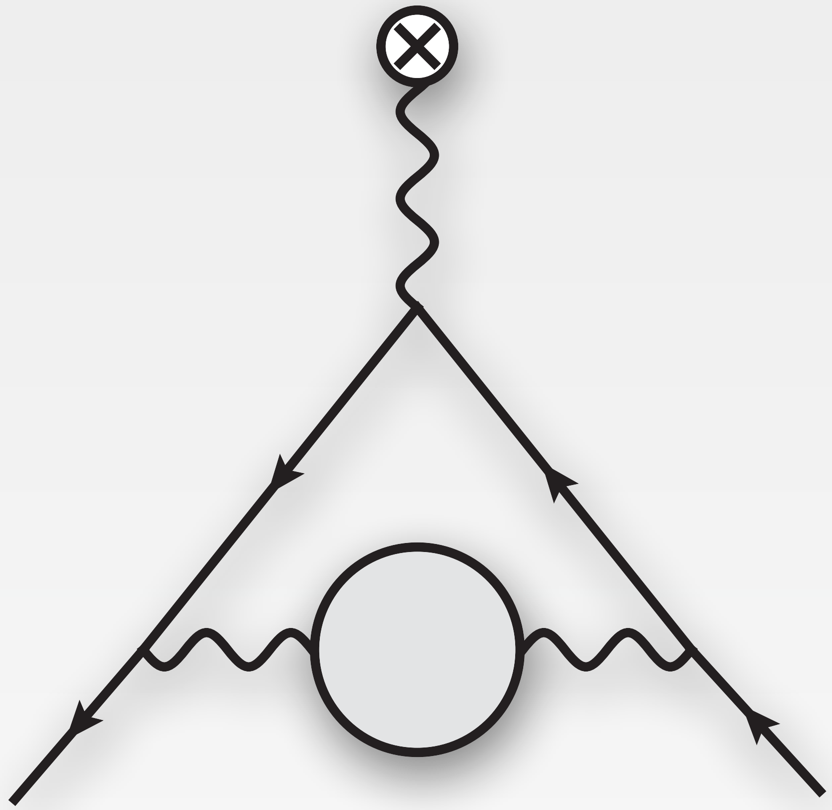

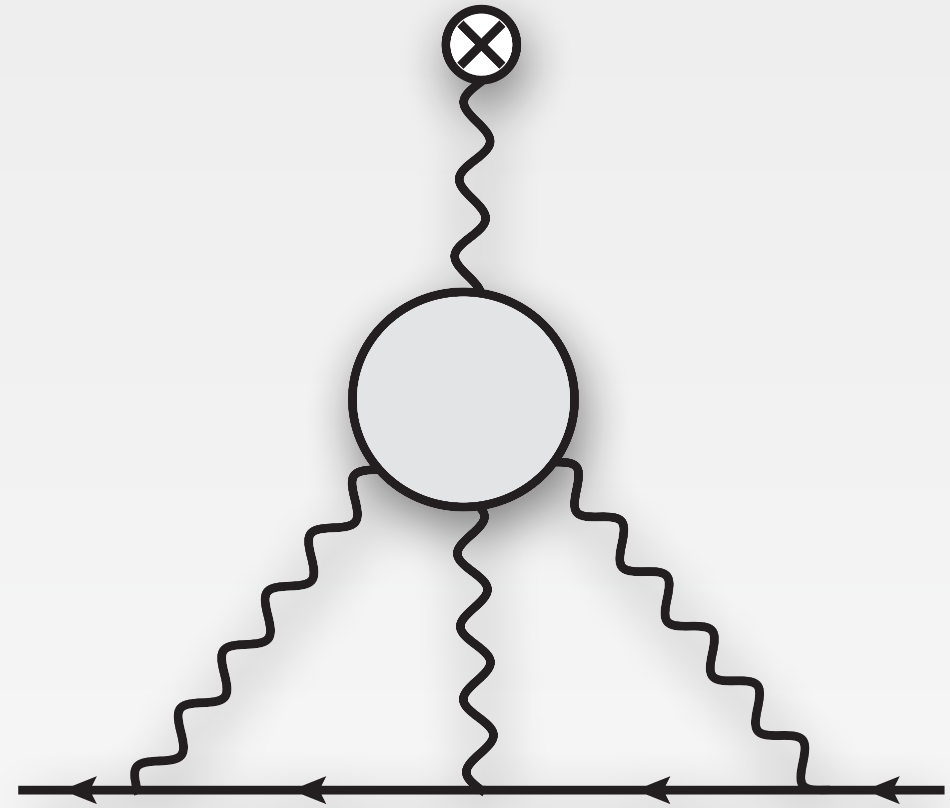

Determining the hadronic contribution to g-2 through lattice simulations

Antonin Portelli (RBC-UKQCD)

22nd of April 2016
INT Workshop 16-1, Seattle

Work done in collaboration with:

<u>BNL & RBRC:</u>

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C. Lehner

<u>CERN:</u>

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L. Jin

<u>U. of Connecticut:</u>

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R. Hudspith

R. Lewis

K. Maltman

- The anomalous magnetic moment of the muon
- Lattice methodology
- The connected strange HVP
- The disconnected HVP
- Outlook & perspectives

The anomalous magnetic moment of the muon

General definition

- Landé g-factor: $\boldsymbol{\mu} = g\frac{e}{2m}\mathbf{S}$
- Anomalous magnetic moment: deviation from the classical value 2: $a = \frac{g-2}{2}$
- Crowning achievement of QED and in general QFT
- QED vertex function:

$$\Gamma^{\mu}\left(k^2\right) = \gamma^{\mu} F_1\left(k^2\right) + \frac{i\sigma^{\mu\nu}k_{\nu}}{2m} F_2\left(k^2\right)$$

- a is then given by $a = F_2(0)$

Discrepancy in the muon case

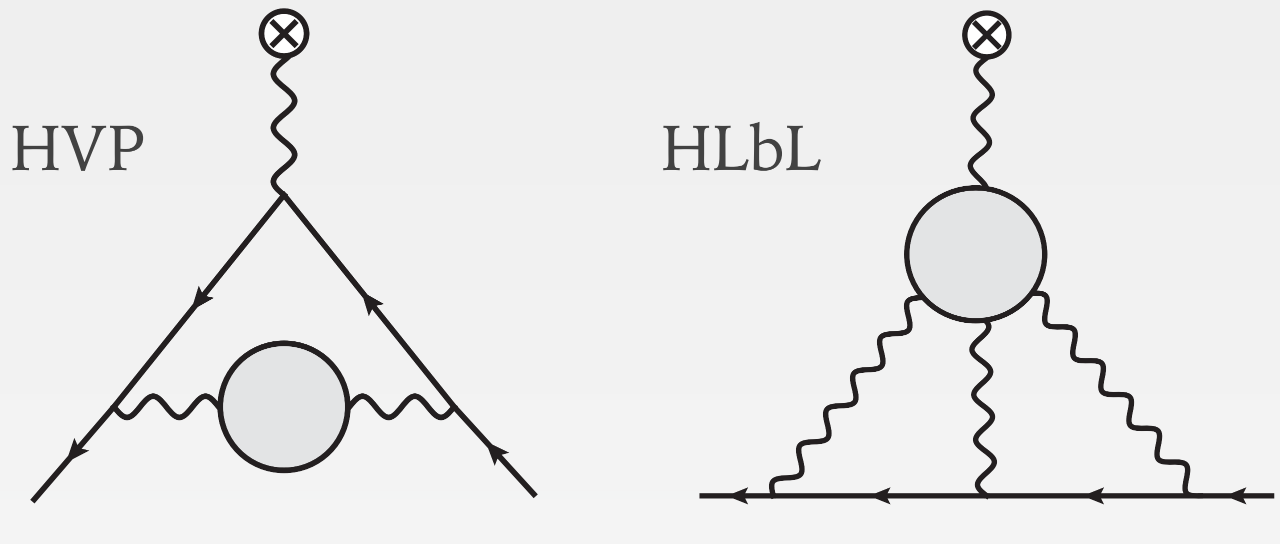
$$a_{\mu,\text{exp.}} = 116592089(54)(33) \cdot 10^{-11}$$
$$a_{\mu,\text{SM}} = 116591802(2)(42)(26) \cdot 10^{-11}$$

[PDG 2015]

EW LO Had. NLO Had.

