Toward a new effective interaction in energy density functional theory

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INT workshop

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IPN
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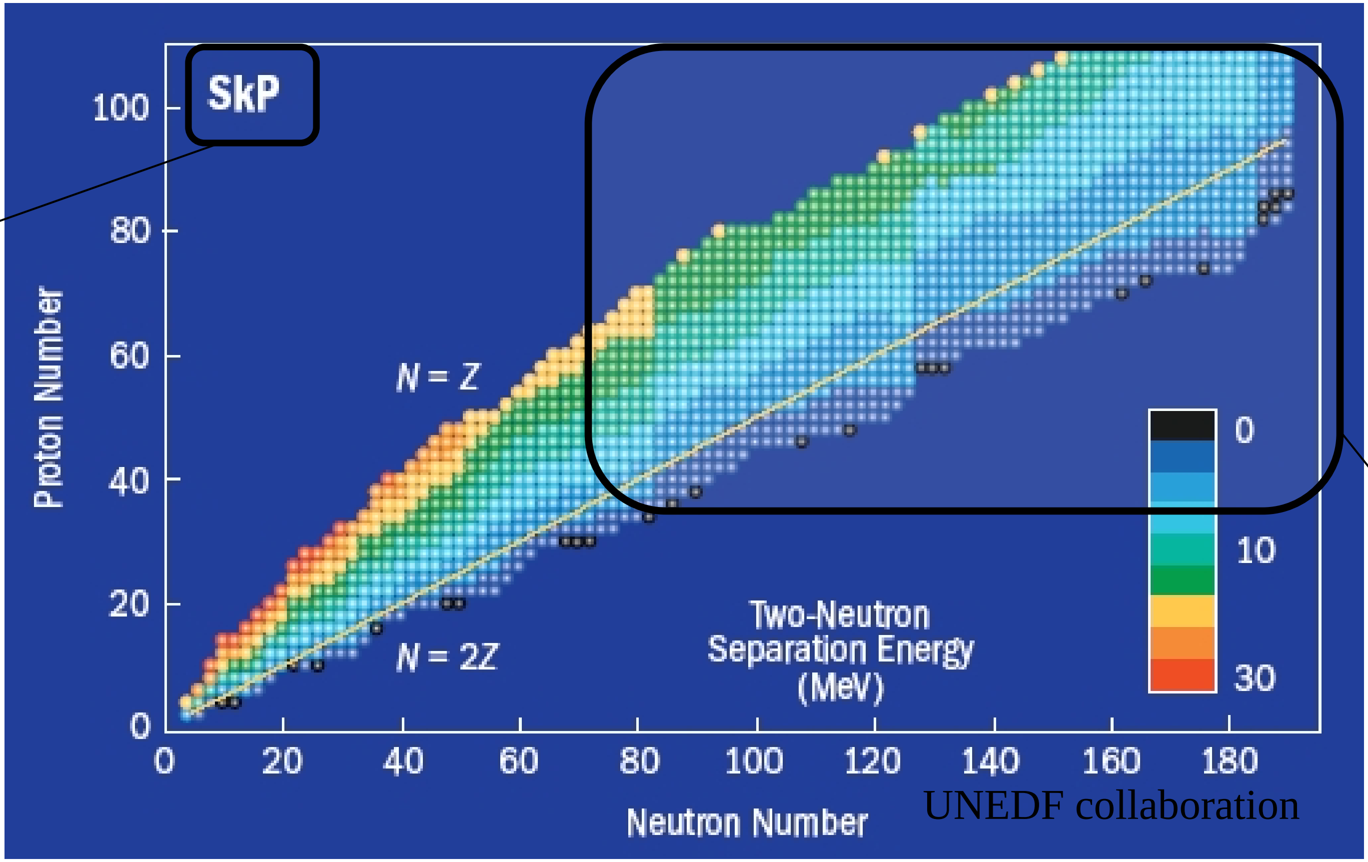
ORSAY

Outline

- Beyond mean field approach. (C.J. Yang, M. Grasso, X. Roca-Maza, G. Colo, and K. Moghrabi, arXiv:1604.06278)
- Renormalizability. (C.J. Yang, M. Grasso, K. Moghrabi, U van Kolck, coming soon!)
- Low density limit. (C.J. Yang, M. Grasso, D Lacroix, arXiv:1604.06587)
- Conclusion and future work.

Motivation



Skyrme-type interaction works o.k. (able to do the fitting in EDF framework)

No way to get with ab-initio!

Need to think about other expansion (than on NN d.o.f.).

Present status of EDF

- Energy density functional (EDF) framework gives reasonable results at mean field, when sufficient amount of parameters (~10) are included.
- Include more parameters won't necessarily help.

→ Limited predictive power.

- Maybe the correct theory has a structure where different terms appears at different order.

→ Need to go beyond mean field to perform the test.

Interaction & mean field EoS

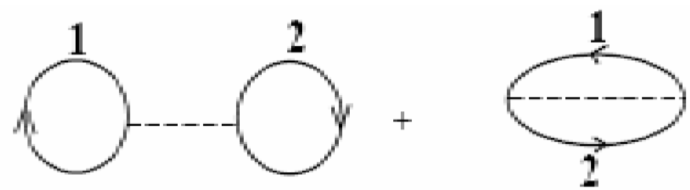
Interaction: Skyrme without spin-orbit

$$v = \underbrace{t_0(1+x_0P_\sigma)}_{S-wave\ \mathrm{O}(0)} + \frac{1}{2}\underbrace{t_1(1+x_1P_\sigma)(k'^2+k^2)}_{S-wave\ \mathrm{O}(\mathrm{q}^2)} + \underbrace{t_2(1+x_2P_\sigma)\mathbf{k}'\cdot\mathbf{k}}_{p-wave\ \mathrm{O}(\mathrm{q}^2)}$$

$$+\frac{1}{6}\underbrace{t_3(1+x_3P_\sigma)\rho^\alpha}_{s-wave,\ \text{higher body}}. \qquad \mathrm{P}_\sigma = \frac{1}{2}(1+\sigma_1\cdot\sigma_2)$$

No pion! Like pionless EFT, except for the density-dependent term.

$$EoS: \quad \frac{E}{A} \propto \frac{1}{\rho}\int_0^{k_{F1}} d^3\mathbf{k}_1 \int_0^{k_{F2}} d^3\mathbf{k}_2 v$$



$$\left(\mathbf{k} = \frac{\mathbf{k}_1\text{-}\mathbf{k}_2}{2}, \mathbf{k}' = \frac{\mathbf{k}_1\text{-}\mathbf{k}_2}{2} + \mathbf{q}\right)$$

2^{nd} order correction (symmetric & neutron matter)

$$\frac{E}{A} = \underbrace{\frac{E^{(0)}}{A}}_{\text{mean field}} + \underbrace{\frac{E^{(2)}}{A}}_{2^{nd}\ \text{order}} + \ldots$$

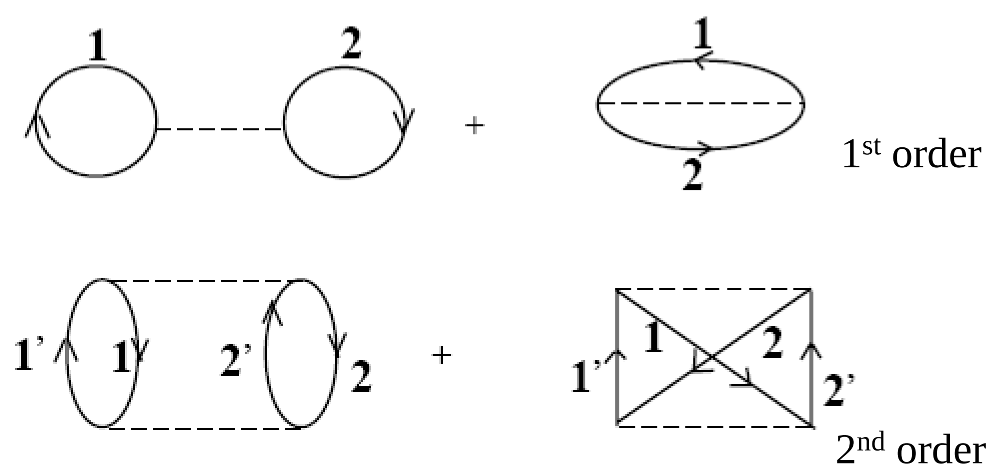
$$\frac{E^{(2)}_{sym}}{A} = \frac{-3m^*}{64\pi k_F^3 (2\pi)^6} \sum_{S,T} (2T+1)(2S+1) \int_{C_I} d^3\mathbf{k}_1 \int d^3\mathbf{k}_2 \int d^3\mathbf{q} [vGv]$$

$$G = \frac{1}{q^2 + \mathbf{q} \bullet (\mathbf{k_1} - \mathbf{k_2})}$$

Contour of integral (C_I):

$$|\mathbf{k}_{1,2}| \in [0, k_{F_{1,2}}]$$

$$|\mathbf{k_1} + \mathbf{q}| > k_{F_1}, |\mathbf{k_2} - \mathbf{q}| > k_{F_2}$$



Previous attempts

1. t_0-t_3 model, done by: K. Moghrabi, M. Grasso, G. Colo, and N.V. Giai, Phys. Rev. Lett. 105, 262501 (2010).

