# Magnetic Properties of Light Nuclei from Lattice QCD

INT-Program **16-1**
***Nuclear Physics from Lattice QCD***

B C Tiburzi
18 May 2016

My work funded by NSF

Done in collaboration with ***Nuclear Physics Lattice QCD*** = NPLQCD

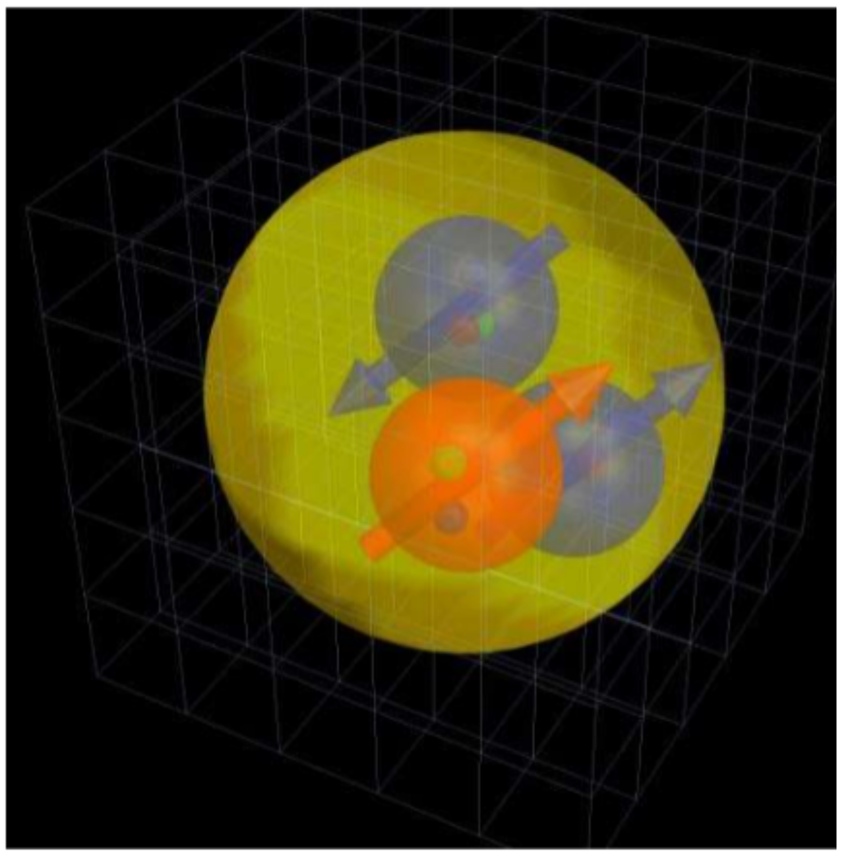
# Magnetic Moments of Light Nuclei

Home Physics General Physics February 2, 2015

PHYS ORG

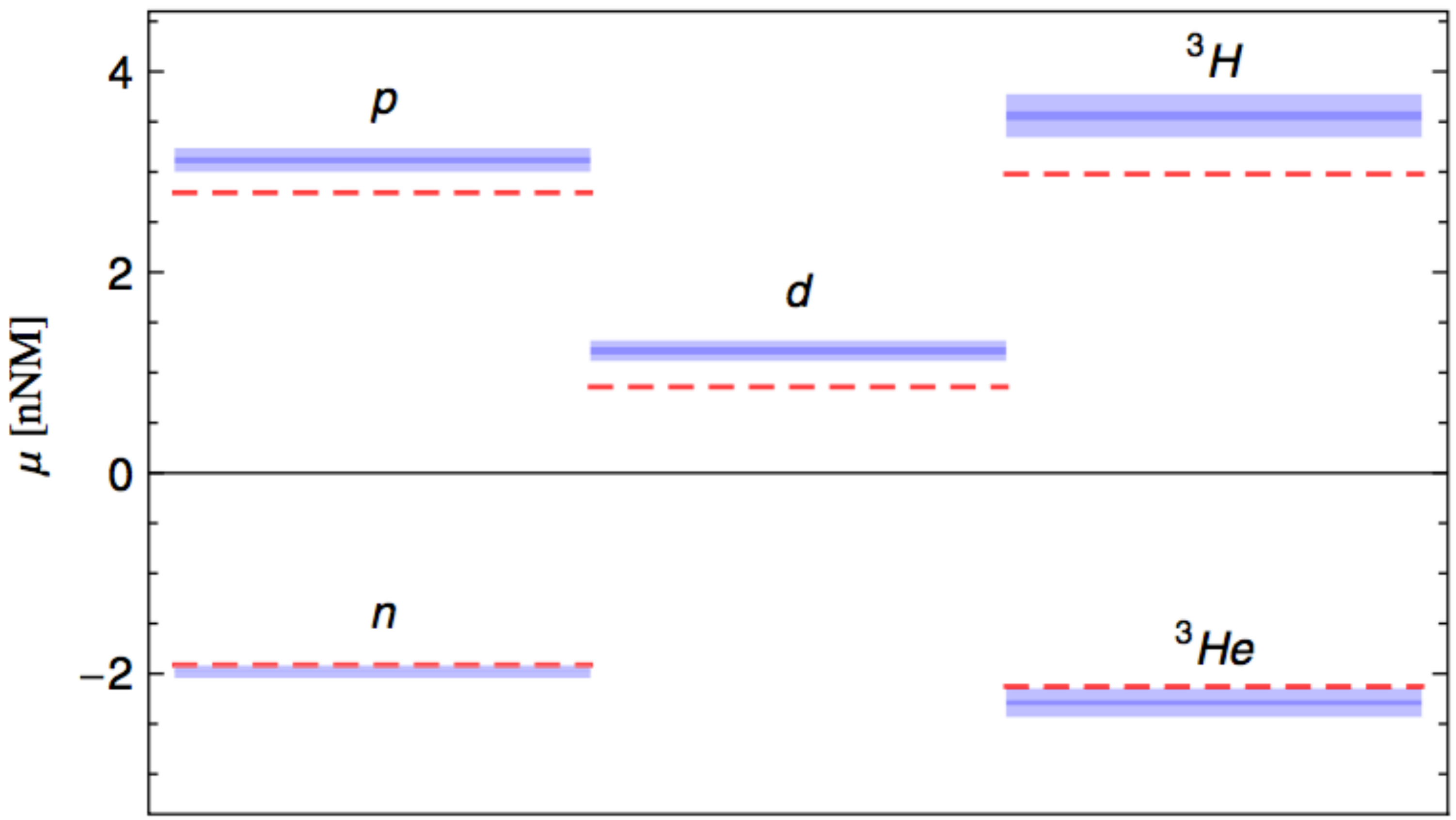
## Pinpointing the magnetic moments of nuclear matter

February 2, 2015 by Kathy Kincade



Artist's impression of a triton, the atomic nucleus of a tritium atom. The image show a red neutron with quarks inside; the arrows indicate the alignments of the spins. Credit: William Detmold, MIT

A team of nuclear physicists has made a key discovery in its quest to shed light on the structure and behavior of subatomic particles.

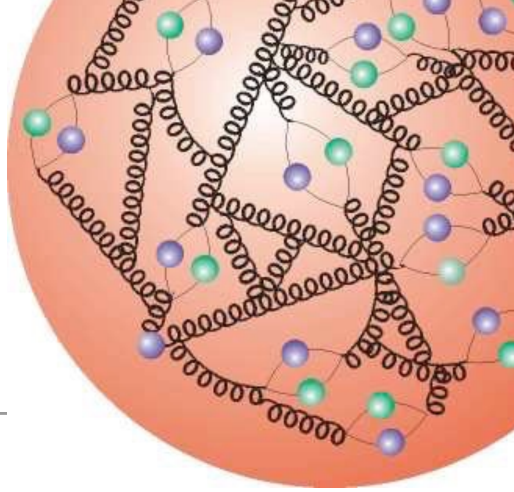



**Beane, *et al.* (NPLQCD), Phys.Rev. Lett.113, 2014.**

**First Computation**: $m_u = m_d = (m_s)_{\text{phys}}$ $m_\pi \sim 800\,\mathtt{MeV}$

# *Grand Overview*

## Electroweak Interactions: Nucleons and Nuclei



- *Lattice QCD* continues to sharpen our knowledge of The Standard Model (e.g. *CKM* extraction, *K* —> $\pi\ \pi$ )
- Nucleons and light nuclei present ~~challenge~~ opportunity
- QCD relevant for high-precision low-energy experiments


Magnetic properties of light nuclei

NPLQCD

# Quark Interactions to Nuclear Physics

Ken Wilson
*1936-2013*

- ***Textbook***: gauge theories defined in perturbation theory

- ***QCD***: short distance perturbative, long distance non-perturbative

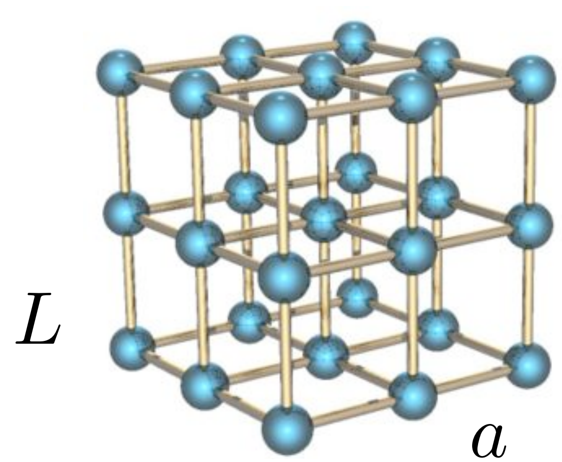
$$\overline{q}\,(\not{D} + m_q)\,q + \frac{1}{4} G_{\mu\nu} G_{\mu\nu} \quad \xrightarrow{\text{Many Technicalities}} \quad M_N \quad \delta_{NN}(k) \quad \epsilon_b(D)$$

Non-perturbative definition of asymptotically free gauge theories

Strong interaction observables

`One step:`
$$\int [\mathcal{D}A_\mu]\, e^{-S_{\mathrm{YM}}(A_\mu)} \approx \frac{1}{N_{\mathrm{cfg}}} \sum_{\{A_\mu\}} e^{-S_{\mathrm{YM}}(A_\mu)}$$
`stat. evaluation`

`Another step:`




`sys. approx.`
$$U_\mu(x) = e^{igaA_\mu(x)} \in SU(3)$$

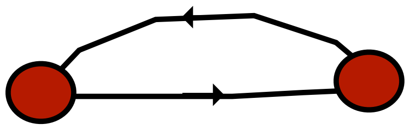
`Quarks:`
$$\frac{1}{\not{D}(U_\mu) + m_q}$$

Quark electroweak interactions fortunately perturbative … $J_\mu = \overline{q}\gamma_\mu q$

# Particle Physics (B=0) vs. Nuclear Physics (B>0)

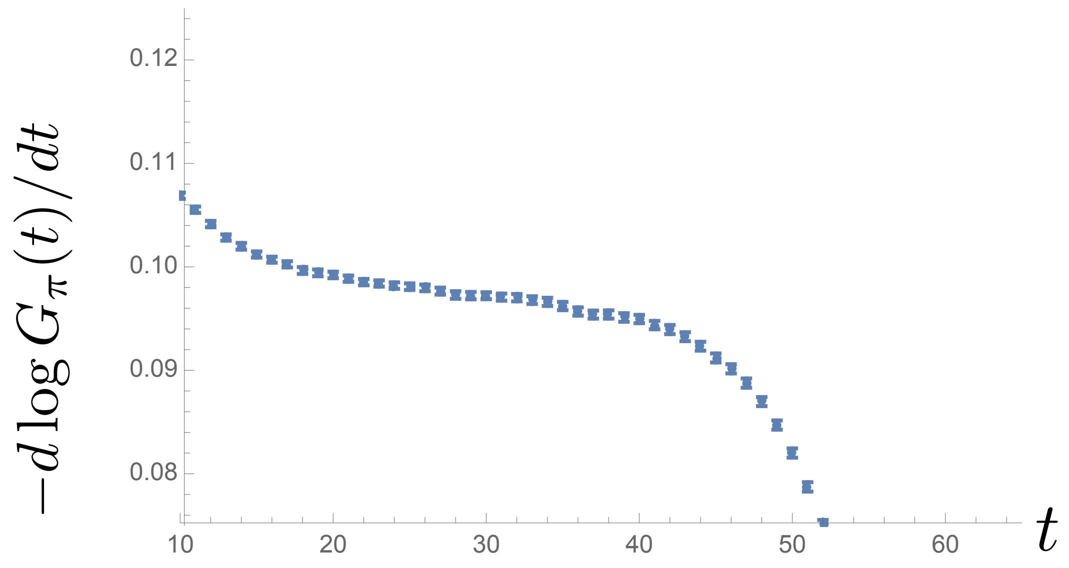
## Pion Correlation Function

Signal $\sum_{\{A_\mu\}} \langle q\overline{q}(t) q\overline{q}(0)\rangle \sim e^{-m_\pi t}$



Noise^2 $\sum_{\{A_\mu\}} \langle q\overline{q}(t) q\overline{q}(t) q\overline{q}(0) q\overline{q}(0)\rangle \sim e^{-2m_\pi t}$

Signal/Noise $\sim \text{const}$



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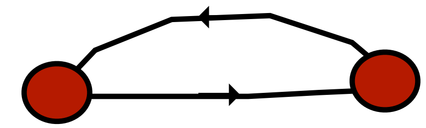

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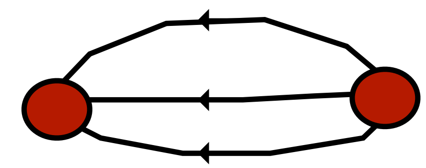
Signal/Noise

