Lattice-QCD Studies of the H-Dibaryon

Hartmut Wittig

PRISMA Cluster of Excellence, Institute for Nuclear Physics and Helmholtz Institute Mainz

INT Program INT-16-1
Nuclear Physics from Lattice QCD
29 March 2016

HELMHOLTZ | ASSOCIATION
Helmholtz Institute Mainz

PRISMA

SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

Introduction — The H-Dibaryon

VOLUME 38, NUMBER 5 PHYSICAL REVIEW LETTERS 31 JANUARY 1977

Perhaps a Stable Dihyperon*

R. L. Jaffe†

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305, and Department of Physics and Laboratory of Nuclear Science,‡ Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 1 November 1976)

In the quark bag model, the same gluon-exchange forces which make the proton lighter than the $\Delta(1236)$ bind six quarks to form a stable, flavor-singlet (with strangeness of -2) $J^P=0^+$ dihyperon (H) at 2150 MeV. Another isosinglet dihyperon (H^*) with $J^P=1^+$ at 2335 MeV should appear as a bump in $\Lambda\Lambda$ invariant-mass plots. Production and decay systematics of the H are discussed.

- MIT bag model predicts di-hyperon state (*H*) with

$$I=0,\ S=-2,\ J^P=0^+$$

and a mass of $m_H = 2150\,\mathrm{MeV}$

- *H*-Dibaryon must decay weakly

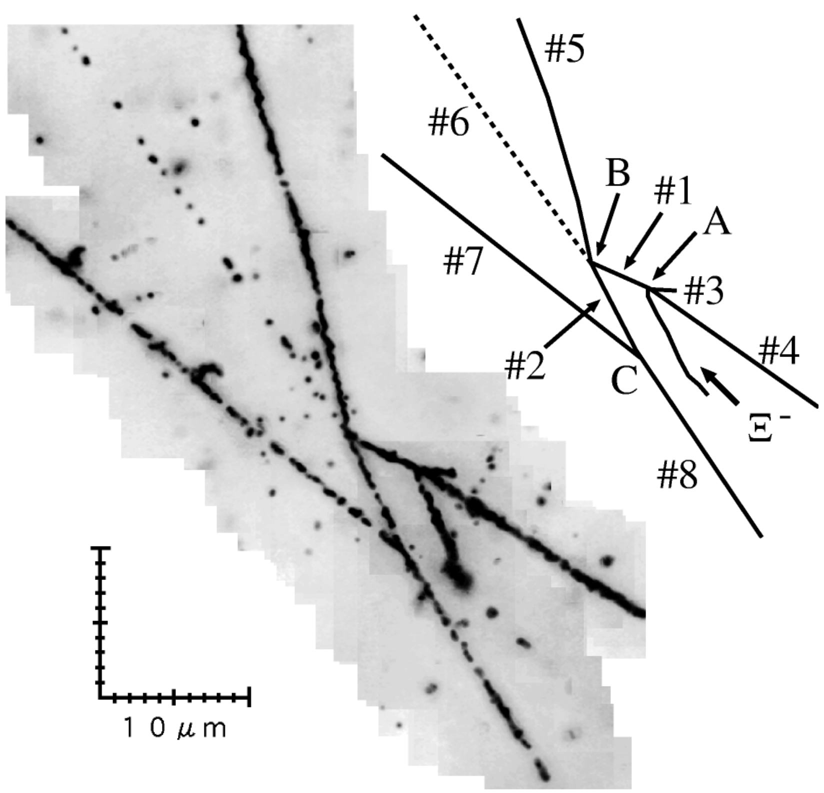
Experimental Searches

VOLUME 87, NUMBER 21 PHYSICAL REVIEW LETTERS 19 NOVEMBER 2001

Observation of a $^{\ 6}_{\Lambda\Lambda}$He Double Hypernucleus (E373@KEK):

A double-hyperfragment event has been found in a hybrid-emulsion experiment. It is identified uniquely as the sequential decay of $^{\ 6}_{\Lambda\Lambda}$He emitted from a Ξ^- hyperon nuclear capture at rest. The mass of $^{\ 6}_{\Lambda\Lambda}$He and the Λ-Λ interaction energy $\Delta B_{\Lambda\Lambda}$ have been measured for the first time devoid of the ambiguities due to the possibilities of excited states. The value of $\Delta B_{\Lambda\Lambda}$ is $1.01 \pm 0.20^{+0.18}_{-0.11}$ MeV. This demonstrates that the Λ-Λ interaction is weakly attractive.

"Nagara" event



Observation of a $^{\ 6}_{\Lambda\Lambda}$He double-hypernucleus

Binding energy:

$$B_{\Lambda\Lambda} = 7.25 \pm 0.19 \,(^{+0.18}_{-0.11})\,\mathrm{MeV}$$

Interpreted as sequential weak decay of $^{\ 6}_{\Lambda\Lambda}$He

$$m_H > 2m_\Lambda - B_{\Lambda\Lambda} = 2223.7\,\mathrm{MeV}$$ @ 90% CL

Current Status

* H-Dibaryon not firmly established experimentally

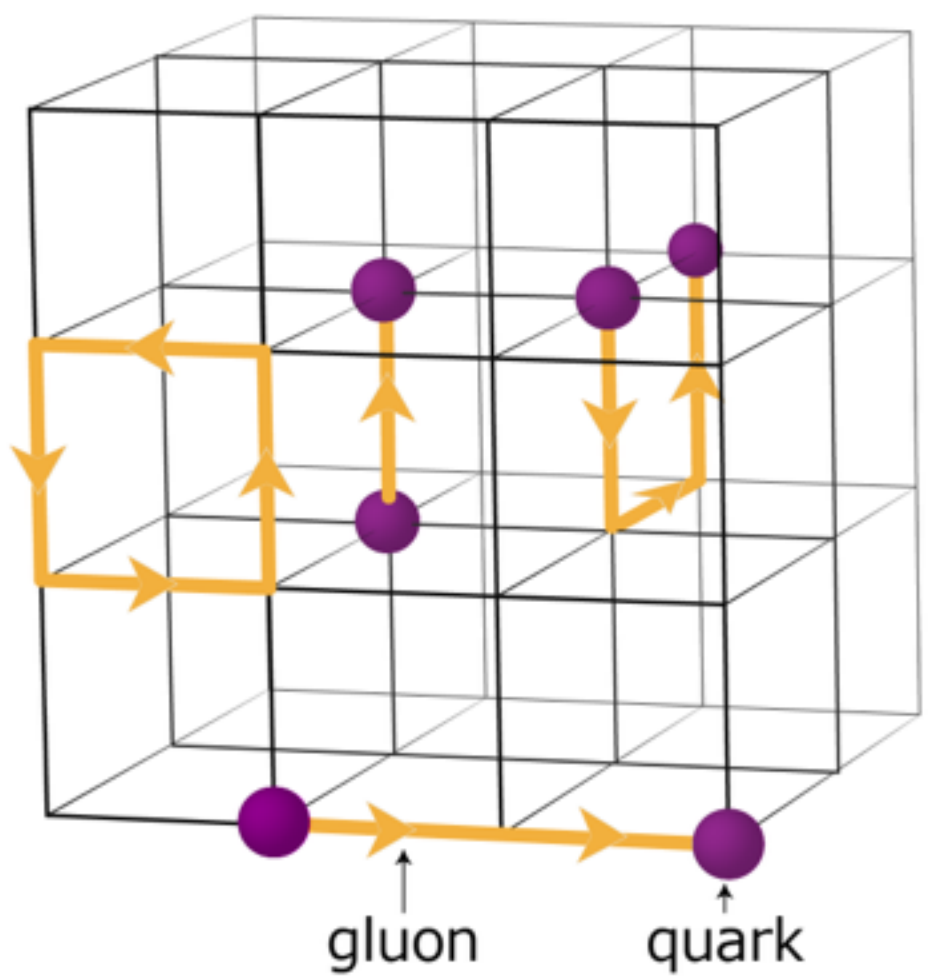
* Is a bound H-Dibaryon a consequence of QCD?

Current Status

* H-Dibaryon not firmly established experimentally

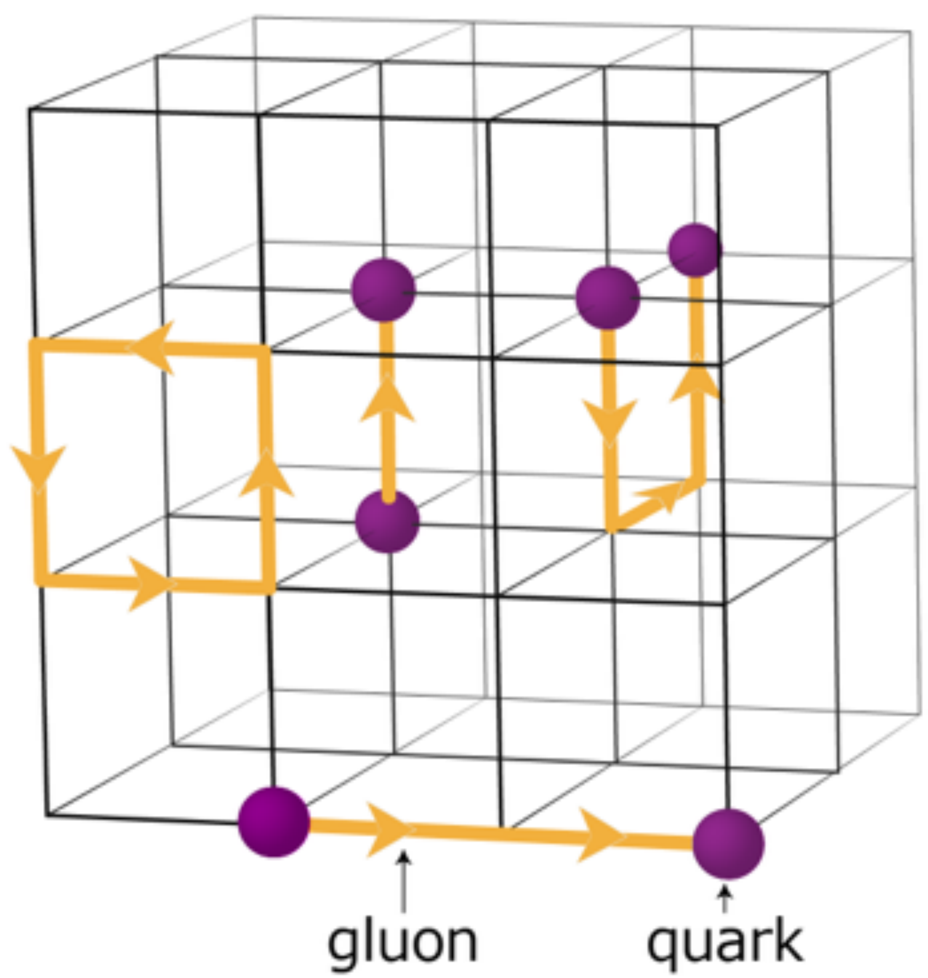
* Is a bound H-Dibaryon a consequence of QCD?

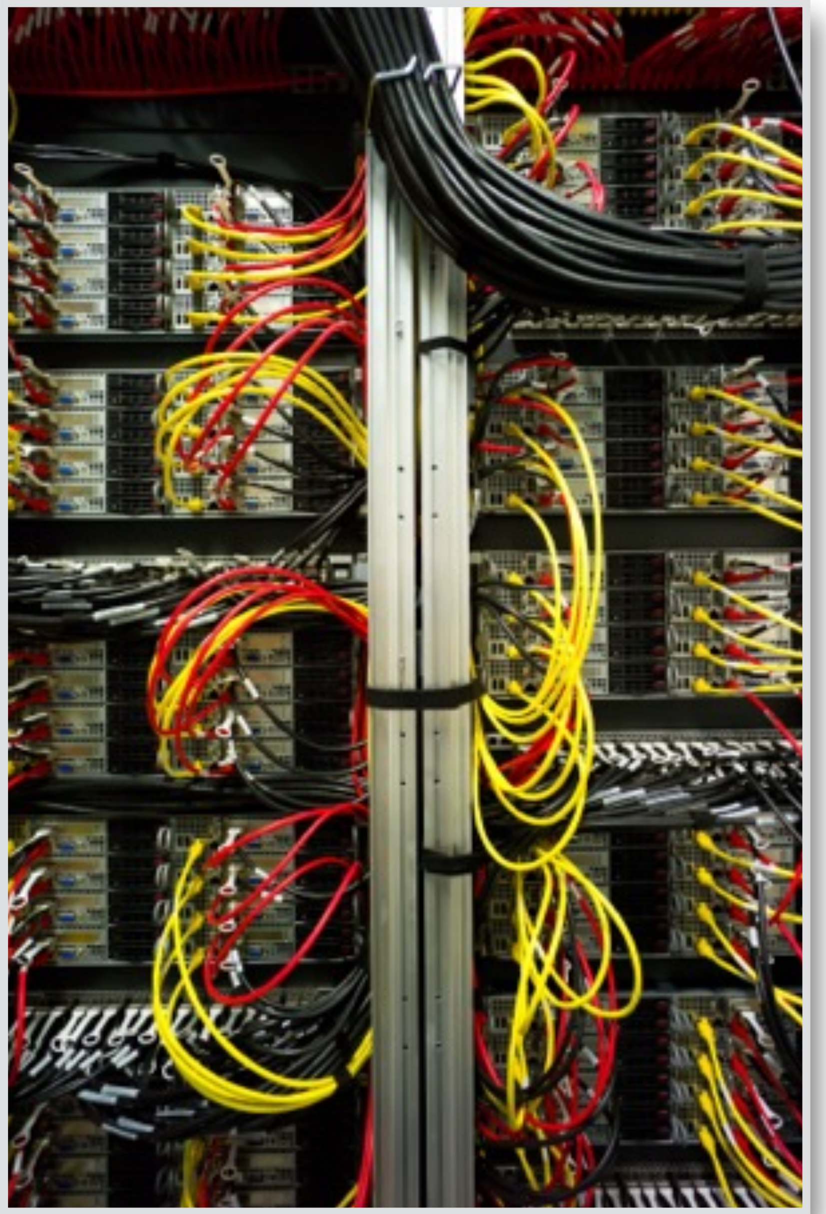
* Try "ab initio" technique: Lattice QCD



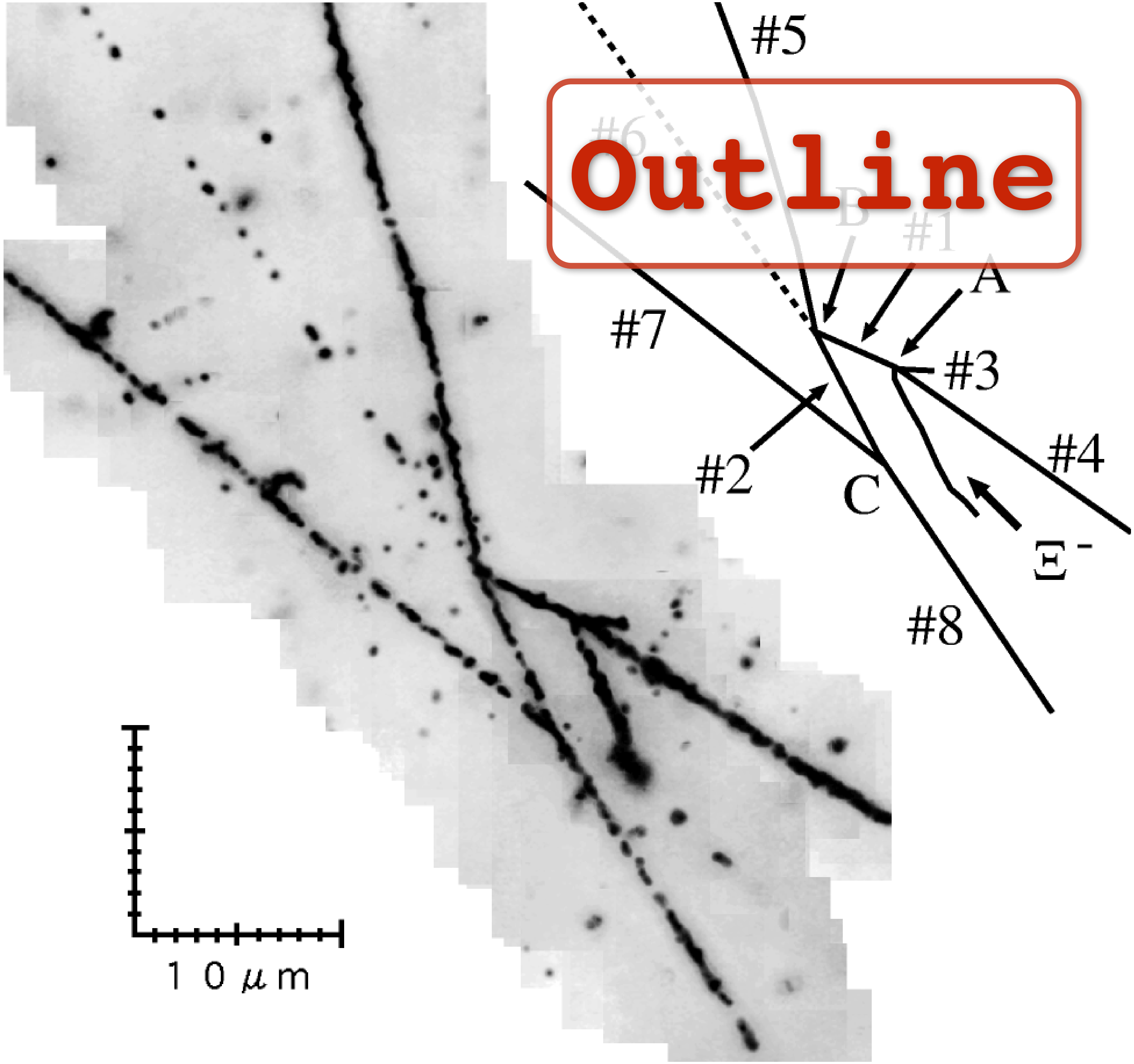
Current Status

* H-Dibaryon not firmly established experimentally
* Is a bound H-Dibaryon a consequence of QCD?
* Try "ab initio" technique: Lattice QCD



