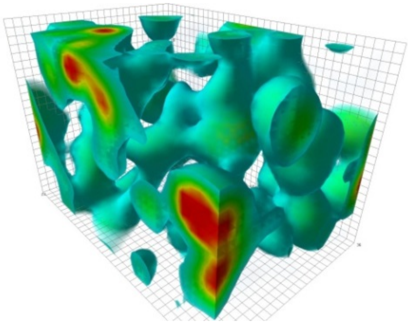
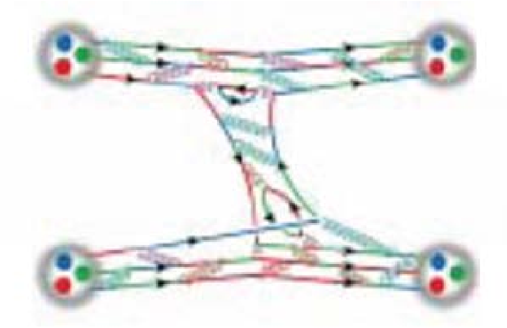
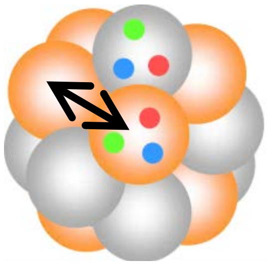
Baryon Interactions from Lattice QCD with physical masses

Takumi Doi
(Nishina Center, RIKEN)

for HAL QCD Collaboration

Hadrons to Atomic nuclei from Lattice QCD (HAL QCD Collaboration)

S. Aoki, D. Kawai, T. Miyamato (YITP)

T. Doi, T.Hatsuda, (RIKEN)

F. Etminan (Univ. of Birjand)

S. Gongyo (Univ. of Tours)

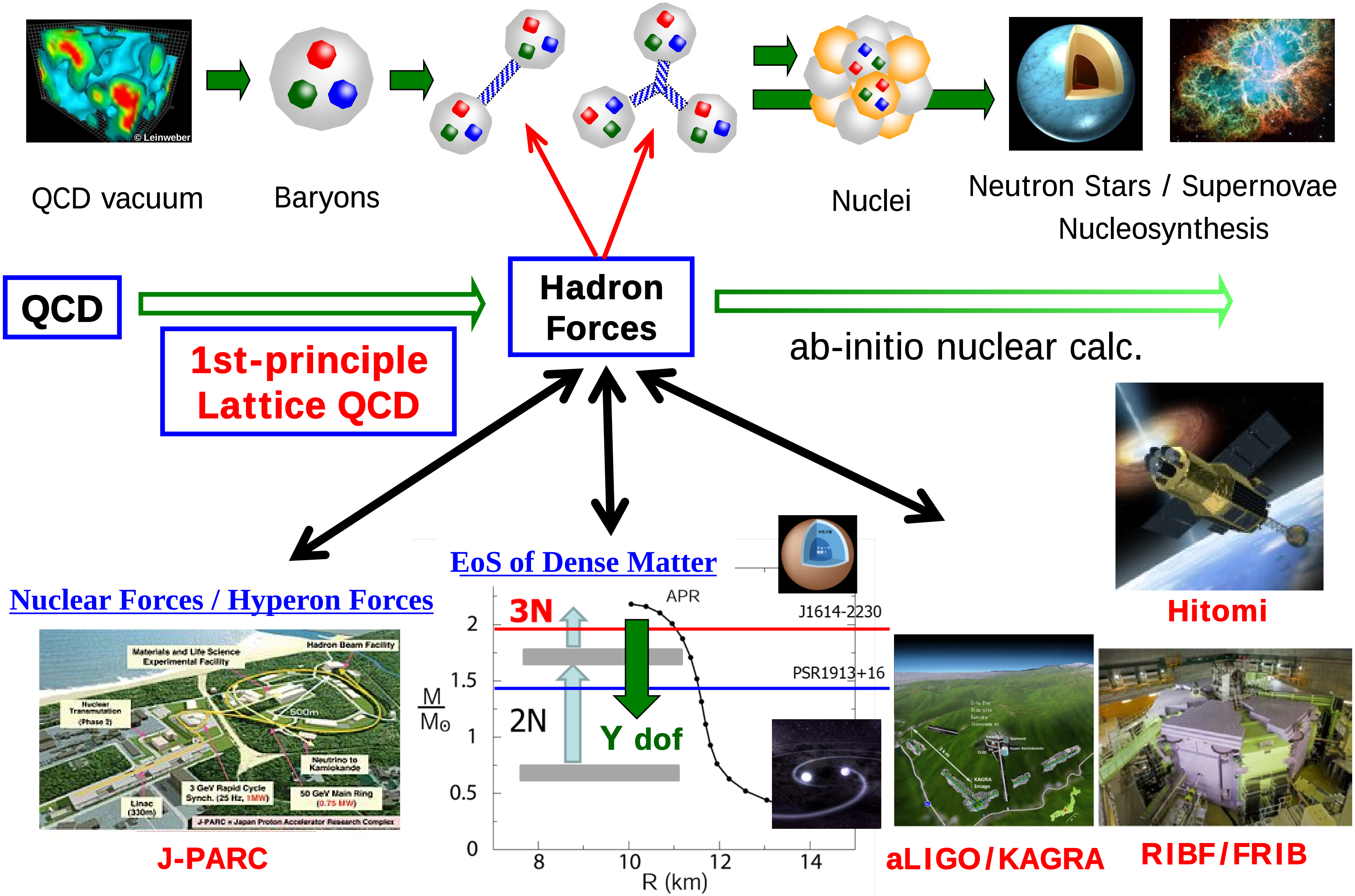
Y. Ikeda, N. Ishii, K. Murano (RCNP)

T. Inoue (Nihon Univ.)

T. Iritani (Stony Brook Univ.)

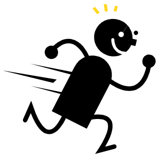
H. Nemura, K. Sasaki (Univ. of Tsukuba)

The journey from Quarks to Universe



The journey from unphysical to physical quark masses

~2012

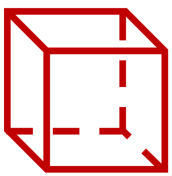


➔ lighter m_q

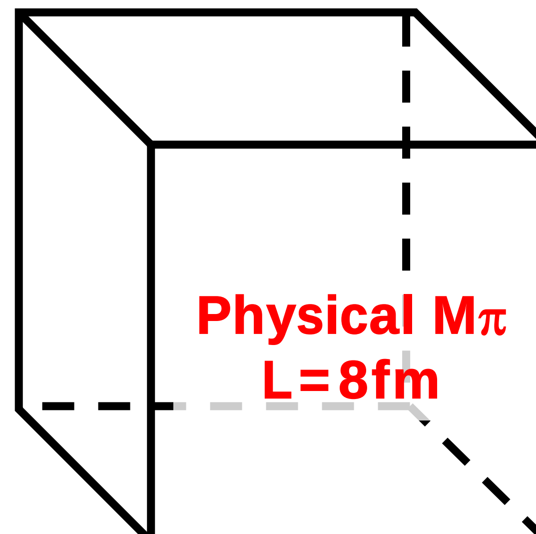
We were here

Phys. point

Mπ=0.4 GeV
L=3fm



K-computer



HPCI Strategic Program Field 5
“The origin of matter and the universe”
FY2010-15

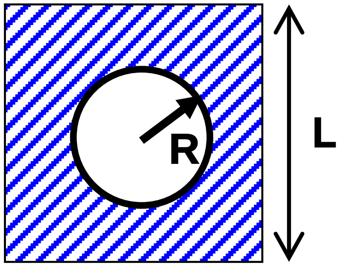
- **Outline**
  - ~~Introduction~~
  - (Theoretical framework) [🡐 S. Aoki's talk]
  - Challenges at physical quark masses
  - Results at physical quark masses
  - Summary / Prospects

[HAL QCD method]

- Nambu-Bethe-Salpeter (NBS) wave function

$$\psi(\vec{r}) = \langle 0|\ N(\vec{r})N(\vec{0})\ |N(\vec{k})N(-\vec{k}); in\rangle$$

$$(\nabla^2 + k^2)\psi(\vec{r}) = 0, \quad r > R$$



  – phase shift at asymptotic region

$$\psi(r) \simeq A\frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$

M.Luscher, NPB354(1991)531
C.-J.Lin et al., NPB619(2001)467
N.Ishizuka, PoS LAT2009 (2009) 119
CP-PACS Coll., PRD71(2005)094504

