# THE THREE NUCLEON SYSTEM AT LEADING ORDER OF CHIRAL EFFECTIVE THEORY

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**INT, UW,Seattle** **2016.04.21.**

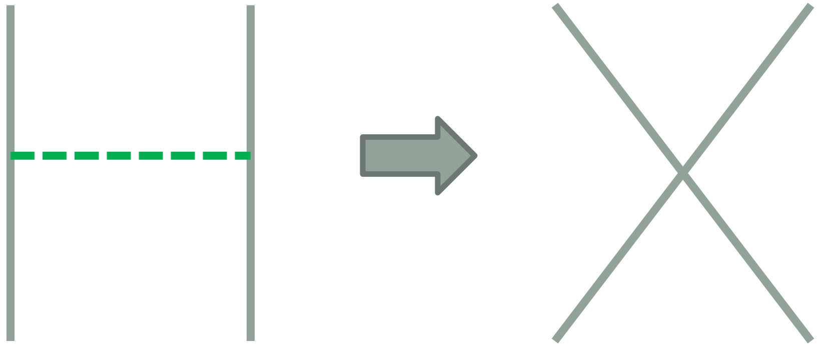
# Outline

- Introduction
  - Chiral Effective Field Theory
  - Weinberg Power counting scheme
  - Problems in non-perturbative renormalization
  - Modified power counting scheme
- Results
  - Two-body case(deuteron, n-p scattering)
  - Three-body case( triton, n-d scattering)
- Summary/Discussion

# Effective Field Theory

- QCD at low energy scale requires either
  - Non-perturbative calculation of QCD (Lattice QCD)
  - Or effective formulation of strong interaction for hadrons.
- (Nuclear) Effective Field Theory

  \- Basic assumption: Low energy dynamics are not sensitive to the short distance detail and we only need to know finite number of LECs at desired accuracy.



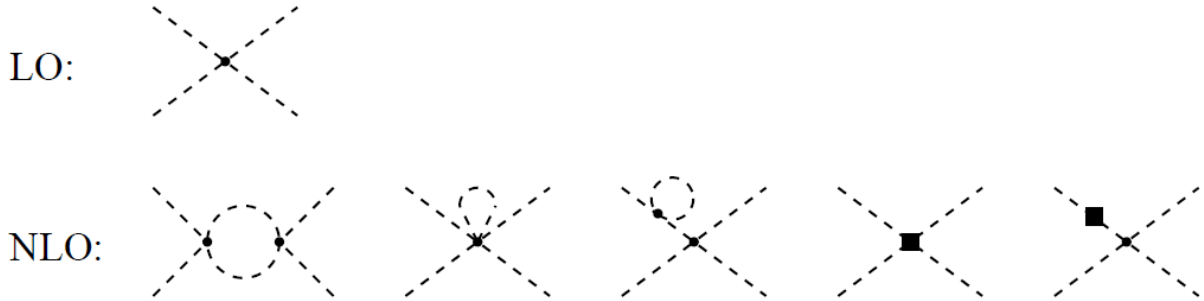
# Chiral Perturbation Theory

- Chiral effective field theory
  - Low energy effective theory of QCD for hadrons
  - Same symmetry and symmetry breaking pattern of QCD : Chiral symmetry
  - Infinite number of operators, diagrams
  - Require truncation scheme or power counting:

    Expansion in power of Q/M_hi, Q/cutoff
  - Require Renormalization Group invariance : cutoff independence
  - expect: inverse power of (residual) cutoff dependence
  - In order to maintain predictive power of EFT, it is necessary to truncate the sum in such a way that the resulting cutoff dependence and truncation error can be decreased systematically as increasing order.

# Chiral Perturbation Theory

- Chiral Perturbation Theory(ChPT): EFT of pions
  - Expansion in power of $\left(\frac{Q}{\Lambda_\chi}\right)^\nu$ $\quad \nu = 2 + 2L + \sum_i V_i \Delta_i \,, \qquad \Delta_i = d_i - 2 \,.$




- All divergences(cutoff dependence) from loop diagrams can be absorbed by counter terms order by order.

# Heavy Baryon Chiral Perturbation Theory

- ChPT+ nucleon: Relativistic theory requires subtle power counting scheme because of heavy nucleon mass scale.
- Heavy Baryon Chiral Perturbation theory: expansion in inverse power of nucleon mass.

$$\delta m_N|_{\text{loop, rel}} \overset{\mathcal{M}\to 0}{=} -\frac{3g_A^2 m^3}{F^2}\left(L(\mu)+\frac{1}{32\pi^2}\ln\frac{m^2}{\mu^2}\right)+\mathcal{O}(d-4)$$

$$\Downarrow$$

$$\delta m_N|_{\text{loop, HB}} = -\frac{3g_A^2 M_\pi^3}{32\pi F^2}$$

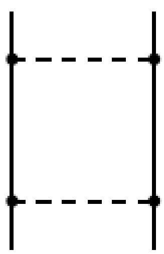
$$\nu = 1 + 2L + \sum_i V_i \Delta_i\,, \qquad \text{with} \qquad \Delta_i = -2 + \frac{1}{2}n_i + d_i$$

- All divergences(cutoff dependence) from loop diagrams can be absorbed by counter terms order by order.

# Weinberg power counting scheme

- However, two nucleon system must be treated non-perturbatively
  - Existence of shallow deuteron bound state
  - Large scattering length in S-wave scattering
  - -> Sum of infinite number of diagrams

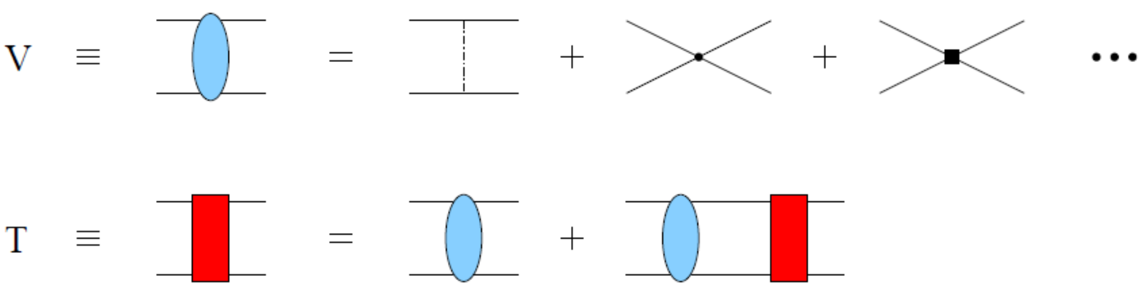
- Infrared enhancement from pure nucleon intermediate state diagram (reducible diagrams)



$$\int \frac{dl^0}{2\pi} \frac{1}{l^0 + \frac{E}{2} - \frac{(\vec{l}+\vec{p})^2}{2m_N} + i\epsilon} \frac{1}{-l^0 + \frac{E}{2} - \frac{(\vec{l}+\vec{p})^2}{2m_N} + i\epsilon}$$

# Weinberg power counting scheme

- Weinberg's suggestion:
  - Compute potential from irreducible diagram
  - Non-perturbative sum of reducible diagrams with kernel




$\nu = 2 - N + 2L + \sum_i V_i \Delta_i \,.$

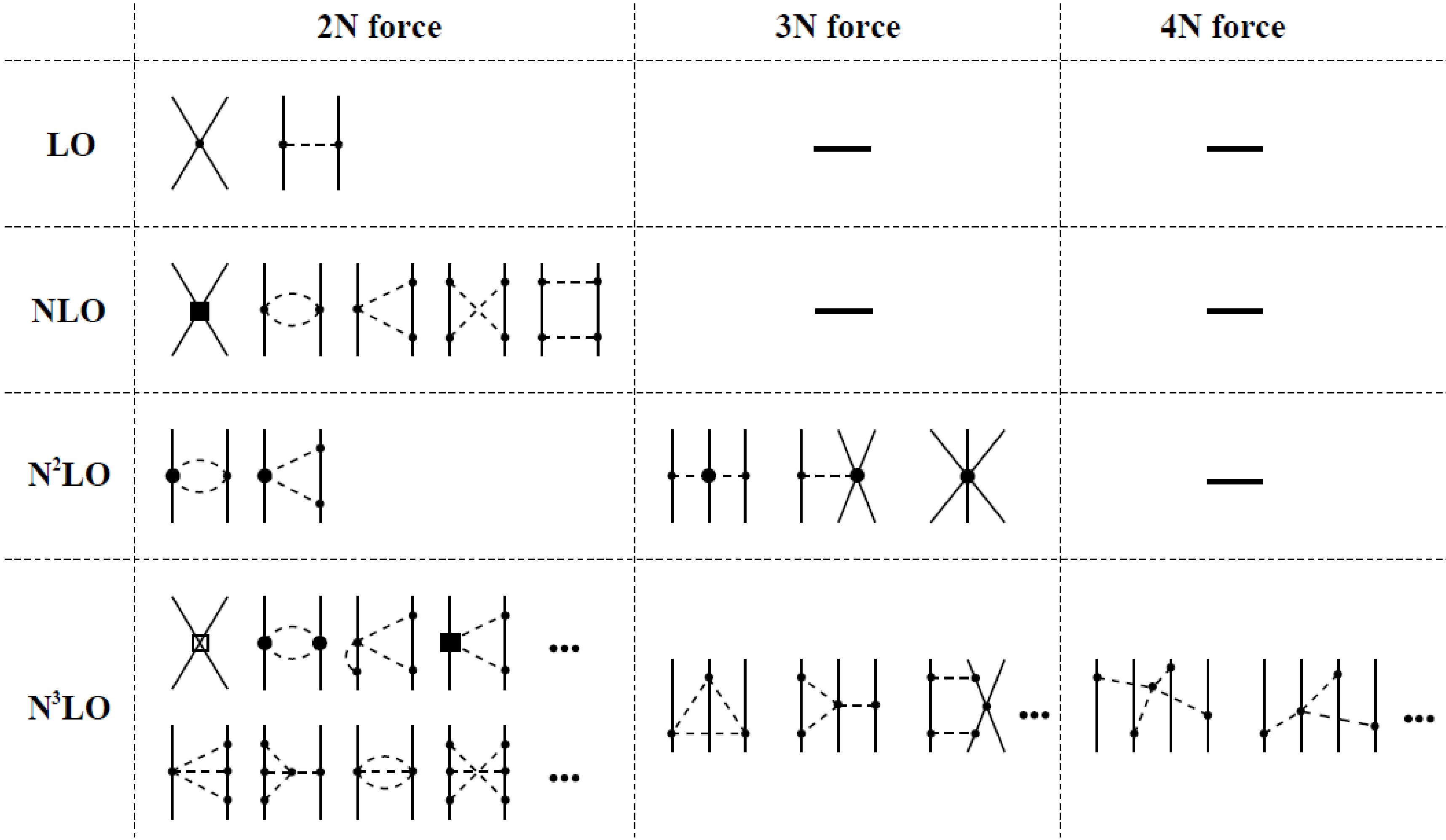
- Naive Dimensional  Analysis of (Heavy Baryon) ChPT

