Precision extraction of QGP properties with quantified uncertainties

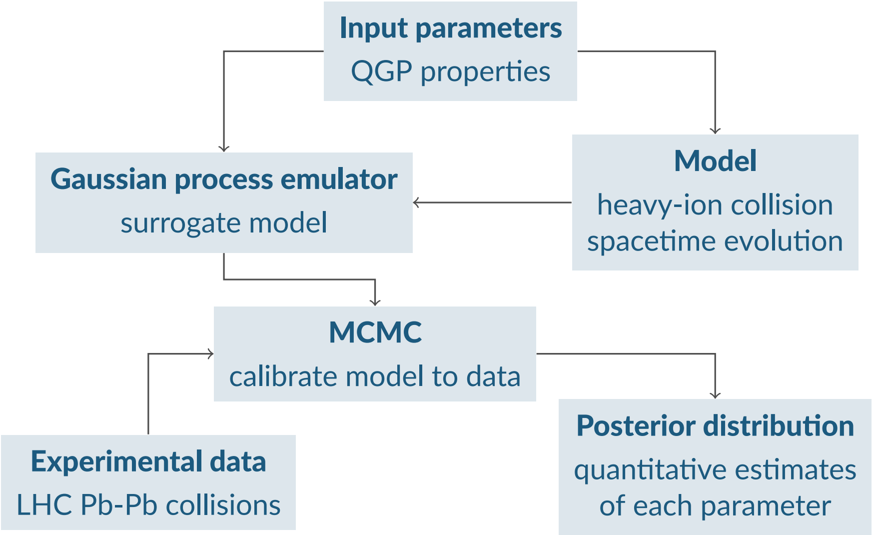
Part II: methodology and results

Jonah E. Bernhard, Steffen A. Bass

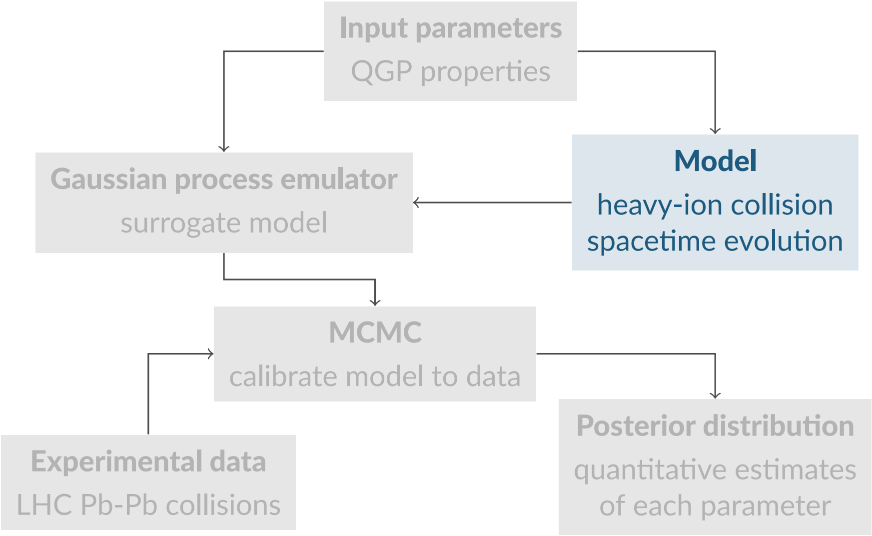
INT workshop: Bayesian methods in nuclear physics
Wednesday, June 15, 2016

arXiv:1605.03954 [nucl-th]

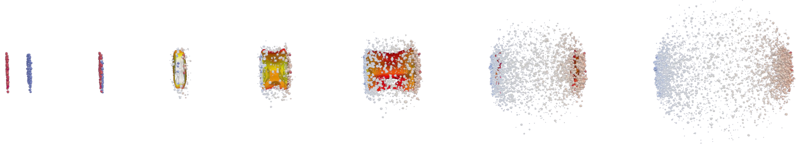
Overview


Input parameters
QGP properties
Gaussian process emulator
surrogate model
Model
heavy-ion collision
spacetime evolution
MCMC
calibrate model to data
Experimental data
LHC Pb-Pb collisions
Posterior distribution
quantitative estimates
of each parameter

Overview


Input parameters
QGP properties
Gaussian process emulator
surrogate model
Model
heavy-ion collision
spacetime evolution
MCMC
calibrate model to data
Experimental data
LHC Pb-Pb collisions
Posterior distribution
quantitative estimates
of each parameter

Heavy-ion collision models



1. Initial conditions $t = 0^+$
   - Entropy deposition
2. (Pre-equilibrium) $t < 1$ fm/c
   - Early-time dynamics and thermalization
3. Hydrodynamics $1 < t < 10$ fm/c
   - Hot and dense quark-gluon plasma
4. Hadronic phase $10 < t < 100$ fm/c
   - Expanding and cooling gas

Initial condition models

Provide initial entropy density
for hydrodynamics

↓

Many different theoretical and
phenomenological approaches

↓

Affects estimates of
QGP properties!

Initial condition models

Provide initial entropy density for hydrodynamics

↓

Many different theoretical and phenomenological approaches

↓

Affects estimates of QGP properties!

