Deconfinement Transition at High Isospin Chemical Potential and Low Temperature

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Outline

- QCD phase diagram.
- Why we should be interested in QCD with finite isospin chemical potential μ_I
- What we already know about this regime
- Low energy effective theory
- First order deconfinement phase transition
- Equation of state and phase diagram

QCD phase diagram: the success so far

- Low μ_B and low temperature → Hadronic phase → handled using effective theory.
- High μ_B and low temperature → Color superconductors → perturbative calculations using QCD.
- Low μ_B and high temperature → QGP → lattice calculations.

QCD phase diagram: the not so successful part.

- Moderate μ_B and low T regime is relevant for neutron star physics.
- In this regime no handle except model calculations.
- Lattice QCD fails due to sign problem.

Finite Isospin (μ_I) regime: benefits

- Learn all we can about QCD with chemical potential when finite μ_B regime is not accessible.
- Tractable using lattice → no sign problem.
- Nonzero μ_I present in neutron stars due to the suppression of proton fraction.

The limit considered in this talk

- QCD with two degenerate flavors of light quarks.
- Asymptotically high μ_I .
- Set μ_B to zero.

Exciting features of this limit

- At low *T*, this limit of QCD is equivalent to *SU(3)* Yang-Mills.
- Expected to undergo a first order deconfinement transition just like *SU(3)* Yang-Mills with changing *T*.
- Scale of this deconfinement transition can be calculated using effective theory.

Exciting features of this limit (contd.)

- Observables in the strongly coupled regime of this limit can be related to the observables of pure YM which is amenable to straightforward lattice calculation.

- For example, the EOS in this limit can be calculated using the EOS calculated in pure YM on lattice.

Exciting feature of this limit (contd.)

- The phase diagram at moderate isospin chemical potential is likely to have either a critical point or a triple point or a phase transition somewhere on the $T=0$ axis.

- Only a lattice calculation can settle this as this regime is beyond the reach of perturbative calculations.

Low Isospin

- Lagrangian at low isospin with matrix pion fields :

$$\mathcal{L}_{\text{eff}} = \frac{f_\pi^2}{4}\mathrm{Tr}\nabla_\nu\Sigma\nabla_\nu\Sigma^\dagger - \frac{m_\pi^2 f_\pi^2}{2}\mathrm{Re}\mathrm{Tr}\Sigma.$$

where,

$$\nabla_i\Sigma = \partial_i\Sigma$$

$$\nabla_0\Sigma = \partial_0\Sigma - \frac{\mu_I}{2}(\tau_3\Sigma - \Sigma\tau_3)$$

- For $\mu_I > m_\pi$, there is a charged pion condensate.

High isospin

- Fermi liquid of anti up quarks and down quarks.
- Attractive interaction at the Fermi surface leading to Cooper pair formation in the $\langle \bar{u}\gamma^5 d \rangle$ channel.
- The condensate has same quantum numbers as the pion condensate.

High isospin

- The condensate is color neutral → no Meissner screening for gluons.
- At temperatures below the gap no Debye screening either for the gluons.
- At low *T* the quarks are gapped → only pure gluodynamics *(SU(3))*.

High Isospin

- However $\mathscr{L} = \frac{-F^2}{4g^2}$ is not the complete picture.

- Despite being bound in color singlet Cooper pairs, the quarks can partially screen the gluons altering the chromo dieletric constant ϵ of the system.

High Isospin

- ϵ and λ (chrormo magnetic permeability) can be calculated by integrating out the quarks around the Fermi surface.
- The deconfinement scale is related to ϵ
- Our results for the deconfinement scale as a function of μ_I should be compared with future lattice calculations.

Similarity with two flavor color superconductor (2SC phase)

- Moderate μ_B : two quarks forming a condensate that breaks color SU(3) down to SU(2).
- Five of the eight gluons acquire Meissner mass. Three remain massless.
- Pure SU(2) gluodynamics at temperature below Debye mass. The quarks alter the chromo dielectric constant of the system.

