Effective theory of ^{3}H and ^{3}He

Sebastian König

in collaboration with H.W. Grießhammer, H.-W. Hammer, and U. van Kolck

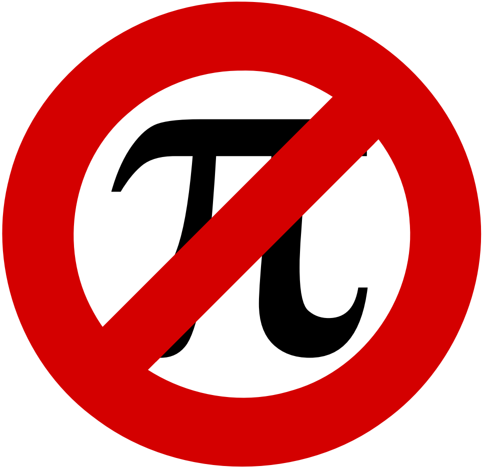
INT Program 16-1, University of Washington

Seattle, WA

April 15, 2016

SK *et al.*, J. Phys. G **43** 055106 (2016), 1508.05085 [nucl-th]

Removing pions



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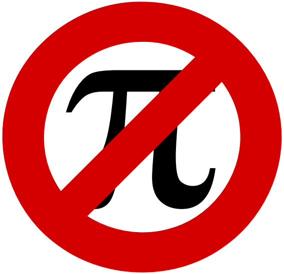
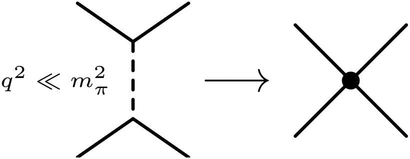
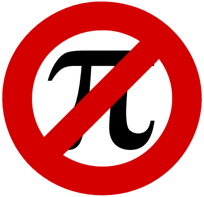
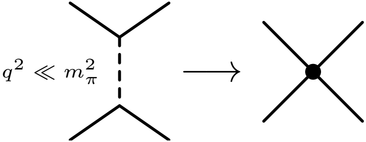




$q^2 \ll m_\pi^2$

Removing pions

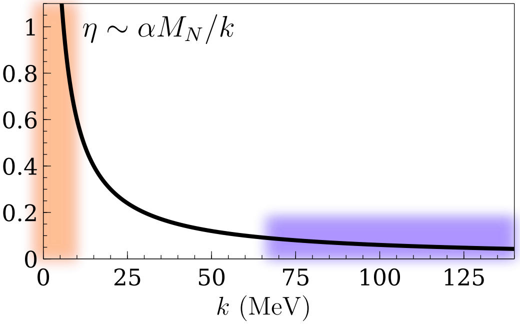




pions are not resolved at sufficiently low energies!

Motivation

- **scaling of Coulomb contributions**



nonperturbative → perturbative

- **three-body forces**

$$H(\Lambda) = \overbrace{H_{0,0}(\Lambda) + H_{0,1}(\Lambda)}^{nd+pd} + \overbrace{H_{0,1}^{(\alpha)}(\Lambda)}^{pd \text{ only}}$$

Vanasse, Egolf, Kerin, SK, Springer, PRC **89** 064003 (2014)

Motivation

Why pionless EFT?

- conceptually clean and (reasonably) simple
- allows for a fully perturbative treatment of higher orders
- cutoff can be made arbitrarily large
- still clearly connected to QCD!

Why talk about it here?

- applied to analyze and extrapolate lattice results

  Barnea *et al.* PRL **114** 052501 (2015)

  Kirscher *et al.* PRC **92** 054002 (2015)
- lattice QCD used to extract extract pionless LECs: $np \to d\gamma$

  Beane *et al.* (NPLQCD) PRL **115** 132001 (2015)
- study general EFT questions (renormalization with Coulomb)

Outline

Introduction

Coulomb effects in $^3\mathrm{He}$

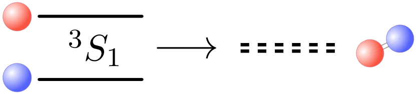
Divergences

A new expansion

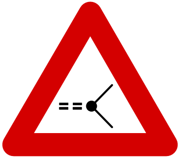
Summary

Two-body sector

Introduce dibaryon fields. . .

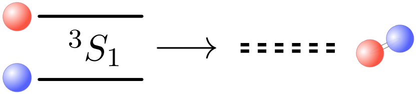


Kaplan (1996/97), Bedaque, van Kolck (1997/98)



Two-body sector

Introduce dibaryon fields...



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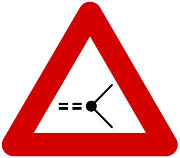
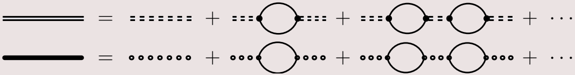


...and resum bubble-insertions to all orders!

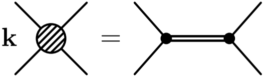
Full dibaryon propagators

$^3S_1$: $\Delta_d =$

$^1S_0$: $\Delta_t =$



$$\Delta_d(k) \sim \frac{\mathrm{i}}{\underbrace{k\cot\delta_d}_{=-\gamma_d+\frac{\rho_d}{2}(k^2+\gamma_d^2)+\cdots} - \mathrm{i}k}$$



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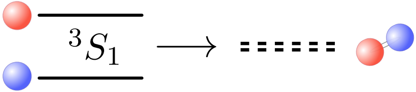

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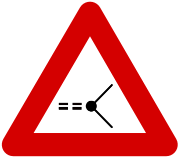

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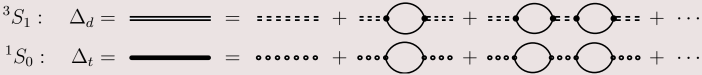

$$\Delta_d(k) \sim \frac{\mathrm{i}}{\underbrace{k\cot\delta_d}_{=-\gamma_d+\frac{\rho_d}{2}(k^2+\gamma_d^2)+\cdots} - \mathrm{i}k}$$

$$\gamma_d\rho_d \sim Q/\Lambda_{\not\pi} = \mathcal{O}(1/3)$$

Propagator renormalization

Full dibaryon propagators

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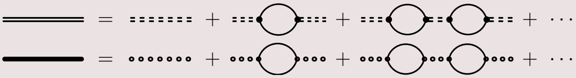
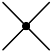
$^1S_0: \quad \Delta_t =$



$$i\Delta_d^{(0)}(p_0,\mathbf{p}) = \frac{-i}{\sigma_d^{(0)} + y_d^2 I_0(p_0,\mathbf{p})}$$

$$I_0(p_0,\mathbf{p}) = M_N \int^{\Lambda} d^3q \frac{1}{M_N p_0 - \mathbf{p}^2/4 - \mathbf{q}^2 + i\varepsilon}$$

$$= -\frac{M_N}{4\pi}\left(\frac{2\Lambda}{\pi} + \underbrace{\sqrt{\frac{\mathbf{p}^2}{4} - M_N p_0 - i\varepsilon}}_{\to -ik}\right) + \mathcal{O}(1/\Lambda)$$

- absorb linear divergence: $\sigma_d^{(0)} = \frac{2\Lambda}{\pi} - \gamma_d$
- $1/\Lambda$ effects are neglected (equivalent: PDS, $2\Lambda/\pi \to \mu_R$)
- without dibaryons: resummation of  $\sim C_0 = -4\pi/(M_N\sigma)$

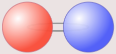
Three-body sector

Nucleon

- spin $^1/_2$
- isospin $^1/_2$

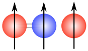
Deuteron

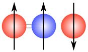
- spin 1
- isospin 0



↪ **two S-wave channels:**

$$\mathbf{1}\otimes\frac{\mathbf{1}}{\mathbf{2}}=\frac{\mathbf{3}}{\mathbf{2}}\Big(\sim\uparrow\uparrow\uparrow\Big)\oplus\overbrace{\frac{\mathbf{1}}{\mathbf{2}}\Big(\sim\uparrow\uparrow\downarrow+\cdots\Big)}^{\text{spin doublet}\to{}^3\mathrm{H},{}^3\mathrm{He}}$$





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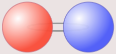

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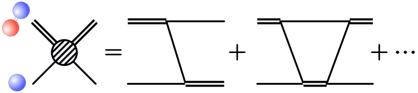
—quartet channel—



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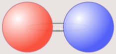

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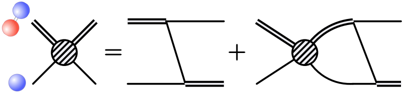

$\hookrightarrow$ **solve integral equations to get phase shifts and binding energies**

Three-body sector

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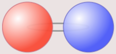

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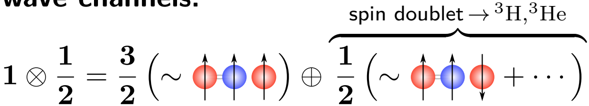

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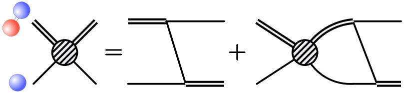
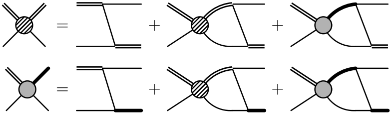
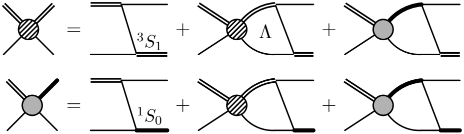


—doublet channel—



↪ **solve integral equations to get phase shifts and binding energies**

The triton



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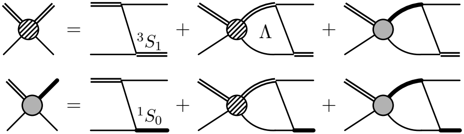
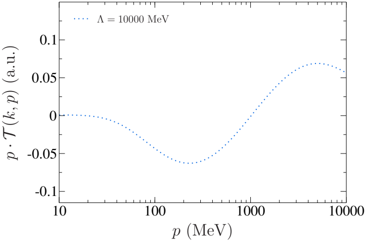
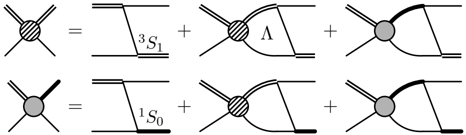
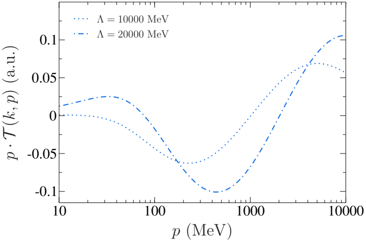

=
$^3S_1$
+
Λ
+
=
$^1S_0$
+
+

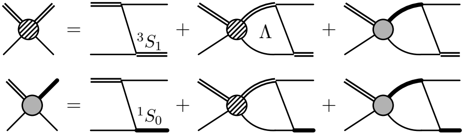

Λ = 10000 MeV
$p \cdot \mathcal{T}(k,p)$ (a.u.)
0.1
0.05
0
-0.05
-0.1
10
100
1000
10000
p (MeV)

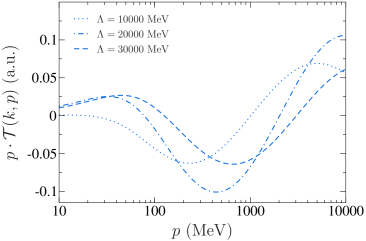
The triton


$^3S_1$
Λ
$^1S_0$


Λ = 10000 MeV
Λ = 20000 MeV
$p \cdot \mathcal{T}(k,p)$ (a.u.)
0.1
0.05
0
-0.05
-0.1
10
100
1000
10000
p (MeV)

The triton


=
$^3S_1$
+
Λ
+
=
$^1S_0$
+
+


Λ = 10000 MeV
Λ = 20000 MeV
Λ = 30000 MeV
$p \cdot \mathcal{T}(k,p)$ (a.u.)
0.1
0.05
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10000
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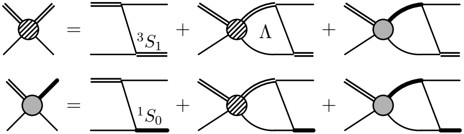
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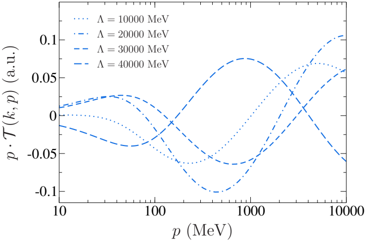
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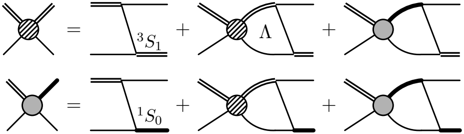
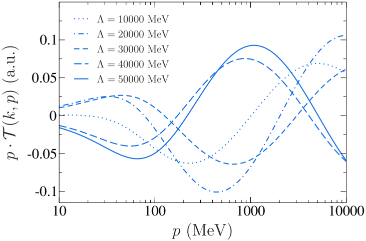


$=$
$^3S_1$
$+$
Λ
$+$
$=$
$^1S_0$
$+$
$+$



$\Lambda = 10000$ MeV
$\Lambda = 20000$ MeV
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$\Lambda = 40000$ MeV
$\Lambda = 50000$ MeV
$p \cdot \mathcal{T}(k,p)$ (a.u.)
0.1
0.05
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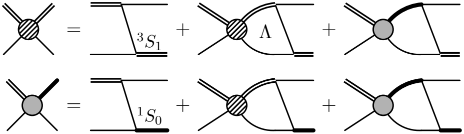
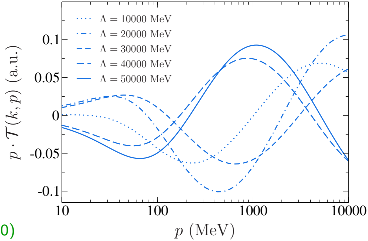


amplitude has no well-defined limit!

Bedaque *et al.*, NPA **676** 357 (2000)



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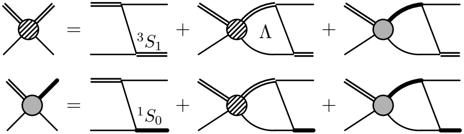

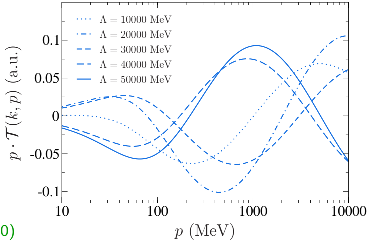

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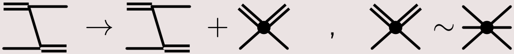
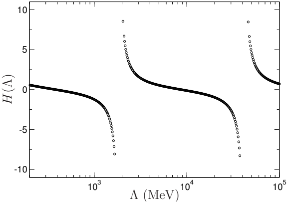
Three-body force promotion

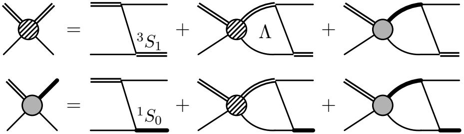
already at LO:

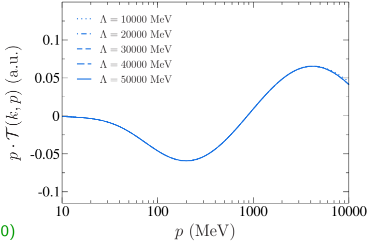




- independent of spin and isospin
  $\rightarrow SU(4)$-symmetry
- coupling runs in RG limit cycle
- **makes amplitude cutoff-independent**

The triton





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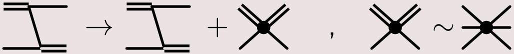

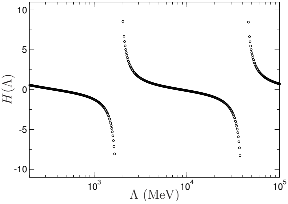

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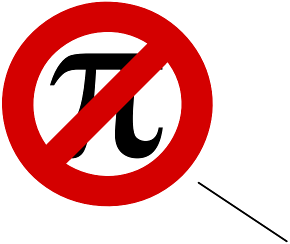
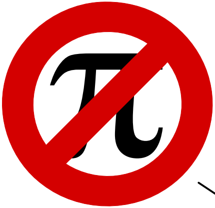
Universality



large scattering length

Braaten, Hammer
Phys. Rept. **428** 259 (2006)

Universality



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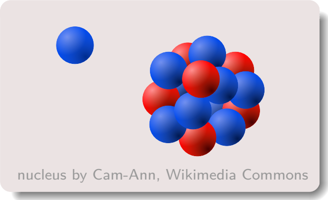
Phys. Rept. **428** 259 (2006)

halo EFT



nucleus by Cam-Ann, Wikimedia Commons

van Kolck, NPA **645**, 273 (1999)
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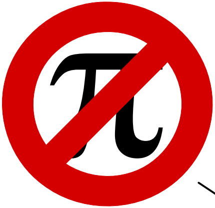
Zhang *et al.*, 1507.07239 [nucl-th]

Universality



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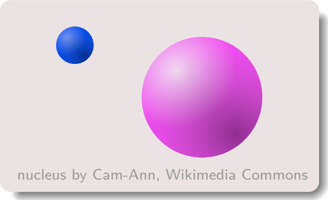

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large scattering length

Braaten, Hammer
Phys. Rept. **428** 259 (2006)

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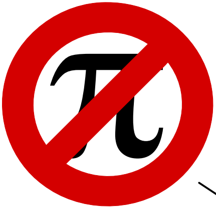


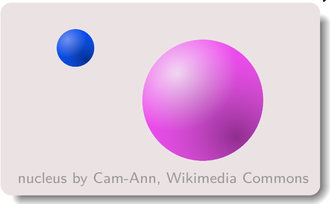
cold atoms

- tunable scattering length!
- Efimov effect!

large scattering length

Braaten, Hammer
Phys. Rept. **428** 259 (2006)

halo EFT



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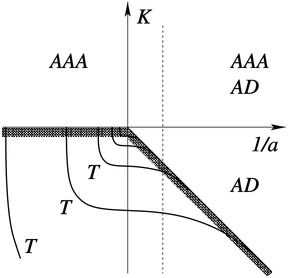
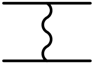


Coulomb contributions

Coulomb photons:



$$\sim (\mathrm{ie})\,\frac{\mathrm{i}}{\mathbf{q}^2}\,(\mathrm{ie}) \longrightarrow (\mathrm{ie})\,\frac{\mathrm{i}}{\mathbf{q}^2+\lambda^2}\,(\mathrm{ie})$$

$\mathcal{O}(\alpha)$ diagrams

Coulomb contributions

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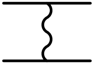

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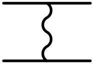
$\mathcal{O}(\alpha)$ diagrams

only generated by dibaryon kinetic term!

$\hookrightarrow$ **higher-order correction** $\sim \rho_d$

Coulomb subtraction

Coulomb photons:



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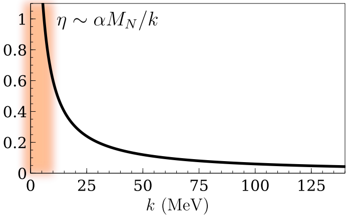
- long (infinite) range → very strong at small momentum transfer



Coulomb nonperturbative for $\eta \sim 1$

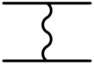
important for p-d scattering length!

↪ SK, Hammer, PRC **90** 034005 (2014)

$$C_\eta^2\, k \cot \delta_{\mathrm{diff}}(k) + \alpha\mu\, h(\eta) = -\frac{1}{a_{p\text{-}d}} + \cdots$$

(modified ERE)

Coulomb subtraction

Coulomb photons:  $\sim (\mathrm{ie})\,\frac{\mathrm{i}}{\mathbf{q}^2}\,(\mathrm{ie}) \longrightarrow (\mathrm{ie})\,\frac{\mathrm{i}}{\mathbf{q}^2+\lambda^2}\,(\mathrm{ie})$

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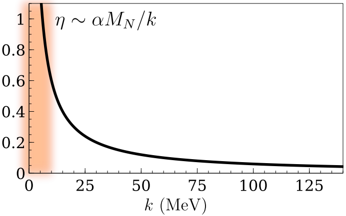

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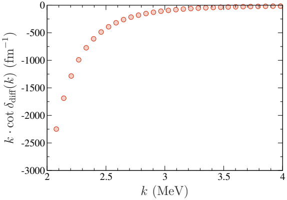
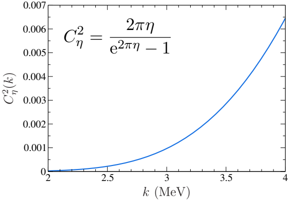


×

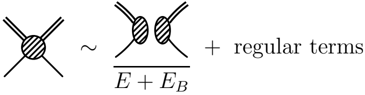


= **finite value**

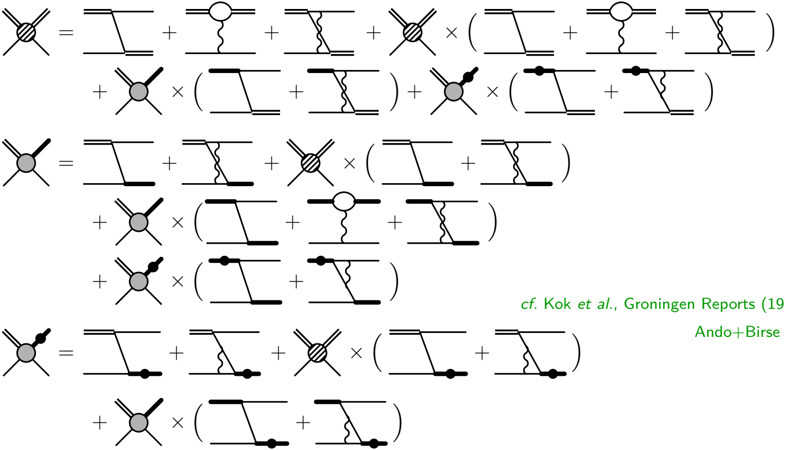
The helion and the counterterm

He-3 binding energy

bound-state ↔ pole!



↪ calculate ^{3}He binding energy!

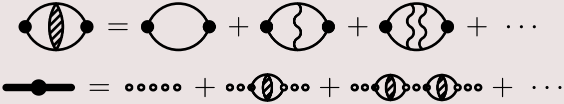


cf. Kok *et al.*, Groningen Reports (1979/81)

Ando+Birse (2010)

Coulomb effects in the proton–proton channel

Coulomb-dressed propagator



$$\Delta_{t,pp}(p) \sim \frac{1}{-1/a_{p-p} - \alpha M_N H(\eta)} \quad , \quad \eta = \alpha M_N / \left(2\mathrm{i}\sqrt{\mathbf{p}^2/4 - M_N p_0 - \mathrm{i}\varepsilon}\right)$$

Kong, Ravndal (1999)

↪ **Coulomb-modified effective range expansion**

Bethe (1949)

cf. Ando, Birse (2010)

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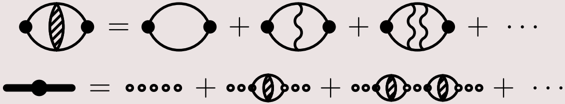

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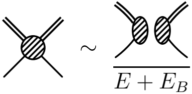
Note: two divergences absorbed into a single parameter!

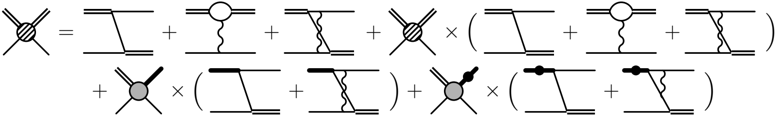
He-3 binding energy

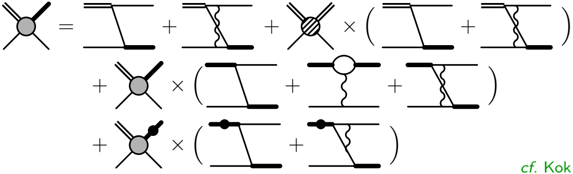
bound-state ↔ pole!



+ regular terms

↪ calculate ^{3}He binding energy!





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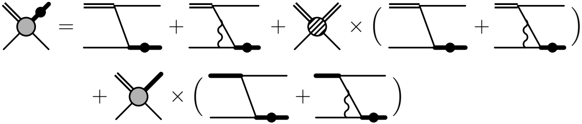
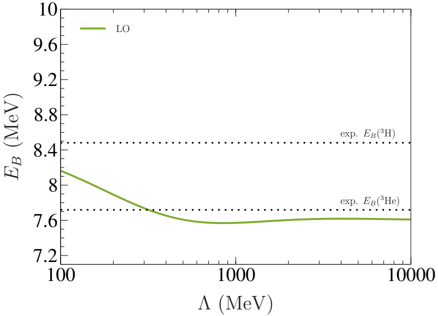


He-3 beyond leading order



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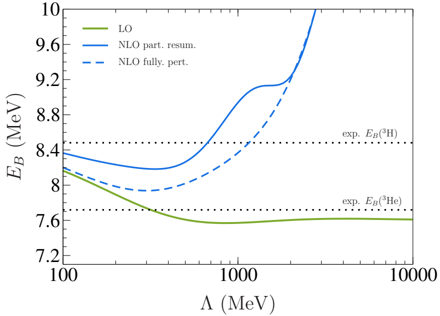
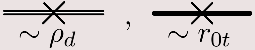


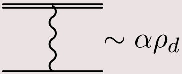
NLO corrections

- effective ranges



$\sim \rho_d$, $\sim r_{0t}$

- dibaryon-photon coupling



$\sim \alpha\rho_d$

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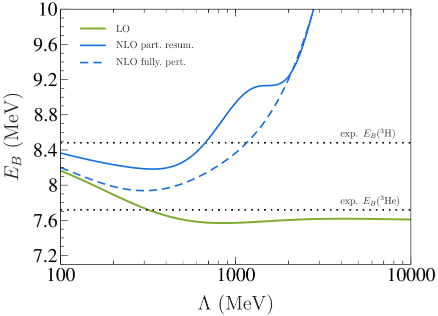

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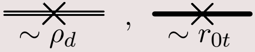

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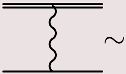


$\sim \alpha\rho_d$

- NLO result is not cutoff stable $\hookrightarrow$ incomplete renormalization!
- refitting the three-body force to $E_B(^3\mathrm{He})$ gives stable p-d phase shifts!

SK, Ph.D. thesis (2013)

SK *et al.*, JPG **42** 045101 (2015)

- form of new p-d specific counterterm can be derived analytically!

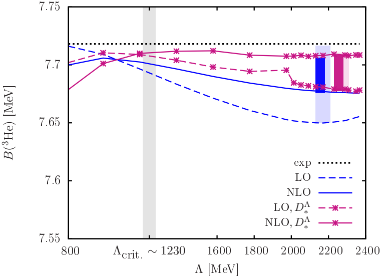
  $\rightsquigarrow$ three body-force $H(\Lambda) = H_{0,0}(\Lambda) + H_{0,1}(\Lambda) + H_{0,1}^{(\alpha)}(\Lambda)$

Vanasse, Egolf, Kerin, SK, Springer, PRC **89** 064003 (2014)

Counterterm controversy

A recent paper does not find a new counterterm at NLO!

Kirscher+Gazit, PLB **755** (2016) 253, 1510.00118 [nucl-th]



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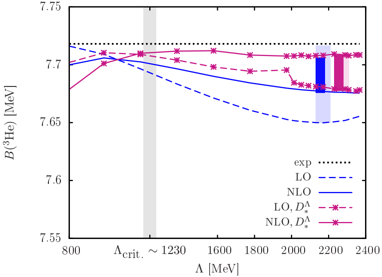

Vanasse *et al.*

- momentum-space formalism
- sharp cutoff (non-local)
- can take Λ arbitrarily large

Kirscher and Gazit

- configuration-space (R)RGM
- Gaussian regulators (local)
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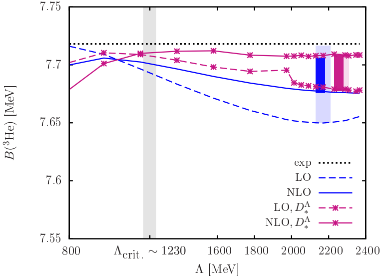

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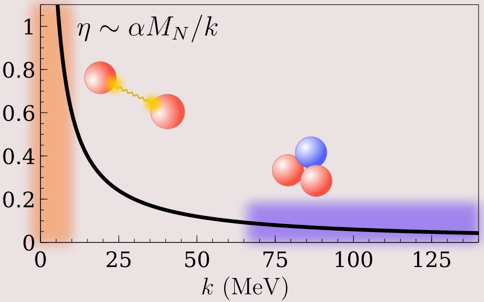
power counting $\leftrightarrow$ regulators?

Coulomb corrections for nuclei

How much Coulomb should we really iterate?

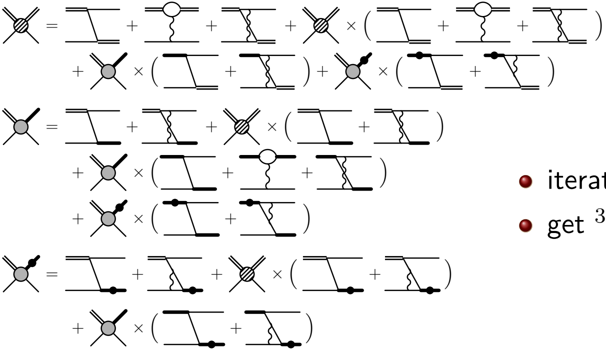
Coulomb regimes

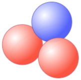
trinucleon binding momentum ~ 80 MeV ...



$\hookrightarrow$ **should Coulomb not be a small perturbative correction?**

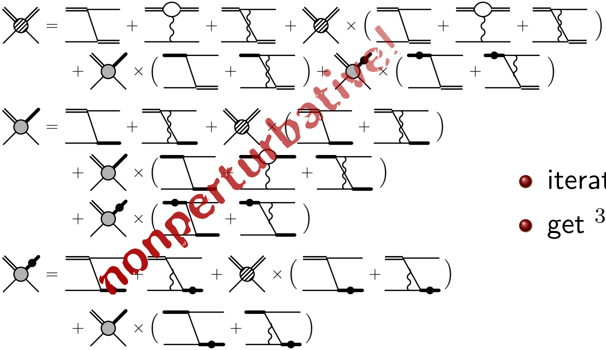
Nonperturbative vs. perturbative and helium

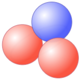




- iterate $\mathcal{O}(\alpha)$ diagrams. . .
- get $^3\mathrm{He}$ pole directly

Nonperturbative vs. perturbative and helium





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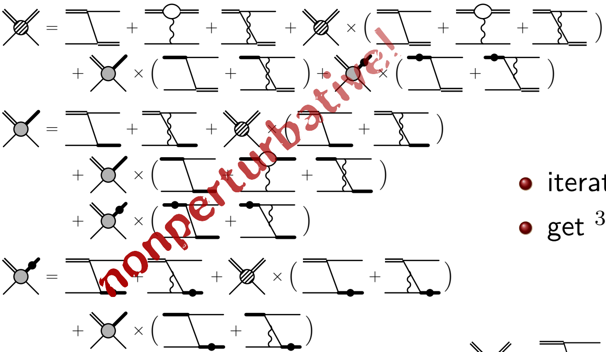
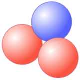
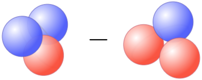


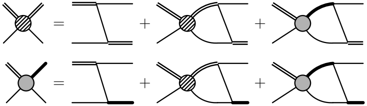


- iterate $\mathcal{O}(\alpha)$ diagrams. . .
- get $^3\mathrm{He}$ pole directly

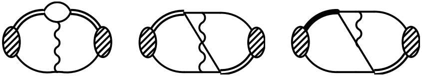
- use trinucleon wavefunctions
- fully perturbative in α!

ΔE :



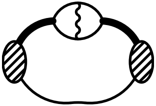


$$\Delta E = \langle\psi|V_C|\psi\rangle$$



SK, Grießhammer, Hammer, J. Phys. G **42** 045101 (2015)

Coulomb bubble divergence



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- . . . but this divergence comes from the photon-bubble subdiagram!

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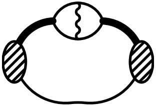

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Strategy

SK, Grießhammer, Hammer, van Kolck, JPG **43** 055106 (2016), 1508.05085 [nucl-th]

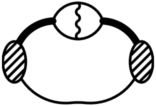
1. isolate divergence: 
2. **take the leading-order $^1S_0$ in the unitarity limit!**

   $$a_{^1S_0} = -23.7 \approx \infty \rightsquigarrow 1/a_{^1S_0} \approx 0$$

3. include divergent diagram together with finite $a_{^1S_0}$

    $+$  $=$ finite

Coulomb bubble divergence



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$$\Delta_{t,pp}(p_0,\mathbf{p}) = \frac{-\mathrm{i}}{\underbrace{\sigma_{t,pp} - \frac{2\Lambda}{\pi} + \alpha M_N\left(\log\frac{2\Lambda}{\alpha M_N} - C_E\right)}_{=1/a_C} - \alpha M_N H(\eta)}$$

cf. Kong, Ravndal (1999)

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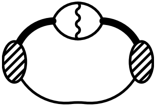

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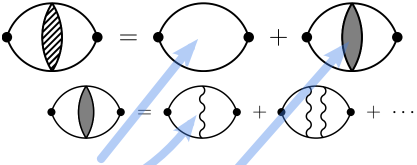
Important to isolate divergence for consistent renormalization!

Dressed bubble integral

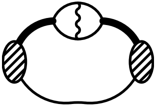
$J_0(k)$
$= G_C(k^2/M_N;\mathbf{0},\mathbf{0})$



$$\hat{G}_C = \hat{G}_0^{(+)} + \hat{G}_0^{(+)}\,\hat{V}_C\,\hat{G}_C = \hat{G}_0^{(+)} + \hat{G}_0^{(+)}\,\hat{T}_C\,\hat{G}_0^{(+)}$$

$$\hat{T}_C = \hat{V}_C + \hat{V}_C\,\hat{G}_0^{(+)}\,\hat{T}_C$$

Coulomb bubble divergence



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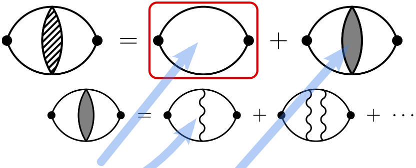

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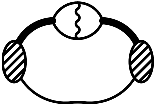

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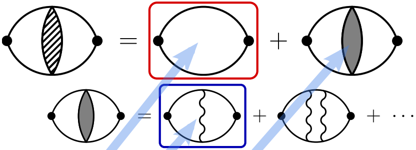

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A new expansion

Take the leading-order singlet channel in the unitarity limit!

$a_t = -23.4 \approx \infty$!

$\sigma_t = \sigma_t^{(0)} + \sigma_t^{(1)}$

$$\mathrm{i}\Delta_t^{(0)}(p_0,\mathbf{p})\frac{-\mathrm{i}}{\sigma_t + y_t^2 I_0(p_0,\mathbf{p})} \to \frac{-\mathrm{i}}{\sqrt{\frac{\mathbf{p}^2}{4} - M_N p_0 - \mathrm{i}\varepsilon}}$$

$$\boxed{\sigma_t^{(0)} - 2\Lambda/\pi = 0}$$

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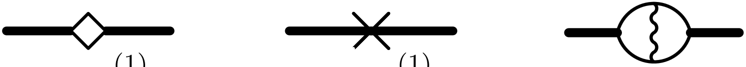
- scattering length: $\sigma_t^{(1)} \sim -1/a_t$
- effective range: $c_t^{(1)} = \dfrac{M_N r_{0t}}{2}$



$\sim \ -\mathrm{i}\sigma_{t(,pp)}^{(1)}$ $\qquad$ $\sim \ -\mathrm{i}c_t^{(1)}$

$$\mathrm{i}\Delta_t^{(1)}(p_0,\mathbf{p}) = \mathrm{i}\Delta_t^{(0)}(p_0,\mathbf{p}) \times \left[-\mathrm{i}\sigma_t^{(1)} - \mathrm{i}c_t^{(1)}\left(p_0 - \frac{\mathbf{p}^2}{4M_N}\right)\right] \times \mathrm{i}\Delta_t^{(0)}(p_0,\mathbf{p})$$

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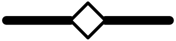

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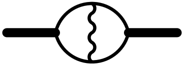