- $\sim 3.6\sigma$ discrepancy
- The experimental error should be reduced by a factor 4 in the next years (FNAL experiment)
- Theoretical error completely dominated by hadronic uncertainties

Hadronic contributions



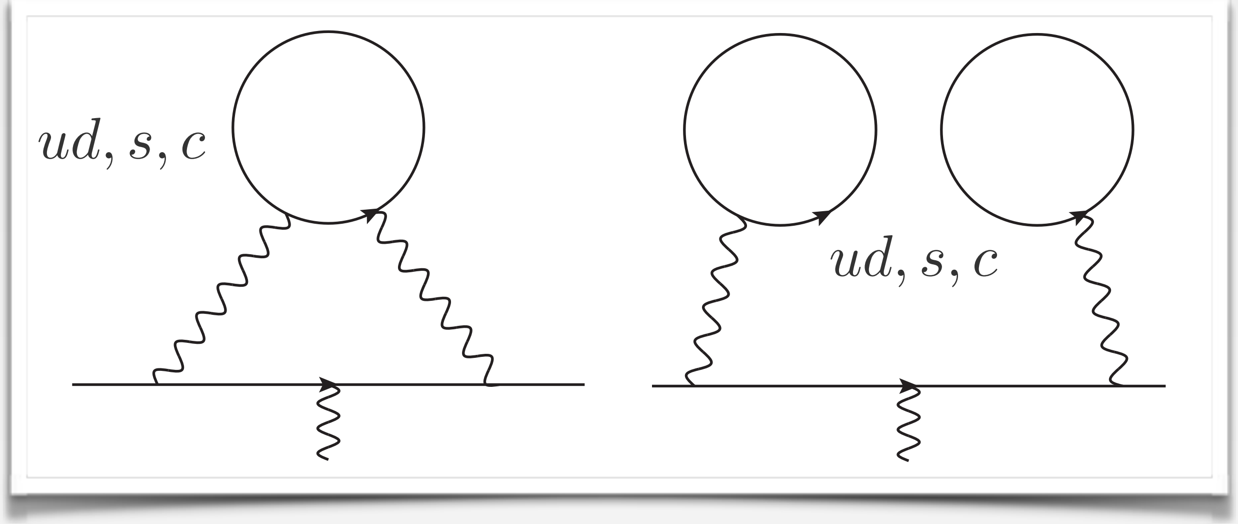
- Order α^2: hadronic vacuum polarisation (HVP)
- Order α^3:
  - hadronic light-by-light (HLbL) scattering
  - QED corrections to the HVP

Hadronic contributions

HVP LO	$6932(42)(3) \times 10^{-11}$	[Davier *et al.* 2011]
HLbL	$105(26) \times 10^{-11}$	[Glasgow Consensus 2007]
HVP NLO	$-98.4(0.6) \times 10^{-11}$	[PDG 2015]

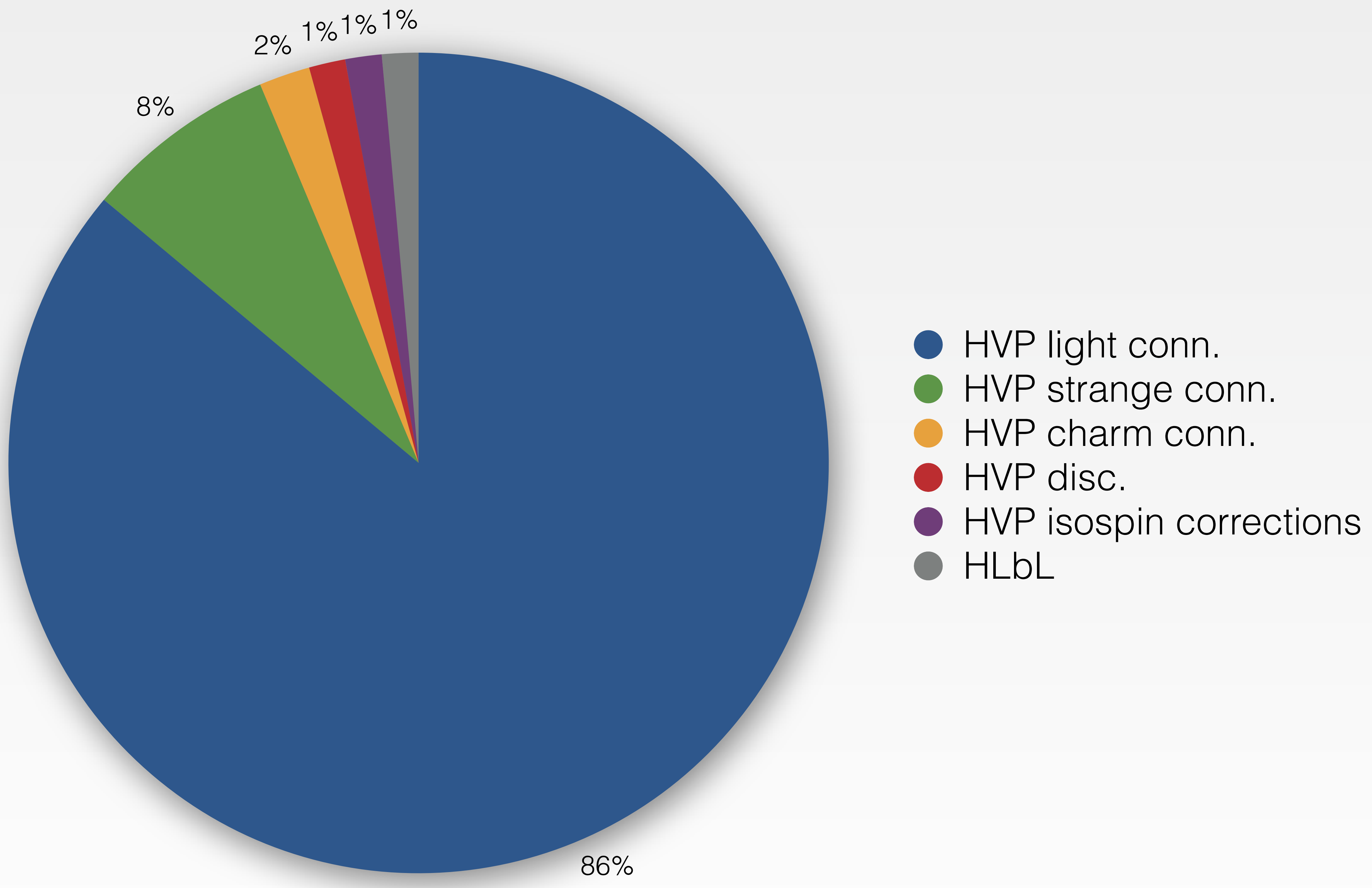
- HVP: dispersive analysis using $e^+e^- \to$ had. data
- HLbL: models
- Quantities dominated by non-perturbative QCD
- **Lattice calculations can increase precision and reliability. Possible target: per-mil precision**
- This talk focuses on the LO HVP

HVP diagram separation



- Different separations: flavours and topologies
- Unphysical separations, but **different numerical properties**
- Using **taylor-made methods** for each term helps reaching high precision on the total
- This talk: **strange connected** and **all flavours disconnected**