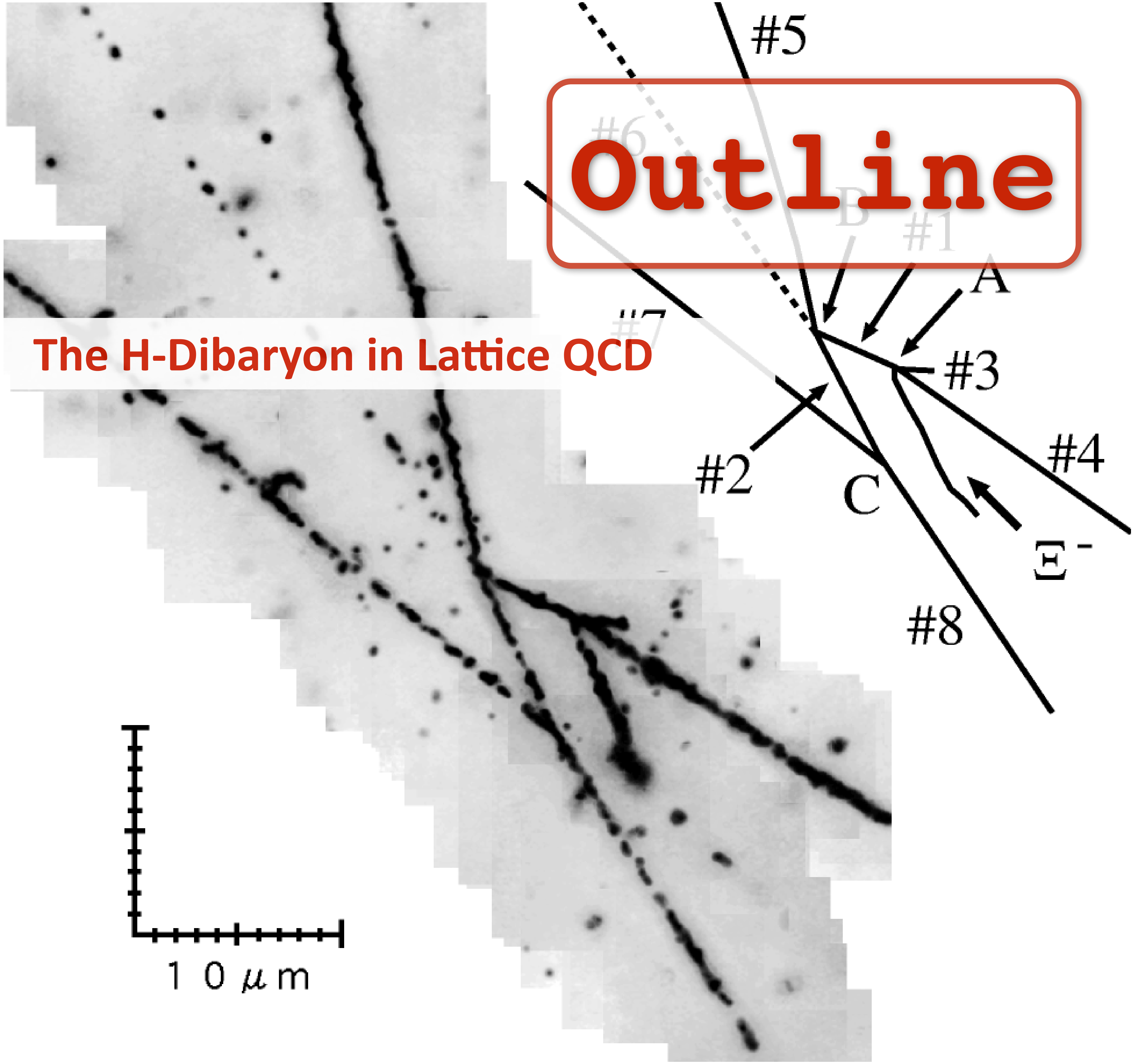


"Clover" @ Mainz



Outline

Outline

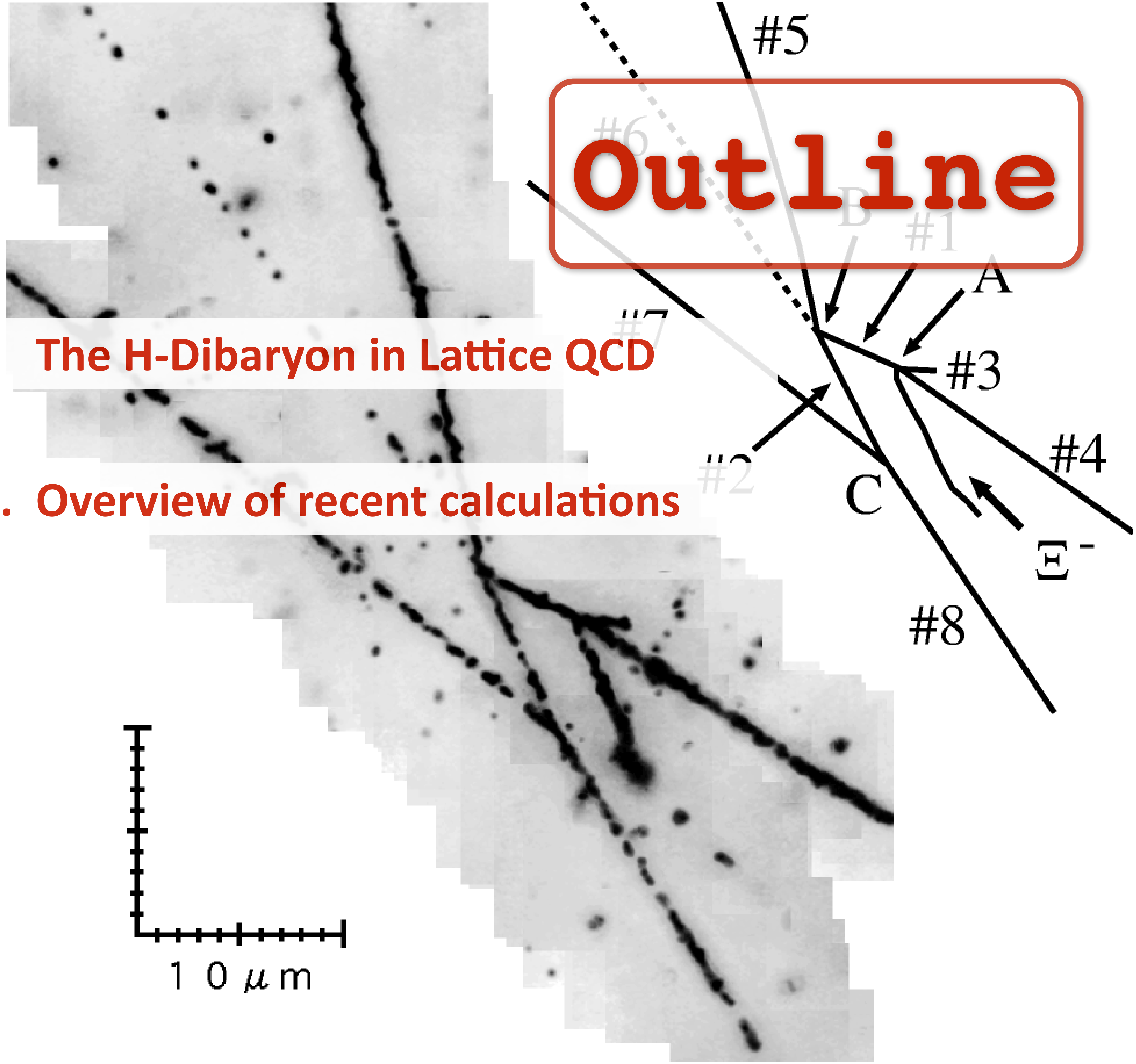


I. The H-Dibaryon in Lattice QCD

Outline

I. The H-Dibaryon in Lattice QCD

II. Overview of recent calculations

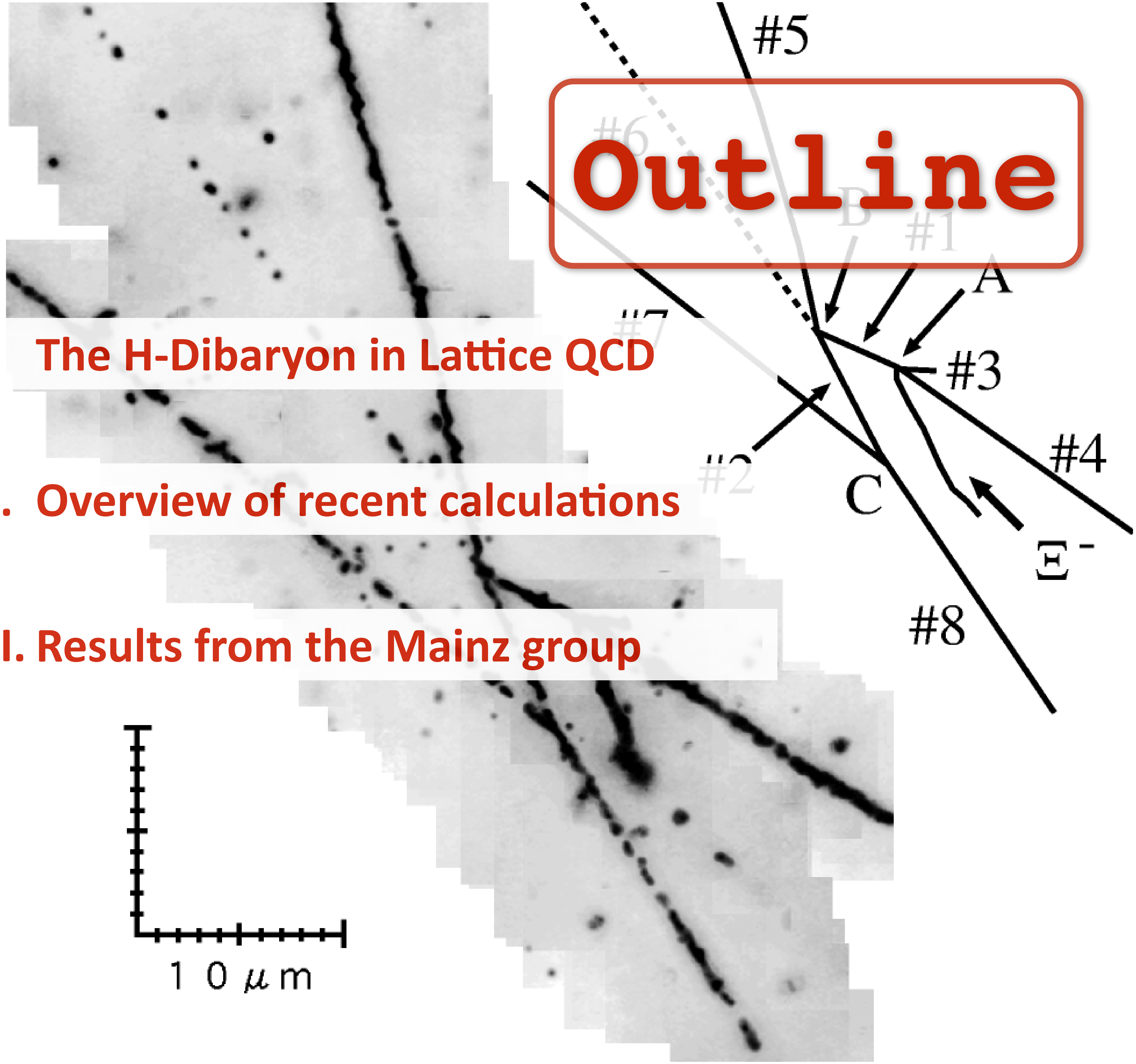


Outline

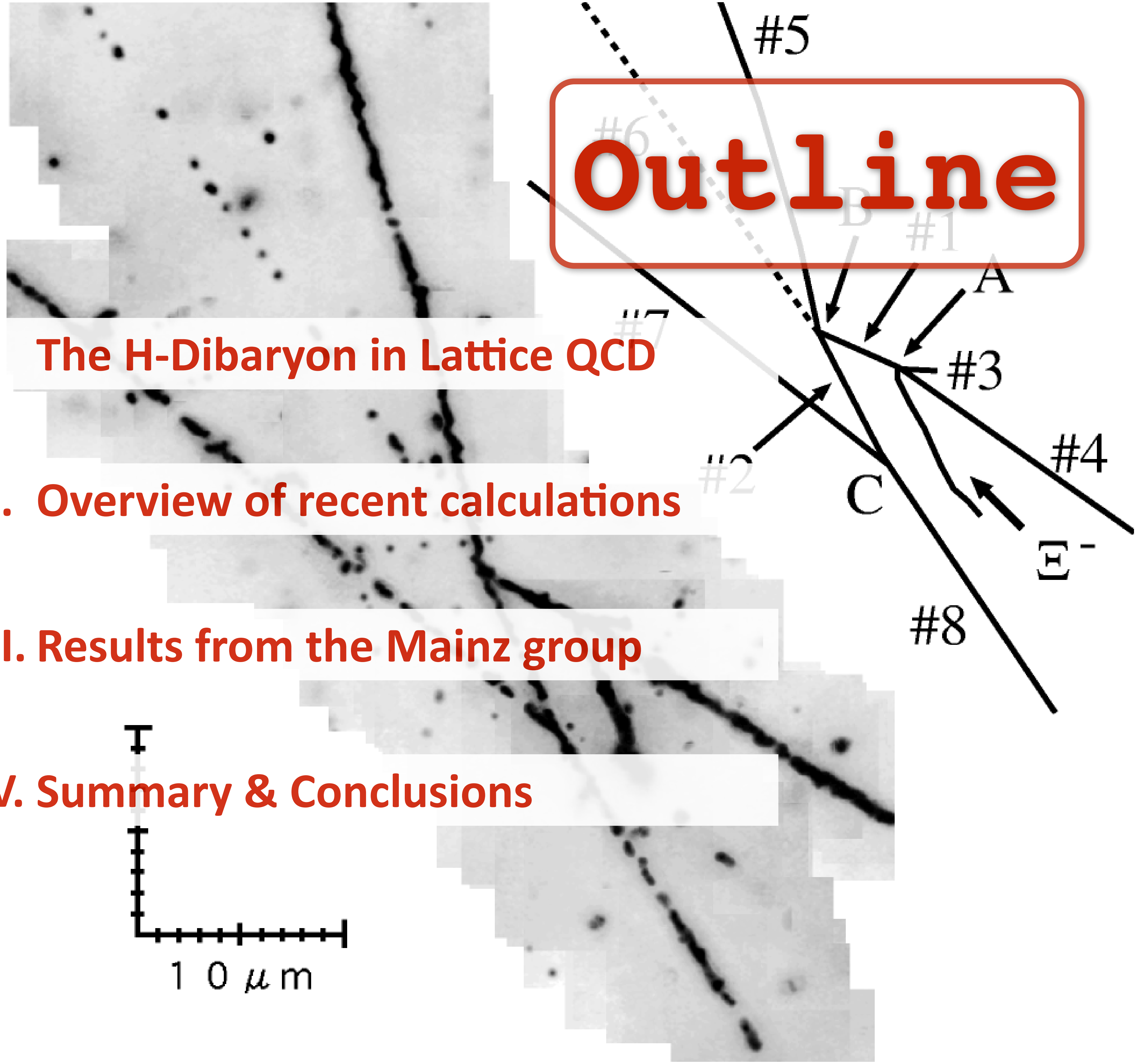
I. The H-Dibaryon in Lattice QCD

II. Overview of recent calculations

III. Results from the Mainz group



Outline



I. The H-Dibaryon in Lattice QCD

II. Overview of recent calculations

III. Results from the Mainz group

IV. Summary & Conclusions

Beyond Perturbation Theory: Lattice QCD

* Non-perturbative treatment; regularised Euclidean functional integrals

  Lattice spacing: $a, \quad x_\mu = n_\mu a, \quad a^{-1} = \Lambda_{\rm UV}$

  Finite volume: $L^3 \cdot T$

$$\langle\Omega\rangle = \frac{1}{Z}\int \prod_{x,\mu} dU_\mu(x)\,\Omega\,\prod_f \det\left(D\!\!\!/^{\rm lat} + m_f\right)\,{\rm e}^{-S_{\rm G}[U]}$$

* Stochastic evaluation of $\langle\Omega\rangle$ via Markov process

  Strong growth of numerical cost near physical m_u, m_d

* Pion mass, i.e. lightest mass in pseudoscalar channel:

$$\underset{(2001)}{\approx 500\,{\rm MeV}} \quad \longrightarrow \quad \underset{(2016)}{\approx 130-200\,{\rm MeV}}$$

Systematic effects

* Lattice artefacts: $$\left\langle\frac{m_N}{f_\pi}\right\rangle^{\text{lat}} = \left\langle\frac{m_N}{f_\pi}\right\rangle^{\text{cont}} + O(a^p), \quad p \geq 1$$

Systematic effects

- Lattice artefacts: $$\left\langle \frac{m_N}{f_\pi} \right\rangle^{\text{lat}} = \left\langle \frac{m_N}{f_\pi} \right\rangle^{\text{cont}} + O(a^p), \quad p \geq 1$$

Systematic effects

* Lattice artefacts: $$\left\langle\frac{m_N}{f_\pi}\right\rangle^{\text{lat}}=\left\langle\frac{m_N}{f_\pi}\right\rangle^{\text{cont}}+O(a^p),\quad p\geq 1$$

Systematic effects

* Lattice artefacts: $$\left\langle\frac{m_N}{f_\pi}\right\rangle^{\text{lat}} = \left\langle\frac{m_N}{f_\pi}\right\rangle^{\text{cont}} + O(a^p), \quad p \geq 1$$

Systematic effects

* Lattice artefacts: $$\left\langle \frac{m_N}{f_\pi} \right\rangle^{\text{lat}} = \left\langle \frac{m_N}{f_\pi} \right\rangle^{\text{cont}} + O(a^p), \quad p \geq 1$$

Systematic effects

* Lattice artefacts: $$\left\langle\frac{m_N}{f_\pi}\right\rangle^{\text{lat}} = \left\langle\frac{m_N}{f_\pi}\right\rangle^{\text{cont}} + O(a^p), \quad p \geq 1$$

