Exotic Glue in the Nucleus?

Double Helicity Flip Gluon Operators from Lattice QCD

Phiala Shanahan

Collaborator: Will Detmold

April 27, 2016

Outline

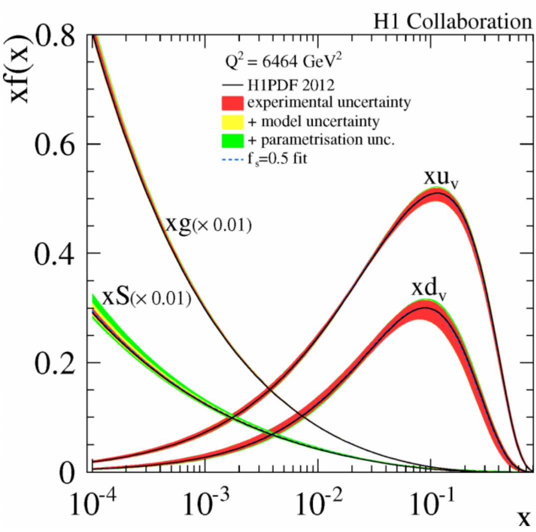
Motivation

Understanding gluons in hadron and nuclear structure is

- Important
  - e.g., Dominance of gluon PDF at low x

Gluons are Important in Hadron Structure

Parton distribution function in the proton



(parton momentum fraction)

Motivation

Understanding gluons in hadron and nuclear structure is

- Important
  - e.g., Dominance of gluon PDF at low x
- Hard
  - No direct expt. measurement of glue in a nucleus (yet)
  - Gluon probed only indirectly in electron scattering from hadrons/nuclei (does not couple to photon)
  - Drell-Yan more direct but messy

Motivation

Understanding gluons in hadron and nuclear structure is

- Important
  - e.g., Dominance of gluon PDF at low x
- Hard
  - No direct expt. measurement of glue in a nucleus (yet)
  - Gluon probed only indirectly in electron scattering from hadrons/nuclei (does not couple to photon)
  - Drell-Yan more direct but messy
- Topical
  - Electron-Ion Collider
  - JLab 12 GeV (lesser extent)

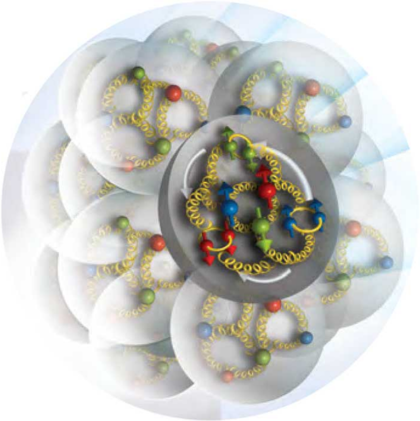
Significant Experimental Progress Expected

Motivation

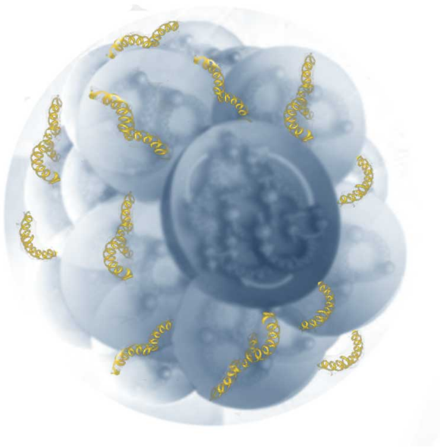
Understanding gluons in hadron and nuclear structure is

- Important
  - e.g., Dominance of gluon PDF at low x
- Hard
  - No direct expt. measurement of glue in a nucleus (yet)
  - Gluon probed only indirectly in electron scattering from hadrons/nuclei (does not couple to photon)
  - Drell-Yan more direct but messy
- Topical
  - Electron-Ion Collider
  - JLab 12 GeV (lesser extent)

‘Exotic’ Glue in the Nucleus



'Exotic' Glue in the Nucleus



'Exotic' Glue

Contributions to gluon observables that are not from nucleon degrees of freedom.

Exotic glue operator:

operator in nucleon $= 0$

operator in nucleus $\neq 0$

Outline

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Jaffe and Manohar (1989)

Leading-twist, double-helicity-flipping structure function $\Delta(x, Q^2)$ sensitive to exotic glue in the nucleus

- Clear signature for exotic glue in nuclei with spin ≥ 1:
  NO analogous twist-2 quark PDF $\rightarrow$ unambiguous
- Experimentally measurable (JLab LOI 2016)
- Moments are calclable on the lattice

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Jaffe and Manohar (1989)

Leading-twist, double-helicity-flipping structure function $\Delta(x, Q^2)$ sensitive to exotic glue in the nucleus

- Clear signature for exotic glue in nuclei with spin ≥ 1:
  NO analogous twist-2 quark PDF $\rightarrow$ unambiguous
- Experimentally measurable (JLab LOI 2016)
- Moments are calclable on the lattice

First Lattice Study

- First moment of $\Delta(x, Q^2)$ in spin-1 ϕ (or ρ) meson

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Jaffe and Manohar (1989)

Leading-twist, double-helicity-flipping structure function $\Delta(x, Q^2)$ sensitive to exotic glue in the nucleus

- Clear signature for exotic glue in nuclei with spin ≥ 1:
  NO analogous twist-2 quark PDF $\rightarrow$ unambiguous
- Experimentally measurable (JLab LOI 2016)
- Moments are calclable on the lattice

First Lattice Study

- First moment of $\Delta(x, Q^2)$ in spin-1 ϕ (or ρ) meson

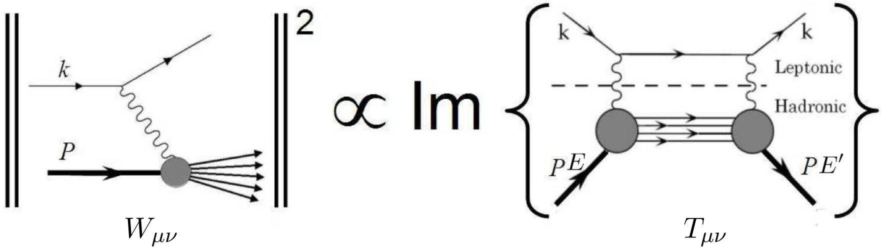
PRELIMINARY

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Hadronic tensor for inelastic lepton scattering from a polarized spin-one target:

$$\frac{1}{4\pi}\int d^4x\, e^{iq\cdot x}\langle p, E'|[j_\mu(x), j_\nu(0)]|p, E\rangle$$

Optical theorem relates squared amplitude of DIS process to imaginary part of forward scattering amplitude:

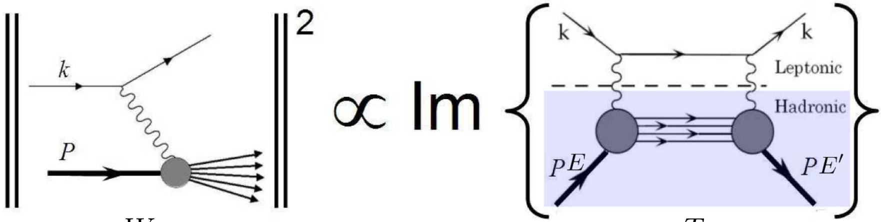


Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Hadronic tensor for inelastic lepton scattering from a polarized spin-one target:

$$\frac{1}{4\pi}\int d^4x\, e^{iq\cdot x}\langle p, E'|[j_\mu(x), j_\nu(0)]|p, E\rangle$$

Optical theorem relates squared amplitude of DIS process to imaginary part of forward scattering amplitude:



$W_{\mu\nu}$ $T_{\mu\nu}$

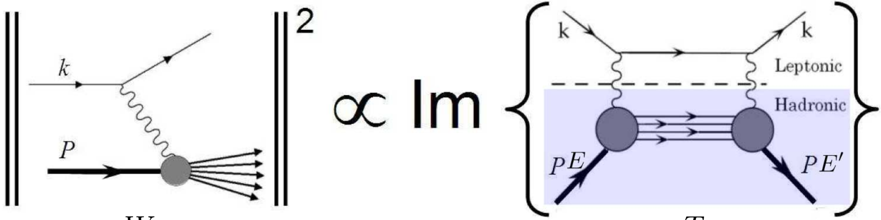
Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Hadronic tensor for inelastic lepton scattering from a polarized spin-one target:

Target Polarization Vector

$$\frac{1}{4\pi}\int d^4x\, e^{iq\cdot x}\langle p, \underline{E}'|[j_\mu(x), j_\nu(0)]|p, \underline{E}\rangle$$

Optical theorem relates squared amplitude of DIS process to imaginary part of forward scattering amplitude:



$W_{\mu\nu}$ $T_{\mu\nu}$

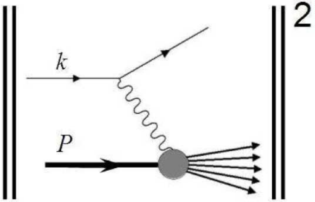
Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Hadronic tensor for inelastic lepton scattering from a polarized spin-one target:

Target Polarization Vector

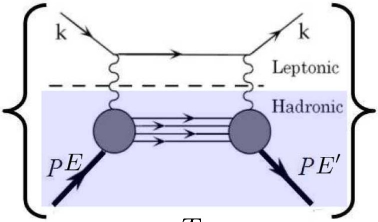
$$\frac{1}{4\pi}\int d^4x\, e^{iq\cdot x}\langle p, \underline{E}'|[j_\mu(x), j_\nu(0)]|p, \underline{E}\rangle = \underline{E}'^{*\alpha}\underline{E}^{\beta}W_{\mu\nu,\alpha\beta}$$

Optical theorem relates squared amplitude of DIS process to imaginary part of forward scattering amplitude:



$W_{\mu\nu}$

$\propto \mathrm{Im}$



$T_{\mu\nu}$

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Hadronic tensor for inelastic lepton scattering from a polarized spin-one target:

Target Polarization Vector

$$\frac{1}{4\pi}\int d^4x\, e^{iq\cdot x}\langle p, \underline{E}'|[j_\mu(x), j_\nu(0)]|p, \underline{E}\rangle = \underline{E}'^{*\alpha}\underline{E}^{\beta}W_{\mu\nu,\alpha\beta}$$

Helicity projection:

$$W_{\mu\nu,\alpha\beta} = \sum_{hH,h'H'} P(hH, h'H')_{\mu\nu,\alpha\beta}A_{hH,h'H'}$$

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Hadronic tensor for inelastic lepton scattering from a polarized spin-one target:

Target Polarization Vector

$$\frac{1}{4\pi}\int d^4x\, e^{iq\cdot x}\langle p, \underline{E}'|[j_\mu(x), j_\nu(0)]|p, \underline{E}\rangle = \underline{E}'^{*\alpha}\underline{E}^{\beta}W_{\mu\nu,\alpha\beta}$$

Helicity projection:

Helicity projection operators

$$W_{\mu\nu,\alpha\beta} = \sum_{hH,h'H'} P(hH, h'H')_{\mu\nu,\alpha\beta}A_{hH,h'H'}$$

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Hadronic tensor for inelastic lepton scattering from a polarized spin-one target:

Target Polarization Vector

$$\frac{1}{4\pi}\int d^4x\, e^{iq\cdot x}\langle p, \underline{E}'|[j_\mu(x), j_\nu(0)]|p, \underline{E}\rangle = \underline{E}'^{*\alpha}\underline{E}^{\beta}W_{\mu\nu,\alpha\beta}$$

Helicity projection:

Helicity projection operators   Target helicity

$$W_{\mu\nu,\alpha\beta} = \sum_{hH,h'H'} P(hH, h'H')_{\mu\nu,\alpha\beta} A_{hH,h'H'}$$

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Hadronic tensor for inelastic lepton scattering from a polarized spin-one target:

Target Polarization Vector

$$\frac{1}{4\pi}\int d^4x\, e^{iq\cdot x}\langle p, \underline{E}'|[j_\mu(x), j_\nu(0)]|p, \underline{E}\rangle = \underline{E}'^{*\alpha}\underline{E}^{\beta}W_{\mu\nu,\alpha\beta}$$

Helicity projection:

Helicity projection operators

Target helicity
Photon helicity

$$W_{\mu\nu,\alpha\beta} = \sum_{hH,h'H'} P(hH, h'H')_{\mu\nu,\alpha\beta} A_{hH,h'H'}$$

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Hadronic tensor for inelastic lepton scattering from a polarized spin-one target:

Target Polarization Vector

$$\frac{1}{4\pi}\int d^4x\, e^{iq\cdot x}\langle p, \underline{E}'|[j_\mu(x), j_\nu(0)]|p, \underline{E}\rangle = \underline{E}'^{*\alpha}\underline{E}^{\beta}W_{\mu\nu,\alpha\beta}$$

Helicity projection:

Helicity projection operators

Target helicity
Photon helicity

$$W_{\mu\nu,\alpha\beta} = \sum_{hH,h'H'} P(hH, h'H')_{\mu\nu,\alpha\beta} A_{hH,h'H'}$$

Double helicity flip amplitude:

$$\Delta(x, Q^2) = A_{+-,-+} = A_{-+,+-}$$

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Hadronic tensor for inelastic lepton scattering from a polarized spin-one target:

Target Polarization Vector

$$\frac{1}{4\pi}\int d^4x\, e^{iq\cdot x}\langle p, \underline{E}'|[j_\mu(x), j_\nu(0)]|p, \underline{E}\rangle = \underline{E}'^{*\alpha}\underline{E}^{\beta}W_{\mu\nu,\alpha\beta}$$

Double helicity flip piece of structure function:

$$W^{\Delta=2}_{\mu\nu,\alpha\beta} = \frac{1}{2}\left\{ \left[\left(E'^*_\mu - \frac{q\cdot E'^*}{\kappa\nu}\left(p_\mu - \frac{M^2}{\nu}q_\mu\right)\right)\left(E_\nu - \frac{q\cdot E}{\kappa\nu}\left(p_\nu - \frac{M^2}{\nu}q_\nu\right)\right) + (\mu\leftrightarrow\nu)\right]\right.$$
$$\left. - \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} + \frac{q^2}{\kappa\nu^2}\left(p_\mu - \frac{\nu}{q^2}q_\mu\right)\left(p_\nu - \frac{\nu}{q^2}q_\nu\right)\right]\left[E'^*\cdot E + \frac{M^2}{\kappa\nu^2}q\cdot E'^*\, q\cdot E\right]\right\}\Delta(x, Q^2)$$

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Hadronic tensor for inelastic lepton scattering from a polarized spin-one target:

Target Polarization Vector

$$\frac{1}{4\pi}\int d^4x\, e^{iq\cdot x}\langle p, \underline{E}'|[j_\mu(x), j_\nu(0)]|p, \underline{E}\rangle = \underline{E}'^{*\alpha}\underline{E}^{\beta}W_{\mu\nu,\alpha\beta}$$

Double helicity flip piece of structure function:

$$W^{\Delta=2}_{\mu\nu,\alpha\beta} = \frac{1}{2}\left\{\left[\left(E'^*_\mu - \frac{q\cdot E'^*}{\kappa\nu}\left(p_\mu - \frac{M^2}{\nu}q_\mu\right)\right)\left(E_\nu - \frac{q\cdot E}{\kappa\nu}\left(p_\nu - \frac{M^2}{\nu}q_\nu\right)\right) + (\mu\leftrightarrow\nu)\right]\right.$$
$$\left. - \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} + \frac{q^2}{\kappa\nu^2}\left(p_\mu - \frac{\nu}{q^2}q_\mu\right)\left(p_\nu - \frac{\nu}{q^2}q_\nu\right)\right]\left[E'^*\cdot E + \frac{M^2}{\kappa\nu^2}q\cdot E'^*\, q\cdot E\right]\right\}\Delta(x, Q^2)$$

Double Helicity Flip

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Hadronic tensor for inelastic lepton scattering from a polarized spin-one target:

Target Polarization Vector

$$\frac{1}{4\pi}\int d^4x\, e^{iq\cdot x}\langle p, \underline{E}'|[j_\mu(x), j_\nu(0)]|p, \underline{E}\rangle = \underline{E}'^{*\alpha}\underline{E}^{\beta}W_{\mu\nu,\alpha\beta}$$