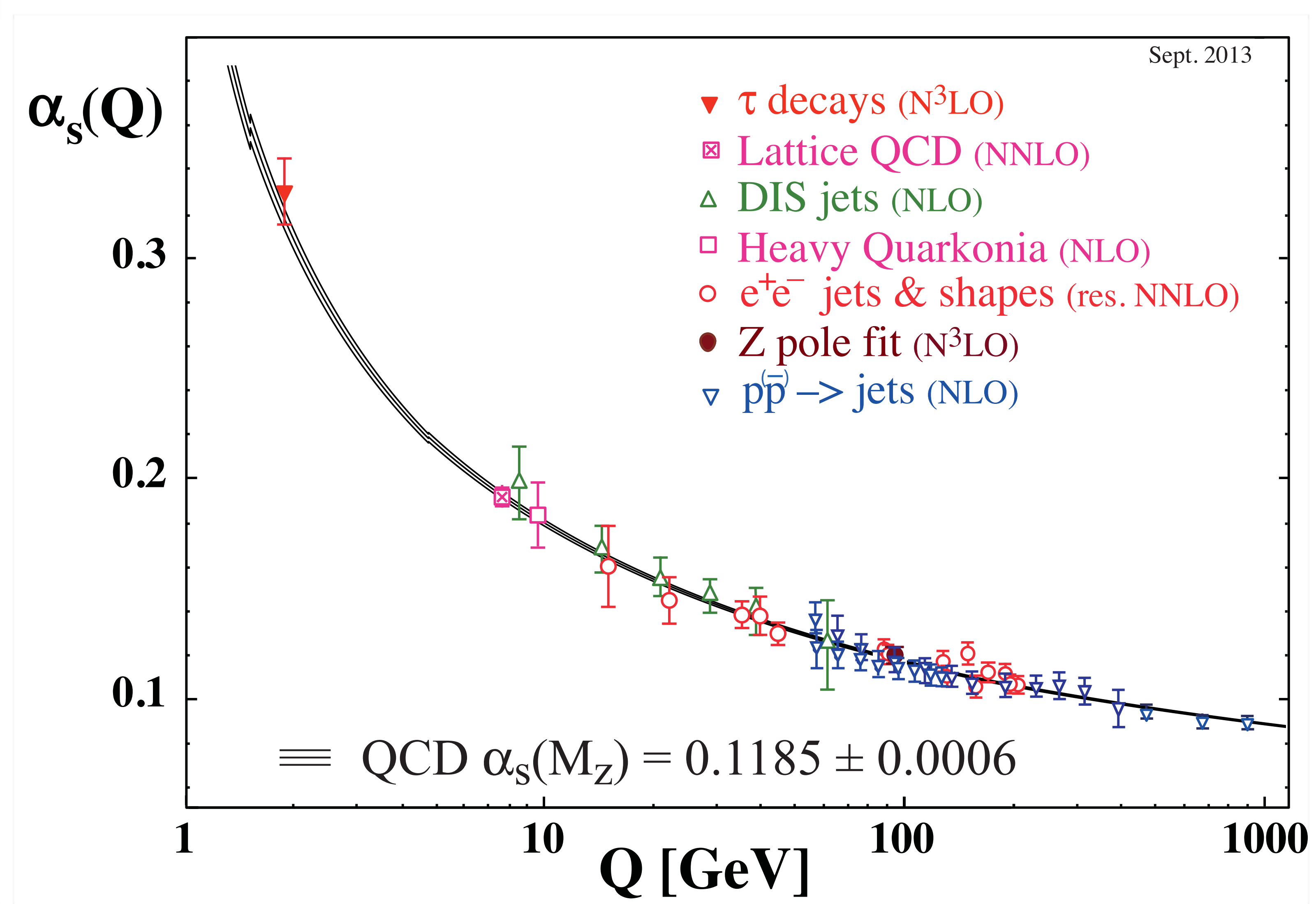
Hadronic contribution summary



Disclaimer: these are of course just rough estimates

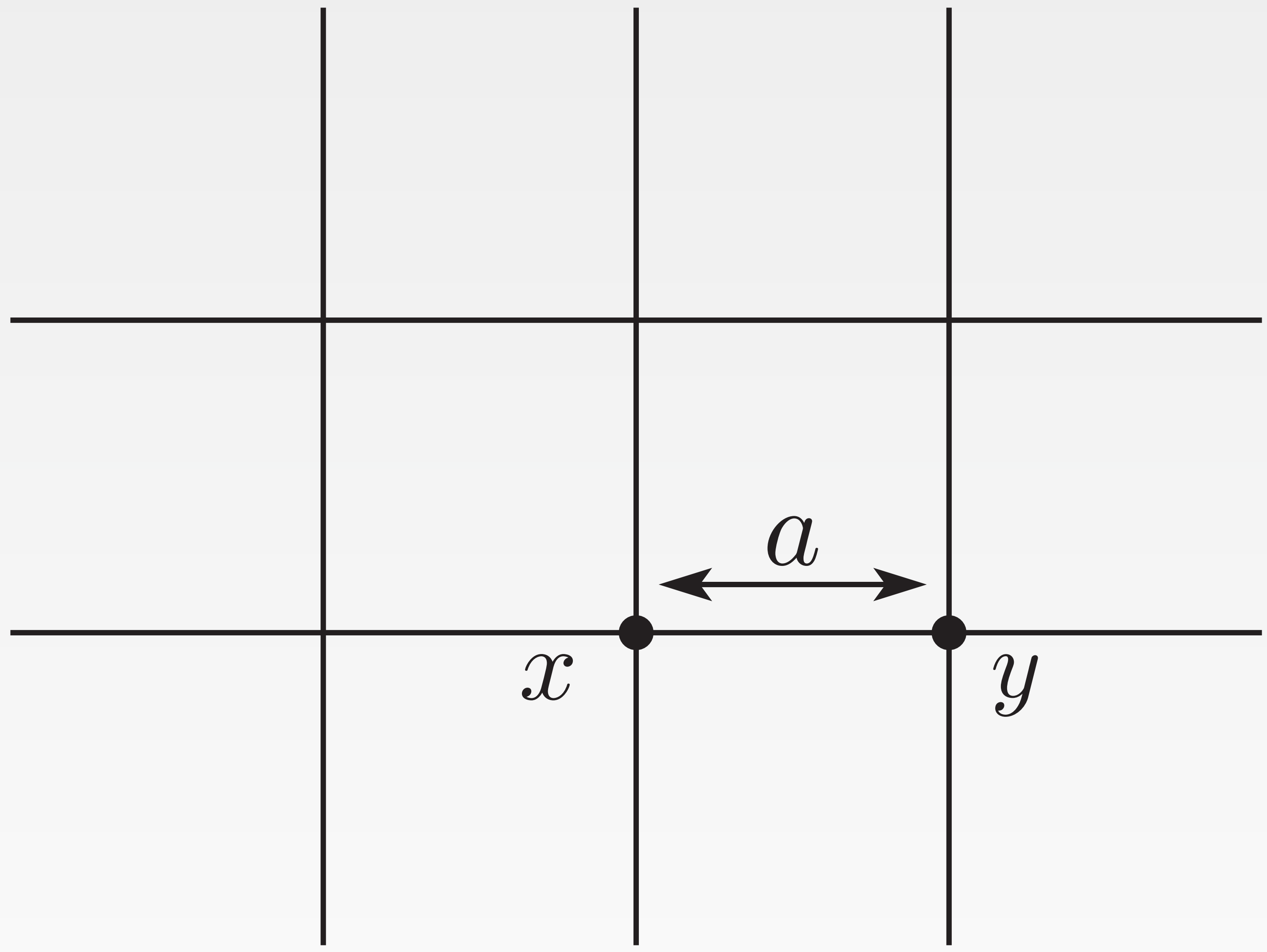
Lattice methodology

QCD at low energies



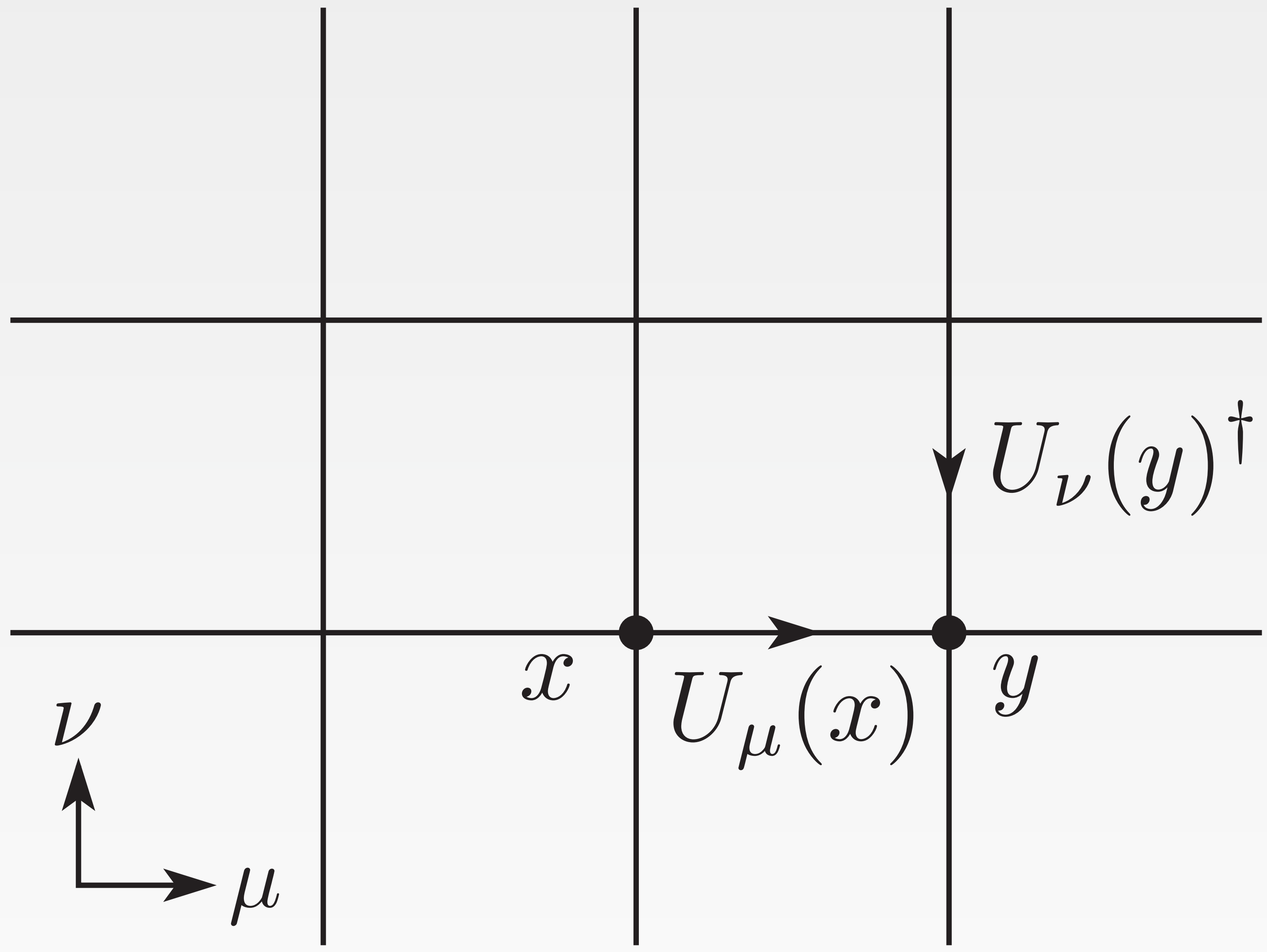
- QCD is **strongly non-perturbative** at GeV scale
- Non-perturbative phenomena like **colour confinement** dominate