$\sim \mathrm{const}$

Baryons are statistically noisy

## Nucleon Correlation Function

Signal $\sum_{\{A_\mu\}} \langle qqq(t)\overline{qqq}(0)\rangle \sim e^{-Mt}$

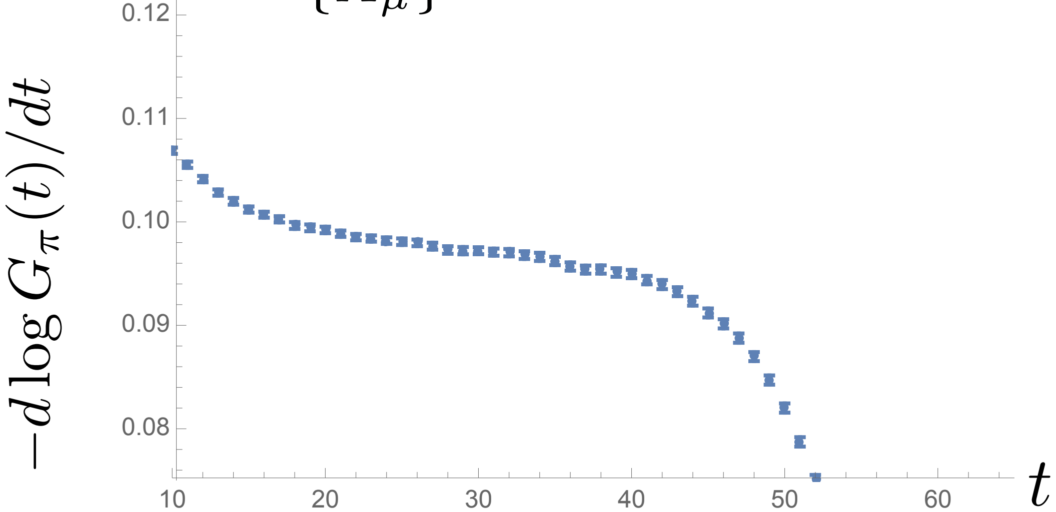


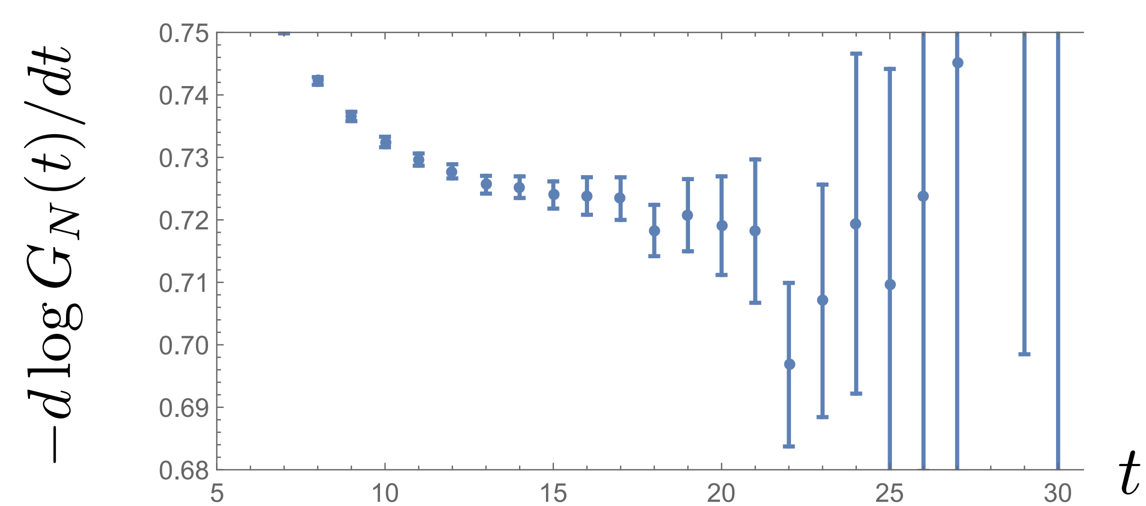
Noise^2 $\sum_{\{A_\mu\}} \langle qqq(t)\overline{qqq}(t) qqq(0)\overline{qqq}(0)\rangle \sim e^{-3m_\pi t}$

Scales exponentially with B in asymptotic time limit

Signal/Noise

$\sim e^{-(M-\frac{3}{2}m_\pi)t}$

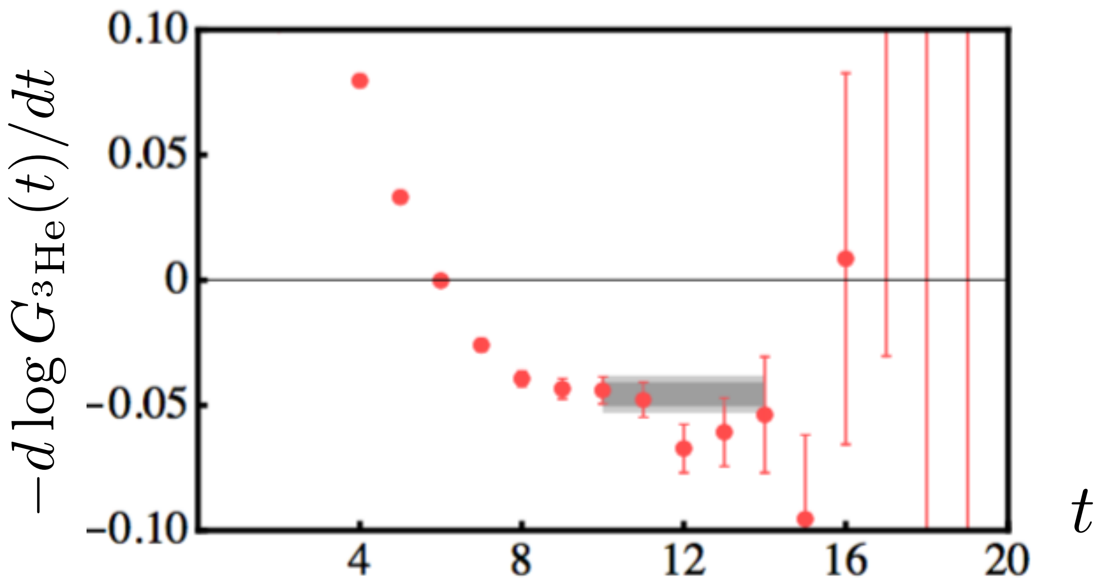


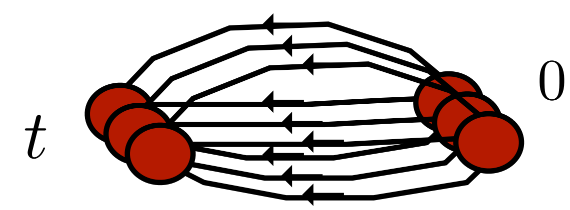
# Nuclear Physics @ $m_\pi$ =800 and 450 MeV

Beane, Chang, Cohen, Detmold, Lin, Luu, Orginos, Parreño, Savage, Walker-Loud **PRD87** (2013)
Orginos, Parreño, Savage, Beane, Chang, Detmold **PRD92** (2015)

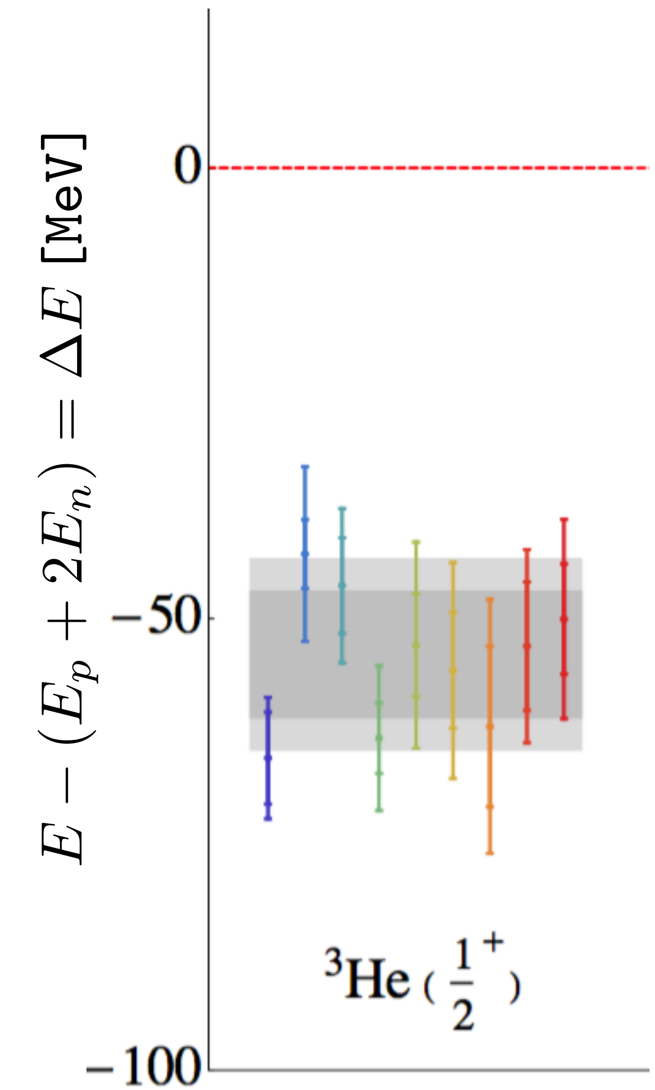
**Spectrum**




**e.g. $m_\pi$ = 800** $G_{^3\mathrm{He}}(t)$




$$\sum_{\{A_\mu\}} \langle qqqqqqqqq(t)\overline{qqqqqqqqq}(0)\rangle$$



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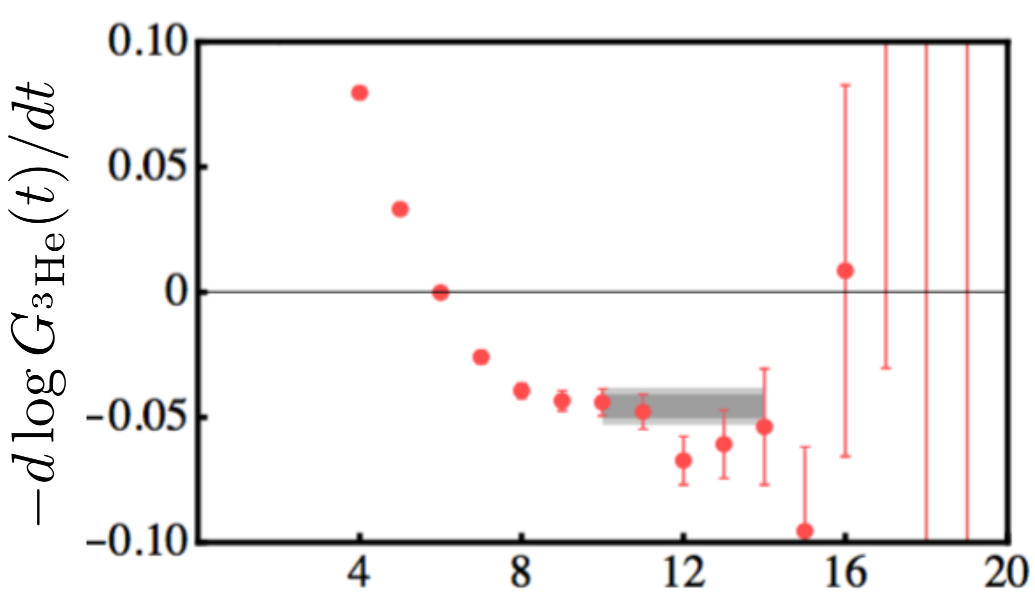
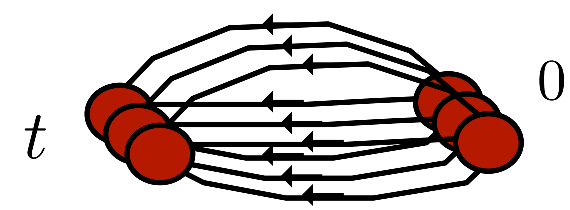
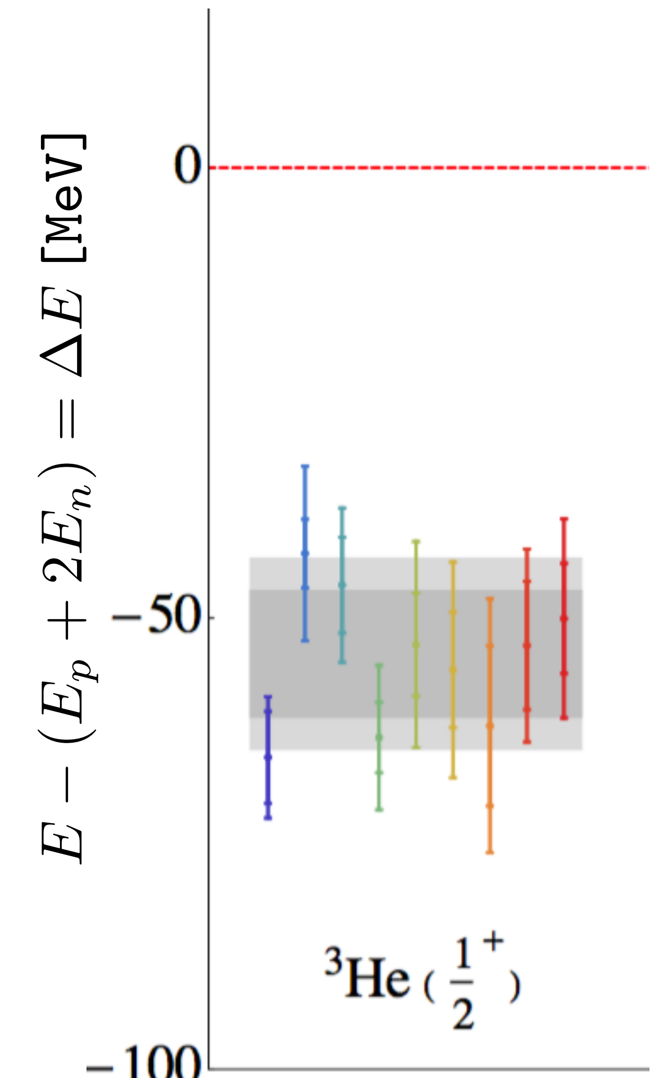



