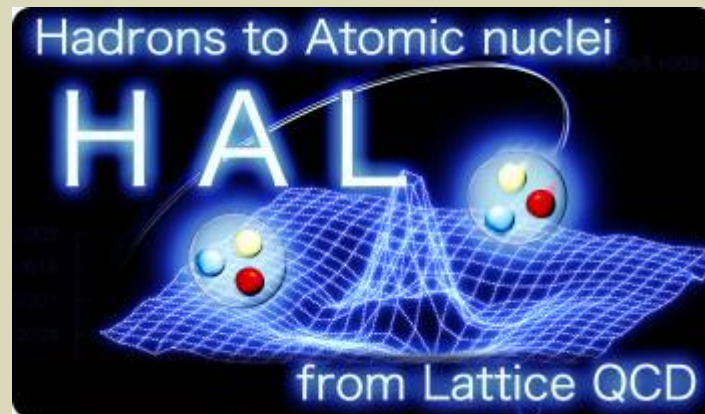


Hyperon–nucleon interaction from lattice QCD and hypernuclear few–body problem

H. Nemura¹,

for HAL QCD Collaboration

S. Aoki², T. Doi³, F. Etminan⁴, S. Gongyo⁵, T. Hatsuda³,
Y. Ikeda⁶, T. Inoue⁷, T. Iritani⁸, N. Ishii⁶, D. Kawai²,
T. Miyamoto², K. Murano⁶, and K. Sasaki²,



¹*University of Tsukuba,*

²*Kyoto University,* ³*RIKEN,* ⁴*University of Birjand,*

⁵*University of Tours,* ⁶*Osaka University,*

⁷*Nihon University,* ⁸*Stony Brook University*

Outline

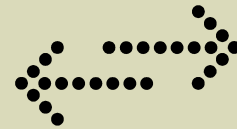
- ⊗ Introduction
- ⊗ Hypernuclear few-body problem
- ⊗ Brief introduction of HAL QCD method
- ⊗ Preliminary results of LN-SN potentials at nearly physical point
 - ⊗ LN-SN($I=1/2$), central and tensor potentials
 - ⊗ SN($I=3/2$), central and tensor potentials
- ⊗ Effective block algorithm for various baryon-baryon channels [arXiv:1510.00903(hep-lat)]
- ⊗ Four-nucleon bound state problem using a lattice NN potential at heavier pion mass about 470MeV
- ⊗ Summary

Plan of research

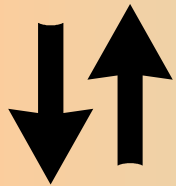


J-PARC,
JLab, GSI, MAMI, ...
YN scattering,
hypernuclei

QCD



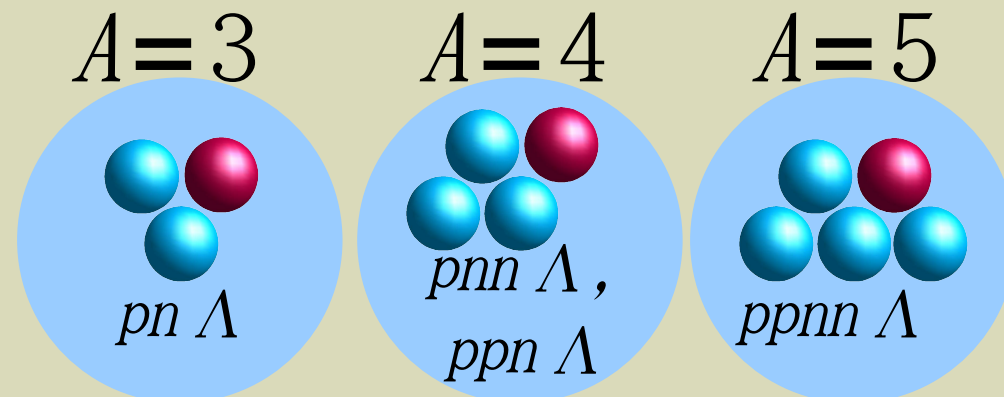
Baryon interaction



Structure and reaction of
(hyper)nuclei

Equation of State (EoS)
of nuclear matter

Neutron star and
supernova



Talks from HAL QCD in the INT Program INT-16-1

13 Apr, S.Aoki,

“Overview of the HAL QCD potential method and recent results”

18 Apr, T.Iritani,

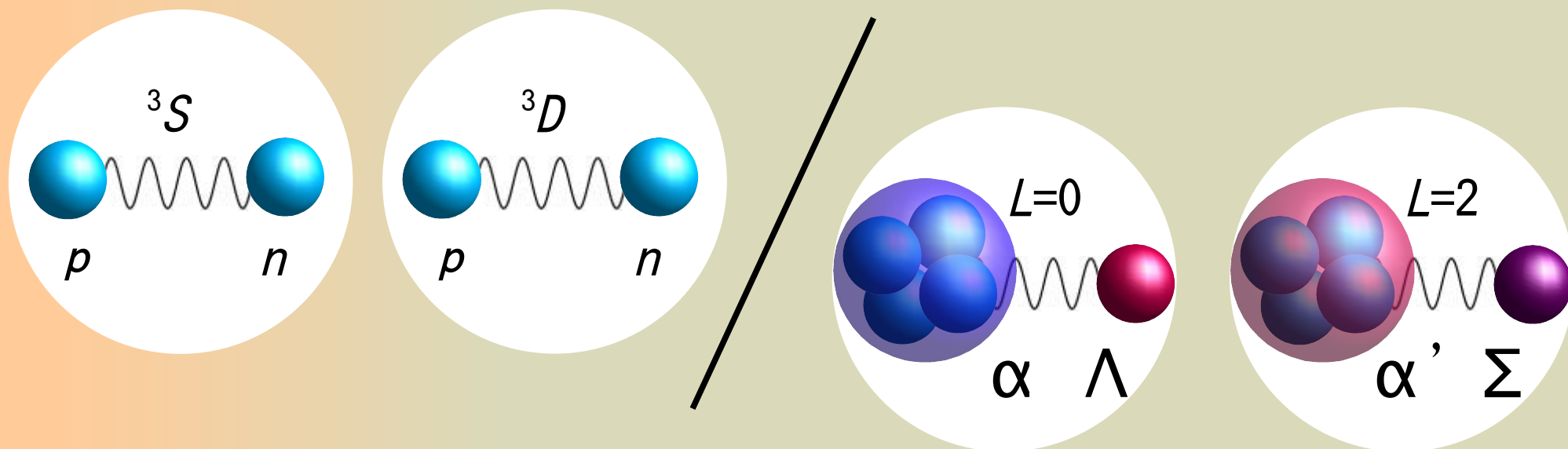
“Baryon interactions from Luscher’s finite volume method and HAL QCD method”

25 Apr, T.Doii,

“Baryon interactions from Lattice QCD with physical masses”

4 May, HN, This talk

Comparison between $d=p+n$ and $\text{core}+Y$



| | $\langle T_S \rangle$ (MeV) | $\langle T_D \rangle$ (MeV) | $\langle V_{NN}(\text{central}) \rangle$ (MeV) | $\langle V_{NN}(\text{tensor}) \rangle$ (MeV) | $\langle V_{NN}(\text{LS}) \rangle$ (MeV) |
|----------------------------|-------------------------------------|---|---|---|--|
| AV8 | 8.57 | 11.31 | -4.46 | -16.64 | -1.02 |
| G3RS | 10.84 | 5.64 | -7.29 | -11.46 | 0.00 |
| | $\langle T_{Y-c} \rangle_{\Lambda}$ | $\langle T_{Y-c} \rangle_{\Sigma} + \Delta \langle H_C \rangle$ | $\langle V_{YN}(\text{のこり}) \rangle$ | $2 \langle V_{\Lambda N - \Sigma N}(\text{tensor}) \rangle$ | |
| ${}^{\Lambda}_5\text{He}$ | 9.11 | 3.88+4.68 | -0.86 | -19.51 | |
| ${}^{\Lambda}_4\text{H}^*$ | 5.30 | 2.43+2.02 | 0.01 | -10.67 | |
| ${}^{\Lambda}_4\text{H}$ | 7.12 | 2.94+2.16 | -5.05 | -9.22 | |

FY calculation with and w/o 3NF

- ☉ Three nucleon force does not change the B_Λ so much.

☉ A. Nogga, *et al.*, PRL**88**, 172501 (2002).

TABLE II. NN and $3N$ interaction dependence of the ${}^4_\Lambda\text{He}$ SE's E_{sep}^Λ and the 0^+-1^+ splitting Δ . We show results for different combinations of YN , NN , and $3N$ forces (YNF , NNF , and $3NF$). All energies are given in MeV.

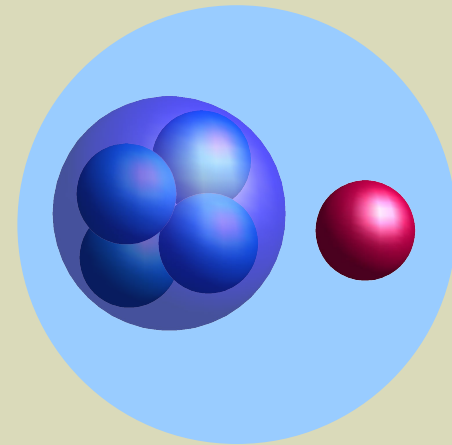
| YNF | NNF | $3NF$ | $E_{\text{sep}}^\Lambda(0^+)$ | $E_{\text{sep}}^\Lambda(1^+)$ | Δ |
|-------|----------|-------|-------------------------------|-------------------------------|----------|
| SC97e | Bonn B | ... | 1.66 | 0.80 | 0.84 |
| SC97e | Nijm 93 | ... | 1.54 | 0.72 | 0.79 |
| SC97e | Nijm 93 | TM | 1.56 | 0.70 | 0.82 |
| SC89 | Bonn B | ... | 2.25 | ... | ... |
| SC89 | Nijm 93 | ... | 2.14 | 0.02 | 2.06 |
| SC89 | Nijm 93 | TM | 2.19 | ... | ... |

What is realistic picture of hypernuclei?

⊗ $B(\text{total}) = B(^4\text{He}) + B_{\Lambda}({}_{\Lambda}^5\text{He})$

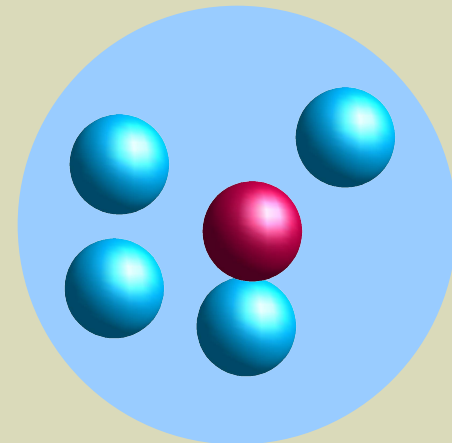
⊗ A conventional picture:

$$\begin{aligned} B(\text{total}) &= B(^4\text{He}) + B_{\Lambda}({}_{\Lambda}^5\text{He}) \\ &= 28 + 3 \text{ MeV}. \end{aligned}$$

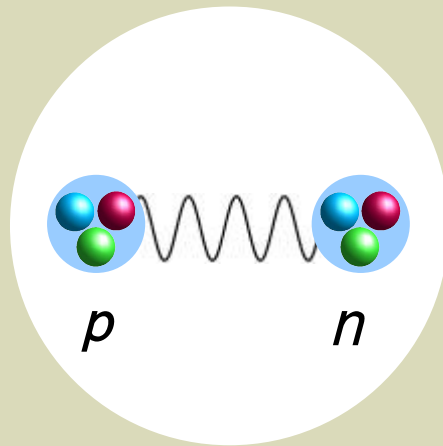


⊗ A (probably realistic) picture:

$$\begin{aligned} B(\text{total}) &= (B(^4\text{He}) - \Delta E_c) + (B_{\Lambda}({}_{\Lambda}^5\text{He}) + \Delta E_c) \\ &= 24 + 7 \text{ MeV}. \end{aligned}$$



Lattice QCD calculation



Multi-hadron on lattice

i) basic procedure:

