RUB

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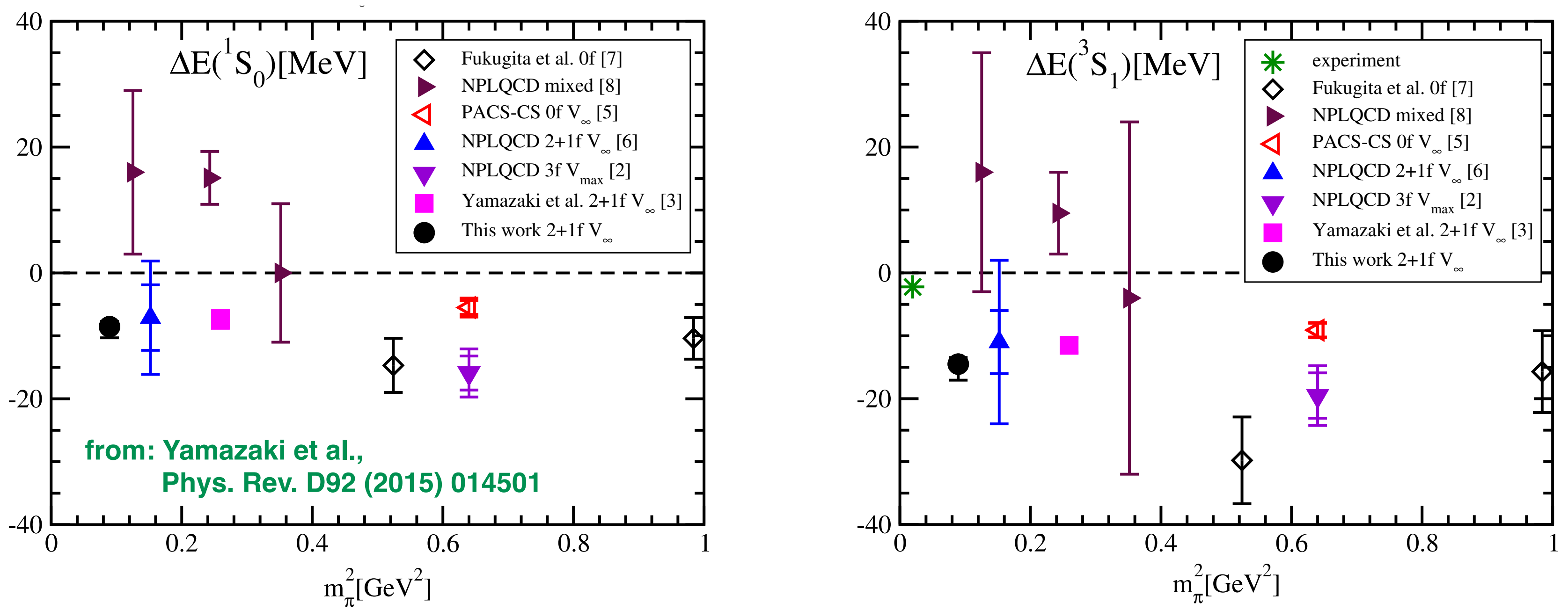
Confronting lattice-QCD results for the NN system with low-energy theorems

in collaboration with Vadim Baru, Arseniy Filin and Jambul Gegelia

- Introduction
- Low-energy theorems for NN scattering
- Testing conjectured linear extrapolation $M_\pi r^{3S1} = A + B M_\pi$
- Implications for the NPLQCD results at $M_\pi \sim 450$ MeV
- Summary

Bundesministerium für Bildung und Forschung
DFG Deutsche Forschungsgemeinschaft
erc European Research Council

Lattice-QCD results for NN scattering observables



Further, the HAL QCD Collaboration claims [by first generating the NN potential] weaker attraction in both $^1S_0$ and $^3S_1$-$^3D_1$ channels and no bound states for $M_\pi > 411$ MeV Ishii et al.'12

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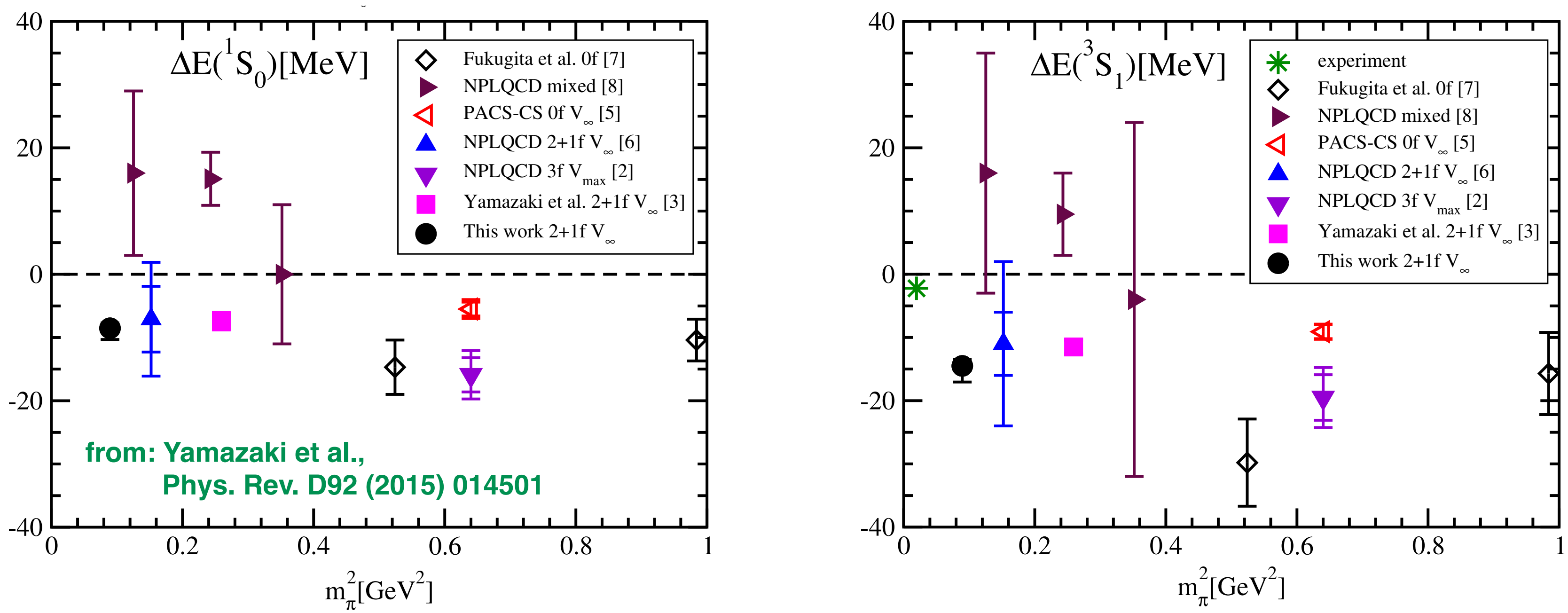

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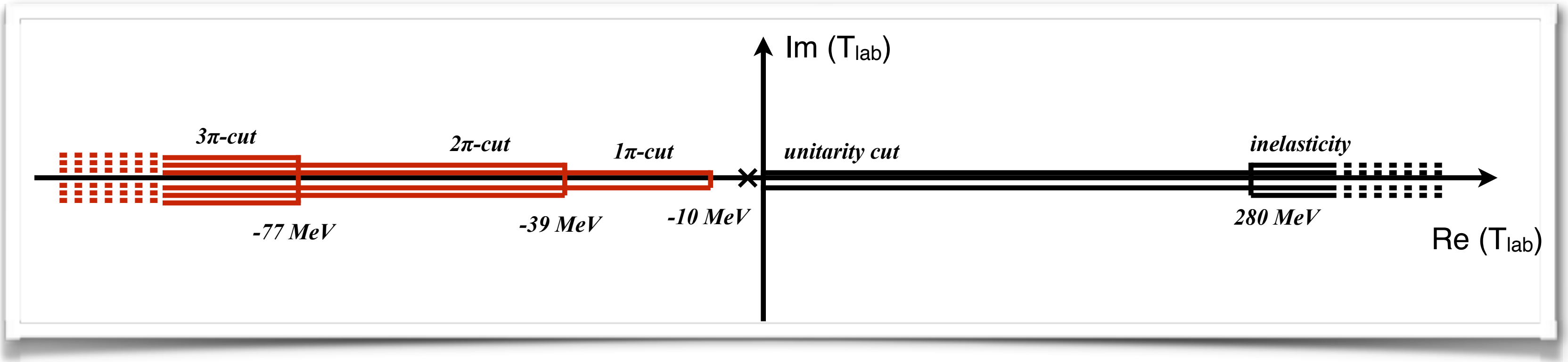
(Some) tools to extrapolate and/or check consistency of lattice QCD results:

- pionless EFT: extrapolate in the number of nucleons [large M_π] Barnea, Kirscher, van Kolck, …
- chiral EFT: extrapolate in M_π (and the number of nucleons) [small M_π]
  Beane, Savage, EE, Glöckle, Meißner, Gegelia, Soto, Chen, …
- Low-Energy Theorems (LETs) for the NN system: extrapolate in energy at fixed M_π
  Baru, EE, Filin, Gegelia

Low-energy theorems
for nucleon-nucleon scattering

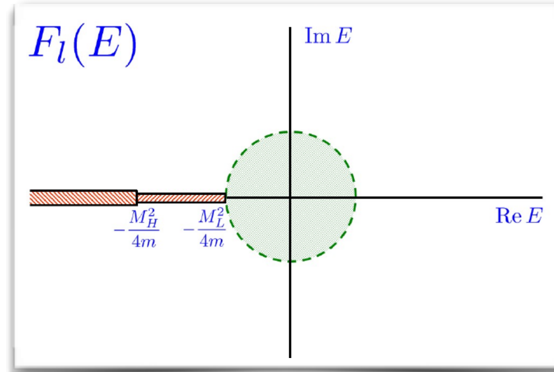
Long-range interactions govern the low-energy behavior of the amplitude and imply correlations between coefficients in the ERE which may be regarded as Low Energy Theorems



For a reconstruction of the amplitude based on dispersion relations + unitarity constraints see: Gasparyan, Lutz, EE, EPJA49 (13) 115; Albalodejo, Oller, PRC84 (11) 054009

