Strongly interacting neutrons: from few to many

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Seattle, WA
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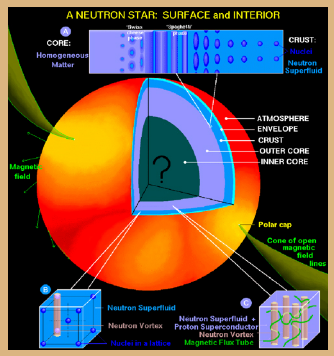
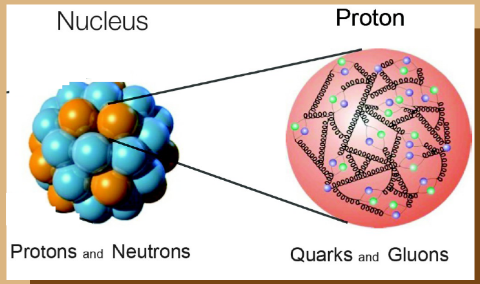
Outline

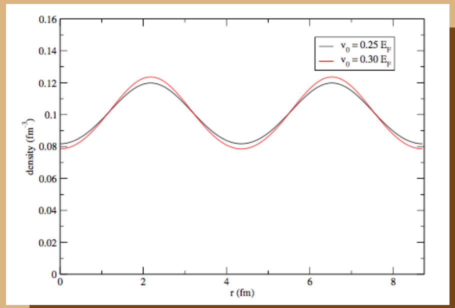


Credit: Dany Page

Motivation



Nuclear background



Recent results

Key questions

1. **What is the nature of the nuclear force that binds protons and neutrons into stable and rare isotopes?**

2. **What is the origin of simple patterns in complex nuclei?**

3. **How did visible matter come into being and how does it evolve?**

FRIB: Opening New Frontiers in Nuclear Science (2012), also LRP and NRC dec.

Quotes on degrees of freedom

“The underlying physical laws necessary for the mathematical theory of a large part of physics and the whole of chemistry are thus completely known**, and the difficulty is only that the exact application of these laws leads to equations much too complicated to be soluble.**”

– Paul Dirac

“To understand macroscopic properties of matter based on understanding these microscopic laws is just unrealistic. Even though the microscopic laws are, in a strict sense, controlling what happens at the larger scale, they are not the right way to understand that.”

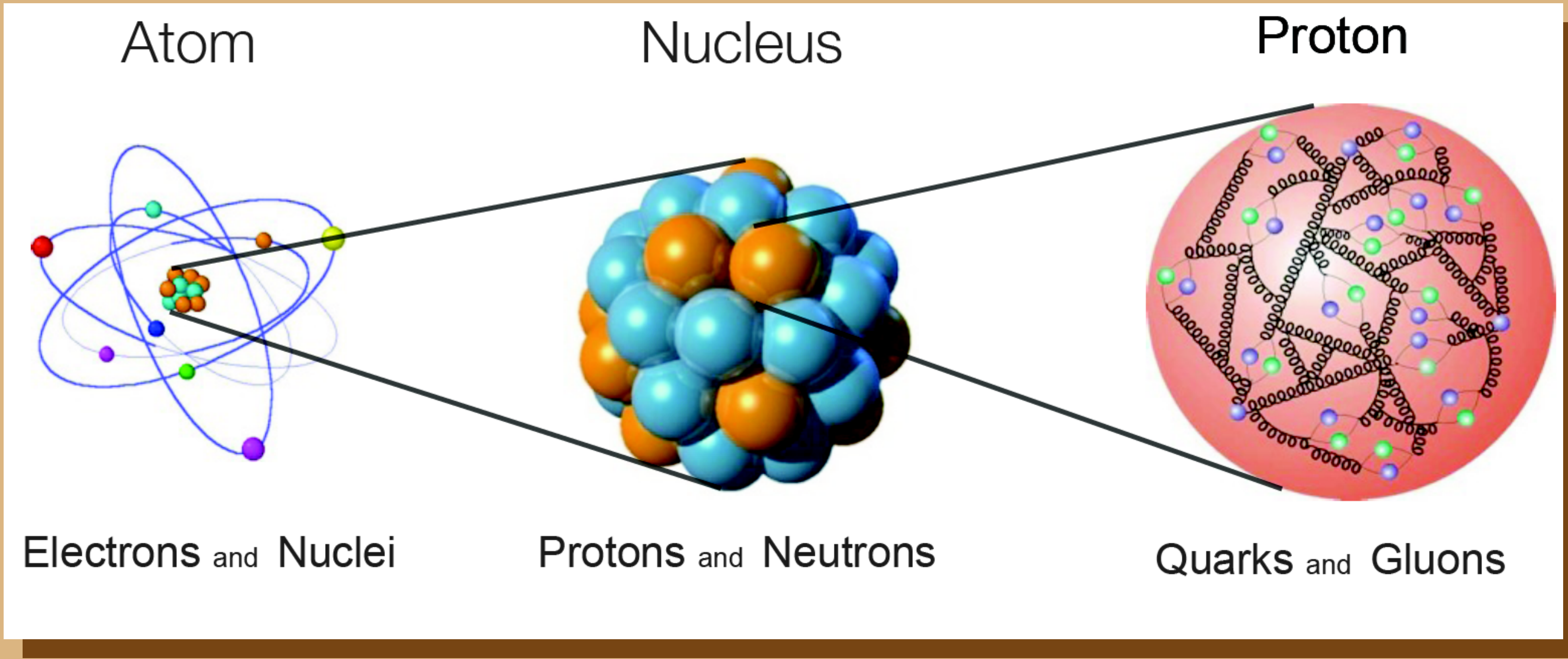
– John Schwarz

“only a fool would imagine that one should try to understand the properties of waves in the ocean in terms of Feynman-diagram calculations in the standard model, even if the latter understanding is possible 'in principle'.”

– Tom Banks

Degrees of freedom

So what does *from first principles* mean?

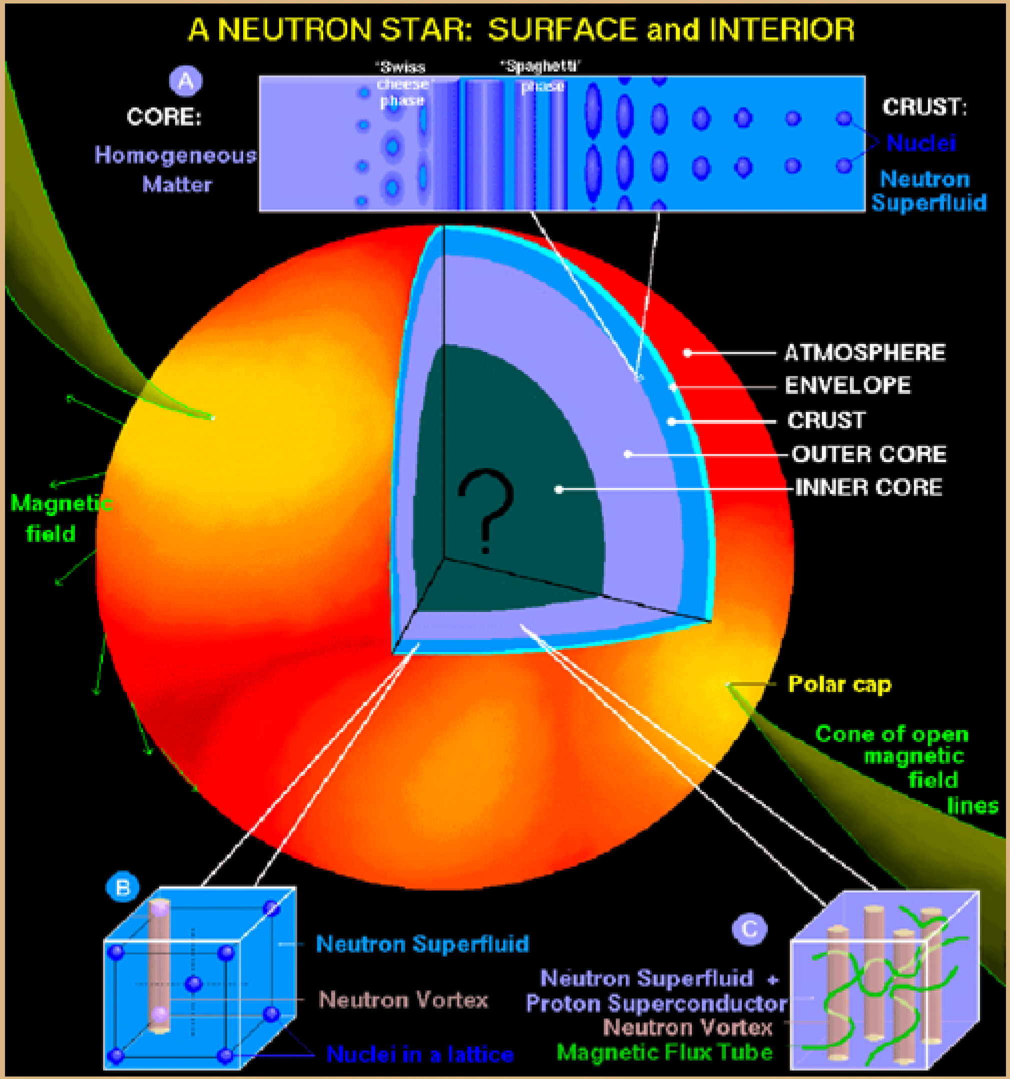


Steven Weinberg's Third Law of Progress in Theoretical Physics:

You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you'll be sorry!

