



Universal aspects of weakly bound two-neutron halo nuclei

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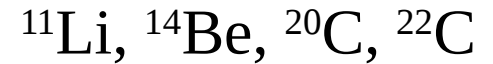
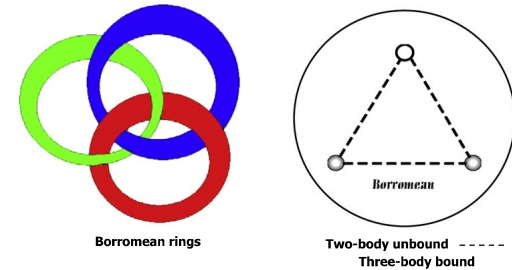
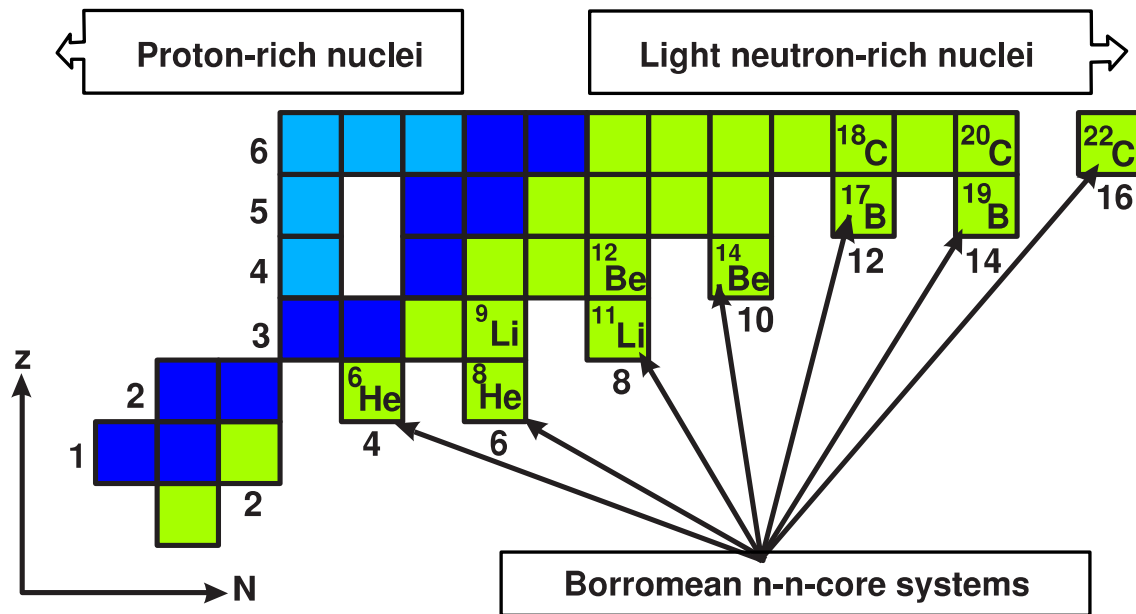
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INT program "Nuclear Physics from lattice QCD"
April 29, 2016



Light-neutron rich nuclei



C.A. Bertulani, Nuclear Physics in a Nutshell, Princeton University Press, 2007.

TF, Delfino, Tomio, Yamashita, “Universal aspects of light halo nuclei”
 Prog. Part. Nucl. Phys. 67 (2012) 939

Tanihata, Savajols Kanungo. “Recent experimental progress in nuclear halo
 structure studies Prog. Part. Nucl. Phys. 68 (2012) 215”

Zinner, Jensen. ”Comparing and contrasting nuclei and cold atomic
 gases”. J. Phys. G: Nucl. Part. Phys. 40 (2013) 053101

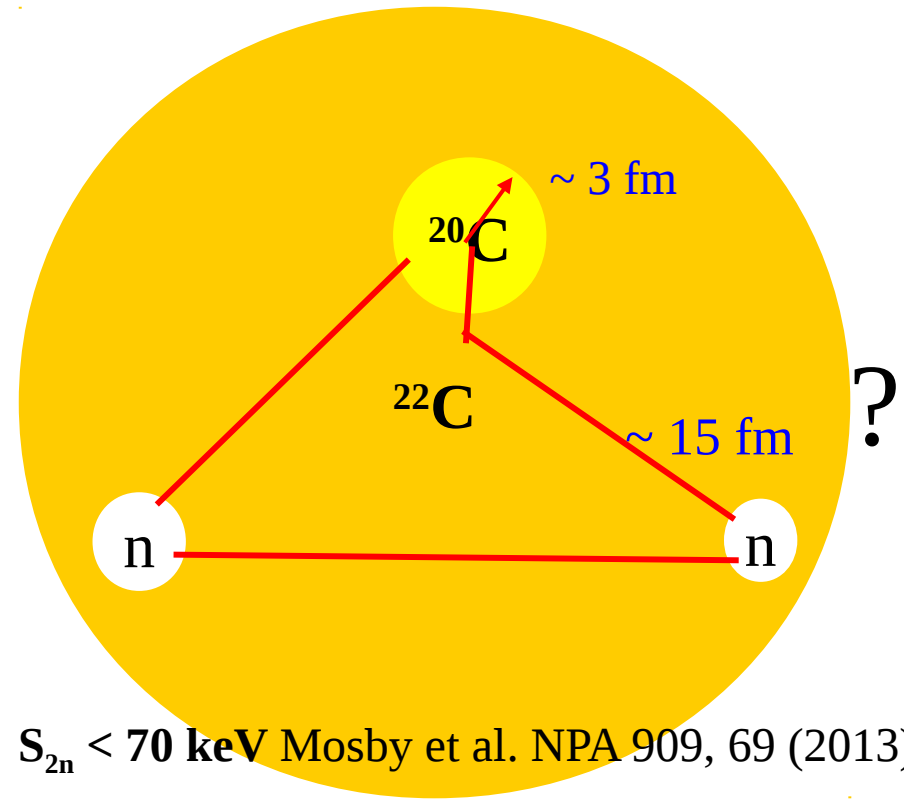
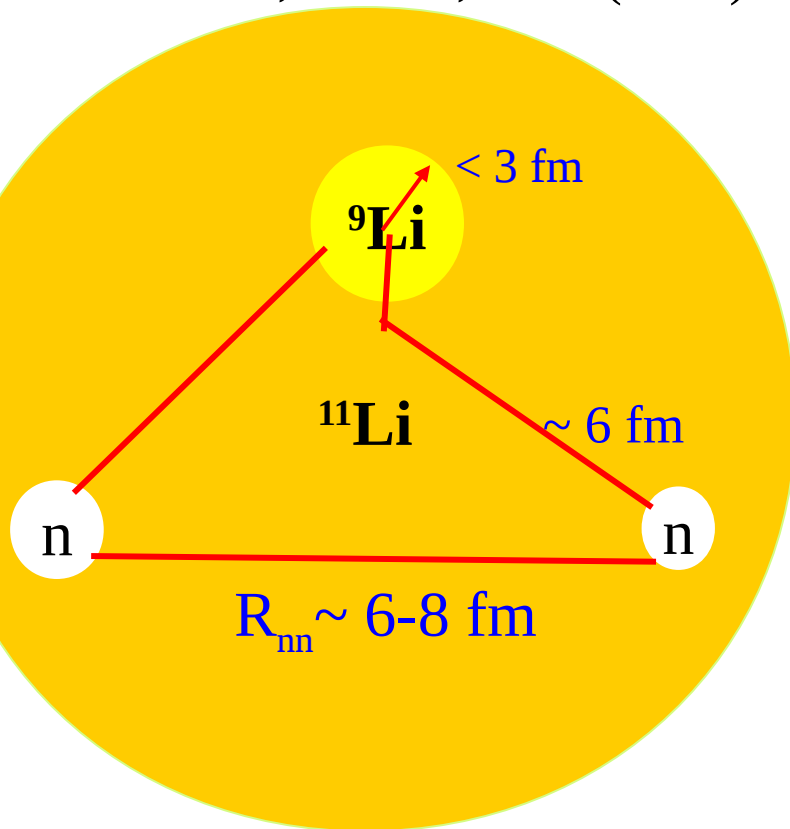
Neutron-neutron-core model for light halo nuclei

Borromean systems: Two-body subsystems unbound

^{11}Li : $S_{2n} = 369 \text{ keV}$ - Smith et al. PRL101, 202501 (2008)

Tanihata et al., PRL55, 2676 (1985)

Tanaka et al. PRL104, 062701 (2010)



$S_{2n} < 70 \text{ keV}$ Mosby et al. NPA 909, 69 (2013)

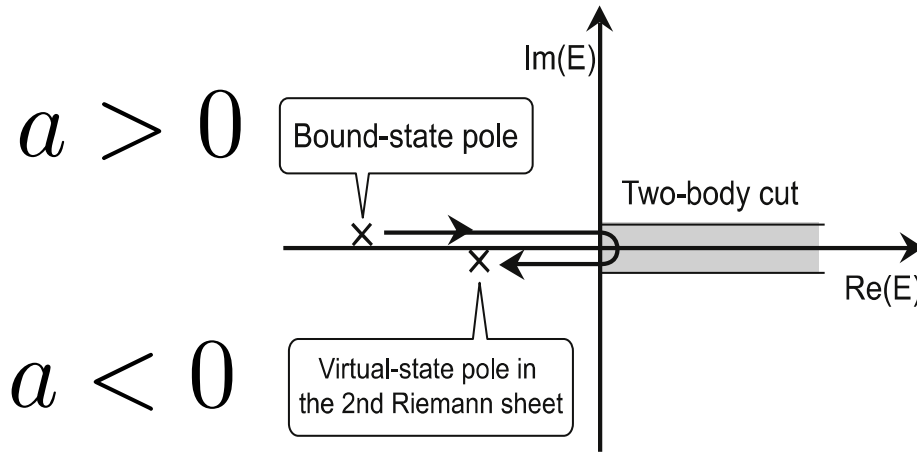
K Riisager Phys. Scr. T152 (2013) 014001 suggests a smaller size!

Core mom. dist. from RIKEN - Kobayashi et al PRC86, 054604 (2012)

(S-wave interaction) Two-body s-wave phase-shift (large scatt. lengths)

$$k \cot(\delta) = -\frac{1}{a} + \frac{r_0}{2} k^2 + \dots$$

$|a| \gg r_0$



- 1S_0 nn **virtual** state: $E_{nn}^{virtual} = -143$ keV ($a = -17$ fm)
- S-wave n-core state:
virtual ($^{10}\text{Li} \sim -50$ keV) or **bound** ($^{19}\text{C} \sim -580$ keV)

Light Halo-nuclei are examples in recent Nuclear Physics studies

Weakly-Bound Quantum Systems: very Large and Dilute

$$(E - H_0)\psi = 0$$

Almost everywhere the wave-function is an eigenstate of H_0

-- short-range force – classically forbidden region --

Physics: symmetry, scaling properties and dimension (& mass ratios)

→ Universality (model independence)

Light halo-nuclei, such as ^{11}Li , ^{14}Be , ^{20}C , ^{22}C , can be described as a three-body n-n-core mass-imbalanced system, where at least one of the two-body subsystem (n-n) is unbound.

Generalization: “The few scales of nuclei and nuclear matter”
Delfino, TF, Timóteo, Tomio. PLB 634 (2006) 185

Limiting case: what is the physics of a contact interaction?