ERE, MERE and LETs

Two-range potential $V(r) = V_L(r) + V_S(r),\ M_L^{-1} \gg M_H^{-1}$



$$S_l = e^{2i\delta_l(k)} = 1 - i\left(\frac{km}{8\pi^2}\right) T_l(k)\,, \quad T_l(k) = -\frac{16\pi^2}{m}\frac{k^{2l}}{F_l(k) - ik^{2l+1}}$$

effective range function, $F_l \equiv k^{2l+1}\cot\delta_l$

$F_l(k^2)$ is a real meromorphic function of k^2 for $|k| < M_L/2$

$\longrightarrow$ ERE: $F_l = -\frac{1}{a} + \frac{1}{2}rk^2 + v_2k^4 + v_3k^6 + v_4k^8 + \ldots$

Landau, Smorodinsky '44; Blatt, Jackson'49; Bethe'49

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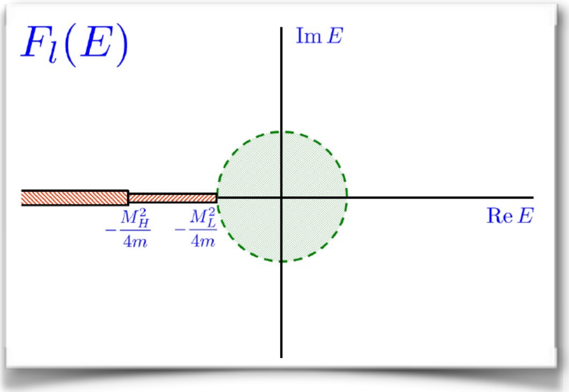

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Generalization to the modified ERE by „subtracting“ effects due to the long-range force

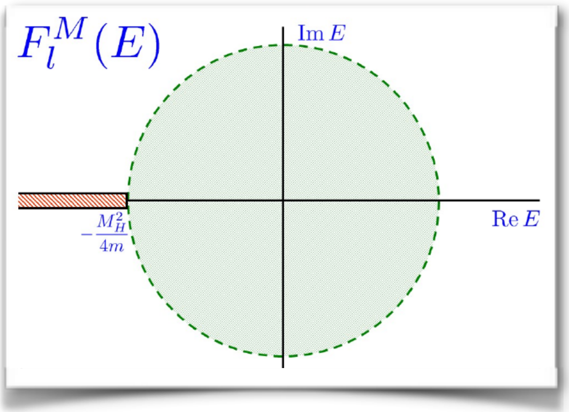
van Haeringen, Kok PRA 26 (1982) 1218

$$F_l^M(k^2) \equiv M_l^L(k) + \frac{k^{2l+1}}{|f_l^L(k)|^2}\cot\left[\delta_l(k) - \delta_l^L(k)\right]$$

$$f_l^L(k) = \lim_{r\to 0}\left(\frac{l!}{(2l)!}(-2ikr)^l f_l^L(k,r)\right)$$

Jost function for $V_L(r)$ — Jost solution for $V_L(r)$

$$M_l^L(k) = Re\left[\frac{(-ik/2)^l}{l!}\lim_{r\to 0}\left(\frac{d^{2l+1}}{dr^{2l+1}}\frac{r^l f_l^L(k,r)}{f_l^L(k)}\right)\right]$$



Per construction, F_l^M reduces to F_l for $V_L = 0$ and is a real meromorphic function for $|k| < M_H/2$

ERE, MERE and LETs

Example: proton-proton scattering

$$F_C(k^2) = C_0^2(\eta)\, k\, \cot[\delta(k) - \delta^C(k)] + 2k\,\eta\, h(\eta) = -\frac{1}{a^M} + \frac{1}{2} r^M k^2 + v_2^M k^4 + \ldots$$

where $\underbrace{\delta^C \equiv \arg \Gamma(1 + i\eta)}_{\text{Coulomb phase shift}}$, $\eta = \frac{m}{2k}\alpha$, $\underbrace{C_0^2(\eta) = \frac{2\pi\eta}{e^{2\pi\eta} - 1}}_{\text{Sommerfeld factor}}$, $h(\eta) = \text{Re}\Big[\underbrace{\Psi(i\eta)}_{\text{Digamma function}}\Big] - \ln(\eta)$

$\Psi(z) \equiv \Gamma'(z)/\Gamma(z)$

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MERE and low-energy theorems

Long-range forces impose correlations between the ER coefficients (low-energy theorems)

Cohen, Hansen '99; Steele, Furnstahl '00

The emergence of the LETs can be understood in the framework of MERE:

$$\underbrace{F_l^M(k^2)}_{\text{meromorphic for } k^2 < (M_H/2)^2} \equiv M_l^L(k) + \frac{k^{2l+1}}{|f_l^L(k)|^2} \cot\left[\delta_l(k) - \delta_l^L(k)\right]$$

can be computed if the long-range force is known

– approximate $F_l^M(k^2)$ by first 1,2,3,… terms in the Taylor expansion in k^2/M_H^2

– calculate all "light" quantities

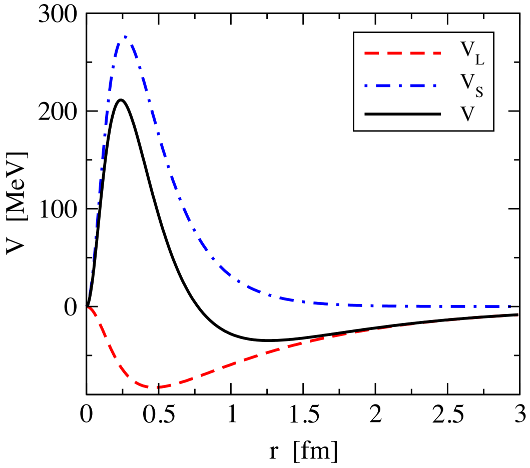
– reconstruct $\delta_l^L(k)$ and predict all coefficients in the ERE

Toy model: Low-energy theorems

$$V(r) = \underbrace{v_L e^{-M_L r} f(r)}_{V_L} + \underbrace{v_H e^{-M_H r} f(r)}_{V_H}$$

where $f(r) = \frac{(M_H r)^2}{1 + (M_H r)^2}$

and $M_L = 1.0\,,\ v_L = -0.875\,,\ M_H = 3.75\,,\ v_H = 7.5$ (all in fm^{-1})



ERE and MERE

	a	r	v_2	v_3	v_4
F_0 [fm^n]	5.458	2.432	0.113	0.515	−0.993
F_0^M [M_S^{-n}]	1.710	−1.063	−0.434	−0.680	2.624

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Low-Energy Theorems

	LO	NLO	NNLO	"Exp"
r				2.432197161
v_2				0.112815751
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v_3	0.61(12)			0.51529
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for an analytic example, see EE, Gegelia, EPJ A41 (2009) 341

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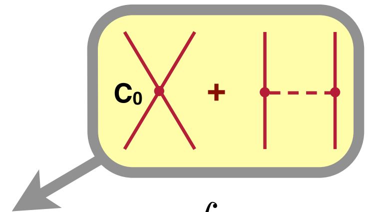
LETs for NN S-waves: Implementation

We implement the LETs by calculating NN amplitude at LO within the renormalizable approach

EE, Gegelia, Phys. Lett. B716 (2012) 338, see Jambul's talk



Kadyshevsky equation: $T\,(\vec{p},\vec{p}\,',k\,) = V\,(\vec{p}\,,\vec{p}\,') + \int d^3q\; V\left(\vec{p},\vec{q}\,\right)\; G(k,q)\; T\left(\vec{q},\vec{p}\,',k\right)$

where the Green's function is given by $G(k,q) = \frac{m_N^2}{2(2\,\pi)^3}\,\frac{1}{\left(\vec{q}^{\,2}+m_N^2\right)\left(E_k - \sqrt{\vec{q}^{\,2}+m_N^2}+i\epsilon\right)}$

The Kadyshevsky equation is solved numerically for a given Λ. The constant C_0 is expressed in terms of the scattering length a (renormalization) and the limit $\Lambda \to \infty$ is taken.

⟶ phase shifts, binding energy & all other coefficients in the ERE are predicted as functions of the scattering length [LETs]

Notice: for the purpose of the LETs, the non-relativistic approach with a finite cutoff would also do the job…

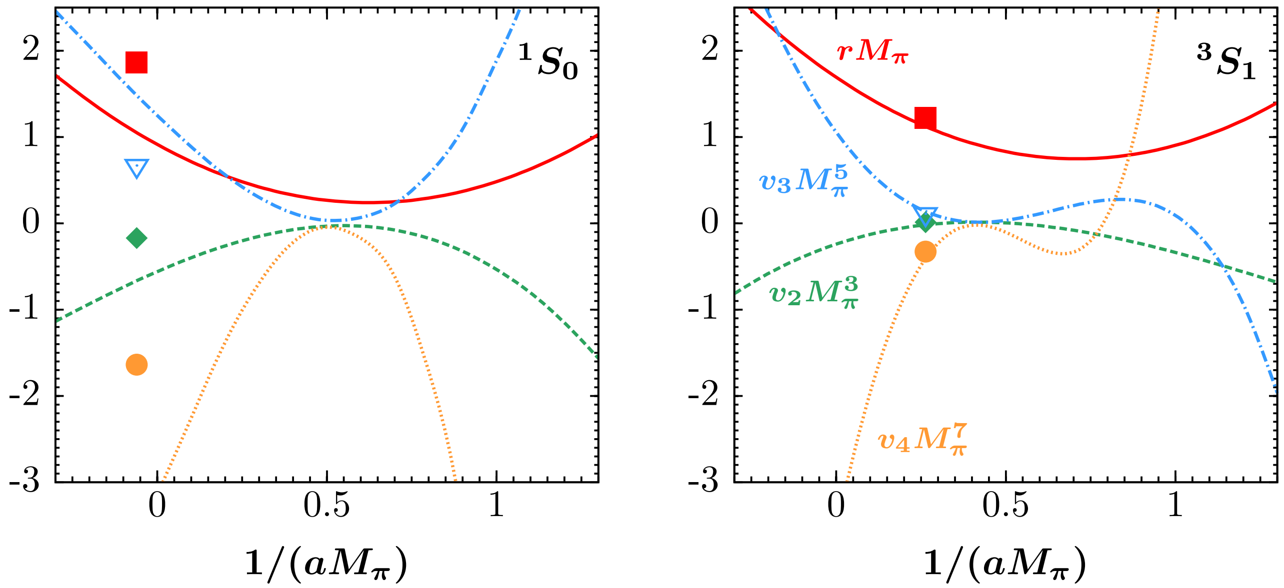
Leading-order LETs for NN S-waves

$^1S_0$ partial wave	a [fm]	r [fm]	v_2 [fm^3]	v_3 [fm^5]	v_4 [fm^7]
NLO KSW Cohen, Hansen '99	fit	fit	-3.3	18	-108
LO Weinberg	fit	1.50	-1.9	8.6(8)	$-37(10)$
Nijmegen PWA	-23.7	2.67	-0.5	4.0	-20