Alternative: parametric models

↓

Mimic theory calculations

↓

Simultaneously characterize initial conditions and QGP medium

T$_R$ENTo: parametric IC model

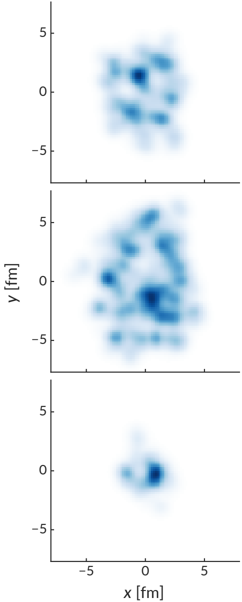
Ansatz

Entropy density proportional to
generalized mean of local nuclear density

$$s \propto \left(\frac{T_A^p + T_B^p}{2} \right)^{1/p}$$

$p \in (-\infty, \infty)$ = tunable parameter

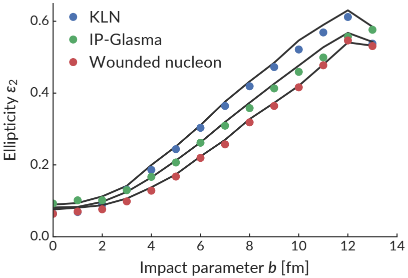
$p = +1$	$p = 0$	$p = -1$
$\frac{T_A + T_B}{2}$	$\sqrt{T_A T_B}$	$\frac{2 T_A T_B}{T_A + T_B}$



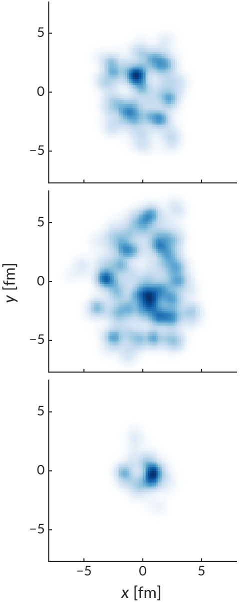
T_RENTo: parametric IC model

$$s \propto \left(\frac{T_A^p + T_B^p}{2} \right)^{1/p}$$

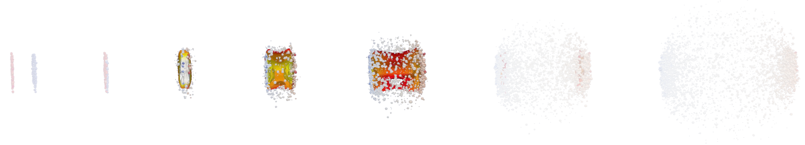
Compare to geometry of other models
Lines = T$_R$ENTo with different p values



Mimics and interpolates other models!



Hydrodynamics



Viscous relativistic hydrodynamics

Energy and momentum conservation + dissipative corrections
Equation of state from lattice QCD (HotQCD collaboration)

Transport coefficients:

- Shear viscosity (linear increase in QGP phase)

$$(\eta/s)(T) = (\eta/s)_{\text{min}} + (\eta/s)_{\text{slope}}(T - T_c), \quad T_c = 154\text{ MeV}$$

- Bulk viscosity (peak near 180 MeV, exponential decrease)

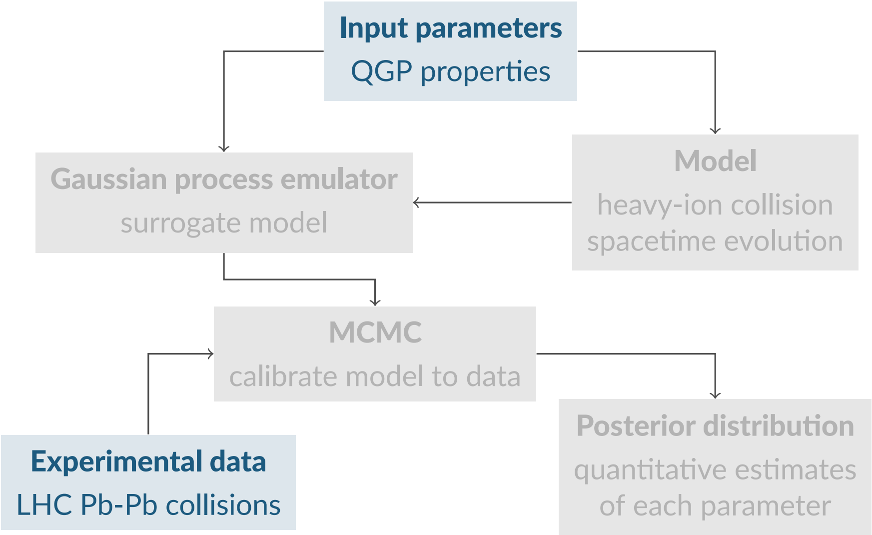
$$(\zeta/s)(T) = (\zeta/s)_{\text{norm}} \times f(T)$$

Hadronic phase

Ultra-relativistic quantum molecular dynamics (UrQMD)

- Switch from hydrodynamics to particles at T_{switch}
  - Temperature window where both models are valid?
- Solves Boltzmann equation with Monte Carlo methods
- Simulates scatterings and decays
- Non-equilibrium breakup and freeze-out