-> perturbative Power counting for nuclear system

$$\nu = 4 - N + 2(L - C) + \sum_i V_i \Delta_i \,, \qquad \Delta_i = d_i + \frac{1}{2} n_i - 2$$

-> Explain the potential order 2B > 3B > 4B

# Weinberg power counting scheme



# Chiral EFT potential

- Perturbative Renormalization in kernel:
  - Divergence(cutoff dependence) from loop diagrams can be absorbed by contact counter terms order by order.
- Non-perturbative Renormalization of amplitude:
  - LS equation is usually divergent and require regularization
  - Cutoff dependence have to be removed by renormalization

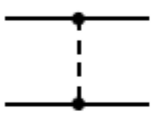
$$T(\boldsymbol{p}',\boldsymbol{p}) = V(\boldsymbol{p}',\boldsymbol{p}) + \int d^3\boldsymbol{k} V(\boldsymbol{p}',\boldsymbol{k}) \frac{M}{\boldsymbol{p}^2 - \boldsymbol{k}^2 + i\epsilon} T(\boldsymbol{k},\boldsymbol{p}).$$

$$V(\vec{p}\,',\vec{p}) \longmapsto V(\vec{p}\,',\vec{p})\, e^{-(p'/\Lambda)^{2n}}\; e^{-(p/\Lambda)^{2n}}$$

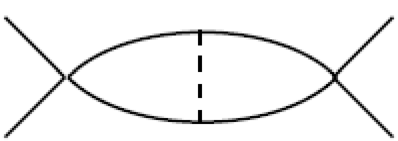
# Problem in Weinberg power counting

- Implicit assumption:
  - counter terms in potential are also enough to absorb cutoff dependence from iteration of one-pion exchange diagrams.
  - Wrong.

- Example: D.B.Kaplan, M.J. Savage, M.B.Wise , NPB478,629(1996)



$$V_{2N}^{(0)} = -\frac{g_A^2}{4F_\pi^2}\frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{\vec{q}^{\,2}+M_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + C_S + C_T\vec{\sigma}_1\cdot\vec{\sigma}_2$$

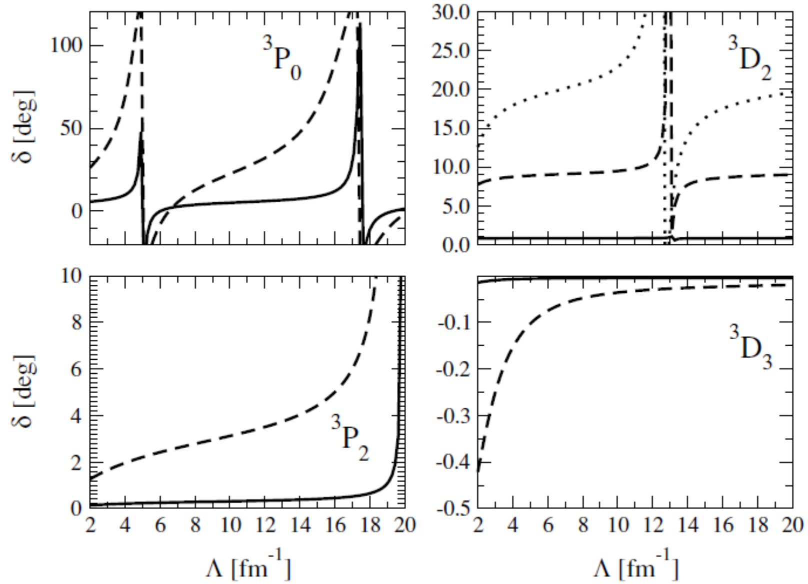


$$-\frac{1}{\epsilon}\frac{g_A^2 m_\pi^2 M^2}{256\pi^2 F_\pi^2}C^2$$

Divergence of
Leading order Diagram
Cannot be absorbed by
Leading order counter term

# Problem in Weinberg power counting

- Example: A. Nogga, R.G.E.Timmermans, U. van Kolck
- PRC72,054006(2005)
  - In addition to the 1S0, 3S1 counter terms at LO, counter terms for 3P0, 3P2,3D2 are required at LO to absorb cutoff dependence.



# Modified power counting

- Origin of problem: singular tensor force in OPE
- Nogga, Timmerman, van Kolck (PRC72(2005)054006)
  - Weinberg's power counting is okay for singlet and repulsive tensor channel : leading order counter term can absorb cutoff dependence
  - However, attractive tensor channels show limit cycle like cutoff dependence (3P0, 3P2, 3D2)
  - Need to promote higher order counter terms to leading order

# Modified power counting

This work basically reproduce NTvK results in two-body and studies cutoff dependence in 3-body system.
Does three-body require modification in power counting? (like pionless EFT?)

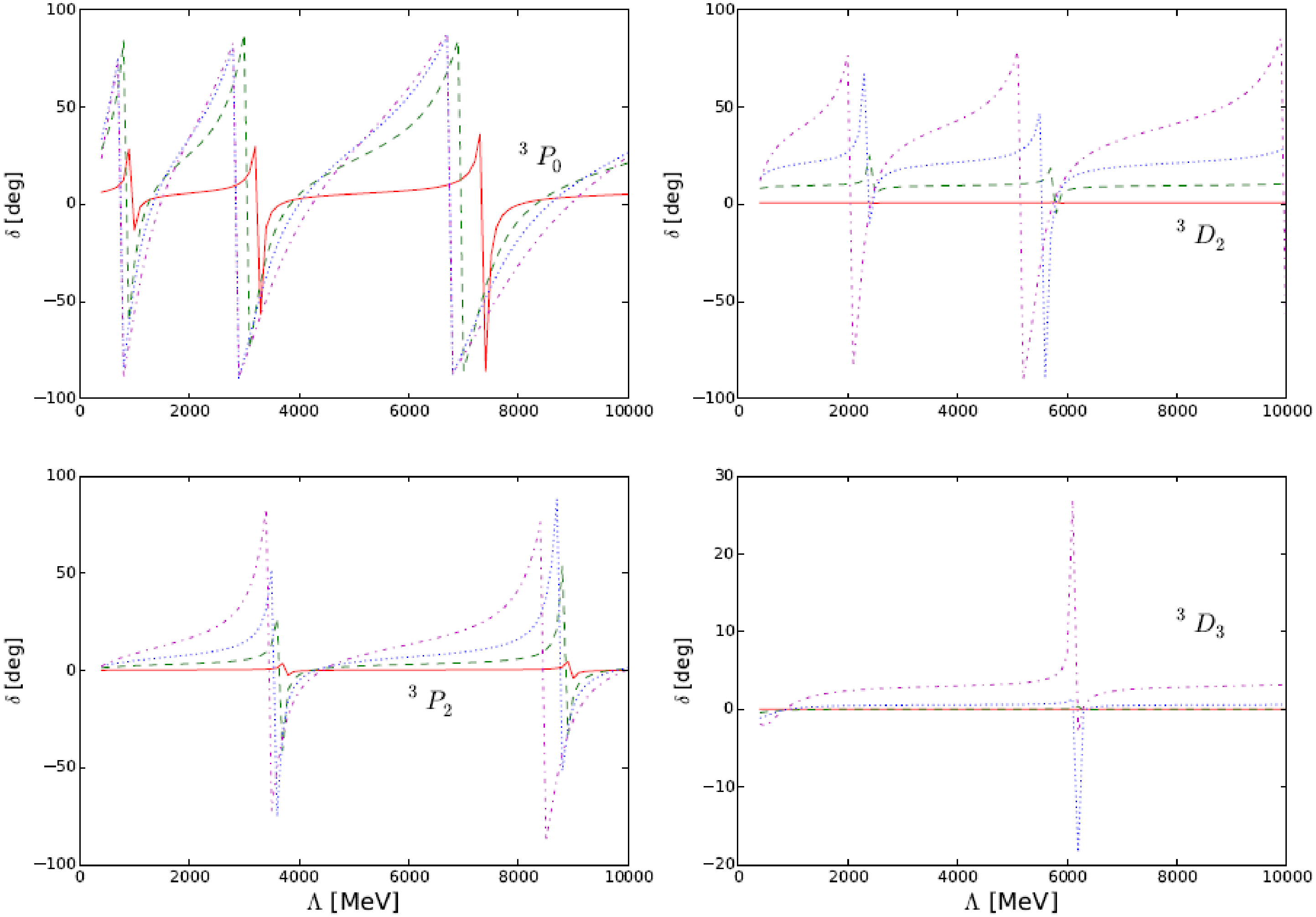
LO potential

$$V_{1\pi}^{(0)} = -\frac{g_A^2}{4f_\pi^2}\tau_1 \cdot \tau_2 \frac{\boldsymbol{\sigma}_1 \cdot \boldsymbol{q}\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}}{q^2 + m_\pi^2}$$

$$V_{ct}^{(0)} = \tilde{C}_{^1S_0} + \tilde{C}_{^3S_1} + C_{^3P_0} p' p + C_{^3P_2} p' p + D_{^3D_2} p'^2 p^2 + D_{^3D_3} p'^2 p^2$$