$^3S_1$ partial wave	a [fm]	r [fm]	v_2 [fm^3]	v_3 [fm^5]	v_4 [fm^7]
NLO KSW Cohen, Hansen '99	fit	fit	-0.95	4.6	-25
LO Weinberg	fit	1.60	-0.05	0.8(1)	$-4(1)$
Nijmegen PWA	5.42	1.75	0.04	0.67	-4.0

- perturbative inclusion of pions (KSW approach) fail
- $^1S_0$ channel: limited predictive power of the LETs due to the weakness of the OPEP; taking into account the range correction (NLO) leads to improvement
  EE, Gasparyan, Gegelia, Krebs, EPJA 51 (2015) 71
- $^3S_1$ channel: LETs work as advertised (strong tensor part of the OPEP)

Next-to-leading-order LETs for NN S-waves



Even in the $^3S_1$ channel, the accuracy is insufficient when using effective range as input:

$$r = 1.75 \text{ fm [input]} \quad \longrightarrow \quad a = 7.16 \text{ fm}, \quad B_d = 1.1 \text{ MeV [LET predictions]}$$

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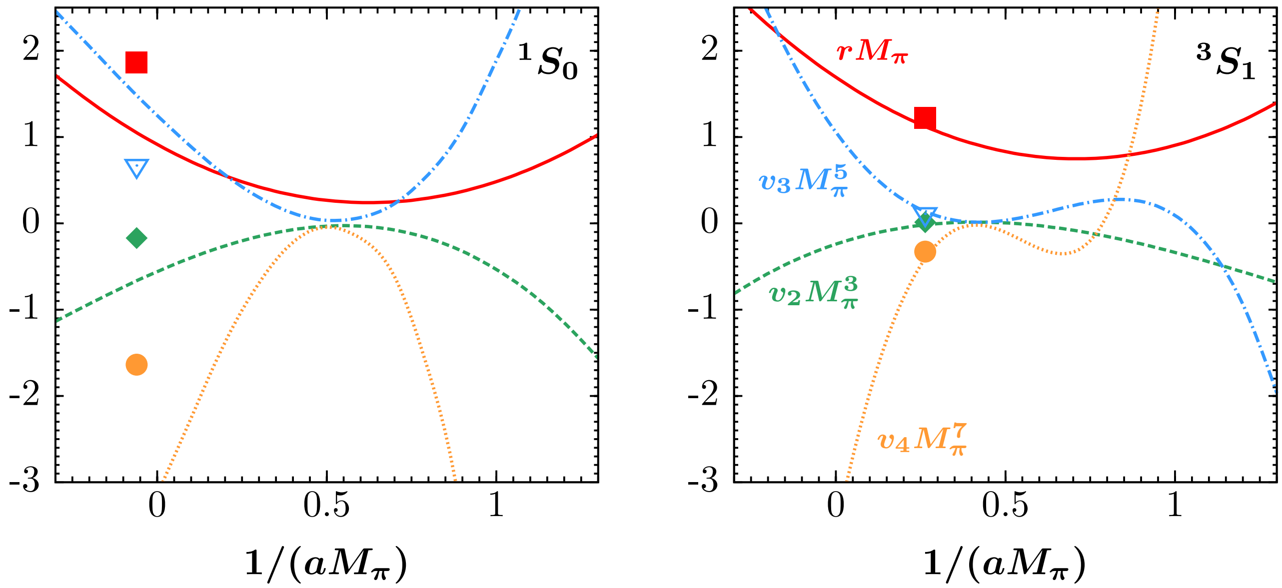

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$$r = 1.75 \text{ fm [input]} \quad \longrightarrow \quad a = 7.16 \text{ fm}, \quad B_d = 1.1 \text{ MeV [LET predictions]}$$

→ go to NLO LETs by including the (modified) effective range correction modeled via

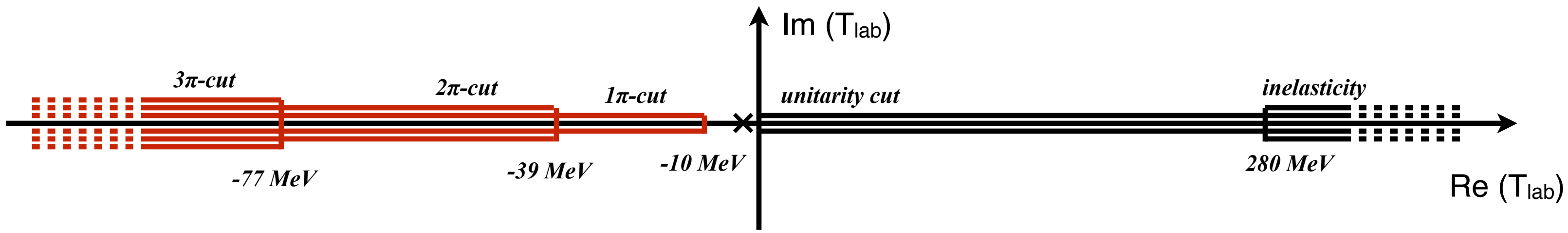
$$V^{\text{NLO}} = V^{\text{LO}} + \beta \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + M^2}, \quad M = 700\,\text{MeV}$$

	a [fm]	r [fm]	v_2 [fm^3]	v_3 [fm^5]	v_4 [fm^7]
LO LET	5.42*	1.60	−0.05	0.82	−5.0
NLO LET	5.42*	1.75*	0.06	0.70	−4.0
Nijmegen PWA	5.42	1.75	0.04	0.67	−4.0

*Fit parameter.

For the $^1S_0$ channel see: EE, Gasparyan, Gegelia, Krebs, EPJA 51 (2015) 71

LETs at unphysical pion mass



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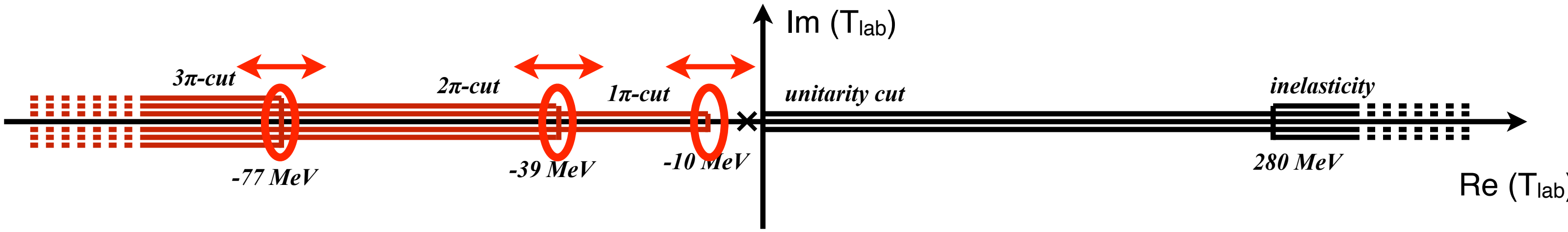

- When going to unphysical pion masses, the main change in the left-hand singularities is due to threshold shifts (explicit M_π-dependence)

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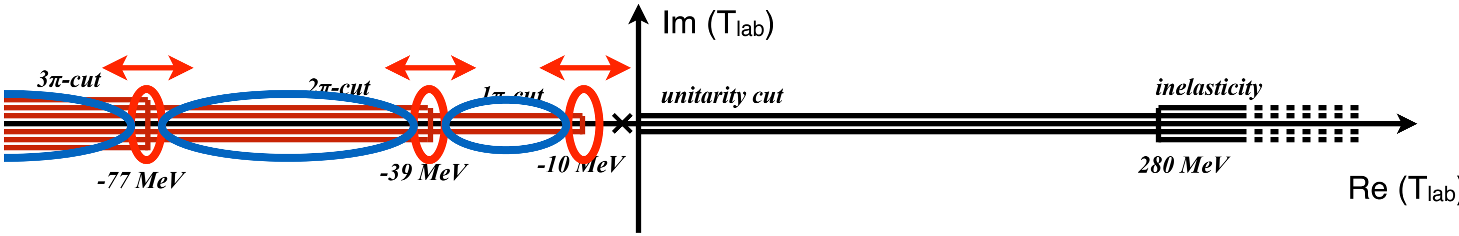
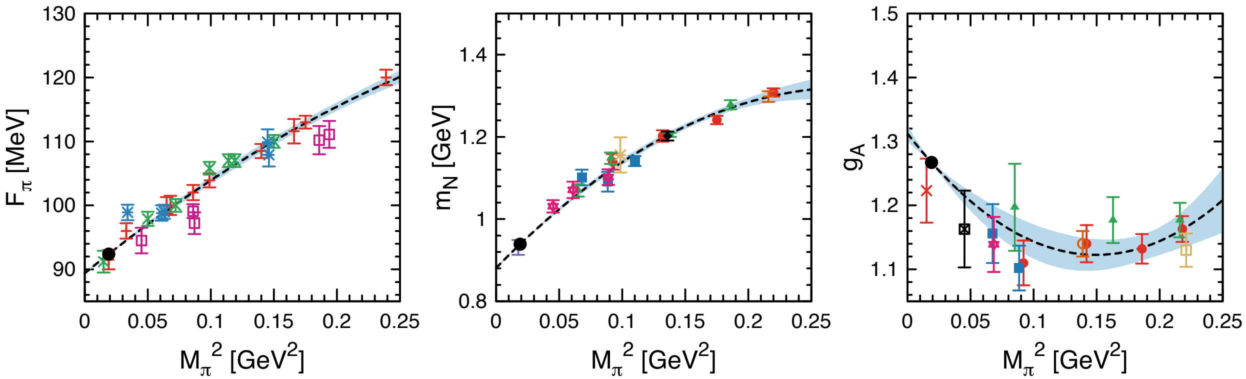


- When going to unphysical pion masses, the main change in the left-hand singularities is due to threshold shifts (explicit M_π-dependence)

- We also include changes in discontinuity across the left-hand cuts (M_π-dependence of g_A, F_π) and M_π-dependence of the nucleon mass



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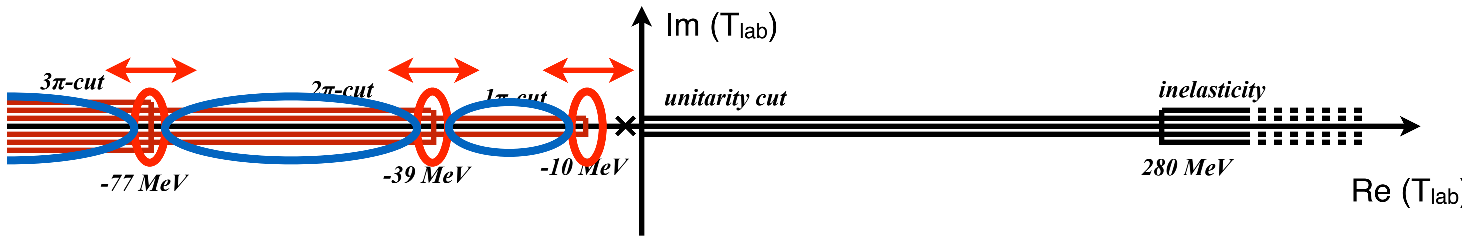

