NN interaction in chiral EFT - general considerations

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Outline

- Motivation for the talk;
- Renormalization
- Low energy chiral EFT;
- Lorentz invariance and low energy EFT;
- Renormalizable version of Weinberg's approach;
- Summary;

EFT approach to few-nucleon systems was suggested in

S. Weinberg, Phys. Lett. B **251**, 288 (1990).

Implementation started with
C. Ordonez, L. Ray and U. van Kolck, Phys. Rev. Lett. **72**, 1982 (1994)
long ago ... in last millennium ...

...

"Since then there has been a large amount of work on chiral effective nuclear forces, but I am not satisfied with the theoretical foundations under this work, and believe that new ideas are needed."
David B. Kaplan: INT, Research page

My talk is a kind of introduction to Evgeny Epelbaum's talk on Thurthday ...

It is NOT my aim to convince anybody in anything! ...

I just express my point of view ...

Instead of stating the problem and suggesting a possible solution I will try to build up my considerations ...

Renormalization

Something very simple and very familiar ...

Consider a theory
$Theory(g_i)$
depending on parameters g_i, with $i = 1, ..., N$.

We fix these parameters from N experiments and try to predict the results of other experiments.

In the language of theory: calculate "reference" physical quantities

$$\begin{aligned}
\sigma_1(\mu_1) &= f_1(g_i, \mu_1)\,, \\
\sigma_2(\mu_2) &= f_2(g_i, \mu_2)\,, \\
&\dots \\
\sigma_N(\mu_N) &= f_N(g_i, \mu_N)\,,
\end{aligned}$$

Express g_i as functions of $\sigma_i(\mu_i)$

$$g_i = \phi_i(\sigma_j, \mu_k)$$

and substitute in all other quantities

$$\begin{aligned}
\sigma_{N+1}(E) &= F_{N+1}(\sigma_i, \mu_j, E)\,, \\
\sigma_{N+2}(E) &= F_{N+2}(\sigma_i, \mu_j, E)\,, \\
&\dots
\end{aligned}$$

Renormalization is expressing all physical quantities in terms of other physical quantities (instead of the original "bare" parameters).

Renormalization itself is not directly related to divergences!

In QFT coefficients of expansions in g_i are divergent.
In some theories divergences disappear after renormalization.

Such theories are called renormalizable!

A self-consistent EFT has to be renormalizable in terms of an infinite number of parameters, perturbatively, as well as non-perturbatively!

We are NOT dealing with perturbatively un-renormalizable, however non-perturbatively finite theories! — We do not seem to have them!

Renormalization is perturbative if we are using perturbation theory and non-perturbative if solving theory non-perturbatively!

In EFT perturbative expansions of renormalized non-perturbative expressions reproduce renormalized perturbative series, wherever this expansion exists!

Renormalization versus "peratization" - toy model

E. Epelbaum and J. Gegelia, “Regularization, renormalization and ’peratization’ in effective field theory for two nucleons,” Eur. Phys. J. A **41**, 341 (2009).

A toy model below demonstrates problems which may appear when taking the cutoff very large without subtracting all divergences - getting rid off them by using renormalization .

Consider the following equation in five space-time dimensions

$$T\left(\vec{p},\vec{q}\right) = V\left(\vec{p},\vec{q}\right) + m\int \frac{d^4\vec{k}}{\left(2\,\pi\right)^4}\, V(\vec{p},\vec{k})\, \frac{1}{mE - k^2 + i\,0^+}\, T(\vec{k},\vec{q})\,,$$

with

$$V(\vec{p},\vec{q}) = V_C + \frac{\alpha}{\left[\left(\vec{p}-\vec{q}\right)^2 + M^2\right]^2} = C + V_\pi(\vec{p},\vec{q}) + V_{NLO} + \cdots\,,$$

where V_C is contact interaction with LO term - C; m, α and M are constant parameters and E is the energy.

The solution for LO equation can be written in a closed form

$$T(\vec{p},\vec{q}) = T_\pi(\vec{p},\vec{q}) - \frac{\Psi_\pi(\vec{p})\Psi_\pi(\vec{q})}{1/C - G_p}\,,$$

where finite quantities T_π, $\Psi_\pi(\vec{p})$ and $\Psi_\pi(\vec{q})$ are given by

$$\begin{aligned} T_\pi\left(\vec{p},\vec{q}\right) &= V_\pi\left(\vec{p},\vec{q}\right) + m\int \frac{d^4\vec{k}}{(2\,\pi)^4}\,\frac{V_\pi(\vec{p},\vec{k})\,T_\pi(\vec{k},\vec{q})}{mE - k^2 + i\,0^+}\,, \\ \Psi_\pi(\vec{q}) &= 1 + m\int \frac{d^4\vec{k}}{(2\,\pi)^4}\,\frac{1}{mE - k^2 + i\,0^+}\,T_\pi(\vec{k},\vec{q})\,, \end{aligned}$$

On the other hand, G_p contains divergences and is given by

$$\begin{aligned} G_p &= m \int \frac{d^4\vec{k}}{(2\pi)^4} \frac{1}{p^2 - k^2 + i\,0^+} \\ &\quad + m^2 \int \frac{d^4\vec{k}_1}{(2\pi)^4} \frac{d^4\vec{k}_2}{(2\pi)^4} \frac{T_\pi(\vec{k}_1, \vec{k}_2)}{\left(mE - k_1^2 + i\,0^+\right)\left(mE - k_2^2 + i\,0^+\right)} \,. \end{aligned}$$

Using cutoff regularization G_p can be written as

$$G_p = a\,\Lambda^2 + (b\,M^2 + c\,mE)\ln\frac{\Lambda}{\mu} + G_p^f(\mu) + \mathcal{O}\left(\frac{mE}{\Lambda^2}\right),$$

where a, b, c are some constants factors and G_p^f is finite, Λ-independent part.

We can absorb $a\Lambda^2 + b\,M^2 \ln(\Lambda/\mu)$ in the renormalization of $1/C$.
However the divergent part $c\,mE \ln(\Lambda/\mu)$ remains.
We still can take the $\Lambda \to \infty$ limit and get a finite result

$$T(\vec{p},\vec{q}) = T_\pi(\vec{p},\vec{q})\,.$$

On the other hand, using the BPHZ renormalization, we subtract *all* divergences and then take $\Lambda \to \infty$ and obtain

$$T(\vec{p},\vec{q}) = T_\pi(\vec{p},\vec{q}) - \frac{\Psi_\pi(\vec{p})\Psi_\pi(\vec{q})}{1/C_R(\mu) - G_p^f(\mu)}\,,$$

where C_R is the renormalized coupling.

Mismatch between two results is apparent.

This kind of "*non-perturbative renormalization*" is more in the spirit of "peratization", see, e.g.,

W. Güttinger, R. Penzl, E. Pfaffelhuber, Peratization of Unrenormalizable Field Theories, Annals of physics: 33, 246-271 (1965)

and references therein.

Renormalization group and NN scattering

The NN scattering amplitude is obtained by solving the equation

$$T = V + \int VGT.$$

We apply cutoff regularization and assume cutoff dependence of the potential such, that T is cutoff independent

$$T = V^{\Lambda} + \int^{\Lambda} V^{\Lambda} GT.$$

V^{Λ} depends on N parameters c_i which we fix from N "reference" physical quantities – $c_i^R(\mu)$, where $\mu = \{\mu_1, \mu_2, \ldots, \mu_N\}$.

The solution to the integral equation depends on Λ explicitly due to the cutoff of the integral and implicitly through c_i:

$$T(E) = F(\Lambda, c_i, E). \tag{1}$$

However when we substitute

$$c_i = f_i(\Lambda, c_j^R(\mu), \mu) \tag{2}$$

in Eq. (1), we obtain Λ-independent result

$$T(E) = \tilde{F}(c_i^R(\mu), E, \mu), \tag{3}$$

which does not depend on μ due to the cancelation of the explicit and implicit dependence.
By substituting Eq. (2) in V^Λ we obtain

$$V^\Lambda = \Phi(\Lambda, c_i^R(\mu), \mu) = V^R(c_i^R(\mu), \mu) + \delta V^\Lambda(\Lambda, c_i^R(\mu), \mu), \tag{4}$$

where V^R is the renormalized potential and δV^Λ - counter term(s).

V^Λ does not depend on μ therefore by differentiating with μ_i we obtain Gell-Mann-Low RG equations

$$\frac{\partial}{\partial \mu_i} V^R = -\frac{\partial}{\partial \mu_i} \delta V^\Lambda(\Lambda, c_i^R(\mu), \mu) \equiv \beta_i(c_j^R, \mu). \tag{5}$$

On the other hand, as V^R does not depend on Λ, by differentiating with Λ we obtain Wilsonian RG equation

$$\frac{\partial}{\partial \Lambda} V^\Lambda = \frac{\partial}{\partial \Lambda} \delta V^\Lambda(\Lambda, c_i^R(\mu), \mu). \tag{6}$$

If there are only logarithmit divergences then, for large values of Λ and $\mu_i = \mu_0$, δV^R depends only on $\ln(\mu_0/\Lambda)$. In such cases there is a clear connection between the two RGs - up to notations they are identical.

While the whole amplitude does not depend on μ, the relative importance of c_i^R depends on the choice of the ren. scheme.

In any perturbative expansion of the amplitude the relative size of different contributions (i.e., power counting) essentially depends on the choice of the renormalization scheme.

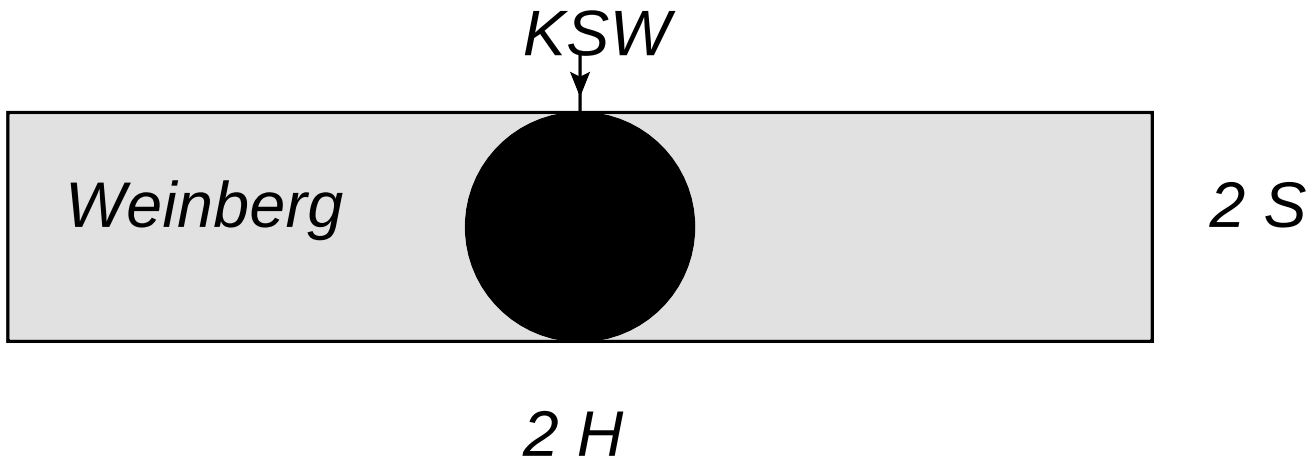
By taking say $\mu_i \equiv 0$, as is done by using dimensional regularization for power-law divergences, one only hides this dependence!

The behaviour of c_i^R as functions of μ_i can be obtained by solving Gel-Mann-Low RG equations.

Can we obtain equivalent information by analysing Wilsonian RG equations?

Birse, McGovern, Richardson, ...

The space of the Gel-Mann-Low RG scale parameters is multidimensional, Wilsonian RG, applied so far for NN EFT, stays one-dimensional!



Space of RG parameters, H-hard scale, S-soft scale.

KSW counting corresponds to the expansion of the potential around the non-trivial fixed point.

Weinberg's power counting DOES NOT correspond to the expansion around the trivial fixed point!

Low energy chiral EFT

- Chiral EFT aims at reproducing the S-matrix of QCD in low-energy region by providing with a systematic expansion of physical quantities in powers of (small scale(s)/ large scale).
- Most general Lagrangian of the EFT of Hadrons with symmetries of QCD gives the most general S-matrix with these symmetries.
- To obtain S-matrix of QCD one needs to fix properly the renormalized parameters of EFT $\cdots$
- $\cdots$ a finite number of them to achieve a finite accuracy!
- $\cdots$ EFT $\neq$ QCD.
  QCD has only one Large scale - EFT has many of them!
  QCD calculates physical quantities in terms of fundamental parameters, EFT only relates physical quantities to each other at low-energies, like

$$\sigma_1(E) = F(E, \sigma_2(\mu_i), \sigma_3(\mu_i), \cdots, \mu).$$

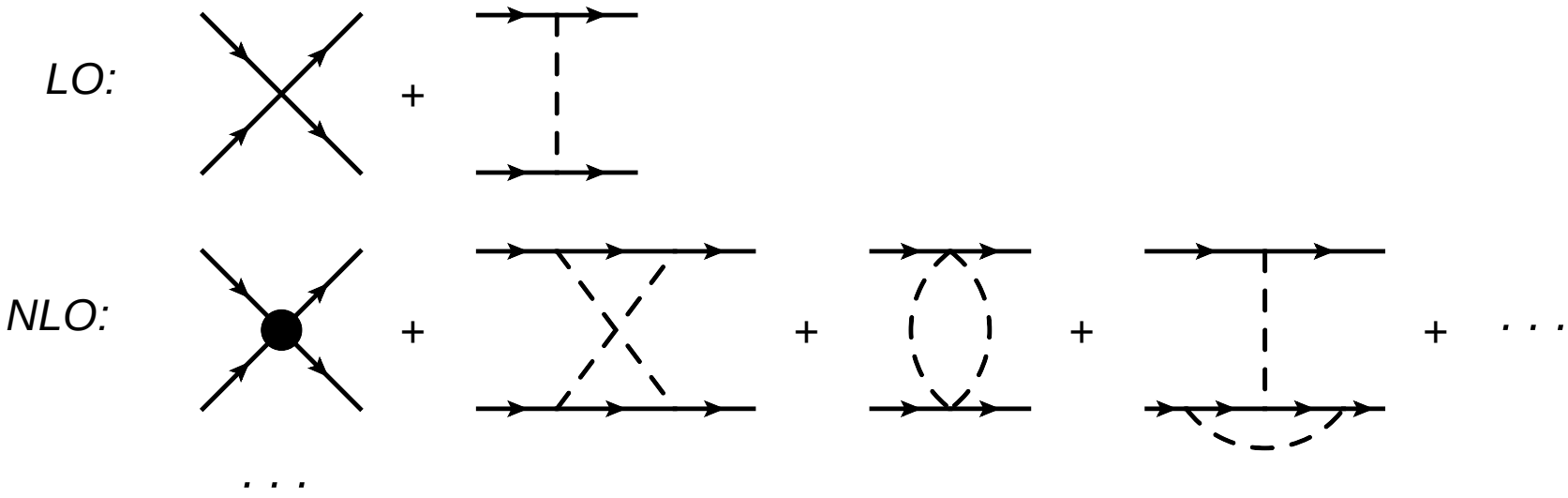
What to do?

- Write down the most general effective Lagrangian.
- Consider *all* Feynman diagrams contributing to the process in question.
- Renormalize/subtract loop diagrams.
- Apply power counting to renormalized diagrams.
- Sum up all *renormalized* diagrams contributions up to the given order.

- An infinite number of diagrams contribute in the amplitude for $N > 1$ nucleons at any fixed order.
- Weinberg suggested power counting for the effective potential.

  S. Weinberg, Phys. Lett. B **251**, 288 (1990).
- N-nucleon effective potential is defined as the sum of N-nucleon irreducible TOPT diagrams.

- Only a finite number of diagrams contribute at any given order.



- Amplitudes are obtained by solving integral equations

$$T = V + V\,G\,T\,.$$

or Schrödinger equation

Implemented in
C. Ordonez, L. Ray and U. van Kolck, Phys. Rev. Lett. **72**, 1982 (1994).
...

A comment on the power counting for the potential

According to the Weinberg's power counting, the potential is given as

$$V = V_0 + Q^2 V_2 + \ldots,$$

and the resolvent operator (Green's function) of

$$T = V + VGT$$

has the order $G \sim Q^0$.

For systems close to the unitary limit T is of the order Q^{-1} (instead of the $T \sim Q^0$ in the natural case)

Contradiction with Weinberg's counting?

Let us take a closer look:

$$V = V_{LO} + \tilde{V},$$

where $\tilde{V}$ is of higher order (by Q^2) and decompose

$$T = T_{LO} + \tilde{T},$$

where T_{LO} is the solution to the equation

$$T_{LO} = V_{LO} + V_{LO}\, G\, T_{LO},$$

that is we have

$$T_{LO} = (1 - V_{LO}\, G)^{-1}\, V_{LO}.$$

The case $T_{LO} \sim Q^{-1}$ has two different realisations:

$$V_{LO} \sim Q^{-1}, \;\; G \sim Q^{1}, \;\; 1 - V_{LO}\, G \sim Q^{0} \; - - - \; \textit{KSW}$$

D. B. Kaplan, M. J. Savage, and M. B. Wise, Phys. Lett. B **424**, 390 (1998).

$$V_{LO} \sim Q^{0}, \;\; G \sim Q^{0}, \;\; 1 - V_{0}\, G \sim Q^{1} \; - - - \; \textit{Weinberg}$$

For $\tilde{T}$ we have

$$\tilde{T} = \tilde{V} + V_0 G \tilde{T} + \tilde{V} G T_0 + \tilde{V} G \tilde{T},$$

that is

$$(1 - V_0 G)\tilde{T} = \tilde{V} + \tilde{V} G T_0 + \tilde{V} G \tilde{T}.$$

Clearly, $\tilde{T} \sim Q^0$.

The above expansion of the amplitude in orders of small parameter exactly matches the series induced by the ERE

$$\begin{aligned} T &= \frac{4\pi}{m_N (1/a + i p)} + \frac{2\pi p^2 r_E}{m_N (1/a + i p)^2} \\ &+ \frac{\pi p^4}{m_N (1/a + i p)^2} \left[4 v_2 + \frac{r_E^2}{1/a + i p} \right] + \cdots \end{aligned}$$

for the case of the large scattering length.

In pionless EFT ($^1S_0$ PW)

$$V_{LO} = C.$$

The renormalized amplitude

$$T_{LO}(p) = \frac{1}{1/C_R(\mu) + \frac{m_N}{4\pi}(\mu + i\,p)}.$$

For $\mu \sim Q^0$, $C_R \sim Q^0$ – Weinberg's counting.

For $\mu \sim Q$, $C_R \sim Q^{-1}$ – KSW counting.

Analogously for higher orders and PWs...

Weinberg's power counting equally well reproduces the expansion of the amplitude for both, "natural" and unnaturally large scattering lengths...

and most importantly – for the intermediate case, in which it is difficult to assign the scattering length to be "natural" or unnaturally large!

Problem: Weinberg's power counting inconsistent with renormalization – (even) LO potential is not renormalizable

Solutions:

Kaplan, Savage+Wise '98, ..., Nogga, Timmermans+van Kolck '05, ...

Epelbaum, Glöckle, Meißner 98, ...

. . .

Lorentz invariance and low energy EFT

Is Lorentz invariance important at low energies?

INT Program 13-1a

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van Kolck:

Root of the problem:
pion exchanges (long-ranged, contribute to waves higher than S)
are singular (sensitive to short-range physics, require counterterms)
This has NOthing to do with relativity...
(For the opposite opinion, see Epelbaum + Gegelia '12)

Non-relativistic theory should be adequate at low energies, however ...

Per definition, non-relativistic expansion means:

1. Lorentz invariant effective Lagrangian – Lorentz invariance is a fundamental symmetry!
2. Quantum corrections.
3. Regularization (Λ) and renormalization.
4. Non-relativistic expansion (expansion in $1/m$) of renormalized quantities.

On the other hand, non-relativistic EFT:

1. Lorentz-invariant EFT Lagrangian – expanded in $1/m \Rightarrow$ non-relativistic EFT Lagrangian.
2. Quantum corrections.
3. Regularization (Λ), Renormalization.
4. $\Lambda \to \infty$ after renormalization.
5. Renormalized quantities are given as non-relativistic series.

- Proper non-relativistic expansion:
  first – calculation of quantum corrections,
  after – $1/m$ expansion.
- Non-relativistic EFT:
  first – expansion in $1/m$,
  after – calculation of quantum corrections.
- Expansions in $1/m$ and calculation of quantum corrections are not commutative!
- Difference (”error”) can be compensated by adding terms in non-relativistic EFT Lagrangian. A finite number of terms needed at any fixed order in one nucleon sector.

  J. Gegelia and G. Japaridze, Phys. Rev. D **60**, 114038 (1999).
- Due to solving an integral equation, an infinite number of compensating terms needed in NN sector already at LO.

Solutions:

- Keep $\Lambda \lesssim m$ – successfully implemented by various groups:

  E. Epelbaum, W. Gloeckle and U. -G. Meissner, ... D. R. Entem and R. Machleidt ...

- Take into account an infinite number of compensating terms of non-relativistic EFT Lagrangian:

  - Realized in KSW approach (perturbative pions)
  D. B. Kaplan, M. J. Savage and M. B. Wise, Phys. Lett. B **424**, 390 (1998) ...

  - Problematic if pions are included non-perturbatively!

- Prove that an infinite number of compensating terms can be safely dropped – has (can)not been done!

- Use the original Lorentz invariant Lagrangian without $1/m$ expansion ... (even better:) *and* keep $\Lambda \lesssim m$

Effective Lagrangian of pions and nucleons

Standard effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \cdots$$

is organized as an expansion in powers of pion masses and derivatives acting on fields.

Lowest-order mesonic Lagrangian

$$\mathcal{L}_2 = \frac{F^2}{4}\text{Tr}\left[\partial_\mu U \left(\partial^\mu U\right)^\dagger\right] + \frac{F^2 M^2}{4}\text{Tr}\left(U^\dagger + U\right),$$

U - (2×2) matrix containing Goldstone bosons.
F - pion-decay constant in the chiral limit.
M - the pion mass at LO.

The lowest-order Lorentz-invariant Lagrangian in one nucleon sector

$$\mathcal{L}^{(1)}_{\pi N} = \bar{\psi}\left(i\not{D} - m + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu\right)\psi,$$

$$D_\mu = \partial_\mu + \frac{1}{2}\left[u^\dagger\partial_\mu u + u\partial_\mu u^\dagger\right], \quad u^2 = U, \quad u_\mu = i\,u^\dagger\partial_\mu U u^\dagger.$$

LO effective NN Lagrangian:

$$\begin{aligned}\mathcal{L}_{NN} &= C_S^a\,\bar{\Psi}\tau^a\Psi\;\bar{\Psi}\tau^a\Psi + C_T^a\,\bar{\Psi}\tau^a\sigma_{\mu\nu}\Psi\;\bar{\Psi}\tau^a\sigma^{\mu\nu}\Psi \\ &+ C_{AV}^a\bar{\Psi}\tau^a\gamma_5\gamma_\mu\Psi\;\bar{\Psi}\tau^a\gamma_5\gamma^\mu\Psi + C_V^a\bar{\Psi}\tau^a\gamma_\mu\Psi\;\bar{\Psi}\tau^a\gamma^\mu\Psi,\end{aligned}$$

where the summation from 0 to 3 over a is implied with τ^i ($i = 1, 2, 3$) - the Pauli (isospin) matrices and τ^0 - the unit matrix. $C_I^1 = C_I^2 = C_I^3$ for all I.

Renormalizable version of Weinberg's approach:

E. Epelbaum and J. Gegelia, Phys. Lett. B **716**, 338 (2012).

Lorentz invariant effective Lagrangian:

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\pi} + \mathcal{L}_{\pi{\rm N}} + \mathcal{L}_{\rm NN} + \cdots \, .$$

Use the time-ordered PT.

NN potential $V :=$ sum of $2\,{\rm N}$-irreducible diagrams.

Off-shell amplitude T satisfies:

$$T = V + V\,G\,T \, .$$

G - two-nucleon propagator.

Expand

$$\begin{aligned} T &= T_0 + T_1 + T_2 + \cdots , \\ G &= G_0 + G_1 + G_2 + \cdots , \\ V &= V_0 + V_1 + V_2 + \cdots , \end{aligned}$$

and solve T order by order.

At leading order:

$$T_0 = V_0 + V_0\, G_0\, T_0 \,.$$

Using T_0 calculate the NLO amplitude:

$$T_1 = V_1 + T_0\, G_0\, V_1 + V_1\, G_0\, T_0 + T_0\, G_0\, V_1\, G_0\, T_0 + T_0\, G_1\, T_0 \,.$$

Using T_0 and T_1 calculate the NNLO amplitude T_2 etc.

LO equation in COM frame

$$
\begin{aligned}
T_0\left(\vec{p}\,',\vec{p}\right) &= V_0\left(\vec{p}\,',\vec{p}\right) - \frac{m^2}{2}\int \frac{d^3\vec{k}}{(2\,\pi)^3}\, V_0\left(\vec{p}\,',\vec{k}\right) \\
&\times \frac{1}{\left(\vec{k}^2+m^2\right)\left(p_0-\sqrt{\vec{k}^2+m^2}+i\,\epsilon\right)}\, T_0\left(\vec{k},\vec{p}\right), \\
V_0 &= V_C + V_\pi .
\end{aligned}
$$

LO potential ($\vec{q}=\vec{p}\,'-\vec{p}$)

$$
V_0(\vec{p}\,',\vec{p}) = C_S + C_T\,\sigma_1\cdot\sigma_2
$$

$$
-\left(\frac{g_A}{2\,F_\pi}\right)^2 \frac{(\vec{\tau}_1\cdot\vec{\tau}_2)(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})}{\sqrt{\vec{q}^2+M_\pi^2}\,(\sqrt{\vec{p}\,'^2+m^2}+\sqrt{\vec{p}^2+m^2}+\sqrt{\vec{q}^2+M_\pi^2}-2p_0-i\,\epsilon)}
$$

$p_0=\sqrt{m^2+p^2}$ with p - three-momentum of incoming nucleons.

V. G. Kadyshevsky, Nucl. Phys. B 6, 125 (1968).

- Milder UV behavior than in LS equation.
- Iterations of LO equation generate only overall logarithmic divergences.
- All divergences absorbed in parameters of the LO potential $\Rightarrow$ LO equation is perturbatively renormalizable.

Non-perturbative regime: for $k \to \infty$ approximate

$$\frac{p_0 + \sqrt{k^2 + m^2}}{\left(k^2 + m^2\right)\,\left(p^2 - k^2 + i\,\epsilon\right)} \to \frac{1}{k\,\left(p^2 - k^2 + i\,\epsilon\right)}\,,$$

and obtain in PW basis

$$T^{sj}_{l'l}\left(p', p\right) = V^{sj}_{l'l}\left(p', p\right) + \frac{m^2}{2} \sum_{l''} \int_0^\infty \frac{dk\,k}{(2\,\pi)^3}\,\frac{V^{sj}_{l'l''}\left(p', k\right)\,T^{sj}_{l''l}\left(k, p\right)}{p^2 - k^2 + i\,\epsilon}.$$

It has the form of PW LS equation in $2 + 1$ dimensions.

Corresponding OPE potential behaves as $\sim \frac{1}{r^2}$ for $r \to 0$.

More singular $\sim \frac{1}{r^3}$ UV behavior in non-relativistic EFT (HBChPT) is an artefact of that formulation.

LO PW equations have unique solutions, except ${}^3P_0$ PW.

${}^3P_0$ PW equation has the same behavior as S-TM equation:

G. V. Skornyakov and Ter-Martirosyan, Sov. Phys. JETP **4**, 648 (1957).

Analogously to

P. F. Bedaque, H. W. Hammer and U. van Kolck, Phys. Rev. Lett. **82**, 463 (1999)

we included a counter-term $\frac{c(\Lambda)\, p\, p'}{\Lambda^2}$ at LO.

By adding *symmetry-preserving* terms to the standard BChPT Lagrangian, we can obtain a modified nucleon propagator

$$S_N^{\Lambda}(p) = \frac{1}{(\not{p} - m + i\epsilon)} \prod_{j=1}^{N_\Psi} \frac{\Lambda_{\Psi j}^2}{\Lambda_{\Psi j}^2 + m^2 - p^2 - i\epsilon}.$$

For the additional terms of the Lagrangian we choose

$$\mathcal{L}_{\pi N}^{\text{reg}} = \sum_{n=1}^{N_\Psi} \frac{Y_n}{2} \left[\bar{\Psi} \left(i\gamma_\mu D^\mu - m \right) \left(D^2 + m^2 \right)^n \Psi + h.c. \right].$$

D. Djukanovic, M. R. Schindler, J. Gegelia and S. Scherer, Phys. Rev. D **72**, 045002 (2005)

By using field transformations our effective Lagrangian with additional terms can be brought in the canonical form. That is, any Λ-dependence of physical quantities can be absorbed in the redefinition of the parameters of the standard effective Lagrangian.

Using the effective Lagrangian we obtain the rules of the TOPT. It is convenient to take the additional terms of the following form

$$\begin{aligned}
\mathcal{L}^{\rm reg}_{\pi N} &= \frac{1}{2\Lambda^2}\left[\bar\Psi\left(i\gamma_\mu D^\mu - m\right)\left(D^2+m^2\right)\Psi + h.c.\right] \\
&\equiv -\frac{1}{2\Lambda^2}\left[\bar\Psi\left(i\gamma_\mu D^\mu - m\right)\vec{D}^2\Psi + h.c.\right] \\
&+ \frac{1}{2\Lambda^2}\left[\bar\Psi\left(i\gamma_\mu D^\mu - m\right)\left(D_0^2+m^2\right)\Psi + h.c.\right]. \qquad (7)
\end{aligned}$$

Contribution of the first term of Eq. (7) in the nucleon two-point function we include non-perturbatively in the nucleon propagator while the second term is treated perturbatively.

The corresponding propagator has the form

$$S_N^{\Lambda}(p) = \frac{\Lambda^2}{(\not{p} - m + i\epsilon)\left(\Lambda^2 + \vec{p}^{\,2}\right)}.$$

To generate the rules of the TOPT we first draw all relevant Feynman diagrams, assign the momenta to propagator lines and perform the integration over the zeroth components of the loop momenta.

As a result we are lead to the diagrams of the TOPT.

Taking $\Lambda \sim$ hard scale corresponds to the choice of an appropriate renormalization scheme.
Lepage, Gegelia, Epelbaum, Glöckle, Meißner, van Kolck (of last millenium)... Beane, Kaplan, Vourinen ...

Summary

- "The reports of my death have been greatly exaggerated."
  Weinberg's power counting

  ...
  Mark Twain

  A renormalizable EFT for NN:
- NN scattering using Lorentz invariant Lagrangian and TOPT.
- LO amplitude obtained by solving an integral equation.
- Corrections calculated perturbatively.

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BS

To demonstrate the modified UV behavior in HB formalism consider

$$I = \frac{4\,i}{(2\,\pi)^4}\int \frac{d^4k\,\theta(\Lambda-|\vec{k}|)}{\left[k^2-m^2+i\,0^+\right]\left[(P-k)^2-m^2+i\,0^+\right]}$$

with $P=(2\sqrt{m^2+\vec{p}^{\,2}},\vec{0}\,)$.

The results for $\Lambda > |\vec{p}|$:

$$I = \frac{|\vec{p}|\,\ln\frac{\Lambda\sqrt{m^2+\vec{p}^{\,2}}+|\vec{p}|\sqrt{\Lambda^2+m^2}}{m\sqrt{\Lambda^2-\vec{p}^{\,2}}}}{\pi^2\sqrt{m^2+\vec{p}^{\,2}}} - \frac{\ln\frac{\Lambda+\sqrt{\Lambda^2+m^2}}{m}}{\pi^2} - \frac{i|\vec{p}|}{2\pi\sqrt{m^2+\vec{p}^{\,2}}}\,.$$

Expand first in $1/\Lambda$ and subsequently in $1/m$:

$$I = -\frac{i|\vec{p}|}{2\pi m} - \frac{\ln\frac{\Lambda}{m}}{\pi^2} - \frac{\ln 2}{\pi^2} + \mathcal{O}\left(\frac{1}{\Lambda^2}, \frac{1}{m^2}\right).$$

Next expand first in $1/m$ and after in $1/\Lambda$:

$$I = -\frac{i|\vec{p}|}{2\pi m} - \frac{\Lambda}{\pi^2 m} + \frac{\vec{p}^{\,2}}{\pi^2 \Lambda m} + \mathcal{O}\left(\frac{1}{m^2}, \frac{1}{\Lambda^2}\right).$$

HB approach corresponds to the second expansion.

In PT (e.g. πN scattering) one compensates the mismatch by adding terms in the effective non-relativistic Lagrangian.

When re-summing iterations to all orders (e.g. for NN), one needs to include contributions of an infinite number of terms.

Otherwise, in HB approach one is not allowed to take $\Lambda > m$.