**e.g. $m_\pi$ = 800** $G_{^3\mathrm{He}}(t)$




$$\sum_{\{A_\mu\}} \langle qqqqqqqqq(t)\overline{qqqqqqqqq}(0)\rangle$$

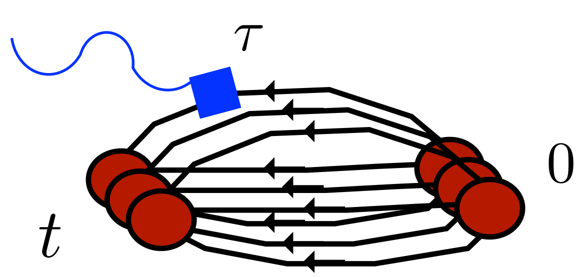



**Nature and properties of these states?**

$$\sum_{\{A_\mu\}} \langle qqqqqqqqq(t)[\overline{q}\gamma_\mu q](\tau)\overline{qqqqqqqqq}(0)\rangle \overset{t-\tau\gg1,\,\tau\gg1}{\sim} \langle{}^3\mathrm{He}|J_\mu|{}^3\mathrm{He}\rangle$$

Ground state requires

$$t-\tau,\ \tau \gtrsim 10$$




Current technology

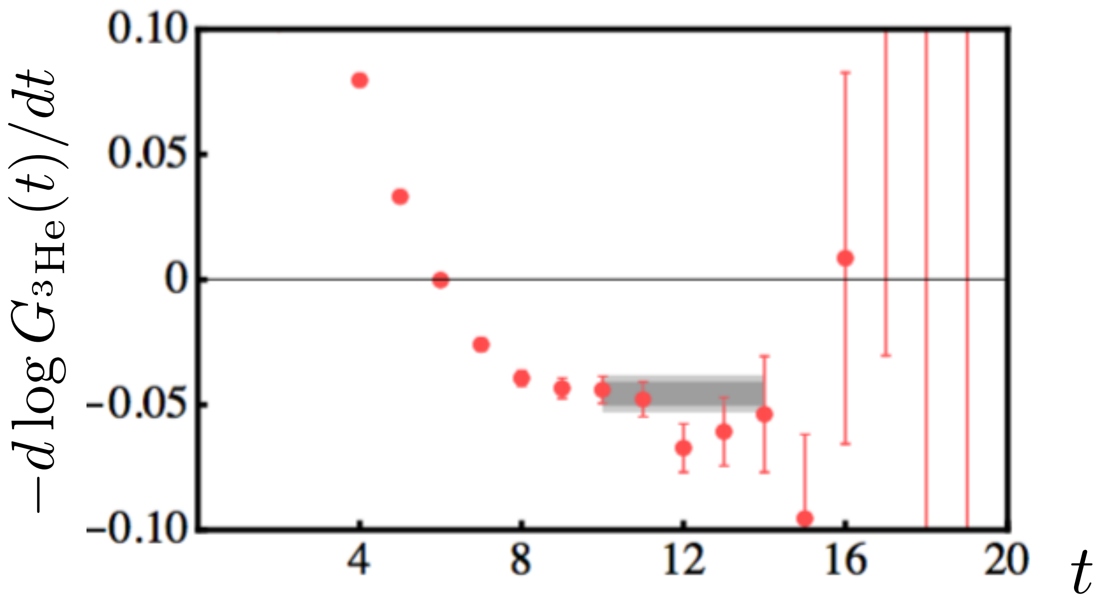
$$t > 20$$

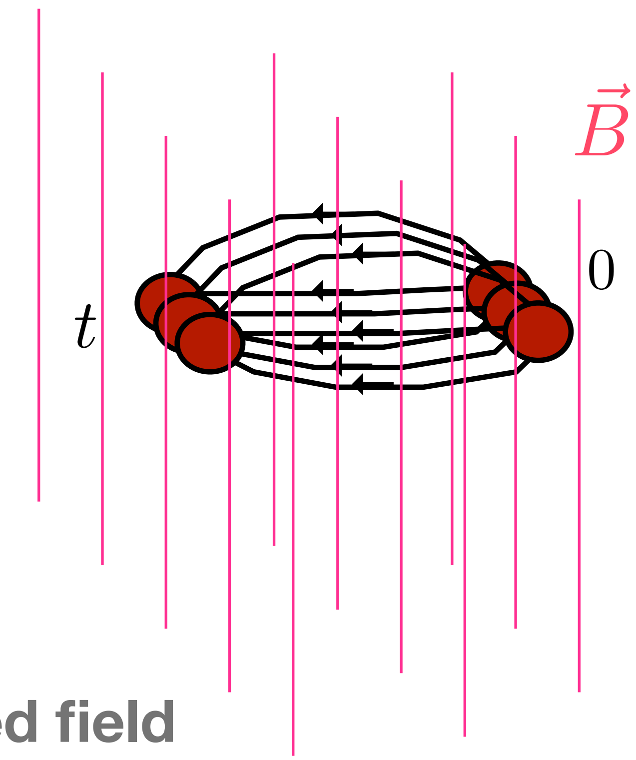
**Nuclear Matrix Elements from QCD?** **. . . not yet attempted**

Better sources
Greater statistics

# Nuclear Properties @ $m_\pi$ =800?

**Spectrum responds to external fields:** **e.g. uniform magnetic fields**




$$G_{^3\mathrm{He}}(t)_{\vec{B}} = \sum_{\{A_\mu\}} \langle qqqqqqqqq(t)\overline{qqqqqqqqq}(0)\rangle_{\vec{B}}$$

**Compute spectrum as a function of applied field**

I). In weak enough fields, can utilize same sources

II). Need roughly same statistics for each field strength

III). Requires fitting the field-strength dependence

IV). Limited number of properties for a given type of field

## Practical Solution:

*Lattice QCD + Classical Fields*


Beane, *et al.* **PRL:113** (2014)
Beane, *et al.* **PRL:115** (2015)
Chang, *et al.* **PRD:92** (2015)
Detmold, *et al.* **PRL:116** (2016)

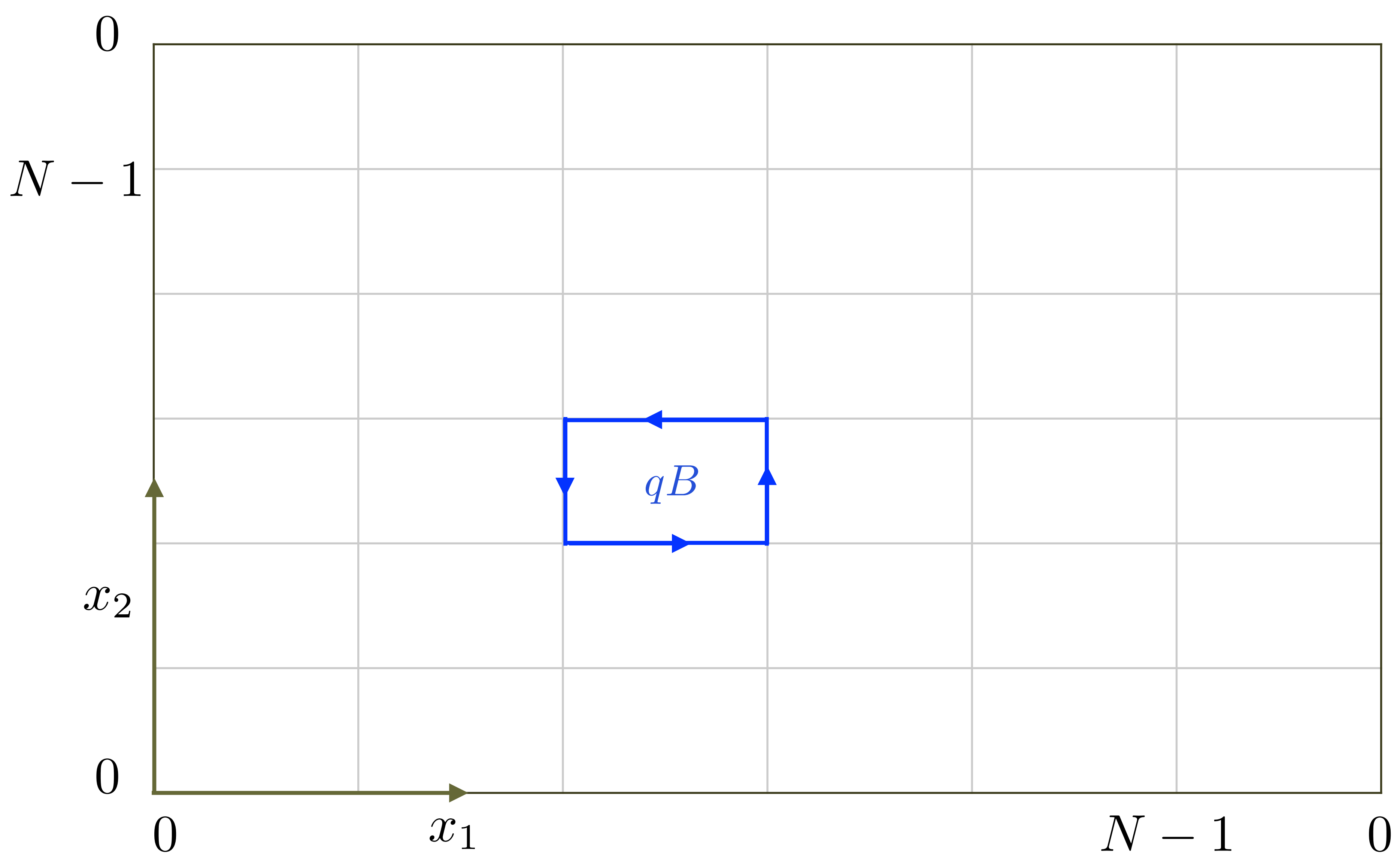
Gauge links:

$U_\mu(x) = e^{igG_\mu(x)} \in SU(3)$

$U_\mu^{\text{e.m.}}(x) = e^{iqA_\mu(x)} \in U(1)$

# Magnetic Field on a Periodic Lattice

**Seek uniform B-field** $U_\mu(x) = e^{-iqx_2 B\delta_{\mu 1}}$




$$U_1(x)U_2(x+\hat{i})U_2^\dagger(x+\hat{i}+\hat{j})U_1^\dagger(x+\hat{j}) = e^{iqF_{12}} = e^{iqB}$$

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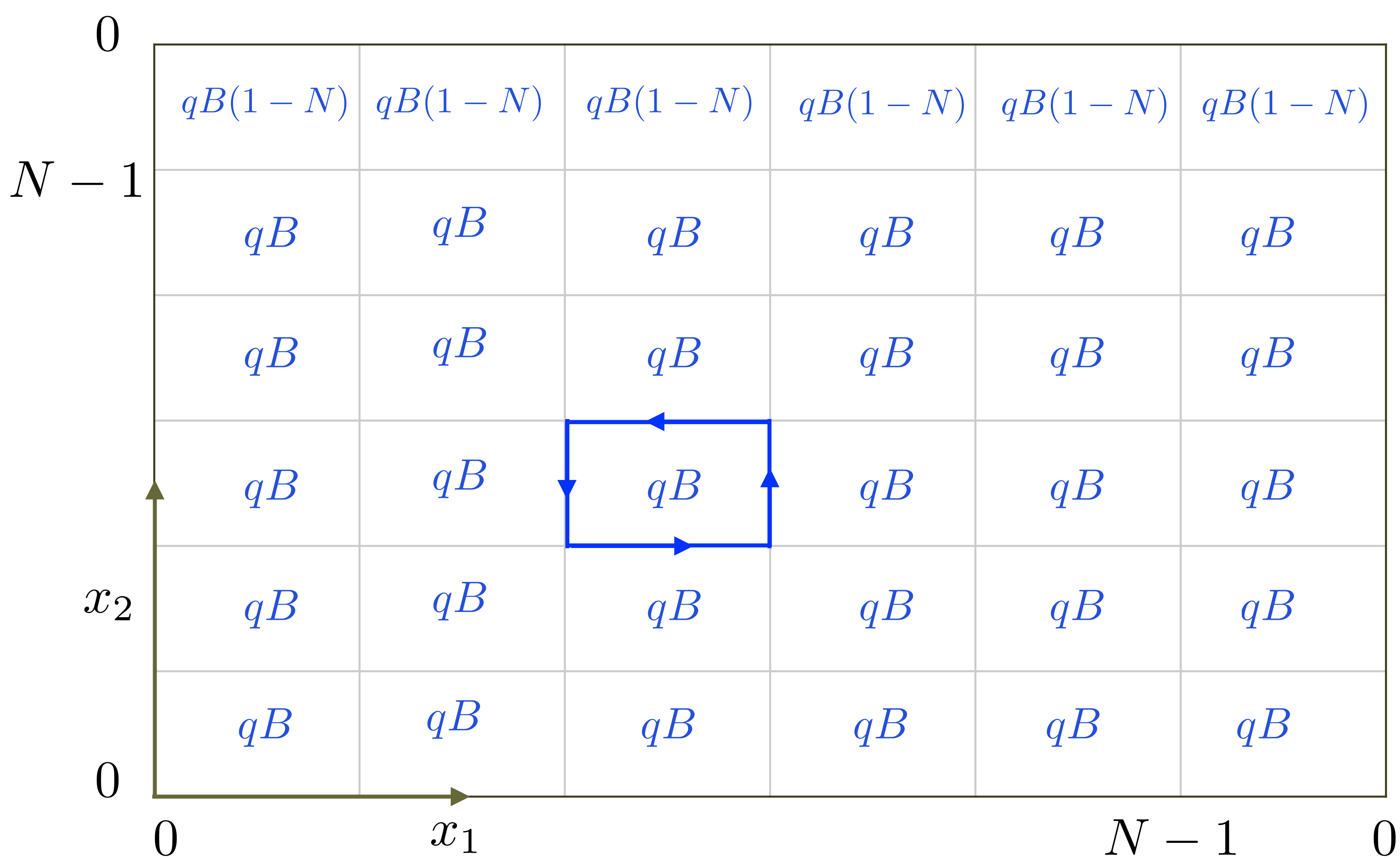