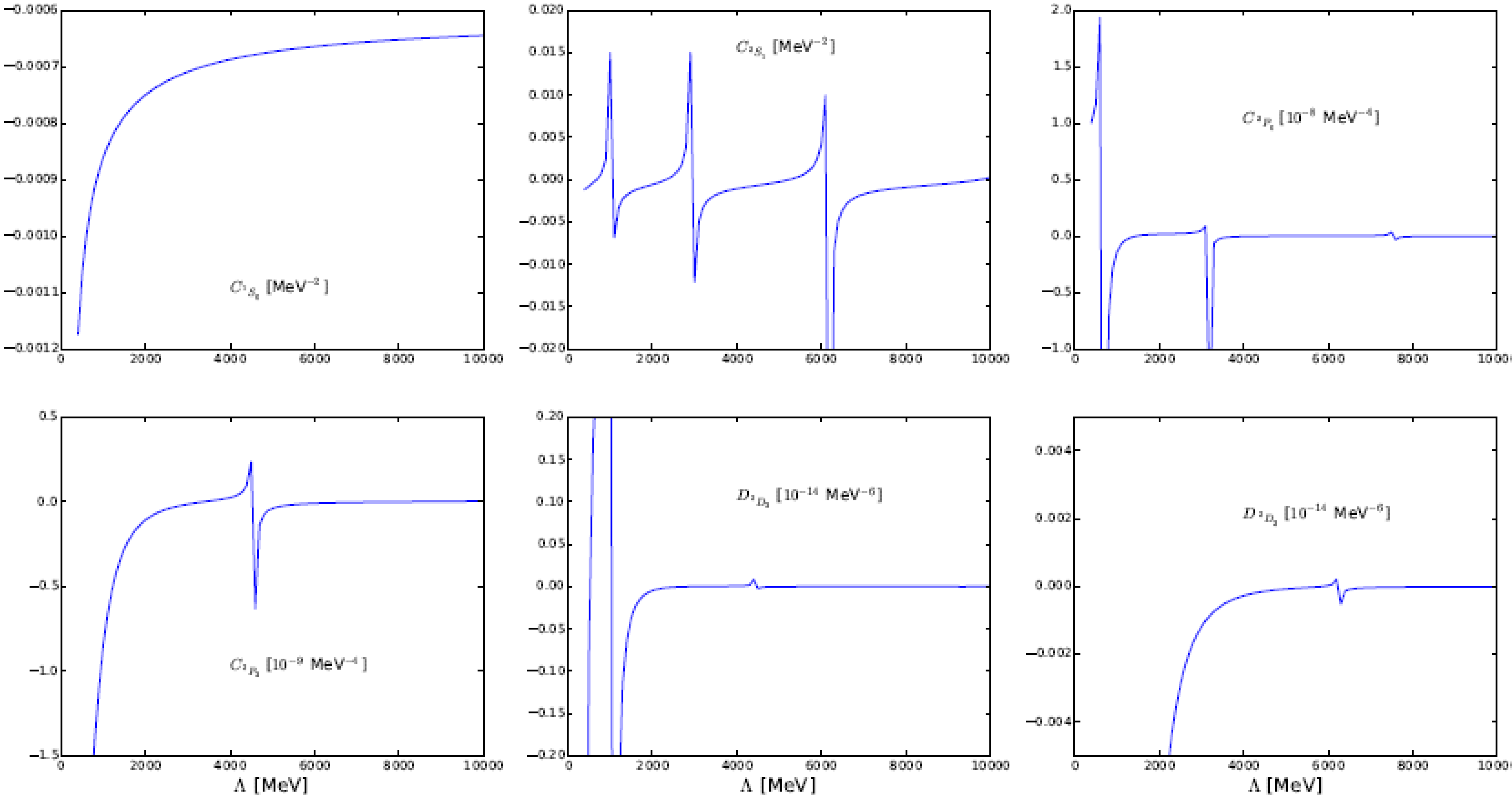
All LECs are fitted to PWA93 phase shifts at T_L= 10 MeV