2. Full Skyrme (no spin-orbit):

K. Moghrabi, M. Grasso, G. Colo, X. Roco-Maza, Phys. Rev. C 85, 044323 (2012).

K. Moghrabi, M. Grasso, Phys. Rev. C 86, 044319 (2012).

↓

Some mistakes

3. Full Skyrme (no density-dep.): N.Kaiser, J. Phys. G 42,095111(2015)

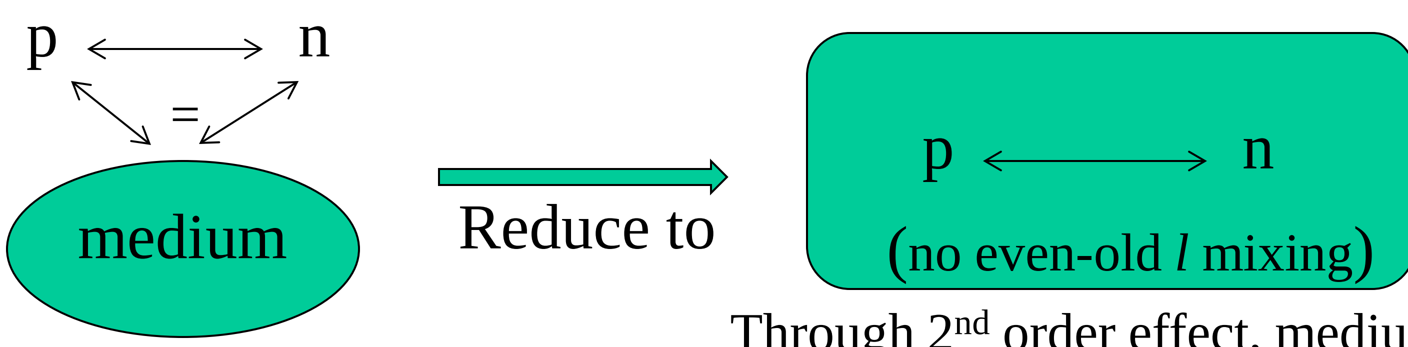
Details: partial-waves mixing

In general, need to sum over contribution from nn, pp and np.

If $k_{F_1} \neq k_{F_2}$, G no longer symmetric under the contour C_I!

$$\textit{Thus,} \int_{C_I} d^3\mathbf{q}\, d^3\mathbf{k}_1\, d^3\mathbf{k}_2 [v_l G v_{l'}] \neq 0. \Rightarrow \textit{In general any partial-waves can mix!}$$

Symmetric or pure n: $k_{F1}=k_{F2}$



Through 2nd order effect, medium (G) act as 3rd particle, but does not destroy the original parity of the interaction.

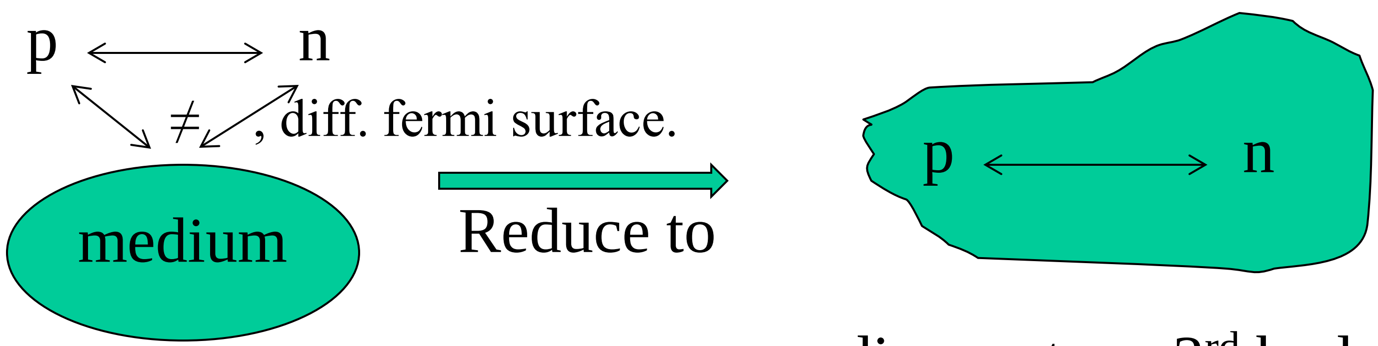
Details: partial-waves mixing

In asymmetric matter, need to sum over contribution from nn, pp and np.

For np part, $k_{F_1} \neq k_{F_2}$, G no longer symmetric under the contour C_I!

$$\textit{Thus,} \int_{C_I} \mathrm{d}^3\mathbf{q}\,\mathrm{d}^3\mathbf{k}_1\mathrm{d}^3\mathbf{k}_2[v_l G v_{l'}] \neq 0. \Rightarrow \textit{any partial-waves can mix!}$$

Asymmetric case: $k_{F1} \neq k_{F2}$



medium acts as 3rd body (by G),

and *mixes even-odd partial-wave*.

Results for nuclear matter

In agreement with N.Kaiser, J. Phys. G 42,095111(2015)

$$\frac{\Delta E^{(2)}_{sym(l=0)}}{A} = -\frac{mk_F^4}{110880\hbar^2\pi^4}\left\{\begin{array}{l} \left[\begin{array}{c} -6534 + 1188ln[2] + 3564\lambda - 19602\lambda^3 - 5940\lambda^5 \\ +(1782 - 20790\lambda^4)ln[\frac{\lambda-1}{\lambda+1}] \\ +(24948\lambda^5 - 5940\lambda^7)ln[\frac{\lambda^2-1}{\lambda^2}] \end{array}\right]\widetilde{T}^2_{03} \\ + \left[\begin{array}{c} - \quad 14696 + 2112ln[2] + 5280\lambda - 2860\lambda^3 \\ -48840\lambda^5 - 18480\lambda^7 + (2640 - 55440\lambda^6)ln[\frac{\lambda-1}{\lambda+1}] \\ +(71280\lambda^7 - 18480\lambda^9)ln[\frac{\lambda^2-1}{\lambda^2}] \end{array}\right] k_F^2\widetilde{T}_{03}\widetilde{T}_1 \\ + \left[\begin{array}{c} -9886 + 1128ln[2] + \ 2520\lambda + 147\lambda^3 - 3654\lambda^5 \\ -35280\lambda^7 - 15120\lambda^9 + (1260 - 41580\lambda^8)ln[\frac{\lambda-1}{\lambda+1}] \\ +(55440\lambda^9 - 15120\lambda^{11})ln[\frac{\lambda^2-1}{\lambda^2}] \end{array}\right] k_F^4\widetilde{T}^2_1 \end{array}\right\}$$