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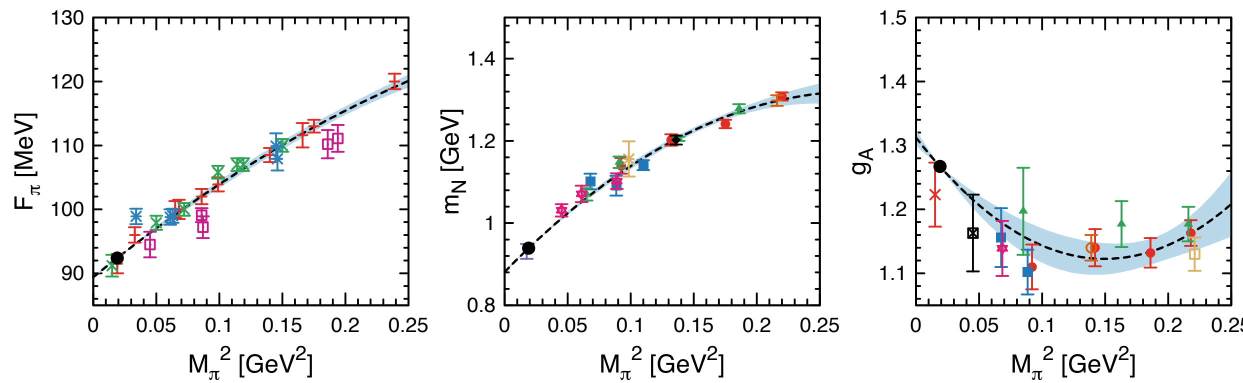
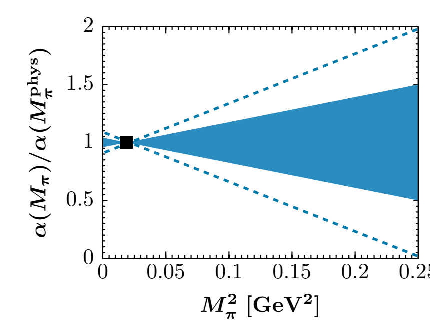


- For NLO LETs, we need to know M_π-dependence of the subleading short-range term (a higher-order effect in EFT)

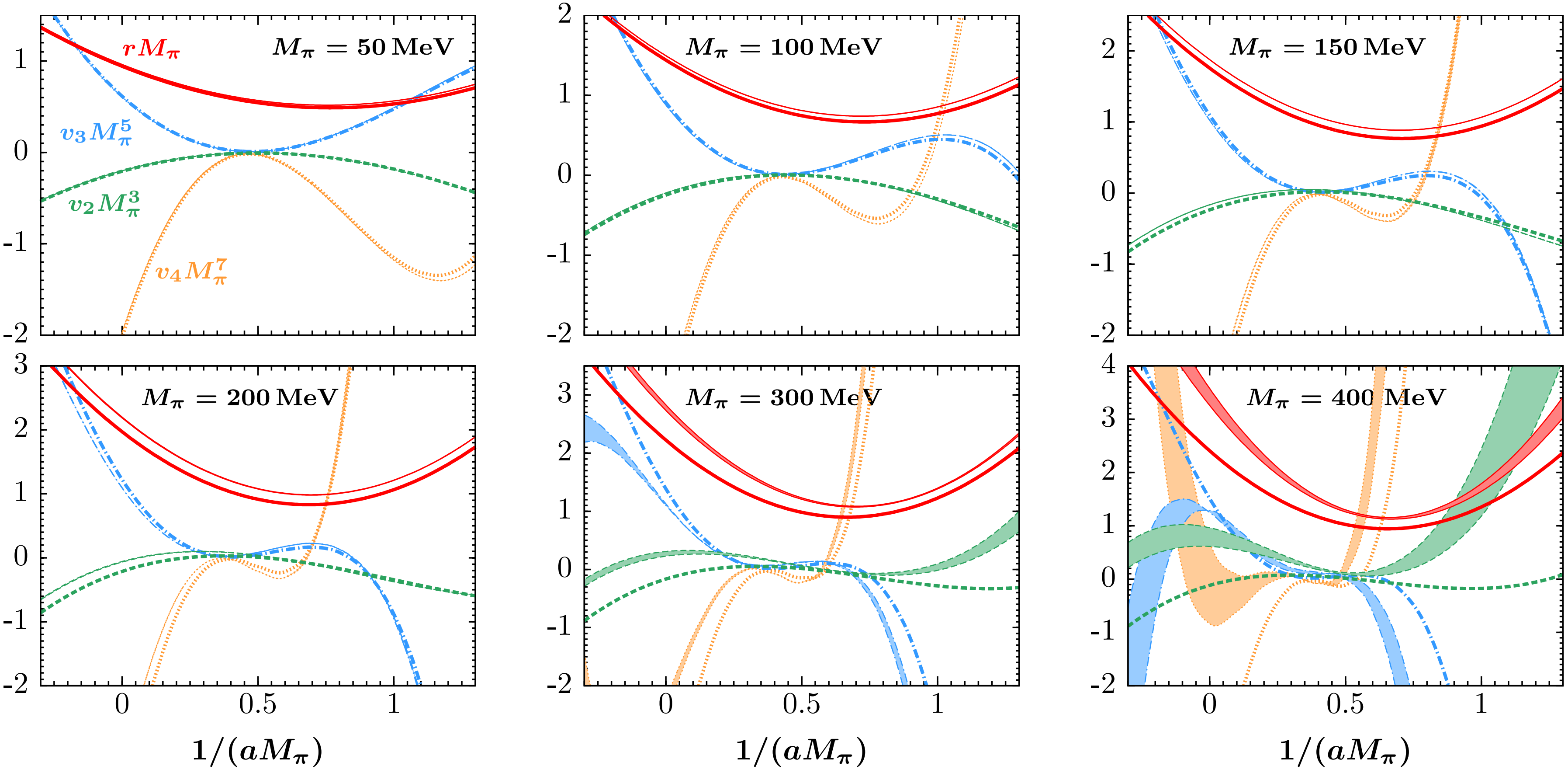
$$V_{\mathrm{NLO}} = \beta \, \frac{\vec{\sigma}_1 \cdot \vec{q} \; \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + M^2}$$

We allow β to vary: $\delta\beta(M_\pi = 500\ \mathrm{MeV}) = \pm 50\%$ (±100%)



NLO LETs for the $^3S_1$ partial wave

LETs at nonphysical pion masses (at NLO, δβ = 0.5)



- good convergence and accuracy of the LETs for low values of M_π (below 200 MeV)
- sizable uncertainty at pion masses above 400 MeV (even at NLO)

Intermediate summary

The take-away message so far:

- LETs = manifestations of the longest-range interaction due to 1π-exchange in the (energy dependence of the) NN scattering amplitude
- Good/fair predictive power in the $^3S_1$-$^3D_1$ / $^1S_0$ channels
- Work at any M_π (provided the OPEP is the longest-range interaction)
- Matching to lattice: Need a single amount of information to fix the short-range physics at a given M_π in order to reconstruct T(k)

LETs vs chiral EFT: Similar in spirit but not quite the same…

- Not a χ-extrapolation in the usual sense: given an observable $X(M_\pi)$, can predict $Y(M_\pi)$, $Z(M_\pi)$,…. No assumptions are made about $C_0 = f(M_\pi)$!
- No reliance on the M_π-expansion; infinite M_π-limit well-defined; M_π-dependence of m_N, g_A, F_π taken directly from lattice QCD
- Accuracy & applicability range limited by the second-lowest non-analyticity in the amplitude (2π-exchange, heavy-meson exchange, Δ, …)

Application 1:
testing the conjectured linear extrapolation for $M_\pi r$: $M_\pi r = A + B M_\pi$

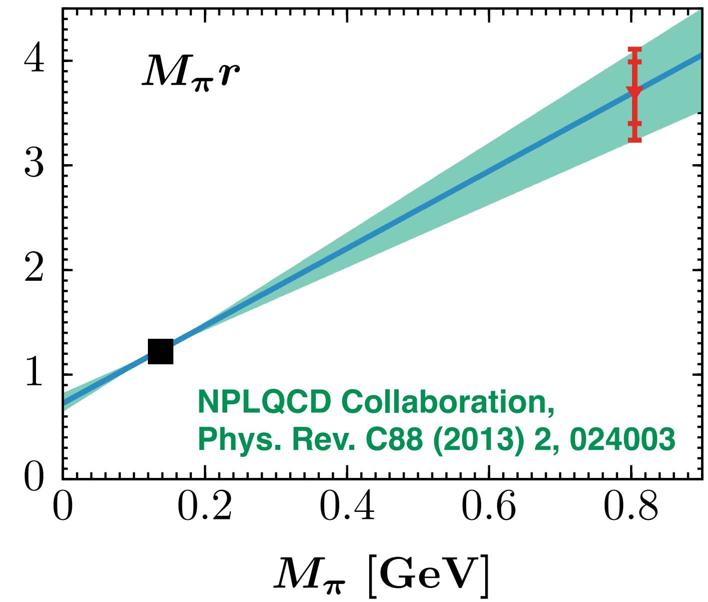
Baru, EE, Filin, Gegelia, Phys. Rev. C92 (2015) 014001

- fix C_0 at a given value of M_π from the condition

$$r(C_0,\ M_\pi) = r_{\rm extrapol}(M_\pi)$$

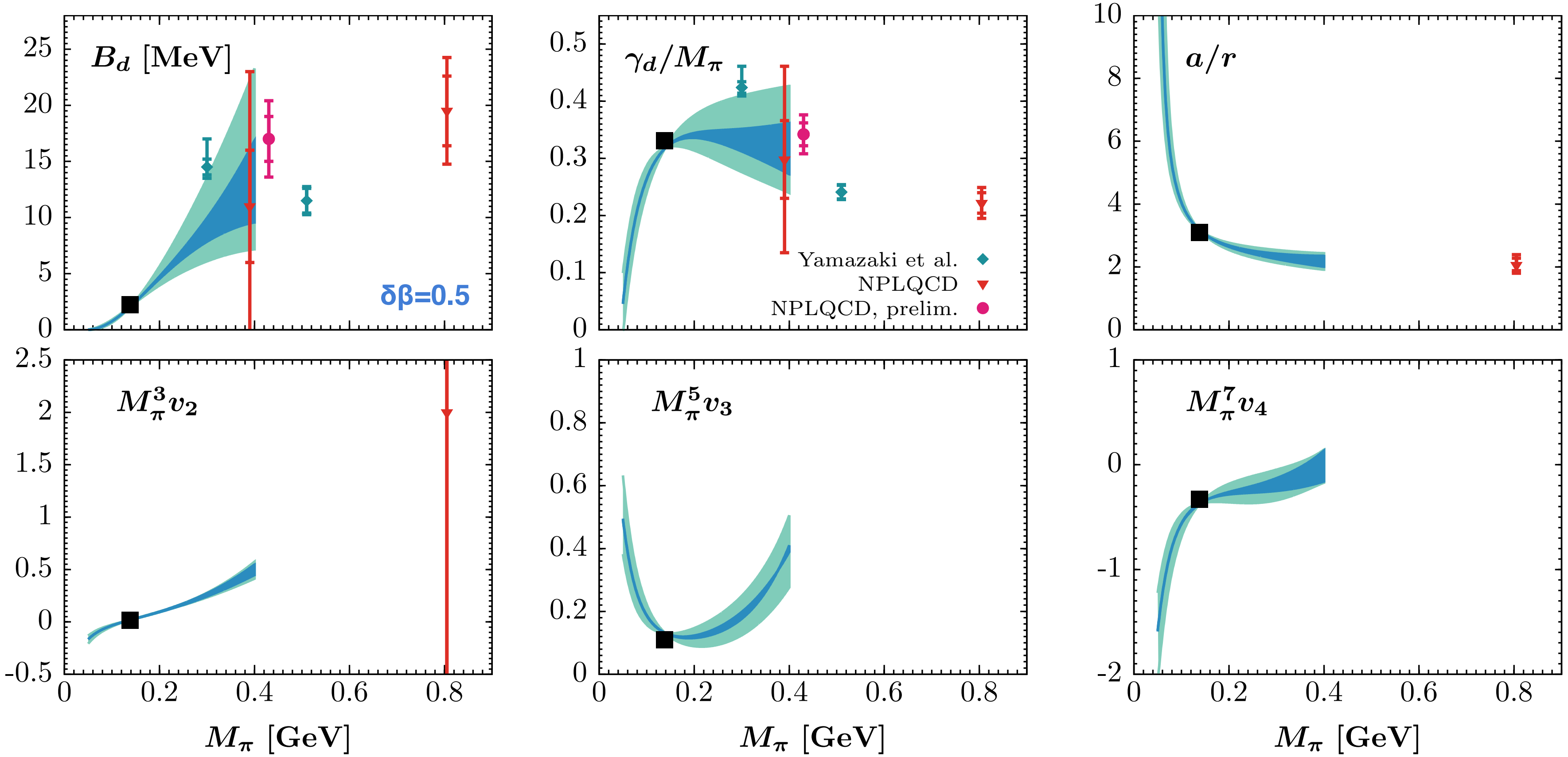
- make predictions for

$$a(M_\pi),\ B_d(M_\pi),\ v_2(M_\pi),\ v_3(M_\pi),\ v_4(M_\pi)$$



LETs for the $^3S_1$ partial wave

Predicted chiral extrapolations based on the LETs + linear M_π-dependence of $M_\pi r$



→ results seem to be internally consistent!

LETs for the $^3S_1$ partial wave

We can also try an extrapolation of $M_\pi r$ which is linear in M_π^2

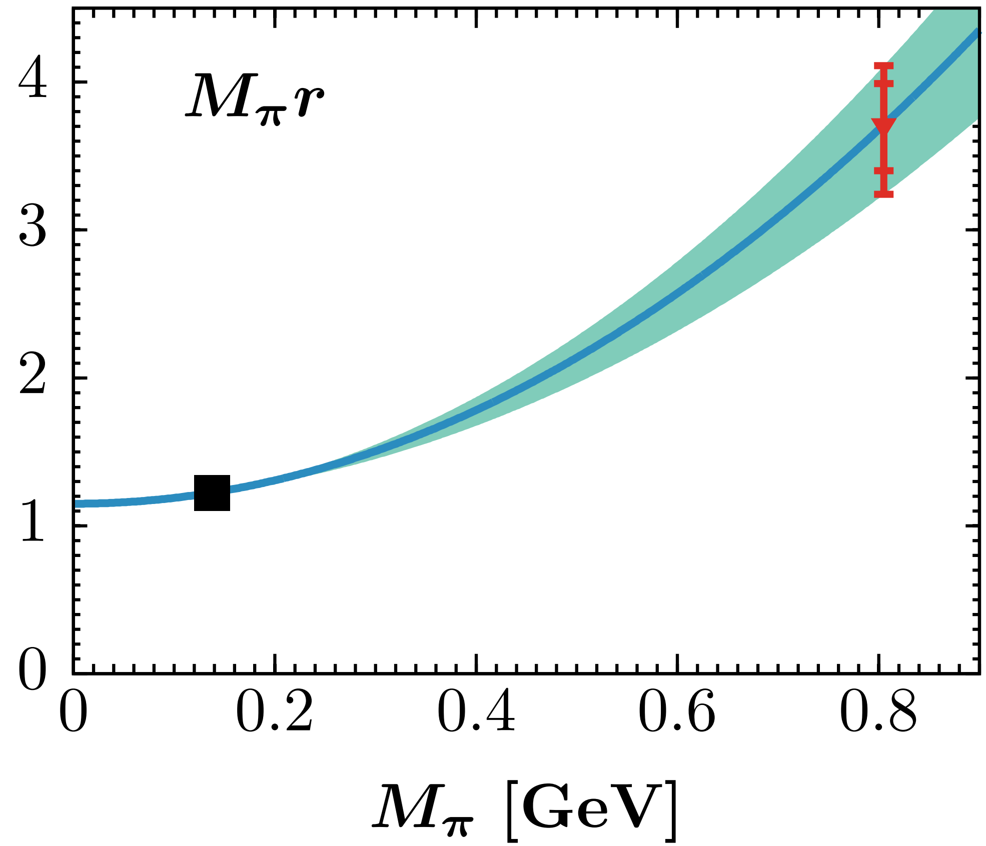
$$M_\pi r_{\text{extrapol}} = A + B M_\pi^2,$$

with

$$A = 1.149^{+0.009+0.011}_{-0.009-0.009}$$

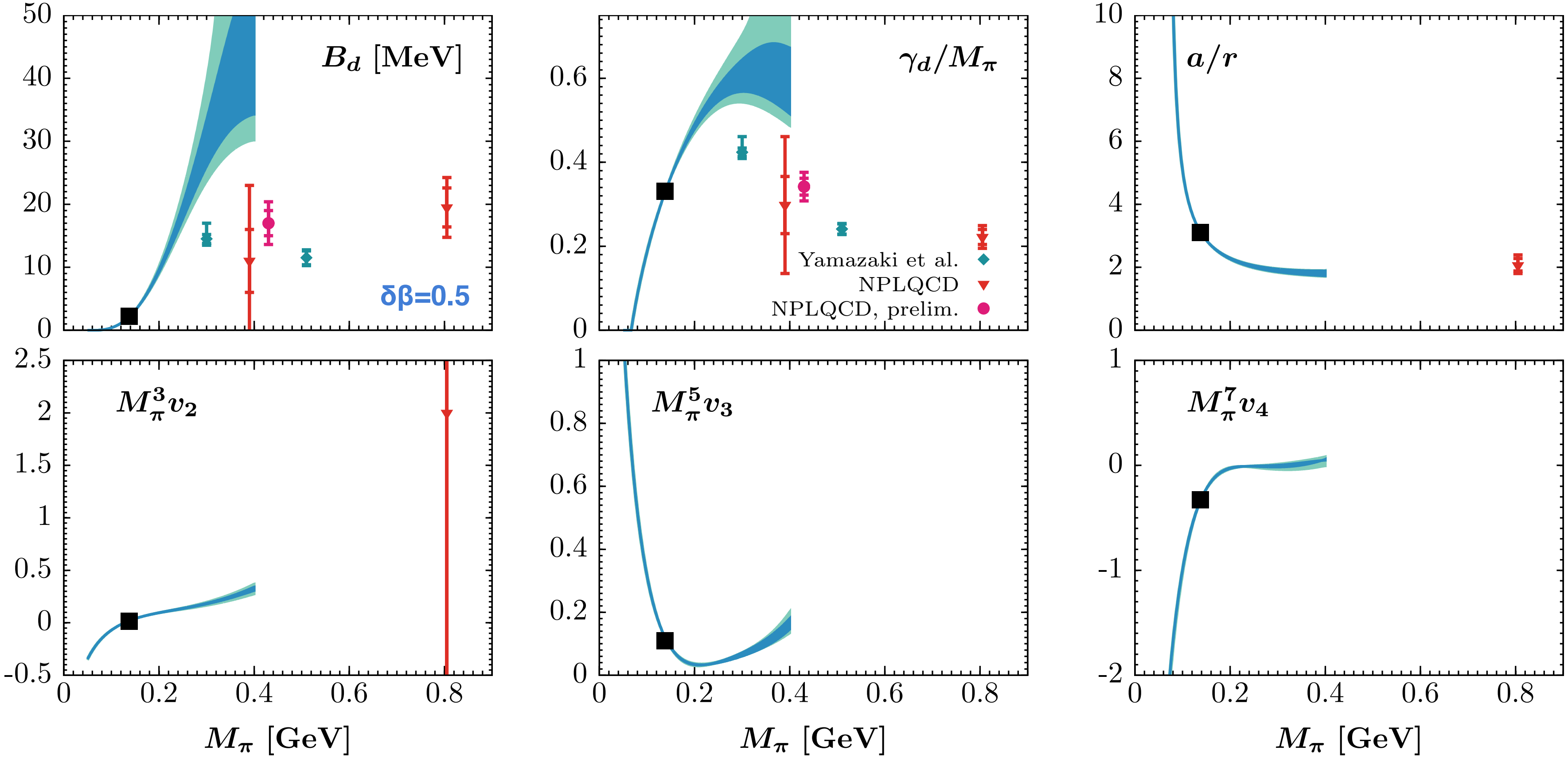
$$B = 3.95^{+0.45+0.45}_{-0.49-0.55}\ \text{GeV}^{-2}$$

ans see what comes out for the binding energy and the ERE coefficients



LETs for the $^3S_1$ partial wave

Predicted chiral extrapolations based on the LETs + linear M_π^2-dependence of $M_\pi r$