Key system

Neutron stars as ultra-dense matter laboratories

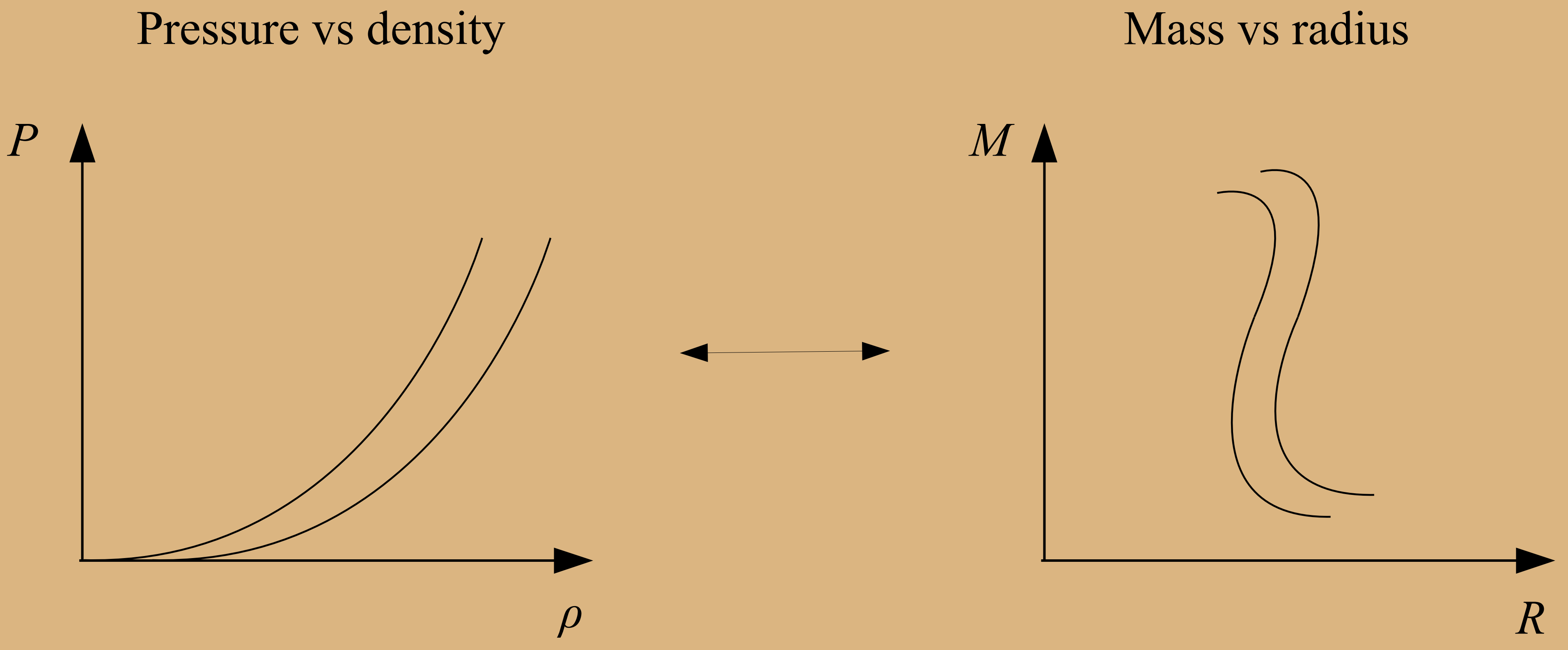


Credit: Dany Page

- Ultra-dense: 1.4 solar masses (or more) within a radius of 10 kilometres
- Terrestrial-like (outer layers) down to exotic (core) behaviour
- Observationally probed, i.e., not experimentally accessible

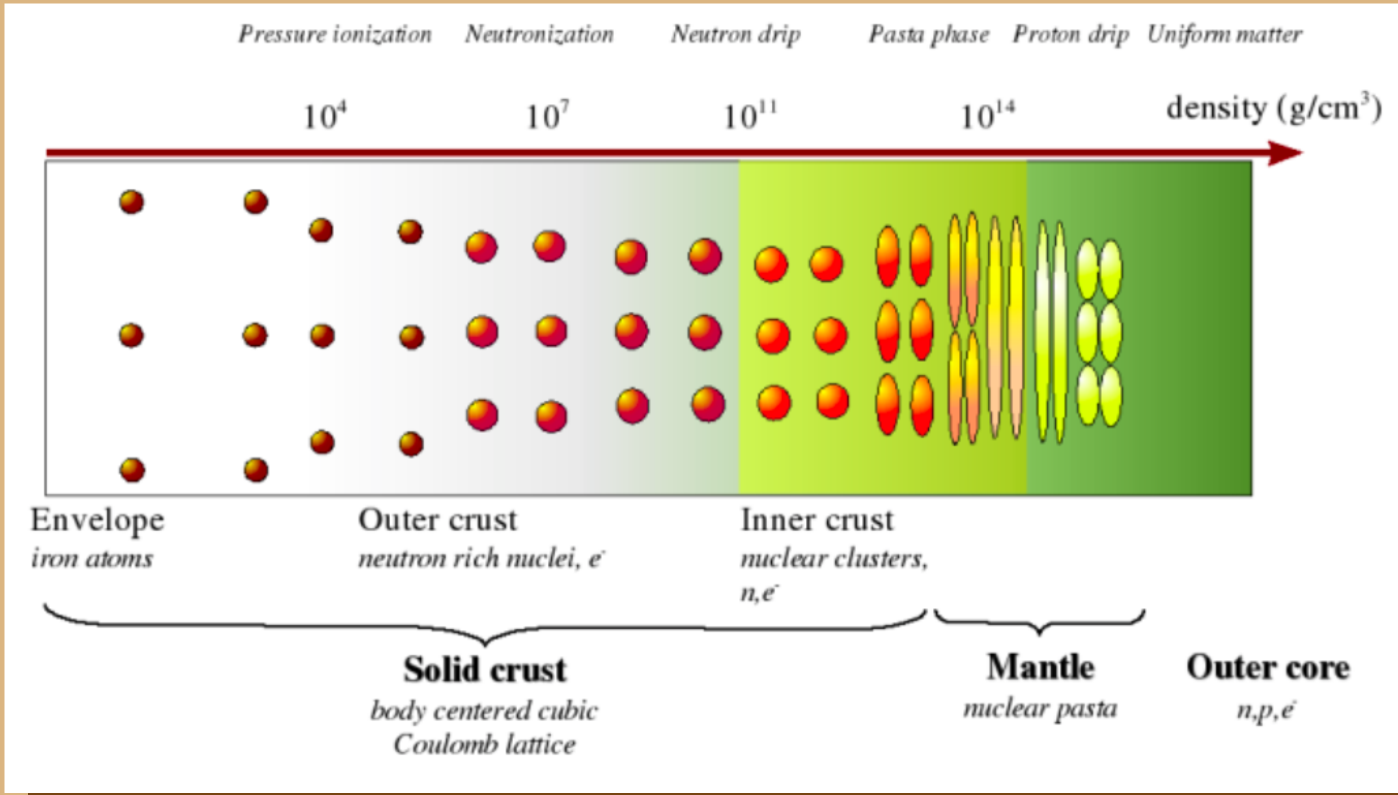
Uncertainty estimates

Tolman-Oppenheimer-Volkoff equations (or Hartle-Thorne, etc)



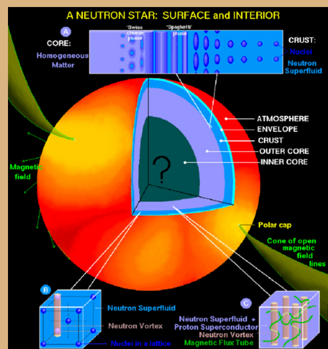
Modern goal: systematic theoretical error bars

Neutron stars: not just neutrons



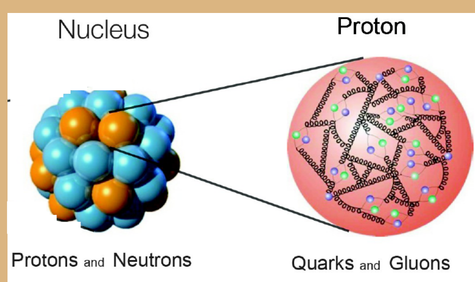
- Indeed, mostly neutrons, but not exclusively
- Non-spherical configurations: *nuclear pasta*
- Deconfined nucleon matter throughout
- Possibly even more exotic in the core

Outline

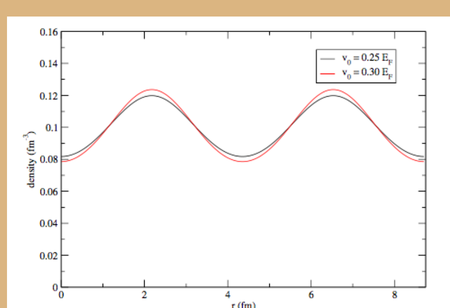


Credit: Dany Page

Motivation



Nuclear background



Recent results

Nuclear interactions 1

Historically

“Effective Interactions” were employed in the context of mean-field theory.

Phenomenological

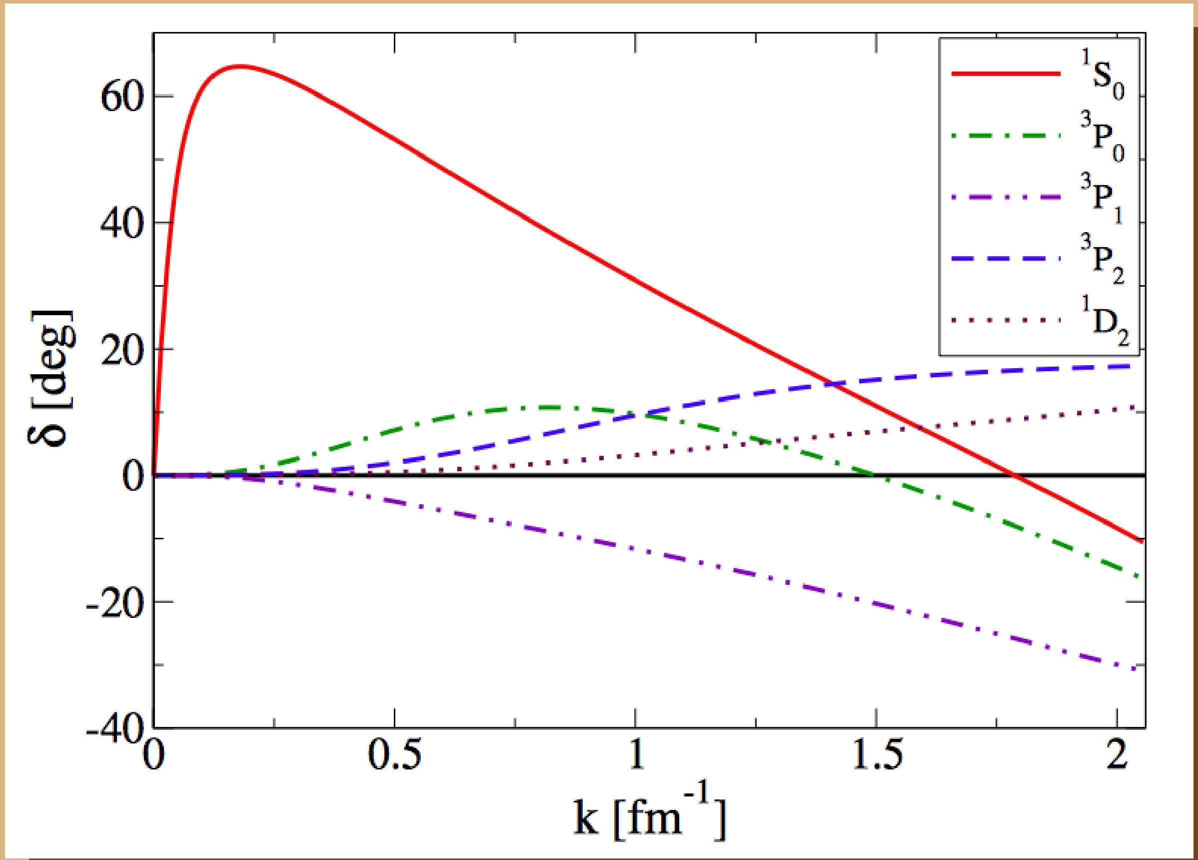
NN interaction fit to N-body experiment

Non-microscopic

NN interaction does not claim to (and will not) describe np scattering

Nuclear physics is difficult

Scattering phase shifts: different "channels" have different behavior.