Lattice gauge action



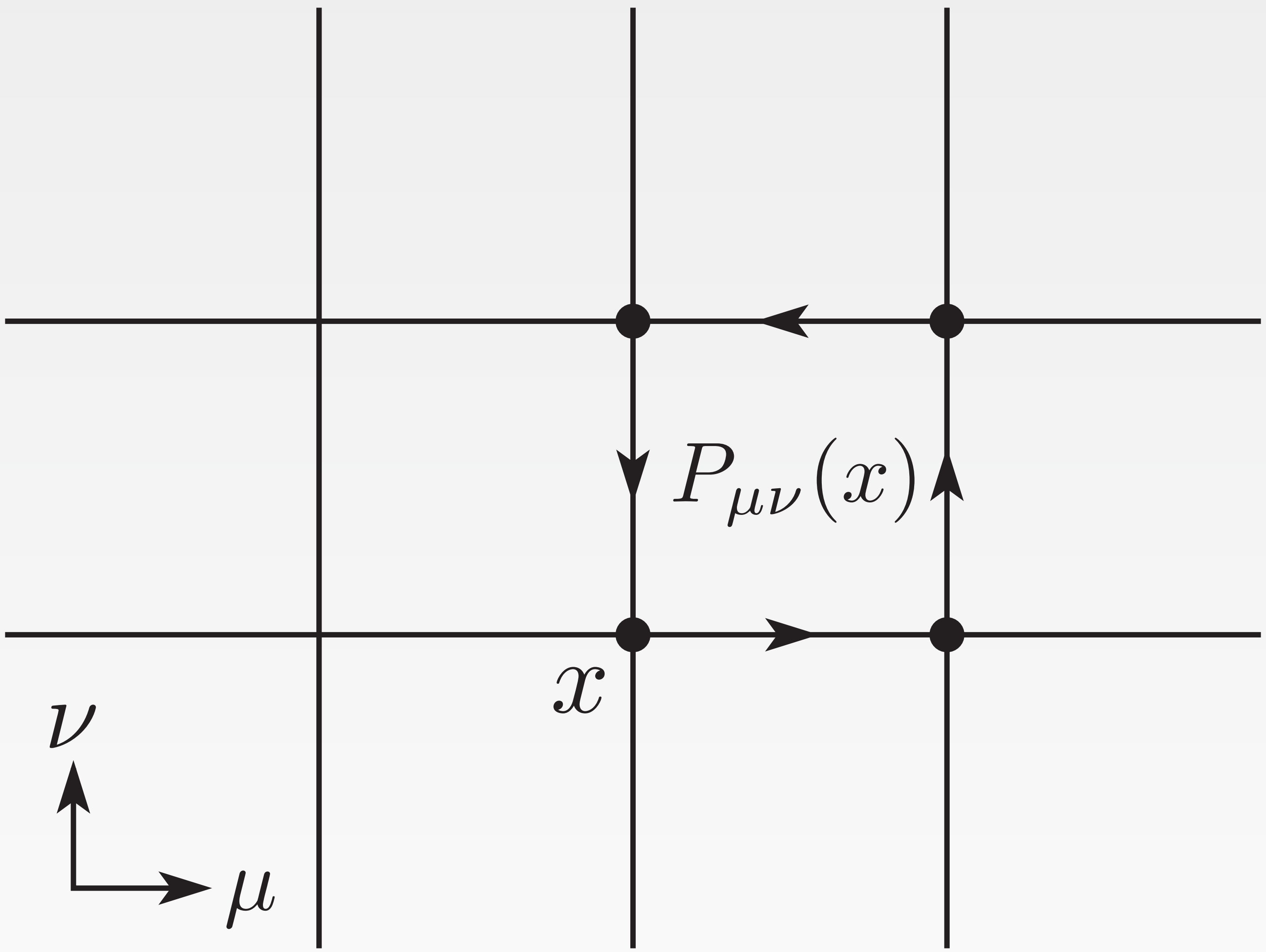
Euclidean space-time: 4D periodic hypercubic lattice

Lattice gauge action



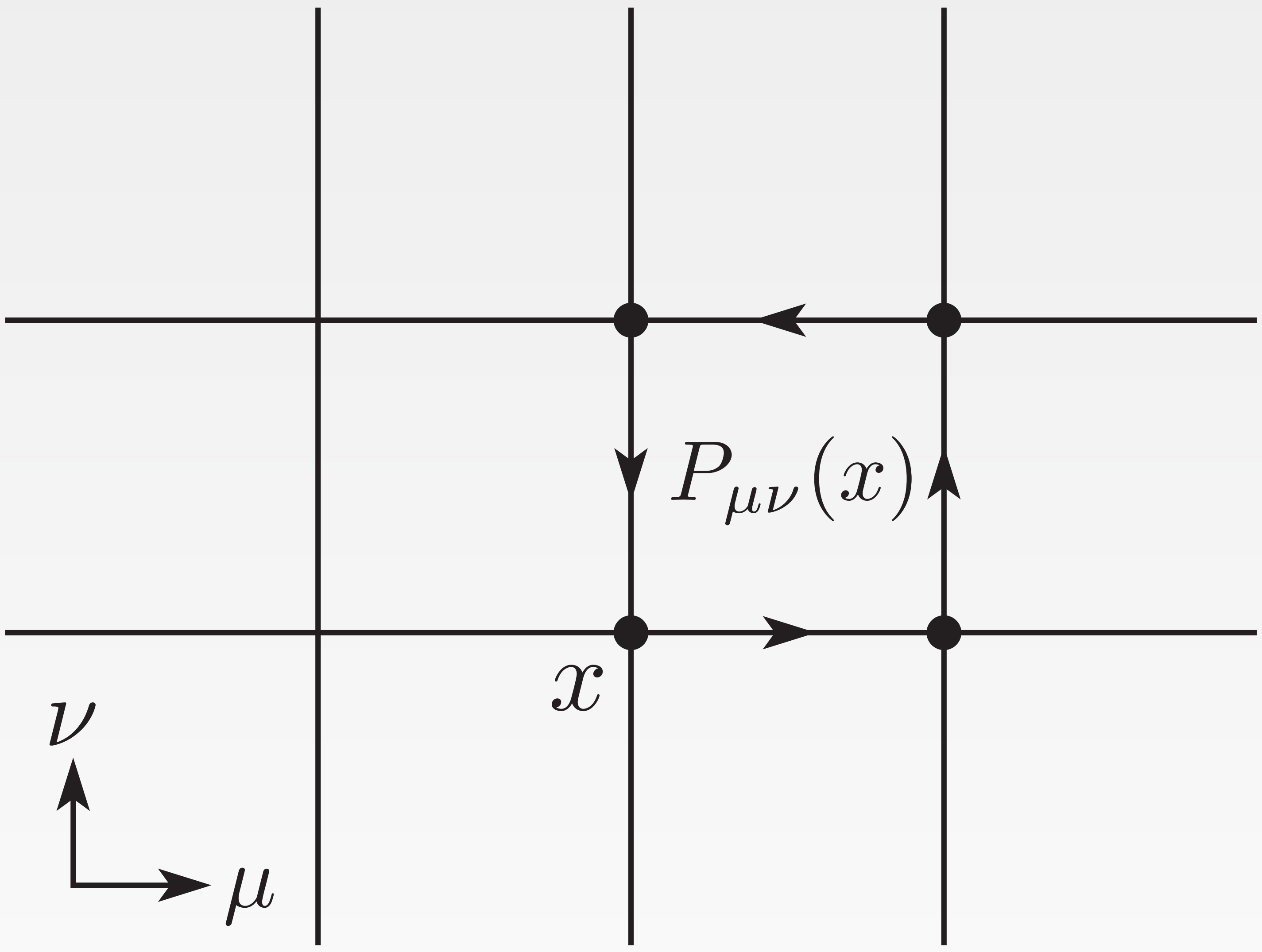
Gauge variables: straight Wilson lines over one lattice spacing

Lattice gauge action



Most elementary gauge invariant quantity: the plaquette $P_{\mu\nu}$

Lattice gauge action



Wilson gauge action:

$$S_{\text{gauge}} = \frac{\beta}{N_c} \sum_{x,\nu>\mu} \Re\,\mathrm{tr}[1 - P_{\mu\nu}(x)] = \frac{\beta g^2}{8N_c} a^4 \sum_{x,\mu,\nu} F_{\mu\nu} F_{\mu\nu} + \mathrm{O}(a^2)$$

Lattice fermion action

- The naive discrete Dirac action contains 16 degenerate fermion “flavours” which survive in the continuum limit. It is the so-called **doublers problem**