Diverge as Λ^5

$$\frac{\Delta E^{(2)}_{sym(l=1)}}{A} = -\frac{mk_F^8}{73920\hbar^2\pi^4}\left\{\left[\begin{array}{c} -1033 + 156ln[2] + 420\lambda + 140\lambda^3 - 840\lambda^5 \\ -5880\lambda^7 - 2520\lambda^9 + (-210 + 6930\lambda^8)ln[\frac{\lambda-1}{\lambda+1}] \\ +(9240\lambda^9 - 2520\lambda^{11})ln[\frac{\lambda^2-1}{\lambda^2}] \end{array}\right]\widetilde{T}^2_2\right\},$$

$$\frac{\Delta E^{(2)}_{neutr(l=0)}}{A} = -\frac{mk_{F_N}^4}{166320\hbar^2\pi^4}\left\{\begin{array}{l} \left[\begin{array}{c} -6534 + 1188ln[2] + 3564\lambda - 19602\lambda^3 - 5940\lambda^5 \\ +(1782 - 20790\lambda^4)ln[\frac{\lambda-1}{\lambda+1}] \\ +(24948\lambda^5 - 5940\lambda^7)ln[\frac{\lambda^2-1}{\lambda^2}] \end{array}\right] T^2_{03} \\ + \left[\begin{array}{c} - \quad 14696 + 2112ln[2] + 5280\lambda - 2860\lambda^3 \\ -48840\lambda^5 - 18480\lambda^7 + (2640 - 55440\lambda^6)ln[\frac{\lambda-1}{\lambda+1}] \\ +(71280\lambda^7 - 18480\lambda^9)ln[\frac{\lambda^2-1}{\lambda^2}] \end{array}\right] k_{F_N}^2 T_{03}T_1 \\ + \left[\begin{array}{c} -9886 + 1128ln[2] + \ 2520\lambda + 147\lambda^3 - 3654\lambda^5 \\ -35280\lambda^7 - 15120\lambda^9 + (1260 - 41580\lambda^8)ln[\frac{\lambda-1}{\lambda+1}] \\ +(55440\lambda^9 - 15120\lambda^{11})ln[\frac{\lambda^2-1}{\lambda^2}] \end{array}\right] k_{F_N}^4 T^2_1 \end{array}\right\}$$

Diverge as Λ^5

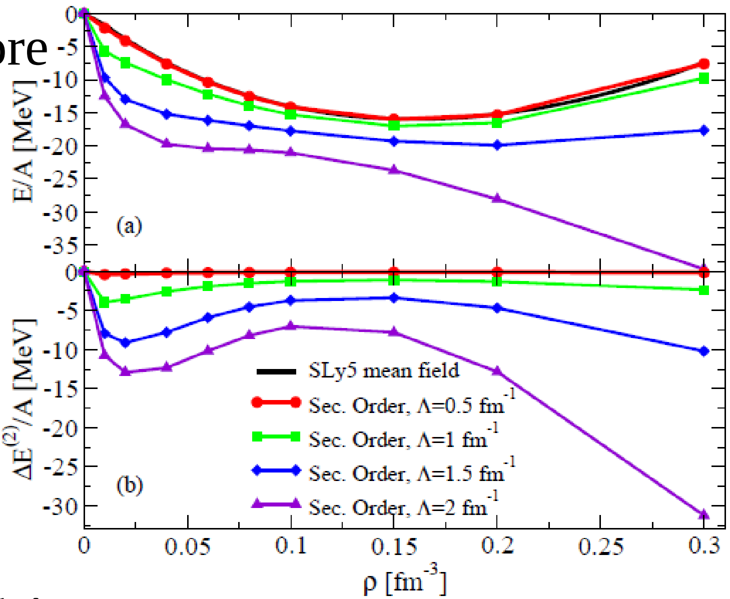
$$\frac{\Delta E^{(2)}_{neutr(l=1)}}{A} = -\frac{mk_{F_N}^8}{110880\hbar^2\pi^4}\left\{\left[\begin{array}{c} -1033 + 156ln[2] + 420\lambda + 140\lambda^3 - 840\lambda^5 \\ -5880\lambda^7 - 2520\lambda^9 + (-210 + 6930\lambda^8)ln[\frac{\lambda-1}{\lambda+1}] \\ +(9240\lambda^9 - 2520\lambda^{11})ln[\frac{\lambda^2-1}{\lambda^2}] \end{array}\right] T^2_2\right\},$$

Renormalization I

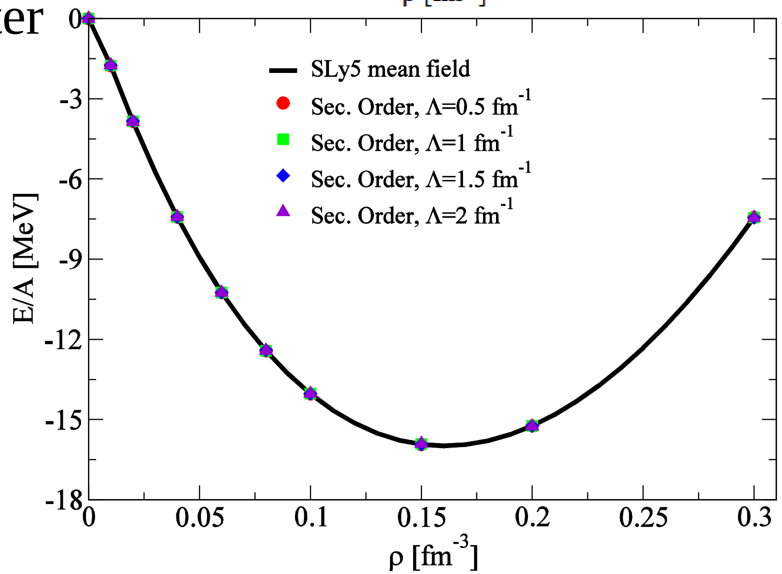
- Pick a Λ, and simply re-adjust the 9 (skyrme) parameters.

