Ab initio nuclear theory's symbiosis with relativistic quantum field theory

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Nuclear Physics from Lattice QCD
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“I worry that there is not enough research on approaches to solving QCD that could be complementary to Monte Carlo simulations, such as the lack of any comparable research built upon light-front QCD.”

“…there needs to be more attention to research on light-front QCD as a complement to research on lattice gauge theory.”

K.G. Wilson, “The Origins of Lattice Gauge Theory,” Nuclear Physics B, Suppl., **140** (2005) p3

Comparison of No-Core Shell Model (NCSM) with Basis Light-Front Quantization (BLFQ)	**NCSM**	**BLFQ**
Select kinematics & Derive 2-, 3-, 4- body Hamiltonian	Non-relativistic & EFT, meson exch, . .	Relativistic & QED, QCD, . . .
Adopt a single-particle basis convenient for symmetries	3D HO, HF, nat. orb., . . .	2D HO+ DLCQ, . . .
Enumerate Fock-space basis subject to symmetry constraints	Slater Deter.	Slater Deter. & Permanents
Renormalize & evaluate many-particle H in that basis	SRG, OLS, . . .	SRG, OLS, . . .
Diagonalize H (Lanczos)	Yes	Yes
Iterate previous 2 steps for sector-dependent renormalization	- - - - -	Yes
Evaluate observables using eigenvectors of H	wavefunctions	Light-Front amplitudes
Repeat previous 4 steps for new values of regulator(s)	λ, Λ	λ, Λ, K, m_g
Extrapolate to infinite matrix limit & remove all regulators	Yes	Yes
Compare with experiment or predict new experimental results	14C, 14F, . . .	Charmonia, . . .

No-Core Shell Model (NCSM) - Configuration Interaction (CI) method

Barrett, Navrátil, Vary, *Ab initio no-core shell model*, PPNP69, 131 (2013)

Given a Hamiltonian operator

$$\hat{\mathbf{H}} = \sum_{i<j} \frac{(\vec{p}_i - \vec{p}_j)^2}{2\,m\,A} + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

solve the eigenvalue problem for wavefunction of A nucleons

$$\hat{\mathbf{H}}\,\Psi(r_1, \ldots, r_A) = \lambda\,\Psi(r_1, \ldots, r_A)$$

- Expand eigenstates in basis states $|\Psi\rangle = \sum a_i |\Phi_i\rangle$
- Diagonalize Hamiltonian matrix $H_{ij} = \langle\Phi_j|\hat{\mathbf{H}}|\Phi_i\rangle$
- No-Core CI: all A nucleons are treated the same
- Complete basis $\longrightarrow$ exact result
- In practice
  - truncate basis
  - study behavior of observables as function of truncation

Nuclei represent strongly interacting, self-bound, open systems with multiple scales – a computationally hard problem whose solution has potential impacts on other fields

Question: What controls convergence/uncertainties of observables?
Answer: Characteristic infrared (IR) and ultraviolet (UV) scales of the operators.

In a plane-wave basis:
- λ = lowest momentum scale - can be zero (e.g. T_{rel}, r^2, B(EL), . . .)
- Λ = highest momentum scale - can be infinity (e.g. T_{rel}, hard-core V_{NN})

In a harmonic-oscillator basis with N_{max} truncation (Max total # oscillator quanta):

$$\lambda \approx \sqrt{\hbar\Omega / N_{\text{max}}}$$

$$\Lambda \approx \sqrt{\hbar\Omega \, N_{\text{max}}}$$

What are examples of the other physically relevant scales in nuclear physics?
- Interaction scales (total binding, Fermi momentum, SRCs, one-pion exchange, . . .)
- Leading dissociation scale (halos, nucleon removal energy, . . .)
- Collective motion, clustering scales (Q, B(E2), giant modes, . . .)

Dirac's Forms of Relativistic Dynamics

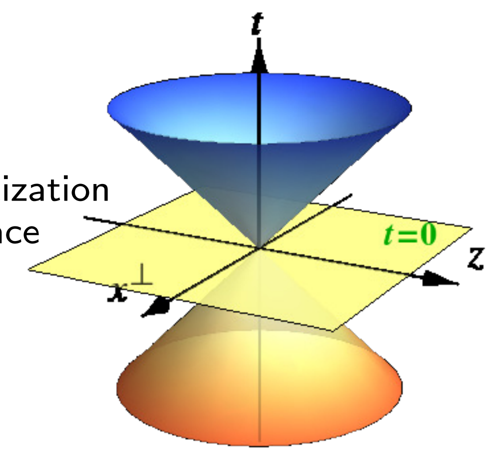
[Dirac, Rev.Mod.Phys. '49]

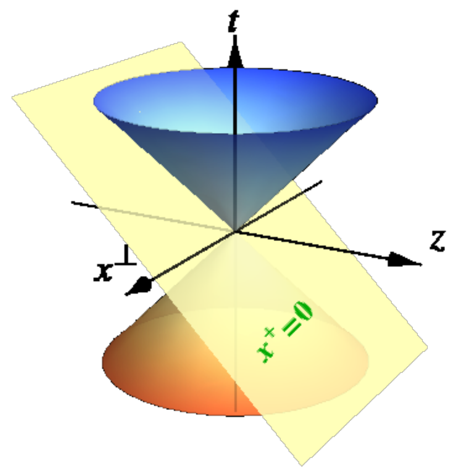
Front form defines QCD on the light front (LF) $x^+ \triangleq t + z = 0$.

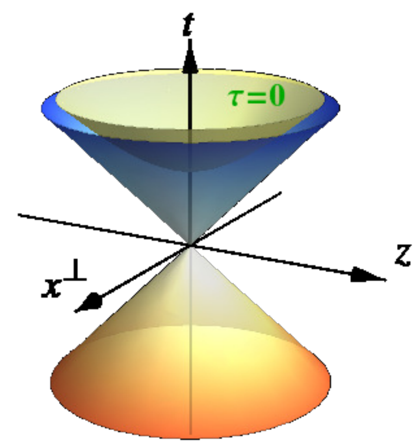
$$P^\pm \triangleq P^0 \pm P^3,\ \vec{P}^\perp \triangleq (P^1, P^2),\ x^\pm \triangleq x^0 \pm x^3,\ \vec{x}^\perp \triangleq (x^1, x^2),\ E^i = M^{+i},$$
$$E^+ = M^{+-},\ F^i = M^{-i},\ K^i = M^{0i},\ J^i = \tfrac{1}{2}\epsilon^{ijk}M^{jk}.$$

	instant form	front form	point form
time variable	$t = x^0$	$x^+ \triangleq x^0 + x^3$	$\tau \triangleq \sqrt{t^2 - \vec{x}^2 - a^2}$

quantization surface







	instant form	front form	point form
Hamiltonian	$H = P^0$	$P^- \triangleq P^0 - P^3$	P^μ
kinematical	$\vec{P}, \vec{J}$	$\vec{P}^\perp, P^+, \vec{E}^\perp, E^+, J_z$	$\vec{J}, \vec{K}$
dynamical	$\vec{K}, P^0$	$\vec{F}^\perp, P^-$	$\vec{P}, P^0$
dispersion relation	$p^0 = \sqrt{\vec{p}^2 + m^2}$	$p^- = (\vec{p}_\perp^2 + m^2)/p^+$	$p^\mu = m v^\mu \ (v^2 = 1)$

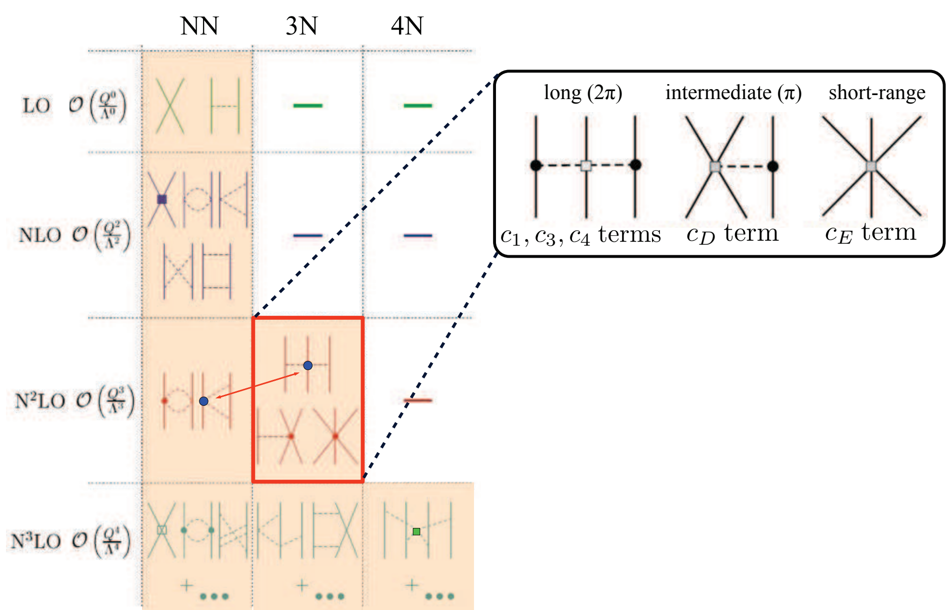
Nuclear interaction from chiral perturbation theory

- Strong interaction in principle calculable from QCD
- Use chiral perturbation theory to obtain effective A-body interaction from QCD
  Entem and Machleidt, PRC68, 041001 (2003)
  - controlled power series expansion in Q/Λ_χ with $\Lambda_\chi \sim 1$ GeV
  - natural hierarchy for many-body forces

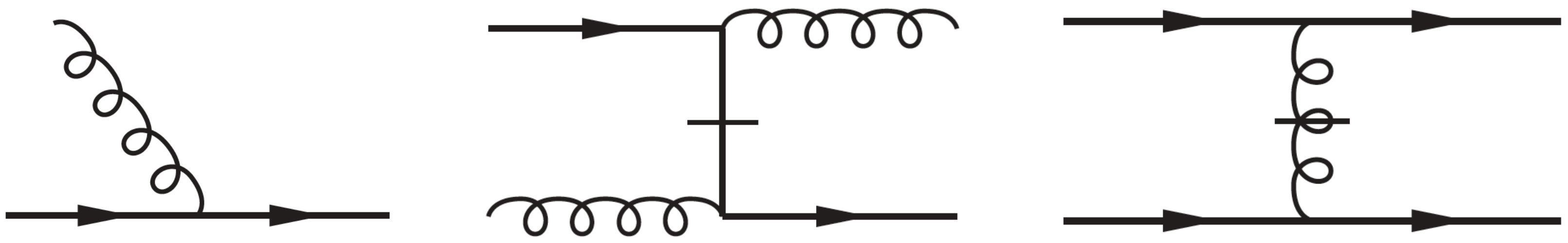
$$V_{NN} \gg V_{NNN} \gg V_{NNNN}$$

  - in principle no free parameters
    - in practice a few undetermined parameters
  - renormalization necessary

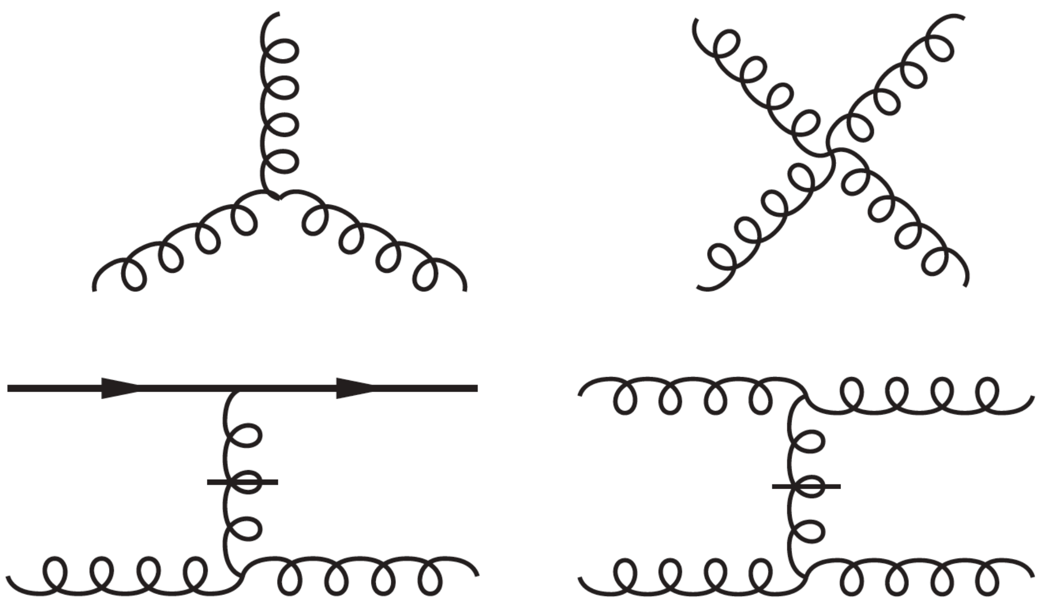
Leading-order 3N forces in chiral EFT



Light Front (LF) Hamiltonian Defined by its Elementary Vertices in LF Gauge



QED & QCD



QCD

Discretized Light Cone Quantization

Pauli & Brodsky c1985

⇩

Basis Light Front Quantization*

$$\phi(\vec{x}) = \sum_{\alpha} \left[f_\alpha(\vec{x}) a_\alpha^+ + f_\alpha^*(\vec{x}) a_\alpha \right]$$

where $\{a_\alpha\}$ satisfy usual (anti-) commutation rules.