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- **Ginsparg-Wilson fermions**: solution featuring a lattice version of chiral symmetry

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- One cannot have a local action, no doublers and chiral symmetry. *(Nielsen-Ninomiya theorem)*

- **Ginsparg-Wilson fermions**: solution featuring a lattice version of chiral symmetry

- Implementation used here: **Möbius domain-wall fermions**

Path integral calculation

$$\langle O \rangle = \frac{1}{\mathscr{Z}} \int \mathrm{d}\psi \mathrm{d}\overline{\psi} \mathrm{d}U_\mu \, O[\psi, \overline{\psi}] \exp(-S_{\mathrm{LQCD}}[\psi, \overline{\psi}, U_\mu])$$

Path integral calculation

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Lattice action is quadratic in the quark fields: integration can be done through Wick's theorem

Path integral calculation

$$\langle O \rangle = \frac{1}{\mathscr{Z}} \int \mathrm{d}U_\mu \, O[(D_\mathrm{W} + M)^{-1}] \det(D_\mathrm{W} + M) \exp(-S_\mathrm{gauge}[U_\mu])$$

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Probability weight in the integral

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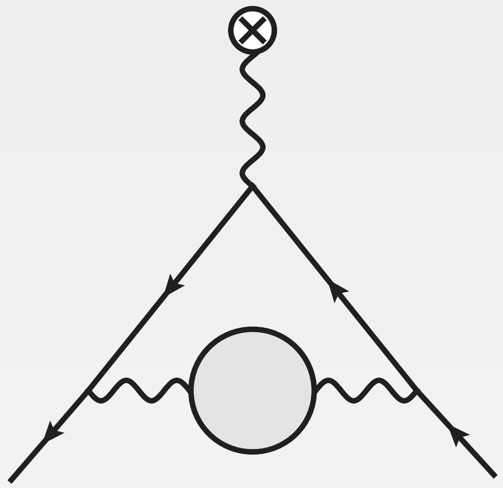
Probability weight in the integral

Monte-Carlo computation

HVP vertex loop integral



- EM current 2-point function in Euclidean space-time:

$$\Pi_{\mu\nu}(q) = \int \mathrm{d}^4x \, \langle 0 | \, \mathrm{T}[J_\mu(x) J_\nu(0)] \, |0\rangle \, e^{iq\cdot x} = (\delta_{\mu\nu} q^2 - q_\mu q_\nu)\Pi(q^2)$$

- Renormalisation: $\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$

- Vertex loop integral:

$$a_\mu^{(2)\mathrm{had.}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^{+\infty} \mathrm{d}q^2 \, \hat{\Pi}(q^2) f(q^2)$$

where $f(q^2)$ is a know function of q^2

HVP and momentum quantisation

- $\Pi_{\mu\nu}(q)$ can be computed directly on the lattice
- Finite volume: momentum quantisation
- $a_\mu^{(2)\mathrm{had.}}$ completely dominated by momenta around:

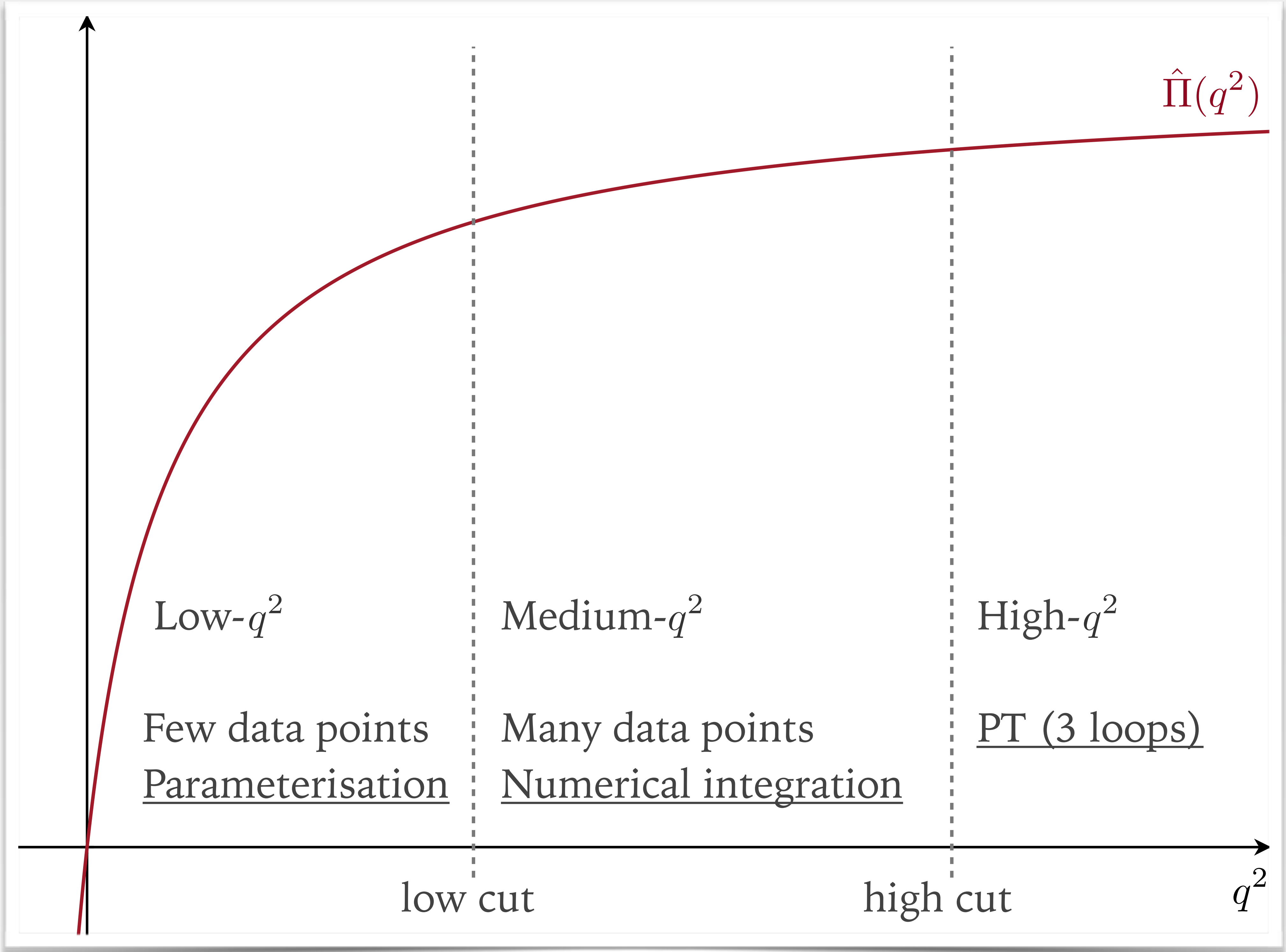
$$q^2 \sim m_\mu^2 \sim (100\ \mathrm{MeV})^2$$

- Typical finite-volume quantum:

$$(2\pi/10\ \mathrm{fm})^2 \sim (125\ \mathrm{MeV})^2$$

- Problem generally circumvented by parameterising the HVP form factor in the low-q^2 region

Hybrid method



proposed in [M. Golterman *et al.*, PRD 90(7), p. 074508, 2014]

Low-q^2 : Padé parametrisation

- Inspired by the spectral decomposition
- Padé $[N-1, N]$ ($2N-1$ parameters):

$$\Pi(q^2) = \Pi(0) + q^2 \sum_{j=1}^{N} \frac{a_j}{q^2 + b_j}$$

- Padé $[N, N]$ ($2N$ parameters):

$$\Pi(q^2) = \Pi(0) + q^2 \left(\sum_{j=1}^{N} \frac{a_j}{q^2 + b_j} + c \right)$$

- Rigorously converges to the true $\Pi(q^2)$
  [C. Aubin *et al.*, PRD 86(5), p. 54509, 2012]

Low-q^2 : conformal polynomials

- For some energy threshold E, analytical expansion:

$$\hat{\Pi}(q^2) = \sum_{n=1}^{+\infty} p_n w(q^2)^n \qquad w(q^2) = \frac{1 - \sqrt{1 + q^2/E^2}}{1 + \sqrt{1 + q^2/E^2}}$$

- Truncation ($N + 1$ parameters):

$$\Pi(q^2) = \Pi(0) + \sum_{n=1}^{N} p_n w(q^2)^n$$

proposed in [M. Golterman *et al.*, PRD 90(7), p. 074508, 2014]

Low-q^2 : parameter matching

- Traditional goodness-of-fit χ^2 minimisation

- Moments method: matching the Taylor expansion at $q^2 = 0$ to the moments of the correlation function

- Moments method applied by solving numerically:

$$\partial_q^n \left(\sum_{t,\mathbf{x}} C_{\mu\nu}(t,\mathbf{x}) e^{iqt} \right) \Big|_{q=0} = \partial_q^n \Pi_{\mu\nu}(q) \Big|_{q=0}$$

$$C_{\mu\nu} = \langle J_\mu(t,\mathbf{x}) J_\nu(0) \rangle$$

for some discrete differentiation operator ∂_q

extension of [HPQCD, PRD 89(1), 2014]

