Ab initio calculations for non-strange and strange few-baryon systems

- Applied ab initio methods
- Non-strange sector: continuum observables
- Strange sector: hypernuclear bound states

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- Non-strange sector: continuum observables
  Resonances
  S-Factor in presence of Coulomb potential

- Strange sector: hypernuclear bound states
  Benchmark calculation with other
  ab initio methods

Applied ab initio methods

- A-body system: A position vectors $\mathbf{r}_i$, removal of center of mass coordinate
  leads to (A-1) Jacobi vectors $\boldsymbol{\eta}_i$ $(\eta_i, \theta_i, \phi_i)$
- Expansion of ground-state wave function or LIT state on a complete set:
  Hyperspherical Harmonics (HH)
- 3(A-1) coordinates of HH basis: hyperradius ρ, 3(A-1) angular coordinates
  Θ_i and ϕ_i , (A-2) hyperspherical angles: 1 hyperradius + (3A – 4) angles
- HH basis states: eigen states of grand-angular momentum operator
  depending on the (3A – 4) angles times a hyperradial basis state
- Different HH versions: normally symmetrized basis states, but also a nonsymmetrized HH (NSHH) basis is possible
- Acceleration of convergence: effective interaction (EIHH)
  Short-range two-body correlations (CHH)

Applied ab initio methods

- Solve Schrödinger or LIT equation with N basis states and increase N up to the point that a sufficient convergence is obtained

LIT method

The LIT of a function $R(E)$ is defined as follows

$$\Rightarrow \quad L(\sigma) = \int dE\, \mathcal{L}(E,\sigma)\, R(E)\,,$$

where the kernel $\mathcal{L}$ is a Lorentzian,

$$\Rightarrow \quad \mathcal{L}(E,\sigma) = \frac{1}{(E-\sigma_R)^2 + \sigma_I^2}$$

For inclusive reactions the LIT $L(\sigma)$ is calculated by solving an equation of the form

$$\boxed{(H-\sigma)\,\tilde{\Psi} = S\,,}$$

where H is the Hamiltonian of the system under consideration and S is an asymptotically vanishing source term related to the operator inducing the specific reaction.

The solution $\tilde{\Psi}$ is localized and the LIT is given by

$$\boxed{L(\sigma) = \langle \tilde{\Psi} | \tilde{\Psi} \rangle\,.}$$

Alternative way:

$$L(\sigma) = -\frac{1}{\sigma_I} Im(\langle S | \frac{1}{\sigma_R + i\sigma_I - H} | S \rangle)\,.$$

The source term S for inclusive reactions has the form

$$\Rightarrow \quad |S\rangle = \theta|0\rangle\ ,$$

where the operator θ induces a specific electroweak reaction.

The corresponding response function is given by

$$\Rightarrow \quad R(E_f) = \int dE_f |\langle f|\theta|0\rangle|^2 \delta(E_f - E_0 - \omega)$$

Ingredients of the solution of the LIT equation via an expansion on a basis of dimension N:

N eigenstates with eigenenergies

ϕ_n E_n

and strength

$$S_n = |\langle \phi_n|\theta|0\rangle|^2$$

leading to the following LIT

$$L(\sigma) = \sum_{i=1}^{N} \frac{S_n}{(\sigma_R - E_n)^2 + \sigma_I^2}$$

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LIT states

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leading to the following LIT

$$\Rightarrow \quad L(\sigma) = \sum_{i=1}^{N} \frac{S_n}{(\sigma_R - E_n)^2 + \sigma_I^2}$$

Inversion of the LIT

- LIT is calculated for a fixed σ_I in many σ_R points

- Express the searched response function formally on a basis set with N basis basis functions $f_n(E)$ and open coefficients c_n with correct threshold behaviour for the $f_n(E)$ (e.g., $f_n = f_{thr}(E) \exp(-\alpha E/n)$)

- Make a LIT transform of the basis functions and determine coefficents c_n by a fit to the calculated LIT

- Increase N up to the point that a sufficient convergence is obtained (uncontrolled oscillations should not be present)

Resonances

^{4}He isoscalar monopole resonance

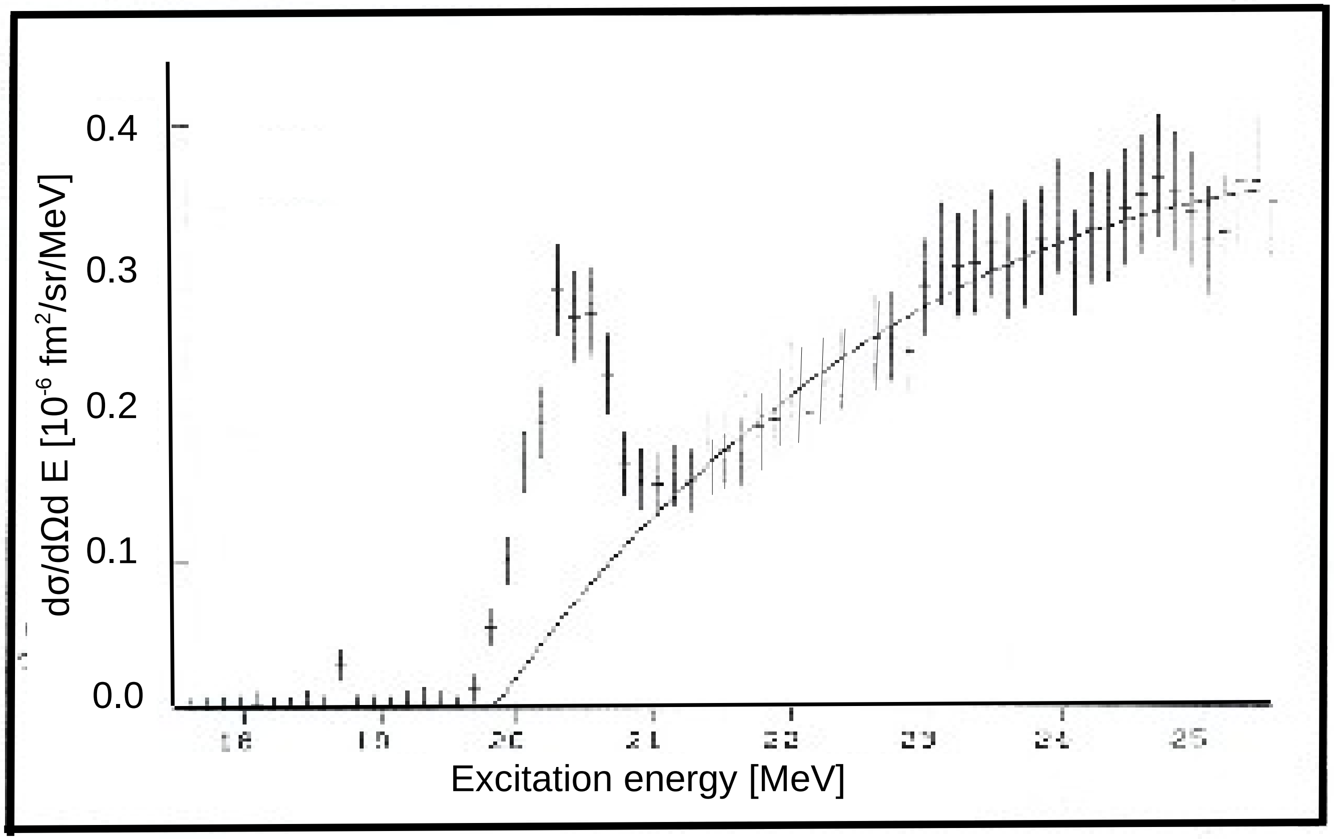
Isoscalar monopole response function $M(q, E_f = E_0 + \omega)$

with transition operator $$\boxed{\theta(q) = \frac{G_E^s(q^2)}{2} \sum_{i=1}^{A} j_0(qr_i)}$$

$G_E^s(q^2)$: nucleon isoscalar electric form factor
j_0: spherical Bessel function of 0^{th} order

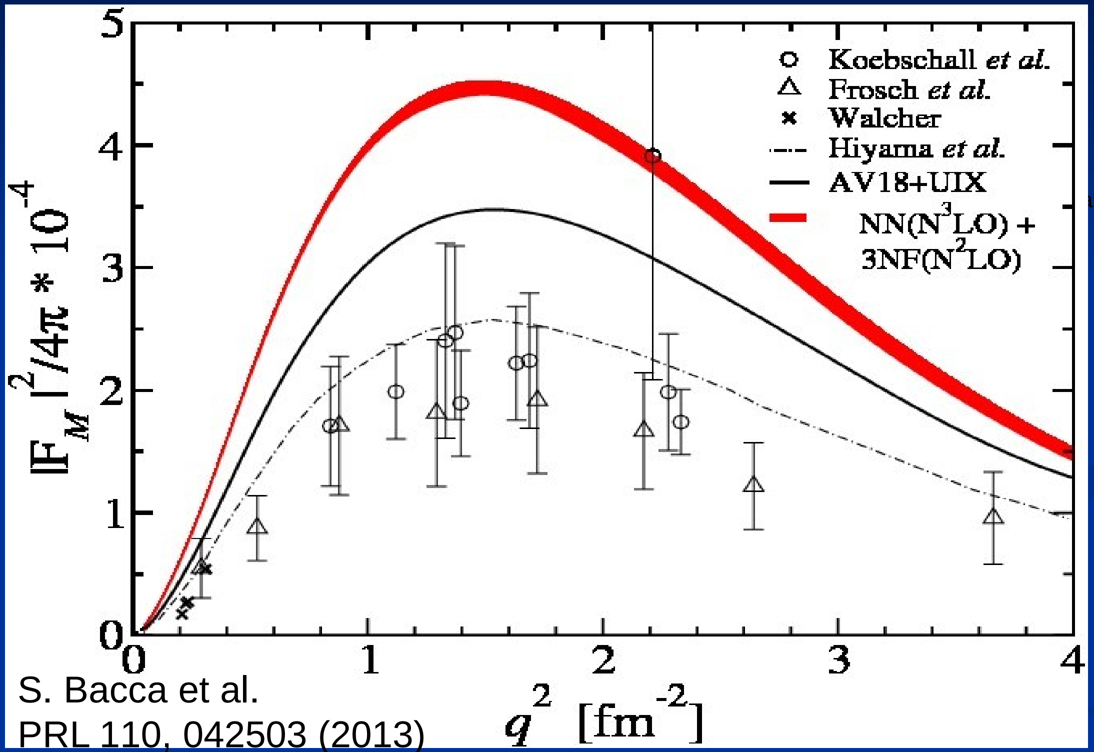
0^+ Resonance in the ^{4}He compound system

G. Köbschall et al./ Quasi bound state in ^{4}He - Nucl. Phys. A405, 648 (1983)



Resonance at E_R = -8.2 MeV, i.e. above the ^{3}H-p threshold. Strong evidence in electron scattering off ^{4}He, Γ = 270±50 keV

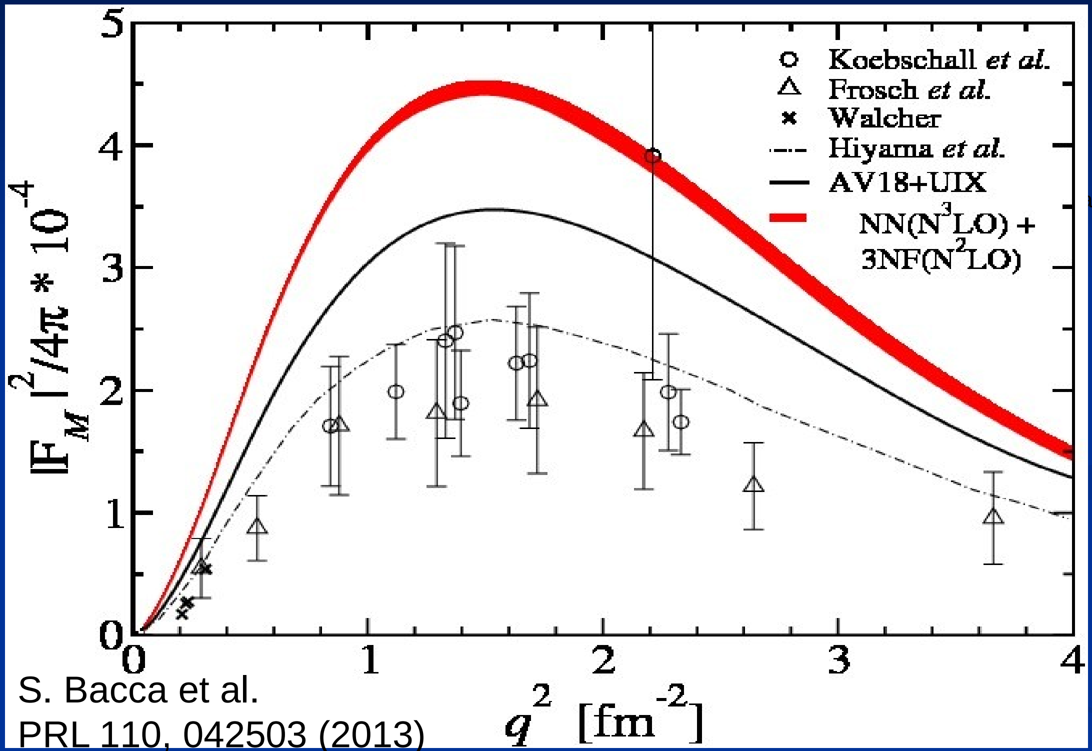
Comparison to experimental results



LIT/EIHH Calculation for AV18+UIX and Idaho-N3LO+N2LO

dash-dotted: AV8' + central 3NF (Hiyama et al.)