Any potential that reproduces them must be spin (and isospin) dependent

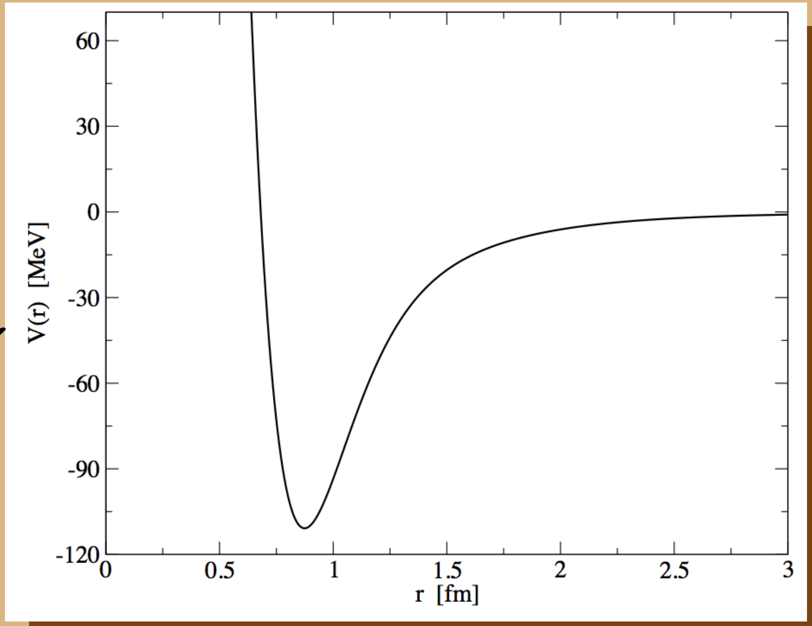
Nuclear interactions 2

Different approach: phenomenology treats NN scattering without connecting with the underlying level

$$V_2 = \sum_{j<k} v_{jk} = \sum_{j<k}\sum_{p=1}^{8} v_p(r_{jk}) O^{(p)}(j,k)$$

$$O^{p=1,8}(j,k) = (1, \sigma_j \cdot \sigma_k, S_{jk}, \mathbf{L}_{jk} \cdot \mathbf{S}_{jk}) \otimes (1, \tau_j \cdot \tau_k)$$

Such potentials are hard, making them non-perturbative at the many-body level (which is a problem for most methods on the market).



Softer, momentum-space formulations like CD-Bonn very popular

How to go beyond?

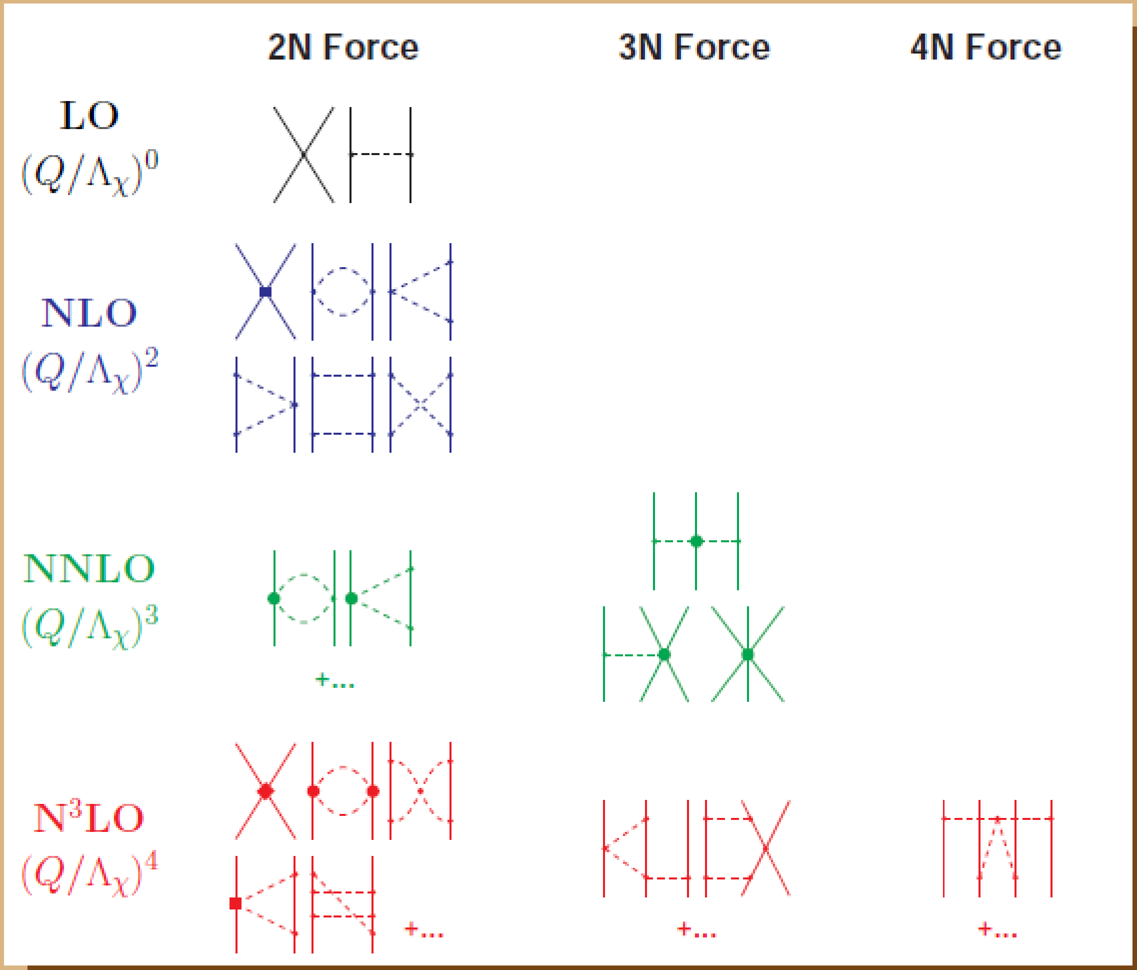
Historically, fit NN interaction to N-body experiment

Parallel approach, fit NN interaction to 2-body experiment, ignoring underlying level of quarks and gluons

Natural goal: fit NN interaction to 2-body experiment, without ignoring underlying level

Chiral effective field theory

Nuclear interactions 3



- Attempts to connect with underlying theory (QCD)
- Systematic low-momentum expansion
- Consistent many-body forces
- Low-energy constants from experiment or lattice QCD
- Until recently non-local in coordinate space, so unused in continuum QMC
- Power counting's relation to renormalization still an open question

Chiral EFT with Quantum Monte Carlo

PRL **111,** 032501 (2013) PHYSICAL REVIEW LETTERS week ending 19 JULY 2013

Quantum Monte Carlo Calculations with Chiral Effective Field Theory Interactions

A. Gezerlis,[1,2,*] I. Tews,[1,2] E. Epelbaum,[3] S. Gandolfi,[4] K. Hebeler,[5] A. Nogga,[6] and A. Schwenk[2,1]

PRL **112,** 221103 (2014) PHYSICAL REVIEW LETTERS week ending 6 JUNE 2014

Quantum Monte Carlo Calculations of Neutron Matter with Nonlocal Chiral Interactions

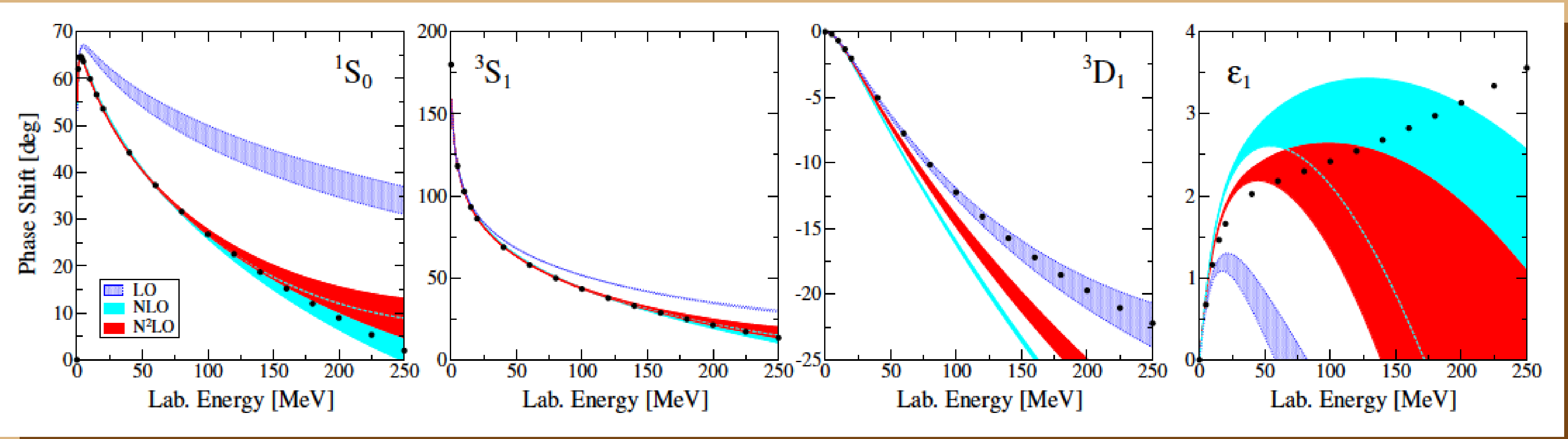
Alessandro Roggero,[1,2,*] Abhishek Mukherjee,[3,†] and Francesco Pederiva[1,2,‡]

PRL **113,** 182503 (2014) PHYSICAL REVIEW LETTERS week ending 31 OCTOBER 2014

Auxiliary-Field Quantum Monte Carlo Simulations of Neutron Matter in Chiral Effective Field Theory

G. Wlazłowski,[1,2] J. W. Holt,[2] S. Moroz,[2] A. Bulgac,[2] and K. J. Roche[2,3]

Local chiral EFT



A. Gezerlis, I. Tews, E. Epelbaum, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, Phys. Rev. Lett. **111**, 032501 (2013).

A. Gezerlis, I. Tews, E. Epelbaum, M. Freunek, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, Phys. Rev. C **90**, 054323 (2014).

