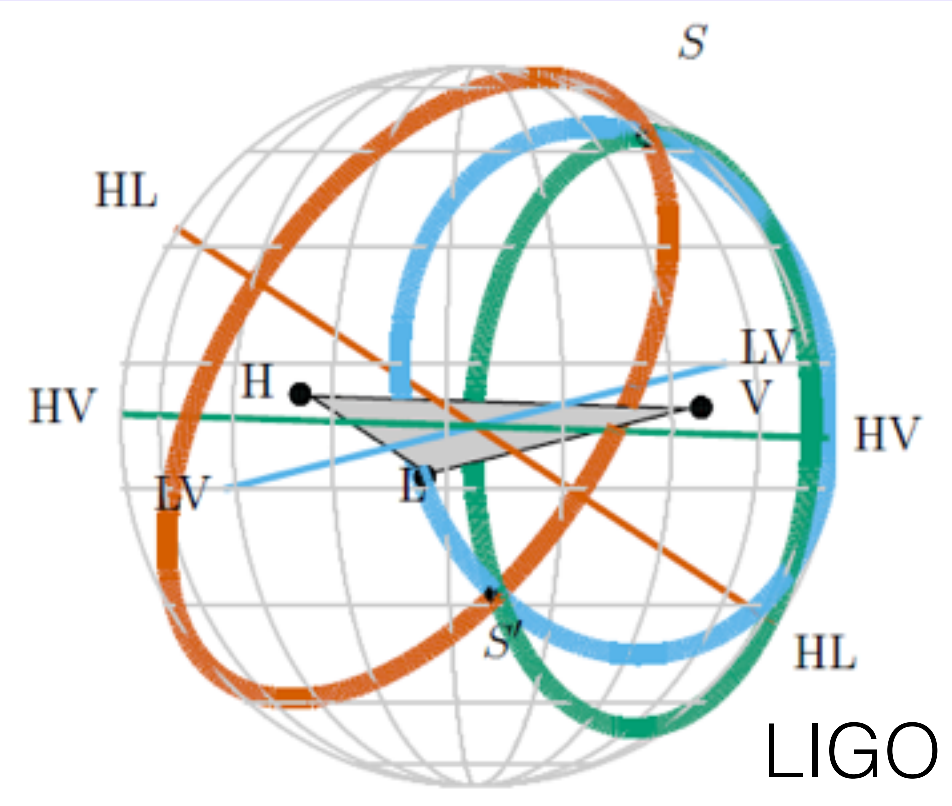
Model jumps

Pilot runs

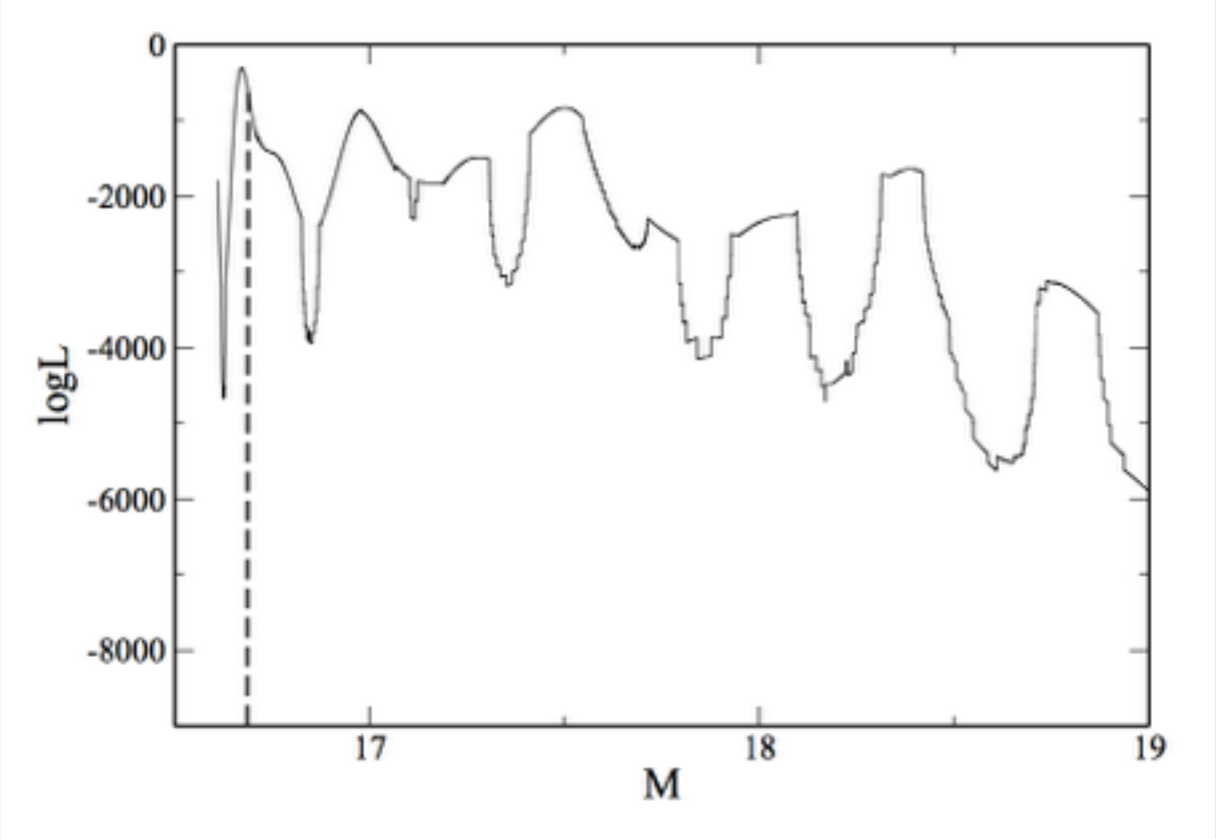


Proposal Distribution

Sky jumps



Customized



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