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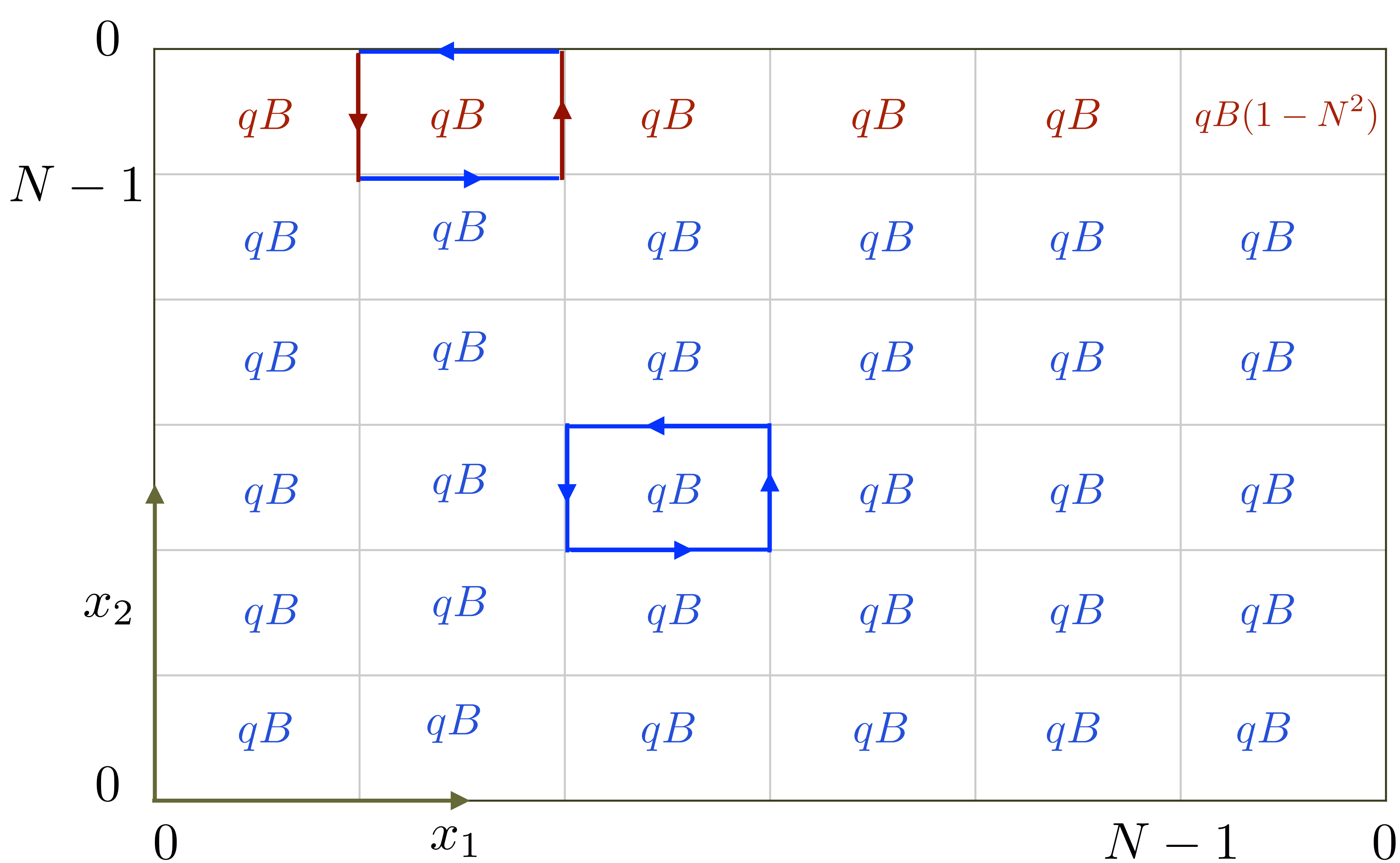



Flux quantization

$$qB = \frac{2\pi}{N^2} n_\Phi$$

$$N = 32$$

We choose:

$$n_\Phi = +3, -6, +12$$

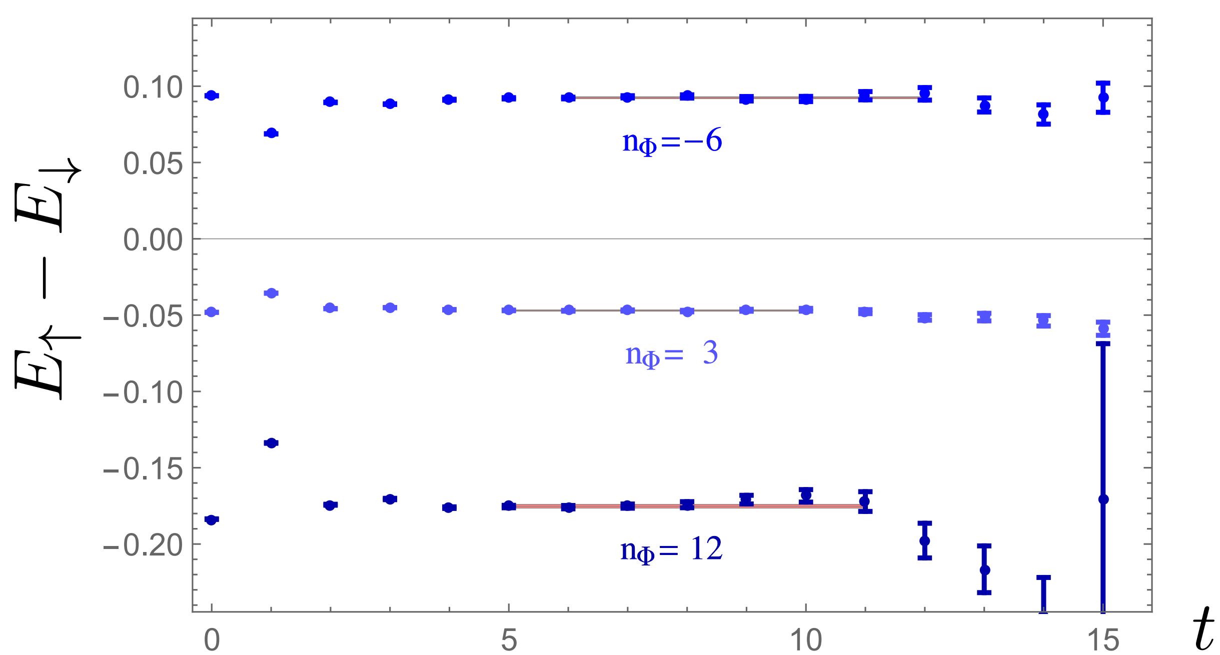
$$U_1(x)U_2(x+\hat{i})U_2^\dagger(x+\hat{i}+\hat{j})U_1^\dagger(x+\hat{j}) = e^{iqF_{12}} = e^{iqB}$$

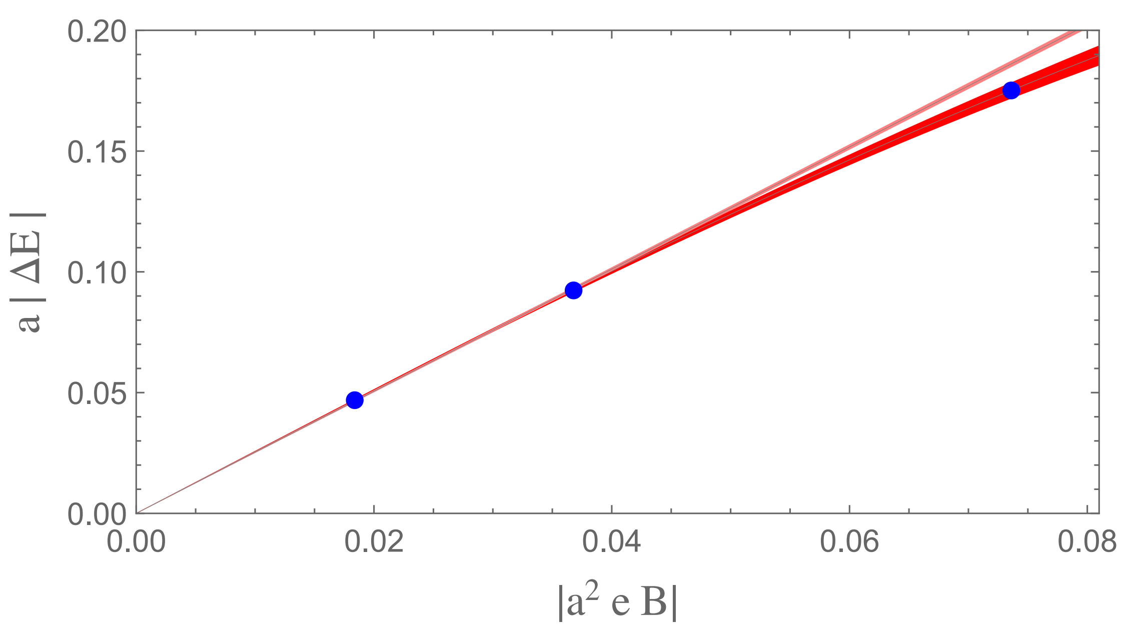
# Magnetic Moments of Octet Baryons

**Compute Zeeman Effect** using Lattice QCD + Uniform Magnetic fields $E(B, J_z)$

$\gtrsim 10^{19}\,\mathtt{G}$

Proton $m_\pi \sim 800\,\mathtt{MeV}$




Units!

$\mu_p = 2.560(09)(52)\ \mathtt{[LatM]}$

$$\mathtt{[LatM]} = \frac{e\,a}{2}$$

$$a = 0.145(2)\,\mathtt{fm}$$

$\mu_p = 1.770(06)(36)(19)\ \mathtt{[NM]}$

$$\mathtt{[NM]} = \frac{e}{2M_N}$$

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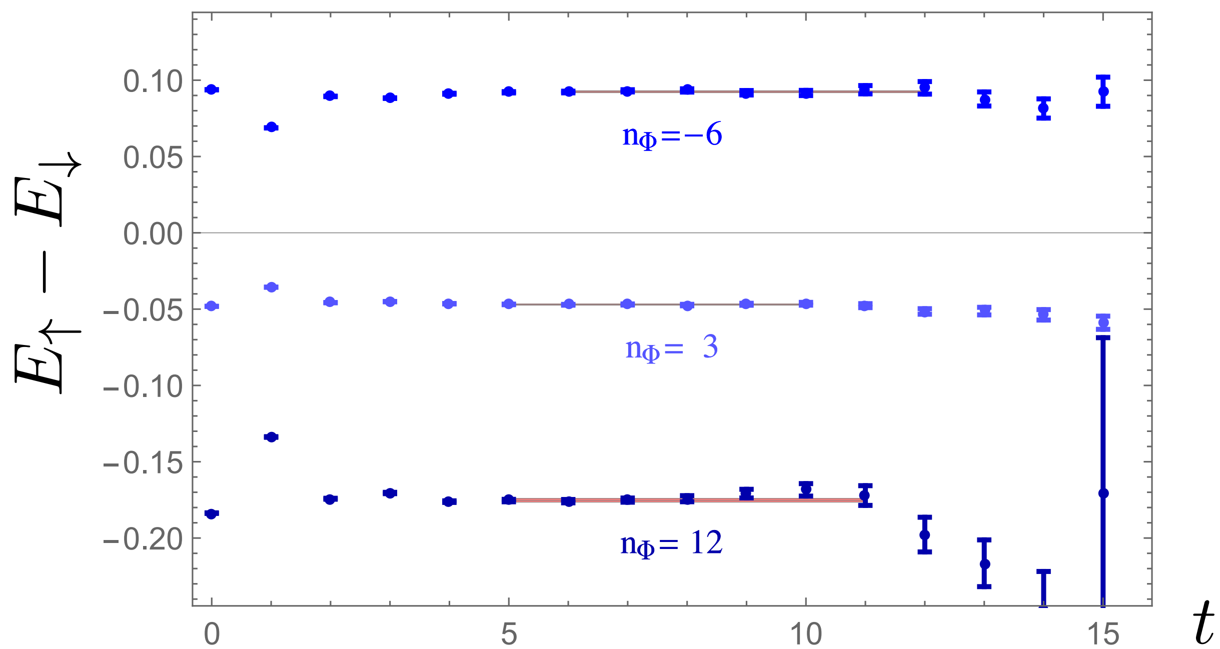
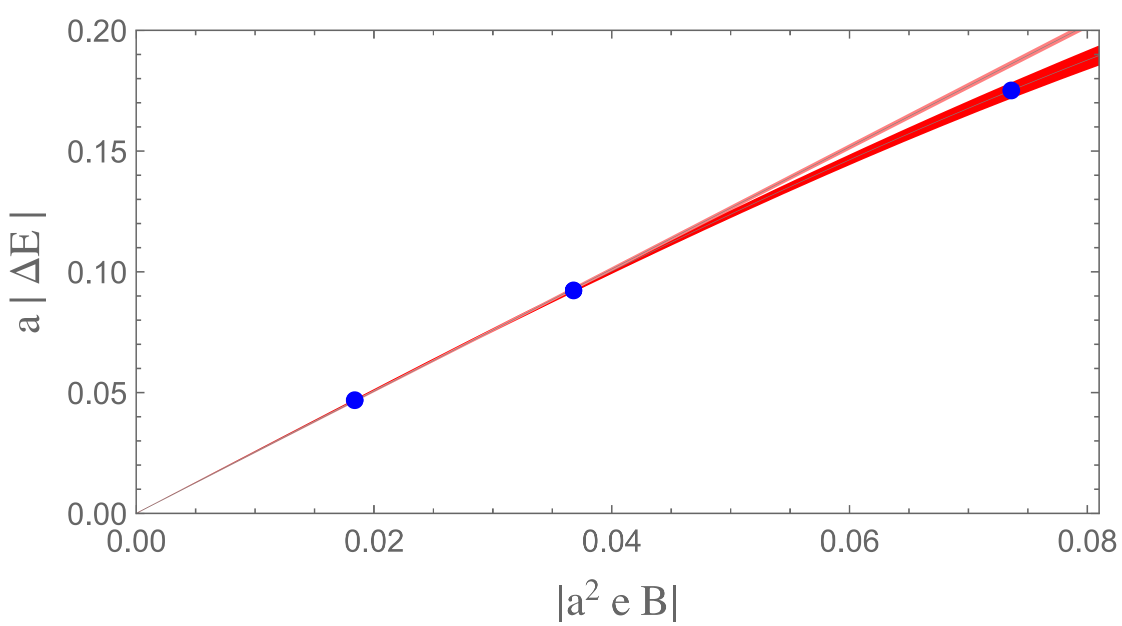