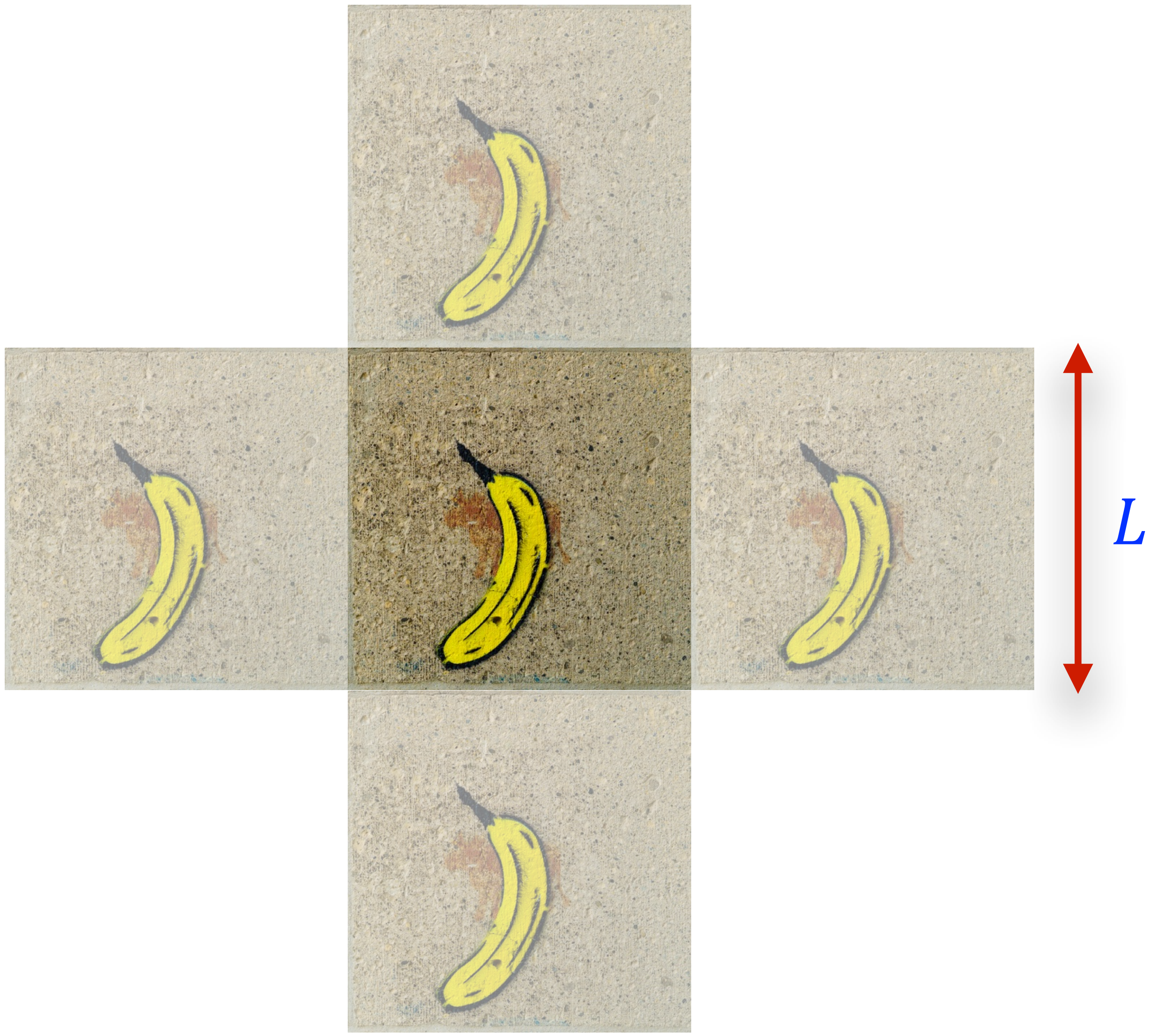
  $\longrightarrow$ extrapolate to continuum limit from $a \approx 0.05 - 0.12\,\text{fm}$

Systematic effects

* Lattice artefacts: $\left\langle \frac{m_N}{f_\pi} \right\rangle^{\text{lat}} = \left\langle \frac{m_N}{f_\pi} \right\rangle^{\text{cont}} + O(a^p), \quad p \geq 1$

  $\longrightarrow$ extrapolate to continuum limit from $a \approx 0.05 - 0.12\,\text{fm}$

* Finite volume effects



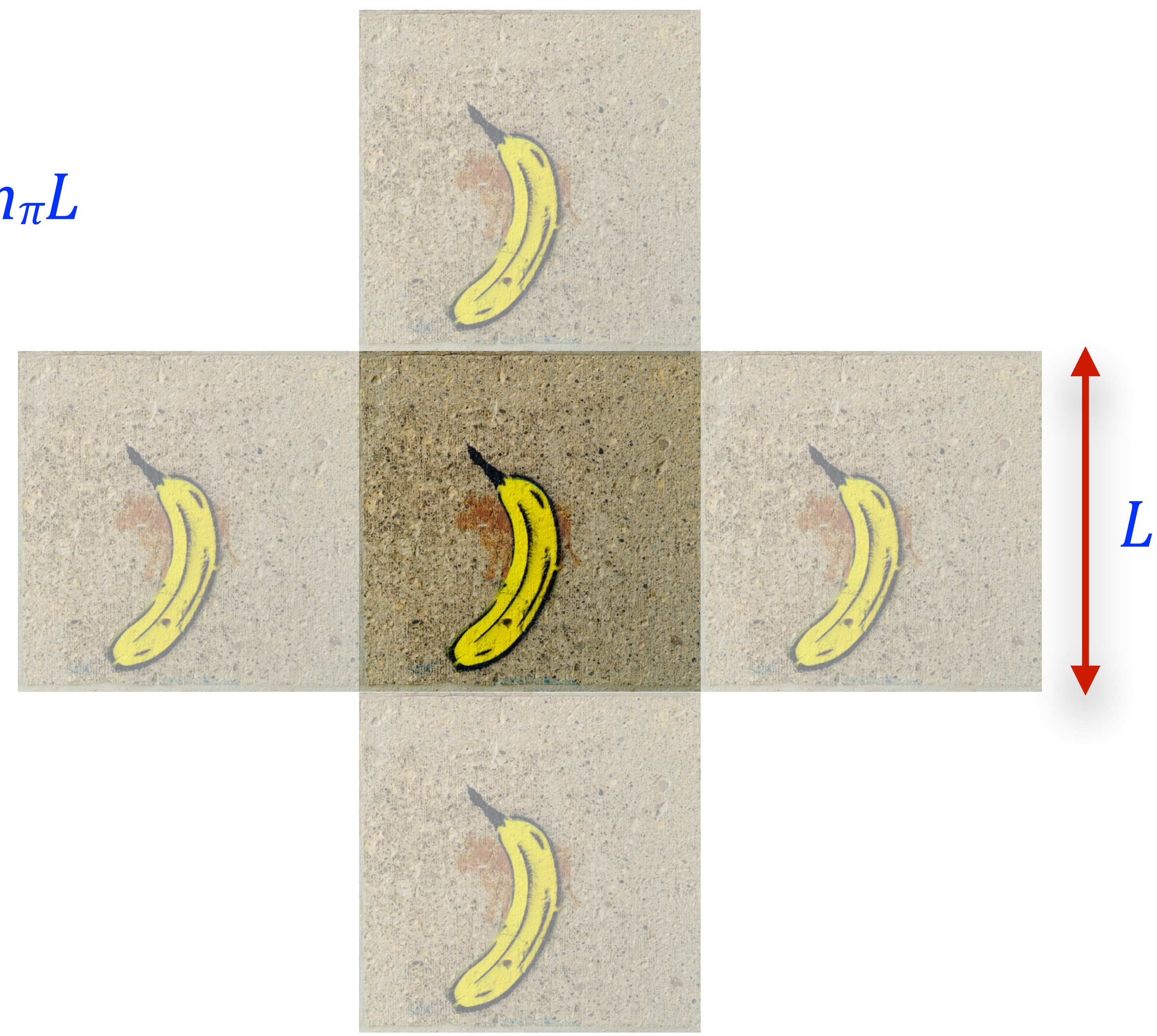
Systematic effects

* Lattice artefacts: $$\left\langle \frac{m_N}{f_\pi} \right\rangle^{\text{lat}} = \left\langle \frac{m_N}{f_\pi} \right\rangle^{\text{cont}} + O(a^p), \quad p \geq 1$$

  $\longrightarrow$ extrapolate to continuum limit from $a \approx 0.05 - 0.12\,\text{fm}$

* Finite volume effects

  Exponentially suppressed in $m_\pi L$



Systematic effects

* Lattice artefacts: $$\left\langle \frac{m_N}{f_\pi} \right\rangle^{\text{lat}} = \left\langle \frac{m_N}{f_\pi} \right\rangle^{\text{cont}} + O(a^p), \quad p \geq 1$$

  $\longrightarrow$ extrapolate to continuum limit from $a \approx 0.05 - 0.12\,\text{fm}$

* Finite volume effects
  - Empirically: $m_\pi L \geq 4$ sufficient for many purposes
  - Could be more severe for multi-baryon systems
  - Provide information on scattering phase shifts

Systematic effects

* Lattice artefacts: $\left\langle \frac{m_N}{f_\pi} \right\rangle^{\text{lat}} = \left\langle \frac{m_N}{f_\pi} \right\rangle^{\text{cont}} + O(a^p), \quad p \geq 1$

  $\longrightarrow$ extrapolate to continuum limit from $a \approx 0.05 - 0.12\,\text{fm}$

* Finite volume effects
  - Empirically: $m_\pi L \geq 4$ sufficient for many purposes
  - Could be more severe for multi-baryon systems
  - Provide information on scattering phase shifts

* Unphysical quark masses
  - Chiral extrapolation to physical values of m_u, m_d

Systematic effects

- ✱ Lattice artefacts: $$\left\langle\frac{m_N}{f_\pi}\right\rangle^{\text{lat}} = \left\langle\frac{m_N}{f_\pi}\right\rangle^{\text{cont}} + O(a^p), \quad p \geq 1$$

  $\longrightarrow$ extrapolate to continuum limit from $a \approx 0.05 - 0.12\,\text{fm}$

- ✱ Finite volume effects
  - Empirically: $m_\pi L \geq 4$ sufficient for many purposes
  - Could be more severe for multi-baryon systems
  - Provide information on scattering phase shifts

- ✱ Unphysical quark masses
  - Chiral extrapolation to physical values of m_u, m_d

- ✱ Inefficient sampling of SU(3) group manifold
  - Simulations become trapped in topological sectors as $a \rightarrow 0$
  - Use open boundary conditions in time direction *[Lüscher & Schaefer, 2012]*

The H-Dibaryon in Lattice QCD

* Spectrum extracted from correlation functions:

$$\sum_{\vec{x},\vec{y}} \left\langle O_{\rm had}(y) O_{\rm had}^{\dagger}(x) \right\rangle = Z_0\, {\rm e}^{-E_0(y_0-x_0)} + Z_1\, {\rm e}^{-E_1(y_0-x_0)} + \ldots$$

$O_{\rm had}(x)$: interpolating operator for a given hadron;

projects on all states with the same quantum numbers

nucleon : $O_{\rm N} = \epsilon_{abc} \left(u^a C \gamma_5 d^b \right) u^c$

The H-Dibaryon in Lattice QCD

* Spectrum extracted from correlation functions:

$$\sum_{\vec{x},\vec{y}} \left\langle O_{\rm had}(y) O_{\rm had}^{\dagger}(x) \right\rangle = Z_0\, {\rm e}^{-E_0(y_0-x_0)} + Z_1\, {\rm e}^{-E_1(y_0-x_0)} + \ldots$$

ground state energy

$O_{\rm had}(x)$: interpolating operator for a given hadron;
projects on all states with the same quantum numbers

nucleon : $O_{\rm N} = \epsilon_{abc} \left(u^a C \gamma_5 d^b \right) u^c$

The H-Dibaryon in Lattice QCD

* Spectrum extracted from correlation functions:

$$\sum_{\vec{x},\vec{y}} \left\langle O_{\rm had}(y) O_{\rm had}^{\dagger}(x) \right\rangle = Z_0\, \mathrm{e}^{-E_0(y_0-x_0)} + Z_1\, \mathrm{e}^{-E_1(y_0-x_0)} + \ldots$$

ground state energy

$O_{\rm had}(x)$: interpolating operator for a given hadron;

projects on all states with the same quantum numbers

nucleon : $O_{\rm N} = \epsilon_{abc} \left(u^a C \gamma_5 d^b \right) u^c$

* Ground state dominates at large Euclidean times: $y_0 - x_0 \to \infty$

The H-Dibaryon in Lattice QCD

* Spectrum extracted from correlation functions:

$$\sum_{\vec{x},\vec{y}} \left\langle O_{\rm had}(y) O_{\rm had}^{\dagger}(x) \right\rangle = Z_0\, \mathrm{e}^{-E_0(y_0-x_0)} + Z_1\, \mathrm{e}^{-E_1(y_0-x_0)} + \ldots$$

ground state energy

$O_{\rm had}(x)$: interpolating operator for a given hadron;
projects on all states with the same quantum numbers

nucleon : $O_{\rm N} = \epsilon_{abc} \left(u^a C\gamma_5 d^b \right) u^c$

* Ground state dominates at large Euclidean times: $y_0 - x_0 \to \infty$

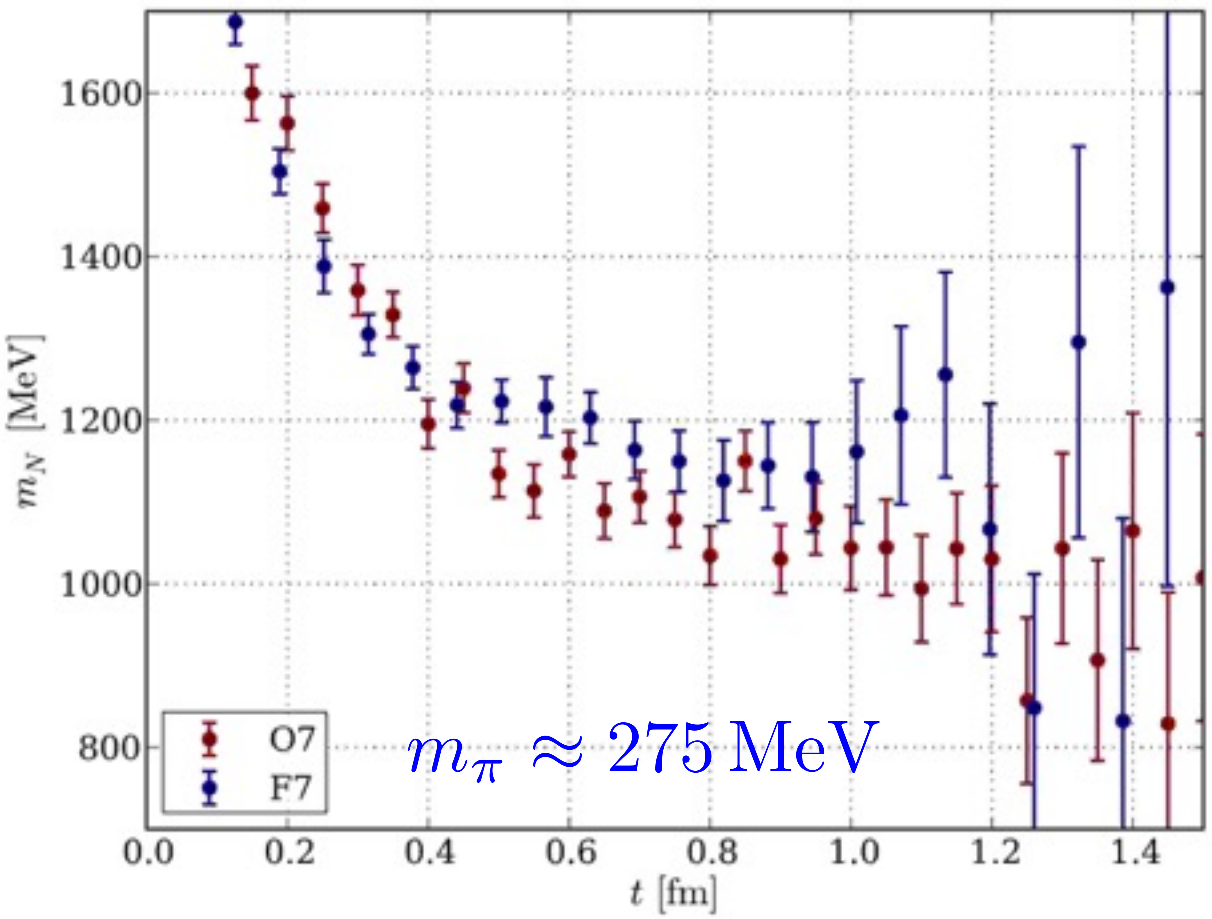
* Excited states are sub-leading contributions

The H-Dibaryon in Lattice QCD

* Noise problem of baryonic correlation functions:

  Exponential growth of noise-to-signal ratio

  Nucleon at rest: $R_{\mathrm{NS}}(x_0) = \mathrm{e}^{(m_{\mathrm{N}} - \frac{3}{2} m_\pi) x_0}$



[Capitani et al., arXiv:1504.04628]

- Excited state contributions die out slowly
- Ground state dominates only for $a \geq 0.5\,\mathrm{fm}$
- Precise calculations require very large statistics

The H-Dibaryon in Lattice QCD

- **Lüscher Method:** Finite-volume effects provide physical information
- Two-particle binding momentum:

$$p^2 = \frac{1}{4}(E^2 - \vec{P}\cdot\vec{P}) - m_\Lambda^2 \qquad E,\ m_\Lambda:$$ determined in finite volume

- Relation to scattering phase shifts in infinite volume:

$$p\cot\delta_0(p) = \frac{2}{\gamma L\sqrt{\pi}}\mathcal{Z}_{0,0}(1,q^2), \quad q = \frac{pL}{2\pi}$$

[Lüscher 1991, Rummukainen & Gottlieb 1995]

$$\mathcal{Z}_{0,0}(1,q^2) = \frac{1}{\sqrt{4\pi}}\left\{\sum_{q^2\neq n^2}^{\Lambda_n}\frac{1}{q^2-n^2} - 4\pi\Lambda_n\right\}$$

The H-Dibaryon in Lattice QCD

* **Lüscher Method:** Finite-volume effects provide physical information

* Two-particle binding momentum:

$$p^2 = \frac{1}{4}(E^2 - \vec{P}\cdot\vec{P}) - m_\Lambda^2$$

$E,\ m_\Lambda:$ determined in finite volume

* Relation to scattering phase shifts in infinite volume:

$$p\cot\delta_0(p) = \frac{2}{\gamma L\sqrt{\pi}}\mathcal{Z}_{0,0}(1,q^2), \quad q = \frac{pL}{2\pi}$$

[Lüscher 1991, Rummukainen & Gottlieb 1995]

$$\mathcal{Z}_{0,0}(1,q^2) = \frac{1}{\sqrt{4\pi}}\left\{\sum_{q^2\neq n^2}^{\Lambda_n}\frac{1}{q^2-n^2} - 4\pi\Lambda_n\right\}$$

* Scattering amplitude: $\mathcal{A} \propto \dfrac{1}{p\cot\delta_0(p) - ip}$

Pole corresponds to binding energy in infinite volume

Recent lattice calculations: Overview

Collaboration	Method	$m_\pi\,[\mathrm{MeV}]$	N_f	References
HALQCD	Baryon-baryon potential; Nambu-Bethe-Salpeter wave function	470–1170	3	*Phys Rev Lett 106 (2011) 162002* *Nucl Phys A881 (2012) 28*
NPLQCD	Two-point correlation functions	806	3	*Phys Rev D87 (2013) 034506*
	Two-point correlation functions	230, 390	2+1	*Phys Rev Lett 106 (2011) 162001* *Mod Phys Lett A26 (2011) 2587*
Mainz	Two-point correlation functions	450–1000	2	*PoS LATTICE2013 (2014) 440* *PoS LATTICE2014 (2015) 107* *arXiv:1511.01849 [hep-lat]*

