Electroweak structure of light nuclei

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2016 INT Program: Nuclear Physics from Lattice QCD - Seattle, WA

∗ in collaboration with ∗
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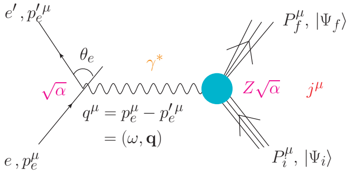
PRC78(2008)064002 - PRC80(2009)034004 - PRC84(2011)024001 - PRC87(2013)014006 - PRC87(2013)035503 - PRL111(2013)062502 - PRC90(2014)024321 - JPhysG41(2014)123002

Ab initio calculations of light nuclei

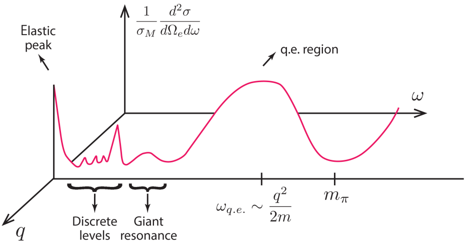
Aim: understand nuclei in terms of interactions between individual nucleons



Electromagnetic reactions are a powerful tool to test our theoretical models

- coupling constant $\alpha \sim 1/137$ allows for a perturbative treatment of the EM interaction $\rightarrow$ single photon γ exchange suffices
- calculated x-sections $\propto |\langle\Psi_f|j^{\mu}|\Psi_i\rangle|^2$ with j^{μ} nuclear EM currents $\rightarrow$ clear connection between measured x-sections and calculated properties of nuclear targets
- EXPT data (in most cases) known with great accuracy $\rightarrow$ viable EXPT constraints on theories
- For few-nucleon systems, the many-body problem can be solved exactly or within controlled approximations

Electromagnetic probes to test predictive power of nuclear theories/models



- In this talk we primarily focus on:
  EM ground state properties and transitions between low-lying states

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* Validate our theoretical understanding and control of nuclear EM structure and reactions is an essential prerequisite for studies on: *

⇒ Weak induced reactions, *e.g.*, ν-nucleus interactions
(major progress by A. Lovato, S. Gandolfi *et al.*)

⇒ Larger nuclear systems

- From LQCD to EFTs to AIM calculations,
  - input to AIM: nucleonic form factors ***
  - input to AIM: strong and electroweak LECs entering many-body nuclear operators (*e.g.*, g_A, L_1, ...) ***
  - calculations of quantities that are not easily accessed experimentally can help calibrate AIM (three-body forces, radiative captures at low energies...)

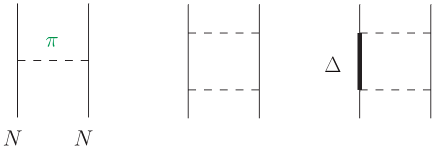
The Basic Model: Nuclear Potentials

- The nucleus is a system made of A non-relativistic interacting nucleons, its energy is given by

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i<j} \upsilon_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

where υ_{ij} and V_{ijk} are 2- and 3-nucleon interaction operators

- Realistic υ_{ij} and V_{ijk} interactions are based on EXPT data fitting and fitted parameters subsume underlying QCD
- Realistic potentials at large inter-particle distances are described in terms of one-pion-exchange, range $\sim 1/m_\pi$. Other mechanisms are, *e.g.*, two-pion exchange, range $\sim 1/2m_\pi$; Δ-excitations ...

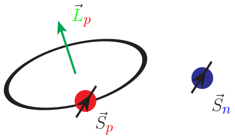


- Potentials utilized in these sets of calculations to generate nuclear wave functions $|\Psi_i\rangle$ solving $H|\Psi_i\rangle = E_i|\Psi_i\rangle$ are: [AV18+UIX], [AV18+IL7], [NN(N3LO)+3N(N2LO)]

- Current and charge operators describe the interaction of nuclei with external fields. They are expanded as a sum of 1−, 2−, ... nucleon operators:

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \ldots , \qquad \mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \ldots$$

- In Impulse Approximation IA nuclear EM currents are expressed in terms of those associated with individual protons and nucleons, *i.e.*, ρ_i and $\mathbf{j}_i$

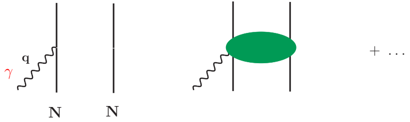


- IA picture is however incomplete; Historical evidence is the 10% underestimate of the *np* radiative capture 'fixed' by incorporating corrections from two-body meson-exchange EM currents - Riska&Brown 1972

The Basic Model: Nuclear Electromagnetic Currents

- Current and charge operators describe the interaction of nuclei with external fields. They are expanded as a sum of 1−, 2−, ... nucleon operators:

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \ldots , \qquad \mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \ldots$$



- Longitudinal EM current operator **j** linked to the nuclear Hamiltonian via continuity eq. (**q** momentum carried by the external EM probe γ)

$$\mathbf{q} \cdot \mathbf{j} = [H, \rho] = [t_i + \upsilon_{ij} + V_{ijk}, \rho]$$

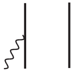
* Meson-exchange currents MEC follow once meson-exchange mechanisms are implemented to describe nuclear forces - Villars&Miyazawa 40ies

These days we have:

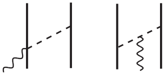
  - Highly sophisticated MEC projected out realistic potentials
  - EM currents derived from χEFTs

χEFT EM current up to $n = 1$ (or up to N3LO)

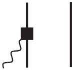
LO : $\mathbf{j}^{(-2)} \sim e Q^{-2}$



NLO : $\mathbf{j}^{(-1)} \sim e Q^{-1}$



N^{2}LO : $\mathbf{j}^{(-0)} \sim e Q^{0}$



* Two-body charge operators enter at N3LO and do not depend on LECs *

- LO = IA
  N2LO = IA(relativistic-correction)
- Strong contact LECs at N3LO fixed from fits to *np* phases shifts
  PRC**68**, 041001 (2003)
- Unknown EM LECs enter the N3LO contact and tree-level currents

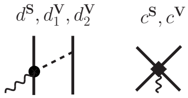
- No three-body EM currents at this order !!!
- NLO and N3LO loop-contributions lead to purely isovector operators

N^{3}LO: $\mathbf{j}^{(1)} \sim e Q$

unknown LEC′s →

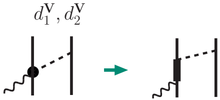
PRC78(2008)064002, PRC80(2009)034004, PRC84(2011)024001

χEFT EM currents at N3LO: fixing the EM LECs



Five LECs: d^S, d_1^V, and d_2^V could be determined by pion photo-production data on the nucleon

Isovector



d_2^V and d_1^V are known assuming Δ-resonance saturation

Left with 3 LECs: Fixed in the $A = 2 - 3$ nucleons' sector

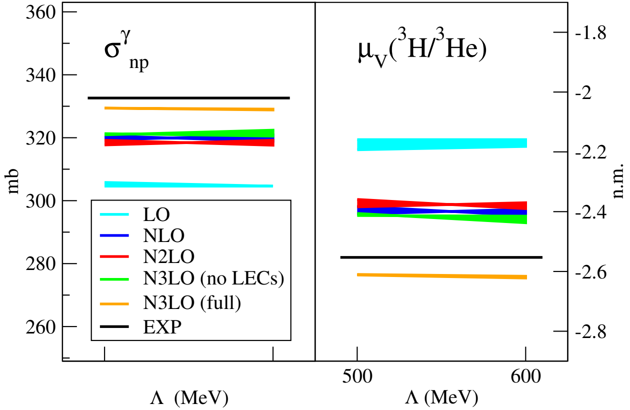
- Isoscalar sector:
  - d^S and c^S from EXPT μ_d and $\mu_S(^3\text{H}/^3\text{He})$
- Isovector sector:
  - model I = c^V from EXPT $npd\gamma$ xsec.