Overview


Input parameters
QGP properties
Gaussian process emulator
surrogate model
Model
heavy-ion collision
spacetime evolution
MCMC
calibrate model to data
Experimental data
LHC Pb-Pb collisions
Posterior distribution
quantitative estimates
of each parameter

Input parameters

Initial condition parameters

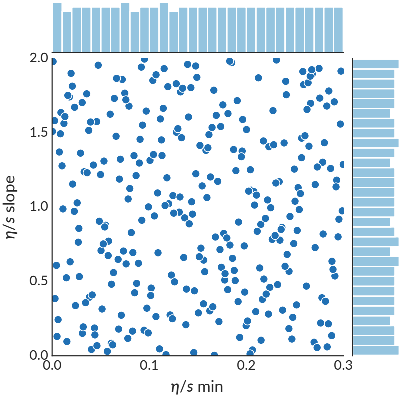
- Normalization factor
- Entropy deposition p
- Gaussian nucleon width w
- Multiplicity fluctuation k

QGP medium parameters

- η/s min and slope
- ζ/s norm
- Hydro $\rightarrow$ particles T_{switch}

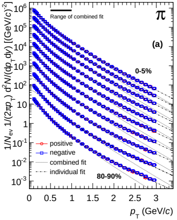
Latin hypercube design

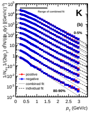
300 semi-random, space-filling parameter points

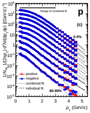


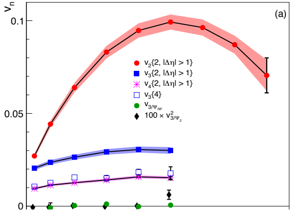
Observables

- Pion, kaon, and proton yields dN/dy
  - Overall particle production and species ratios
- Mean transverse momentum $\langle p_T \rangle$
  - Magnitude of radial expansion
- Anisotropic flow coefficients v_n
  - Azimuthal momentum anisotropy





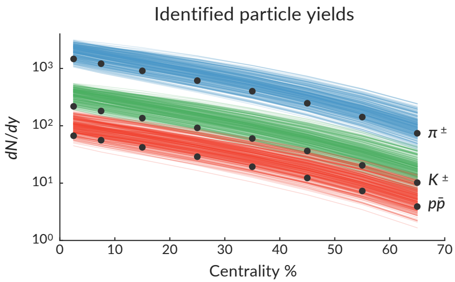


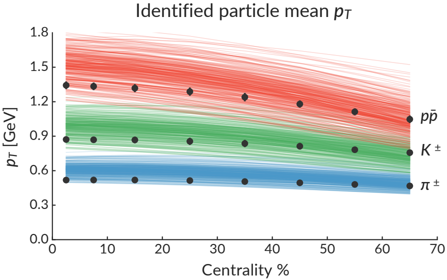


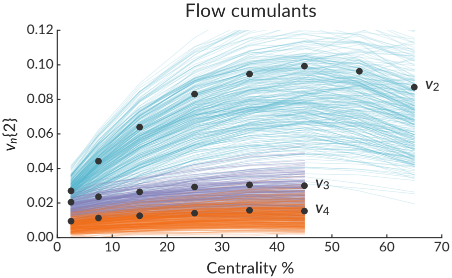
All experimental data from the ALICE collaboration at the LHC
Pb-Pb collisions at $\sqrt{s} = 2.76$ TeV

Training data

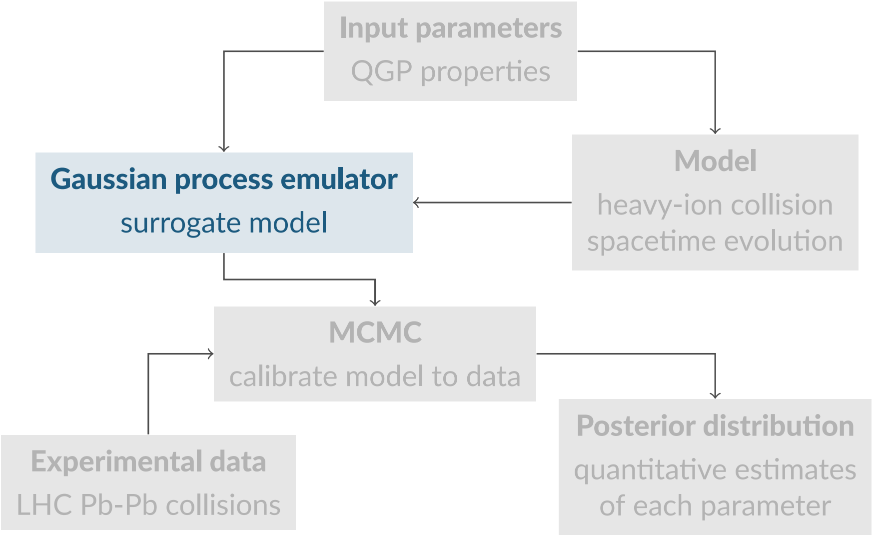
- Model calculations at each design point
- To be used as training data for emulator