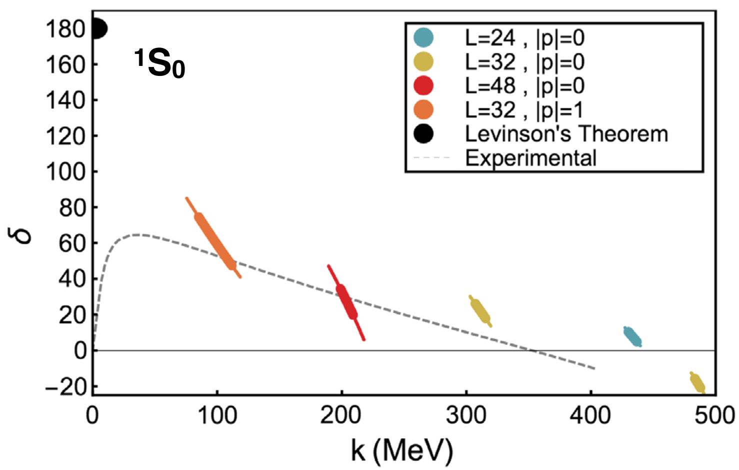


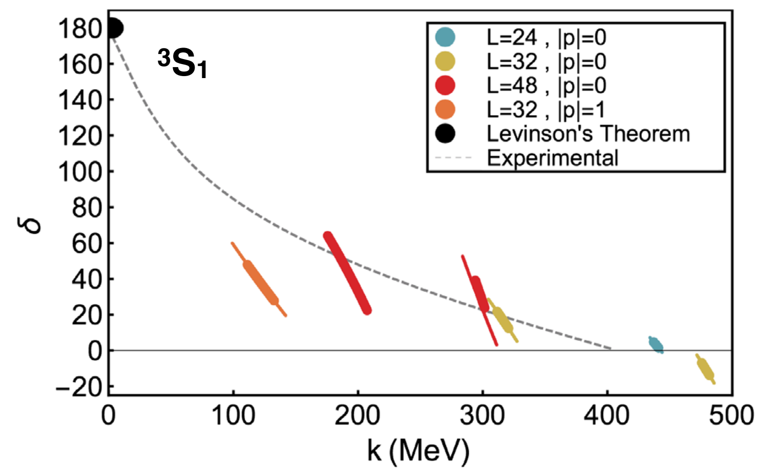
→ seems to be inconsistent with the trend in B_d…

Application 2:

NPLQCD at M_π ~ 450 MeV meets LETs

Baru, EE, Filin, to appear





$$B_{nn} = 12.5\left(^{+3.2}_{-4.9}\right)\ \text{MeV}$$

$$\left(M_\pi a^{(^1S_0)}\right)^{-1} = 0.021\left(^{+0.028}_{-0.036}\right)\left(^{+0.032}_{-0.063}\right)$$

$$M_\pi r^{(^1S_0)} = 6.7\left(^{+1.0}_{-0.8}\right)\left(^{+2.0}_{-1.3}\right)$$

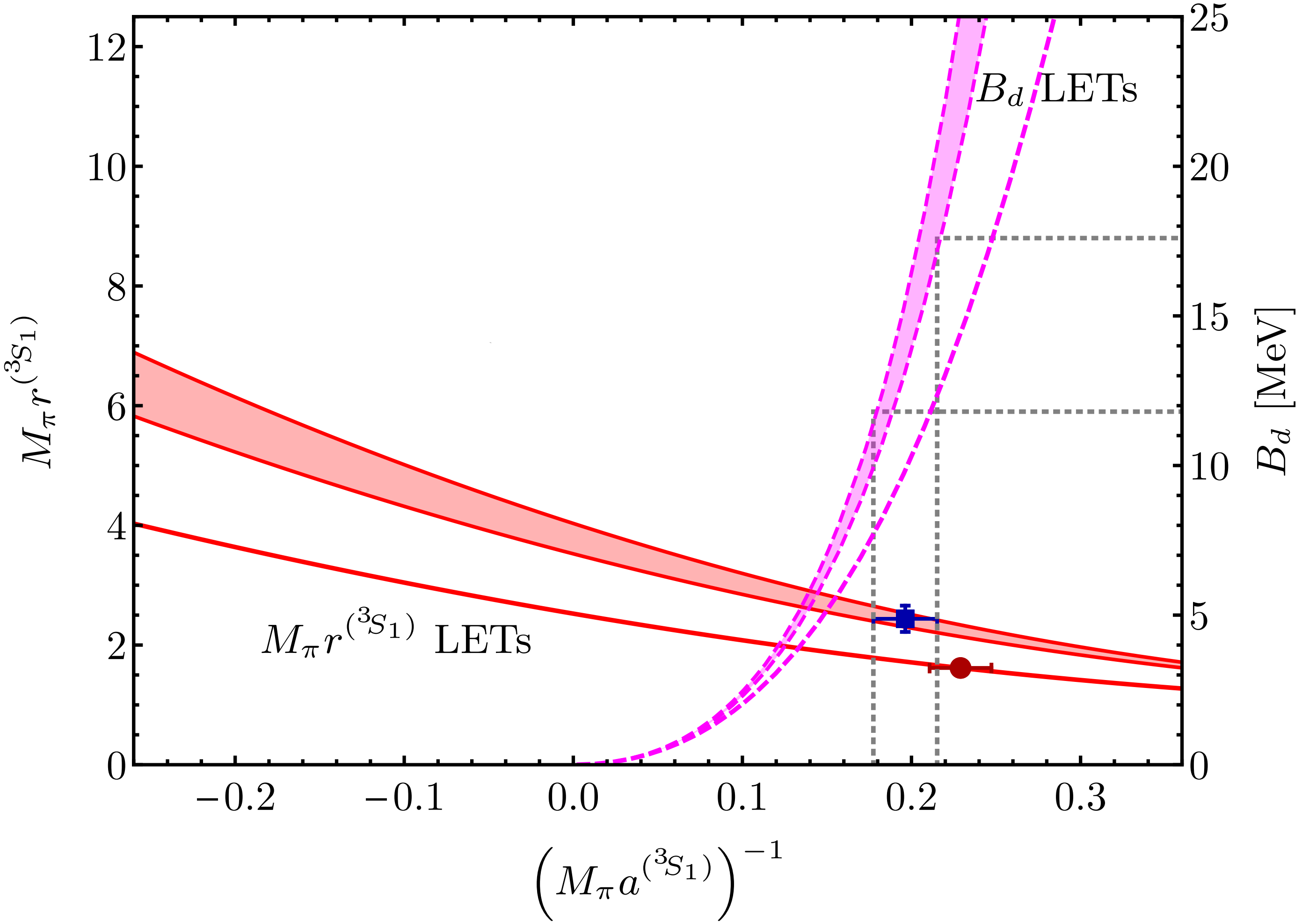
$$B_d = 14.4\left(^{+3.2}_{-2.6}\right)\ \text{MeV}$$

$$\left(M_\pi a^{(^3S_1)}\right)^{-1} = -0.04\left(^{+0.07}_{-0.10}\right)\left(^{+0.08}_{-0.17}\right)$$

$$M_\pi r^{(^3S_1)} = 7.8\left(^{+2.2}_{-1.5}\right)\left(^{+3.5}_{-1.7}\right)$$

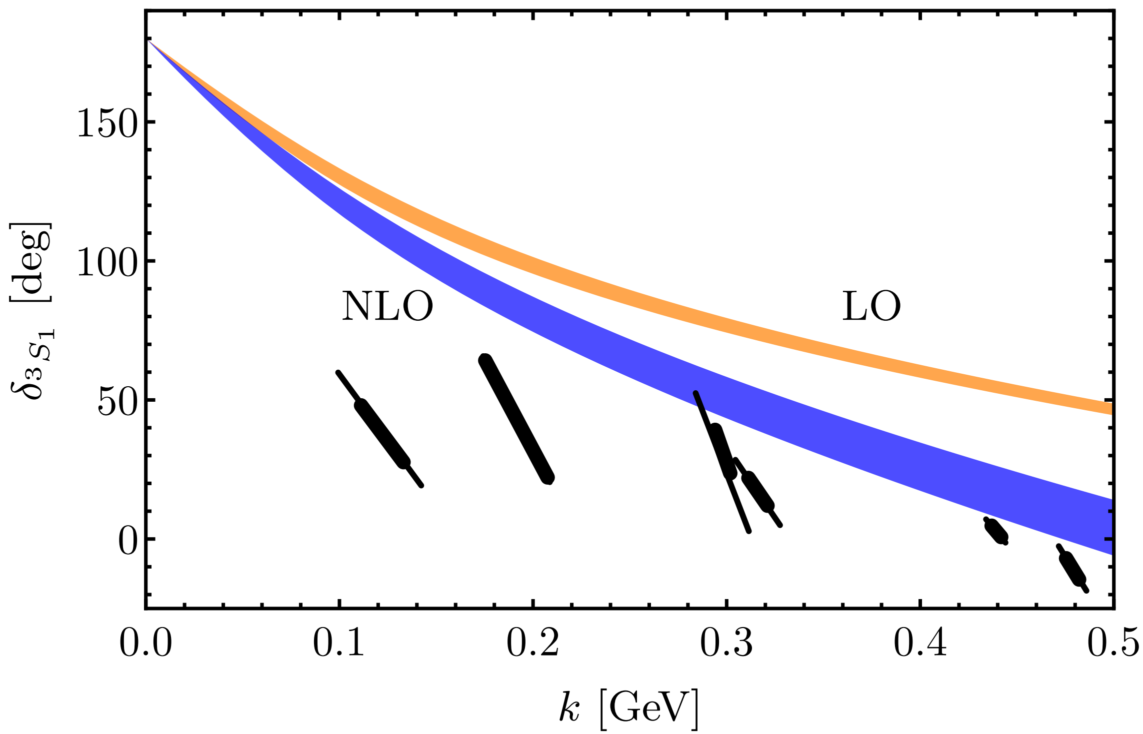
from: Orginos, Parreno, Savage, Beane, Chang, Detmold, Phys. Rev. D92 (2015) 114512