    or
  - model II = c^V from EXPT $\mu_V(^3\text{H}/^3\text{He})$ m.m. $\leftarrow$ our choice

Λ	NN/NNN	$10 \times d^S$	c^S	c^V
600	AV18/UIX (N3LO/N2LO)	–2.033 (3.231)	5.238 (11.38)	-1.025(-11.69)

Predictions with χEFT EM currents for $A = 2$–3 systems

np capture xsec. (using model II) / μ_V of $A = 3$ nuclei (using model I)
bands represent nuclear model dependence (N3LO/N2LO – AV18/UIX)

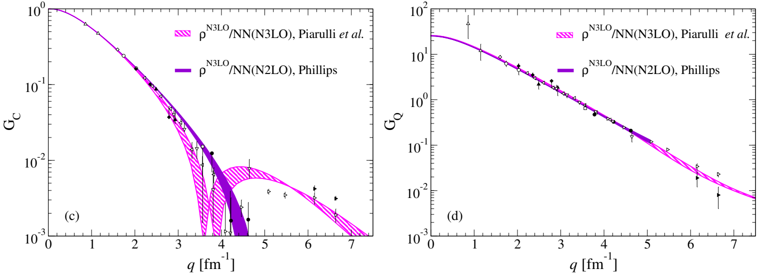


- $npd\gamma$ xsec. and $\mu_V(^3\text{H}/^3\text{He})$ m.m. are within 1% and 3% of EXPT
- Two-body currents important to reach agreement with exp data
- Negligible dependence on the cutoff entering the regulator $exp(-(k/\Lambda)^4)$

PRC87(2013)014006

Applications:
EM form factors of nuclei with $A = 2$ and 3

Predictions with χEFT EM Currents for the Deuteron Charge and Quadrupole f.f.'s

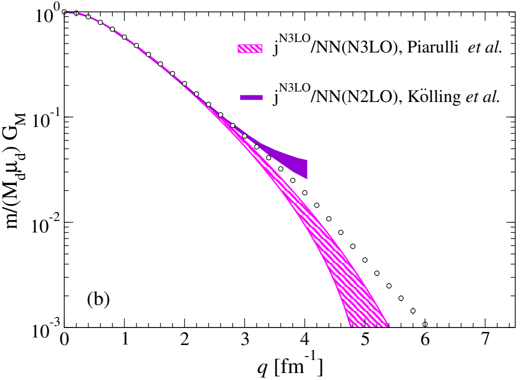


Λ MeV	$< r_d >$ (fm)	$< r_d >$ EXP	Q_d (fm^2)	Q_d (fm^2) EXP
500	1.976	1.9734(44)	0.285	0.2859(3)
600	1.968		0.282	

- Calculations include nucleonic f.f.'s taken from EXPT data
- Sensitivity to the cutoff used to regularize divergencies in the matrix elements is given by the bands' thickness

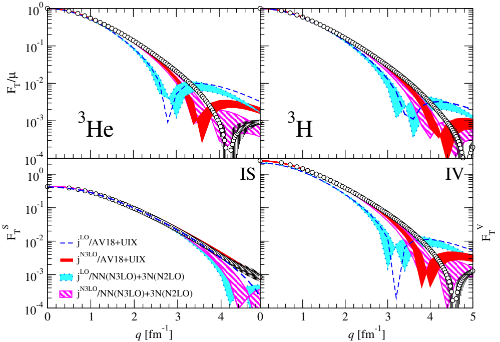
J.Phys.G34(2007)365 & PRC87(2013)014006

Predictions with χEFT EM Currents for the Deuteron Magnetic f.f.



PRC86(2012)047001 & PRC87(2013)014006

Predictions with χEFT EM Currents for ^{3}He and ^{3}H Magnetic f.f.'s



LO/N3LO with AV18+UIX – LO/N3LO with χ-potentials NN(N3LO)+3N(N2LO)

- ^{3}He/^{3}H m.m.'s used to fix EM LECs; $\sim$ 15% correction from two-body currents
- Two-body corrections crucial to improve agreement with EXPT data

	^{3}He $< r >_{EXP} = 1.976 \pm 0.047$ fm		^{3}H $< r >_{EXP} = 1.840 \pm 0.181$ fm	
Λ	500	600	500	600
LO	2.098 (2.092)	2.090 (2.092)	1.924 (1.918)	1.914 (1.918)
N3LO	1.927 (1.915)	1.913 (1.924)	1.808 (1.792)	1.794 (1.797)

PRC87(2013)014006

Benchmark calculations of ^{3}He Zemach Moments*

Quote: Precise moments are useful observables for the comparison with theoretical calculations, … in particular for light nuclei where very accurate *ab initio* calculations can be performed. I. Sick - PRC90(2014)064002

$$\langle r \rangle_{(2)} \propto -\int_0^\infty \frac{dq}{q^2}\left[G_E G_M - 1\right], \qquad \langle r^3 \rangle_{(2)} \propto \int_0^\infty \frac{dq}{q^4}\left[G_E^2 - 1 + q^2 R^2/3\right]$$

	VMC(IA)	VMC(TOT)	GFMC(IA)	GFMC(TOT)	EXPT
$\langle r \rangle_{(2)}$	2.522	2.477	2.504	2.454	2.528 ± 0.016 fm
$\langle r^3 \rangle_{(2)}$	27.40	n.a.	29.30	n.a.	28.15 ± 0.70 fm^3
$\langle r_{\rm ch}^2 \rangle^{1/2}$	1.967	n.a.	1.970	n.a.	1.973 ± 0.014 fm
$\langle r_{\rm m}^2 \rangle^{1/2}$	2.000	1.962	2.019	1.942	1.976 ± 0.047 fm
$\langle r_{\rm ch}^4 \rangle$	19.8	n.a.	30.0	n.a.	32.9 ± 1.60 fm^4
$\langle \mu \rangle$	-1.775	-2.134	-1.767	-2.129	-2.127 μ_N

* collaboration with
Nir Nievo, Chen Ji, Sonia Bacca, Maria Piarulli and Bob Wiringa

Preliminary!!!

Calculations with EM Currents from χEFT with π's and N's

- Park, Min, and Rho *et al.* (1996)

  applications to:
  magnetic moments and M1 properties of A=2–3 systems, and
  radiative captures in A=2–4 systems by Song, Lazauskas, Park *at al.*
  (2009-2011) within the hybrid approach

  ......
  * Based on EM χEFT currents from NPA596(1996)515

- Meissner and Walzl (2001);
  Kölling, Epelbaum, Krebs, and Meissner (2009–2011)

  applications to:
  d and ^{3}He photodisintegration by Rozpedzik *et al.* (2011); e-scattering (2014);
  d magnetic f.f. by Kölling, Epelbaum, Phillips (2012);
  radiative $N-d$ capture by Skibinski *et al.* (2014)

  ......
  * Based on EM χEFT currents from PRC80(2009)045502 & PRC84(2011)054008 and consistent χEFT potentials from UT method

- Phillips (2003-2007)

  applications to deuteron static properties and f.f.'s

  ......