Furthermore, $f_\alpha(\vec{x})$ are arbitrary except for conditions:

Orthonormal: $\int f_\alpha(\vec{x}) f_{\alpha'}^*(\vec{x}) d^3x = \delta_{\alpha\alpha'}$

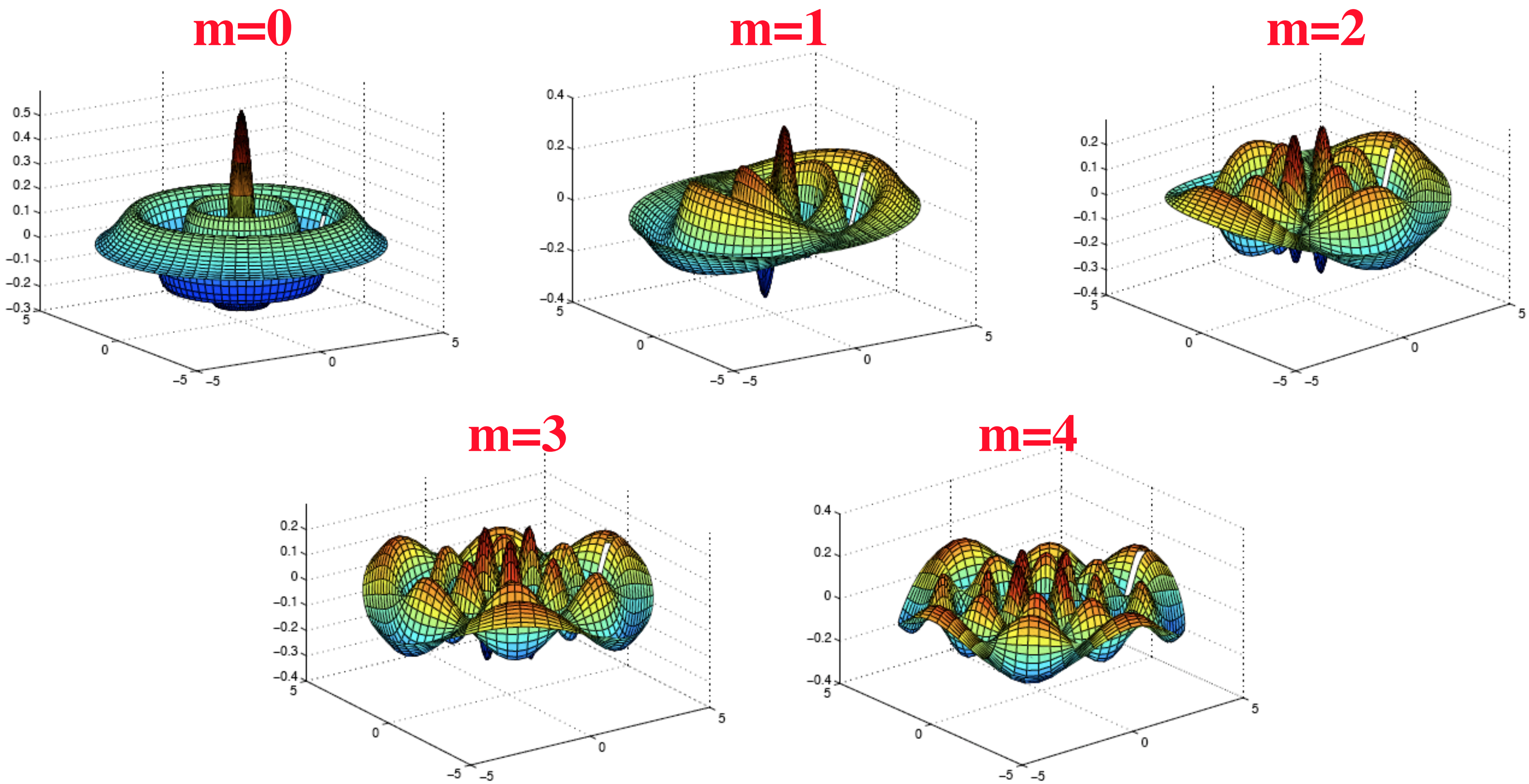
Complete: $\sum_{\alpha} f_\alpha(\vec{x}) f_\alpha^*(\vec{x}') = \delta^3(\vec{x} - \vec{x}')$

=> Wide range of choices for $f_a(\vec{x})$ and our initial choice is

$$f_\alpha(\vec{x}) = N e^{ik^+x^-} \Psi_{n,m}(\rho,\varphi) = N e^{ik^+x^-} f_{n,m}(\rho) \chi_m(\varphi)$$

*J.P. Vary, H. Honkanen, J. Li, P. Maris, S.J. Brodsky, A. Harindranath, G.F. de Teramond, P. Sternberg, E.G. Ng and C. Yang, PRC 81, 035205 (2010). ArXiv:0905:1411

Set of Transverse 2D HO Modes for n=4



J.P. Vary, H. Honkanen, J. Li, P. Maris, S.J. Brodsky, A. Harindranath,
G.F. de Teramond, P. Sternberg, E.G. Ng and C. Yang, PRC 81, 035205 (2010)

BLFQ Highlights

BLFQ introduced:
J. P. Vary, H. Honkanen, Jun Li, P. Maris, S. J. Brodsky, A. Harindranath, G. F. de Teramond, P. Sternberg, E. G. Ng, C. Yang, Phys. Rev. C 81, 035205 (2010); arXiv: 0905.1411

Successfully applied to QED test cases – electron in a trap:
H. Honkanen, P. Maris, J.P. Vary, S.J. Brodsky, Phys. Rev. Lett. 106, 061603 (2011); arXiv: 1008.0068

Introduced non-peturbative scattering in time-dependent BLFQ (tBLFQ):
X. Zhao, A. Ilderton, P. Maris and J. P. Vary, Phys. Rev. D 88, 065014 (2013); arXiv: 1303.3237

tBLFQ successfully applied with time and space-dependent external fields:
X. Zhao, A. Ilderton, P. Maris and J. P. Vary, Phys. Letts. B 726, 856 (2013); arXiv: 1309.5338

Improvements to BLFQ for QED test cases: trap independence, renormalization, . . .
X. Zhao, H. Honkanen, P. Maris, J.P. Vary, S.J. Brodsky, Phys. Letts. B 737, 65 (2014); arXiv:1402.4195

Positronium at strong coupling:
P. Wiecki, Y. Li, X. Zhao, P. Maris and J.P. Vary, Phys. Rev. D 91, 105009 (2015); arXiv 1404.6234

FFs and GPDs evaluated for positronium at strong coupling:
L. Adhikari, Y. Li, X. Zhao, P. Maris, J. P. Vary and A. Abd El-Hady, Phys. Rev. C93, 055202 (2016); arXiv: 1602.06027

Heavy Quarkonium in Holographic basis:
Y. Li, P. Maris, X. Zhao and J. P. Vary, Phys. Letts. B 758, 118 (2016); arXiv:1509.07212

NCSM
Symmetries & Constraints

All $J \geq J_z$ states in one calculation

Baryon number $\sum_i b_i = B = A$

Charge $\sum_i q_i = Q = Z$

Angular momentum projection (M-scheme) $\sum_i (m_i + s_i) = J_z$

Mode regulator (3D HO) $\sum_i (2n_i + l_i + 3/2) \leq N_{max}$

Optional Fock-Space Truncation

$H \to H + \lambda H_{CM}$

Finite basis regulators

Preserve Galilean boost invariance

BLFQ
Symmetries & Constraints

Baryon number	$\sum_i b_i = B$
Charge	$\sum_i q_i = Q$
Angular momentum projection (M-scheme)	$\sum_i (m_i + s_i) = J_z$
Longitudinal momentum (Bjorken sum rule)	$\sum_i x_i = \sum_i \frac{k_i}{K} = 1$
Transverse mode regulator (2D HO)	$\sum_i (2n_i + \lvert m_i \rvert + 1) \leq N_{max}$

Global Color Singlets (QCD)

Light Front Gauge

Optional Fock-Space Truncation

$H \to H + \lambda H_{CM}$

All $J \geq J_z$ states in one calculation

Finite basis regulators

Preserve transverse boost invariance

Light-Front Regularization and Renormalization Schemes

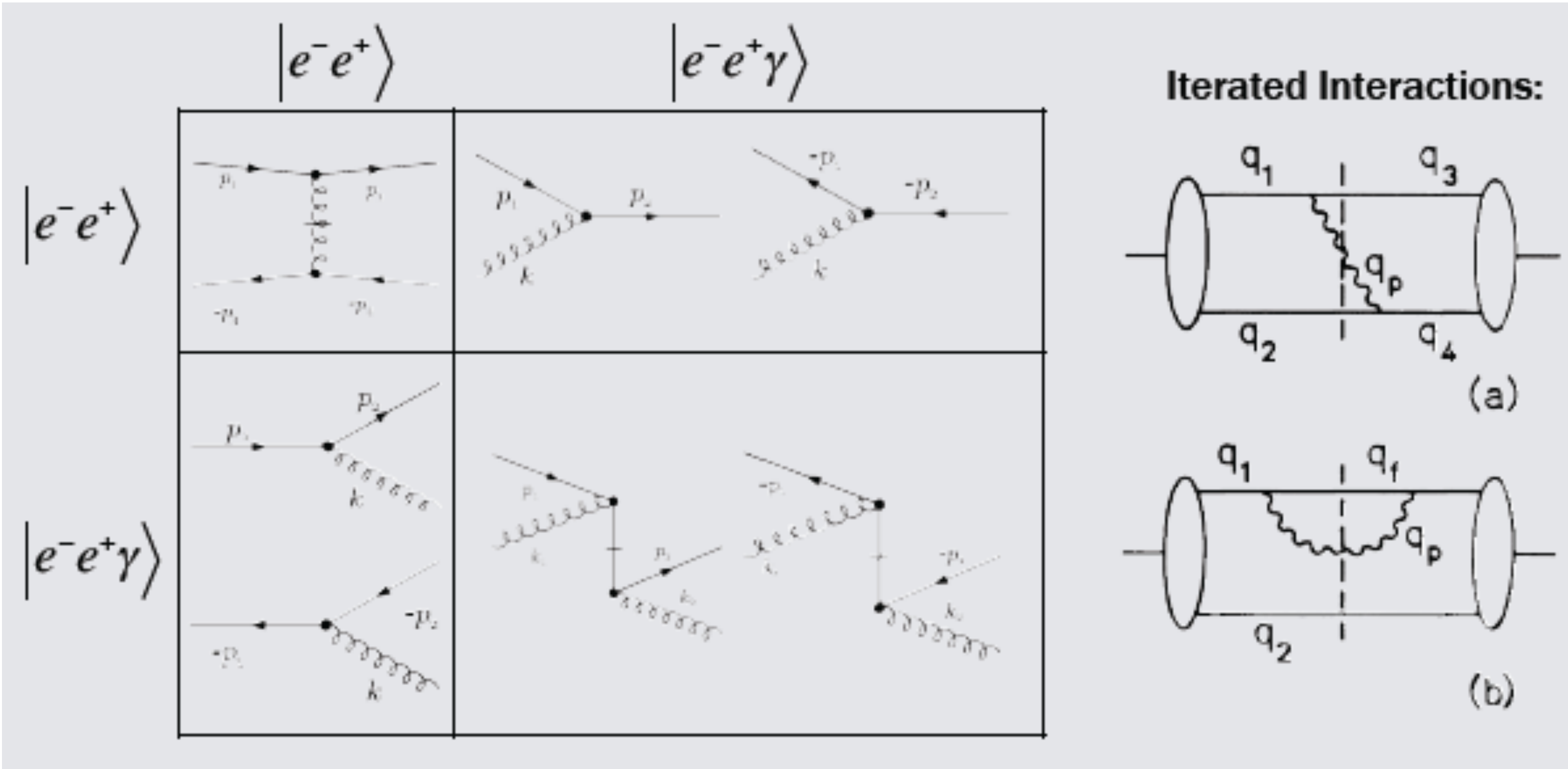
1. Regulators in BLFQ (2-D HO params, K)
2. Additional Fock space truncations (if any)
3. Counterterms identified/tested*
4. Sector-dependent renormalization**
5. SRG & OLS in NCSM*** - adapted to BLFQ (future)

*D. Chakrabarti, A. Harindranath and J.P. Vary,
Phys. Rev. D **69**, 034502 (2004)
*P. Wiecki, Y. Li, X. Zhao, P. Maris and J.P. Vary,
Phys. Rev. D **91**, 105009 (2015)

**V. A. Karmanov, J.-F. Mathiot, and A. V. Smirnov,
Phys. Rev. D **77**, 085028 (2008); Phys. Rev. D **86**, 085006 (2012)
**Y. Li, V.A. Karmanov, P. Maris and J.P. Vary,
Phys. Letts. B. 748, **278** (2015); arXiv: 1504.05233

***B.R. Barrett, P. Navratil and J.P. Vary,
Prog. Part. Nucl. Phys. **69**, 131 (2013)

Positronium



- **Neglect instantaneous interactions when corresponding dynamical exchange is not present in model space**