Double helicity flip piece of structure function:

$$W^{\Delta=2}_{\mu\nu,\alpha\beta} = \frac{1}{2}\Big\{\Big[\Big(E'^*_\mu - \frac{q\cdot E'^*}{\kappa\nu}\Big(p_\mu - \frac{M^2}{\nu}q_\mu\Big)\Big)\Big(E_\nu - \frac{q\cdot E}{\kappa\nu}\Big(p_\nu - \frac{M^2}{\nu}q_\nu\Big)\Big) + (\mu\leftrightarrow\nu)\Big] - \Big[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} + \frac{q^2}{\kappa\nu^2}\Big(p_\mu - \frac{\nu}{q^2}q_\mu\Big)\Big(p_\nu - \frac{\nu}{q^2}q_\nu\Big)\Big]\Big[E'^*\cdot E + \frac{M^2}{\kappa\nu^2}q\cdot E'^*\, q\cdot E\Big]\Big\}\Delta(x, Q^2)$$

Double Helicity Flip

Double Helicity Flip Structure Function

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Relate to matrix elements of operator (next slide) using
Operator Product Expansion

$$\mathcal{T}_{\mu\nu}(q) \equiv i \int d^4x \, e^{iq\cdot x} T(j_\mu(x) j_\nu(0))$$

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Relate to matrix elements of operator (next slide) using
Operator Product Expansion

$$\mathcal{T}_{\mu\nu}(q) \equiv i \int d^4x \, e^{iq\cdot x} T(j_\mu(x) j_\nu(0))$$

Showing only double helicity flip part: tower of gluonic operators

$$\frac{1}{2}\mathcal{T}_{\mu\nu}(q) = \ldots + \sum_{n=2,4,\ldots} \frac{2^n q^{\mu_1} \ldots q^{\mu_n}}{(Q^2)^n} C_n(Q^2) \mathcal{O}_{\mu\nu\mu_1\ldots\mu_n}$$

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Relate to matrix elements of operator (next slide) using
Operator Product Expansion

$$\mathcal{T}_{\mu\nu}(q) \equiv i \int d^4x \, e^{iq\cdot x} T(j_\mu(x) j_\nu(0))$$

Showing only double helicity flip part: tower of gluonic operators

$$\frac{1}{2}\mathcal{T}_{\mu\nu}(q) = \ldots + \sum_{n=2,4,\ldots} \frac{2^n q^{\mu_1} \ldots q^{\mu_n}}{(Q^2)^n} C_n(Q^2) \boxed{\mathcal{O}_{\mu\nu\mu_1\ldots\mu_n}}$$

where

$$\boxed{\mathcal{O}_{\mu\nu\mu_1\ldots\mu_n}} = S\left[G_{\mu\mu_1} \overleftrightarrow{D}_{\mu_3} \ldots \overleftrightarrow{D}_{\mu_n} G_{\nu\mu_2}\right]$$

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Relate to matrix elements of operator (next slide) using
Operator Product Expansion

$$\mathcal{T}_{\mu\nu}(q) \equiv i \int d^4x\, e^{iq\cdot x} T(j_\mu(x) j_\nu(0))$$

Showing only double helicity flip part: tower of gluonic operators

$$\frac{1}{2}\mathcal{T}_{\mu\nu}(q) = \ldots + \sum_{n=2,4,\ldots} \frac{2^n q^{\mu_1} \ldots q^{\mu_n}}{(Q^2)^n} C_n(Q^2) \boxed{\mathcal{O}_{\mu\nu\mu_1\ldots\mu_n}}$$

where

Symmetrize and trace subtract in $\mu_1, \ldots, \mu_n$

$$\boxed{\mathcal{O}_{\mu\nu\mu_1\ldots\mu_n}} = \underline{S}\left[G_{\mu\mu_1} \overleftrightarrow{D}_{\mu_3} \ldots \overleftrightarrow{D}_{\mu_n} G_{\nu\mu_2} \right]$$

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

$$\begin{aligned}\langle pE'|\mathcal{O}_{\mu\nu\{\mu_1\ldots\mu_n\}}-\mathrm{Tr}|pE\rangle \\ = (-2i)^{n-2} S\left[(p_\mu E'^*_{\mu_1} - p_{\mu_1} E'^*_{\mu})(p_\nu E_{\mu_2} - p_{\mu_2} E_\nu)\right. \\ \left. + (\mu \leftrightarrow \nu)\right] p_{\mu_3}\ldots p_{\mu_n} A_n(Q^2)\ldots,\end{aligned}$$

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

$$\langle pE' | \mathcal{O}_{\mu\nu\{\mu_1\ldots\mu_n\}} - \mathrm{Tr} | pE \rangle$$

Symmetrize and trace subtract in $\mu_1, \ldots, \mu_n$

$$= (-2i)^{n-2} \underline{S} \left[(p_\mu E'^*_{\mu_1} - p_{\mu_1} E'^*_\mu)(p_\nu E_{\mu_2} - p_{\mu_2} E_\nu) + (\mu \leftrightarrow \nu) \right] p_{\mu_3} \ldots p_{\mu_n} A_n(Q^2) \ldots,$$

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

$$\langle pE' | \mathcal{O}_{\mu\nu\{\mu_1 \ldots \mu_n\}} - \mathrm{Tr} | pE \rangle$$

Symmetrize and trace subtract in $\mu_1, \ldots, \mu_n$

$$= (-2i)^{n-2} \underline{S} \left[(p_\mu E'^{*}_{\mu_1} - p_{\mu_1} E'^{*}_{\mu})(p_\nu E_{\mu_2} - p_{\mu_2} E_\nu) + (\mu \leftrightarrow \nu) \right] p_{\mu_3} \ldots p_{\mu_n} \boxed{A_n(Q^2)} \ldots,$$

Reduced Matrix Element

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

$$\langle pE' | \mathcal{O}_{\mu\nu\{\mu_1 \ldots \mu_n\}} - \text{Tr} | pE \rangle$$

Symmetrize and trace subtract in $\mu_1, \ldots, \mu_n$

$$= (-2i)^{n-2} \underline{S} \left[(p_\mu E'^*_{\mu_1} - p_{\mu_1} E'^*_{\mu})(p_\nu E_{\mu_2} - p_{\mu_2} E_\nu) + (\mu \leftrightarrow \nu) \right] p_{\mu_3} \ldots p_{\mu_n} \boxed{A_n(Q^2)} \ldots,$$

Reduced Matrix Element

Dispersion relation for helicity flip part of $\mathcal{T}_{\mu\nu}$ (previous slide) and analytic continuation give **moments**:

$$\int_0^1 dx x^{n-1} \Delta(x, Q^2) = \frac{\alpha_s(Q^2)}{3\pi} \frac{A_n(Q^2)}{n+2}, \quad n = 2, 4, 6 \ldots,$$

Unpolarized scattering: symmetric piece of hadronic tensor $W_{\mu\nu}$, $\rightarrow$ even n

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

$$\langle pE' | \mathcal{O}_{\mu\nu\{\mu_1 \ldots \mu_n\}} - \mathrm{Tr} | pE \rangle$$

Symmetrize and trace subtract in $\mu_1, \ldots, \mu_n$

$$= (-2i)^{n-2} \underline{S} \left[(p_\mu E'^*_{\mu_1} - p_{\mu_1} E'^*_{\mu})(p_\nu E_{\mu_2} - p_{\mu_2} E_\nu) + (\mu \leftrightarrow \nu) \right] p_{\mu_3} \ldots p_{\mu_n} \boxed{A_n(Q^2)} \ldots,$$

Reduced Matrix Element

Dispersion relation for helicity flip part of $\mathcal{T}_{\mu\nu}$ (previous slide) and analytic continuation give **moments**:

Moment of Structure Function

$$\boxed{\int_0^1 dx x^{n-1} \Delta(x, Q^2)} = \frac{\alpha_s(Q^2)}{3\pi} \frac{A_n(Q^2)}{n+2}, \quad n = 2, 4, 6 \ldots,$$

Unpolarized scattering: symmetric piece of hadronic tensor $W_{\mu\nu}$, $\rightarrow$ even n