Low-q^2 : zero-mode subtraction

- In infinite volume: $\Pi_{\mu\nu}(0) = \int \mathrm{d}^4x\, C_{\mu\nu}(x) = 0$
- In a periodic finite volume it does not have to be the case (constant, harmonic contributions)
- On can define a subtracted tensor:

$$\Pi_{\mu\nu}(q) = \overline{\Pi}_{\mu\nu}(q) - \Pi_{\mu\nu}(0)$$

- In practice it improves the signal at low q^2 because of correlations with the zero-mode

Sine cardinal interpolation

- Naive idea: using the Fourier transform with a continuous momentum variable:

$$\tilde{\Pi}_{\mu\nu}(q) = \sum_{t,\mathbf{x}} C_{\mu\nu}(t,\mathbf{x}) e^{iqt}$$

cf. [D. Bernecker and H. B. Meyer, EPJA 47(11), pp. 148–16, 2011]
[X. Feng *et al.*, PRD 88(3), p. 034505, 2013]

- $\tilde{\Pi}_{\mu\nu}(q)$ interpolates the discrete values of $\Pi_{\mu\nu}(q)$
- The interpolation error is $\mathrm{O}(e^{-M_\pi T})$
  [L. Del Debbio & A.P., to appear]
- Can be seen as a generalisation of Shannon's sampling theorem

The connected strange HVP (based on 10.1007/JHEP04(2016)037)

JHEP

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Lattice calculation of the leading strange quark-connected contribution to the muon $g-2$

The RBC/UKQCD collaboration

T. Blum,[a] P.A. Boyle,[b] L. Del Debbio,[b] R.J. Hudspith,[c] T. Izubuchi,[d,e] A. Jüttner,[f] C. Lehner,[d] R. Lewis,[c] K. Maltman,[g,h] M. Krstić Marinković,[f,i] A. Portelli[b,f] and M. Spraggs[f]

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[e] *RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, U.S.A.*

JHEP04(2016)

RBC-UKQCD simulations

	48I	64I
V	96×48^3	128×64^3
a^{-1} (GeV)	1.729(4)	2.358(7)
M_π (MeV)	139.2(4)	139.2(5)
M_K (MeV)	499.0(1)	507.6(2)

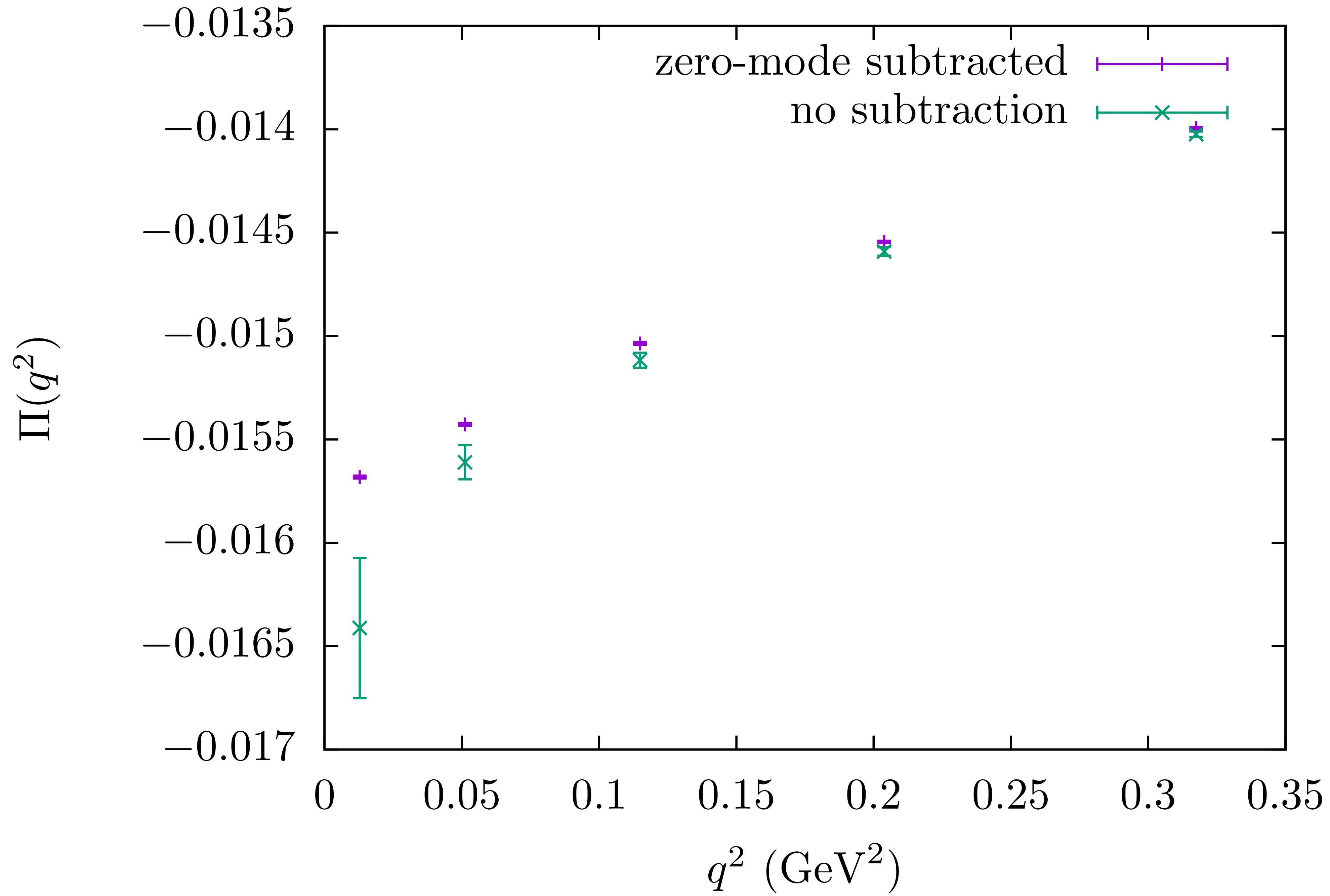
[RBC-UKQCD, arXiv:1411.7017]

- Möbius domain-wall fermions with Iwasaki gauge action
- 2+1 flavours at physical quark masses
- 2 lattice spacings, 1 volume $\sim (5.8 \text{ fm})^3$