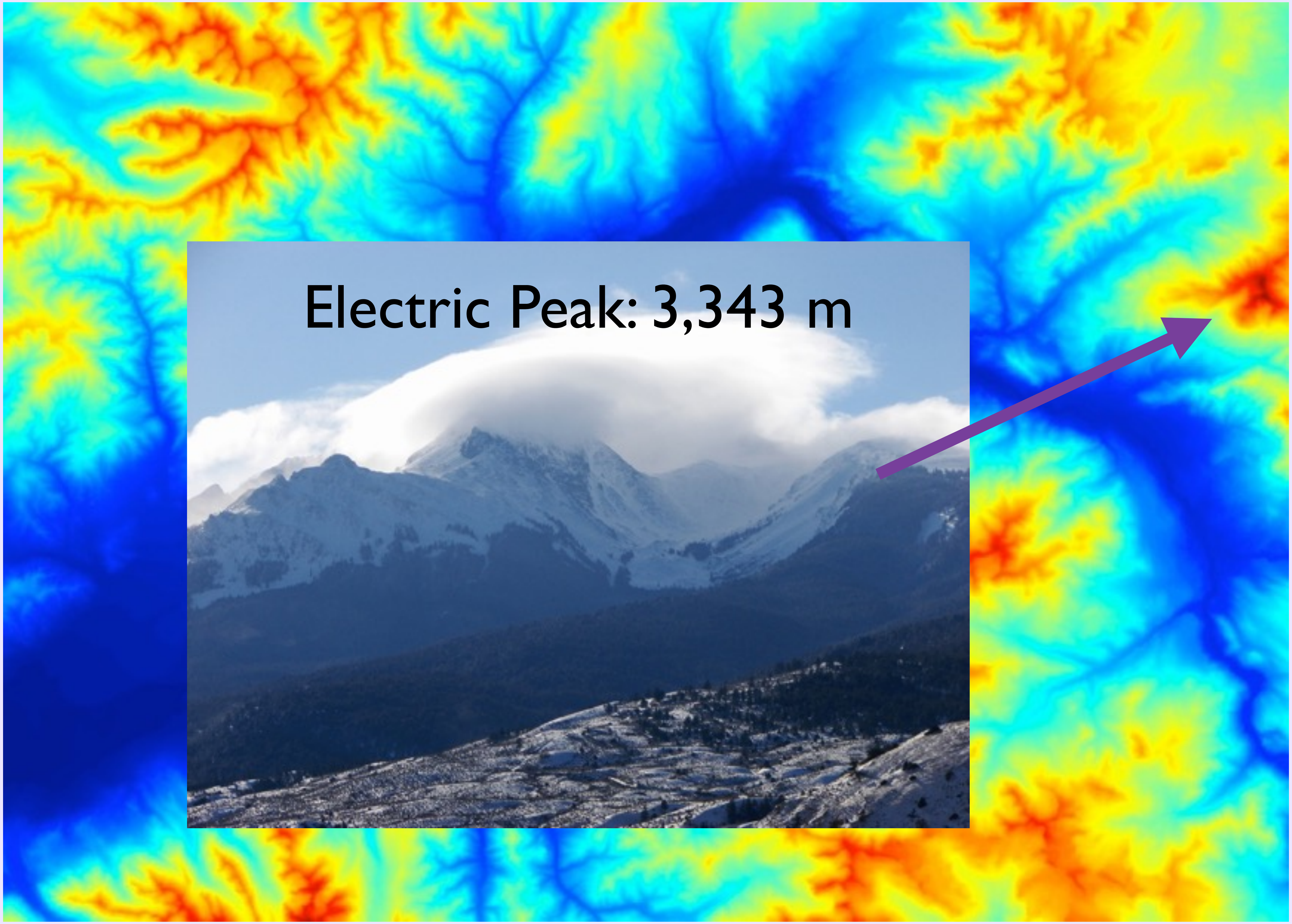
Finding the Highest Peak in Gallatin Range



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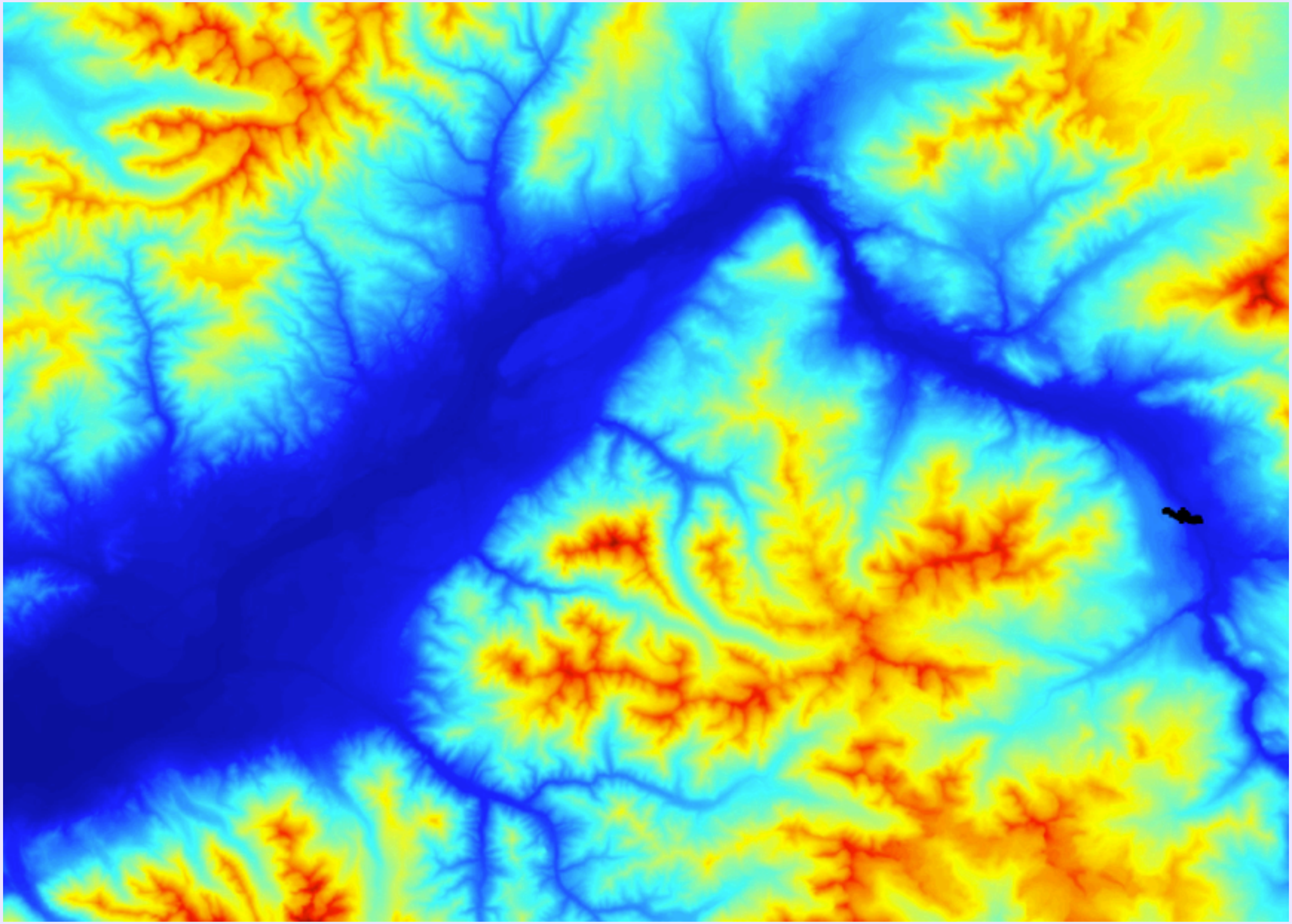
MCMC



