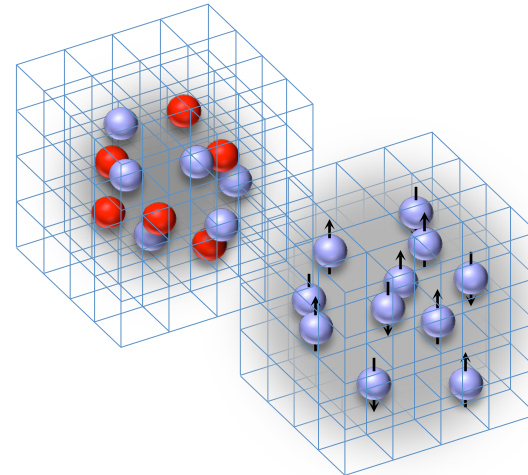


Recent Results in Nuclear Lattice Effective Field Theory

Dean Lee
North Carolina State University
Nuclear Lattice EFT Collaboration

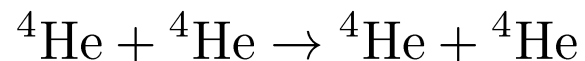
Nuclear Physics from Lattice QCD
Institute for Nuclear Theory
University of Washington
May 9, 2016



Outline

Lattice effective field theory

Adiabatic projection method



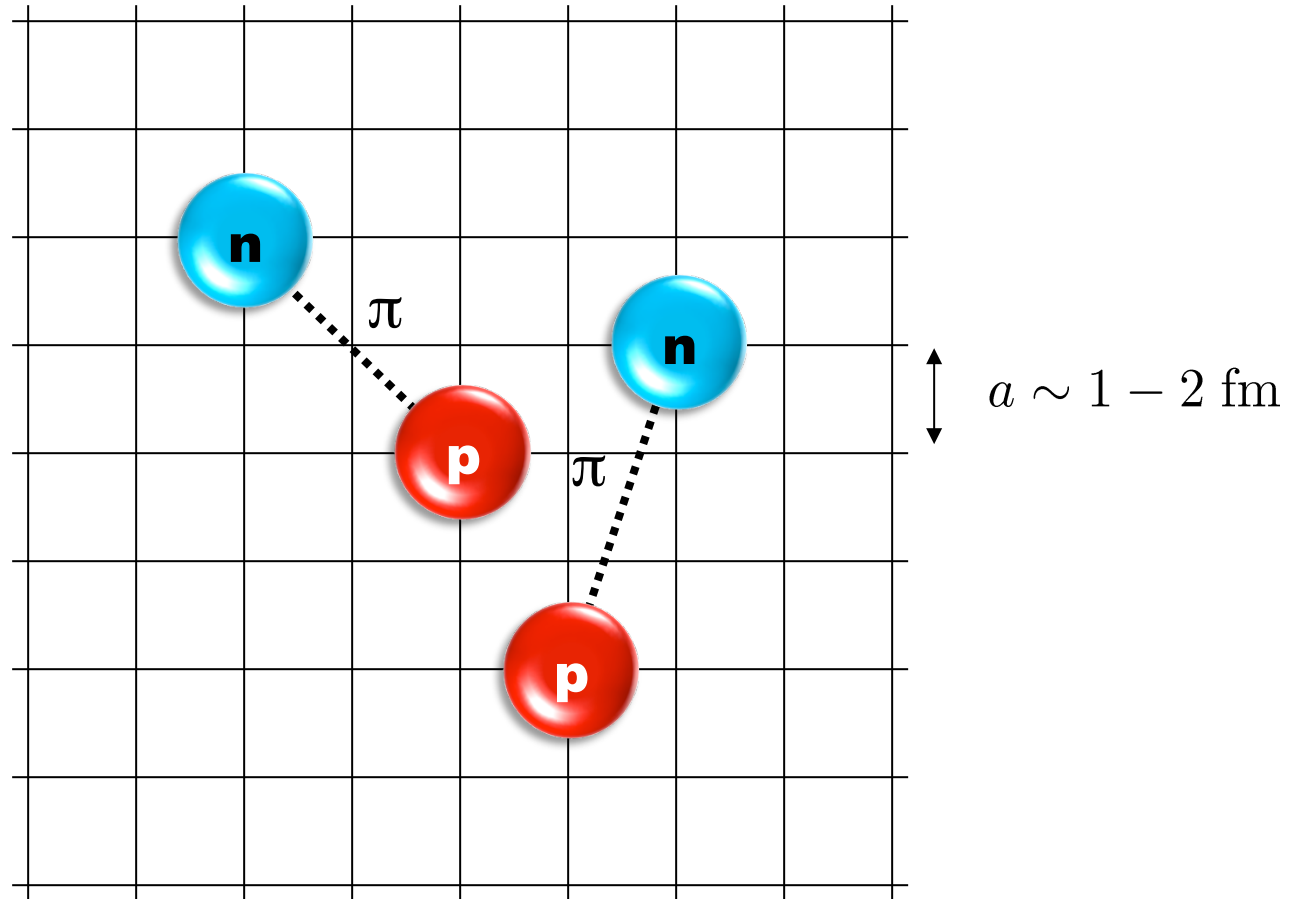
Designing new lattice interactions

Quantum phase transition

Applications

Summary and outlook

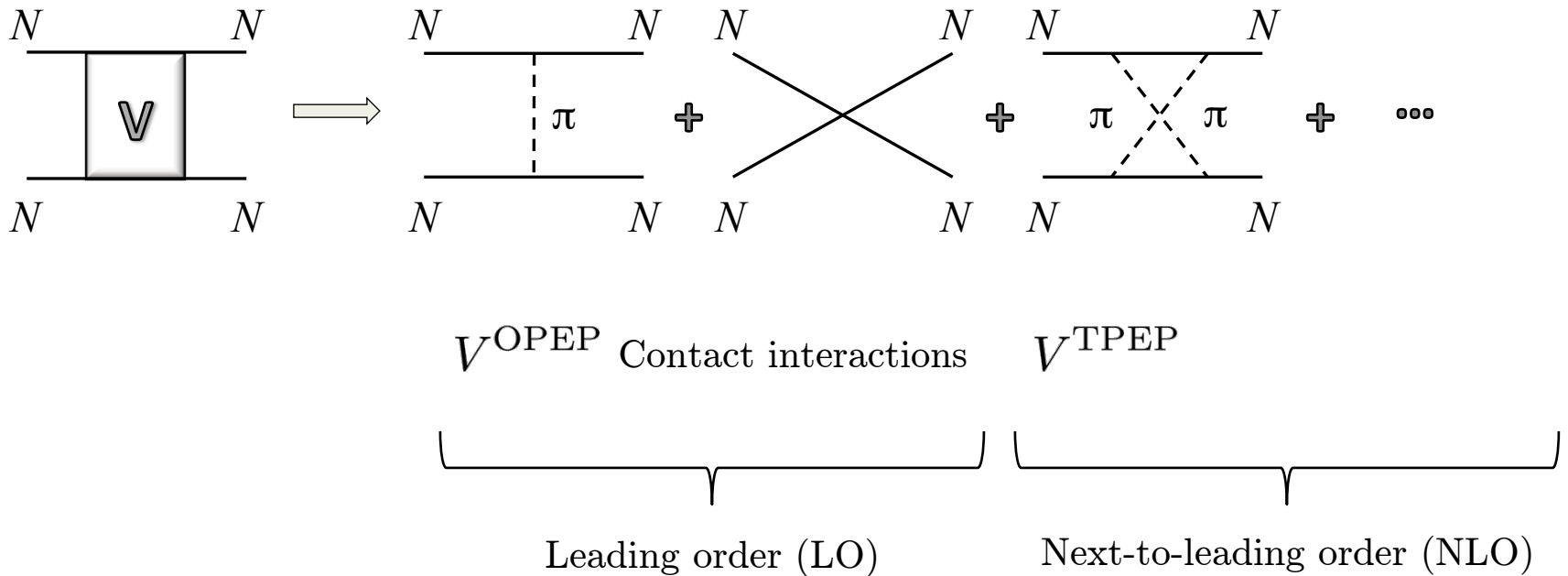
Lattice chiral effective field theory



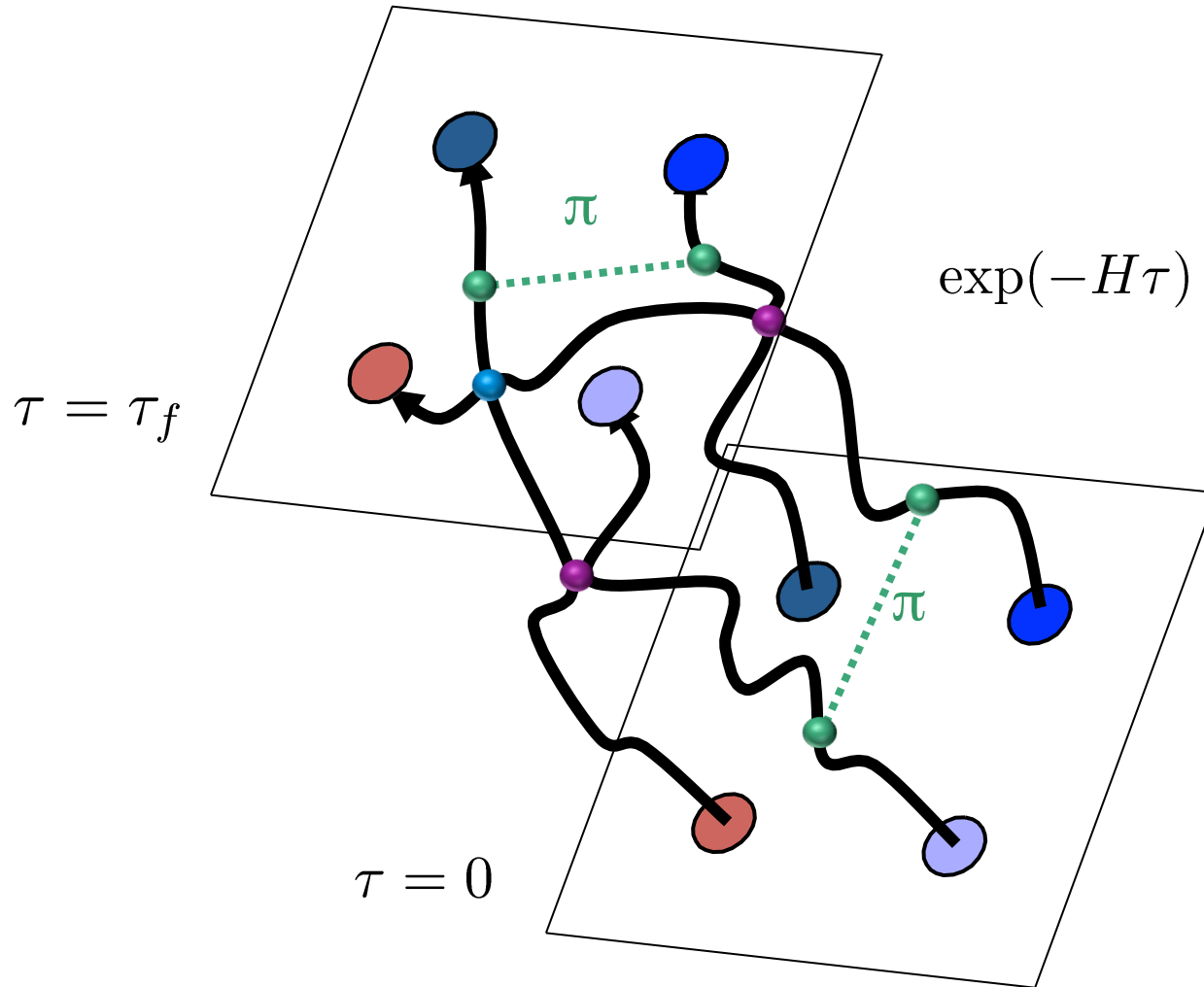
Review: D.L, Prog. Part. Nucl. Phys. 63 117-154 (2009)