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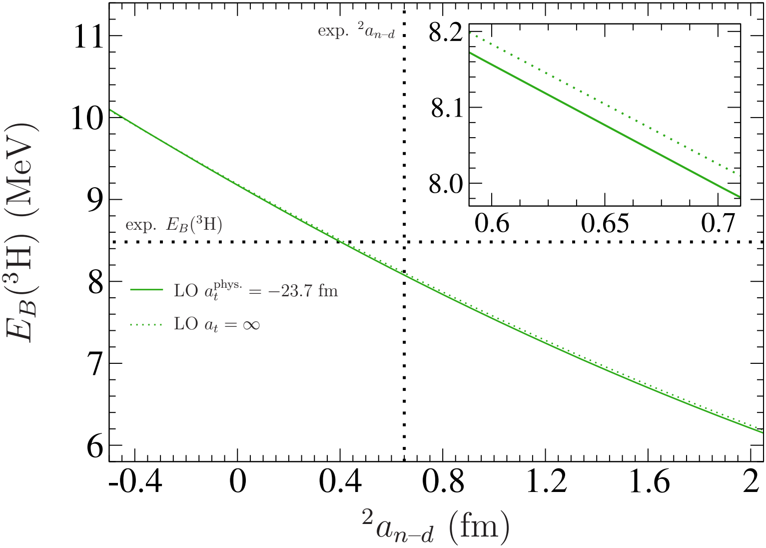
- new ${}^1S_0$ LO is isospin-symmetric and parameter-free
- **allows matching between perturbative and non-perturbative Coulomb regimes**

Phillips line



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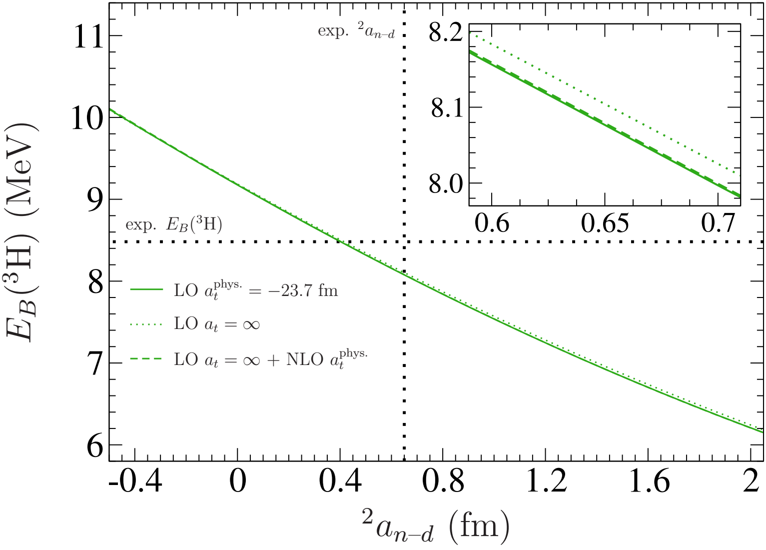

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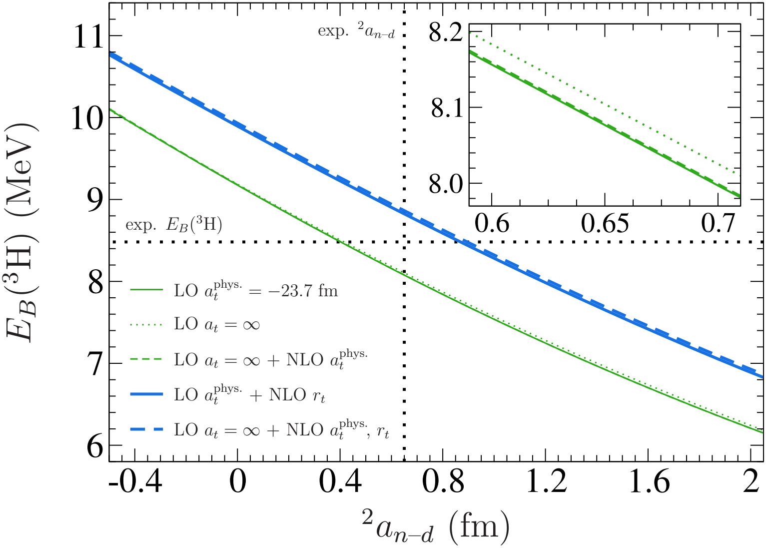
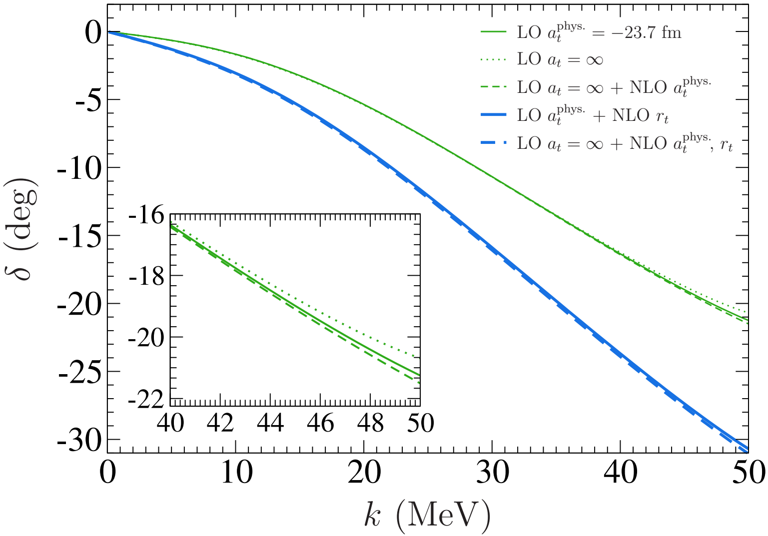


Doublet-channel phase shift



New perturbative scheme

Leading order

- standard NN spin-triplet (pionless) amplitude (parameter γ_d)
- unitary NN spin-singlet amplitude (parameter-free)
- contact three-body force (parameter $\Lambda_* \rightarrow E_B(^3\mathrm{H})$ or $^2a_{\mathrm{n-d}}$)

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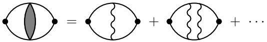
Next-to-leading order

- effective range in the NN spin-triplet channel (parameter ρ_d)
- isospin-symmetric range in the NN spin-singlet channel (parameter r_t)
- scattering-length correction to unitarity in the NN spin-singlet np and nn channels (parameter a_t),
- scattering-length correction to unitarity in the NN pp channel (parameter $a_{p\text{–}p}$)
- one-photon exchange (parameter $\alpha = 1/137$)

Two-body sector

Consider two regimes in the p–p sector:

1. $Q \sim \alpha M_N \hookrightarrow$ Coulomb non-perturbative!