Comparison to experimental results



S. Bacca et al.
PRL 110, 042503 (2013)

Observable is strongly dependent on potential model

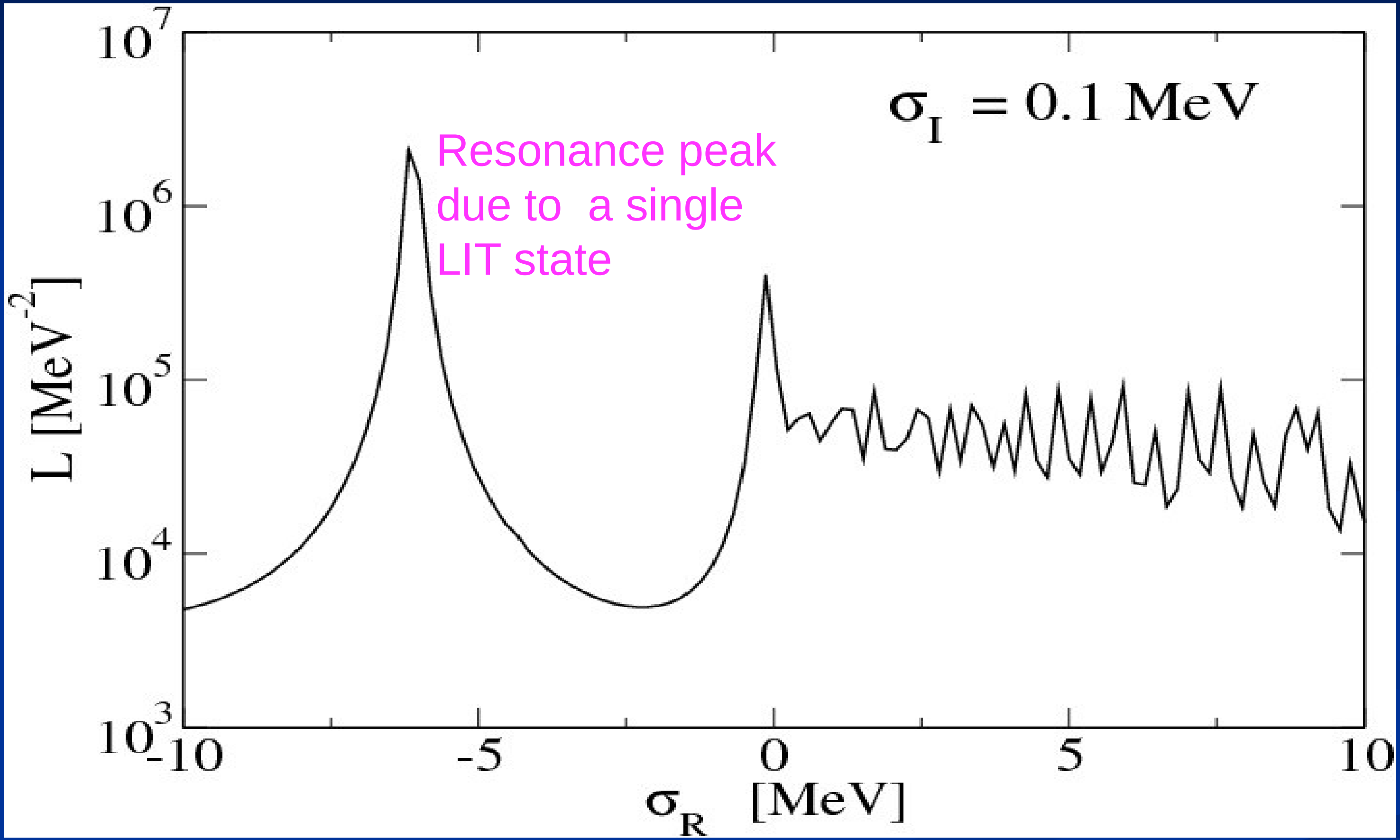
Why were we unable to determine the width of the 4He isoscalar monopole resonance?

To answer this let us check our very first LIT calculation from 1997:

^{4}He(e,e') inelastic longitudinal response function

with a central NN potential

Unpublished result from a CHH calculation with the TN potential (V. Efros, WL, G. Orlandini, PRL 78,432 (1997))



To study the problem better let us consider first instead of a four-body reaction a simpler three-body reaction:

$$^3He + \gamma \longrightarrow d + p$$

at low energies

LIT calculation with central MTI/III NN potential in unretarded dipole approximation

Aim: Increase low-energy density of LIT states

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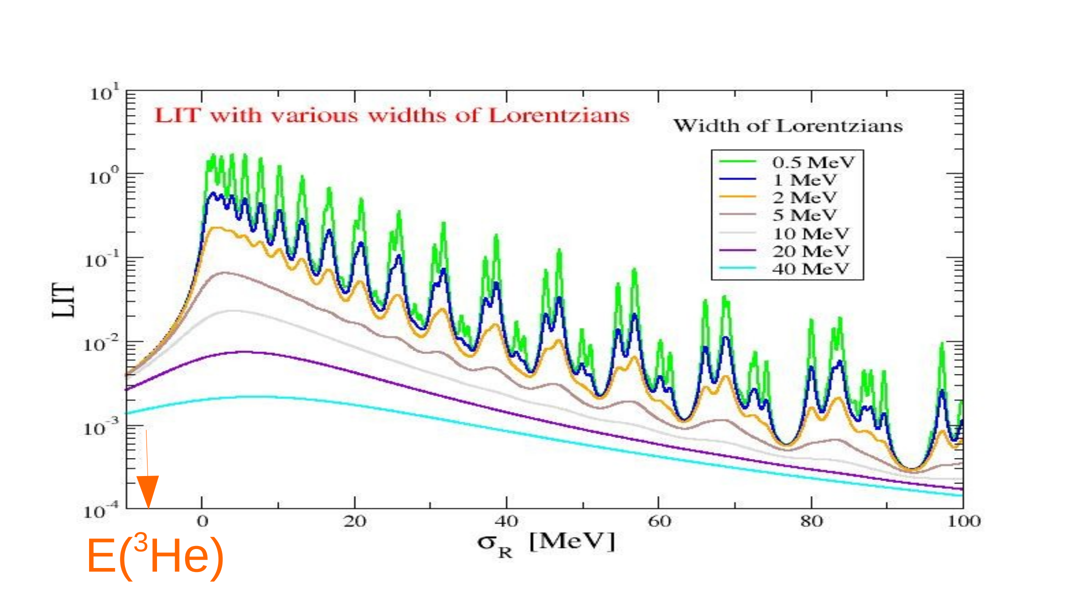
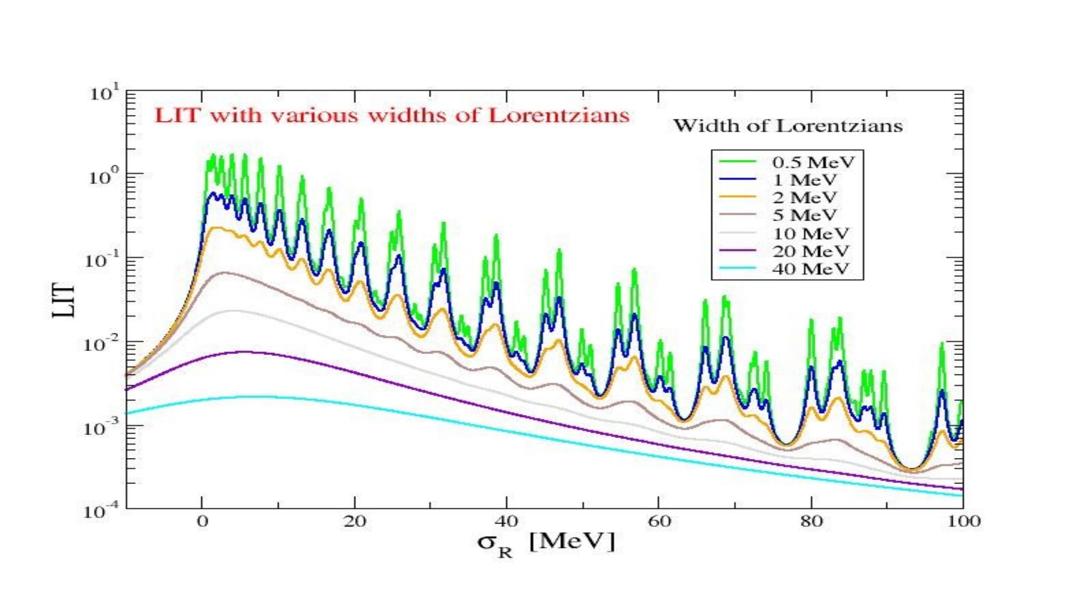
Also note: hyperradial basis states consist in an expansion on Laguerre polynomials times a spatial cutoff $\exp(-\rho/b)$
Increase of b shifts spectrum to lower energies



30 hyperspherical
31 hyperradial
⇒ 930 basis states
b = 0.6 fm



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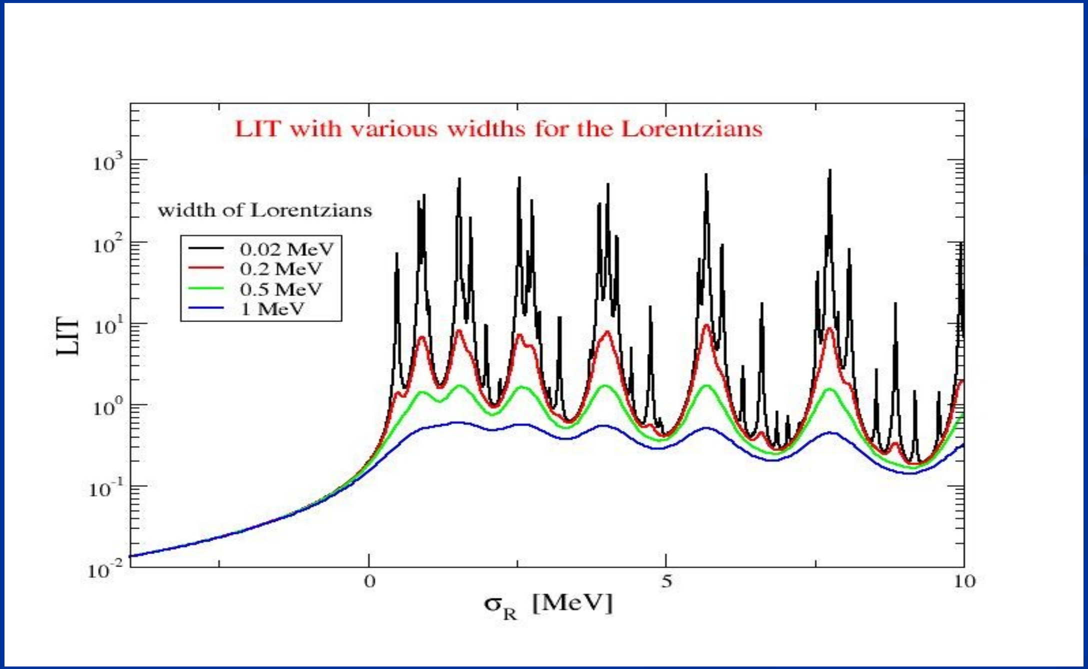

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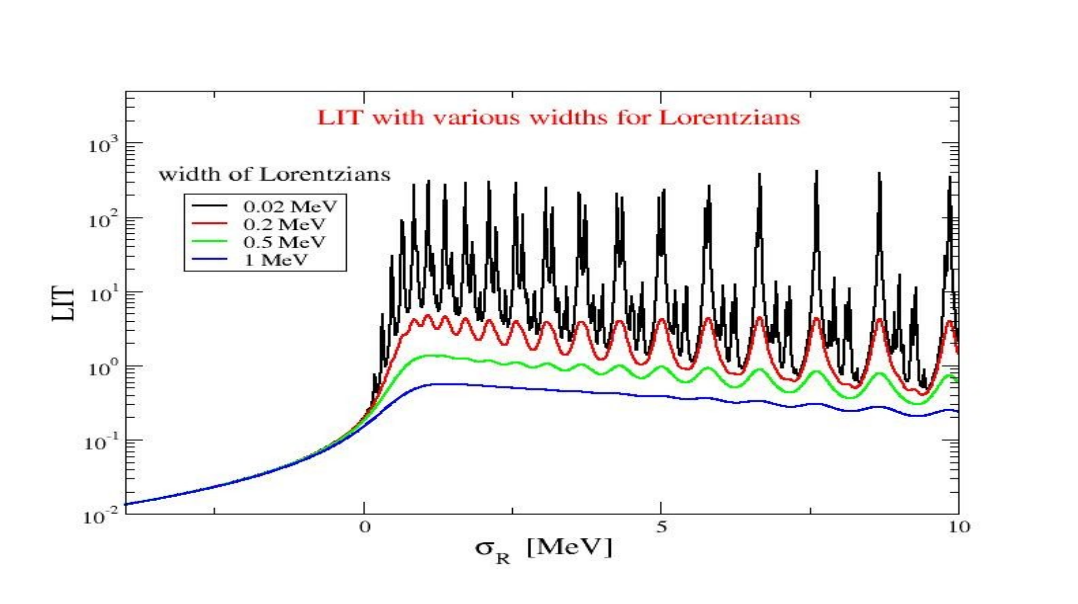
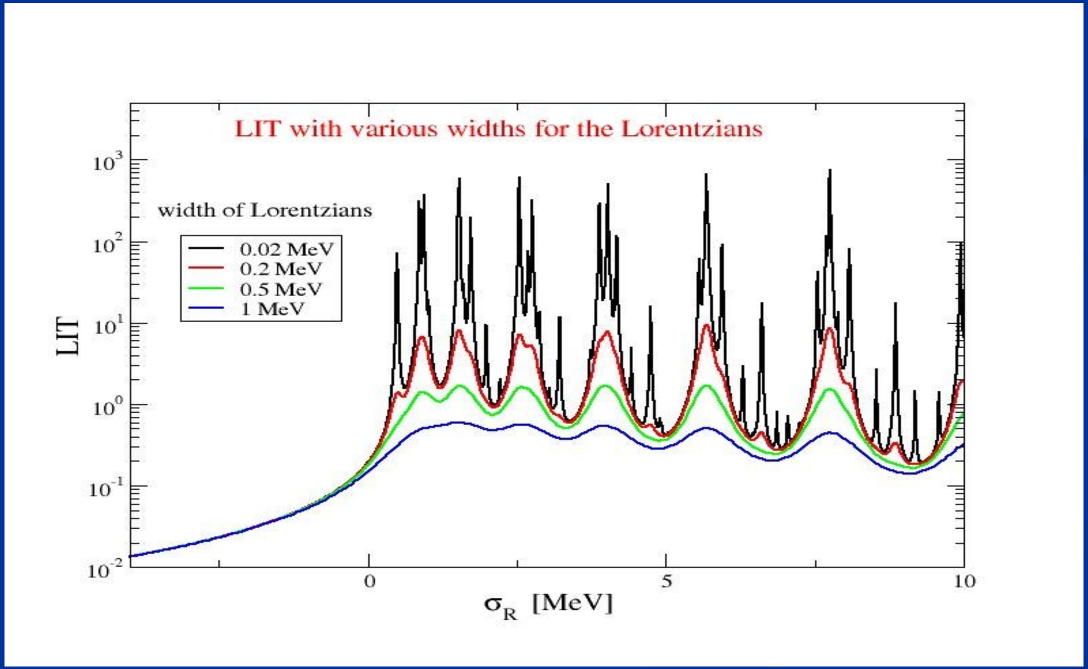