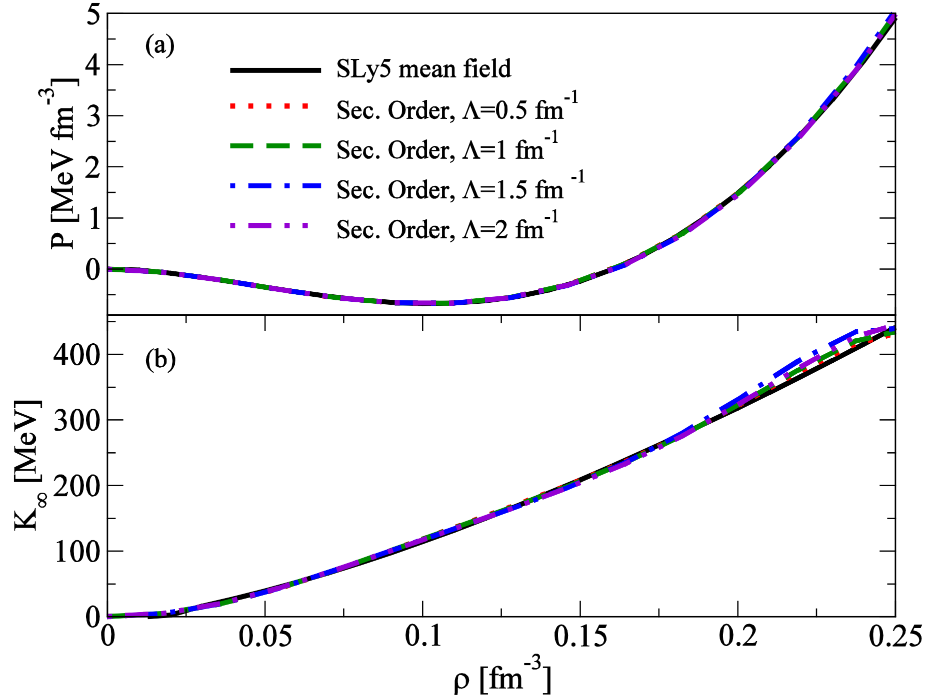
Symmetric matter

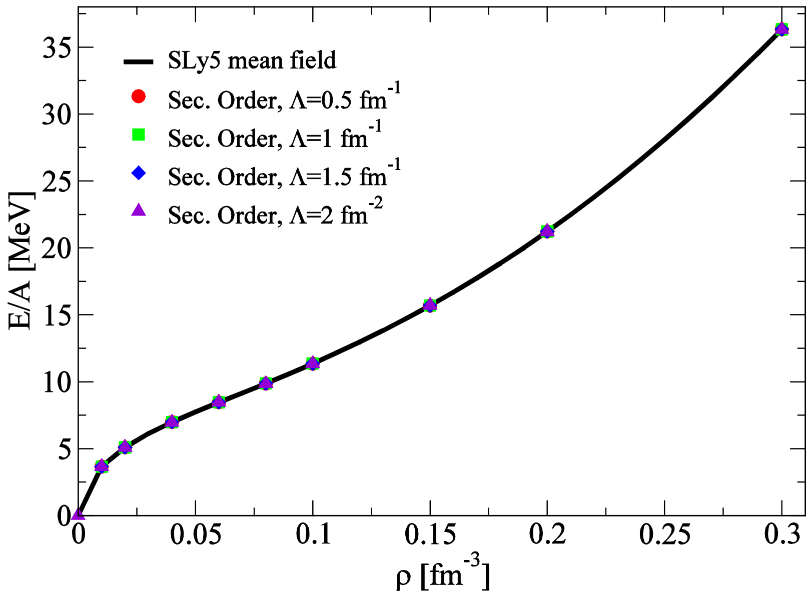
before



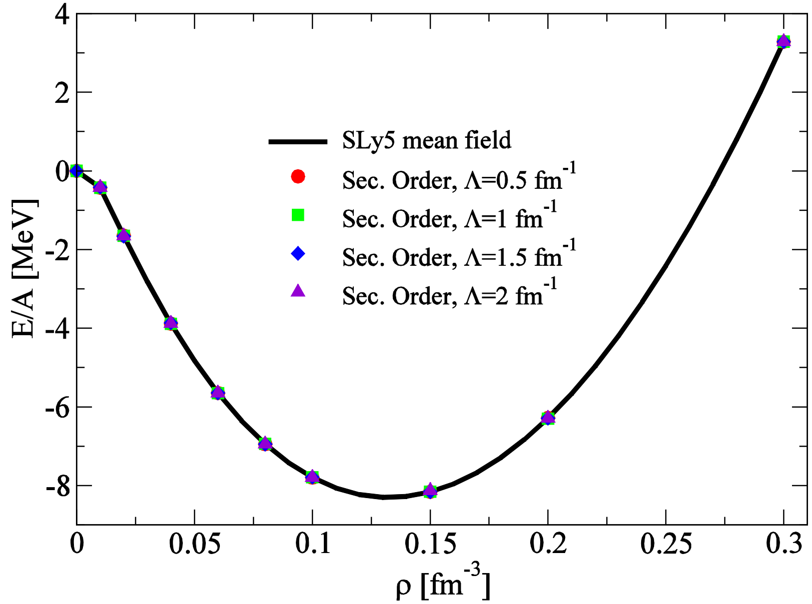
after





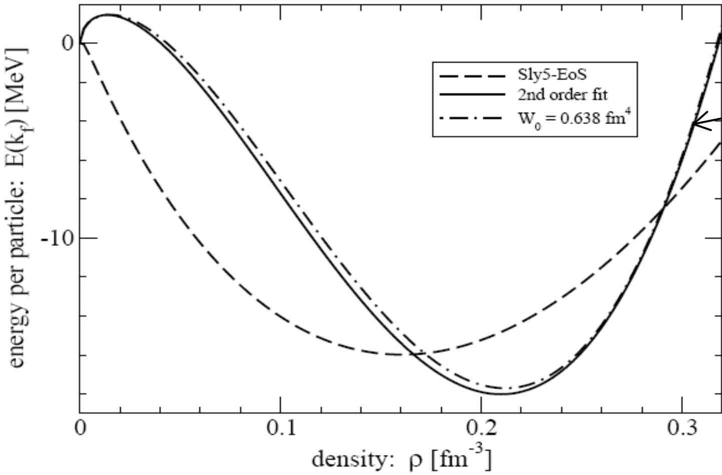


Pure neutron matter

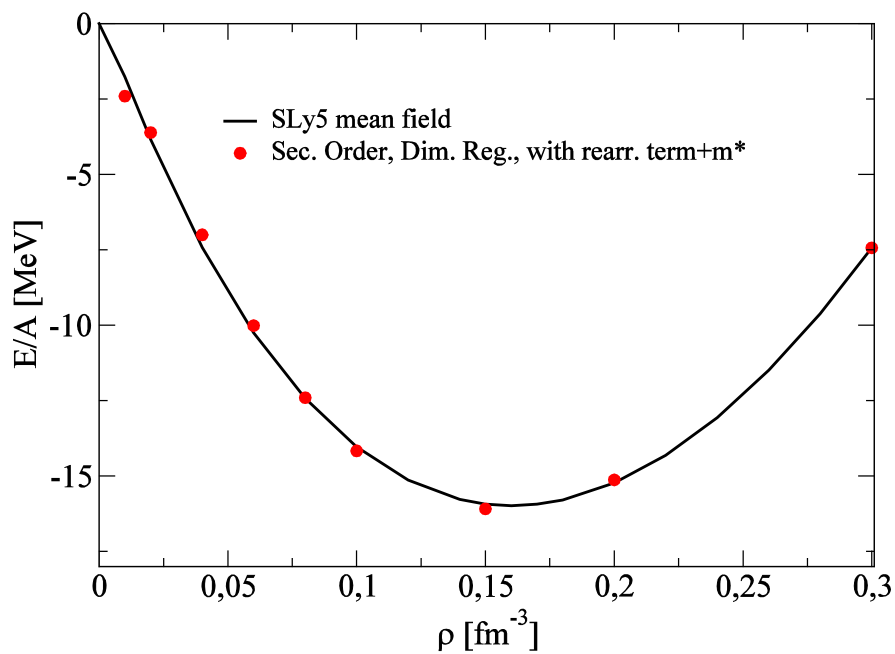


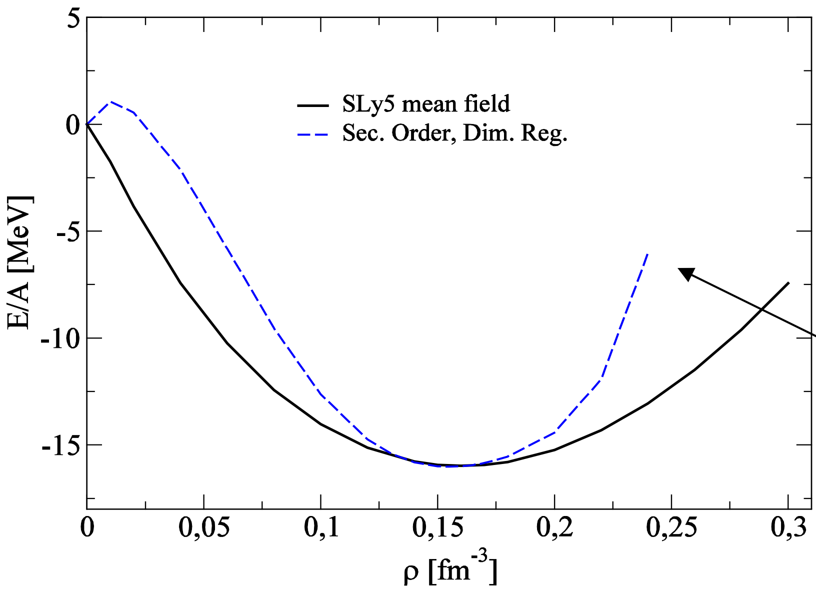
Asymmetric matter (δ=0.5)

Keep only the finite part
(Dimensional regularization)