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

$$\langle pE' | \mathcal{O}_{\mu\nu\{\mu_1 \ldots \mu_n\}} - \mathrm{Tr} | pE \rangle$$
$$= (-2i)^{n-2} \underline{S} \left[(p_\mu E'^*_{\mu_1} - p_{\mu_1} E'^*_\mu)(p_\nu E_{\mu_2} - p_{\mu_2} E_\nu) + (\mu \leftrightarrow \nu) \right] p_{\mu_3} \ldots p_{\mu_n} \boxed{A_n(Q^2)} \ldots,$$

Symmetrize and trace subtract in $\mu_1, \ldots, \mu_n$

Reduced Matrix Element

Dispersion relation for helicity flip part of $\mathcal{T}_{\mu\nu}$ (previous slide) and analytic continuation give **moments**:

Moment of Structure Function

Reduced Matrix Element

$$\boxed{\int_0^1 dx x^{n-1} \Delta(x, Q^2)} = \frac{\alpha_s(Q^2)}{3\pi} \frac{\boxed{A_n(Q^2)}}{n+2}, \qquad n = 2, 4, 6 \ldots,$$

Unpolarized scattering: symmetric piece of hadronic tensor $W_{\mu\nu}$, $\rightarrow$ even n

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Parton model interpretation

For a target in the infinite momentum frame polarized in the $\hat{x}$ direction perpendicular to its momentum,

$$\Delta(x, Q^2) \propto \int_x^1 \frac{dy}{y^3} \left(g_{\hat{x}}(y, Q^2) - g_{\hat{y}}(y, Q^2) \right)$$

$g_{\hat{x},\hat{y}}(y, Q^2)$: probability of finding a gluon with momentum fraction y linearly polarized in the $\hat{x}$, $\hat{y}$ direction

"How much more momentum of transversely polarized particle carried by gluons aligned rather than perpendicular to it in the transverse plane"

Outline

Lattice Operators

First moment of $\Delta(x, Q^2)$

$\Updownarrow$

Matrix elt. of $\mathcal{O}_{\mu\nu\mu_1\mu_2} = S[G_{\mu\mu_1} G_{\nu\mu_2}]$

Lattice Operators

First moment of $\Delta(x, Q^2)$

$\Updownarrow$

Matrix elt. of $\mathcal{O}_{\mu\nu\mu_1\mu_2} = S[G_{\mu\mu_1} G_{\nu\mu_2}]$

- Relate $\mathcal{O}_{\mu\nu\mu_1\mu_2}$ to Euclidean operator
- Find linear combs. of Euclidean operator (with different indices) that
  1. Transform irriducibly under appropriate representations of $H(4)$
  2. Don't mix with same or lower-dimensional quark or gluon operators
     - 3 irreps. of dimension 2, 6, 2, i.e., 10 basis vectors
- Lattice simulation of matrix element in ϕ meson (spin-1)

Lattice Details

Luscher-Weisz gauge action with a clover-improved quark action with one level of stout link smearing

L/a	T/a	β	am_l	am_s
24	64	6.1	-0.2800	-0.2450
a (fm)	L (fm)	T (fm)	m_π (MeV)	m_K (MeV)
0.1167(16)	2.801(29)	7.469(77)	450(5)	596(6)
m_ϕ (MeV)	$m_\pi L$	$m_\pi T$	N_{cfg}	N_{src}
1040(3)	6.390	17.04	1042	10^5

- All ϕ polarization states ($\{1,2,3\}$ or $\{+,-,0\}$)
  - on-diagonal
  - off-diagonal
- Momenta up to (1,1,1) in lattice units
- HYP smearing of gauge fields in operator (2-5 steps)

Lattice Details

Luscher-Weisz gauge action with a clover-improved quark action with one level of stout link smearing

L/a	T/a	β	am_l	am_s
24	64	6.1	-0.2800	-0.2450
a (fm)	L (fm)	T (fm)	m_π (MeV)	m_K (MeV)
0.1167(16)	2.801(29)	7.469(77)	450(5)	596(6)
m_ϕ (MeV)	$m_\pi L$	$m_\pi T$	N_{cfg}	N_{src}
1040(3)	6.390	17.04	1042	10^5

- All ϕ polarization states ($\{1,2,3\}$ or $\{+,-,0\}$)
  - on-diagonal
  - off-diagonal
- Momenta up to (1,1,1) in lattice units
- HYP smearing of gauge fields in operator (2-5 steps)

Lattice Details

Luscher-Weisz gauge action with a clover-improved quark action with one level of stout link smearing

L/a	T/a	β	am_l	am_s
24	64	6.1	-0.2800	-0.2450
a (fm)	L (fm)	T (fm)	m_π (MeV)	m_K (MeV)
0.1167(16)	2.801(29)	7.469(77)	450(5)	596(6)
m_ϕ (MeV)	$m_\pi L$	$m_\pi T$	N_{cfg}	N_{src}
1040(3)	6.390	17.04	1042	10^5

- All ϕ polarization states ($\{1, 2, 3\}$ or $\{+, -, 0\}$)
  - on-diagonal
  - off-diagonal
- Momenta up to (1,1,1) in lattice units
- HYP smearing of gauge fields in operator (2-5 steps)

Lattice Details

Luscher-Weisz gauge action with a clover-improved quark action with one level of stout link smearing

L/a				am_s
24				-0.2450
a (fm)				m_K (MeV)
0.1167(16)				596(6)
m_ϕ (MeV)	$m_\pi L$	$m_\pi T$	N_{cfg}	N_{src}
1040(3)	6.390	17.04	1042	10^5

SYSTEMATICS IGNORED

- Quark mass effects
- Volume effects
- Discretization effects
- Renormalization (for now)

- All ϕ polarization states ($\{1, 2, 3\}$ or $\{+, -, 0\}$)
  - on-diagonal
  - off-diagonal
- Momenta up to (1,1,1) in lattice units
- HYP smearing of gauge fields in operator (2-5 steps)

Extraction of A_2

$$\langle pE'|\mathcal{O}_{\mu\nu\{\mu_1\ldots\mu_n\}} - \mathrm{Tr}|pE\rangle$$

Symmetrize and trace subtract in $\mu_1, \ldots, \mu_n$

$$= (-2i)^{n-2} \underline{S} \left[(p_\mu E'^*_{\mu_1} - p_{\mu_1} E'^*_\mu)(p_\nu E_{\mu_2} - p_{\mu_2} E_\nu) + (\mu \leftrightarrow \nu) \right] p_{\mu_3} \ldots p_{\mu_n} \boxed{A_n(Q^2)} \ldots,$$

Reduced Matrix Element

Extraction of A_2

$$\langle pE' | \mathcal{O}_{\mu\nu\{\mu_1 \ldots \mu_n\}} - \mathrm{Tr} | pE \rangle$$

Symmetrize and trace subtract in $\mu_1, \ldots, \mu_n$

$$= (-2i)^{n-2} \underline{S} \left[(p_\mu E'^*_{\mu_1} - p_{\mu_1} E'^*_\mu)(p_\nu E_{\mu_2} - p_{\mu_2} E_\nu) + (\mu \leftrightarrow \nu) \right] p_{\mu_3} \ldots p_{\mu_n} \boxed{A_n(Q^2)} \ldots,$$

Reduced Matrix Element

factors of m and p

$$\left[\frac{C^{EE'}_{3\mathrm{pt}}}{C^{EE'}_{2\mathrm{pt}}} \right] (t_{\mathrm{sink}}, \tau) \propto \boxed{A_2}, \qquad 0 \ll \tau \ll t_{\mathrm{sink}}$$

Extraction of A_2: 3pt/2pt ratio

Some choice of irrep. and basis vector
ϕ momentum $(0,0,0)$

$$
\begin{array}{c} \\ \rho_0 \\ \rho_+ \\ \rho_- \end{array}
\begin{array}{c}
\begin{array}{ccc} \quad\rho_0 \quad & \quad\rho_+\quad & \quad\rho_-\quad \end{array} \\
\begin{pmatrix}
\frac{2m^2 A_2}{\sqrt{3}} & 0 & 0 \\
0 & -\frac{m^2 A_2}{\sqrt{3}} & 0 \\
0 & 0 & -\frac{m^2 A_2}{\sqrt{3}}
\end{pmatrix}
\end{array}
$$