J. E. Lynn, J. Carlson, E. Epelbaum, S. Gandolfi, **A. Gezerlis**, K. E. Schmidt, A. Schwenk, I. Tews, Phys. Rev. Lett. **113**, 192501 (2014)

I. Tews, S. Gandolfi, **A. Gezerlis**, A. Schwenk, Phys. Rev. C **93**, 024305 (2016)

J. E. Lynn, I. Tews, J. Carlson, S. Gandolfi, **A. Gezerlis**, K. E. Schmidt, A. Schwenk, I. Tews, Phys. Rev. Lett. **116**, 062501 (2016)

**But even with the interaction in place,
how do you solve the many-body problem?**

Nuclear many-body problem

$$H\Psi = E\Psi$$

where $$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \cdots$$

so

$$H\Psi(\mathbf{r}_1, \cdots, \mathbf{r}_A; s_1, \cdots, s_A; t_1, \cdots, t_A) = E\Psi(\mathbf{r}_1, \cdots, \mathbf{r}_A; s_1, \cdots, s_A; t_1, \cdots, t_A)$$

i.e. $2^A \binom{A}{Z}$ complex coupled second-order differential equations

Nuclear many-body methods

Phenomenological (fit to A-body experiment)

Ab initio (fit to few-body experiment)

Nuclear many-body methods

Phenomenological (fit to A-body experiment)

- **Shell model**
  mainstay of nuclear physics, still very important
- **Hartree-Fock/Hartree-Fock-Bogoliubov (HF/HFB)**
  mean-field theory, a priori inapplicable, unreasonably effective
- **Energy-density functionals (EDF)**
  like mean-field but with wider applicability

Nuclear many-body methods

Ab initio (fit to few-body experiment)

- **Quantum Monte Carlo (QMC)**
  stochastically solve the many-body problem “exactly”

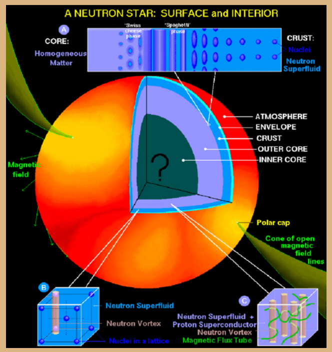
- **Perturbative Theories (PT)**
  first few orders only

- **Resummation schemes (e.g. SCGF)**
  selected class of diagrams up to infinite order

- **Coupled cluster (CC)**
  generate np-nh excitations of a reference state

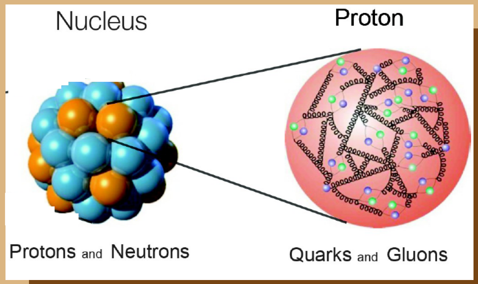
- **No-core shell model (NCSM)**
  fully ab initio, in contradistinction to traditional SM

Outline

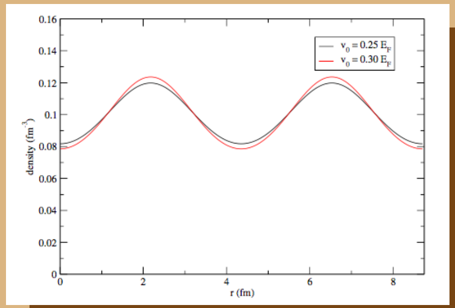


Credit: Dany Page

Motivation



Nuclear background



Recent results

Main methods employed (by me)

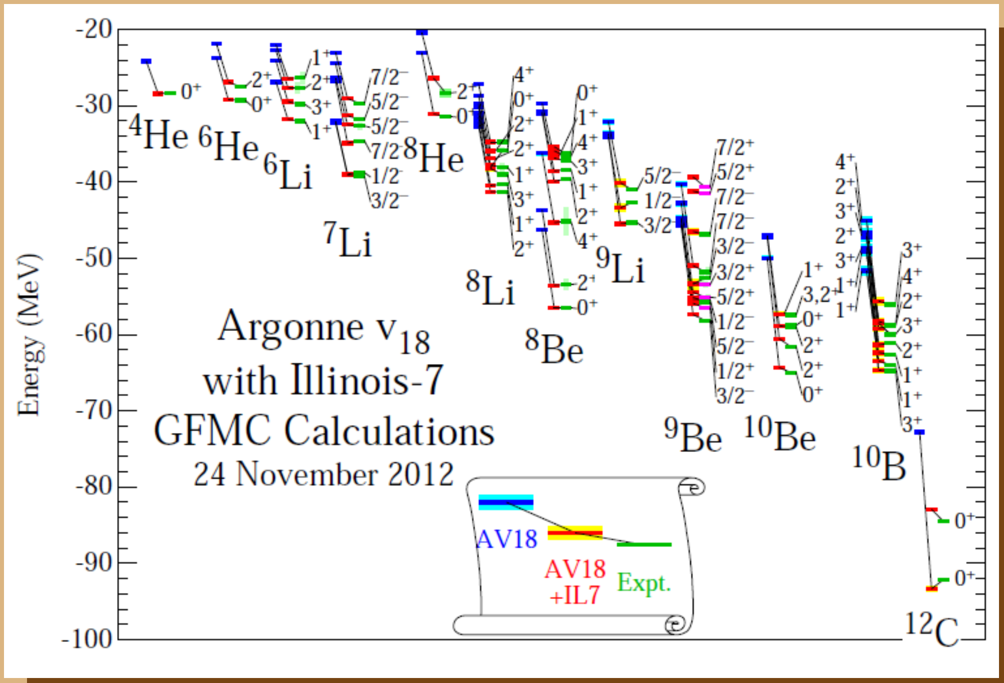
Two complementary methods

Quantum Monte Carlo

- Microscopic
- Computationally demanding (3N particle coordinates + spins)
- Limited to smallish N

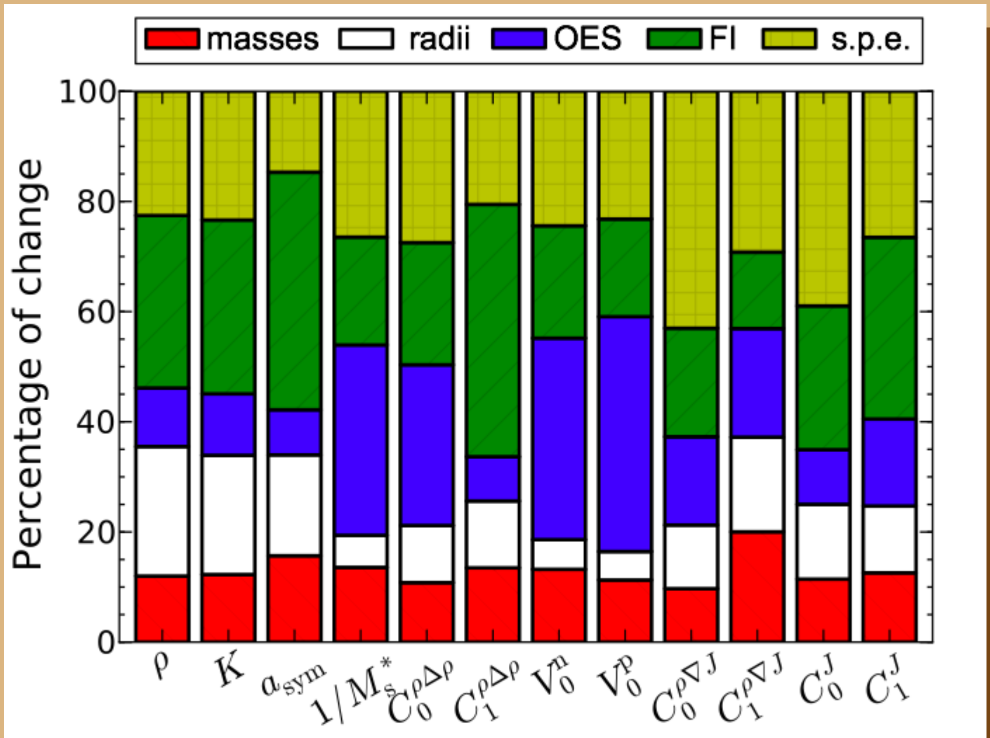
$$\Psi(\tau \to \infty) = \lim_{\tau \to \infty} e^{-(\mathcal{H} - E_T)\tau} \Psi_V$$

$$\to \alpha_0 e^{-(E_0 - E_T)\tau} \Psi_0$$



Credit: Steve Pieper

Two complementary methods



M. Kortelainen et al, Phys. Rev. C **89**, 89, 054314 (2014)

Density Functional Theory

- More phenomenological (to date, but see major developments)
- Easier in crude form (orbitals → density → energy density)
- Can do any large N

$$E = \int d^3r \, \{\mathcal{E}[\rho(\mathbf{r})] + \rho(\mathbf{r})V_{\text{ext}}(\mathbf{r})\}$$

Two complementary methods

Quantum Monte Carlo

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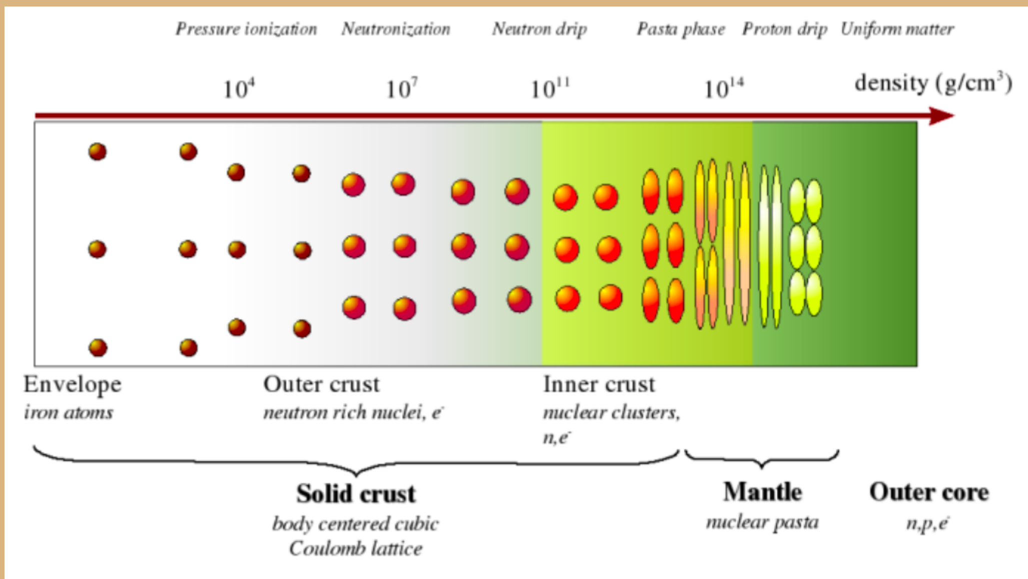
Research Strategies

i) Use QMC as a benchmark with which to compare DFT results
ii) Constrain DFT with QMC, then use DFT to make predictions