Moving on to larger nuclear systems:
magnetic moments and transitions in $A \leq 10$ nuclei

Green's function Monte Carlo

A trial w.f. Ψ_V is obtained by minimizing the H = T + AV18 + IL7 expectation value

$$E_V = \frac{\langle\Psi_V|H|\Psi_V\rangle}{\langle\Psi_V|\Psi_V\rangle} \geq E_0$$

Ψ_V is further improved it by "filtering" out the remaining excited state contamination:

$$\Psi(\tau) = \exp[-(H-E_0)\tau]\Psi_V = \sum_n \exp[-(E_n-E_0)\tau]a_n\psi_n$$

$$\Psi(\tau\to\infty) = a_0\psi_0$$

Evaluation of $\Psi(\tau)$ is done stochastically (Monte Carlo method) in small time steps $\Delta\tau$ using a Green's function formulation.
In practice, we evaluate a "mixed" estimates

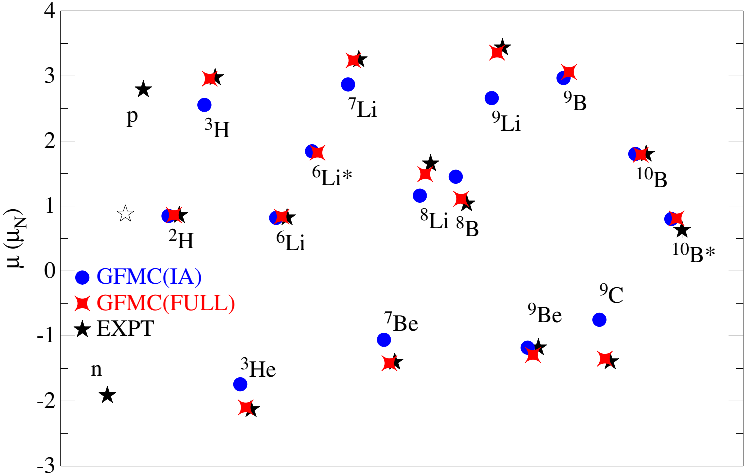
$$\langle O(\tau)\rangle = \frac{{}_f\langle\Psi(\tau)|O|\Psi(\tau)\rangle_i}{\langle\Psi(\tau)|\Psi(\tau)\rangle} \approx \langle O(\tau)\rangle^i_{\text{Mixed}} + \langle O(\tau)\rangle^f_{\text{Mixed}} - \langle O\rangle_V$$

$$\langle O(\tau)\rangle^i_{\text{Mixed}} = \frac{{}_f\langle\Psi_V|O|\Psi(\tau)\rangle_i}{{}_f\langle\Psi_V|\Psi(\tau)\rangle_i} \; ; \; \langle O(\tau)\rangle^f_{\text{Mixed}} = \frac{{}_f\langle\Psi(\tau)|O|\Psi_V\rangle_i}{{}_f\langle\Psi(\tau)|\Psi_V\rangle_i}$$

[Wiringa *et al.* PRC51(1995)38 + Piper *et al.* PRC64(2001)014001]

Magnetic Moments in $A \leq 10$ Nuclei

Predictions for $A > 3$ nuclei

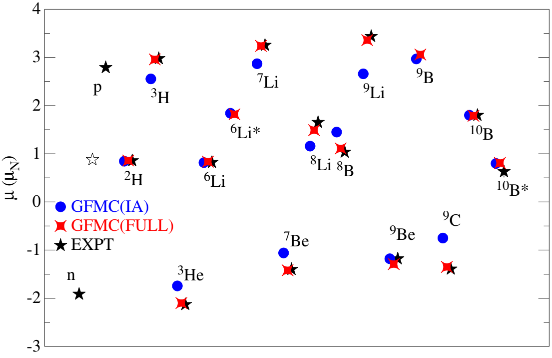


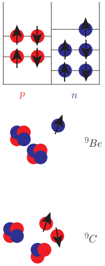
- $\mu(\text{IA}) = \mu_N \sum_i [(L_i + g_p S_i)(1+\tau_{i,z})/2 + g_n S_i (1-\tau_{i,z})/2]$
- GFMC calculations based on H = T + AV18 + IL7

PRC87(2013)035503

Magnetic Moments in $A \leq 10$ Nuclei - bis

Predictions for $A > 3$ nuclei



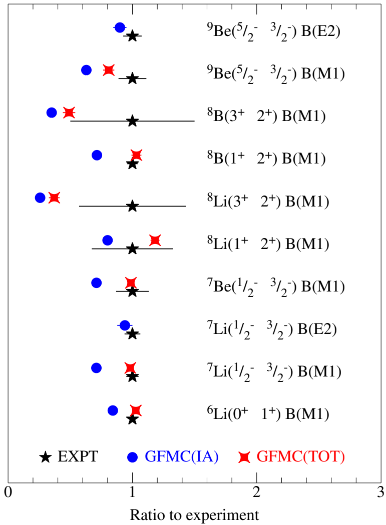


- $\mu_N(\text{IA}) = \sum_i [(L_i + g_p S_i)(1 + \tau_{i,z})/2 + g_n S_i (1 - \tau_{i,z})/2]$
- ^{9}C (^{9}Li) dominant spatial symmetry [s.s.] = [432] = [α,^{3}He(^{3}H),$pp(nn)$] $\rightarrow$ Large MEC
- ^{9}Be (^{9}B) dominant spatial symmetry [s.s.] = [441] = [$\alpha,\alpha,n(p)$]

PRC87(2013)035503

EM Transitions in $A \leq 9$ Nuclei

- Two-body EM currents bring the theory in a better agreement with the EXP
- Significant correction in $A = 9$, $T = 3/2$ systems. Up to $\sim 40\%$ correction found in ^{9}C m.m.
- Major correction ($\sim 60 - 70\%$ of total MEC) is due to the one-pion-exchange currents at NLO – purely isovector



One M1 prediction: ^{9}Li$(1/2 \to 3/2)$*

$\Gamma(\text{IA}) = 0.59(2)$ eV
$\Gamma(\text{TOT}) = 0.79(3)$ eV

+ a number of B(E2)s in IA

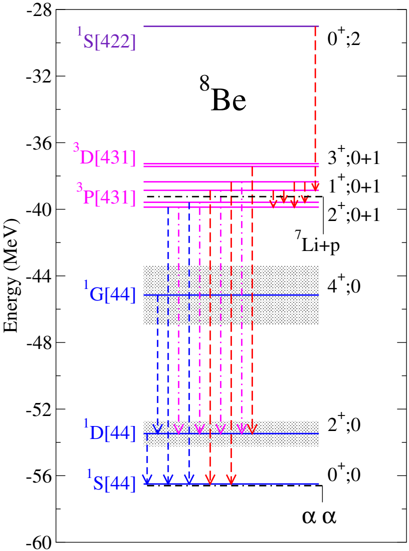
*Ricard-McCutchan *et al.* TRIUMF proposal 2014 - ongoing data analysis

PRC87(2013)035503

^{8}Be Energy Spectrum

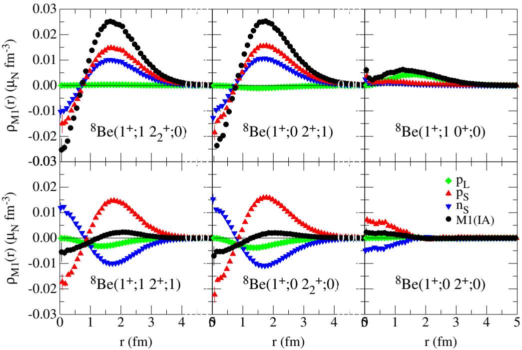
- 2^+ and 4^+ broad states at ~ 3 MeV and ~ 11 MeV
- isospin-mixed states at ~ 16 MeV, ~ 17 MeV, ~ 19 MeV
- M1 transitions
- E2 transitions
- E2 + M1 transitions