Derivation of the effective Lagrangian from the microscopic theory

- Microscopic Lagrangian:

$$\mathscr{L} = \overline{\psi}(i\gamma^\mu D_\mu + \mu_I \gamma^0 \tau_3)\psi - \frac{1}{4}(F^a_{\mu\nu})^2$$

where $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$ $D_\mu = \partial_\mu - igA^a_\mu t^a$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu$$

- Rewrite using $\tilde{\psi} \equiv \begin{pmatrix} u \\ \tilde{d} \end{pmatrix}$ $\tilde{d} = \gamma_5 d$

- Find the full Fermion propagator using the gap equation.

$$G = \begin{pmatrix} G^+ & \Sigma^- \\ \Sigma^+ & G^- \end{pmatrix}$$

where

$$G^-(k) = \sum_{e=\pm} \frac{-ik_0 + (\mu - ek)}{-k_0^2 - (\epsilon_k^e)^2} \lambda_{\mathbf{k}}^{-e} \gamma^0$$

$$G^+(k) = \sum_{e=\pm} \frac{-ik_0 - (\mu - ek)}{-k_0^2 - (\epsilon_k^e)^2} \lambda_{\mathbf{k}}^{+e} \gamma^0$$

$$\Sigma^-(k) = i \sum_{e=\pm} \frac{\Delta^e \lambda_k^e}{-k_0^2 - (\epsilon_k^e)^2}$$

$$\Sigma^+(k) = -i \sum_{e=\pm} \frac{\Delta^e \lambda_k^{-e}}{-k_0^2 - (\epsilon_k^e)^2}$$

$$(\epsilon_k^e)^2 = (|\mathbf{k}| - e\mu)^2 + (\Delta^e)^2 \quad e = \pm$$

$$\lambda_{\mathbf{p}}^{\pm} \equiv \frac{1}{2}(1 \pm \gamma_0 \boldsymbol{\gamma} \cdot \hat{\boldsymbol{p}})$$

Low energy effective action

- The effective action:

$$S_{glue} = \sum_a \frac{1}{g^2} \int d^4q \left(\frac{\epsilon}{2} E_a^2 - \frac{1}{2\lambda} B_a^2 \right)$$

where $E_i^a = F_{0i}^a$ $B_k^a = \epsilon_{ijk} F_{ij}$

- The polarization tensor can be written as

$$\Pi_{ab}^{00}(q_0, \mathbf{q}) = -(\epsilon - 1)|\mathbf{q}|^2 \delta_{ab}$$

$$\Pi_{ab}^{ij} \frac{\delta^{ij}}{2} \frac{\delta_{ab}}{8} = -(\epsilon - 1) q_0^2$$

$$\Pi_{ab}^{ij}(q_0, \mathbf{q}) \left(-\delta^{ik} - \frac{q^i q^k}{|\mathbf{q}|^2} \right) = \left(\frac{1}{\lambda} - 1 \right) |\mathbf{q}|^2 \delta_{ab} \delta^{jk}$$

One loop polarization can be calculated as

$$\Pi^{ij}_{ab}(q) = g^2 T \sum_{k_0} \int \frac{d^3k}{(2\pi)^3} \mathrm{Trace}\left[\gamma^i t_a G^+(k) \gamma^j t_b G^+(k-q)\right.$$
$$+\gamma^i t_a G^-(k) \gamma^j t_b G^-(k-q)$$
$$+\gamma^i t_a \Sigma^-(k) \gamma^j t_b \Sigma^+(k-q)$$
$$\left.+\gamma^i t_a \Sigma^+(k) \gamma^j t_b \Sigma^-(k-q)\right]$$

In the limit $T \to 0$ and around q_0 and $|\mathbf{q}| \to 0$

$$\Pi^{00}_{aa} = -\frac{g^2 \mu_I^2 |\mathbf{q}|^2}{18\pi^2 (\Delta)^2} \qquad \Pi^{ij}_{ab} = \frac{g^2 \mu_I^2 q_0^2}{18\pi^2 (\Delta)^2} \delta^{ij} \delta_{ab}$$

leads to $\lambda = 1 \qquad \epsilon = 1 + \frac{g^2 \mu_I^2}{18\pi^2 (\Delta)^2}$

- The effective action can be recast as

$$S = -\frac{1}{4(g'')^2}\int d^4x''(F'')^2$$

using $t_0'' = \frac{t_0}{\sqrt{\epsilon}}$ $A_0^{a''} = \sqrt{\epsilon}A_0^a$ $g'' = \frac{g}{\epsilon^{1/4}}$