* NPLQCD and HALQCD find bound H-dibaryon for $m_\pi \geq 400\,\mathrm{MeV}$

HALQCD Collaboration

* Obtain baryon-baryon potential from Nambu-Bethe-Salpeter amplitude computed on the lattice

$$G_4(\vec{r}, t-t_0) = \left\langle 0 \left| (BB)^{(\alpha)}(\vec{r},t)(\overline{BB})^{(\alpha)}(t_0) \right| 0 \right\rangle = \phi(\vec{r},t)\,\mathrm{e}^{-2M(t-t_0)}$$

$(BB)^{(\alpha)}(\vec{r},t)$	:	2-baryon interpolating operator; flavour irrep. α
$\phi(\vec{r},t)$	:	NBS wave function
M	:	single baryon mass

* Determine potential via $$V(r) = \frac{[-H_0 - (\partial/\,\partial t)]\,\phi(\vec{r},t)}{\phi(\vec{r},t)}$$

* Solve the Schrödinger equation to determine binding energies and scattering phase shifts

HALQCD Collaboration

* Details of the calculation:

  $N_\mathrm{f} = 3$ i.e. mass-degenerate *u, d, s* quarks

  Single lattice spacing: $a = 0.121(2)\,\mathrm{fm}$

  5 pion masses in the range: $m_\pi = 469 - 1171\,\mathrm{MeV}$

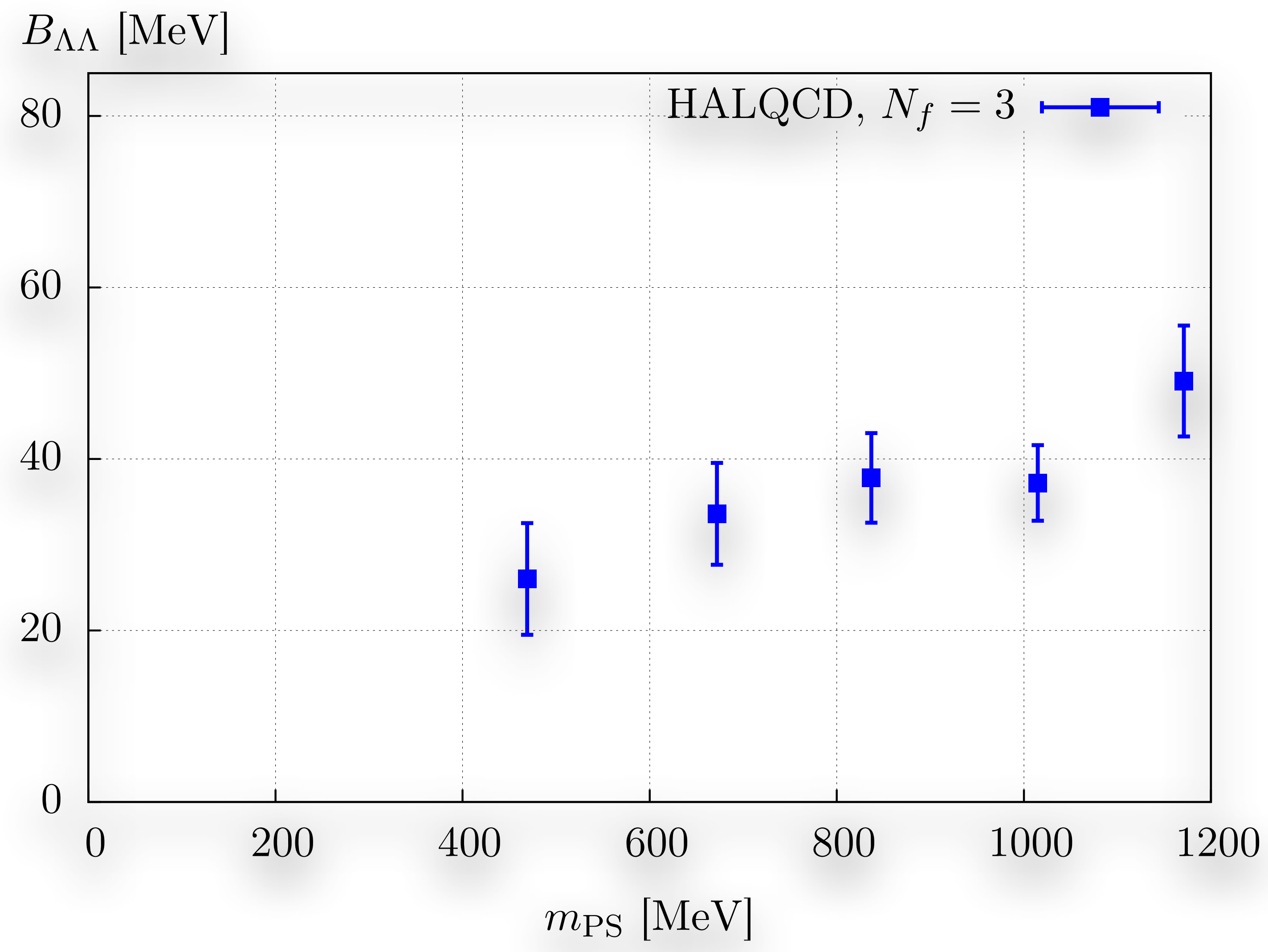
  Volumes: $L = 3.87,\ 2.90,\ 1.97\,\mathrm{fm}$

  Statistics: $O(500)$ gauge configurations per ensemble

  $O(8000)$ “measurements” for each pion mass

HALQCD Collaboration

* Chiral behaviour of H-dibaryon binding energies in SU(3) limit:



(statistical and systematic errors combined)

* SU(3) breaking effects not accounted for

Variational method

* Consider set of interpolating operators: $O_i, \quad i = 1, \ldots, N$

  Matrix correlator: $$C_{ij}(t) = \sum_{\vec{x}} \left\langle O_i(\vec{x}, t + t_0) O_j^\dagger(\vec{x}_0, t_0) \right\rangle$$

* Variational method: solve Generalised Eigenvalue Problem (GEVP):

  $$\mathbf{C}(t + \Delta t) v_n(t) = \lambda_n(t) \mathbf{C}(t) v_n(t) \qquad E_{\text{eff}}(t) = -\partial_t \log \lambda_n(t)$$

* Operator basis:
  - Apply "smearing" to interpolating operators at source and sink
  - Use 6-quark and two-baryon operators
  - Project onto irreducible representations in flavour space

Variational basis in the dibaryon channel

- Six-quark operators:

$$[rstuvw] = \epsilon_{ijk}\epsilon_{lmn}\left(s^a C\gamma_5 P_+ t^b\right)\left(v^l C\gamma_5 P_+ w^m\right)\left(r^k C\gamma_5 P_+ u^n\right)$$

$$H^{(\mathbf{1})} = \frac{1}{48}\left([sudsud] - [udusds] - [dudsus]\right)$$

$$H^{(\mathbf{27})} = \frac{1}{48\sqrt{3}}\left(2[sudsud] + [udusds] - [dudsus]\right)$$

- Momentum-projected two-baryon operators:

$$B_\alpha \equiv [rst]_\alpha = \epsilon_{ijk}\left(s^i C\gamma_5 t^j\right) r^k_\alpha$$

$$(BB)(\vec{p}_1,\vec{p}_2;t) = \sum_{\vec{x},\vec{y}} \mathrm{e}^{i\vec{p}_1\cdot\vec{x}}\mathrm{e}^{i\vec{p}_2\cdot\vec{y}} B_1(\vec{x},t) C\gamma_5 P_+ B_2(\vec{y},t)$$

Construct $(BB)^{(\mathbf{1})},\ (BB)^{(\mathbf{8})},\ (BB)^{(\mathbf{27})}$

NPLQCD Collaboration

- Compute energy levels in Di-baryon channel from two-point correlation functions of multi-baryon operators:

$$O_{\rm H}^{\rm 2baryon} = \epsilon_{ijk}\left\{\left(r^i C\gamma_5 P_+ s^j\right) t^k\right\}(\vec{x},t)\,\epsilon_{lmn}\left\{\left(u^l C\gamma_5 P_+ v^m\right) w^n\right\}(\vec{y},t)$$

- Binding energy: $B_{\Lambda\Lambda} = 2m_\Lambda - E_{\Lambda\Lambda}$

- Details of the calculation: *[Beane et al, Phys Rev D87 (2013) 034506]*

  $N_{\rm f} = 3$ i.e. mass-degenerate *u, d, s* quarks

  Single lattice spacing: $a = 0.145(2)\,{\rm fm}$

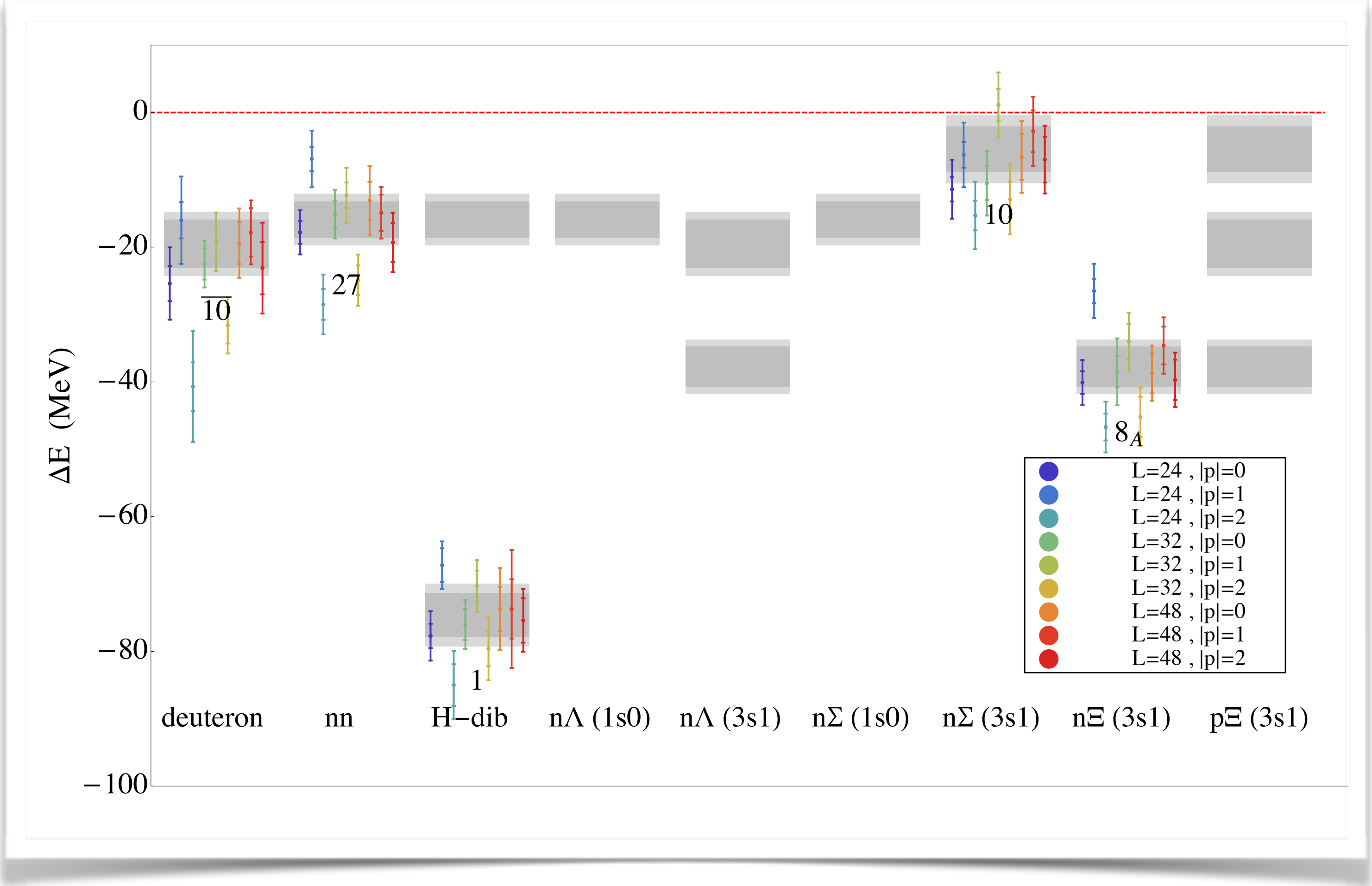
  Single pion mass: $m_\pi = 807\,{\rm MeV}$

  Volumes: $L = 3.4,\ 4.5,\ 6.7\,{\rm fm}$

  Statistics: $O(10^5)$ “measurements” per ensemble

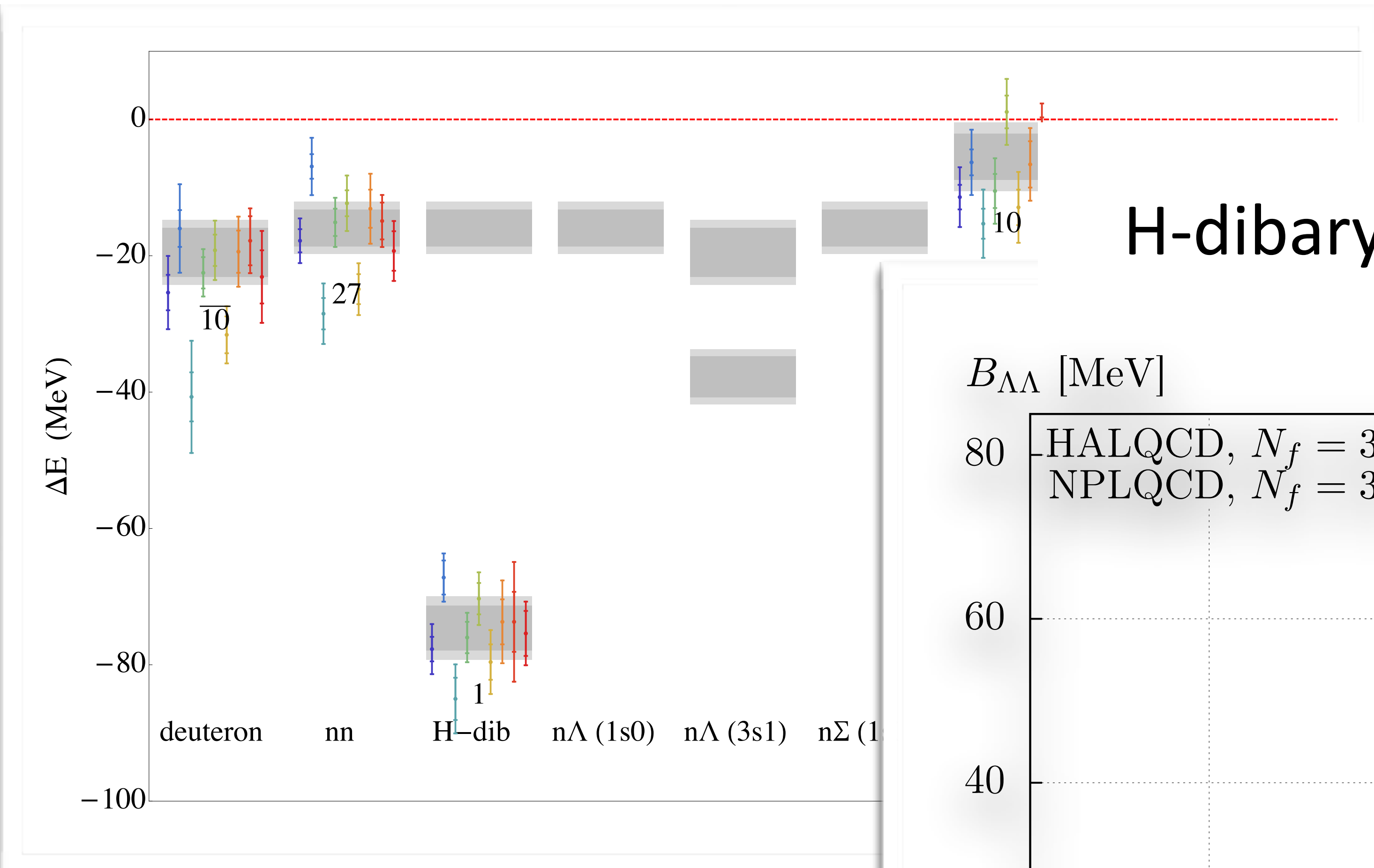
NPLQCD Collaboration

Binding energies of various two-baryon systems:

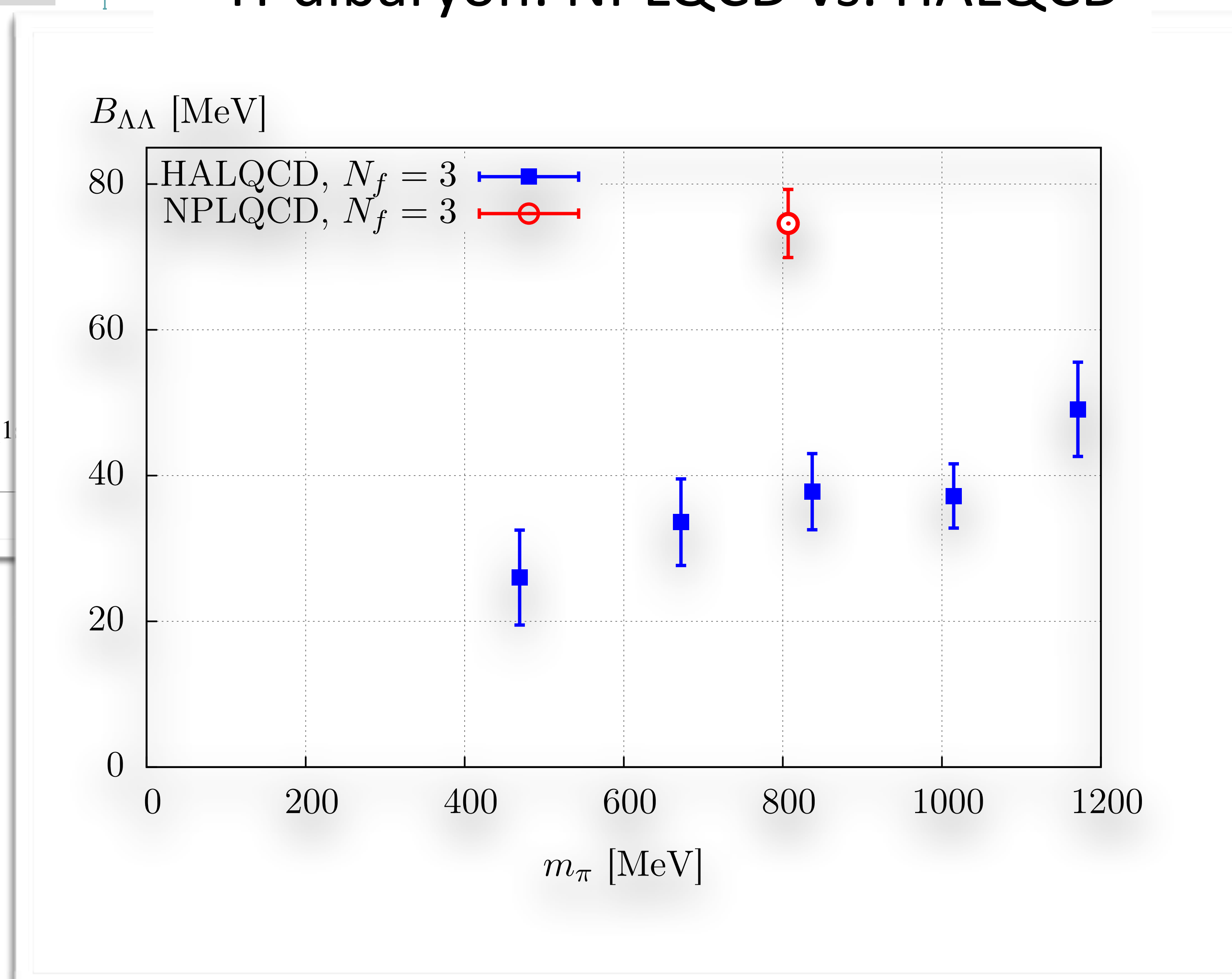


NPLQCD Collaboration

Binding energies of various two-baryon systems:



H-dibaryon: NPLQCD vs. HALQCD



NPLQCD Collaboration

* Details of the calculation:

[Beane et al, Phys Rev Lett (2011) 162001, Mod Phys Lett A26 (2011) 2587]

$N_f = 2+1,$ anisotropic action: $a_s/a_t = 3.50(3)$

Single lattice spacing: $a_s = 0.123(1)\,\text{fm}$

Pion masses: $m_\pi = 389, 230\,\text{MeV}$

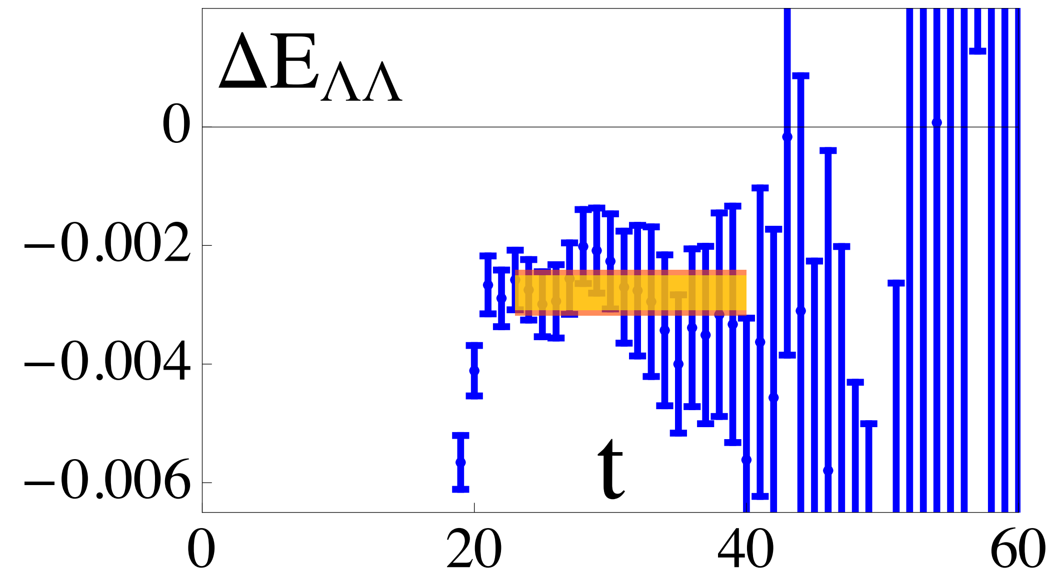
Volumes: $L = 3.9,\ 3.0,\ 2.5,\ 2.0\,\text{fm}$

Statistics: $O(10^5)$ "measurements" per ensemble

* Variational approach:

source: six-quark operators

sink: two-baryon operators



The Mainz Dibaryon Project

* In collaboration with:

  A. Francis, J. Green, M. Hansen, P. Junnarkar, Ch. Miao, T. Rae

* $N_f = 2$ flavours of $O(a)$ improved Wilson fermions; quenched strange quark

The Mainz Dibaryon Project

- In collaboration with:

  A. Francis, J. Green, M. Hansen, P. Junnarkar, Ch. Miao, T. Rae

- $N_f = 2$ flavours of $O(a)$ improved Wilson fermions; quenched strange quark

Run	a [fm]	L/a	L [fm]	m_π [MeV]	$m_\pi L$	N_{cfg}	N_{src}	N_{meas}
E1	0.063	32	2.02	1000	10.2	168	128	43 008
N1	0.050	48	2.45	830	10.1	100	128	25 600
A1	0.079	32	2.53	770	9.9	286	128	73 216
E5	0.063	32	2.03	451	4.6	1990	32	127 360

The Mainz Dibaryon Project

* In collaboration with:

  A. Francis, J. Green, M. Hansen, P. Junnarkar, Ch. Miao, T. Rae

* $N_f = 2$ flavours of $O(a)$ improved Wilson fermions; quenched strange quark

Run	a [fm]	L/a	L [fm]	m_π [MeV]	$m_\pi L$	N_{cfg}	N_{src}	N_{meas}
E1	0.063	32	2.02	1000	10.2	168	128	43 008
N1	0.050	48	2.45	830	10.1	100	128	25 600
A1	0.079	32	2.53	770	9.9	286	128	73 216
E5	0.063	32	2.03	451	4.6	1990	32	127 360

* Ensembles E1, N1, A1: $m_s = m_{light}$, i.e. SU(3)-symmetric

Noise reduction

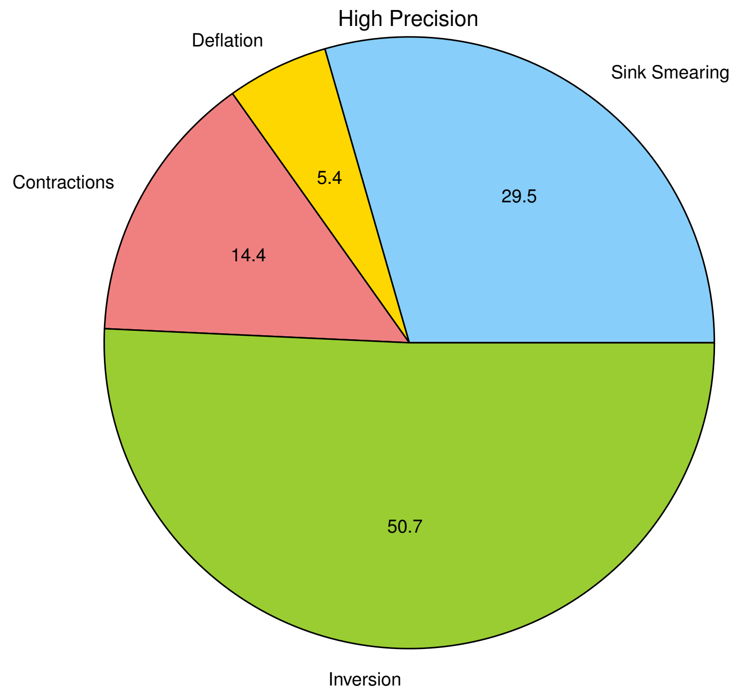
* **All-mode-averaging** *[Blum et al, Phys Rev D88 (2013) 094503*

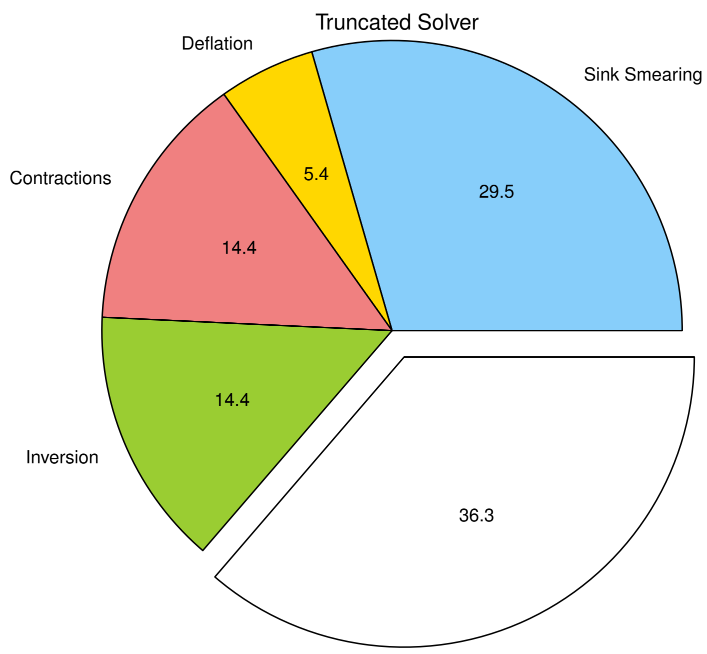
Combine low-precision solver with bias correction:

$$O = O_{\vec{x}_0} - O_{\vec{x}_0}^{(\text{apprx})} + \frac{1}{N_{\Delta\vec{x}}} \sum_{\Delta\vec{x}} O_{\vec{x}_0+\Delta\vec{x}}^{(\text{apprx})}$$

Variance reduction:

$$2(1-r) + \frac{1}{N_{\Delta\vec{x}}}, \quad r = \text{Corr}\left(O_{\vec{x}+\Delta\vec{x}}^{(\text{apprx})}, O_{\vec{x}+\Delta\vec{x}'}^{(\text{apprx})}\right)$$





GEVP Setup

* Ensembles $\mathrm{E1}$, $\mathrm{N1}$, $\mathrm{A1}$— SU(3)-symmetric situation:

  source: 6-quark operator $H^{(\mathbf{1})}$with two different smearing levels

  sink: 6-quark and multi-baryon operators

* Ensemble $\mathrm{E5}$ — estimate SU(3)-breaking effects:

  source: 6-quark operators $H^{(\mathbf{1})}, H^{(\mathbf{27})}$, different smearing levels

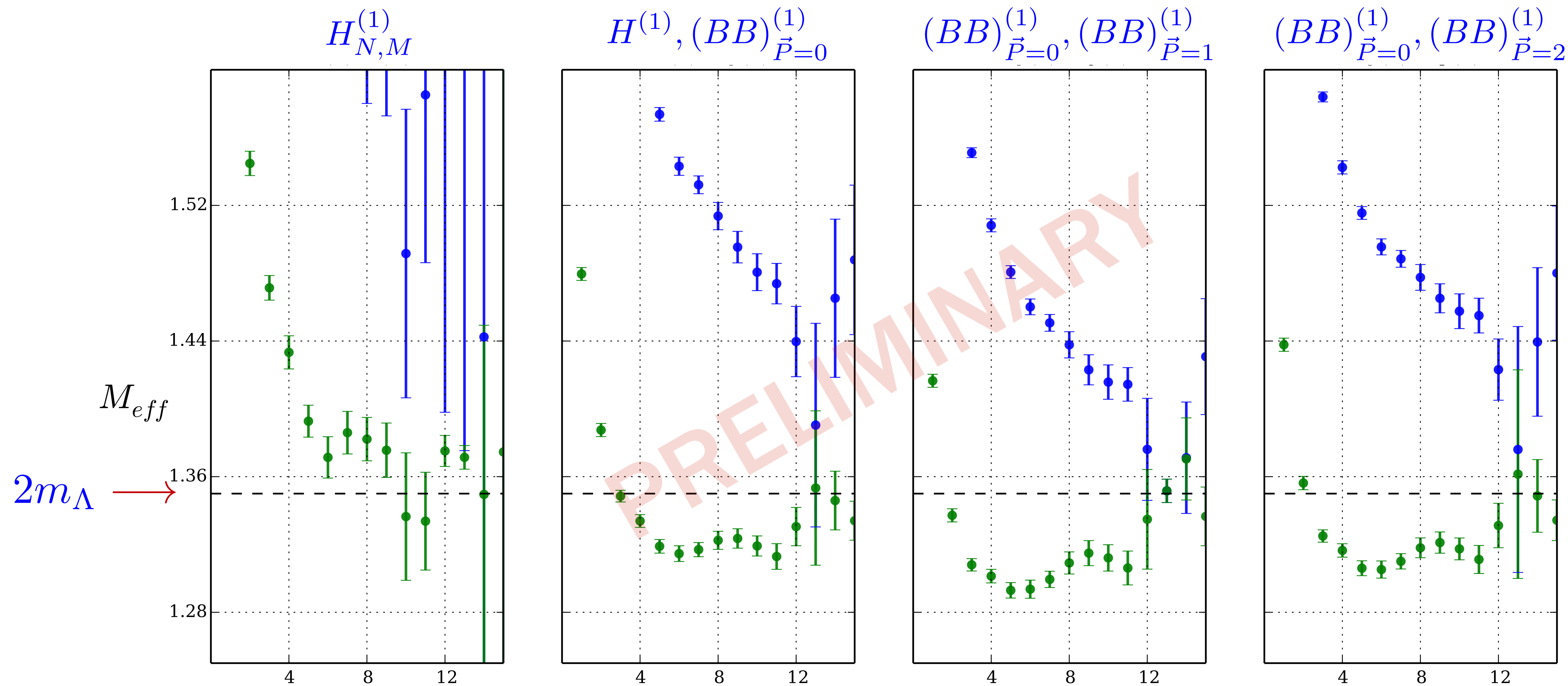
  sink: $H^{(\mathbf{1})}$ and $H^{(\mathbf{27})}$ or $(BB)^{(\mathbf{1})}$ and $(BB)^{(\mathbf{27})}$

* Asymmetric GEVP: use different sink operators to check for systematics

* Different total momentum — rest frame vs. moving frame

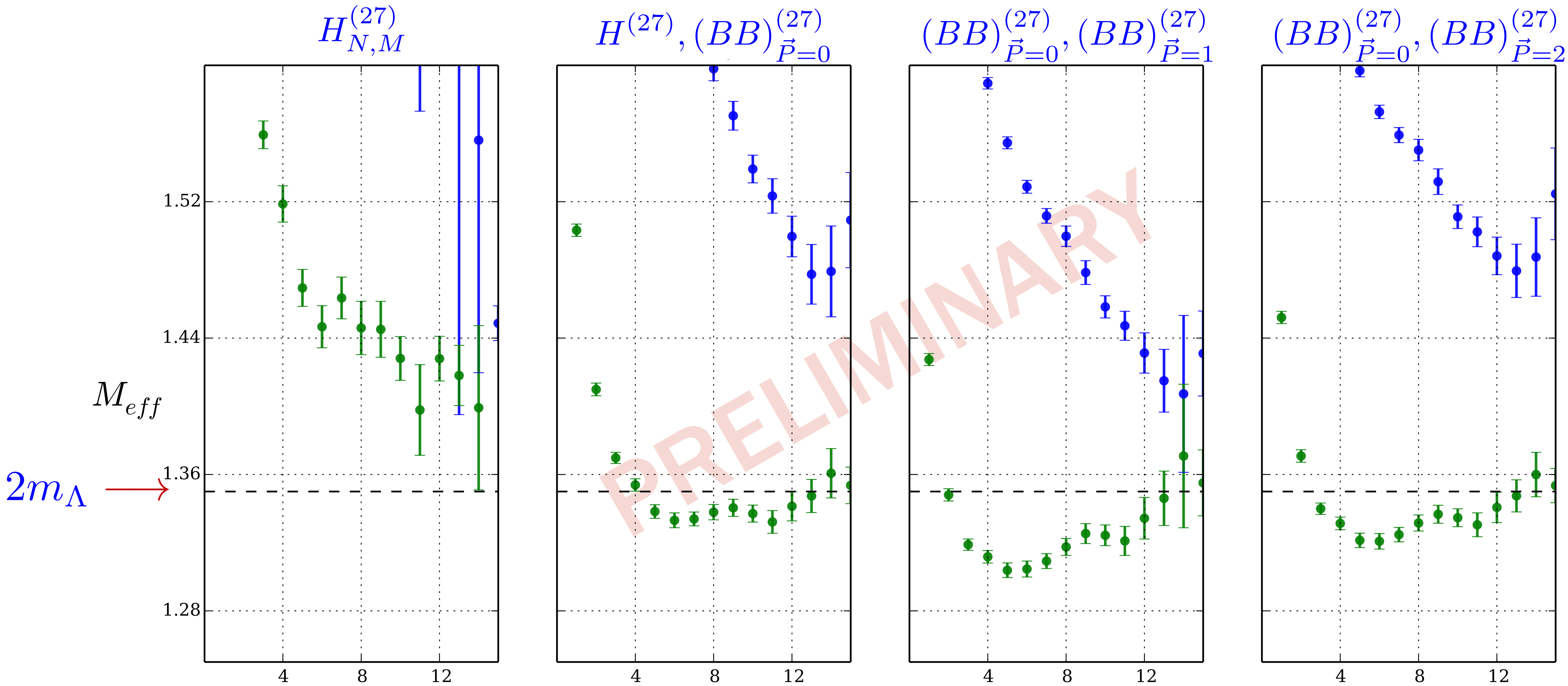
SU(3)-symmetric case

- Ensemble E1 (m_π = 1000 MeV)
- Energy levels probed by different sink operators — singlet channel:



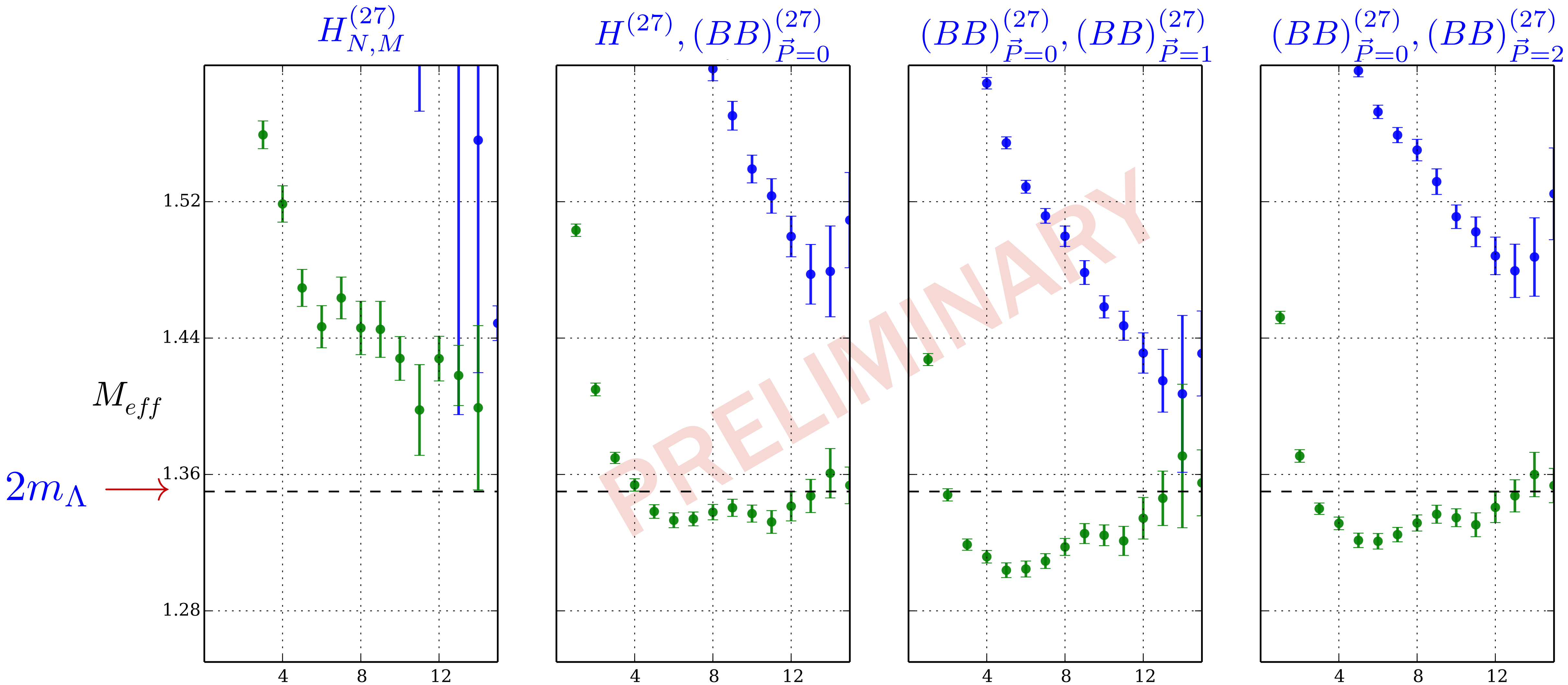
SU(3)-symmetric case

* Ensemble E1 (m_π = 1000 MeV)
* Energy levels probed by different sink operators — 27-plet:



SU(3)-symmetric case

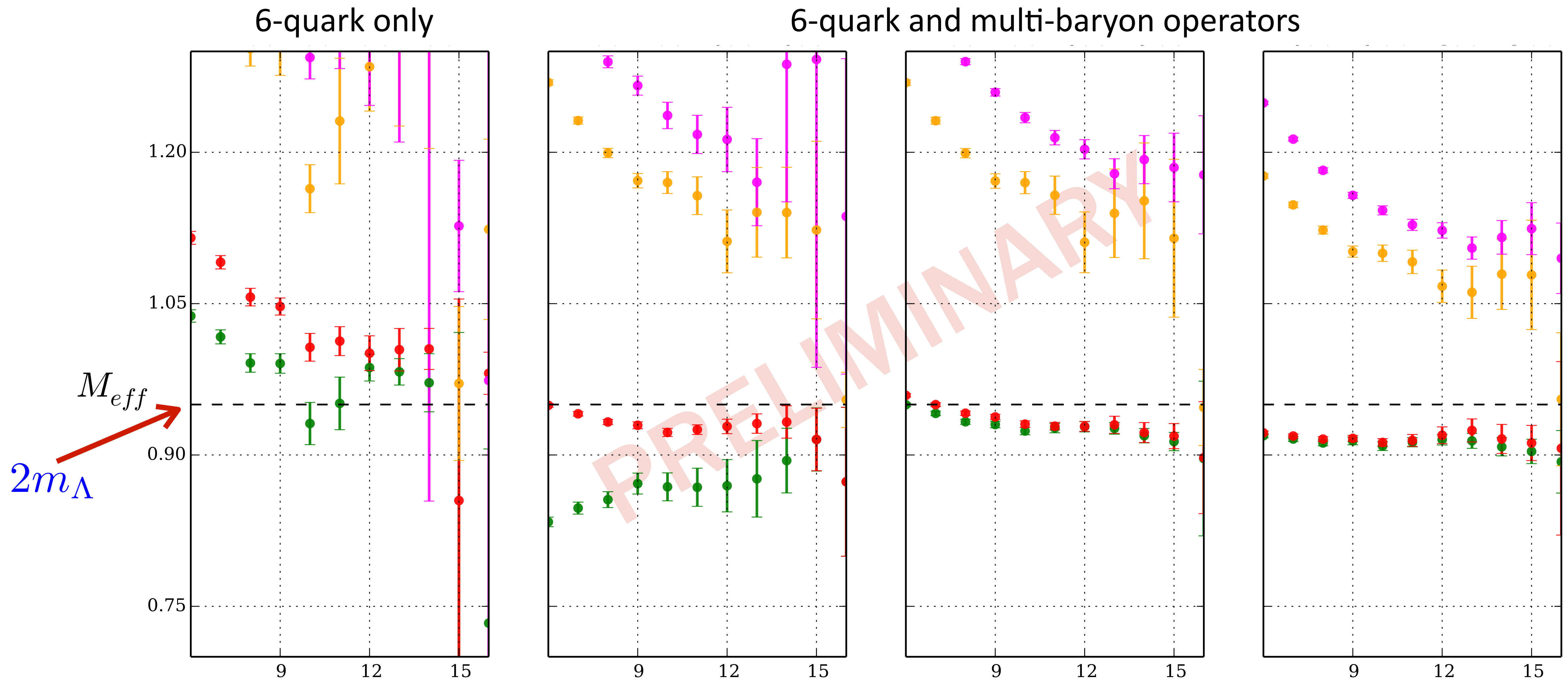
* Ensemble E1 (m_π = 1000 MeV)
* Energy levels probed by different sink operators — 27-plet:



* Similar results for lighter pion masses and different lattice spacings

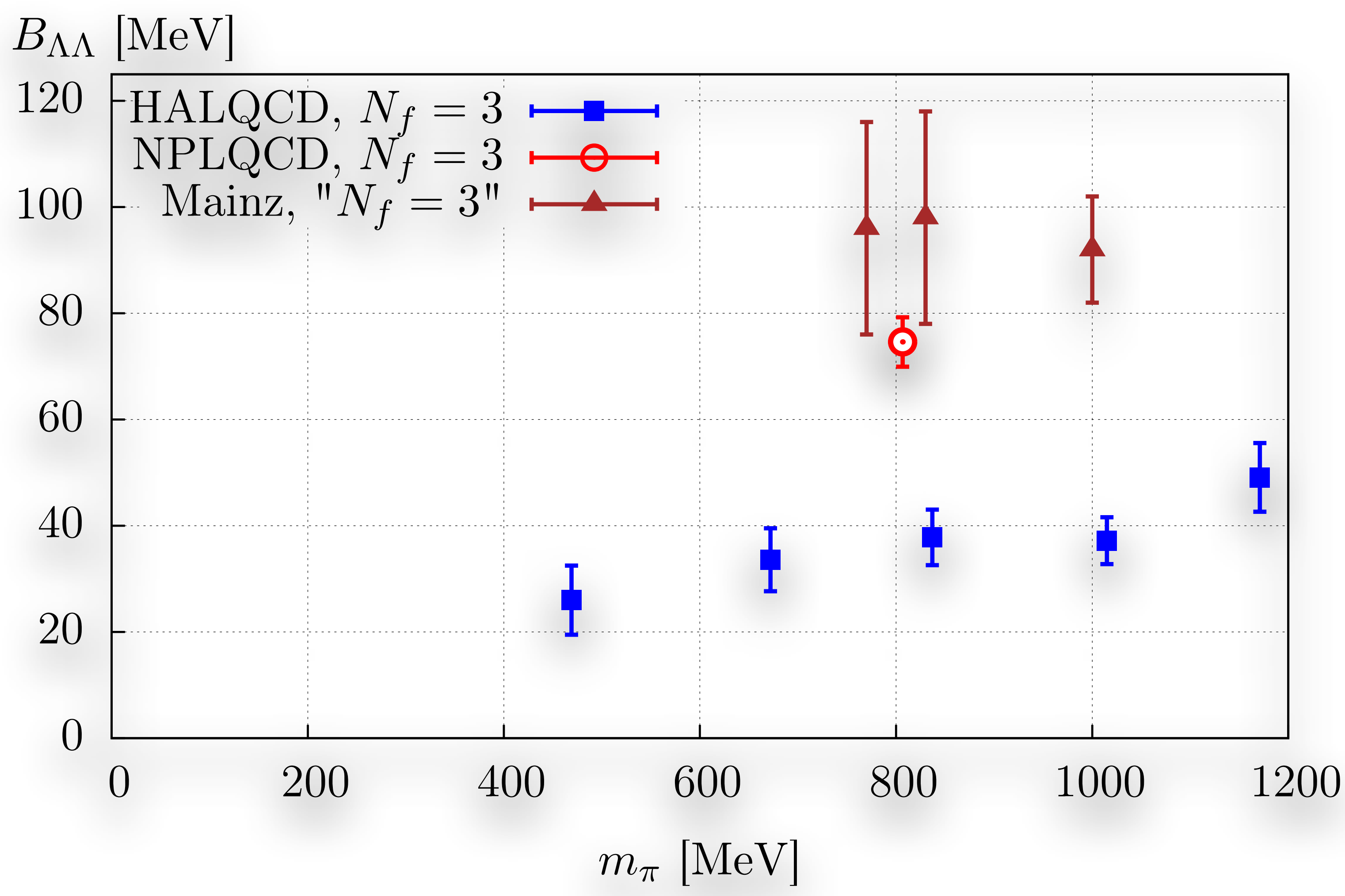
SU(3)-nondegenerate case

- Ensemble E5 (m_π = 451 MeV), $m_s \neq m_{u,d}$
- Energy levels determined from 4x4 GEVP



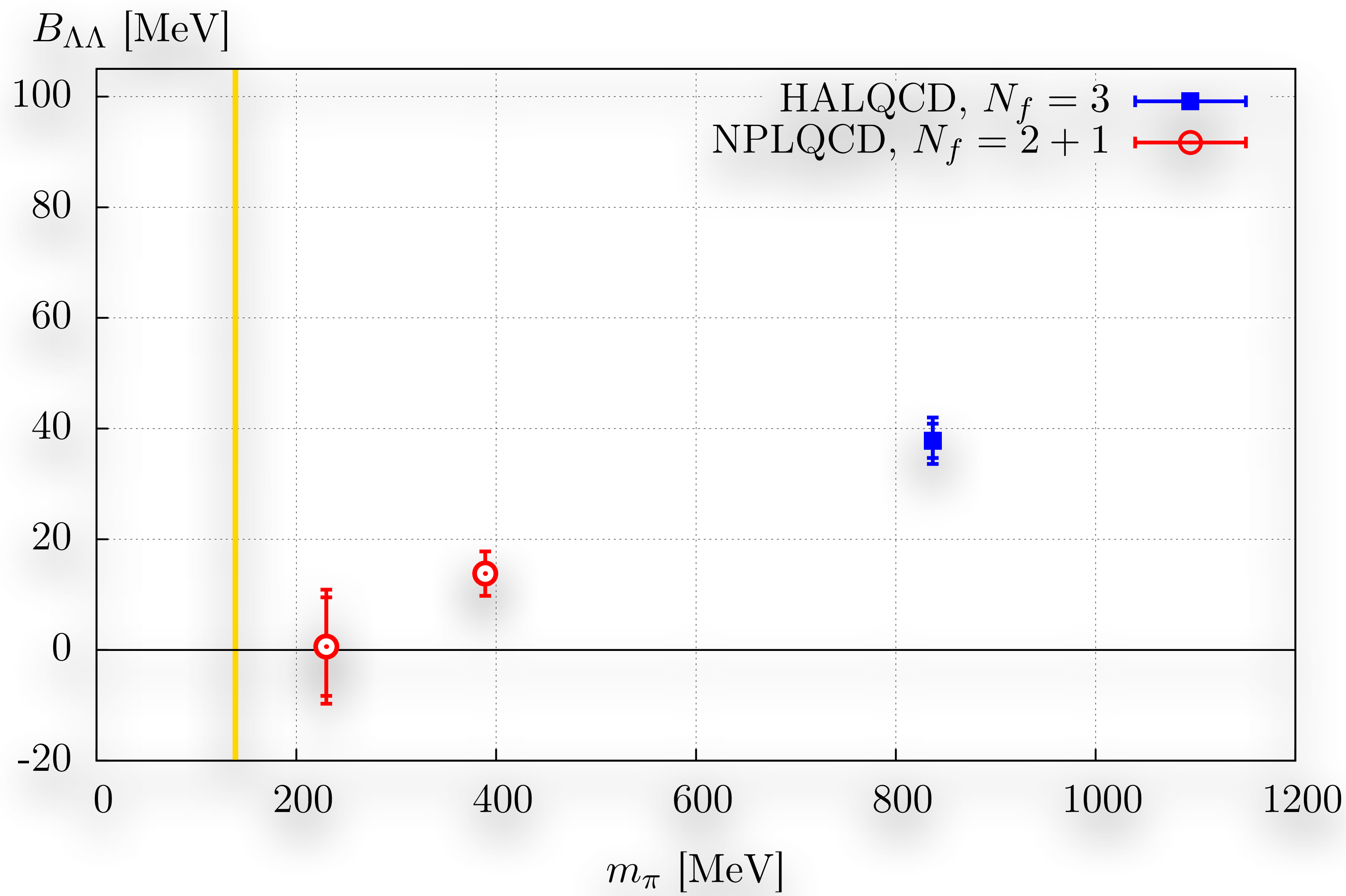
Comparison — chiral behaviour

- Bound dibaryon observed; multi-baryon operators provide better overlap onto ground state
- SU(3)-symmetric case:



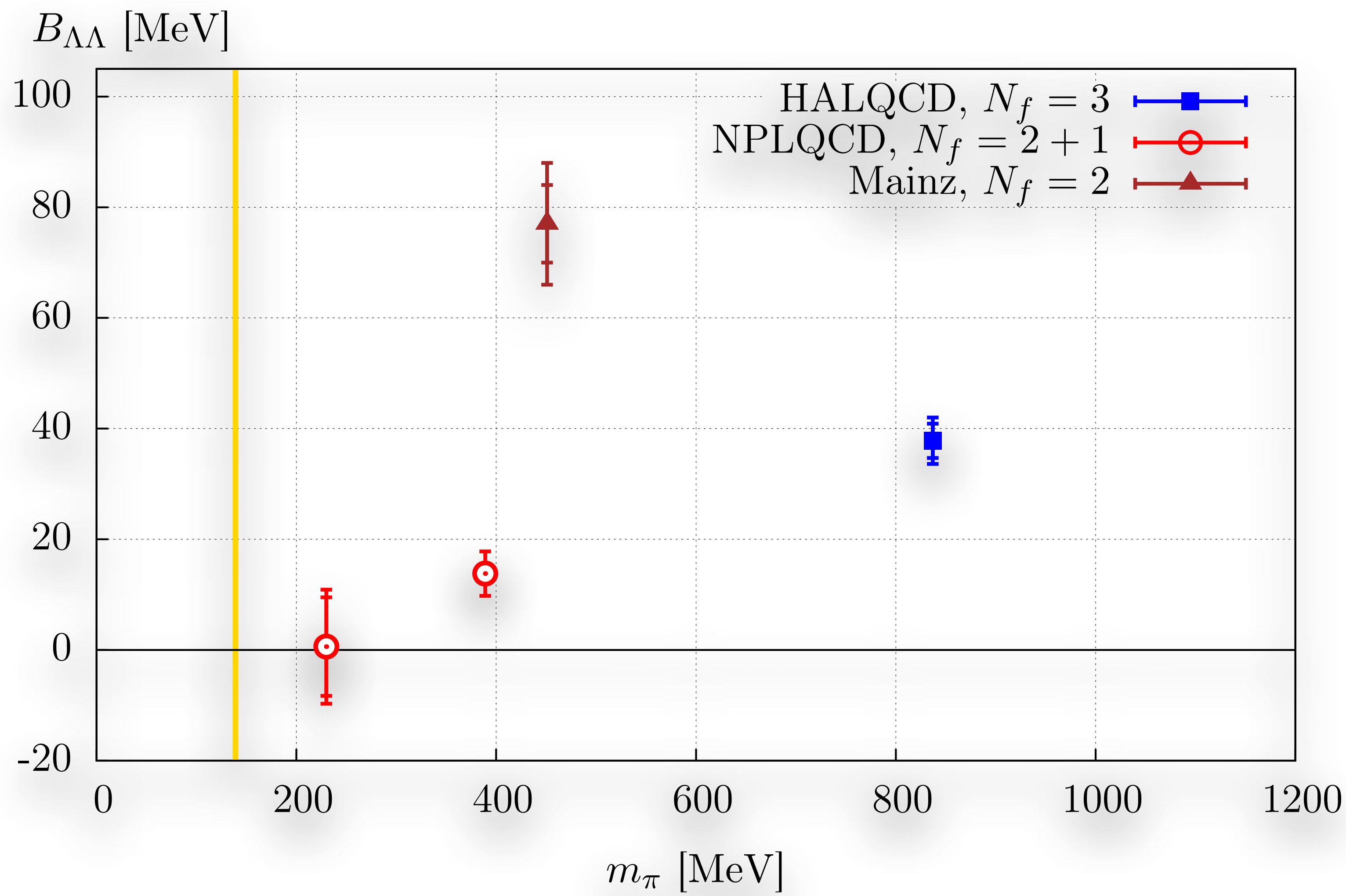
Comparison — chiral behaviour

* Bound dibaryon observed; multi-baryon operators provide better overlap onto ground state
* SU(3)-non-degenerate case:



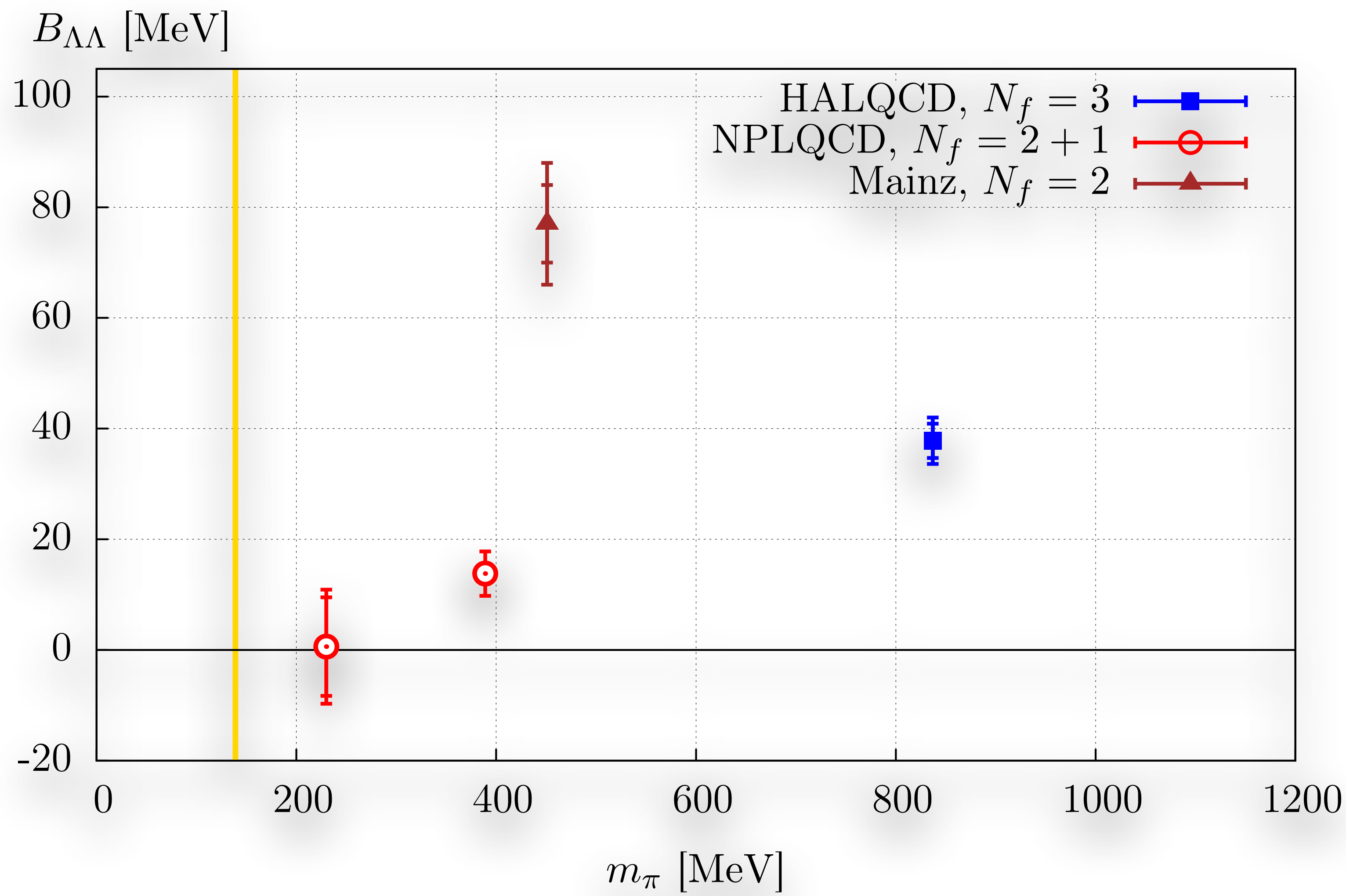
Comparison — chiral behaviour

* Bound dibaryon observed; multi-baryon operators provide better overlap onto ground state
* SU(3)-non-degenerate case:



Comparison — chiral behaviour

* Bound dibaryon observed; multi-baryon operators provide better overlap onto ground state
* SU(3)-non-degenerate case:



* Caveat: binding energy estimated relative to non-interacting Λ's

Scattering phase shifts

* Lüscher recap: $$p\cot\delta_0(p) = \frac{2}{\gamma L\sqrt{\pi}}\mathcal{Z}_{0,0}(1,q^2), \qquad q = \frac{pL}{2\pi}$$

* Binding momentum p: $$p^2 = \tfrac{1}{4}(E^2 - \vec{P}^2) - m_\Lambda^2$$

Scattering phase shifts

* Lüscher recap: $$p\cot\delta_0(p) = \frac{2}{\gamma L\sqrt{\pi}}\mathcal{Z}_{0,0}(1,q^2), \quad q = \frac{pL}{2\pi}$$

* Binding momentum p: $$p^2 = \frac{1}{4}(E^2 - \vec{P}^2) - m_\Lambda^2$$

Determine in lattice calculation

Scattering phase shifts

- Lüscher recap: $$p\cot\delta_0(p)=\frac{2}{\gamma L\sqrt{\pi}}\mathcal{Z}_{0,0}(1,q^2),\qquad q=\frac{pL}{2\pi}$$

- Binding momentum p: $$p^2=\frac{1}{4}(E^2-\vec{P}^2)-m_\Lambda^2$$

Determine in lattice calculation

- Locate pole of the scattering amplitude:

$$\mathcal{A}\propto\frac{1}{p\cot\delta_0(p)-ip}$$

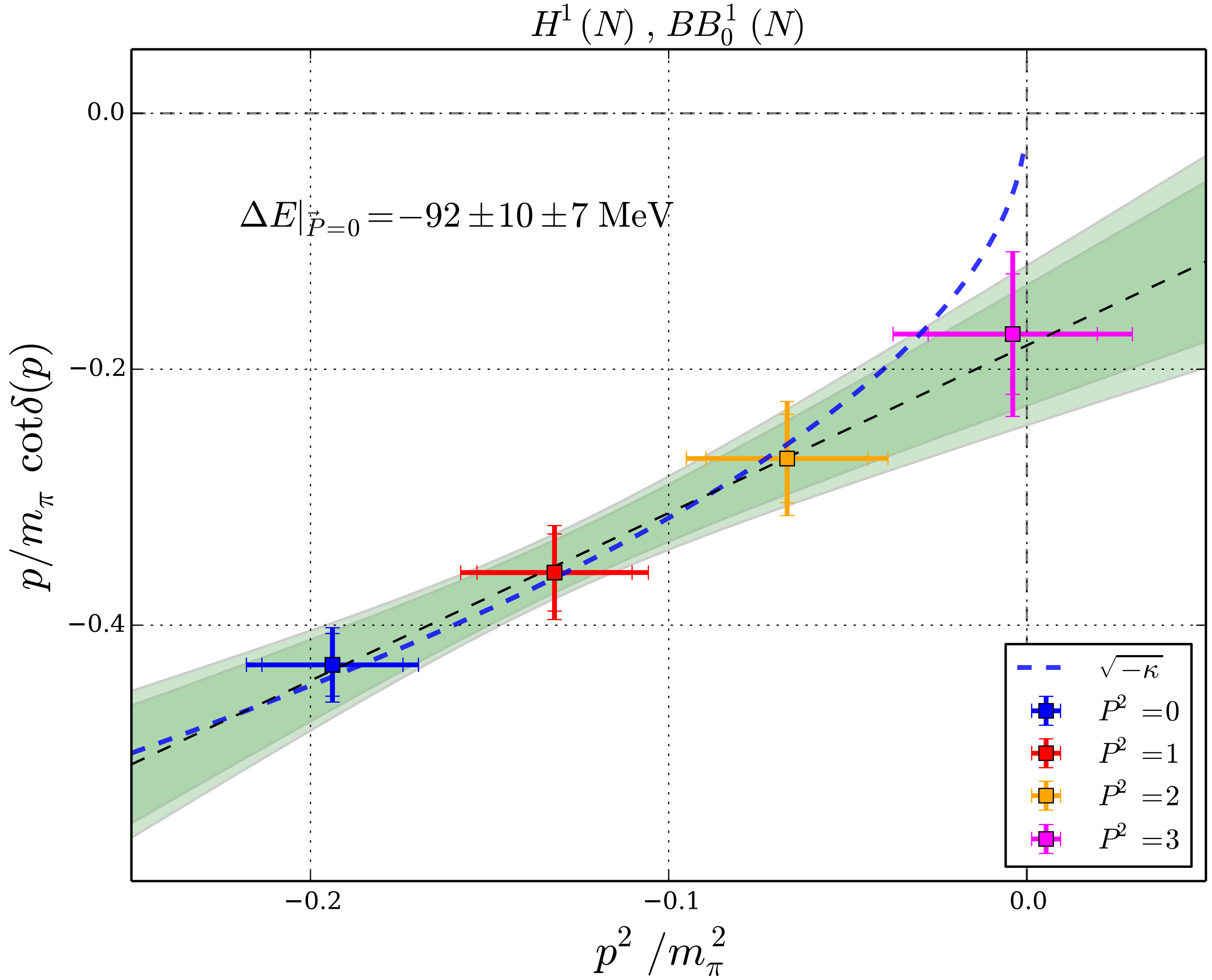
Scattering phase shifts

* Lüscher recap: $p\cot\delta_0(p) = \dfrac{2}{\gamma L\sqrt{\pi}}\mathcal{Z}_{0,0}(1,q^2), \quad q = \dfrac{pL}{2\pi}$

* Binding momentum p: $p^2 = \frac{1}{4}(E^2 - \vec{P}^2) - m_\Lambda^2$

* Locate pole of the scattering amplitude:

$$\mathcal{A} \propto \frac{1}{p\cot\delta_0(p) - ip}$$



Scattering phase shifts

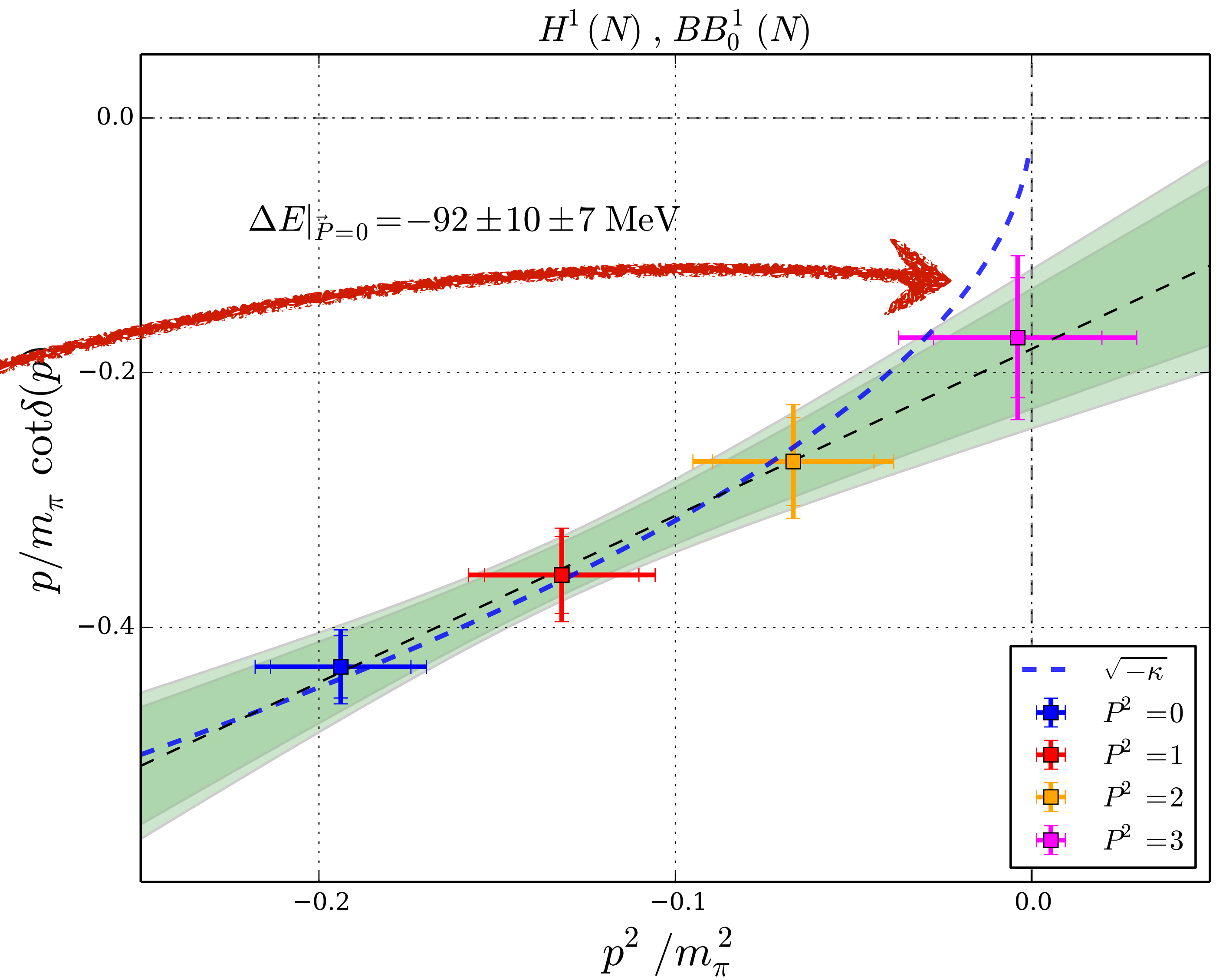
- Lüscher recap: $p\cot\delta_0(p) = \frac{2}{\gamma L\sqrt{\pi}}\mathcal{Z}_{0,0}(1,q^2), \quad q = \frac{pL}{2\pi}$

- Binding momentum p: $p^2 = \frac{1}{4}(E^2 - \vec{P}^2) - m_\Lambda^2$

- Locate pole of the scattering amplitude:

$$\mathcal{A} \propto \frac{1}{p\cot\delta_0(p) - ip}$$

$$-\sqrt{-p^2}$$



Scattering phase shifts

* Lüscher recap: $$p\cot\delta_0(p) = \frac{2}{\gamma L\sqrt{\pi}}\mathcal{Z}_{0,0}(1,q^2), \quad q = \frac{pL}{2\pi}$$

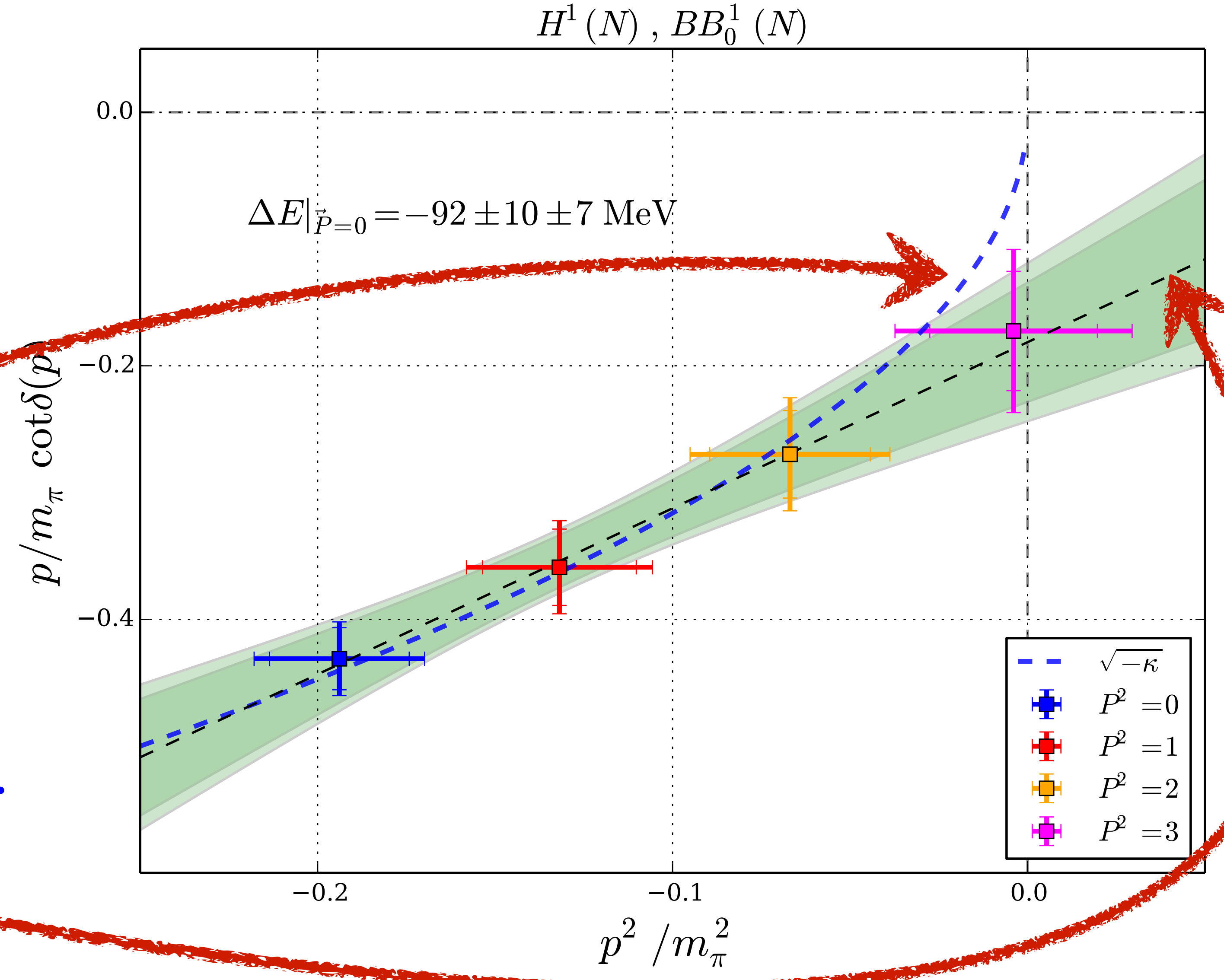
* Binding momentum p: $$p^2 = \frac{1}{4}(E^2 - \vec{P}^2) - m_\Lambda^2$$