# Results : 2 body scattering

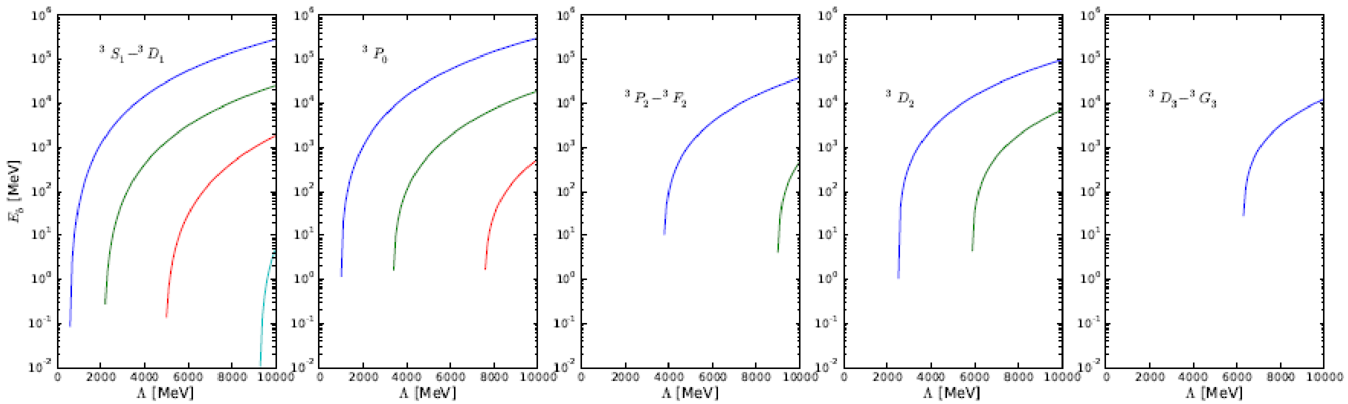



10 MeV (red solid line), 50 MeV (green dashed line), 100 MeV ( blue dotted line), and 200 MeV (magenta dot dashed line)

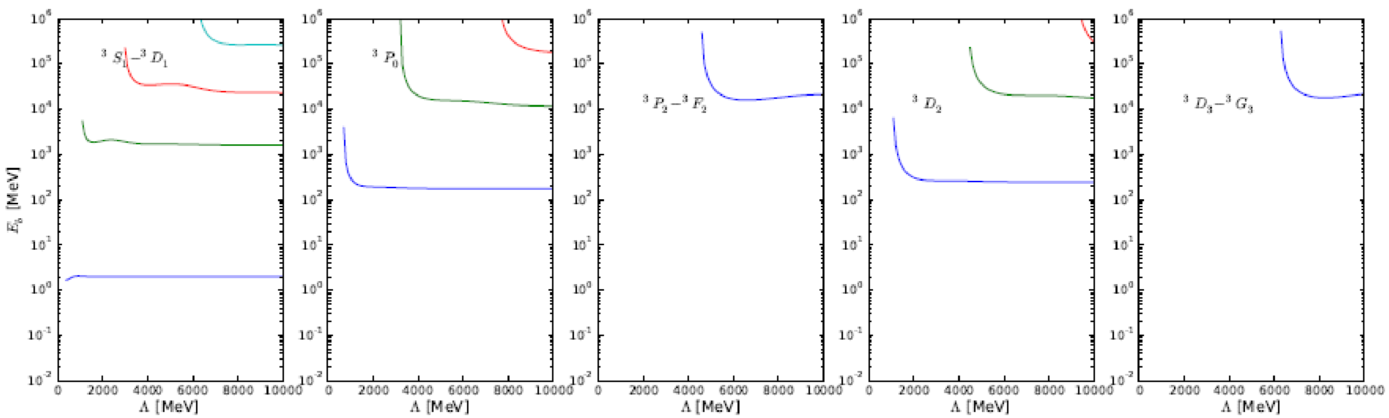
# Counter terms



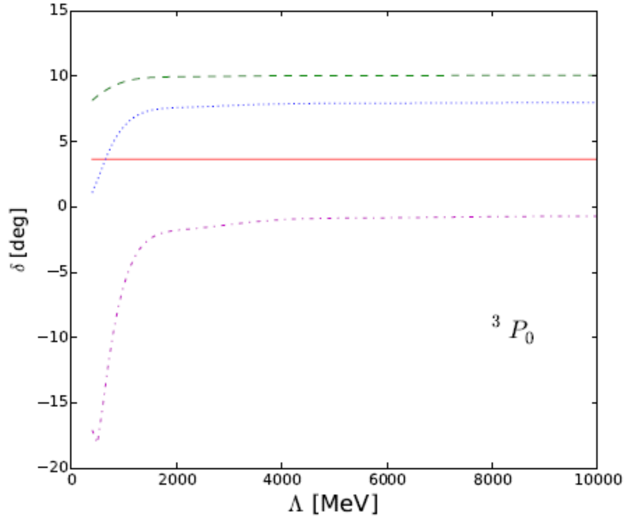
# Results : 2 body bound state

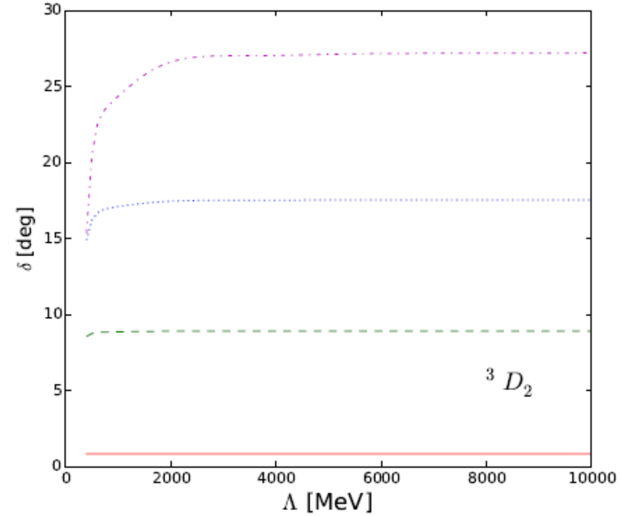


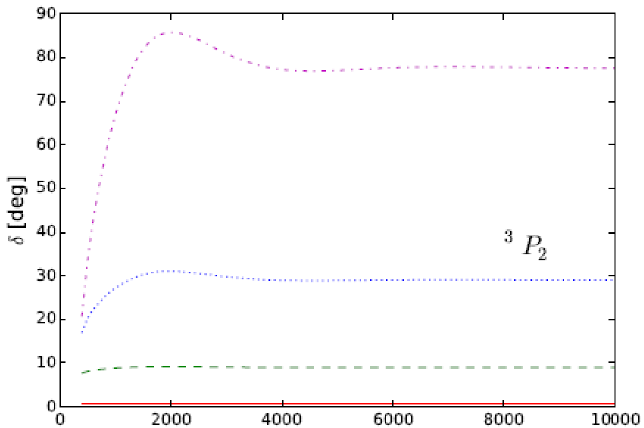