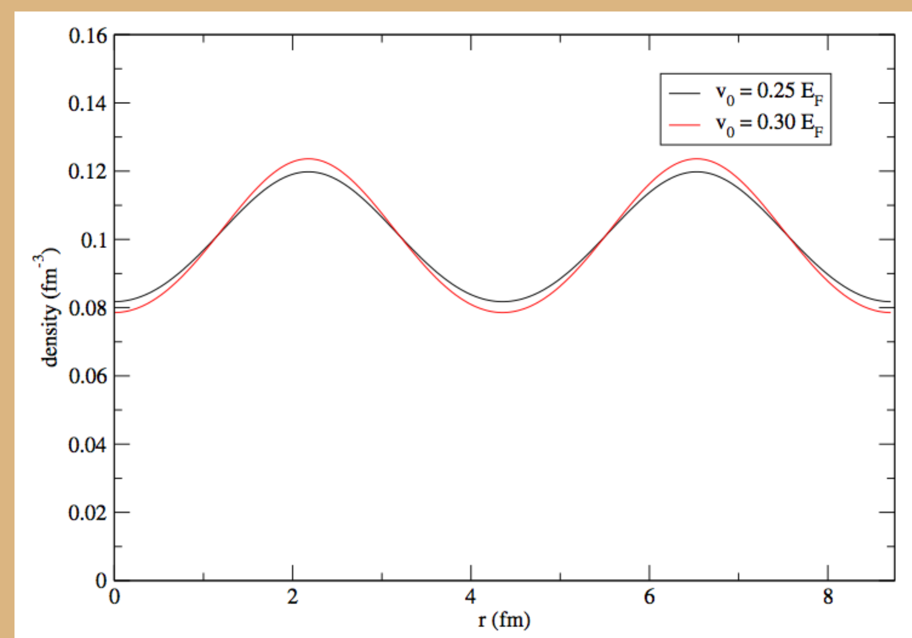
Beyond pure matter

Neutron star crusts inhomogeneous



See also: S. Moroni, D. M. Ceperley, G. Senatore, Phys. Rev. Lett. **69**, 1837 (1992); **75**, 689 (1995)

M. Buraczynski and A. Gezerlis, Phys. Rev. Lett. **116**, 152501 (2016)



Problem setup

Hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \sum_i 2v_q \cos(\mathbf{q}\cdot\mathbf{r}_i)$$

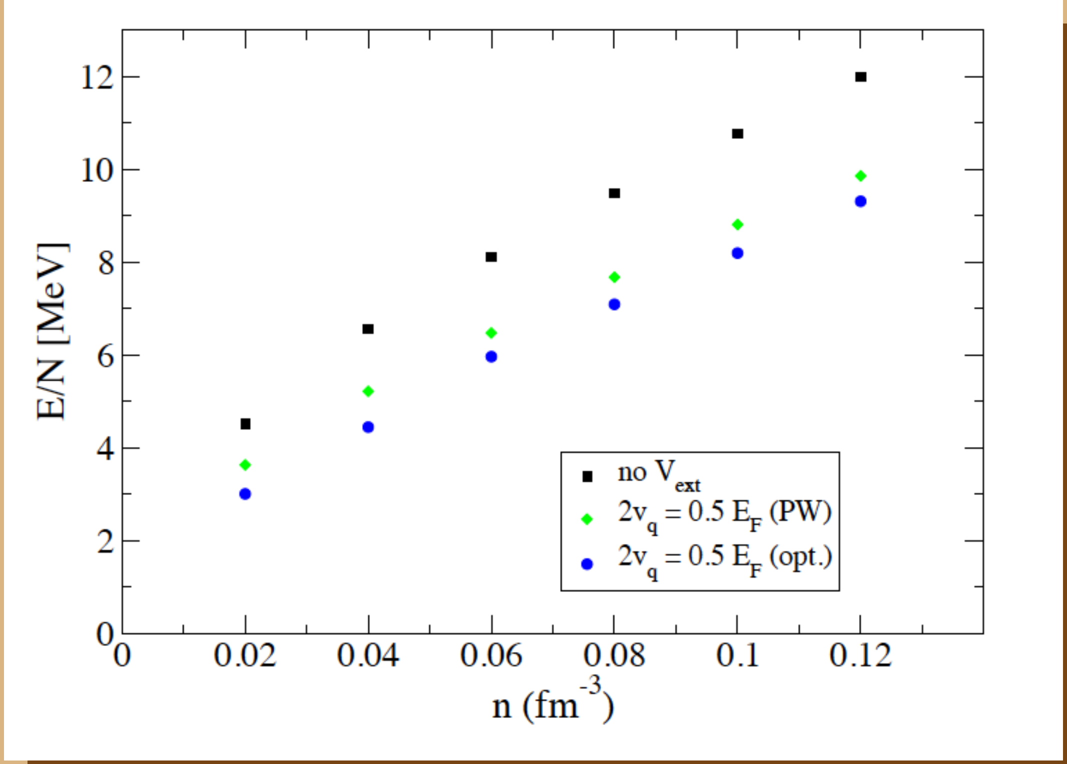
Trial wave function

$$|\Psi_T\rangle = \prod_{i<j} f(r_{ij})\ \mathcal{A}\left[\prod_i |\phi_i, s_i\rangle\right]$$

single-particle orbitals:

- plane waves
- Mathieu functions

One periodicity, one strength



- Periodic potential in addition to nuclear forces
- Energy trivially decreased
- Considerable dependence on wave function (physics input)
- Microscopic input for energy-density functionals

M. Buraczynski and A. Gezerlis,
Phys. Rev. Lett. **116**, 152501 (2016)

NEUTRONS

Background on DFT

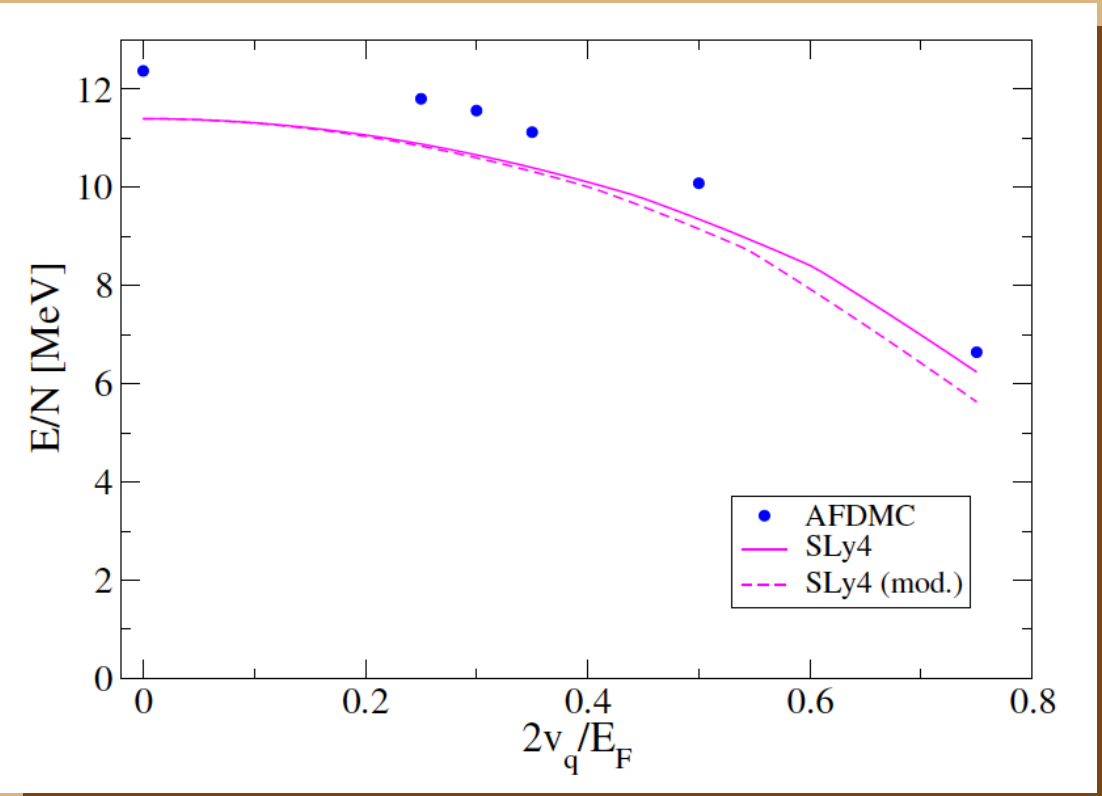
Standard functional in PNM

$$\mathcal{E} = \frac{\hbar^2}{2m}\tau + s_1 n^2 + s_2 n^{\sigma+2} + s_3 n\tau + s_4 (\nabla n)^2$$

Skyrme functional in isospin representation

$$\mathcal{E}_{\text{Skyrme}} = \sum_{T=0,1} \left[(C_T^{n,a} + C_T^{n,b} n_0^{\sigma}) n_T^2 + C_T^{\Delta n} (\nabla n_T)^2 + C_T^{\tau} n_T \tau_T \right]$$

One periodicity, many strengths



M. Buraczynski and A. Gezerlis,
Phys. Rev. Lett. **116**, 152501 (2016)

n = 0.10 fm^{-3}

- Try to disentangle bulk from isovector gradient contribution (homogeneous EOSs also differ)

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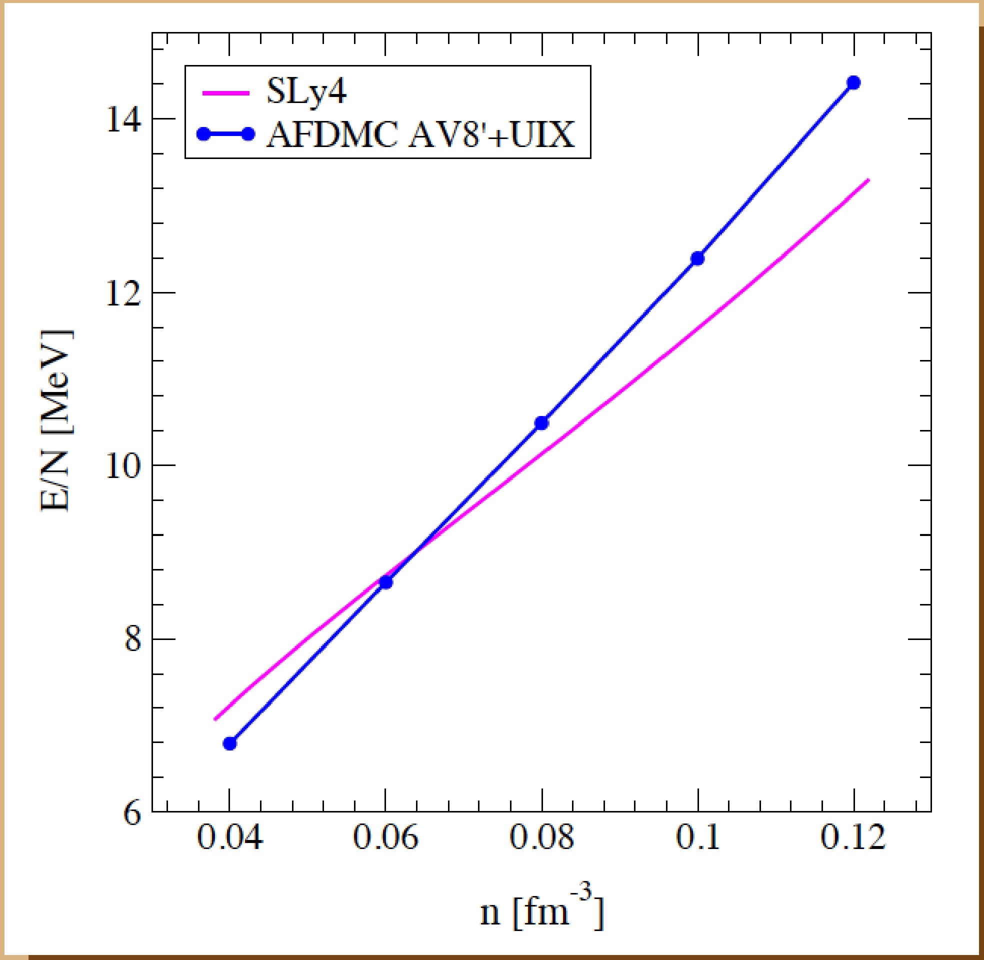
Homogeneous equation of state



(homogeneous EOSs also differ)

- What about strength-dependence at different densities?