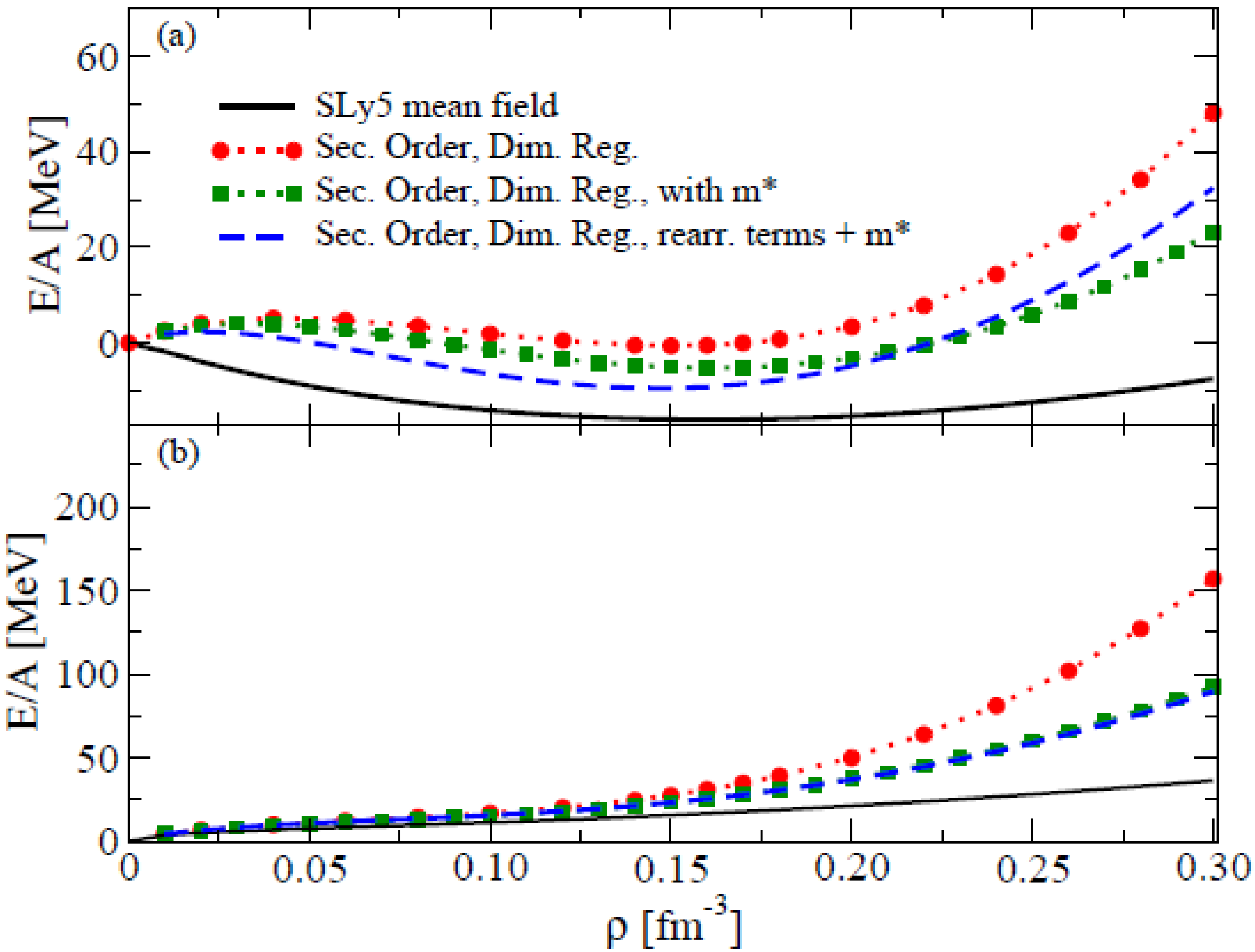
Without density-dep.(t_3) term, Kaiser 2015



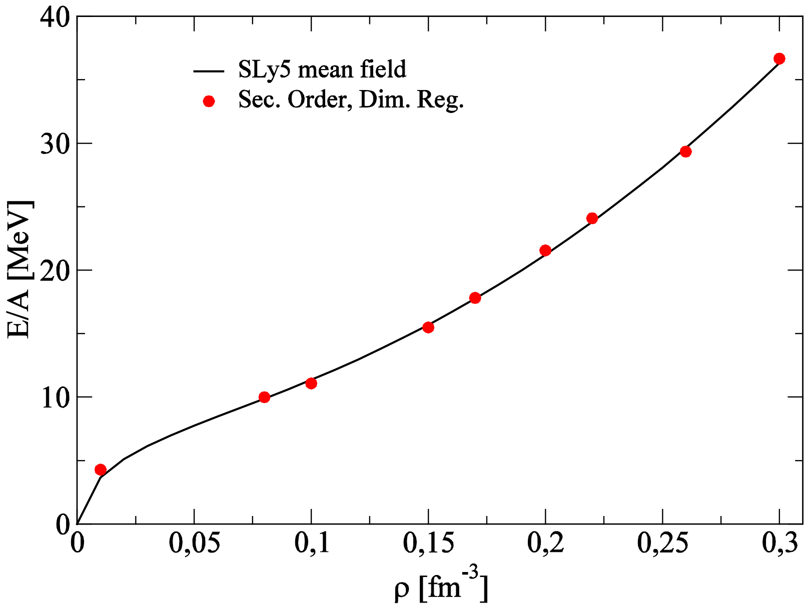


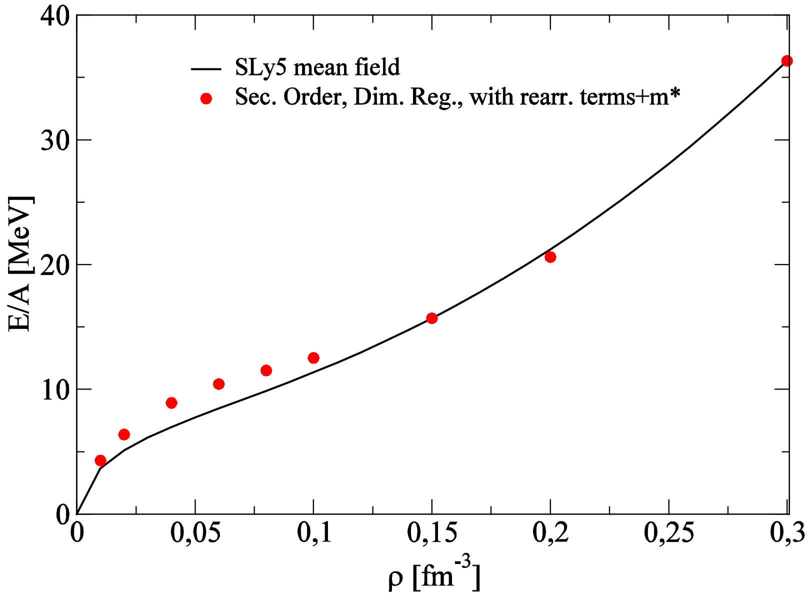
With density-dep., without rearrangement term

Before renormalization



Pure neutron matter (DR)





PART II: RENORMALIZABILITY

- When $\Lambda \to \infty$, how the 2nd order terms behaves?

$$\frac{\Delta E_f^{(2)}(k_F)}{A} = \frac{3m}{2\pi^4\hbar^2} k_F^4 \left[A_0 + A_1 T_3 k_F^{3\alpha} + A_2 T_3^2 k_F^{6\alpha} + A_3 k_F^2 + A_4 T_3 k_F^{2+3\alpha} + A_5 k_F^4\right],$$

Converge terms

$$\frac{\Delta E_a^{(2)}(k_F,\lambda)}{A} = -\frac{m}{8\pi^4\hbar^2} \lambda k_F^3 \left[B_0(\lambda) + B_1(\lambda) T_3 k_F^{3\alpha} + B_2(\lambda) k_F^2\right],$$

Diverge, k_F-dep appears in MF

$$\frac{\Delta E_d^{(2)}(k_F,\lambda)}{A} = -\frac{m}{8\pi^4\hbar^2} \lambda k_F^3 \left[C_0 T_3^2 k_F^{6\alpha} + C_1 T_3 k_F^{2+3\alpha} + C_2 k_F^4\right],$$

Diverge, k_F-dep *not* in MF

- Idea: Absorb the Λ-divergence in 2nd order into mean field terms with the same k_F-dependence.

$$\frac{\Delta E_f^{(2)}(k_F)}{A} = \frac{3m}{2\pi^4\hbar^2} k_F^4 \left[A_0 + A_1 T_3 k_F^{3\alpha} + A_2 T_3^2 k_F^{6\alpha} + A_3 k_F^2 + A_4 T_3 k_F^{2+3\alpha} + A_5 k_F^4\right],$$

converge

$$\frac{\Delta E_a^{(2)}(k_F,\lambda)}{A} = -\frac{m}{8\pi^4\hbar^2} \lambda k_F^3 \left[B_0(\lambda) + B_1(\lambda) T_3 k_F^{3\alpha} + B_2(\lambda) k_F^2\right],$$

Diverge, k_F-dep appears in MF

$$\frac{\Delta E_d^{(2)}(k_F,\lambda)}{A} = -\frac{m}{8\pi^4\hbar^2} \lambda k_F^3 \left[C_0 T_3^2 k_F^{6\alpha} + C_1 T_3 k_F^{2+3\alpha} + C_2 k_F^4\right],$$

Diverge, k_F-dep *not* in MF

↓

eliminate by setting α=1/3 and t_1=t_2=0, or seting t_1=t_2=t_3=0.