With Counter terms

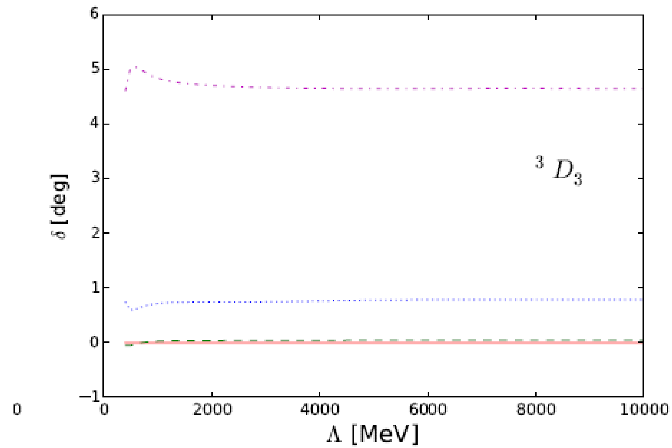


# Results : 2 body scattering(cutoff)

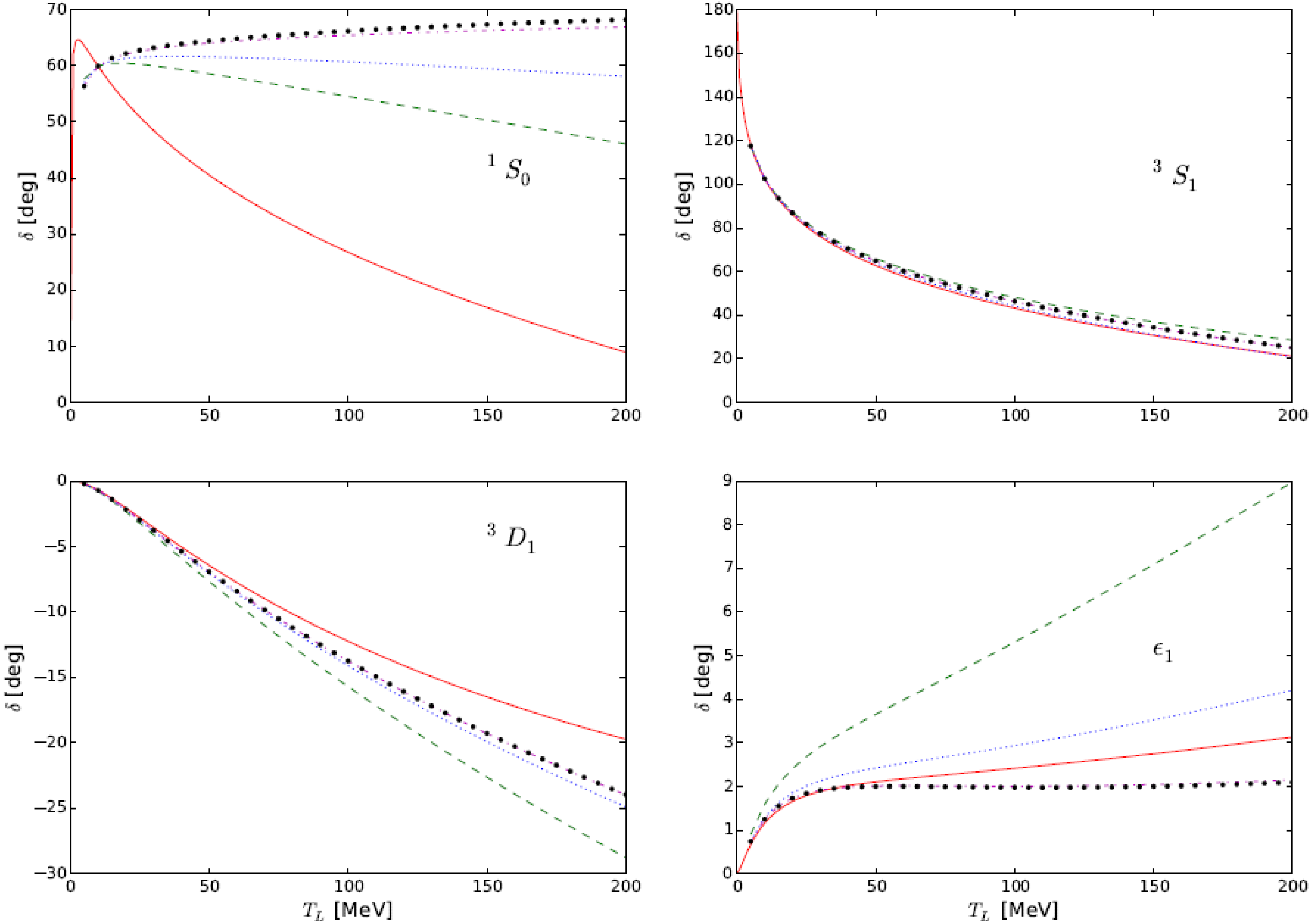


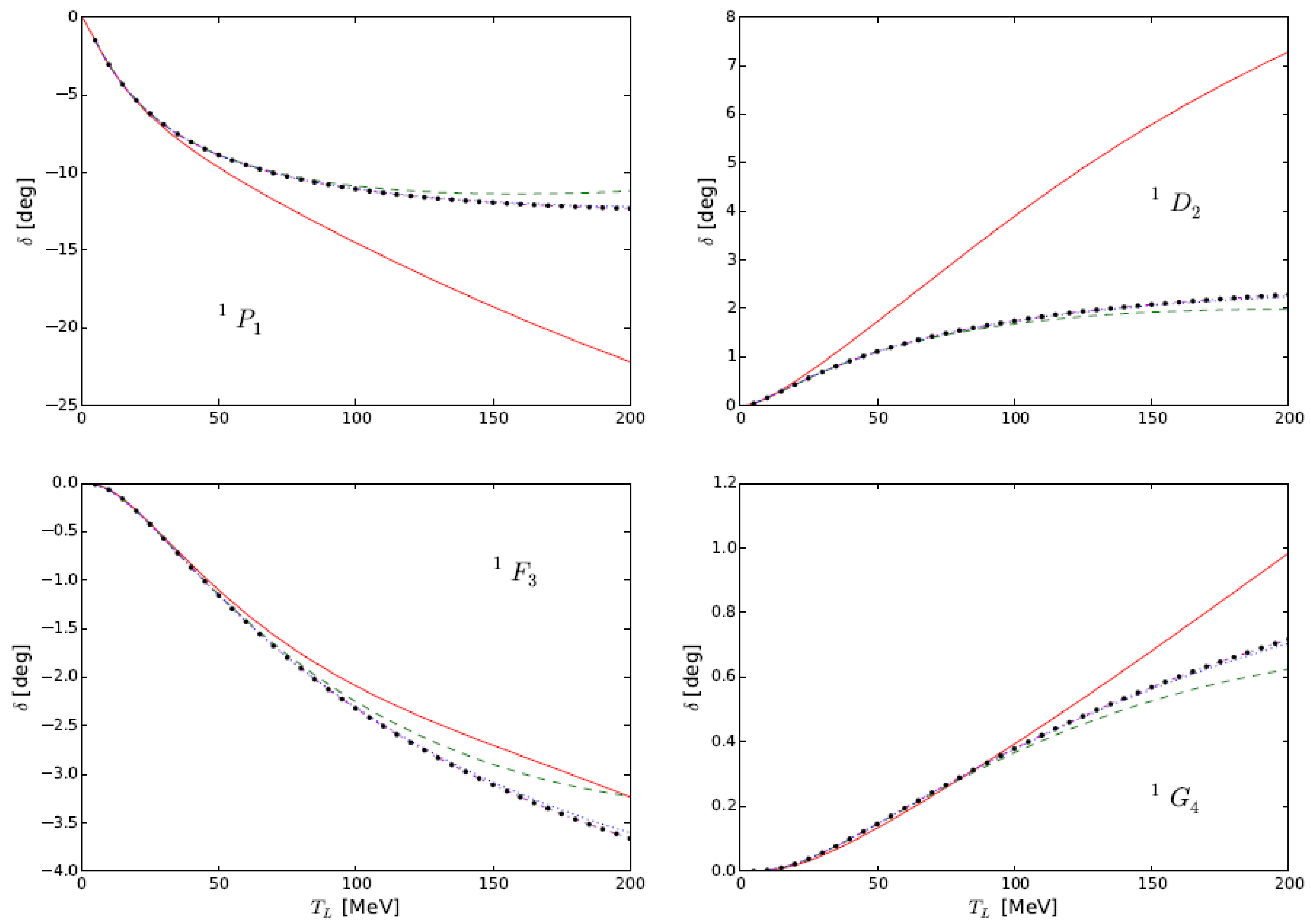



# Results : 2 body scattering(energy)

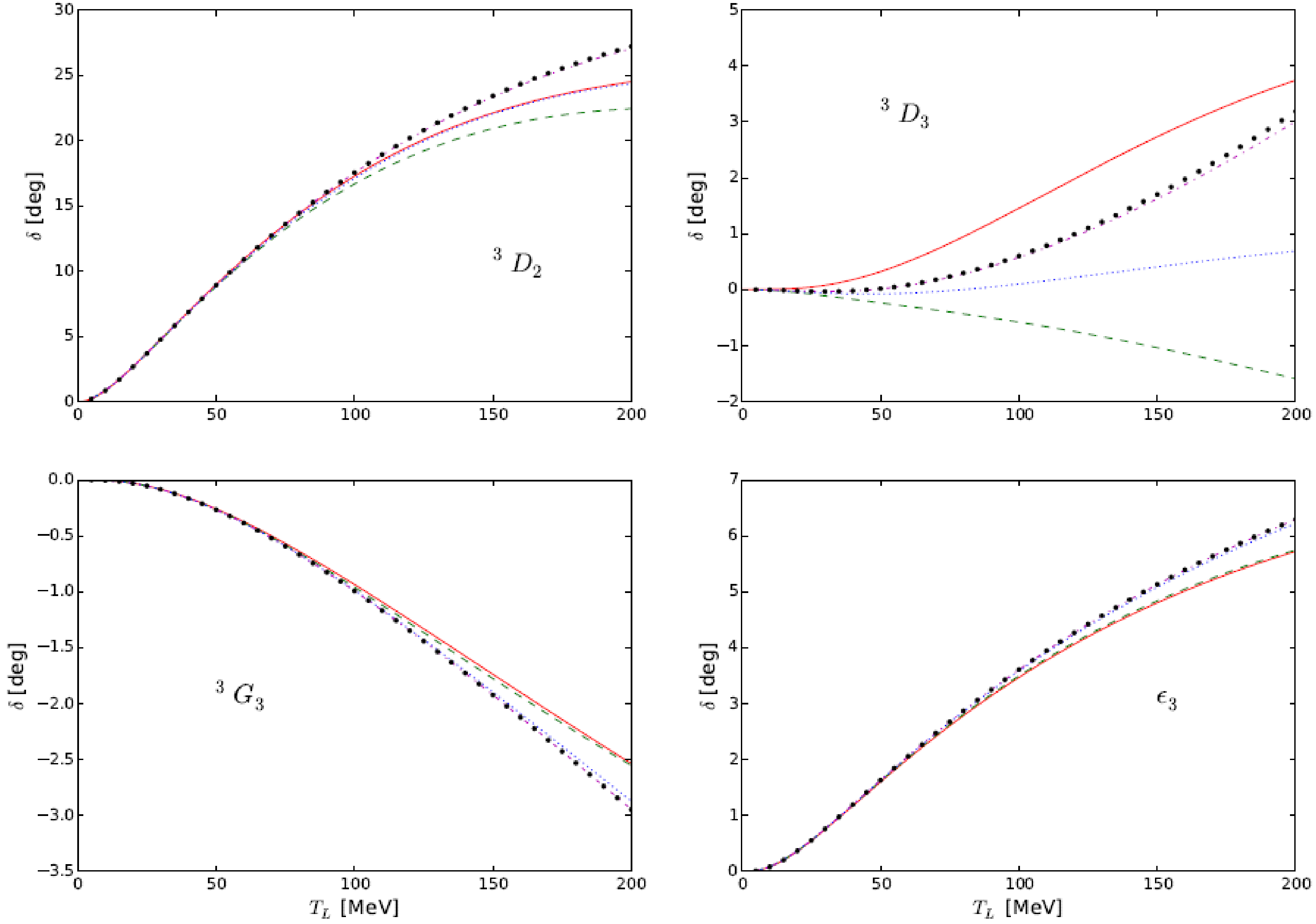



PWA93 data(red solid line), cutoff values 600 MeV(green dashed line), 1 GeV( blue dotted line), 4 GeV (magenta dot dashed line) and 10 GeV(black dots).