40 hyperspherical
51 hyperradial
⇒ 2040 basis states
b = 1 fm



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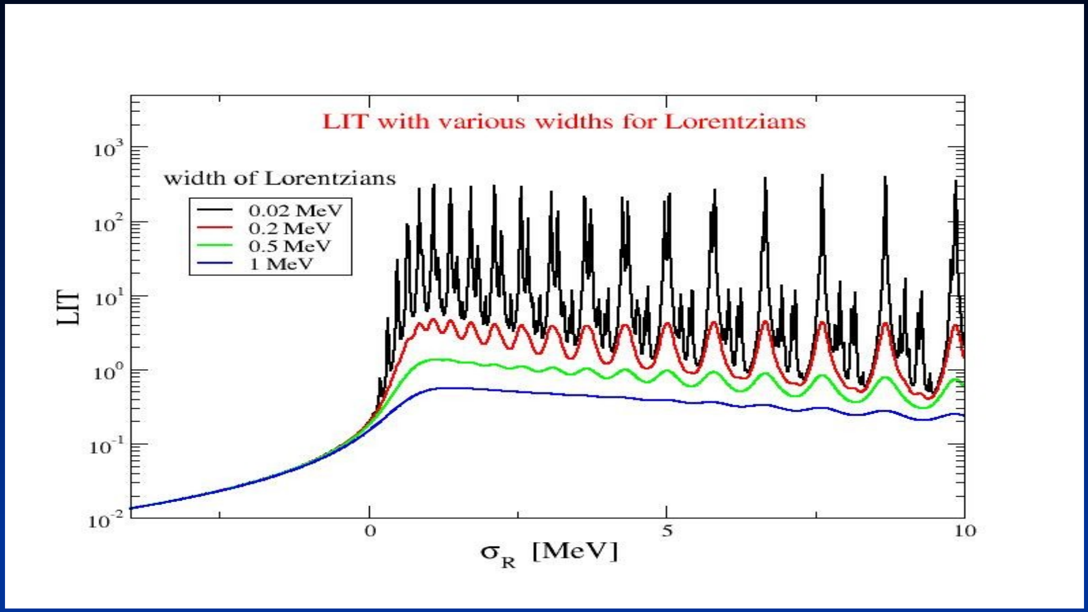

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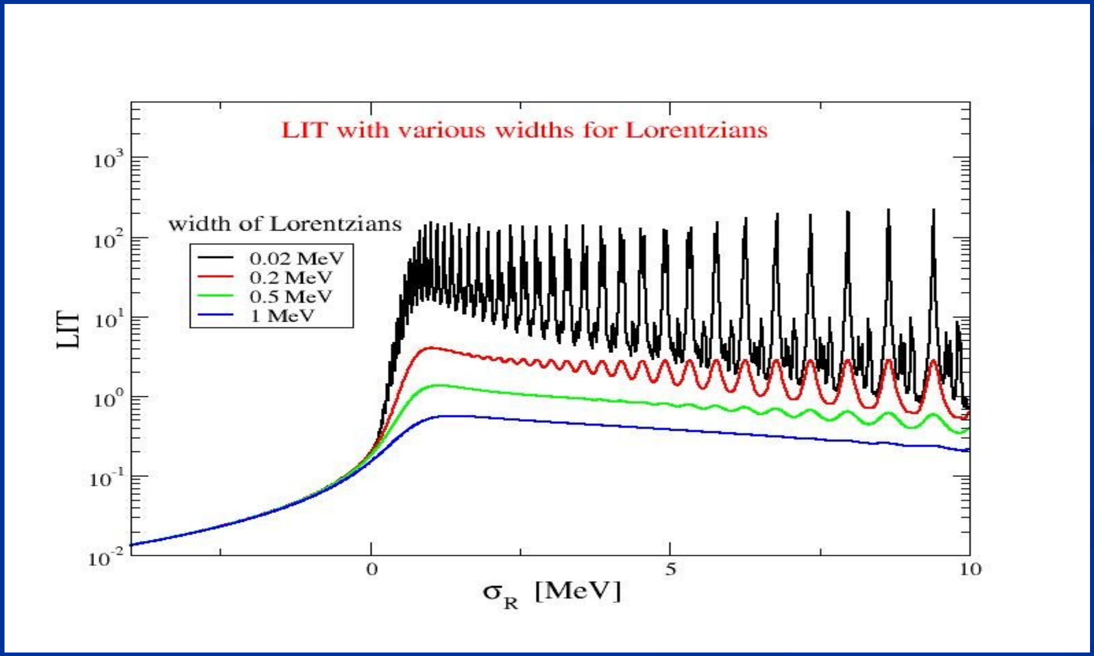

40 hyperspherical
76 hyperradial
⇒ 3040 basis states
b = 2 fm

Observation

The LIT is a method with a controlled resolution

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But in present LIT calculation below three-body breakup threshold
not a single LIT state! Similar problem as in the previous four-body case

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Solution: use instead of the HH basis a somewhat modified basis

New A-body basis

Note one of the (A-1) Jacobi vectors can be written in the following form:

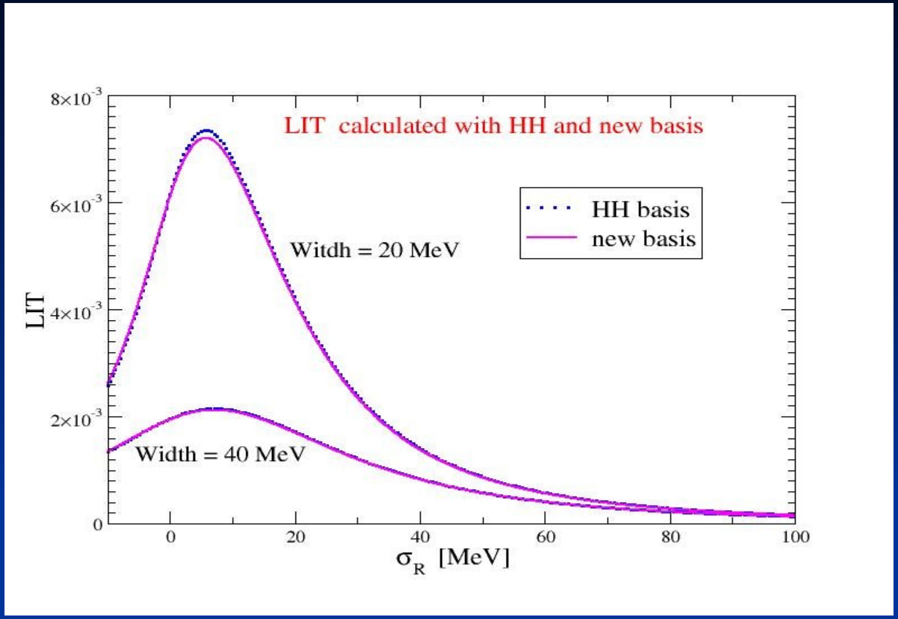
$$\boldsymbol{\eta} = \mathbf{r}_A - \mathbf{R}_{cm}(1,2,\ldots,A\text{-}1)$$

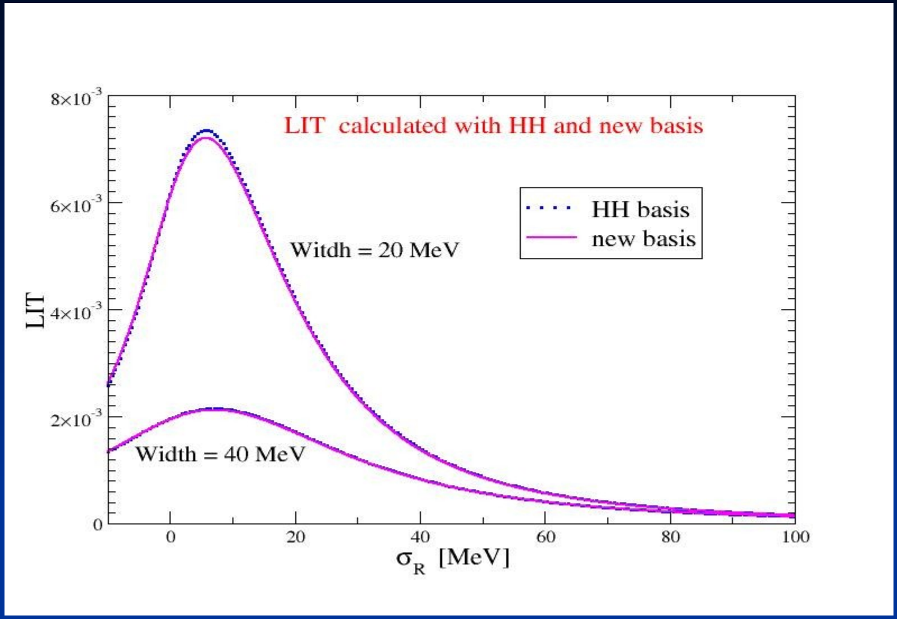
This is the coordinate one would use for the scattering of a nucleon with a (A-1)-nucleon system. In other words the relevant coordinate for a two-body breakup. Therefore

A-body HH basis ⟶ (A-1)-body HH basis times expansion on $\boldsymbol{\eta}$

radial part: Laguerre polynomials

angular part: $Y_{LM}(\theta_\eta,\phi_\eta)$


LIT calculated with HH and new basis
HH basis
new basis
Witdh = 20 MeV
Width = 40 MeV
LIT
σR [MeV]
8×10-3
6×10-3
4×10-3
2×10-3
0
0
20
40
60
80
100


LIT calculated with HH and new basis
HH basis
new basis
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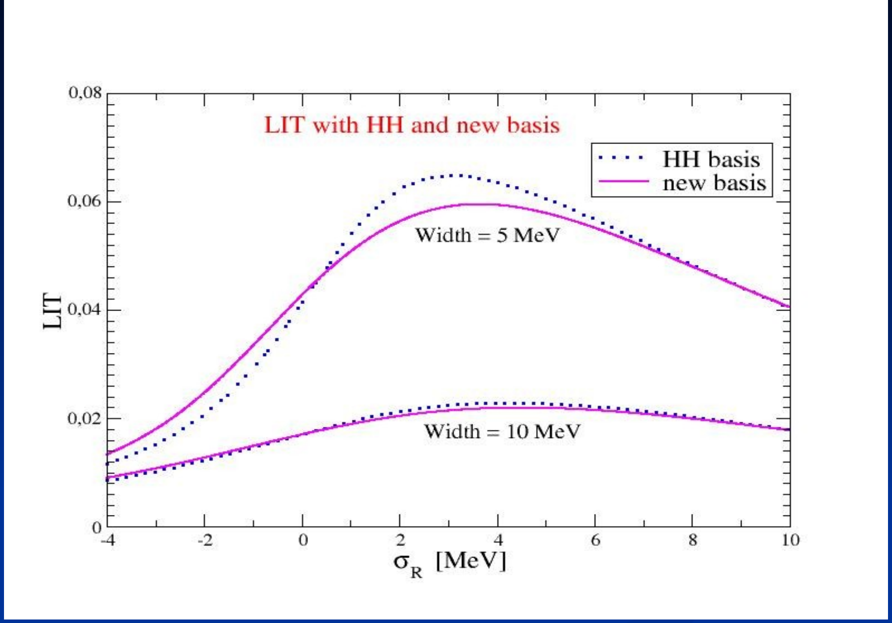
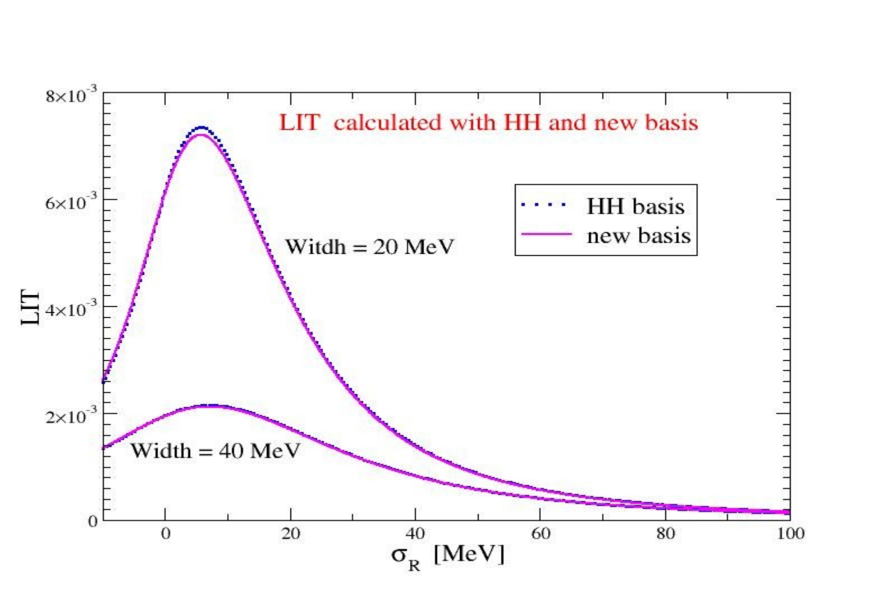
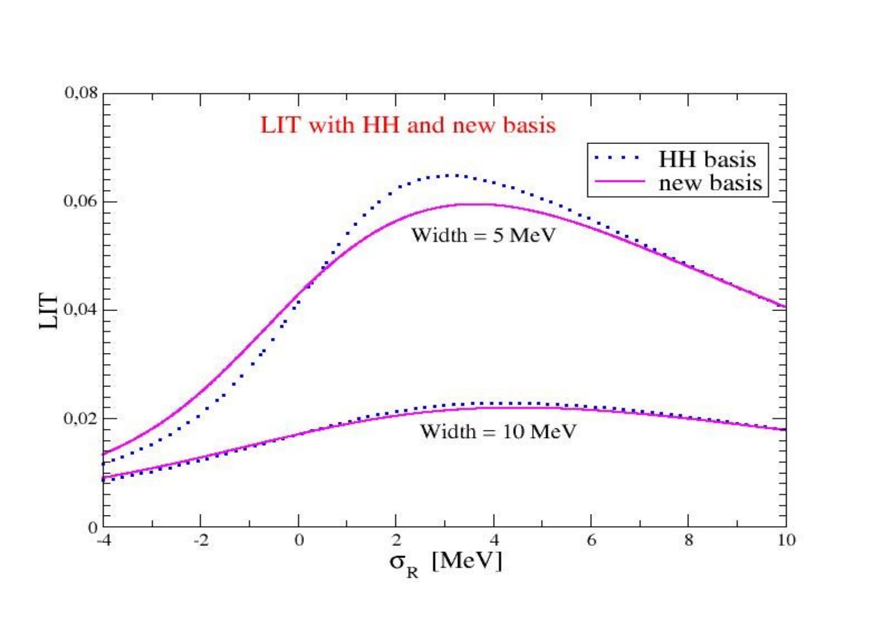
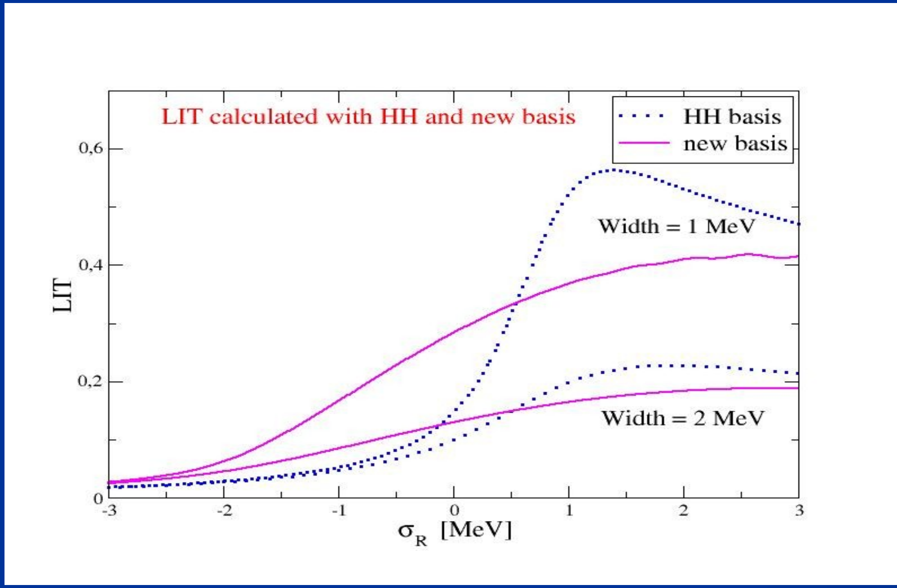
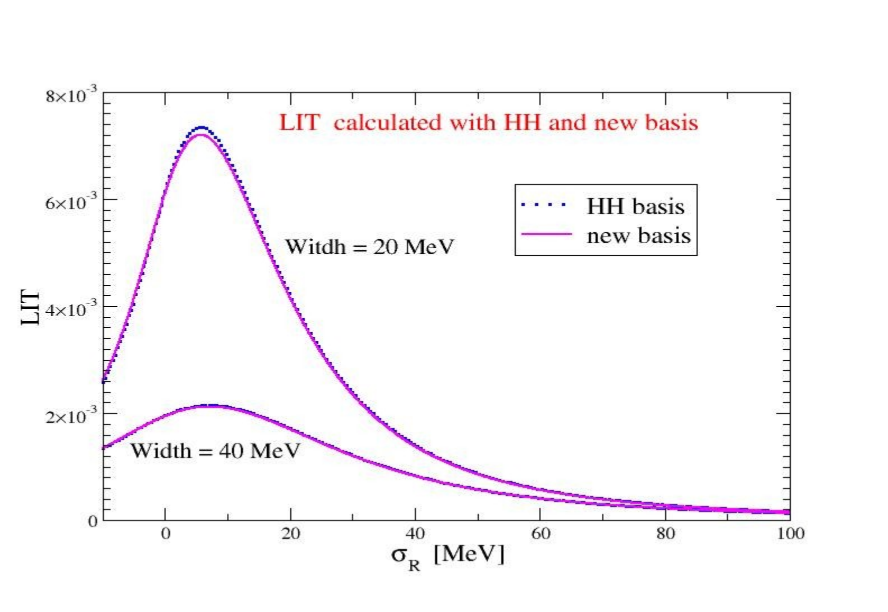
Width = 40 MeV
LIT
σR [MeV]

