Nuclear Reactions in Lattice EFT

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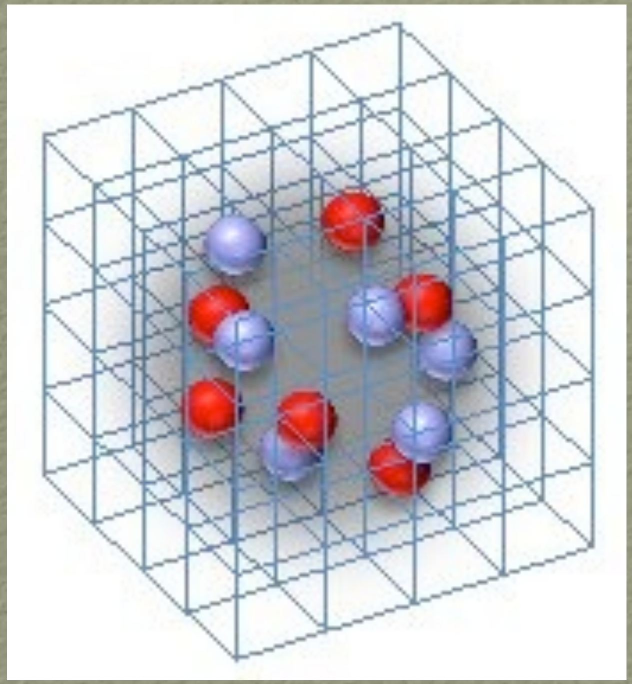
Mississippi State University



Nuclear Lattice EFT Collaboration

Evgeny Epelbaum (Bochum)
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Thomas Luu (Jülich)
Ulf-G. Meißner (Bonn)

Nuclear Physics from Lattice QCD, Institute for Nuclear Theory
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Outline

- Background and motivation
- Weakly bound systems at low energy
- Adiabatic projection method - proof of concept
- neutron capture
- proton-proton fusion
- n-d doublet channel, connection to lattice QCD

Motivations

Astrophysics

- Low energy reactions dominate
- Need accurate cross sections but hard to measure experimentally
- Model-independent theoretical calculations important

Theoretical

- Weakly bound systems are fun
- First principle calculation

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Use Effective Field Theory

EFT: THE LONG AND SHORT OF IT

- Identify degrees of freedom

$$\mathcal{L} = c_0\, \mathrm{O}^{(0)} + c_1\, \mathrm{O}^{(1)} + c_2\, \mathrm{O}^{(2)} + \ldots \quad \text{expansion in } \frac{Q}{\Lambda}$$

Hide UV ignorance
- short distance

IR explicit
- long distance

- Determine c_n from data (elastic, inelastic)

- EFT : ERE + currents + relativity

Not just Ward-Takahashi identity

M. Savage's talk on two-body current

A low energy example

$^7\mathrm{Li}(n,\gamma)^8\mathrm{Li}$

- Isospin mirror systems $^7\mathrm{Li}(n,\gamma)^8\mathrm{Li} \leftrightarrow {}^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$
- Inhomogeneous BBN

Whats the theoretical error?

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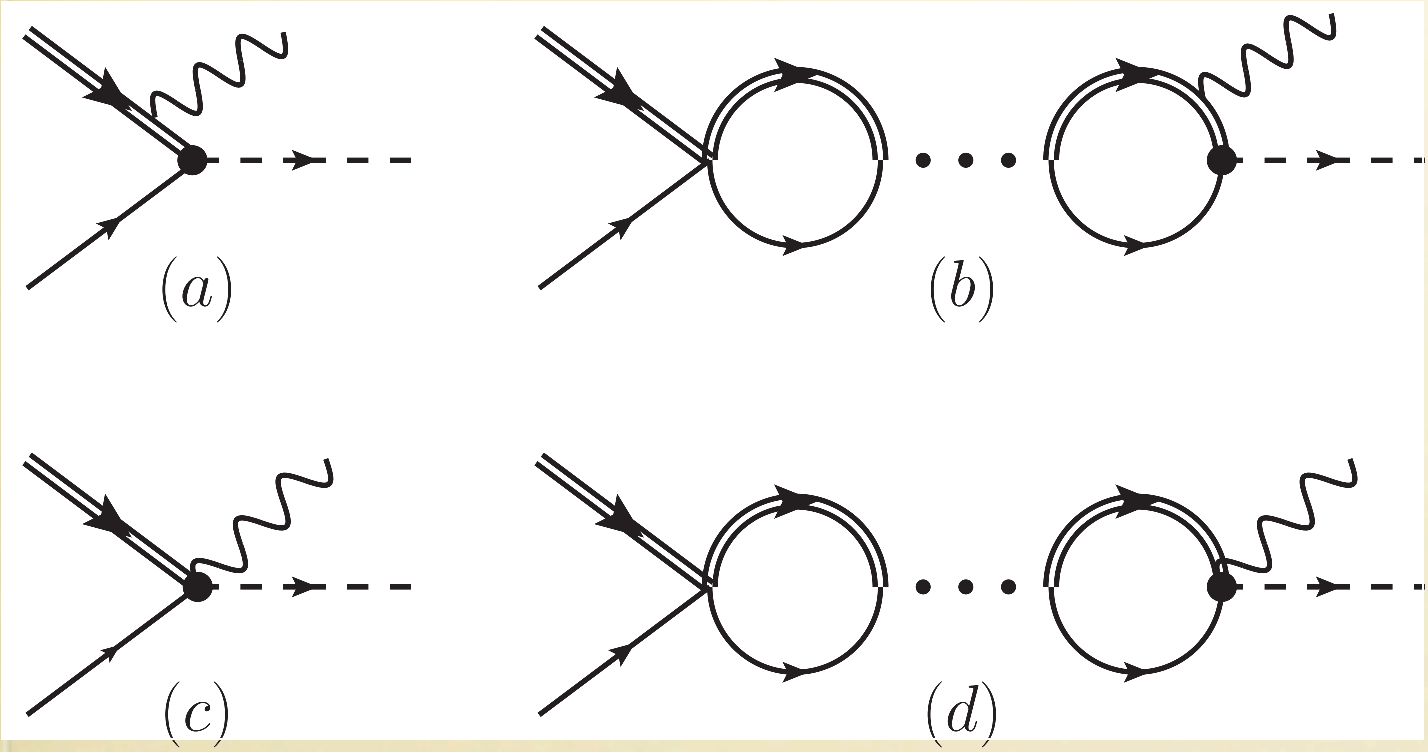

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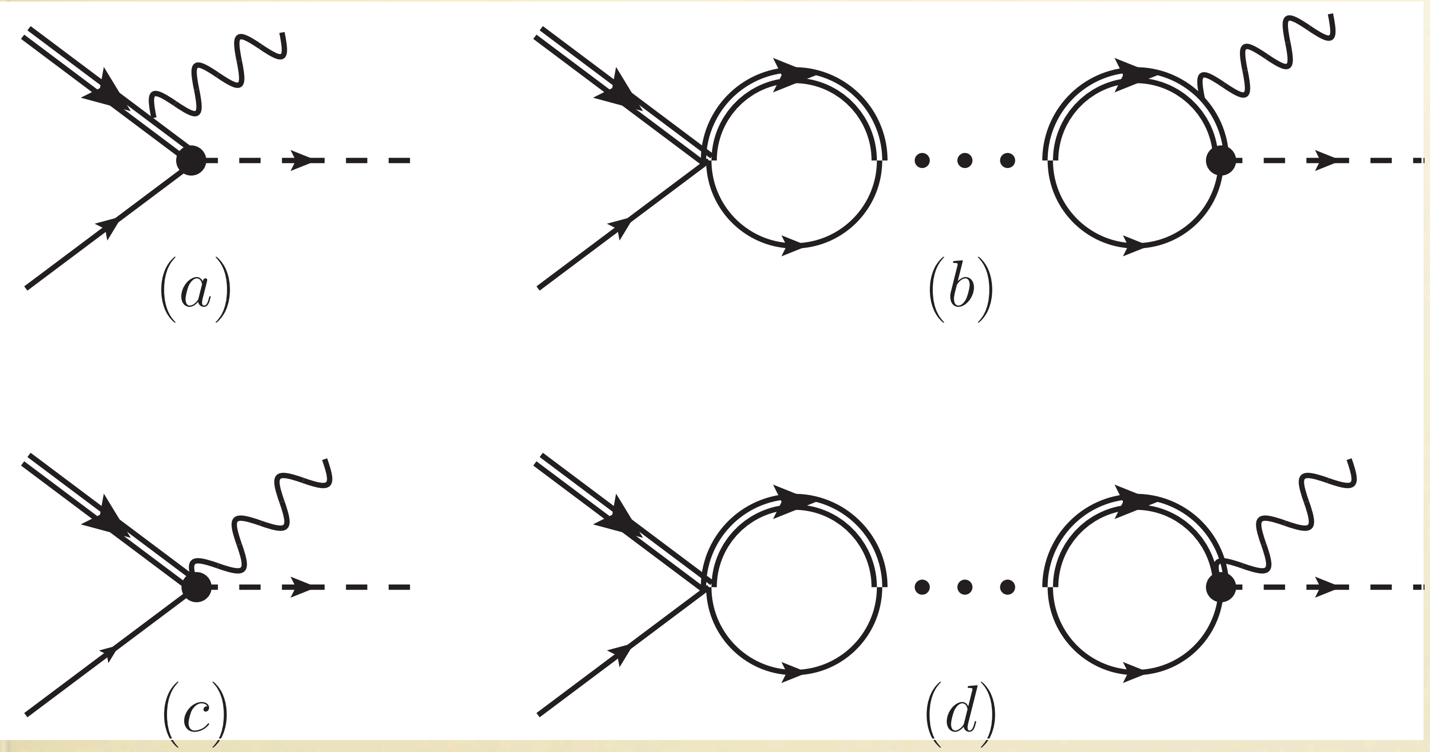

Asymptotic normalization

$$\sqrt{Z} = \frac{1}{\sqrt{2}}\sqrt{\frac{1}{1+3\gamma/r_1}}$$

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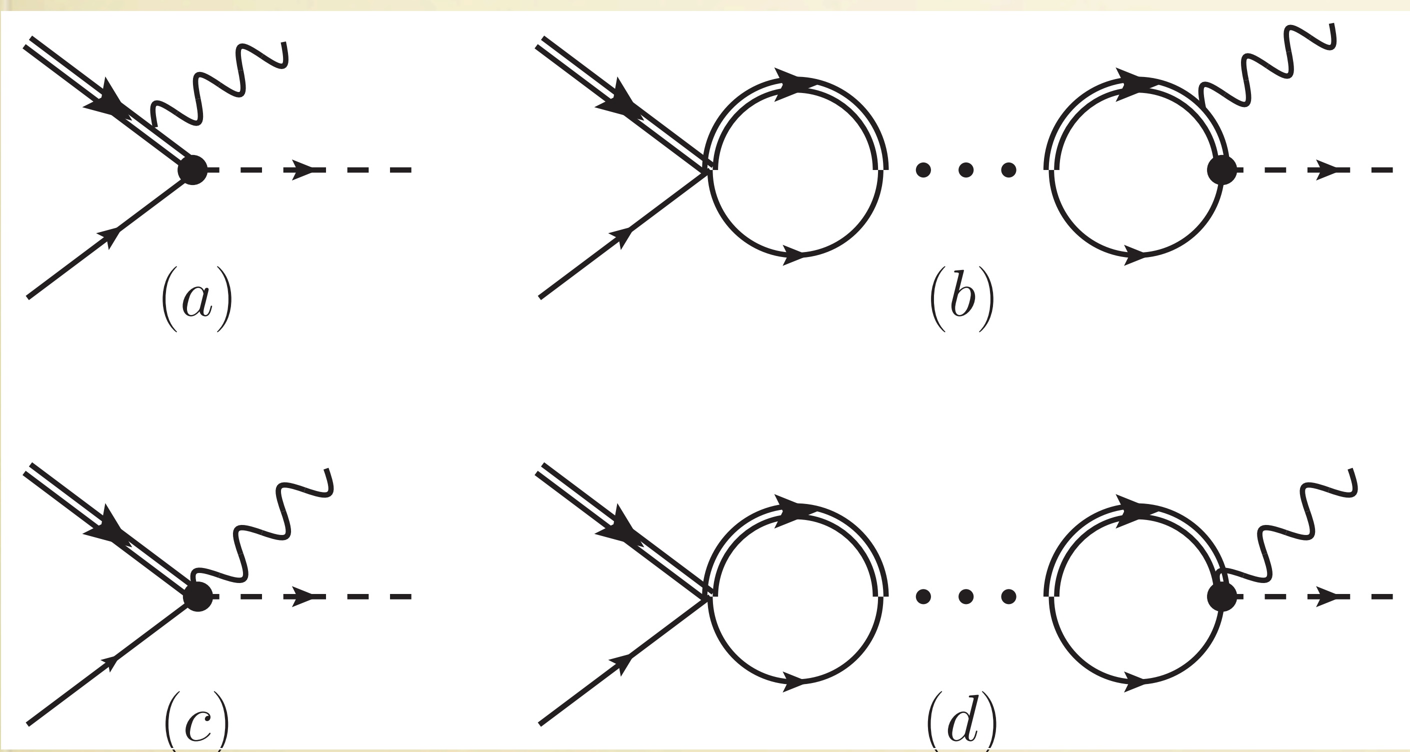

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Need effective range r1 and binding energy at leading order

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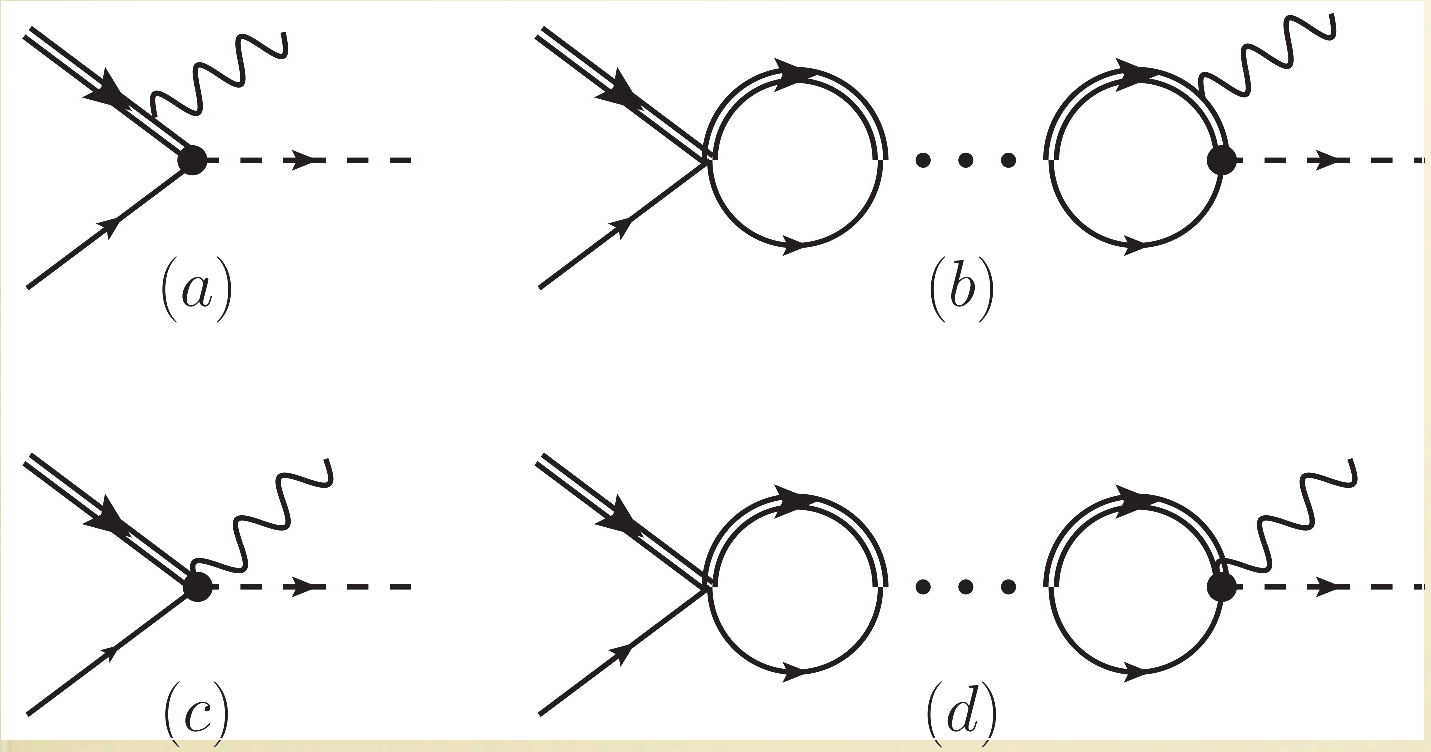