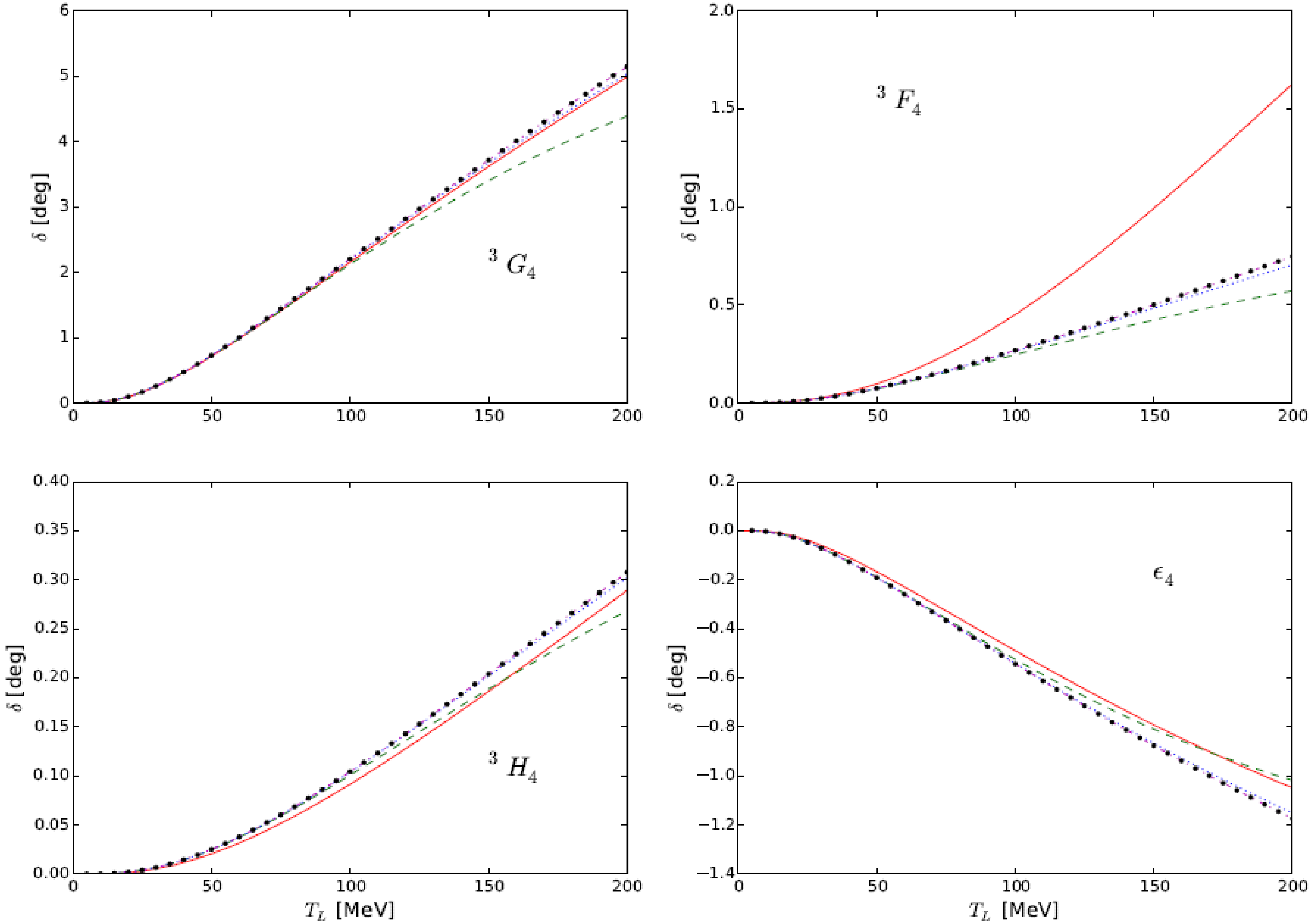
# Results : 2 body scattering



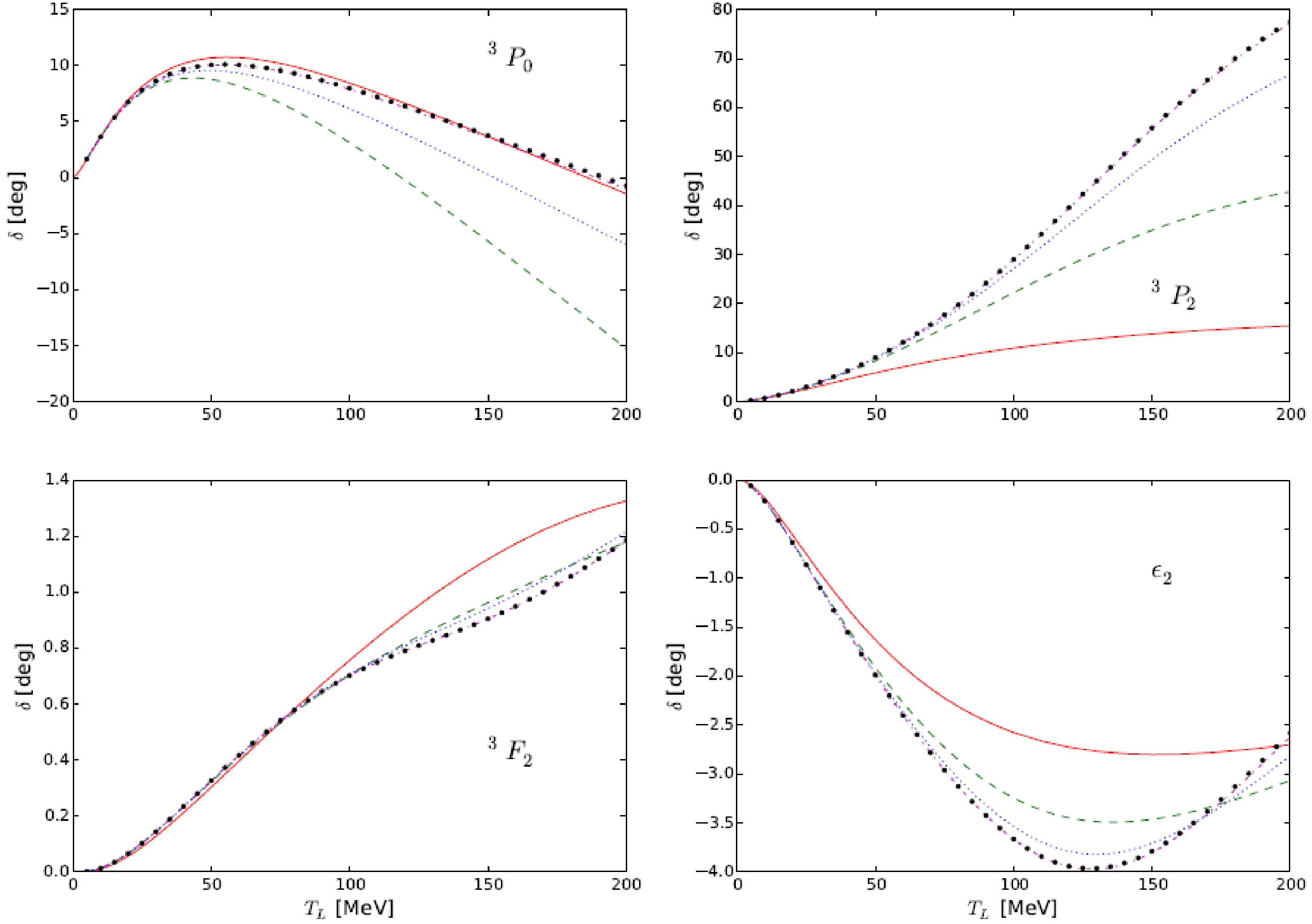
# Results : 2 body scattering



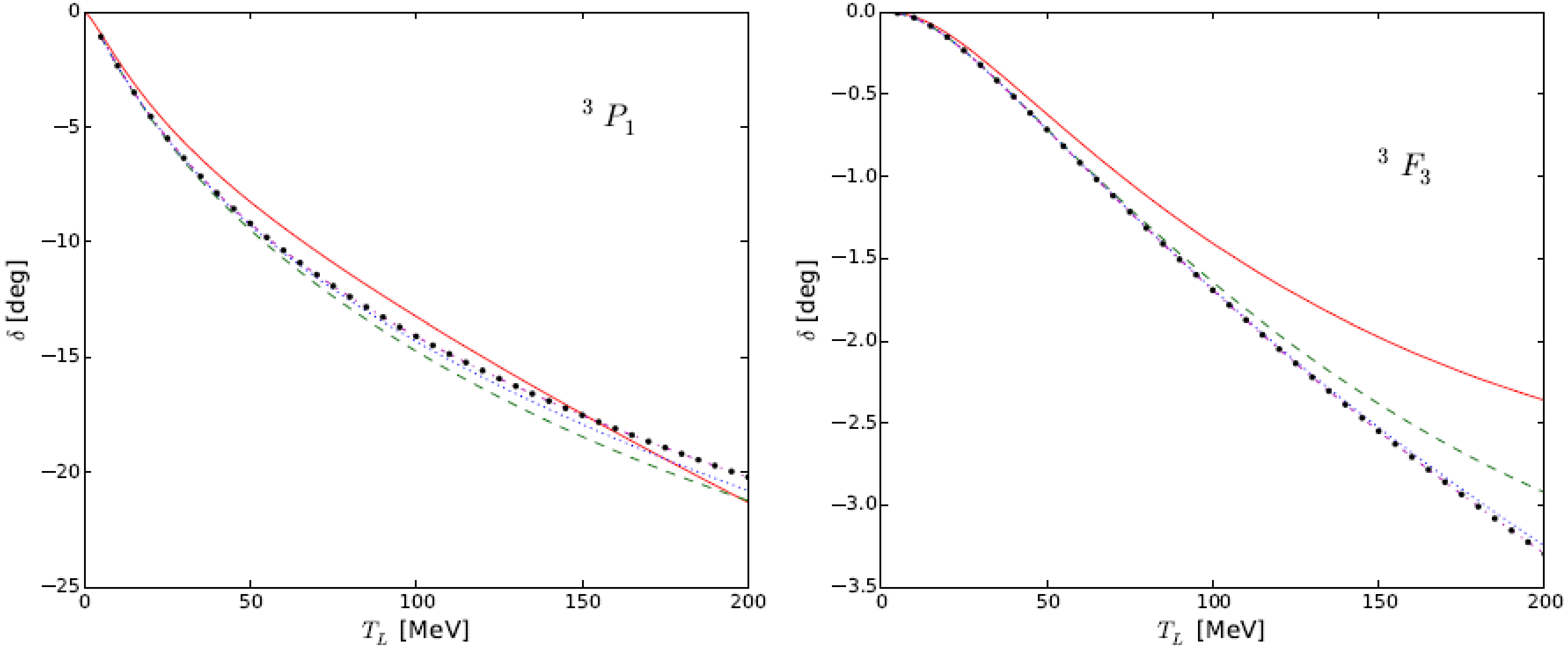
# Results : 2 body scattering



# Results : 2 body scattering

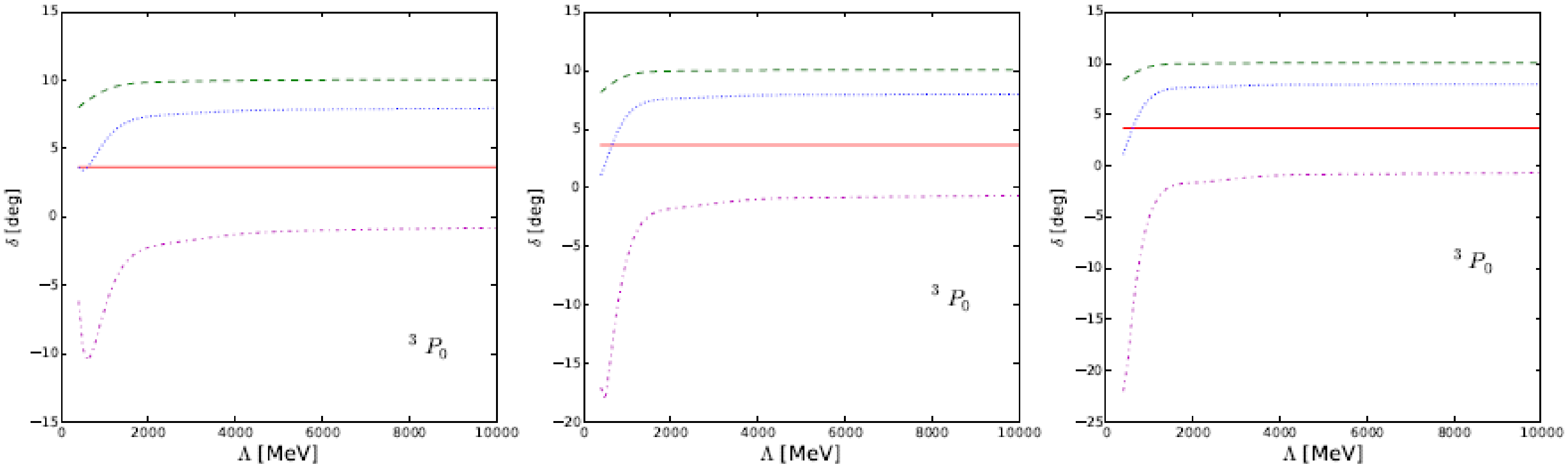


# Results : 2 body scattering



# Results : 2 body scattering(regulator)

Regulator dependence




$$V^{\Lambda}(\boldsymbol{p}',\boldsymbol{p}) = f_{\Lambda}(\boldsymbol{p}')V(\boldsymbol{p}',\boldsymbol{p})f_{\Lambda}(\boldsymbol{p})$$

$$f_{\Lambda}(\boldsymbol{p}) = \exp\left(-\frac{p^n}{\Lambda^n}\right), \quad n = 2, 4, 6.$$

# Results : 3 body

- We obtain 3-body wave function by solving Faddeeve equation in configuration space
- Fourier transform potential from momentum space to configuration space
- Removed unphysical deep bound states in 3-body calculation.