Exact factorization of CM motion

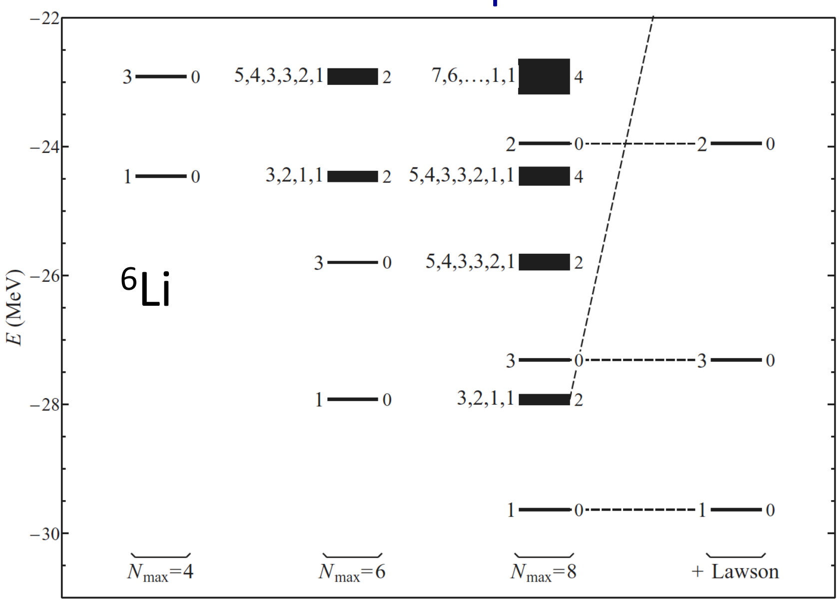
Preserves Galilean invariance in NCSM and transverse boost invariance in BLFQ

$$H \to H + \lambda_{\text{CM}} H_{\text{CM}} \quad \text{with} \quad H_{\text{CM}} = \frac{1}{2} M \Omega^2 R_{\text{CM}}^2$$

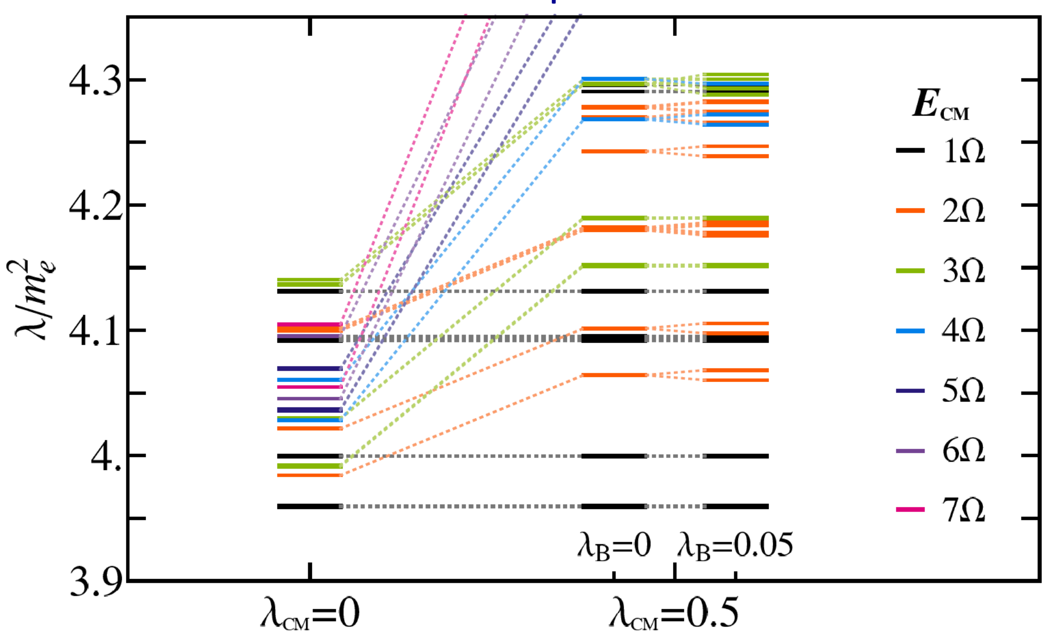
$$R_{\text{CM}} = 3D \text{ CM coordinate (NCSM)}$$

$$R_{\text{CM}} = 2D \text{ transverse CM coordinate (BLFQ)}$$

NCSM example: ^{6}Li



BLFQ example: Positronium



M. A. Caprio, P. Maris and J. P. Vary,
Phys. Rev. C 86, 034312 (2012); arXiv:1208.4156

Y. Li, P.W. Wiecki, X. Zhao, P. Maris and J.P. Vary,
Proceedings NTSE-2013, 136 (2014), arXiv: 1311.2980
http://www.ntse-2013.khb.ru/Proc/Yli.pdf

- **Projection operators P and Q**

$$P^2 |\Psi_i\rangle = M_i^2 |\Psi_i\rangle$$

$$H \equiv P^2$$

$$PHP \equiv H_{00}$$
$$PHQ \equiv H_{01}$$
$$QHP \equiv H_{10}$$
$$QHQ \equiv H_{11}$$

	$\vert e^- e^+\rangle$	$\vert e^- e^+ \gamma\rangle$
$\vert e^- e^+\rangle$	*PHP*	*PHQ*
$\vert e^- e^+ \gamma\rangle$	*QHP*	*QHQ*

Positronium:
P. Weicki, et al., PRD **91**, 105009 (2015)

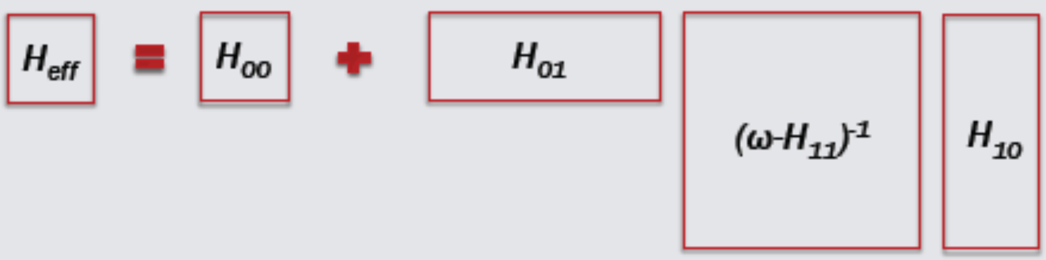
- **Eigenvalue equation can be rewritten:**

$$H_{\text{eff}}(\omega)|\psi_i(\omega)\rangle_0 = \tilde{M}_i^2(\omega)|\psi_i(\omega)\rangle_0$$

$$H_{\text{eff}}(\omega) \equiv H_{00} + H_{01}\frac{1}{\omega - H_{11}}H_{10}$$

$$\omega = M_i^2$$

$$H_{\text{eff}}(\omega) \equiv H_{00} + H_{01}\frac{1}{\omega - H_{11}}H_{10}$$



- **In special case where H_{11} is diagonal**

$$\langle f|H_{\text{eff}}(\omega)|i\rangle = \langle f|H_{00}|i\rangle + \sum_{|n\rangle}\frac{\langle f|H_{01}|n\rangle\langle n|H_{10}|i\rangle}{\omega - \langle n|H_{11}|n\rangle}$$

- **Resolvent operator $(\omega\text{-}H_{11})^{-1}$ not diagonal in H.O. basis**
  - **Work in momentum space initially**

- **Consider just second term of effective interaction**

$$\langle f | H_{eff}^{(2)}(\omega) | i \rangle = \sum_{|n\rangle} \frac{\langle f | H_{01} | n \rangle \langle n | H_{10} | i \rangle}{\omega - E_n}$$

  - **Sum reduces to sum over polarization states of photon**

$$\sum_{\lambda_g} \varepsilon_\mu(k_g, \lambda_g)\, \varepsilon_\nu^*(k_g, \lambda_g) = -g_{\mu\nu} + (k_{g,\mu}\eta_\nu + k_{g,\nu}\eta_\mu)/k_g^\kappa \eta_\kappa$$

$$\eta^\mu = (\eta^+, \eta_\perp, \eta^-) = (0, 0_\perp, 2)$$
$$; \ \eta^2 \equiv \eta^\mu \eta_\mu = 0$$
$$k\eta = k^+$$

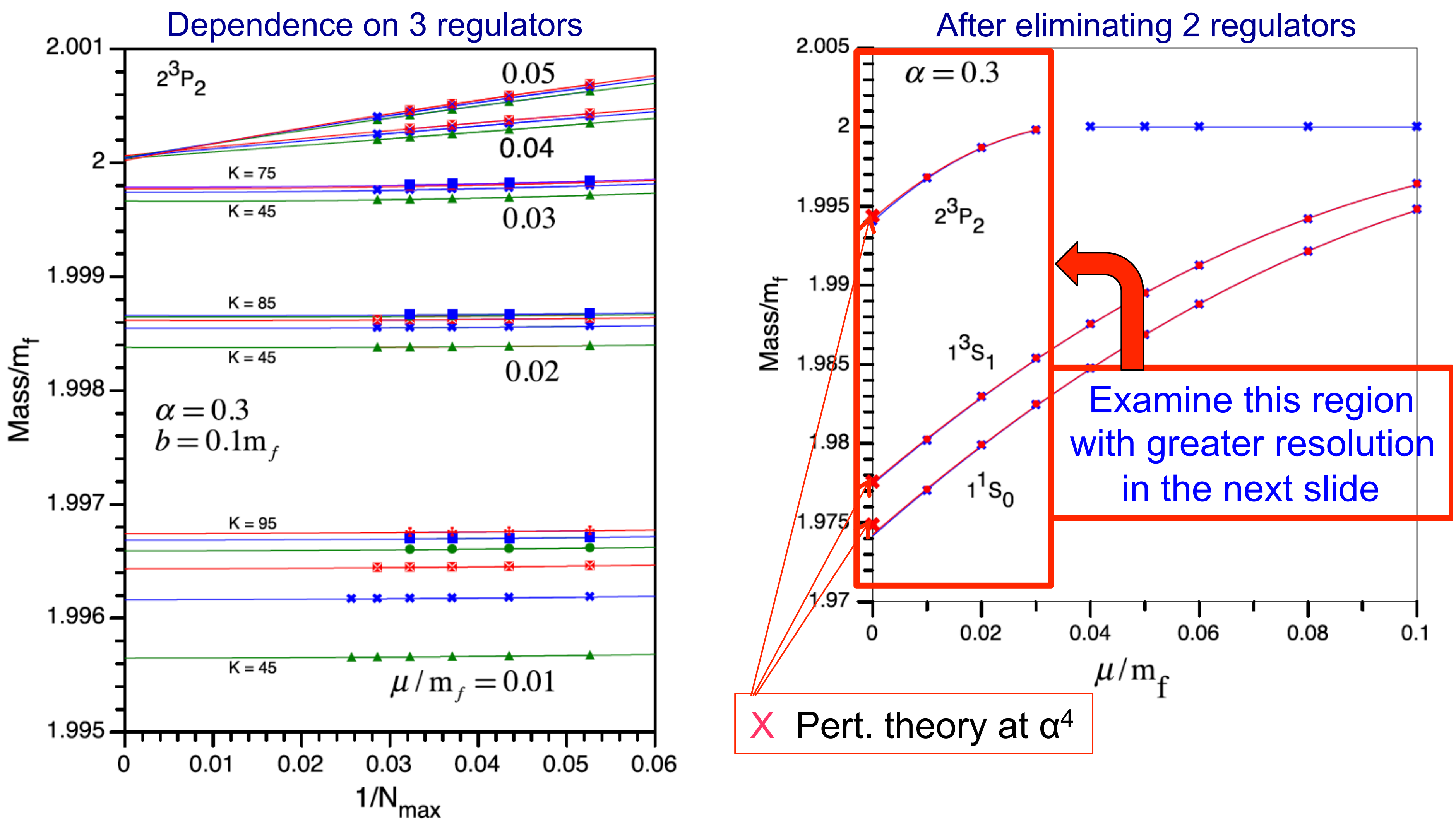
  - **Expected cancellation is achieved only if:**

$$\omega = \frac{E_i + E_f}{2}$$

With this choice the IR divergence of the instantaneous graph is cancelled exactly by IR divergent part of the effective interaction. Henceforward, these cancelling divergences are dropped from the calculations = a counterterm prescription.