Chiral effective field theory

Construct the effective potential order by order



Euclidean time projection

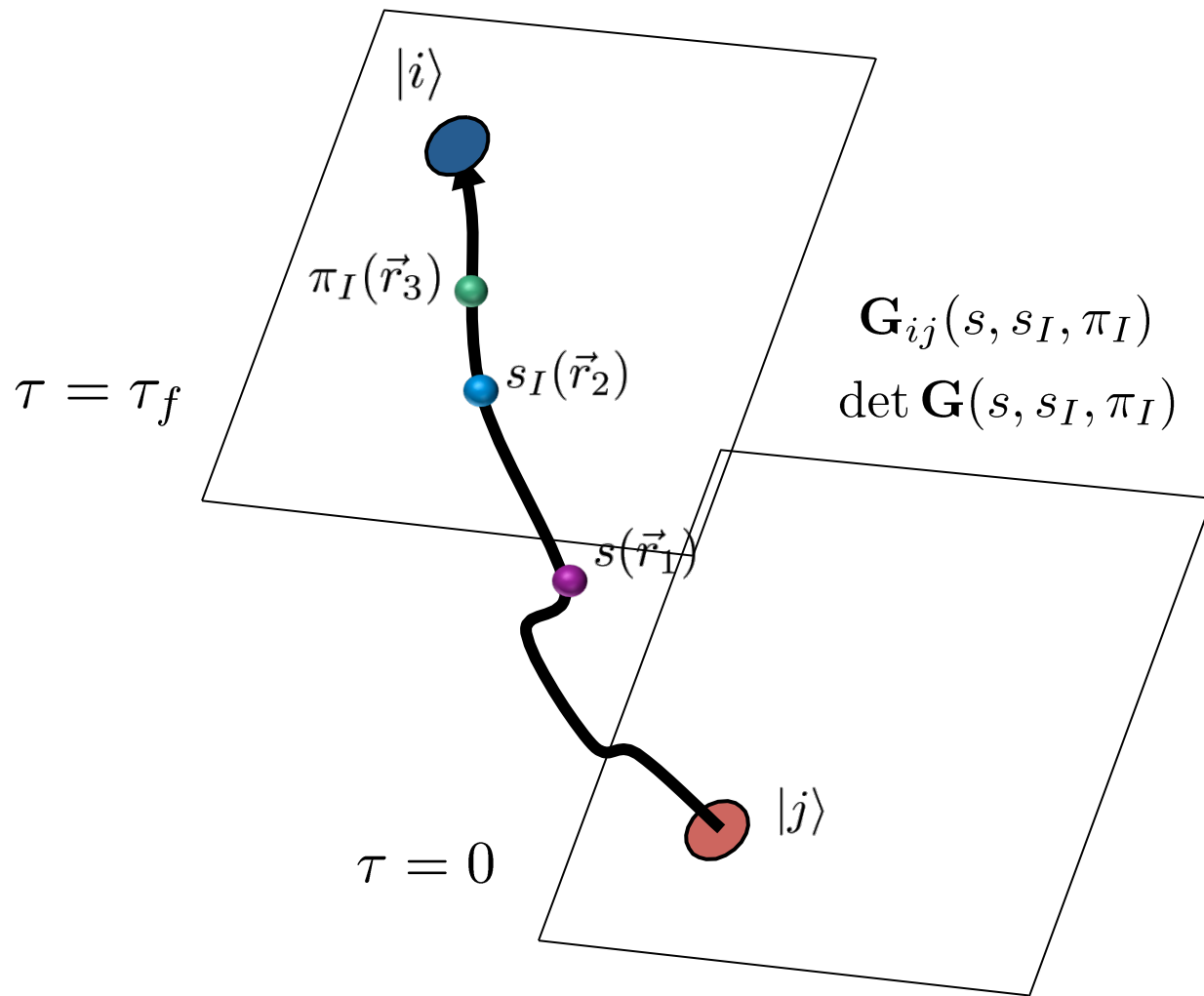


Auxiliary field method

We can write exponentials of the interaction using a Gaussian integral identity

$$\exp \left[-\frac{C}{2} (N^\dagger N)^2 \right] \quad \times \quad (N^\dagger N)^2$$
$$= \sqrt{\frac{1}{2\pi}} \int_{-\infty}^{\infty} ds \exp \left[-\frac{1}{2} s^2 + \sqrt{-C} s (N^\dagger N) \right] \quad \rangle \quad s N^\dagger N$$

We remove the interaction between nucleons and replace it with the interactions of each nucleon with a background field.



Adiabatic projection method

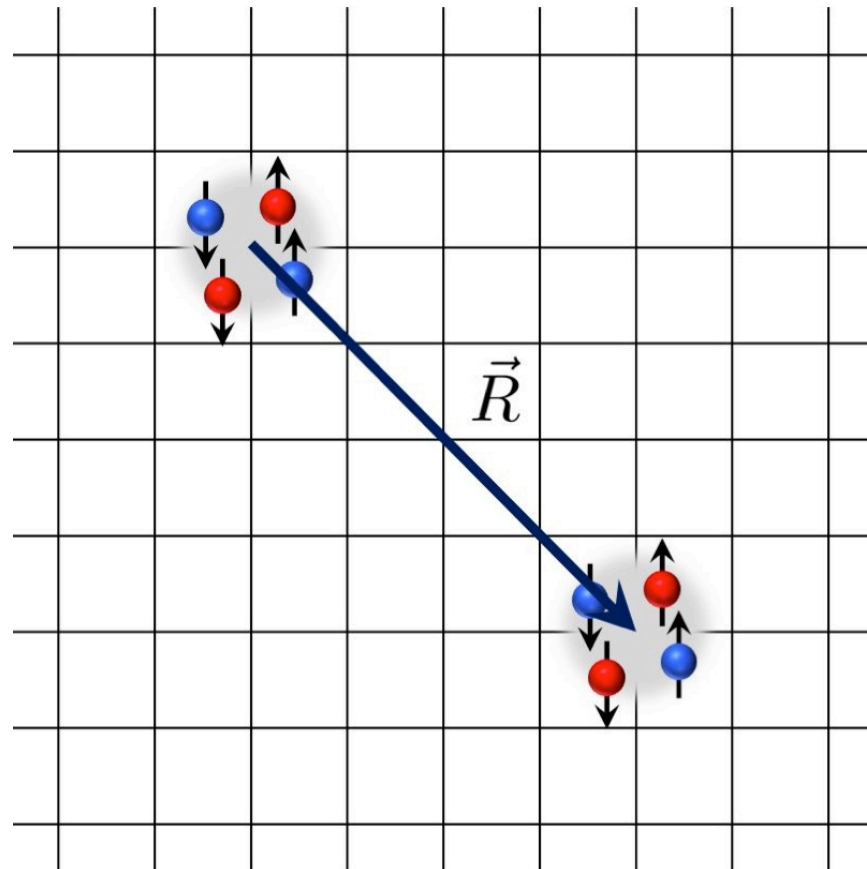
The adiabatic projection method is an *ab initio* method for nuclear scattering and reactions.

Strategy is to divide the problem into two parts. In the first part, we use Euclidean time projection and lattice Monte Carlo to derive an *ab initio* low-energy cluster Hamiltonian, called the adiabatic Hamiltonian (adiabatic transfer matrix for nonzero temporal lattice spacing).

In the second part, we use the adiabatic Hamiltonian to compute scattering phase shifts or reaction amplitudes.

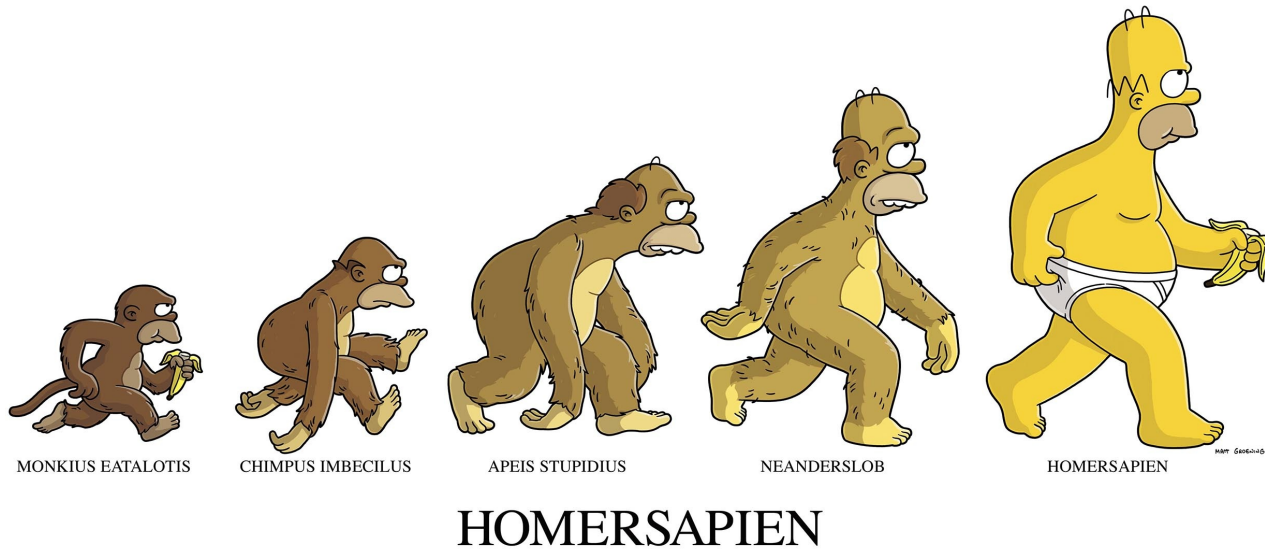
Start with localized cluster states for all possible separation vectors \vec{R}

$$|\vec{R}\rangle = \sum_{\vec{r}} |\vec{r} + \vec{R}\rangle_1 \otimes |\vec{r}\rangle_2$$