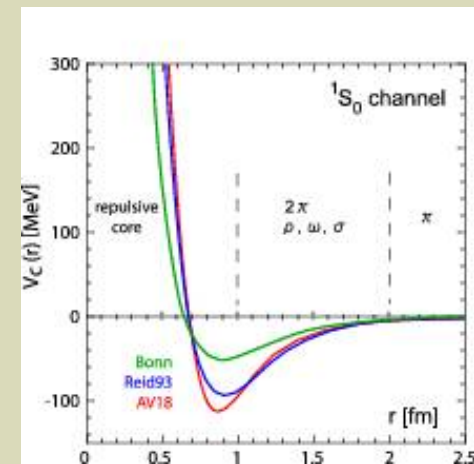
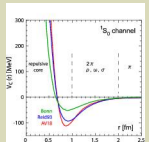
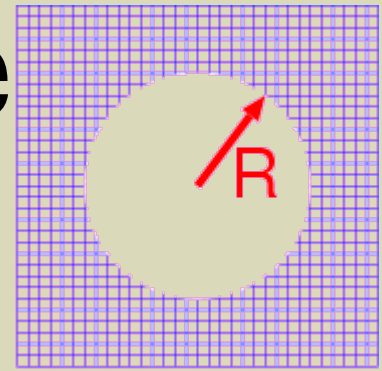
asymptotic region

\longrightarrow phase shift

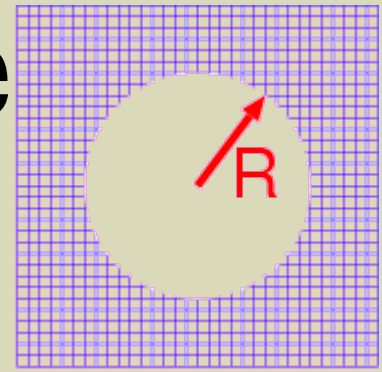
ii) HAL's procedure:

interacting region

\longrightarrow potential



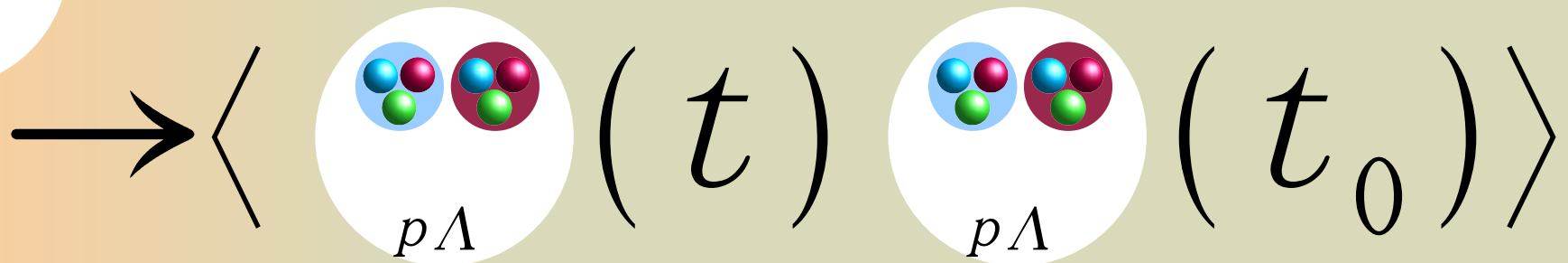
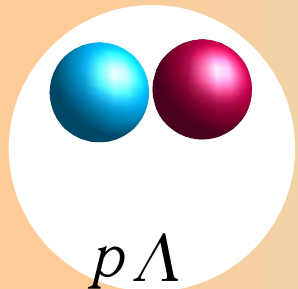
Multi-hadron on lattice



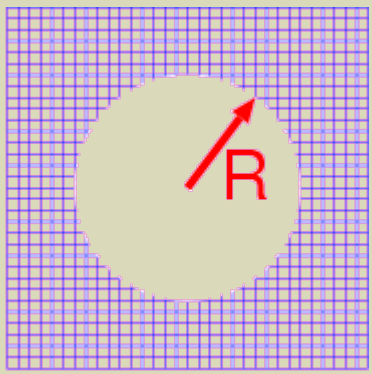
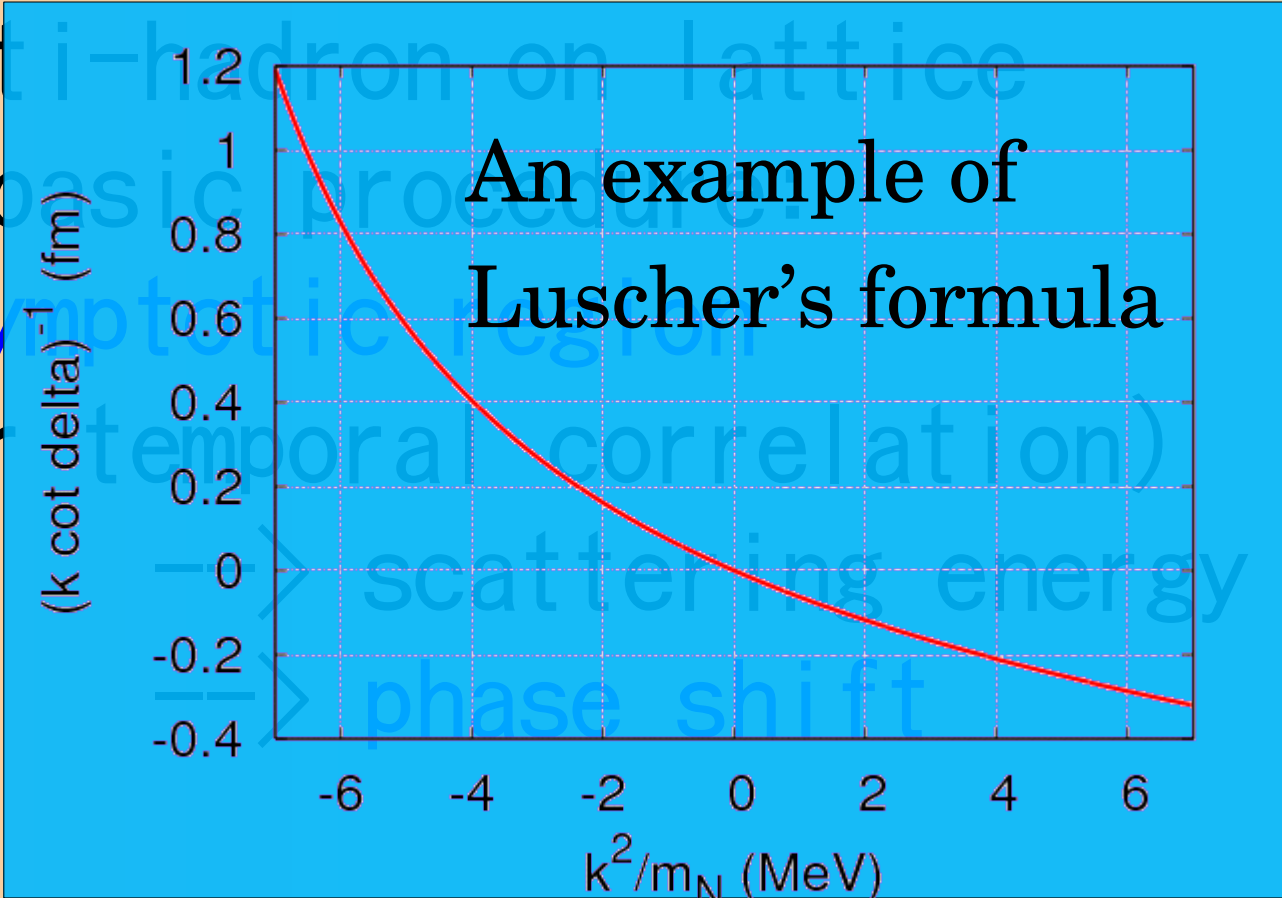
Lattice QCD simulation

$$L = -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + \bar{q} \gamma^\mu (i \partial_\mu - g t^a A_\mu^a) q - m \bar{q} q$$

$$\begin{aligned} \langle O(\bar{q}, q, U) \rangle &= \int dU d\bar{q} dq e^{-S(\bar{q}, q, U)} O(\bar{q}, q, U) \\ &= \int dU \det D(U) e^{-S_v(U)} O(D^{-1}(U)) \\ &= \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N O(D^{-1}(U_i)) \end{aligned}$$



Mult
 i) b
 asy
 (or



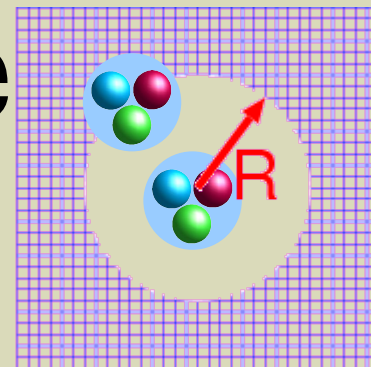
$$E = \frac{k^2}{2\mu}$$

$$k \cot \delta_0(k) = \frac{2}{\sqrt{\pi} L} Z_{00}(1; (kL/(2\pi))^2) = \frac{1}{a_0} + O(k^2)$$

$$Z_{00}(1; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n} \in \mathbb{Z}^3} \frac{1}{(n^2 - q^2)^s} \quad \Re s > \frac{3}{2}$$

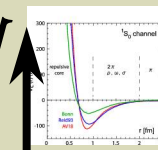
Luscher, NPB354, 531 (1991).
 Aoki, et al., PRD71, 094504 (2005).

Multi-hadron on lattice

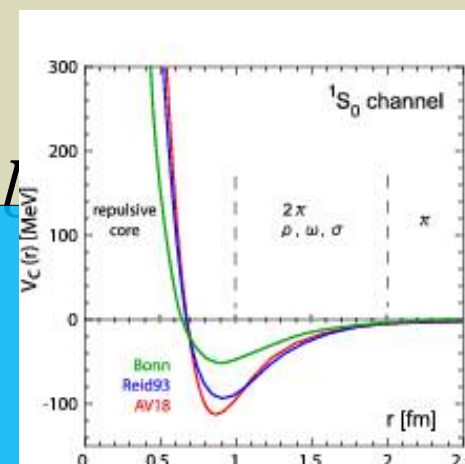


Lattice QCD simulation

$$L = -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + \bar{q} \gamma^\mu (i \partial_\mu - g t^a A_\mu^a) q - m \bar{q} q$$



$$\begin{aligned} \langle O(\bar{q}, q, U) \rangle &= \int dU d\bar{q} dq e^{-S(\bar{q}, q, U)} O(\bar{q}, q) \\ &= \int dU \det D(U) e^{-S_V(U)} O(D^{-1}(U)) \end{aligned}$$



$$F_{\alpha\beta}^{(JM)}(\vec{r}, t - t_0)$$

$$\rightarrow \left\langle \left(\text{p} \Lambda \right) (\vec{r}, t) \left(\text{p} \Lambda \right) (t_0) \right\rangle$$

Calculate the scattering state

Multi-hadron on lattice

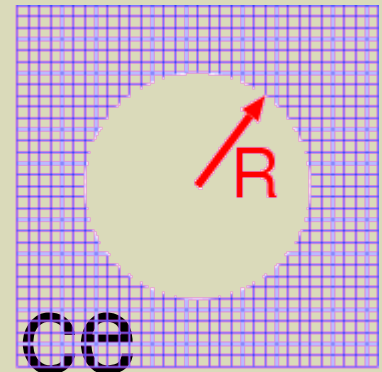
ii) HAL's procedure:

make better use of the lattice

output ! (wave function)

interacting region

→ potential



Ishii, Aoki, Hatsuda,
PRL99, 022001 (2007);
ibid., PTP123, 89 (2010).

NOTE:

- > Potential is not a direct experimental observable.
- > Potential is a useful tool to give (and to reproduce) the physical quantities. (e.g., phase shift)

Multi-hadron on lattice

ii) HAL's procedure:

make better use of the lattice

output ! (wave function)

interacting region

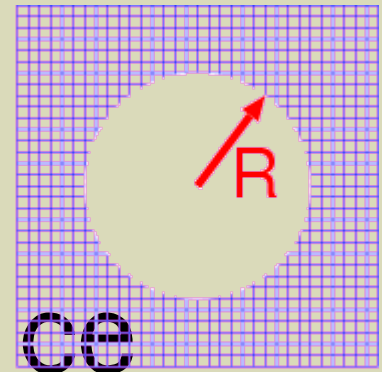
→ potential

Ishii, Aoki, Hatsuda,
PRL99, 022001 (2007);
ibid., PTP123, 89 (2010).