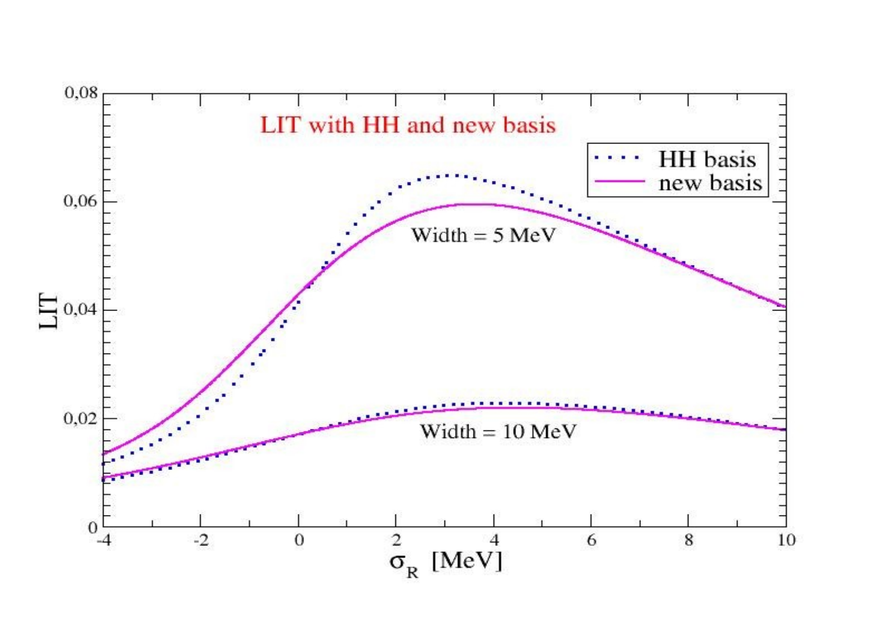

LIT with HH and new basis
HH basis
new basis
Width = 5 MeV
Width = 10 MeV
LIT
σR [MeV]

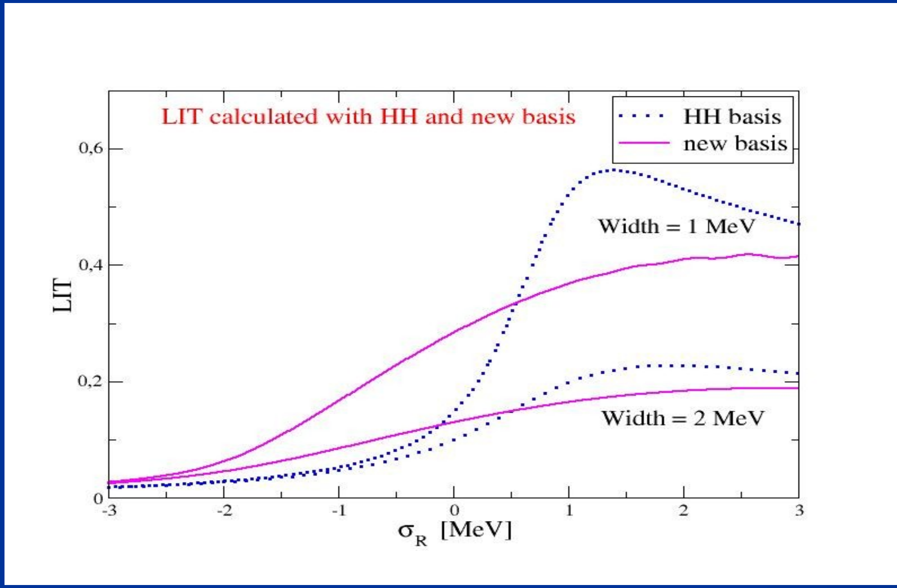

LIT calculated with HH and new basis
HH basis
new basis
Witdh = 20 MeV
Width = 40 MeV
LIT
σR [MeV]

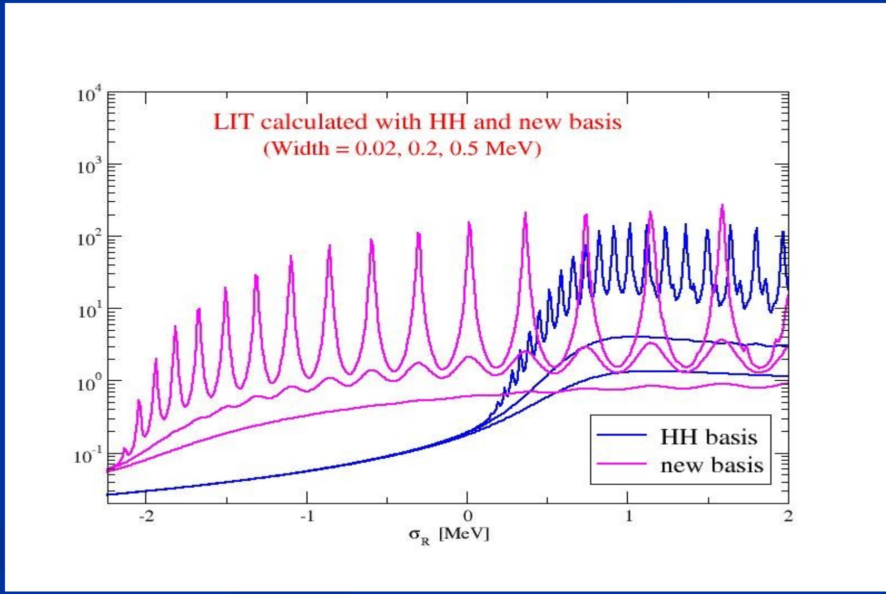

LIT with HH and new basis
HH basis
new basis
Width = 5 MeV
Width = 10 MeV
LIT
σR [MeV]


LIT calculated with HH and new basis
HH basis
new basis
Width = 1 MeV
Width = 2 MeV
LIT
σR [MeV]


LIT calculated with HH and new basis
HH basis
new basis
Witdh = 20 MeV
Width = 40 MeV
LIT
σR [MeV]


LIT with HH and new basis
HH basis
new basis
Width = 5 MeV
Width = 10 MeV
LIT
σR [MeV]

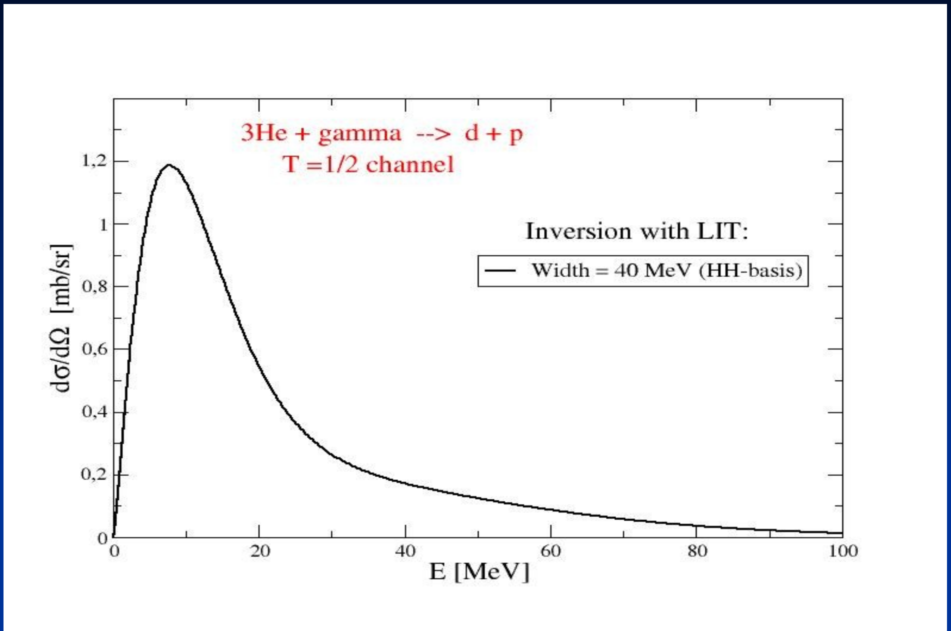

LIT calculated with HH and new basis
HH basis
new basis
Width = 1 MeV
Width = 2 MeV
LIT
σR [MeV]

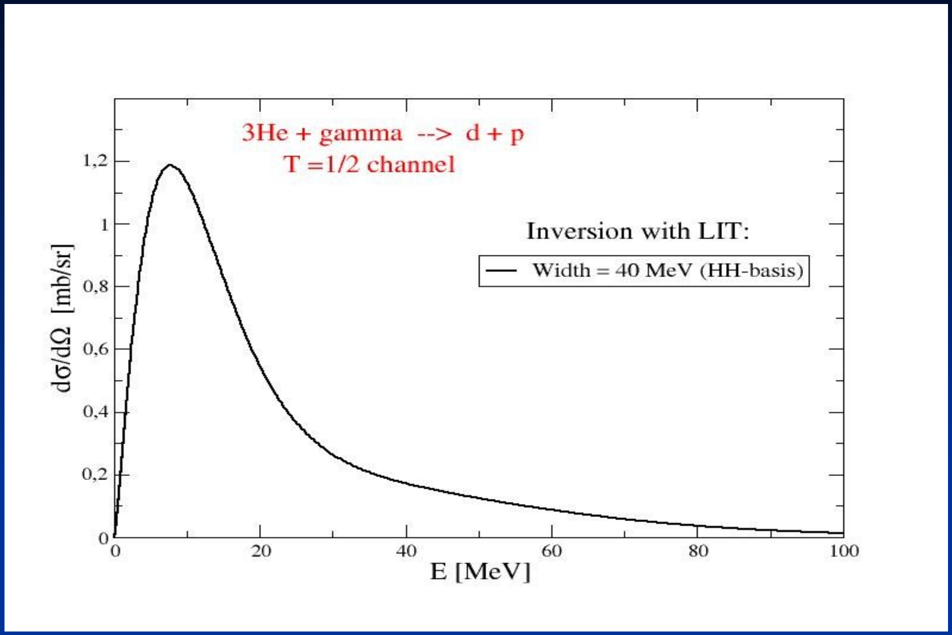

LIT calculated with HH and new basis
(Width = 0.02, 0.2, 0.5 MeV)
HH basis
new basis
σR [MeV]