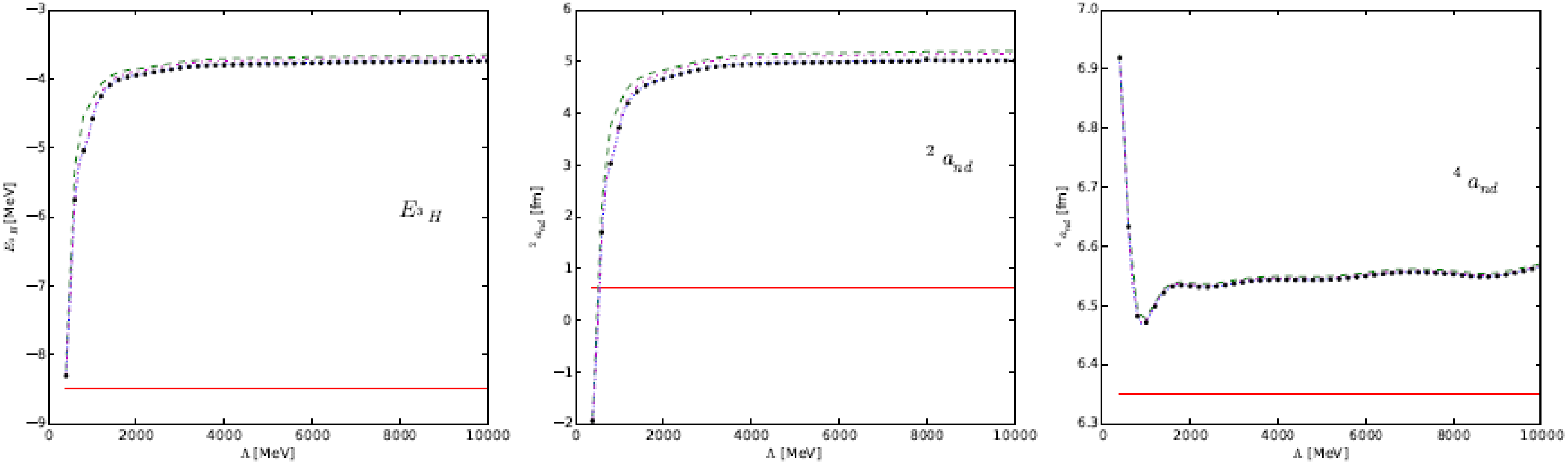
$$(E - H_0 - V_{ij})\,\psi_k = V_{ij}(\psi_i + \psi_j),$$

$$\psi_k^{\pm} = \sum_{\alpha} \frac{F_{\alpha}^{\pm}(x_k, y_k)}{x_k y_k} \left| \left(l_x\,(s_i s_j)_{s_x}\right)_{j_x} (l_y s_k)_{j_y} \right\rangle_{JM} \otimes \left|(t_i t_j)_{t_x}\, t_k\right\rangle_{TT_z}$$

# Results : 3 body

- Partial wave dependence: jmax value dependence

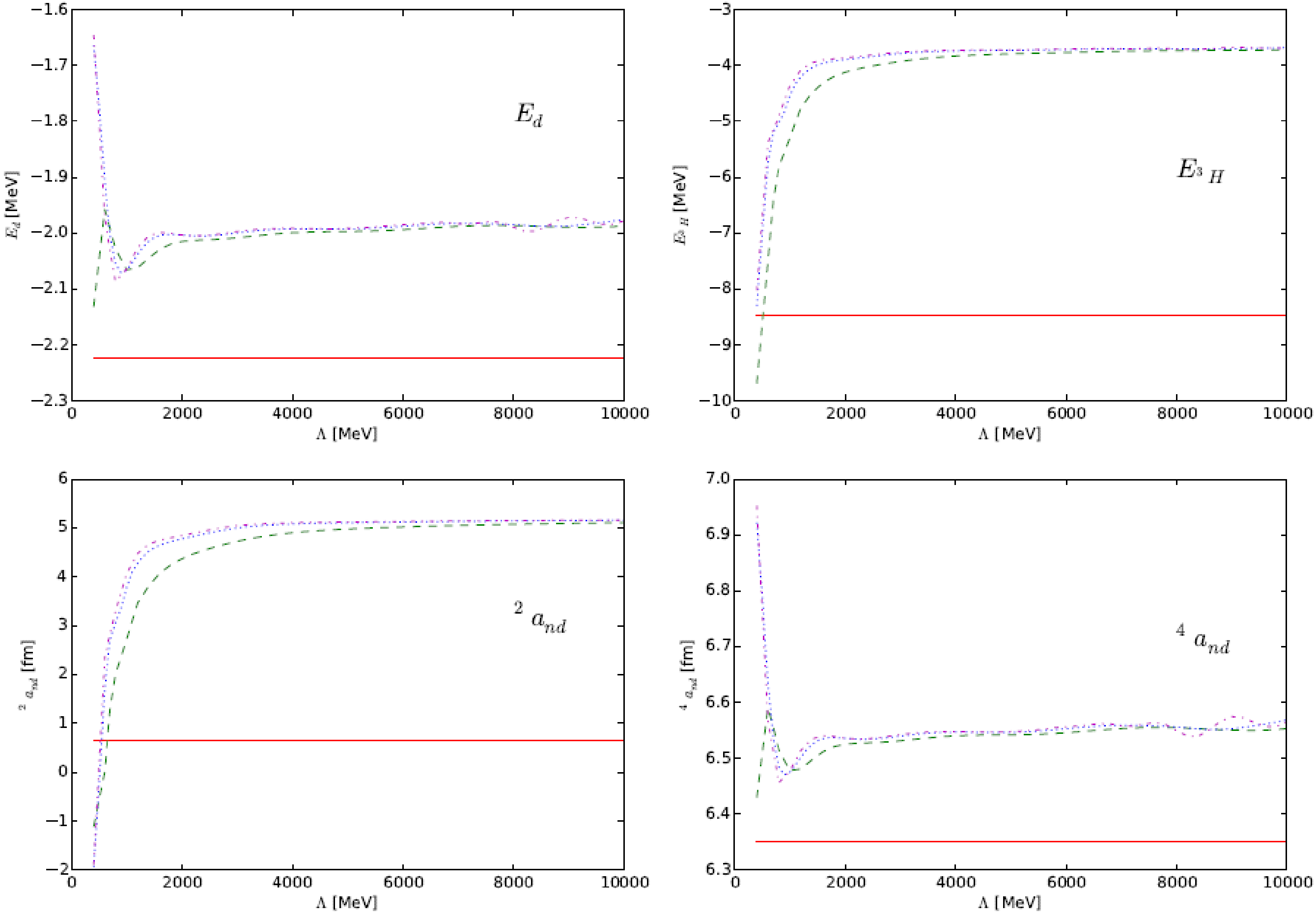
$$V_{j_x}(\boldsymbol{x}) = 0 \text{ if } j_x > j_{max}$$




jmax = 1 (green dashed line), jmax = 2 ( blue dotted line),
jmax = 3 (magenta dot dashed line),and jmax = 4 (black dots).