$J^\pi;T$	GFMC	Iso-mixed	Experiment
0^+	-56.3(1)		-56.50
2^+	+ 3.2(2)		+ 3.03(1)
4^+	+11.2(3)		+11.35(15)
$2^+;0$	+16.8(2)	+16.746(3)	+16.626(3)
$2^+;1$	+16.8(2)	+16.802(3)	+16.922(3)
$1^+;1$	+17.5(2)	+17.67	+17.640(1)
$1^+;0$	+18.0(2)	+18.12	+18.150(4)
$3^+;1$	+19.4(2)	+19.10	+19.07(3)
$3^+;0$	+19.9(2)	+19.21	+19.235(10)



PRL111(2013)062502 & PRC90(2014)024321

One-body M1 transitions densities

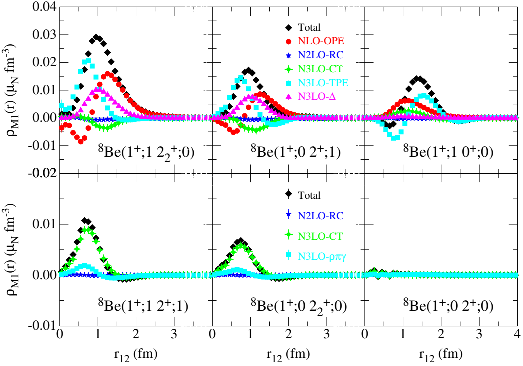


- [s.s.]-conserving transitions are enhanced due to overlap between large components of the initial and final w.f.'s
- Isospin-conserving transitions are suppressed w.r.t. isospin-changing transitions due to a cancellation between proton and neutron spin magnetization terms

$$\mathrm{M1(IA)} = \mu_N \sum_i [(L_i + g_p S_i)(1+\tau_{i,z})/2 + g_n S_i (1-\tau_{i,z})/2]$$

PRC90(2014)024321

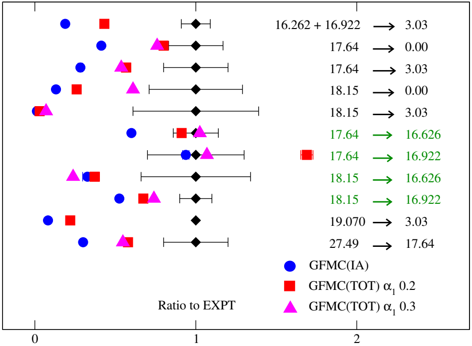
Two-body M1 transitions densities



$(J_i,T_i) \rightarrow (J_f,T_f)$	IA	NLO-OPE	N2LO-RC	N3LO-TPE	N3LO-CT	N3LO-Δ	MEC
$(1^+;1) \rightarrow (2_2^+;0)$	2.461 (13)	0.457 (3)	-0.058 (1)	0.095 (2)	-0.035 (3)	0.161 (21)	0.620 (5)

PRC90(2014)024321

M1 Transition Widths / EXPT

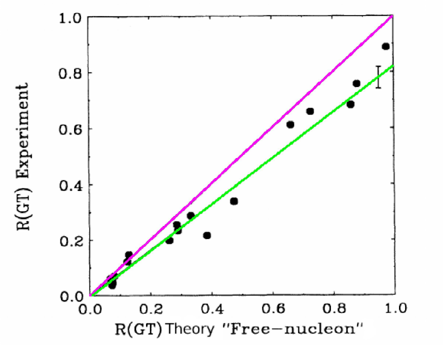


- Predictions for [s.s.]-conserving transitions are in fair agreement with EXPT
- The theoretical description for this system is unsatisfactory, however, MEC provide a $\sim 20 - 30\%$ correction to the calculated matrix elements improving the agreement with EXPT data

PRC90(2014)024321

Beta-decay rates for $A \leq 10$ nuclei

Theory vs Experiment: Quenching



$3 \leq A \leq 18$

Fig. from Chou *et al.* PRC47(1993)163

perfect agreement

theory > experiment

temporary fix:

$g_A^{\rm eff} \simeq 0.70\, g_A$

Quenching origin: *i*) better w.f.'s and/or *ii*) many body currents are required

$\beta\pm$ - $(J_i^\pi, T_i) \to (J_f^\pi, T_f)$	simple w.f.'s	IA	IA+MEC	Experiment
^{3}H(1/2$^+$,1/2) $\to$ ^{3}He(1/2$^+$,1/2)	2.449	2.2765(1)		2.357(10)*
^{6}He(0$^+$,1) $\to$ ^{6}Li(1+,0)	2.449	2.150	2.187	2.182*
^{7}Be(3/2$^-$,1/2) $\to$ ^{7}Li(3/2−,1/2)	2.582	2.292	2.395	2.290*
^{10}C(0$^+$,1) $\to$ ^{10}B(1+,0)	2.449	2.024	2.076	1.862*

Preliminary!!!

- in collaboration with B. Wiringa, S. Gandolfi, R. Schiavilla, J. Carlson

* data from TUNL compilations

* data from Suzuki *et al.* PRC67(2003)044302

Summary

The microscopic description of nuclei successfully reproduces EXPT data provided that many-body effects in nuclear interactions and EM currents are accounted for.

J.Phys.G41(2014)123002 - S.Bacca&S.P.

- Two-body EM currents from χEFT tested in $A \leq 10$ nuclei
- Two-body corrections can be sizable and improve on theory/EXPT agreement
- EM structure of $A = 2$–3 nuclei well reproduced with chiral charge and current operators for $q \lesssim 3m_\pi$
- $\sim 40\%$ two-body correction found in ^{9}C's m.m.
- $\sim$ 20-30% corrections found in M1 transitions in low-lying states of ^{8}Be

Outlook

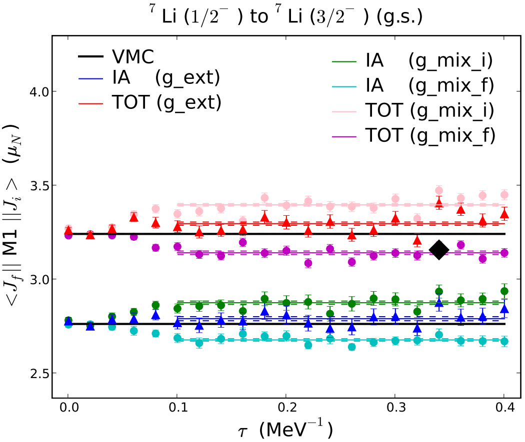
The microscopic description of nuclei successfully reproduces EXPT data provided that many-body effects in nuclear interactions and EM currents are accounted for.
J.Phys.G41(2014)123002 - S.Bacca&S.P.

* EM structure and dynamics of light nuclei
  - Charge and magnetic form factors of $A \leq 10$ systems
  - M1/E2 transitions in light nuclei
  - Radiative captures, photonuclear reactions ...
  - Fully consistent χEFT calculations with 'MEC' for $A > 4$
  - Role of Δ-resonances in 'MEC' (EM current consistent with the chiral 'Δ-full' NN potential developed by M. Piarulli et al. PRC91(2015)024003)

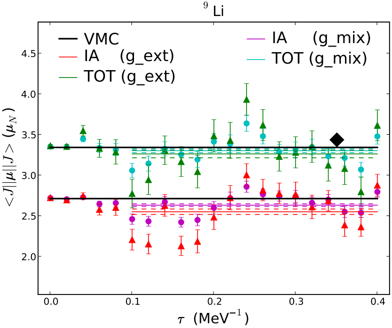
* Electroweak structure and dynamics of light nuclei
  - ν-nucleus scattering J. Carlson, S. Gandolfi, B. Wiringa, R. Schaivilla
  - Test axial currents (chiral and conventional) in light nuclei (A. Baroni et al.PRC93(2016)015501)
  - Many-body effects in ν-d pion-production at threshold (in preparation)

