INT Program INT-16-1

Nuclear Physics from Lattice QCD

Week 7

Electroweak structure of A=2, 3 nuclei

האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

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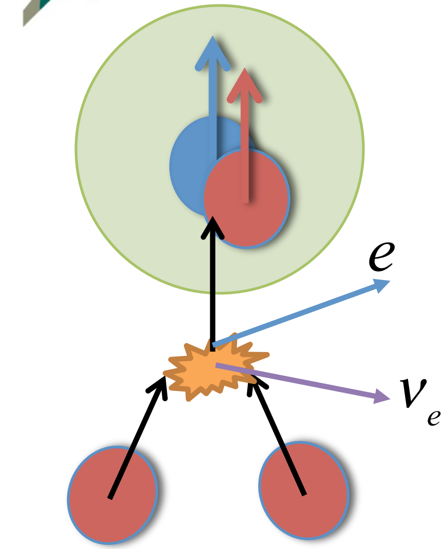
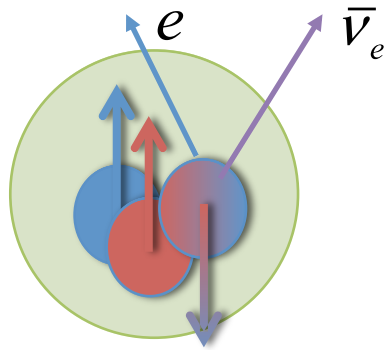
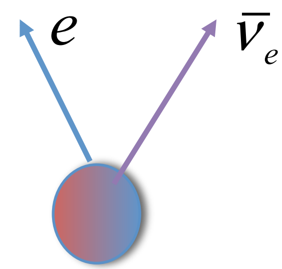
Low energy electroweak observables A≤ 3 nuclei



$p+p \to d+\nu_e+e^+$



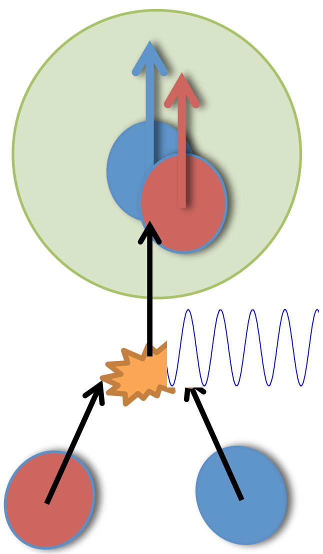
$^3\text{H} \to {}^3\text{He}+e^-+\bar{\nu}_e$



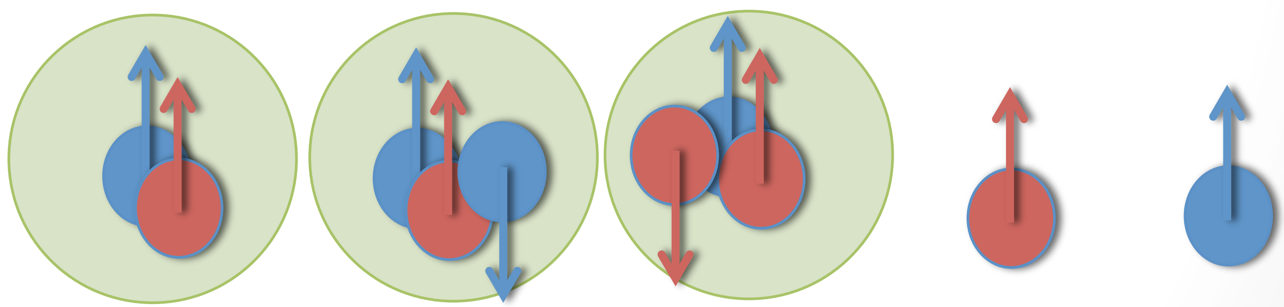
$n \to p+e^-+\bar{\nu}_e$

Weak observables

EM observables

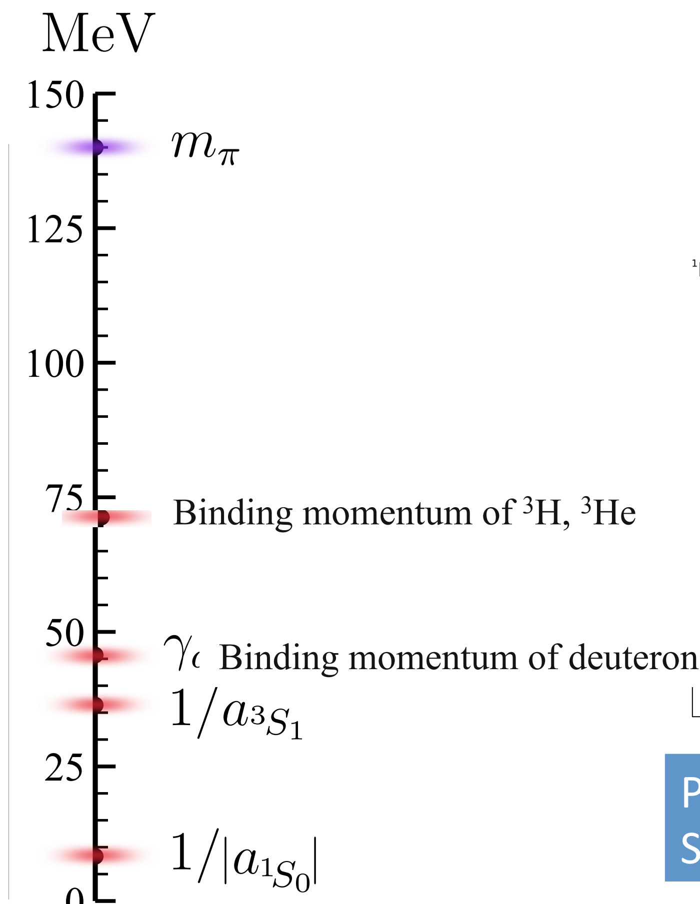


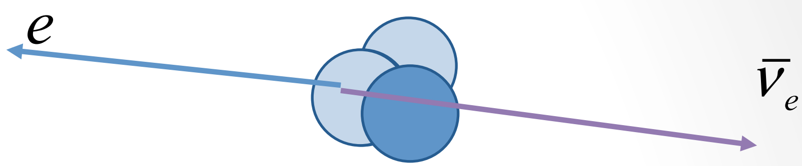
$n+p \to d+\gamma$



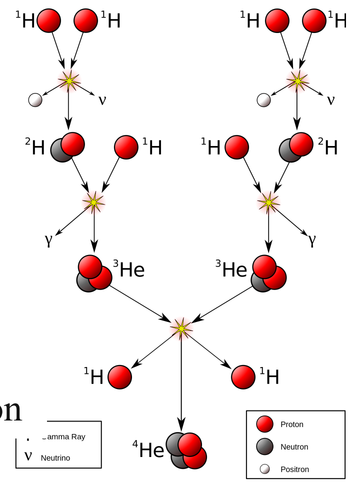
μ_d $\mu_{^3\text{H}}$ $\mu_{^3\text{He}}$ μ_p μ_n

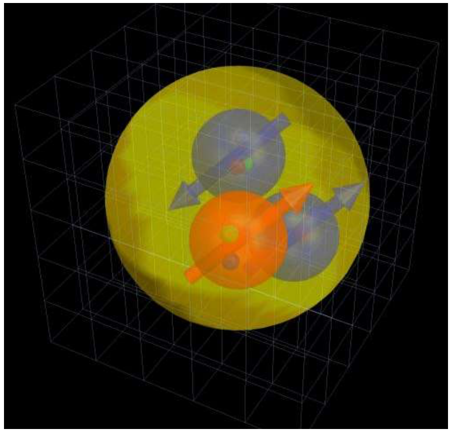
Low energy electroweak observables of light nuclei





Nuclear *beta*-decays



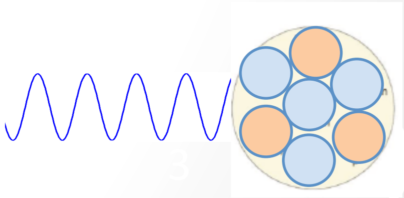


Proton-proton
Solar fusion cycle

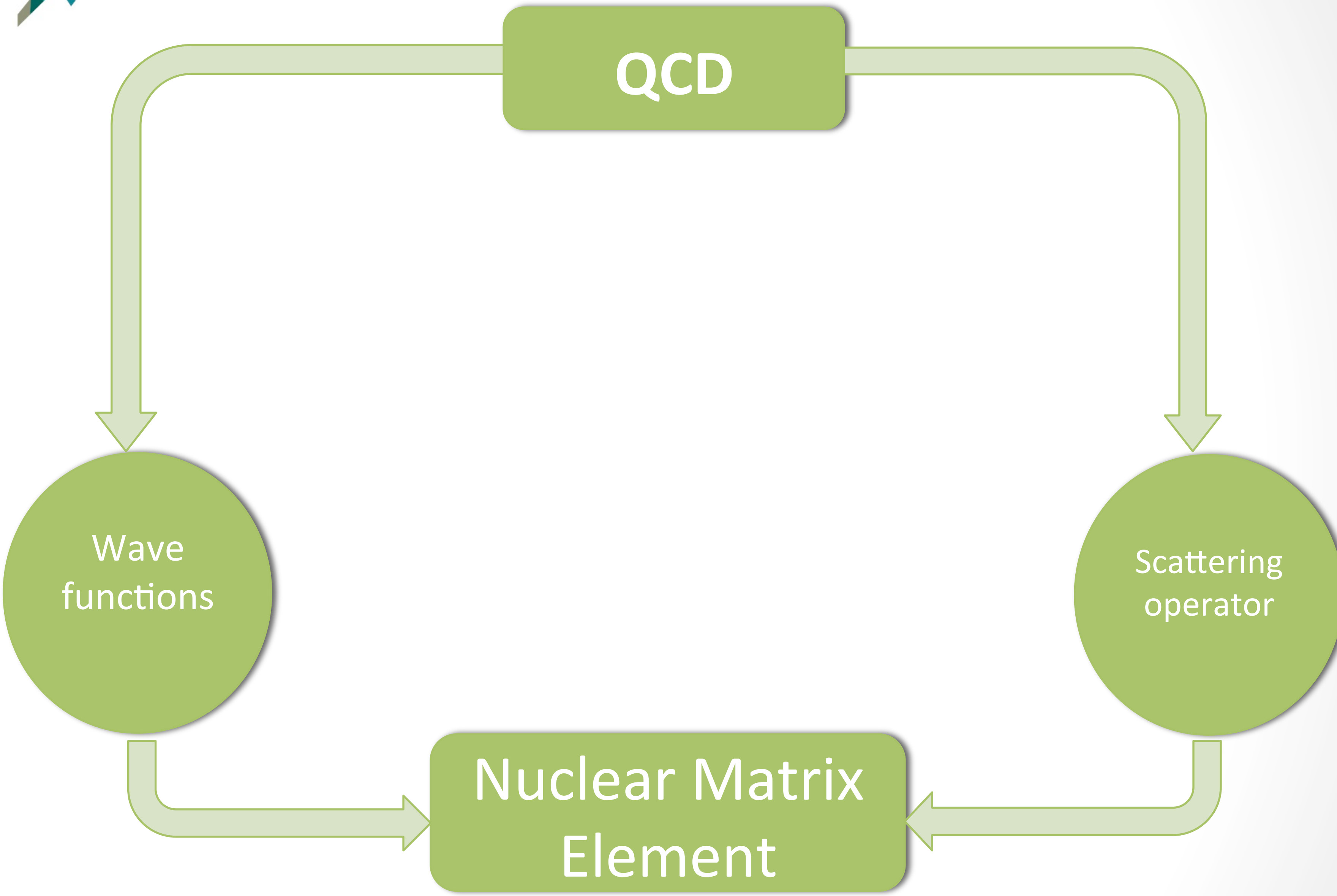
Nuclear magnetic moments

- $a_{^3S_1} \approx 5.4 \text{ fm}, a_{^1S_0} \approx -23.7 \text{ fm} \gg 1/m_\pi \approx 1.4 \text{ fm}$
- effective ranges $(1.8 \text{ fm}, 2.7 \text{ fm})$ are natural

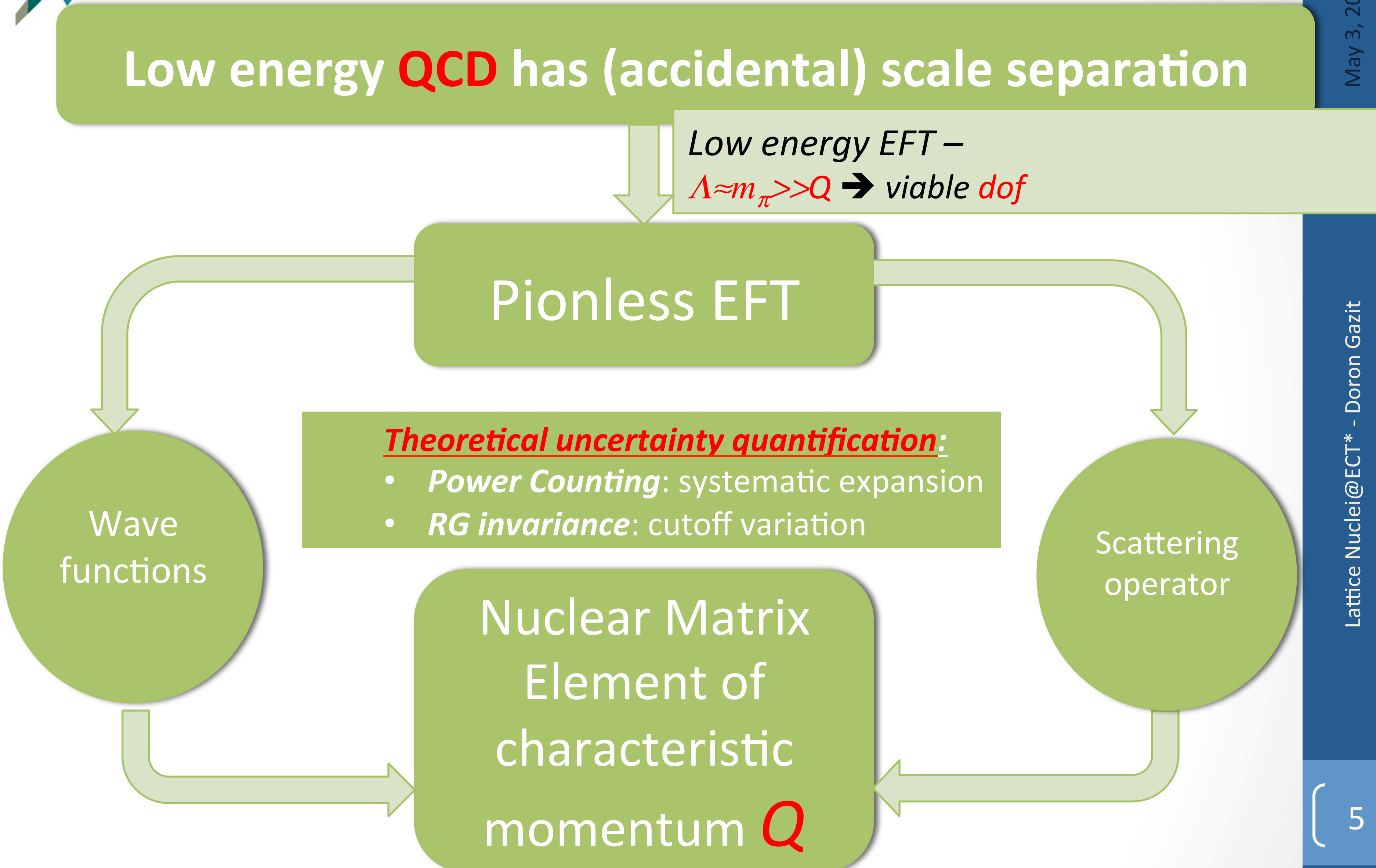
- Vanishing, or small, energy/momentum transfer: $q,\ q_0 << m_p$
- Typical momentum for $A \leq 4$: $Q_{typ} \approx \sqrt{2m_N B/A} << m_\pi$
- Effective field theory without pions.



The interaction of a nucleus with an external probe



Nucleus interaction with a probe, EFT point of view:



Leutwyler, Gasser, Weinberg, Kaplan, Savage, Wise, van-Kolck, Birse, Meissner, Epelbaum, Machleidt, Chen, Park, Rho,…

The interaction of a nucleus with an external probe

"QCD"

↓ Non-physical m_p, sign problem, statistics, configurations, finite volume…

Lattice formulation in a background field

↓

Nuclear Matrix Element

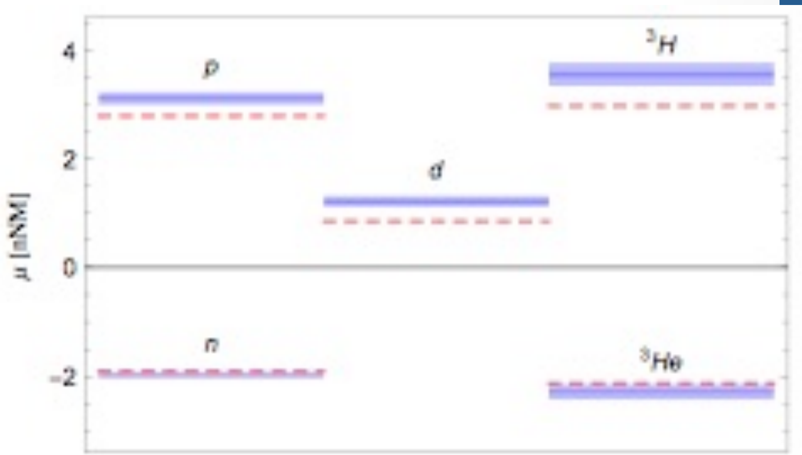
Application in the EM sector (1)

Magnetic moments of light nuclei from lattice quantum chromodynamics

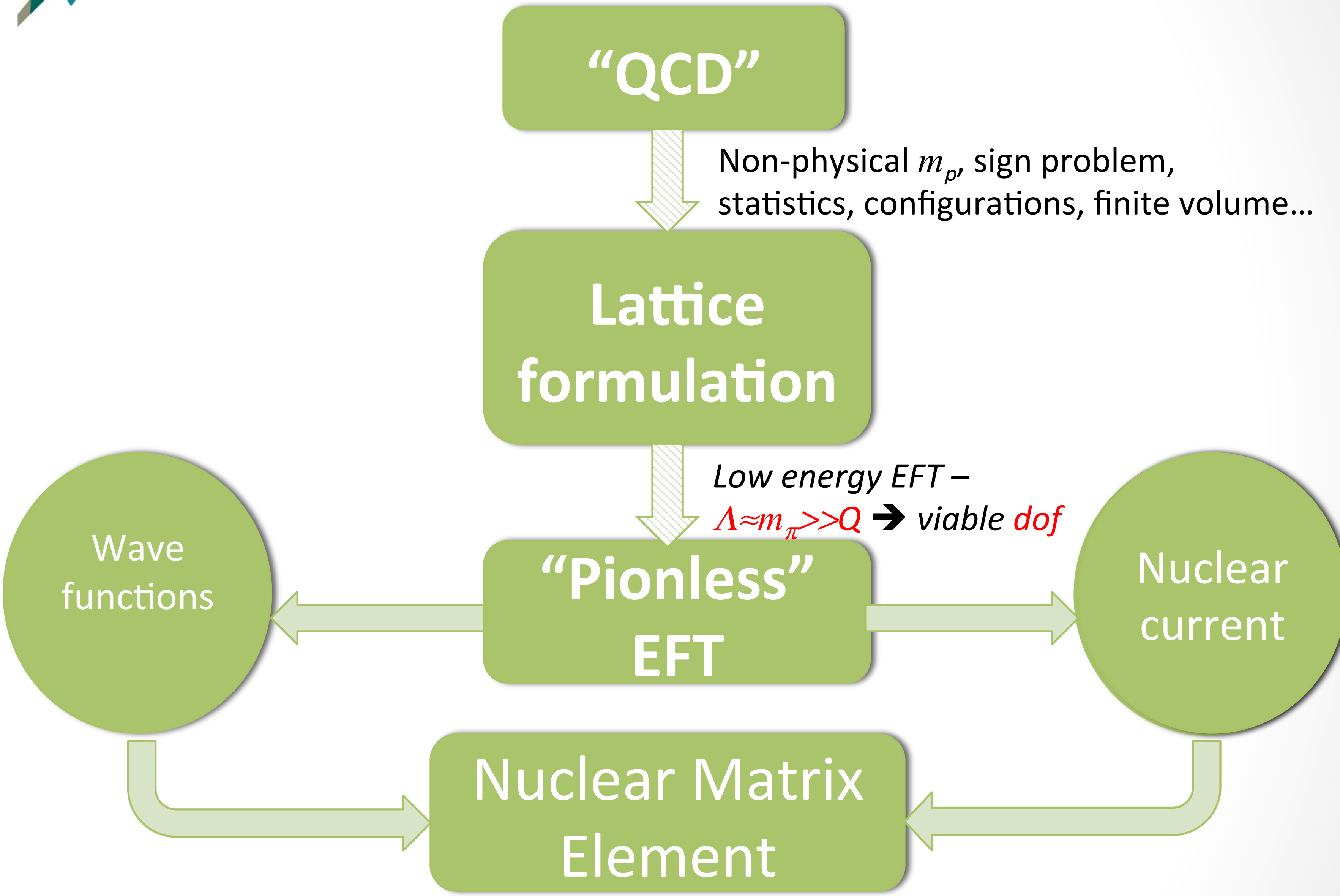
S.R. Beane,[1] E. Chang,[1,2] S. Cohen,[1,2] W. Detmold,[3] H.W. Lin,[1]
K. Orginos,[4,5] A. Parreño,[6] M.J. Savage,[1,2] and B.C. Tiburzi[7,8,9]
(NPLQCD Collaboration)

$$\delta E^{(B)} = -2\mu\,|\mathbf{B}| + \gamma\,|\mathbf{B}|^3$$

for $m_\pi = 806\,\mathrm{MeV/c^2}$



The interaction of a nucleus with an external (weak) probe



Application in the EM sector (2)

PRL **115,** 132001 (2015) PHYSICAL REVIEW LETTERS week ending 25 SEPTEMBER 2015

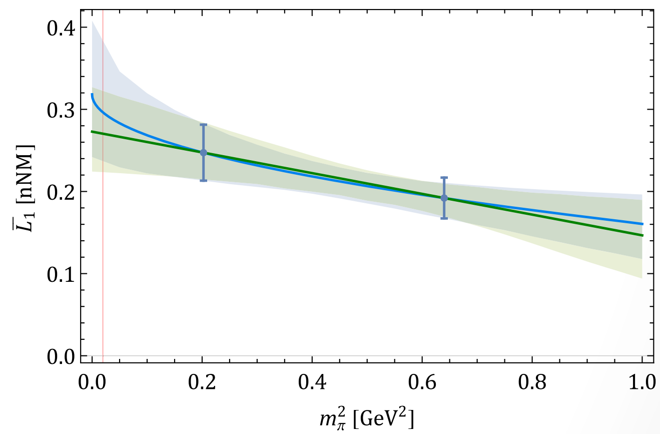
Ab initio Calculation of the _np_ → _dγ_ Radiative Capture Process

Silas R. Beane,[1] Emmanuel Chang,[2] William Detmold,[3] Kostas Orginos,[4,5] Assumpta Parreño,[6] Martin J. Savage,[2] and Brian C. Tiburzi[7,8,9]

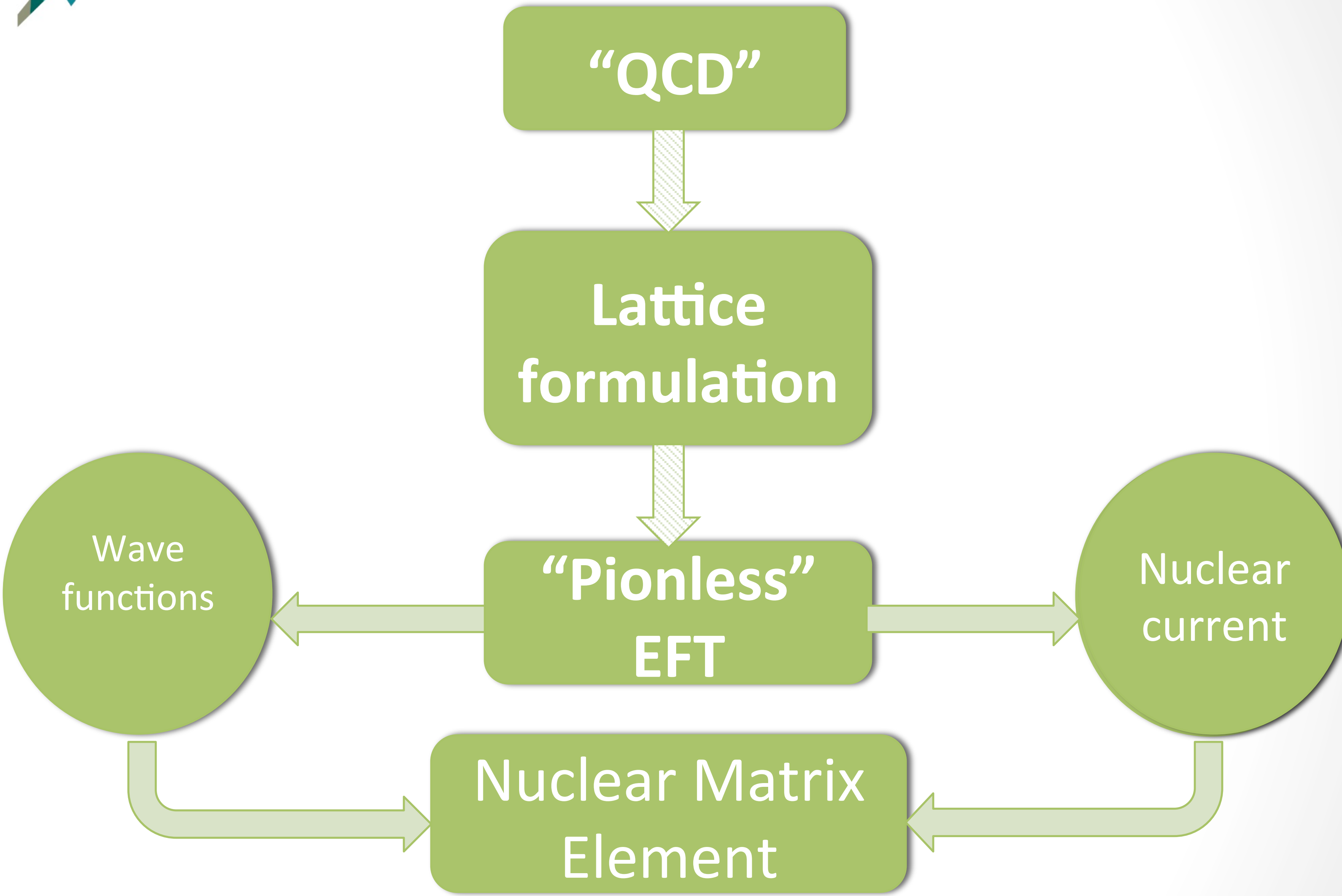
(NPLQCD Collaboration)

$$\Delta E_{^3S_1,^1S_0}(\mathbf{B}) = 2(\kappa_1 + \gamma_0 Z_d^2 \tilde{l}_1)\frac{e}{M}|\mathbf{B}| + \mathcal{O}(|\mathbf{B}|^2),$$

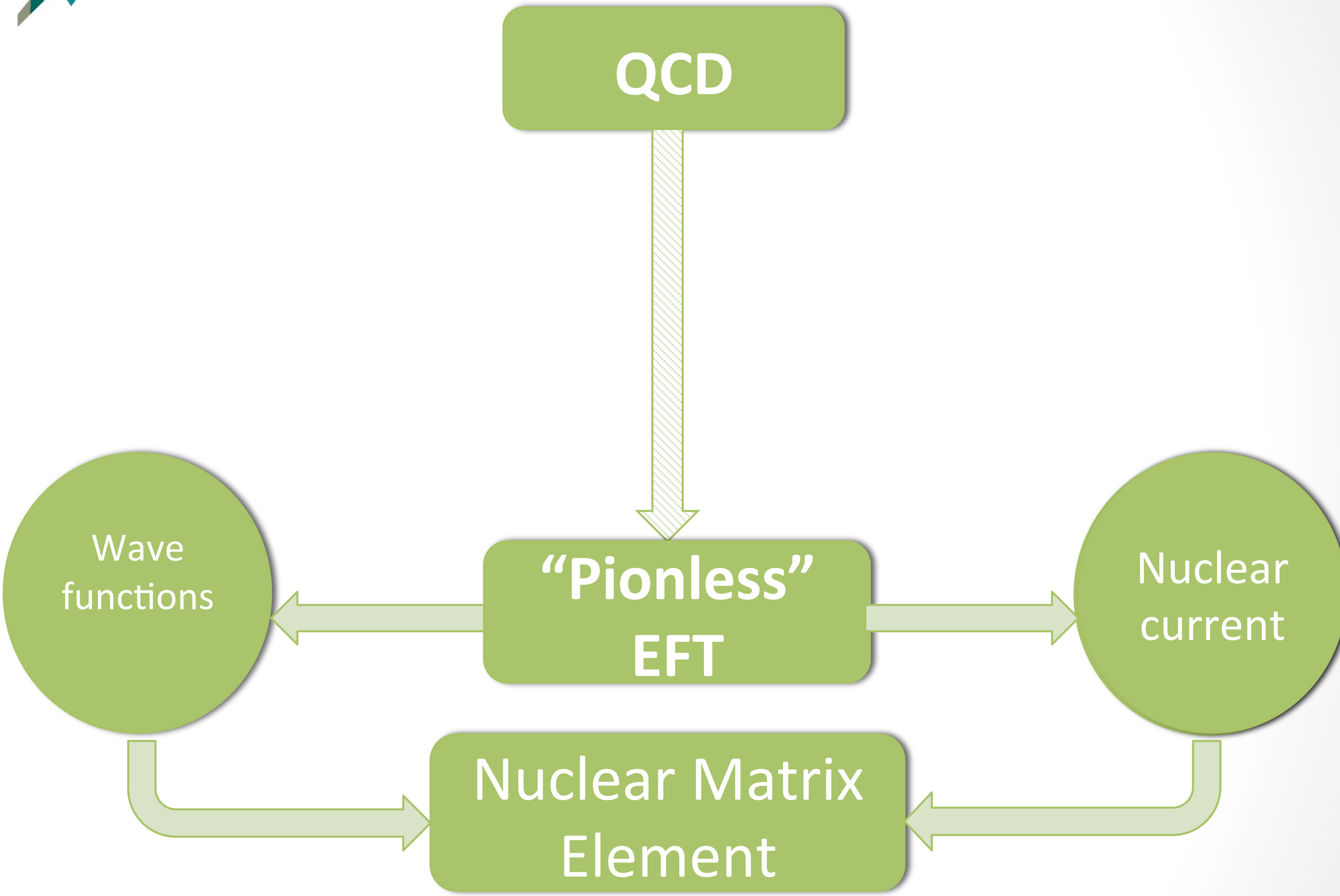
$$l_1^{\mathrm{lqcd}} = -4.41\left(\begin{array}{c}+15\\-16\end{array}\right)\ \mathrm{fm}.$$



The interaction of a nucleus with an external (weak) probe

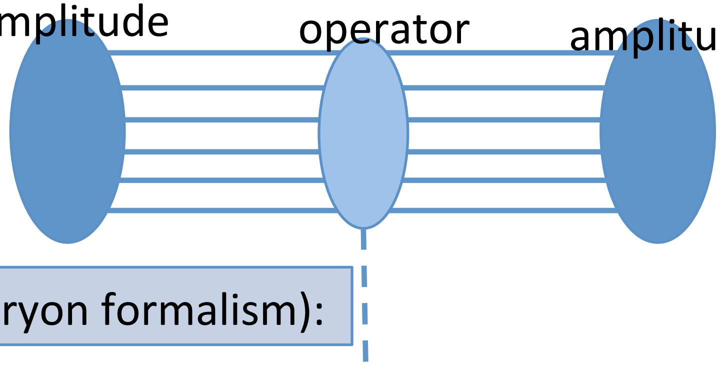


The interaction of a nucleus with an external (weak) probe



Outline

- EM:
  - Calculate A=2, 3 magnetic moments, as well as n+p capture, using consistent pionless EFT up to next-to-leading order (NLO).
    - Use A=3 magnetic moments to fix the EFT, and postdict A=2 observables.
    - Use A=2 observables to fix the EFT, and postdict A=3 observables.
  - Show stability of the calculation, naturalness, power counting.
  - ***Confront with nature and lattice data***.
  - Give ***reliable*** uncertainty estimate, especially for an n+p→d+γ postdiction, and check its validity!
- Weak interaction:
  - Calculate ^3H decay and p+p fusion, using consistent pionless EFT up to next-to-leading order (NLO).
    - Use measured 3H decay rate to predict p+p fusion.
  - Give ***reliable*** uncertainty estimate!



Amplitudes (in a dibaryon formalism):

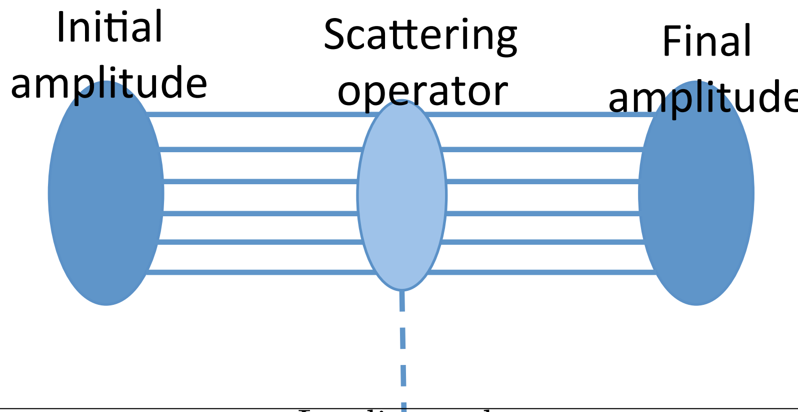
$$\mathcal{L} = N^\dagger \left(iD_0 + \frac{D^2}{2M_N} \right) N - t^{i\dagger} \left[\sigma_t + \left(iD_0 + \frac{D^2}{4M_N} \right) \right] t^i - s^{A\dagger} \left[\sigma_s + \left(iD_0 + \frac{D^2}{4M_N} \right) \right] s^A +$$

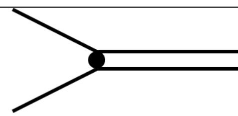
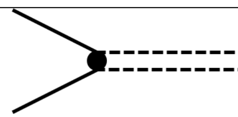
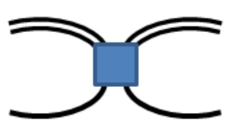
$$y_t \left[t^{i\dagger} \left(N^T P_t^i N \right) + h.c \right] + y_s \left[s^{A\dagger} \left(N^T P_s^A N \right) + h.c \right],$$

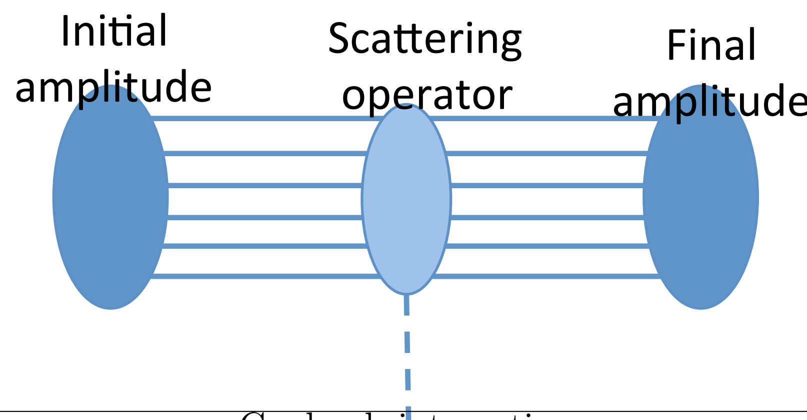
$$y_{t,s}^2 = \frac{8\pi}{M_N^2 \rho_{t,s}}$$

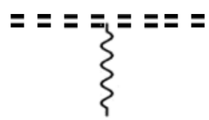
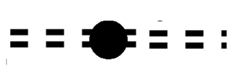
$$\sigma_{t,s} = \frac{2}{M_N \rho_{t,s}} \left(\frac{1}{a_{t,s}} - \mu \right)$$

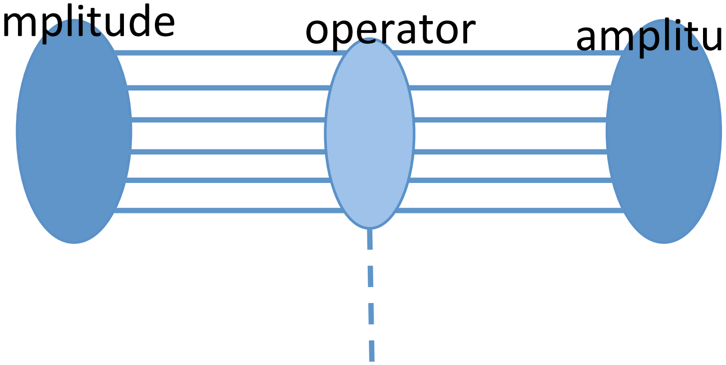
Kaplan (1996), Beane, Savage (1999)



Leading order			
(a)	single nucleon line		${}_iS^{\alpha a}_{\beta b} = i\delta^a_b\delta^\alpha_\beta \left(q_0 - \frac{\mathbf{q}^2}{2m} + i\varepsilon\right)^{-1}$
(b)	LO spin-triplet dibaryon propagator eq. (17)		$i\mathcal{D}^{ij}_t(q_0,\mathbf{q}) = i\delta^{ij}\frac{4\pi}{M_N y_t^2}\left(\frac{1}{a_t} - \sqrt{-M_N q_0 + \frac{\mathbf{q}^2}{4}}\right)^{-1}$
(c)	LO spin-singlet dibaryon propagator eq. (17)		$i\mathcal{D}^{AB}_s(q_0,\mathbf{q}) = i\delta^{AB}\frac{4\pi}{M_N y_s^2}\left[\frac{1}{a_s} - \sqrt{-M_N q_0 + \frac{\mathbf{q}^2}{4}}\right]^{-1}$
(d)	LO pp propagator eq. (61)		$i\mathcal{D}^{AB}_{pp}(q_0,\mathbf{q}) = i\delta^{AB}\frac{4\pi}{M_N y_t^2}\left[\frac{1}{a_p} + 2\kappa\Phi\left(\kappa/q'\right)\right]^{-1}$
(e)	spin-triplet dibaryon vertex eq. (10)		$-2iy_t\frac{1}{\sqrt{8}}\sigma^2\tau^2\sigma^i$
(f)	spin-singlet dibaryon vertex eq. (10)		$-2iy_s\frac{1}{\sqrt{8}}\sigma^2\tau^2\tau^A$
(g)	three nucleons vertex eq. (23)		$\frac{2H(\Lambda)}{\Lambda^2}$



Coulomb interaction			
(h)	Coulomb propagator eq. (50)		$\frac{i}{q^2+\lambda^2}$
(i)	photon-nucleon vertex		α
(j)	photon-dibaryon vertex		α
Next to leading order			
(k)	NLO spin-triplet Effective range correction eq. (21)		$-\delta^{ij}\frac{4\pi}{M_N y_t^2}\frac{\rho_t}{2}\frac{\sqrt{-M_N q_0+\frac{\mathbf{q}^2}{4}}+\frac{1}{a_t}}{\frac{1}{a_t}-\sqrt{-M_N q_0+\frac{\mathbf{q}^2}{4}}}$
(l)	NLO spin-singlet Effective range correction eq. (22)		$-\delta^{AB}\frac{4\pi}{M_N y_s^2}\frac{\rho_s}{2}\frac{-M_N q_0+\frac{\mathbf{q}^2}{4}}{\left(\frac{1}{a_s}-\sqrt{-M_N q_0+\frac{\mathbf{q}^2}{4}}\right)^2}$
(m)	NLO Coulomb Effective range correction eq. (65)		$-\delta^{AB}\frac{4\pi}{M_N y_s^2}\frac{\rho_C}{2}\frac{\mathbf{q}^2/4-M_N q_0}{\left[\frac{1}{a_p}-2\kappa\Phi\left(\kappa/q'\right)\right]^2}$



Deutron normalization (ANC): $$Z_d^{-1} = i \frac{\partial}{\partial_{p_0}} \frac{1}{i\mathcal{D}_t(p_0,p)}\Big|_{p_0=\frac{\gamma_t^2}{M_N},p=0}$$

Range expansion: $$Z_d = \frac{1}{1-\gamma_t\rho_t} = \underbrace{1}_{\text{LO}} + \underbrace{\gamma_t\rho_t}_{NLO} + \underbrace{(\gamma_t\rho_t)^2}_{N^2LO} + \underbrace{(\gamma_t\rho_t)^3}_{N^3LO} + ... = 1.69$$

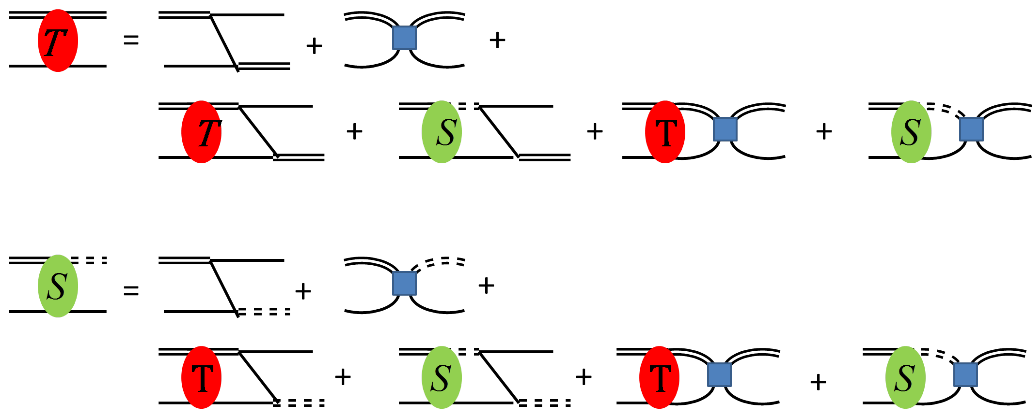
Z-parameterization: $$Z_d = \underbrace{1}_{\text{LO}} + \underbrace{Z_d - 1}_{NLO} + \underbrace{0}_{N^2LO} + \underbrace{0}_{N^3LO} + ... = 1.69$$

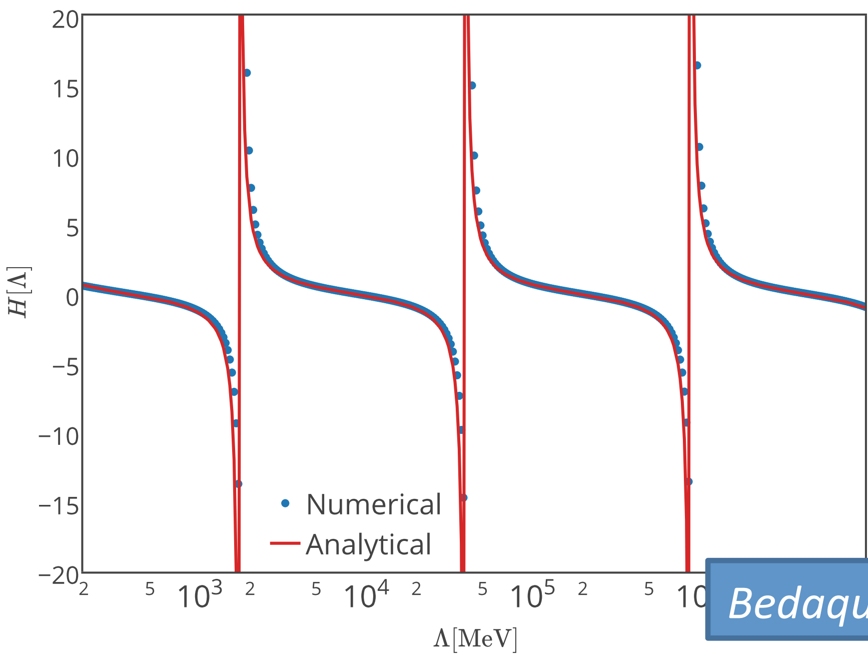
These are two alternatives to arrange the EFT expansion. The difference between their predictions is (one) measure of the theoretical uncertainty due to higher order corrections.

Phillips, Rupak, Savage, Phys. Lett. **B473**, 209 (2000)
Grießhammer, Nucl. Phys. **A744**, 192 (2004)

^{3}H:

- A=3 Efimov effect: *triton at LO has strong cutoff dependence* ➔ *add 3-body contact at LO.*





Regularization:
-- loop integrals cutoff at finite Λ.
-- each cycle is characterized by
the appearance of a new bound state.

Bedaque, Hammer, van Kolck (1999), Vansse (2014)

^{3}H-^{3}He binding energy difference:

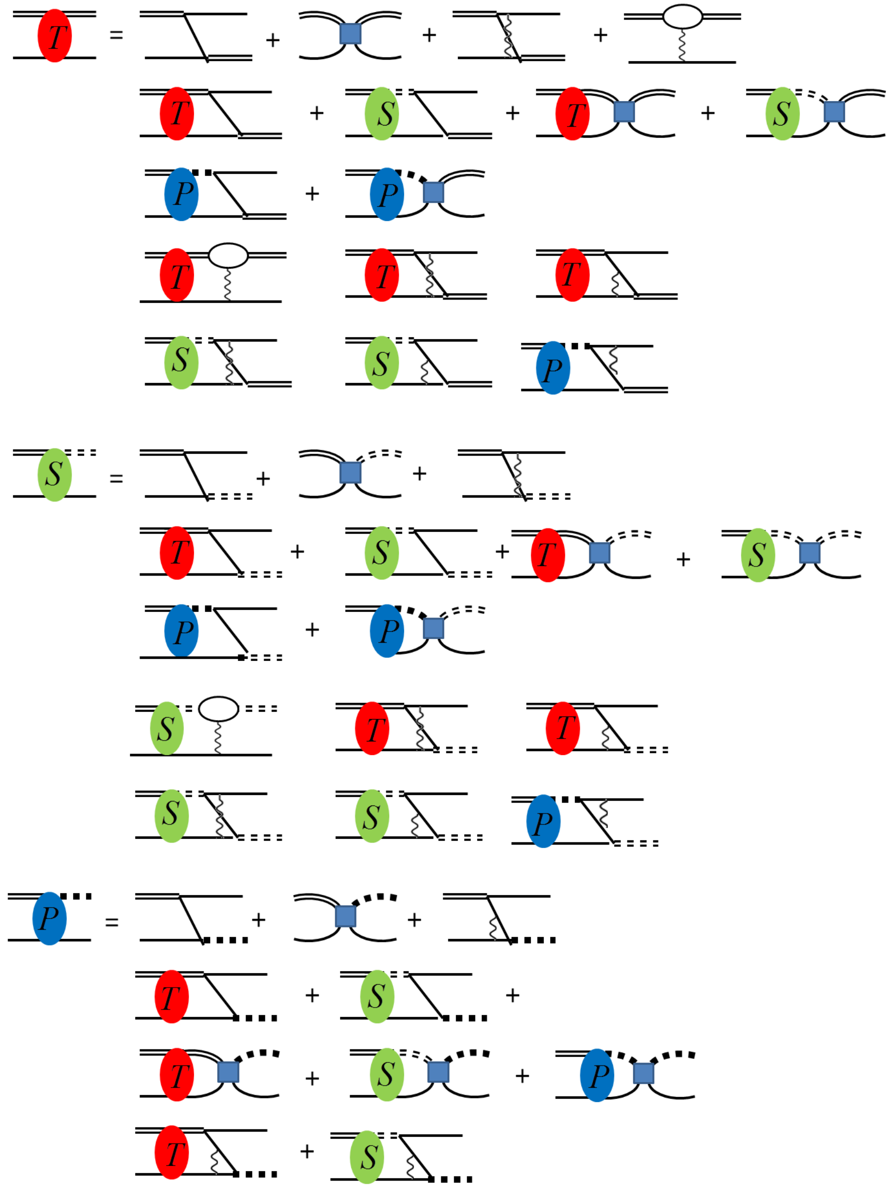
- Since the typical momentum is $Q \geq \sqrt{M_N E^B_{^3\mathrm{He}}} \simeq 85\mathrm{MeV}$, then the Coulomb interaction is perturbative:

$$\eta(Q) = \frac{\alpha M_N}{2Q} << 1$$

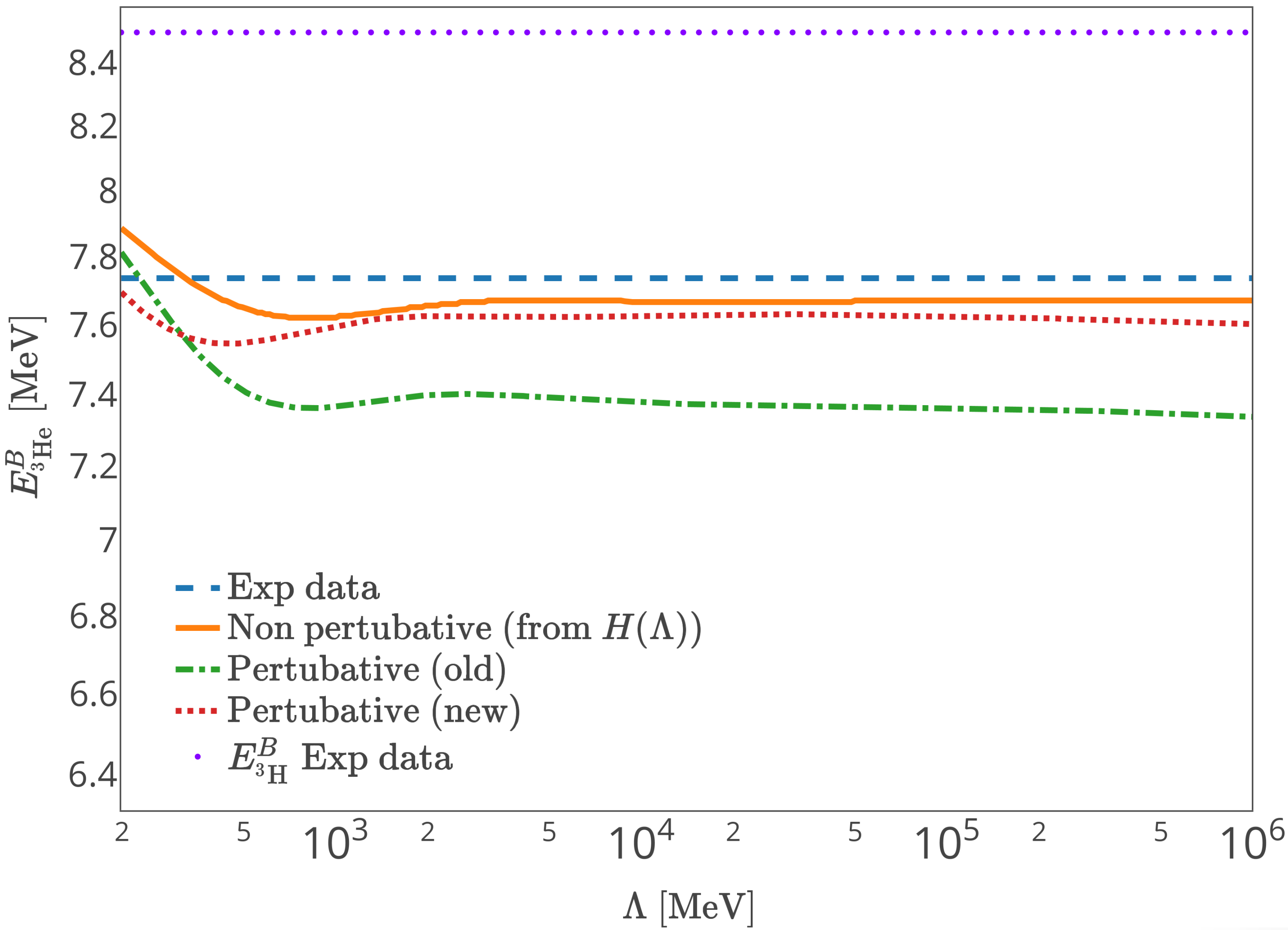
- The pp propogator always has to be renormalized (as Q can be low).
- two ways to find the A=3 b.e. difference:
  - Find the pole of a non-perturbative solution of the homogenous Fadeev equations with Coulomb (i.e., 3He w.f.).
  - Since Coulomb is perturbative in ^{3}He, one can calculate the energy shift in the one photon approximation.

König, Hammer (2011-15), Vannase et al (2014), König et al (2014-2016)

Faddeev eq. for ^{3}He



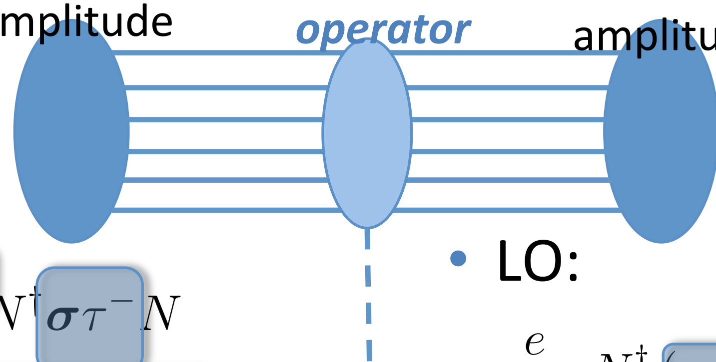
^3H-^3He binding energy difference:



König, Hammer (2011-15), Vannase et al (2014), König et al (2014-2016)

NLO corrections

- NLO corrections to the amplitude include effective range (or Z_d) insertions.
- For ^3H, no need in a new 3NF (just a renormalization of the LO 3NF).
- For ^3He, NLO leads to a new isospin breaking force at NLO (Vanasse et al 2014).
  - See König et al (2016) for one possible solution.
- Normalization of amplitude is unchanged for ^3H, and changes insignificantly for ^3He.



Weak

EM

- LO:

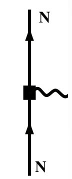
$$N^\dagger \frac{\boldsymbol{\tau}^-}{2} N + \frac{g_A}{2} N^\dagger \boldsymbol{\sigma}\tau^- N$$

Fermi

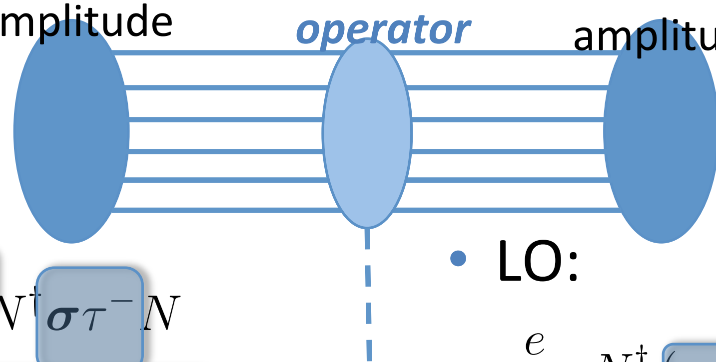
Gamow-Teller

- LO:

$$\frac{e}{2M_N} N^\dagger \left(\kappa_0 + \kappa_1 \tau_3\right) \sigma \cdot B N$$



Single nucleon interaction



- LO:

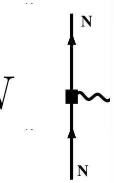
$$N^\dagger \frac{\boldsymbol{\tau}^-}{2} N + \frac{g_A}{2} N^\dagger \boldsymbol{\sigma}\tau^- N$$

Fermi

Gamow-Teller

- LO:

$$\frac{e}{2M_N} N^\dagger (\kappa_0 + \kappa_1 \tau_3)\, \sigma \cdot B N$$



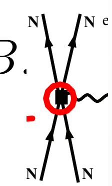
Single nucleon interaction

- NLO (correction to GT):

$$-L_{1A}(t^\dagger s + s^\dagger t)$$

- NLO:

$$-L_1'(t^\dagger s + s^\dagger t)\cdot B + L_2'(t^\dagger t)\cdot B.$$



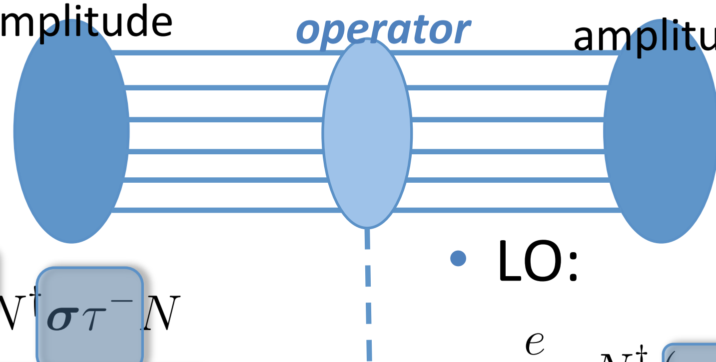
Simultaneous interaction with two nucleons coupled to singlet (s) / triplet (t)

Initial amplitude

Scattering operator

Final amplitude

Weak



EM

- LO:

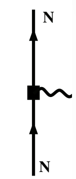
$$N^\dagger \frac{\tau^-}{2} N + \frac{g_A}{2} N^\dagger \boldsymbol{\sigma}\tau^- N$$

Fermi

Gamow-Teller

- LO:

$$\frac{e}{2M_N} N^\dagger (\kappa_0 + \kappa_1 \tau_3)\, \sigma \cdot B N$$



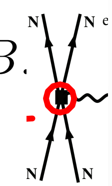
Single nucleon interaction

- NLO (correction to GT):

$$-L'_{1A}(t^\dagger s + s^\dagger t)$$

- NLO:

$$-L'_1(t^\dagger s + s^\dagger t)\cdot B + L'_2(t^\dagger t)\cdot B.$$



Simultaneous interaction with two nucleons coupled to singlet (s) / triplet (t)

$$L'_{1A} = g_A \left[\frac{1}{2}\frac{\rho_t + \rho_s}{\sqrt{\rho_t \rho_s}} - L_{1A}\frac{1}{2\pi\sqrt{\rho_t\rho_s} g_A} \left(\mu - \frac{1}{a_t}\right)\left(\mu - \frac{1}{a_s}\right)\right]$$

$$L'_1 = \frac{e}{2M_N}\left[-\frac{1}{2}\frac{\rho_t + \rho_s}{\sqrt{\rho_t\rho_s}}(\kappa_p - \kappa_n) + L_1 \frac{M_N}{\pi\sqrt{\rho_t\rho_s}}\left(\mu - \frac{1}{a_t}\right)\left(\mu - \frac{1}{a_s}\right)\right]$$

$$L'_2 = \frac{e}{2M_N}\left[L_2 \frac{2M_N}{\pi\rho_t}\left(\mu - \frac{1}{a_t}\right)^2 - (\kappa_p + \kappa_n)\right]$$

Should be cutoff invariant, nu +photon bremsstralung

n+p→d+γ

For low neutron energy: $\sigma = \frac{\alpha\left(\gamma_t^2+p^2\right)^3}{M_N^2 p} Y^2$

With:

$$Y = \sqrt{\frac{\pi}{\gamma_t}} \frac{\sqrt{Z_d}}{M_N} \left(\kappa_p - \kappa_n\right) \left[\left(\frac{1}{\gamma_t} - a_s\right) + \frac{a_s}{4}\left(\rho_t + \rho_s\right) - a_s \frac{M_N L_1}{2\pi\left(\kappa_p - \kappa_n\right)} \left(\mu - \frac{1}{a_t}\right)\left(\mu - \frac{1}{a_s}\right)\right]$$

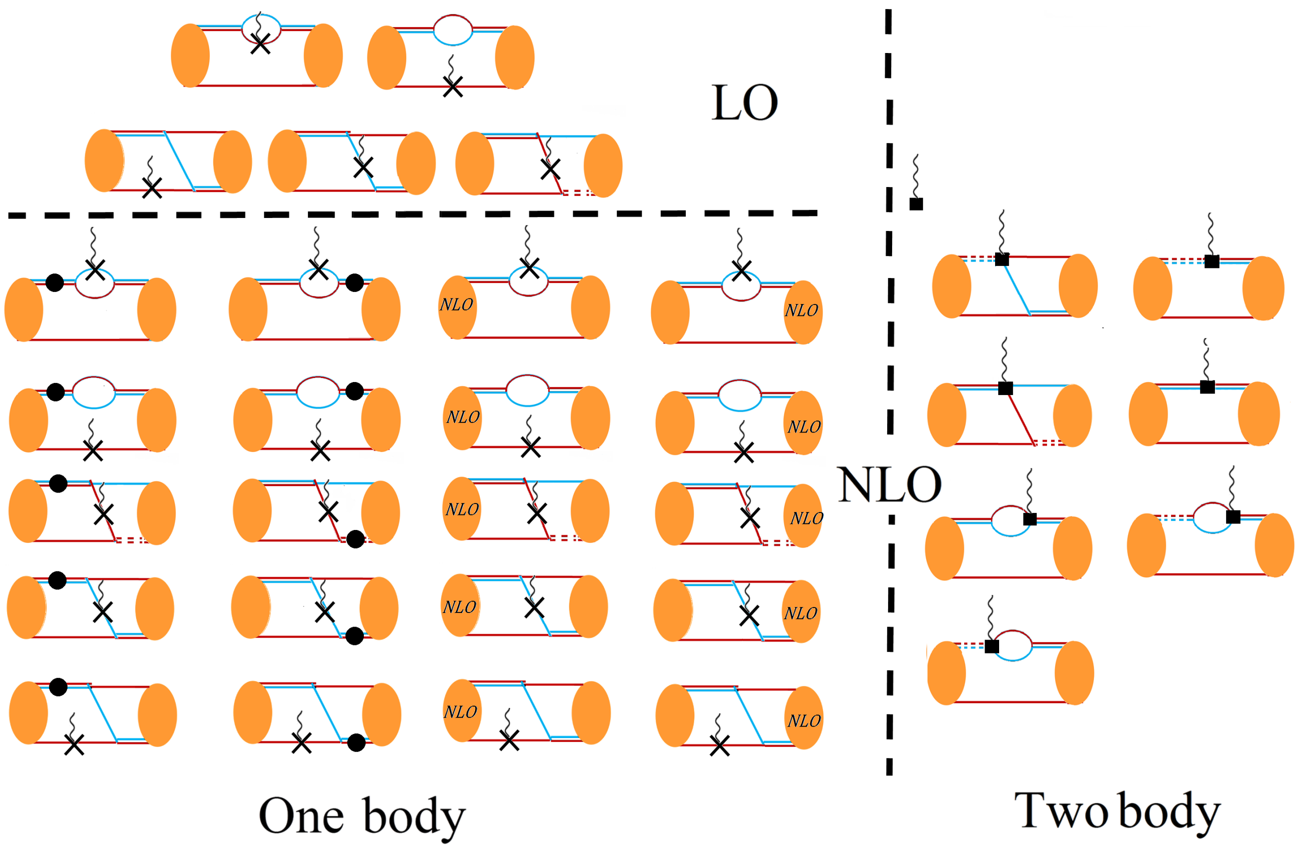
Remember: $Z_d^{LO} = 1$ and $Z_d^{NLO} = (1+\gamma\rho_t)$

$$\sigma_{np} = \frac{\pi\alpha\gamma_t^5 a_s^2\left(\kappa_p - \kappa_n\right)^2}{M_N^4 p} \left[\left(1 - \frac{1}{\gamma_t a_s}\right)\left(1 + \frac{1}{2}\gamma_t\rho_t\right) - \frac{\gamma_t}{4}\left(\rho_t + \rho_s\right) + \frac{\gamma_t M_N}{2\pi\left(\kappa_p - \kappa_n\right)} L_1 \left(\mu - \frac{1}{a_t}\right)\left(\mu - \frac{1}{a_s}\right)\right]^2$$

Magnetic moments

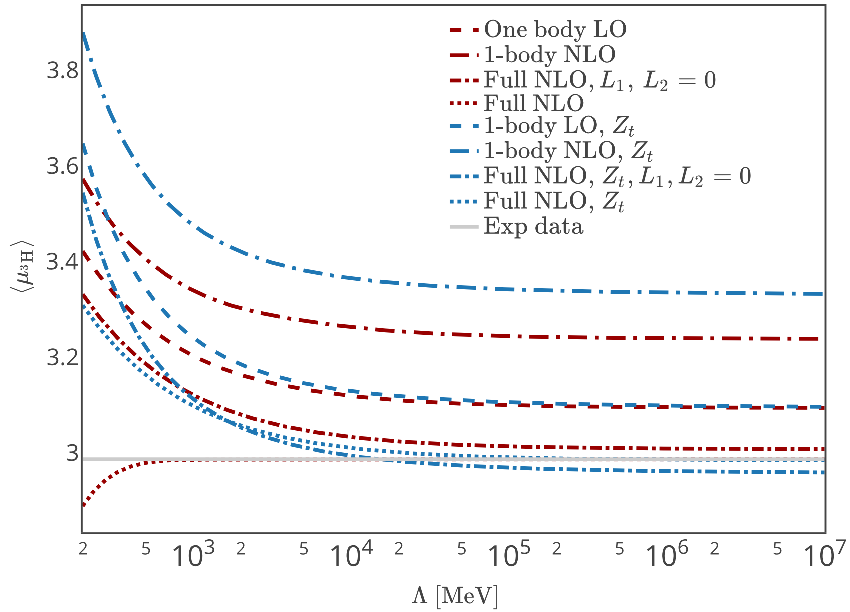
Deutron: $$\mu_d = \frac{e}{2M_N}(\kappa_n+\kappa_p)\left[1+L_2\frac{2M_N\gamma_t}{\pi(\kappa_n+\kappa_p)}\left(\mu-\frac{1}{a_t}\right)^2\right]$$

A=3:

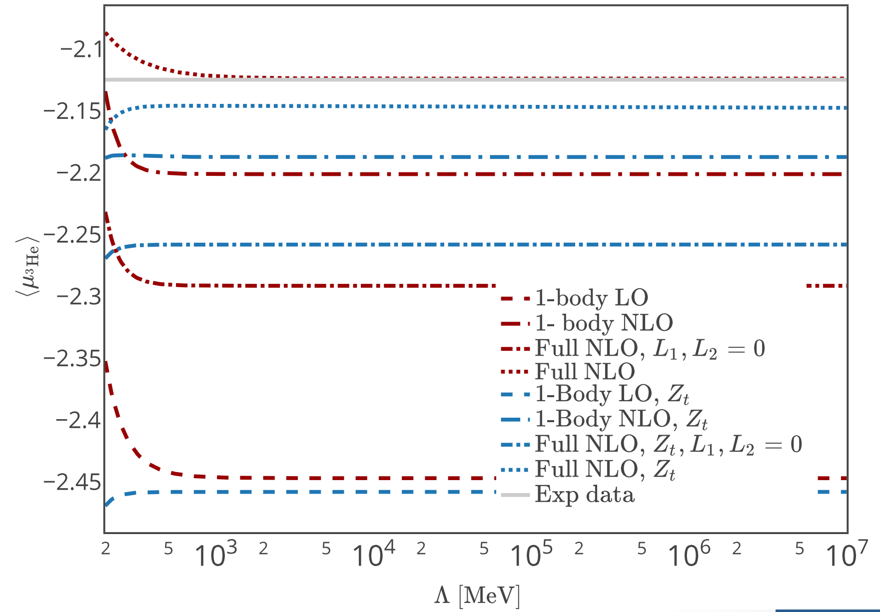


A=3 magnetic moments calculations:

^{3}H



^{3}He

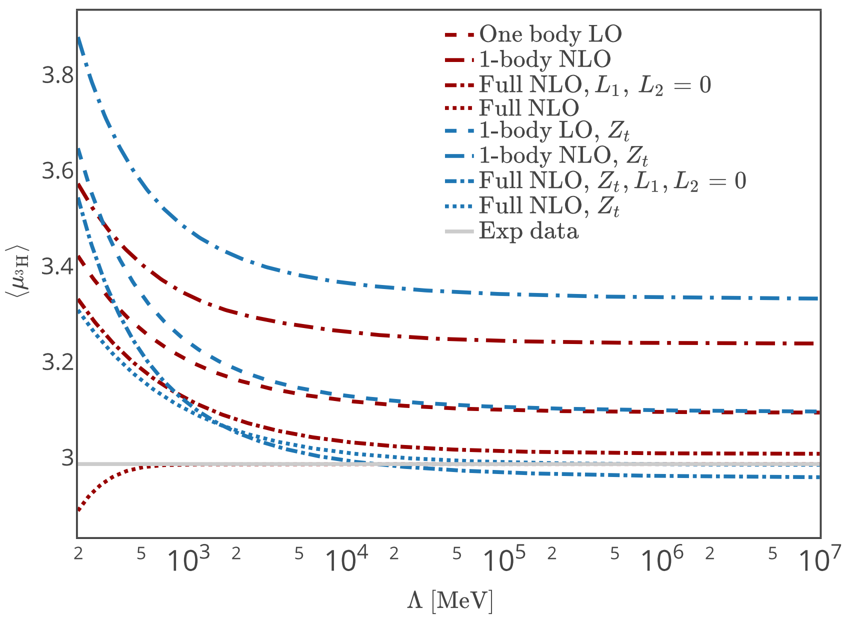


- All NLO contributions of the same order of magnitude 5-10% – Natural NLO contributions.
- Cutoff independence.
- When L_1 and L_2 are fixed **from A=2 observables**:

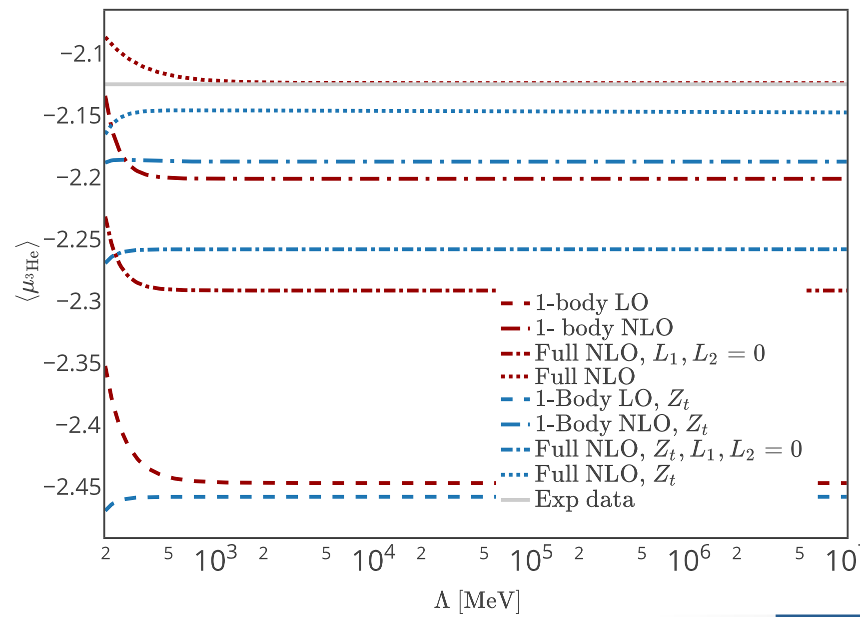
LO:	$\mu^{LO}_{^3\mathrm{H}} = 3.09 \pm_{Z_d} 0.01$	$\mu^{LO}_{^3\mathrm{He}} = -2.455 \pm_{Z_d} 0.005$
NLO:	$\mu^{NLO}_{^3\mathrm{H}} = 3.005 \pm_{Z_d} 0.01$	$\mu^{NLO}_{^3\mathrm{He}} = -2.13 \pm_{Z_d} 0.01$
exp:	$\mu^{\exp}_{^3\mathrm{H}} = 2.9789\ldots$	$\mu^{\exp}_{^3\mathrm{He}} = -2.1276\ldots$

A=3 magnetic moments calculations:

^{3}H



^{3}He



- When L_1 and L_2 are fixed **from A=3 magnetic moments**:

LO:	$\mu_d^{LO} = 0.8798$	$\sigma_{np}^{LO} = 298.2\,\mathrm{mb}$
NLO:	$\mu_d^{NLO} = 0.8617 \pm_{Z_d} 0.0002$	$\sigma_{np}^{NLO} = 335(Z_d) - 320(\rho)$
exp:	$\mu_d^{\exp} = 0.8574\ldots$	$\sigma_{np}^{\exp} = 334.2 \pm 0.5\mathrm{mb}\ldots$

Repeat using NPLQCD m.m. calculations

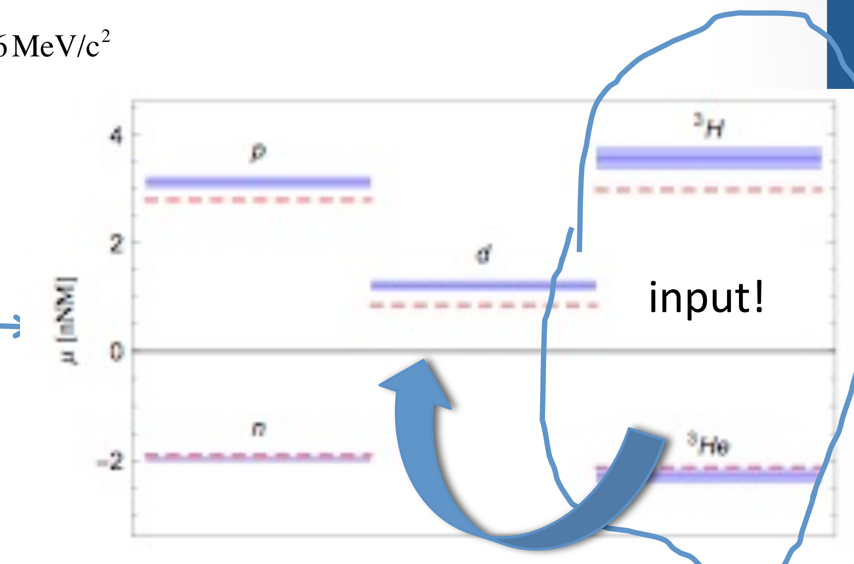
NPLQCD

Magnetic moments of light nuclei from lattice quantum chromodynamics

S.R. Beane,[1] E. Chang,[1,2] S. Cohen,[1,2] W. Detmold,[3] H.W. Lin,[1]
K. Orginos,[4,5] A. Parreño,[6] M.J. Savage,[1,2] and B.C. Tiburzi[7,8,9]
(NPLQCD Collaboration)

$$\delta E^{(B)} = -2\mu\,|\mathbf{B}| + \gamma\,|\mathbf{B}|^3$$

$for\;\; m_\pi = 806\,\mathrm{MeV/c^2}$



Postdiction from A=3 NPLQCD calcs:	$\mu_d^{NLO} = 0.90..{*}$
NPLQCD result:	$\mu_d^{NPLQCD} = 1.218(38)(87)$

*-preliminary

Using NPLQCD l_1 in our counting

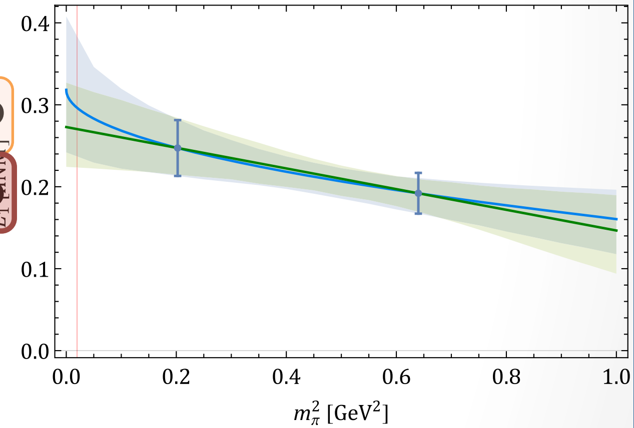
PRL **115,** 132001 (2015) PHYSICAL REVIEW LETTERS week ending 25 SEPTEMBER 2015

***Ab initio* Calculation of the $np \to d\gamma$ Radiative Capture Process**

Silas R. Beane,[1] Emmanuel Chang,[2] William Detmold,[3] Kostas Orginos,[4,5] Assumpta Parreño,[6]
Martin J. Savage,[2] and Brian C. Tiburzi[7,8,9]

(NPLQCD Collaboration)

$$\Delta E_{^3S_1,^1S_0}(\mathbf{B}) = 2(\kappa_1 + \gamma_0 Z_d^2 \tilde{l}_1)\frac{e}{M}|\mathbf{B}| + \mathcal{O}(|\mathbf{B}|^2),$$



Rho paramet. $l_1 = -3.934\,fm \;\; --> \;\; \sigma_{np} = 322.9\,\text{mb}$

Z-paramet. $l_1 = -5.48\,fm \;\; --> \;\; \sigma_{np} = 342.6\,\text{mb}$

This could be regarded as a measure of the NPLQCD uncertainty in predicting n+p fusion, due to the EFT Expansion.

Using NPLQCD l_1 in our counting

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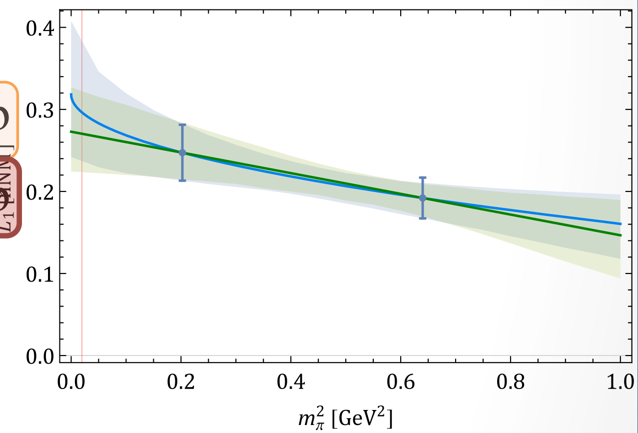

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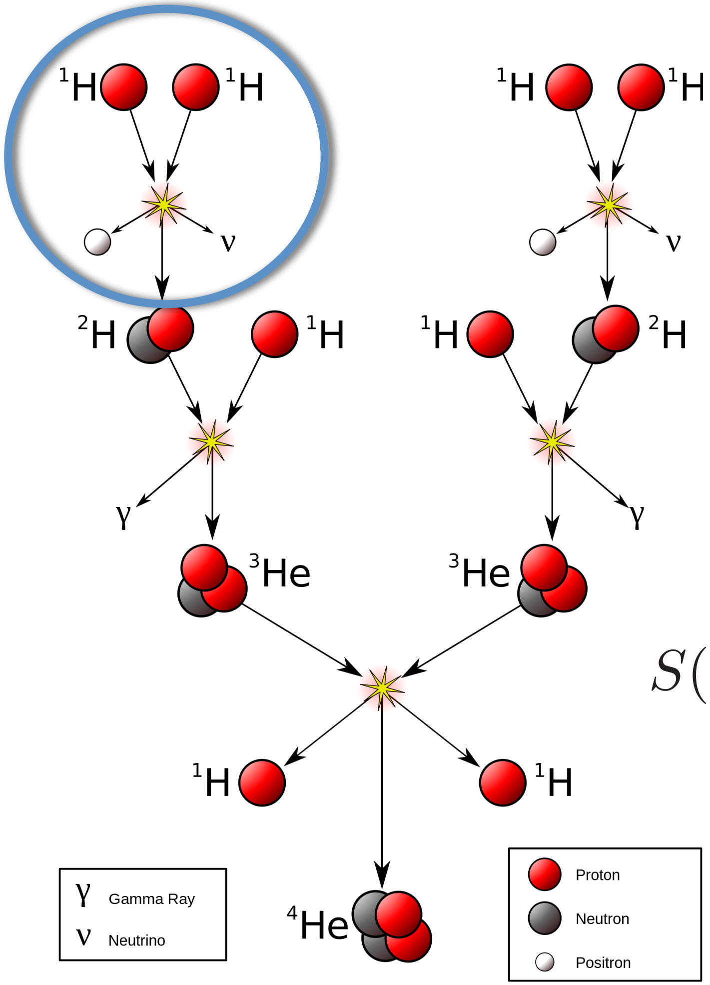
Weak proton-proton fusion in the Sun

SFII – Adelberger et al., Rev. Mod. Phys. 83, 195 (2011)



- Cannot be measured terrestrially – depends on theory
- Very low proton-proton relative momentum (***E_{rel}~6 keV***).
- Needed accuracy: ~1%.

$$\sigma(E) = \frac{S(E)}{E} \exp[-2\pi\eta(E)]$$

$$S(E) = S(0) + S'(0)E + S''(0)E^2/2 + \cdots$$

Weak proton-proton fusion in the Sun – theory standards

SFII – Adelberger et al., Rev. Mod. Phys. 83, 195 (2011)

$$4.01(1 \pm 0.009) \times 10^{-25}\ \mathrm{MeV\,b} \quad \text{potential models,}$$
$$4.01(1 \pm 0.009) \times 10^{-25}\ \mathrm{MeV\,b} \quad \text{EFT}^*,$$
$$3.99(1 \pm 0.030) \times 10^{-25}\ \mathrm{MeV\,b} \quad \text{pionless EFT.}$$

SFII recommended value (2011): $S_{11}(0) = 4.01(1 \pm 0.009) \times 10^{-25}\ \mathrm{MeV\,b}.$

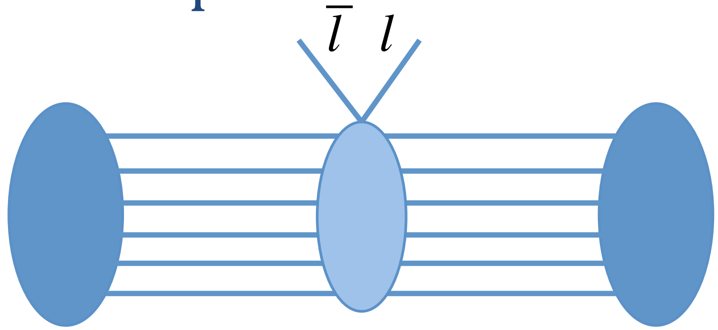
Modern cEFT calculation by Marcucci et al., Phys. Rev. Lett. (2013):
Use consistent ^3H decay-rate to constrain consistently axial MEC (DG, Quaglioni, Navratil, PRL 2009), and predict pp-fusion rate.

$$S(0) = (4.030 \pm 0.006) \times 10^{-23}\ \mathrm{MeV\ fm}^2$$

Including: p-wave contribution (+0.005%), full EM (-0.0025-(-0.0075)%), difference between 500 and 600 MeV cutoff and potential models.

Chen, Butler (2001-3), Ando et al (2005-8), Rupak (2014)

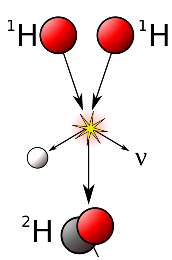
Pionless EFT description of weak interaction at low-energies

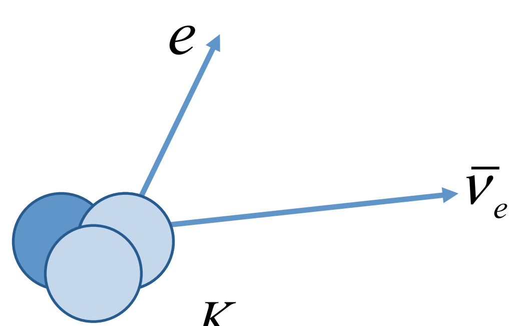


Initial amplitude | Weak Interaction | Final amplitude

$$\left\langle \psi_i \middle| \mathcal{J}_\mu \middle| \psi_f \right\rangle$$

$$\mathcal{J}_\mu^\pm = \frac{\tau_\pm}{2}\left(\mathcal{V}_\mu^\pm - \mathcal{A}_\mu^\pm\right)$$

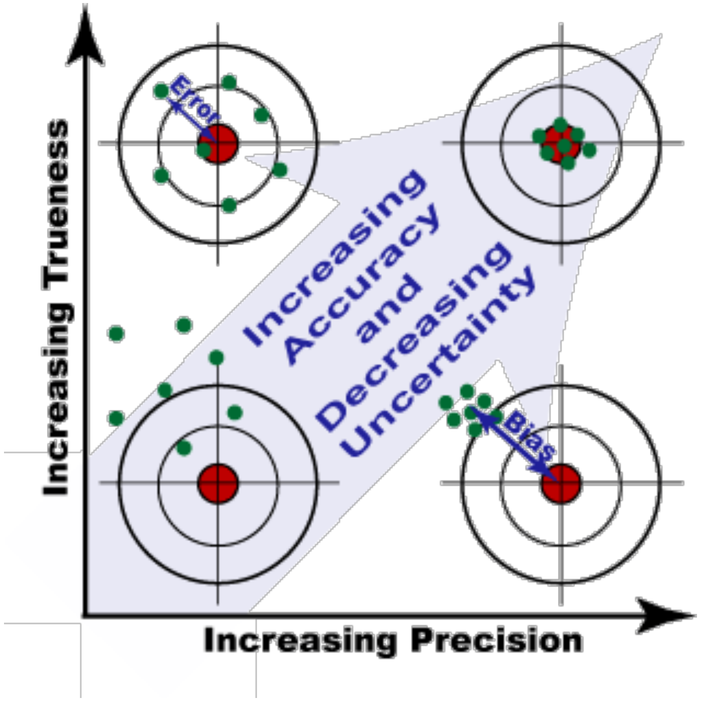




$$\left\langle pp \middle| \mathcal{A}_\mu^- \middle| {}^2\mathrm{H} \right\rangle$$

$$ft = \frac{K}{G_F^2 V_{ud}^2 \left[\left| \left\langle {}^3\mathrm{H} \middle\| \mathcal{V}_\mu^+ \middle\| {}^3\mathrm{He} \right\rangle \right|^2 + \frac{f_A}{f_V} \left| \left\langle {}^3\mathrm{H} \middle\| \mathcal{A}_\mu^+ \middle\| {}^3\mathrm{He} \right\rangle \right|^2 \right]}$$

Precision, Uncertainty, and predictions



Advantages of ⌀EFT UQ for proton-proton fusion:

1. Small number of parameters.
2. Two NLO ⌀EFT set-ups.
3. A “cheat-sheet” in the electromagnetic sector.
4. Cutoff independence up to infinity.

We revisit the pp-fusion problem within pionless EFT, fixing the unknown LEC using triton decay.

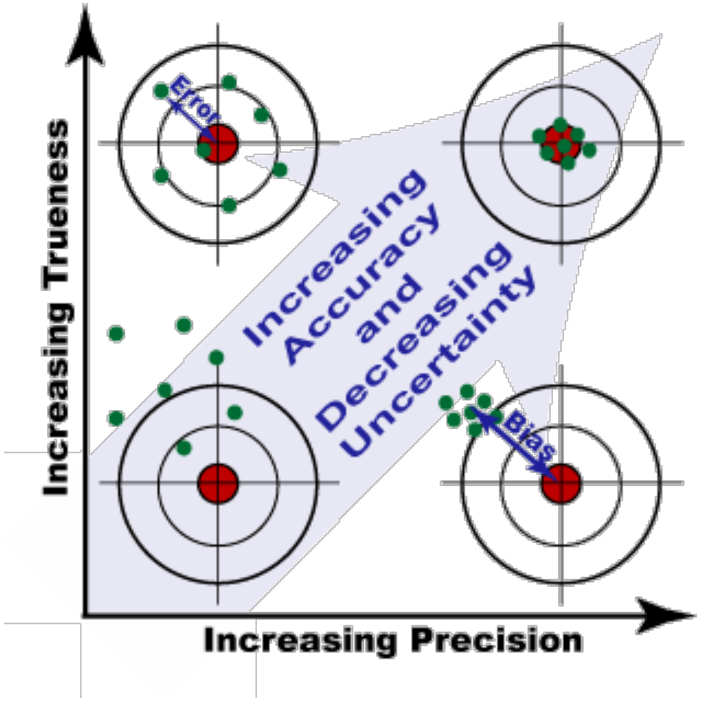
A fully perturbative pionless EFT A=2, 3 calculation @NLO

- LO Parameters:
  - nn and 2-np Scattering lengths: $^3S_1$, $^1S_0$.
  - pp scattering length.
  - Fine structure constant.
  - Three body force strength to prevent Thomas collapse.

- NLO parameters:
  - 2 effective ranges.
  - Renormalizations of pp and 3NF.
  - (isospin dependent 3NF to prevent logarithmic divergence in the binding energy of ^{3}He).

- ***Weak Interaction: LO (g_A – 1 body), NLO (L_{1A} – 2 body)***
- ***EM Interaction: LO (k_S , k_V) – 1 body), NLO (L_1 , L_2– 2 body)***

Precision, Uncertainty, and predictions



Advantages of $\not{\pi}$EFT UQ for proton-proton fusion:

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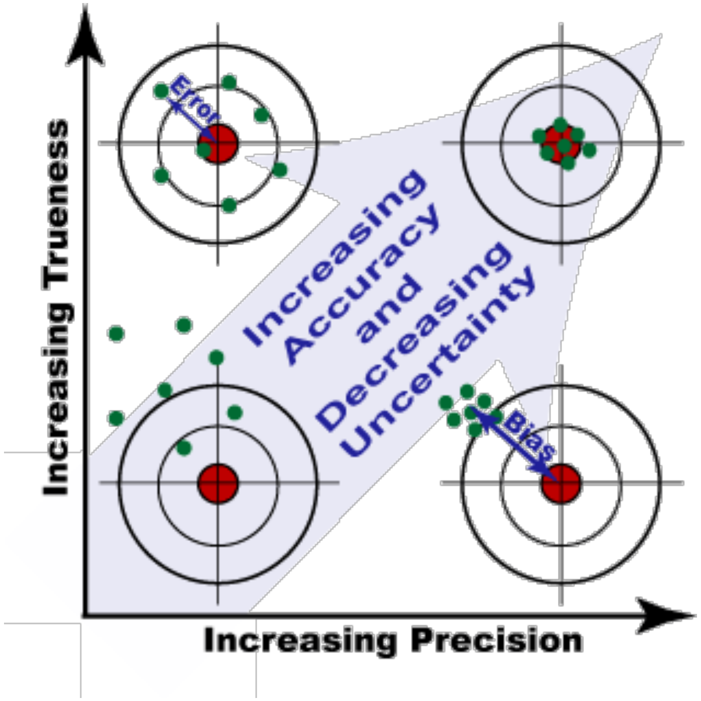
Precision, Uncertainty, and predictions



Advantages of ∅EFT UQ for proton-proton fusion:

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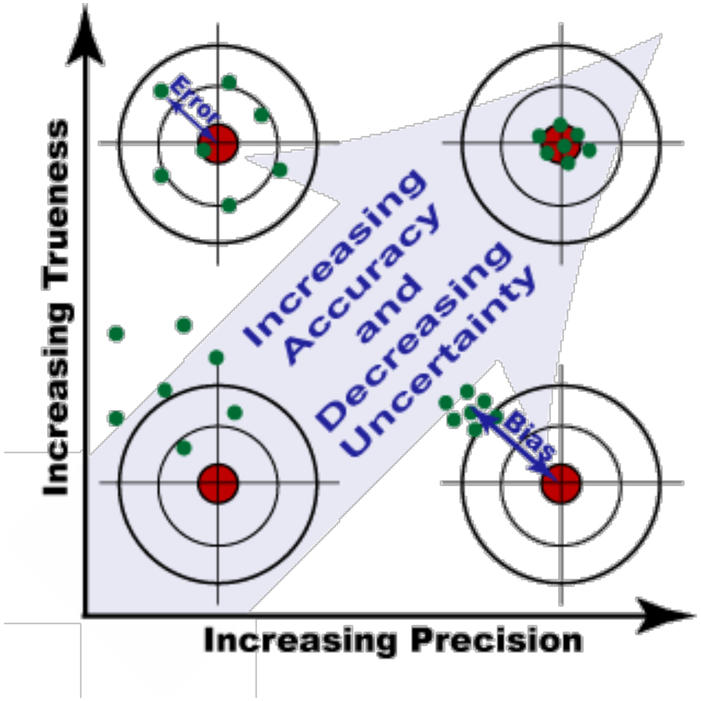
Precision, Uncertainty, and predictions



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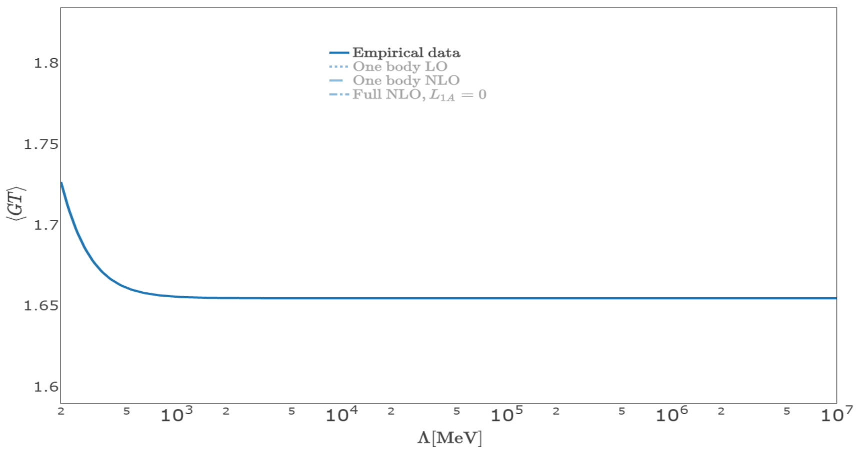
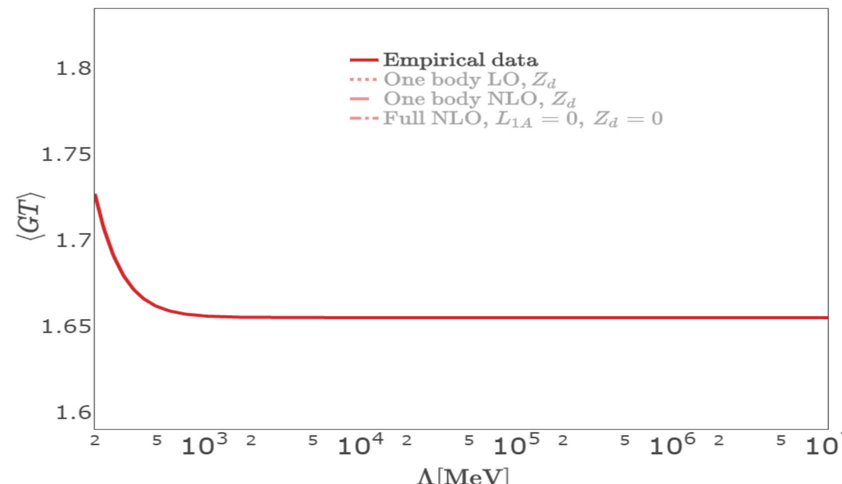
Triton decay – GT cutoff independence

Rho-parameterization **Z-parameterization**

$$ft = \frac{K}{G_F^2 V_{ud}^2 \left[\left| \left\langle {}^3\mathrm{H} \left\| \mathcal{V}_\mu^+ \right\| {}^3\mathrm{He} \right\rangle \right|^2 + \frac{f_A}{f_V} \left| \left\langle {}^3\mathrm{H} \left\| \mathcal{A}_\mu^+ \right\| {}^3\mathrm{He} \right\rangle \right|^2 \right]}$$





"Empirical" extraction of GT (using calculated F strength)

Triton decay – GT cutoff independence

Rho-parameterization **Zed-parameterization**

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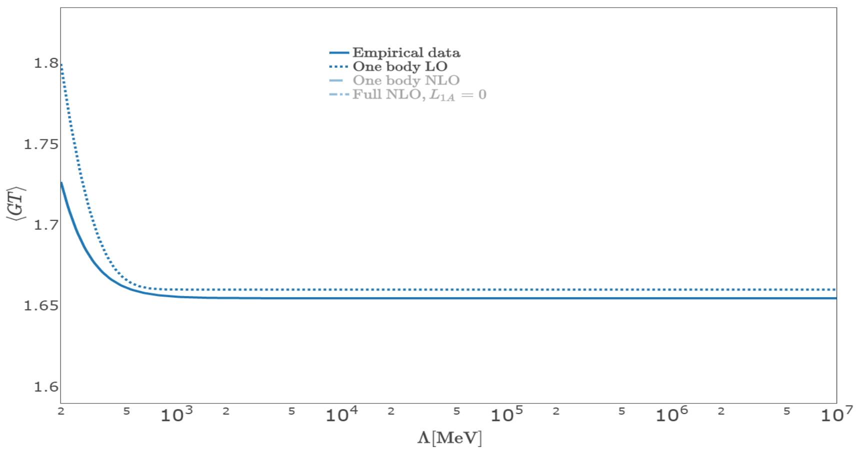

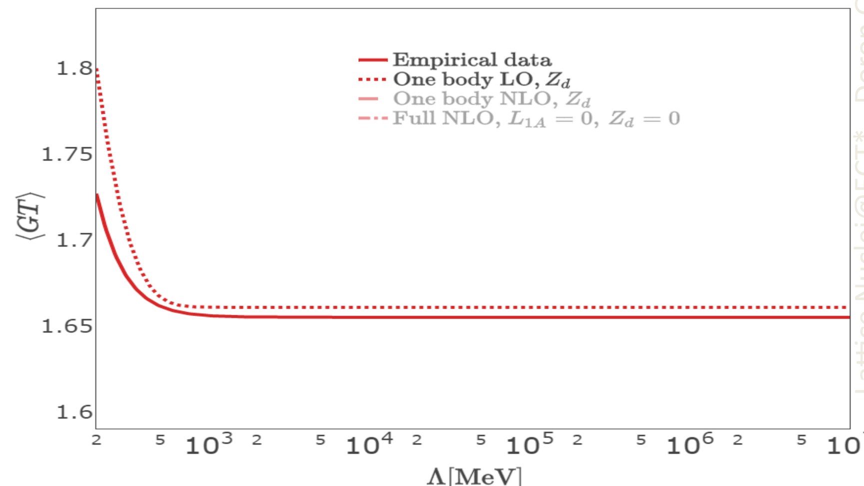

Adding the LO 1-body contribution

Triton decay – GT cutoff independence

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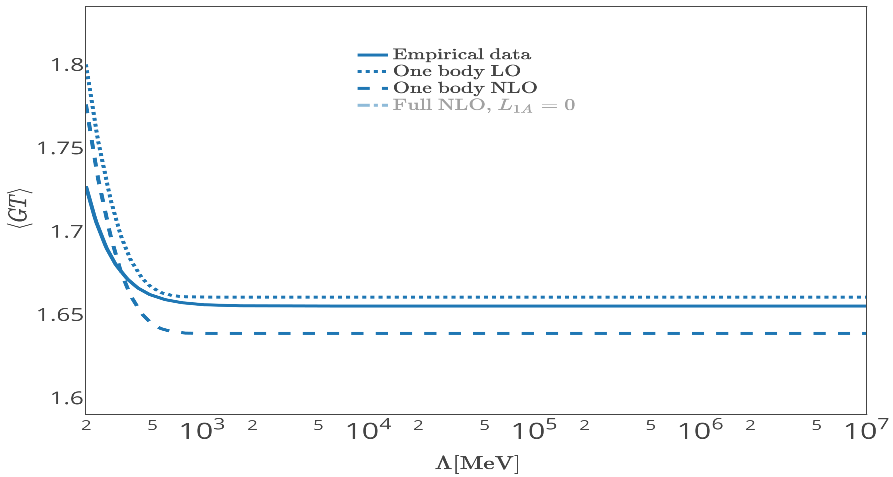

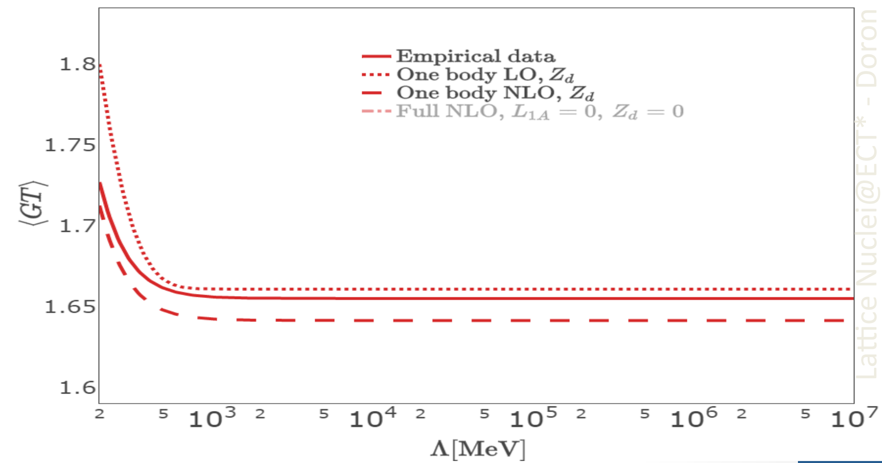

Adding the NLO 1-body contributions

Triton decay – GT cutoff independence

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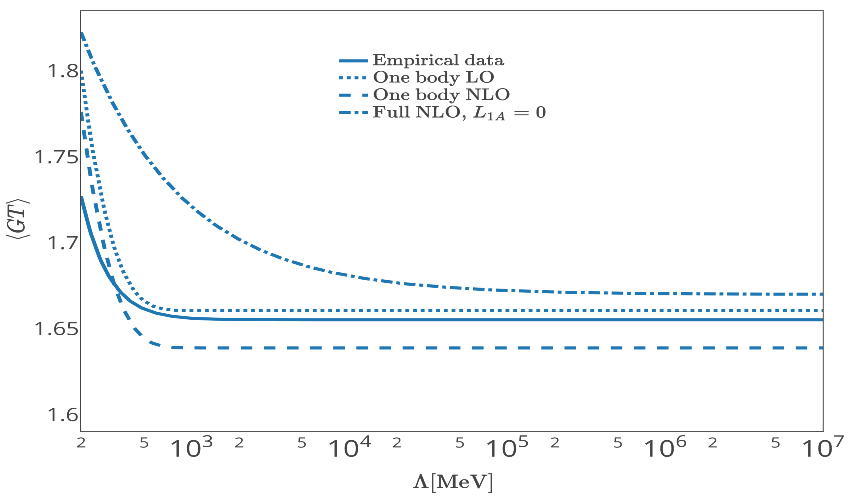
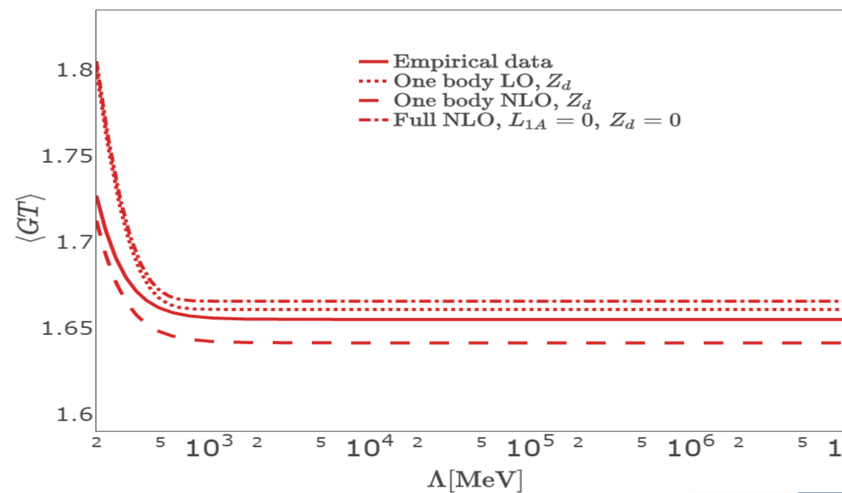




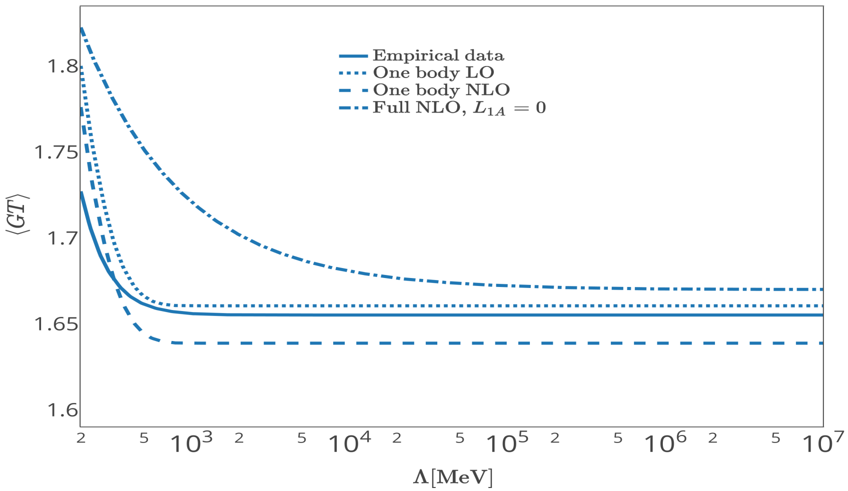
Adding all contribution, but L_{1A}

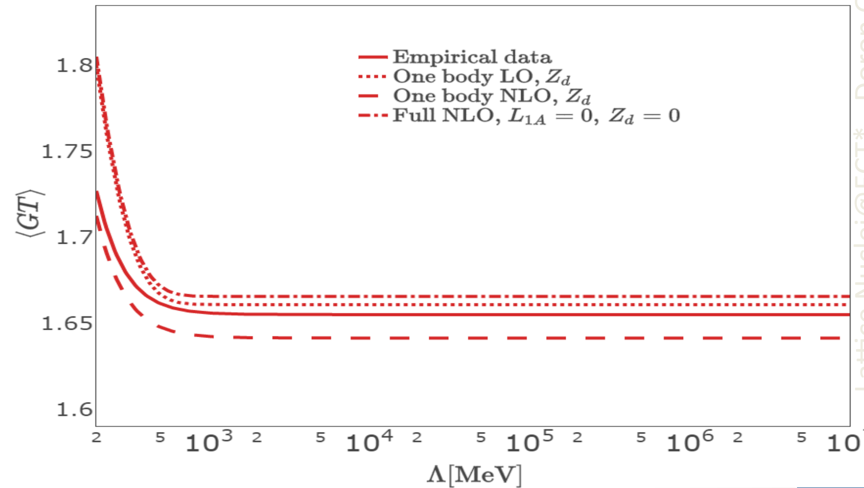
1st estimate of theoretical uncertainty:
All NLO contributions are of the same order,
one can estimate higher order effects as the NLO contribution.

Triton decay – GT cutoff independence

Rho-parameterization **Zed-parameterization**

$$ft = \frac{K}{G_F^2 V_{ud}^2 \left[\left| \left\langle {}^3\mathrm{H} \right\| \mathcal{V}_\mu^+ \left\| {}^3\mathrm{He} \right\rangle \right|^2 + \frac{f_A}{f_V} \left| \left\langle {}^3\mathrm{H} \right\| \mathcal{A}_\mu^+ \left\| {}^3\mathrm{He} \right\rangle \right|^2 \right]}$$





Adding all contributi

Translates to ±2% difference in pp fusion

1st estimate of theoretical uncertainty:
All NLO contributions are of the same order,
one can estimate higher order effects as the NLO contribution.

So… is 3% too big to be called precision physics?

$$S_{pp}(g_A = 1.2695) = \left(3.83(\rho) - 3.99(Z)\right) \pm 0.07 \pm 0.04$$

$$S_{pp}(g_A = 1.275) = \left(3.96(\rho) - 4.12(Z)\right) \pm 0.07 \pm 0.04$$

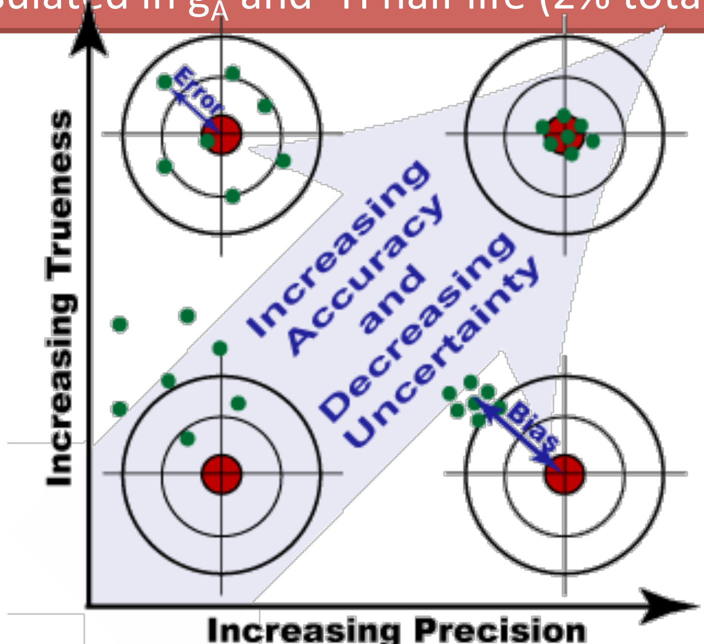
g_A systematic uncertainty

theoretical uncertainty

g_A stat. unc.

^3H halflife syst. unc.

i.e., theoretical uncertainty of the same order of systematic experimental error encapsulated in g_A and ^3H half life (2% total).



Summary

- Pionless EFT reproduces low-energy electroweak observables to a very good precision (~1%), even at NLO, and allows reliable uncertainty estimates.
- A coherent use of pionless EFT allows to estimate model uncertainty and higher order contribution.
- Pionless EFT allows assessing Lattice QCD calculations.
- Based on the EM sector, a theoretical prediction for pp fusion:

$$S_{pp}(g_A = 1.2701) = 4.01 \pm_{theory} 0.08 \pm_{g_A(1\sigma)} 0.07 \pm_{^3\text{H half life}} 0.04$$

$$S_{pp}(g_A = 1.275) = 4.12 \pm_{theory} 0.08 \pm_{g_A(1\sigma)} 0.07 \pm_{^3\text{H half life}} 0.04$$

- Better determination of g_A is necessary!
- (^{3}H half life is also an open exp. issue).