NPLQCD meets LETs: The $^3S_1$ channel



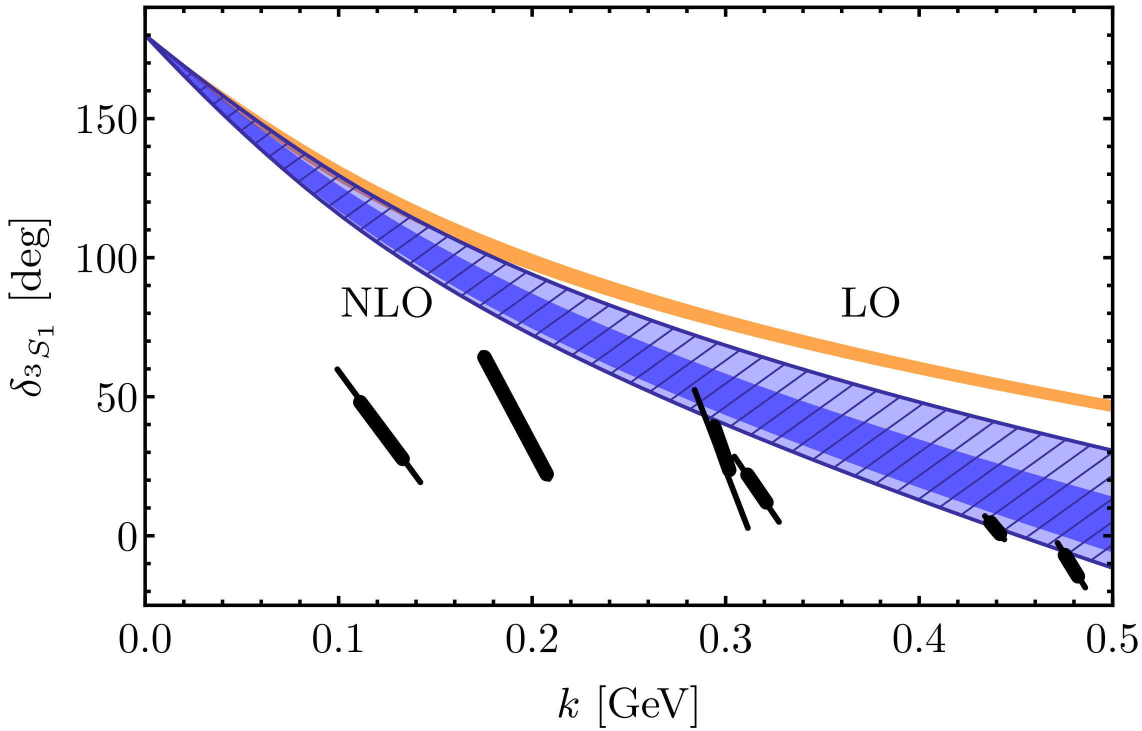
Once the scattering length is fixed, all other quantities (e.g. phase shifts) are predicted in a parameter-free way

NPLQCD meets LETs: The $^3S_1$ channel



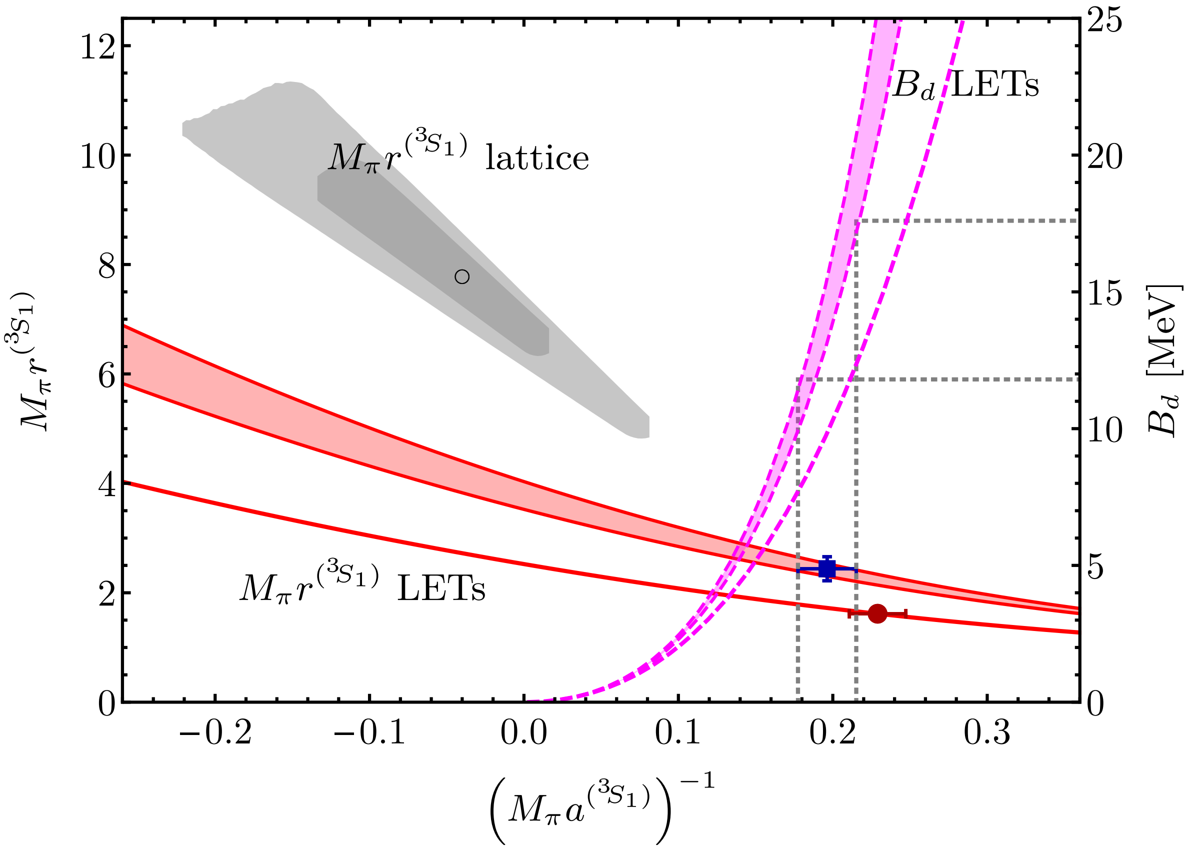
- Reasonably good convergence of the LETs
- Lattice phase shifts at two lowest energies seem to be inconsistent with B_d

NPLQCD meets LETs: The $^3S_1$ channel



- Reasonably good convergence of the LETs
- Lattice phase shifts at two lowest energies seem to be inconsistent with B_d
- A more conservative uncertainty estimation ($\delta\beta = 1$) does not help…

NPLQCD meets LETs: The $^3S_1$ channel



- Consequently, different results for the scattering length and effective range:

NPLQCD: $(M_\pi a^{(^3S_1)})^{-1} = -0.04(^{+0.07}_{-0.10})(^{+0.08}_{-0.17})$, $\quad M_\pi r^{(^3S_1)} = 7.8(^{+2.2}_{-1.5})(^{+3.5}_{-1.7})$

statistics systematics

NLO LETs: $(M_\pi a^{(^3S_1)})^{-1} = 0.196(^{+0.014}_{-0.013})(^{+0.018}_{-0.008})$, $\quad M_\pi r^{(^3S_1)} = 2.44(^{+0.08}_{-0.08})(^{+0.21}_{-0.47})$

error in B_d uncertainty of the LETs ($\delta\beta=1$)

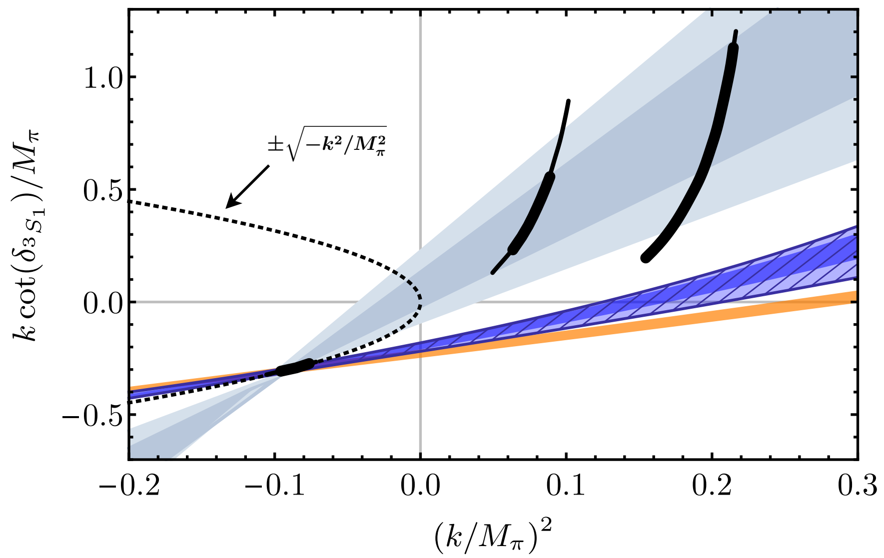
NPLQCD meets LETs: The $^3S_1$ channel

(If true), the very large effective range,

$$r^{(^3S_1)} \sim 8M_\pi^{-1}$$

would suggest:

- either the interaction range (much) longer than that of $V_{1\pi}$
- or the appearance of a pole in $k\cot\delta$ near threshold



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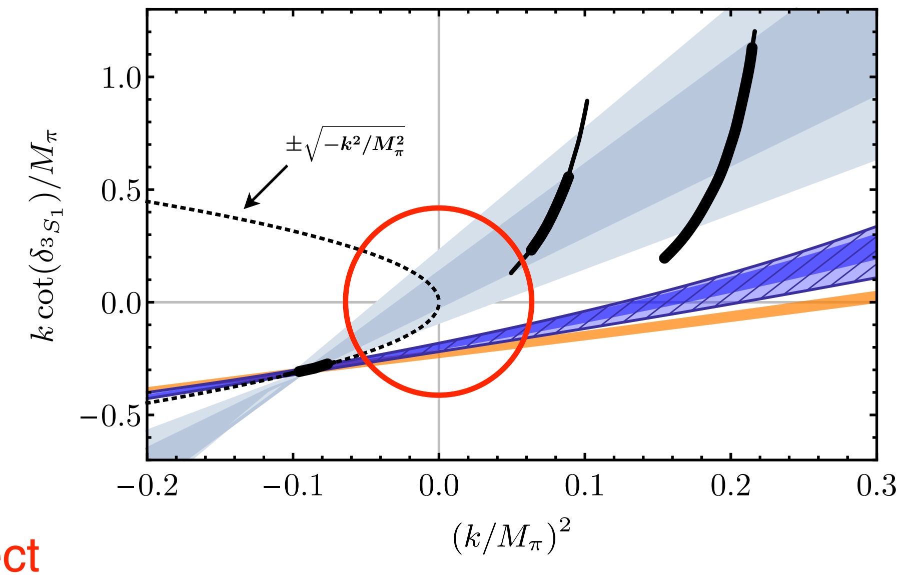

In both cases, there is no reason to expect the approximation $k\cot\delta \simeq -\frac{1}{a} + \frac{1}{2}rk^2$ to be valid for $|k| \gtrsim 2/r \sim M_\pi/4$.