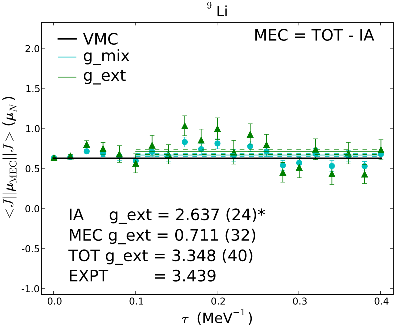
EXTRA SLIDES

Example of GFMC propagation: M1 Transition in $A = 7$



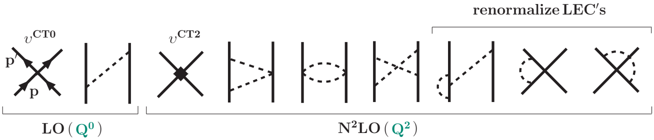
Examples of GFMC propagation: Magnetic moment in $A = 9$





Reduce noise by increasing the statistic for the IA results

NN Potential at NLO (or $Q^{n=2}$)

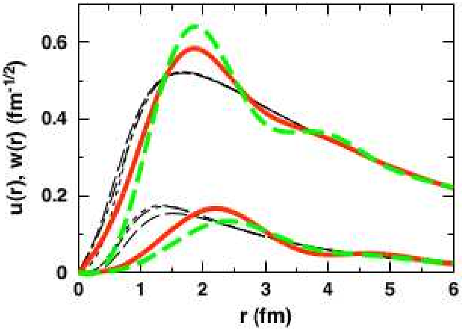


- Contact potential at LO (or $Q^{n=0}$) depends on 2 LECs
- Contact potential at NLO (or $Q^{n=2}$) depends on 7 additional LECs

NN potentials with π's and *N*'s

* van Kolck *et al.* (1994–96)
* Kaiser, Weise *et al.* (1997–98)
* Epelbaum, Glöckle, Meissner (1998–2015)
* Entem and Machleidt (2002–2015) ←
* …

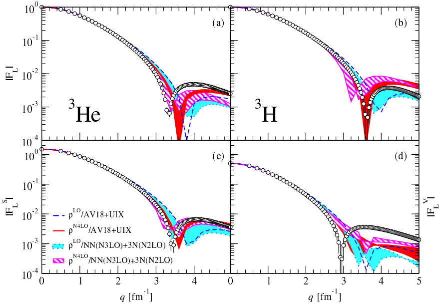
Deuteron wave functions



from Entem&Machleidt 2011 Review

- Entem&Machleidt N3LO
- Epelbaum *et al.* 2005
- black lines = conventional potentials, *i.e.* AV18, CD-Bonn, Nijm-I

^{3}He and ^{3}H charge f.f.'s

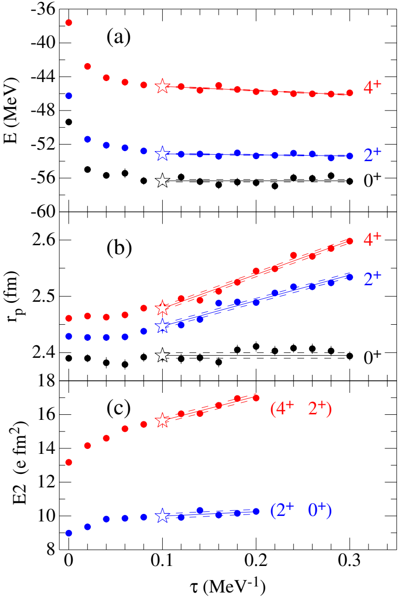


- Excellent agreement up to $q \simeq 2$ fm^{-1}
- N3LO and N4LO comparable

	^{3}He $< r >_{EXP} = 1.959 \pm 0.030$ fm		^{3}H $< r >_{EXP} = 1.755 \pm 0.086$	
Λ	500	600	500	600
LO	1.966 (1.950)	1.958 (1.950)	1.762 (1.743)	1.750 (1.743)
N4LO	1.966 (1.950)	1.958 (1.950)	1.762 (1.743)	1.750 (1.743)

E2 transitions in ^{8}Be

- 2^+ and 4^+ broad rotational states at ~ 3 MeV and ~ 11 MeV
- $4^+ \rightarrow 2^+$ transition recently measured at BARC*, Mumbai
- Calculational challenge: 2^+ and 4^+ states tend to break up into two α as τ increases
- Results obtained by linear fitting the GFMC points and extrapolating at $\tau = 0.1$ MeV where stability is observed in the g.s. energy propagation



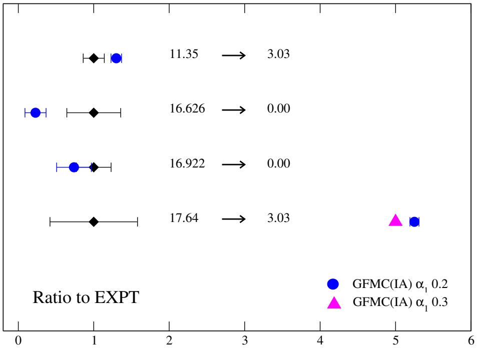
$J^\pi;T$	E [MeV]	B(E2) [e^2 fm^4]
0^+	-56.3(1)	
2^+	+ 3.2(2)	20.0 (8)– [$2^+ \rightarrow 0^+$]
4^+	+11.2(3)	27.2(15)– [$4^+ \rightarrow 2^+$]*

*Bhabha Atomic Research Centre

*EXPT B(E2) = 21 ± 2.3 e^2 fm^4

E2 transition widths / EXPT

- We attempt to evaluate a number of E2 transitions (predictions not shown in the figure)
- Complications are due to large cancellations among large m.e.'s $\rightarrow$ E2s very sensitive to small components
- One more complication: make sure that the first and second $(J^\pi, T) = (2^+, 0)$ states are orthogonal



  * We orthogonalize the second $(J^\pi, T) = (2^+, 0)$ via

$$|\Psi^{2_2^+}(\mathrm{ortho})\rangle_G = |\Psi^{2_2^+}\rangle_G - {}_G\langle\Psi^{2_2^+}|\Psi^{2^+}\rangle_V|\Psi^{2^+}\rangle_G$$

Anomalous magnetic moment of ^{9}C

Mirror nuclei spin expectation value

- Charge Symmetry Conserving (CSC) picture ($p \longleftrightarrow n$) $^\diamond$

$$< \sigma_z >= \frac{\mu(T_z=+T)+\mu(T_z=-T)-J}{(g_s^p+g_s^n-1)/2} = \frac{2\mu(\mathrm{IS})-J}{0.3796}$$

- For $A=9$, $T=3/2$ mirror nuclei: ^{9}C and ^{9}Li
EXP $<\sigma_z>= 1.44$ while THEORY $<\sigma_z>\sim 1$ (assuming CSC)
possible cause: Charge Symmetry Breaking (CSB)
- Three different predictions for $<\sigma_z>$ with CSC w.f.'s (*) and CSB w.f.'s

$<\sigma_z>$	Symmetry	IA	TOT	EXP
CSB	^{9}Li$(\frac{3}{2}^-;\frac{3}{2})$,^{9}C$(\frac{3}{2}^-;\frac{3}{2})$	1.05(1)	1.31(11)	1.44
CSC	^{9}Li$(\frac{3}{2}^-;\frac{3}{2})$,^{9}C$(\frac{3}{2}^-;\frac{3}{2})$*	0.95 (11)	1.00 (11)	
CSC	^{9}Li$(\frac{3}{2}^-;\frac{3}{2})$*,^{9}C$(\frac{3}{2}^-;\frac{3}{2})$	1.00 (1)	1.05 (1)	

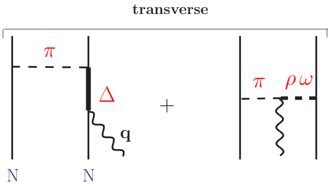
- Need both CSB in the w.f.'s and MEC!