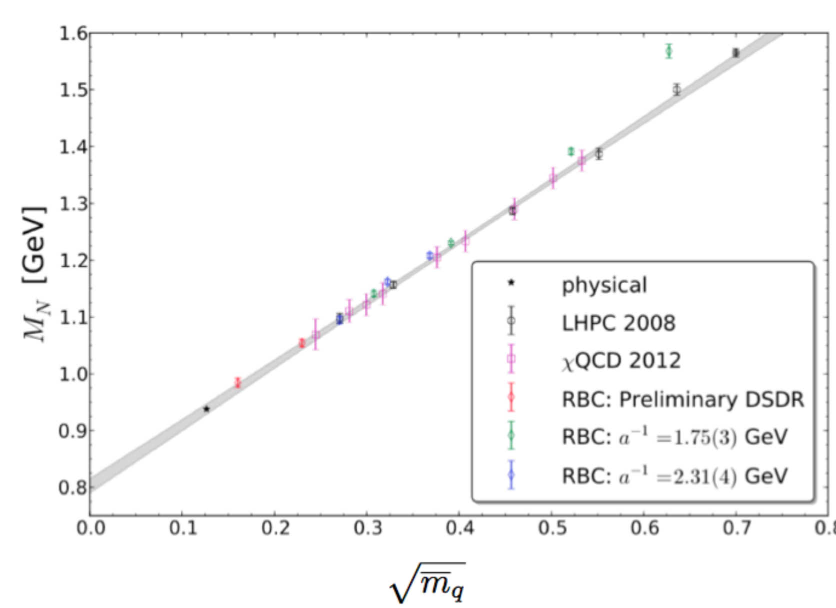




$$\mu_p = 1.770(06)(36)(19)\ \mathtt{[NM]}$$

$$\mathtt{[NM]} = \frac{e}{2M_N}$$

**Ruler Mass Rule** *(Walker-Loud, LHPC)*

$$M_N(m_\pi) = 800\ \mathtt{MeV} + m_\pi \sim 1,600\,\mathtt{MeV}$$




$$\mathtt{[nNM]} = \frac{e}{2M_N(m_\pi)}$$

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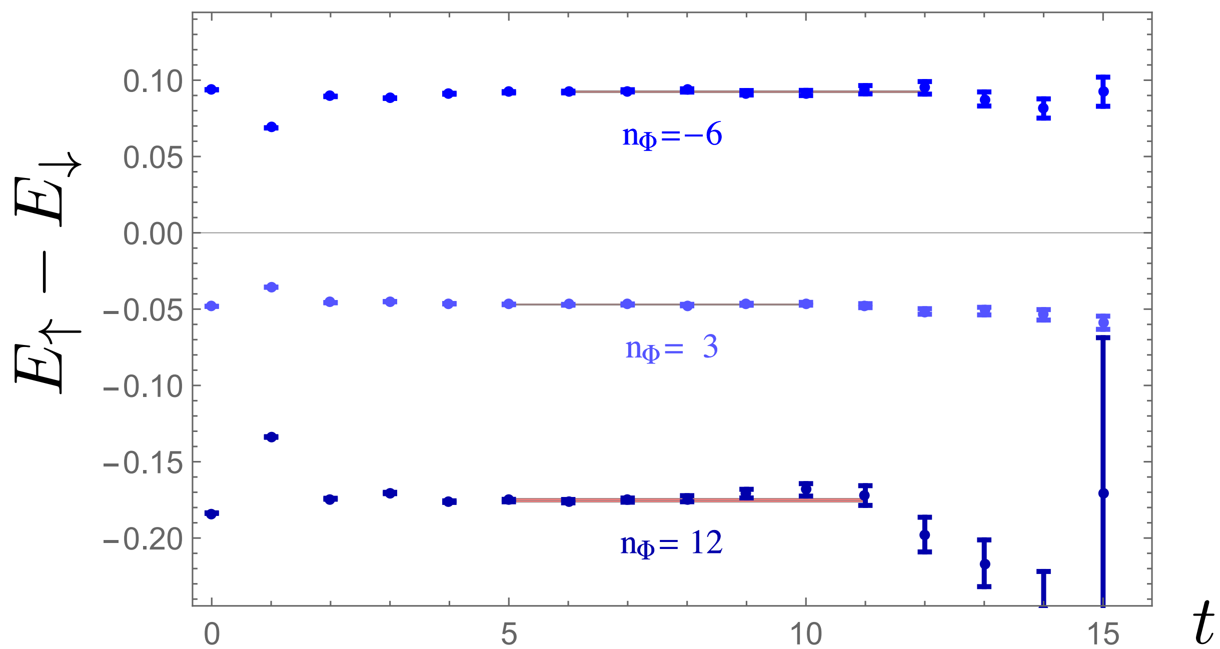


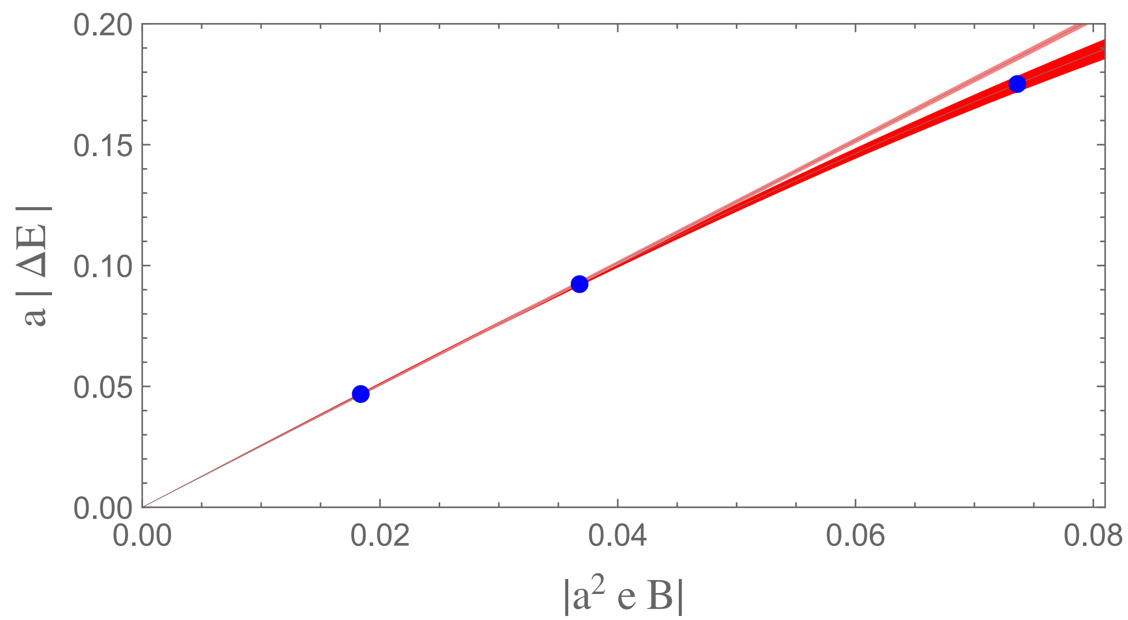


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$$\mathtt{[NM]} = \frac{e}{2M_N}$$

Natural nucleon magnetons

$$\mathtt{[nNM]} = \frac{e}{2M_N(m_\pi)}$$

$$\mu_p = 3.087(10)(62)\ \mathtt{[nNM]}$$

**Dirac** part is short-distance & guaranteed to $\mathcal{O}(a^2\Lambda^2_{\rm QCD})$

$$\delta\mu_p = 2.087(10)(62)\ \mathtt{[nNM]}$$

$$\delta\mu_p^{\rm exp} = 1.7929...\ \mathtt{[NM]}$$

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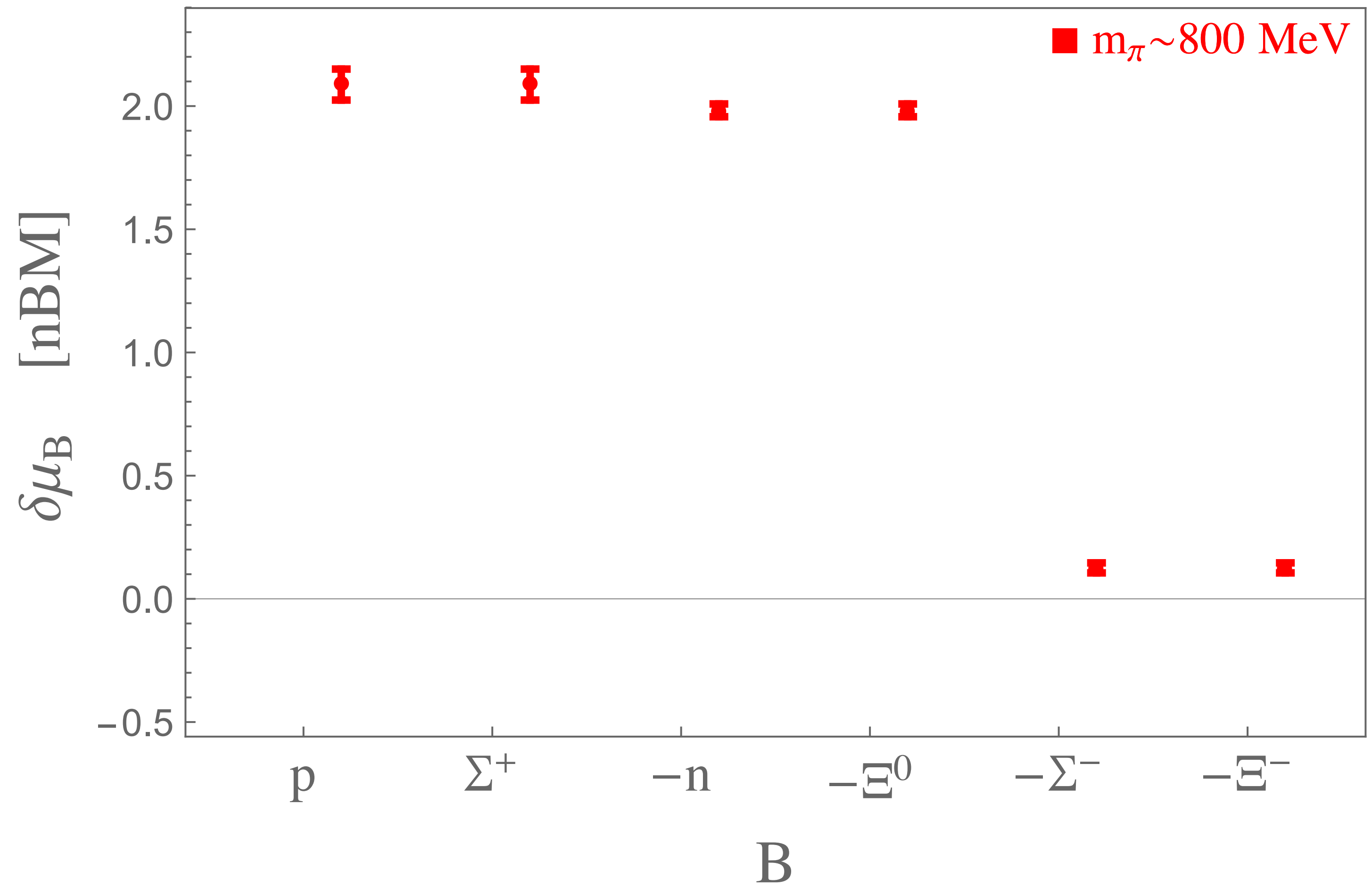
**Compute Zeeman Effect** using Lattice QCD + Uniform Magnetic fields




$$U(3)_F \xrightarrow{Q} U(1)_U \times U(1)_{D+S} \times SU(2)_{U-\text{spin}}$$

Natural baryon magnetons

$$[\texttt{nBM}] = \frac{e}{2M_B(m_\pi)}$$

Anomalous magnetic moments

$$\delta\mu_B\ [\texttt{nBM}] = \mu_B\ [\texttt{nBM}] - Q_B$$

U-spin

$$\begin{pmatrix} d \\ s \end{pmatrix} \xrightarrow{SU(2)} U \begin{pmatrix} d \\ s \end{pmatrix}$$