⇒

> Phase shift

> Nuclear many-body problems



In lattice QCD calculations, we compute the normalized four-point correlation function

$$R_{\alpha\beta}^{(J,M)}(\vec{r}, t-t_0) = \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X} + \vec{r}, t) B_{2,\beta}(\vec{X}, t) \overline{\mathcal{J}_{B_3 B_4}^{(J,M)}}(t_0) \right| 0 \right\rangle / \exp\{-(m_{B_1} + m_{B_2})(t-t_0)\},$$

$$p = \varepsilon_{abc} (u_a C \gamma_5 d_b) u_c, \quad n = -\varepsilon_{abc} (u_a C \gamma_5 d_b) d_c, \quad (2)$$

$$\Sigma^+ = -\varepsilon_{abc} (u_a C \gamma_5 s_b) u_c, \quad \Sigma^- = -\varepsilon_{abc} (d_a C \gamma_5 s_b) d_c, \quad (3)$$

$$\Sigma^0 = \frac{1}{\sqrt{2}} (X_u - X_d), \quad \Lambda = \frac{1}{\sqrt{6}} (X_u + X_d - 2X_s), \quad (4)$$

$$\Xi^0 = \varepsilon_{abc} (u_a C \gamma_5 s_b) s_c, \quad \Xi^- = -\varepsilon_{abc} (d_a C \gamma_5 s_b) s_c, \quad (5)$$

where

$$X_u = \varepsilon_{abc} (d_a C \gamma_5 s_b) u_c, \quad X_d = \varepsilon_{abc} (s_a C \gamma_5 u_b) d_c, \quad X_s = \varepsilon_{abc} (u_a C \gamma_5 d_b) s_c, \quad (6)$$

An improved recipe for NY potential:

☉cf. Ishii (HAL QCD), PLB712 (2012) 437.

- ☉Take account of not only the spatial correlation but also the temporal correlation in terms of the R-correlator:

$$-\frac{1}{2\mu} \nabla^2 R(t, \vec{r}) + \int d^3 r' U(\vec{r}, \vec{r}') R(t, \vec{r}') = -\frac{\partial}{\partial t} R(t, \vec{r})$$

$\rightarrow \frac{k^2}{2\mu} R(t, \vec{r})$

$$U(\vec{r}, \vec{r}') = V_{NY}(\vec{r}, \nabla) \delta(\vec{r} - \vec{r}')$$

- ☉A general expression of the potential:

$$\begin{aligned} V_{NY} = & V_0(r) + V_\sigma(r) (\vec{\sigma}_N \cdot \vec{\sigma}_Y) \\ & + V_T(r) S_{12} + V_{LS}(r) (\vec{L} \cdot \vec{S}_+) \\ & + V_{ALS}(r) (\vec{L} \cdot \vec{S}_-) + O(\nabla^2) \end{aligned}$$

Determination of baryon-baryon potentials at nearly physical point

Effective block algorithm for various baryon-baryon calculations

arXiv:1510.00903(hep-lat)

Generalization to the various baryon-baryon channels strangeness S=0 to -4 systems

$$\langle pn\bar{p}\bar{n} \rangle, \quad (4.1)$$

$$\begin{aligned} &\langle p\Lambda\bar{p}\bar{\Lambda} \rangle, \quad \langle p\Lambda\bar{\Sigma}^+n \rangle, \quad \langle p\Lambda\bar{\Sigma}^0p \rangle, \\ &\langle \Sigma^+n\bar{p}\bar{\Lambda} \rangle, \quad \langle \Sigma^+n\bar{\Sigma}^+n \rangle, \quad \langle \Sigma^+n\bar{\Sigma}^0p \rangle, \\ &\langle \Sigma^0p\bar{p}\bar{\Lambda} \rangle, \quad \langle \Sigma^0p\bar{\Sigma}^+n \rangle, \quad \langle \Sigma^0p\bar{\Sigma}^0p \rangle, \end{aligned} \quad (4.2)$$

$$\begin{aligned} &\langle \Lambda\Lambda\bar{\Lambda}\bar{\Lambda} \rangle, \quad \langle \Lambda\Lambda\bar{p}\bar{\Xi}^- \rangle, \quad \langle \Lambda\Lambda\bar{n}\bar{\Xi}^0 \rangle, \quad \langle \Lambda\Lambda\bar{\Sigma}^+\bar{\Sigma}^- \rangle, \quad \langle \Lambda\Lambda\bar{\Sigma}^0\bar{\Sigma}^0 \rangle, \\ &\langle p\bar{\Xi}^-\bar{\Lambda}\bar{\Lambda} \rangle, \quad \langle p\bar{\Xi}^-\bar{p}\bar{\Xi}^- \rangle, \quad \langle p\bar{\Xi}^-\bar{n}\bar{\Xi}^0 \rangle, \quad \langle p\bar{\Xi}^-\bar{\Sigma}^+\bar{\Sigma}^- \rangle, \quad \langle p\bar{\Xi}^-\bar{\Sigma}^0\bar{\Sigma}^0 \rangle, \quad \langle p\bar{\Xi}^-\bar{\Sigma}^0\bar{\Lambda} \rangle, \\ &\langle n\bar{\Xi}^0\bar{\Lambda}\bar{\Lambda} \rangle, \quad \langle n\bar{\Xi}^0\bar{p}\bar{\Xi}^- \rangle, \quad \langle n\bar{\Xi}^0\bar{n}\bar{\Xi}^0 \rangle, \quad \langle n\bar{\Xi}^0\bar{\Sigma}^+\bar{\Sigma}^- \rangle, \quad \langle n\bar{\Xi}^0\bar{\Sigma}^0\bar{\Sigma}^0 \rangle, \quad \langle n\bar{\Xi}^0\bar{\Sigma}^0\bar{\Lambda} \rangle, \\ &\langle \Sigma^+\bar{\Sigma}^-\bar{\Lambda}\bar{\Lambda} \rangle, \quad \langle \Sigma^+\bar{\Sigma}^-\bar{p}\bar{\Xi}^- \rangle, \quad \langle \Sigma^+\bar{\Sigma}^-\bar{n}\bar{\Xi}^0 \rangle, \quad \langle \Sigma^+\bar{\Sigma}^-\bar{\Sigma}^+\bar{\Sigma}^- \rangle, \quad \langle \Sigma^+\bar{\Sigma}^-\bar{\Sigma}^0\bar{\Sigma}^0 \rangle, \quad \langle \Sigma^+\bar{\Sigma}^-\bar{\Sigma}^0\bar{\Lambda} \rangle, \\ &\langle \Sigma^0\bar{\Sigma}^0\bar{\Lambda}\bar{\Lambda} \rangle, \quad \langle \Sigma^0\bar{\Sigma}^0\bar{p}\bar{\Xi}^- \rangle, \quad \langle \Sigma^0\bar{\Sigma}^0\bar{n}\bar{\Xi}^0 \rangle, \quad \langle \Sigma^0\bar{\Sigma}^0\bar{\Sigma}^+\bar{\Sigma}^- \rangle, \quad \langle \Sigma^0\bar{\Sigma}^0\bar{\Sigma}^0\bar{\Sigma}^0 \rangle, \\ &\quad \langle \Sigma^0\bar{\Lambda}\bar{p}\bar{\Xi}^- \rangle, \quad \langle \Sigma^0\bar{\Lambda}\bar{n}\bar{\Xi}^0 \rangle, \quad \langle \Sigma^0\bar{\Lambda}\bar{\Sigma}^+\bar{\Sigma}^- \rangle, \quad \langle \Sigma^0\bar{\Lambda}\bar{\Sigma}^0\bar{\Lambda} \rangle, \end{aligned} \quad (4.3)$$

$$\begin{aligned} &\langle \Xi^-\bar{\Lambda}\bar{\Xi}^-\bar{\Lambda} \rangle, \quad \langle \Xi^-\bar{\Lambda}\bar{\Sigma}^-\bar{\Xi}^0 \rangle, \quad \langle \Xi^-\bar{\Lambda}\bar{\Sigma}^0\bar{\Xi}^- \rangle, \\ &\langle \Sigma^-\bar{\Xi}^0\bar{\Xi}^-\bar{\Lambda} \rangle, \quad \langle \Sigma^-\bar{\Xi}^0\bar{\Sigma}^-\bar{\Xi}^0 \rangle, \quad \langle \Sigma^-\bar{\Xi}^0\bar{\Sigma}^0\bar{\Xi}^- \rangle, \\ &\langle \Sigma^0\bar{\Xi}^-\bar{\Xi}^-\bar{\Lambda} \rangle, \quad \langle \Sigma^0\bar{\Xi}^-\bar{\Sigma}^-\bar{\Xi}^0 \rangle, \quad \langle \Sigma^0\bar{\Xi}^-\bar{\Sigma}^0\bar{\Xi}^- \rangle, \end{aligned} \quad (4.4)$$

$$\langle \Xi^-\bar{\Xi}^0\bar{\Xi}^-\bar{\Xi}^0 \rangle. \quad (4.5)$$

Make better use of the computing resources!

Almost physical point lattice QCD calculation using $N_F=2+1$ clover fermion + Iwasaki gauge action

- ⊗ APE–Stout smearing ($\rho=0.1$, $n_{\text{stout}}=6$)
- ⊗ Non-perturbatively $O(a)$ improved Wilson Clover action at $\beta=1.82$ on $96^3 \times 96$ lattice

- ⊗ $1/a = 2.3 \text{ GeV}$ ($a = 0.085 \text{ fm}$)

- ⊗ Volume: $96^4 \rightarrow (8\text{fm})^4$

- ⊗ $m_\pi = 145\text{MeV}$, $m_K = 525\text{MeV}$

- ⊗ DDHMC(ud) and UVPHMC(s) with preconditioning

- ⊗ K.-I. Ishikawa, et al., PoS LAT2015, 075;
arXiv:1511.09222 [hep-lat].

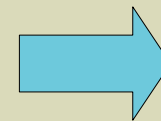


- ⊗ NBS wf is measured using wall quark source with Coulomb gauge fixing, spatial PBD and temporal DBC; #stat=207configs x 4rotation x Nsrc
(Nsrc=4 \rightarrow 20 \rightarrow 96 (2015FY))

LN-SN potentials at nearly physical point

The methodology for coupled-channel V is based on:
 Aoki, et al., Proc. Japan Acad. B87 (2011) 509.
 Sasaki, et al., PTEP 2015 (2015) no.11, 113B01.
 Ishii, et al., JPS meeting, March (2016).