Extraction of A_2: 3pt/2pt ratio

Some choice of irrep. and basis vector

ϕ momentum $(0, 0, 0)$

$$\begin{array}{c} \\ \rho_0 \\ \rho_+ \\ \rho_- \end{array} \begin{array}{c} \begin{array}{ccc} \quad\rho_0 \quad & \quad \rho_+ \quad & \quad \rho_- \end{array} \\ \begin{pmatrix} \frac{2m^2 A_2}{\sqrt{3}} & 0 & 0 \\ 0 & -\frac{m^2 A_2}{\sqrt{3}} & 0 \\ 0 & 0 & -\frac{m^2 A_2}{\sqrt{3}} \end{pmatrix} \end{array}$$

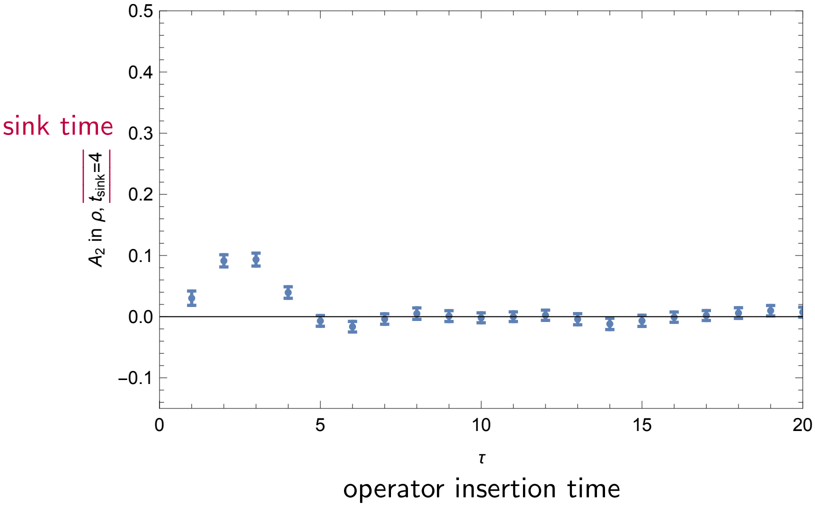
Extraction of A_2: 3pt/2pt ratio

Some choice of irrep. and basis vector

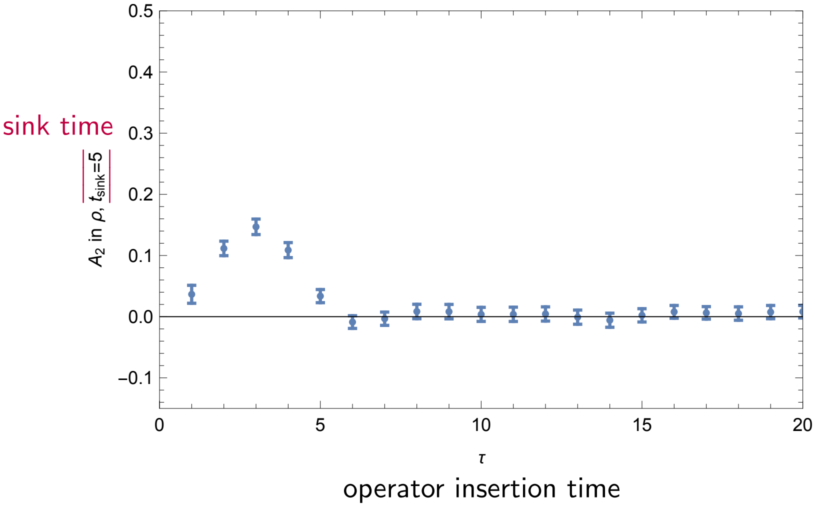
ϕ momentum (p,p,p)

$$
\begin{array}{c}
\\ \rho_0 \\ \rho_+ \\ \rho_-
\end{array}
\begin{array}{ccc}
\rho_0 & \rho_+ & \rho_- \\
\multicolumn{3}{c}{
\begin{pmatrix}
\frac{2\left(m^3+\sqrt{m^2+3p^2}m^2+4p^2m+2p^2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{3}\left(m+\sqrt{m^2+3p^2}\right)} & \frac{(1-i)p^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{6}\left(m+\sqrt{m^2+3p^2}\right)} & -\frac{(1+i)p^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{6}\left(m+\sqrt{m^2+3p^2}\right)} \\
\frac{(1+i)p^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{6}\left(m+\sqrt{m^2+3p^2}\right)} & -\frac{\left(m^3+\sqrt{m^2+3p^2}m^2+4p^2m+2p^2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{3}\left(m+\sqrt{m^2+3p^2}\right)} & -\frac{2ip^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{3}\left(m+\sqrt{m^2+3p^2}\right)} \\
-\frac{(1-i)p^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{6}\left(m+\sqrt{m^2+3p^2}\right)} & \frac{2ip^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{3}\left(m+\sqrt{m^2+3p^2}\right)} & -\frac{\left(m^3+\sqrt{m^2+3p^2}m^2+4p^2m+2p^2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{3}\left(m+\sqrt{m^2+3p^2}\right)}
\end{pmatrix}}
\end{array}
$$