Inversions

Implement correct threshold behaviour for $^3He + \gamma \longrightarrow d + p$

Due to Coulomb potential: usual Gamow factor


3He + gamma --> d + p
T =1/2 channel
Inversion with LIT:
Width = 40 MeV (HH-basis)
dσ/dΩ [mb/sr]
E [MeV]
0
0,2
0,4
0,6
0,8
1
1,2
0
20
40
60
80
100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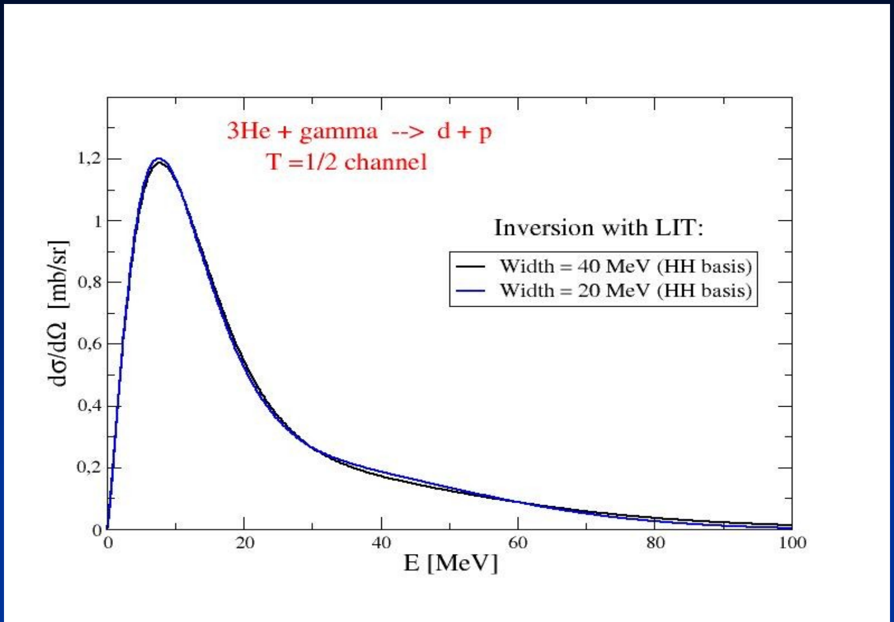
3He + gamma --> d + p
T =1/2 channel
Inversion with LIT:
Width = 40 MeV (HH basis)
Width = 20 MeV (HH basis)
dσ/dΩ [mb/sr]
E [MeV]
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0,2
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0,8
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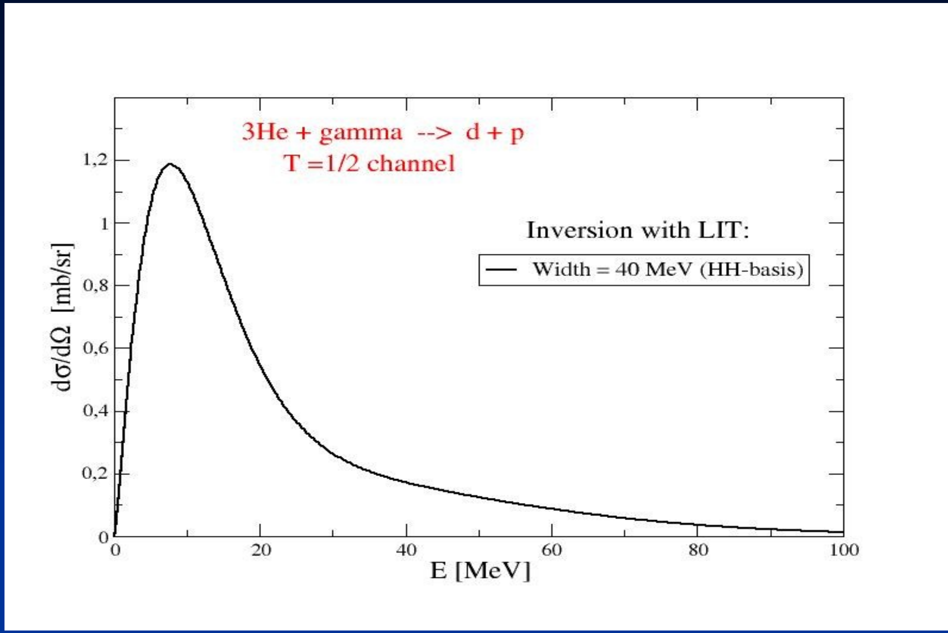
3He + gamma --> d + p
T =1/2 channel
Inversion with LIT:
Width = 40 MeV (HH-basis)
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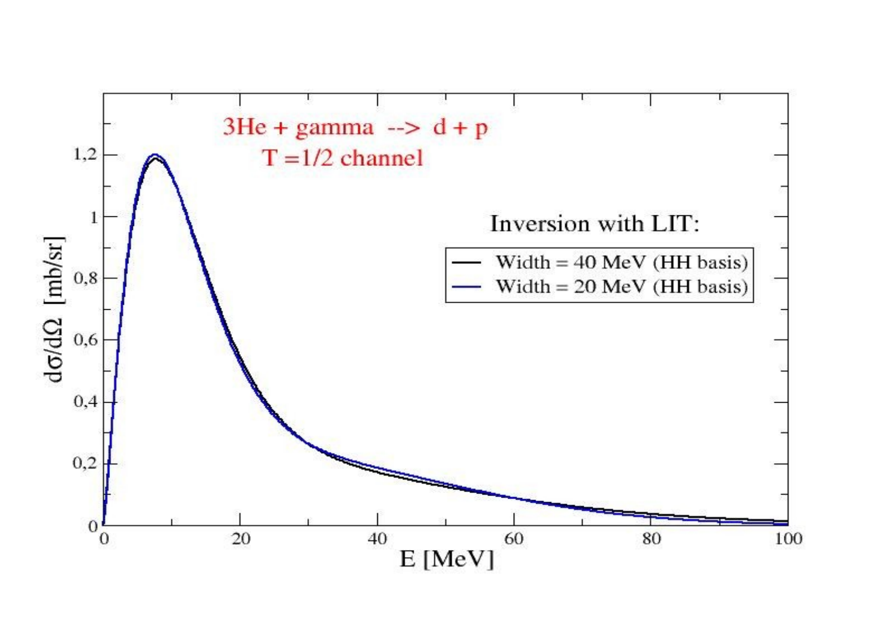
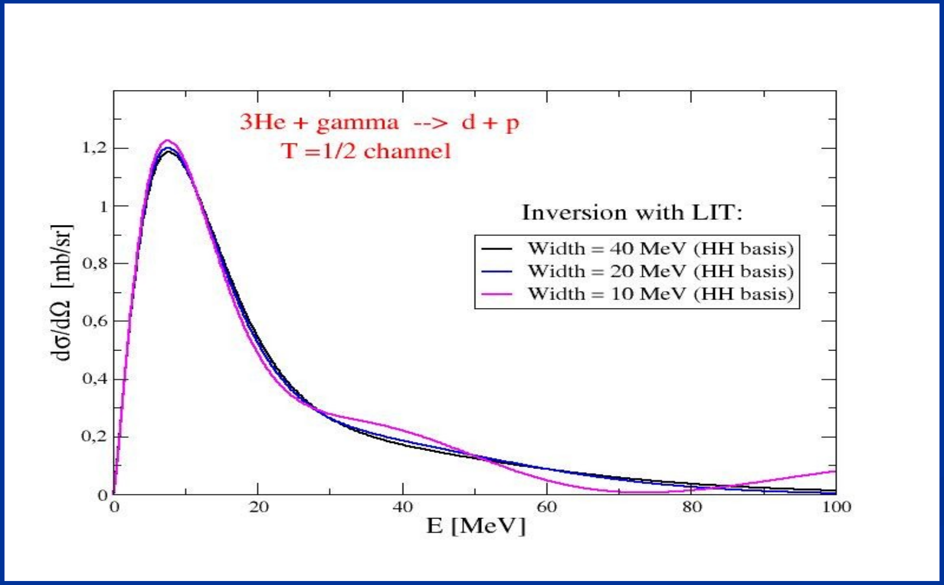
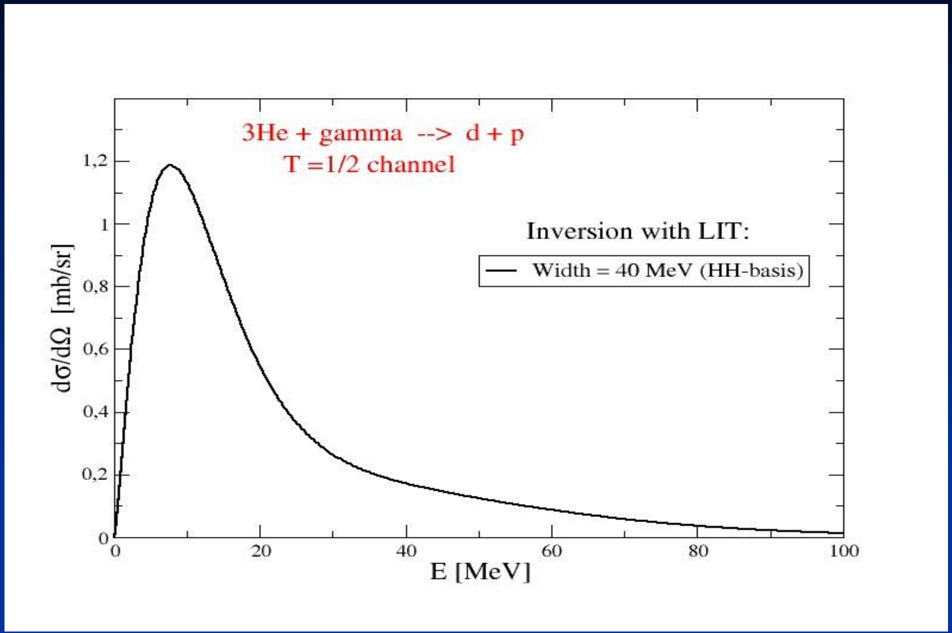
E [MeV]


3He + gamma --> d + p
T =1/2 channel
Inversion with LIT:
Width = 40 MeV (HH basis)
Width = 20 MeV (HH basis)
dσ/dΩ [mb/sr]
E [MeV]


3He + gamma --> d + p
T =1/2 channel
Inversion with LIT:
Width = 40 MeV (HH basis)
Width = 20 MeV (HH basis)
Width = 10 MeV (HH basis)
dσ/dΩ [mb/sr]
E [MeV]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Inversion with LIT:
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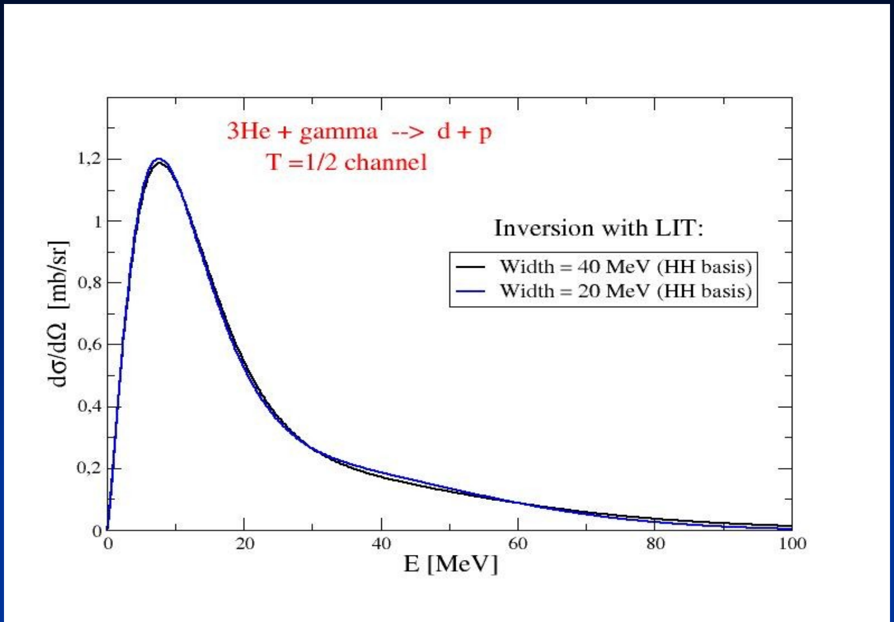
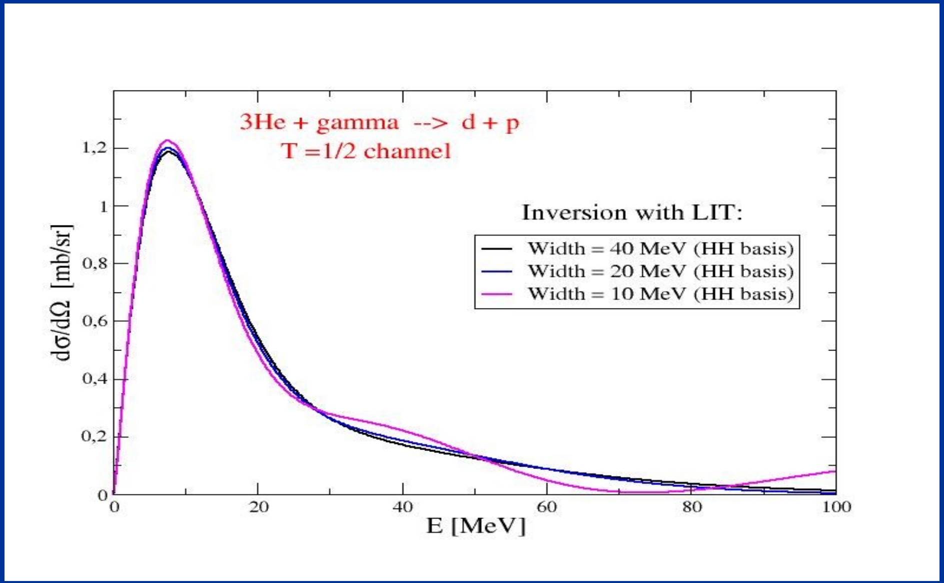
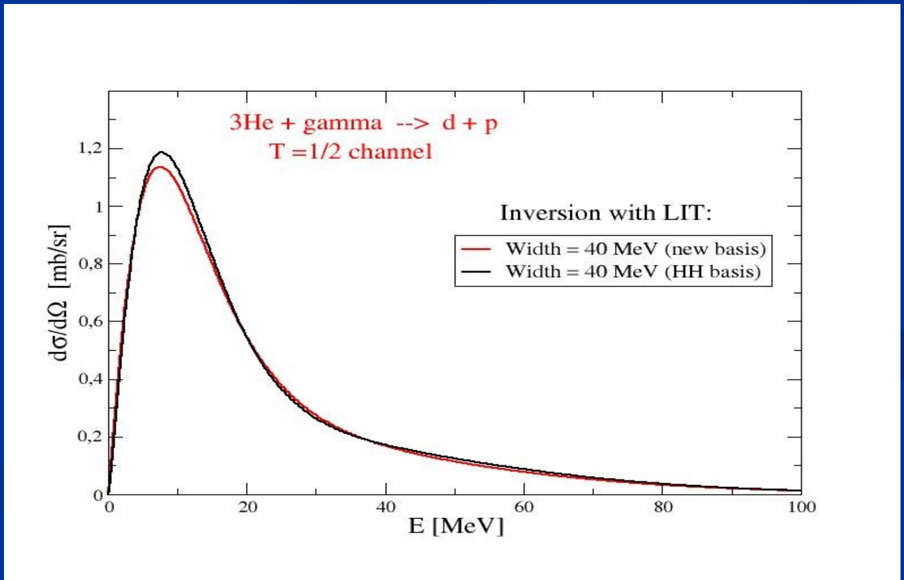
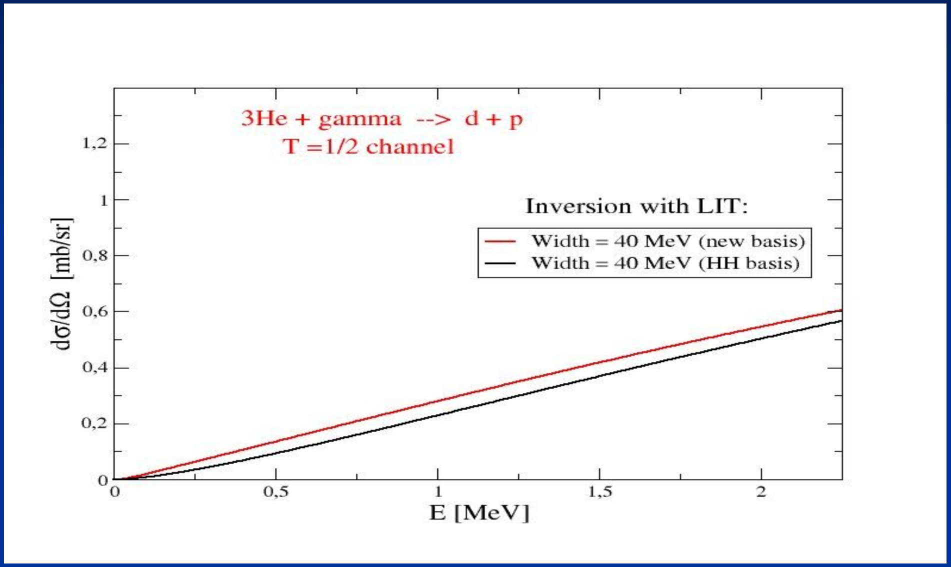
E [MeV]