#stat: (this/scheduled in FY2015) < 0.05



0.2 for

$\Lambda N - \Sigma N (I = 1/2)$

$V_C({}^1S_0)$

$V_C({}^3S_1 - {}^3D_1)$

$V_T({}^3S_1 - {}^3D_1)$

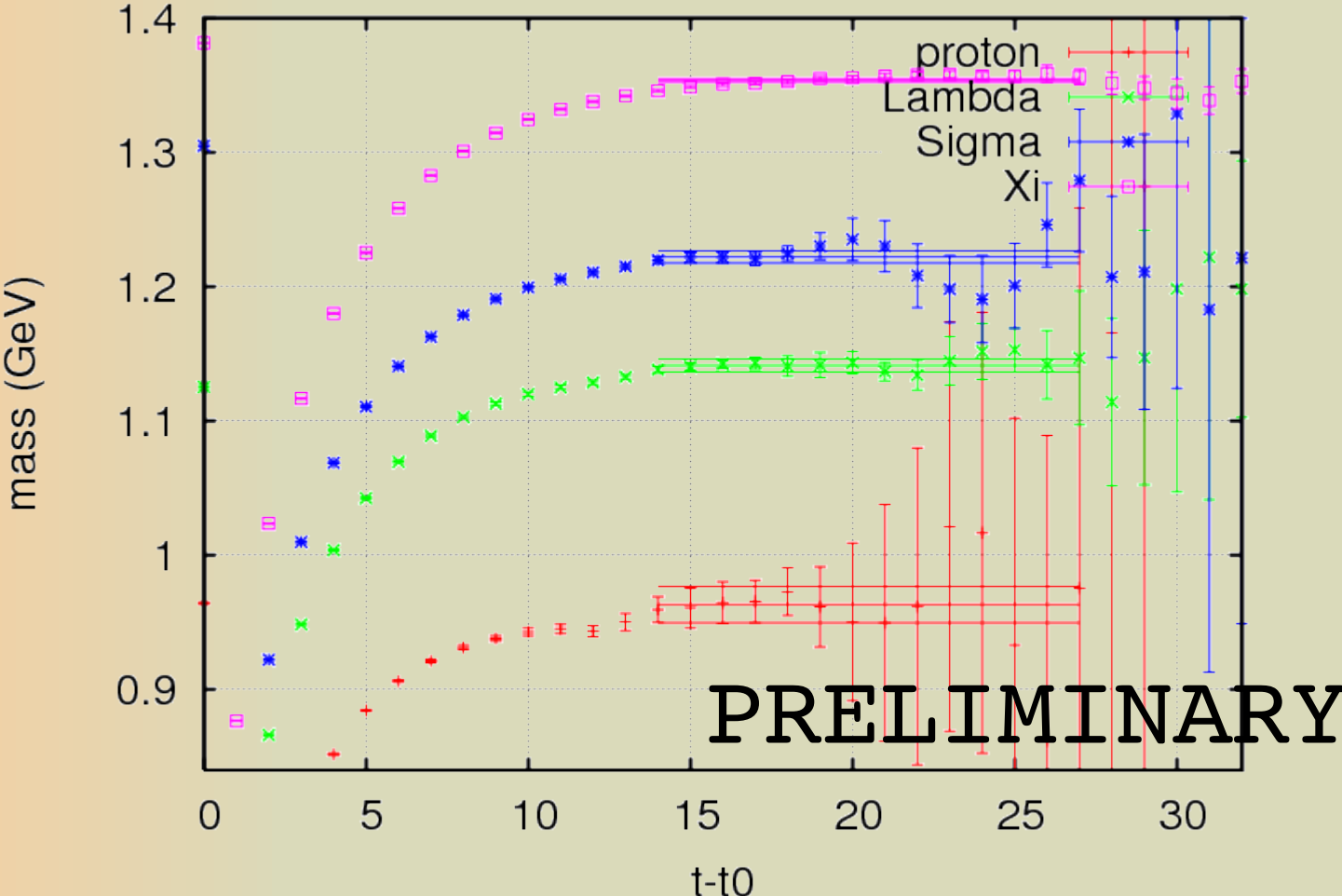
$\Sigma N (I = 3/2)$

$V_C({}^1S_0)$

$V_C({}^3S_1 - {}^3D_1)$

$V_T({}^3S_1 - {}^3D_1)$

Effective mass plot of the single baryon' s correlation function

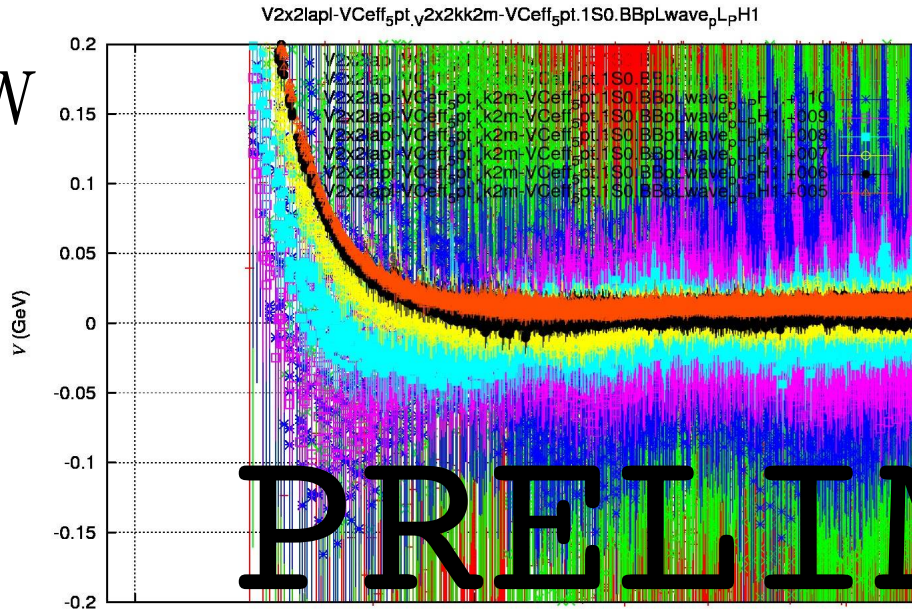


Very preliminary result of LN potential at the physical point

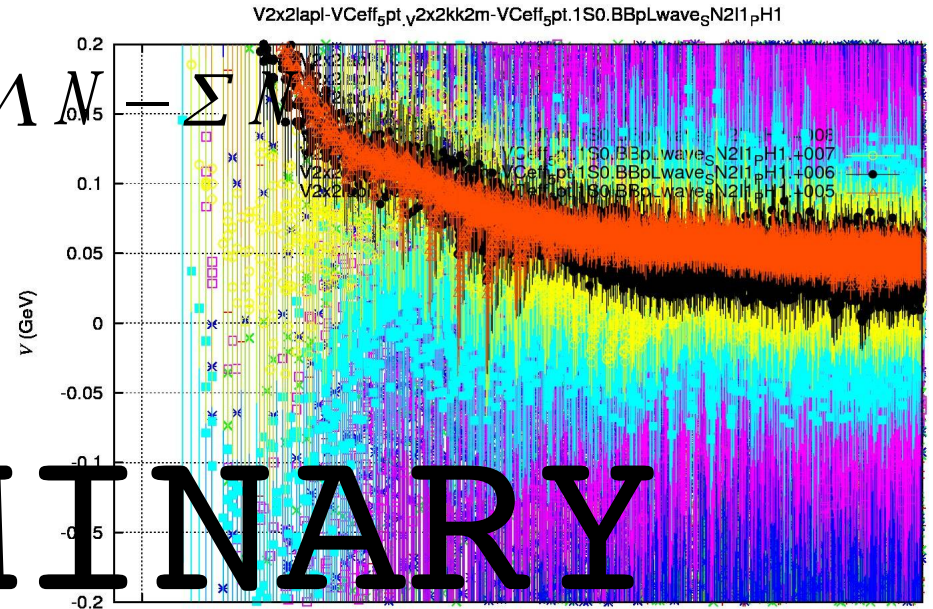
$$V_c({}^1S_0)$$

$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r}, t) = \int d^3r' U(\vec{r}, \vec{r}') R(\vec{r}', t) + O(k^4) = V_{LO}(\vec{r}) R(\vec{r}, t) + \dots \quad (8)$$

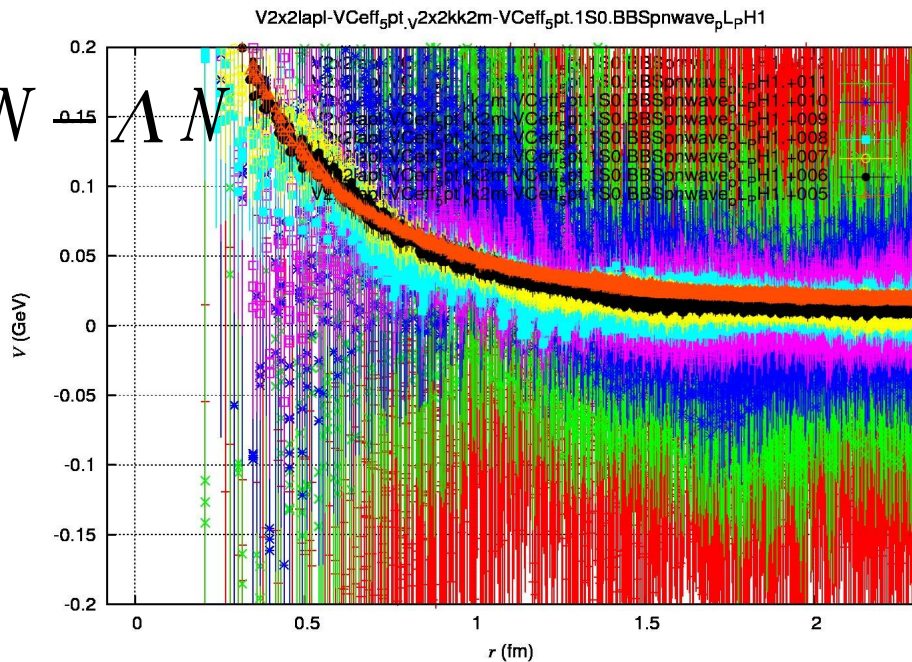
ΛN



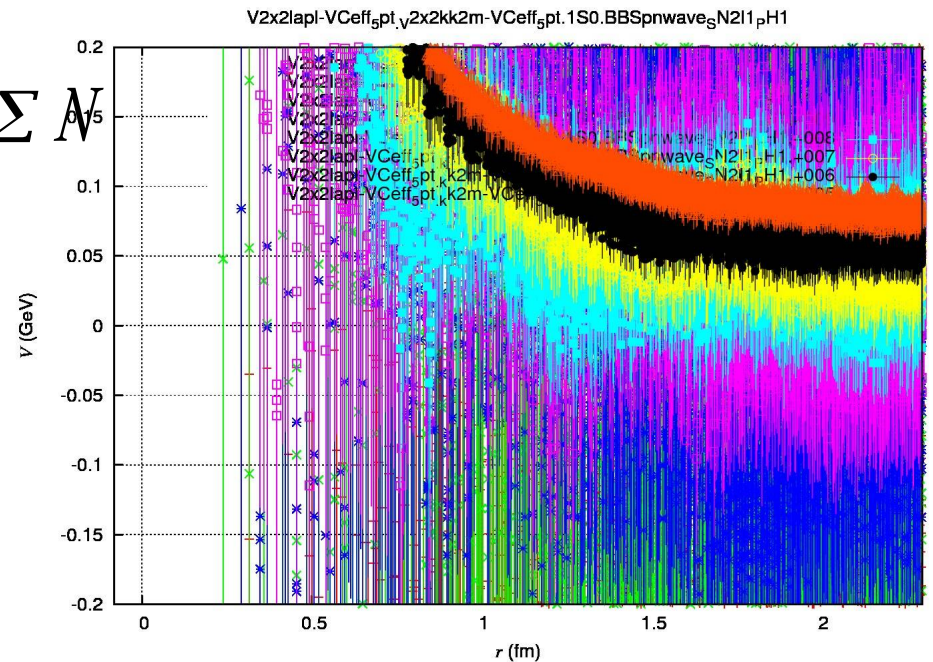
$\Lambda N - \Sigma N$



ΣN



ΣN



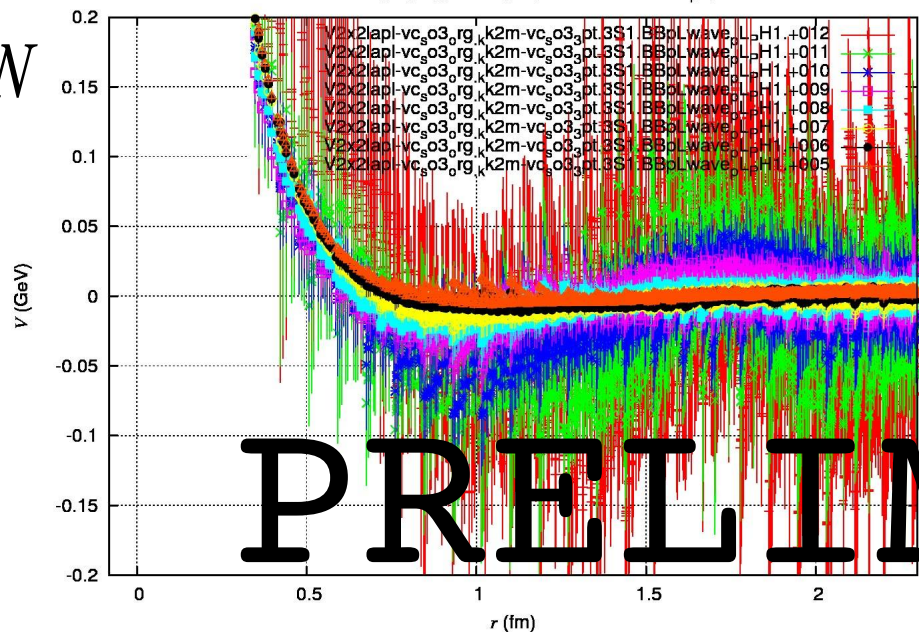
PRELIMINARY

Very preliminary result of LN potential at the physical point

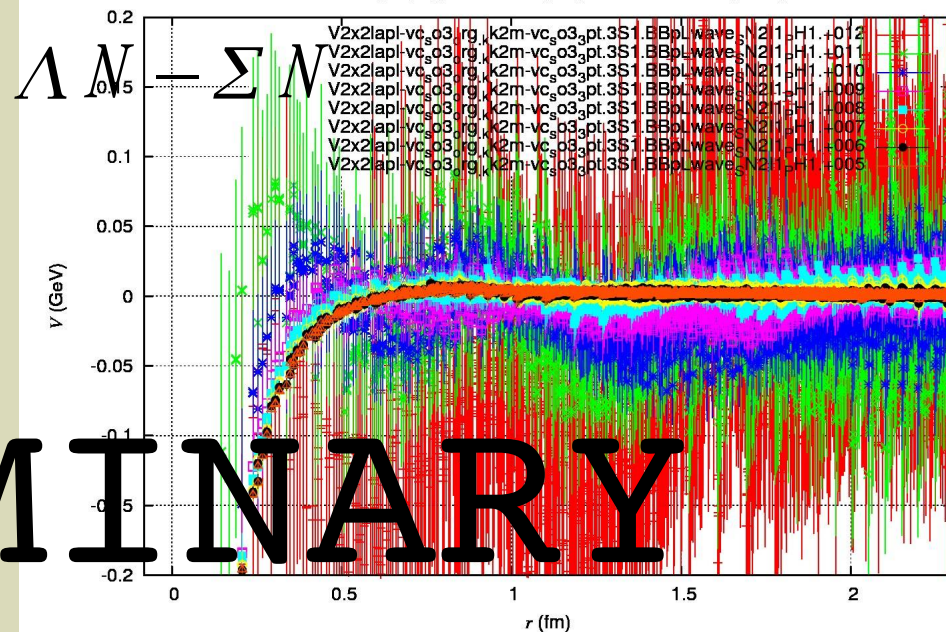
$$V_C({}^3S_1 - {}^3D_1)$$

$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r}, t) = \int d^3r' U(\vec{r}, \vec{r}') R(\vec{r}', t) + O(k^4) = V_{LO}(\vec{r}) R(\vec{r}, t) + \dots \quad (8)$$