Extraction of A_2: 3pt/2pt ratio

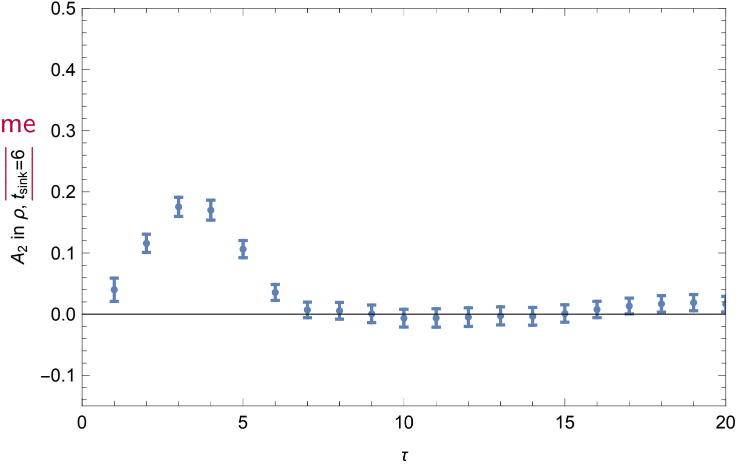


Extraction of A_2: 3pt/2pt ratio



Extraction of A_2: 3pt/2pt ratio

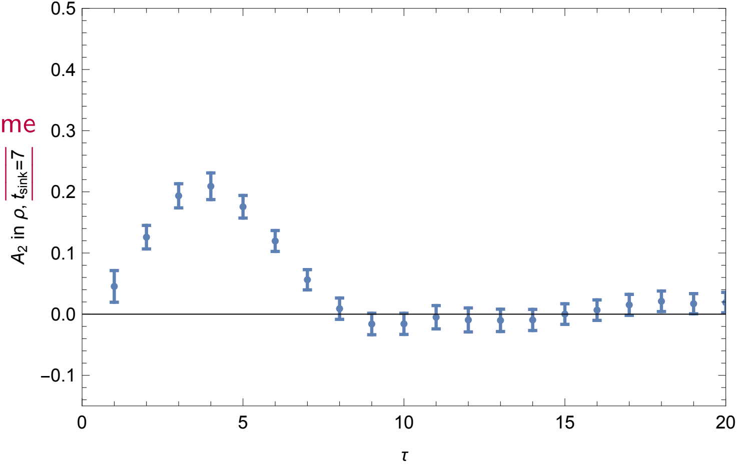
sink time



operator insertion time

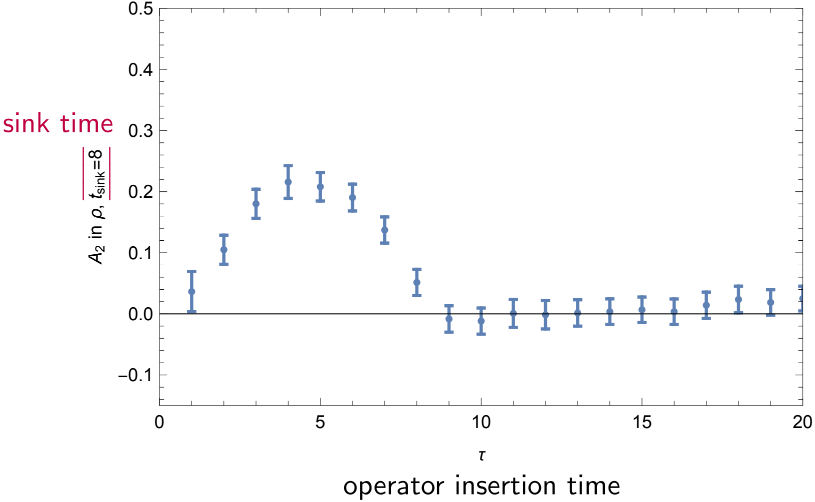
Extraction of A_2: 3pt/2pt ratio

sink time



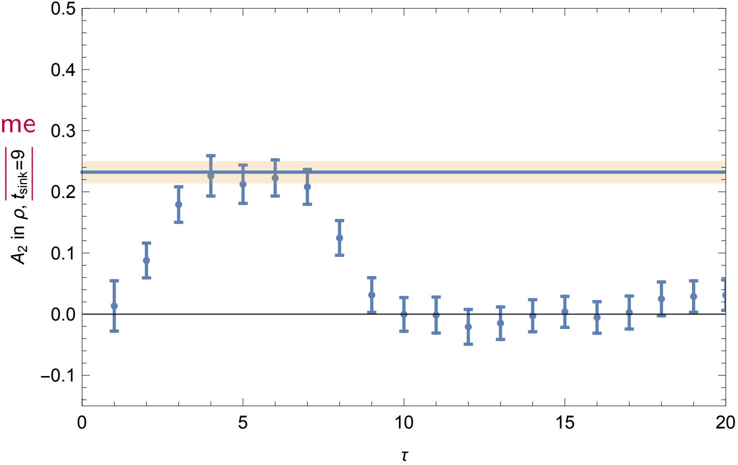
operator insertion time

Extraction of A_2: 3pt/2pt ratio



Extraction of A_2: 3pt/2pt ratio

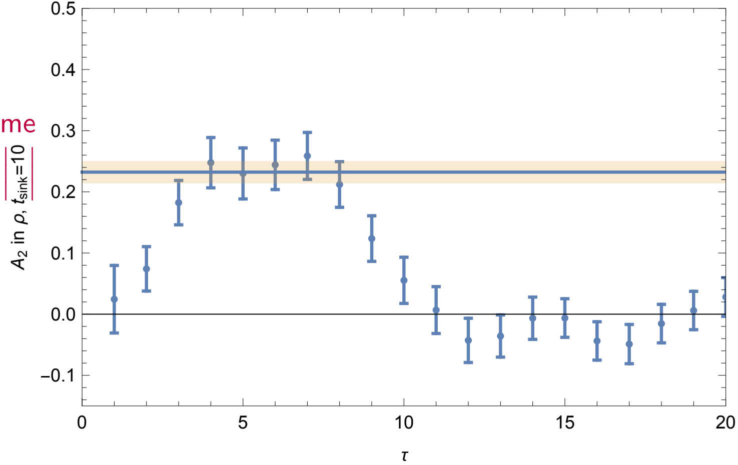
sink time



operator insertion time

Extraction of A_2: 3pt/2pt ratio

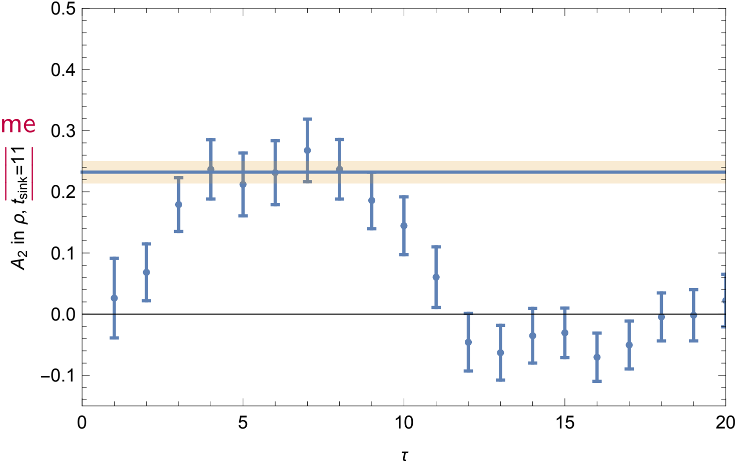
sink time



operator insertion time

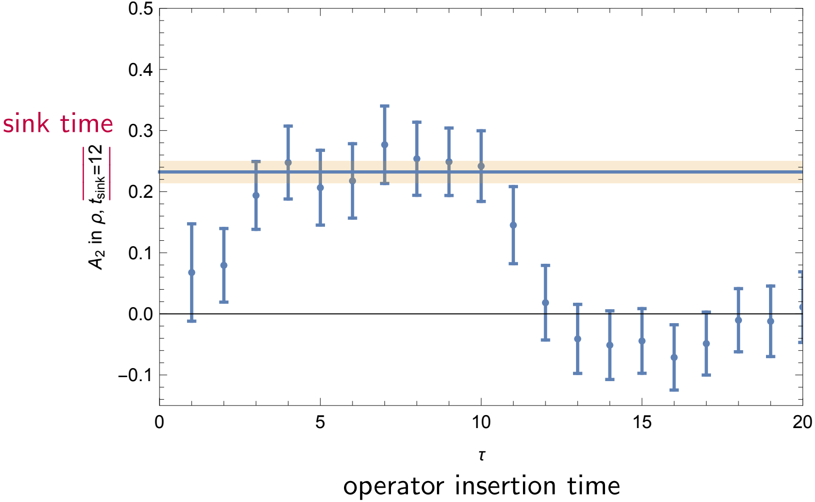
Extraction of A_2: 3pt/2pt ratio

sink time

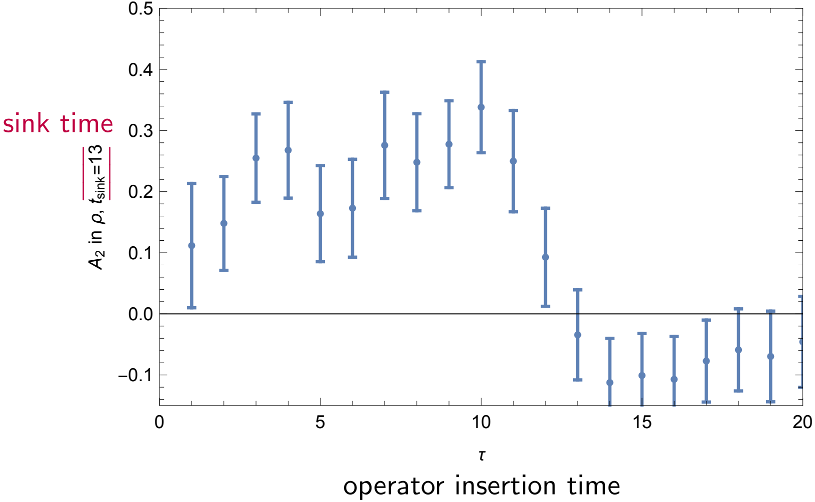


operator insertion time

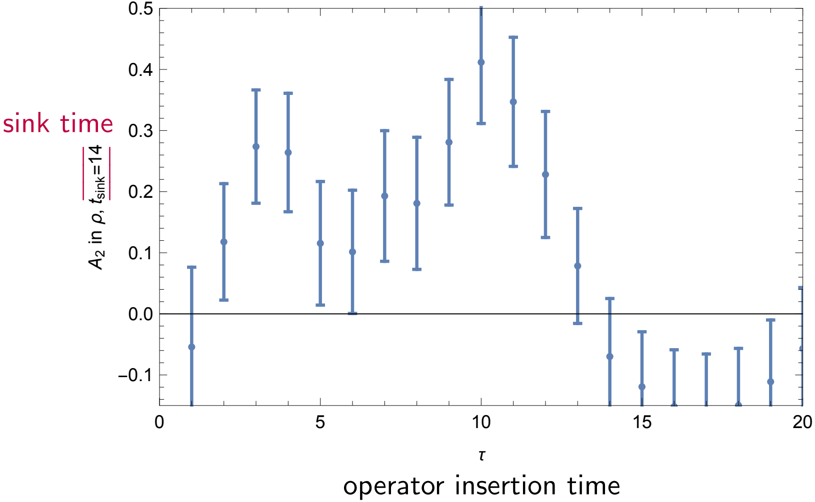
Extraction of A_2: 3pt/2pt ratio



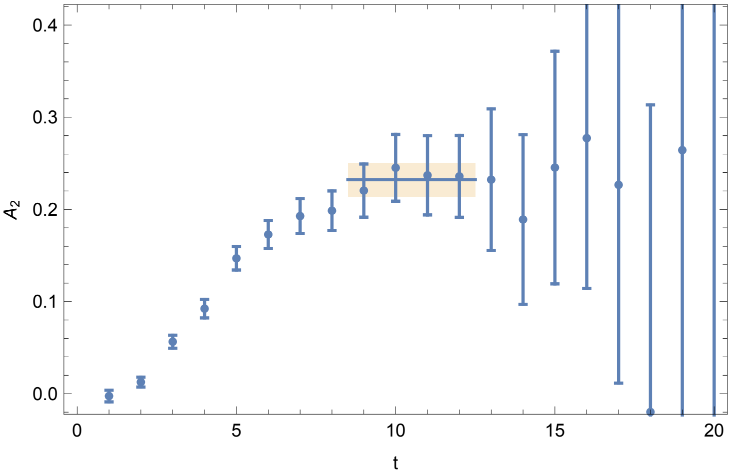
Extraction of A_2: 3pt/2pt ratio



Extraction of A_2: 3pt/2pt ratio