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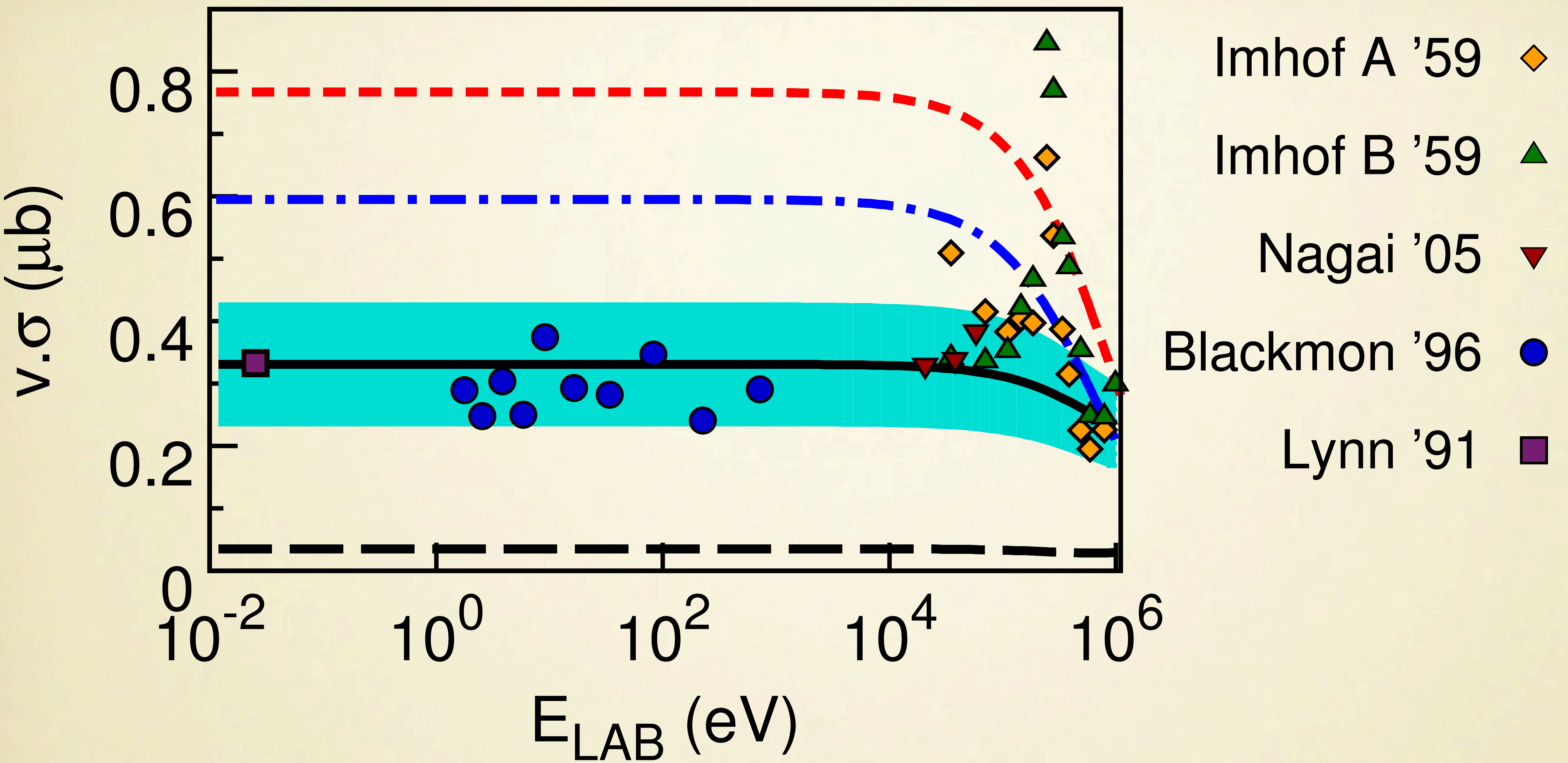
Two EFT operators



Red: Tombrello

Blue: Davids-Typel

Black: EFT

Rupak, Higa; PRL 106, 222501 (2011)

Fernando,Higa, Rupak; EPJA 48, 24 (2012)

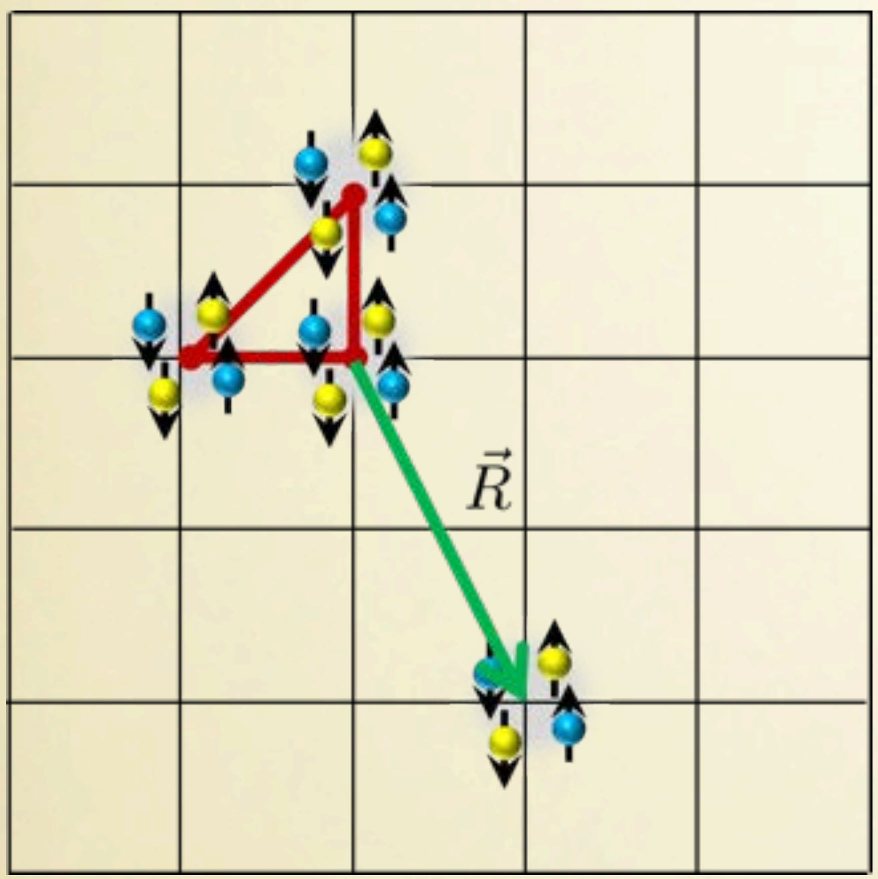
RADCAP: Bertulani

CDXS+: Typel

Reactions in lattice EFT

- Consider: $a(b, \gamma)c$; $a(b, c)d$
- Need effective "cluster" Hamiltonian -- acts in cluster coordinates, spins,etc.
- Calculate reaction with cluster Hamiltonian. Many possibilities --- traditional methods, continuum EFT, lattice method

Adiabatic Projection Method



Initial state $|\vec{R}\rangle$

Evolved state $|\vec{R}\rangle_\tau = e^{-\tau H}|\vec{R}\rangle$

D. Lee's talk
U.-G. Meißner's talk

$${}_\tau\langle\vec{R}'|H|\vec{R}\rangle_\tau$$

Energy measurements in cluster basis.
Divide by the norm matrix as these are not orthogonal basis $[N_\tau]_{\vec{R},\vec{R}'} = {}_\tau\langle\vec{R}|\vec{R}'\rangle_\tau$

Microscopic Hamiltonian $L^{3(A-1)}$
Cluster Hamiltonian L^3 ← smaller matrices in practice!!
-- acts on the cluster CM and spins

Proof of Concept

- n-d scattering in the quartet channel
  - Low energy EFT is known, only two body
  - Shallow deuteron (large coupling)

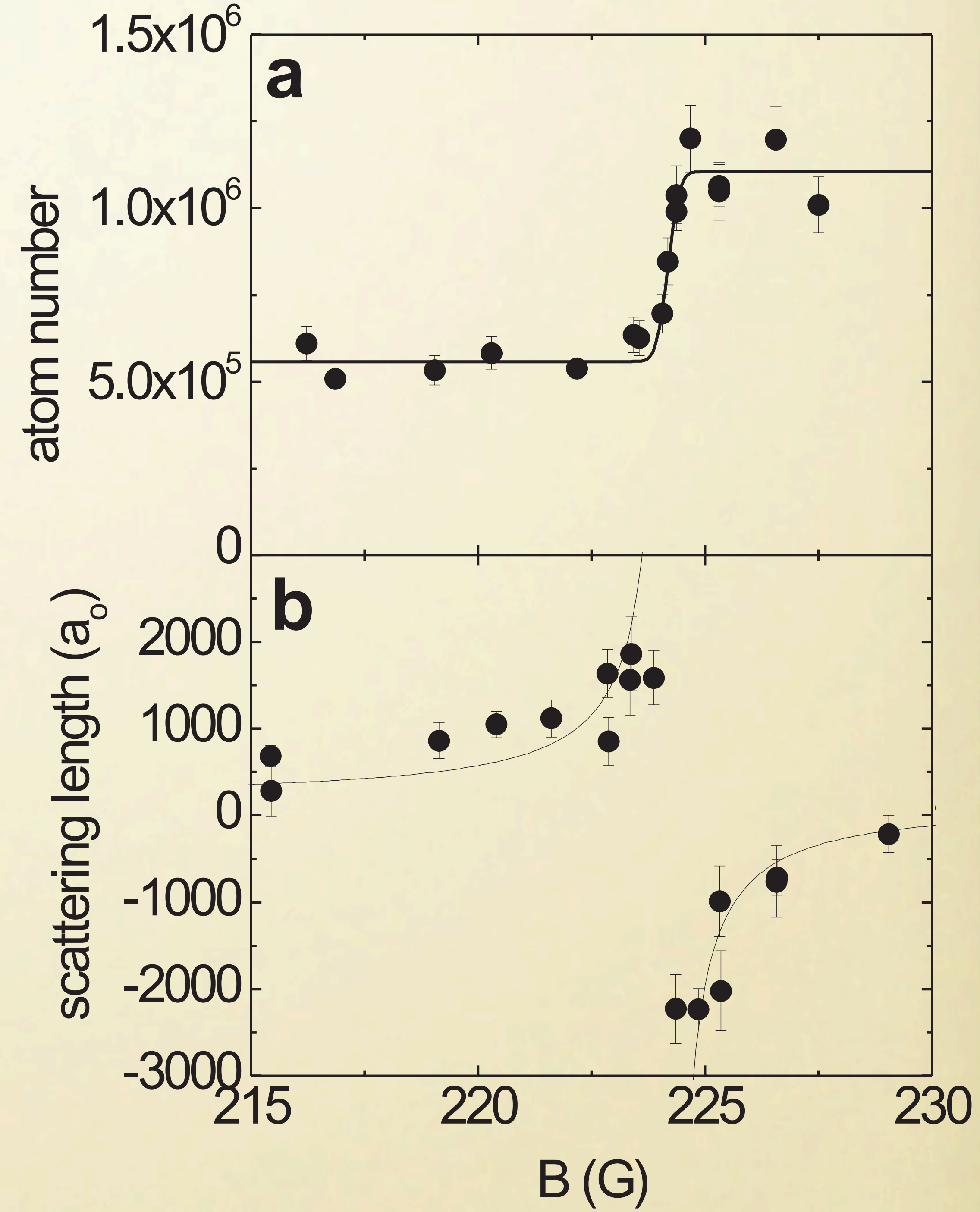
Spin-1/2 Fermions

$$a_{nn} \sim -19 \text{ fm}, \; R \sim 1.4 \text{ fm}$$

$$a_{np} \sim -24 \text{ fm}, \; R \sim 1.4 \text{ fm}$$

Potassium-40

Regal et. al, Nature 2003



Tuning scattering lengths in lattice QCD with magnetic field

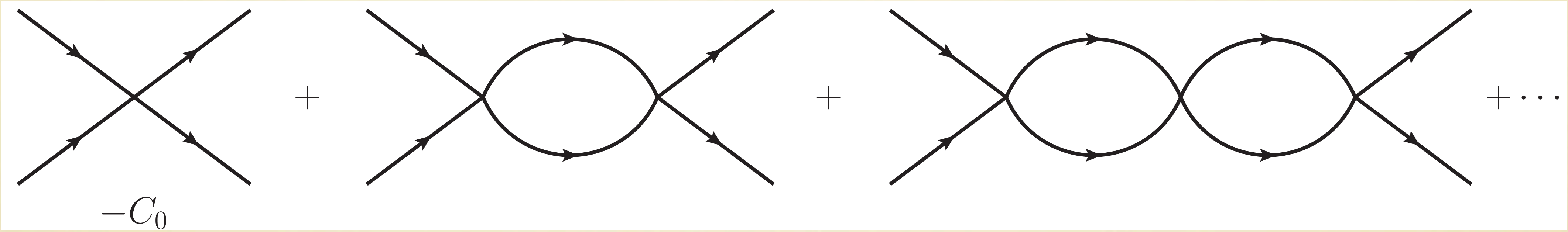
Weakly Bound Systems

$$i\mathcal{A}(p) = \frac{2\pi}{\mu}\frac{i}{p\cot\delta_0 - ip} = \frac{2\pi}{\mu}\frac{i}{-1/a + \frac{r}{2}p^2 + \cdots - ip}$$

--- Large scattering length $a >> 1/\Lambda$

$$i\mathcal{A}(p) \approx -\frac{2\pi}{\mu}\frac{i}{1/a + ip}\left[1 + \frac{1}{2}\frac{rp^2}{1/a+ip} + \cdots\right]$$