$^\diamond$ Utsuno – PRC**70**, 011303(R) (2004)

Currents from nuclear interactions $*$- Marcucci *et al.* PRC**72**, 014001 (2005)

- Current operator **j** constructed so as to satisfy the continuity equation with a realistic Hamiltonian
- Short- and intermediate-behavior of the EM operators inferred from the nuclear two- and three-body potentials

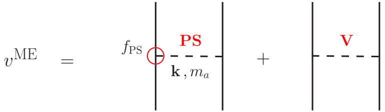
$$\mathbf{j} = \mathbf{j}^{(1)} + \mathbf{j}^{(2)}(v) + \mathbf{j}^{(3)}(V)$$



$*$ also referred to as Standard Nuclear Physics Approach (SNPA) currents

- Long range part of $\mathbf{j}(\upsilon)$ corresponds to OPE seagull and pion-in-flight EM currents

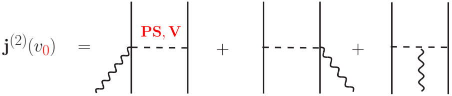
Currents from nuclear interactions - Marcucci *et al.* PRC**72**, 014001 (2005)



- Exploiting the meson exchange (ME) mechanism, one assumes that the static part υ_0 of υ is due to pseudoscalar (PS) and vector (V) exchanges
- $\upsilon^{\rm ME}$ is expressed in terms of 'effective propagators' υ_{PS}, υ_V, υ_{VS}, fixed such to reproduce υ_0, for example

$$\upsilon_{PS} = [\upsilon^{\sigma\tau}(k) - 2\upsilon^{t\tau}(k)]/3$$

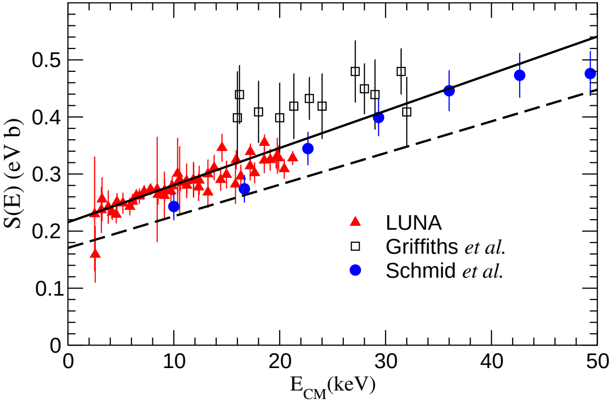
  with $\upsilon^{\sigma\tau}$ and $\upsilon^{t\tau}$ components of υ_0
- The current operator is obtained by taking the non relativistic reduction of the ME Feynman amplitudes and replacing the bare propagators with the 'effective' ones



Currents from nuclear interactions - Marcucci *et al.* PRC**72**, 014001 (2005)

Satisfactory description of a variety of nuclear EM properties [see Marcucci *et al.* (2005) and (2008)]

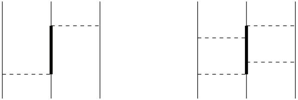
^{2}H(p,γ)^{3}He capture



- Isoscalar magnetic moments are a few % off (10% in A=7 nuclei) courtesy of R.B.Wiringa

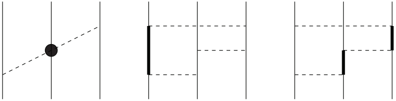
THREE-NUCLEON POTENTIALS

Urbana $V_{ijk} = V_{ijk}^{2\pi P} + V_{ijk}^{R}$



Carlson, Pandharipande, & Wiringa, NP **A401**, 59 (1983)

Illinois $V_{ijk} = V_{ijk}^{2\pi P} + V_{ijk}^{2\pi S} + V_{ijk}^{3\pi\Delta R} + V_{ijk}^{R}$



Pieper, Pandharipande, Wiringa, & Carlson, PRC **64**, 014001 (2001)

Illinois-7 has 4 strength parameters fit to 23 energy levels in $A \leq 10$ nuclei.
In light nuclei we find (thanks to large cancellation between $\langle K \rangle$ & $\langle v_{ij} \rangle$):

$$\langle V_{ijk} \rangle \sim (0.02 \text{ to } 0.07)\ \langle v_{ij} \rangle \sim (0.15 \text{ to } 0.5)\ \langle H \rangle$$

We expect $\langle \mathcal{V}_{ijkl} \rangle \sim 0.05\ \langle V_{ijk} \rangle \sim (0.01 \text{ to } 0.03)\ \langle H \rangle \sim 1$ MeV in ^{12}C .

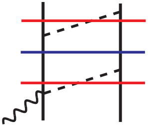
Transition amplitude in time-ordered perturbation theory

$$T_{\rm fi} = \langle f \mid T \mid i \rangle = \langle f \mid H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\eta} H_1 \right)^{n-1} \mid i \rangle$$

$$= \langle f \mid H_1 \mid i \rangle + \sum_{|I\rangle} \langle f \mid H_1 \mid I \rangle \frac{1}{E_i - E_I} \langle I \mid H_1 \mid i \rangle + \dots$$

- A contribution with N interaction vertices and L loops scales as

$$\underbrace{e \left(\prod_{i=1}^{N} Q^{\alpha_i - \beta_i/2} \right)}_{H_1 \,\text{scaling}} \times \underbrace{Q^{-(N-N_K-1)} \, Q^{-2N_K}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integration}}$$



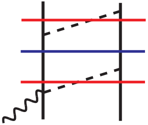
α_i = number of derivatives in H_1 and β_i = number of π's at each vertex

N_K = number of pure nucleonic intermediate states

- Due to the chiral expansion, the transition amplitude $T_{\rm fi}$ can be expanded as

$$T_{\rm fi} = T^{\rm LO} + T^{\rm NLO} + T^{\rm N2LO} + \dots \quad \text{and} \quad T^{\rm NnLO} \sim (Q/\Lambda_\chi)^n T^{\rm LO}$$

Power counting



- N_K energy denominators scale as Q^{-2}

$$\frac{1}{E_i - E_I}|I\rangle = \frac{1}{E_i - E_N}|I\rangle \sim Q^{-2}|I\rangle$$

- $(N - N_K - 1)$ energy denominators scale Q^{-1} in the static limit; they can be further expanded in powers of $(E_i - E_N)/\omega_\pi \sim Q$

$$\frac{1}{E_i - E_I}|I\rangle = \frac{1}{E_i - E_N - \omega_\pi}|I\rangle \sim -\Big[\underbrace{\frac{1}{\omega_\pi}}_{Q^{-1}} + \underbrace{\frac{E_i - E_N}{\omega_\pi^2}}_{Q^0} + \underbrace{\frac{(E_i - E_N)^2}{\omega_\pi^3}}_{Q^1} + \dots\Big]|I\rangle$$

- Terms accounted into the Lippmann-Schwinger Eq. are subtracted from the reducible amplitude
- EM operators depend on the off-the-energy shell prescription adopted for the non-static OPE and TPE potentials
- Ultimately, the EM operators are unitarily equivalent: Description of physical systems is not affected by this ambiguity

Magnetic moment at N^3LO

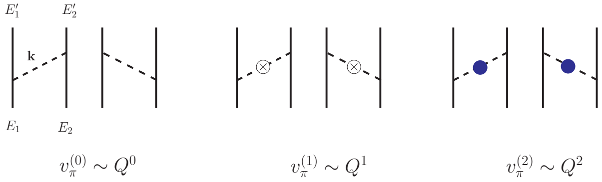
- Magnetic moment operator due to two-body current density $\mathbf{J}(\mathbf{x})$

$$\boldsymbol{\mu}(\mathbf{R},\mathbf{r}) = \frac{1}{2}\left[\mathbf{R}\times\int \mathrm{d}\mathbf{x}\,\mathbf{J}(\mathbf{x}) + \int \mathrm{d}\mathbf{x}\,(\mathbf{x}-\mathbf{R})\times\mathbf{J}(\mathbf{x})\right]$$