* Locate pole of the scattering amplitude:

$$\mathcal{A} \propto \frac{1}{p\cot\delta_0(p) - ip}$$

$$-\sqrt{-p^2}$$

$$p\cot\delta_0(p) = A + Bp^2 + \ldots$$



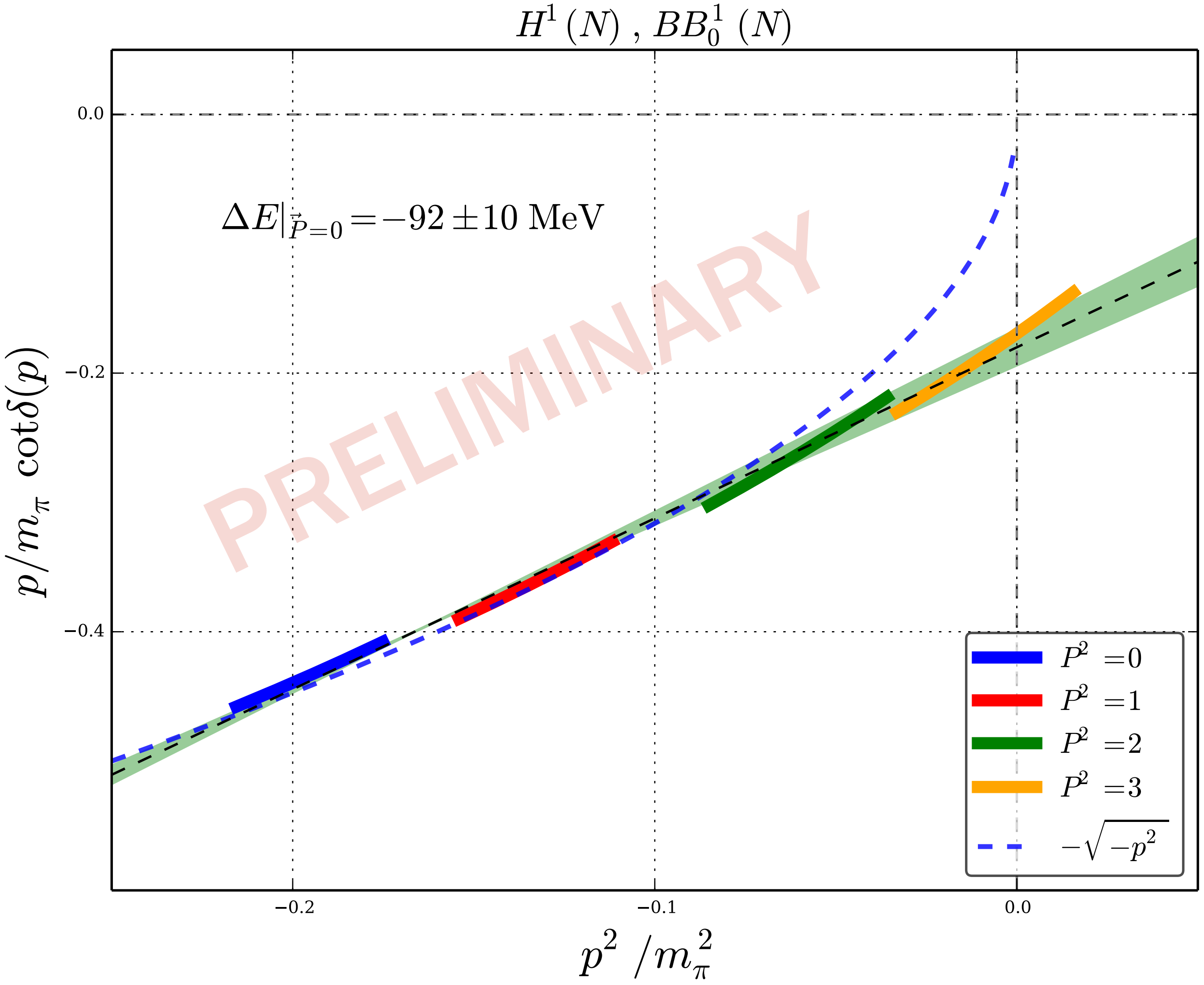
Scattering phase shifts

* Lüscher recap: $p\cot\delta_0(p) = \frac{2}{\gamma L\sqrt{\pi}}\mathcal{Z}_{0,0}(1,q^2), \quad q = \frac{pL}{2\pi}$

* Binding momentum p: $p^2 = \frac{1}{4}(E^2 - \vec{P}^2) - m_\Lambda^2$

* Locate pole of the scattering amplitude:

$$\mathcal{A} \propto \frac{1}{p\cot\delta_0(p) - ip}$$



Scattering phase shifts

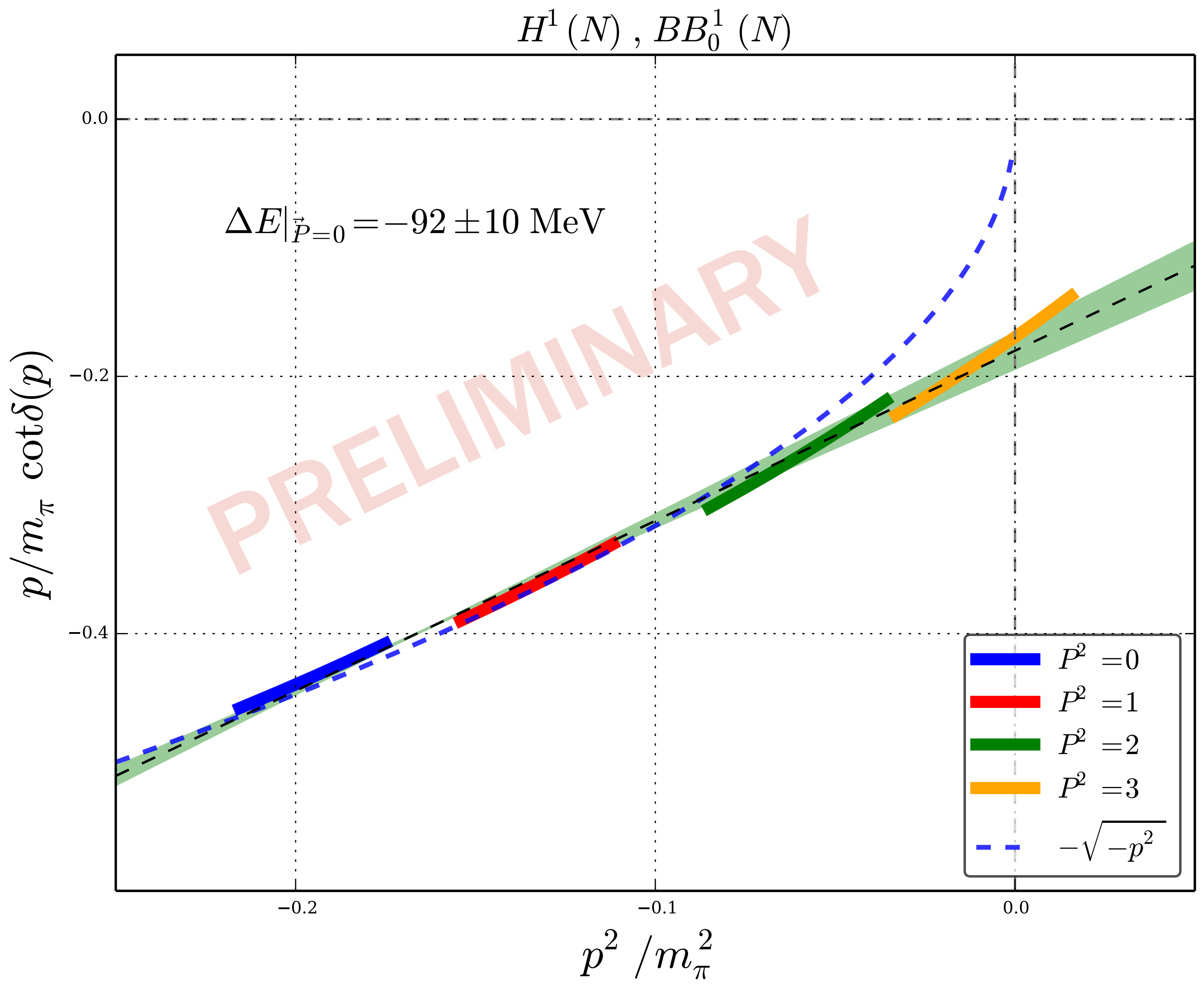
* Lüscher recap: $$p\cot\delta_0(p) = \frac{2}{\gamma L\sqrt{\pi}}\mathcal{Z}_{0,0}(1,q^2), \quad q=\frac{pL}{2\pi}$$

* Binding momentum p: $$p^2 = \tfrac{1}{4}(E^2-\vec{P}^2)-m_\Lambda^2$$

* Locate pole of the scattering amplitude:

$$\mathcal{A} \propto \frac{1}{p\cot\delta_0(p)-ip}$$

* Difficult to determine intersection of the two curves



Summary and Outlook

Summary and Outlook

* Lattice calculations of the spectrum in the H-Dibaryon channel is technically demanding
* Bound H-Dibaryon found for unphysically large pion masses
* Multi-baryon operators crucial for isolating the ground state
* Chiral behaviour of binding energies and SU(3)-breaking effects to be investigated

Summary and Outlook

* Lattice calculations of the spectrum in the H-Dibaryon channel is technically demanding

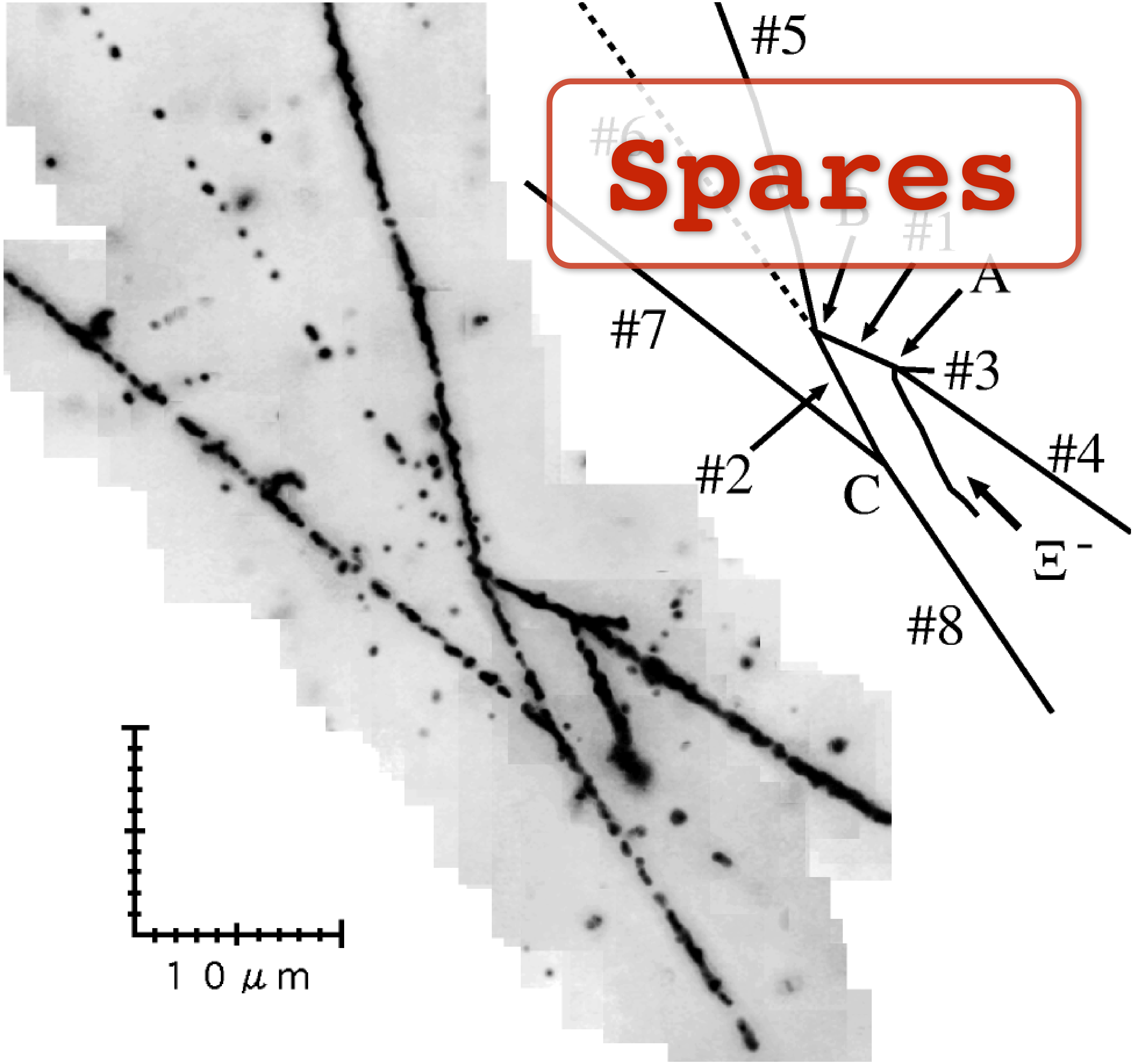
* Bound H-Dibaryon found for unphysically large pion masses

* Multi-baryon operators crucial for isolating the ground state

* Chiral behaviour of binding energies and SU(3)-breaking effects to be investigated

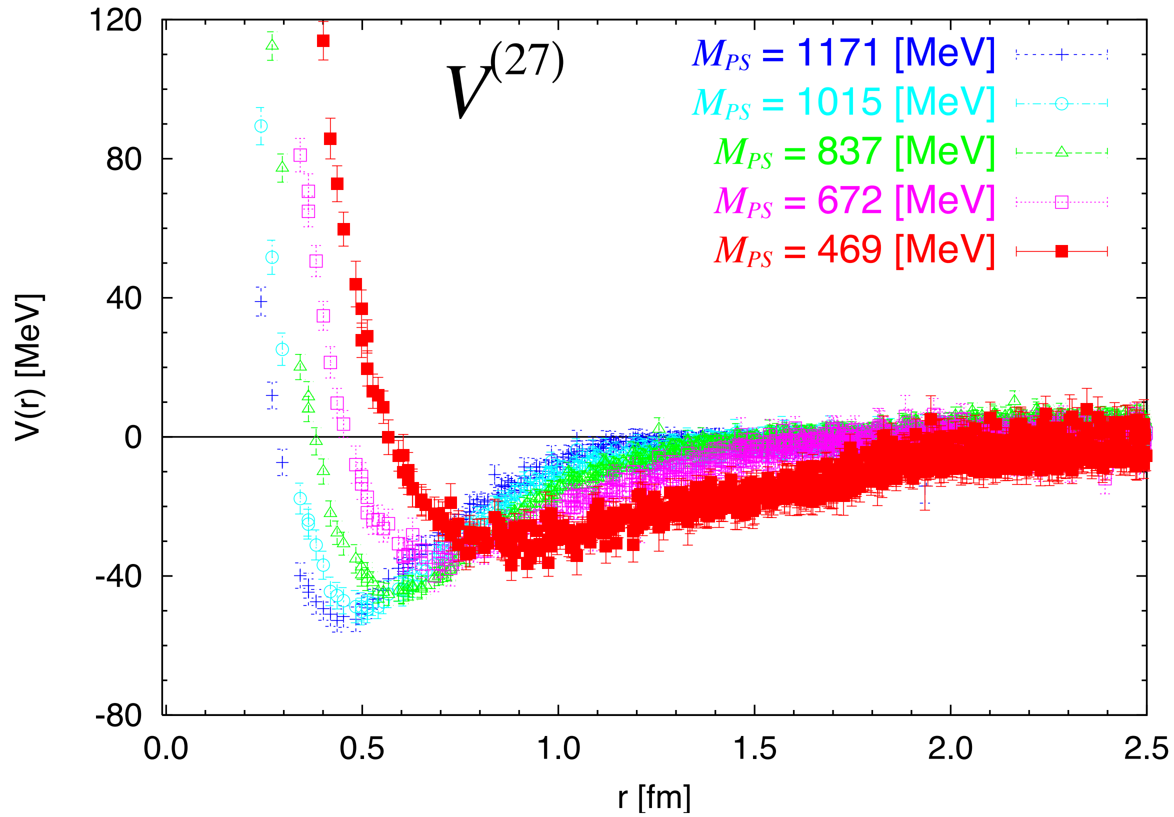
* Future plans: process ensembles with $N_f = 2+1$ generated by CLS:
  - SU(3)-symmetric point at physical strange quark mass
  - Different volumes
  - Numerical improvements

Spares

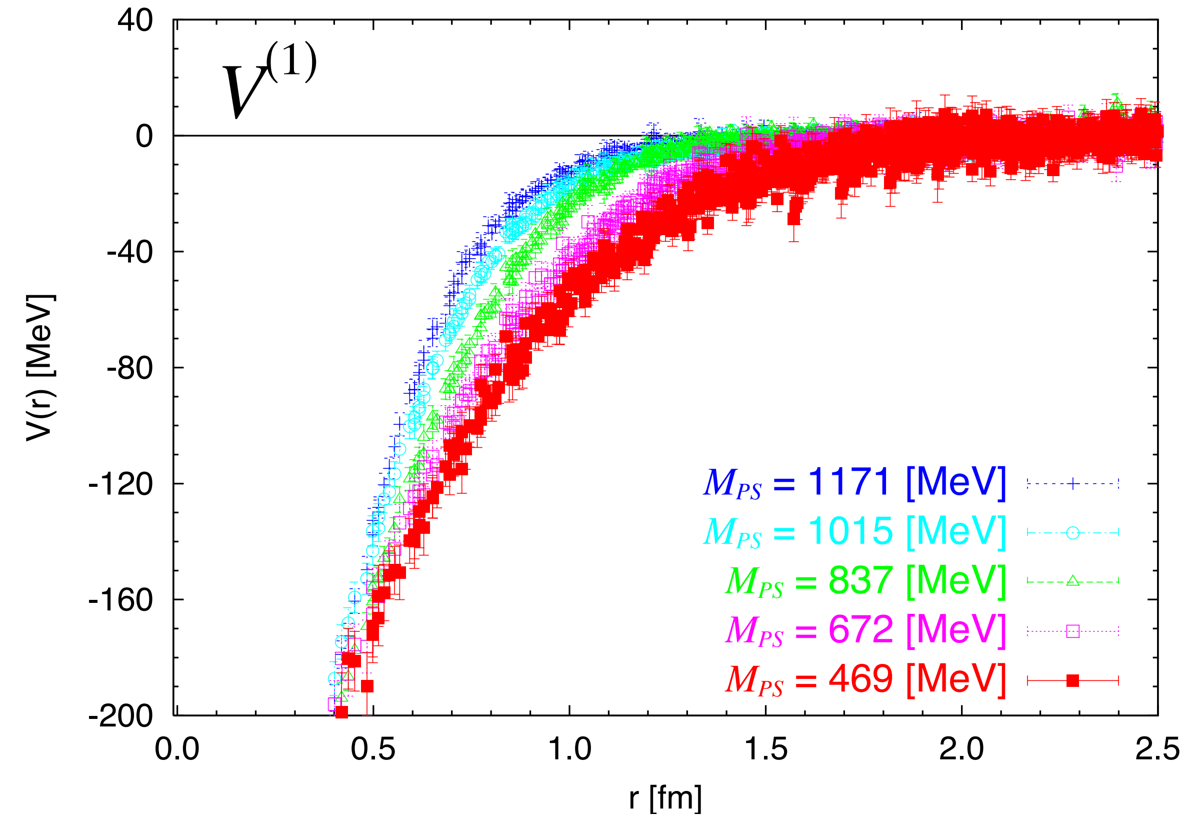


HALQCD Collaboration

NN potential (27-plet)



Flavour-singlet potential



- Repulsive core
- Attractive well at intermediate and long distances

- Attractive core
- Mild pion mass dependence
- Parameterise potential as input for Schrödinger equation

NPLQCD Collaboration

* Binding energy in infinite volume limit:

[Beane et al, Phys Rev Lett (2011) 162001, Mod Phys Lett A26 (2011) 2587]

$$B_{\Lambda\Lambda}^{\infty} = \begin{cases} 13.2(1.8)_{\rm stat}\,(4.0)_{\rm syst}\,\text{MeV}, & m_\pi = 389\,\text{MeV} \\ 0.6(8.9)_{\rm stat}\,(10.3)_{\rm syst}\,\text{MeV}, & m_\pi = 230\,\text{MeV} \end{cases}$$

* Chiral behaviour: