Evgeny Epelbaum, Ruhr-University Bochum Nuclear physics from QCD, INT Seattle,

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Confronting lattice-QCD results for the NN system with low-energy theorems

in collaboration with Vadim Baru, Arseniy Filin and Jambul Gegelia

- **Introduction**
- **Low-energy theorems for NN scattering**
- **Testing conjectured linear extrapolation** $M_\pi r^{3S1} = A + B M_\pi r^{1S1}$
- **Implications for the NPLQCD results at Mπ ~ 450 MeV**
- **Summary**

Lattice-QCD results for NN scattering observables \mathcal{F}_1 , and the 1S1 \mathcal{F}_2 as \mathcal{F}_3 as \mathcal{F}_4 as \mathcal{F}_5 as \mathcal{F}_6 notation the 1S0 NNNN \mathcal{F}_6 for NN scattering observables channel.

 $\frac{1}{\sqrt{2}}$

Further, the HAL QCD Collaboration claims [by first generating the NN potential] weaker attraction in both ${}^{1}S_{0}$ and ${}^{3}S_{1}$ - ${}^{3}D_{1}$ channels and no bound states for M_{π} > 411 MeV Ishii et al.'12 Further, the HAL OCD Colloberation algime. -arrie F_{ref} concreting the NNI netertiall weekey rst generating the inn potential] weaker $R_{\rm IS}$ and no bound states for $m_\pi > 4$ in the V-fishing fall 12

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Some) tools to extrapolate and/or check t_{max} to approximate the experimental value of t_{max} (Some) tools to extrapolate and/or check consistency of lattice QCD results:

- pionless EFT: extrapolate in the number of nucleons [large M_{π}] Barnea, Kirscher, van Kolck, ... negative values, except for those of Ref. [8] with large
	- $\frac{1}{\sqrt{1-\frac{1$ • chiral EF I: extrapolate in M_π (and the Beane, Savage, EE, Glöckle, Meißner, Gegelia, Soto, Chen, ... \mathbf{v} ber of nucleons) [smail W_{π}] λ hen, λ , $\$ channel. Open and contained symbols denote the quenches of pupples. Computer the quenches the $\frac{1}{2}$ chiral EFT: extrapolate in M $_{\pi}$ (and the number of nucleons) [small M $_{\pi}$]
Bears Savare EE Gläckle Meißear Geselie Sete Chen E_{S} and E_{S} and the infinite volume $\sum_{i=1}^n E_i$
- **interval manufal manufal manufal manufal manufal manufal manufal manufal manufal method when** \bullet LOW-ENERGY THEORENS (LETS) FOR THE Baru, EE, Filin, Gegelia α is no significant volume dependence of α • Low-Energy Theorems (LETs) for the NN system: extrapolate in energy at fixed M_{π}

Low-energy theorems for nucleon-nucleon scattering

Long-range interactions govern the low-energy behavior of the amplitude and imply correlations between coefficients in the ERE which may be regarded as Low Energy Theorems

For a reconstruction of the amplitude based on dispersion relations + unitarity constraints see: Gasparyan, Lutz, EE, EPJA49 (13) 115; Albalodejo, Oller, PRC84 (11) 054009

ERE, MERE and LETs *X*(*n*) **C** *n*₂ *and*¹ **CT**₂ *X*(*n*) (9) *X(CDE and LETA*

⇤*^b* = 600 *...* 400 MeV (10)

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Two-range potential
$$
V(r) = V_L(r) + V_S(r)
$$
, $M_L^{-1} \gg M_H^{-1}$ $F_l(E)$

$$
S_l = e^{2i\delta_l(k)} = 1 - i \left(\frac{km}{8\pi^2}\right) T_l(k), \quad T_l(k) = -\frac{16\pi^2}{m} \frac{k^{2l}}{F_l(k) - ik^{2l+1}}
$$

effective range function, $F_l \equiv k^{2l+1} \cot \delta_l$

 $F_l(k^2)$ is a real meromorphic function of k^2 for $|k| < M_{L/2}$ *n* a real meromorphic function of k^{2i} *^F^l* ⌘ *^k*²*l*+1 cot *^l*

ERE: $F_l = -\frac{1}{a} + \frac{1}{2}rk^2 + v_2k^4 + v_3k^6 + v_4k^8 + ...$ Landa $+$ 1 2

 $rk^2+v_2k^4+v_3k^6+v_4k^8+\ldots$ Landau, Smorodinsky '44; Blatt, Jackson'49; Bethe'49