- Sachs' and translationally invariant magnetic moments

$$\begin{aligned}
\boldsymbol{\mu}_{\mathrm{Sachs}}(\mathbf{R},\mathbf{r}) &= -i\,\frac{\mathbf{R}}{2}\times\int \mathrm{d}\mathbf{x}\,\mathbf{x}\,[\rho(\mathbf{x})\,,\,\upsilon_{12}] \\
\boldsymbol{\mu}_{\mathrm{T}}(\mathbf{r}) &= -\frac{i}{2}\int_{\mathbf{k}} \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}}\,\nabla_q\times\mathbf{j}(\mathbf{q},\mathbf{k})\Big|_{\mathbf{q}=0}
\end{aligned}$$

OPEP beyond the static limit



On-the-energy-shell, non-static OPEP at N2LO (Q^2) can be equivalently written as

$$\begin{aligned}
\upsilon_\pi^{(2)}(\nu = 0) &= \upsilon_\pi^{(0)}(\mathbf{k}) \, \frac{(E_1' - E_1)^2 + (E_2' - E_2)^2}{2\,\omega_k^2} \\
\upsilon_\pi^{(2)}(\nu = 1) &= -\upsilon_\pi^{(0)}(\mathbf{k}) \, \frac{(E_1' - E_1)(E_2' - E_2)}{\omega_k^2} \\
\upsilon_\pi^{(0)}(\mathbf{k}) &= -\frac{g_A^2}{F_\pi^2} \, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \, \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{k} \, \boldsymbol{\sigma}_2 \cdot \mathbf{k}}{\omega_k^2}
\end{aligned}$$

$\upsilon_\pi^{(2)}(\nu)$ corrections are different off-the-energy-shell $(E_1 + E_2 \neq E_1' + E_2')$

- TPE contributions are affected by the choice made for the parameter ν

From amplitudes to potentials

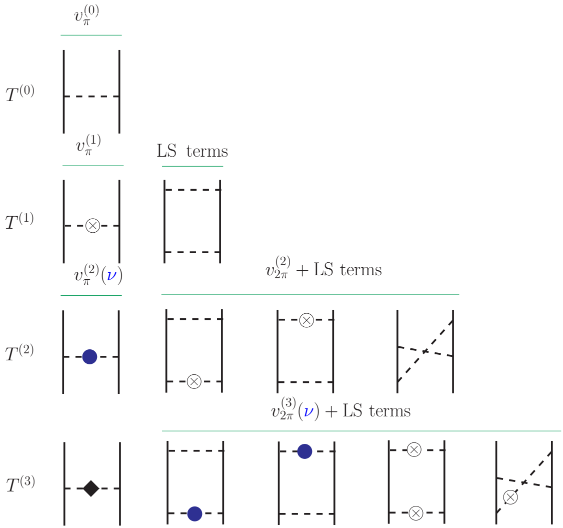
The two-nucleon potential $\upsilon = \upsilon^{(0)} + \upsilon^{(1)} + \upsilon^{(2)} + \ldots$ (with $\upsilon^{(n)} \sim Q^n$) is iterated into the Lippmann-Schwinger (LS) equation *i.e.*

$$\upsilon + \upsilon\, G_0\, \upsilon + \upsilon\, G_0\, \upsilon\, G_0\, \upsilon + \ldots\,, \qquad\qquad G_0 = 1/(E_i - E_I + i\,\eta)$$

$\upsilon^{(n)}$ is obtained subtracting from the transition amplitude $T_{\rm fi}^{(n)}$ terms already accounted for into the LS equation

$$\begin{aligned}
\upsilon^{(0)} &= T^{(0)}\,, \\
\upsilon^{(1)} &= T^{(1)} - \left[\upsilon^{(0)}\, G_0\, \upsilon^{(0)}\right]\,, \\
\upsilon^{(2)} &= T^{(2)} - \left[\upsilon^{(0)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)}\right] - \left[\upsilon^{(1)}\, G_0\, \upsilon^{(0)} + \upsilon^{(0)}\, G_0\, \upsilon^{(1)}\right]\,, \\
\upsilon^{(3)}(\nu) &= T^{(3)} - \left[\upsilon^{(0)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)}\right] - \left[\upsilon^{(1)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)} + \text{permutations}\right] \\
&\quad \underbrace{- \left[\upsilon^{(1)}\, G_0\, \upsilon^{(1)}\right] - \left[\upsilon^{(2)}(\nu)\, G_0\, \upsilon^{(0)} + \upsilon^{(0)}\, G_0\, \upsilon^{(2)}(\nu)\right]}_{\text{LS terms}}
\end{aligned}$$

From amplitudes to potentials: an example with OPE and TPE only



- To each $\upsilon_\pi^{(2)}(\nu)$ corresponds a $\upsilon_{2\pi}^{(3)}(\nu)$

Unitary equivalence of $\upsilon_\pi^{(2)}(\nu)$ and $\upsilon_{2\pi}^{(3)}(\nu)$

- Different off-the-energy-shell parameterizations lead to unitarily equivalent two-nucleon Hamiltonians

$$H(\nu) = t^{(-1)} + \upsilon_\pi^{(0)} + \upsilon_{2\pi}^{(2)} + \upsilon_\pi^{(2)}(\nu) + \upsilon_{2\pi}^{(3)}(\nu)$$

  $t^{(-1)}$ is the kinetic energy, $\upsilon_\pi^{(0)}$ and $\upsilon_{2\pi}^{(2)}$ are the static OPEP and TPEP

- The Hamiltonians are related to each other via

$$H(\nu) = \mathrm{e}^{-iU(\nu)} H(\nu=0)\, \mathrm{e}^{+iU(\nu)} \,, \qquad i\,U(\nu) \simeq i\,U^{(0)}(\nu) + i\,U^{(1)}(\nu)$$

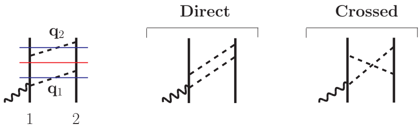
  from which it follows

$$H(\nu) = H(\nu=0) + \left[t^{(-1)} + \upsilon_\pi^{(0)}\,,\, i\,U^{(0)}(\nu)\right] + \left[t^{(-1)}, i\,U^{(1)}(\nu)\right]$$

- Predictions for physical observables are unaffected by off-the-energy-shell effects

Technical issue II - Recoil corrections at N^{3}LO

$$\mathbf{j}^{\mathrm{N^3LO}} \quad = $$



- Reducible contributions

$$\begin{aligned}\mathbf{j}_{\mathrm{red}} \quad &\sim \quad \int \upsilon^{\pi}(\mathbf{q}_2)\,\frac{1}{E_i - E_I}\,\mathbf{j}^{\mathrm{NLO}}(\mathbf{q}_1) \\ &- \quad \int 2\,\frac{\omega_1 + \omega_2}{\omega_1\,\omega_2}\,V_{\pi NN}(2,\mathbf{q}_2)\,V_{\pi NN}(2,\mathbf{q}_1)\,V_{\pi NN}(1,\mathbf{q}_2)\,V_{\gamma\pi NN}(1,\mathbf{q}_1)\end{aligned}$$