EFT non-perturbative

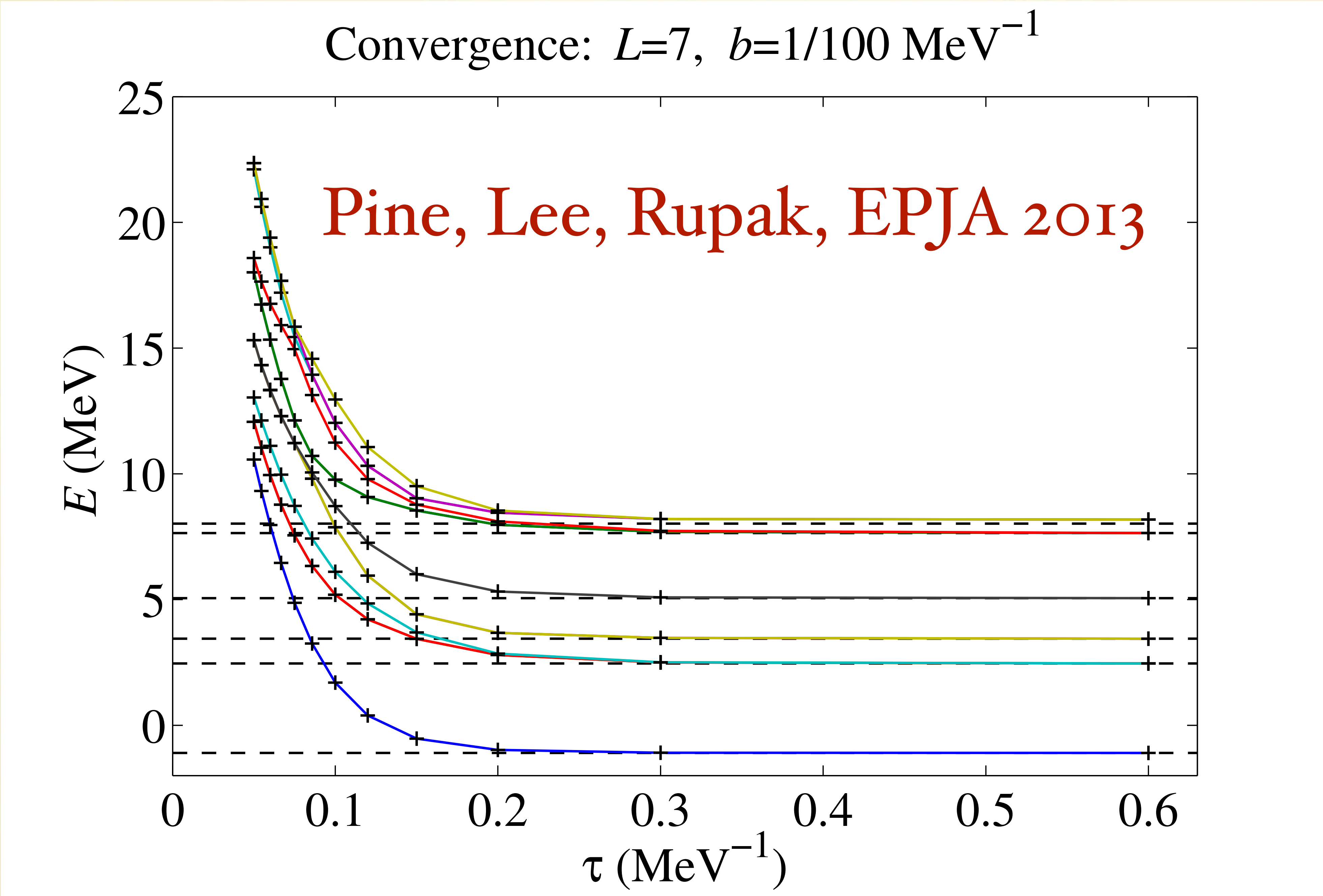


$$i\mathcal{A}(p) = \frac{-i}{\frac{1}{C_0} + i\frac{\mu}{2\pi}p} \Rightarrow C_0 = \frac{2\pi a}{\mu}$$

large coupling

Weinberg '90
Bedaque, van Kolck '97
Kaplan, Savage, Wise '98

Neutron-Deuteron System



- grouping R found efficient, more later

$\sim 30 \times 30$

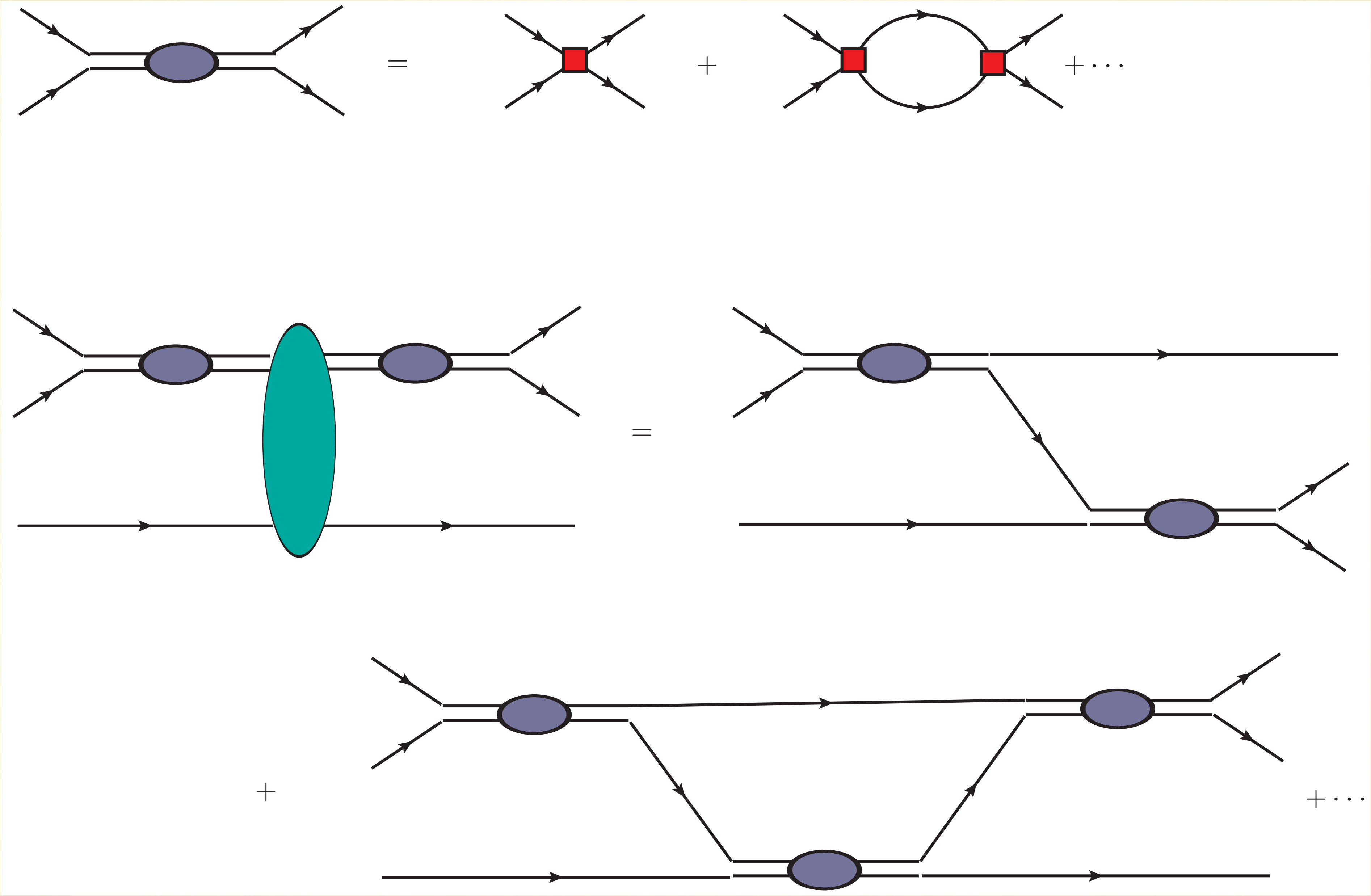
Lüscher's Method

$$p \cot \delta = \frac{1}{\pi L} S(\eta), \quad \eta = \left(\frac{pL}{2\pi} \right)^2$$

$$E_{\text{fd}} = \frac{p^2}{2\mu^*} - B + \tau(p)[B - B_L]$$

- effective mass
- topological factor (Bour, Hammer, Lee, Meißner 2013)

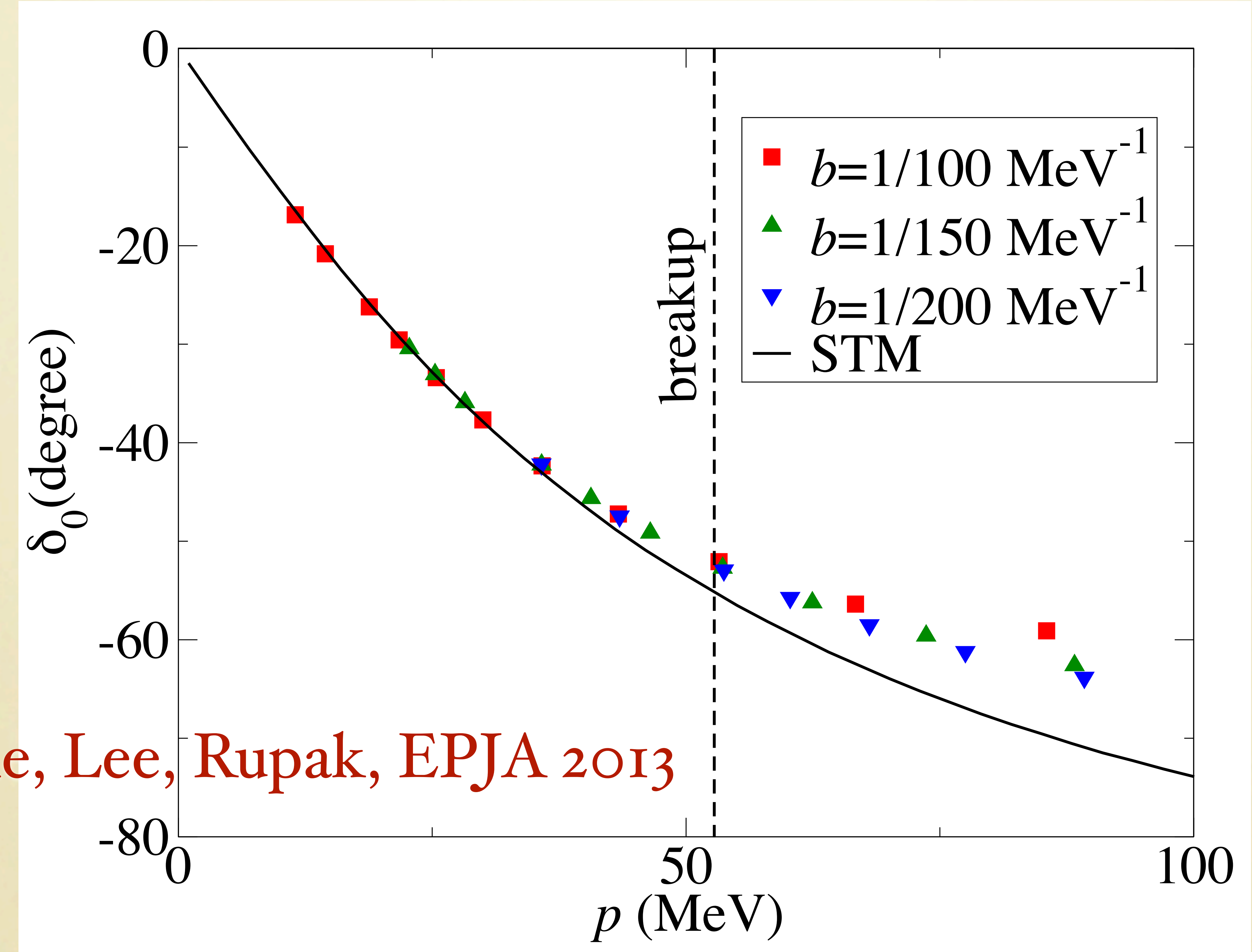
Neutron-Deuteron System



$$T(p) = h(p) + \int dq K(p,q) T(q)$$

$$T(p) = \frac{2\pi}{\mu} \frac{1}{p\cot\delta - ip}$$

Neutron-Deuteron Phase Shift

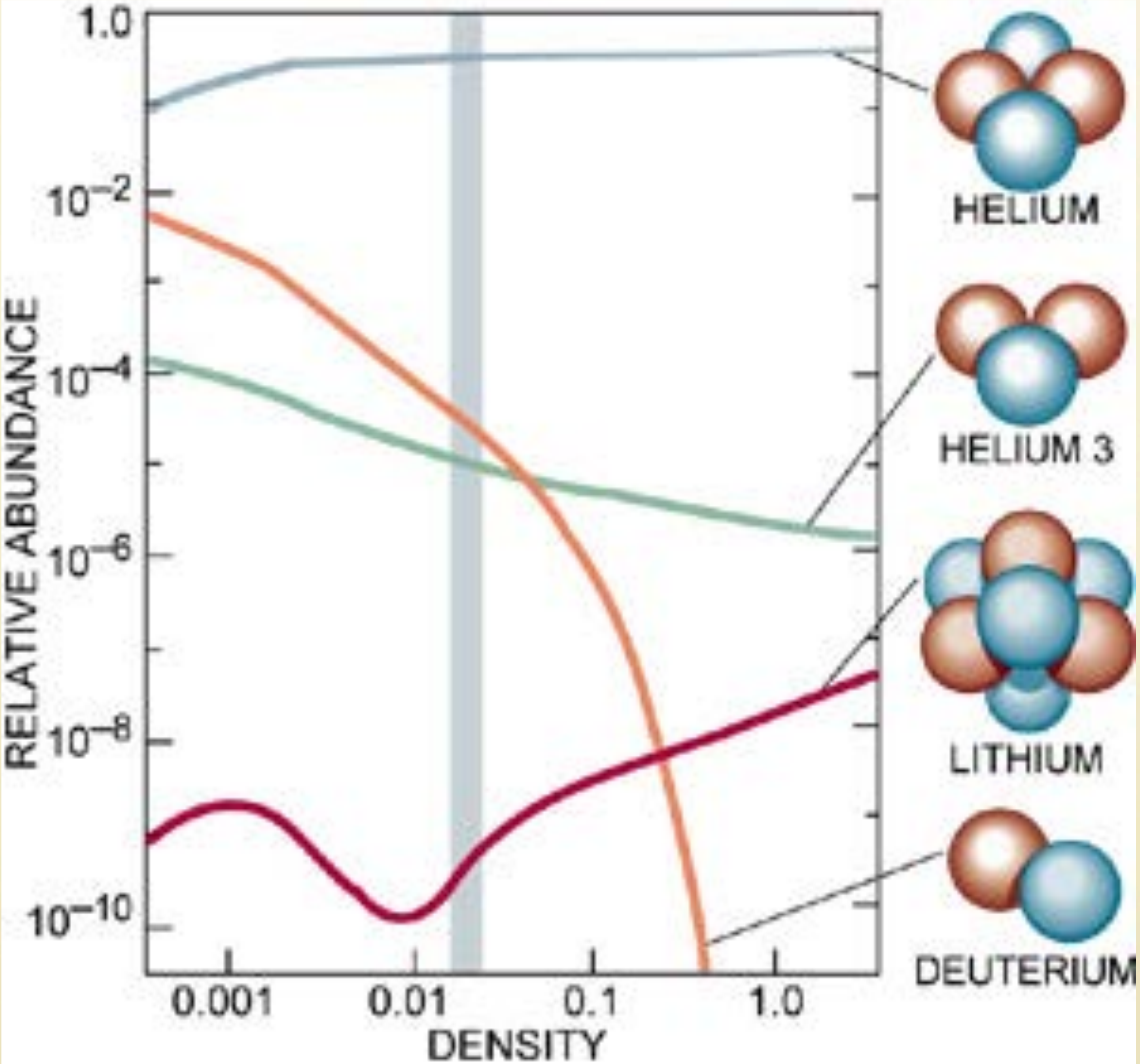


Pine, Lee, Rupak, EPJA 2013

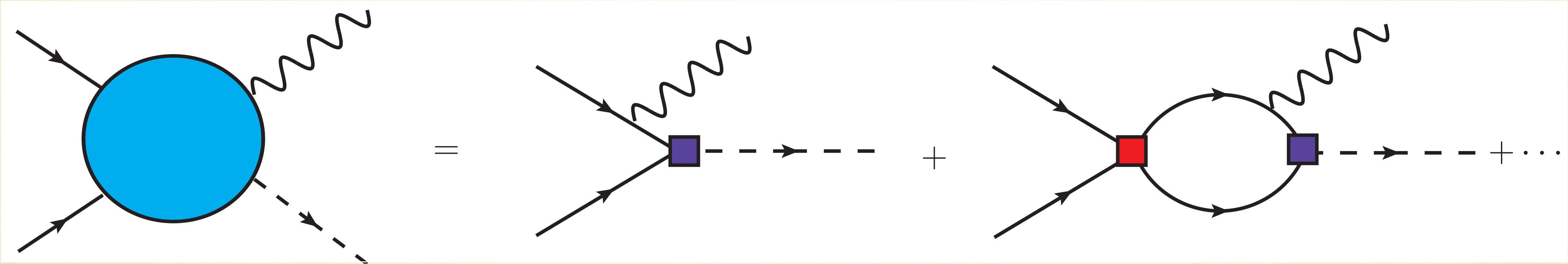
Now, what can I do with an adiabatic Hamiltonian?