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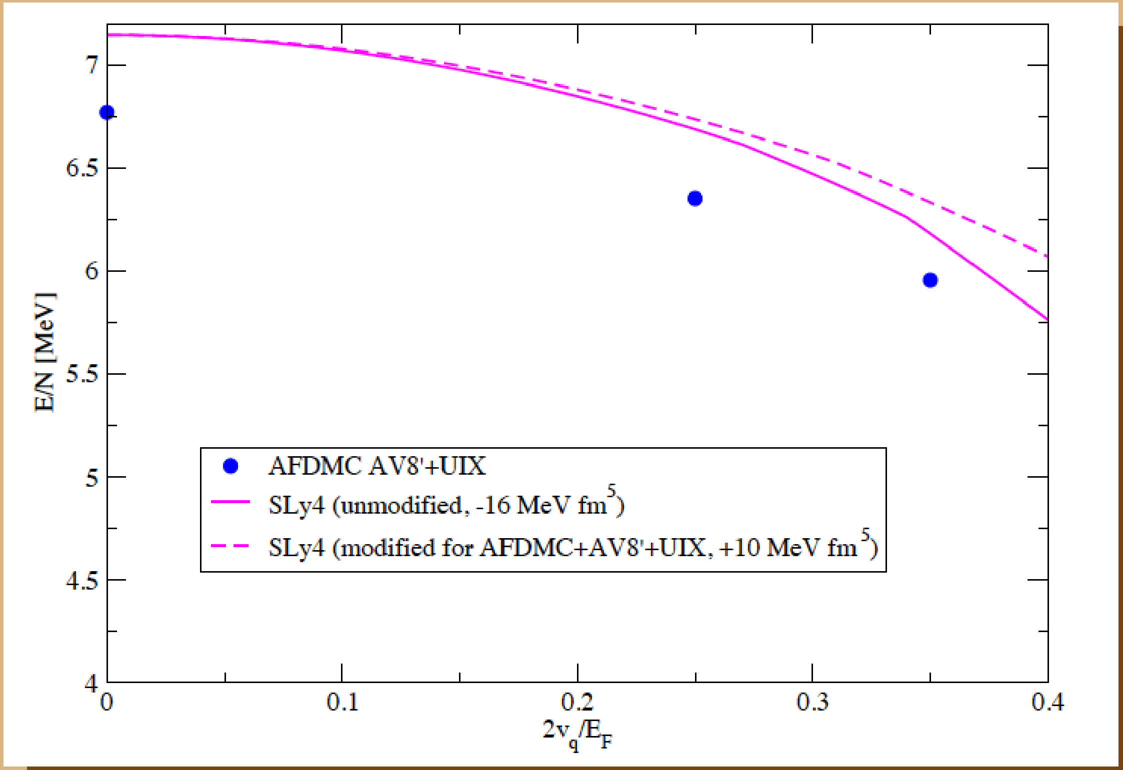
One periodicity, many strengths

n = 0.04 fm^{-3}



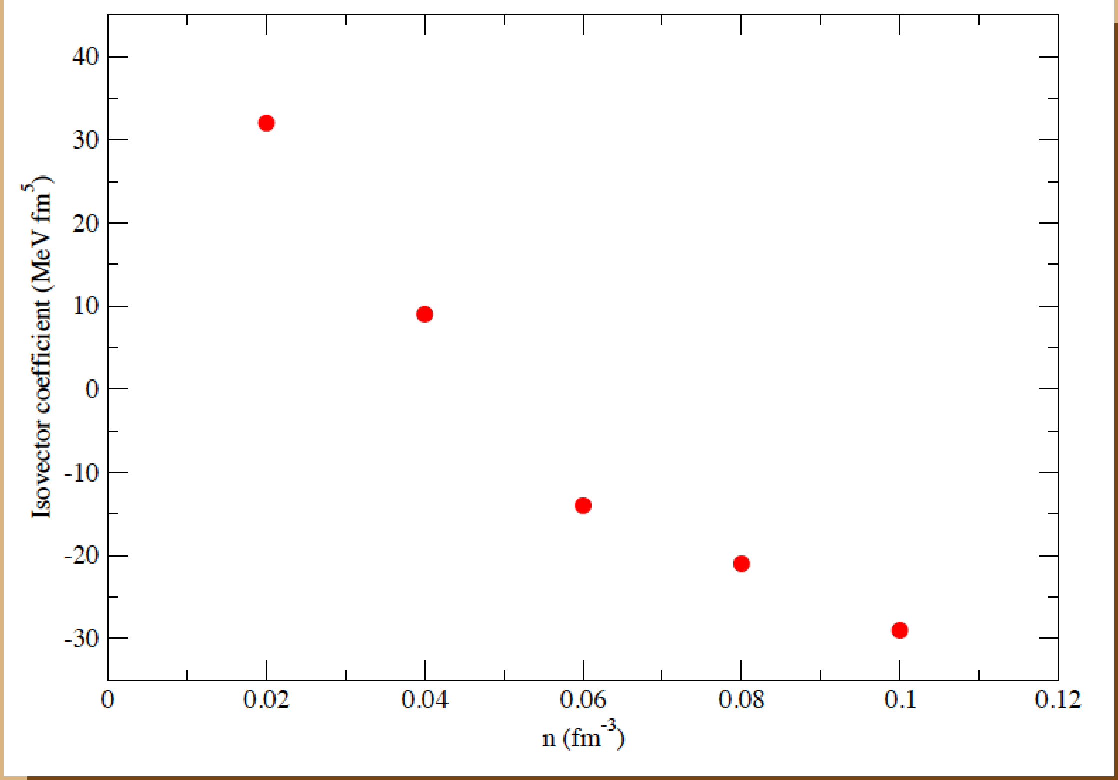
M. Buraczynski and A. Gezerlis, *in preparation*

- Repeat exercise at lower density
- Homogeneous relation is reversed
- Now with adjusted isovector gradient coefficient

NEUTRONS

One periodicity, many strengths

Many densities



M. Buraczynski and A. Gezerlis, *in preparation*

- Find density-dependent isovector coefficient, analogously to what is seen with DME (Holt, Kaiser)
- Isovector coefficient even changes sign

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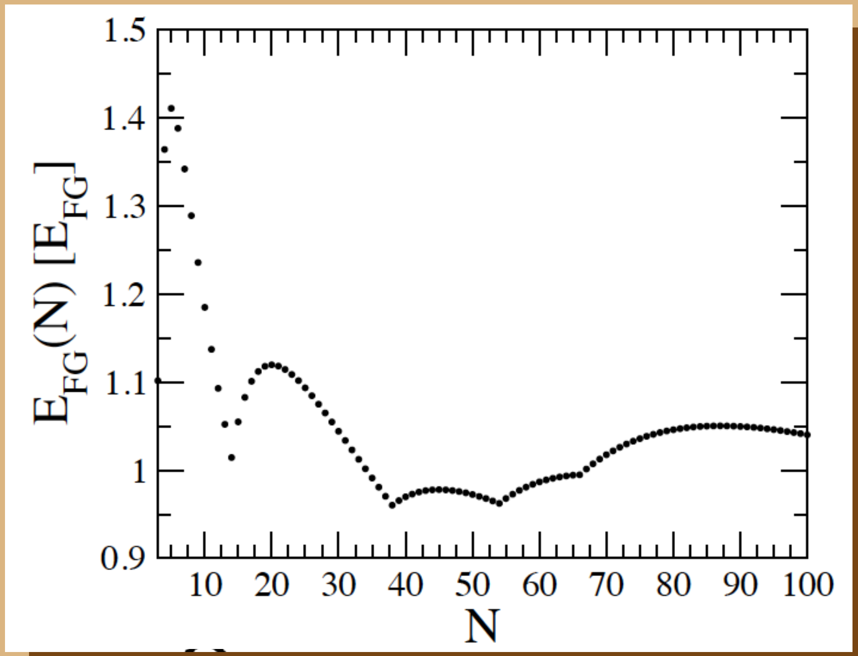
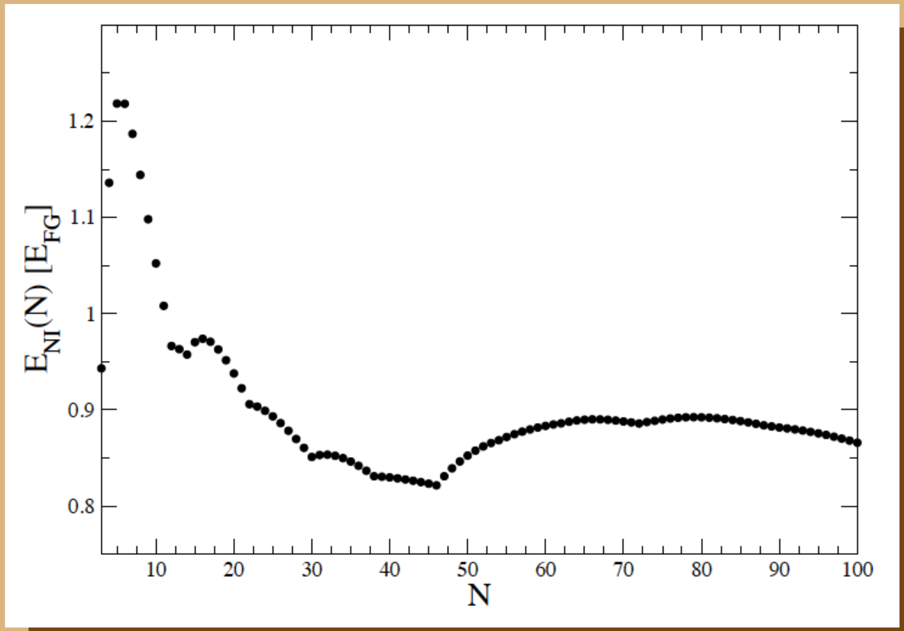
Finite-size effects