- Idea: Absorb the Λ-divergence in 2nd order into mean field terms with the same k_F-dependence.

$$\frac{\Delta E_f^{(2)}(k_F)}{A} = \frac{3m}{2\pi^4\hbar^2} k_F^4 \left[A_0 + A_1 T_3 k_F^{3\alpha} + A_2 T_3^2 k_F^{6\alpha} + A_3 k_F^2 + A_4 T_3 k_F^{2+3\alpha} + A_5 k_F^4\right],$$

converge

$$\frac{\Delta E_a^{(2)}(k_F, \lambda)}{A} = -\frac{m}{8\pi^4\hbar^2} \lambda k_F^3 \left[B_0(\lambda) + B_1(\lambda) T_3 k_F^{3\alpha} + B_2(\lambda) k_F^2\right],$$

Diverge, k_F-dep appears in MF

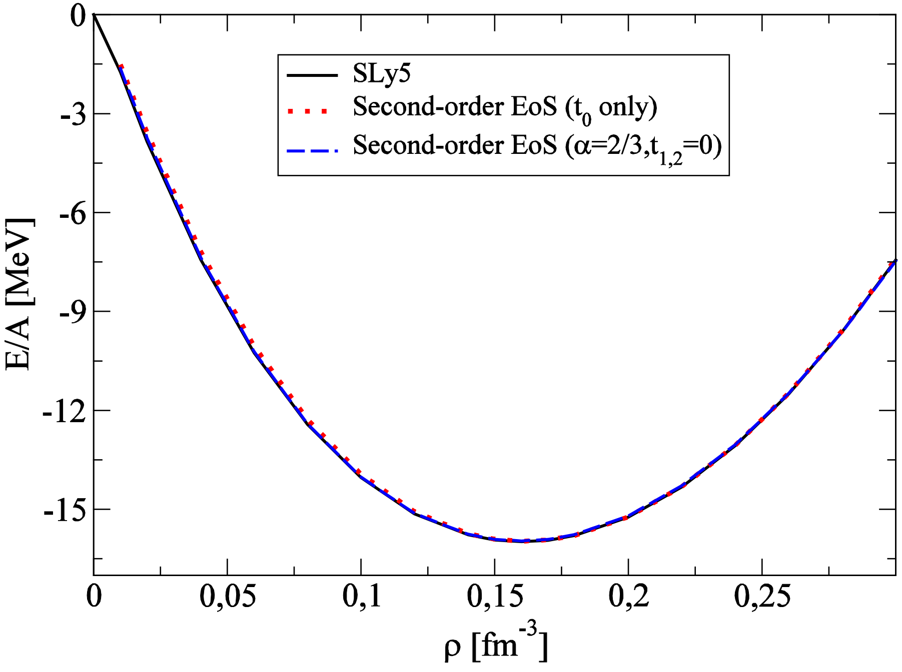
↓

Treatment 1: Absorb divergence into redefinition of parameters.

Treatment 2: Add counter terms correspond to each divergence.

Results

- Treatment 1 doesn't work (cannot obtain reasonable fit).
- Treatment 2 works.



Lesson

- The leading order quite possible just contains only t_0-t_3 terms.
- However, the regulator dependence tells us the power counting cannot be established in this way.

Part III: Matching the low density limit

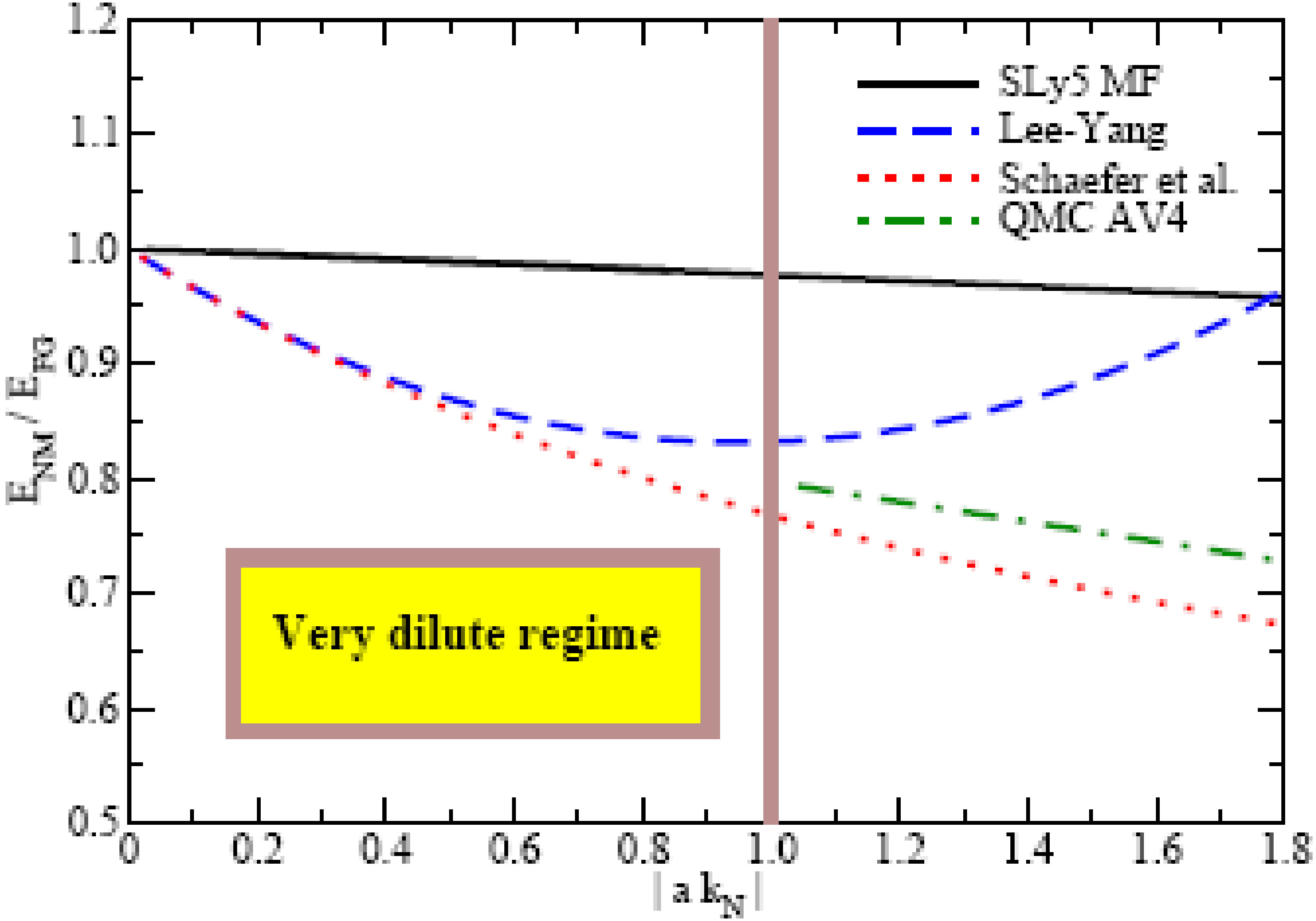
Neutron matter at very low-ρ

Lee & Yang formula (1957) describes the dilute system.