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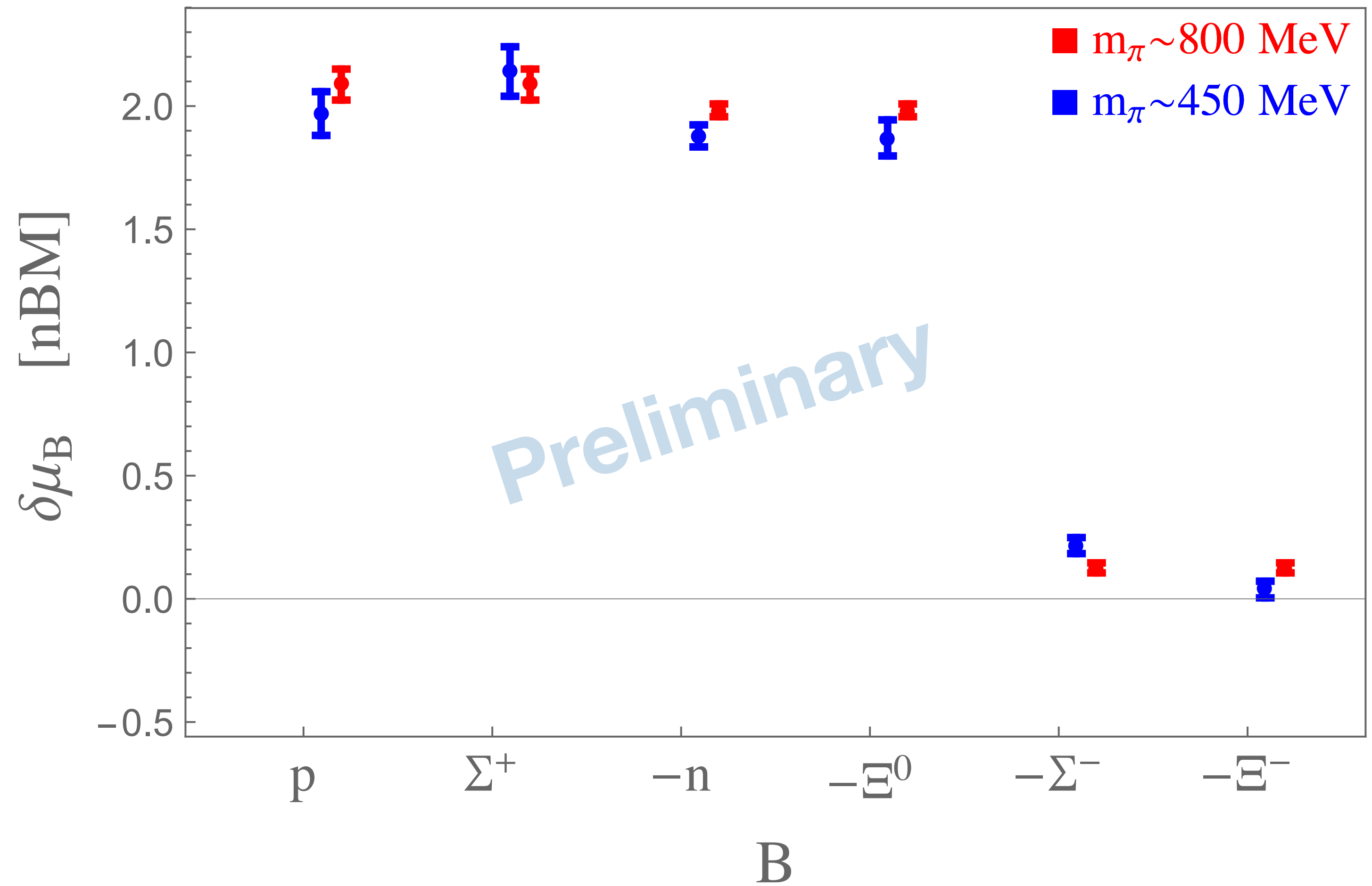


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$$U(2)_I \times U(1)_S \xrightarrow{Q} U(1)_B \times U(1)_{I_3} \times U(1)_S \qquad m_\pi \sim 450\,\texttt{MeV}$$

[Actually more complicated, our sea quarks are neutral]

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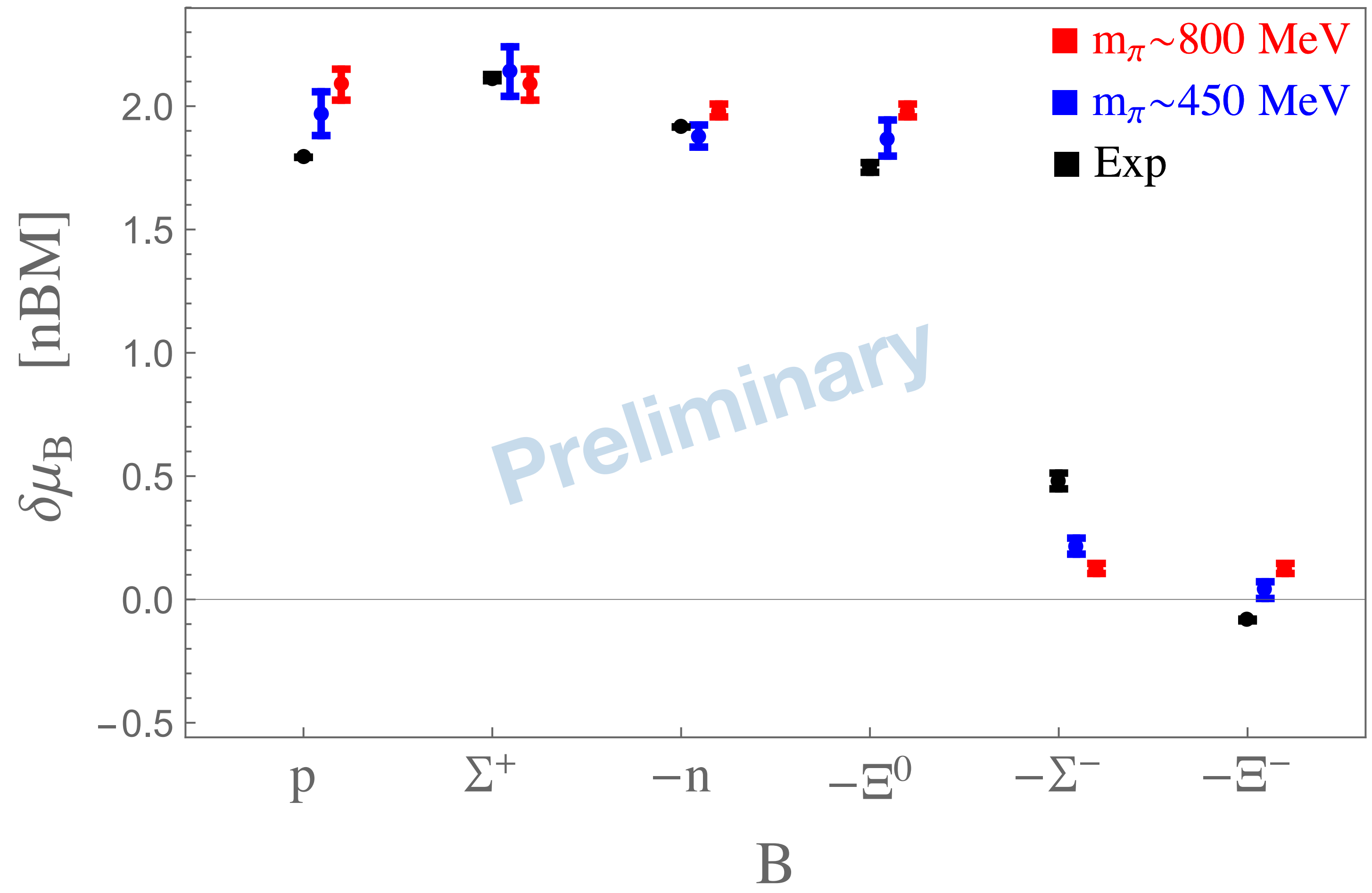


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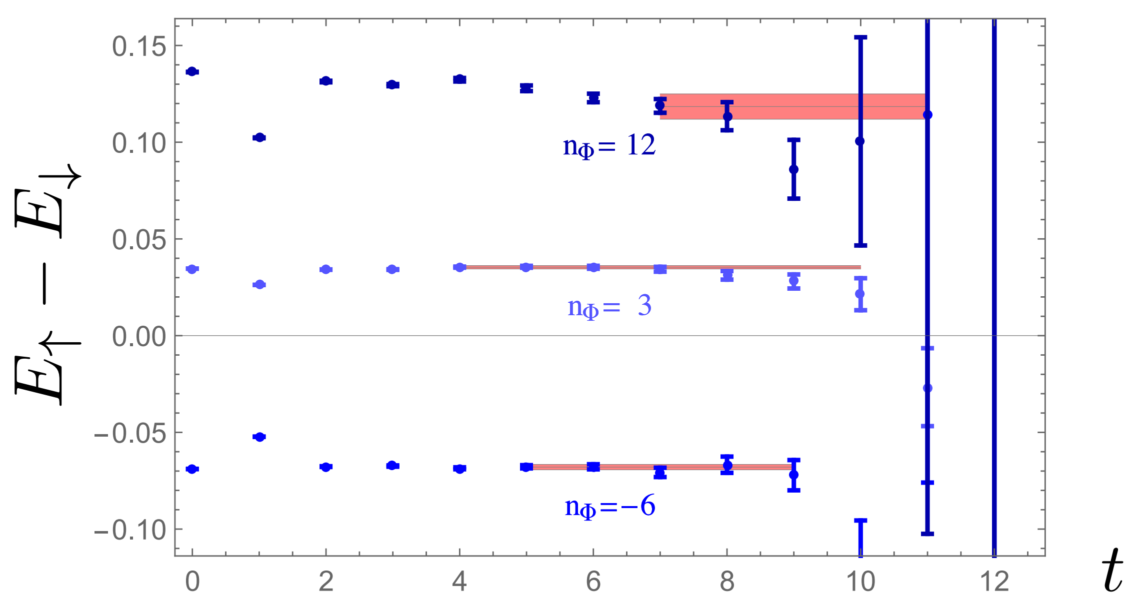
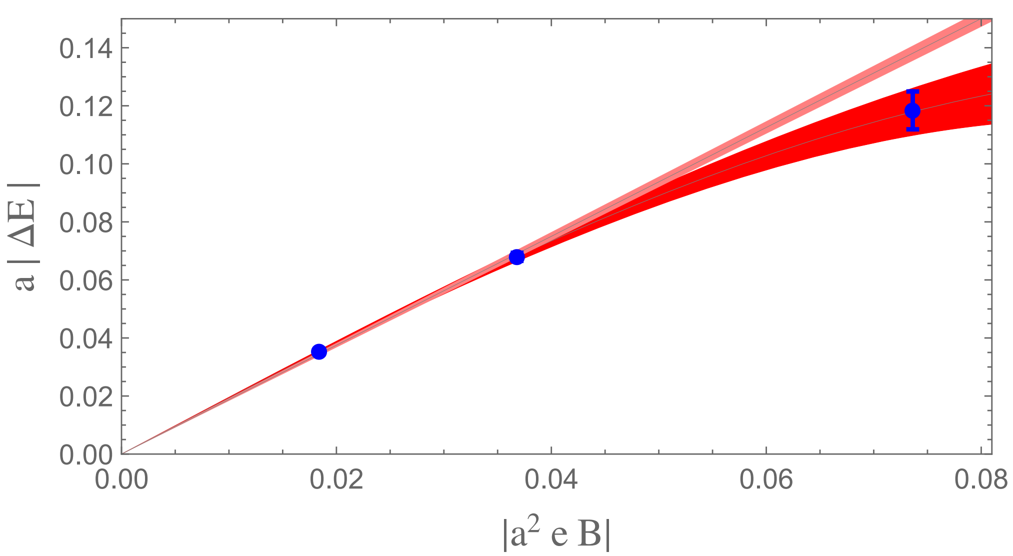
$^3\mathrm{He}$