Overview


Input parameters
QGP properties
Gaussian process emulator
surrogate model
Model
heavy-ion collision
spacetime evolution
MCMC
calibrate model to data
Experimental data
LHC Pb-Pb collisions
Posterior distribution
quantitative estimates
of each parameter

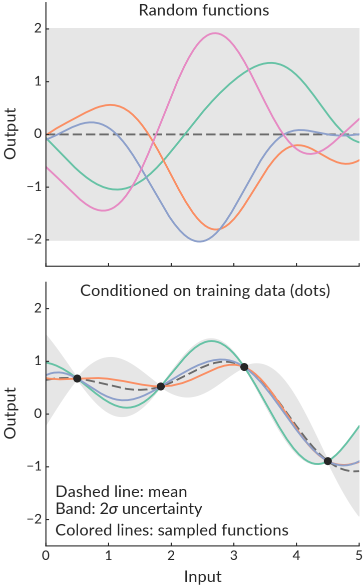
Gaussian process emulator

Gaussian process:

- Stochastic function: maps inputs to normally-distributed outputs
- Specified by mean and covariance functions

As a model emulator:

- Non-parametric interpolation
- Predicts *probability distributions*
  - Narrow near training points, wide in gaps
- Fast surrogate to actual model



Multivariate output

Many highly correlated outputs
$\rightarrow$ **principal component analysis**

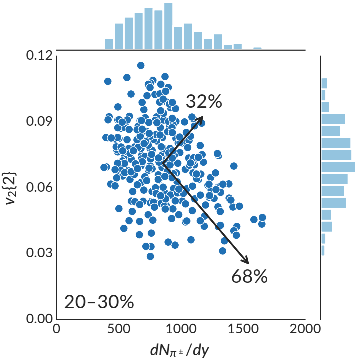
PCs = eigenvectors of sample covariance matrix

$$Y^{\mathsf{T}} Y = U \Lambda U^{\mathsf{T}}$$

Transform data into orthogonal, uncorrelated linear combinations

$$Z = \sqrt{m}\, Y U$$

Emulate each PC independently



Multivariate output

Many highly correlated outputs
$\rightarrow$ **principal component analysis**

PCs = eigenvectors of sample covariance matrix

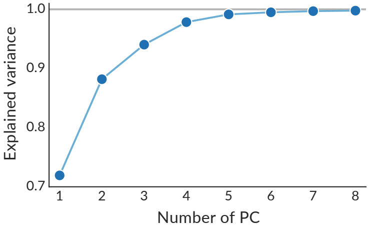
$$Y^\intercal Y = U \Lambda U^\intercal$$

Transform data into orthogonal, uncorrelated linear combinations

$$Z = \sqrt{m}\, Y U$$

Emulate each PC independently

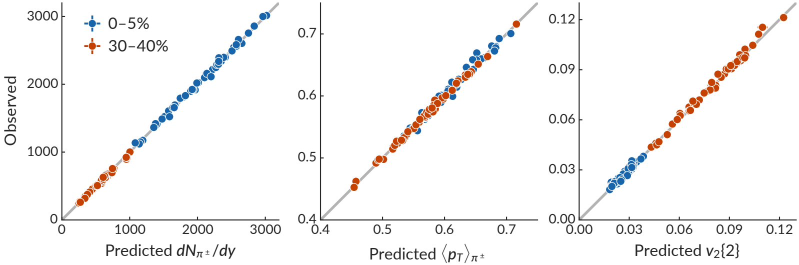
68 outputs $\rightarrow$ 8 PCs



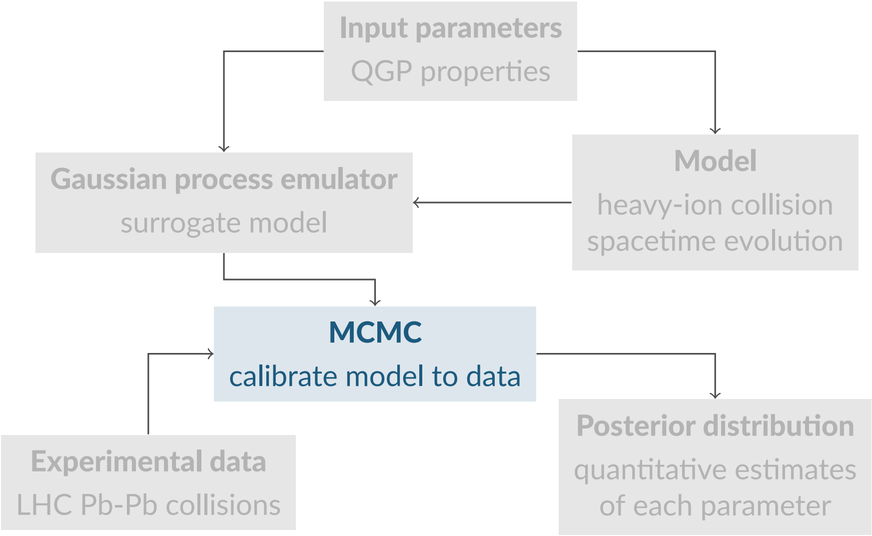
Validation

Independent 50-point validation design

Run full model and predict with emulator



Overview


Input parameters
QGP properties
Gaussian process emulator
surrogate model
Model
heavy-ion collision
spacetime evolution
MCMC
calibrate model to data
Experimental data
LHC Pb-Pb collisions
Posterior distribution
quantitative estimates
of each parameter