3He + gamma --> d + p
T =1/2 channel
Inversion with LIT:
Width = 40 MeV (HH basis)
Width = 20 MeV (HH basis)
dσ/dΩ [mb/sr]
E [MeV]


3He + gamma --> d + p
T =1/2 channel
Inversion with LIT:
Width = 40 MeV (HH basis)
Width = 20 MeV (HH basis)
Width = 10 MeV (HH basis)
dσ/dΩ [mb/sr]
E [MeV]


3He + gamma --> d + p
T =1/2 channel
Inversion with LIT:
Width = 40 MeV (new basis)
Width = 40 MeV (HH basis)
dσ/dΩ [mb/sr]
E [MeV]

Energy region up to three-body breakup threshold



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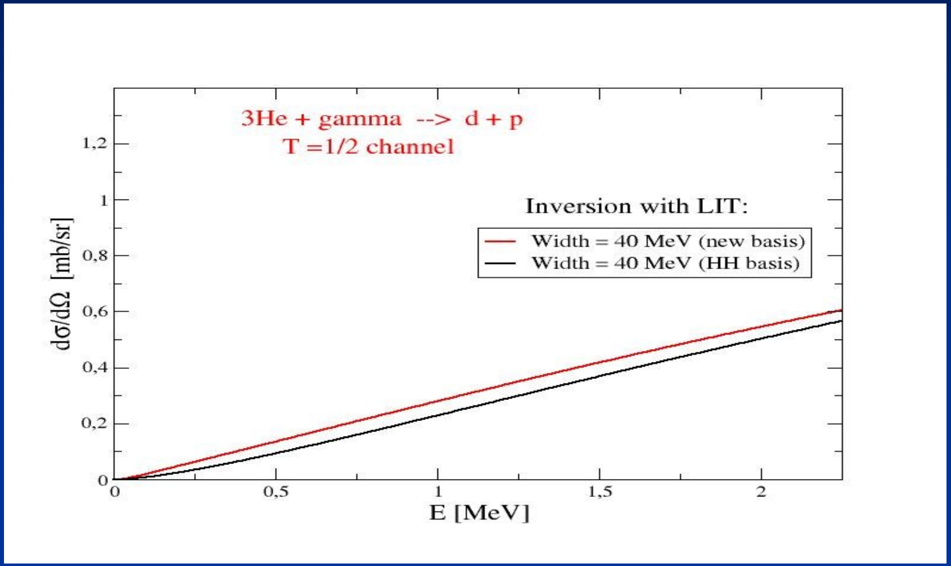

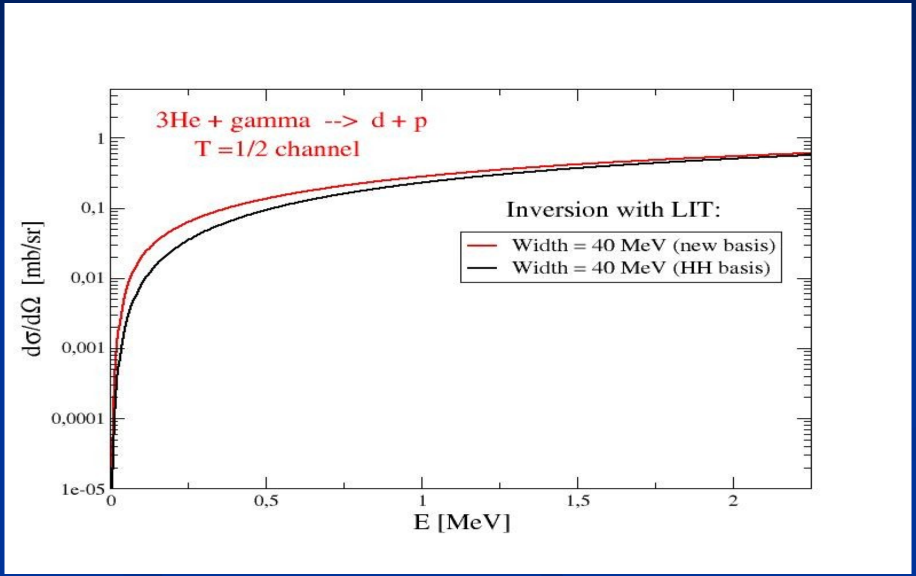

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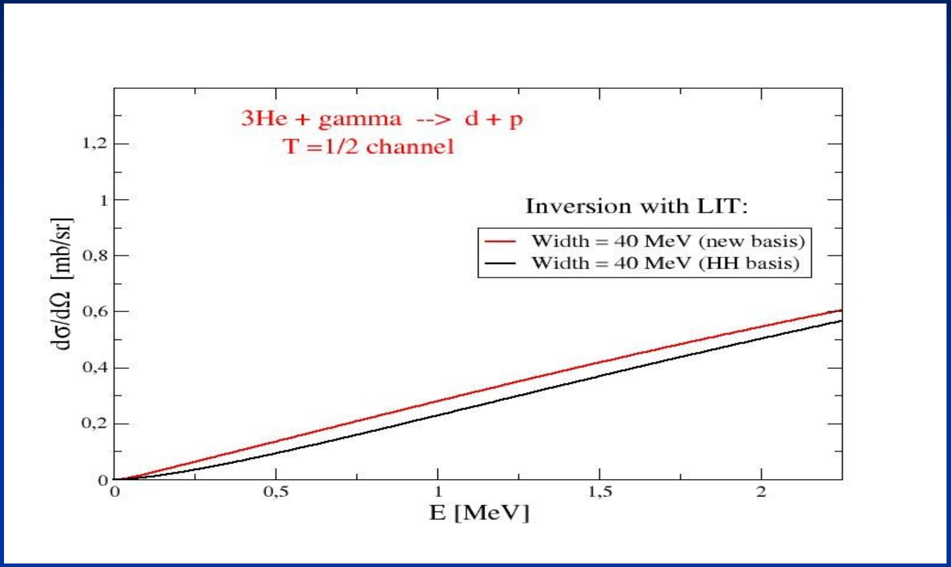
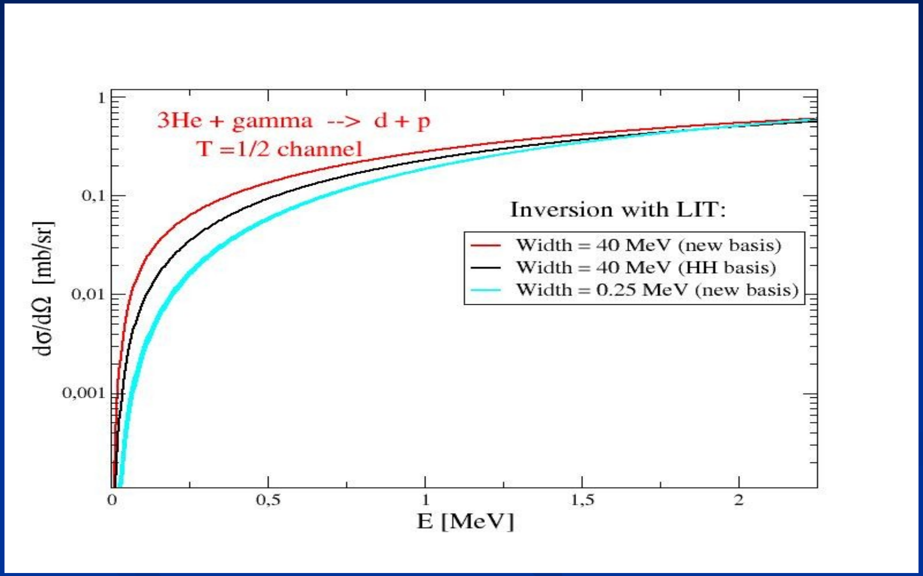
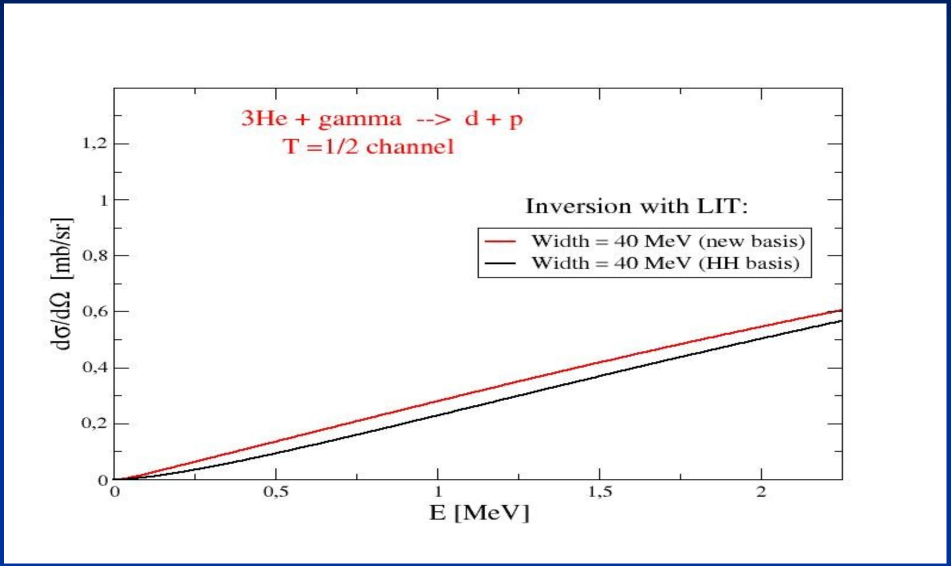
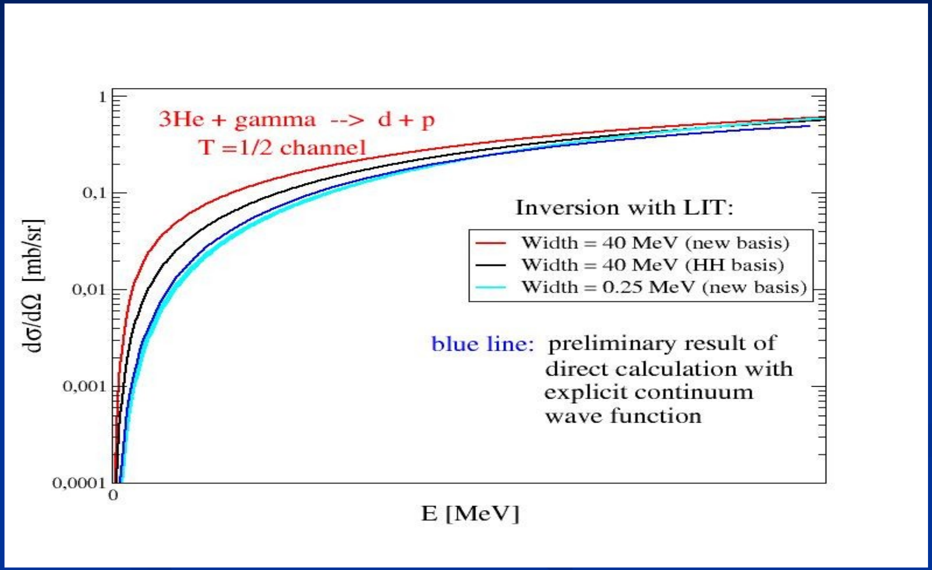