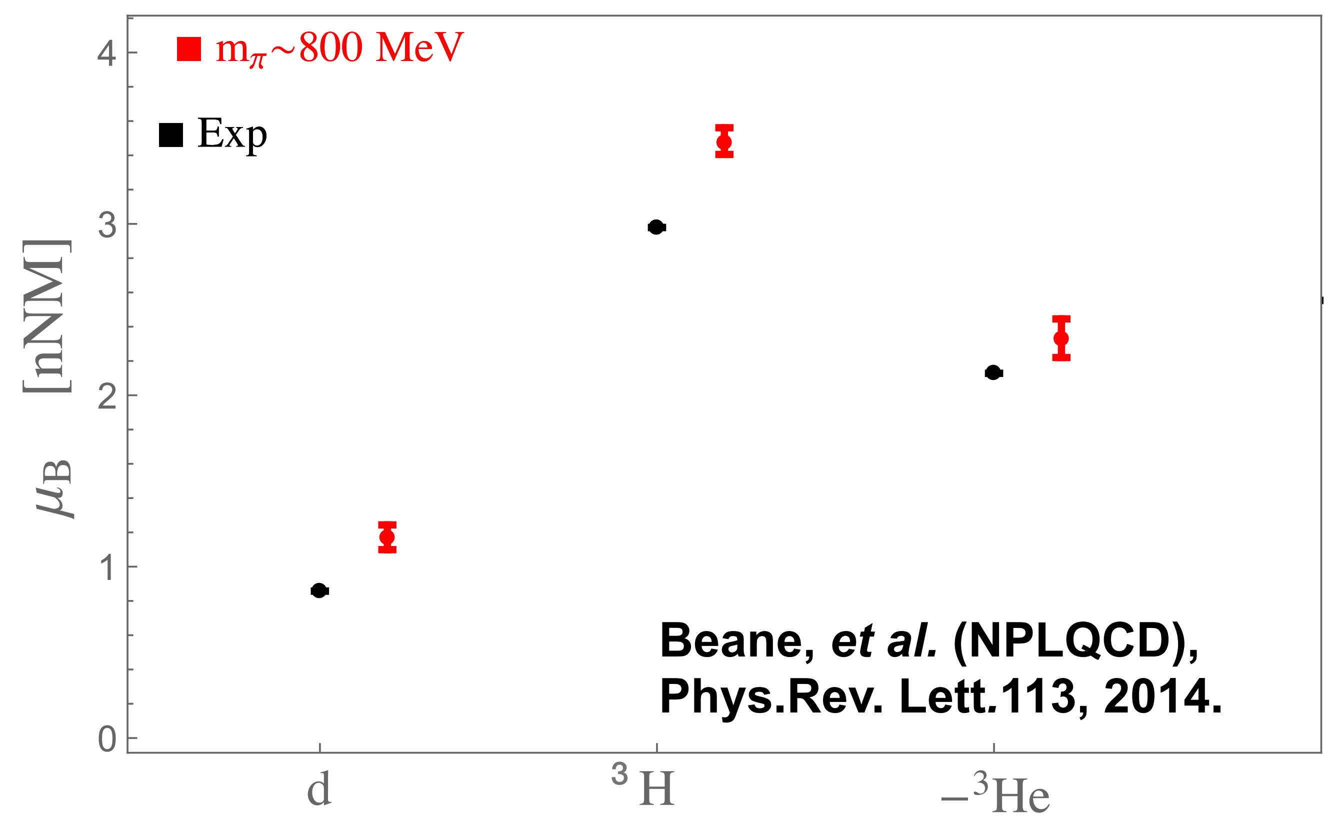




$m_\pi \sim 800\,\mathtt{MeV}$

$$[\mathtt{nNM}] = \frac{e}{2M_N(m_\pi)}$$

$$\mu_{^3\mathrm{He}} = -2.33(03)(11)\ [\mathtt{nNM}]$$

$$\mu^{\mathrm{exp}}_{^3\mathrm{He}} = -2.1276...\ [\mathtt{NM}]$$



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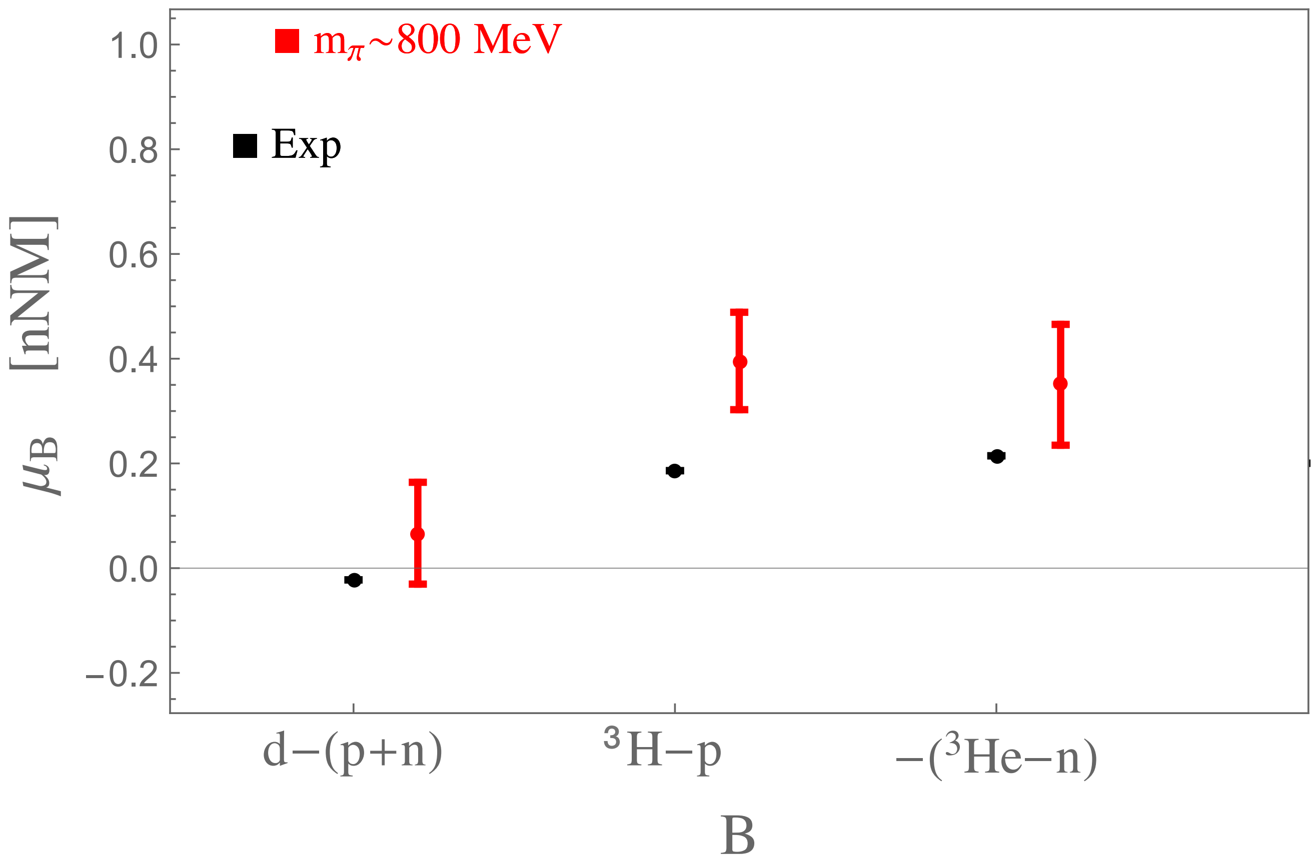


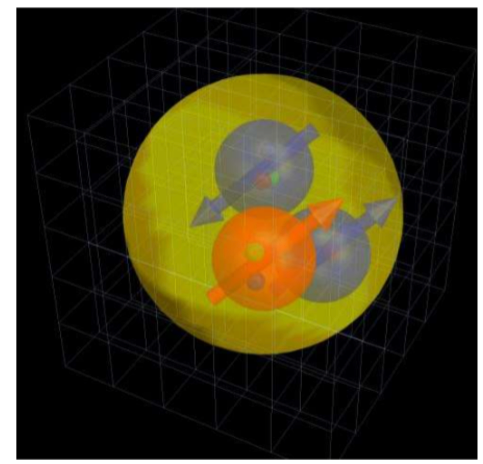

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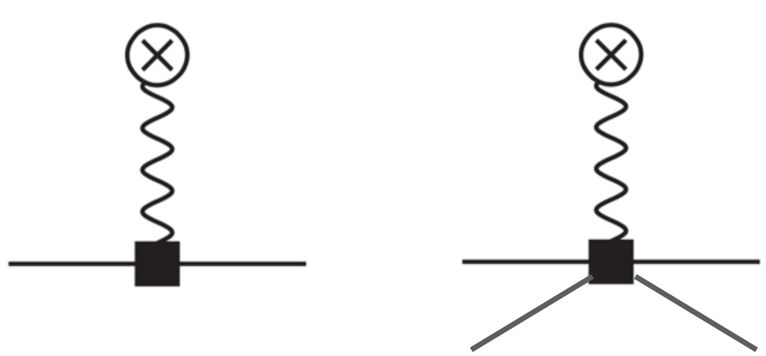
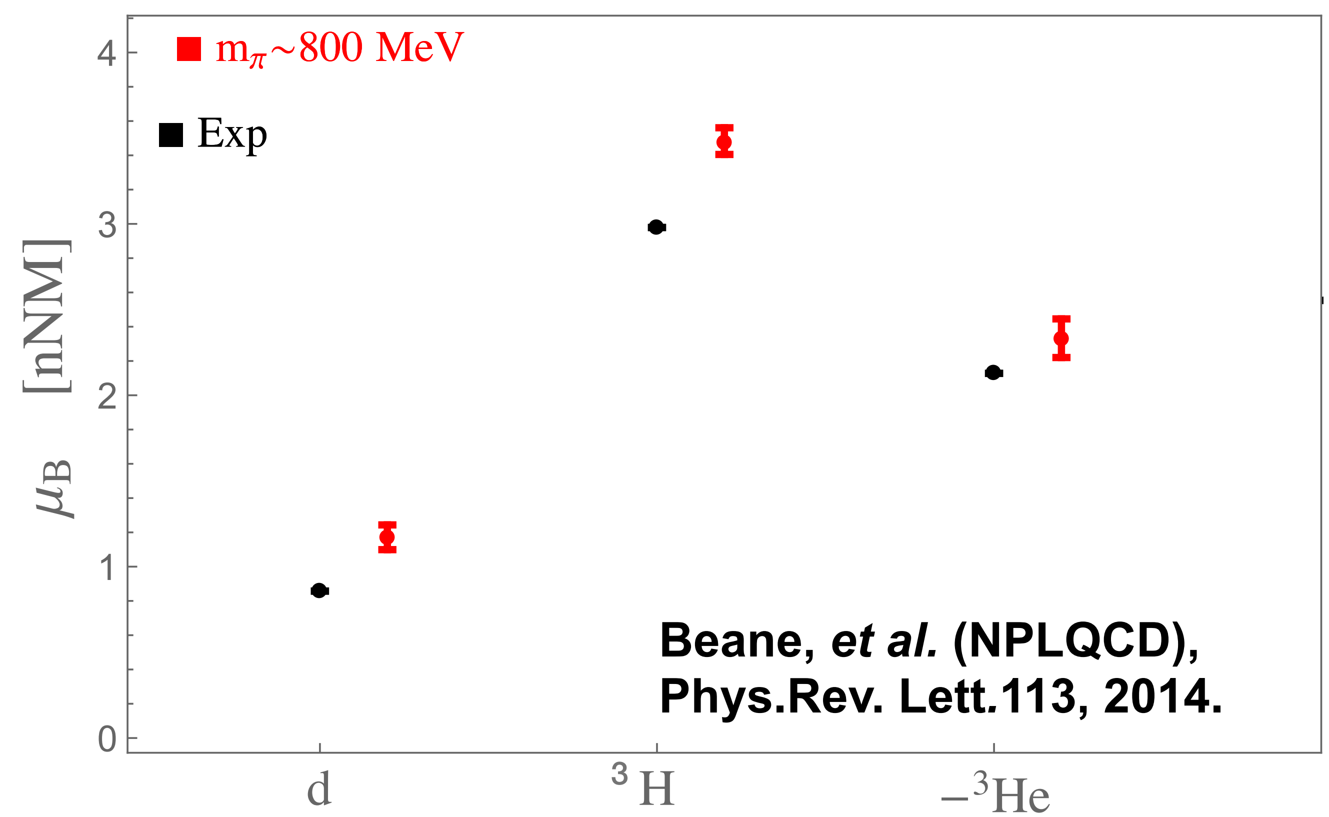





**Beane, *et al.* (NPLQCD), Phys.Rev. Lett.113, 2014.**

# First "Nuclear Reaction" from QCD

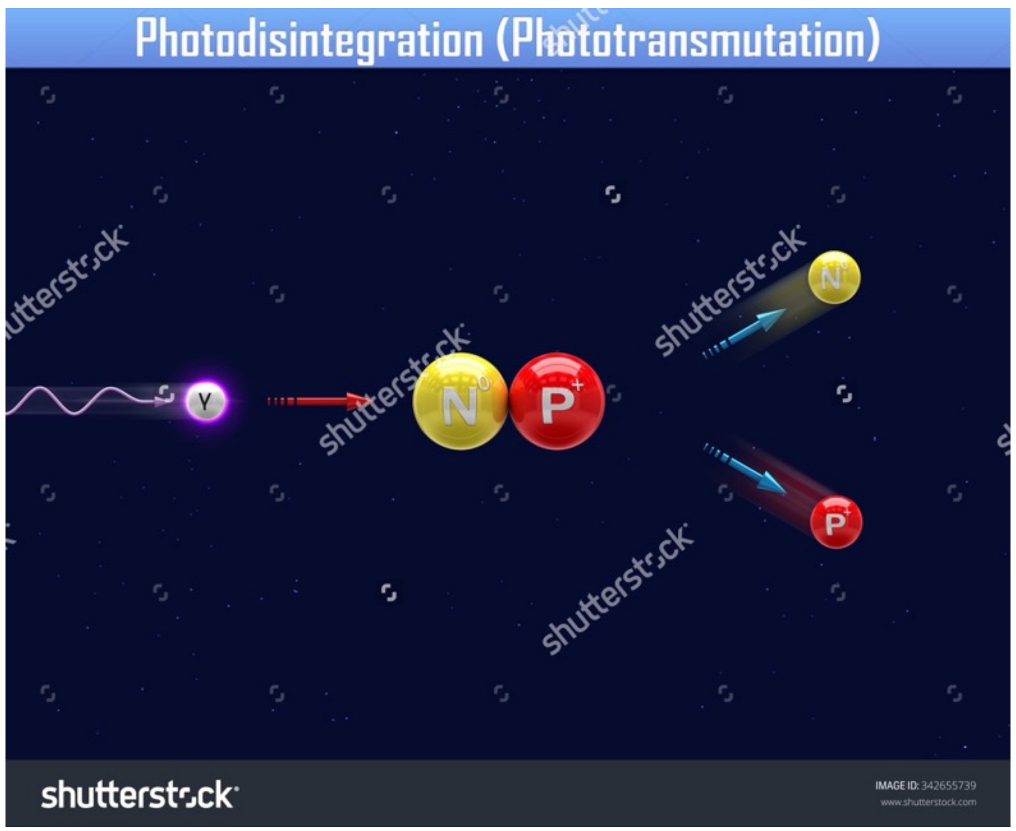
Dominant `M1` transition @ Low Energy

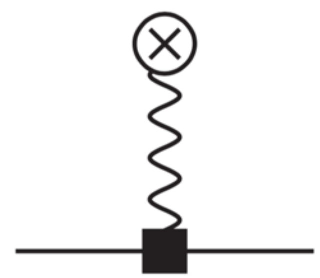
$$n + p \rightarrow d + \gamma \qquad \gamma^* + d \rightarrow n + p$$