Free non-interacting gas



Modulated non-interacting gas



M. Buraczynski and A. Gezerlis, *in preparation*

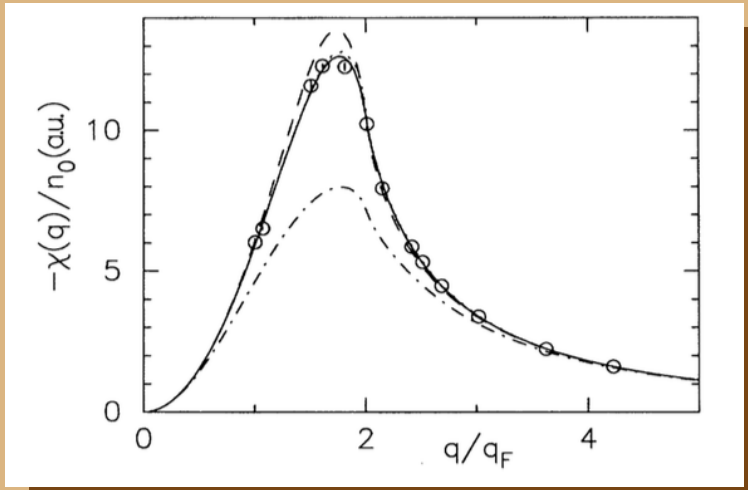
Neutron matter density response

Non-interacting gas: Lindhard function

$$\chi_L = -\frac{mq_F}{2\pi^2\hbar^2}\left[1 + \frac{q_F}{q}\left(1 - \left(\frac{q}{2q_F}\right)^2\right)\ln\left|\frac{q+2q_F}{q-2q_F}\right|\right]$$

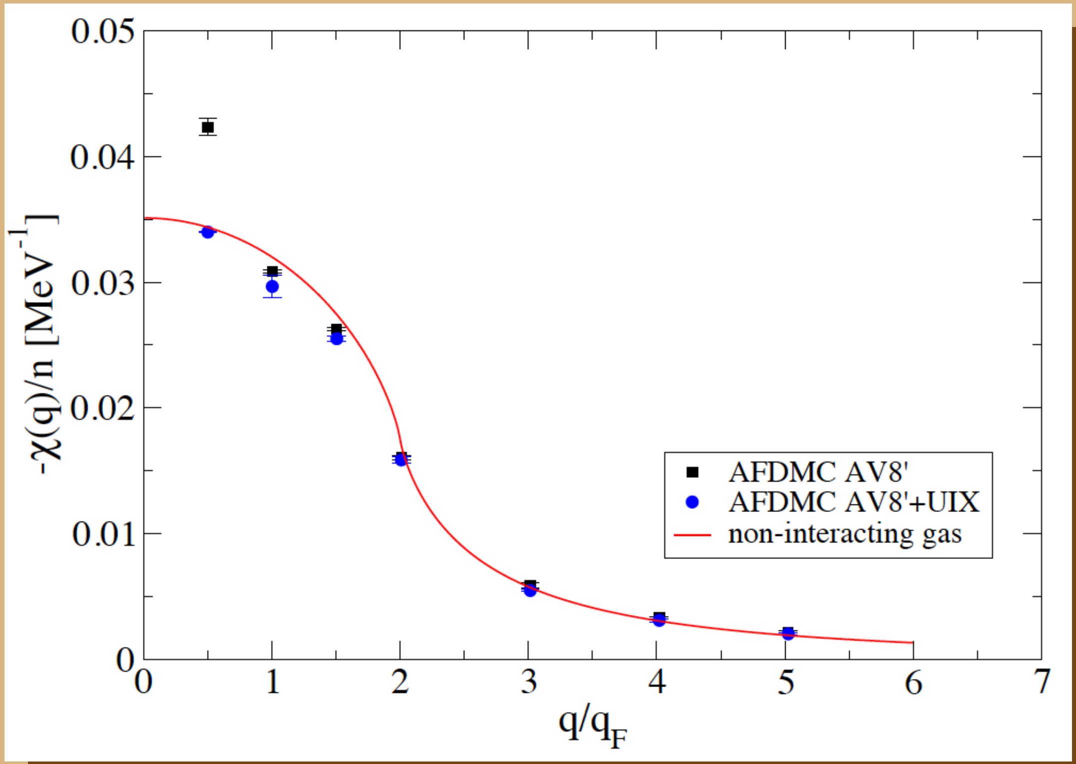
$$\frac{E_{\rm tot}}{N} = \frac{E_0}{N} + \frac{\chi(q)}{n_0}v_q^2 + C_4 v_q^4 + \cdots$$

Three-dimensional electron gas



S. Moroni, D. M. Ceperley, G. Senatore, Phys. Rev. Lett. **75**, 689 (1995)

Many periodicities, many strengths



n = 0.10 fm^{-3}

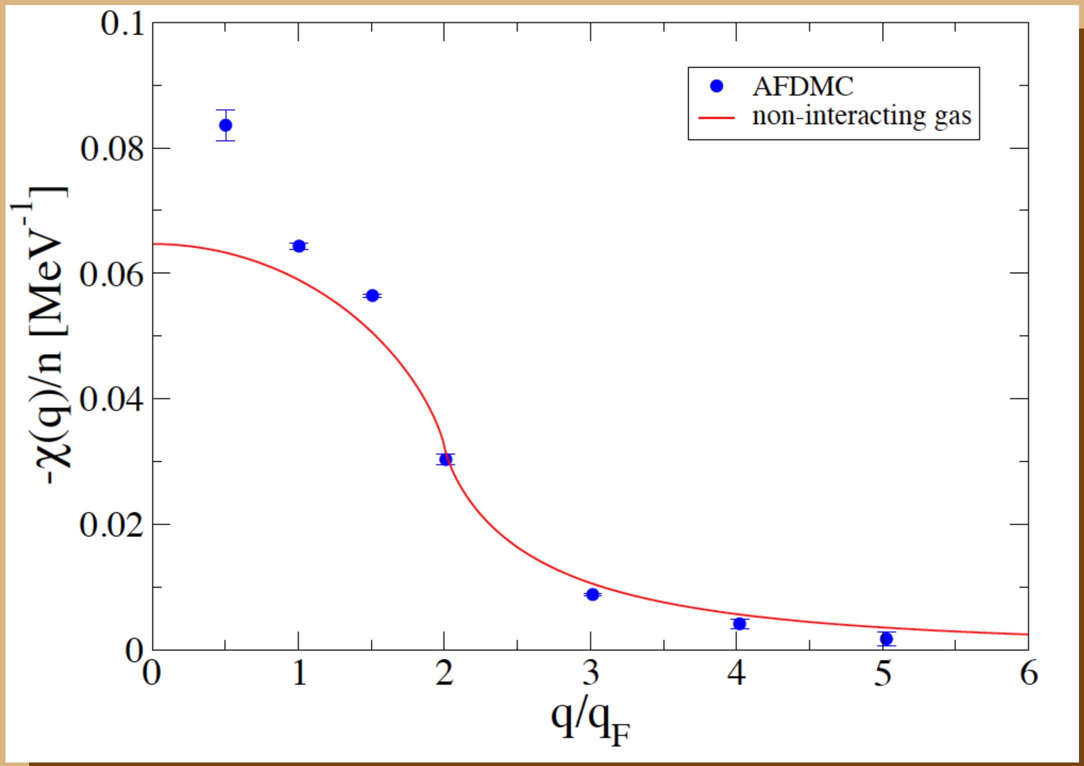
- First ever ab initio density-density response for neutron matter
- Neither Lindhard nor Coulomb
- Results on this plot derived from several strengths and periodicities

M. Buraczynski and A. Gezerlis, Phys. Rev. Lett. **116**, 152501 (2016)

M. Buraczynski and A. Gezerlis, *in preparation*

NEUTRONS

Many periodicities, many strengths



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M. Buraczynski and A. Gezerlis, Phys. Rev. Lett. **116**, 152501 (2016)

M. Buraczynski and A. Gezerlis, *in preparation*

Two neutrons in a finite volume

P. Klos, J. E. Lynn, I. Tews, S. Gandolfi, **A. Gezerlis**, H.-W. Hammer, M. Hoferichter, and A. Schwenk, submitted to Phys. Rev. C, arXiv:1604.01387

Lüscher's formula

$$p \cot \delta_0(p) = \frac{1}{\pi L} S\left(\left(\frac{Lp}{2\pi}\right)^2\right)$$

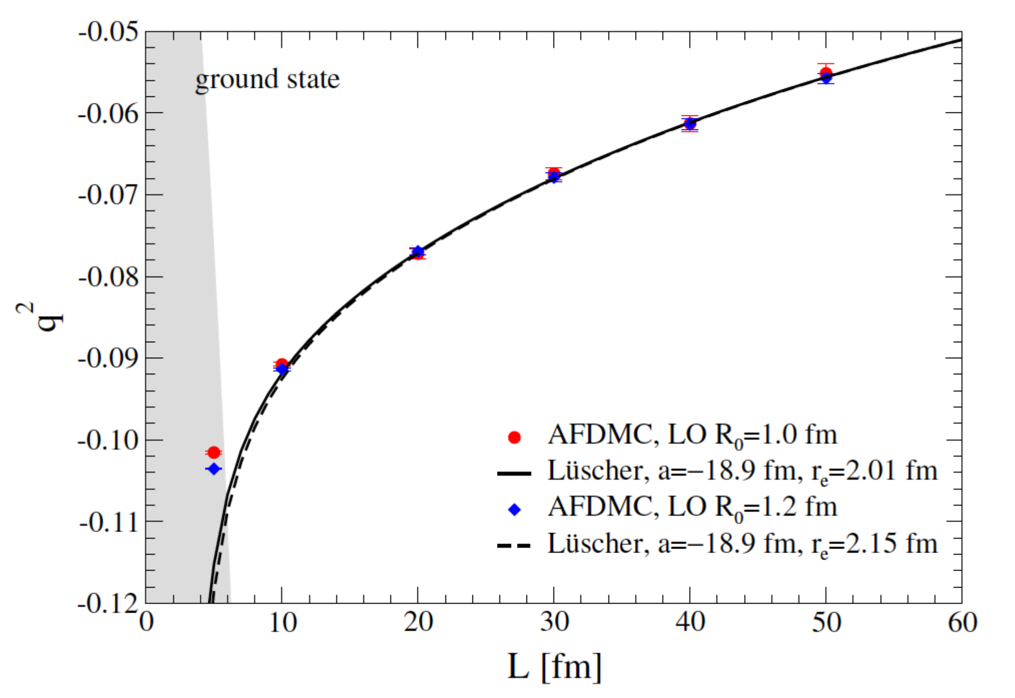
where

$$S(\eta) = \lim_{\Lambda \to \infty} \left(\sum_{|\mathbf{j}|<\Lambda} \frac{1}{\mathbf{j}^2 - \eta} - 4\pi\Lambda \right)$$