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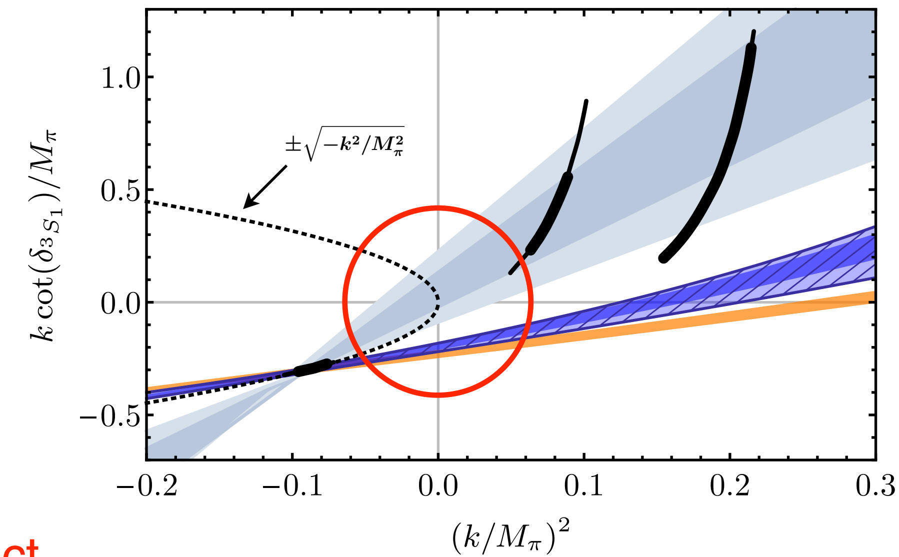


In both cases, there is no reason to expect the approximation $k \cot\delta \simeq -\frac{1}{a} + \frac{1}{2} r k^2$ to be valid for $|k| \gtrsim 2/r \sim M_\pi/4$.

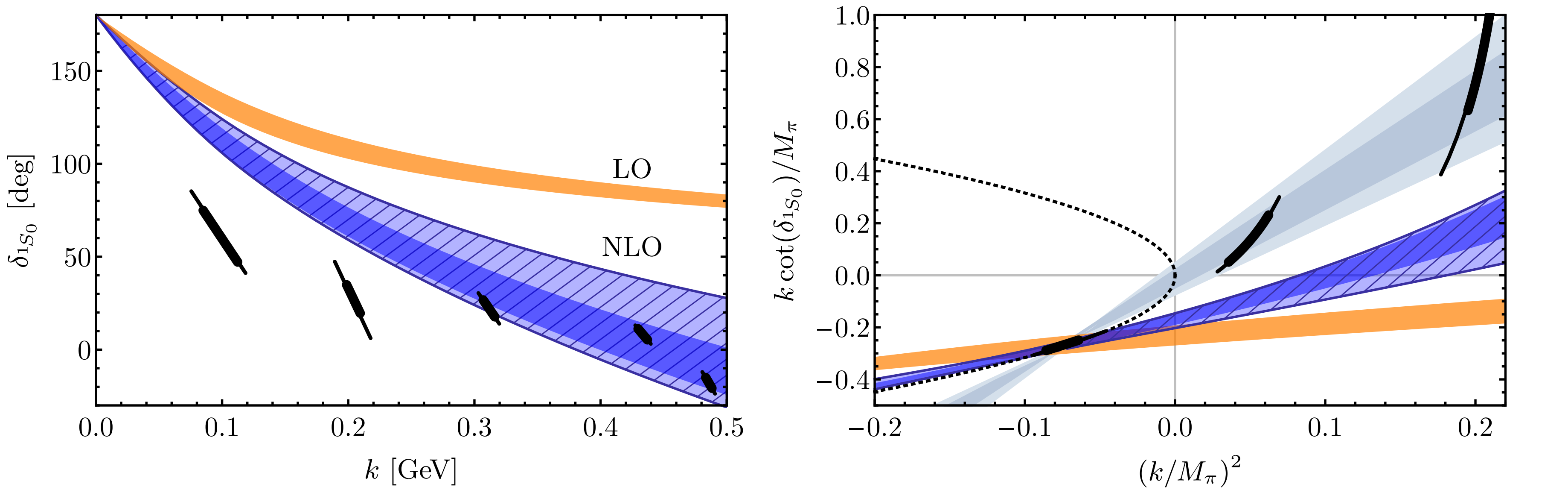
Moreover, the second, deeper bound state is (normally) to be viewed as an artifact of the effective range approximation:

$$-\frac{1}{a} + \frac{1}{2} r k^2 = 0 \quad \longrightarrow \quad k_{1,2} = \frac{i}{r}\left(1 \pm \sqrt{1 - \frac{2r}{a}}\right) \quad \xrightarrow[|r/a| \ll 1]{} \quad \begin{cases} k_1 \simeq \frac{i}{a}\left(1 + \frac{r}{2a}\right) \\ k_2 \simeq i\left(\frac{2}{r} - \frac{1}{a}\right) \end{cases}$$

— physical pion mass: $k_1 \simeq 45i$ MeV (deuteron), $k_2 \simeq 200i$ MeV (artifact)

— NPLQCD solution: $k_1 \simeq -15i$ MeV (virtual state), $k_2 \simeq 135i$ MeV (deuteron)

NPLQCD meets LETs: The $^1S_0$ channel

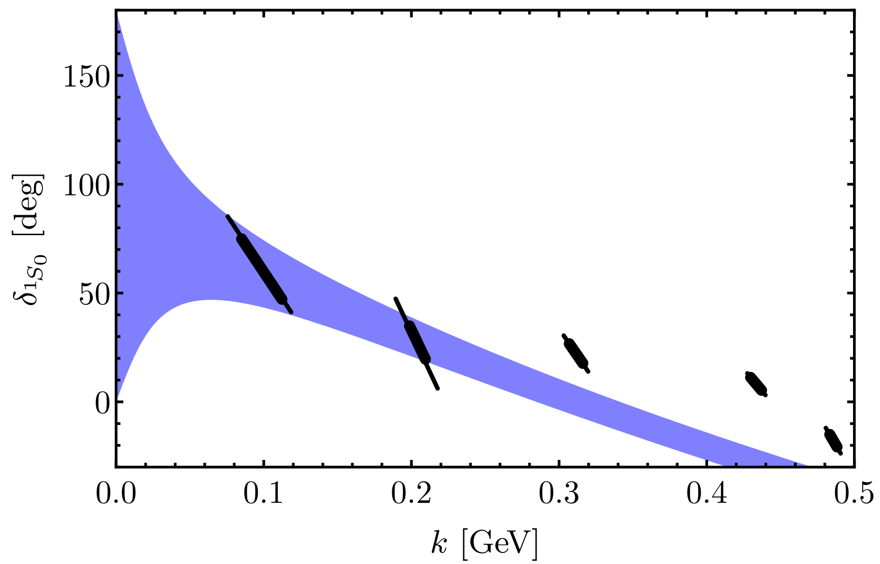


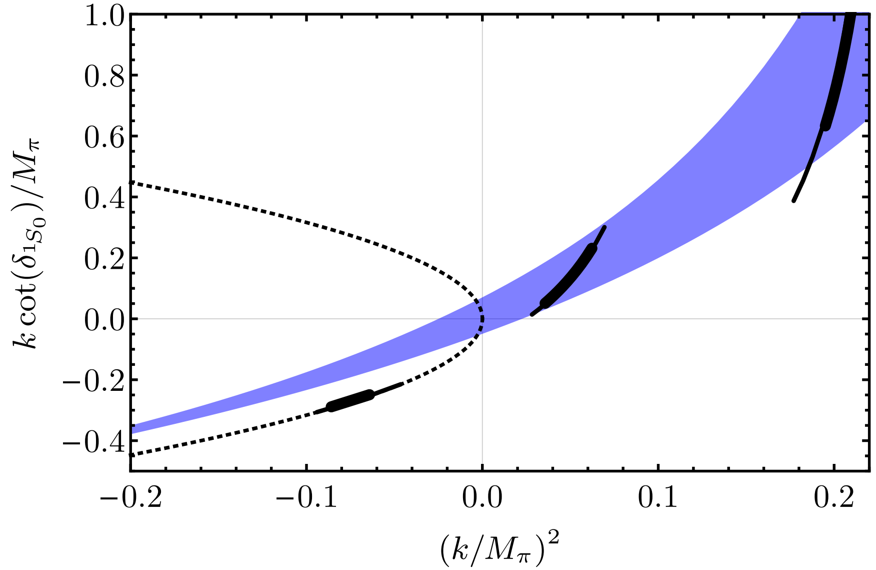
NPLQCD: $(M_\pi a^{(^1S_0)})^{-1} = 0.021(^{+0.028}_{-0.036})(^{+0.032}_{-0.063})$, $M_\pi r^{(^1S_0)} = 6.7(^{+1.0}_{-0.8})(^{+2.0}_{-1.3})$

NLO LETs: $(M_\pi a^{(^1S_0)})^{-1} = 0.175(^{+0.013}_{-0.028})(^{+0.024}_{-0.008})$, $M_\pi r^{(^1S_0)} = 2.86(^{+0.27}_{-0.12})(^{+0.27}_{-0.74})$

- Similar (but somewhat less stringent) conclusions as in the $^3S_1$ partial wave
- Again, the large effective range puts in question the applicability of the effective range approximation at energies of the lattice data
- The NPLQCD „dineutron“ seems to be an artifact; shallow bound/virtual state…

NPLQCD meets LETs: The $^1S_0$ channel





- When using the lattice phase shifts at two lowest energies as input, we cannot accommodate for $\boldsymbol{B_{nn} \simeq 12.5}$ MeV. We find:

$$\color{red}{\boldsymbol{B_{nn} < 0.5\ \text{MeV}, \qquad B_{nn}^{\text{virtual}} < 0.6\ \text{MeV}}}$$

Summary and conclusions

- LETs allow to reconstruct the NN scattering amplitude at fixed M_π using a single observable (e.g. binding energy) as input

  → extrapolations of lattice-QCD results in energy, self-consistency checks

- The linear in M_π dependence of $M_\pi\, r^{(3S1)}$ conjectured by the NPLQCD collaboration based on their $M_\pi \sim 800$ MeV results is consistent with the common trend for B_d

- The newest NPLQCD results at $M_\pi \sim 450$ MeV for the $^1S_0$ / $^3S_1$ phase shifts at the two lowest energies are incompatible with their B_{nn} / B_d energies (within errors). Underestimated systematics for the extraction of phase shifts?

- The NPLQCD determination of the scattering lengths and effective ranges based on the effective range approximation is not self-consistent…

LETs: a useful addition to the lattice QCD toolbox!