V2x2lapi-vc_so3_{rg},k2m-vc_so3_{pt}.3S1.BBpLwave_pLpH1

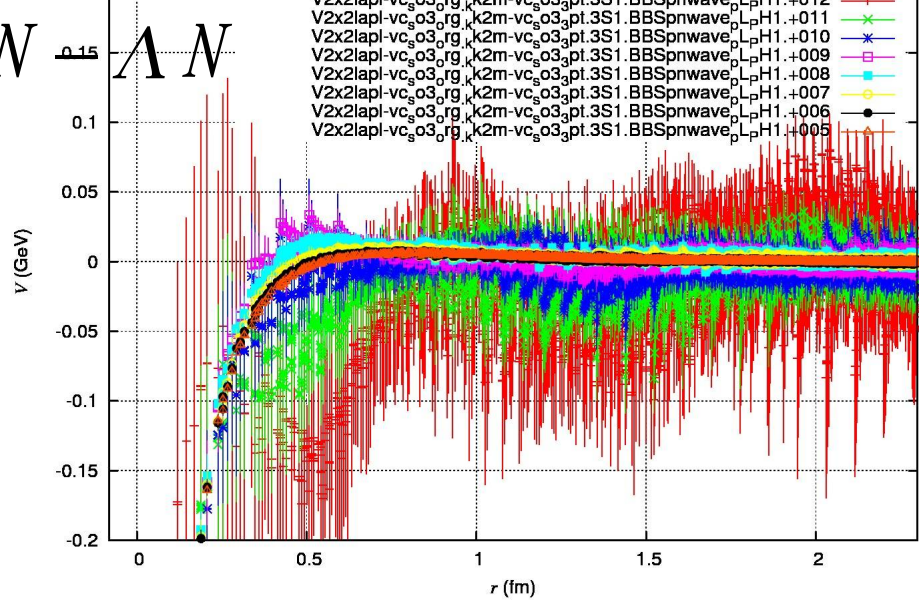


V2x2lapi-vc_so3_{rg},v2x2kk2m-vc_so3_{pt}.3S1.BBpLwave_sN21_pH1

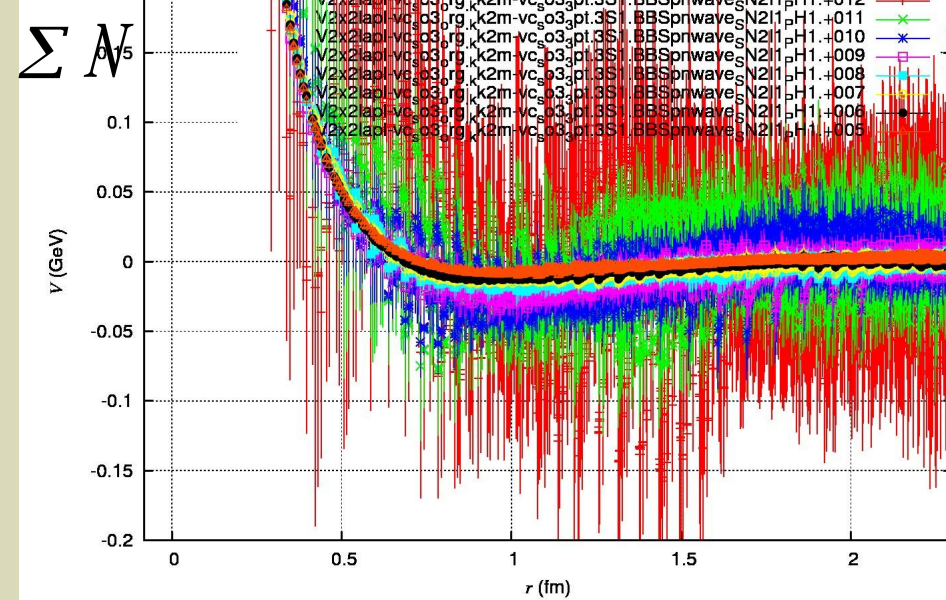


PRELIMINARY

V2x2lapi-vc_so3_{rg},k2m-vc_so3_{pt}.3S1.BBSpnwave_pLpH1



V2x2lapi-vc_so3_{rg},v2x2kk2m-vc_so3_{pt}.3S1.BBSpnwave_sN21_pH1

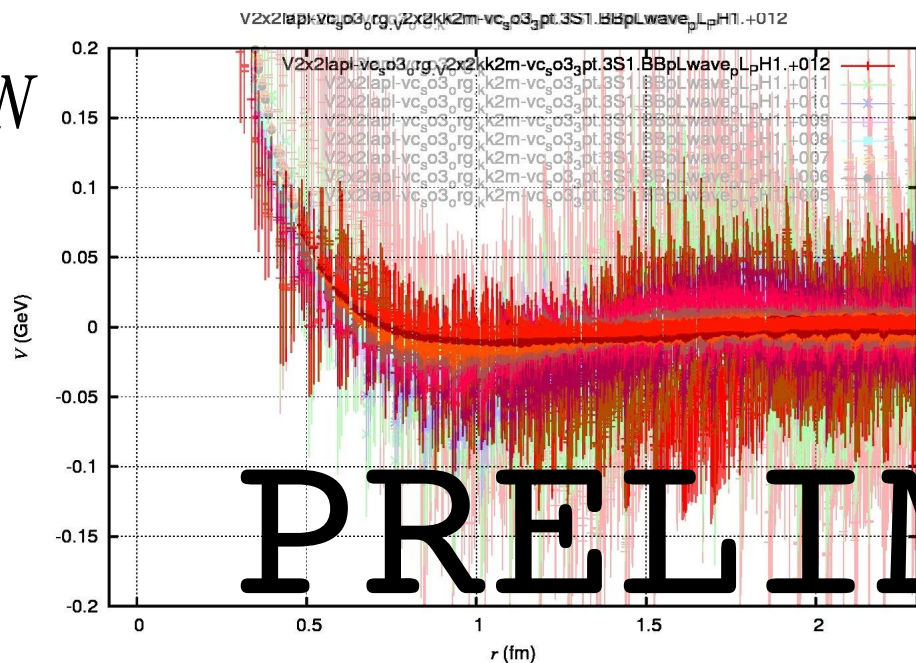


Very preliminary result of LN potential at the physical point

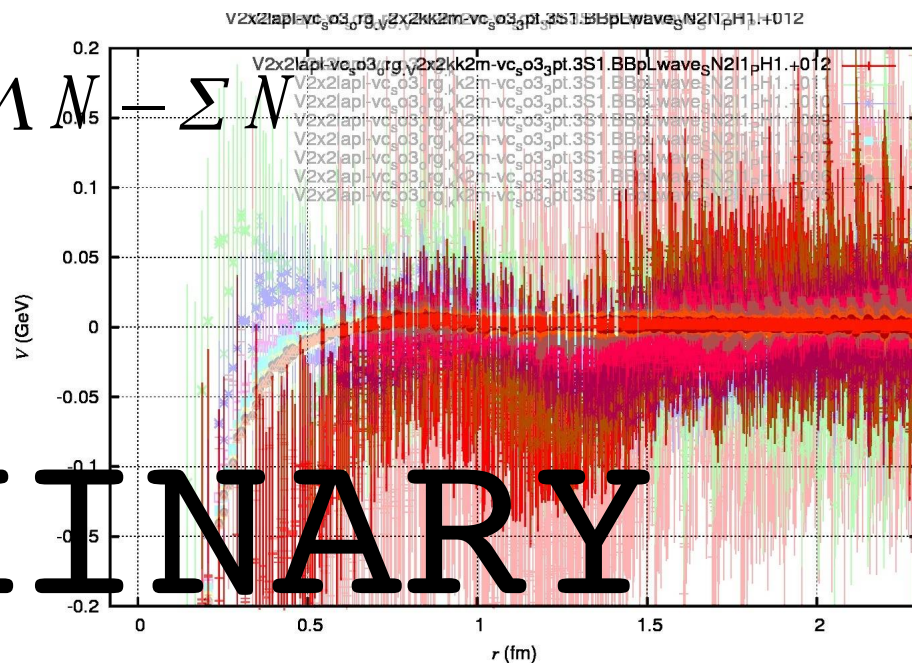
$$V_C({}^3S_1 - {}^3D_1)$$

$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r}, t) = \int d^3r' U(\vec{r}, \vec{r}') R(\vec{r}', t) + O(k^4) = V_{LO}(\vec{r}) R(\vec{r}, t) + \dots \quad (8)$$