Calibration

Assume true parameters $\mathbf{x}_\star$ exist $\to$ find posterior distribution

$$P(\mathbf{x}_\star | X, Y, \mathbf{y}_{\text{exp}}) \propto P(X, Y, \mathbf{y}_{\text{exp}} | \mathbf{x}_\star)\, P(\mathbf{x}_\star)$$

given design X, training data Y, experimental data $\mathbf{y}_{\text{exp}}$

- Flat prior
- Likelihood (in PC space):

$$P(X, Z, \mathbf{z}_{\text{exp}} | \mathbf{x}_\star) \propto \exp\left\{ -\frac{1}{2} (\mathbf{z}_\star - \mathbf{z}_{\text{exp}})^\intercal \Sigma_z^{-1} (\mathbf{z}_\star - \mathbf{z}_{\text{exp}}) \right\}$$

with flat 10% uncertainty on PCs

$$\Sigma_z = \text{diag}(\sigma_z^2\, \mathbf{z}_{\text{exp}}), \quad \sigma_z = 0.10$$

MCMC

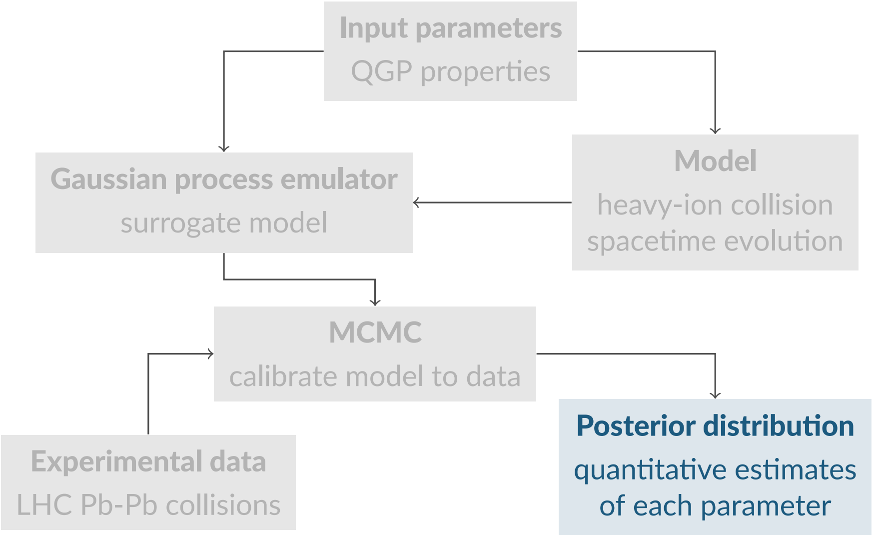
Markov chain Monte Carlo

- Random walk through parameter space weighted by posterior
- Large number of samples
$\rightarrow$ chain equilibrates to posterior distribution

This study

- Emulator serves as stand-in for full model
- Affine-invariant ensemble sampler: many interdependent walkers
- 1000 walkers, 10^6 burn-in steps, 10^7 production steps

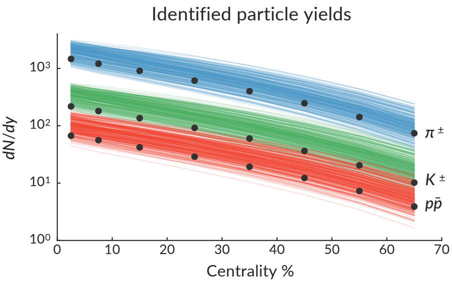
Overview

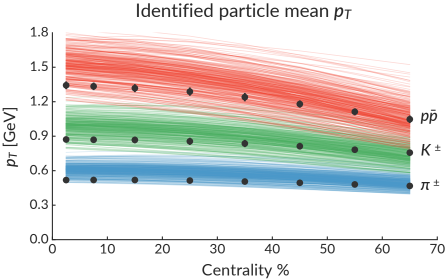


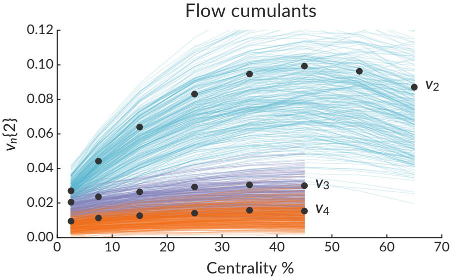
Input parameters
QGP properties
Model
heavy-ion collision
spacetime evolution
Gaussian process emulator
surrogate model
MCMC
calibrate model to data
Experimental data
LHC Pb-Pb collisions
Posterior distribution
quantitative estimates
of each parameter