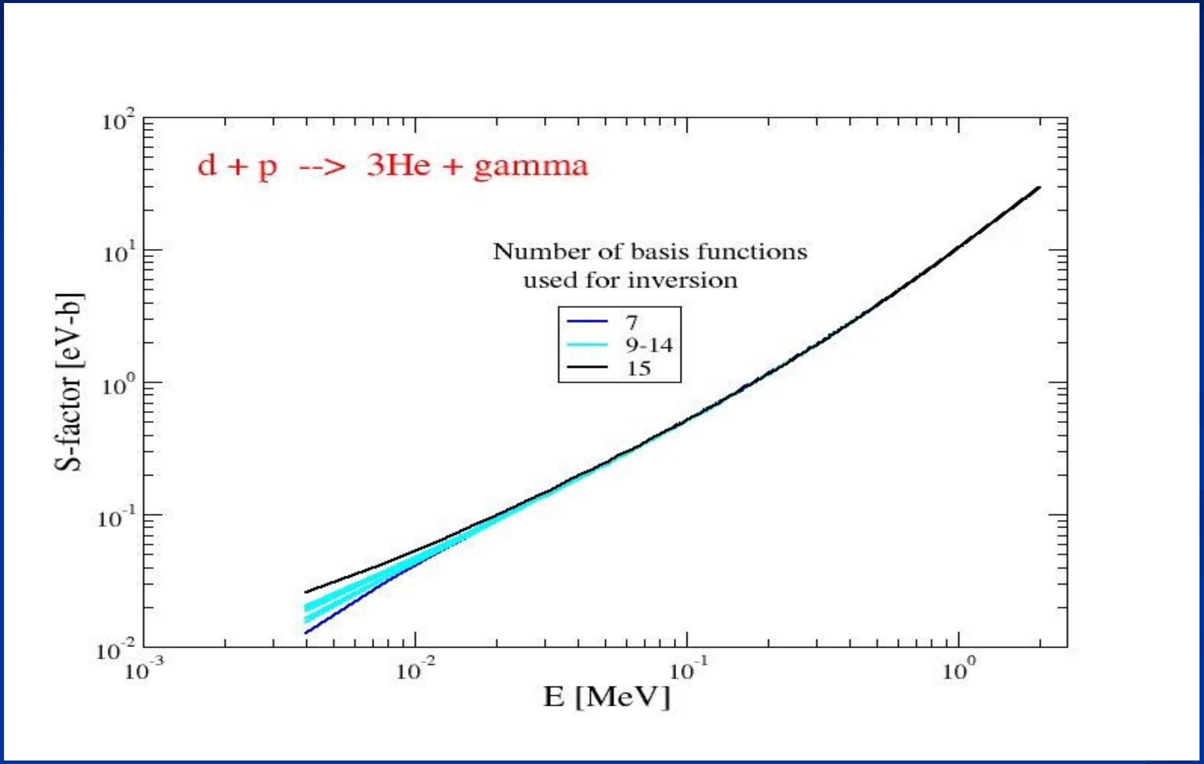


Energy region up to three-body breakup threshold



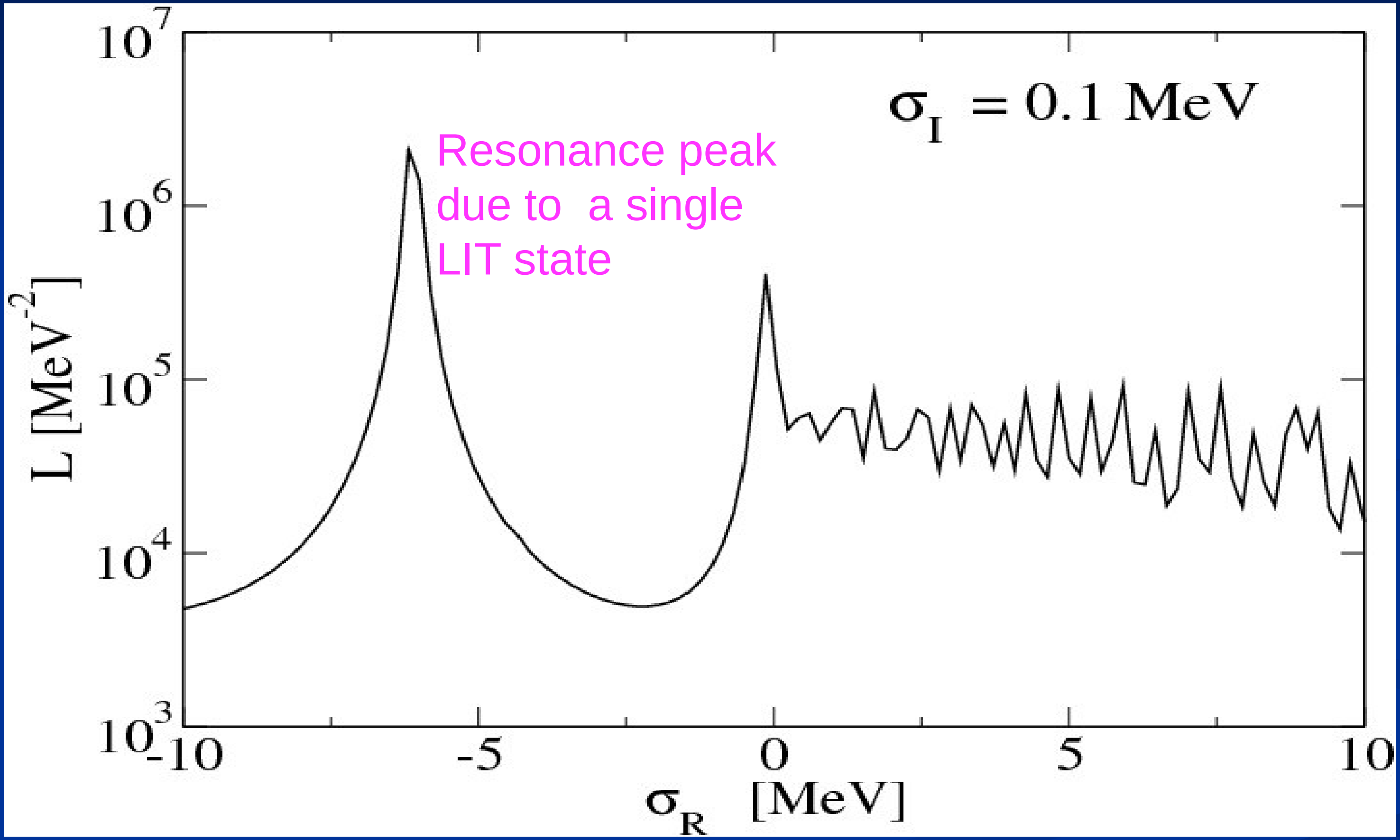


S-factor of reaction $d + p \longrightarrow {}^3He + \gamma$

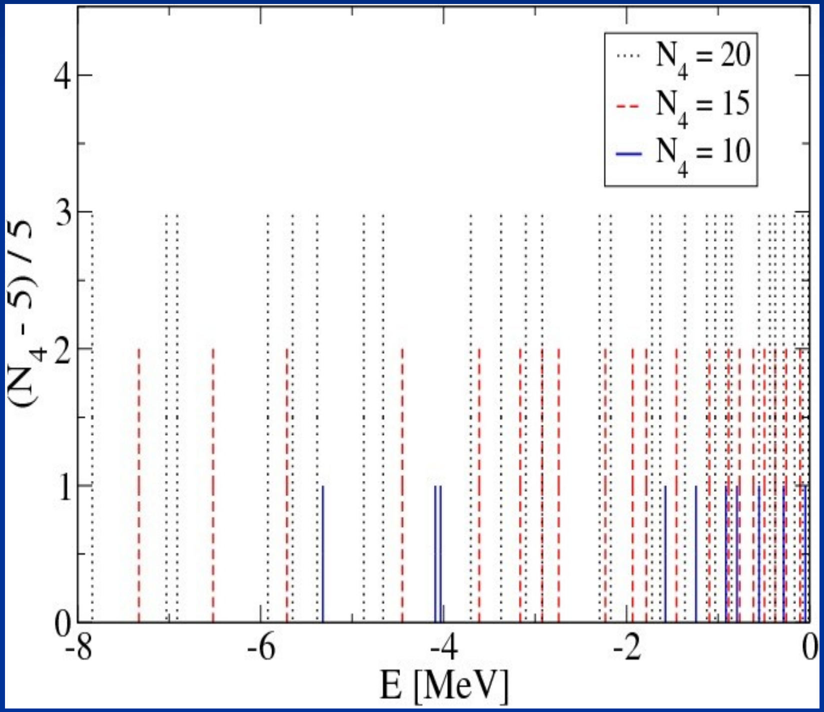


Back to the ^4He resonance

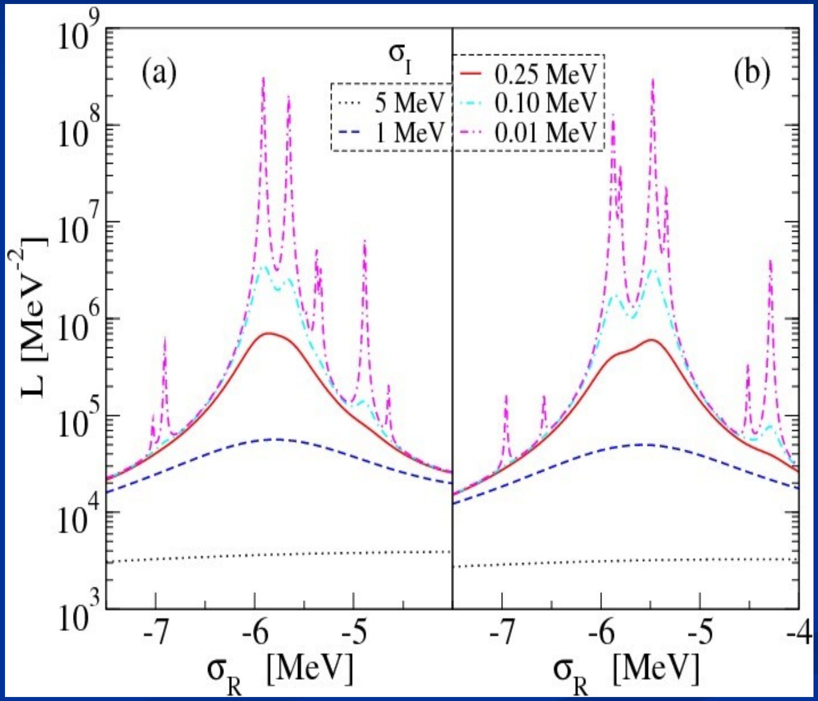
Unpublished result from a CHH calculation with the TN potential (V. Efros, WL, G. Orlandini, PRL 78,432 (1997))



Results with new basis

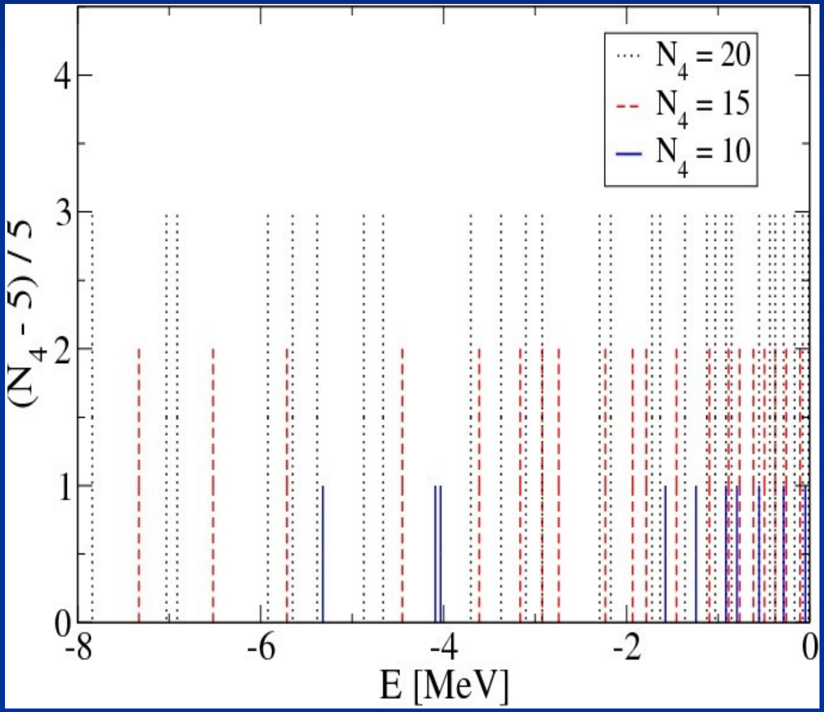


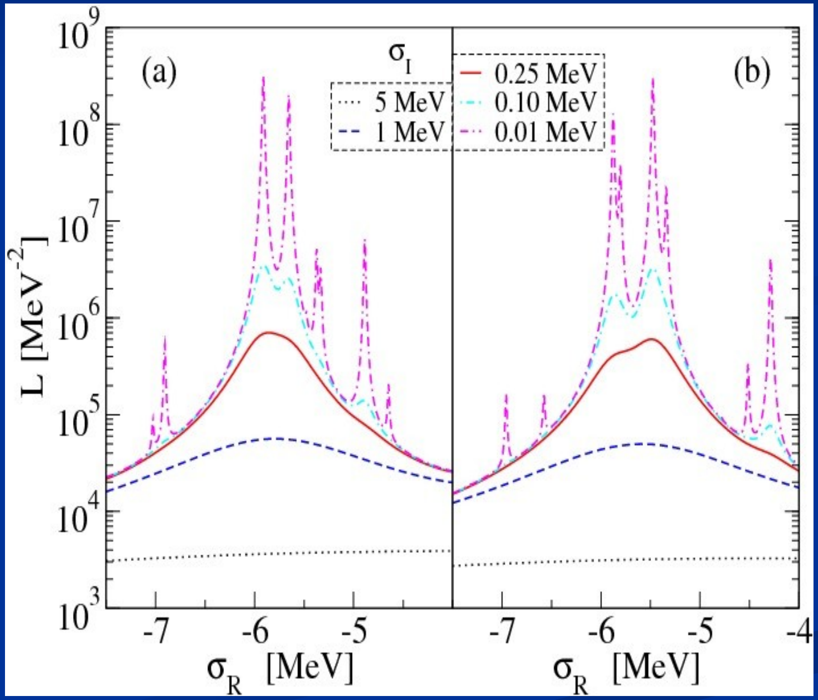
$N_4 = 20$ $N_4 = 21$



LIT

Results with new basis





Inversion: Γ = 180(70) keV

WL, PRC 91, 054001 (2015)

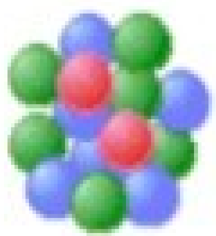
Benchmark calculations for hypernuclei

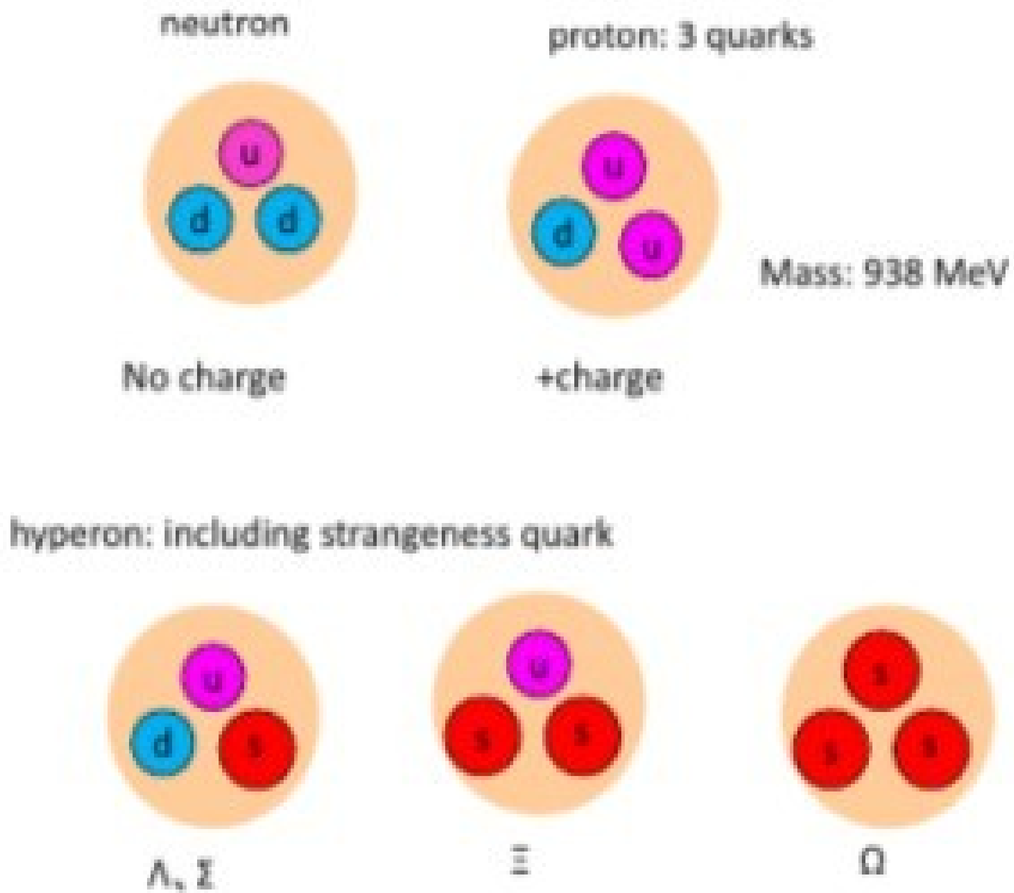
- Quick introduction to hypernuclei
- Short outline of our NSHH method
- Preliminary benchmark results: comparison with

  AFDMC (D. Lonardoni, F. Pederiva)
  Faddeev (A. Nogga)
  (GEM: E. Hiyama)