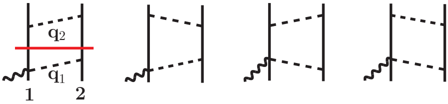
- Irreducible contributions

$$\begin{aligned}\mathbf{j}_{\mathrm{irr}} \quad &= \quad \int 2\,\frac{\omega_1 + \omega_2}{\omega_1\,\omega_2}\,V_{\pi NN}(2,\mathbf{q}_2)\,V_{\pi NN}(2,\mathbf{q}_1)\,V_{\pi NN}(1,\mathbf{q}_2)\,V_{\gamma\pi NN}(1,\mathbf{q}_1) \\ &- \quad \int 2\,\frac{\omega_1^2 + \omega_2^2 + \omega_1\,\omega_2}{\omega_1\,\omega_2(\omega_1 + \omega_2)}\left[V_{\pi NN}(2,\mathbf{q}_2), V_{\pi NN}(2,\mathbf{q}_1)\right]_{-}\,V_{\pi NN}(1,\mathbf{q}_2)V_{\gamma\pi NN}(1,\mathbf{q}_1)\end{aligned}$$

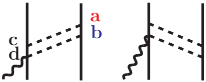
- Observed partial cancellations at N^{3}LO between recoil corrections to reducible diagrams and irreducible contributions

The box diagram: an example at N^{3}LO

Reducible



Irreducible direct



Irreducible crossed

$$\text{direct} = f_d(\omega_1, \omega_2)\, V_a\, V_b\, V_c\, V_d$$

$$\text{crossed} = f_c(\omega_1, \omega_2)\, V_b\, V_a\, V_c\, V_d \qquad V_b V_a = V_a V_b - [V_a, V_b]_-$$

$$\begin{aligned} \text{irreducible} &= [f_d(\omega_1, \omega_2) + f_c(\omega_1, \omega_2)]\, V_a\, V_b\, V_c\, V_d \\ &- f_c(\omega_1, \omega_2)[V_a, V_b]_-\, V_c\, V_d \end{aligned}$$

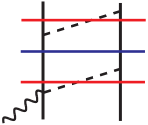
Transition amplitude in time-ordered perturbation theory

$$T_{\rm fi} = \langle f \mid T \mid i \rangle = \langle f \mid H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\eta} H_1 \right)^{n-1} \mid i \rangle$$

$$= \langle f \mid H_1 \mid i \rangle + \sum_{|I\rangle} \langle f \mid H_1 \mid I \rangle \frac{1}{E_i - E_I} \langle I \mid H_1 \mid i \rangle + \dots$$

- A contribution with N interaction vertices and L loops scales as

$$\underbrace{e \left(\prod_{i=1}^{N} Q^{\alpha_i - \beta_i/2} \right)}_{H_1\,\text{scaling}} \times \underbrace{Q^{-(N-N_K-1)}\, Q^{-2N_K}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integration}}$$



α_i = number of derivatives in H_1 and β_i = number of π's at each vertex

N_K = number of pure nucleonic intermediate states

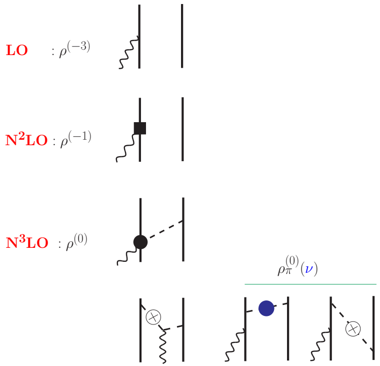
- $(N - N_K - 1)$ energy denominators expanded in powers of $(E_i - E_N)/\omega_\pi \sim Q$

$$\frac{1}{E_i - E_I} |I\rangle = \frac{1}{E_i - E_N - \omega_\pi} |I\rangle \sim - \Bigg[\underbrace{\frac{1}{\omega_\pi}}_{Q^{-1}} + \underbrace{\frac{E_i - E_N}{\omega_\pi^2}}_{Q^0} + \underbrace{\frac{(E_i - E_N)^2}{\omega_\pi^3}}_{Q^1} + \dots \Bigg] |I\rangle$$

- Due to the chiral expansion, the transition amplitude $T_{\rm fi}$ can be expanded as

$$T_{\rm fi} = T^{\rm LO} + T^{\rm NLO} + T^{\rm N2LO} + \dots \quad \text{and} \quad T^{\rm NnLO} \sim (Q/\Lambda_\chi)^n T^{\rm LO}$$

EM charge up to $n = 0$ (or up to N3LO)



- $n = -3$

  $\rho^{(-3)}(\mathbf{q}) = e\,(2\pi)^3\delta(\mathbf{p}_1 + \mathbf{q} - \mathbf{p}'_1)\,(1+\tau_{1,z})/2 + 1 \rightleftharpoons 2$

- $n = -1$:

  $(Q/m_N)^2$ relativistic correction to $\rho^{(-3)}$

- $n = 0$:

  i) 'static' tree-level current (originates from a $\gamma\pi N$ vertex of order $e\,Q$)

  ii) 'non-static' OPE charge operators, $\rho_\pi^{(0)}(\nu)$ depends on $\upsilon_\pi^{(2)}(\nu)$

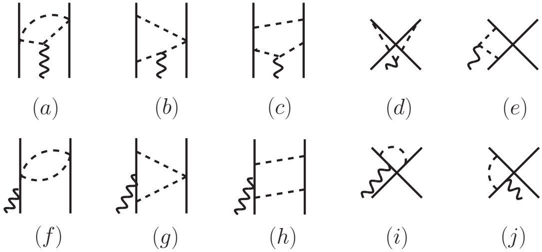
- $\rho_\pi^{(0)}(\nu)$'s are unitarily equivalent

$$\rho_\pi^{(0)}(\nu) \quad = \quad \rho_\pi^{(0)}(\nu = 0) + \left[\rho^{(-3)}\,,\, i\,U^{(0)}(\nu)\right]$$

- No unknown LECs up to this order (g_A, F_π)

EM charge @ $n = 1$ (or N4LO) 1.

N^{4}LO : $\rho^{(1)}$

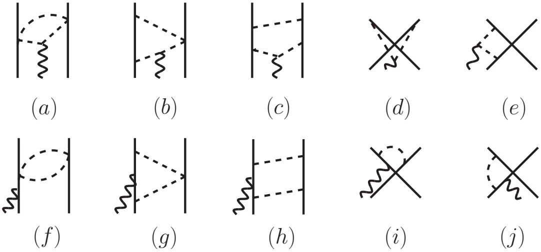


- (a), (f), (d), and (i) vanish
- Divergencies associated with (b) + (g), (c) + (h), and (e) + (j) cancel out—as they must since there are no counter-terms at N4LO
- $\rho_{\rm h}^{(1)}(\nu)$ depends on the parametrization adopted for $\upsilon_\pi^{(2)}(\nu)$ and $\upsilon_{2\pi}^{(3)}(\nu)$
- $\rho_{\rm h}^{(1)}(\nu)$'s are unitarily equivalent

$$\rho_{\rm h}^{(1)}(\nu) \quad = \quad \rho_{\rm h}^{(1)}(\nu=0) + \left[\rho^{(-3)}\,,\, i\,U^{(1)}(\nu)\right]$$

EM charge @ $n = 1$ (or N4LO) 2.

N^{4}LO : $\rho^{(1)}$



- Charge operators (ν-dependent included) up to $n = 1$ satisfy the condition

$$\rho^{(n>-3)}(\mathbf{q}=0) = 0$$

which follows from charge conservation

$$\rho(\mathbf{q}=0) = \int \mathrm{d}\mathbf{x}\,\rho(\mathbf{x}) = e\,\frac{(1+\tau_{1,z})}{2} + 1 \rightleftharpoons 2 = \rho^{(-3)}(\mathbf{q}=0)$$

- $\rho^{(1)}$ does not depend on unknown LECs and it is purely isovector