- The coupling *g''* runs like that of pure YM as the energy scale reached Δ from below at which point the coupling needs to be matched as follows

$$\alpha_s''(\Delta) = \frac{\alpha_s(\mu_I)}{\sqrt{\epsilon}}$$

Confinement

- The new confinement scale can be found as

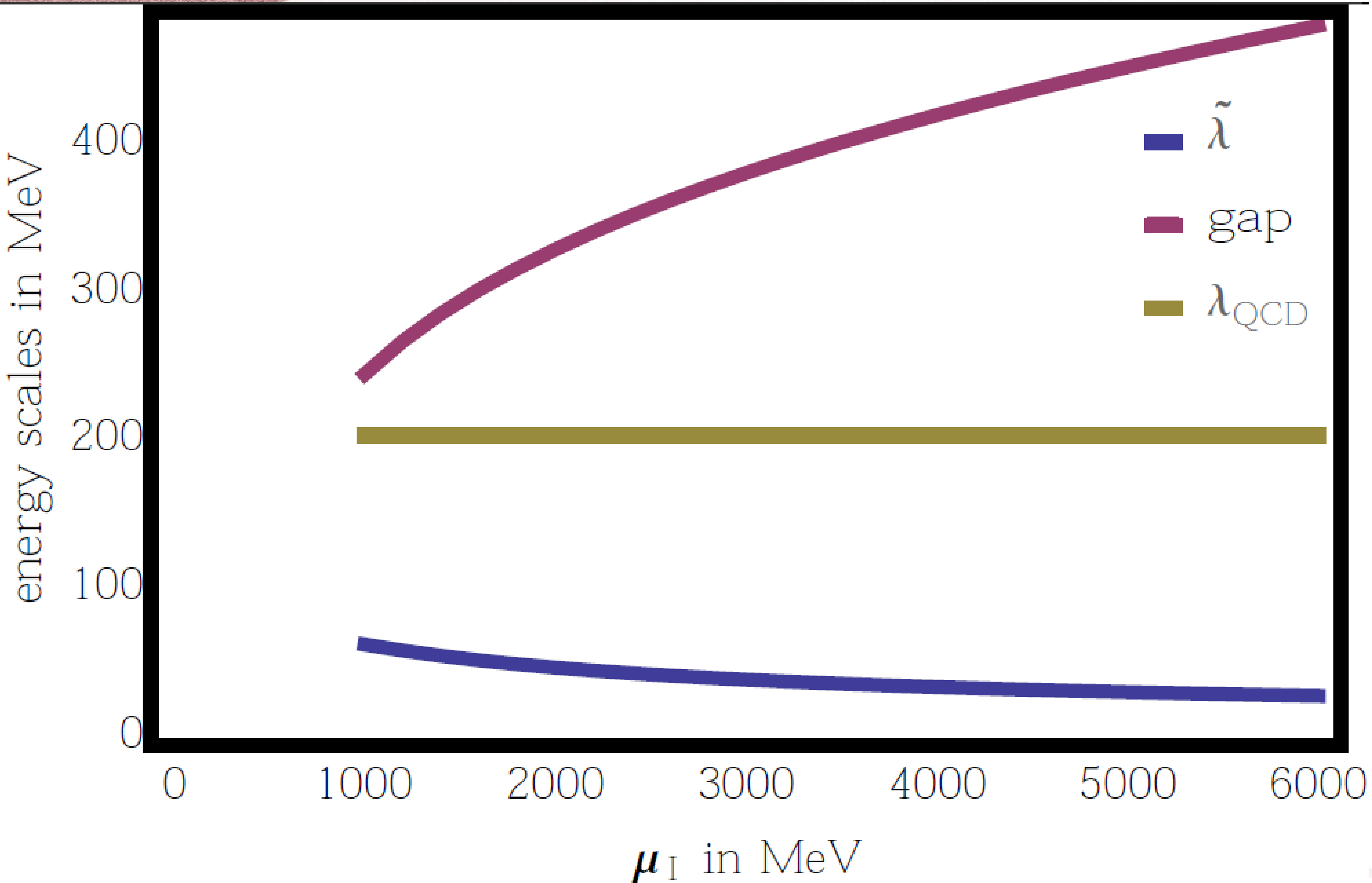
$$\tilde{\lambda} = \Delta \exp\left(-\frac{2\pi}{b_0 \alpha_s''(\Delta)}\right)$$
$$= \Delta \mathrm{Exp}\left[-2\sqrt{2\pi}\frac{\mu_I}{33\Delta\sqrt{\alpha_s(\mu_I)}}\right]$$

where $b_0 = 11$ for pure *SU(3)* YM.