Primordial Deuterium

$$p(n,\gamma)d$$



Warm Up $p(n,\gamma)d$



Exact analytic continuum result

$$\mathcal{M}_C(\epsilon) = \frac{1}{p^2+\gamma^2} - \frac{1}{(1/a + ip_\epsilon)(\gamma - ip_\epsilon)}, \quad p_\epsilon = \sqrt{p^2 + iM\epsilon}$$

When $\epsilon \to 0^+$, $\mathcal{M}_C$ reduces to known **M1** result

Rupak & Lee, PRL 2013

Lattice $p(n,\gamma)d$

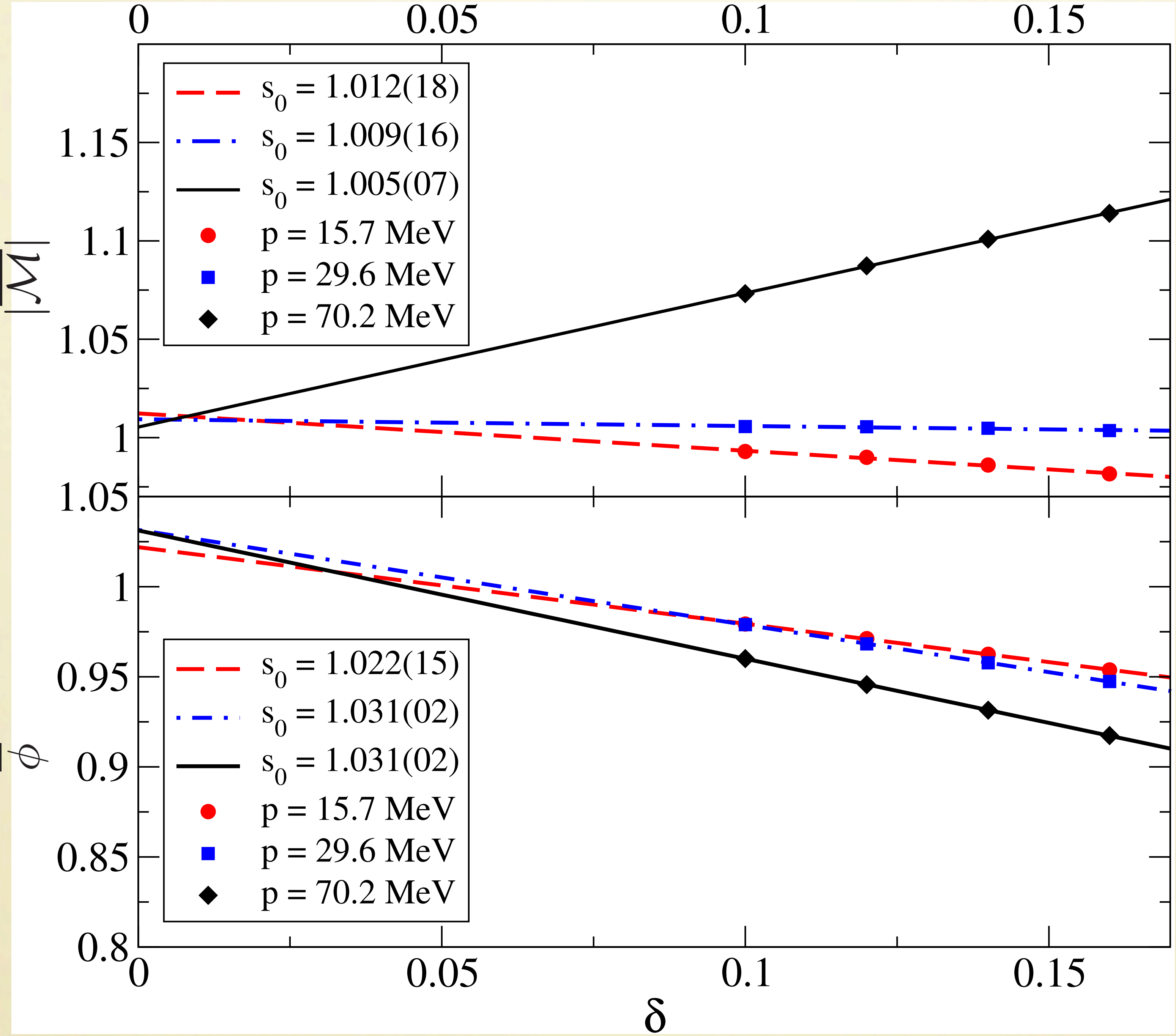
Write $\langle\psi_B|O_{\rm EM}|\psi_i\rangle$ using retarded Green's function

$$\mathcal{M}(\epsilon) = \left(\frac{p^2}{M} - E - i\epsilon\right)\sum_{\boldsymbol{x},\boldsymbol{y}} \psi_B^*(\boldsymbol{y})\langle\boldsymbol{y}|\frac{1}{E-\hat{H}_s+i\epsilon}|\boldsymbol{x}\rangle e^{i\boldsymbol{p}\cdot\boldsymbol{x}}$$

cluster Hamiltonian goes here

LSZ reduction in QM

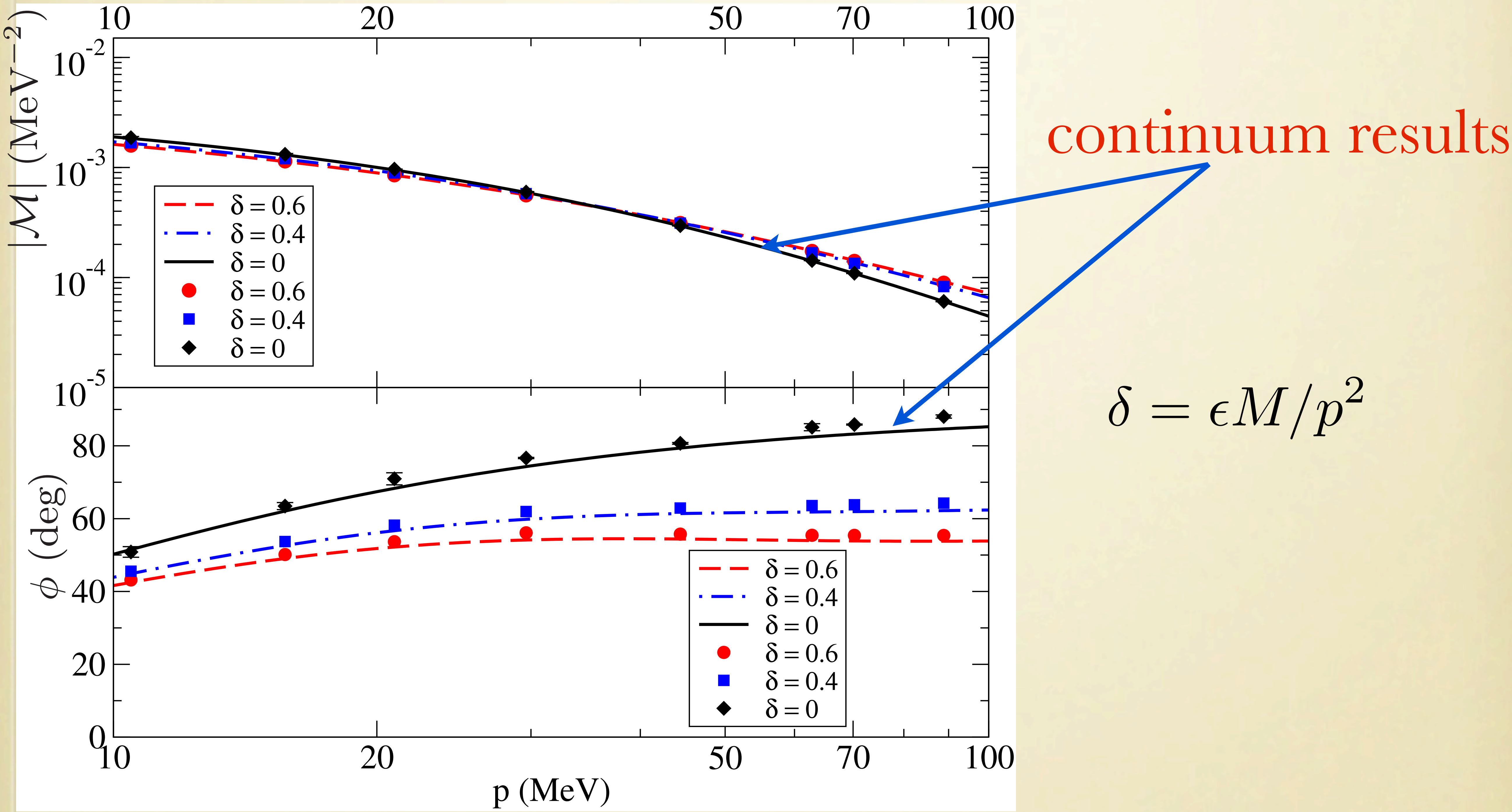
Continuum Extrapolation



$$\delta = \epsilon M/p^2$$

$$\text{Fit} \;\; : s_0 + s_1\delta$$

Capture Amplitude



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Rupak & Lee, PRL 2013

Something still missing ...

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long range Coulomb

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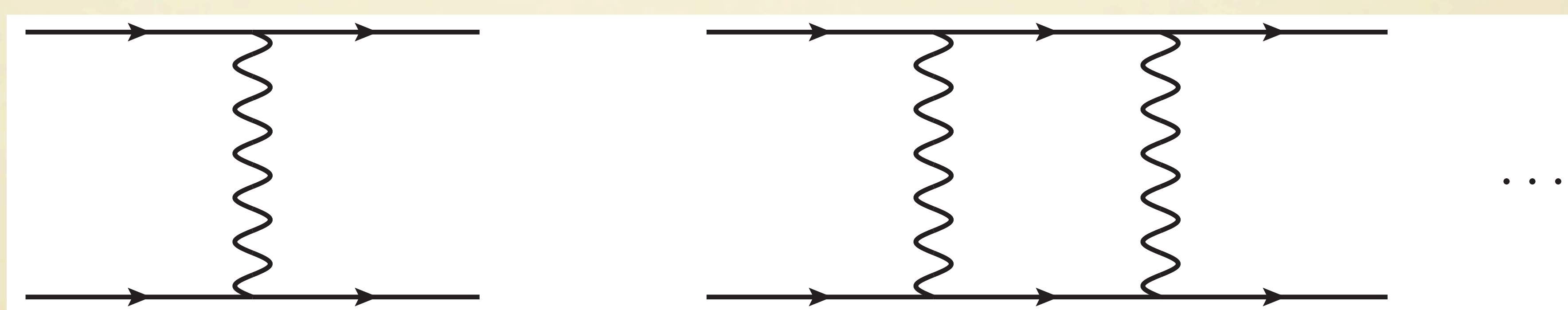
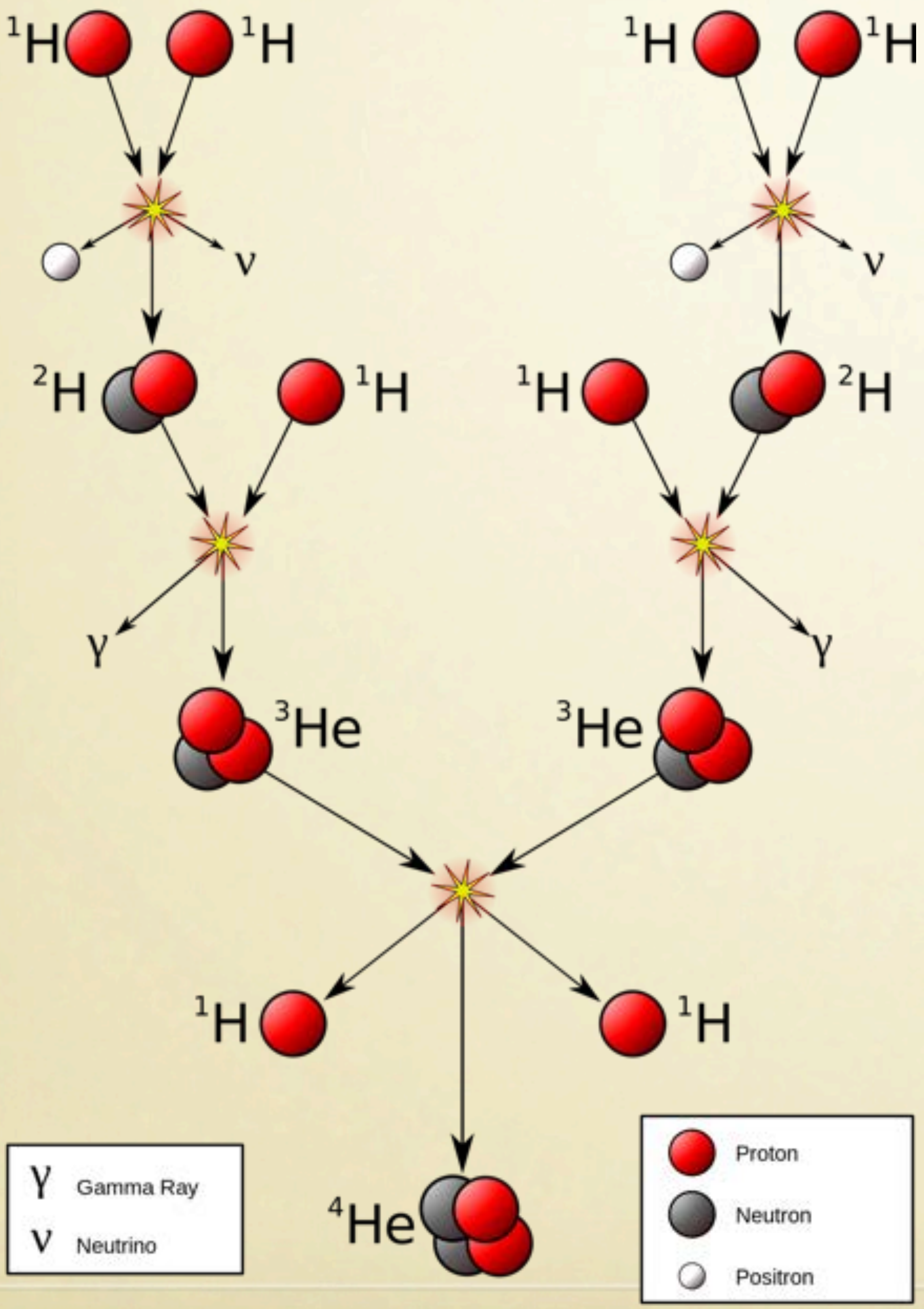


$$\mathcal{O}\left(\frac{\alpha}{p^2}\right) \qquad \mathcal{O}\left(\frac{\alpha}{p^2}\frac{\alpha\mu}{p}\right)$$