Combine with shape-independent approximation

$$p \cot \delta_0(p) = -\frac{1}{a} + \frac{1}{2} r_e p^2$$

Two neutrons in a box: LO



- $q^2 = \dfrac{p^2 L^2}{4\pi^2}$
- Quantum Monte Carlo naturally suited to calculations with periodic boundary conditions
- AFDMC matches Luescher formula when pions unimportant

NEUTRONS

P. Klos, J. E. Lynn, I. Tews, S. Gandolfi, A. Gezerlis, H.-W. Hammer, M. Hoferichter, and A. Schwenk, submitted to Phys. Rev. C, arXiv:1604.01387

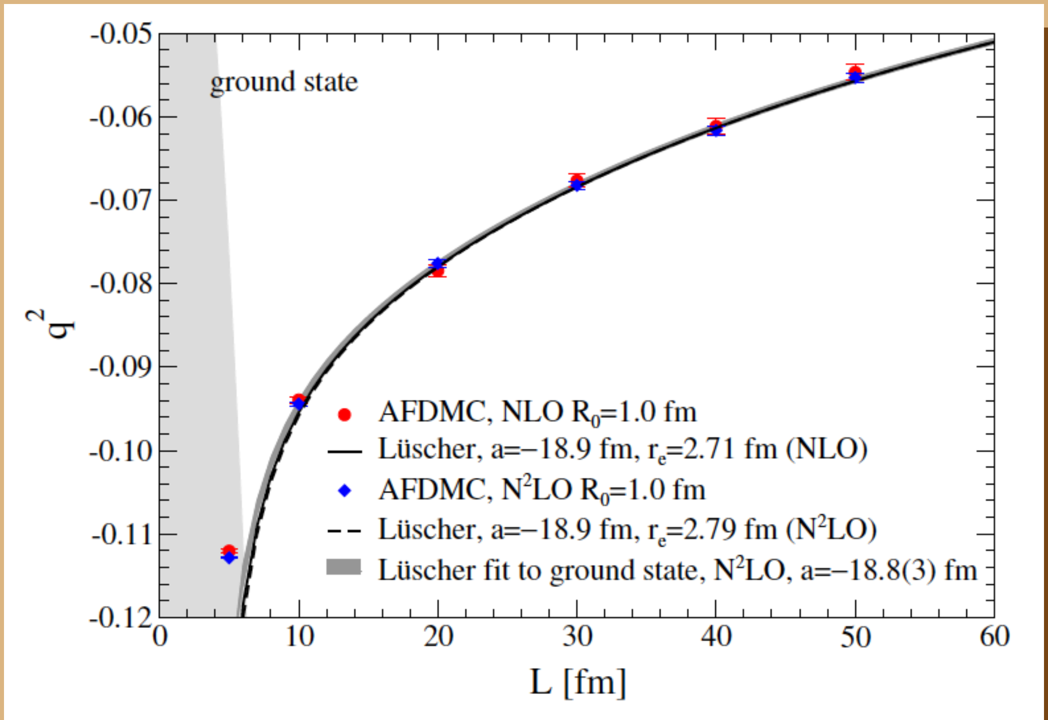
Lüscher's formula

Also possible to invert the process

$$-\frac{1}{a}+\frac{1}{2}r_e p^2=\frac{1}{\pi L}S\left(\left(\frac{Lp}{2\pi}\right)^2\right)$$

Take results for E_i (or p_i^2) at each L_i and extract a and r_e

Two neutrons in a box: NLO & N²LO



- $$q^2 = \frac{p^2 L^2}{4\pi^2}$$
- Quantum Monte Carlo naturally suited to calculations with periodic boundary conditions
- Process can be inverted, to extract scattering parameters from QMC energies

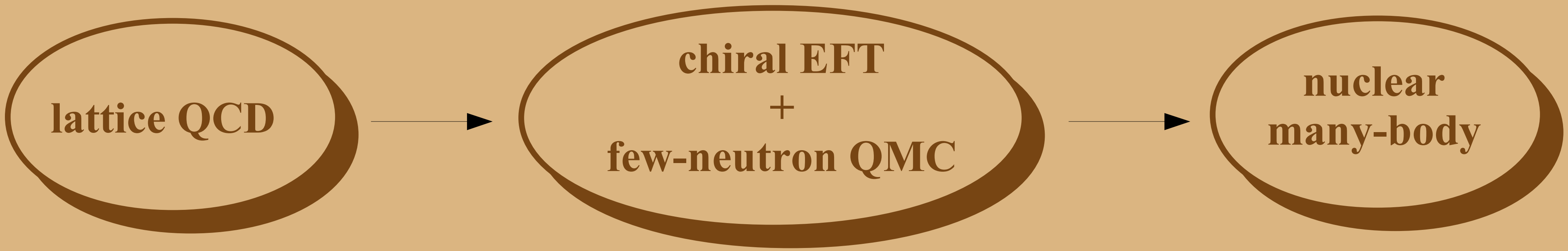
NEUTRONS

P. Klos, J. E. Lynn, I. Tews, S. Gandolfi, A. Gezerlis, H.-W. Hammer, M. Hoferichter, and A. Schwenk, submitted to Phys. Rev. C, arXiv:1604.01387

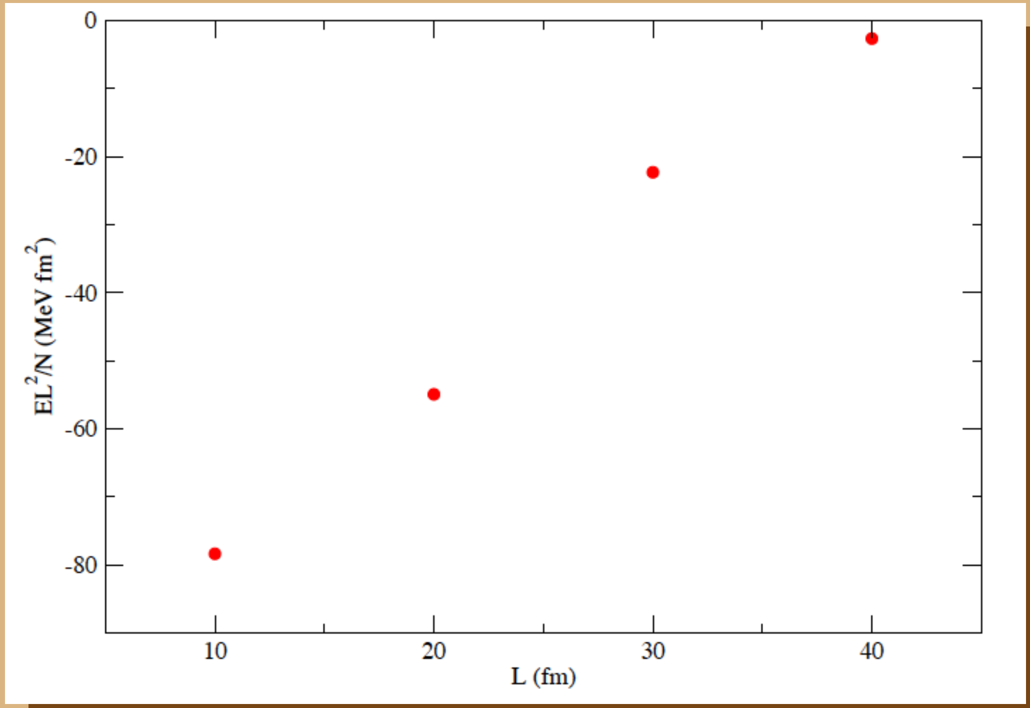
General strategy

Address few-neutron systems:

- Experimental input scarce/nonexistent
- Lüscher's formula and/or handling of pions difficult



Two neutrons in a box: with cosine



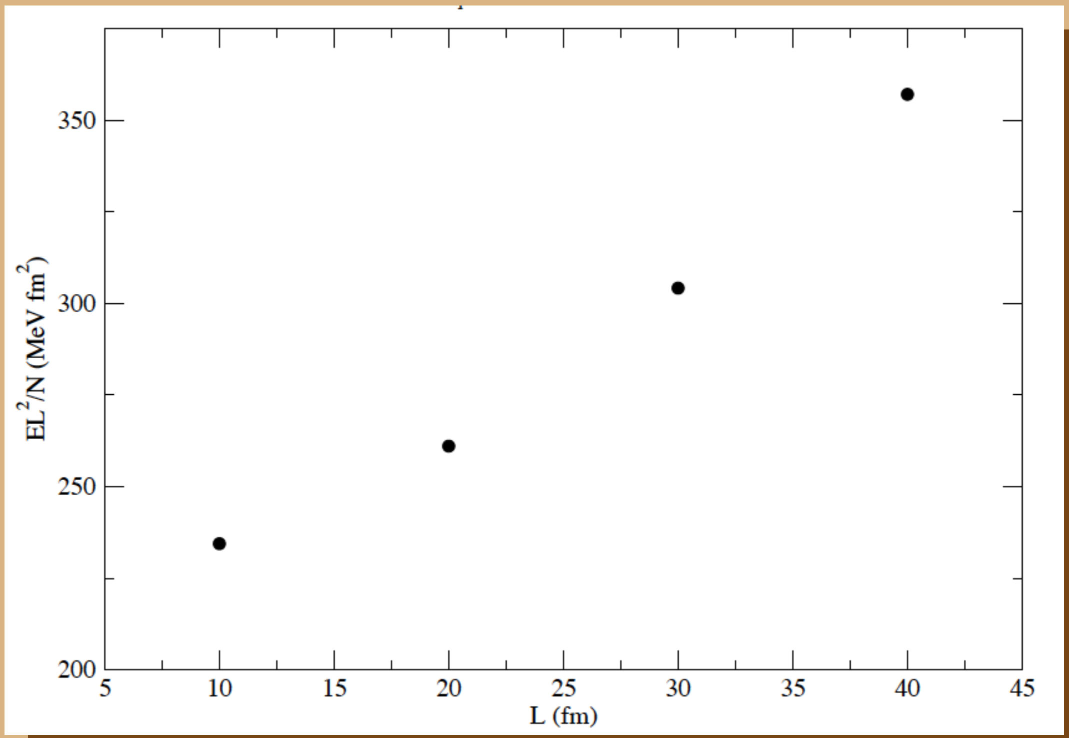
- Quantum Monte Carlo naturally suited to calculations with periodic boundary conditions

- Background field easy to implement (?)

M. Buraczynski and A. Gezerlis, *in preparation*

NEUTRONS

Four neutrons in a box: with cosine



- Quantum Monte Carlo naturally suited to calculations with periodic boundary conditions
- Background field easy to implement (?)

M. Buraczynski and A. Gezerlis, *in preparation*

NEUTRONS

Conclusions

- Rich connections between response and physics of neutron-rich nuclei, cold atoms, solid state
- Exciting time in terms of interplay between nuclear interactions, QCD, and many-body approaches
- Ab initio and phenomenology are mutually beneficial

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