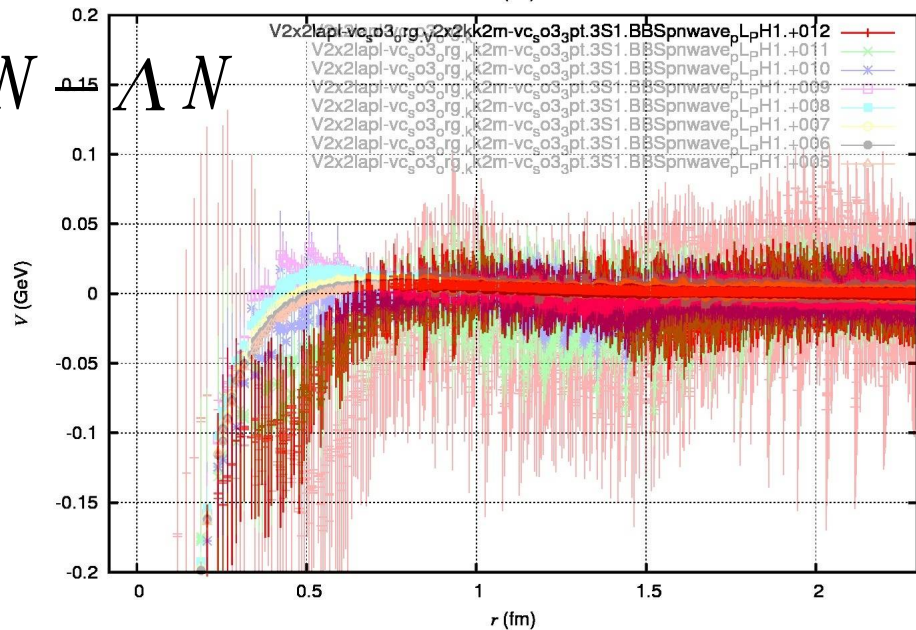
ΛN



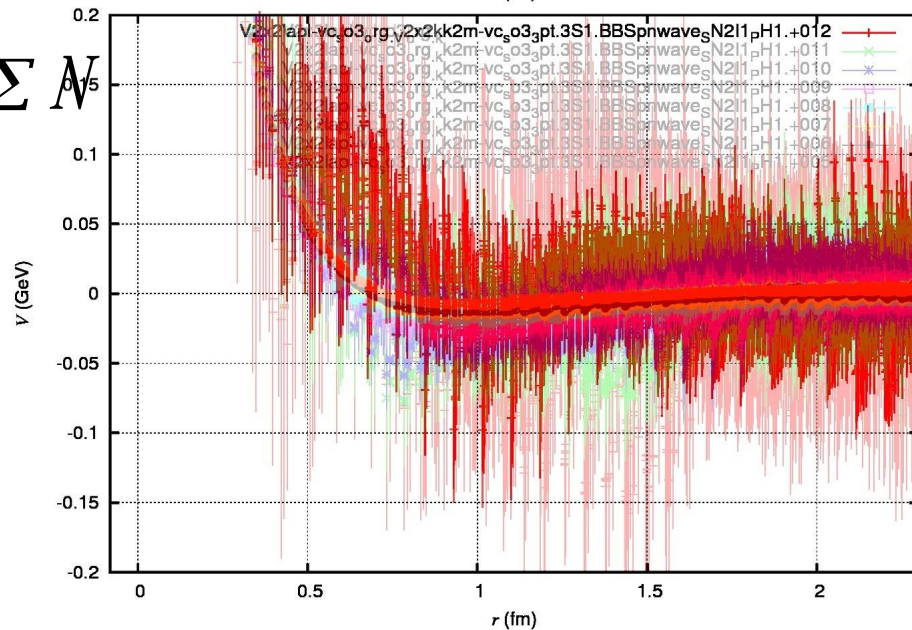
$\Lambda N - \Sigma N$



ΣN



$\Sigma N - \Lambda N$



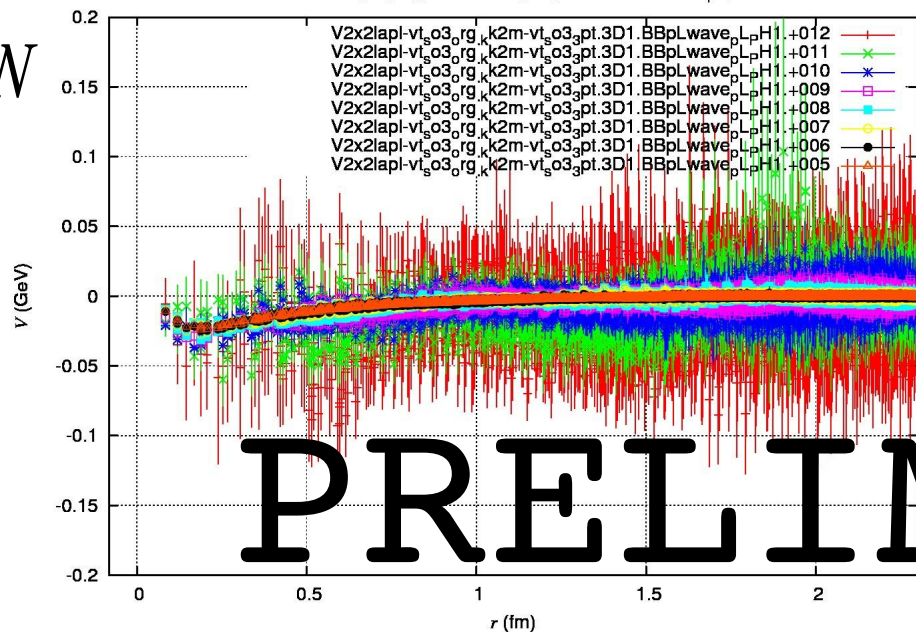
PRELIMINARY

Very preliminary result of LN potential at the physical point

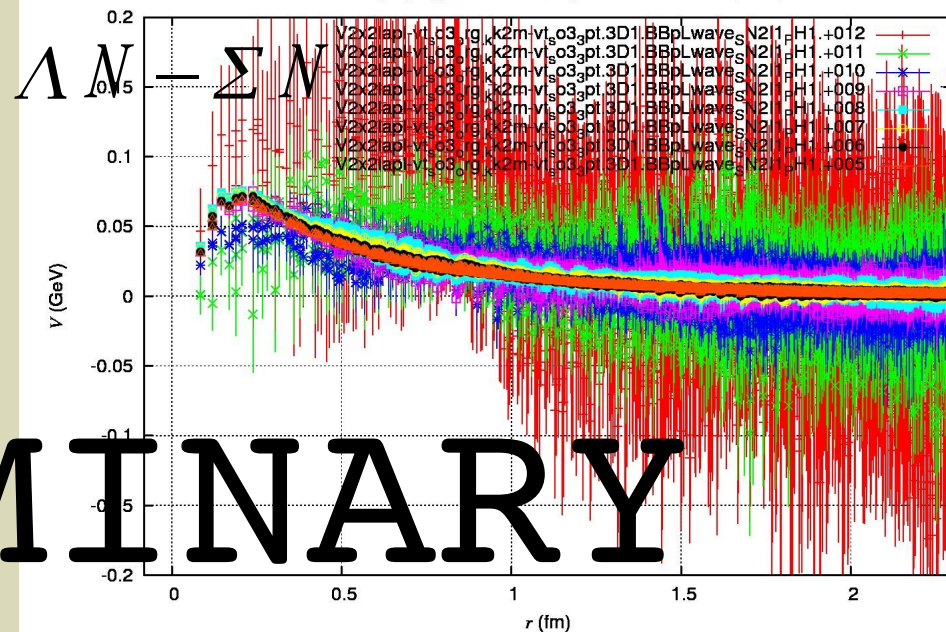
$$V_T({}^3S_1 - {}^3D_1)$$

$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r}, t) = \int d^3r' U(\vec{r}, \vec{r}') R(\vec{r}', t) + O(k^4) = V_{LO}(\vec{r}) R(\vec{r}, t) + \dots \quad (8)$$

V2x2lapi-vt_so₃rg_v2x2kk2m-vt_so₃pt.3D1.BBpLwave_pLpH1

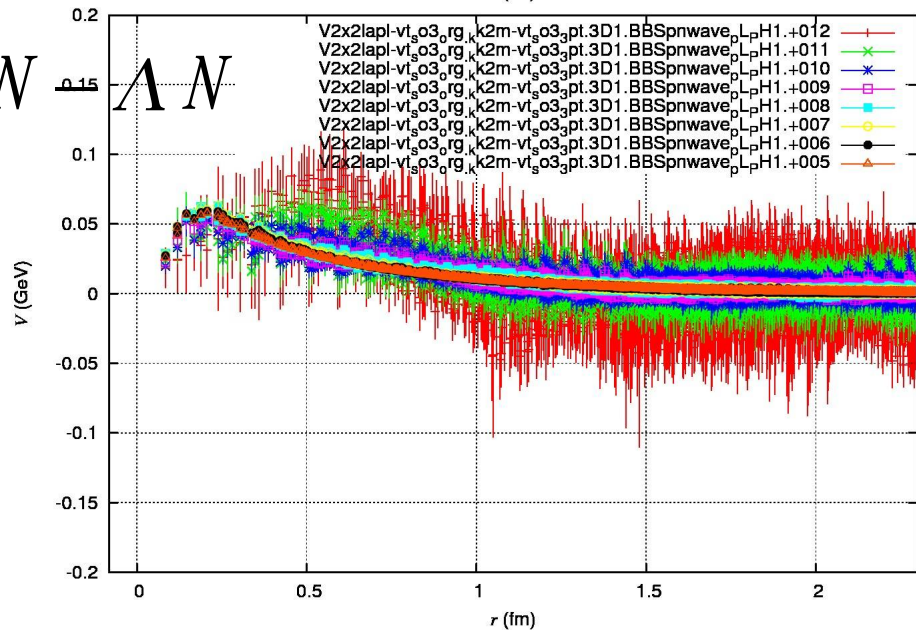


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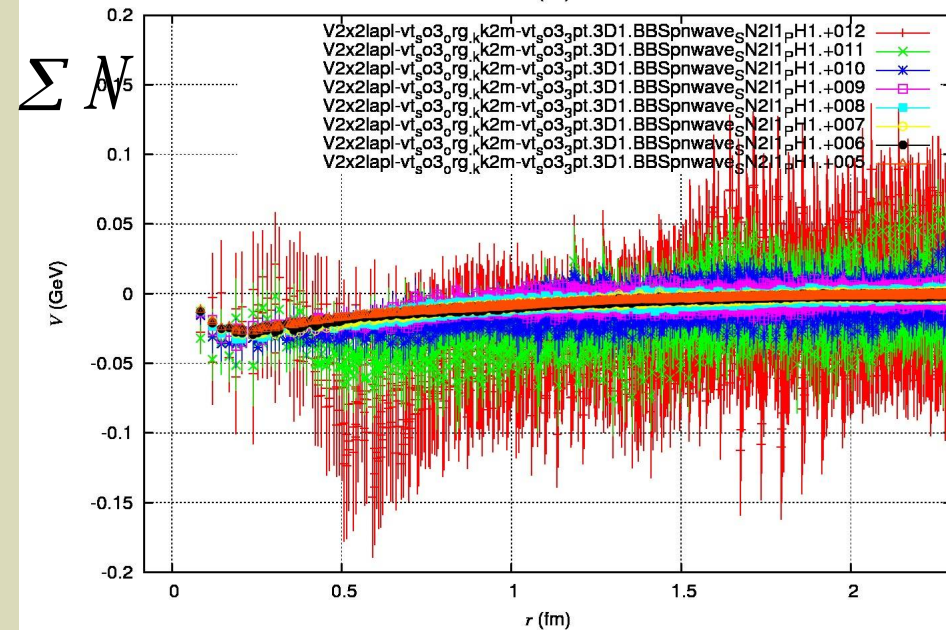


PRELIMINARY

ΣN ΛN



ΣN ΛN

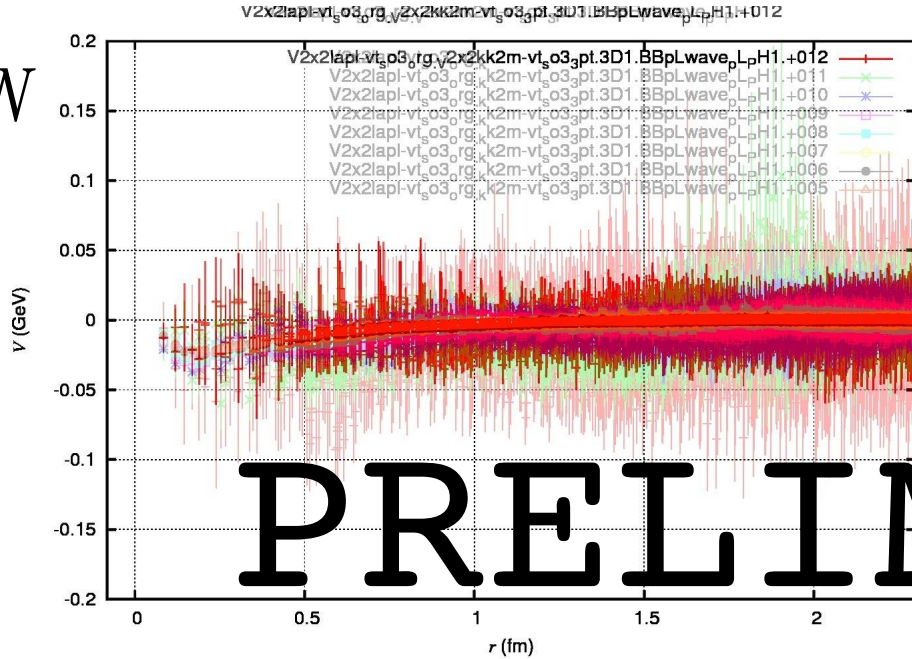


Very preliminary result of LN potential at the physical point

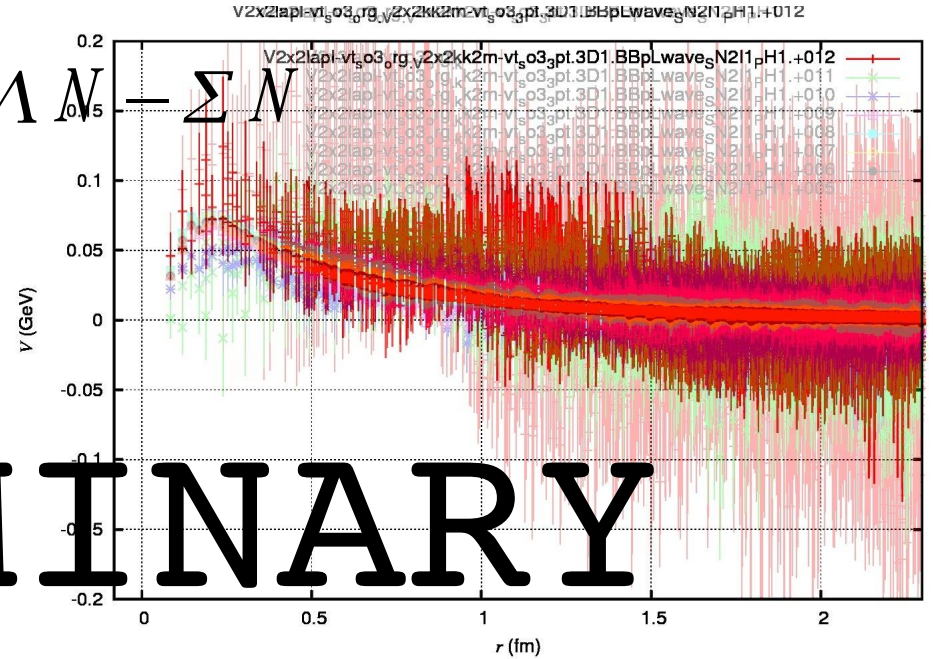
$$V_T({}^3S_1 - {}^3D_1)$$

$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r}, t) = \int d^3r' U(\vec{r}, \vec{r}') R(\vec{r}', t) + O(k^4) = V_{LO}(\vec{r}) R(\vec{r}, t) + \dots \quad (8)$$