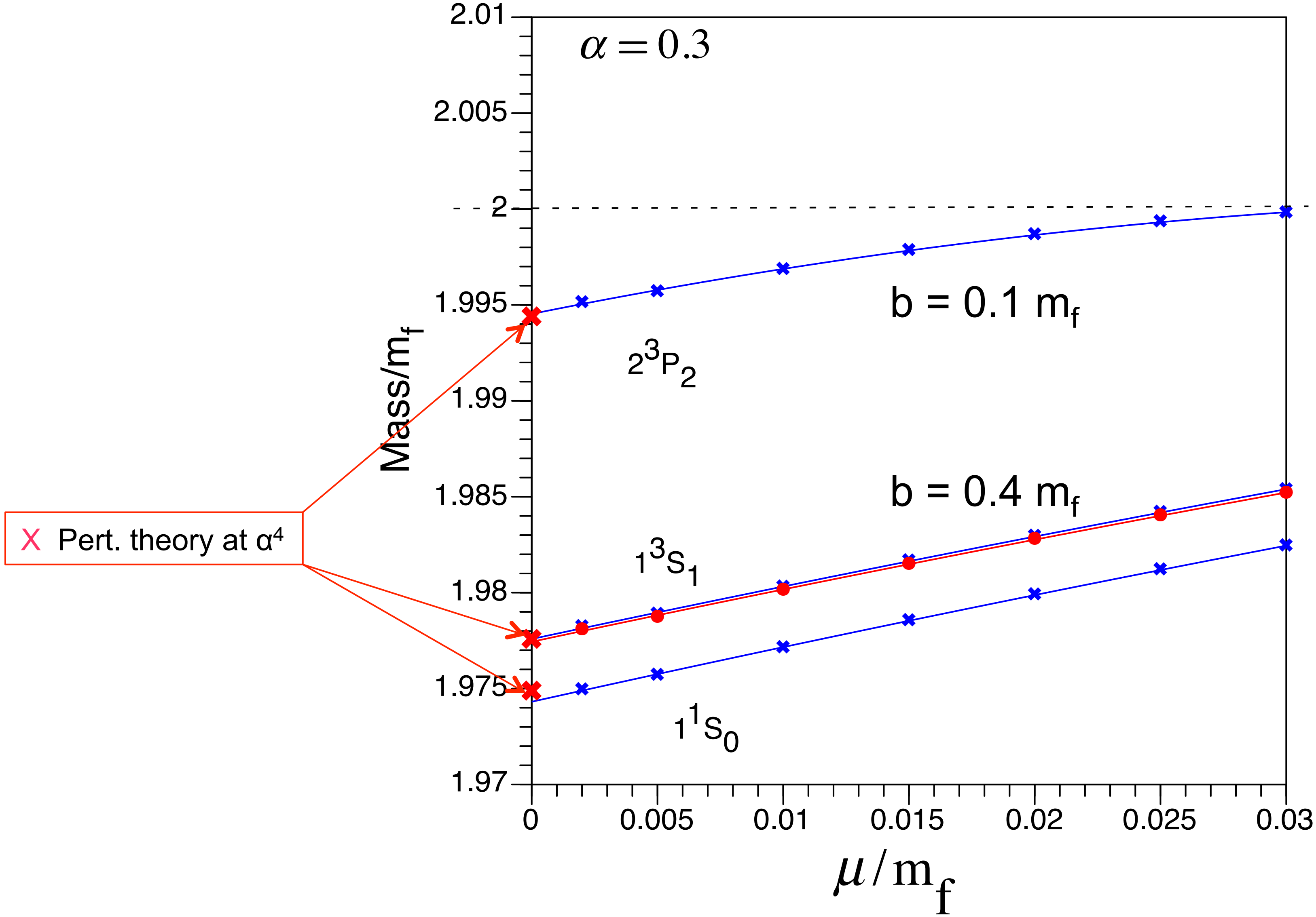
Similarly a counterterm is identified and included for the UV divergence in H_{eff}.

Basis Light-Front Quantization (BLFQ)
Positronium in QED at Strong Coupling (α = 0.3)
Systematic removal of regulators (b = HO momentum scale)



P. Wiecki, Y. Li, X. Zhao, P. Maris and J.P. Vary, Phys. Rev. D **91**, 105009 (2015)

Positronium in QED at Strong Coupling
Covariant Basis Light-Front Quantization (BLFQ)

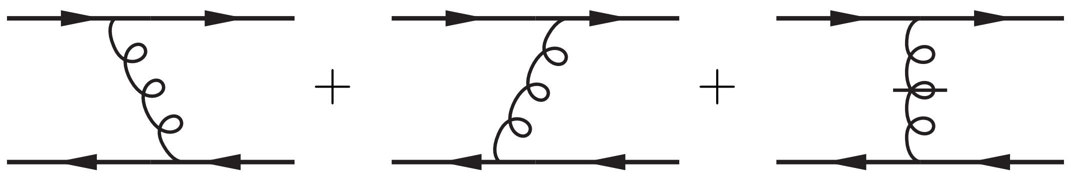


P. Wiecki, Y. Li, X. Zhao, P. Maris and J.P. Vary, Phys. Rev. D **91**, 105009 (2015); & to be published

[YL et al., Phys.Lett.B **758**, 118 (2016)]

Consider the case of heavy quarkonium: charmonium and bottomonium

- Fock sector truncation, effective Hamiltonian method etc [Wilson '74]



$$H_{\text{eff}} = \mathcal{P}H_0\mathcal{P} + \mathcal{P}H\mathcal{Q}\frac{1}{\frac{1}{2}(\epsilon_i + \epsilon_f) - \mathcal{Q}H_0\mathcal{Q}}\mathcal{Q}H\mathcal{P}$$

However, this is only suitable for QCD at short distance (and QED).

- For long-distance physics, we adopt a confining potential inspired by light-front holographic QCD [Brodsky '06, Trawiński '14]

$$V(\zeta_\perp) = \kappa^4\zeta_\perp^2 + \text{const.} \qquad (\zeta_\perp = \sqrt{x(1-x)}r_\perp)$$

  - AdS/QCD: first approximation to QCD inspired by AdS/CFT
  - soft-wall AdS/QCD produces Regge trajectory [Karch '06]
  - LF holography relates AdS/QCD to LF Schrödinger equation
  - successful applications: spectrum, form factors, β-function, ...

Basis Representation

[YL et al., Phys.Lett.B **758**, 118 (2016)]

The Hamiltonian is analytically solvable without the one-gluon exchange:

- Transverse: 2D HO in holographic variables $\phi_{nm}(\vec{k}_\perp/\sqrt{x(1-x)})$
- Longitudinal: $\chi_\ell(x) = x^{\frac{1}{2}\alpha}(1-x)^{\frac{1}{2}\beta}P_\ell^{(\alpha,\beta)}(2x-1)$

$\alpha = 2m_{\bar{q}}(m_q + m_{\bar{q}})/\kappa^2$, $\beta = 2m_q(m_q + m_{\bar{q}})/\kappa^2$, $P_\ell^{(a,b)}(z)$ Jacobi polynomials

- Mass eigenvalues:
  $M^2_{nm\ell} = (m_q + m_{\bar{q}})^2 + 2\kappa^2(2n + |m| + \ell + 3/2) + \frac{\kappa^4}{(m_q+m_{\bar{q}})^2}\ell(\ell+1)$

We adopt these functions (soft-wall LFWFs) as the basis:

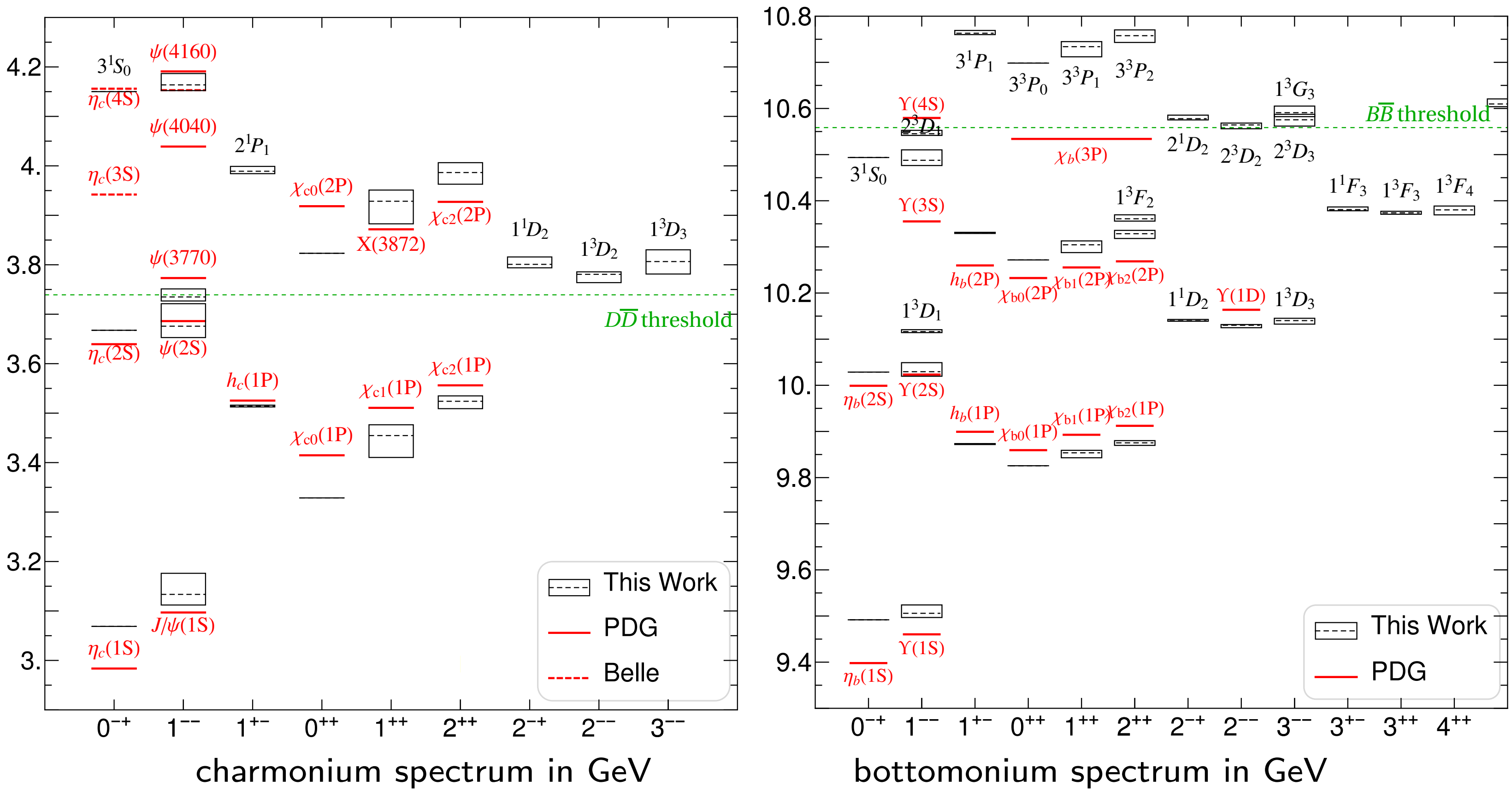
$$\psi_{h/q\bar{q}}(\vec{k}_\perp, x, s, \bar{s}) = \sum_{n,m,l} \Psi_{h/q\bar{q}}(n, m, l, s, \bar{s})\, \phi_{nm}\Big(\frac{\vec{k}_\perp}{\sqrt{x(1-x)}}\Big)\chi_l(x)$$

- implement LF holographic QCD for first approximation
- transverse 2D HO functions are scalable in the many-body sector (factorization of c.m. motion) [Li '13]
- basis truncation: $2n + |m| + 1 \le N_{\max}, l \le L_{\max}$
- quantum number identification (esp. mirror parity) [Soper '72]

We fix α_s and fit κ, m_q to the experimentally measured masses.

Mass Spectroscopy

[YL et al., Phys.Lett.B **758**, 118 (2016)]



Masses show weak m_J dependence due to the violation of rotational symmetry. We use boxes to indicate the spread of masses (dashed bars: averaged masses).

	α_s	μ_g (GeV)	κ (GeV)	m_q (GeV)	$\delta\overline{M}$ (MeV)	$N_{\max} = L_{\max}$
$c\bar{c}$	0.3595	0.02	0.938	1.522	52 (8 states)	24
$b\bar{b}$	0.2500		1.490	4.763	50 (14 states)	

Light-Front Wavefunctions (LFWFs)

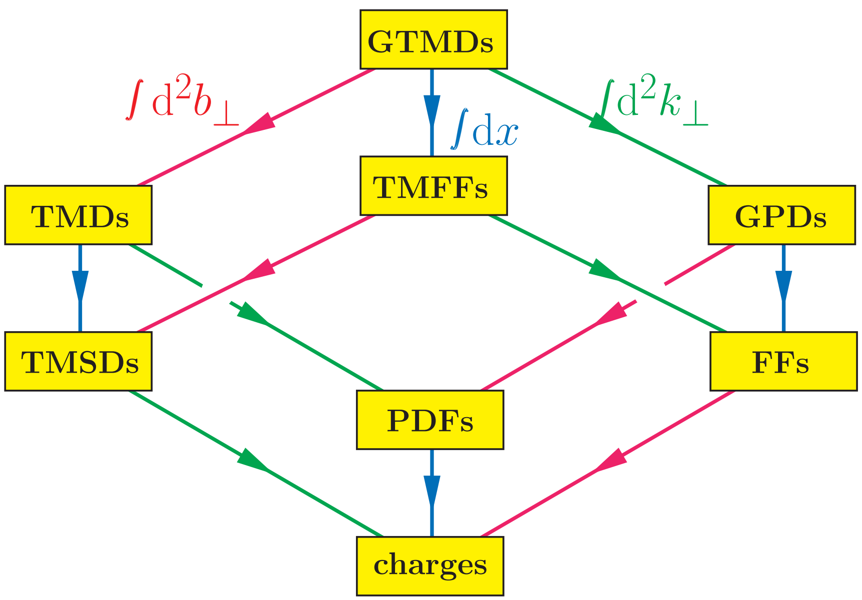
LFWFs provides intrinsic information of the structure of hadrons:

- Form factors (electromagnetic, gravitational ...) [Ji '97&'98]

$$A(q^2) = \sum_n \int \mathrm{d}D_n \sum_{f=1}^{n} x_f \, \psi_n^*(\{\vec{k}'_{i\perp}, x_i, \lambda_i\}_f)\psi_n(\{\vec{k}_{i\perp}, x_i, \lambda_i\}_f)$$

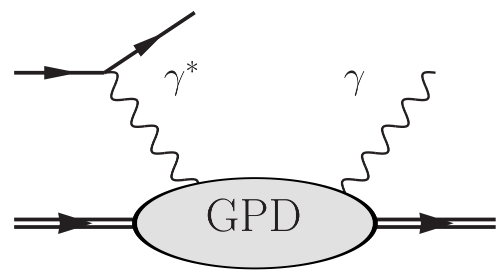
$$\vec{k}'_{i\perp} = \begin{cases} \vec{k}_{i\perp} + (1 - x_i)\vec{q}_\perp, & \text{for struck partons} \\ \vec{k}_{i\perp} - x_i\vec{q}_\perp, & \text{for spectators.} \end{cases}$$

- Distributions (hadron tomography) [Ji '97&'98]



$\vec{k}_\perp \leftrightarrow \vec{r}_\perp$, $\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

[Lorce & Pasquini '11]



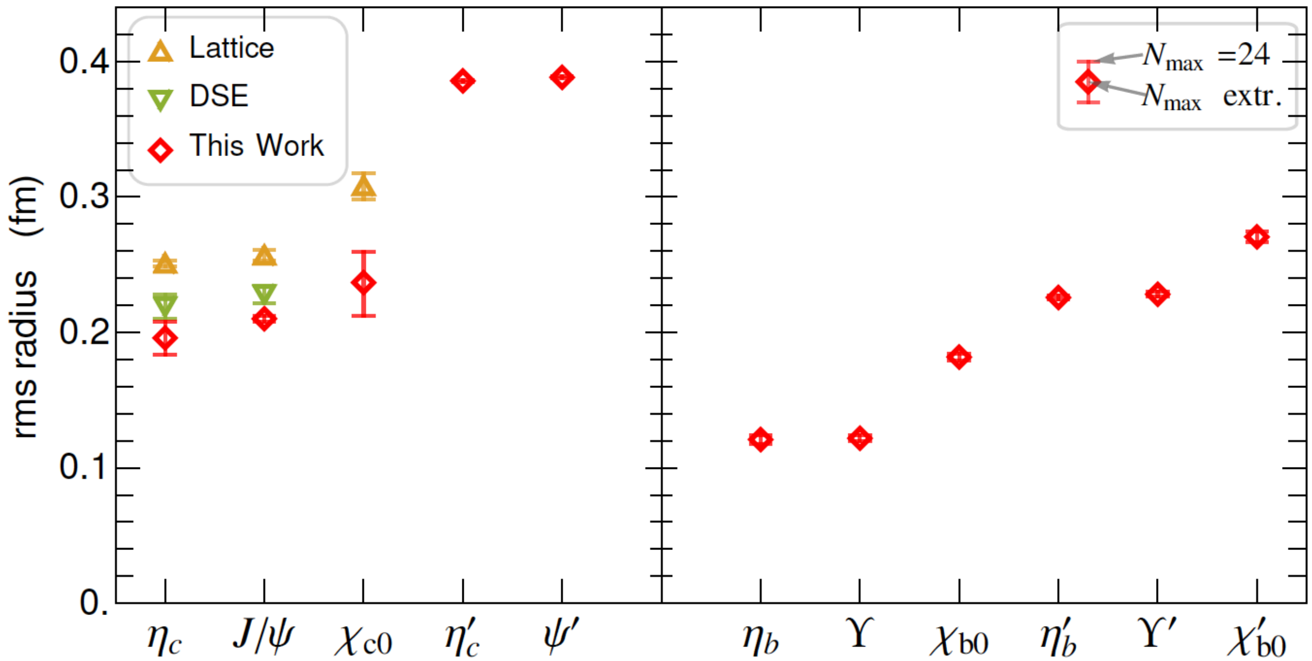
Charge Radii

[YL et al., Phys. Lett. B **758,** 118 (2016);
[JPV et al., Few Body Sys. DOI 10.1007/s00601-016-1117-x (2016)]

The charge radius:

$$\langle r^2 \rangle = -6 \frac{\partial}{\partial Q^2} G_0(Q^2)\big|_{Q^2 \to 0}.$$

- test long-distance physics (cf. decay constants)



[DSE: Maris '07; Lattice: Dudeck '06]

Hadron Tomography

[Adhikari et al., Phys.Rev.C **93**, 055202 (2016)]

- Generalized parton distributions (GPDs) [Ji '97 & '98]

$$H(x,\zeta,t) = \tfrac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ixP^+z^-} \langle P'|\overline{\psi}(-\tfrac{1}{2}z)\gamma^+\psi(+\tfrac{1}{2}z)|P\rangle\Big|_{z^+=z^\perp=0}$$

$q = P' - P$, $\zeta = q^+/P^+$, $t = q^2$.

  - DVCS, SIDIS, ..., spin physics

- Impact parameter dependent GPDs: [Burkardt '01]

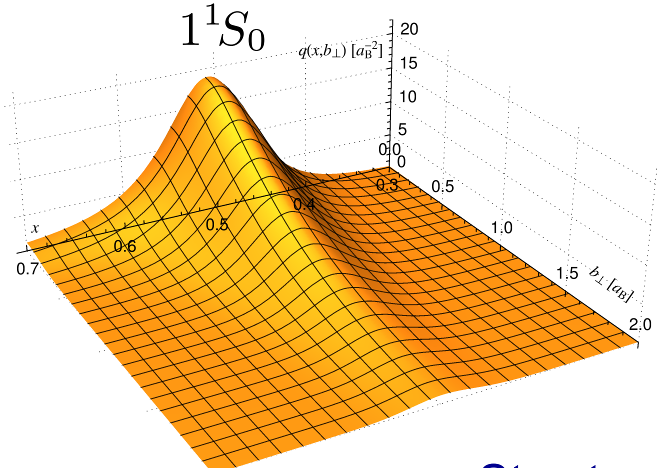
$$q(x,\vec{b}_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp} H(x,\zeta=0,t=-\Delta_\perp^2).$$

  - partonic interpretation: $\int \mathrm{d}^2 b_\perp \int_0^1 \mathrm{d}x \,\big|q(x,\vec{b}_\perp)\big|^2 = 1.$

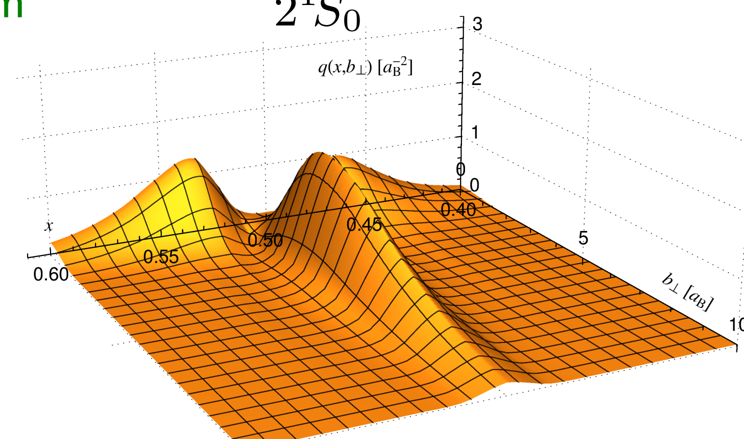
- Light-front wavefunction representation [Brodsky '01, Diehl '03]

positronium

1^1S_0



2^1S_0



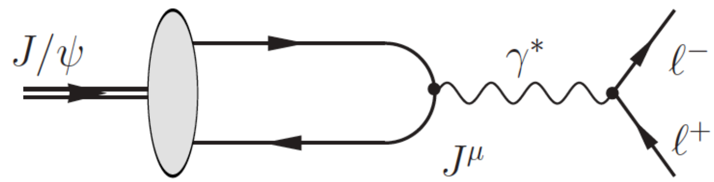
Stay tuned for quarkonia GPDs

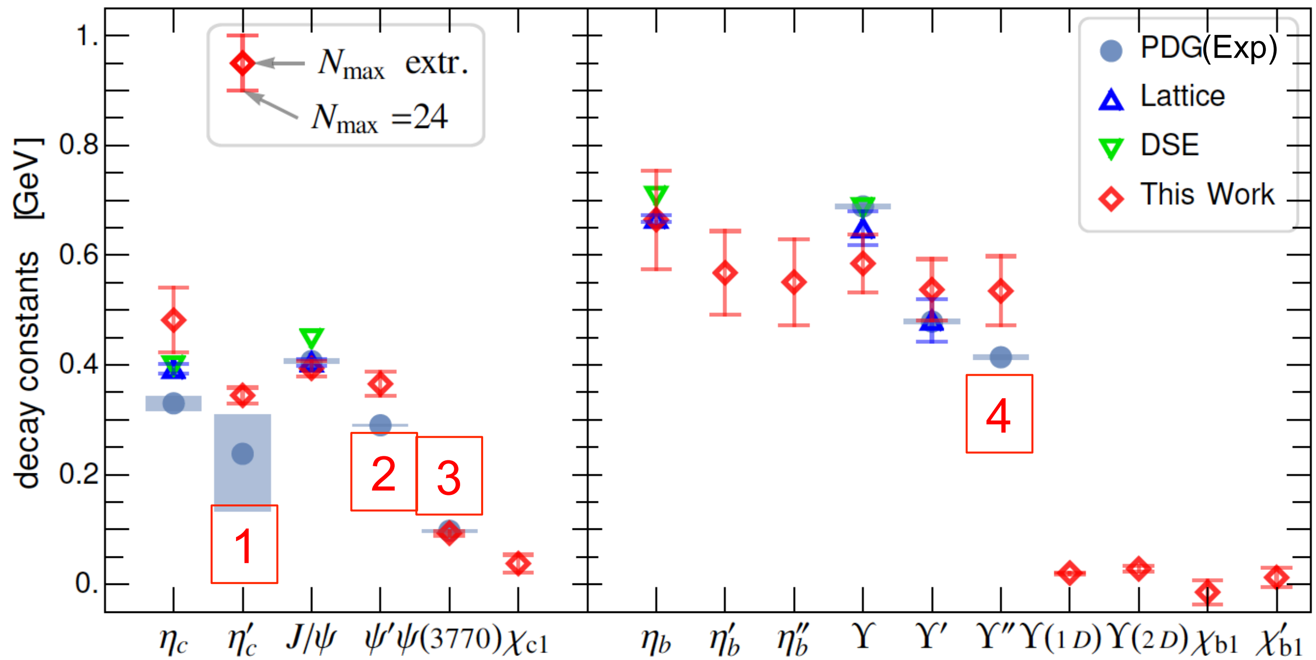
Decay Constants

[YL et al., Phys.Lett.B **758**, 118 (2016)]

$$\langle 0|\overline{\psi}\gamma^\mu\gamma^5\psi|P(p)\rangle = ip^\mu f_P,$$

$$\langle 0|\overline{\psi}\gamma^\mu\psi|V_\lambda(p)\rangle = e^\mu_\lambda(p) m_V f_V$$





Results
8 Exp
5 Lattice
4 DSE
16 BLFQ

Cases 1-4
Exp-BLFQ
alone

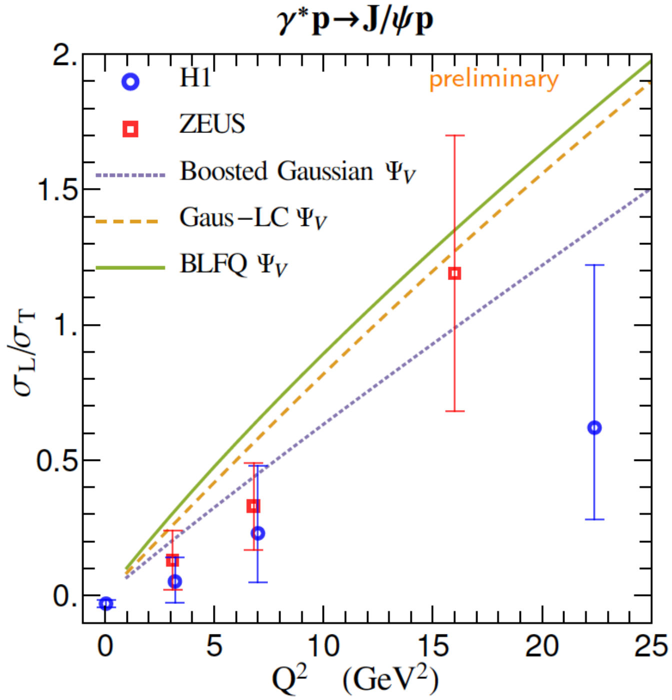
[DSE: Blank '11, Lattice: HPQCD, '10–'15]

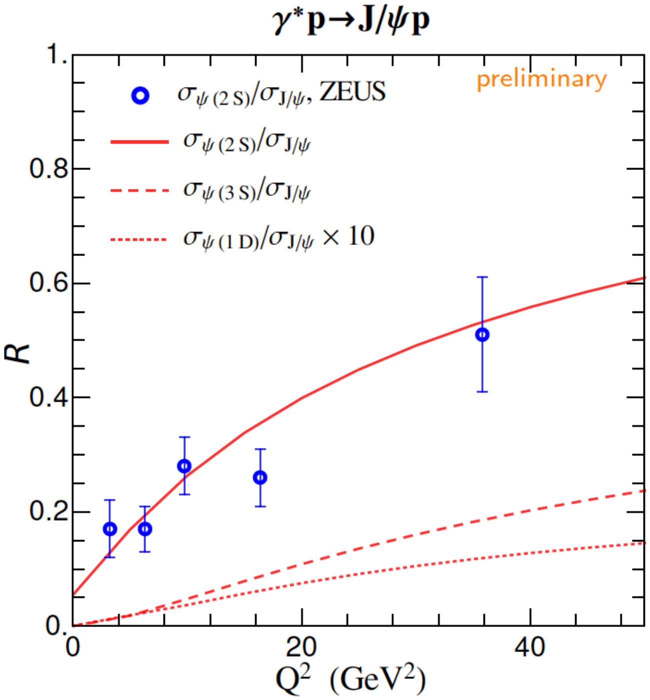
- Test "wavefunction at the origin" (cf. charge radius)
- Results are in reasonable agreement with experimental measurements as well as Lattice and DSE calculations where available.
- Results were extrapolated from $N_{\rm max} = L_{\rm max} = 8, 16, 24$, and there is some residual regulator dependence.