Training data

Model calculations
at each design point

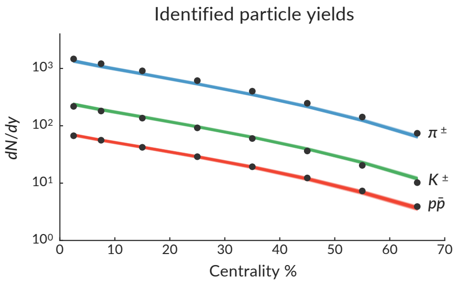


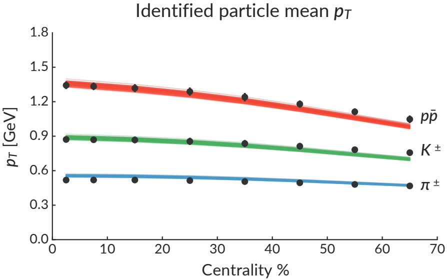


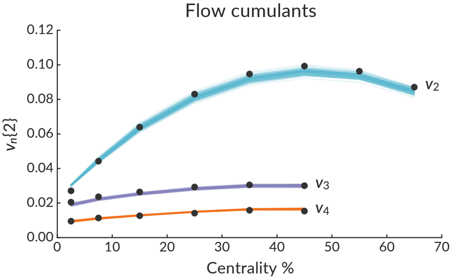


Posterior samples

Model calculations
at each design point
↓
Emulator predictions
from calibrated posterior

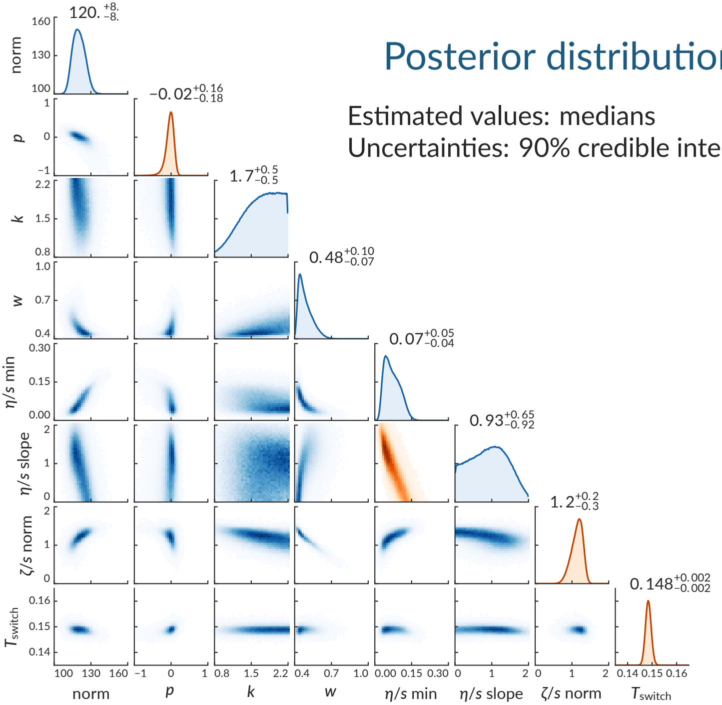






Posterior distribution

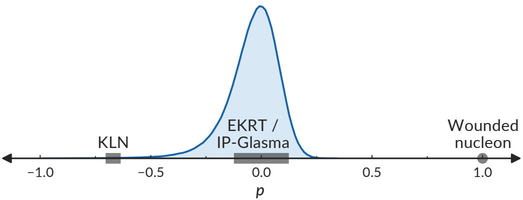
Estimated values: medians
Uncertainties: 90% credible intervals



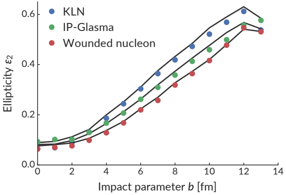
Constraining initial conditions

TRENTo ansatz:

$$s \propto \left(\frac{T_A^p + T_B^p}{2} \right)^{1/p}$$



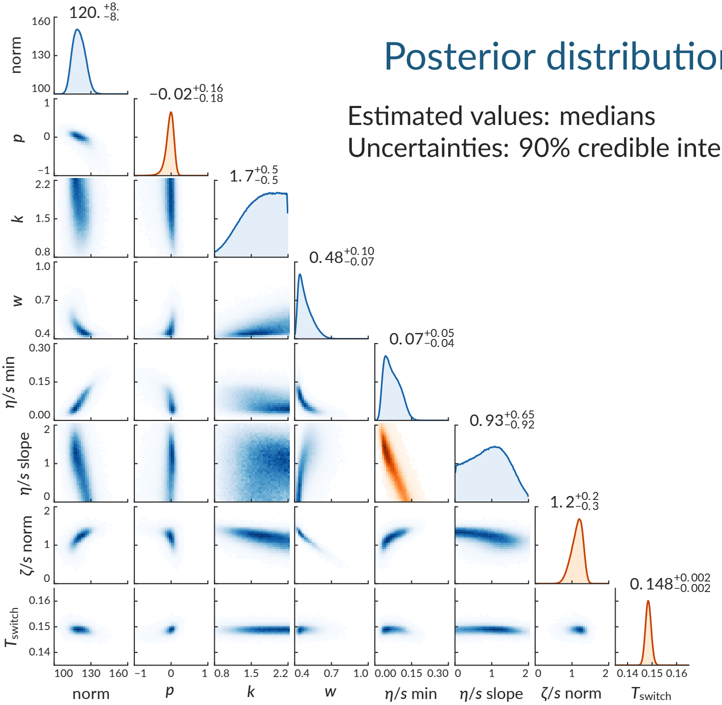
Mimics other models:



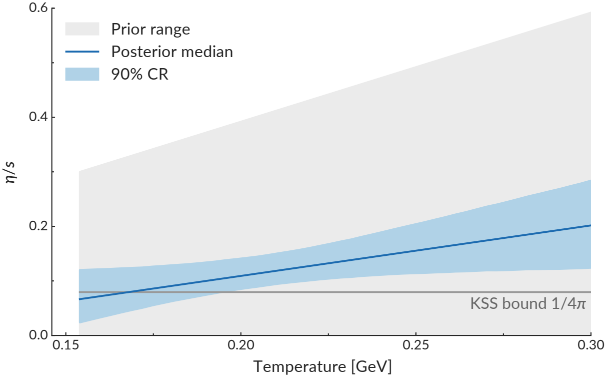
- Entropy deposition approx. proportional to geometric mean of nuclear density: $s \sim \sqrt{T_A T_B}$
- Confirms success / failure of existing models

Posterior distribution

Estimated values: medians
Uncertainties: 90% credible intervals



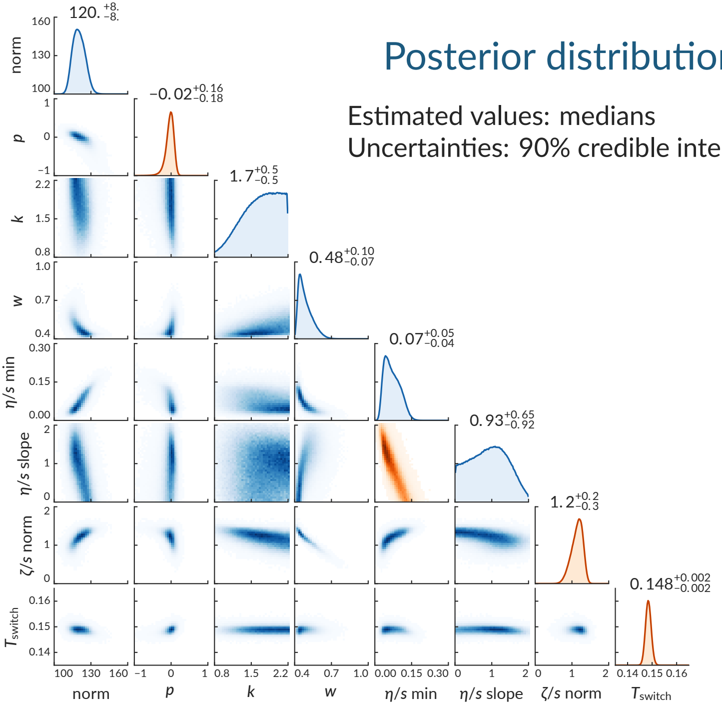
Estimate of $(\eta/s)(T)$

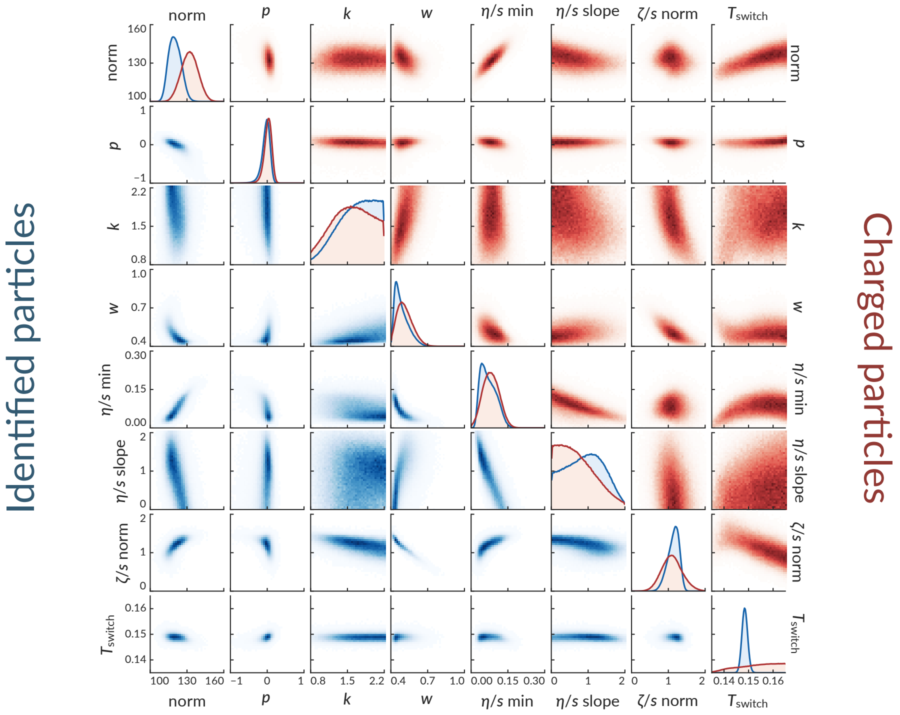


- First systematic, quantitative estimate of T-dependent η/s
- "Handle" near 200 MeV $\rightarrow$ need multiple beam energies!