$$\frac{E_{NM}}{A} = \frac{\hbar^2 k_N^2}{2m}\left[\underbrace{\frac{3}{5}}_{K.E.} + \underbrace{\frac{2}{3\pi}(k_N a)}_{\textit{fixed}\text{ to } t_0 \text{ term}} + \underbrace{\frac{4}{35}(11-2\ln 2)(k_N a)^2}_{\textit{automatically}\text{ recover in } 2^{nd} \text{ of } t_0} + \underbrace{O(k_N^3)}_{\textit{higher}\text{ order}}\right]$$

- The 2nd order EoS automatically recover the $(11\text{-}2\ln2)(k_N a)^3$ term.
- If take physical value of a=-18.9 fm, then *impossible* to fit pure neutron matter EoS outside region $k_N a \ll 1$ (adding t1, t2, t3 terms doesn't help).
- $(k_N a)$ needs to be re-summed. (Steele (2000), Schafer (2005), Kaiser (2011))

$$\frac{E_{NM}}{N} = \frac{\hbar^2 k_N^2}{2m}\left[\frac{3}{5} + \frac{2}{3\pi}\frac{k_N a}{1 - 6k_N a(11-2\ln 2)/(35\pi)}\right]$$



SLy5 MF
Lee-Yang
Schaefer et al.
QMC AV4
Very dilute regime
E_{NM} / E_{FG}
$| a\, k_N |$
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8

This work: Resumed-inspired functional

$$V=\frac{B_\beta}{1-R_\beta\rho^{1/3}+\underbrace{C_\beta\rho^{2/3}}_{\text{higher order in L\&Y to be resumed*}}}+\underbrace{D_\beta\rho^{2/3}}_{velocity-dep\ \text{term*}}+\underbrace{F_\beta\rho^\alpha}_{3^+-body}$$

B_β, R_β are fixed to reproduce first two term in Lee & Yang.

$$=> B_\beta=2\pi\frac{\hbar^2}{\mathrm{m}}\frac{\nu-1}{\nu}a_\beta,\ R_\beta=\frac{6}{35\pi}\left(\frac{6\pi^2}{\nu}\right)^{1/3}(11-2\ln 2)a_\beta.$$

(degeneracy: $\nu=2(4)$ for $\beta=\underbrace{0}_{\text{pure n}}\ (\underbrace{1}_{\text{sym}}))$

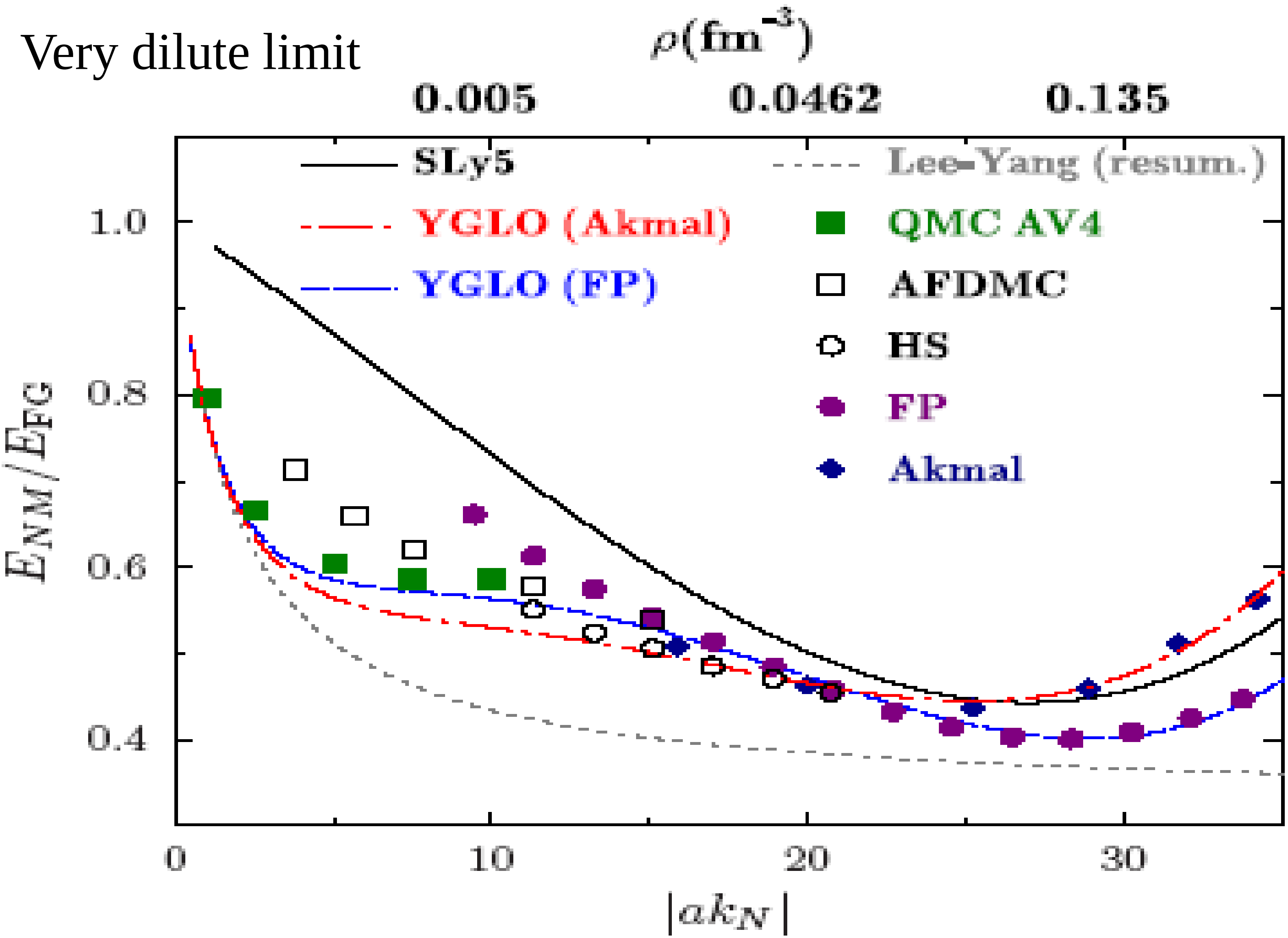
$$a_0=-18.9\mathrm{fm},\quad \underbrace{a_1=-20\ \mathrm{fm}}_{avg.\ \text{of}\ a_{nn},\ a_{pp},\ a_{np}\ \text{in}\ ^1S_0}\ .$$

$$\frac{E}{A}=KE_\beta+\frac{B_\beta\rho}{1-R_\beta\rho^{1/3}+C_\beta\rho^{2/3}}+D_\beta\rho^{5/3}+F_\beta\rho^{\alpha+1}$$

Results

Able to describe pure neutron matter from very-dilute-limit to twice saturation density.