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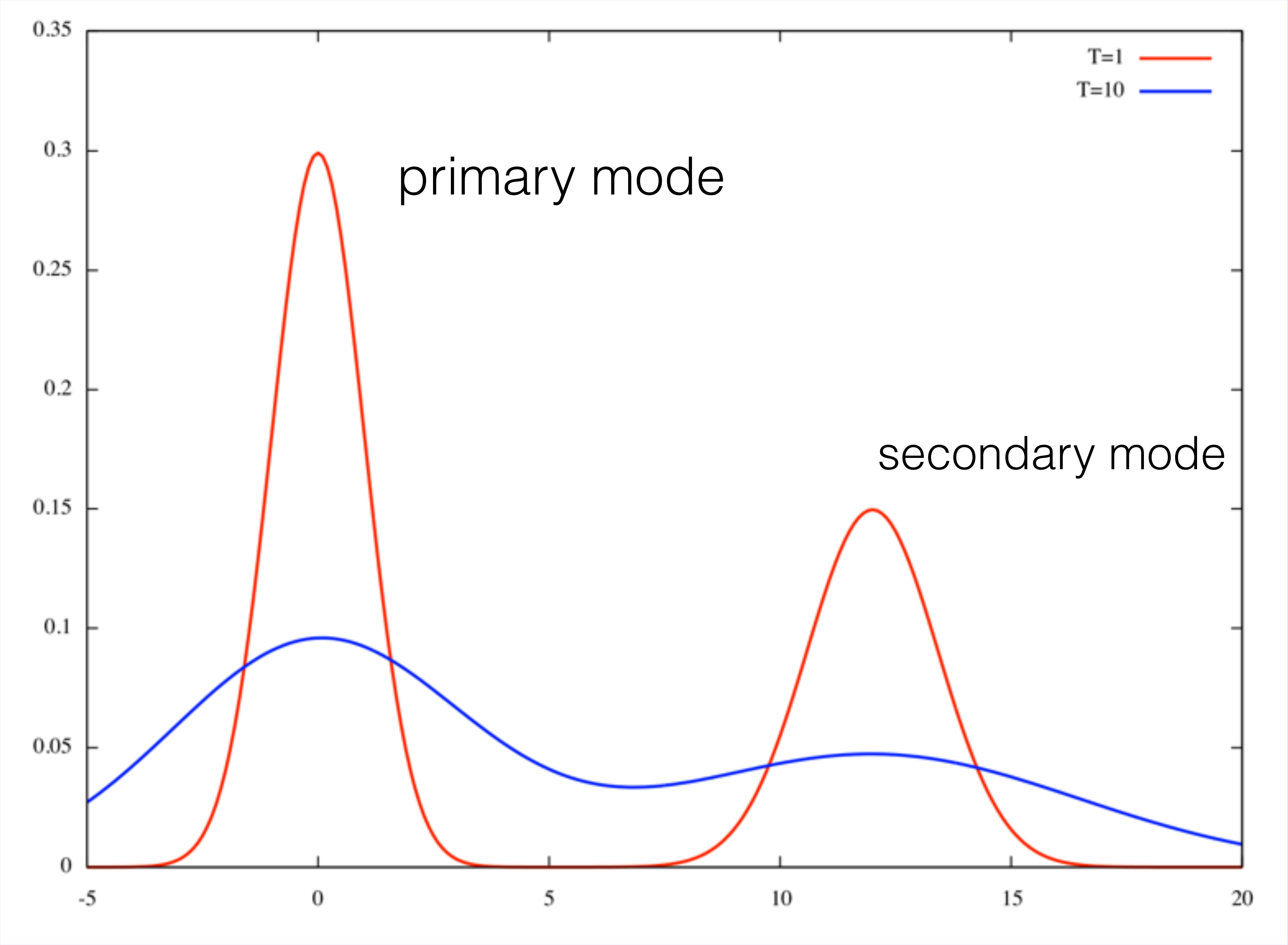
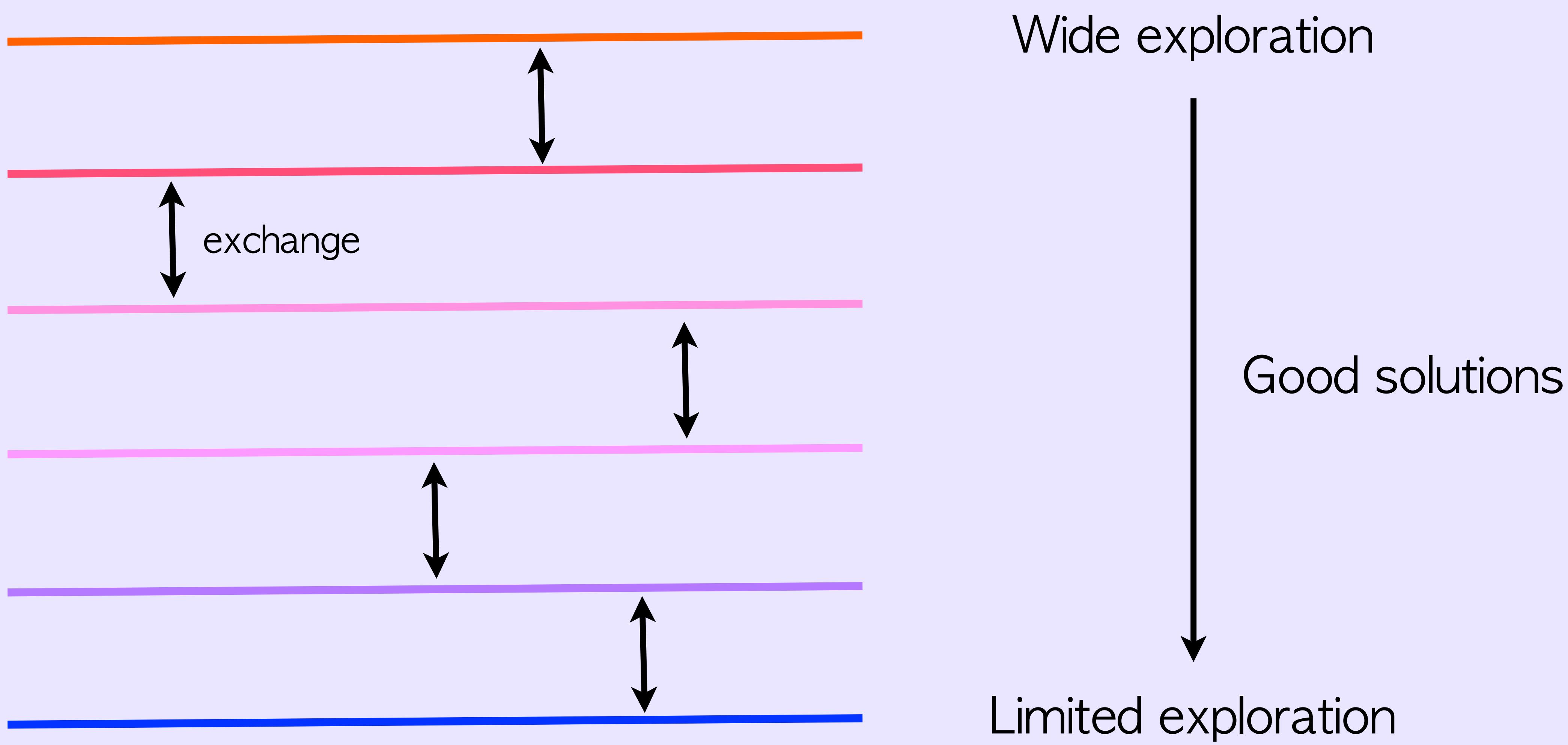
Multi-Modal Distributions