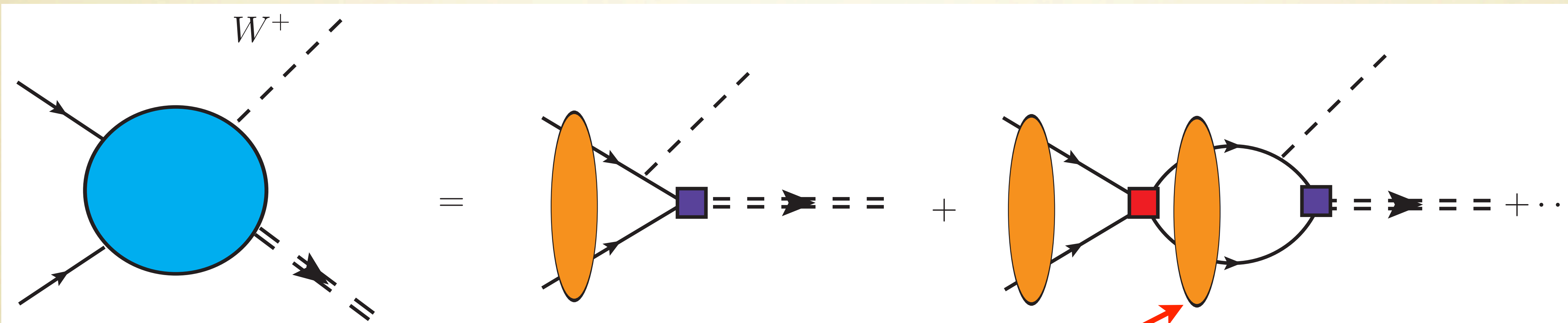
Proton-Proton Fusion



$$p + p \rightarrow d + e^{+} + \nu_e$$

long and steady burning

Proton fusion in continuum EFT



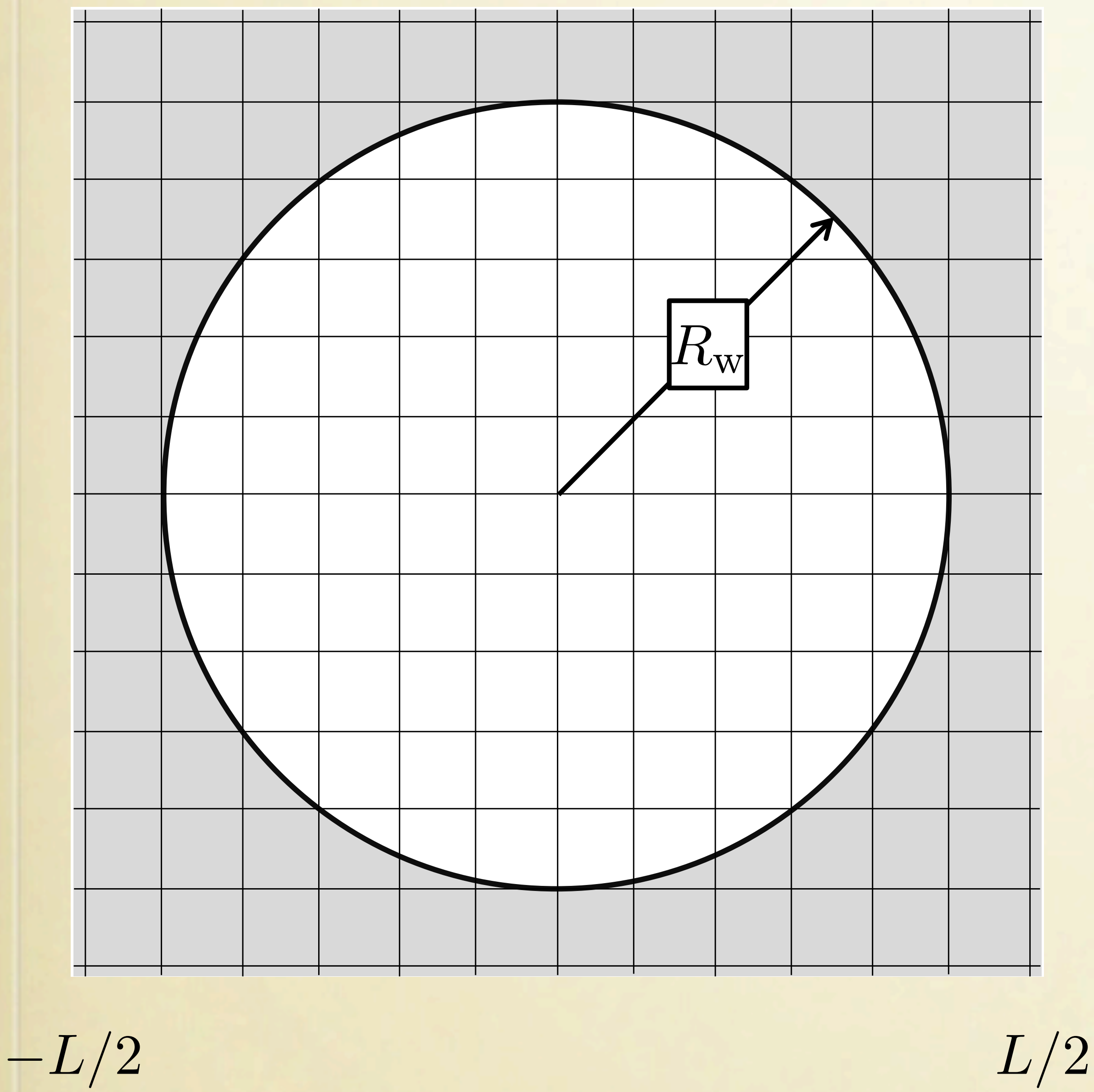
Kong, Ravndal 1999

Coulomb ladder

Peripheral scattering but how to calculate on the lattice?

Consider elastic proton-proton scattering as a warmup

Spherical-wall method



$$\psi_{\text{short}}(r) \propto j_0(kr)\cot\delta_s - n_0(kr),$$
$$\psi_{\text{Coulomb}}(r) \propto F_0(kr)\cot\delta_{sc} + G_0(kr)$$

Adjust from free theory:

$$j_0(k_0 R_w) = 0$$

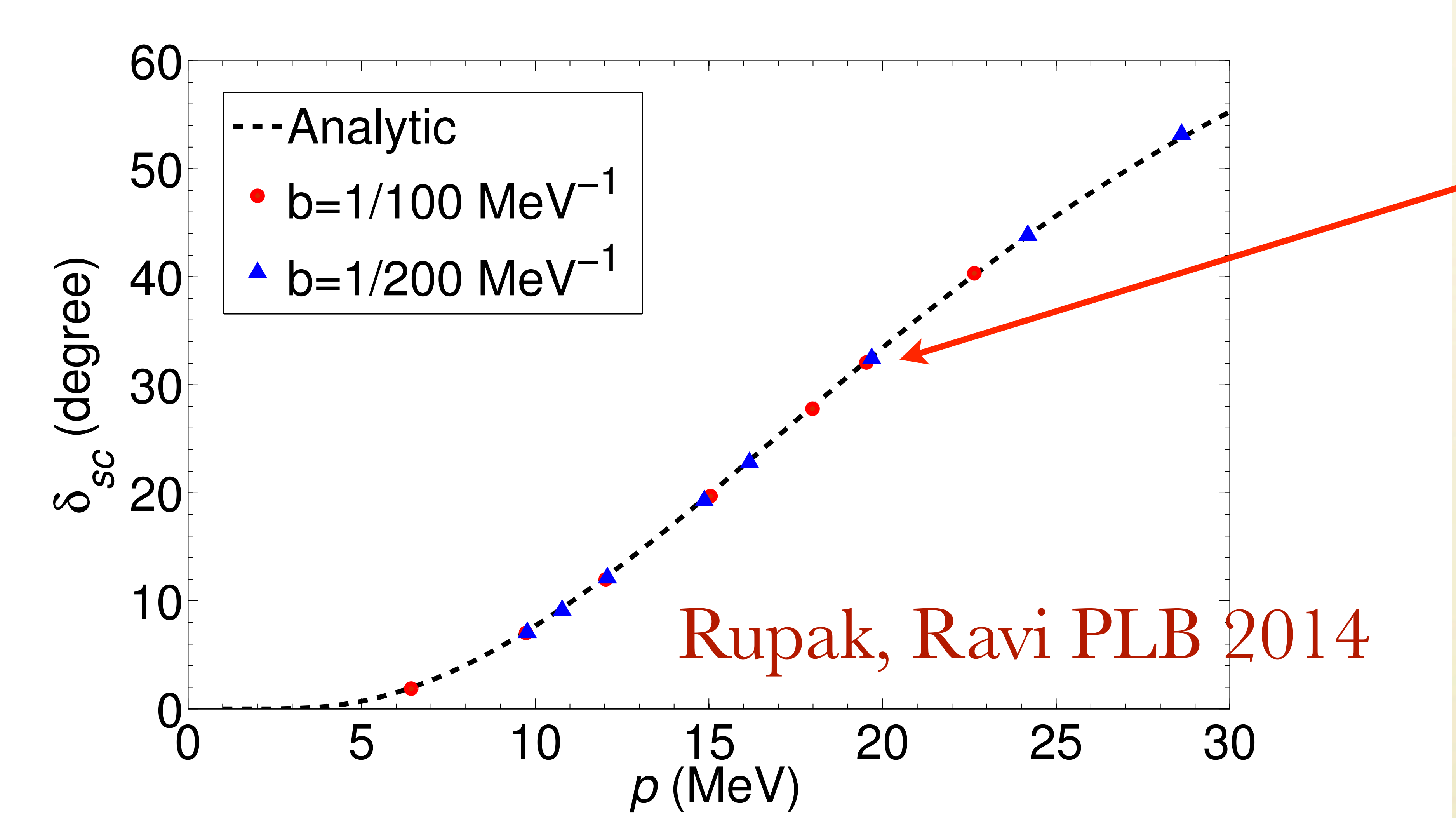
IR scale setting

Hard spherical wall boundary conditions, Borasoy et al. 2007

Carlson et al. 1984

Even older ?

Coulomb Subtracted Phase Shift



Rupak, Ravi PLB 2014

3% error in fits

$$T = T_c + T_{sc}$$

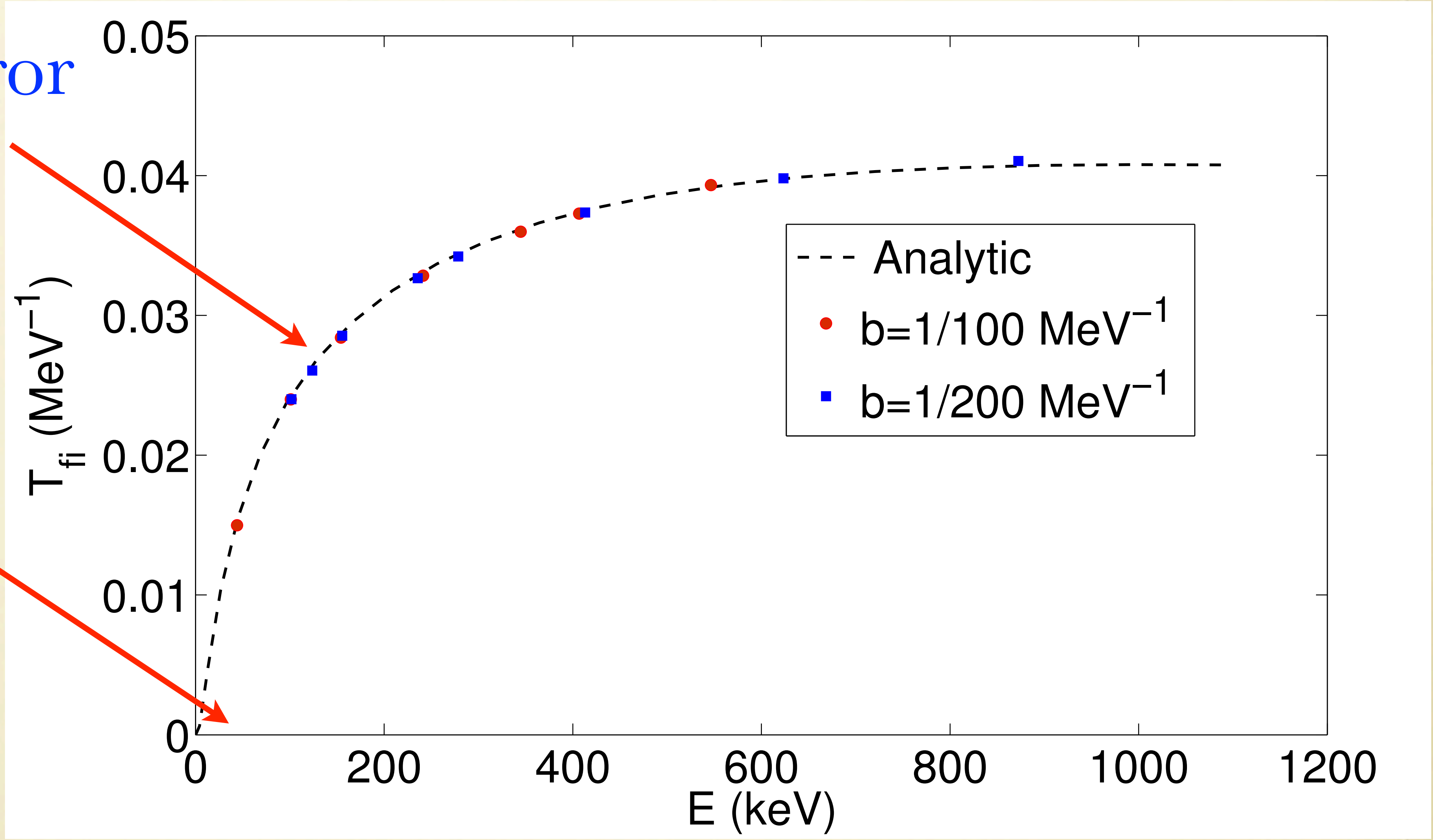
$$T_c \approx \frac{2\pi}{\mu} \frac{e^{2i\sigma} - 1}{2ip}$$