Posterior distribution

Estimated values: medians
Uncertainties: 90% credible intervals

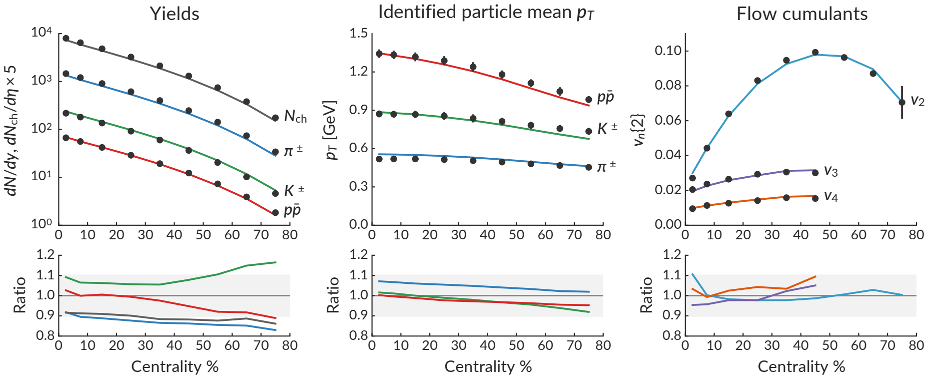




Identified particles
Charged particles
norm
p
k
w
η/s min
η/s slope
ζ/s norm
T_{switch}
100 130 160
−1 0 1
0.8 1.5 2.2
0.4 0.7 1.0
0.00 0.15 0.30
0 1 2
0.14 0.15 0.16

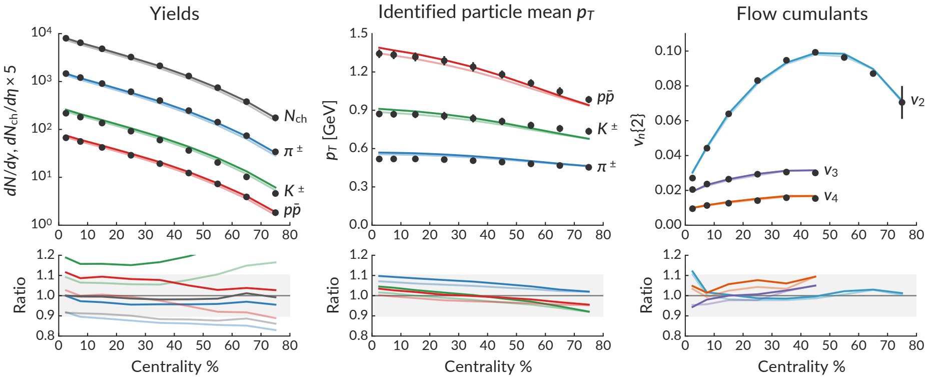
Most probable parameters

norm	120. / 129.	η/s min	0.08
p	0.0	η/s slope	0.85 / 0.75 GeV^{-1}
k	1.5 / 1.6	ζ/s norm	1.25 / 1.10
w	0.43 / 0.49 fm	T_{switch}	0.148 GeV



Most probable parameters

norm	120. / 129.	η/s min	0.08
p	0.0	η/s slope	0.85 / 0.75 GeV^{-1}
k	1.5 / 1.6	ζ/s norm	1.25 / 1.10
w	0.43 / 0.49 fm	T_{switch}	0.148 GeV



Summary

- T_RENTo parametric initial conditions, viscous relativistic hydrodynamics, hadronic afterburner (UrQMD)
- Excellent simultaneous fit to experimental data
- Estimated initial condition and QGP medium properties
  - Entropy deposition $\sim$ geometric mean of nuclear density
  - Relation between η/s min and slope, handle near 200 MeV
  - Finite bulk viscosity
  - T_{switch} constrained by particle ratios only

- Additional beam energies (200 GeV, 2.76 TeV, 5.02 TeV)
- Improve treatment of uncertainty

Gaussian processes

Definition

A Gaussian process is a collection of random variables, any finite number of which have a joint Gaussian distribution.

Stochastic function: $\mathbf{x} \to y$

- $\mathbf{x}$ = n-dimensional input vector
- y = normally distributed output

Specified by

- Mean function $\mu(\mathbf{x})$
- Covariance function $\sigma(\mathbf{x}, \mathbf{x}')$, e.g.:

$$\sigma(\mathbf{x}, \mathbf{x}') = \exp\left(-\frac{|\mathbf{x} - \mathbf{x}'|^2}{2\ell^2}\right)$$

Conditioning a Gaussian process

Given

- training input points X and
- observed training outputs $\mathbf{y}$ at X

the predictive distribution at arbitrary test points X_* is the multivariate-normal distribution

$$\begin{aligned} \mathbf{y}_* &\sim \mathcal{N}(\mu, \Sigma), \\ \mu &= \sigma(X_*, X)\sigma(X, X)^{-1}\mathbf{y}, \\ \Sigma &= \sigma(X_*, X_*) - \sigma(X_*, X)\sigma(X, X)^{-1}\sigma(X, X_*). \end{aligned}$$