Cluster evolution with Euclidean time

$$|\vec{R}\rangle_\tau = \exp(-H\tau)|\vec{R}\rangle$$



Use projection Monte Carlo to propagate cluster wavefunctions in Euclidean time to form dressed cluster states

$$|\vec{R}\rangle_\tau = \exp(-H\tau)|\vec{R}\rangle$$

Evaluate matrix elements of the full microscopic Hamiltonian with respect to the dressed cluster states,

$$[H_\tau]_{\vec{R},\vec{R}'} = {}_\tau\langle\vec{R}|H|\vec{R}'\rangle_\tau$$

Since the dressed cluster states are in general not orthogonal, we construct a norm matrix given by the inner product

$$[N_\tau]_{\vec{R},\vec{R}'} = {}_\tau\langle\vec{R}|\vec{R}'\rangle_\tau$$

The adiabatic Hamiltonian is defined by the matrix product

$$[H_\tau^a]_{\vec{R},\vec{R}'} = \left[N_\tau^{-1/2} H_\tau N_\tau^{-1/2} \right]_{\vec{R},\vec{R}'}$$

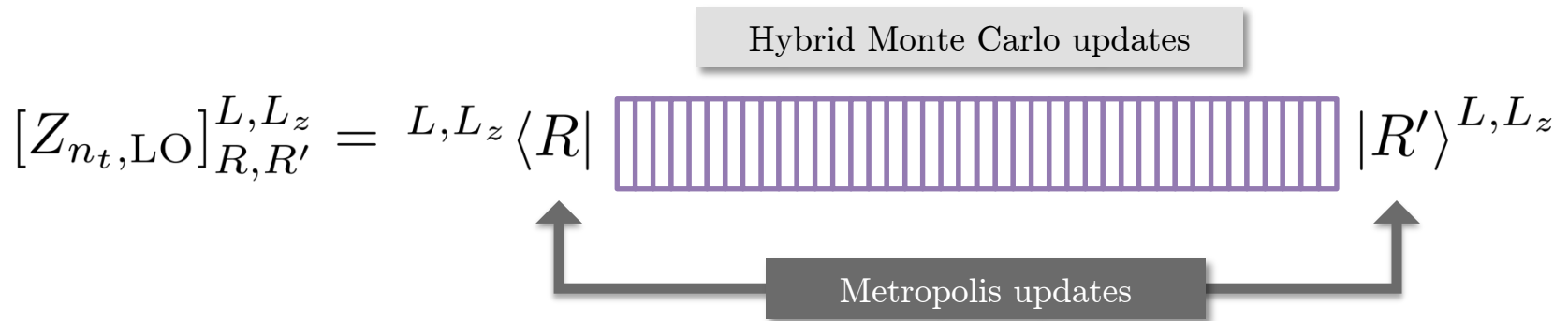
Distortion and polarization of the nuclear wave functions are automatically produced by the Euclidean time projection.

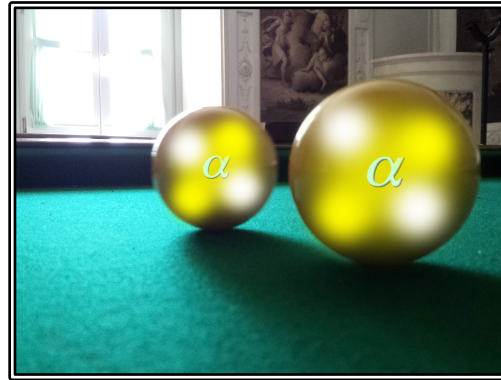
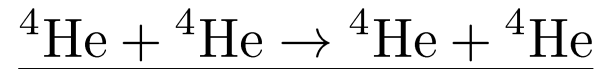
As we increase the projection time, the adiabatic Hamiltonian exactly reproduces the low-energy spectrum of the full microscopic Hamiltonian. We can read off the scattering phase shifts for the asymptotic long-distance properties of the scattering wave functions.

We use projections onto spherical harmonics defined on sets of lattice points with the same distance from the origin.

$$|R\rangle^{L,L_z} = \sum_{\vec{R}'} Y_{L,L_z}(\hat{R}') \delta_{R,|\vec{R}'|} |\vec{R}'\rangle$$

New algorithm developed for auxiliary field updates and initial/final state updates

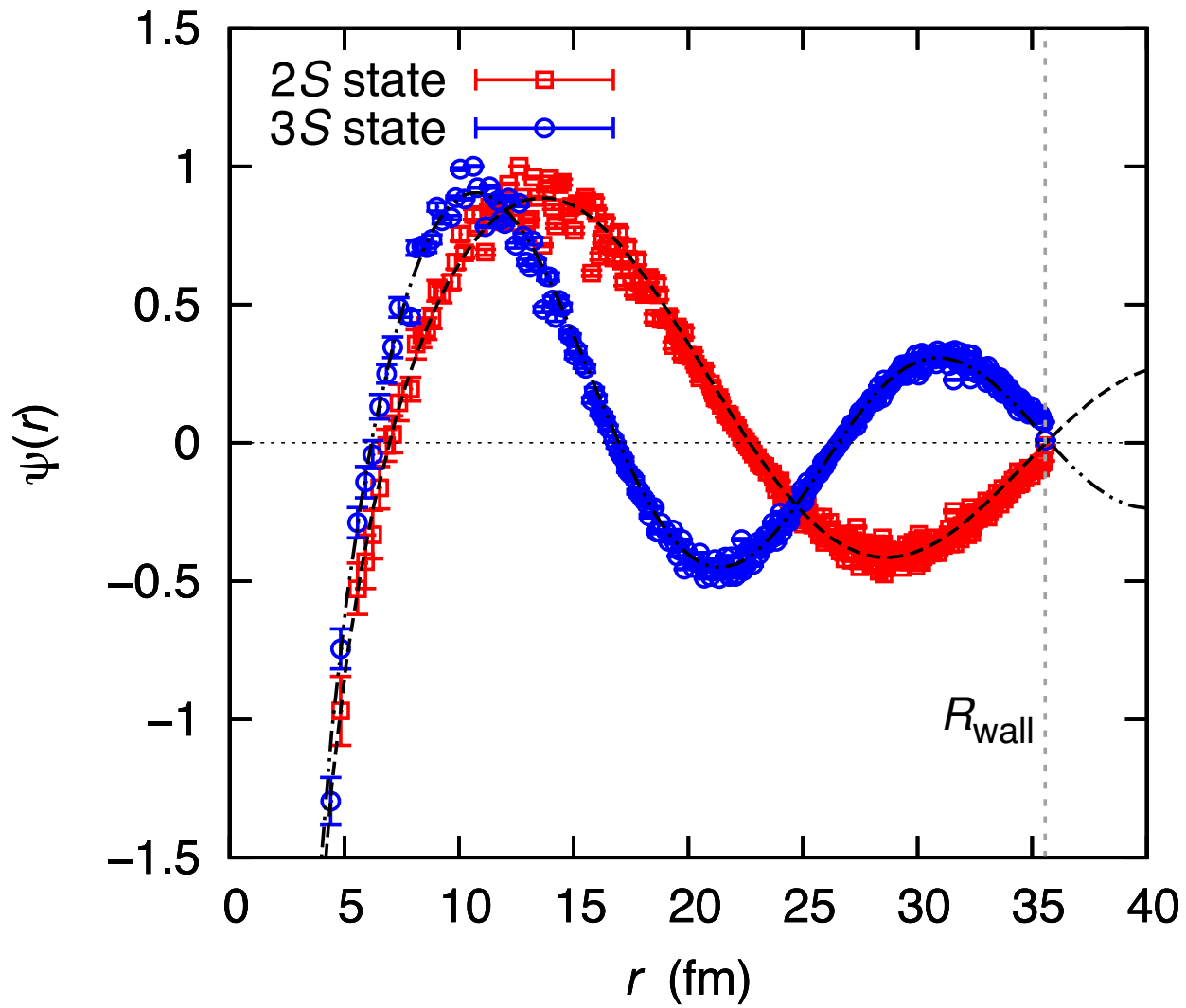




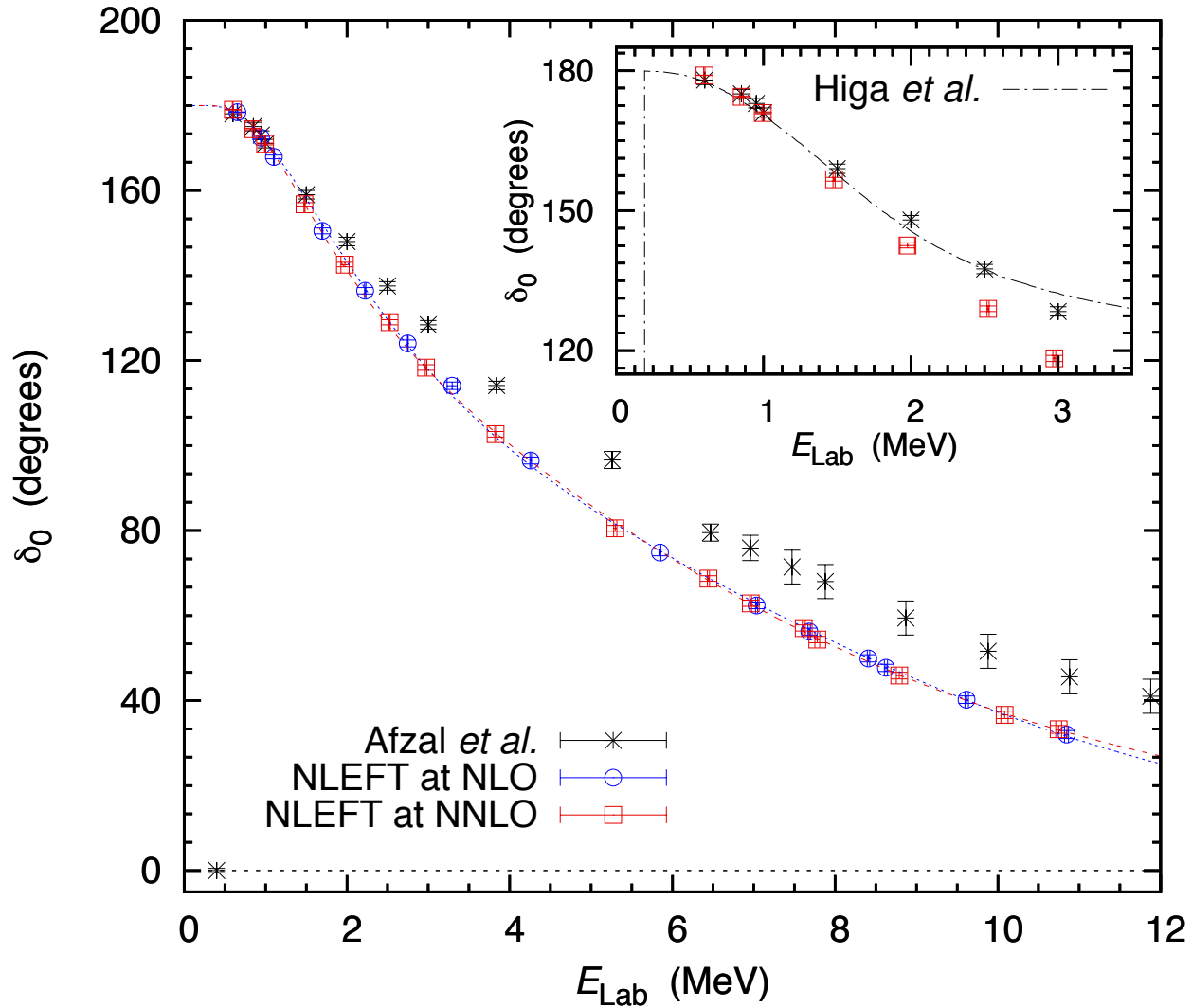
We now present *ab initio* results for alpha-alpha scattering up to NNLO with lattice spacing 1.97 fm.

Using the adiabatic projection method, we performed lattice simulations for the *S*-wave and *D*-wave channels.

Elhatisari, D.L., Rupak, Epelbaum, Krebs, Lähde, Luu, Meißner, Nature 528, 111 (2015)



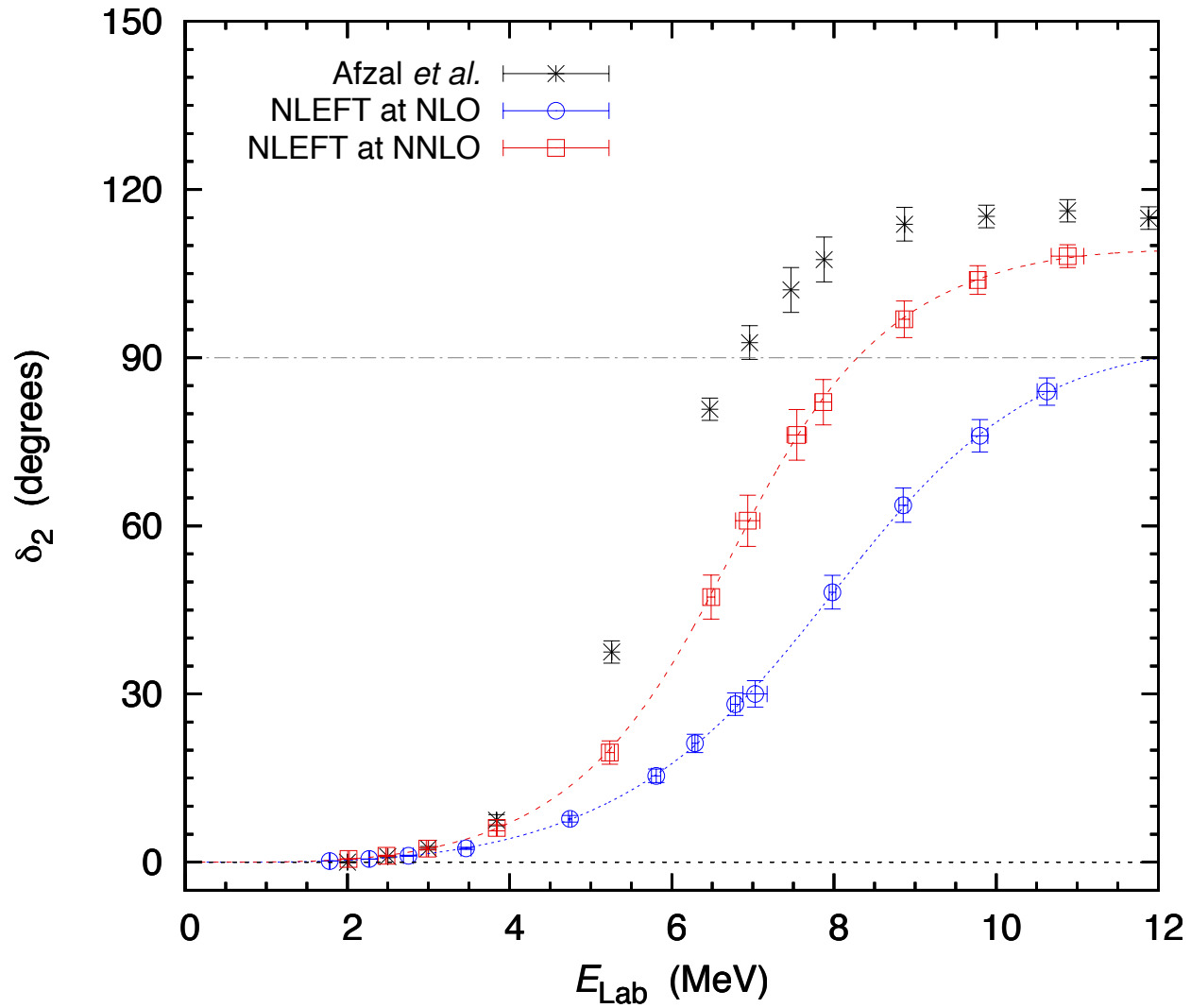
S -wave scattering



Afzal, Ahmad, Ali, RMP 41 247 (1969)

Higa, Hammer, van Kolck, NPA 809 171(2008)

D -wave scattering



More work needs to be done. But alpha processes now appear to be in reach of *ab initio* methods.

For an A_1 -body + A_2 -body scattering or reaction process the computational scaling is typically $\sim (A_1 + A_2)^2$.

For mass and charge transfer processes, we do the same steps but consider coupled channel scattering. For capture reactions, we include one-photon matrix elements and compute overlaps between bound states and scattering states.

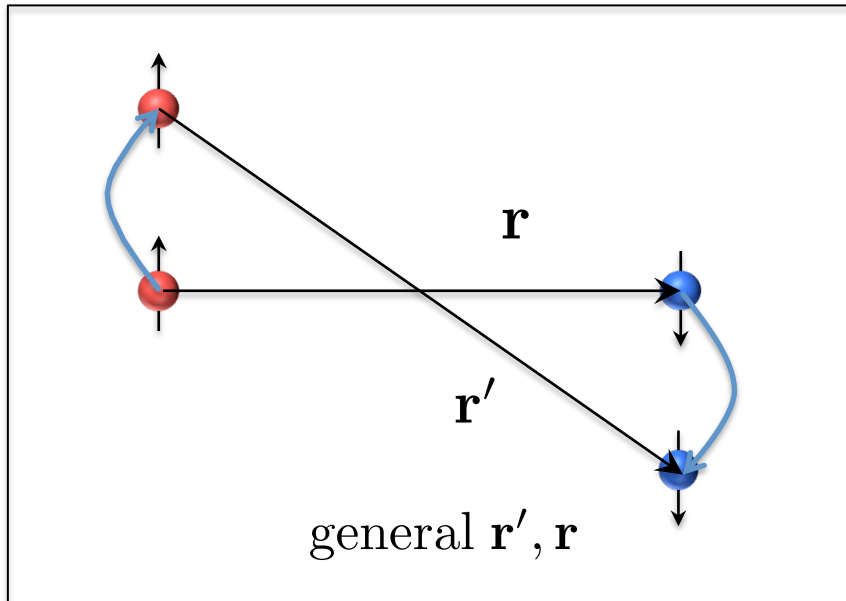
Rupak, D.L., PRL 111 032502 (2013)

For discussion: The adiabatic projection method could be applied to lattice QCD calculations for hadronic scattering and reactions.

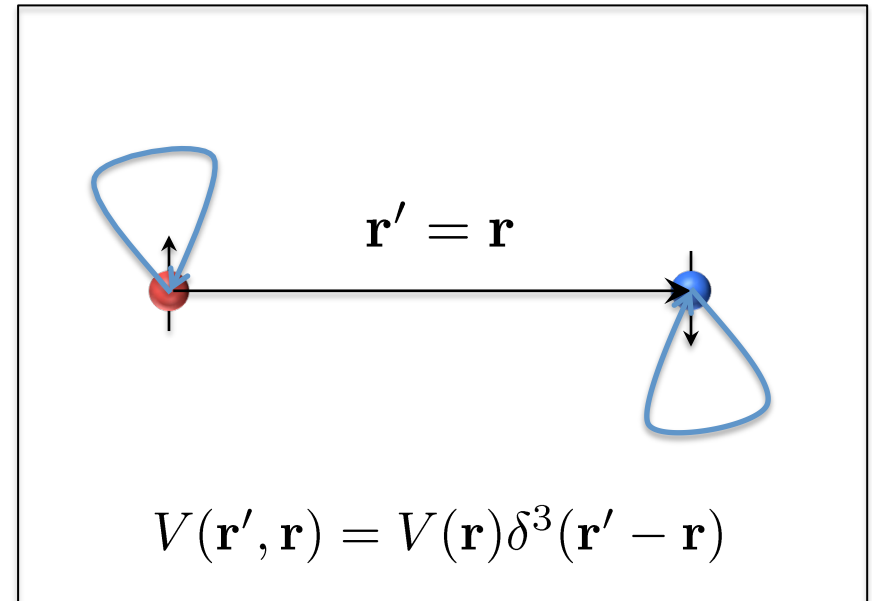
Designing new lattice interactions

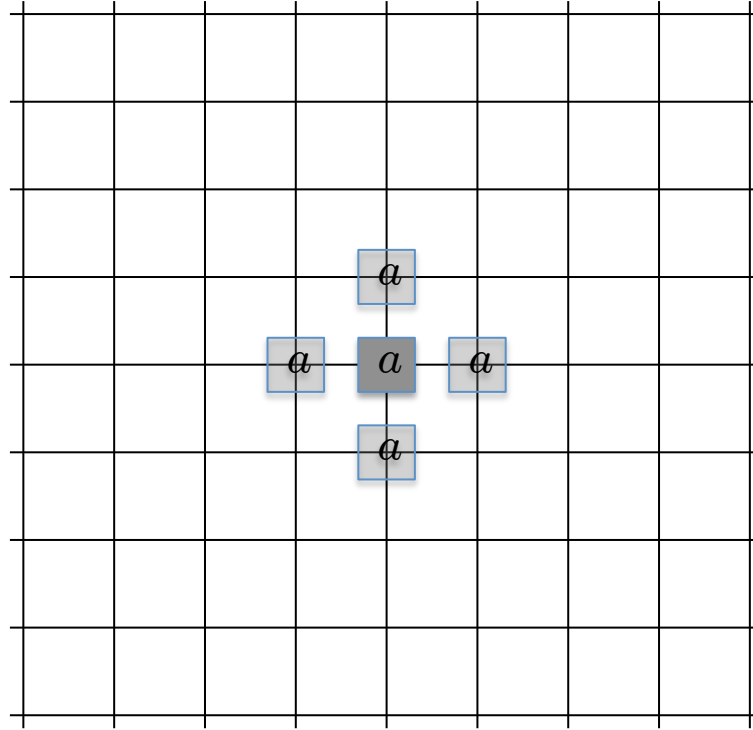
$$V(\mathbf{r}', \mathbf{r})$$

Nonlocal interaction



Local interaction





$$a_{\text{NL}}(\mathbf{n}) = a(\mathbf{n}) + s_{\text{NL}} \sum_{\langle \mathbf{n}' \mathbf{n} \rangle} a(\mathbf{n}')$$

$$a_{\text{NL}}^\dagger(\mathbf{n}) = a^\dagger(\mathbf{n}) + s_{\text{NL}} \sum_{\langle \mathbf{n}' \mathbf{n} \rangle} a^\dagger(\mathbf{n}')$$

Nonlocal density operators

$$\begin{aligned}\rho_{\text{NL}}(\mathbf{n}) &= a_{\text{NL}}^\dagger(\mathbf{n})a_{\text{NL}}(\mathbf{n}) \\ \rho_{I,\text{NL}}(\mathbf{n}) &= a_{\text{NL}}^\dagger(\mathbf{n})[\tau_I]a_{\text{NL}}(\mathbf{n})\end{aligned}$$

Nonlocal S -wave interactions

$$V_{\text{NL}} = \frac{c_{\text{NL}}}{2} \sum_{\mathbf{n}} : \rho_{\text{NL}}(\mathbf{n})\rho_{\text{NL}}(\mathbf{n}) : + \frac{c_{I,\text{NL}}}{2} \sum_{\mathbf{n},I} : \rho_{I,\text{NL}}(\mathbf{n})\rho_{I,\text{NL}}(\mathbf{n}) :$$

We can simulate using auxiliary fields

$$V_{\text{NL}}^s = \sqrt{-c_{\text{NL}}} \sum_{\mathbf{n}} \rho_{\text{NL}}(\mathbf{n})s(\mathbf{n}) + \sqrt{-c_{I,\text{NL}}} \sum_{\mathbf{n},I} \rho_{I,\text{NL}}(\mathbf{n})s_I(\mathbf{n})$$

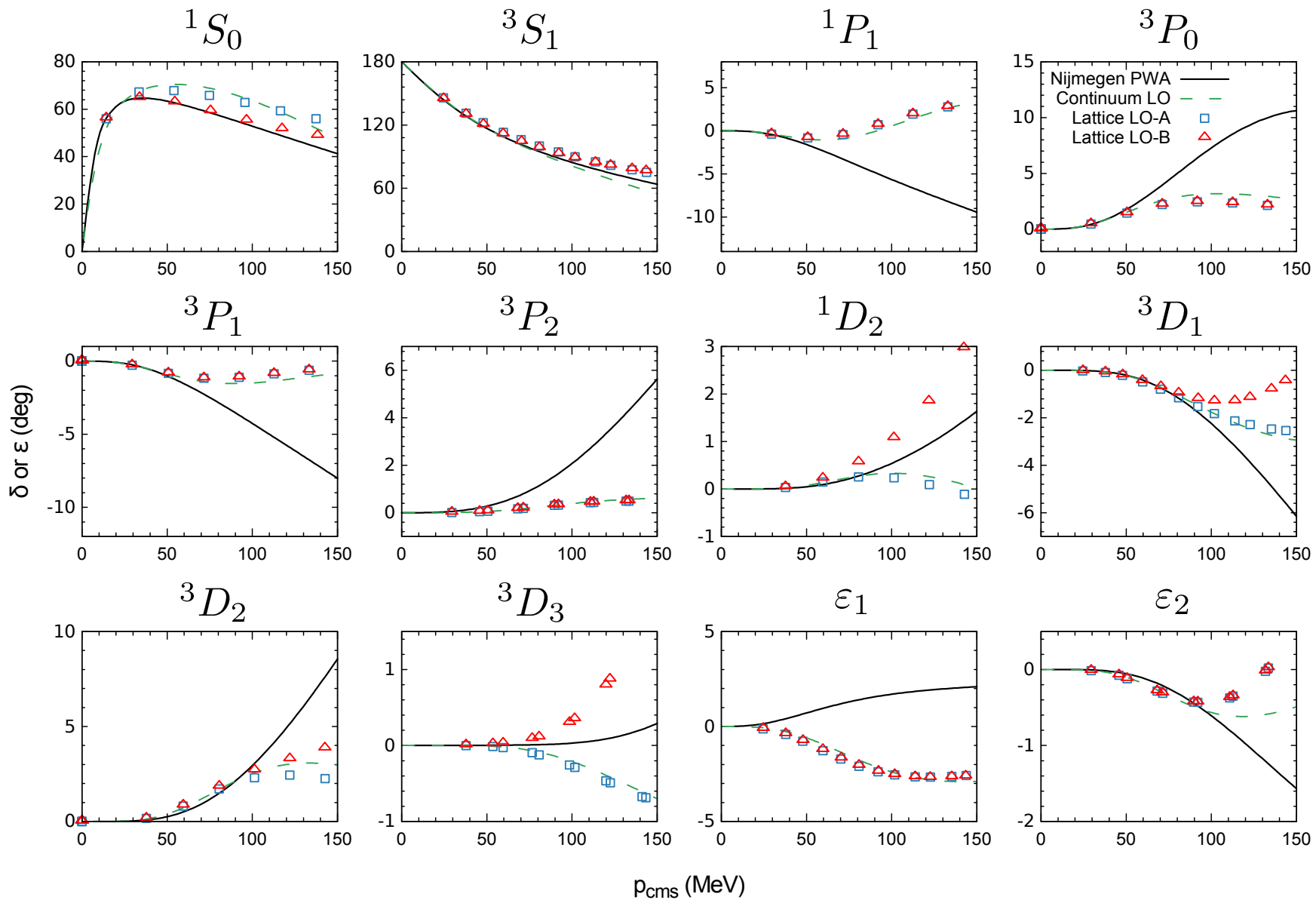
Interaction A at LO (LO + Coulomb)

Nonlocal short-range interactions
One-pion exchange interaction
(+ Coulomb interaction)

Interaction B at LO (LO + Coulomb)

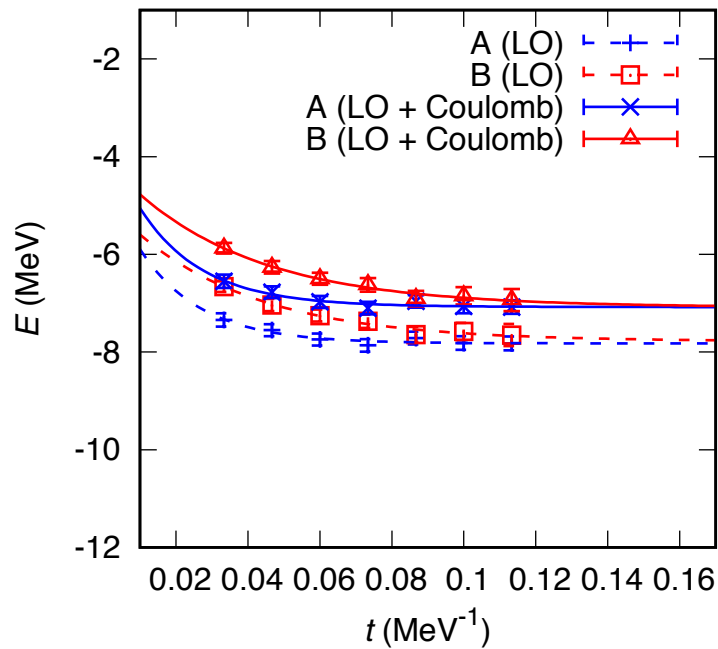
Nonlocal short-range interactions
Local short-range interactions
One-pion exchange interaction
(+ Coulomb interaction)

Details of how the interactions are fitted described later in the discussion.

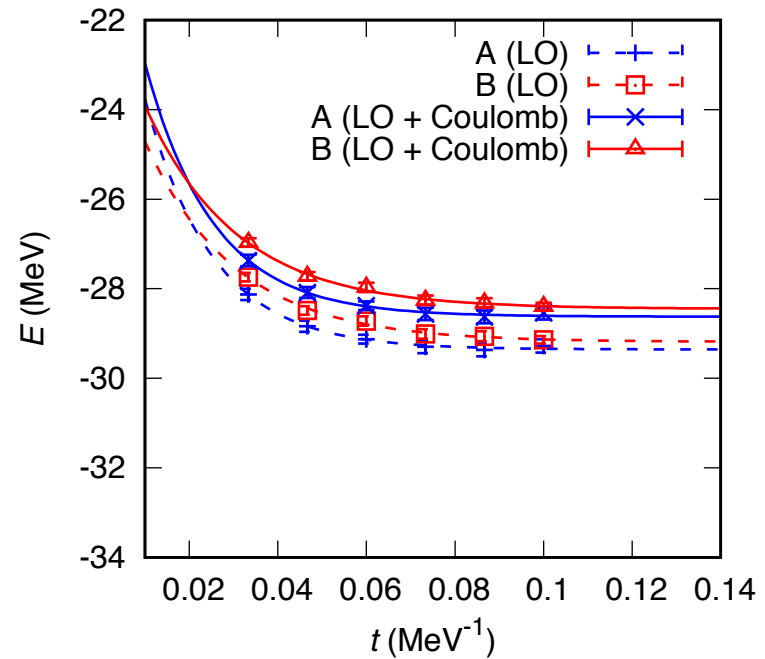


Ground state energies

${}^3\text{He}$

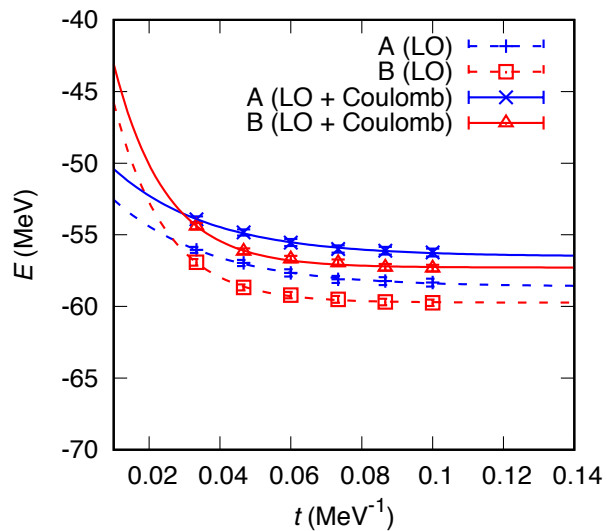
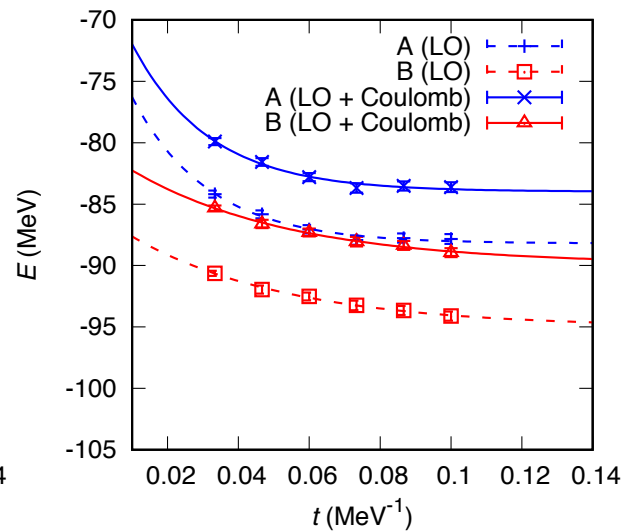
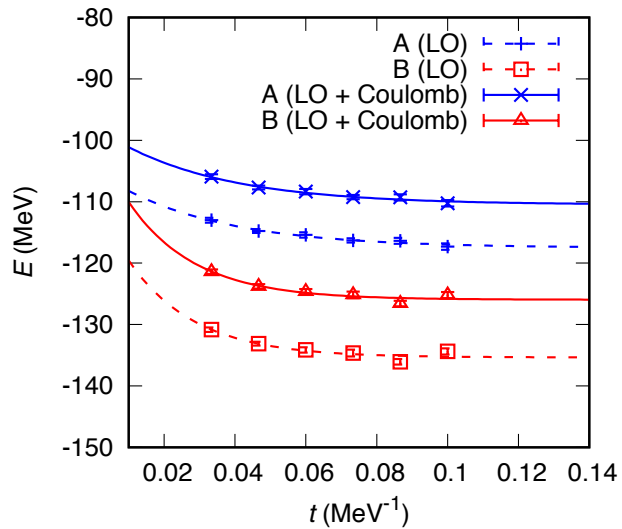
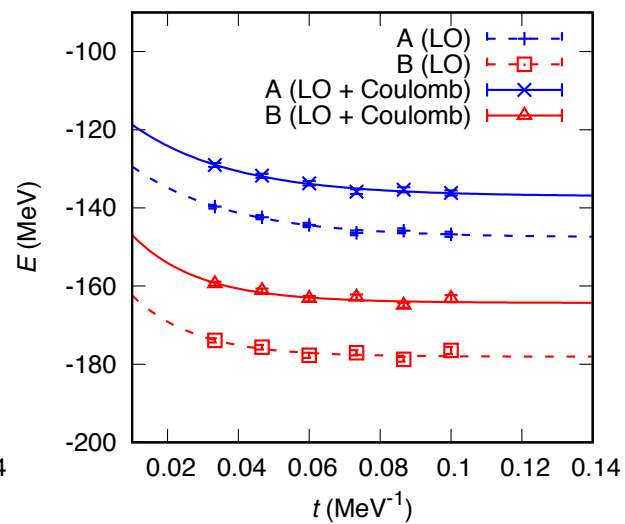


${}^4\text{He}$



Both interactions significantly reduce the Monte Carlo sign oscillation problem, the original motivation for studying the different interactions.

Nucleus	A (LO)	B (LO)	A (LO + Coulomb)	B (LO + Coulomb)	Experiment
${}^3\text{H}$	-7.82(5)	-7.78(12)	-7.82(5)	-7.78(12)	-8.482
${}^3\text{He}$	-7.82(5)	-7.78(12)	-7.08(5)	-7.09(12)	-7.718
${}^4\text{He}$	-29.36(4)	-29.19(6)	-28.62(4)	-28.45(6)	-28.296

^8Be  ^{12}C  ^{16}O  ^{20}Ne 

Nucleus	A (LO)	B (LO)	A (LO + Coulomb)	B (LO + Coulomb)	Experiment
^8Be	-58.61(14)	-59.73(6)	-56.51(14)	-57.29(7)	-56.591
^{12}C	-88.2(3)	-95.0(5)	-84.0(3)	-89.9(5)	-92.162
^{16}O	-117.5(6)	-135.4(7)	-110.5(6)	-126.0(7)	-127.619
^{20}Ne	-148(1)	-178(1)	-137(1)	-164(1)	-160.645

?

Nucleus	A (LO)	B (LO)	A (LO + Coulomb)	B (LO + Coulomb)	Experiment
^8Be	-58.61(14)	-59.73(6)	-56.51(14)	-57.29(7)	-56.591
^{12}C	-88.2(3)	-95.0(5)	-84.0(3)	-89.9(5)	-92.162
^{16}O	-117.5(6)	-135.4(7)	-110.5(6)	-126.0(7)	-127.619
^{20}Ne	-148(1)	-178(1)	-137(1)	-164(1)	-160.645

Nucleus	A (LO)	B (LO)	A (LO + Coulomb)	B (LO + Coulomb)	Experiment
${}^8\text{Be}$	-58.61(14)	-59.73(6)	-56.51(14)	-57.29(7)	-56.591
${}^{12}\text{C}$	-88.2(3)	-95.0(5)	-84.0(3)	-89.9(5)	-92.162
${}^{16}\text{O}$	-117.5(6)	-135.4(7)	-110.5(6)	-126.0(7)	-127.619
${}^{20}\text{Ne}$	-148(1)	-178(1)	-137(1)	-164(1)	-160.645

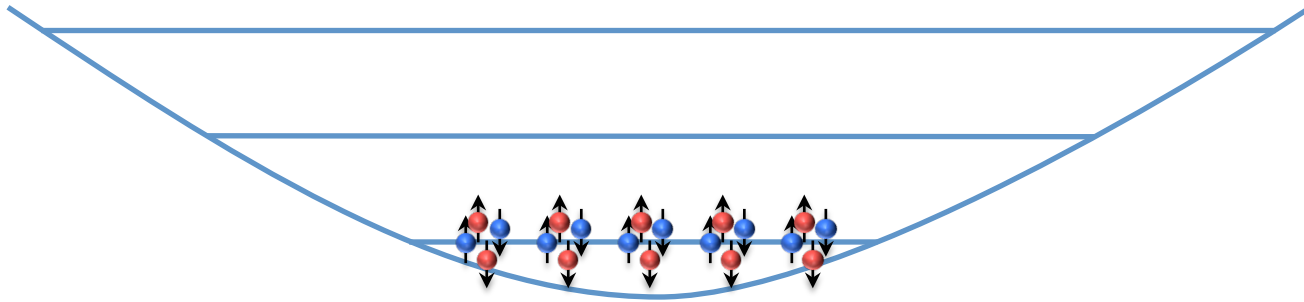
$$\frac{E_{8\text{Be}}}{E_{4\text{He}}} = 1.997(6)$$

$$\frac{E_{12\text{C}}}{E_{4\text{He}}} = 3.00(1)$$

$$\frac{E_{16\text{O}}}{E_{4\text{He}}} = 4.00(2)$$

$$\frac{E_{20\text{Ne}}}{E_{4\text{He}}} = 5.03(3)$$

Bose condensate of alpha particles!



Nucleus	A (LO)	B (LO)	A (LO + Coulomb)	B (LO + Coulomb)	Experiment
^8Be	-58.61(14)	-59.73(6)	-56.51(14)	-57.29(7)	-56.591
^{12}C	-88.2(3)	-95.0(5)	-84.0(3)	-89.9(5)	-92.162
^{16}O	-117.5(6)	-135.4(7)	-110.5(6)	-126.0(7)	-127.619
^{20}Ne	-148(1)	-178(1)	-137(1)	-164(1)	-160.645

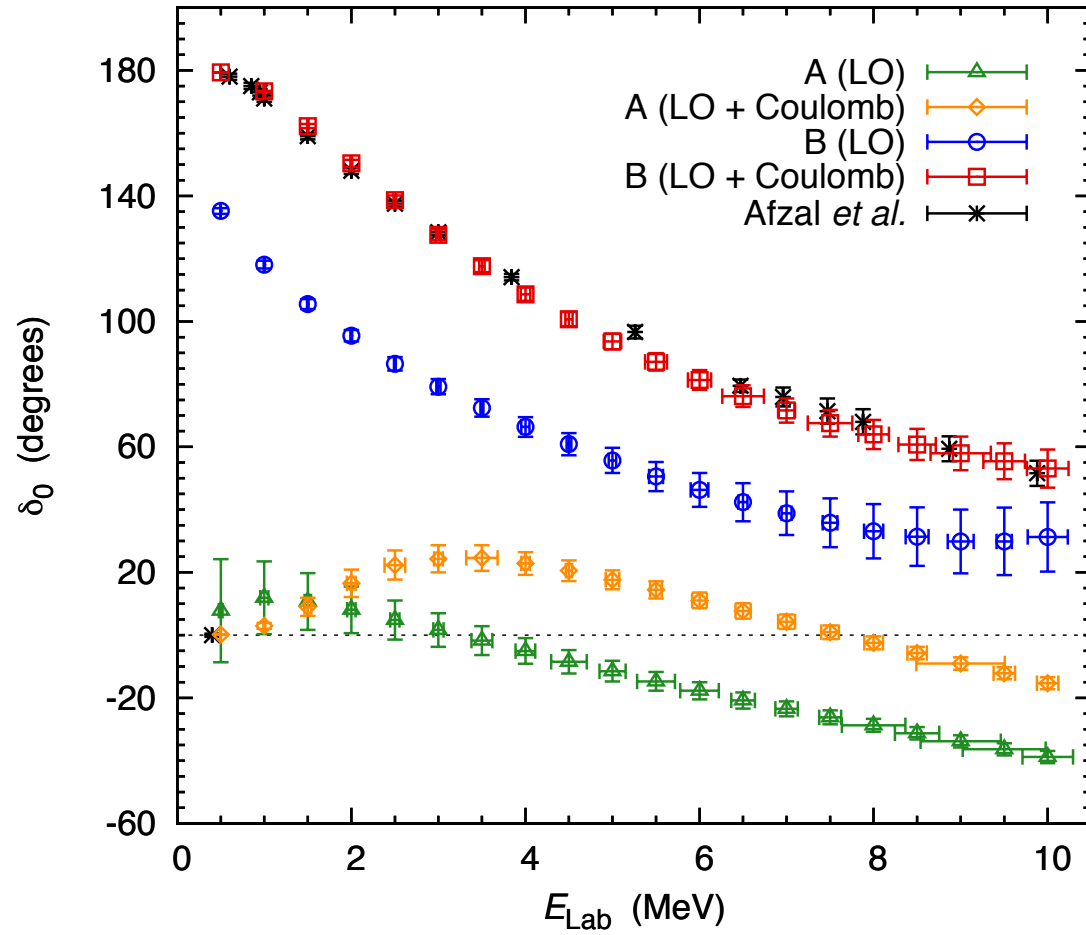
$$\frac{E_{8\text{Be}}}{E_{4\text{He}}} = 1.997(6)$$

$$\frac{E_{12\text{C}}}{E_{4\text{He}}} = 3.00(1)$$

$$\frac{E_{16\text{O}}}{E_{4\text{He}}} = 4.00(2)$$

$$\frac{E_{20\text{Ne}}}{E_{4\text{He}}} = 5.03(3)$$

alpha-alpha S -wave scattering



Interaction B was tuned to the nucleon-nucleon phase shifts, deuteron energy, and alpha-alpha phase shifts.

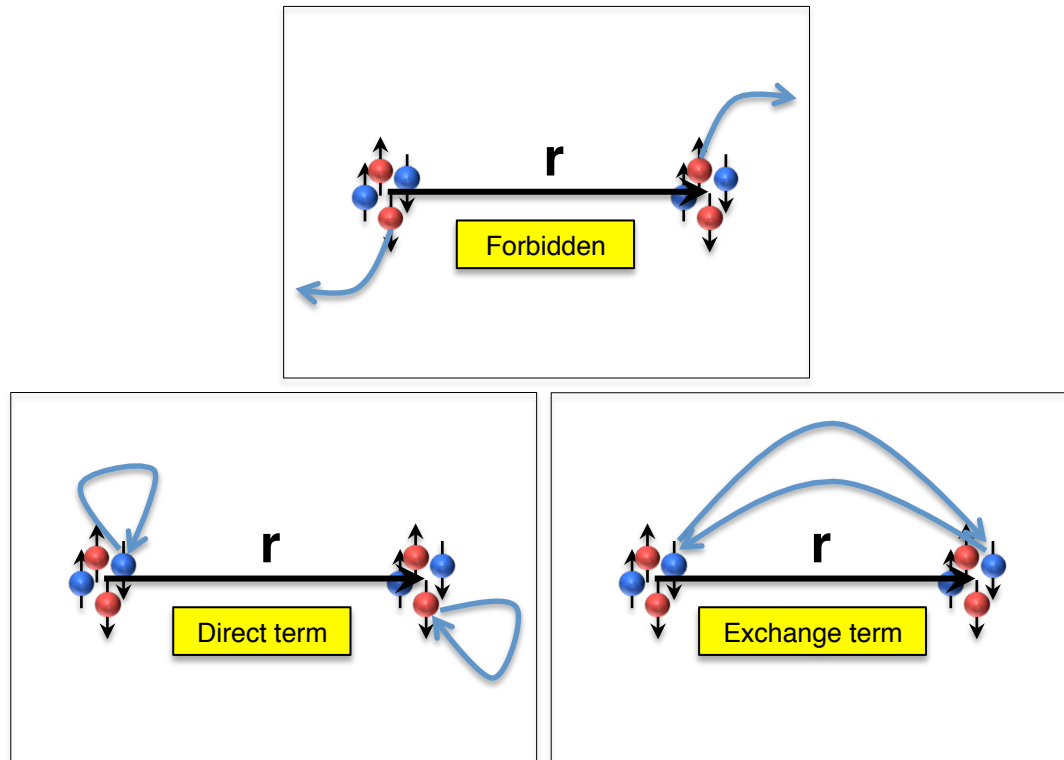
Interaction A was set by starting from interaction B, shutting off all local short-range interactions, and then adjusting the coefficients of the nonlocal short-range interactions to the nucleon-nucleon phase shifts and deuteron energy.

The alpha-alpha interaction is sensitive to the degree of locality of the interaction.

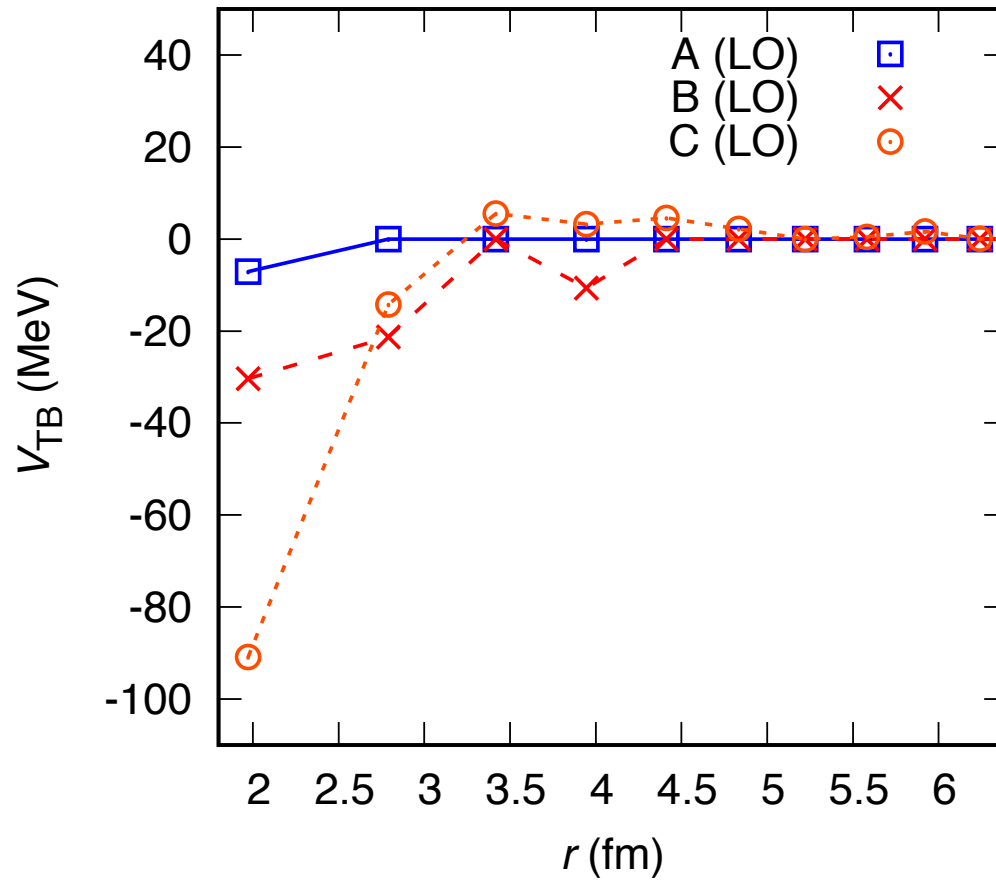
Why is the alpha-alpha interaction sensitive to the degree of locality of the interaction?

Tight-binding approximation

Qualitative picture: Treat the alpha particle radius as a small but nonzero parameter. Consider contributions of the nucleon-nucleon interaction to the effective low-energy alpha-alpha interaction.



Tight-binding potential



Interaction C is the interaction from the first half of this talk

Using the interactions A and B, we can define a one-parameter family of interactions

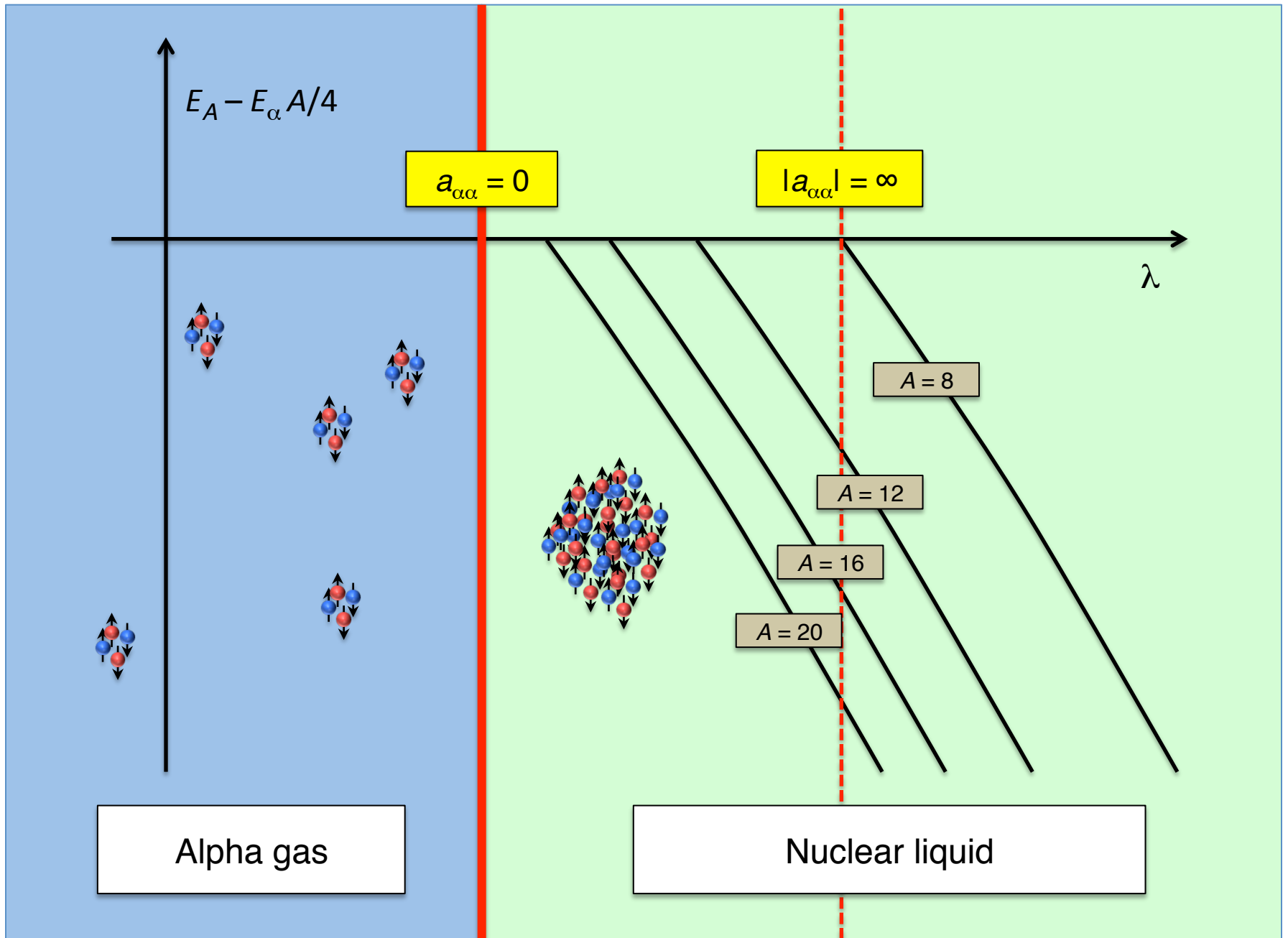
$$V_\lambda = (1 - \lambda)V_A + \lambda V_B$$