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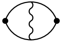

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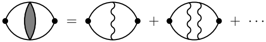

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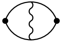

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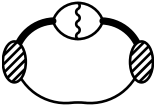

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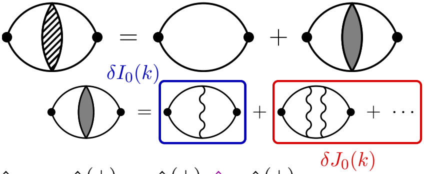

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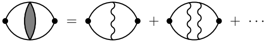

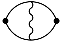

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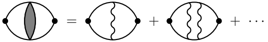

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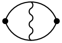

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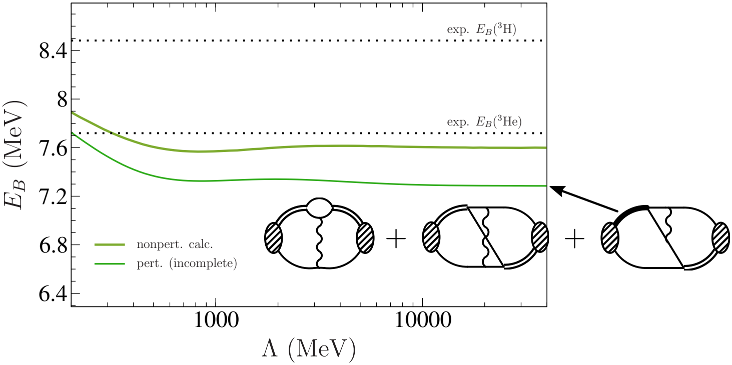
⇝ perturbative prediction for p–p phase shift:

$$k\cot\delta_{t,pp}(k) = -\frac{1}{a_{p-p}} + \alpha M_N C_\Delta + \frac{r_t}{2}k^2 + \alpha M_N \log\left(\frac{\alpha M_N}{2k}\right) + \cdots$$

Three-body sector: perturbative ^{3}He

New expansion ensures renormalized NLO result for $^3\mathrm{He}$!



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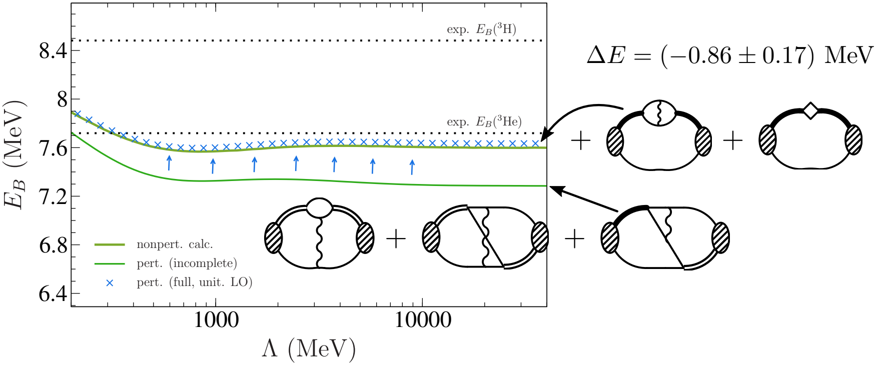

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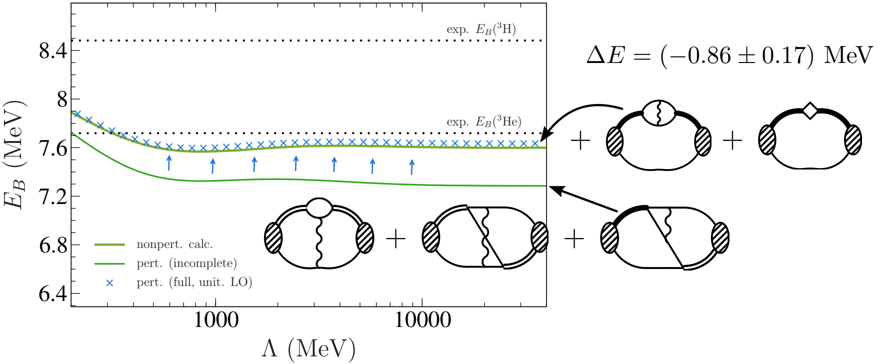

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- Coulomb indeed perturbative in $^3\mathrm{He}$ ✓
- consistent renormalization crucial to achieve this!

Divergence dissection

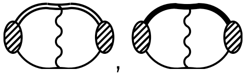
Vanasse, Egolf, Kerin, SK, Springer, PRC **89** 064003 (2014)

Look at structure of p-d three-body force:

$$H_{0,1}^{(\alpha)}(\Lambda) = h_I^{(\alpha)}(\Lambda) + h_\kappa^{(\alpha)}(\Lambda)$$

$$\begin{aligned}h_I^{(\alpha)}(\Lambda) = -\frac{3\pi(1+s_0^2)}{16}\Bigg\{&\boxed{\frac{1}{12}(r_{p\text{–}p}-r_t)\Lambda\Big[1-\cdots\Big]}\\ &+\text{various terms}\sim\log\Lambda,\text{ all}\propto(r_{p\text{–}p}-r_t)\text{ or}\propto(\gamma_{p\text{–}p}-\gamma_t)\Bigg\}/\sin^2(\cdots)\end{aligned}$$

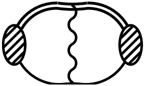
$$h_\kappa^{(\alpha)} = -\frac{\sqrt{3}\kappa\pi(1+s_0^2)}{48}\Big\{\text{various terms}\sim\log\Lambda\text{ that do not vanish in the isospin limit}+\Psi(\Lambda)\Big\}/\sin^2(\cdots)$$



- pieces associated with $r_{p\text{–}p} \neq r_t$ and $\alpha\rho_d$, αr_t
- these have been relagated to a higher order in the new scheme!
- **but an otherwise remaining log-divergence is absent!**

Lessons learned and open questions

1. **Coulomb perturbative in $^3\mathrm{He}$ ✓**
2. **consistent renormalization crucial to achieve this!**
3. **$^1\mathrm{S}_0$ NN channel can be expanded around unitarity limit**
4. **unnecessary iteration can cause spurious divergences**
5. **one-photon exchange generates logarithmic divergences:**



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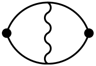

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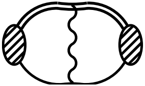

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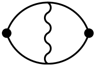

!

Questions

- reason for discrepancy with Kirscher and Gazit?
  (more generally: regulator dependence?)
- how much Coulomb has to be iterated in low-energy p-d scattering? (yet another counting above d-breakup threshold?)