$$T \approx \frac{2\pi}{\mu} \frac{e^{2i(\sigma + \delta_{sc})} - 1}{2ip}$$

Proton-proton fusion

3% fitting error propagates

Gamow peak 6 keV

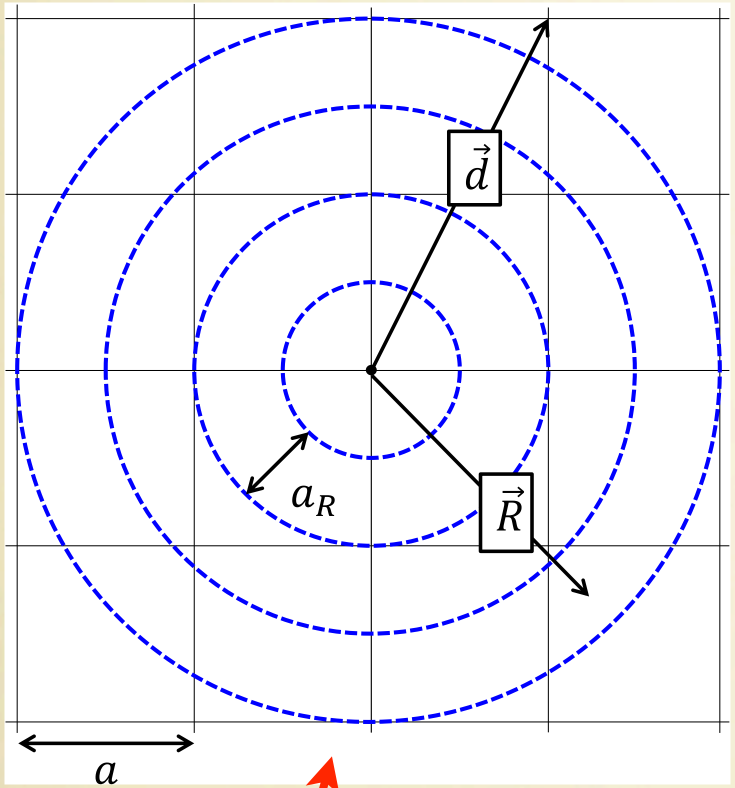


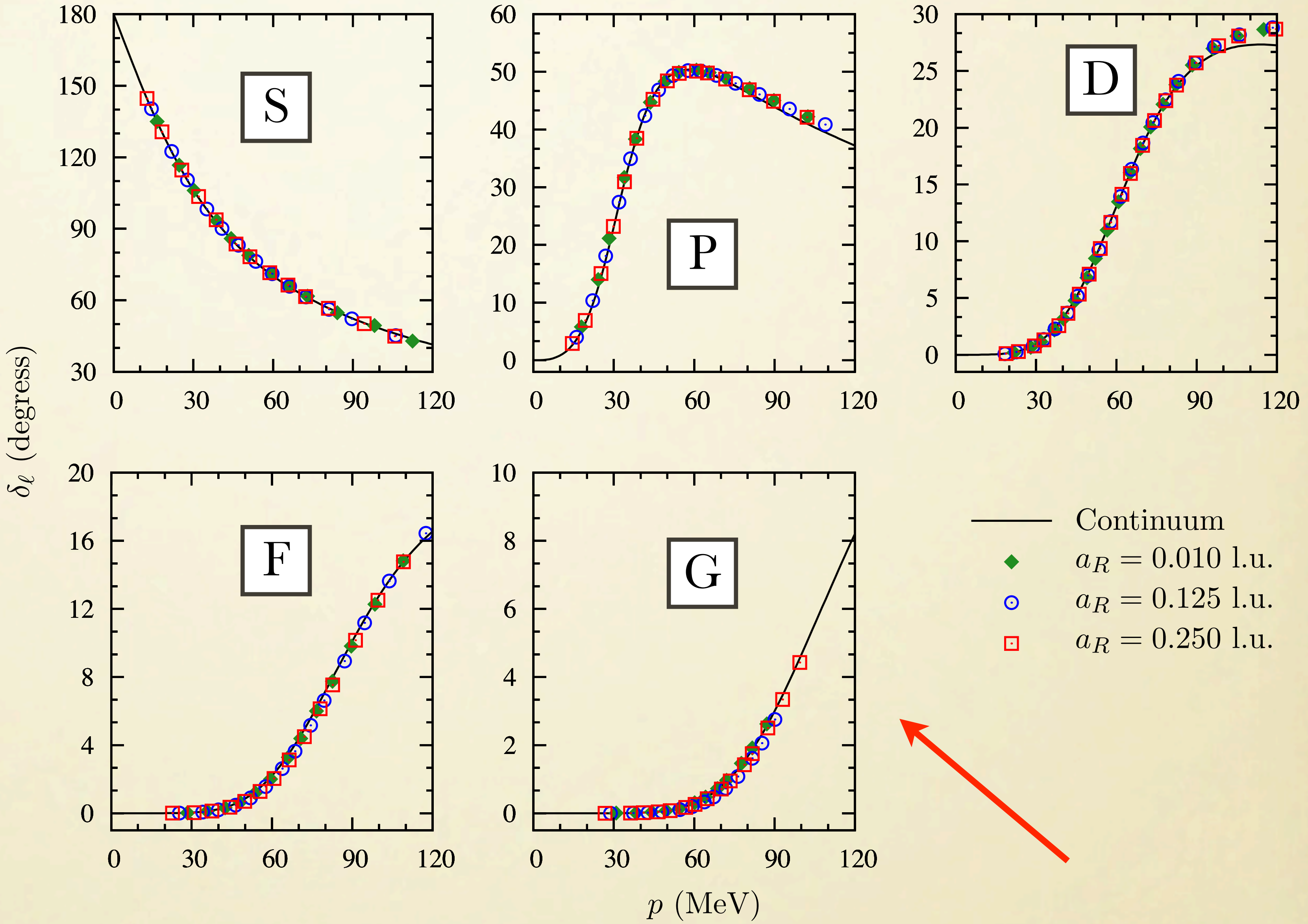
$$\Lambda(p) = \sqrt{\frac{\gamma^2}{8\pi C_\eta^2}}|T_{fi}(p)|$$

$\Lambda_{EFT}(0) \approx 2.51$ Kong, Ravndal 1999

Lattice fit : $\Lambda(0) \approx 2.49 \pm 0.02$ Rupak, Ravi PLB 2014

More on adiabatic Projection



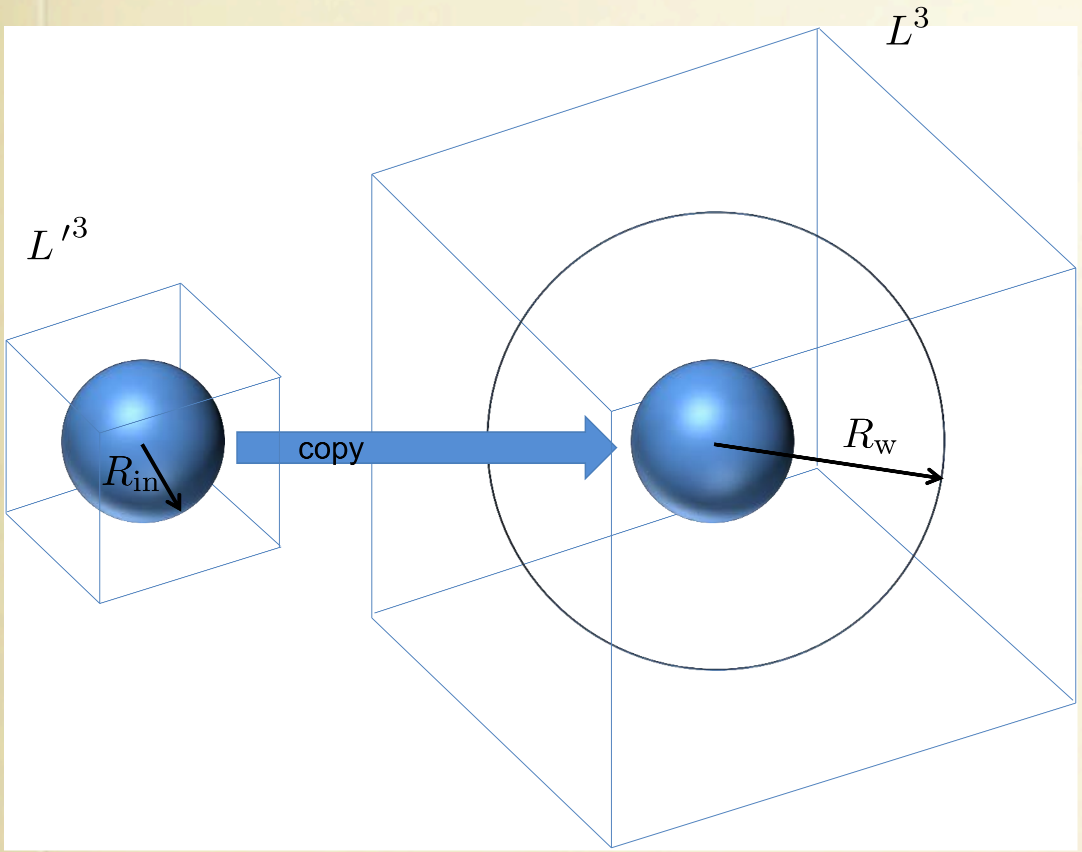


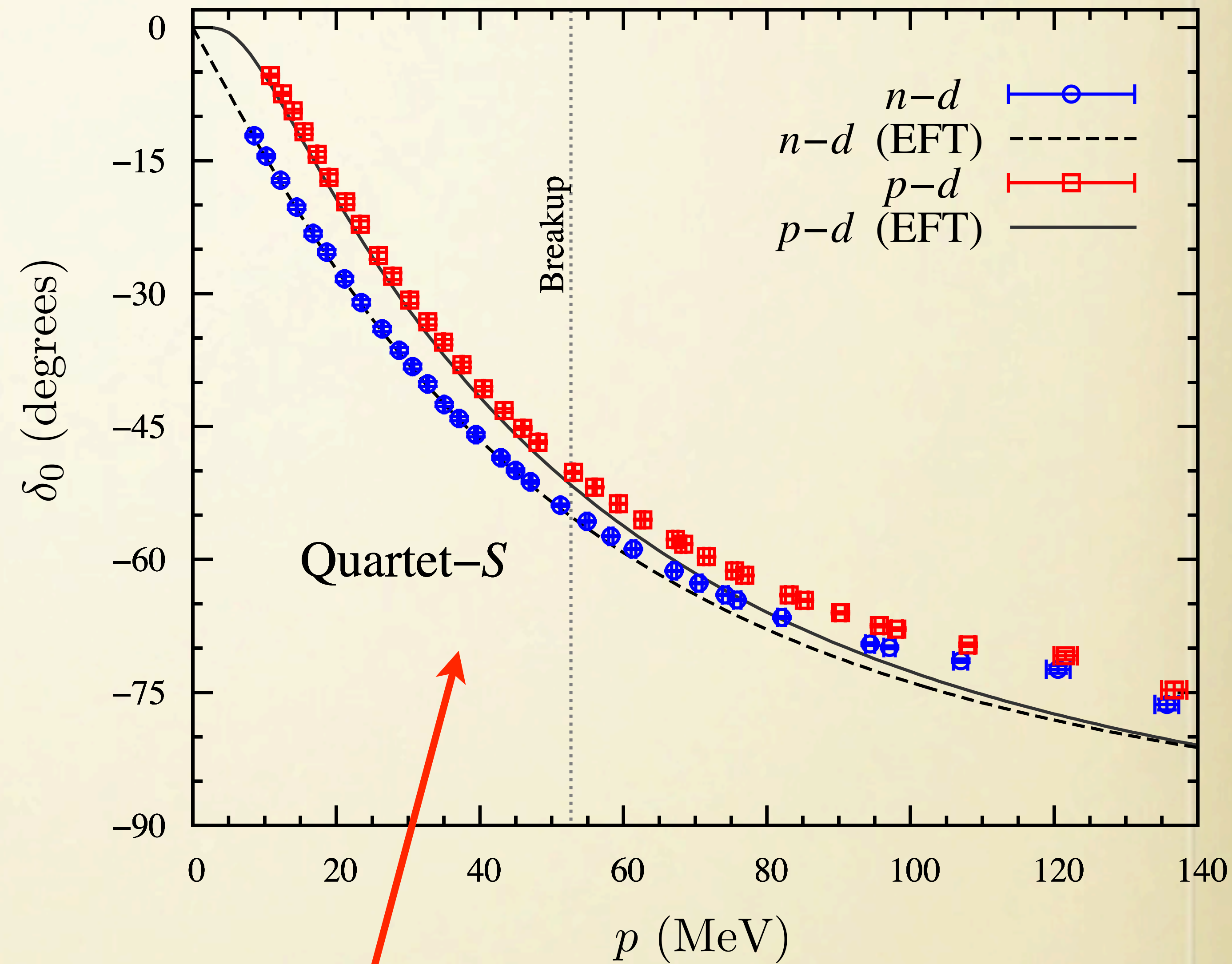
binning used earlier

Toy model, two-body

Elhatisari, Lee, Meißner, Rupak, arXiv 1603.02333

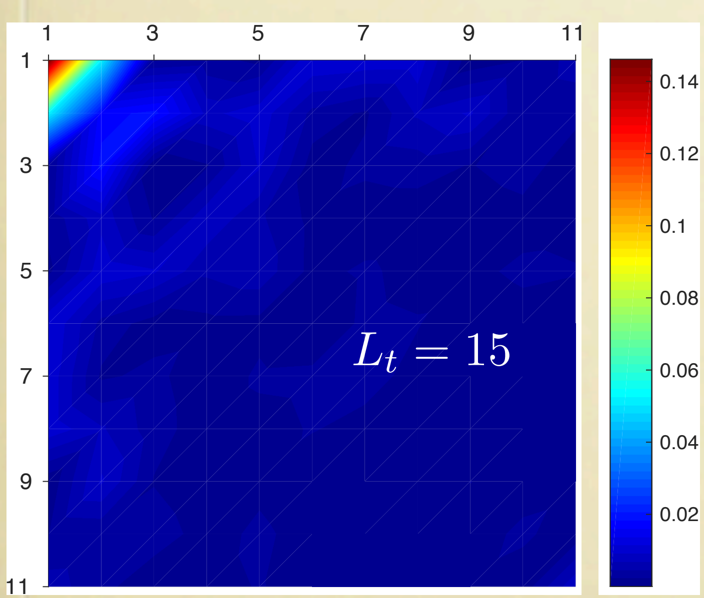
Two-cluster simulation





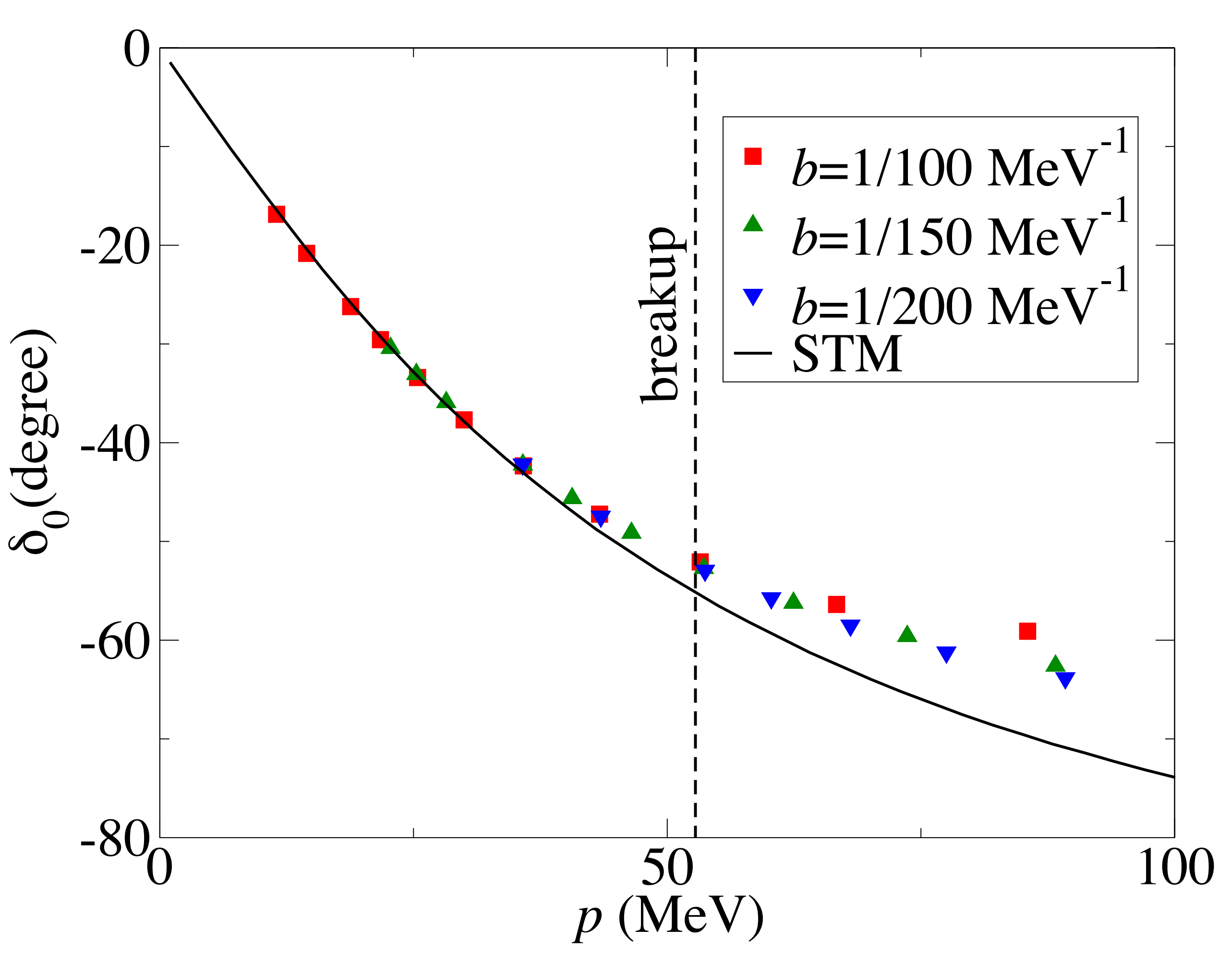
Matching: 15 to 82

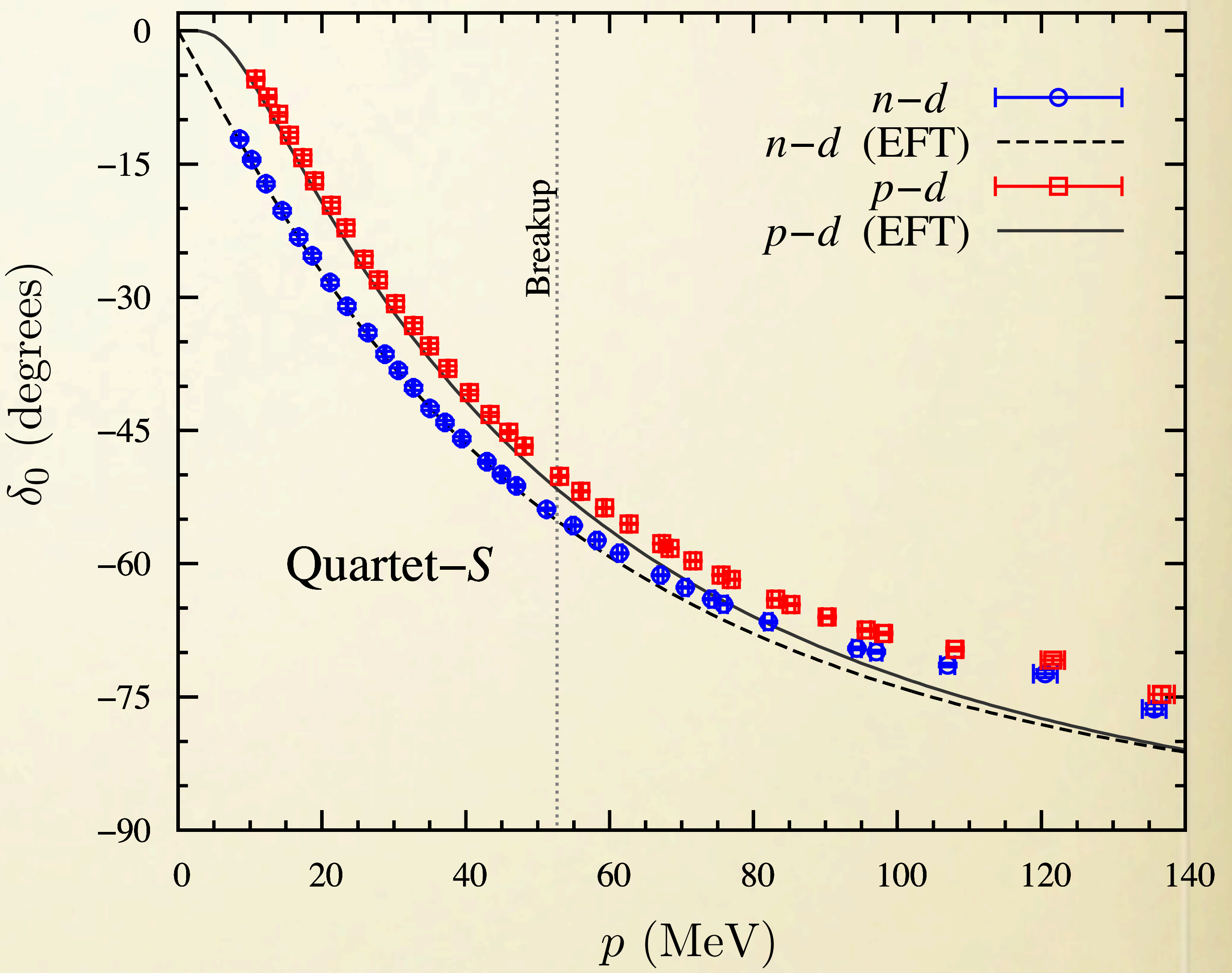
Better lattice results than before



Fermion-dimer

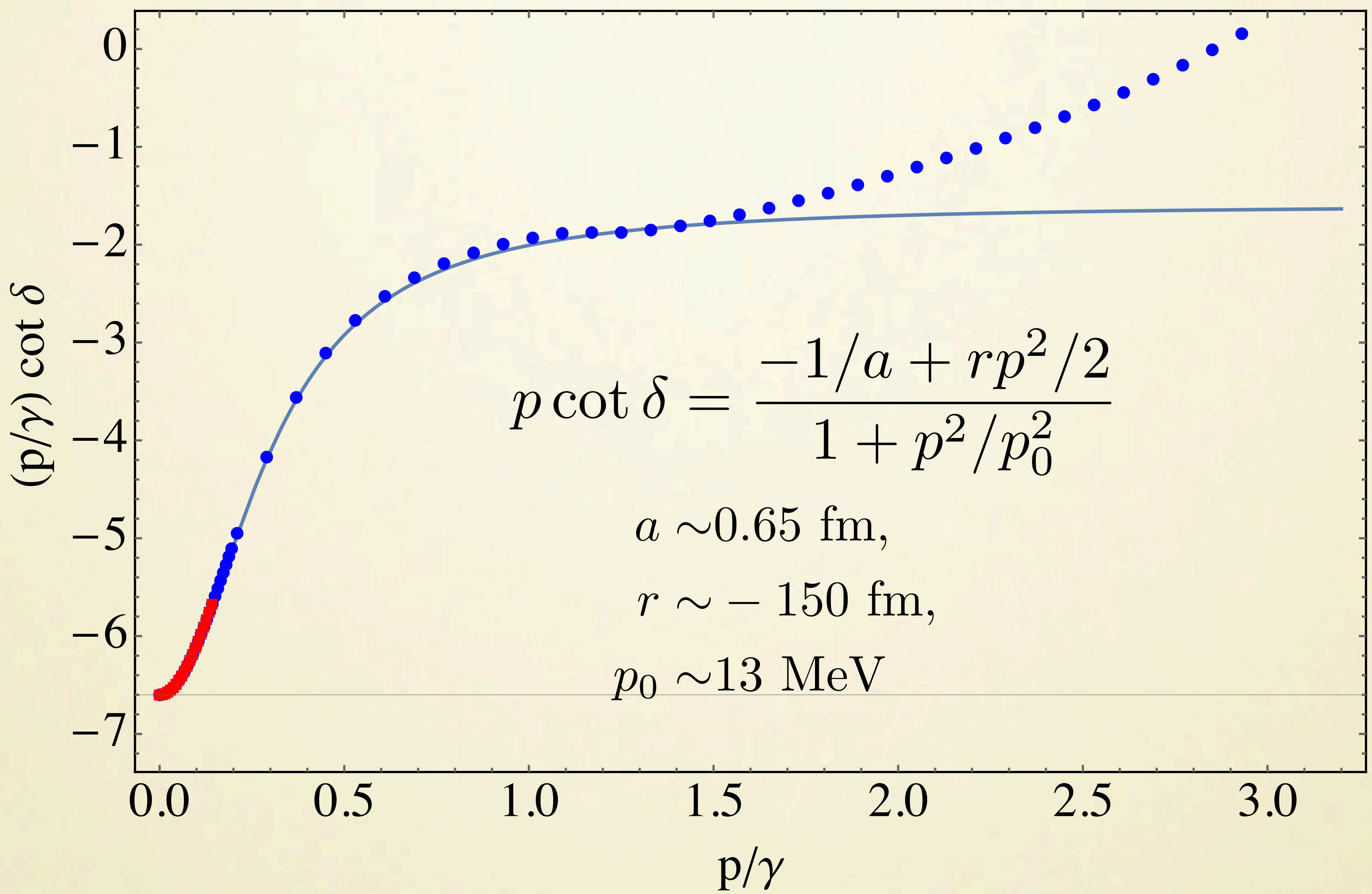
Improvement





Pine, Lee, Rupak (2013) Elhatisari, Lee, Meißner, Rupak (2016)

N-D DOUBLET CHANNEL



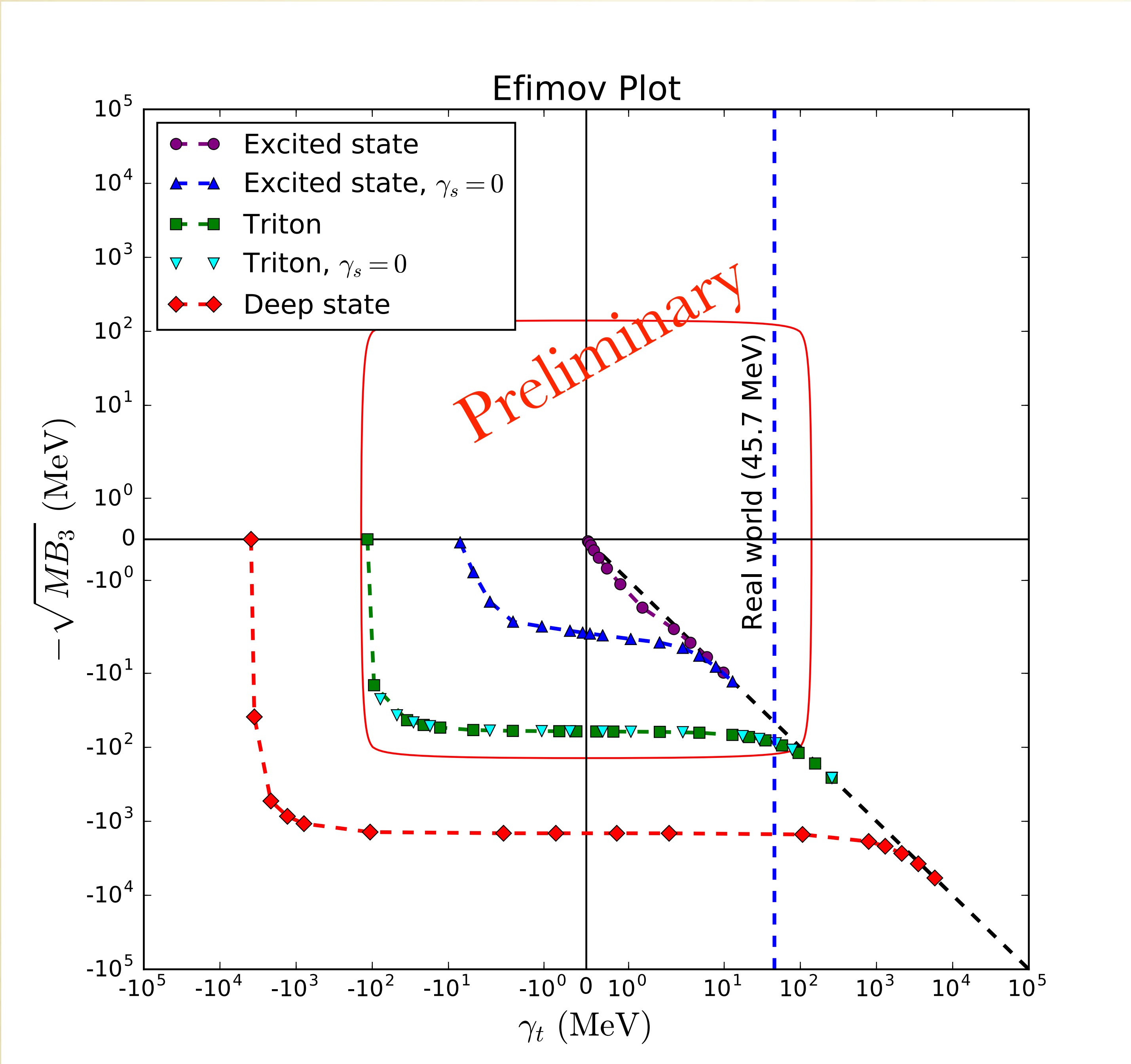
ERE form van Oers & Seagrave (1967)

-what EFT for modified ERE

Virtual state at 0.5 MeV Girard & Fuda (1979)

- Efimov physics

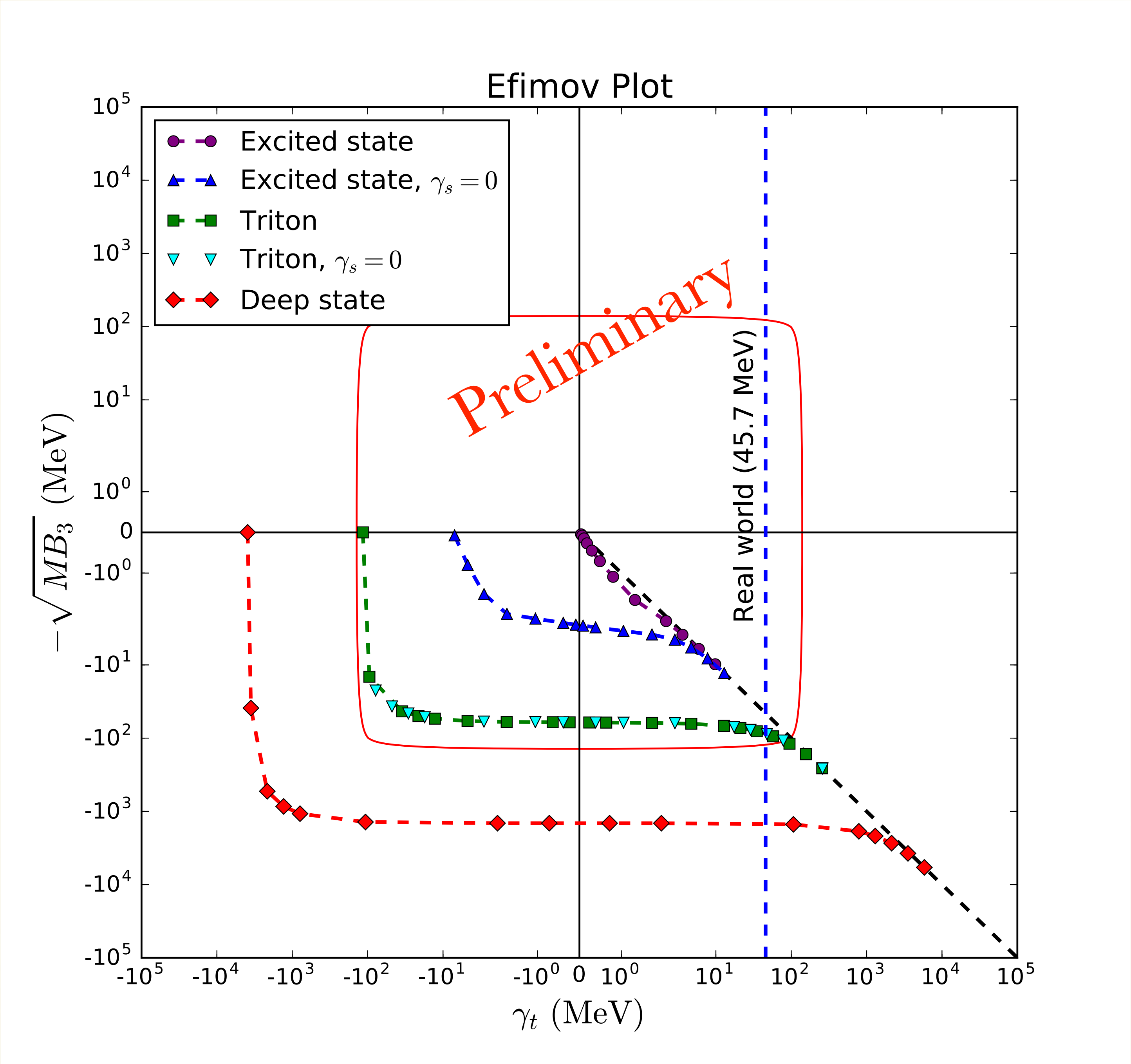
Efimov plot

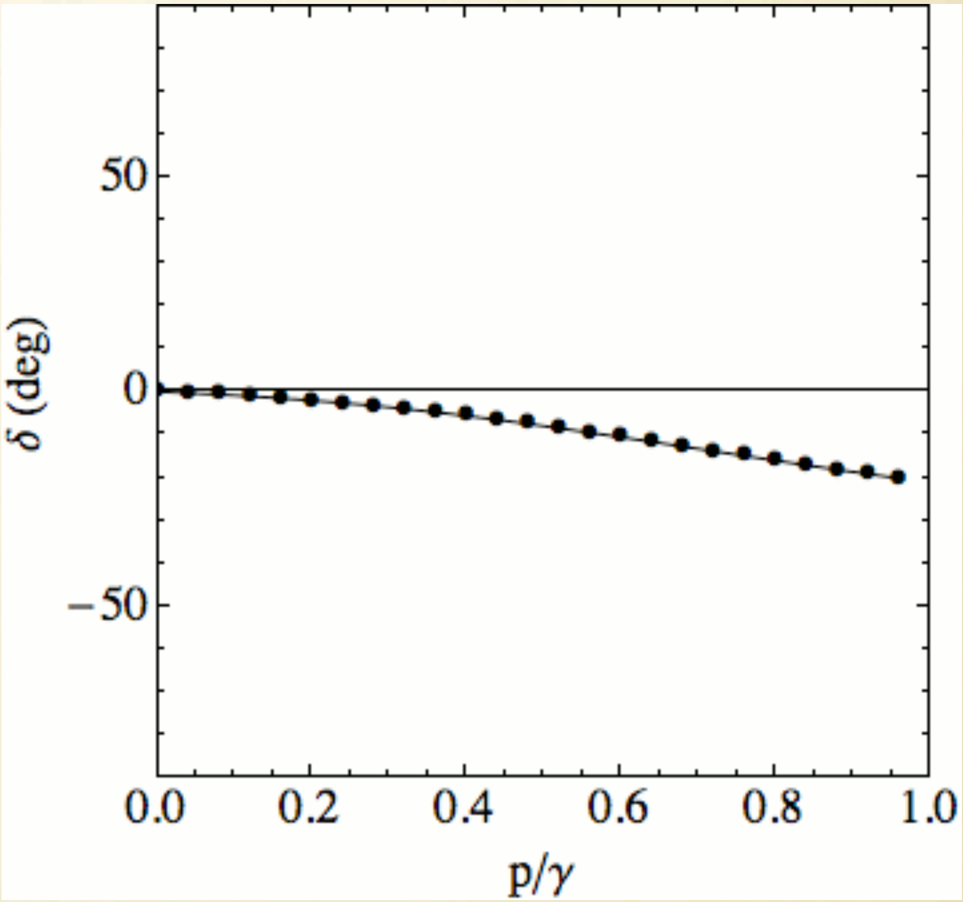


Higa, Rupak, Vaghani, van Kolck

Shallow virtual to bound state

EFIMOV PLOT





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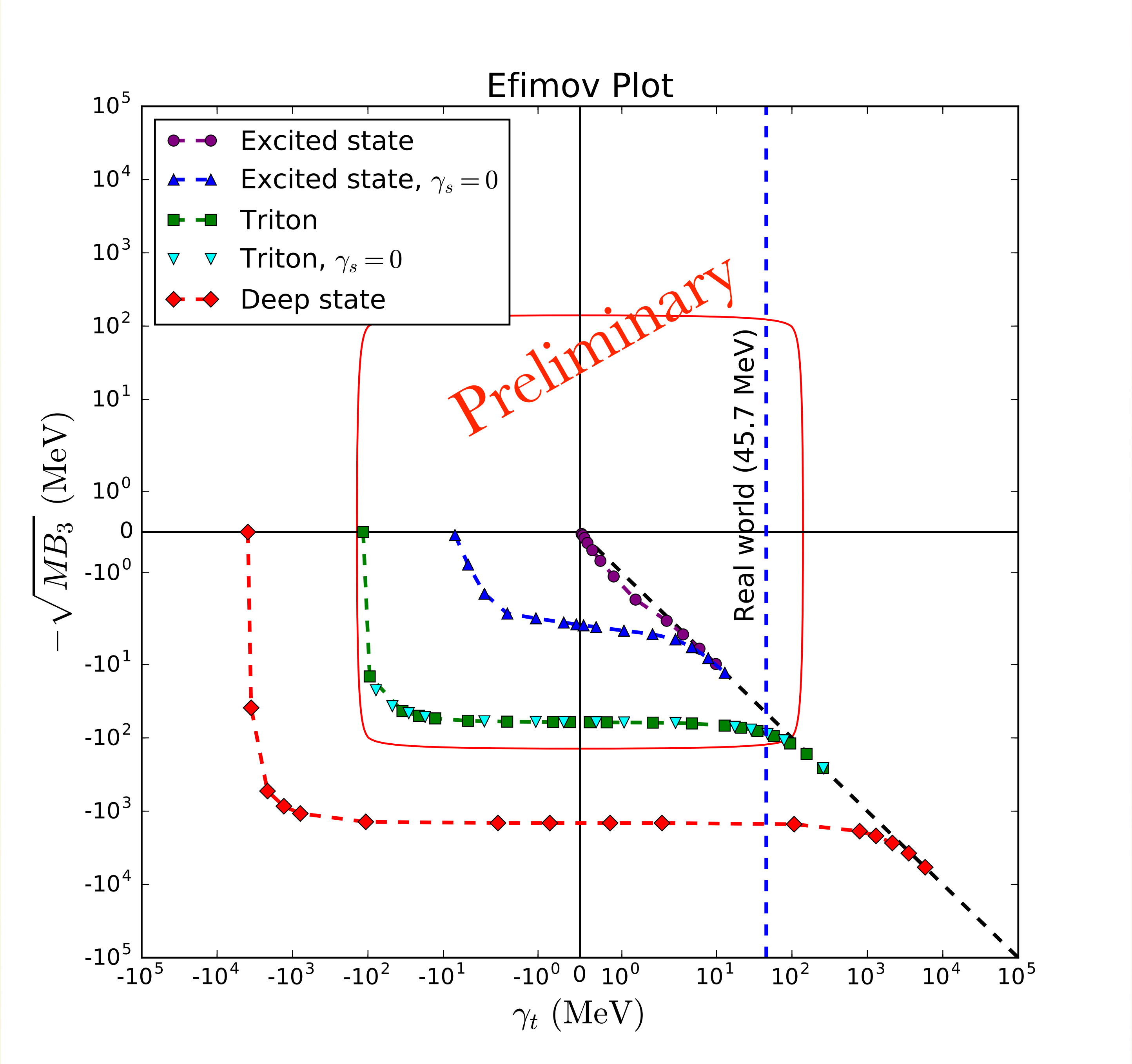
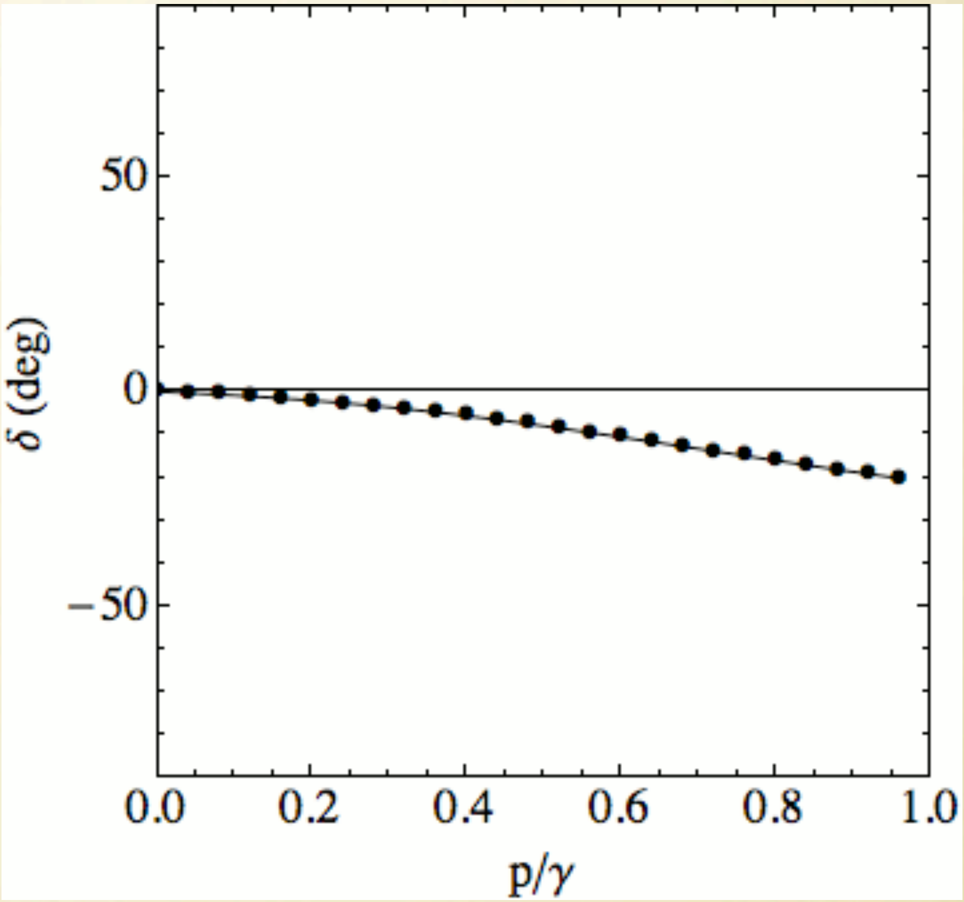


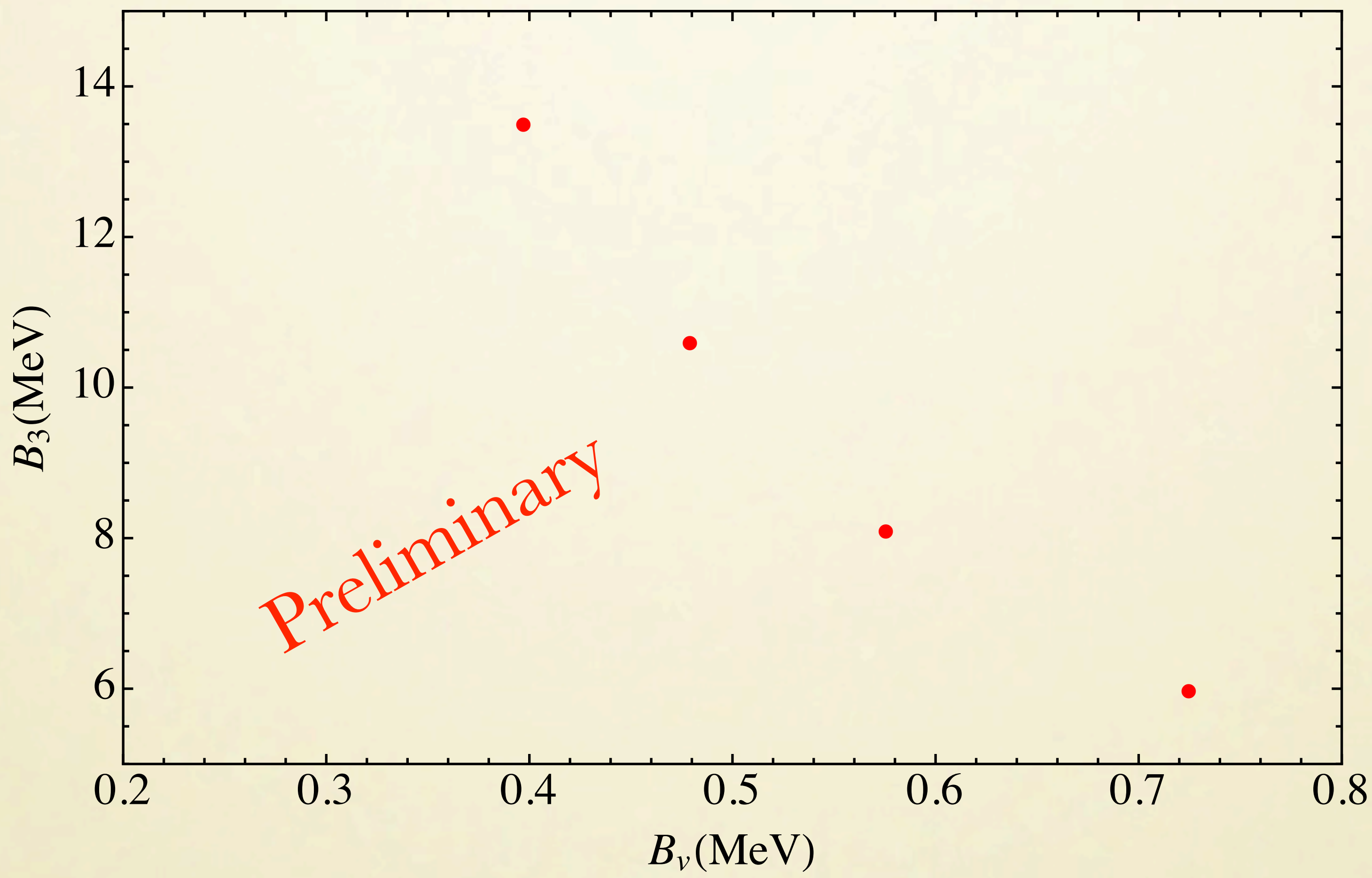


Higa, Rupak, Vaghani, van Kolck

Shallow virtual to bound state

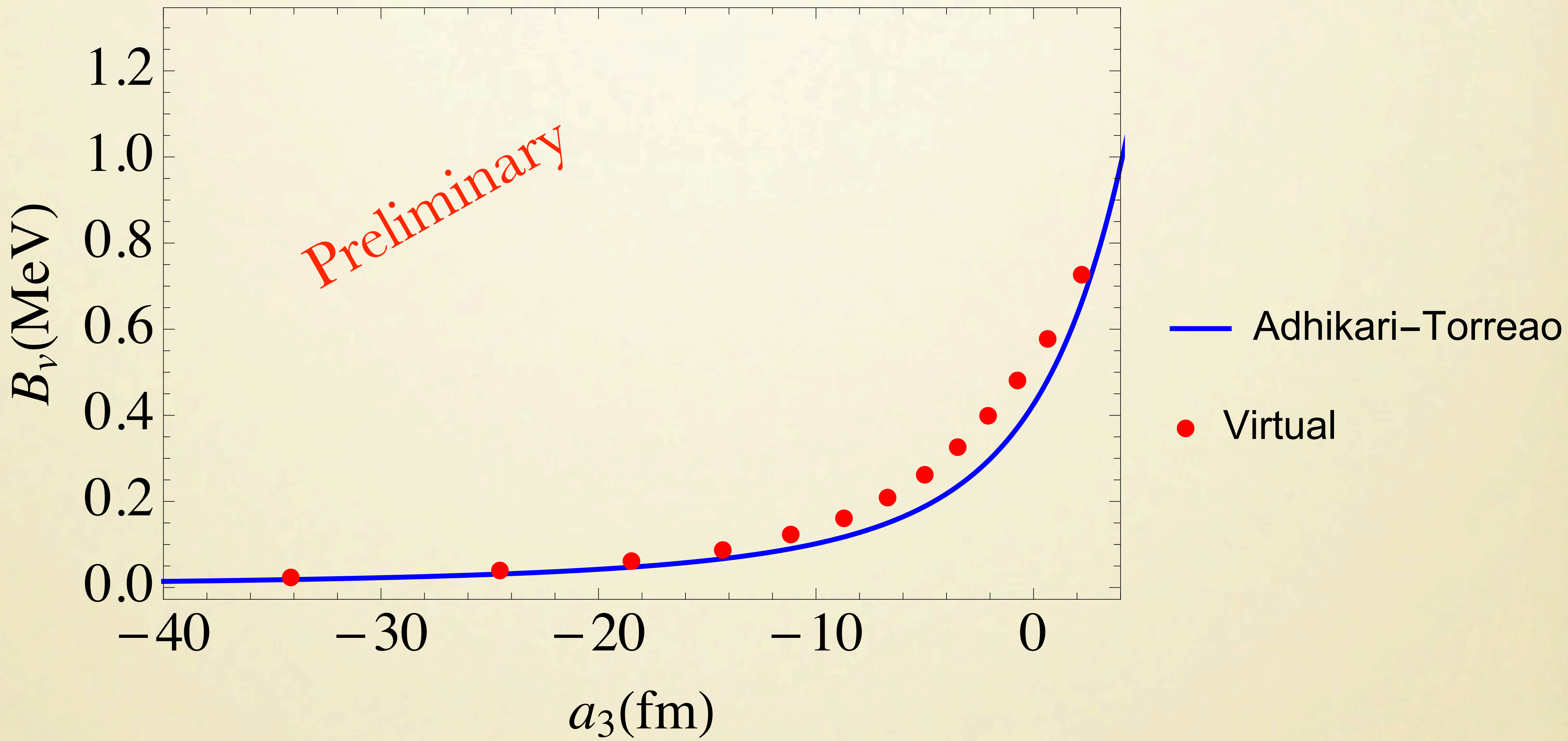
lattice QCD with B field, even with heavy pions?

Phillips-Girard-Fuda



3-body correlation

Adhikari-Torreao



Conclusions

- Adiabatic Projection Method to derive effective two-body Hamiltonian
- Retarded Green's function for problems without long-range Coulomb
- Spherical wall method with adiabatic Hamiltonian when long range Coulomb important
- Efimov physics from lattice QCD?

Thank you