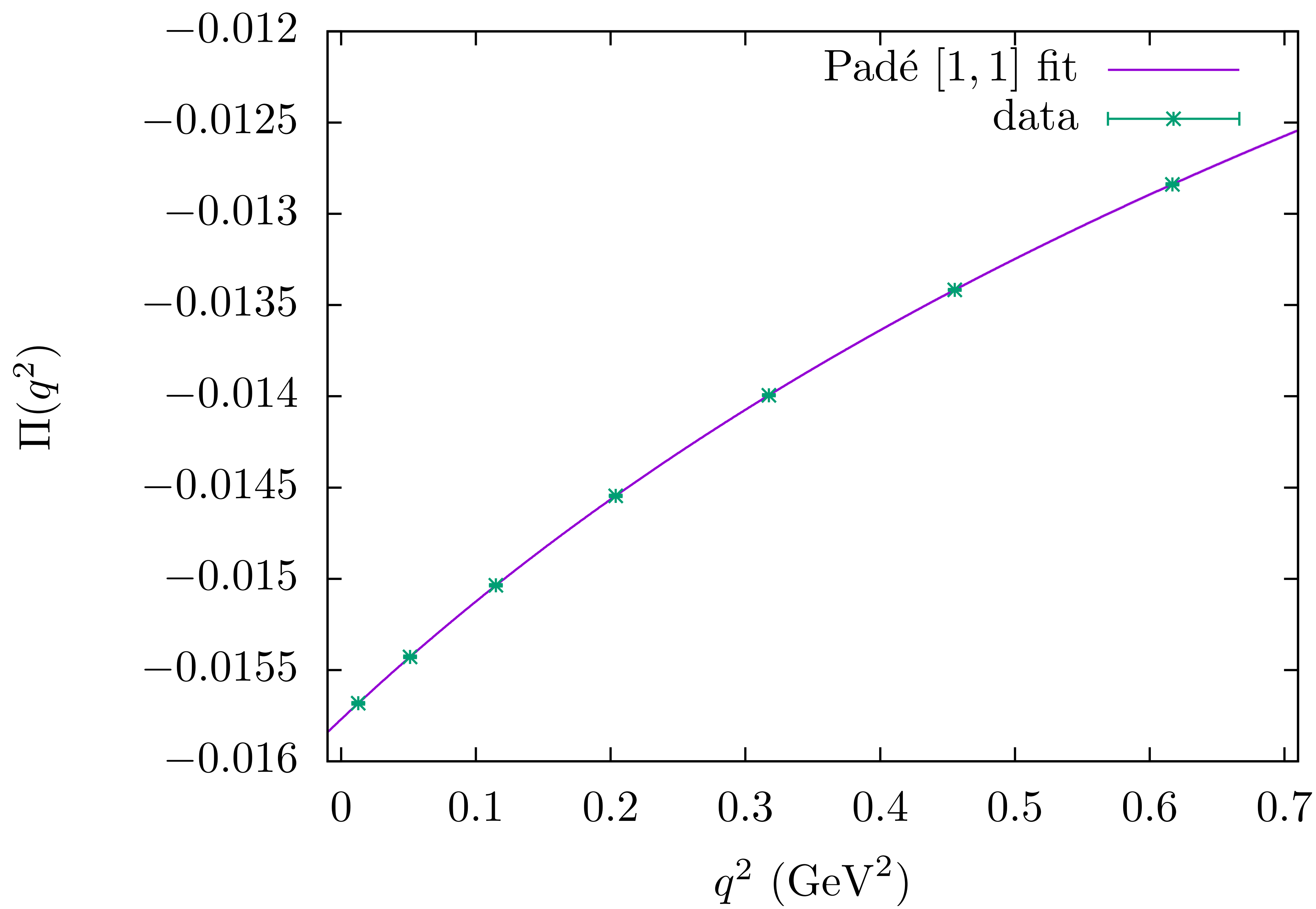
Measurement setup

- Conserved-local current 2-point functions: same WI than in the continuum
- $\mathbb{Z}_2$ wall stochastic sources, many sources per configuration
- 2 valence strange masses per ensemble
- This talk: only strange, connected results
- Physical light correlation functions very noisy

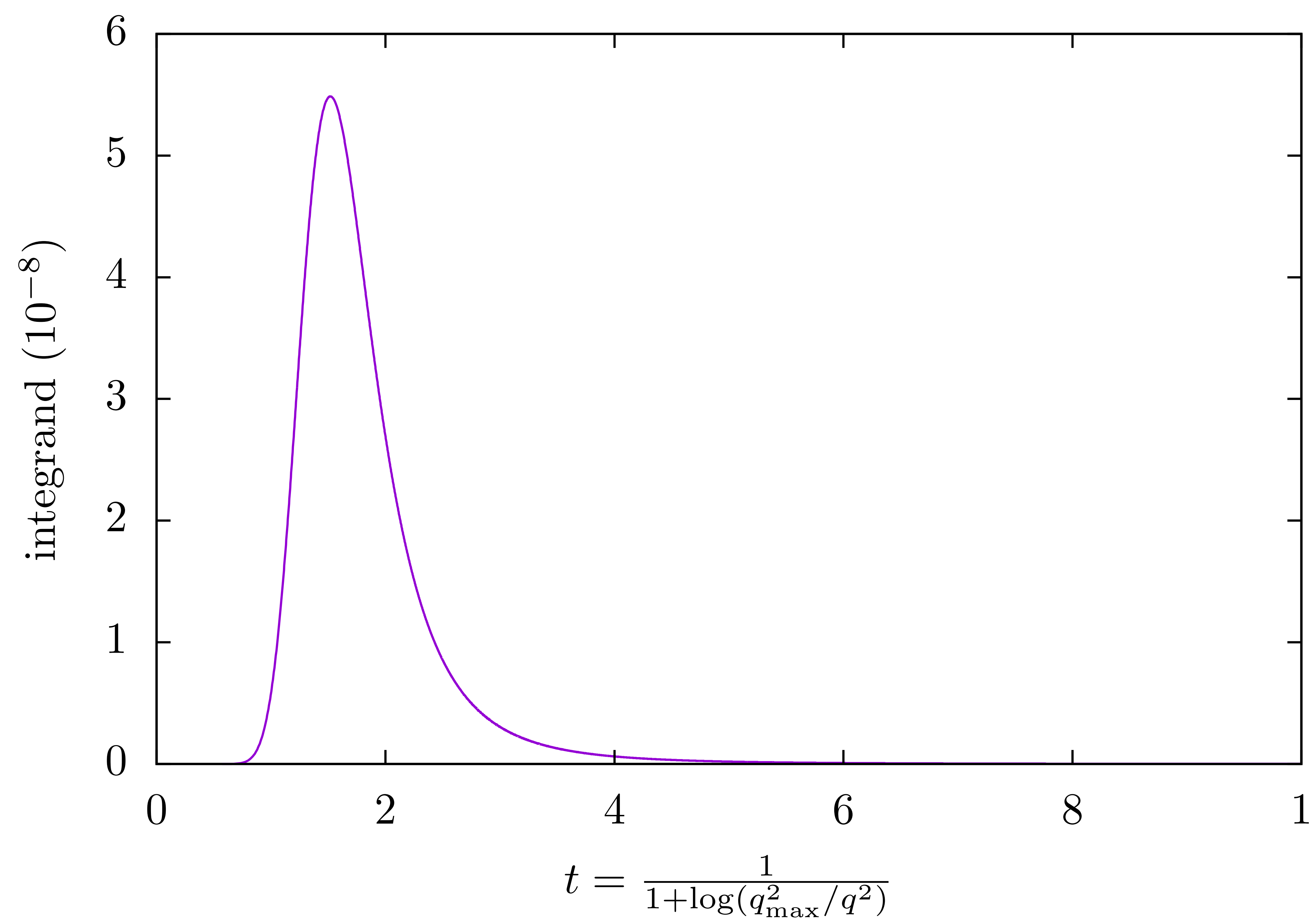
Zero-mode subtraction



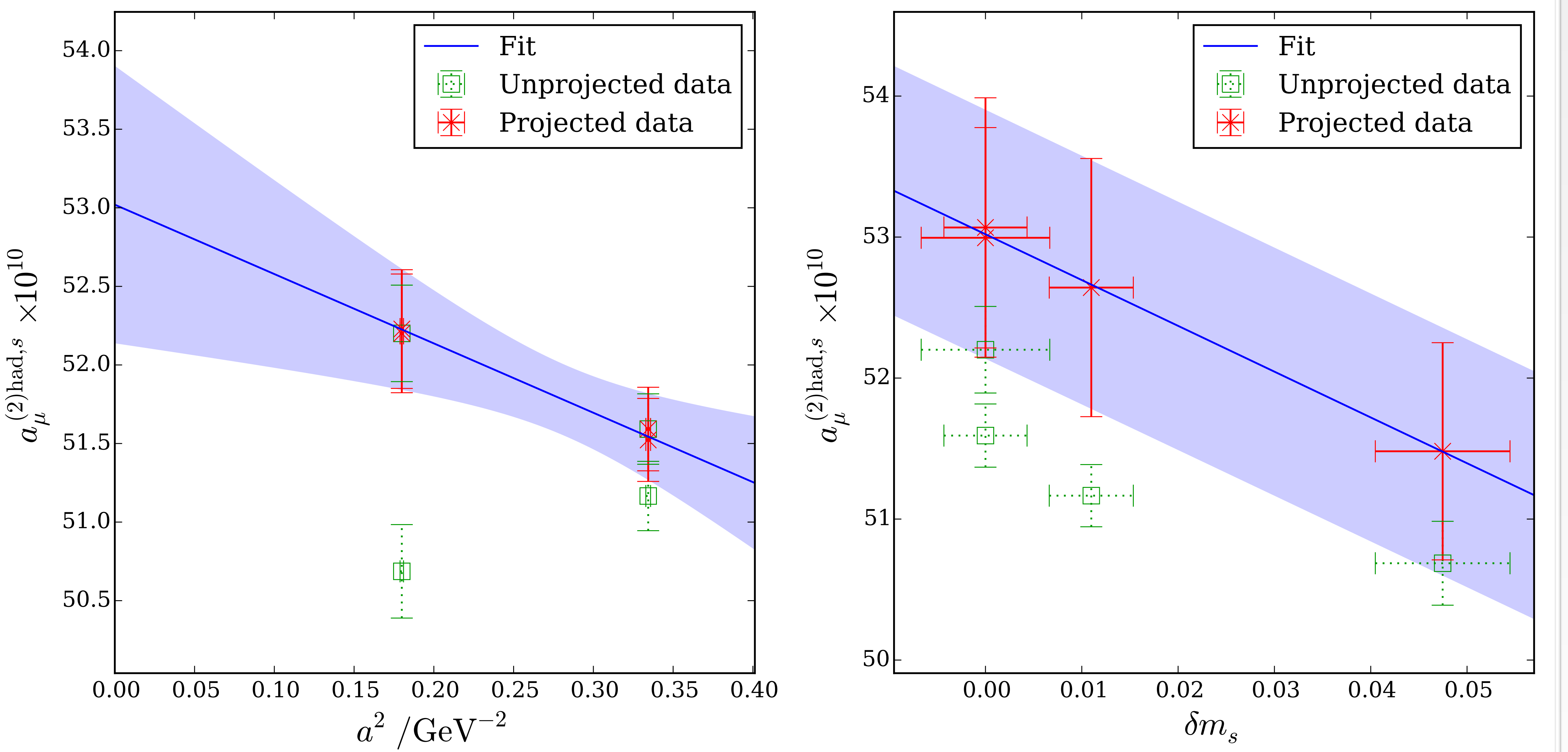
Low-q^2 fit


−0.012
−0.0125
−0.013
−0.0135
−0.014
−0.0145
−0.015
−0.0155
−0.016
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Π(q²)
q² (GeV²)
Padé [1, 1] fit
data