Magnetically Coupled Channels

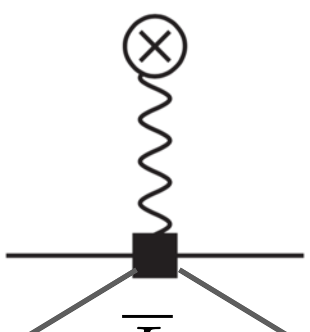
$$|\Delta I| = |\Delta J| = 1 \qquad I_3 = j_z = 0$$

$$\mathbf{C}(t;\mathbf{B}) = \begin{pmatrix} C_{^3S_1,^3S_1}(t;\mathbf{B}) & C_{^3S_1,^1S_0}(t;\mathbf{B}) \\ C_{^1S_0,^3S_1}(t;\mathbf{B}) & C_{^1S_0,^1S_0}(t;\mathbf{B}) \end{pmatrix}$$

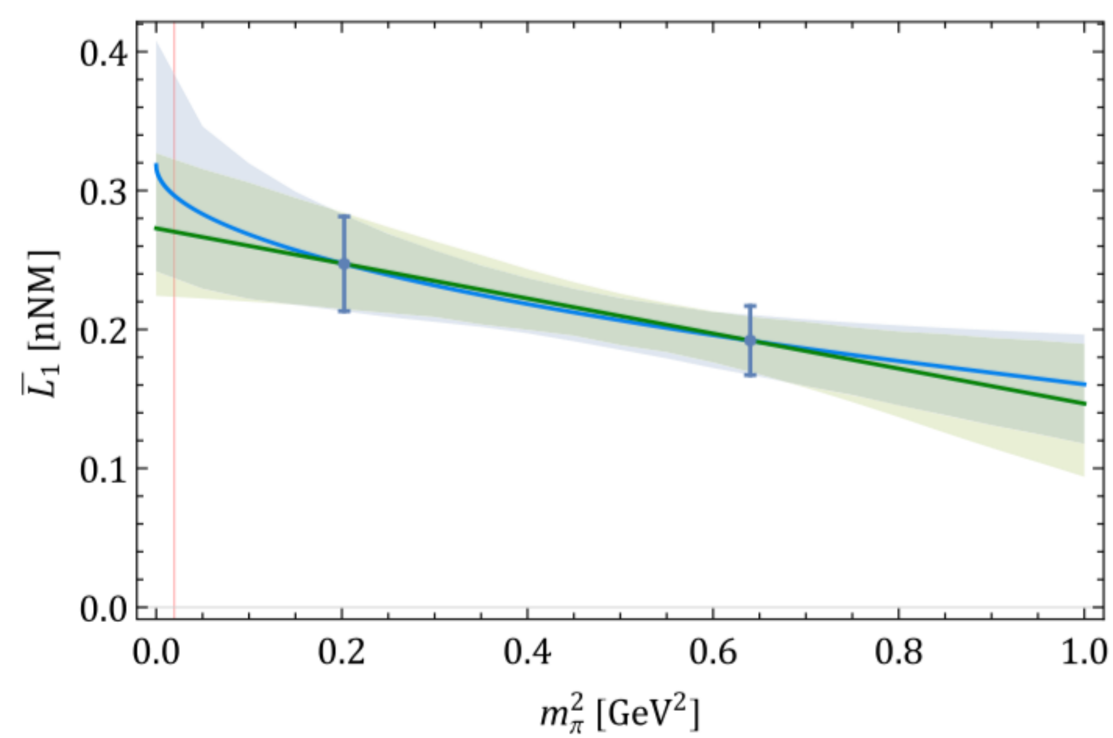


$\mu_1$



$\overline{L}_1$




Two-body contribution isolated & compares favorably with EFT($\not\pi$) phenomenology

Beane, *et al.* (NPLQCD), Phys.Rev. Lett.115, 2015.

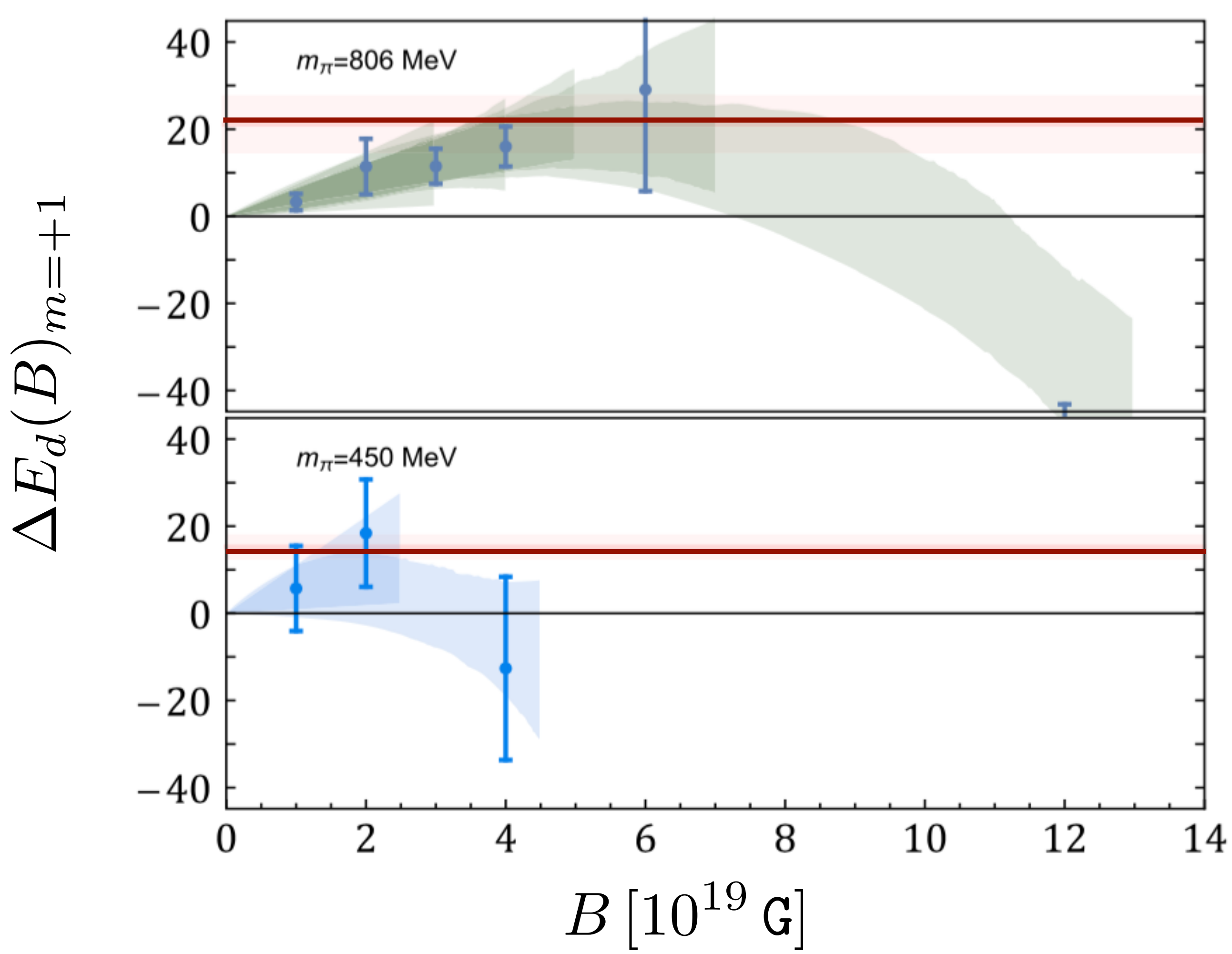
# Extreme Magnetic Environments

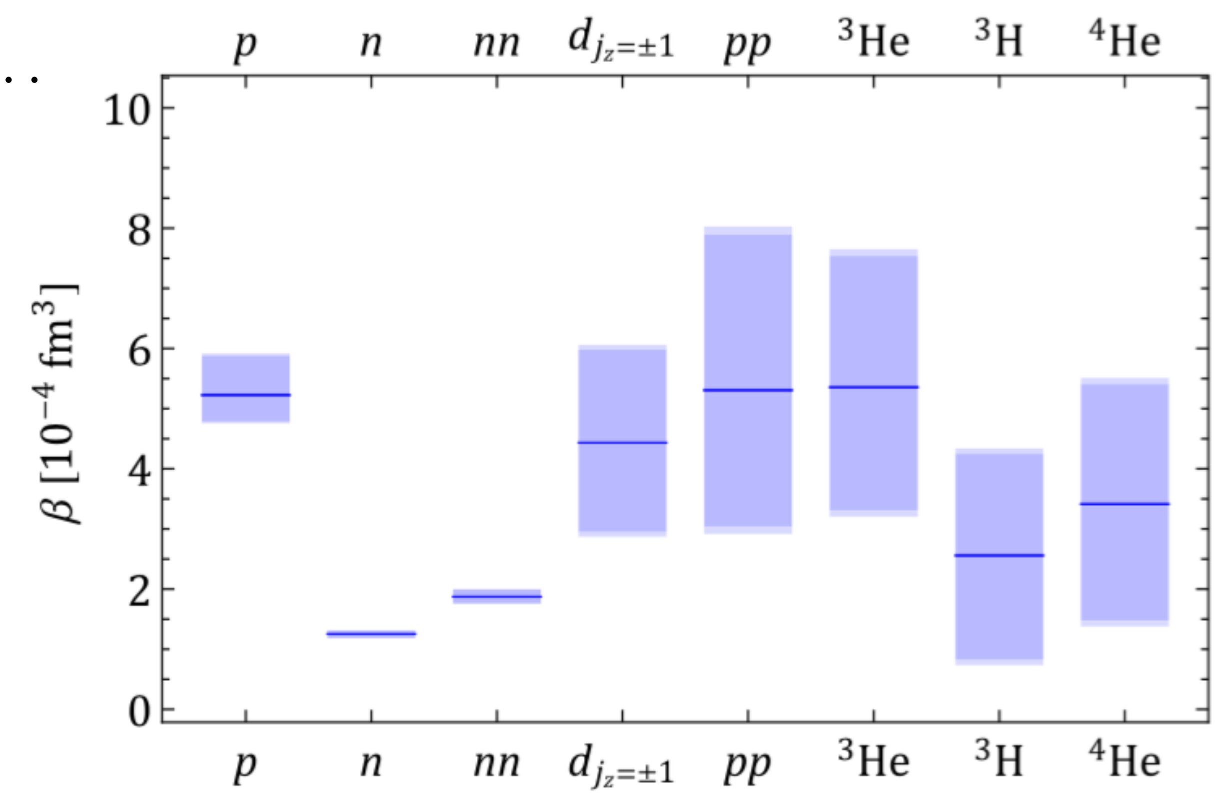
- Beyond Linear: Magnetic Polarizabilities

Chang, ***et al.*** **(NPLQCD), Phys.Rev. D*92*, 2015.**

$$E_{\uparrow+\downarrow}(B) = \sqrt{M^2 + (2n+1)|QB|} - \frac{1}{2}\beta_M B^2 + \cdots$$

Landau identification dominates uncertainties




- Unitary NN Interactions?

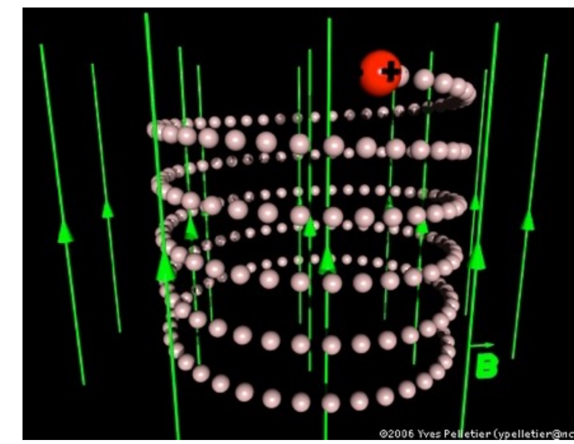
$a_{NN}(B) \to \infty$?

Detmold, ***et al.*** **(NPLQCD), Phys.Rev. Lett.116, 2016.**

# Future Directions

- **Magnetic Structure of Nuclei**

  **Move beyond exploratory studies**: remove systematics, lower pion mass, better treat Landau levels, sea quarks, …




- **Electric Structure of Nuclei**

  **Electric polarizabilities**?
  **EDMs of light nuclei** from θ-term?, BSM sources?

- **Nuclei in other classical fields…**

  Gravitational?, Weak?

**Nuclear Physics from QCD** → **EW Reactions** → **BSM Physics**