$$p(d|\vec{x}) \to p(d|\vec{x})^{1/T}$$

Exchanges



Kirkpatrick, Gelatt, Vecchi (1983)
Swendsen, Wang (1986)

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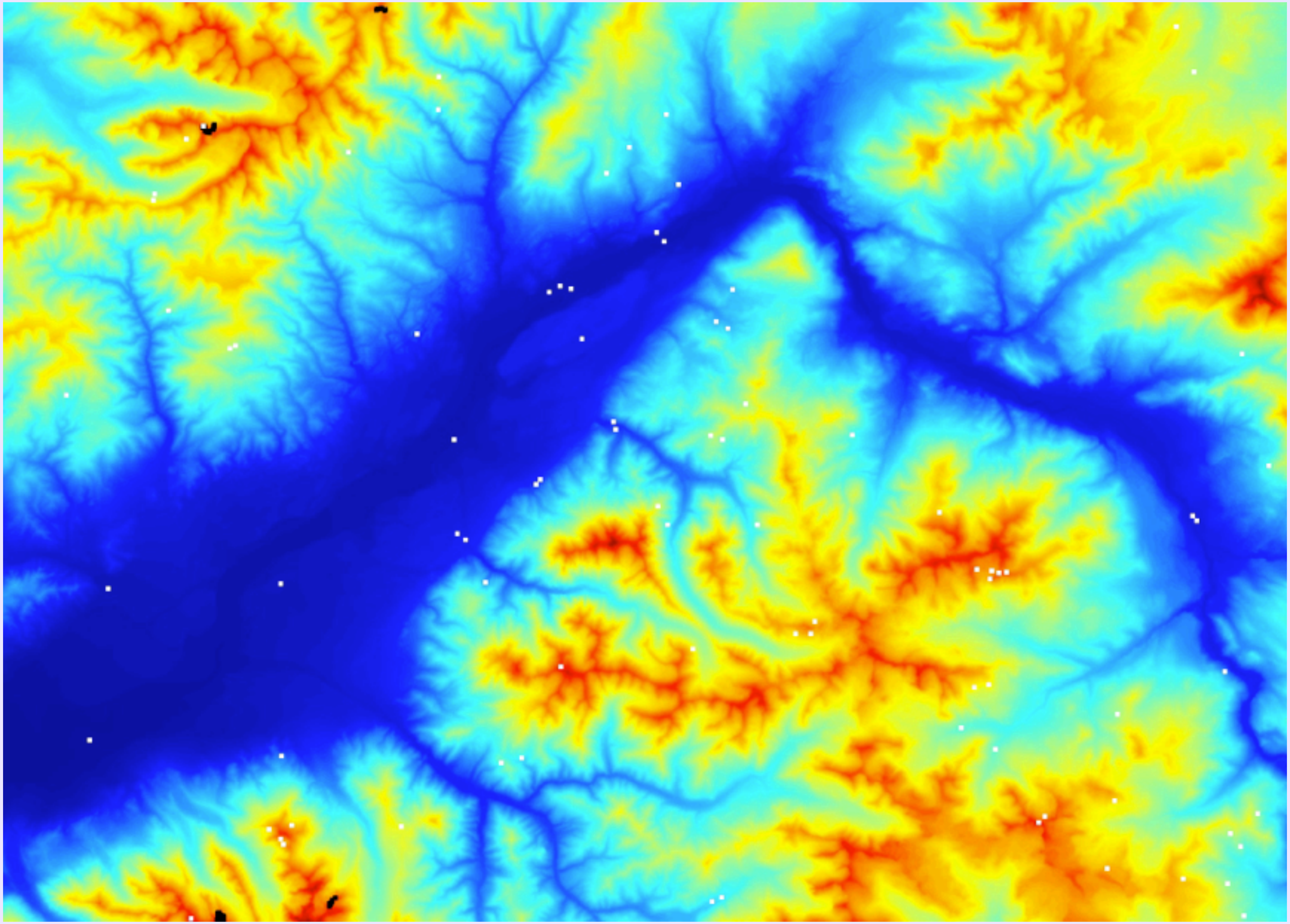
Parallel Tempering



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Cornish

Our Analysis

EoS	Method/Model	Composition
AP4	variational	n, p, e, μ
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ALF4, ALF5	variational	n, p, e, μ, π, Q
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SQM3	MIT bag	Q (u, d, s)

If an EoS **with kaons** fits the data better than an otherwise identical EoS **without kaons**, then we have **detected** kaons in a NS interior

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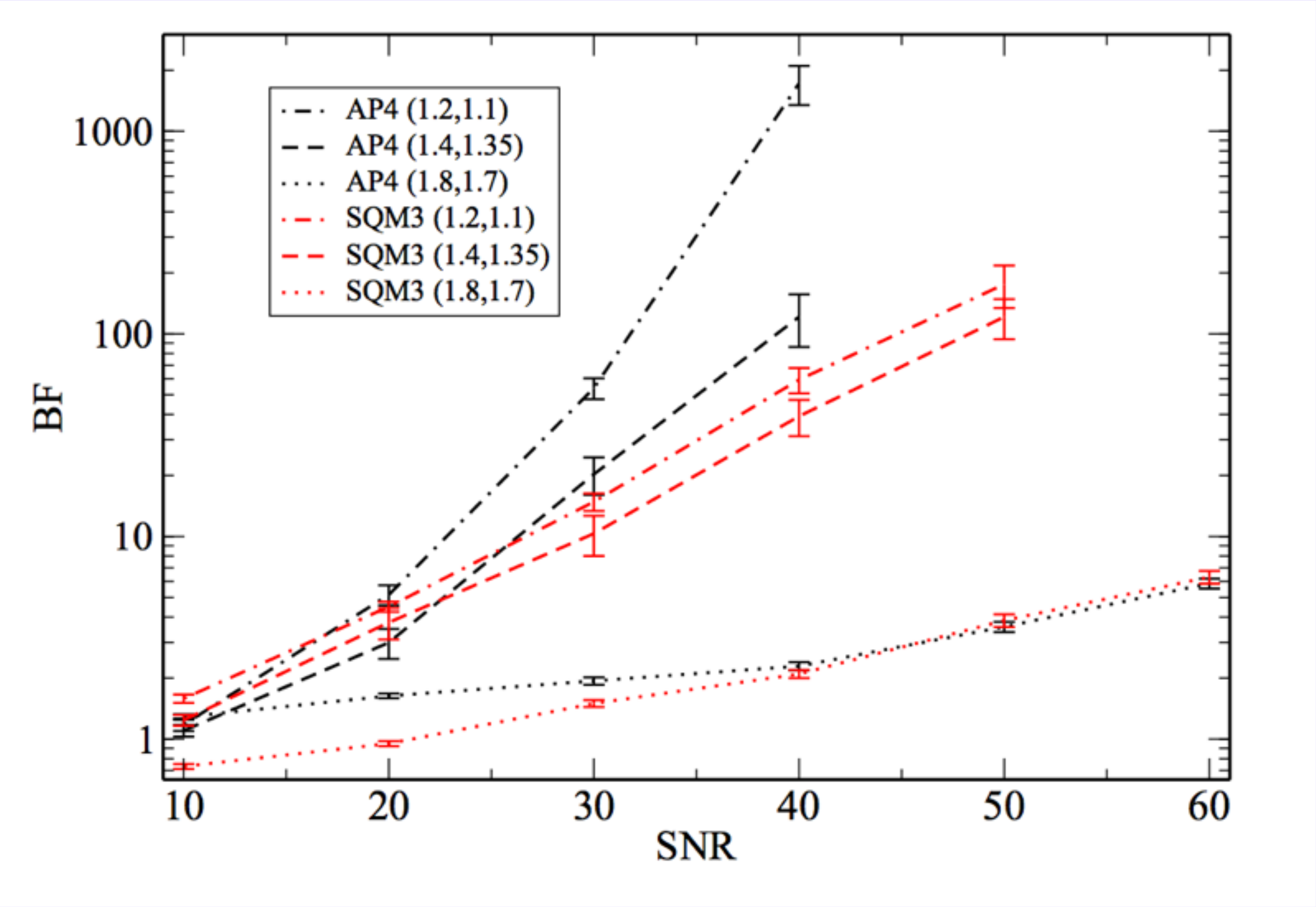
Bayes Factors



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Results

	Strange Quark Stars	Hybrid Quark Stars	Kaons	Hyperons
aLIGO	**Yes!**	**Maybe**	**Unlikely**	**Unlikely**
SNR	**20**	**30-40**	**50-60**	**50-60**
mass	$(1.2, 1.5)M_\odot$	$1.4M_\odot$	$2M_\odot$	$2M_\odot$

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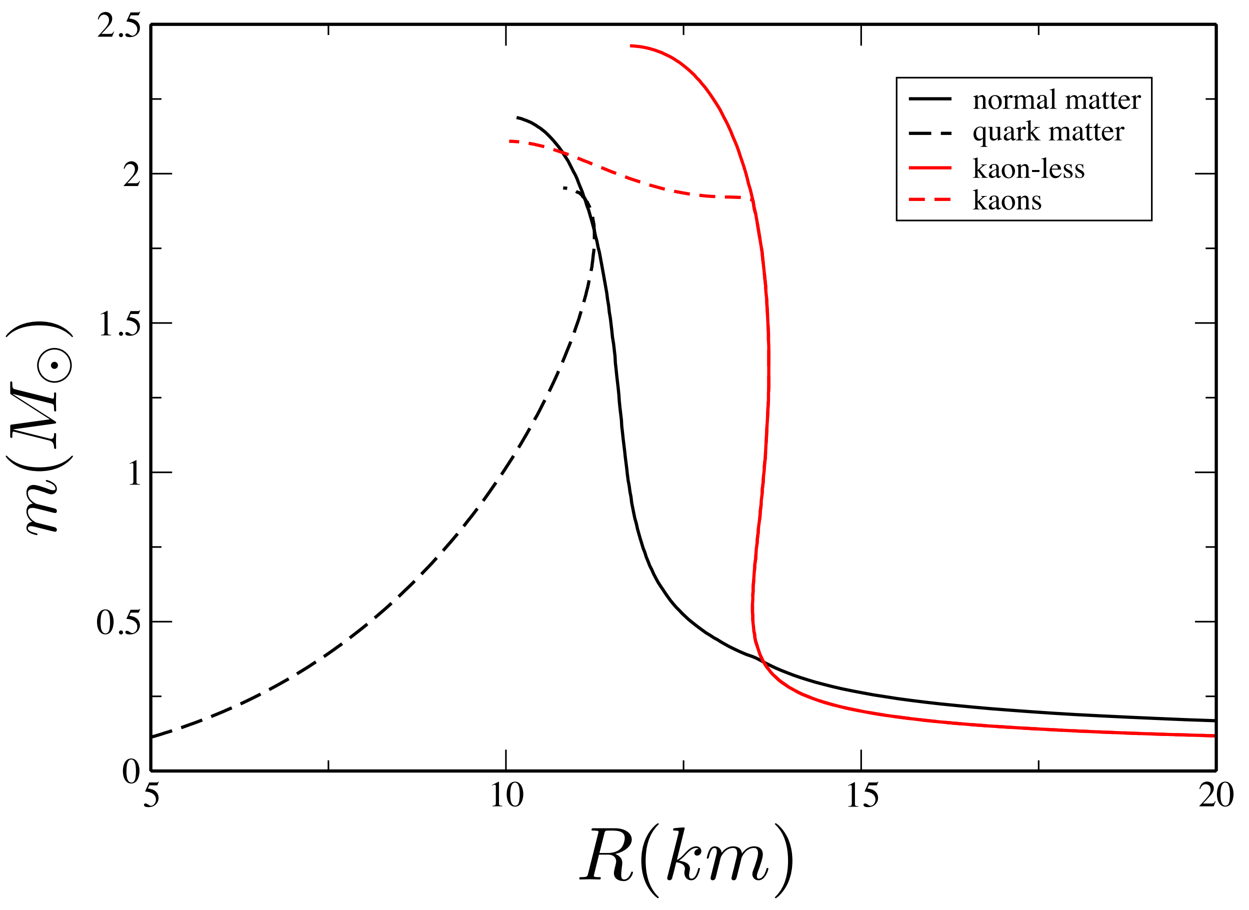
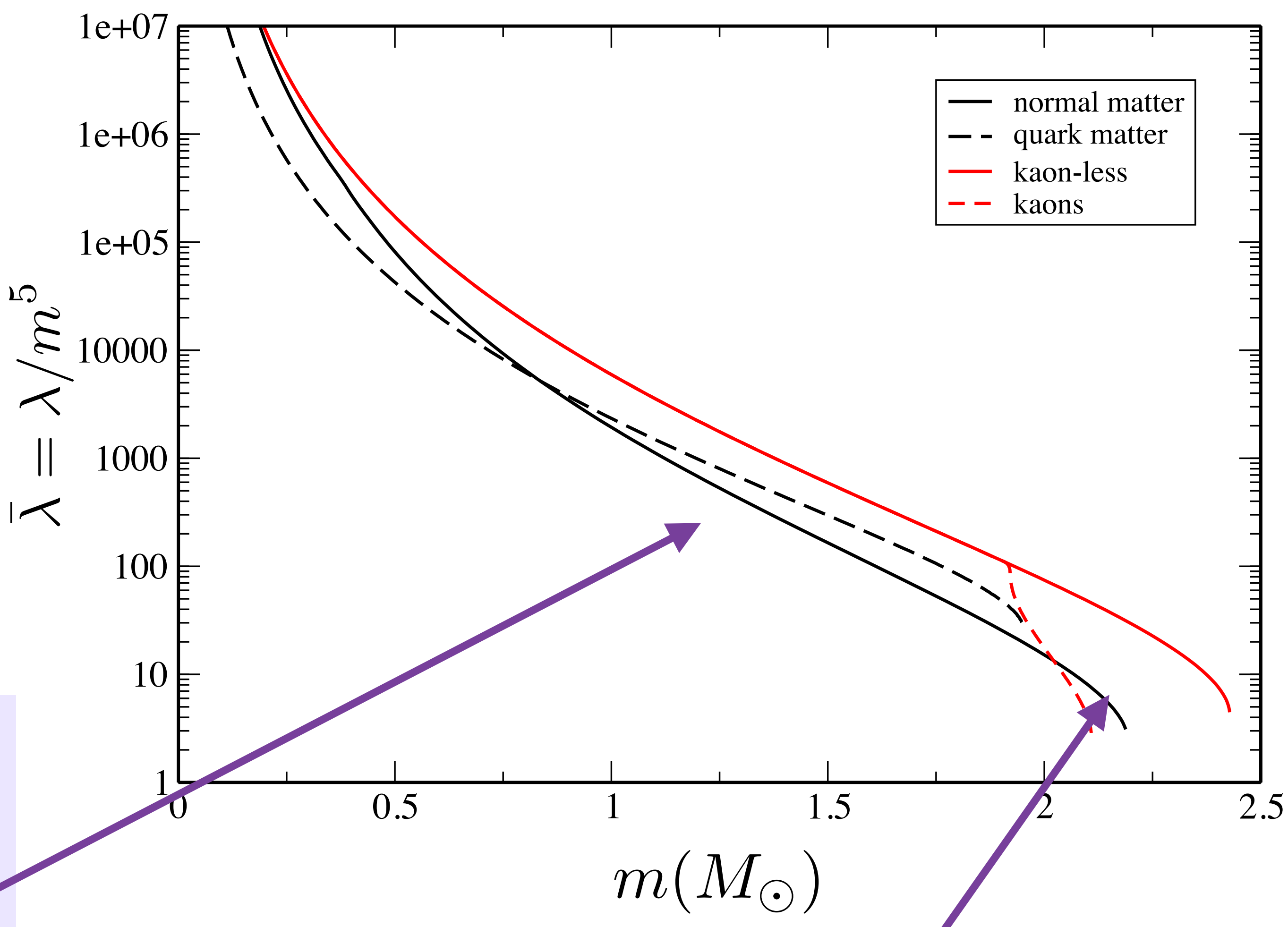
Mass Matters





Directly measure low mass effects

Partial information on high mass effects

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Errors

Systematic Errors: our models might be wrong

- General Relativity might be wrong
- Perturbative models not accurate enough
- Models not accurate astrophysically
- Unkown noise contribution
- Detector Calibration

Statistical Errors: finite signal strength

- Width of the Posterior
- Noise Realization
- Marginalization

Further work

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- Further meaningful comparisons
- Inspiral phase: improve modeling
- Merger phase: modeling

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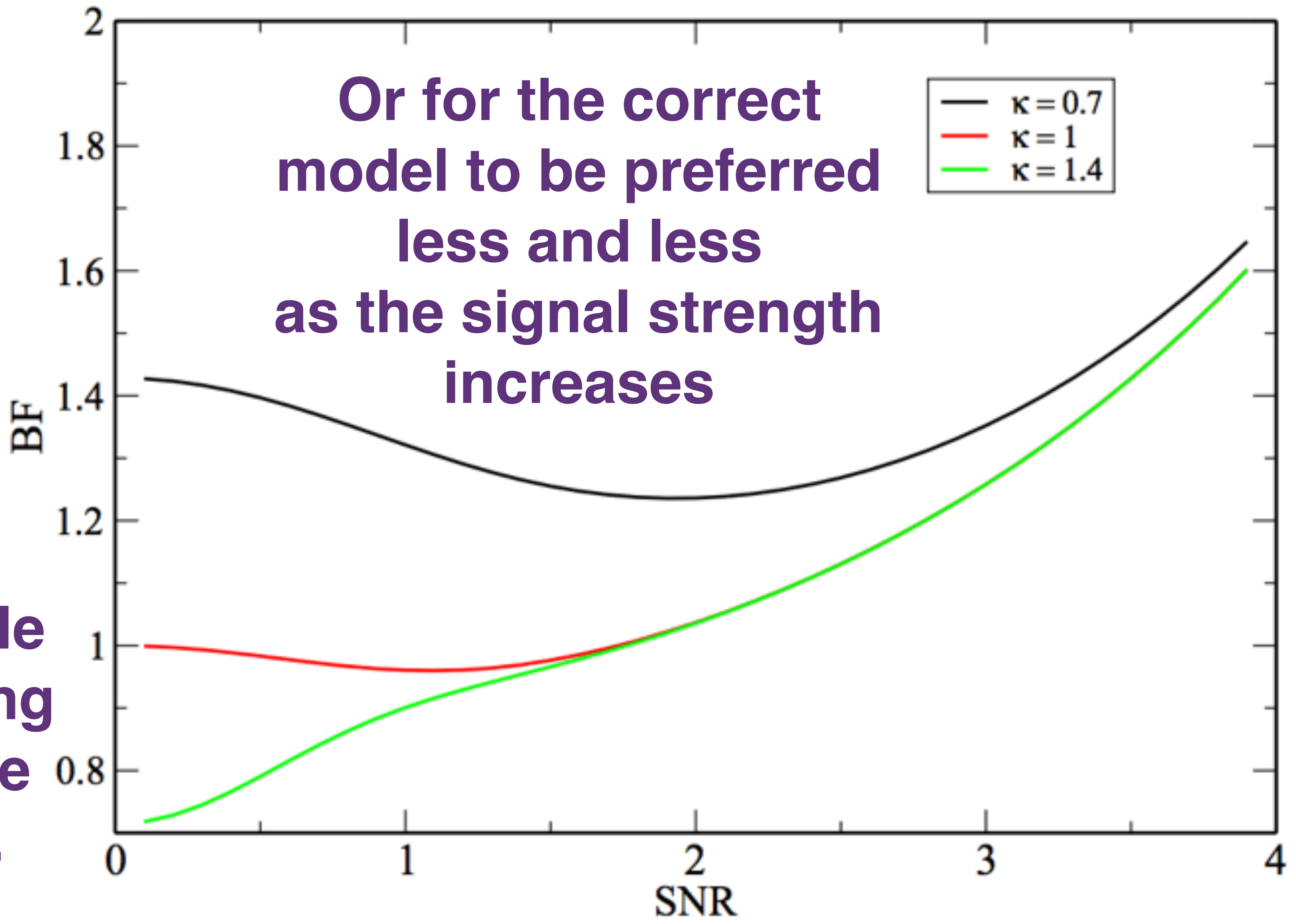
- Efficient trans-model jumps
- Exploration of disfavored models
- Thermodynamic Integration
- Merger phase: unmodeled search

Thank you!

The Edge of the Prior



Or for the correct model to be preferred less and less as the signal strength increases

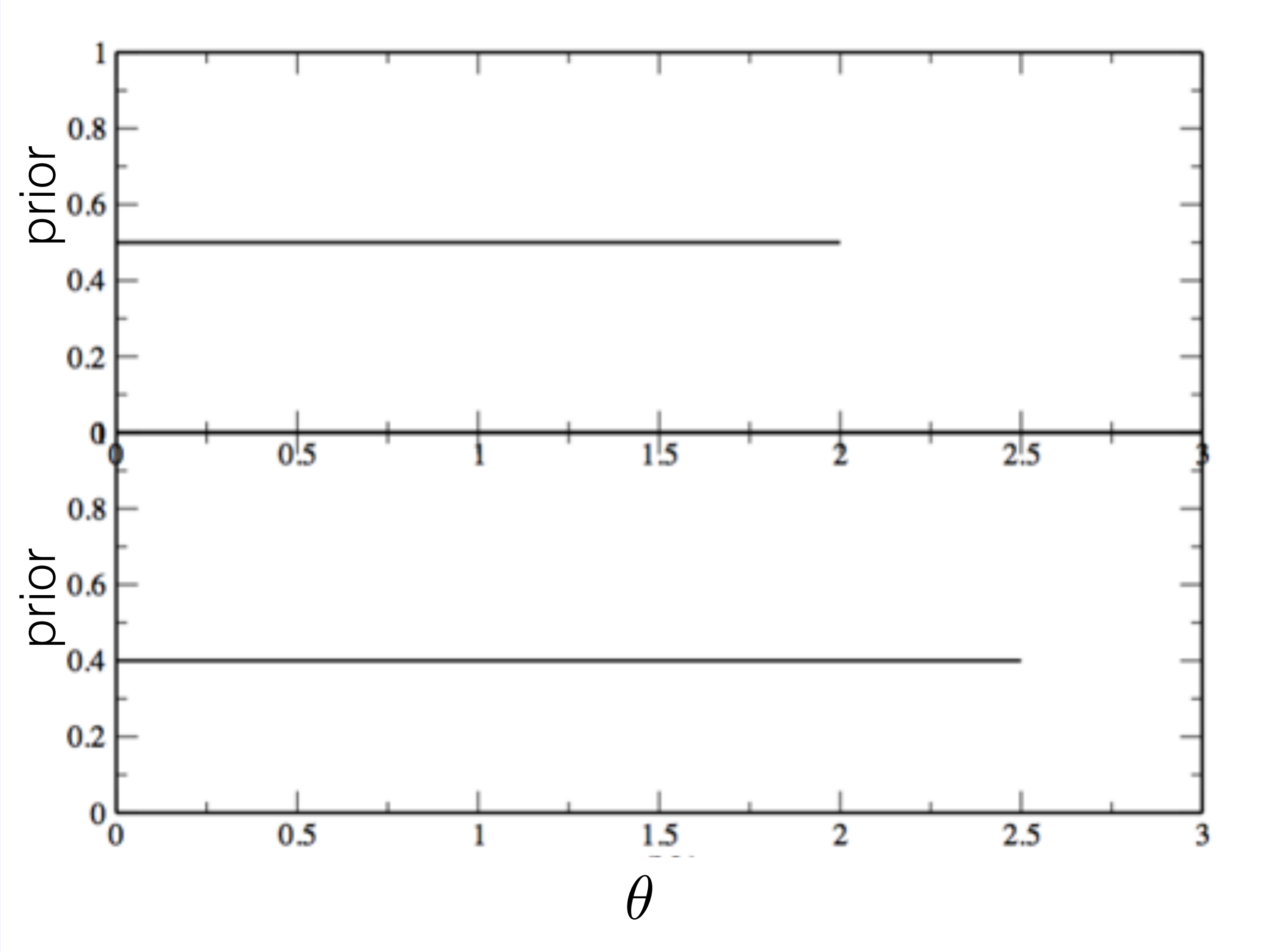
It is possible for the wrong model to be preferred.

Occam Penalty

A model that requires more parameters to fit the data is penalized

$$\frac{\delta\theta}{\Delta\theta}$$

But what if it's the denominator that changes between the various models?



Toy Model

We get N data from a signal $d(f) = f$

Two competing models

$h_1 = af \qquad a \in (0, 2)$

$h_2 = af^{1.5} \qquad a \in (0, 2\kappa)$

Likelihood

$$L_i = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ - \sum^{N} \frac{[d(f) - h_i(f)]^2}{2\sigma^2} \right\}$$

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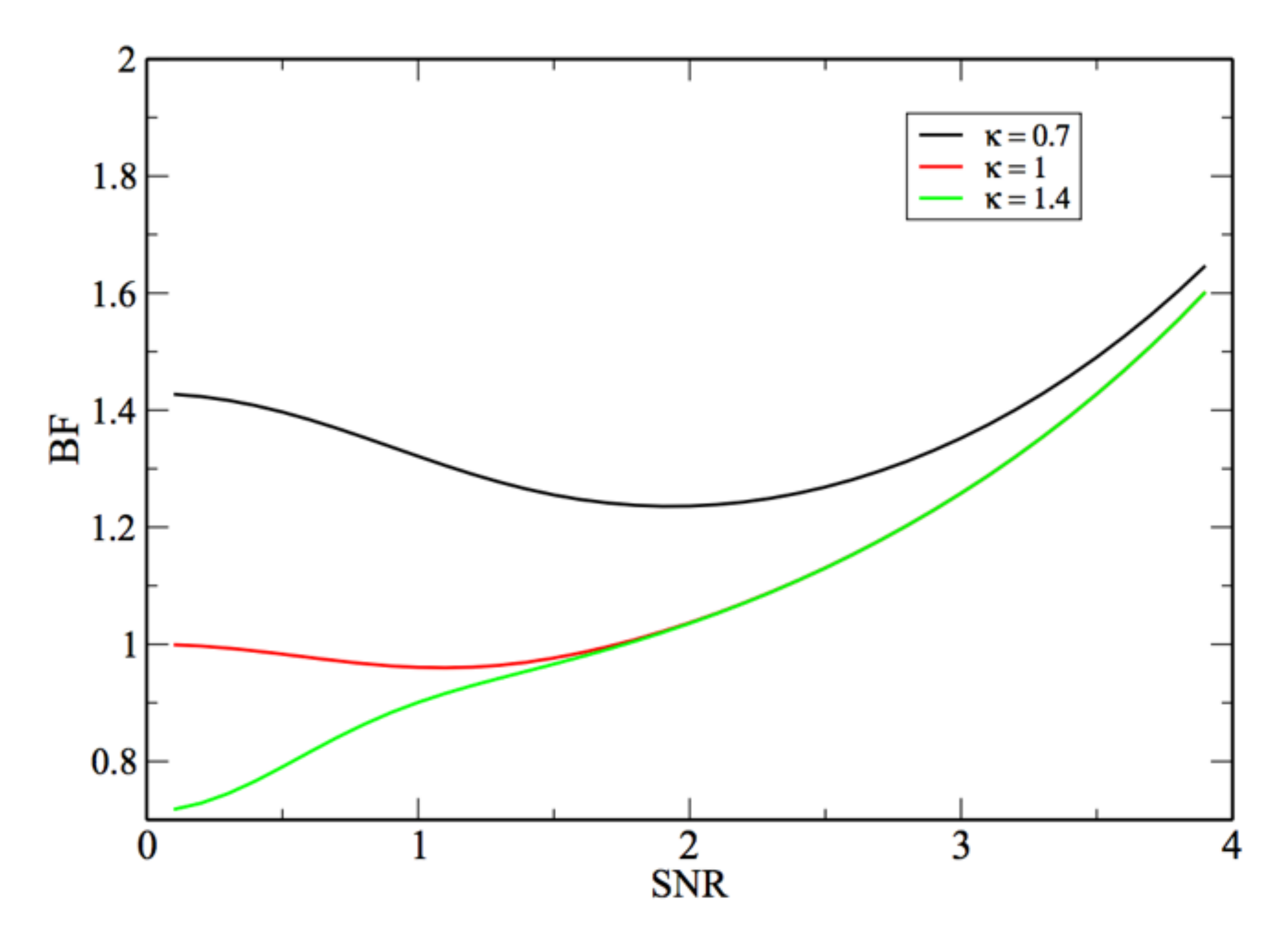
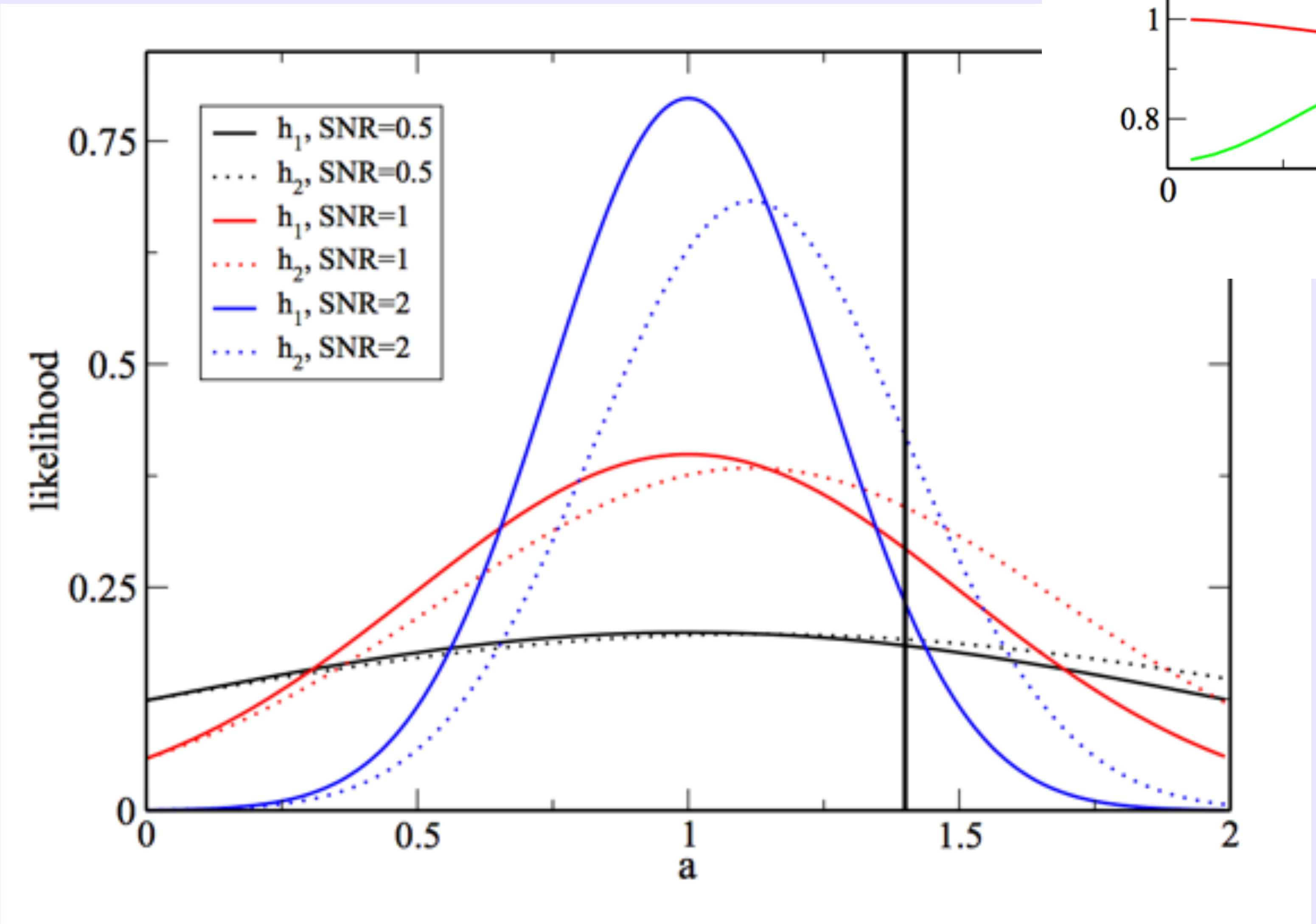
Same Dimensionality, Different Prior Volume

$$h_1 = af \qquad a \in (0, 2)$$

$$h_2 = af^{1.5} \qquad a \in (0, 2\kappa)$$





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