Sine cardinal interpolation



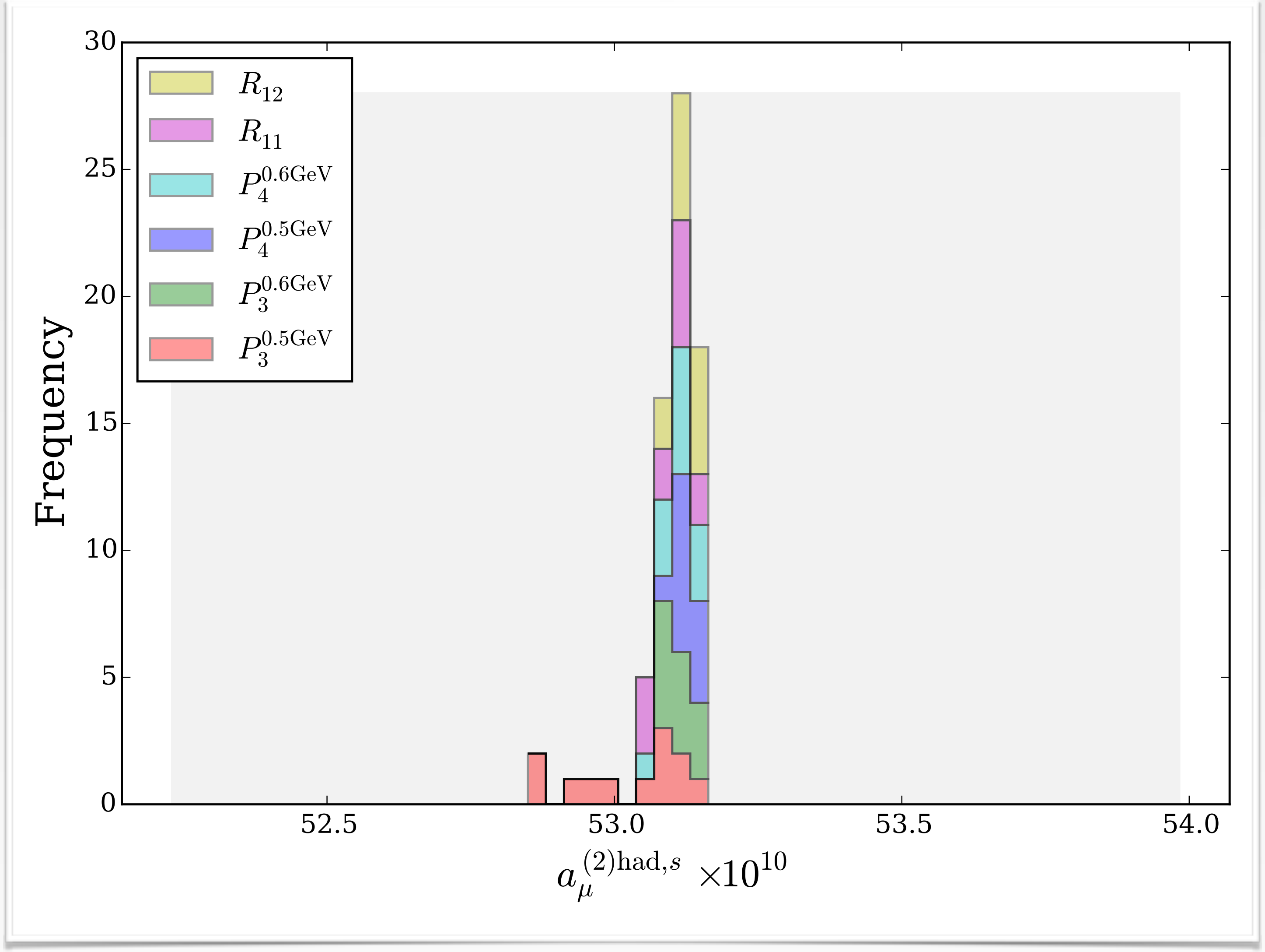
Physical point fit



Systematic error

- Low-q^2:
  - 6 parameterisations
  - 2 matching methods (fit & moments)
  - 3 low cuts (0.5, 0.6 and 0.7 GeV^2)
- Medium-q^2 :
  - 2 interpolation methods (linear & quadratic)
- Plus sine cardinal interpolation
- **73 ways** to obtain a physical value

Systematic errors



- Central value: median of all results
- Statistical error: variance of the central value
- Systematic error: support of the result distribution

inspired by [BMWc, Science 322, pp. 1224-1227, 2008]

Final result

- We obtained:

$$a_\mu^{(2)\text{had.},s} = 531(9)_{\text{stat.}}(1)_{\text{sys.}} \times 10^{-11}$$

- In very good agreement with [HPQCD, PRD 89(1), 2014]:

$$a_\mu^{(2)\text{had.},s} = 534.1(5.9) \times 10^{-11}$$

- Precision is competitive for an O(0.1%) determination of the total HVP contribution

The disconnected HVP (based on arXiv:1512.09054)

Calculation of the hadronic vacuum polarization disconnected contribution to the muon anomalous magnetic moment

T. Blum,[1] P.A. Boyle,[2] T. Izubuchi,[3,4] L. Jin,[5] A. Jüttner,[6] C. Lehner,[3,*]
K. Maltman,[7,8] M. Marinkovic,[9] A. Portelli,[2,6] and M. Spraggs[6]
(RBC and UKQCD Collaborations)
[1] *Physics Department, University of Connecticut, Storrs, CT 06269-3046, USA*
[2] *SUPA, School of Physics, The University of Edinburgh, Edinburgh EH9 3JZ, UK*
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[8] *CSSM, University of Adelaide, Adelaide 5005 SA, Australia*
[9] *CERN, Physics Department, 1211 Geneva 23, Switzerland*
(Dated: December 30, 2015)

We report the first lattice QCD calculation of the hadronic vacuum polarization disconnected contribution to the muon anomalous magnetic moment at physical pion mass. The calculation uses a refined noise-reduction technique which enabled the control of statistical uncertainties at the desired level with modest computational effort. Measurements were performed on the $48^3 \times 96$ physical-pion-mass lattice generated by the RBC and UKQCD collaborations. We find $a_\mu^{\mathrm{HVP\ (LO)\ DISC}} = -9.6(3.3)(2.3) \times 10^{-10}$, where the first error is statistical and the second systematic.

PACS numbers: 12.38.Gc

INTRODUCTION

The anomalous magnetic moment of leptons provides a powerful tool to test relativistic quantum-mechanical effects at tremendous precision. Consider the magnetic dipole moment of a fermion

rent experimental and theoretical determinations of a_μ,

$$a_\mu^{\mathrm{EXP}} - a_\mu^{\mathrm{SM}} = (27.6 \pm 8.0) \times 10^{-10} \text{ [3]}, \quad (25.0 \pm 8.0) \times 10^{-10} \text{ [4]}, \qquad (2)$$

where the experimental measurement is dominated by the BNL experiment E821 [5]. The theoretical prediction [6] is broken down in individual contributions in Tab. I.

Computational strategy

- Closed quark loops **notoriously difficult to compute** in lattice QCD
- Important **flavour SU(3) cancelation**: combining all flavours improve greatly the final precision
- Additional noise reduction achieved using **LMA** and **sparse stochastic sources**
- Contribution to $a_\mu^{(2)\mathrm{had.}}$ computed using the sine cardinal interpolation

Final result

- We obtained:

$$a_\mu^{(2)\text{had.,disc.}} = -96(33)_{\text{stat.}}(23)_{\text{sys.}} \times 10^{-11}$$

- In good agreement with [HPQCD, arXiv:1601.03071]:

$$a_\mu^{(2)\text{had.,disc.}} = 0(90) \times 10^{-11}$$

- Precision is competitive for an $\mathrm{O}(1\%)$ determination of the total HVP contribution

Outlook & perspectives

Outlook

- Clear lattice formulation of the LO HVP contribution to g-2
- Extensive systematic study of low-q^2 parametrisations
- The sine cardinal interpolation provides an precise parameter-free continuous description of $\Pi(q^2)$
- Stochastic sources combined with zero-mode subtraction greatly increases the signal
- Precise determination of the strange connected contribution to the HVP
- First statistically significant determination of the disconnected contribution to the HVP

In progress

- High-precision calculation of the connected charm part
- High-precision calculation of the connected light part
- Higher-precision calculation of the disconnected part
- Isospin breaking contributions
- Hadronic light-by-light
- Total precision of $O(0.1\%)$ probably achievable within the next $\sim$4 years

Thank you!