- The gap is given by $\Delta = b|\mu_I|g(\mu_I)^{-5}\exp(-\frac{3\pi^2}{2g(\mu_I)})$

where $b = 10^4$

Different scales in the problem

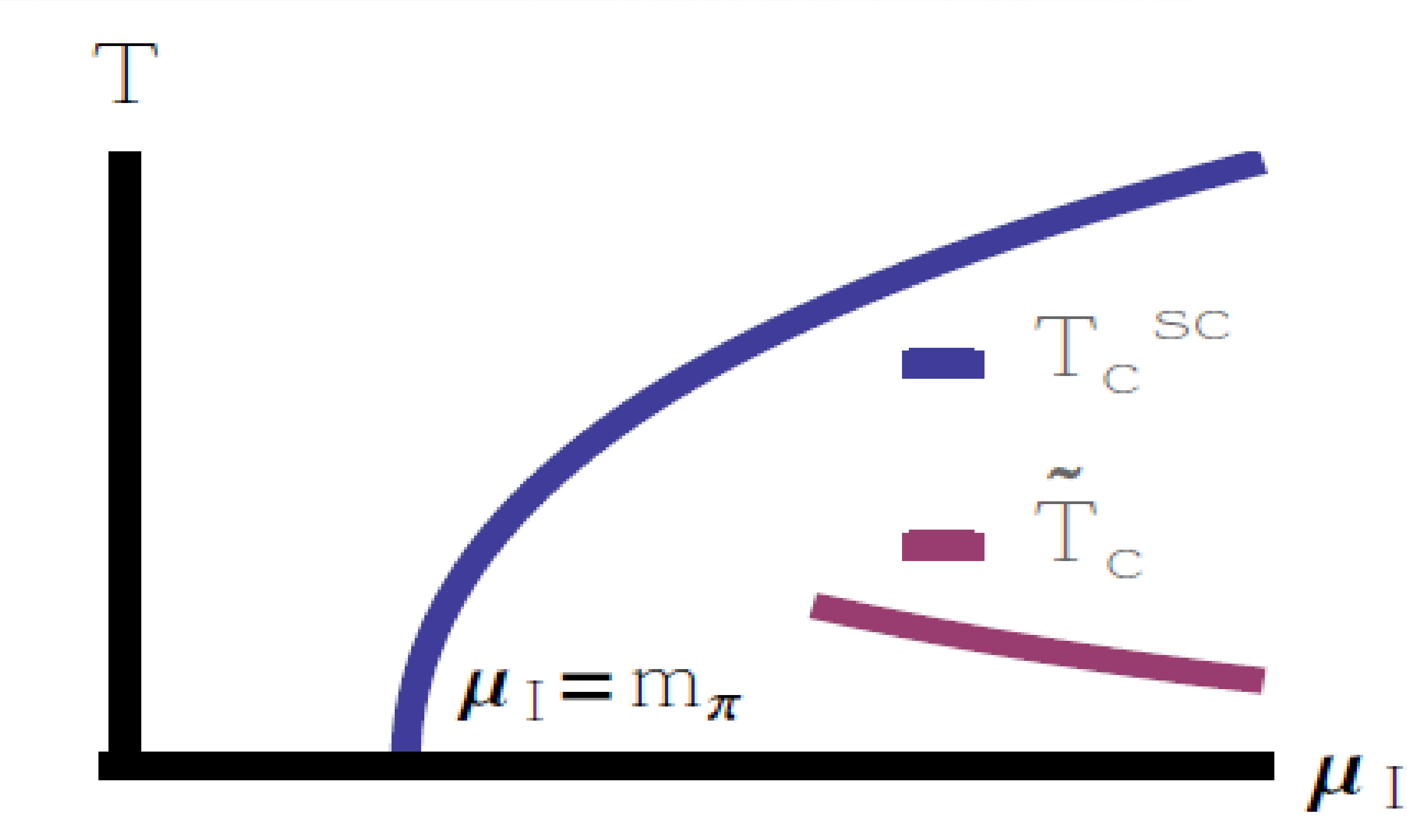


Equation of state

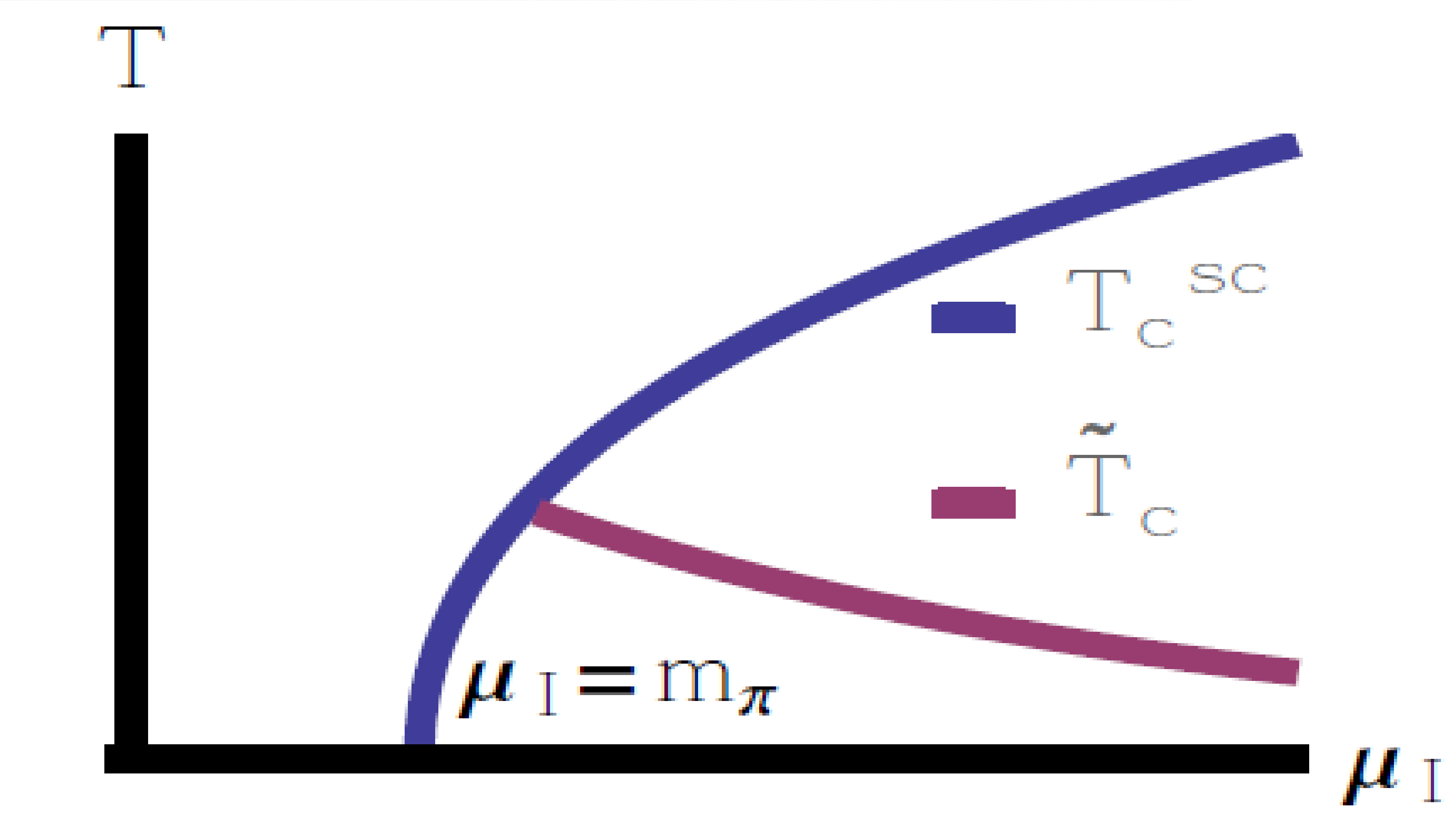
- In our problem, other than the pure YM at low energy we also have a Goldstone mode corresponding to the spontaneous breaking of $$\begin{pmatrix} u \\ d \end{pmatrix} \to e^{i\alpha\tau_3} \begin{pmatrix} u \\ d \end{pmatrix}$$
- The gluons and the Goldstone mode don't interact.
- Hence the pressure at low energy

$$P(T) = T^4 f\left(\frac{T}{\tilde{T}_c}\right) + \frac{\pi^2}{90}\frac{T^4}{v^4}$$