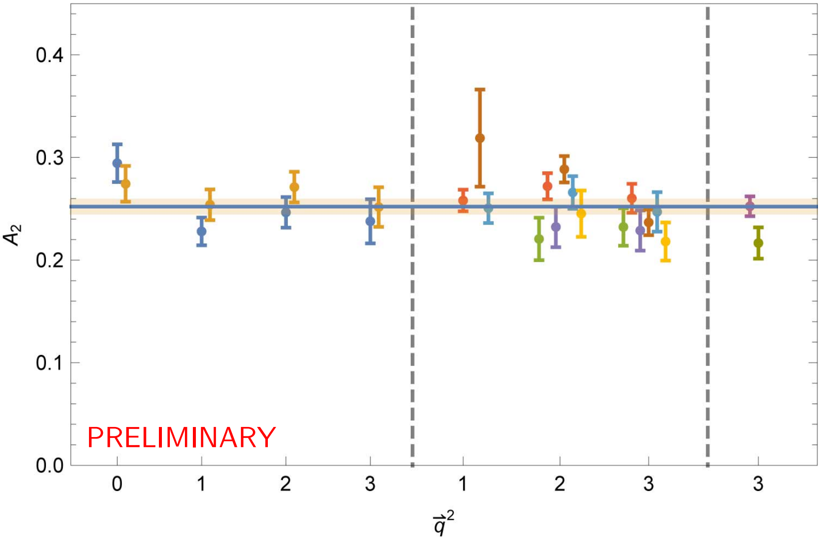
Extraction of A_2: 3pt/2pt ratio



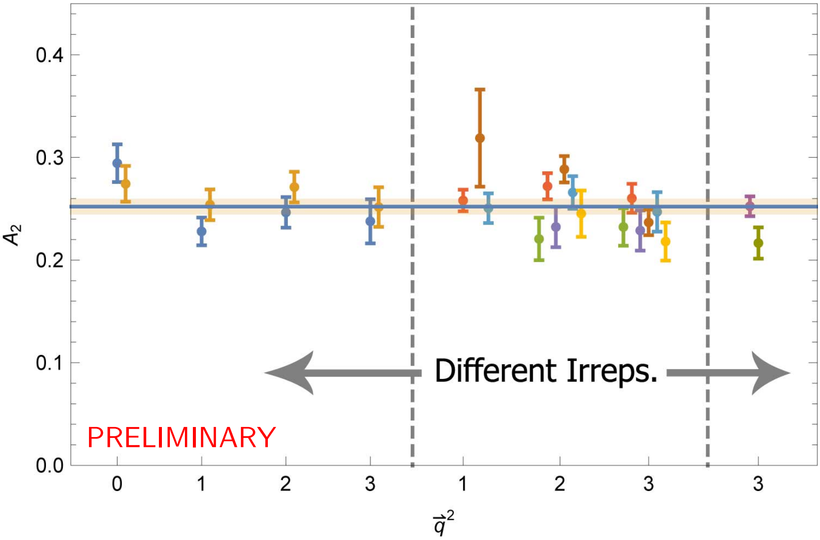
sink time

Outline

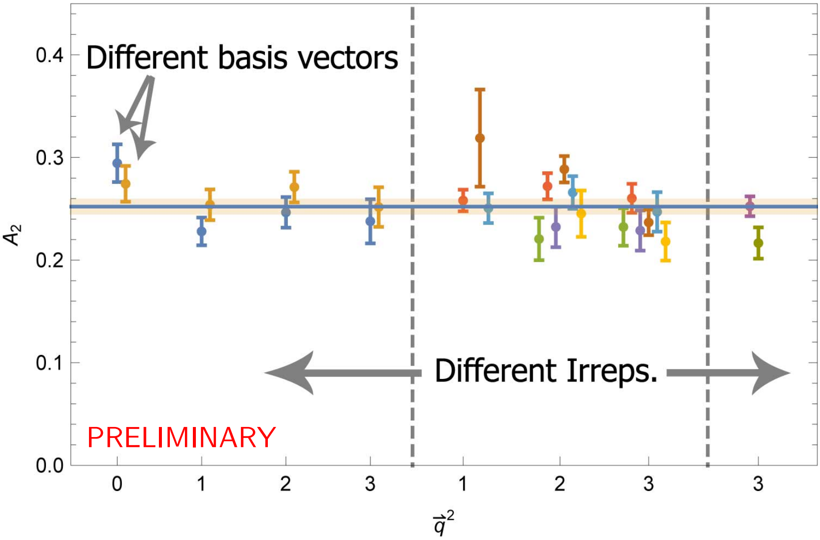
UNRENORMALISED reduced matrix element: ϕ meson



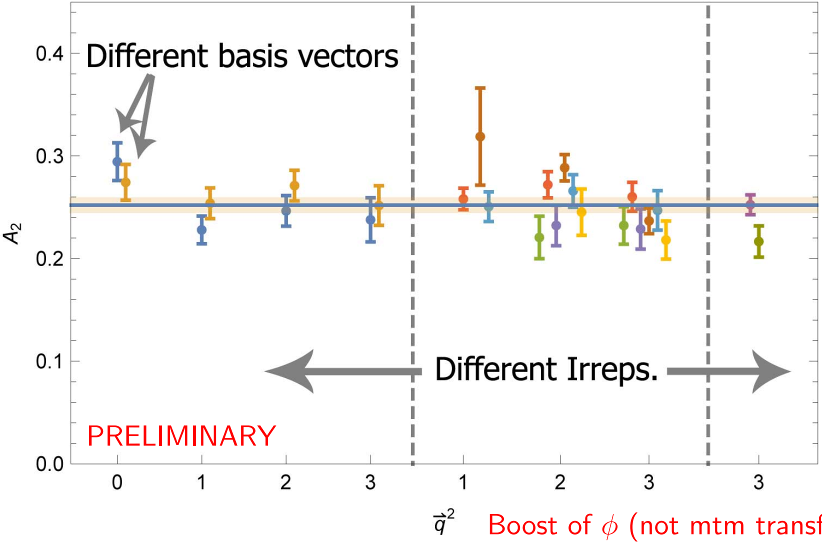
UNRENORMALISED reduced matrix element: ϕ meson



UNRENORMALISED reduced matrix element: ϕ meson



UNRENORMALISED reduced matrix element: ϕ meson



Boost of ϕ (not mtm transfer)

Preliminary Results: ϕ meson

CLEAR NON-ZERO SIGNAL

Proof of principle: similar signal in a nucleus $\Leftrightarrow$ exotic glue

Preliminary Results: ϕ meson

CLEAR NON-ZERO SIGNAL

Proof of principle: similar signal in a nucleus $\Leftrightarrow$ exotic glue

SYSTEMATICS IGNORED

- Quark mass effects
- Volume effects
- Discretization effects
- Renormalization

Renormalisation:

- Gradient flow instead of HYP smearing
- Gradient-flowed lattice perturbation theory
- Non-peturbative renormalisation
  (set gauge-fixed gluon matrix elt. = tree level value)