Diffractive Vector Meson Production

[Chen, in preparation]

The diffractive VM production tests BLFQ over a dynamical range not covered by the mass spectroscopy and decay constants.





- Provides access to excited states that are well constrained by physical observables (mass spectrum, decay constant etc).
- BLFQ LFWFs could help to discern the advantages and limitations of the dipole models (GBW, IP-Sat, b-CGC etc).
- Beyond Eikonal approximation using tBLFQ with Color Glass Condensate

[Li, in progress]

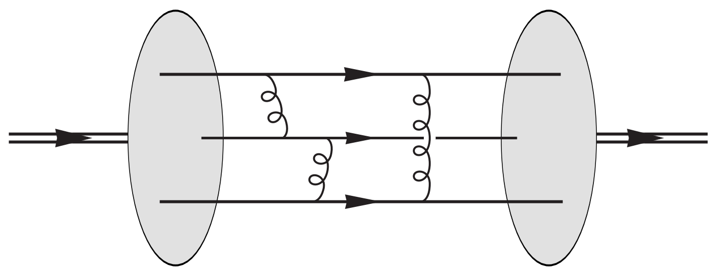
Generalization to Baryons

[work in progress]

The effective interaction can be generalized to the baryon sector:

$$H_{\text{eff}} = \sum_a \frac{\vec{p}_{a\perp}^2 + m_a^2}{x_a} - \vec{P}_\perp^2 + \frac{1}{2}\sum_{a,b} V_{ab}^{(2)} + \frac{1}{6}\sum_{a,b,c} V_{abc}^{(3)} + \cdots$$

- The soft-wall confinement: $V_{\text{SW}} = \frac{1}{2}\sum_{a,b} x_a x_b (\vec{r}_{a\perp} - \vec{r}_{b\perp})^2$.
- The one-gluon exchange



Jacobi coordinates on the light front (three-body example):

longitudinal: $x = x_3, \quad \chi = \frac{x_2}{1-x_3}$;

transverse momenta: $\vec{k}_\perp = (1-x_3)\vec{p}_{3\perp} - x_3(\vec{p}_{1\perp} + \vec{p}_{2\perp}), \quad \vec{\kappa}_\perp = \frac{x_1\vec{p}_{2\perp} - 2\vec{p}_{1\perp}}{x_1+x_2}$;

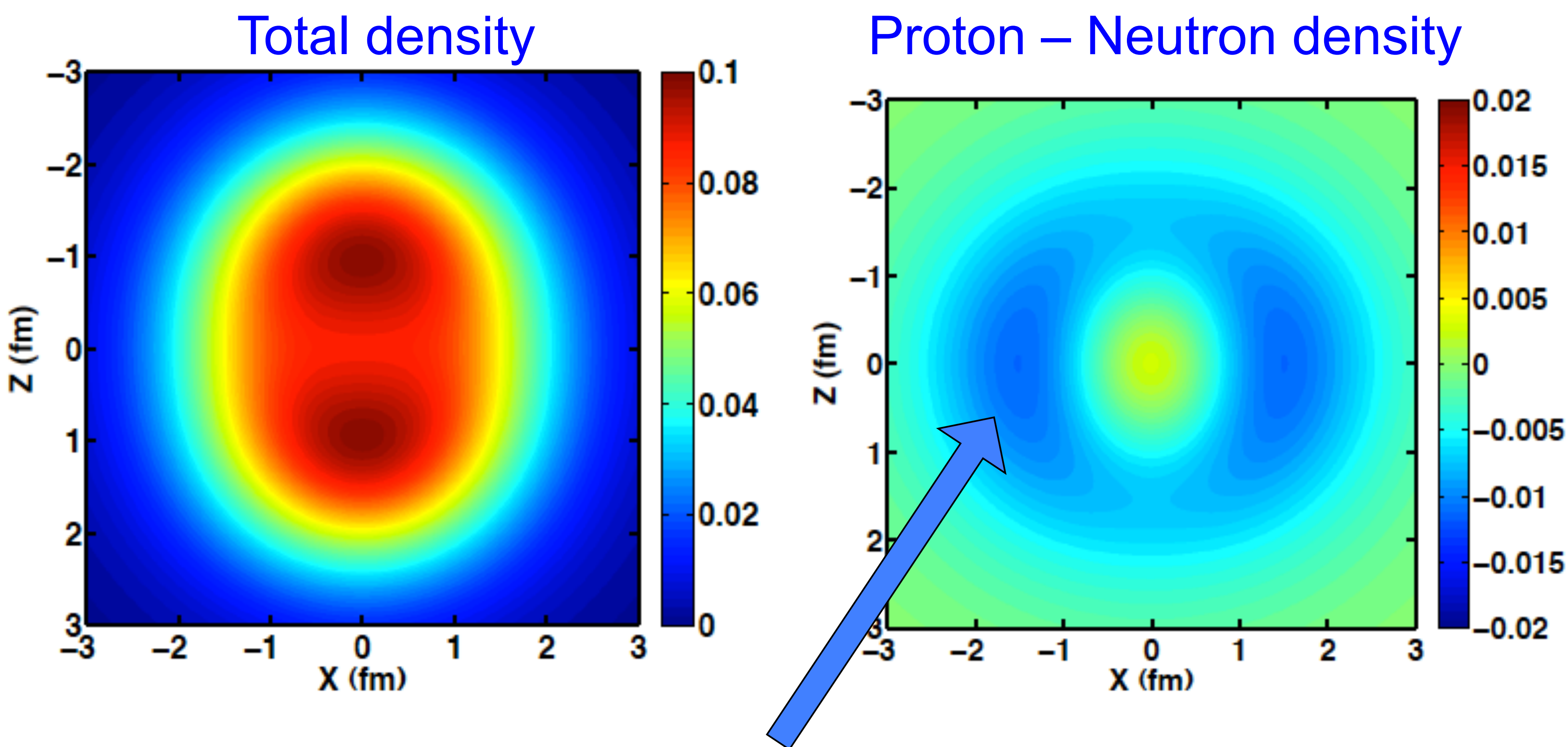
transverse coordinates: $\vec{r}_\perp = \vec{r}_{3\perp} - \frac{x_1\vec{r}_{1\perp} - x_2\vec{r}_{2\perp}}{x_1+x_2}, \quad \vec{\rho}_\perp = \vec{r}_{1\perp} - \vec{r}_{2\perp}$.

- Taking advantage of the kinematical nature of light-front boosts
  $V_{\text{SW}} = \kappa^4 x(1-x)\vec{r}_\perp^2 + \kappa^4(1-x)\chi(1-\chi)\vec{\rho}_\perp^2$
- The longitudinal confinement
  $V_L = -\frac{\kappa^4}{(m_1+m_2+m_3)^2}\Big[\partial_x\big(x(1-x)\partial_x\big) + \frac{1}{1-x}\partial_\chi\big(\chi(1-\chi)\partial_\chi\big)\Big]$

Emergent Phenomena

NCSM: Clustering and Collective Rotational Motion
BLFQ: Spontaneous Symmetry Breaking

9Be Translationally invariant gs density
Full 3D densities = rotate around the vertical axis



Shows that one neutron provides a “ring” cloud around two alpha clusters binding them together

C. Cockrell, J.P. Vary, P. Maris, Phys. Rev. C86, 034325 (2012); C. Cockrell, PhD, Iowa State University

Can we observe a phase transition in Φ^4_{1+1}?

How does a phase transition develop as a function of increased coupling?

What are the observables associated with a phase transition?

What are its critical properties (coupling, exponent, …)?

ϕ^4 in 1+1 Dimensions
DLCQ with Coherent State Analysis

A. Derive the Hamiltonian and quantize it on the light front,
investigate coherent state treatment of vacuum:
A. Harindranath and J.P. Vary, Phys Rev D**36**, 1141(1987)

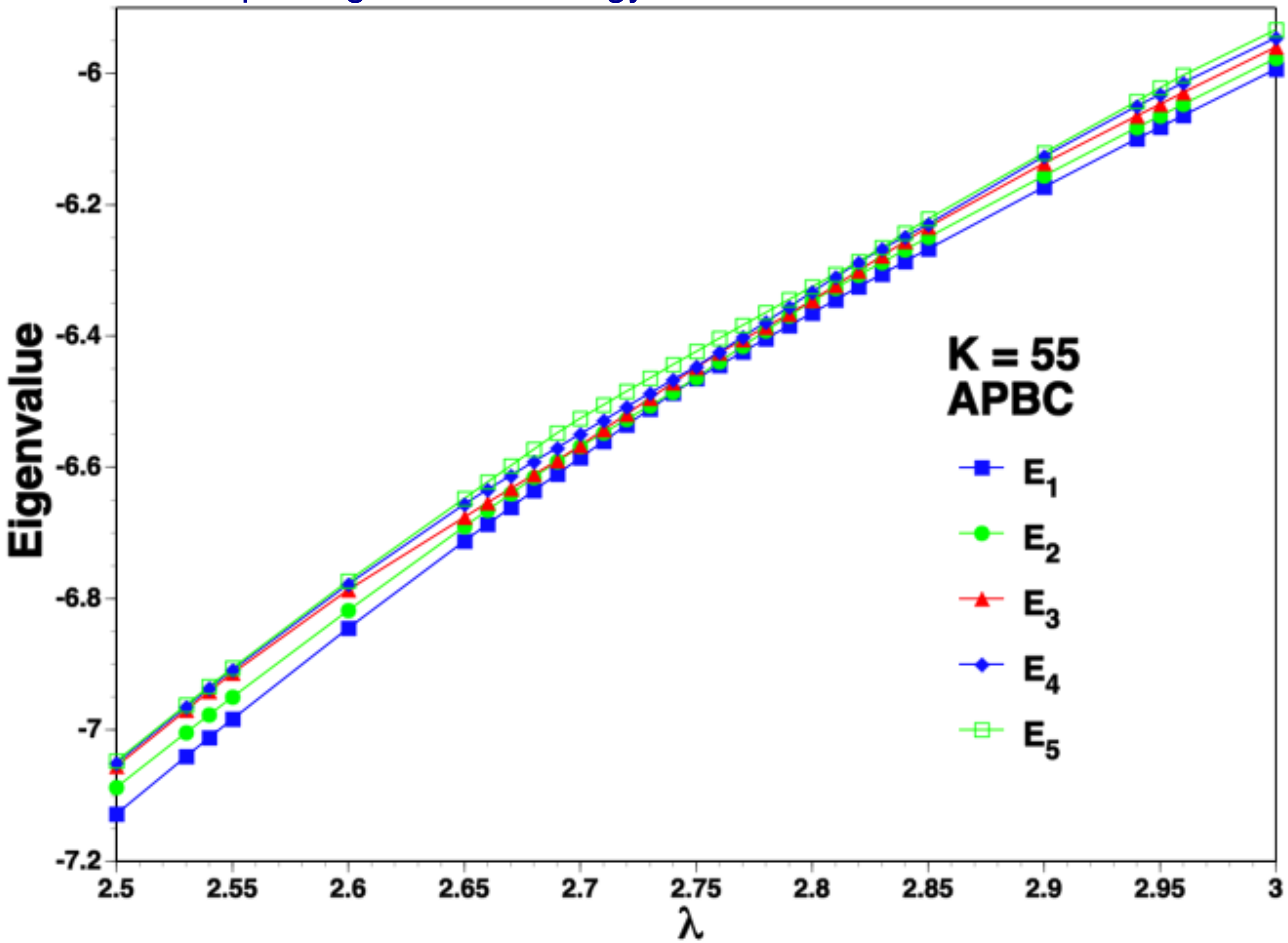
B. Obtain vacuum energy as well as the mass and profile functions (topological properties) of soliton-like solutions ("Kinks") in the symmetry-broken phase:

APBC: SSB = degeneracy ~ Kink ~ coherent state!
Chakrabarti, Harindranath, Martinovic and Vary,
Phys. Letts. B**582**, 196 (2004); hep-th/0309263