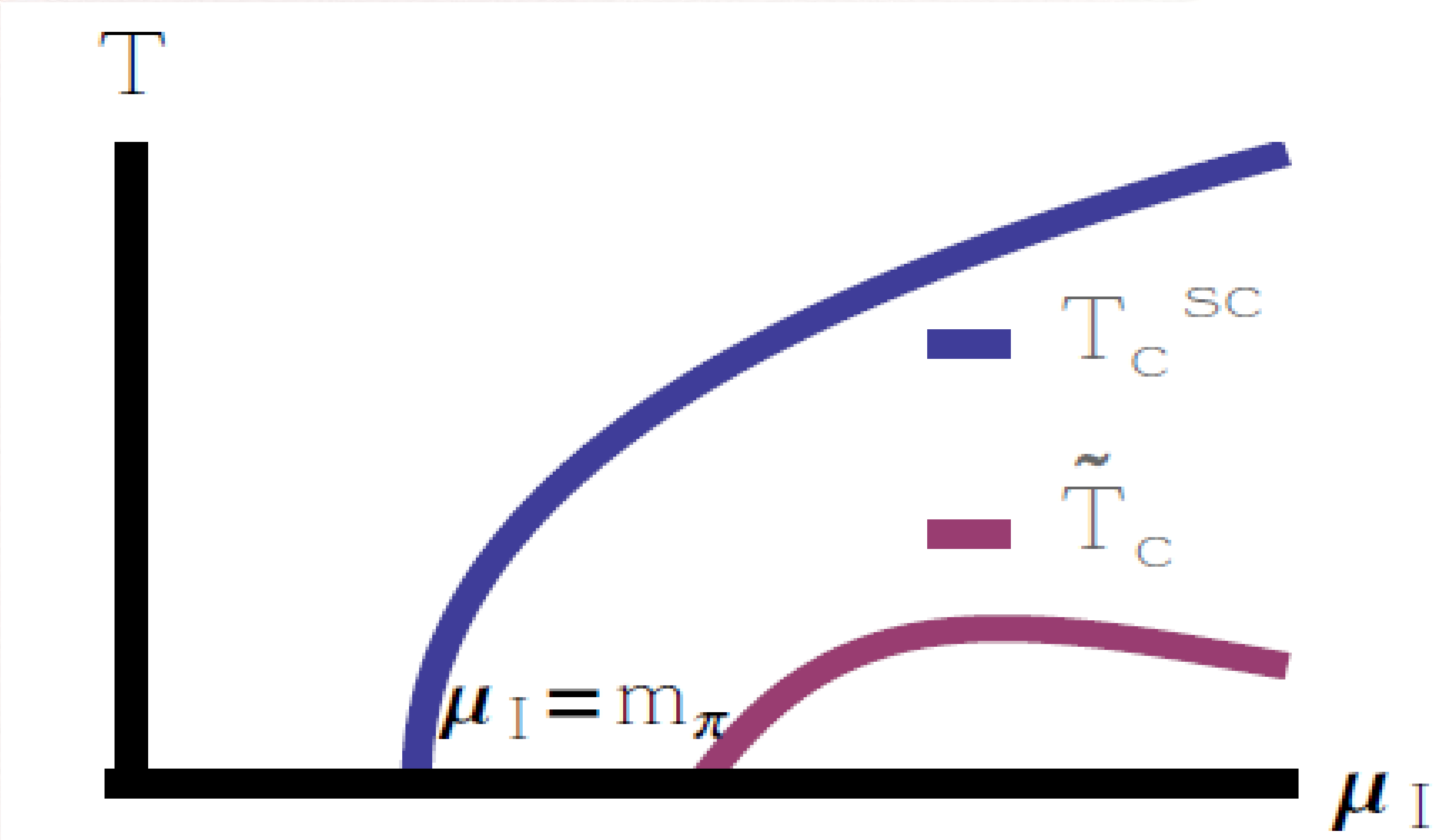
Phase Diagram: Scenario 1



Phase diagram: Scenario 2



Phase diagram: Scenario 3



Summary

- QCD at very high isospin chemical potential undegoes a first order deconfinement transition with increasing temperature.
- We calculate the scale of this deconfinement transition.
- Our prediction for this deconfinement scale as a function of μ_I should be tested using lattice.
- The EOS in the strongly coupled regime of this limit of QCD can be obtained using the EOS of pure YM found using lattice.