Soffer bound analogue

Explore gluon structure of ϕ meson more generally

Soffer bound analogue

Explore gluon structure of ϕ meson more generally

Soffer bound for transversity quark distributions:

$$|\delta q(x)| \leq \frac{1}{2}\left(q(x) + \Delta q(x)\right)$$

Soffer bound analogue

Explore gluon structure of ϕ meson more generally

Soffer bound for transversity quark distributions:

Transversity

$$|\delta q(x)| \leq \frac{1}{2}\left(q(x) + \Delta q(x)\right)$$

Soffer bound analogue

Explore gluon structure of ϕ meson more generally

Soffer bound for transversity quark distributions:

Transversity

Spin-independent

$$|\delta q(x)| \leq \frac{1}{2}\left(q(x) + \Delta q(x)\right)$$

Soffer bound analogue

Explore gluon structure of ϕ meson more generally

Soffer bound for transversity quark distributions:

$$|\delta q(x)| \leq \frac{1}{2}\left(q(x) + \Delta q(x)\right)$$

Transversity

Spin-independent

Spin-dependent

Soffer bound analogue

Explore gluon structure of ϕ meson more generally

Soffer bound for transversity quark distributions:

Transversity

Spin-independent

$$|\delta q(x)| \leq \frac{1}{2}\left(q(x) + \Delta q(x)\right)$$

Spin-dependent

Direct analogue for leading moments of gluon distributions:

$$|A_2| \leq \frac{1}{2} B_2$$

Soffer bound analogue

Explore gluon structure of ϕ meson more generally

Soffer bound for transversity quark distributions:

$$|\delta q(x)| \leq \frac{1}{2}\left(q(x) + \Delta q(x)\right)$$

Transversity

Spin-independent

Spin-dependent

Direct analogue for leading moments of gluon distributions:

$G_{\mu\mu_1}G_{\nu\mu_2}$

$$|A_2| \leq \frac{1}{2}B_2$$

Soffer bound analogue

Explore gluon structure of ϕ meson more generally

Soffer bound for transversity quark distributions:

Transversity

Spin-independent

$$|\delta q(x)| \leq \frac{1}{2}\left(q(x) + \Delta q(x)\right)$$

Spin-dependent

Direct analogue for leading moments of gluon distributions:

$G_{\mu\mu_1}G_{\nu\mu_2}$

$G_{\mu_1\alpha}G_{\mu_2}{}^{\alpha}$

$$|A_2| \leq \frac{1}{2}B_2$$

Soffer bound analogue

Explore gluon structure of ϕ meson more generally

Soffer bound for transversity quark distributions:

Transversity

Spin-independent

$$|\delta q(x)| \leq \frac{1}{2}\left(q(x) + \Delta q(x)\right)$$

Spin-dependent

Direct analogue for leading moments of gluon distributions:

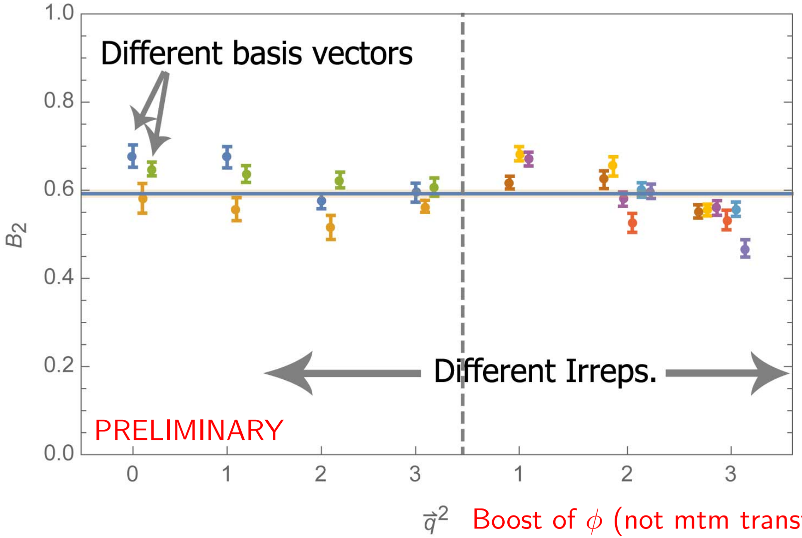
$G_{\mu\mu_1}G_{\nu\mu_2}$

$G_{\mu_1\alpha}G_{\mu_2}{}^{\alpha}$

$$|A_2| \leq \frac{1}{2} B_2$$

$\widetilde{G}_{\mu_1\alpha}G_{\mu_2}{}^{\alpha} \to 0$

UNRENORMALISED reduced matrix element: ϕ meson



Boost of ϕ (not mtm transfer)

Soffer bound analogue

If we imagine approx. the same renormalisation for A_2 and B_2:

$$\overset{G_{\mu\mu_1}G_{\nu\mu_2}}{|A_2|} \leq \frac{1}{2}\overset{G_{\mu_1\alpha}G_{\mu_2}{}^{\alpha}}{B_2}$$

$$0.25 \leq \frac{1}{2}\, 0.6$$

First two moments of quark distributions: Soffer bound saturated to 80% (lattice QCD, Diehl *et al.* 2005)

Outline

Question for Discussion

Given lattice results for light nuclei
$\Rightarrow$ extrapolate to heavier nuclei?

Recall:

- Leading twist, double helicity flip
- Zero in spin < 1 states (nucleons, pions)
  i.e., gluon contributions from non-nucleonic and pionic degrees of freedom

Experimental possibilities:

- JLab 2016 proposal: **Nitrogen**
- EIC: possibly deuteron and other lighter nuclei!

Question for Discussion

deuteron
^{3}He$_\Sigma$
lithium

Given lattice results for light nuclei
$\Rightarrow$ extrapolate to heavier nuclei?

Recall:

- Leading twist, double helicity flip
- Zero in spin < 1 states (nucleons, pions)
  i.e., gluon contributions from non-nucleonic and pionic degrees of freedom

Experimental possibilities:

- JLab 2016 proposal: **Nitrogen**
- EIC: possibly deuteron and other lighter nuclei!

Outline

Summary

NON-ZERO signal for 'exotic glue' operator
in the ϕ (or ρ) meson

Proof of principle: similar signal in a nucleus $\Leftrightarrow$ exotic glue

Explore gluon structure of hadrons more generally
e.g., Soffer bound analogue

Summary

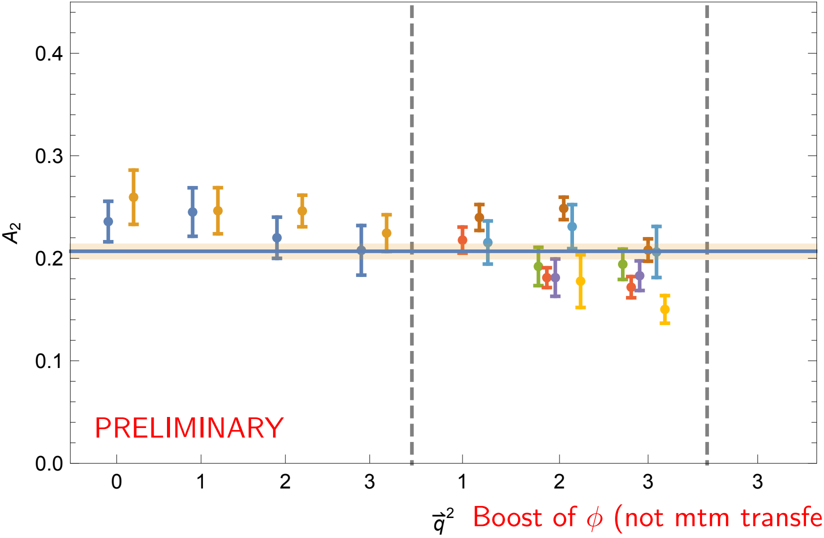
NON-ZERO signal for 'exotic glue' operator
in the ϕ (or ρ) meson

Proof of principle: similar signal in a nucleus $\Leftrightarrow$ exotic glue

Explore gluon structure of hadrons more generally
e.g., Soffer bound analogue

BUT: SYSTEMATICS IGNORED
$\Rightarrow$ no physically meaningful number (yet)

UNRENORMALISED reduced matrix element: ϕ meson, nHYP=2



Boost of ϕ (not mtm transfer)