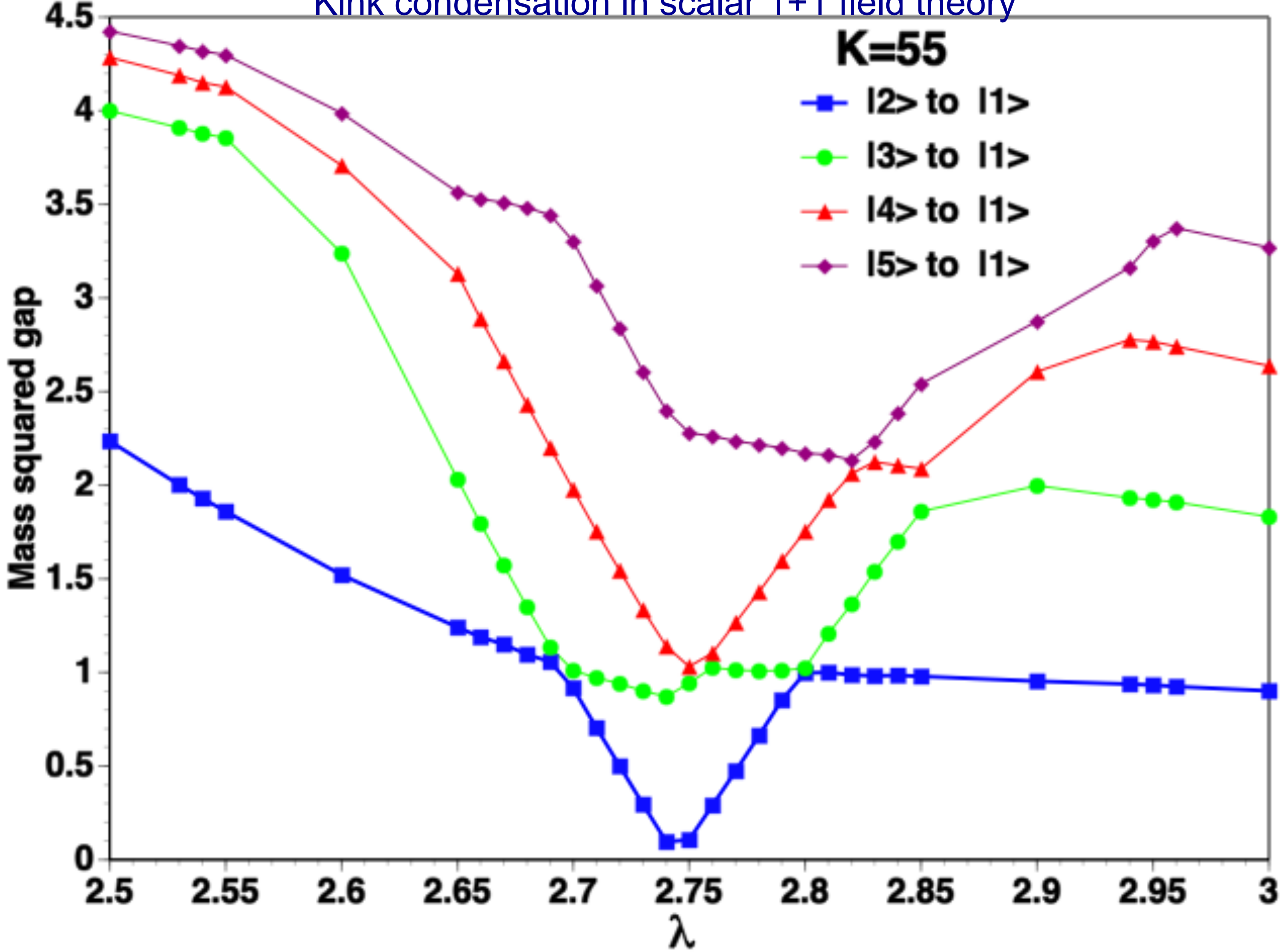
PBC: SSB = degeneracy ~ Kink + Antinkink ~ coherent state!
Chakrabarti, Harindranath, Martinovic, Pivovarov and Vary,
Phys. Letts. B**617**, 92(2005); hep-th/0310290.

C. Demonstrate onset of Kink Condensation at strong coupling (**APBC**)
Chakrabarti, Harindranath and Vary,
Phys. Rev. D**71**, 125012(2005); hep-th/0504094

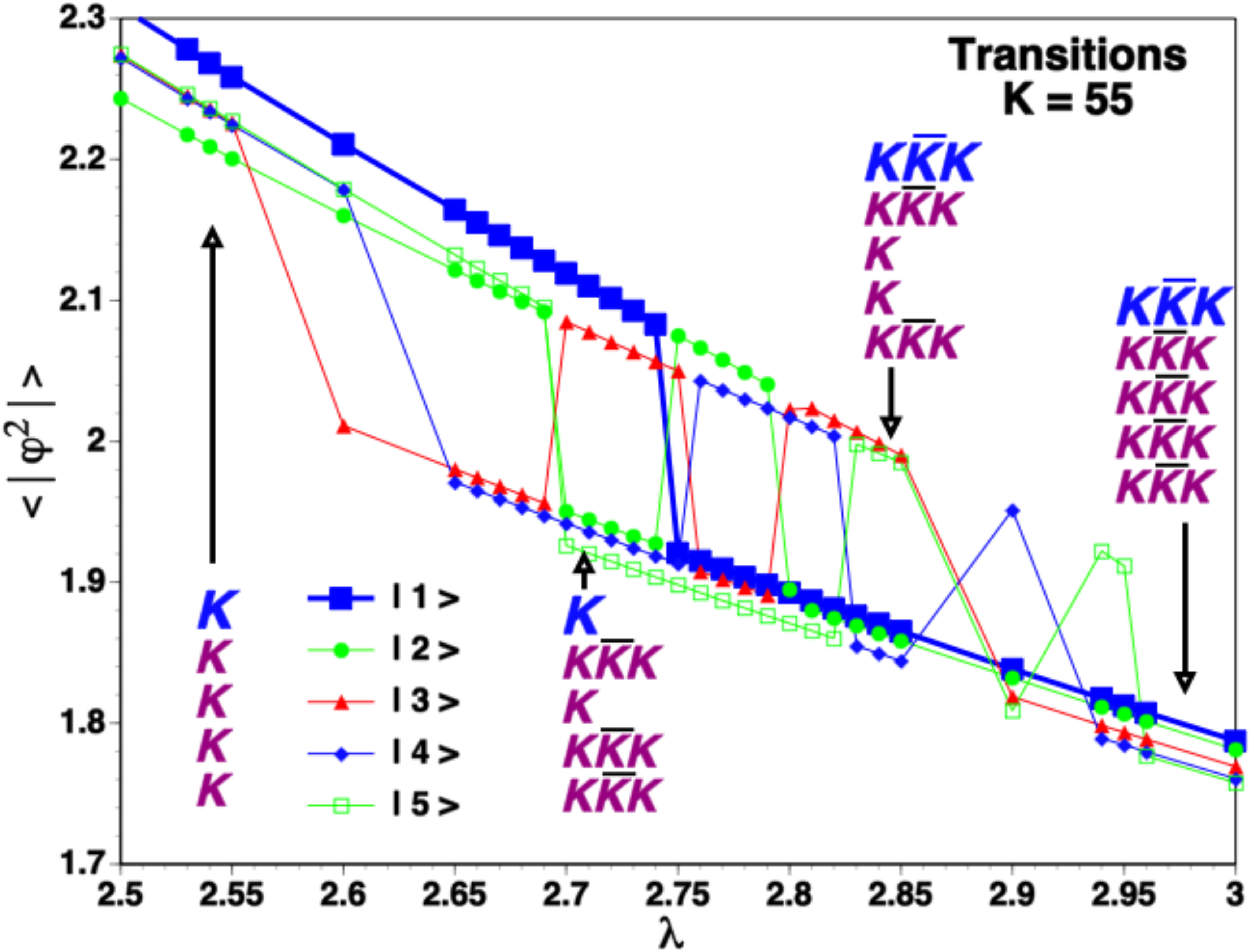
Just plotting the total energy on a course scale reveals little



Spontaneous symmetry breaking in LF quantized Hamiltonian approach
Kink condensation in scalar 1+1 field theory

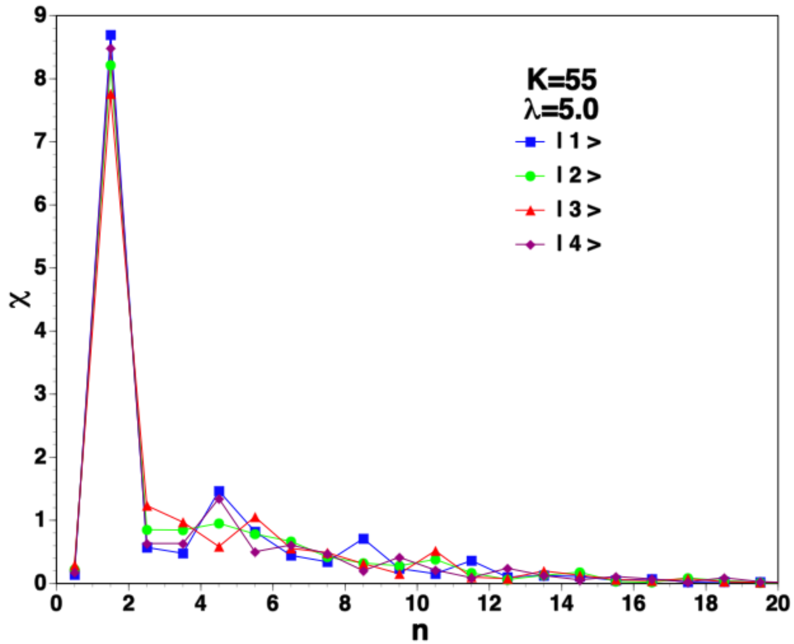


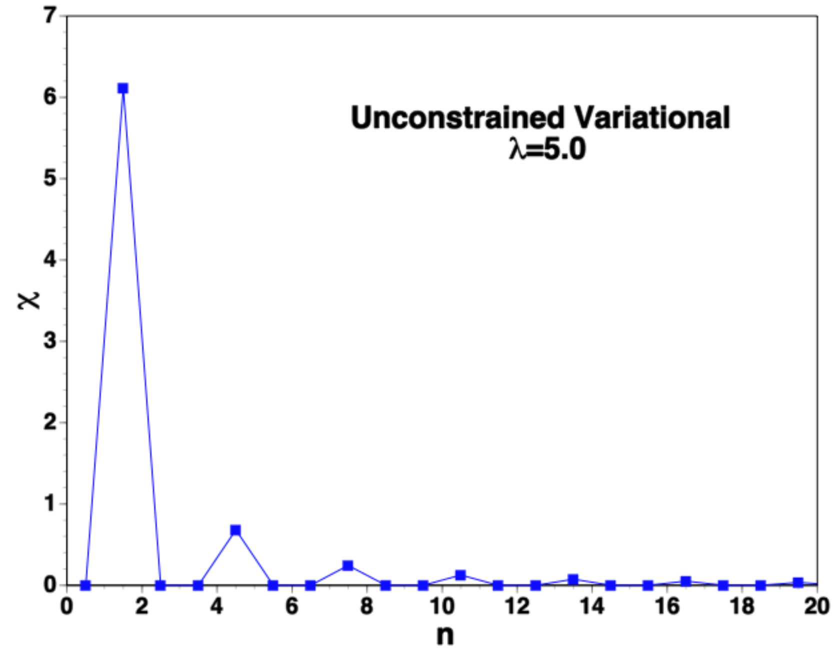
D. Chakrabarti, A. Harindranath and J.P. Vary, *Phys. Rev. D* **71**, 125012(2005); hep-th/05104094.