Nuclei with Strangeness

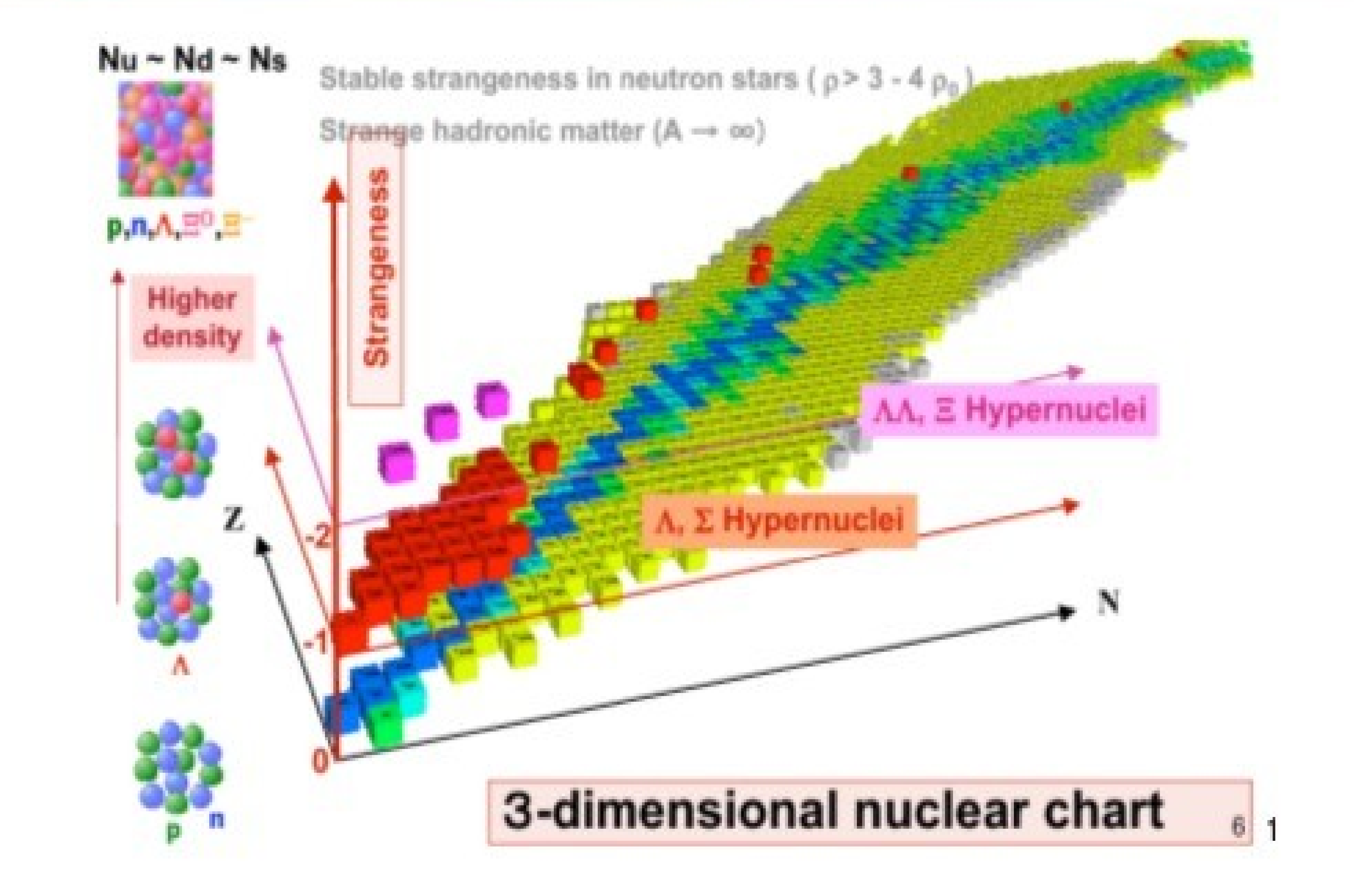
$$m_\Lambda = 1116 \text{ MeV}$$
$$m_{\Sigma^+} = 1189$$
$$m_{\Sigma^0} = 1193$$
$$m_{\Omega^-} = 1673$$





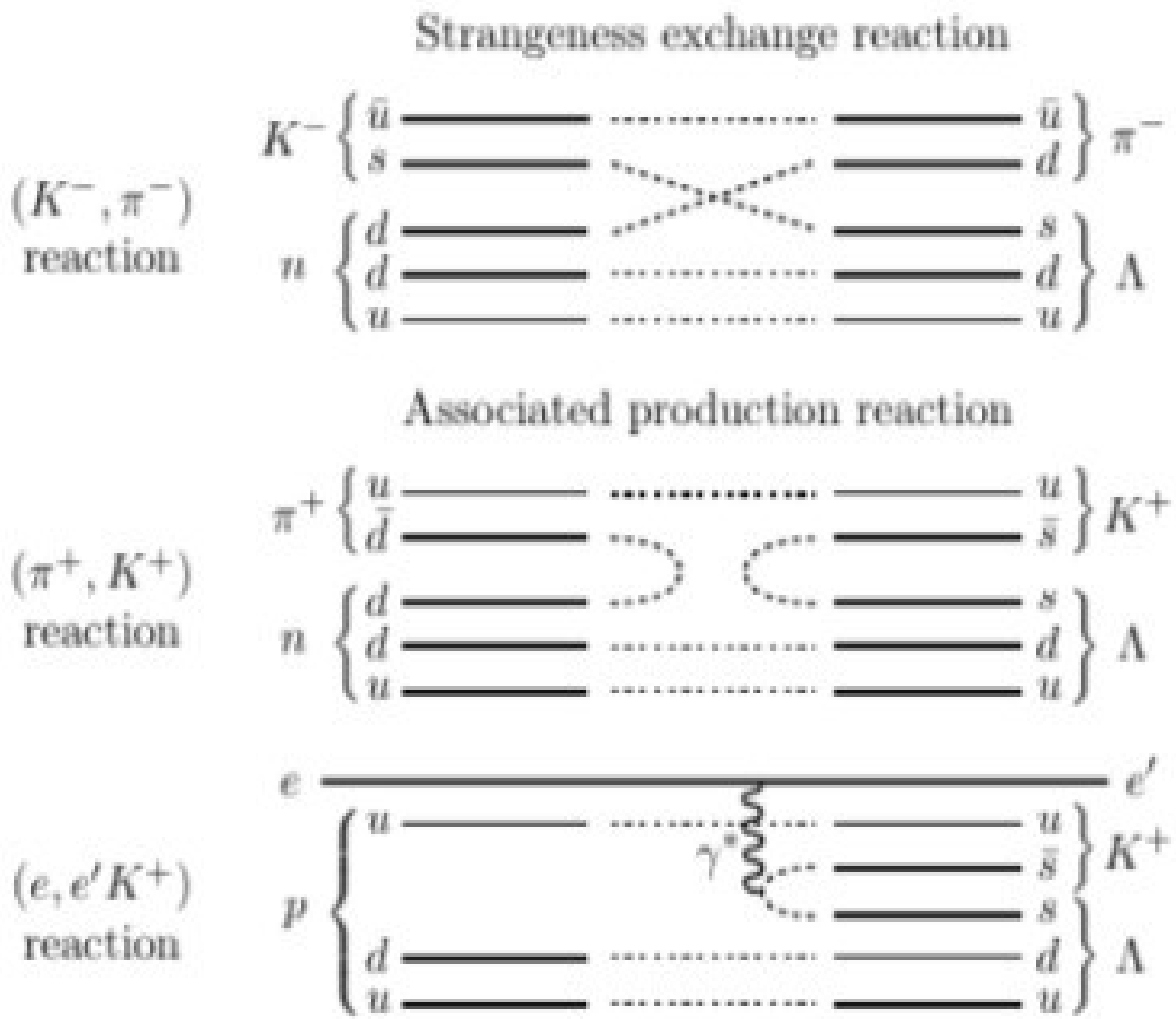
$$\tau_\Lambda = 263\ ps$$
$$\tau_{\Sigma^+} = 80\ ps$$
$$\tau_{\Sigma^0} = 7.4 \cdot 10^{-20} s$$
$$\tau_{\Omega^-} = 82\ ps$$

Hypernuclear Chart



1

Production of Hypernuclei



Experimental Present and Future Perspectives

- Despite exensive investigations, single Λ hypernuclei knowledge is **far** from that of ordinary nuclei;
- Only **one** bound Σ-hypernucleus detected!
- **No** Ξ hypernuclei detected (some indications of weak attraction);
- **No** experimental information about Ω hypernuclei;
- **Four** $\Lambda\Lambda$-hypernuclei energies measured ($^{6}_{\Lambda\Lambda}$He, $^{10}_{\Lambda\Lambda}$Be, $^{12}_{\Lambda\Lambda}$Be, $^{13}_{\Lambda\Lambda}$B);

Non-Symmetrized HH method

Problem: selection of **antisymmetric** states (we deal with fermions):

⇒ We add to Ĥ the **Casimir operator** of the **permutation group $\mathbf{S}_N$**, which selects "by himself" the interesting states:

$$\hat{H}' = \hat{H} + \gamma \hat{C}(A) \quad ; \quad \hat{C}(A) = \sum_{i>j} \hat{P}_{ij}$$

Its action on the vectors:

$$\hat{C}(A)\Psi_s = \frac{A(A-1)}{2}\Psi_s = \lambda_s \Psi_s \ ;$$

$$\hat{C}(A)\Psi_m = \lambda_m \Psi_m \ ;$$

$$\hat{C}(A)\Psi_a = -\frac{A(A-1)}{2}\Psi_a = \lambda_a \Psi_a \ ,$$

⇒ with a proper choice of γ the g.s. energy $\mathbf{E}_A^0$ becomes the **lowest eigenvalue of H′** (similar procedure for exited states).

HYP-NSHH: different particles

Hypernuclei are systems made of **two different species** of particles.

- Different **masses**:
  - mass weighted coordinates $\Rightarrow$ dependence inside **transpositions**:

$$\mathcal{B}_{ij} \rightarrow \mathcal{B}_{ij}(m_1, m_i, m_j) ;$$

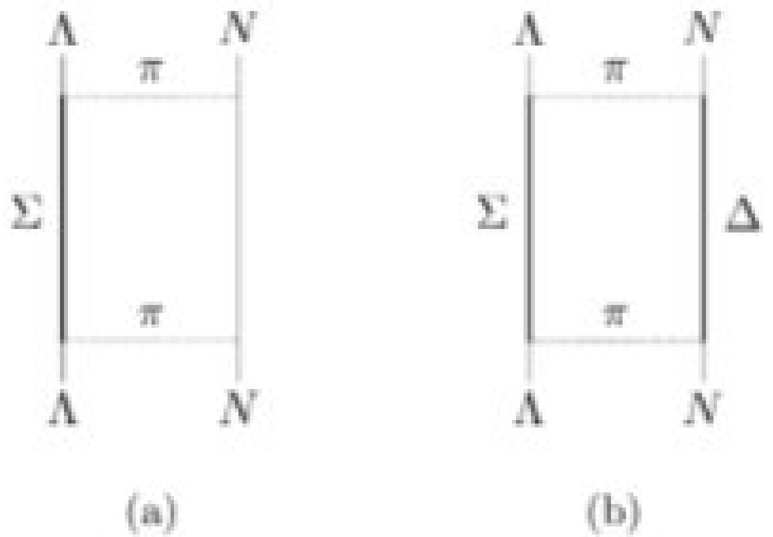
  - relative coordinate **rescaling** - mass dependence inside potential.

- Assuming 2-body potential: 3 types of different **interactions**:
  - **NN** - nuclear core;
  - **YN** - nucleon-hyperon couples;
  - **YY** - hyperonic part.

2-body Bodmer Usmani interaction

The Λ particle has $T = 0$ so there is no OPE term:

$$v_{\Lambda N}(r) = v_0(r) + \frac{v_\sigma}{4} T_\pi^2(r)\, \boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_N \,,$$



$$v_0(r) = \frac{W_c}{1 + e^{\frac{r - \bar{r}}{a}}} - \bar{v} T_\pi^2(r)$$

$$T_\pi(r) = \left[1 + \frac{3}{\mu_\pi r} + \frac{3}{(\mu_\pi r)^2}\right] \frac{e^{-\mu_\pi r}}{\mu_\pi r} (1 - e^{-cr^2})^2$$

Bodmer Usmani benchmark results

$V_{NN} + V_{YN}$	System	AFDMC	NSHH	FY
AV4'	^{2}H	-2.245(15)	[-2.245(1)]	-2.245(1)
AV4'+U	$^{3}_{\Lambda}H$	-2.45(5)	-2.529(1)	-2.537(1)
	B_{Λ}	0.21(5)	0.284(1)	0.292(1)
AV4'	^{3}H	-8.92(5)	-8.983(7)	
AV4'+U	$^{4}_{\Lambda}H$	-11.95(5)	-12.023(3)	
	B_{Λ}	3.03(7)	3.039(8)	
AV4'	^{4}H	-32.85(5)	-32.695(6)	
AV4'+U	$^{5}_{\Lambda}H$	-39.50(5)	-39.543(10)	
	B_{Λ}	6.65(7)	6.848(12)	

NSC97f realistic interaction

We employed the **NSC97f realisitc potential**[3] which simulates the Nijmegen scattering phase shifts:

$$
{}^{S}V_{NY-NY'}(r) = \sum_i \Big({}^{S}V^{C}_{NY-NY'}\, e^{-(r/\beta_i)^2} + {}^{S}V^{T}_{NY-NY'}\, S_{12}\, e^{-(r/\beta_i)^2} + {}^{S}V^{LS}_{NY-NY'}\, \mathbf{LS}\, e^{-(r/\beta_i)^2} \Big)
$$

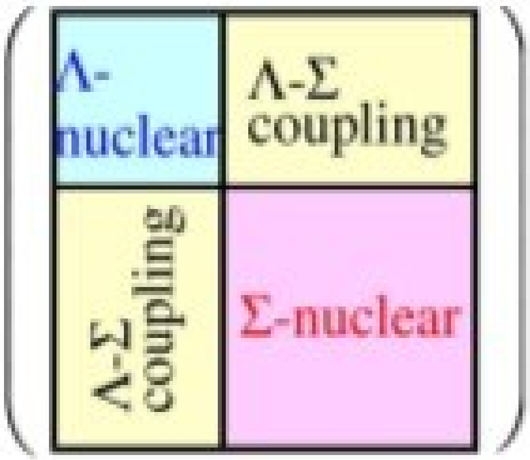
- $Y \rightarrow \Lambda, \Sigma$;
- C $\rightarrow$ central, T $\rightarrow$ tensor, LS $\rightarrow$ spin-orbit;
- gaussian radial functions with fitted paramenters.

Explicit use of **Σ degree of freedom** ⇒ need for **extension** of the HYP-NSHH method.

[3] E.Hiyama et al., Th.A.Rijken, Phys. Rev. C89, 061302 (2014).

Lambda-Sigma mixing

⇒ extension of the basis including Λ/Σ degree of freedom:



- definition of transformation between two Jacobi sets **differing by one mass**;
- **extension of Lee-Suzuki** procedure including Λ/Σ degree of freedom.

NSC97f interaction benchmark

$V_{NN} + V_{YN}$	System	NSHH	FY	GEM
AV8'	^{2}H	[-2.226(1)]	-2.226(1)	
AV8'+NSC97f	$^{3}_{\Lambda}H$	-2.41(2)	-2.415(1)	
	B_{Λ}	0.19(2)	0.189(1)	0.19(1)
AV8'	^{3}H	-7.76(0)		
AV8'+NSC97f	$^{4}_{\Lambda}H$	-10.05(7)		
	B_{Λ}	2.29(7)		2.33

Results were obtained in collaboration with

S. Deflorian and F. Ferrari-Rufino (Trento PhD students)

N. Barnea (Jerusalem), V. Efros (Moscow), G. Orlandini (Trento)