In order to discuss the many-body limit, we turn off the Coulomb interaction and explore the zero-temperature phase diagram.

As a function of λ , there is a quantum phase transition at the point where the alpha-alpha scattering length vanishes.

Stoof, PRA 49, 3824 (1994)

The transition is a first-order transition from a Bose-condensed gas of alpha particles to a nuclear liquid.



Applications

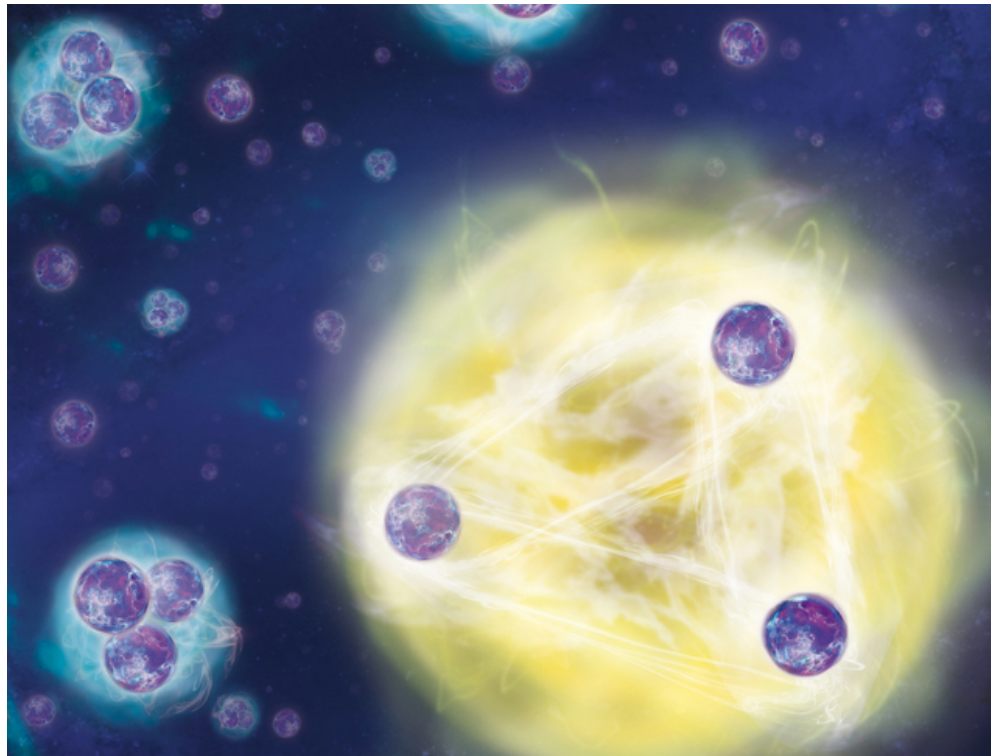
Ab initio chiral effective field theory is an excellent theoretical framework, however there is no guarantee it will work well for nuclei with increasing numbers of nucleons. Cutoff dependence, higher-order corrections, and higher-body forces can become large, rendering the calculation inefficient.

There are an infinite number of different ways to write *ab initio* chiral effective field theory interactions at any given order. While they may look equivalent for the low-energy nucleon-nucleon phase shifts, one can use light nucleus-nucleus scattering data to identify a more likely-to-succeed set of interactions where cutoff dependence, higher-order corrections, and higher-body forces appear to be small. Should be useful for *ab initio* nuclear structure and reaction calculations.

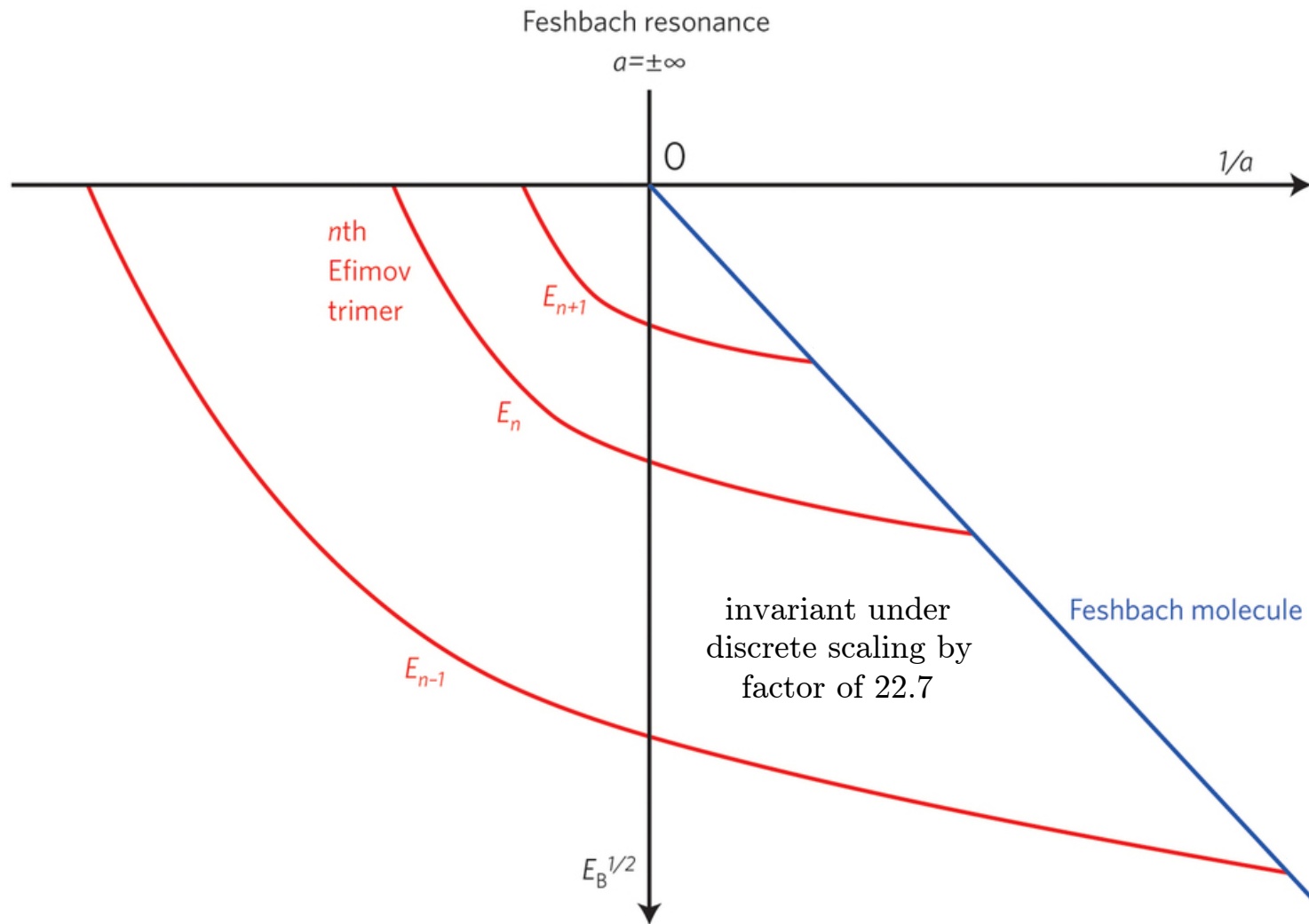
Efimov effect

Efimov, Sov. J. Nucl. Phys. 589 (1971); PRC 47, 1876 (1993)

Universality in three boson system at large scattering length



Credit: Quanta Magazine, Walchover, May 2014



Chin, Wang, Nat. Phys. 11 449 (2015); Braaten, Hammer, Phys. Rept. 259 (2006)

By adjusting the parameter λ in *ab initio* calculations, one can move the energy of any alpha cluster state up and down relative to alpha separation thresholds. This can be used as a new window to view the structure of these exotic nuclear states.

In particular, one can tune the alpha-alpha scattering length to infinity. In the absence of the Coulomb interaction, the Hoyle state of ^{12}C can be continuously deformed into a universal Efimov trimer.

Similarly, in the absence of the Coulomb interaction, the second 0^+ state of ^{16}O can be continuously deformed into a universal Efimov tetramer.

Summary and outlook

We have presented numerical evidence from *ab initio* lattice simulations showing that nature is near a quantum phase transition.

It is a first-order transition from a Bose-condensed gas of alpha particles to a nuclear liquid. Whether one has an alpha-particle gas or nuclear liquid is determined by the strength of the alpha-alpha interactions, and the alpha-alpha interactions depend on the strength and degree of locality of the nucleon-nucleon interactions.

Several applications of this work include practical methods to improve *ab initio* nuclear structure and reaction calculations, a new theoretical window on alpha cluster states, and a connection to the universal physics of Efimov states.