D. Chakrabarti, A. Harindranath and J.P. Vary, *Phys. Rev. D* **71**, 125012(2005); hep-th/05104094.

Compare lowest state's LF momentum distribution at strong coupling with ansatz variational coherent state's

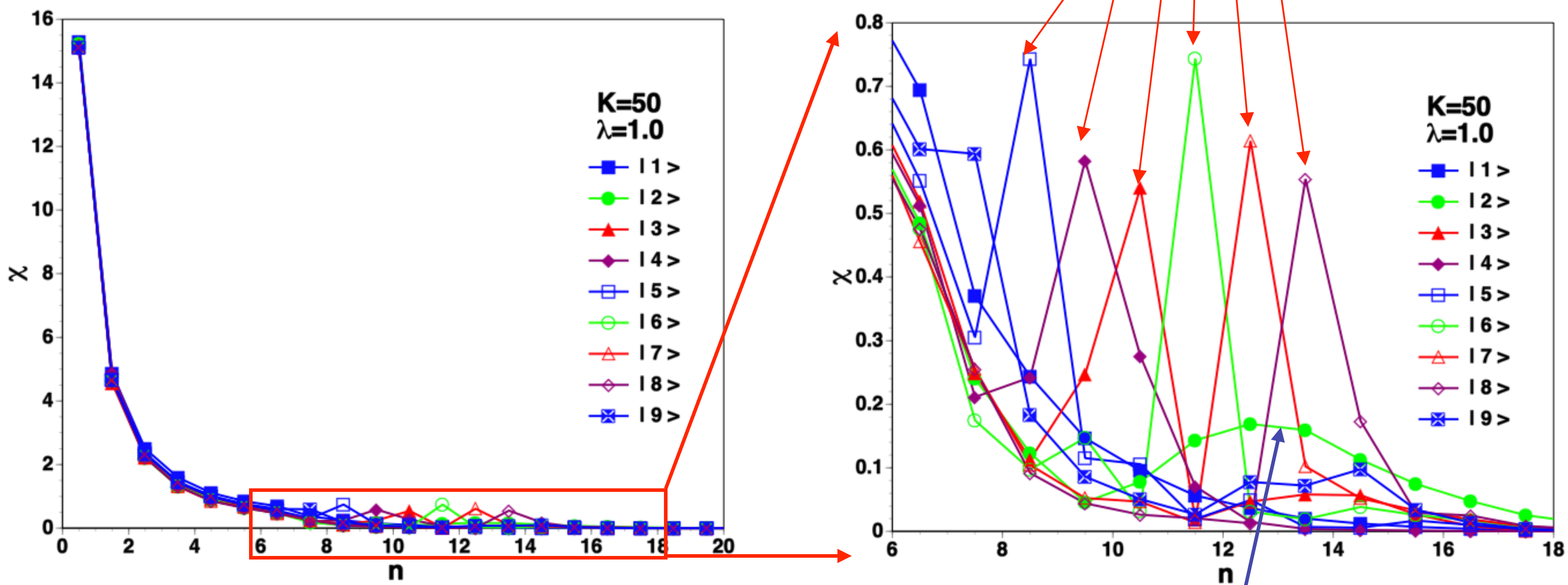




D. Chakrabarti, A. Harindranath and J.P. Vary, *Phys. Rev. D* **71**, 125012(2005); hep-th/05104094.

At weak coupling, kink-boson scattering states and kink collective excitation observed

States 3-8:
single boson in plane wave states
scattering from Kink

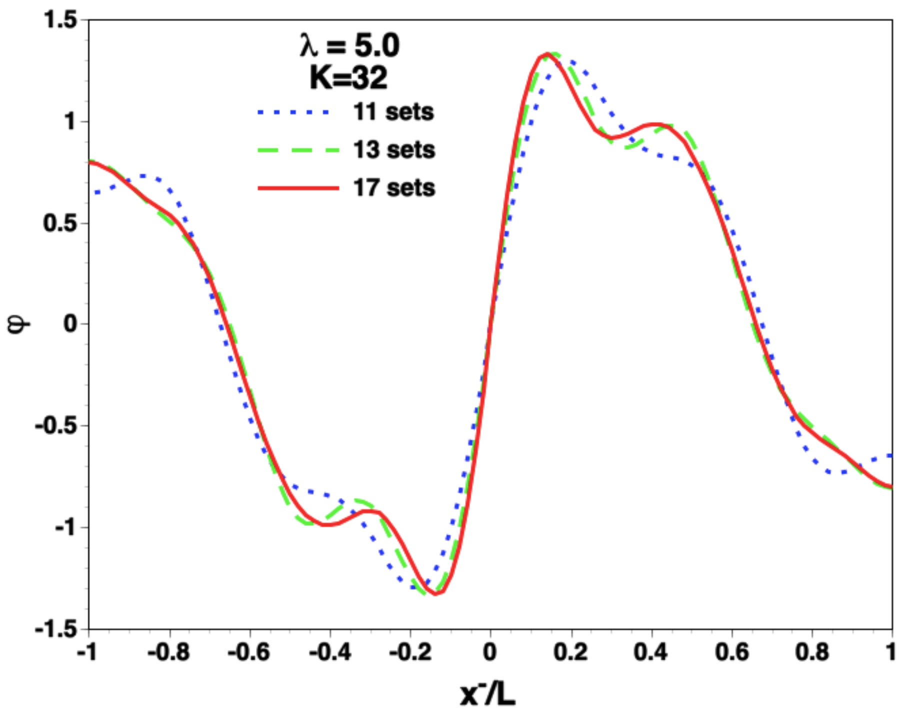


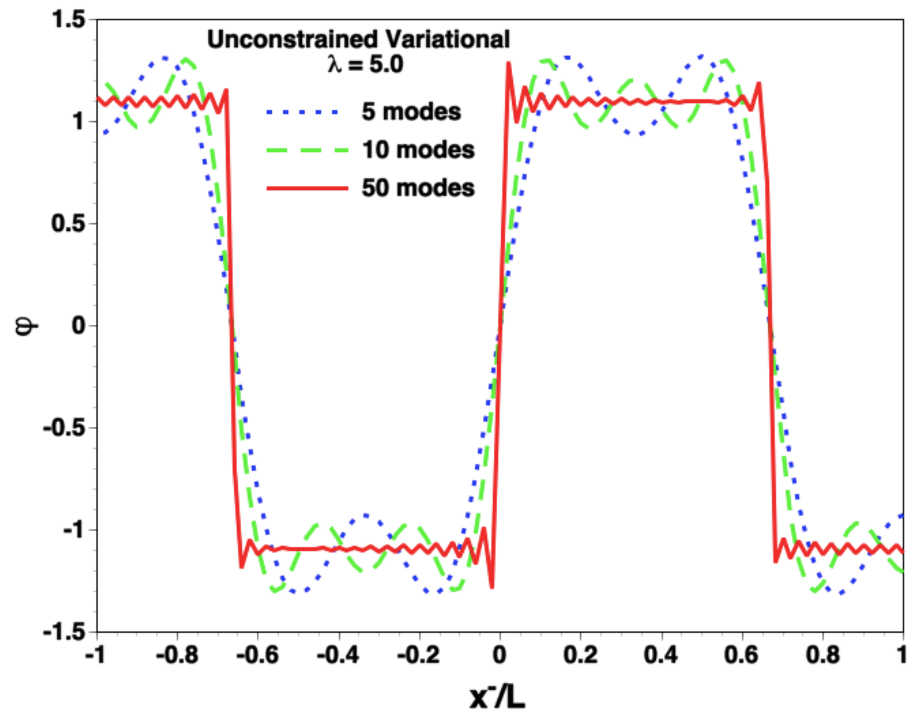
State 2 = Collective Kink excitation

State 9 => 2 boson excitation

D. Chakrabarti, A. Harindranath and J.P. Vary, *Phys. Rev. D* **71**, 125012(2005); hep-th/05104094.

Compare lowest state's topology at strong coupling with ansatz variational coherent state topology

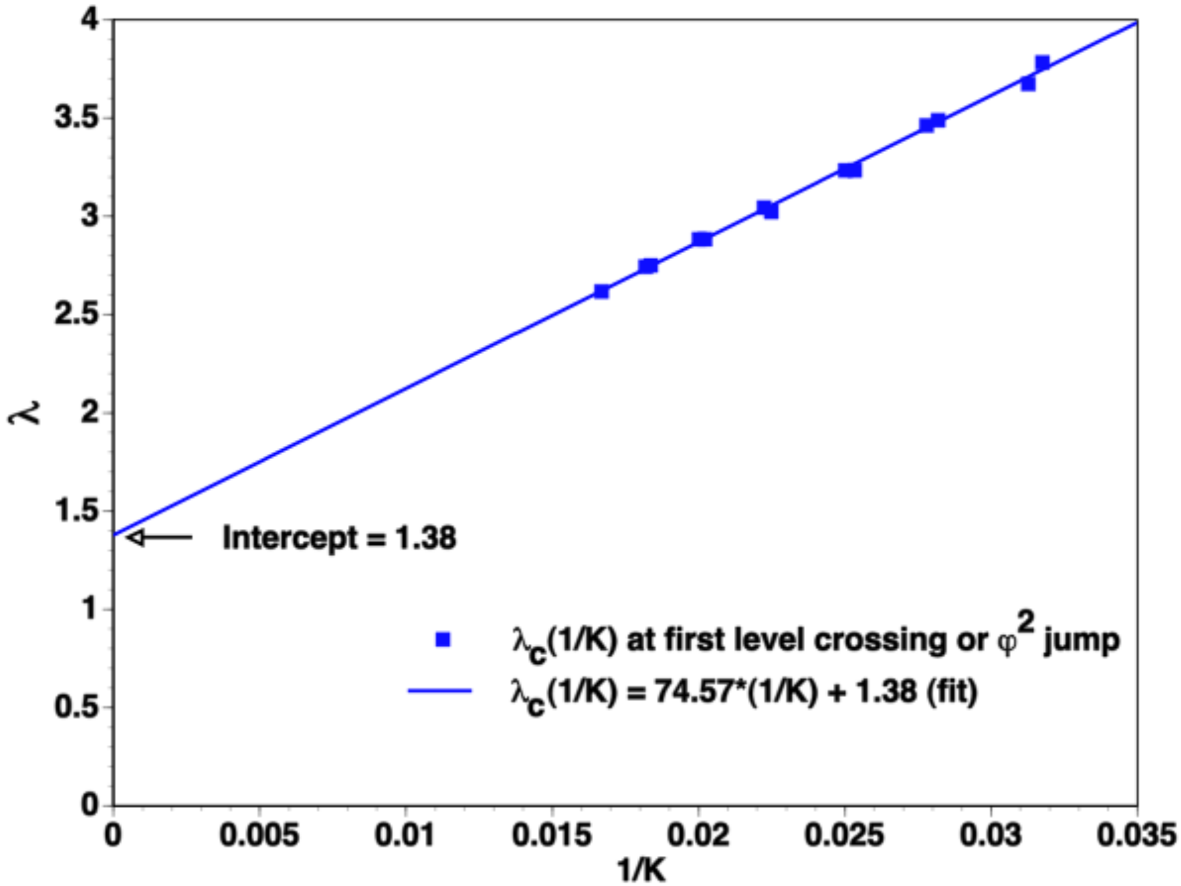




D. Chakrabarti, A. Harindranath and J.P. Vary, *Phys. Rev. D* **71**, 125012(2005); hep-th/05104094.

D. Chakrabarti, A. Harindranath and J.P. Vary, *Phys. Rev. D* **71**, 125012(2005); hep-th/05104094.

Continuum limit of the critical coupling and critical exponent



Analysis of the vanishing mass gap δM^2 yields:

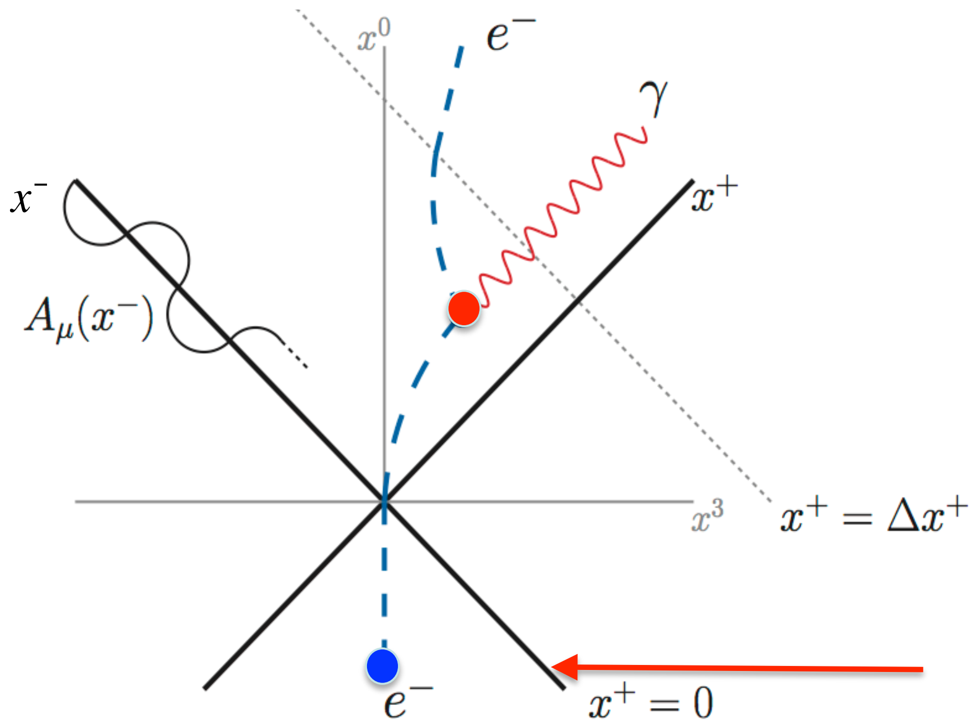
$$\delta M^2 = \left(\left|\lambda - \lambda_c\right|\right)^\nu, \qquad \nu = 1.0$$

Agrees with classical constructive field theory

tBLFQ: Nonlinear Compton Scattering

See Friday's talk by Xingbo Zhao

- Space-time structure



- Two effects: acceleration and radiation

Xingbo Zhao, Anton Ilderton, Pieter Maris and James P. Vary, Phys. Rev. D 88, 065014 (2013); and Phys. Letts B 726, 856 (2013); Guangyao Chao, et al, in preparation

Summary and Outlook

There exist multiple avenues of symbiosis between
ab initio nuclear theory and relativistic quantum field theory

Continuous interchange should be mutually beneficial:

- Many-body theory – bound states and scattering
- Renormalization, regularization and extrapolation
- Uncertainty quantification
- Methods for identifying emergent phenomena
- Efficient utilization of supercomputing resources

This INT program is one example of a productive approach