Three-boson system

Subtle three-body phenomenon in $L_{\text{total}}=0$:

Thomas(1935) – Efimov (1970) effect!

$$|a|/r_0 \rightarrow \infty$$

Adhikari, Delfino,TF,Goldman,Tomio, PRA37 (1988) 3666

One three-body scale is necessary to represent short-range physics !!!!
& discrete scaling

Jensen, Riisager, Fedorov, Garrido, RMP76, 215 (2004)

Braaten, Hammer Phys. Rep.428, 259 (2006)

The Efimov effect

Efimov Physics (1970): Nuclear Physics

Vitaly Efimov

Vol 440/16 March 2006 | doi:10.1038/nature04626

nature

LETTERS

Evidence for Efimov quantum states in an ultracold gas of caesium atoms

T. Kraemer¹, M. Mark¹, P. Waldburger¹, J. G. Danzl¹, C. Chin^{1,2}, B. Engels¹, A. D. Lange¹, K. Pilch¹, A. Jaakkola¹, H.-C. Nägerl¹ & R. Grimm^{1,2}

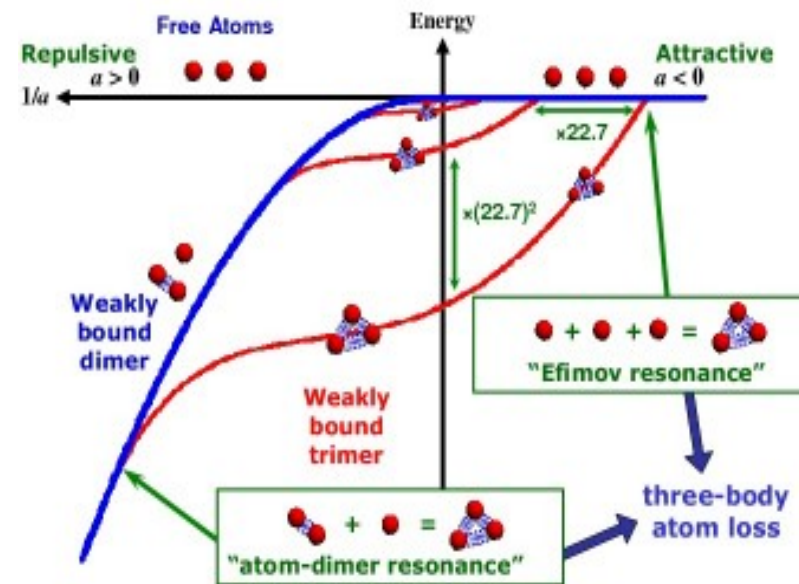
nature physics

LETTERS

PUBLISHED ONLINE: 22 FEBRUARY 2009 | DOI: 10.1038/NPHYS1203

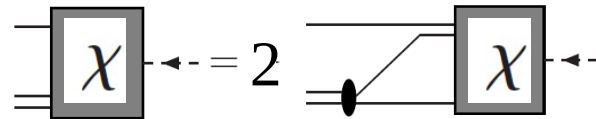
Observation of an Efimov-like trimer resonance in ultracold atom-dimer scattering

S. Knoop^{3*}, F. Feriaino¹, M. Mark¹, M. Berninger², H. Schöbel¹, H.-C. Nägerl¹ and R. Grimm^{1,2}



Zero-range 3-boson equation: Thomas-Efimov effect (3d)

Skorniakov and Ter-Martirosian equations (1956)



$$\text{Diagram 1} = 2 \text{Diagram 2} \quad (\hbar = m = 1)$$

$$\chi(\vec{y}) = \frac{-\pi^{-2}}{\pm \sqrt{\epsilon_2} - \sqrt{\epsilon_3 + \frac{3}{4}y^2}} \int d^3x \left(\frac{1}{\epsilon_3 + y^2 + x^2 + \vec{y} \cdot \vec{x}} - \frac{1}{1 + y^2 + x^2 + \vec{y} \cdot \vec{x}} \right) \chi(\vec{x})$$

$$\epsilon_{\bar{3}} = E_3 / \mu_{(3)}^2 \quad \epsilon_2 = E_2 / \mu_{(3)}^2 \quad \mu_{(3)}^2 = 1$$

Thomas collapse: $\mu_{(3)}^2 \rightarrow \infty$

$$\epsilon_2 = E_2 / \mu_{(3)}^2$$

Efimov effect: $E_2 \rightarrow 0$

Thomas-Efimov effect!

S.K. Adhikari, A. Delfino, T. Frederico, I.D. Goldman, and L. Tomio, Phys. Rev. A **37**, 3666 (1988).

Hamiltonian for the Subtracted 3B equations (3D)

Frederico et al., PPNP 67, 939 (2012)

Subtracted-Faddeev equations 3B:

$$T_k(E) = t_{(ij)} \left(E - \frac{q_k^2}{2m_{ij,k}} \right) [1 + (G_0^{(+)}(E) - G_0(-\mu_3^2)) (T_i(E) + T_j(E))]$$

Adhikari, Frederico, Goldman, PRL 74 (1995) 487

Renormalized Hamiltonian:

$$H_{\mathcal{R}} = H_0 + V_{\mathcal{R}} \quad \frac{\partial V_{\mathcal{R}}}{\partial \mu^2} = 0 \quad \text{and} \quad \frac{\partial H_{\mathcal{R}}}{\partial \mu^2} = 0.$$

$$V_{\mathcal{R}} = H_{\mathcal{R}I}^{(3B)} = \sum_{(ij)} V_{\mathcal{R}(ij)}^{(2B)} + V_{\mathcal{R}}^{(3B)}.$$

EFT 3B interaction: Bedaque, Hammer, van Kolck PRL 82 (1999) 463

RGE & Subtracted Eqs. - TF, Delfino, Tomio, PLB 481 (2000) 143

Scaling function & limit cycle

$$\epsilon_3^{(N)} \equiv \epsilon_3^{(N)} (\pm \sqrt{\epsilon_2}) \quad \xi \equiv \pm \sqrt{\epsilon_2} = \pm (E_2 \epsilon_3^{(N)} / E_3^{(N)})^{1/2}$$

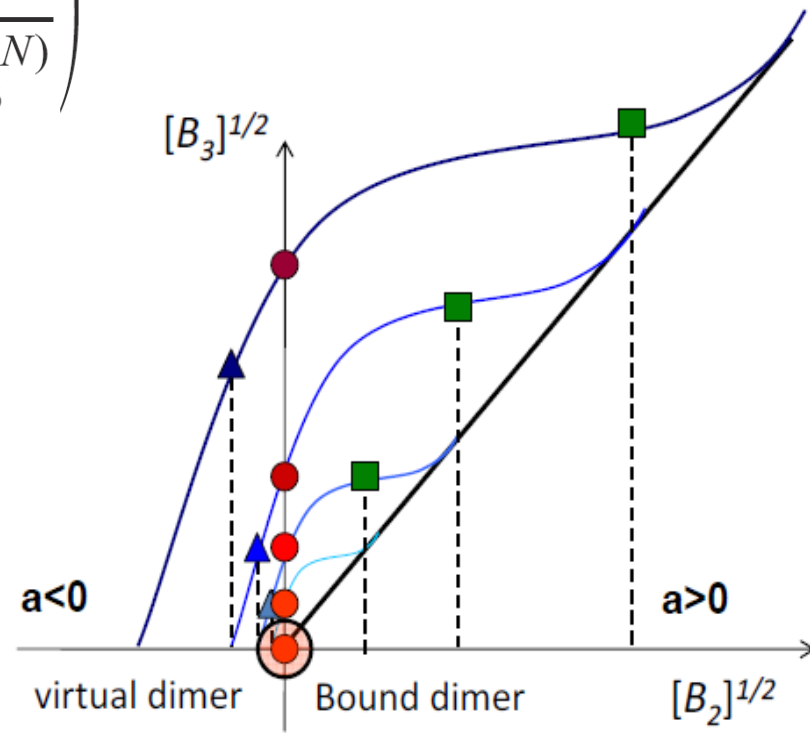
$$\frac{E_3^{(N+1)}}{E_3^{(N)}} = \lim_{N \rightarrow \infty} \frac{\epsilon_3^{(N+1)}(\xi)}{\epsilon_3^{(N)}} = \mathcal{F} \left(\pm \sqrt{\frac{E_2}{E_3^{(N)}}} \right)$$

$$\mathcal{F}(0) = e^{-2\pi/s_0} = 1/515$$

Frederico et al, “**Scaling limit of weakly bound triatomic states**”, PRA60 (1999)R9.

Amorim et al, “**Universal aspects of Efimov states and light halo nuclei**”, PRC56(1997) R2378

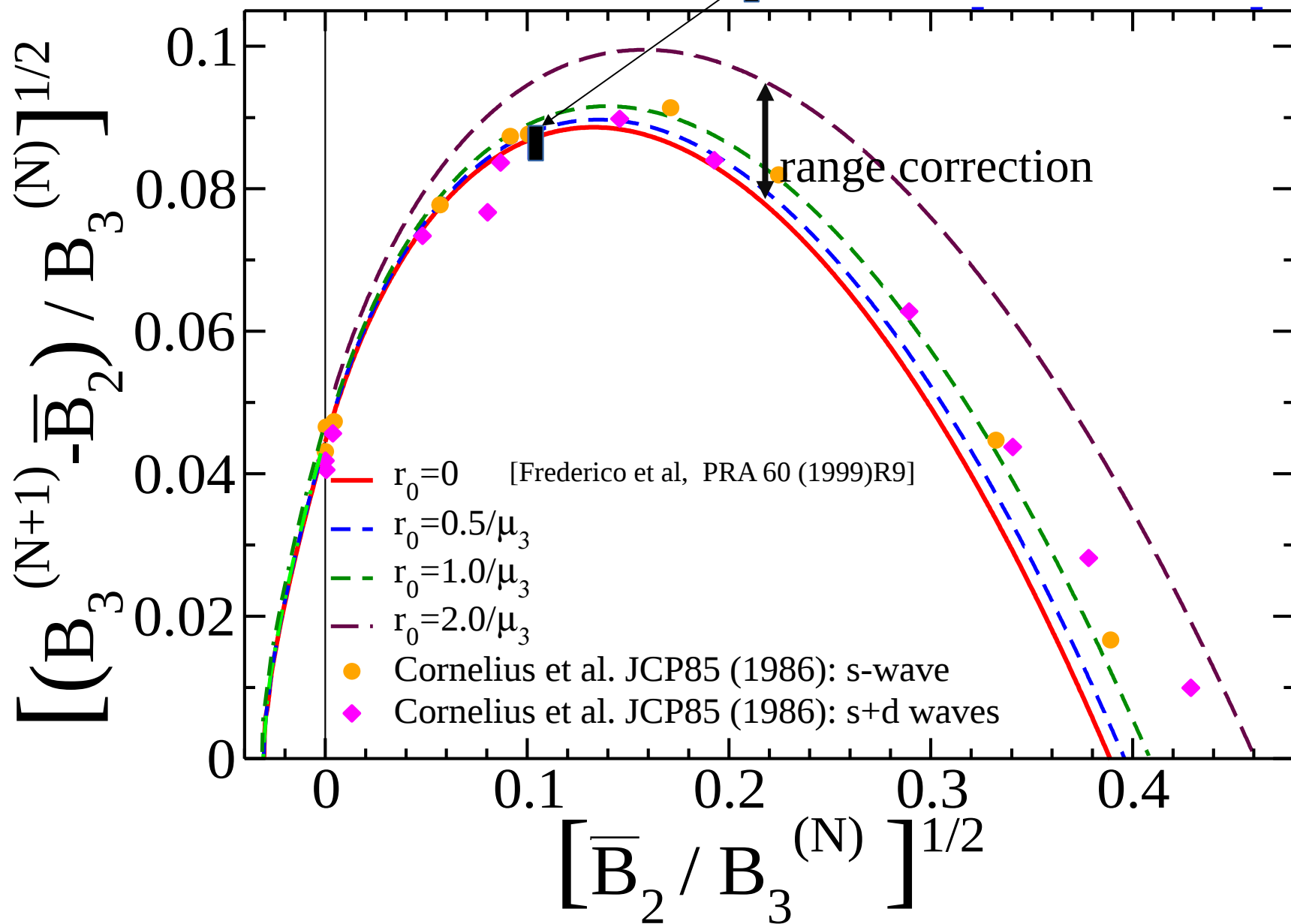
Limit cycle: Mohr et al Ann.Phys. 321 (2006) 225