Very dilute limit

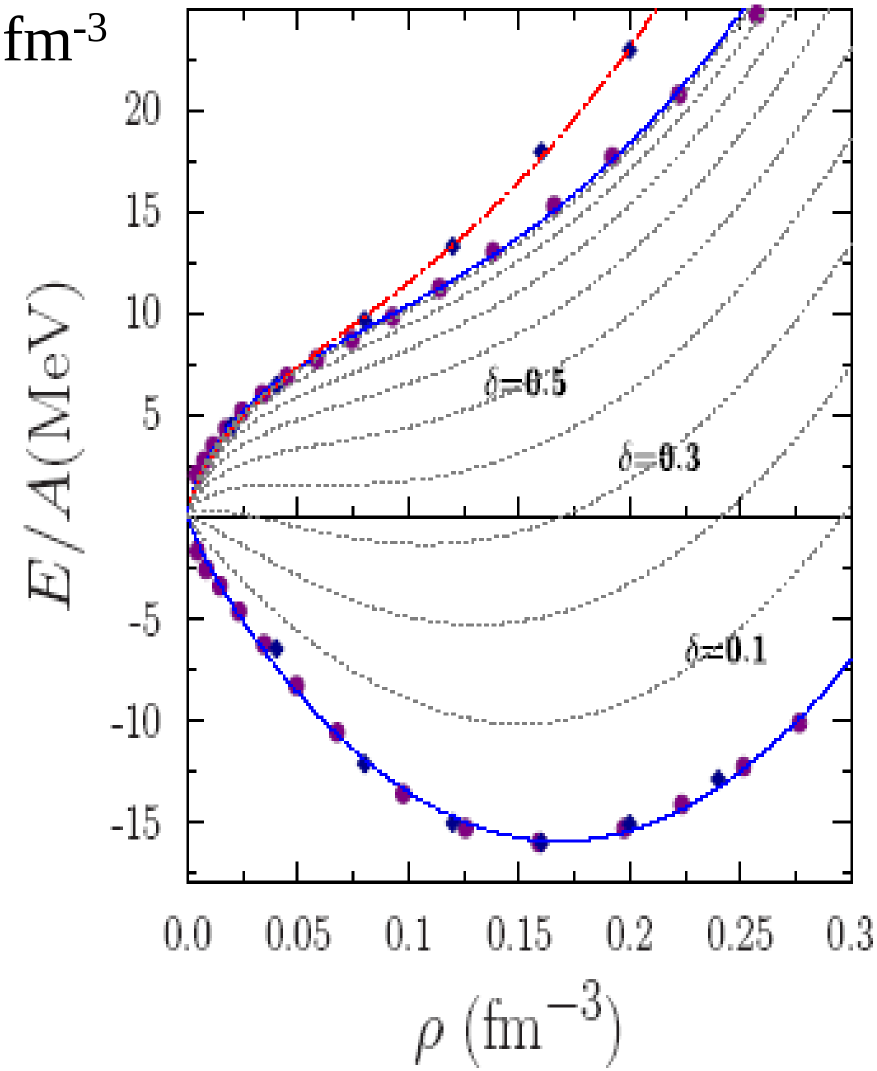


FP: B. Friedman and V. Pandharipande, Nucl. Phys. A361,502 (1981).

Akmal: A. Akmal, V. R. Pandharipande, and D. G. Ravenhall, Phys. Rev. C 58, 1804 (1998).

HS: K. Hebeler and A. Schwenk Phys. Rev. **C 82**, 014314(2010).

Up to ρ=0.3 fm^{-3}



Able to describe both sym and pure neutron matter EoS up to $2\rho_0$ very well with only 4 free parameters each.

Asymmetric case

Before: Lots of models fail

Parabolic approximation

$$\frac{E_\delta}{A}(\rho) = \frac{E_{sym}}{A}(\rho) + S(\rho)\delta^2,$$

$$(\delta = (\rho_N - \rho_p)/(\rho_N + \rho_p))$$

$$L = 3\rho_0 (dS/d\rho)_{\rho=\rho_0}$$

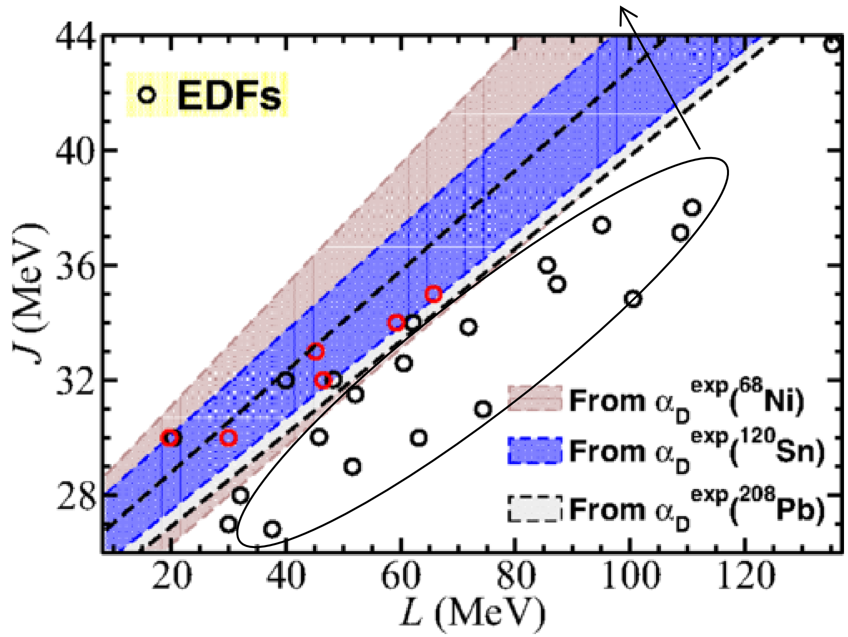


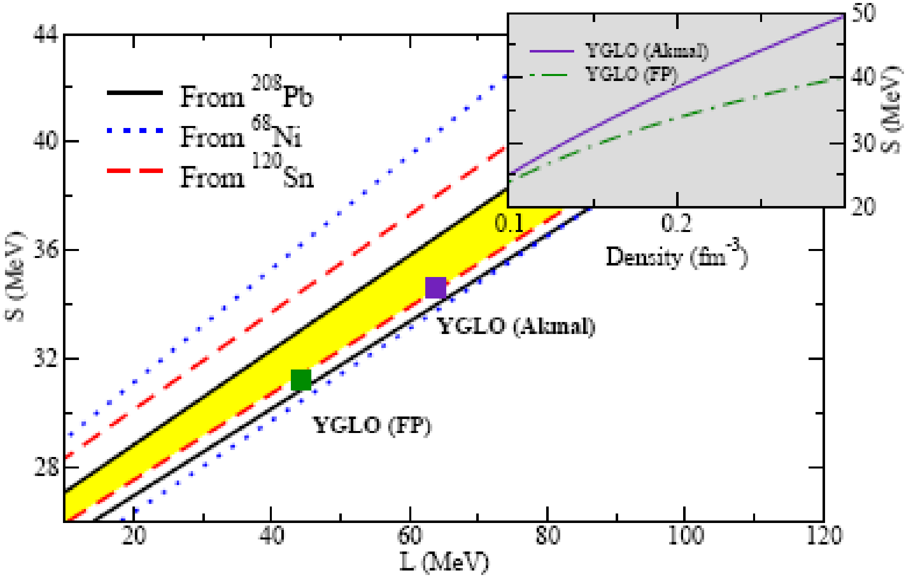
FIG. 4: Symmetry energy at saturation density as a function of its slope L. The black lines delimit the phenomenological area constrained by the experimental determination of the electric dipole polarizability in ^{208}Pb. The blue dotted lines delimit the area constrained by the same measurement in ^{68}Ni, and the red dashed lines refer to the measurement done in ^{120}Sn. The yellow area is the overlap. Inset: density dependence of the Symmetry energy for the two YGLO parametrizations of this work.

X. Roca-Maza, et al., (2015).

Asymmetric case

Our result (prediction)

Satisfies the experimental constraint.



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Summary and conclusion

- 2^{nd} order contribution is considered, and with the rearrangement term included, results can be refitted to standard data very well in both cutoff- and dimensional-regularization.
- Renormalizability suggests that the leading order(MF) likely to be a t_0-t_3 model (to avoid a rapid grows of counter terms).
- Resuming the t_0 part of interaction is necessary to describe low ρ limit correctly.

Future prospects

Try to bridge EFT ideas/techniques to mean field (and beyond) within EDF framework.

Mean field with potential models (effective interaction).
(e.g., Skyrme-type)

2nd order corrections

Higher order corrections

Add new effective interactions?

What is the proper form of it?

Is the improvement systematic?

Renormalization-group analysis
+
power counting check

Goal: Systematic treatment of the interactions.

Thank you!

Simplification for symmetric and pure neutron matter

Mean field: $E \propto \int_0^{k_{F1}} d^3\mathbf{k}_1 \int_0^{k_{F2}} d^3\mathbf{k}_2 v(\mathbf{k}', \mathbf{k}).$

2^{nd} order correction: $E^{(2)} \propto \int_0^{k_{F1}} d^3\mathbf{k}_1 \int_0^{k_{F2}} d^3\mathbf{k}_2 \int_0^{\Lambda} vGv.$

Define $\mathbf{k} = \frac{\mathbf{k}_1 - \mathbf{k}_2}{2}, \mathbf{k}' = \frac{\mathbf{k}_1 - \mathbf{k}_2}{2} + \mathbf{q}$

$$G_{sym,neut} \rightarrow \frac{-m^*}{k'^2 - k^2}.$$