ΛN

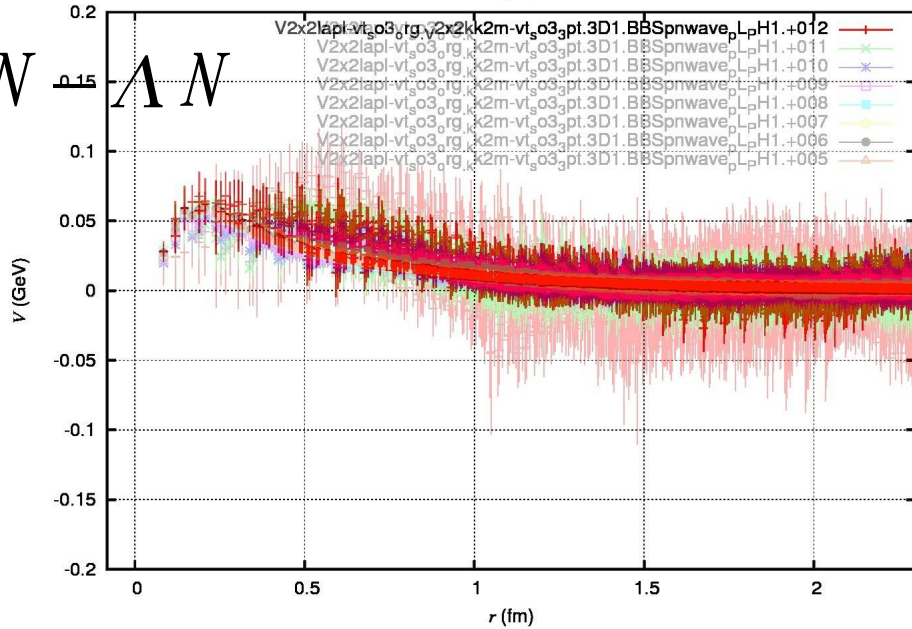


$\Lambda N - \Sigma N$

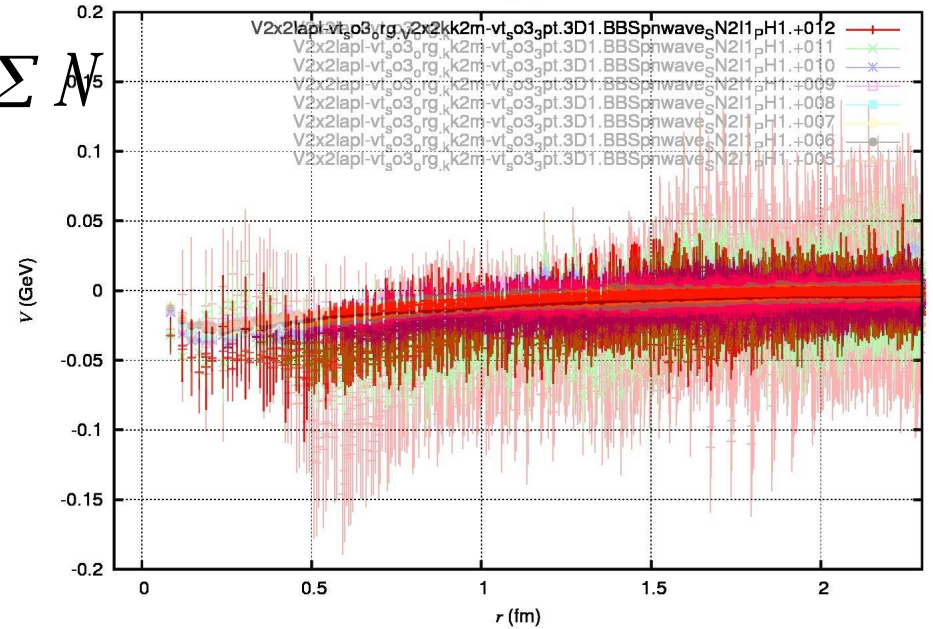


PRELIMINARY

ΣN



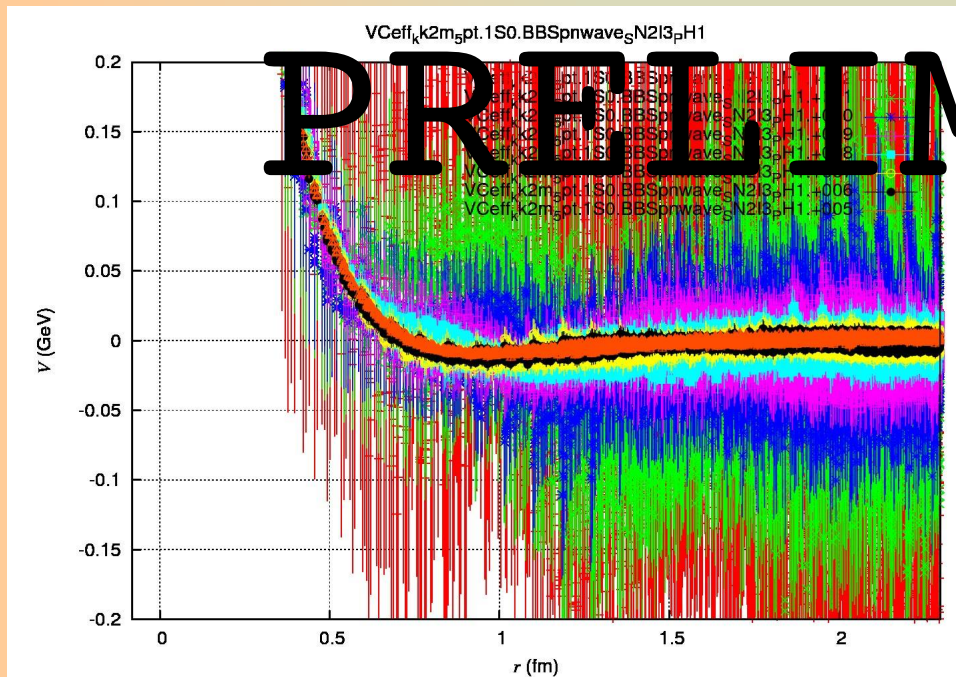
ΣN



Very preliminary result of LN potential at the physical point

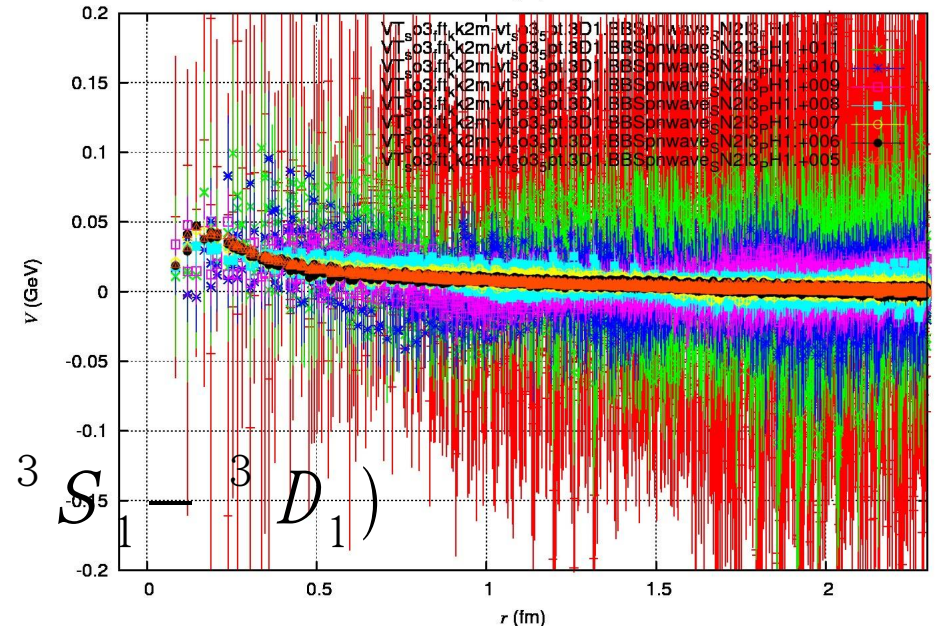
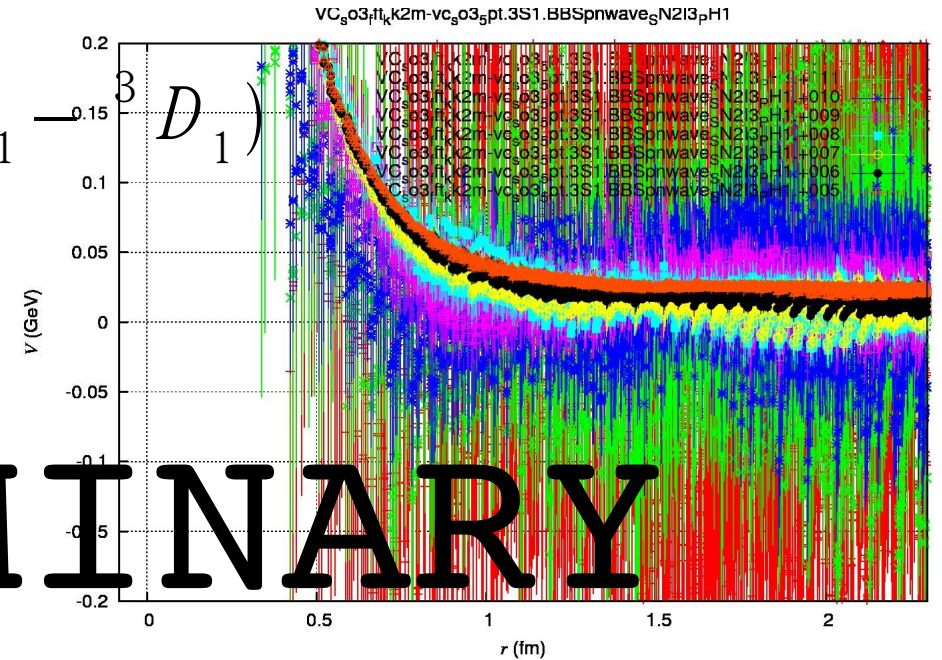
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r}, t) = \int d^3r' U(\vec{r}, \vec{r}') R(\vec{r}', t) + O(k^4) = V_{LO}(\vec{r}) R(\vec{r}, t) + \dots \quad (8)$$