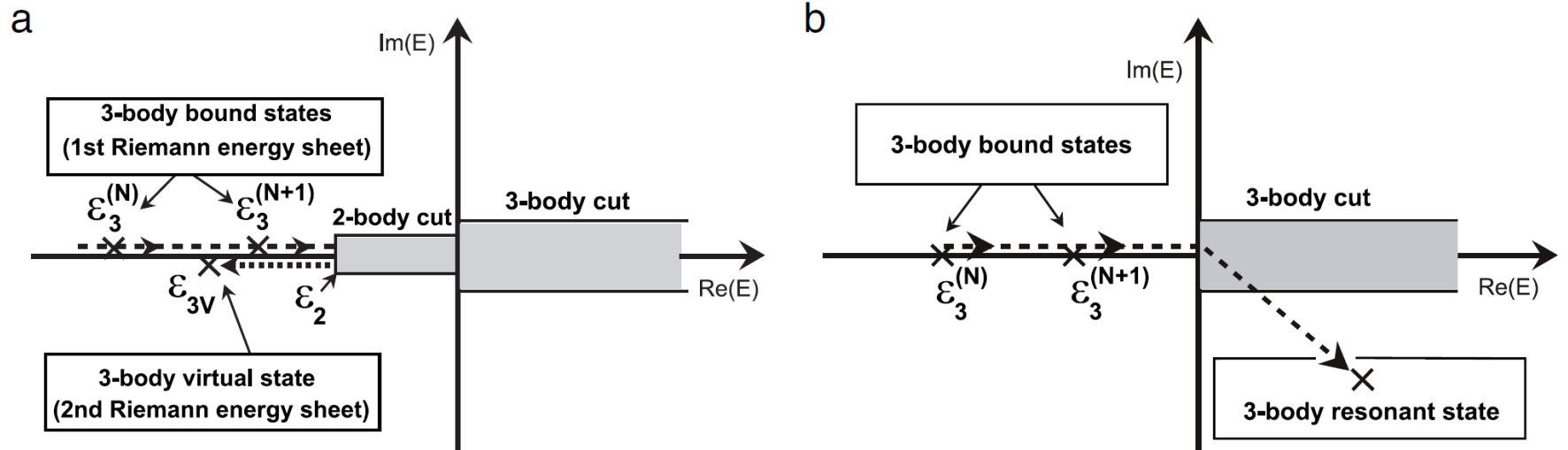
Correlations between observables: Jensen, Fedorov, Yamashita, Hammer, Platter, Gattobigio, Kievsky, Kolganova, Van Kolck, Bedaque, Phillips, ...

Scaling function & Limit Cycle

[Kunitski et al, Science 348 \(2015\) 551](#)



Analytic Structure & Efimov State Trajectory



S.K. Adhikari and L. Tomio, Phys. Rev. C **26**, 83 (1982); S.K. Adhikari, A.C. Fonseca, and L. Tomio, *ibid.* **26**, 77 (1982).

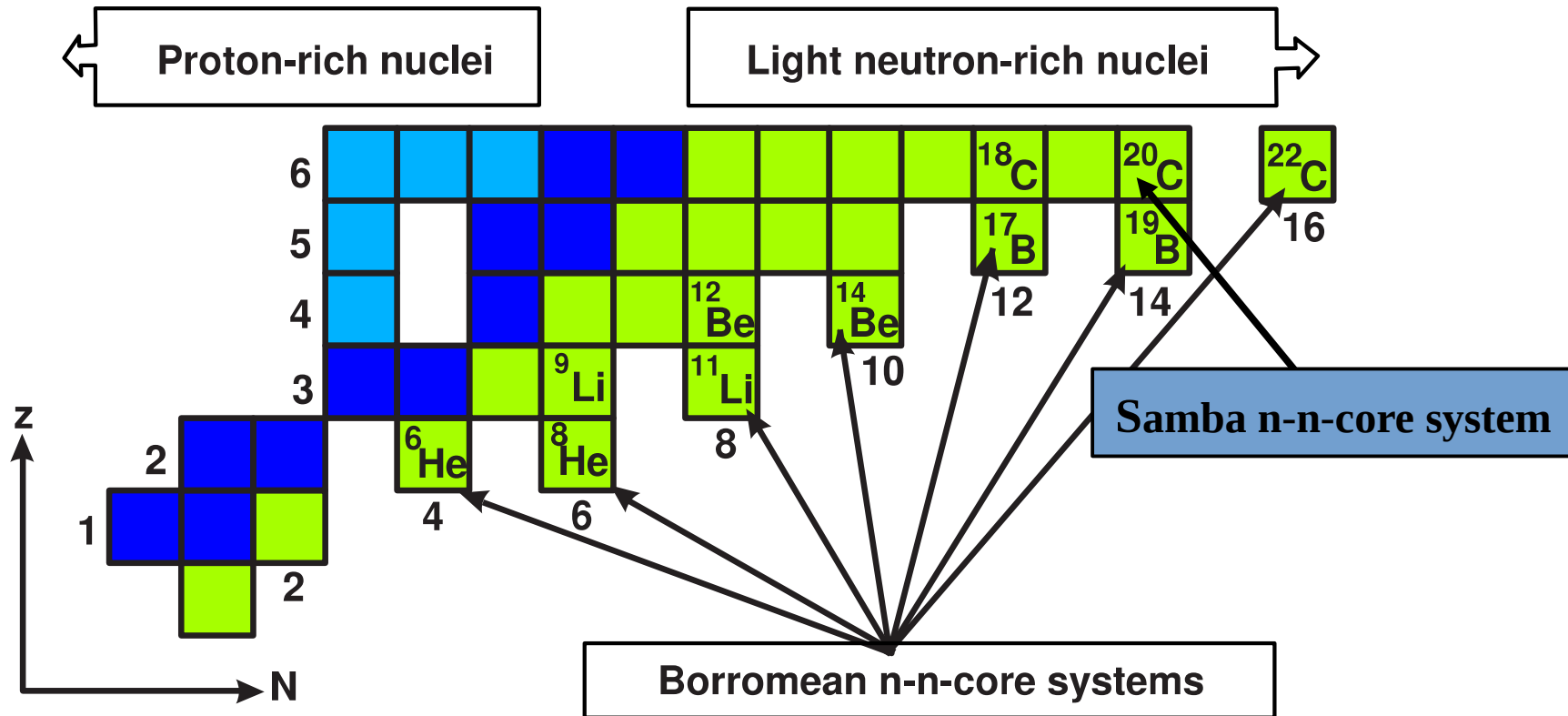
F. Bringas, M.T. Yamashita and T. Frederico, Phys. Rev. A **69**, 040702(R) (2004).

Continuum resonances of Borromean systems: observation in atomic traps!

Resonant 3-body recombination (Innsbruck, Rice, Heidelberg, Bar Ilan, Florence...)

L=0 neutron-neutron-core systems with n-core and n-n subsystems dominated by s-wave low energy states (bound or virtual)

^{11}Li , ^{14}Be , ^{20}C , ^{22}C



Configuration space two-neutron halo wave function (2n spin singlet) L=0

$$H\Psi = \left[-\sum_{i=1}^3 \frac{\hbar^2}{2m_i} \nabla_i^2 + \lambda_{jk} \delta(\mathbf{R}_{jk}) \right] \Psi = -S_{2n} \Psi \quad (\text{C.M.})$$

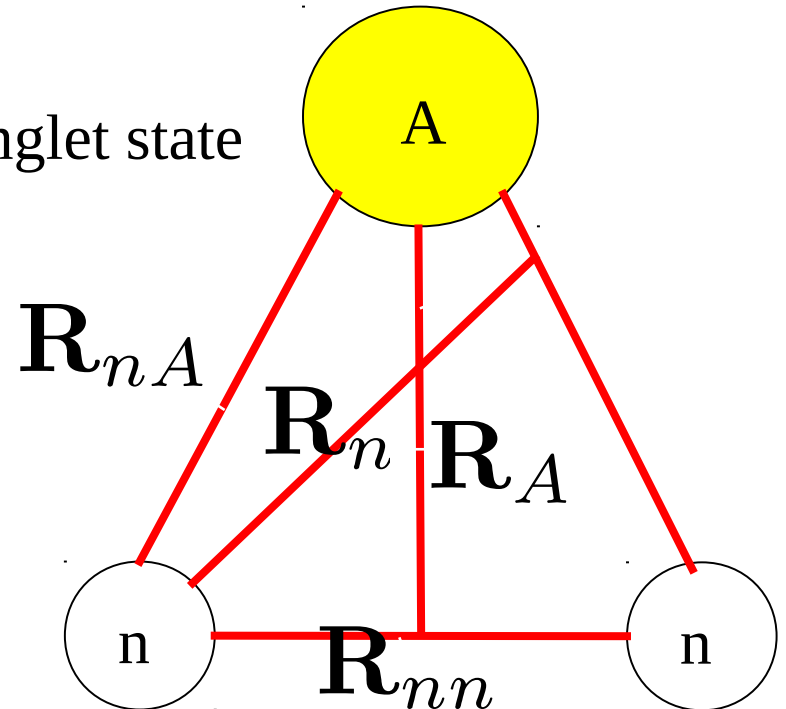
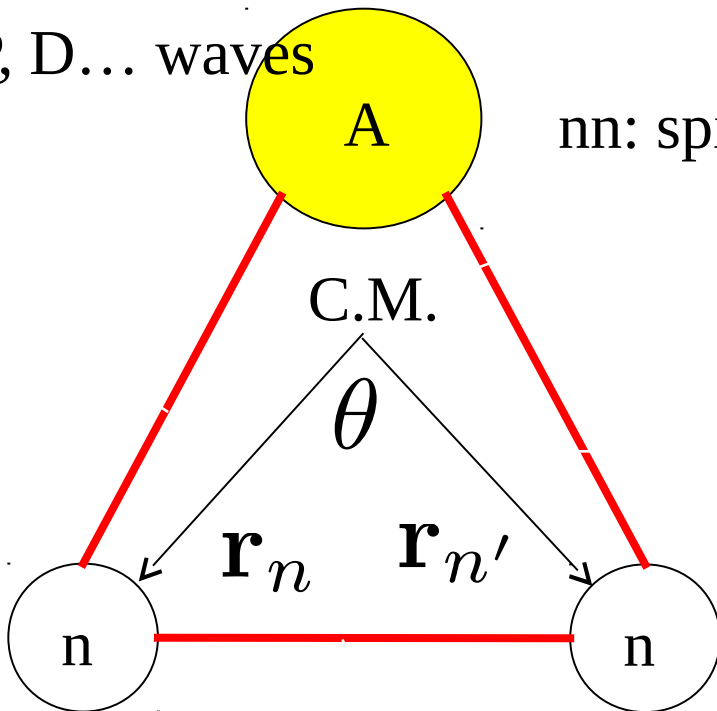
$$\Psi(\mathbf{r}_n, \mathbf{r}_{n'}) = \int d\mathbf{q} \frac{e^{-\kappa_{nn} |\mathbf{R}_{nn}|}}{|\mathbf{R}_{nn}|} e^{i\mathbf{q} \cdot \mathbf{R}_A} \chi_A(\mathbf{q}) + \int d\mathbf{q} \frac{e^{-\kappa_{nA} |\mathbf{R}_{nA}|}}{|\mathbf{R}_{nA}|} e^{i\mathbf{q} \cdot \mathbf{R}_n} \chi_n(\mathbf{q}) + \dots$$

$$\Psi(|\mathbf{r}_n|, |\mathbf{r}_{n'}|, \cos \theta)$$

$$\kappa_{nn} = \sqrt{2\mu_{nn} \left(S_{2n} + \frac{q^2}{2\mu_A} \right)} \quad \text{and} \quad \kappa_{nA} = \sqrt{2\mu_{nA} \left(S_{2n} + \frac{q^2}{2\mu_n} \right)}$$