# Results : 3 body

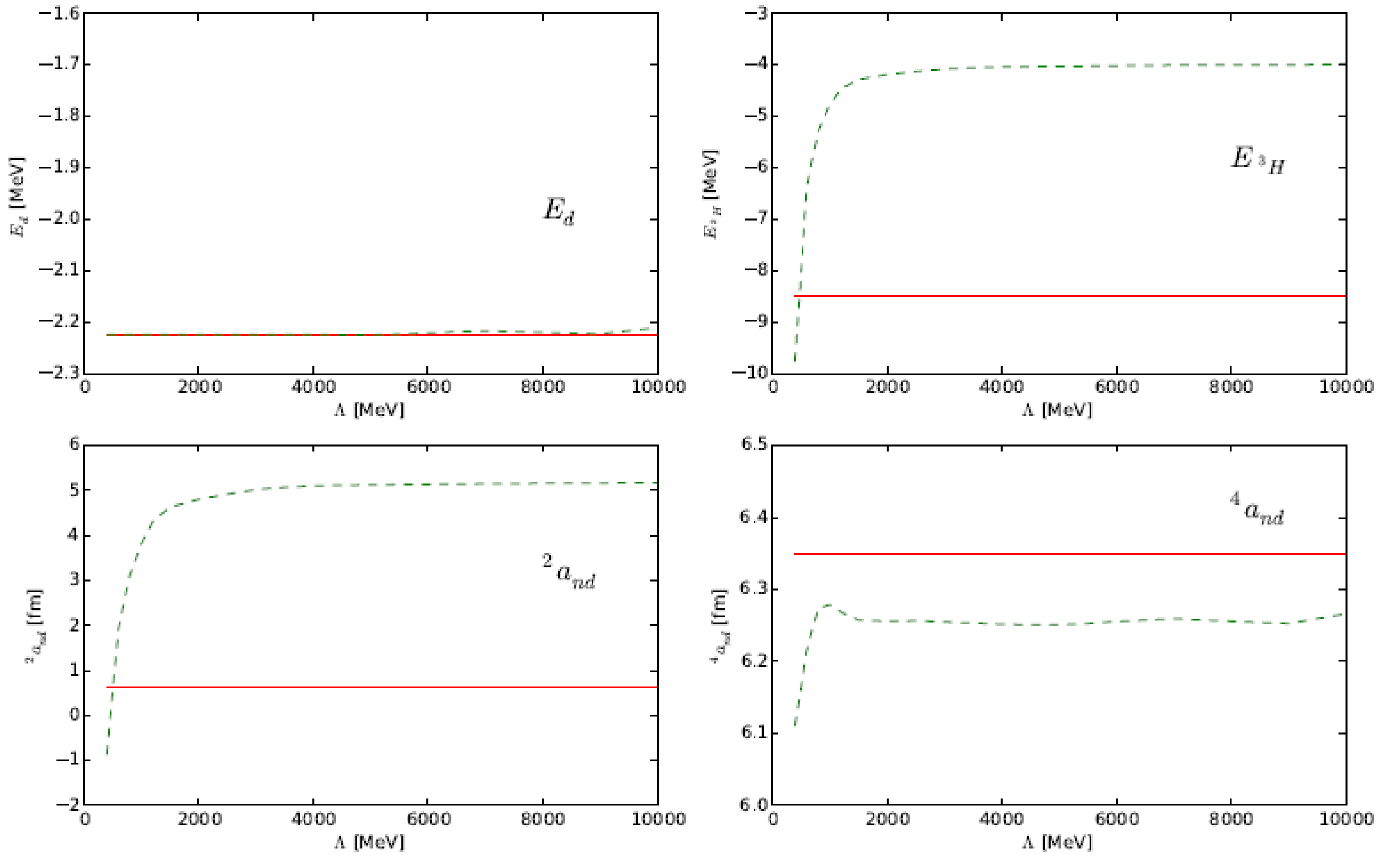
- regulator dependence




n=2
(green dashed line),
n=4regulator
( blue dotted line),
n=6 regulator
(magenta dot dashed
line).

# Results: 3-body (fitting)

- If we fit 3S1 counter term to deuteron binding energy

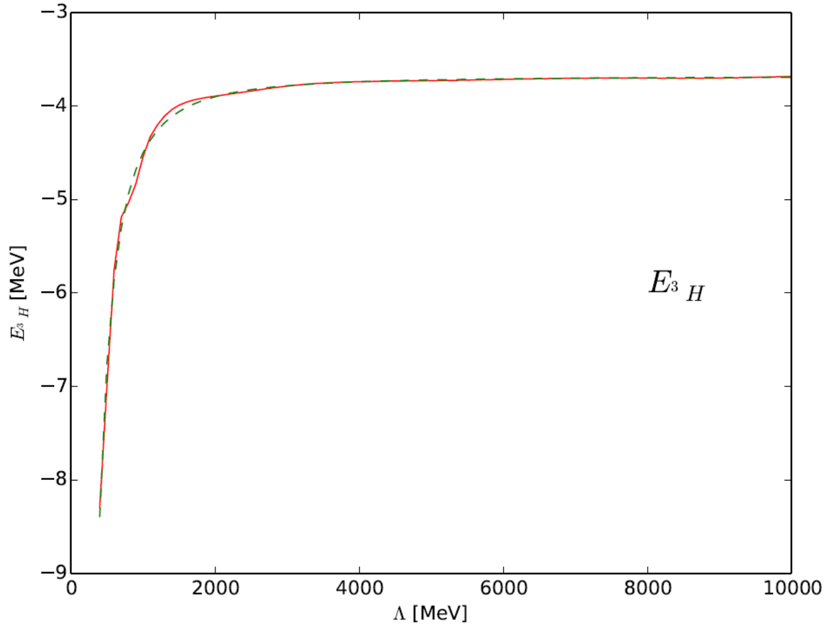


# Summary and Discussion

- We confirmed the results of NTvK in larger cutoff range(~10 GeV)
  - Leading order potential in attractive tensor channels requires additional counter terms to original Weinberg power counting potential and can provide cutoff independent results.
- Triton binding energy and n-d scattering length result shows that it does not require modification of power counting of three body force at leading order.
- The NLO calculation is on-going.

# Summary and discussion

- Fitting cutoff dependence




$$E(\Lambda) = (-3.68 \text{ MeV}) \left(1 + \left(\frac{455 \text{ MeV}}{\Lambda}\right)^{1.91}\right)$$

To have better accuracy to experimental results
and insight on the power of three-body force,
We need to go beyond leading order.

# Preliminary: NLO calculation.

- Long and Yang, PRC86, 024001(2012)

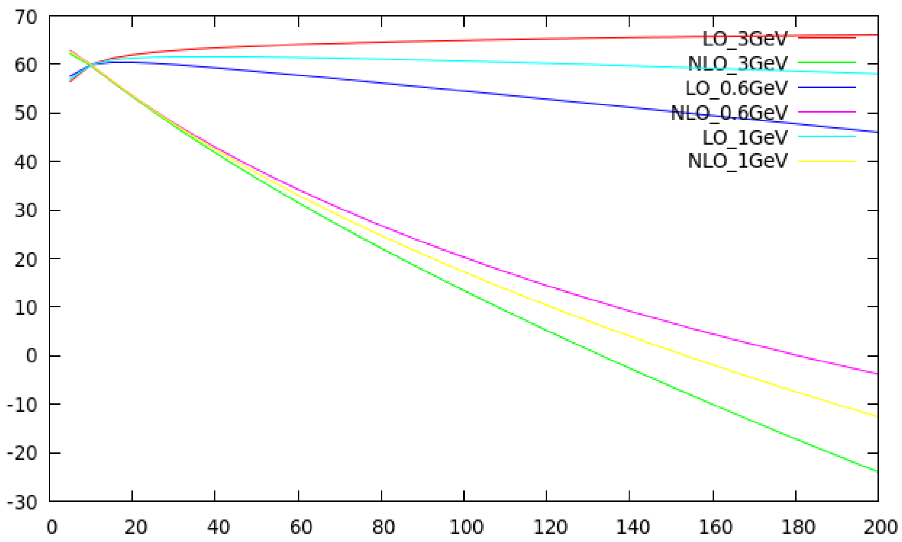
TABLE I. Power counting for pion exchanges and $S$- and $P$-wave counterterms up to $\mathcal{O}(Q^3)$. $p$ ($p'$) is the magnitude of the center-of-mass incoming (outgoing) momentum. The two-by-two matrices are for the coupled channels.

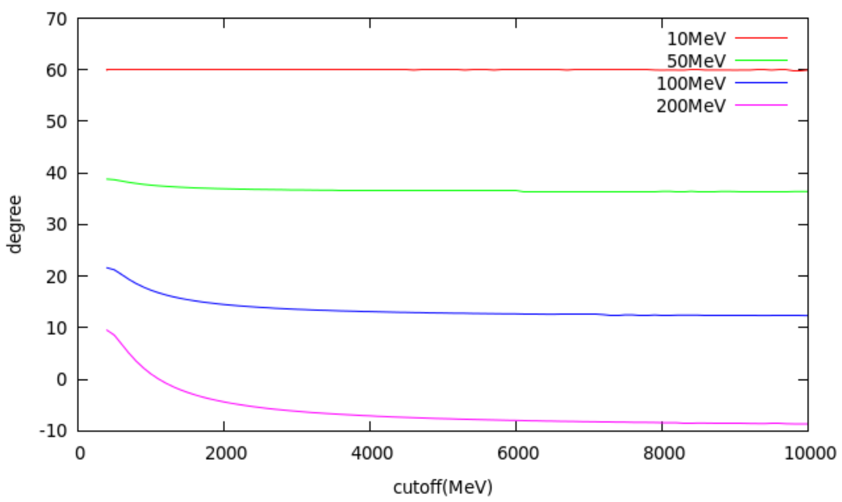
| | |
|---|---|
| $\mathcal{O}(1)$ | OPE, $C_{^1S_0}$, $\begin{pmatrix} C_{^3S_1} & 0 \\ 0 & 0 \end{pmatrix}$, $C_{^3P_0}\, p'p$, $\begin{pmatrix} C_{^3P_2}\, p'p & 0 \\ 0 & 0 \end{pmatrix}$ |
| $\mathcal{O}(Q)$ | $D_{^1S_0}(p'^2 + p^2)$ |
| $\mathcal{O}(Q^2)$ | TPE0, $E_{^1S_0}\, p'^2 p^2$, $\begin{pmatrix} D_{^3S_1}(p'^2 + p^2) & E_{SD}\, p^2 \\ E_{SD}\, p'^2 & 0 \end{pmatrix}$, |
| | $D_{^3P_0}\, p'p(p'^2 + p^2)$, $p'p\begin{pmatrix} D_{^3P_2}(p'^2 + p^2) & E_{PF}\, p^2 \\ E_{PF}\, p'^2 & 0 \end{pmatrix}$, |
| | $C_{^1P_1}\, p'p$, $C_{^3P_1}\, p'p$ |
| $\mathcal{O}(Q^3)$ | TPE1, $F_{^1S_0}\, p'^2 p^2(p'^2 + p^2)$ |

Note: NLO and higher are strictly treated as a perturbation.

# Preliminary: NLO calculation

- Only 1S0 counter term at NLO
- Fit counter terms at E=20 MeV




Is 3-body counter term necessary at NLO? In progress.