$$\Sigma N(I=3/2) \quad V_C({}^3S_1 - {}^3D_1)$$



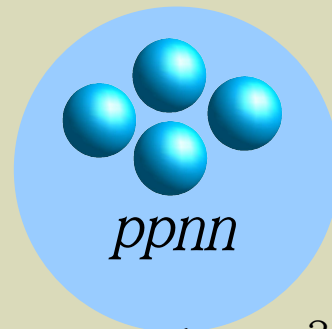
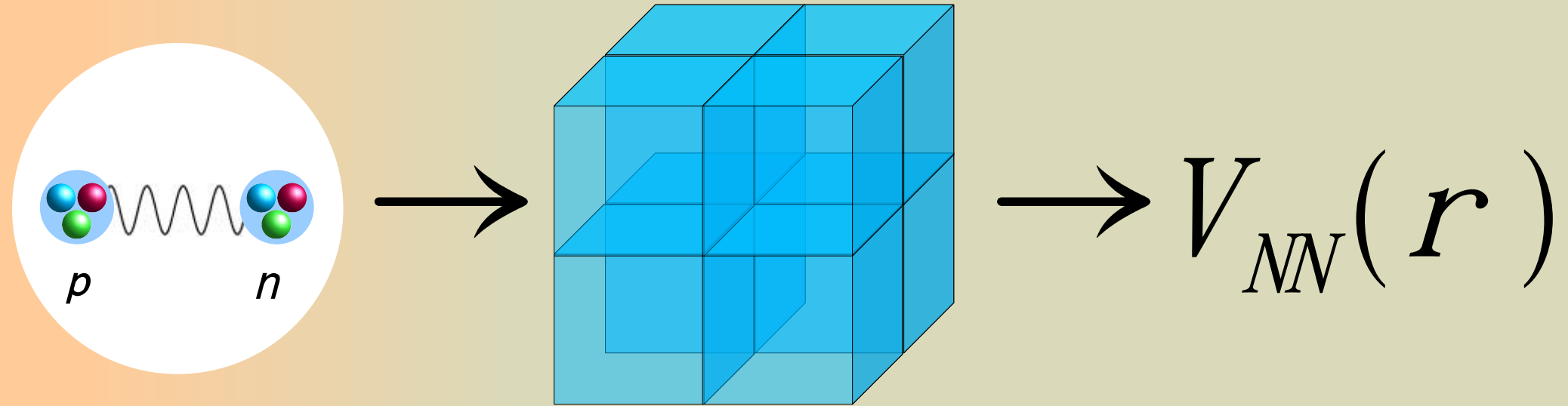
$$V_C({}^1S_0)$$

$$V_T({}^3S_1 - {}^3D_1)$$



PRELIMINARY

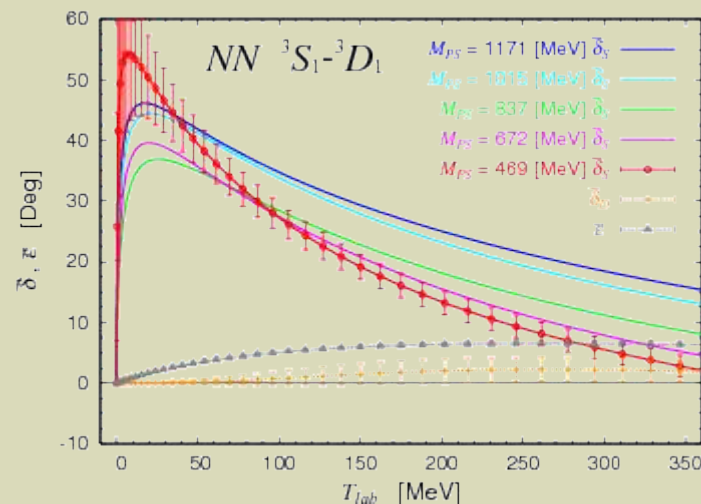
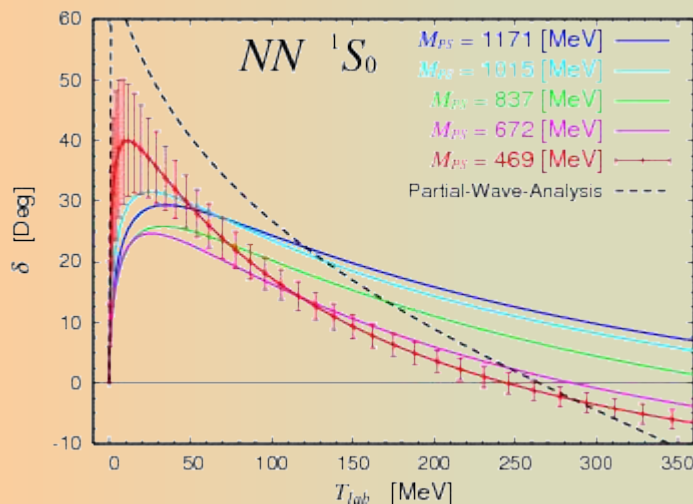
Nuclear few-body problem



$$H = \sum_{i=1}^A \frac{\mathbf{p}_i^2}{2m_N} - T_{c.m.} + \sum_{i < j}^A V_{NN}(r_{ij})$$

Stochastic variational calculation of 4He with using a lattice potential

- For NN potential, we use the SU(3) potential at the lightest quark mass ($m_{ps} = 469 \text{ MeV}$), which has been reported to have a 4N bound state (about 5.1MeV) within a tensor-included effective central potential; NPA881, 28-43 (2011).



Benchmark test calculation of a four-nucleon bound state,

Phys. Rev. C64, 044001 (2001).

TABLE I. The expectation values $\langle T \rangle$ and $\langle V \rangle$ of kinetic and potential energies, the binding energies E_b in MeV, and the radius in fm.

| Method | $\langle T \rangle$ | $\langle V \rangle$ | E_b | $\sqrt{\langle r^2 \rangle}$ |
|--------|---------------------|---------------------|-------------|------------------------------|
| FY | 102.39(5) | -128.33(10) | -25.94(5) | 1.485(3) |
| CRCGV | 102.30 | -128.20 | -25.90 | 1.482 |
| SVM | 102.35 | -128.27 | -25.92 | 1.486 |
| HH | 102.44 | -128.34 | -25.90(1) | 1.483 |
| GFMC | 102.3(1.0) | -128.25(1.0) | -25.93(2) | 1.490(5) |
| NCSM | 103.35 | -129.45 | -25.80(20) | 1.485 |
| EIHH | 100.8(9) | -126.7(9) | -25.944(10) | 1.486 |

Spin-orbit force from lattice QCD



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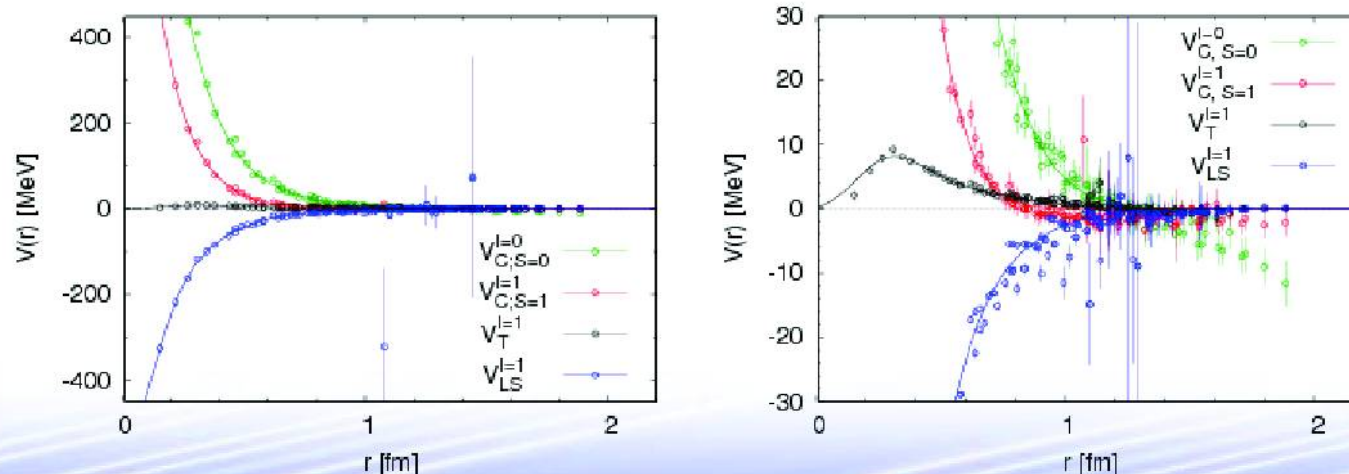
Spin-orbit potential

Scattering phase shift

ABSTRACT

We present a first attempt to determine nucleon-nucleon potentials in the parity-odd sector which appear in the 1P_1 , 3P_0 , 3P_1 , 3F_2 - 3F_2 channels in $N_f=2$ lattice QCD simulations. These potentials are constructed from the Nambu-Bethe-Salpeter wave functions for $J^P=0^-, 1^-$ and 2^- , which correspond to the A_1^- , T_1^- and $T_2^- \oplus E^-$ representation of the cubic group, respectively. We have found a large and attractive spin-orbit potential $V_{LS}(r)$ in the isospin-triplet channel, which is qualitatively consistent with the phenomenological determination from the experimental scattering phase shifts. The potentials obtained from lattice QCD are used to calculate the scattering phase shifts in the 1P_1 , 3P_0 , 3P_1 and 3P_2 - 3F_2 channels. The strong attractive spin-orbit force and a weak repulsive central force in spin-triplet P -wave channels lead to an attraction in the 3P_2 channel, which is related to the P -wave neutron pairing in neutron stars.

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Inoue san's NN potential

- ★ Central and spin-orbit potentials

$$V_{C,LS}(r) = V_1 \exp(-\alpha_1 r^2) + V_2 \exp(-\alpha_2 r^2) - V_3 (1 - \exp(-\alpha_3 r^2))^2 (\exp(-\alpha_4 r)/r)^2$$

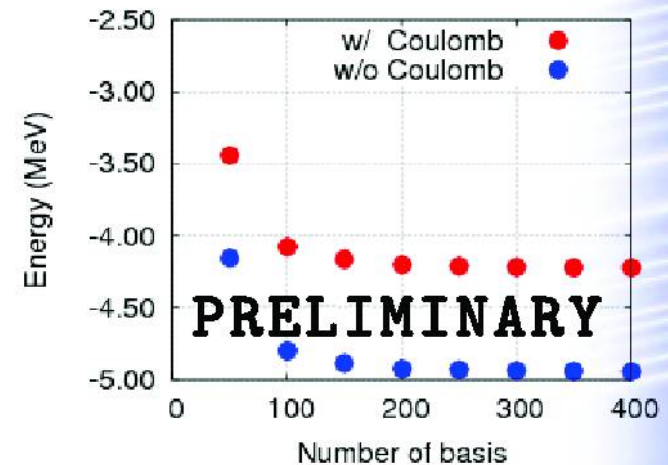
- ★ Tensor potential

$$V_T(r) = V_1 (1 - \exp(-\alpha_1 r^2))^2 \left(1 + \frac{3}{\alpha_2 r} + \frac{3}{(\alpha_2 r)^2}\right) \frac{\exp(-\alpha_2 r)}{r} + V_2 (1 - \exp(-\alpha_3 r^2))^2 \left(1 + \frac{3}{\alpha_4 r} + \frac{3}{(\alpha_4 r)^2}\right) \frac{\exp(-\alpha_4 r)}{r}$$

Results of few-body calculation

★ Inputs:

- $m=1161.0$ MeV,
- $\hbar c = 197.3269602$ MeV fm
- $\hbar c/e^2 = 137.03599976$
- V_{NN} consists of AV8 type operators, determined from $\{1S0, 3S1, 3SD1, 1P1, 3P0, 3P1, 3PF2\}$.
- $V_0, V_\sigma, V_\tau, V_{\sigma\tau}, V_T, V_{T\tau}, V_{LS}^{odd}$ are determined



★ Preliminary results:

- $B(4\text{He})=4.23$ MeV (w/ Coulomb) (old: 4.37MeV)
- Probabilities of (S, P, D) waves = (98.8%, 0.002%, 1.2%)
- cf. roughly speaking (S,P,D) \sim ($<90\%$, $<0.1\%$, $>10\%$) for a realistic NN force
- $B(4\text{He})=4.95$ MeV (w/o Coulomb) (old: 5.09MeV)
- Probabilities of (S, P, D) waves = (98.8%, 0.002%, 1.2%)

Summary

(I-1) Preliminary results of LN-SN potentials at nearly physical point. (Λ -N, Σ -N: central, tensor)

Statistics approaching to 0.2 (=present/scheduled)

Several interesting features seem to be obtained with more high statistics.

(I-2) Effective hadron block algorithm for the various baron-baryon interaction

Paper available from [arXiv:1510.00903(hep-lat)]

(I-3) Four-nucleon bound state with a lattice NN potential at pion mass about 470MeV (+Coulomb potential) has been solved by using the stochastic variational method

The lattice NN potential is represented by AV8-type operators.

The tensor potential is weaker than the phenomenological realistic NN potentials due to the heavier pion mass.

The D-state probability is only about 1.2%.

Future work:

(II-1) Physical quantities including the binding energies of few-body problem of light hypernuclei with the lattice YN potentials