S, P, D... waves

nn: spin singlet state



Scales for the $L=0$ n-n-c system with s-wave zero-range interaction

E_{nn} Energy of the virtual nn system

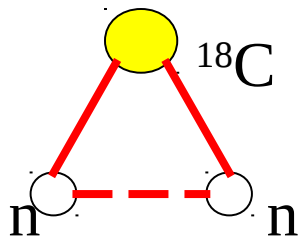
E_{nc} Energy of the bound/virtual nc system

S_{2n} Binding energy of the nnc system

A = mass of the core

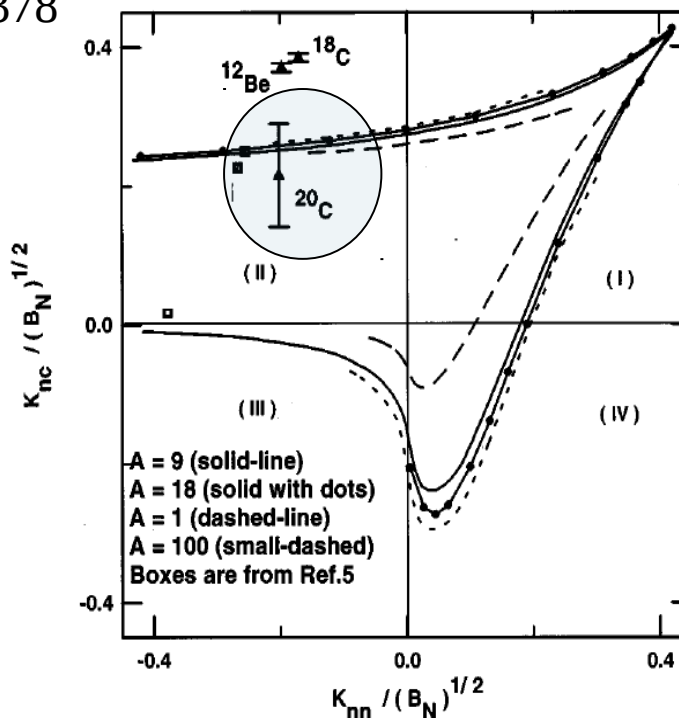
Threshold for an excited Efimov state and trajectory: ^{20}C

Amorim, TF, Tomio PRC56(1997)2378



$$E_3 = 3.5 \text{ MeV}$$

$$E_{nc} = 160 \pm 110 \text{ keV}$$



^{20}C can have a continuum resonance or virtual Efimov state?

Arora, Mazumdar, Bhasin PRC69 (2004)061301(R) Mazumdar, Rau, Bhasin PRL97(2006)062503
Efimov state \rightarrow resonance of $n+^{19}\text{C}$ by changing K_{nc}

^{20}C has a VIRTUAL STATE:
 ^{19}C is bound!

Yamashita, TF, Tomio, PRL99 (2007)269201 &
PLB660(2008)339

If L_{total} is nonzero ?

- Virtual p-wave states of light non Borromean nn halo nucleus
 $E_{\text{virtual}} \sim 1.7 E_{\text{nc}}$ SAMBA type
- Delfino, F, **Hussein**, Tomio et al PRC61, 051301 (2000)

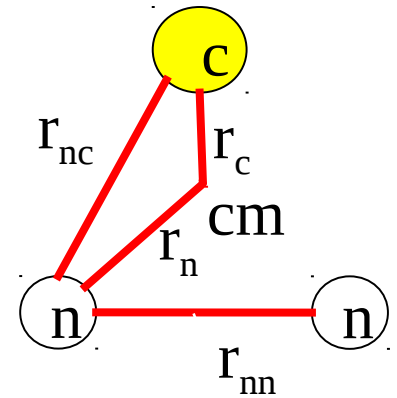
- Pigmy dipole 1^- resonance:
- M. Cubero et al, PRL 109, 262701 (2012) $^{11}\text{Li}+^{208}\text{Pb}$ close the Coulomb barrier $\rightarrow E_{\text{res}}=690 \text{ keV}$ width=0.32 keV
- Fernandez-Garcia et al PRL 110, 142701 (2013) $^{11}\text{Li}+^{208}\text{Pb}$ breakup around the Coulomb barrier

Determined by scattering lengths only!

Root mean square radii

Universal Scaling functions (model independent)

Limit cycles (Efimov, Wilson...)



$$\sqrt{\langle r_{n\gamma}^2 \rangle} S_{2n} = \mathcal{R}_{n\gamma} \left(\pm \sqrt{\frac{|E_{nn}|}{S_{2n}}}, \pm \sqrt{\frac{|E_{nc}|}{S_{2n}}} \right)$$

$$\sqrt{\langle r_{\gamma}^2 \rangle} S_{2n} = \mathcal{R}_{\gamma}^{cm} \left(\pm \sqrt{\frac{|E_{nn}|}{S_{2n}}}, \pm \sqrt{\frac{|E_{nc}|}{S_{2n}}} \right)$$

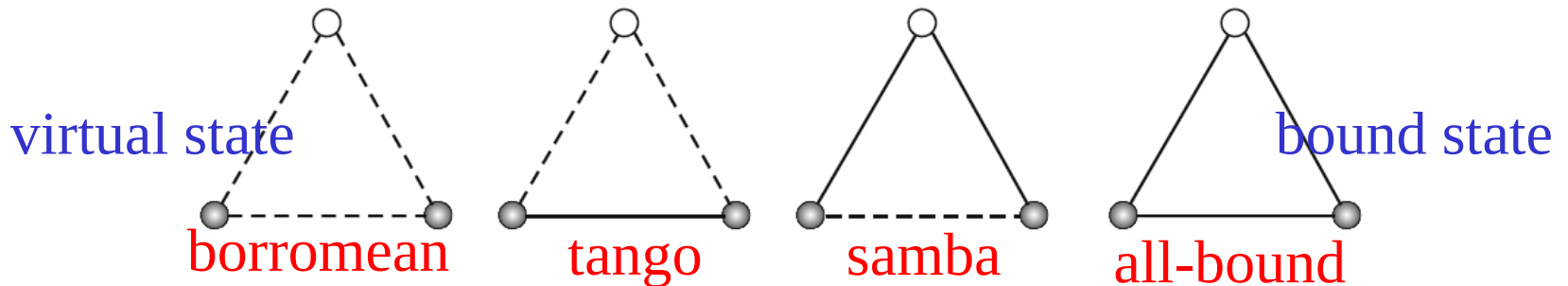
Build constraints!

Usefull for ^{22}C

$\gamma = n$ or c

+ two-body bound state
- two-body virtual state

Yamashita et al NPA735(2004)40

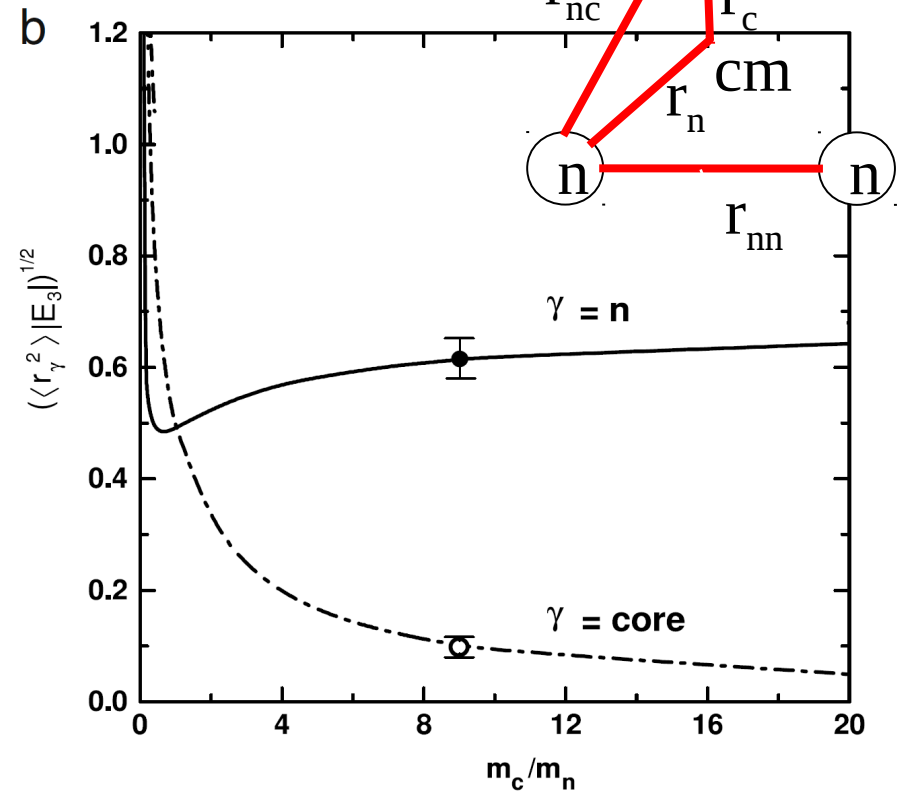
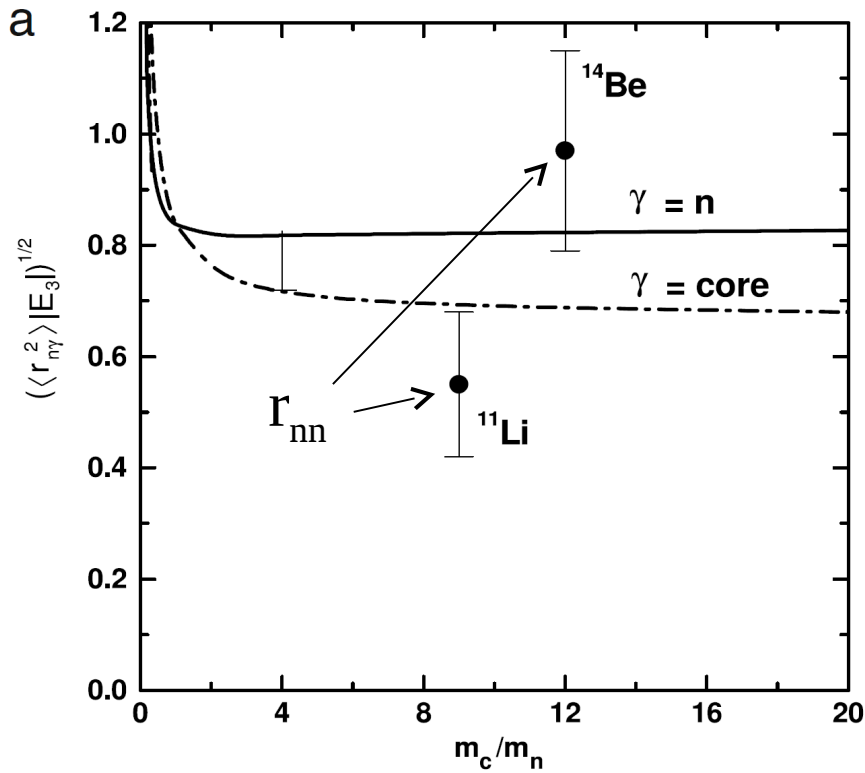


$^{11}\text{Li}, ^{14}\text{Be}, ^{22}\text{C}$

^{20}C

Tango: Robicheaux PRA60(1999)1706

Root mean square radii: Core+neutron+neutron



$S_{2n}[^{11}\text{Li}] = 369.15(65)$ KeV -- Smith et al PRL101(08)202501

Charge radius ^{11}Li [2.217(35) fm] and ^9Li [2.467(37) fm] -- Sanchez et al PRL96(96)03302

neutron halo radius ^{11}Li [6.54(38) fm] -- Egelhof et al EJPA15 (02) 27

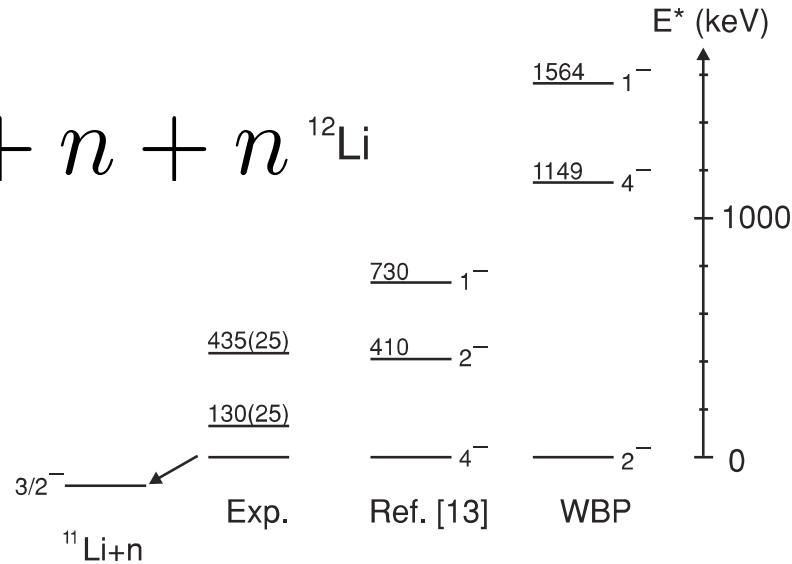
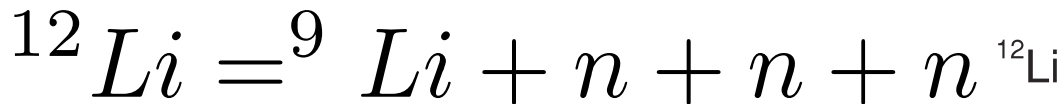
Range corrections EFT: Canham and Hammer NPA 836 (2010) 275

First observation of excited states in ^{12}Li

(n+n+n+core)

C. C. Hall,¹ E. M. Lunderberg,¹ P. A. DeYoung,^{1,*} T. Baumann,² D. Bazin,² G. Blanchon,³ A. Bonaccorso,⁴ B. A. Brown,^{2,5} J. Brown,⁶ G. Christian,^{2,5} D. H. Denby,¹ J. Finck,⁷ N. Frank,^{2,5,†} A. Gade,^{2,5} J. Hinnefeld,⁸ C. R. Hoffman,^{9,10} B. Luther,¹¹ S. Mosby,^{2,5} W. A. Peters,^{2,5,‡} A. Spyrou,^{2,5} and M. Thoennessen^{2,5}

The neutron-unbound ground state and two excited states of ^{12}Li were formed by the two-proton removal reaction from a 53.4-MeV/u ^{14}B beam. The decay energy spectrum of ^{12}Li was measured with the Modular Neutron Array (MoNA) and the Sweeper dipole superconducting magnet at the National Superconducting Cyclotron Laboratory. Two excited states at resonance energies of 250 ± 20 keV and 555 ± 20 keV were observed for the first time and the data are consistent with the previously reported s -wave ground state with a scattering length of $a_s = -13.7$ fm.



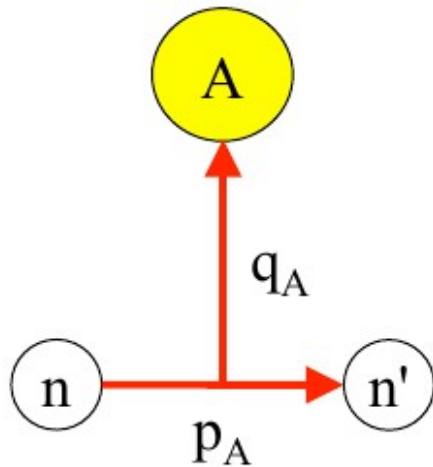
Four-boson scale with s -wave zero-range potential:

Hadizadeh, Yamashita, Tomio, Delfino, TF, Phys. Rev. Lett. 107, 135304 (2011)

BUT Pauli principle kills sensitivity to the 4-body scale!

Neutron-neutron correlation function in ^{11}Li and ^{14}Be

Yamashita, TF, Tomio PRC 72, 011601(R) (2005)



$$C_{nn}(\vec{p}_A) = \frac{\int d^3 q_A |\Phi(\vec{q}_A, \vec{p}_A)|^2}{\int d^3 q_A \rho(\vec{q}'_n) \rho(\vec{q}_n)}$$

$$\vec{q}'_n = \vec{p}_A - \frac{\vec{q}_A}{2} \quad \vec{q}_n = -\vec{p}_A - \frac{\vec{q}_A}{2}$$

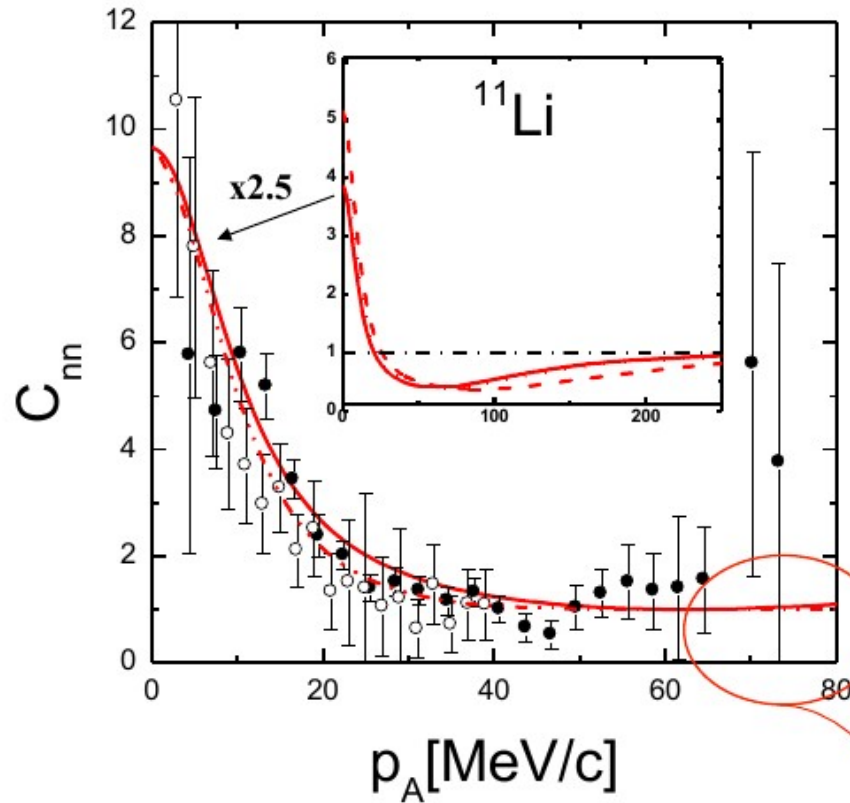
One-body density

$$\rho(\vec{q}_{nA}) = \int d^3 q_{n'A} \left| \Phi \left(-\vec{q}_{nA} - \vec{q}_{n'A}, \frac{\vec{q}_{nA} - \vec{q}_{n'A}}{2} \right) \right|^2$$

$\Phi \equiv \Phi(\vec{q}_A, \vec{p}_A)$ Breakup amplitude including the FSI between the neutrons

$$\Phi = \Psi(\vec{q}_A, \vec{p}_A) + \frac{1/(2\pi^2)}{\sqrt{E_{nn} - ip_A}} \int d^3 p \frac{\Psi(\vec{q}_A, \vec{p})}{p_A^2 - p^2 + i\varepsilon} \quad \Psi \text{ is the three-body wave function}$$

nn-correlation function in ^{11}Li



F. M. Marqués et al.
Phys. Rev. C **64**, 061301 (2001)



M. Petrascu et al.
Nucl. Phys. A **738**, 503 (2004)

— $S_{2n} = 290 \text{ KeV}$
 $E_{nc} = 50 \text{ KeV}$

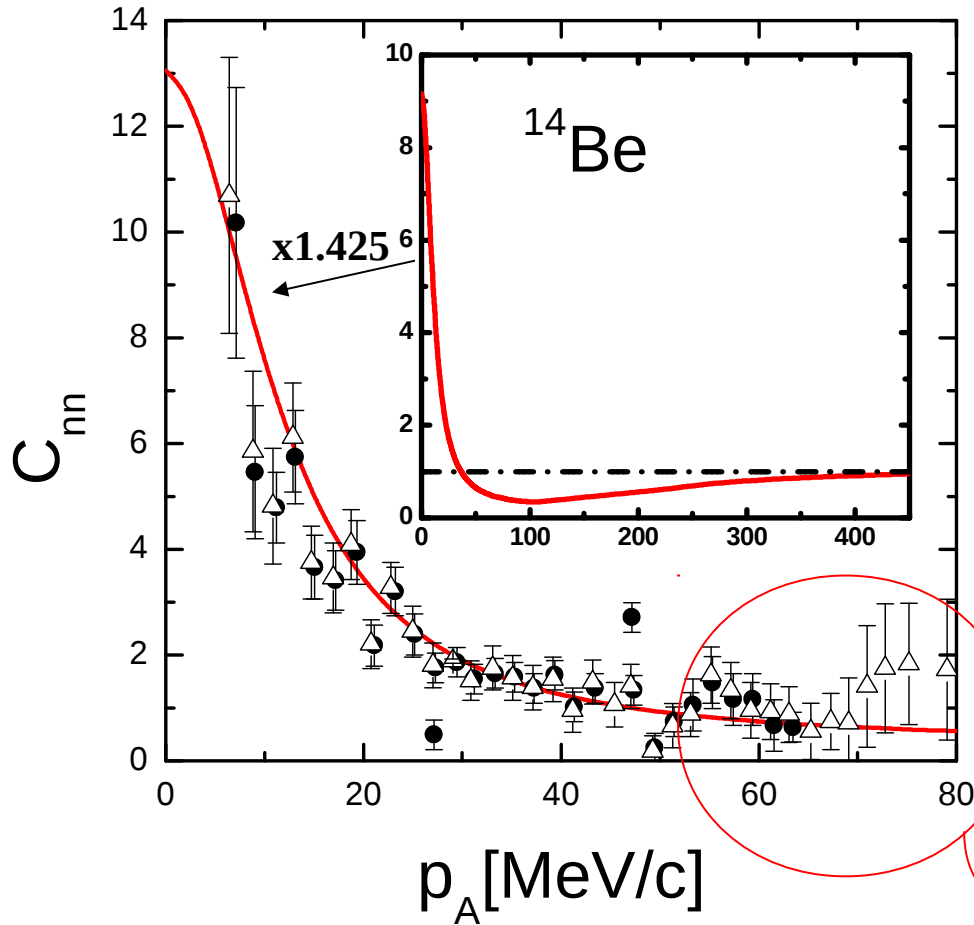
- - - $S_{2n} = 370 \text{ KeV}$
 $E_{nc} = 800 \text{ KeV}$

⋯ $S_{2n} = 370 \text{ KeV}$
 $E_{nc} = 50 \text{ KeV}$

$E_{nn} = 143 \text{ KeV}$

asymptotic region ?

nn-correlation function in ^{14}Be




F. M. Marqués et al.
Phys. Rev. C **64**, 061301 (2001)



F. M. Marqués et al.
Phys. Lett. B **476**, 219 (2000)

$$S_{2n} = 1337 \text{ KeV}$$


$$E_{nc} = 200 \text{ KeV}$$

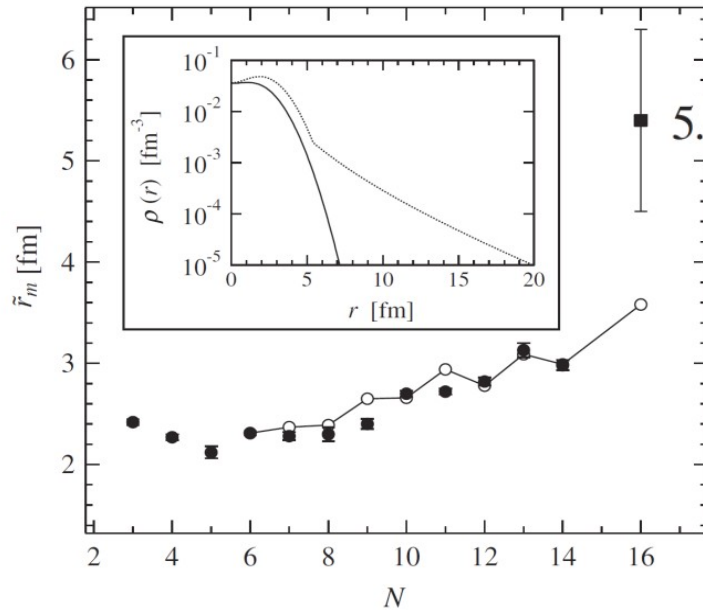
$$E_{nn} = 143 \text{ KeV}$$

asymptotic region ?

$${}^{22}\text{C} = n - n - {}^{20}\text{C}$$

K. Tanaka *et al.*, Phys. Rev. Lett. **104** (2010) 062701

Reaction cross sections (σ_R) for ${}^{19}\text{C}$, ${}^{20}\text{C}$ and the drip-line nucleus ${}^{22}\text{C}$ on a liquid hydrogen target have been measured at around 40A MeV by a transmission method. A large enhancement of σ_R for ${}^{22}\text{C}$ compared to those for neighboring C isotopes was observed. Using a finite-range Glauber calculation under an optical-limit approximation the rms matter radius of ${}^{22}\text{C}$ was deduced to be 5.4 ± 0.9 fm. It does not follow the systematic behavior of radii in carbon isotopes with $N \leq 14$, suggesting a neutron halo. It was found by an analysis based on a few-body Glauber calculation that the two-valence neutrons in ${}^{22}\text{C}$ preferentially occupy the $1s_{1/2}$ orbital.



$$5.4 \pm 0.9 \text{ fm}$$

$$S_{2n} = 420 \pm 940 \text{ keV}$$

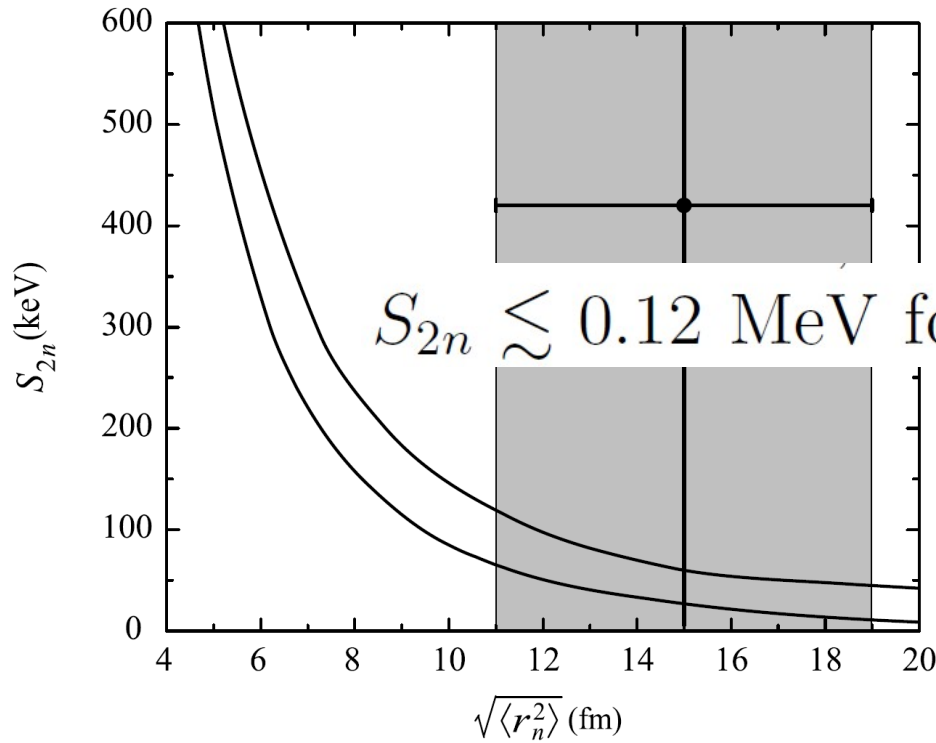
$$\tilde{r}_m^{22}\text{C} \equiv \langle (r_m^{22}\text{C})^2 \rangle^{1/2}$$

$$\tilde{r}_m^{20}\text{C} = 3 \text{ fm}$$

$$\tilde{r}_n^{22}\text{C} = \sqrt{\frac{22}{2}} \sqrt{(\tilde{r}_m^{22}\text{C})^2 - \frac{20}{22} (\tilde{r}_m^{20}\text{C})^2} \approx 15 \pm 3 \text{ fm}$$

$${}^{22}\text{C} = n - n - {}^{20}\text{C}$$

${}^{21}\text{C}$ virtual state energy 0, -100 KeV. $E_{nn} = -143\text{KeV}$



$$S_{2n} = 420 \pm 940 \text{ keV}$$

$S_{2n} \lesssim 0.12 \text{ MeV}$ for ${}^{22}\text{C}$

Yamashita, M de Carvalho, TF, Tomio,
PLB697(2011)90; A&E PLB715(2012)282

$$11 \text{ fm} \leq \sqrt{\langle r_n^2 \rangle} \leq 19 \text{ fm}$$

$$\sqrt{\langle r_{ch}^2({}^{22}\text{C}) \rangle} - \sqrt{\langle r_{ch}^2({}^{20}\text{C}) \rangle} \gtrsim 0.9 \text{ fm}$$

H.T. Fortune, R. Sherr, Phys. Rev. C 85 (2012) 027303.

Acharya, Ji, Phillips PLB723(2013)19 [$S_{2n} < 100 \text{ keV}$] (EFT)

$^{22}\text{C} = n - n - ^{20}\text{C}$ with finite range potentials

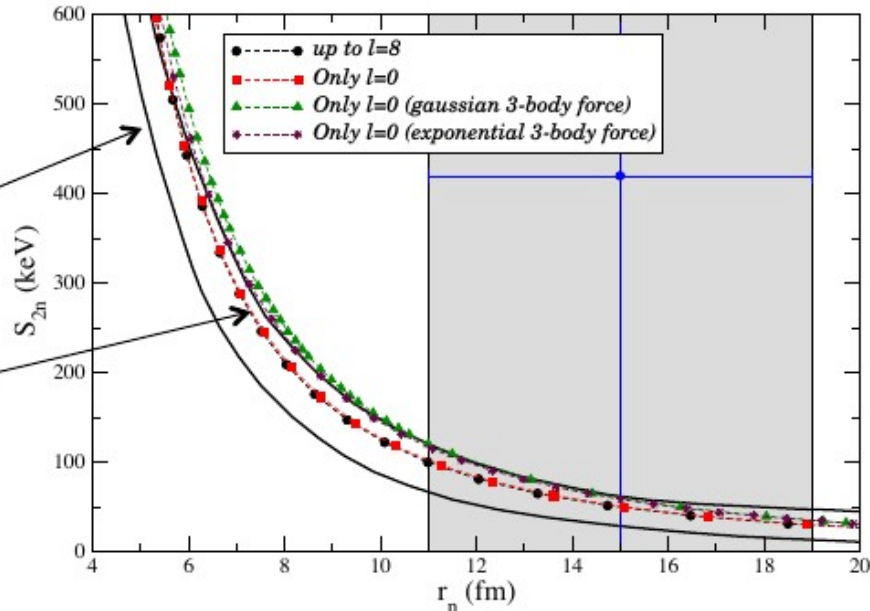
Eduardo Garrido (Madrid)

n - ^{20}C finite-range potential Y. Kucuk and J. A. Tostevin, Phys. Rev. C **89** (2014) 034607

$$V_{\text{central}}(r) = -\frac{V_c}{1 + e^{\frac{r-R}{a}}}; \quad R \text{ is } 1.25 \text{ fm and the diffuseness } a \text{ is } 0.65 \text{ fm.}$$

| | |
|----------------|----------------------|
| V_c (MeV) | 21.0 |
| S_{2b} (MeV) | $-1.5 \cdot 10^{-5}$ |
| a (fm) | 1192 |
| r_e (fm) | 2.88 |

3-body force with Range 1 fm



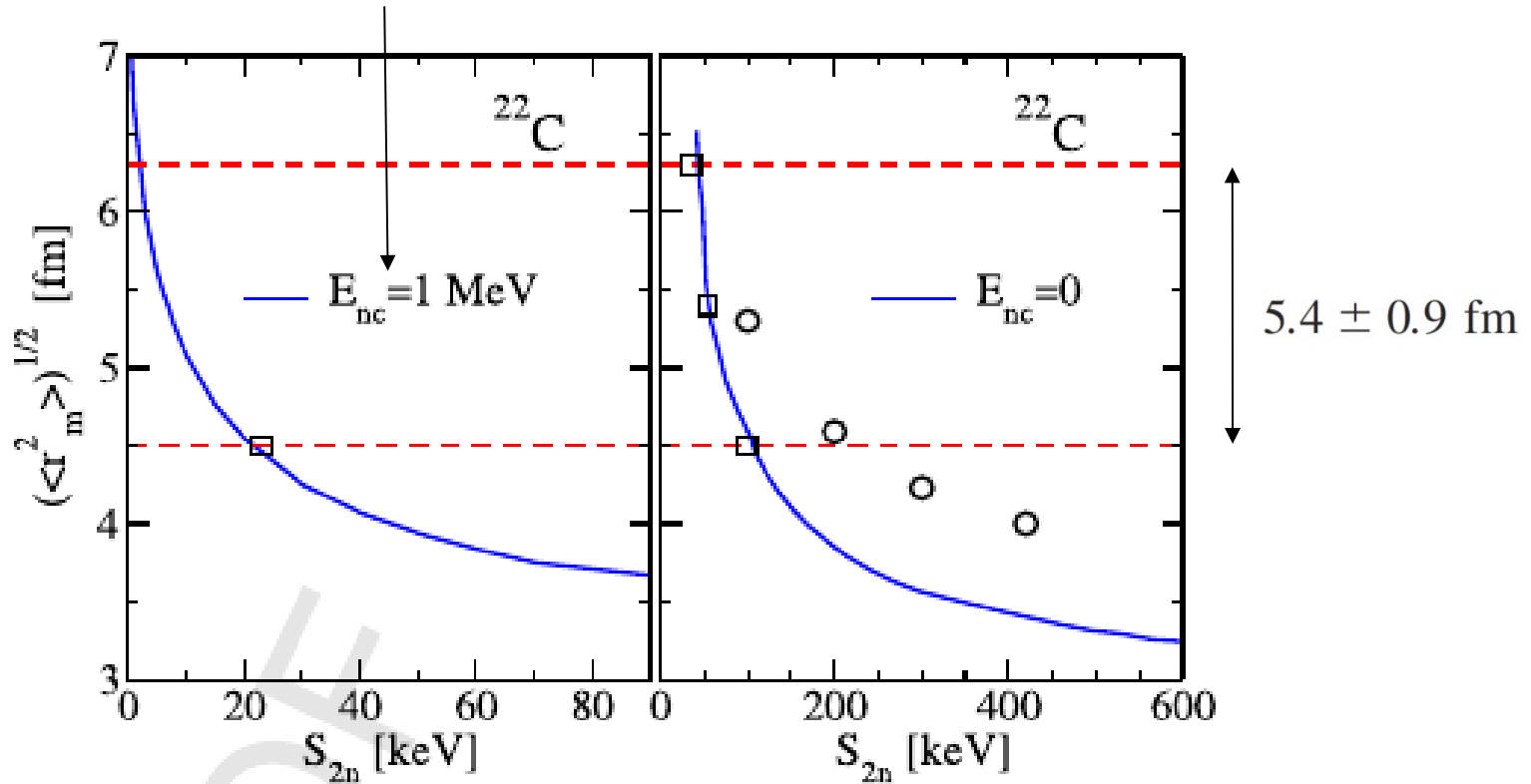
$E_{\text{vl}}[^{21}\text{C}] = -100 \text{ KeV}$

$E_{\text{vl}}[^{21}\text{C}] = 0 \text{ KeV}$

^{21}C virtual state energy $E_{\text{vl}}[^{21}\text{C}]$: 0, -100 KeV. $E_{\text{nn}} = -143 \text{ KeV}$

^{22}C Matter Radius

21C Mosby et al. NPA 909, 69 (2013) $|a_s| < 2.8 \text{ fm}$ (^{21}C virtual state)



$$\sqrt{\langle r_m^2 [^{22}\text{C}] \rangle} = \sqrt{\frac{2}{22} \langle r_n^2 \rangle + \frac{20}{22} \langle r_m^2 [^{20}\text{C}] \rangle}. \quad \sqrt{\langle r_m^2 [^{20}\text{C}] \rangle} = 2.98(5) \text{ fm}$$

[circles] WS potential + core - H. T. Fortune, R. Sherr, PRC 85 (2012) 027303. (^{21}C bound)

[boxes] EFT - B. Acharya, C. Ji, D. R. Phillips, PLB 723 (2013) 196;

B. Acharya, D. R. Phillips, EPJWoC 113(2016) 06013.

$$S_{2n} \lesssim 0.12 \text{ MeV for } ^{22}\text{C}$$

Is S_{2n} consistent with RIKEN data on core momentum recoil distribution?

Kobayashi et al PRC86 (2012) 054604

Core Momentum distribution $nnc = AAB$: ^{11}Li , ^{14}Be , ^{20}C , ^{22}C

L. A. Souza et al PLB757 (2016) 368 & FBS57 (2016)361

$$n(q_B) = \int d^3 p_B |\langle \vec{q}_B \vec{p}_B | \Psi \rangle|^2$$

Yamashita et al PRA **87**, 062702 (2013)

$$\langle \vec{q}_B \vec{p}_B | \Psi \rangle = \frac{\chi_{AA}(q_i) + \chi_{AB}(q_j) + \chi_{AB}(q_k)}{E_3 + H_0} = \frac{\chi_{AA}(q_B) + \chi_{AB}(|\vec{p}_B - \frac{\vec{q}_B}{2}|) + \chi_{AB}(|\vec{p}_B + \frac{\vec{q}_B}{2}|)}{E_3 + H_0}$$

Scaling function (limit cycle) for the width of the distribution

$$\frac{\sigma}{\sqrt{S_{2n}}} = \mathcal{S}_c \left(\pm \sqrt{\frac{E_{nn}}{S_{2n}}}, \pm \sqrt{\frac{E_{nc}}{S_{2n}}}; A \right) \quad \text{FWHM} = 2\sqrt{2 \ln 2} \sigma$$

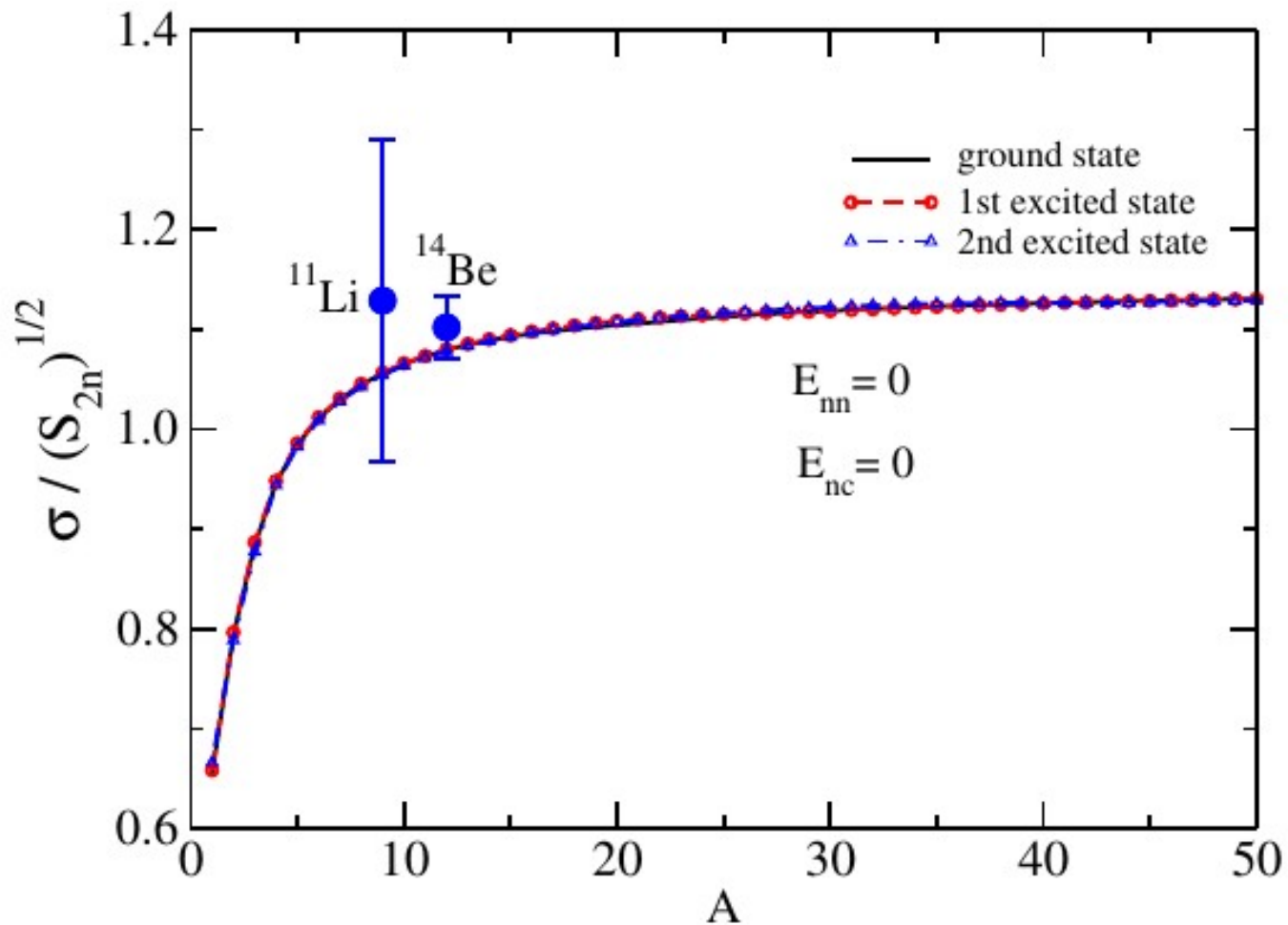


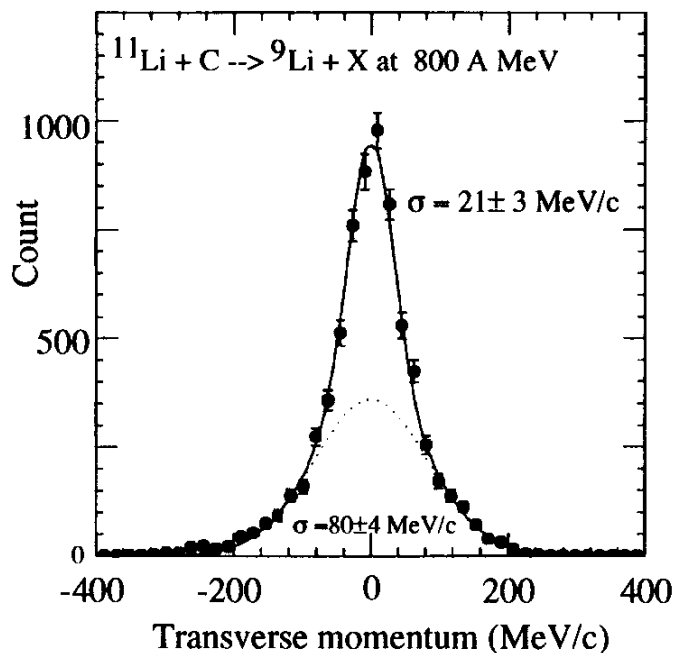
Fig. 1. Scaling plot for the core recoil momentum distribution σ in the Efimov limit as a function of the core mass number A . Experimental widths are from Refs. [1] and [9], for ^{11}Li and ^{14}Be , respectively.

[^{11}Li] I. Tanihata, J. Phys. G 22 (1996) 157;

[^{14}Be] M. Zahar, et al., Phys. Rev. C 48 (1993) R1484.

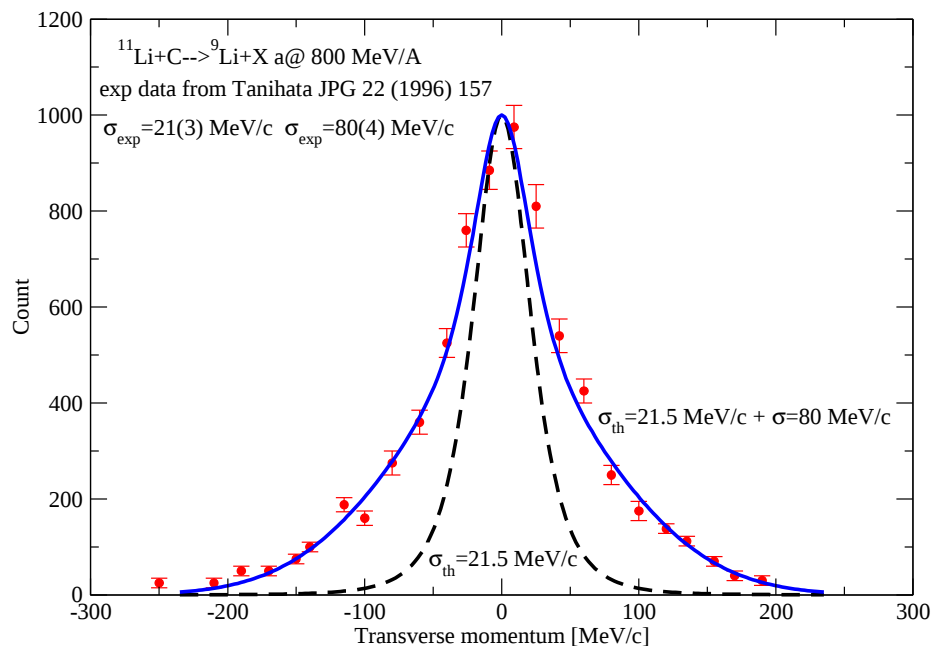
Core Momentum distribution $nnC = AAB$ systems: ^{11}Li , ^{20}C , ^{22}C

^{11}Li



Tanihata JPG22 (1996) 157

$S_{2n}[^{11}\text{Li}] = 369 \text{ KeV}$ $E_{\text{virtual}}[^{10}\text{Li}] = 50 \text{ KeV}$

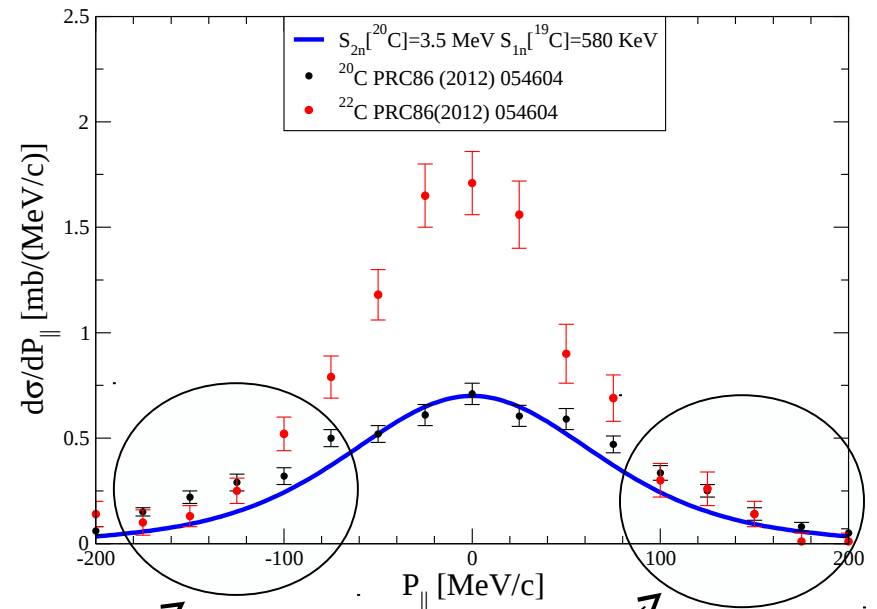
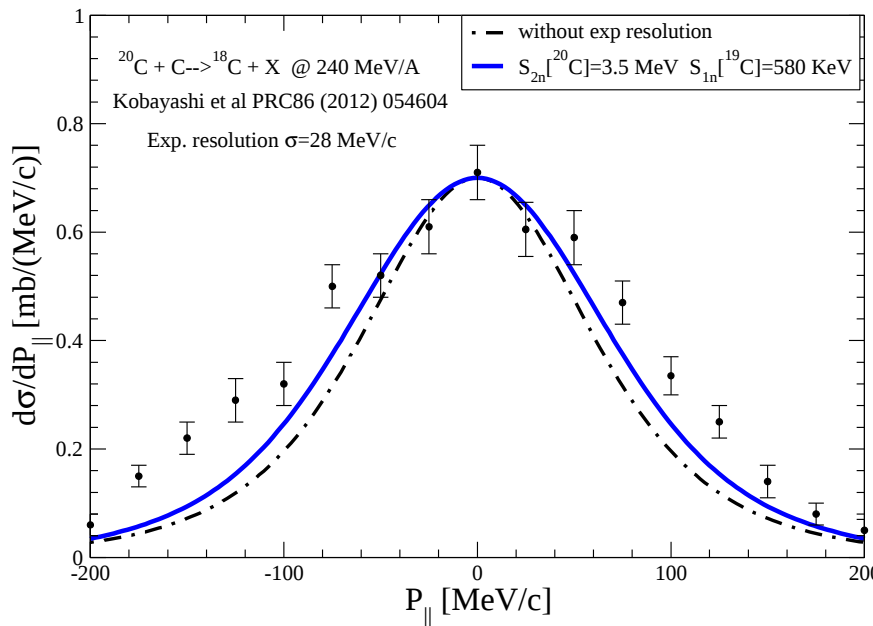


Small range corrections: Canham and Hammer
NPA 836 (2010) 275

RIKEN: Kobayashi et al PRC86 (2012) 054604
Inclusive cross-sections with 2n removal on C target

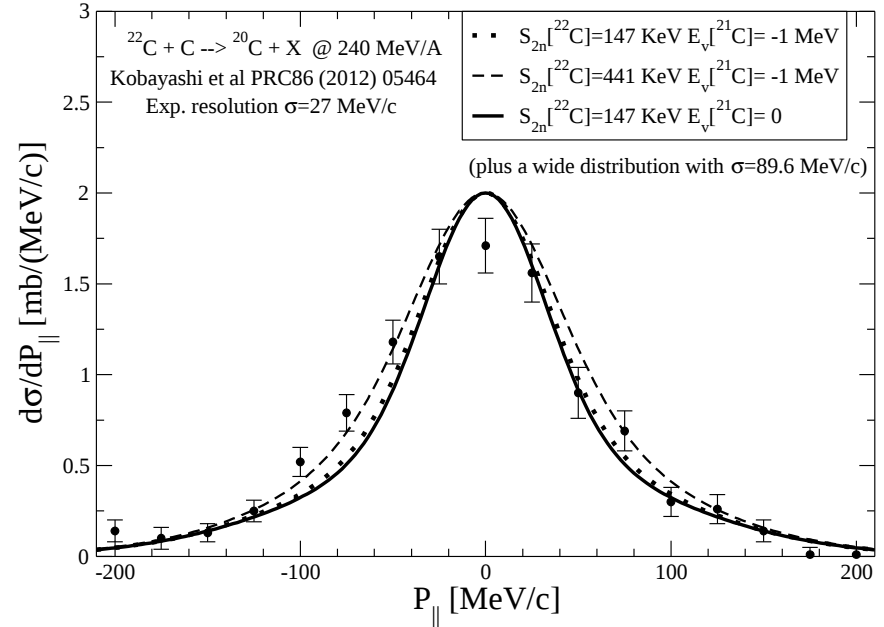
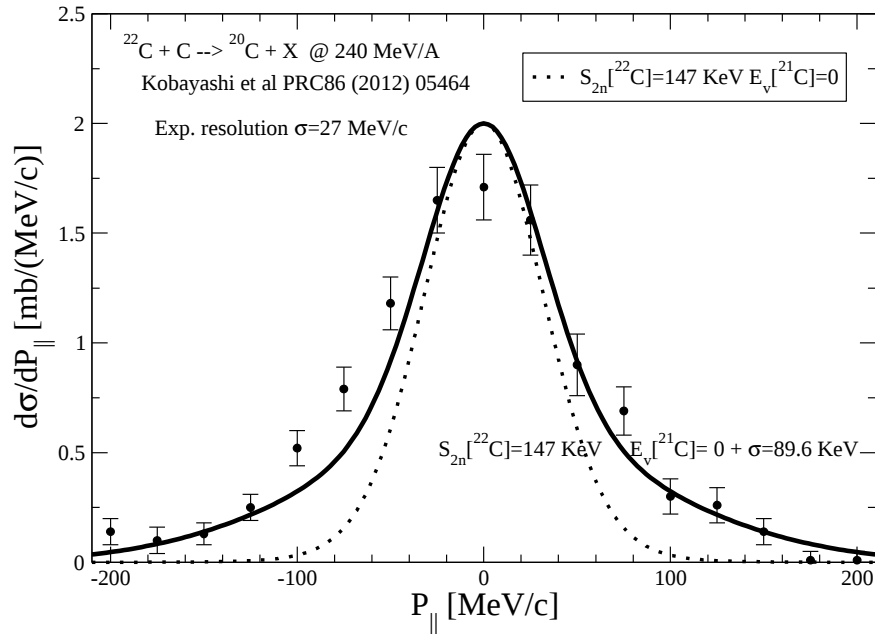
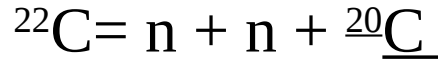
Core momentum distribution

$$^{20}\text{C} = n + n + \text{^{18}C} \quad \text{and} \quad ^{22}\text{C} = n + n + \text{^{20}C}$$



Broad contribution to the momentum distribution of the core in ^{22}C !

Core momentum distribution



$E_v[^{21}\text{C}]=1$ MeV Mosby et al. NPA 909, 69 (2013) – MSU - $|a_s| < 2.8$ fm (^{21}C virtual state)

$$100 \text{ KeV} \lesssim S_{2n}[^{22}\text{C}] \lesssim 400 \text{ KeV}$$

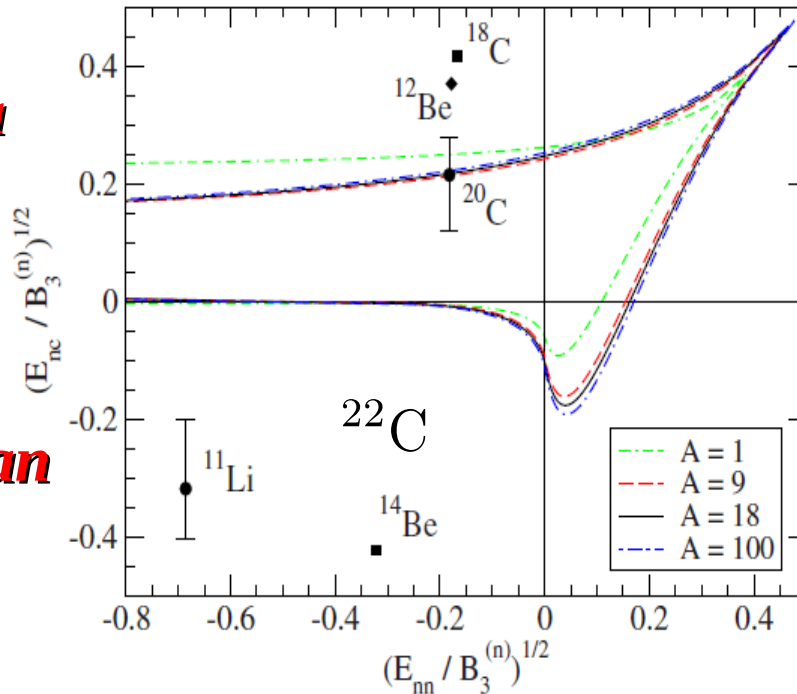
→ $r_n \lesssim 7 \text{ fm} \rightarrow \tilde{r}_m^{^{22}\text{C}} \lesssim 4 \text{ fm}$ $5.4 \pm 0.9 \text{ fm}$

Tanaka et al PRL 104(10)

Threshold for an excited Efimov state: Halo-nuclei

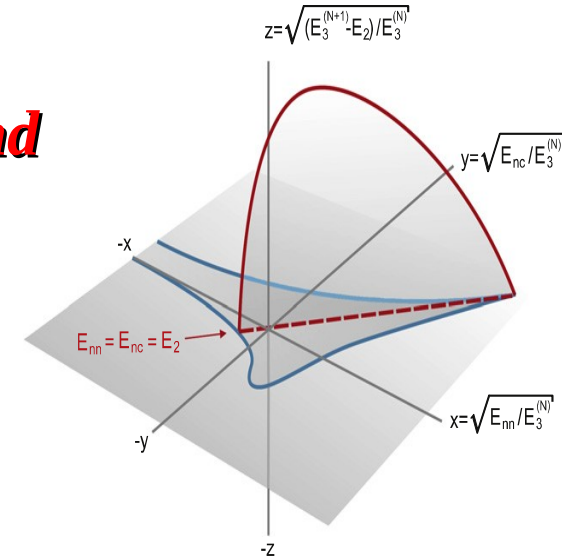
Critical condition for an excited (N+1)-th above the N-th state:

Samba
Borronean



All-bound

Tango



Amorim, TF, Tomio PRC56(1997)2378;

Canham and Hammer EPJA 37 (2008) 367; NPA 836 (2010) 275

^{21}C with a virtual state with energy 1 MeV and $100 \text{ KeV} \lesssim S_{2n}[^{22}\text{C}] \lesssim 400 \text{ KeV}$
 \rightarrow Improbable an excited Efimov state/continuum resonance

Summary

→ Weakly bound & large systems: **few scales regime** in halo nuclei, molecules, trapped atoms
CORRELATIONS BETWEEN OBSERVABLES → CONSTRAINTS!

→ Zero-range model n-n-c system:
suitable to study the structure of S-wave halos in ^{11}Li , ^{14}Be , ^{20}C , ^{22}C

→ Two neutron correlations in ^{11}Li and ^{14}Be well reproduced

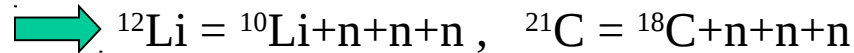
→ Core Momentum distribution in ^{11}Li , ^{14}Be and ^{20}C well reproduced

→ Core Momentum distribution in ^{22}C : **$S_{2n} \sim 100 - 400 \text{ KeV}$**

$$\rightarrow \tilde{r}_m^{22}\text{C} \lesssim 4 \text{ fm}$$

Outlook

→ Neutron halo $> 2n$ (no need of a 4-body scale)...



→ Universality in scattering, one neutron and two neutron transfers → exotic, breakup of halo nuclei & CDCC ...

→ Pigmy resonances in Borromean halos $L_{\text{total}} = 1, 2, 3 \dots$

→ Formation of neutron halo nuclei in neutron rich environment?
How this affect neutron capture? ...

→ Fix the tail of ab-initio calculations...

$$\mathcal{A} \left[\Psi(^9\text{Li}) \times \Psi_{3B}(^9\text{Li} - n - n) \right]$$