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Generalization to the modified ERE by "subtracting" effects due to the long-range force 1 van Haeringen, Kok PRA 26 (1982) 1218

$$
F_l^M(k^2) \equiv M_l^L(k) + \frac{k^{2l+1}}{|f_l^L(k)|^2} \cot\left[\delta_l(k) - \delta_l^L(k)\right]
$$

$$
f_l^L(k) = \lim_{r \to 0} \left(\frac{l!}{(2l)!}(-2ikr)\frac{lf^L(k,r)}{dt}(k,r)\right)
$$
Just function for $V_L(r)$
Just solution for $V_L(k)$
Just solution for $V_L(k)$
Just solution for $V_L(k)$
to $Re\left[\frac{(-ik/2)^l}{l!}\lim_{r \to 0} \left(\frac{d^{2l+1}}{dr^{2l+1}}\frac{r^l f_l^L(k,r)}{f_l^L(k)}\right)\right]$

Per construction, F_l^M reduces to F_l for $V_L=0$ and is a real meromorphic function for $|k| < M_H/2$

ERE, MERE and LETs

Example: proton-proton scattering

$$
F_C(k^2) = C_0^2(\eta) k \cot[\delta(k) - \delta^C(k)] + 2k \eta h(\eta) = -\frac{1}{a^M} + \frac{1}{2} r^M k^2 + v_2^M k^4 + \dots
$$

\nwhere $\delta^C \equiv \arg \Gamma(1 + i\eta)$, $\eta = \frac{m}{2k}\alpha$, $C_0^2(\eta) = \frac{2\pi\eta}{e^{2\pi\eta} - 1}$, $h(\eta) = \text{Re}[\Psi(i\eta)] - \ln(\eta)$
\nCoulomb phase shift
\nSommerteld factor
\nDigamma function $\Psi(z) \equiv \Gamma'(z)/\Gamma(z)$

ERE, MERE and LETs

T = *V* + *V G*0*T* = *V* + *V G*0*V* + *V G*0*V G*0*V* + *...*

*^S*0) = 6*.*7(+1*.*⁰ 0*.*8)(+2*.*⁰ 1*.*3)*.*

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MERE and low-energy theorems

Long-range forces impose correlations between the ER coefficients (low-energy theorems)
Cohan Hansen in Staale Eurotabl in Cohen, Hansen '99; Steele, Furnstahl '00 $\overline{ }$ er \overline{c} *rk*²

The emergence of the LETs can be understood in the framework of MERE:

$$
F_l^M(k^2) \equiv M_l^L(k) + \frac{k^{2l+1}}{|f_l^L(k)|^2} \cot \left[\delta_l(k) - \delta_l^L(k)\right]
$$

meromorphic for
 $k^2 < (M_H/2)^2$

- τ approximate $F^M_l(k^2)$ by first 1,2,3,… terms in the Taylor expansion in k^2/M_H^2
- − calculate all "light" quantities
- $-$ reconstruct $\delta_l^L(k)$ and predict all coefficients in the ERE

$$
V(r) = v_L e^{-M_L r} f(r) + v_H e^{-M_H r} f(r)
$$

$$
V_L
$$

where $f(r) = \frac{(M_{H}r)^{2}}{1 + (M_{H}r)^{2}}$

^L 1.069 m³

 $\overline{}$ and $\overline{}$ multipliers are $\overline{}$

d −13.98/m5
| 3.98/m5 = 13.98/m5 = 13.98/m5

and $M_L = 1.0$, $v_L = -0.875$, $M_H = 3.75$, $v_H = 7.5$ (all in fm-1)

ERE and MERE

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$$

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for an analytic example, see EE, Gegelia, EPJ A41 (2009) 341

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| 3.98/m5 = 13.98/m5 = 13.98/m5

^L 1.069 m³

 $\overline{}$ and $\overline{}$ multipliers are $\overline{}$

LETs for NN S-waves: Implementation kept unexpanded. THE IS IOI INN STWAVES. IIIIPIEIIIEIIIAUUII scattering are expected to be governed by the left-hand cut generated by the OPEP. The OPEP is, however, singular The longest-range part of the NN force is due to the one-pion exchange potential (OPEP). Thus, the LET for NN s for inn s-waves. Implementation **the left-hand cut generation** at the origin and requires regularization and renormalization. Therefore, instead of using the quantum mechanical

 23)-3%.. 23 , 23 , 23 , 23 , 23 , 23 , 23 , 23 , 23 , 23 , 23 , 23 , 23 , 23 , 23 , 23 , 23 , 25 , 25 , 25 , 25 , 25 , 25 , 25 , 25 , 25 , 25 , 25 , 25 , 25 ,

We implement the LETs by calculating NN amplitude at LO within the renormalizable approach EE, Gegelia, Phys. Lett. B716 (2012) 338, see Jambul's talk

 \mathbf{C}_0 \times $+$ $+$ $+$ \parallel introduced originally by Kadyshevsky for the fully o $\mathcal{O}(\sqrt{-s}+1)$ which, has the form $T(\vec{p}, \vec{p}', k) = V(\vec{p}, \vec{p}') + \int d^3q \ V(\vec{p}, \vec{q}) \ G(k, q) \ T(\vec{q}, \vec{p}', k)$ To be specific, we calculate the scattering amplitude *T* by solving the Lippmann-Schwinger-type integral equation \log_2 which, for the case of the fully output of the fully of the fully of the fully of the form of *d*3*d p*, *d*₂*d d*₂*d d*₂*d d*₂*d d*₂*d d*₂*d d*₂*d d*₂*d d*₂*d* *, k*) = *V* (*p ,* ~ *p*~ ⁰ where $\frac{1}{\sqrt{2}}$ Kadyshevsky equation:

removing the ultraviolet cuto↵ should be model and regularization-scheme independent.

 $2(2\pi)^3$ $(\vec{q}^2 +$ $(y + m_N)$ $(E_K$ V_1 $\cdots N$ $e)$ *^E^k* ^p~*^q* ² ⁺ *^m*² $G(k,q) = \frac{m_N^2}{2(2-p)^2}$ $2(2\,\pi)^3$ 1 $\sqrt{\vec{q}^{\; 2} + m_N^2}\left(E_k - \sqrt{\vec{q}^{\; 2} + m_N^2} + i\epsilon\right)$ $\overline{\lambda}$ where the Green's function is given by

The Kadyshevsky equation is solved numerically for a given Λ. The constant C_0 is expressed in terms of the scattering length a (renormalization) and the limit $\Lambda \to \infty$ is taken. \overline{a} p_0 be valuation is solved numerically for a given Λ . The constant C_0 is expressed

Korrektur 3. Ordnung → phase shifts, binding energy & all other coefficients in the ERE are predicted
as functions of the scattering length ILETs1 as functions of the scattering length [LETs]
 \overline{a} \overline{a} *f* \overline{b} and \overline{b} \overline{c} \overline{c} are predicted

Notice: for the purpose of the LETs, the non-relativistic approach with a finite cutoff would also do the job…

Leading-order LETs for NN S-waves

- perturbative inclusion of pions (KSW approach) fail
- **EE, Gasparyan, Gegelia, Krebs, EPJA 51 (2015) 71** \sim 1S_e channel: limited predictive power of the LETs due to the weakness of the OPI perturbatively. Comparison with the values from the values of the Nijmegen Publisher into a conservation with the Nijman and Nijman and Nijman and Nijman and Nijman and Nijman an E E., Gasparyan, Gegena, Krebs, EPJA 51 (2015) 71 $1S₀$ channel: limited predictive power of the LETs due to the weakness of the OPEP; taking into account the range correction (NLO) leads to improvement
- In addition to the predicted energy dependence of the phase shifts, the proper in- $3S₁$ channel: LETs work as advertised (strong tensor part of the OPEP)

0*.*036)(+0*.*⁰³² 0*.*063)*, M*⇡*r*(1*S*0) = 6*.*7(+1*.*⁰ 0*.*8)(+2*.*⁰ 1*.*3) *.* **Next-to-leading-order LETs for NN S-waves**

(*M*⇡*a*(1*S*0)

)

¹ = 0*.*021(+0*.*⁰²⁸

6

Fig. 1: Correlations and the interpretations in a *insufficient* when using effective range as input Even in the ³S₁ channel, the accuracy is insufficient when using effective range as input: $r=1.75\;{\rm fm}\;{\rm [input]}\;\;\longrightarrow\;\;a=7.16\;{\rm fm},\;\;\;B_d=1.1\;{\rm MeV}\;{\rm [LET}\;{\rm predictions}]$

0*.*036)(+0*.*⁰³² 0*.*063)*, M*⇡*r*(1*S*0) = 6*.*7(+1*.*⁰ 0*.*8)(+2*.*⁰ 1*.*3) *.* Next-to-leading-order LETs for NN S-waves ⇡ **R** *′ LETs* $\overline{2}$ **15 101 1111 5-Wave:**

(*M*⇡*a*(1*S*0)

m ! 1

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Fig. 1: Correlations and the interpretations in a *insufficient* when using effective range as input Even in the ${}^{3}S_{1}$ channel, the accuracy is insufficient when using effective range as input: $r=1.75\;{\rm fm}\;{\rm [input]}\;\;\longrightarrow\;\;a=7.16\;{\rm fm},\;\;\;B_d=1.1\;{\rm MeV}\;{\rm [LET}\;{\rm predictions}]$ *X*(*n*) = *X*(0) + *X*(2) + *...* + *X*(*n*) *.* (8)

 \rightarrow go to NLO LETs by including the (modified) effective range correction modeled via

LETs at unphysical pion mass

LETs at unphysical pion mass

When going to unphysical pion masses, the main change in the left-hand singularities is due to threshold shifts (explicit M_{π} -dependence)

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FIG. 2: Cubic polynomial regression fits to lattice QCD data for the pion decay constant *F*⇡, nucleon mass *m^N* and the For NLO LETs, we need to know M_{π} -dependence of the subleading σ in the sumthe Budapest-Marseille-Wuppertal collaboration using Wilson fermions $\frac{1}{2}$ and $\frac{1}{2}$ flavor simulations reported in $\frac{1}{2}$ short-range term (a higher-order effect in EFT)
 $\vec{g}^{1.5}$
 $\vec{g}^{1.5}$ k
0
2 *R* d to know M_π-depe

$$
V_{\mathrm{NLO}} = \beta \, \frac{\vec{\sigma}_1 \cdot \vec{q} \ \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + M^2}
$$

 \mathbf{I} \mathcal{L} We allow β to vary: δβ(M_π=500 MeV) = ±50% (±100%)

NLO LETs for the ³S₁ partial wave

LETs at nonphysical pion masses (at NLO, δβ = 0.5)

 \bullet good convergence and accuracy of the LETs for low values of M $_\pi$ (below 200 MeV)

light-shaded bands depict the results of NLO LETs and reflect the estimated uncertainty due to unknown *M*⇡-dependence of \bullet sizable uncertainty at pion masses above 400 MeV (even at NLO)

Intermediate summary

The take-away message so far:

- \bullet LETs = manifestations of the longest-range interaction due to 1 π -exchange in the (energy dependence of the) NN scattering amplitude
- Good/fair predictive power in the ${}^{3}S_{1}$ - ${}^{3}D_{1}$ / ${}^{1}S_{0}$ channels
- Work at any M_π (provided the OPEP is the longest-range interaction)
- Matching to lattice: Need a single amount of information to fix the short-range physics at a given M_{π} in order to reconstruct $T(k)$

LETs vs chiral EFT: Similar in spirit but not quite the same…

- Not a *χ*-extrapolation in the usual sense: given an observable $X(M_\pi)$, can predict $Y(M_\pi)$, $Z(M_\pi)$,.... No assumptions are made about $C_0 = f(M_\pi)$!
- No reliance on the M_{π} -expansion; infinite M_{π} -limit well-defined; M_{π} -dependence of m_N , g_A , F_π taken directly from lattice QCD
- Accuracy & applicability range limited by the second-lowest non-analyticity in the amplitude (2π -exchange, heavy-meson exchange, Δ , ...)

Application 1: testing the conjectured linear extrapolation 15 for $M_\pi r$: $M_\pi r = A + BM_\pi$ *B***nnication 1** *nn <* 0*.*6 MeV ✓ *r R* ation 1: plication **i** . *V*NLO = ~¹ *· q*~ ~² *· q*~ **for** M_{m} , M_{m} ~¹ *· q*~ ~² *· q*~ *^q*[~] ² ⁺ *^M*² *, V*NLO = **Example 1999**
 Example 1999
 $\frac{1}{2}$ \overline{a} $\overline{ }$ \vee (*M*⇡) $\left(\boldsymbol{r} \boldsymbol{:} \quad \boldsymbol{M_{\pi}} \boldsymbol{r} \right) =$ i
I i
I ⇡ (*M*phys *M*² ⇡ i. j $\overline{1}$ $\overline{1}$ \overline{M} \overline{r} \overline{M} \overline{r} \overline{M} \overline{r} \overline{M} \overline{R} \overline{M} *^q*[~] ² ⁺ *^M*² *,* $\overline{\Lambda}$ 1 \overline{a} ⇡ (*M*phys *M*² ⇡ $\overline{\mathbf{r}}$ *(* $\bm{r} = 1$ $= A + B\Lambda$ $\overline{}$ \backslash $\overline{}$ \blacksquare *A p p* \blacksquare +0*.*011 0*.*009 *B* = 3*.*95+0*.*⁴⁵

*M*²

*M*² Baru, EE, Filin, Ge) gelia, Phys. Rev.
' 01400 Hev. C92

Hev. C92
 (2015)
 01400 $\mathbf 1$ ~¹ *· q*~ ~² *· q*~ *g*2 *A* ~¹ *· q*~ ~² *· q*~ 0*.*49 0*.*⁵⁵ GeV² **Baru, EE, Filin, Gegelia, Phys. Rev. C92 (2015) 014001** *•* $\frac{1}{2}$ + α */* α / *c* α

4*F*²

fix C_0 at a given value of M_π from the condition $\frac{1}{1}$ \bullet fix C_0 at a given value of M_π from the *Condition* ix C_0 at a given value of M_π from the

(*M*phys

q~ ² + *M*²

 $r(C_0, M_\pi) = r_{\rm extrapol}(M_\pi)$ $\alpha_{\mathrm{trapol}}(M_\pi)$

make predictions for

 j

4*F*²

*M*² ⇡

q~ ² + *M*²

 $\overline{\mathcal{M}}$ *,* $a(M_\pi),\,B_d(M_\pi),\,v_2(M_\pi),\,v_3(M_\pi),\,v_4(M_\pi)$

LETs for the ³S₁ partial wave **M E E E** FIG. TO THE ³S1 partial wave suggested in the 3S1 partial wave suggested in Ref. [39]. Solid square and filled triangle t refer to the experimental value and he lattice-QCD result of that work, respectively.

 $\frac{1}{\pi}$ *v*₄

Predicted chiral extrapolations based on the LETs + linear Mπ-dependence of Mπr

and the first three shape parameters in the ³S¹ partial wave assuming the linear *M*⇡-dependence of the e↵ective range as shown → results seem to be internally consistent!

MANUSIME IS STATE: LETs for the ³S₁ partial wave *M*⇡*r* = *A* + *BM*⇡*, M* \blacksquare **LE** is tor

We can also try an extrapolation of M_{π} r which is linear $\frac{1}{2}$ in M_{π^2} 0.000 milion lation of M_{π} r which is linear hich is linear

$$
M_\pi r_{\rm extrapol}=A+BM_\pi^2,
$$

with

0 0.2 0.4 0.6 0.8

```
A = 1.149_{-0.009}^{+0.009}+0.011
                             -0.009A = 1.149_{-0.009}^{+0.009}<sup>-0.009</sup>
                             +0.011
                             +0.011<br>-0.009
```

$$
B=3.95^{+0.45+0.45}_{-0.49-0.55}~\rm GeV^{-2}
$$

ans see what comes out for 1 ans see what comes out for
the binding energy and the ERE coefficients *a* \overline{a} *re binaing chorgy* and *the*
:RE coefficients *M* and the Dirichle Briefly and the state of the Dirichle Brief and the Brief School and the Brief and the Brief Andre

0 0.2 0.4 0.6 0.8

LETs for the ³S₁ partial wave

Predicted chiral extrapolations based on the LETs + linear M^π 2-dependence of Mπr

 \rightarrow seems to be inconsistent with the trend in B_{d...}

Application 2: Experimental data is a set of the set of NPLQCD at $M_\pi \sim 450$ **MeV meets LETs** with uncertainties, with the phase shift calculated at mπ ∼ $\frac{1}{2}$, we define the zeros of the $SU(1)$ at $\rm ML \sim 450$ MeV $s \sim$ as the such as the such as the scattering length and such as the scattering length and s of similar size to the effective range (as expected for maats I H is not met and $\frac{d}{dt}$ is assumed the assumed linear fitting function should function should function should be that have been presented is not yet sufficient to perform a Instead, we present a simplified discussion of the two channels to highlight some of the important features and naligatian 2. constrained by the location of the dineutron pole. At NLO, there are nominally two additional fit parameters, but requiring that the dineutron pole remains unchanged **PLQUD AT M** $_{\pi}$ \sim **450** \mid be directly related to \mathcal{H} . Finally, at NNLO there are the are three are three are three are three are three We further emphasize that the low-energy behavior of the mixing angle found in Ref. [9] and shown in the right of Fig. 5 seems to be at variance with the expected threshold behavior for this quantity, ¯✏¹ ⇠ *^k*³. EE: correct? Irregardless of these di↵erences, the two approaches yield similar numerical results for the ³*S*¹ phase shift and the mini oon al mising angle of momenta. The scattering length and the scattering length and election and election in Ref. (10) for \mathcal{U} at ivi π \sim that paper that the e α ective range, expressed in units of the pion mass, may be approximated by a linear function *M*⇡. While the LETs are certainly beyond their range of applicability at such heavy pion masses, this conjecture was tested using the LETs in our previous work in our previous work in the *Le*Ts in the deuteron binding of the deuteron binding energy of the deuteron binding energy of the deuteron binding energy of the deuteron binding ene was independent with the general trend of lattice data $\frac{1}{2}$ calculated on the lattice at \mathcal{A} in comparison with the predictions based on the LETS at LO and NLO using a second on **Annication O.** AS SHOWN IN THE SHOW ARRIVE AT SIMILAR CONCLUSIONS AS IN THE CASE OF THE CASE OF THE SPIN-**EXECT PREDICTIONS FOR A 450 MEV MEGIS LETS IN A VERY GOOD ALL IS** collaboration, there is a clear discrepancy for the two lowest values of the momentum *k*. In particular, for the lowest As shown in Fig. 6, we arrive at similar conclusions as in the case of the spin-triplet channel. While our NLO **CALCULATED ON THE LATTICE AT ASSESS** the NPLQCD result for the dineutron binding energy *Bnn* as input. For notation see Fig. 3.

effective range, is not feasible, and additional calculations are required in order to accomplish this complish this. However, the complish this complish this complish the \sim accomplish a reliable determination of the chiral nuclear forces from LQCD. Related discussions in the context of **Example 2018** Baru, EE, Filin, to appear and the control of the control of the control of the control of the con α , α , lattice-QCD analysis. Similarly to the ³*S*¹ channel, the predictions of the LETs based on the dineutron binding energy collaboration, there is a clear discrepancy for the two lowest values of the momentum k . In particular, for the lowest values of the low

pionless EFTs for multinucleon systems can be found in

from: Orginos, Parreno, Savage, Beane, Chang, Detmold, Ph We use the NPLQC result in Eq. (3.15) and denote the deuteron in Eq. (3.15) and in Eq. (3.15) and le from: Orginos, Parreno, Savage, Beane, Chang, Detmold, Phys. Rev. D92 (2015) 114512 *^a*(3*S*1) ⁺ 2 from: Orginos, Parreno, Savage, Beane, Chang, Detmold, Phys. Rev. D92 (2015) 1⁻ *k*₂ $\overline{2}$ *x*₂ $\overline{3}$ *x*₂ $\overline{4}$ *x*₂ $\overline{4}$ *x*₂ $\overline{5}$ *x*₂ $\overline{6}$ *x*₂ $\overline{2}$ *x*₂ $\overline{4}$ *x*₂ $\overline{5}$ *x*₂ $\overline{6}$ *x* $\overline{7}$ *x* $\overline{7}$ *x* $\overline{7$ elaborate on the providence on the robustness of the robustness of the robustness of the robustness of the rob approximation is not self-consistent. All arguments given in the previous section apply to the ¹*S*⁰ channel too, even **from: Orginos, Parreno, Savage, Beane, Chang, Detmold, Phys. Rev. D92 (2015) 114512**

NPLQCD meets LETs: The ³S₁ channel

Once the scattering length is fixed, all other quantities (e.g. phase shifts) are predicted in a parameter-free way 3138 and beamening iong... To middle, and only apartmed poly. Phase online, are prodicted in a parameter free wov predicted in a parameter-free way

NPLQCD meets LETs: The 3S₁ channel

- Reasonably good convergence of the LETs \mathcal{L} and convergence of the LETs raphy good convergence of the LETs
- Lattice phase shifts at two lowest energies seem to be inconsistent with B_d parametrization of the S-matrix [41]. Filled black regions correspond to the lattice-QCD calculations. Orange bands show the

NPLQCD meets LETs: The 3S₁ channel

- Reasonably good convergence of the LETs F_1 and convergence of the LETs raphy good convergence or the LETs
- Lattice phase shifts at two lowest energies seem to be inconsistent with B_d result for the deuteron binding energy *B^d* as input. The uncertainty at LO shown by the orange bands is entirely given by the
- A more conservative uncertainty estimation (δβ = 1) does not help…

NPLQCD meets LETs: The ³S₁ channel

● Consequently, different results for the scattering length and effective range:

 $\begin{array}{ccc} \textsf{NPLQCD:} & (M_{\pi} a^{(3S_1)})^{-1}=-0.04(^{+0.07}_{-0.10})(^{+0.08}_{-0.17}), & M_{\pi} r^{(3S_1)} ~=~ 7.8(^{+2.2}_{-1.5})(^{+3.5}_{-1.7}) \end{array}$ $\begin{array}{ccc}\n\lambda & -0.10 \lambda & -0.11 \lambda\n\end{array}$ and $\lambda = 0.11 \lambda$ and $\lambda = 0.11 \lambda$ via the variation of $\lambda = 1.3 \lambda \lambda = 1.1 \lambda$ = 0*.*5 as described in the text. The horizontal dotted lines specify the range of values for *B^d* consistent with the lattice-QCD NLO LETS: $(M_{\pi}a^{(3S_1)})^{-1} = 0.196(^{+0.014}_{-0.013})(^{+0.018}_{-0.008}), \qquad M_{\pi}r^{(3S_1)} = 2.44(^{+0.08}_{-0.08})(^{+0.21}_{-0.47})$ $\sqrt{-0.013}$ $\sqrt{-0.000}$ $\sqrt{-0.000}$ $\sqrt{-0.000}$ $\sqrt{-0.000}$ $\sqrt{-0.000}$ t_{HIO} in D_q and from the grey area around it shows the estimated uncertainty from that F results from the estimate t_{H} NPLQCD: $\hspace{-0.7cm} \begin{array}{rcl} +0.08 \cr -0.17 \end{array}$, $\hspace{0.2cm} M_{\pi} r^{(^3\!S_1)} \hspace{0.1cm} = \hspace{0.1cm} 7.8 \bigl(\begin{array}{c} +2.2 \cr -1.5 \end{array} \bigr) \bigl(\begin{array}{c} +3.5 \cr -1.7 \end{array} \bigr)$ $M_{\pi}r^{(3S_1)} = 2.44(^{+0.08}_{-0.08})(^{+0.21}_{-0.08})$
declinations of statistics systematics error in B_d uncertainty of the LETs (δβ=1)

T = *V* + *V G*0*T* = *V* + *V G*0*V* + *V G*0*V G*0*V* + *...* **NPLQCD meets LETs: The ³S₁ channel**

(If true), the very large effective range,

 $r^{(^3S_1)} \sim 8 M_\pi^{-1}$

T = *V* + *V G*0*T* = *V* + *V G*0*V* + *V G*0*V G*0*V* + *...*

T = *V* + *V G*0*T* = *V* + *V G*0*V* + *V G*0*V G*0*V* + *...* would suggest:

- either the interaction range (much) $-$ enner the interaction range $V_{1\pi}$ *The metal distribution range*
	- $-$ or the appearance of a pole in $k \cot \delta$ near threshold !#*n*

T = *V* + *V G*0*T* = *V* + *V G*0*V* + *V G*0*V G*0*V* + *...* **NPLQCD meets LETs: The ³S₁ channel**

(If true), the very large effective range,

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T = *V* + *V G*0*T* = *V* + *V G*0*V* + *V G*0*V G*0*V* + *...* would suggest:

- either the interaction range (much) $\frac{1}{2}$ longer than that of $V_{1\pi}$ \rightarrow α ₁ α ¹ α ¹
	- $-$ or the appearance of a pole in $k \cot \delta$ near threshold !#*n*

the approximation $k \cot \delta \simeq -\bar{ } + \bar{ } - r k^2$ to be valid for $|k| \geq 2/r \sim M_\pi/4$. calculated on the lattice at $a-2$ in comparison with the predictions based on the predictions based on the predictions based on the predictions based on the LETs of the LETs of the LETs of the predictions based on the LET $rac{1}{2}$ $rac{1}{$ **T** *d p*¹ mation $k \cot \delta \simeq -\frac{1}{k} + \frac{1}{2} r k^2$ to be valid for $|k| \gtrsim 2/r$ In both cases, there is no reason to expect $\frac{1}{2}$ is no reason to expect
 $\frac{1}{2}$, $\frac{1}{2}$, the approximation $k \cot \delta \simeq -\frac{1}{a} + \frac{1}{2}rk^2$ *a* + 2 *X*(*n*) (*p*)*, n* = 0*,* 2*,* 3*,* 4*,...* (7) the approximation $k\cot\delta\simeq -\frac{1}{a}+\frac{1}{2}rk^2$ to be valid for $|k|\gtrsim 2/r\sim M_\pi/4$. 1 *a* $+$ 1 2 rk^2 to be valid for $\; |k| \gtrsim 2/r \sim M_\pi/4$

T = *V* + *V G*0*T* = *V* + *V G*0*V* + *V G*0*V G*0*V* + *... ^k* cot ' ¹ *^a* ⁺ 1 *rk*² *k* cot ' *a* 2 **NPLQCD meets LETs: The ³S₁ channel**

1

*rk*²

1

+

(If true), the very large effective range, $\sqrt{1 + \left(\frac{1}{2} \right)^2 + \left(\frac{1}{2} \right)^2 + \left(\frac{1}{2} \right)^2 + \left(\frac{1}{2} \right)^2}$

$$
r^{(^3S_1)}\sim 8M_\pi^{-1}
$$

T = *V* + *V G*0*T* = *V* + *V G*0*V* + *V G*0*V G*0*V* + *...*

T = *V* + *V G*0*T* = *V* + *V G*0*V* + *V G*0*V G*0*V* + *...* would suggest:

- either the interaction range (much) $\frac{1}{2}$ longer than that of $V_{1\pi}$ \rightarrow α ₁ α ¹ α ¹
	- $\mu_{1\pi}$ or the appearance of a pole in $k \cot \delta$ near threshold !#*n*

the approximation $k \cot \delta \simeq -\bar{ } + \bar{ } - r k^2$ to be valid for $|k| \geq 2/r \sim M_\pi/4$. calculated on the lattice at $a-2$ in comparison with the predictions based on the predictions based on the predictions based on the predictions based on the LETs of the LETs of the LETs of the predictions based on the LET *R2 R*₂ m *R*² *R*₂ m *R*² and *C* asses, there is no reason to expect $rac{1}{2}$ $rac{1}{$ **T** *d p*¹ mation $k \cot \delta \simeq -\frac{1}{k} + \frac{1}{2} r k^2$ to be valid for $|k| \gtrsim 2/r$ $\frac{1}{2}$ is no reason to expect
 $\frac{1}{2}$, $\frac{1}{2}$, the approximation $k\cot\delta \simeq -\frac{1}{a}+\frac{-rk^2}{2}$ α approximation \boldsymbol{k} co *a* + 2 *X*(*n*) (*p*)*, n* = 0*,* 2*,* 3*,* 4*,...* (7) the approximation $k\cot\delta\simeq -\frac{1}{a}+\frac{1}{2}rk^2$ to be valid for $|k|\gtrsim 2/r\sim M_\pi/4$. 1 *a* $+$ 1 2 rk^2 to be valid for $\; |k| \gtrsim 2/r \sim M_\pi/4$ *<u>koncon</u> <mark>o* expe</mark> *i*_{*i*} *± k*²*/M*² ⇡ $\overline{}$ *<i><i>x**I*</sub> /*A* q *k*²*/M*² *|k|* & 2*/r* ⇠ *M*⇡*/*4

Moreover, the second, deeper bound state is (normally) to be viewed as an artifact uncertainty of **B**_d in Eq. (3.15). The NLO data distribution bands correspond to the uncertainty in *Bd* and uncer of the effective range approximation: **with intervals estimation** of the combined in quadrature. The combined in \mathbb{R} deel \sim *f R* ء *a*
ond. deeper bound <u>Ind</u> *normally) to be viewed as an artif* $\frac{1}{2}$ he effective range a *|k|* & 2*/r* ⇠ *M*⇡*/*4 of the effective range approximation: *|k|* & Moreover, the second, deeper bound state is (normally) to be viewed as an artifact
of the effective range approximation: \overline{X} *a* \overline{a} $\frac{a}{b}$ is (norm ially) to be viewed as a<mark>n art</mark>i l, $|$ *k* α be view

$$
-\frac{1}{a} + \frac{1}{2}rk^2 = 0 \longrightarrow k_{1,2} = \frac{i}{r}\left(1 \pm \sqrt{1 - \frac{2r}{a}}\right) \longrightarrow k_{2} \simeq i\left(\frac{2}{r} - \frac{1}{a}\right)
$$

priyologi pion mass, $\kappa_1 = 40i$ me v (deuteron), $\kappa_2 = 200i$ me v (artifact) $\bm{k_1}$ $\bf 45\mathit{i}$ \mathbf{A} *m p*
 MPLOCD sel $\kappa_1 \leq 4$ *i* wie v (deuteron), $\kappa_2 \leq 200$ is - physical pion mass: $k_1 \approx 45i \text{ MeV (deuteron)}, k_2 \approx 200i \text{ MeV (artifact)}$ *k*¹ ' $k_2 \simeq 200i$ MeV (artifact *r k*1*, wev* (artifact) $\simeq 200i$ M

 $\overline{}$ MPI OCD solution: $\overline{h} \approx 15$. MoV (virtual state) $\overline{h} \approx 135$. MeV (deuteren) - NPLQCD solution: $k_1 \simeq -15i \text{ MeV}$ (virtual state), $k_2 \simeq 135i \text{ MeV}$ (deuteron) *,* ⇤*^b* $k_1 \simeq -15$ *a ^VC*(*q*) = ² Z ⇤SFR *dµ µ* ⇢*C*(*µ*) $-$ **NPLQCD solution:** $k_1 \simeq -15i$ MeV (virtual state), $k_2 \simeq 135i$ MeV (deuteron) $k_1 \simeq -15i\;\text{MeV}$ (virtual state), $k_2 \simeq 135i\;\text{MeV}$ (deuteron) *a a aa******af a******<i>a******<i>a******<i>a******<i>a m* (virtual state), k $_2 \simeq 1$ ⌘

NPLQCD meets LETs: The ¹S₀ channel

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- Let $\sum_{i=1}^{\infty}$ 300 meV are in a very good agreement with the phase shift with the phase shifts calculated by the NPL α \bullet Similar (but somewhat less stringent) conclusions as in the ³S₁ partial wave *|k|* & 2*/r* ⇠ *M*⇡*/*4
- mantantal momentum in the large of the the from the induced the number of the shift from the larger than the lar
Analyzed the larger than the larges of the larges than the shift of the shift of the shift of the shift of th • Again, the large effective range puts in question the applicability of the effective range approximation at energies of the lattice data *Bnn <* 0*.*5 MeV*, B*virtual *nn <* 0*.*6 MeV *k*a and *a allie allies* or ne lattic
.
	- The NPLQCD "dineutron" seems to be an artifact; shallow bound/virtual state... $\overline{4}$

NPLQCD meets LETs: The ¹S₀ channel

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callbon using the lattice phase shifte at two lewest energies as input we connet • When using the lattice phase shifts at two lowest energies as input, we cannot accommodate for $\,_{nn} \simeq 12.5 \; \text{MeV}\,$. We find:

 $\sum_{\alpha} \alpha_{\alpha}$ prediction. The apparent state corresponding to the gray bands of the gray bands $B_{nn} < 0.5 \; \text{MeV}, \qquad B^{\text{virtual}}_{nn} < 0.6 \; \text{MeV}$

Summary and conclusions

• LETs allow to reconstruct the NN scattering amplitude at fixed M_{π} using a single observable (e.g. binding energy) as input

> \rightarrow extrapolations of lattice-QCD results in energy, self-consistency checks

- The linear in M_π dependence of M_π r^(3S1) conjectured by the NPLQCD collaboration based on their M_{π} ~ 800 MeV results is consistent with the common trend for B_{d}
- The newest NPLQCD results at M_{π} ~ 450 MeV for the ${}^{1}S_{0}$ / ${}^{3}S_{1}$ phase shifts at the two lowest energies are incompatible with their B_{nn} / B_d energies (within errors). Underestimated systematics for the extraction of phase shifts?
- The NPLQCD determination of the scattering lengths and effective ranges based on the effective range approximation is not self-consistent…

LETs: a useful addition to the lattice QCD toolbox!