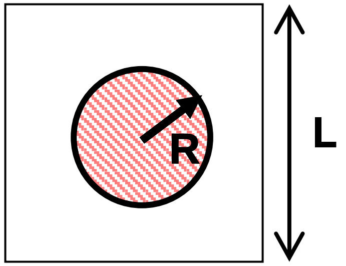
Extended to multi-particle systems

S. Aoki et al., PRD88(2013)014036

- Consider the wave function at "interacting region"

$$(\nabla^2 + k^2)\psi(\boldsymbol{r}) = m\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')\psi(\boldsymbol{r}'), \quad r < R$$

  – U(r,r'): faithful to the phase shift by construction
    - U(r,r'): E-independent, while non-local in general
      – Non-locality ➔ derivative expansion

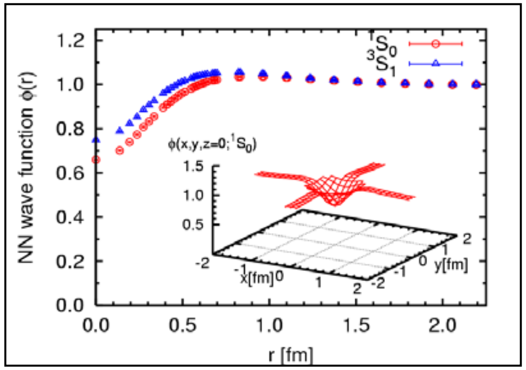


Aoki-Hatsuda-Ishii PTP123(2010)89

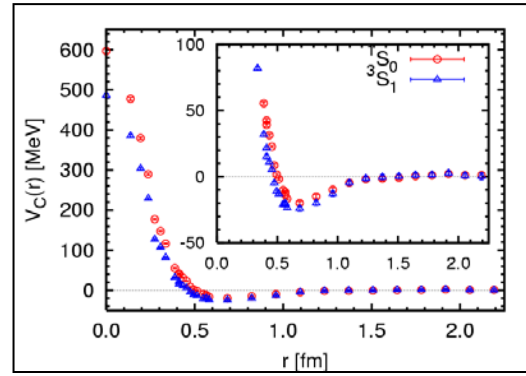
HAL QCD method

Lattice QCD

NBS wave func.



Lat Nuclear Force



$$\psi_{NBS}(\vec{r}) = \langle 0|N(\vec{r})N(\vec{0})|N(\vec{k})N(-\vec{k}), in\rangle$$
$$\simeq e^{i\delta_l(k)} \sin(kr - l\pi/2 + \delta_l(k))/(kr)$$

(at asymptotic region)

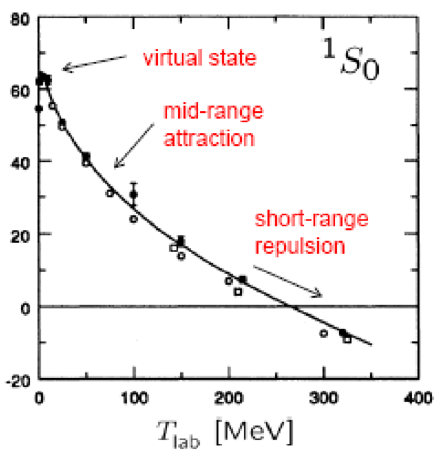
$$(k^2/m_N - H_0)\,\psi(\vec{r}) = \int d\vec{r}\,' U(\vec{r},\vec{r}\,')\psi(\vec{r}\,')$$

E-indep (& non-local) Potential:
Faithful to phase shifts

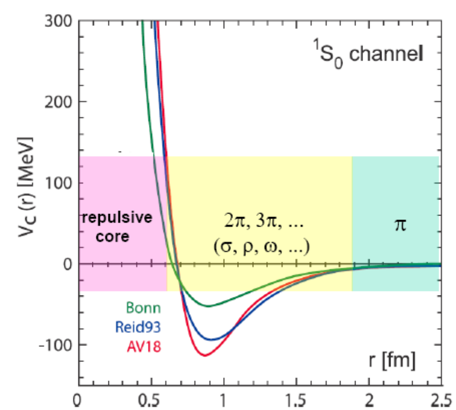
Analog to …

Scattering Exp.

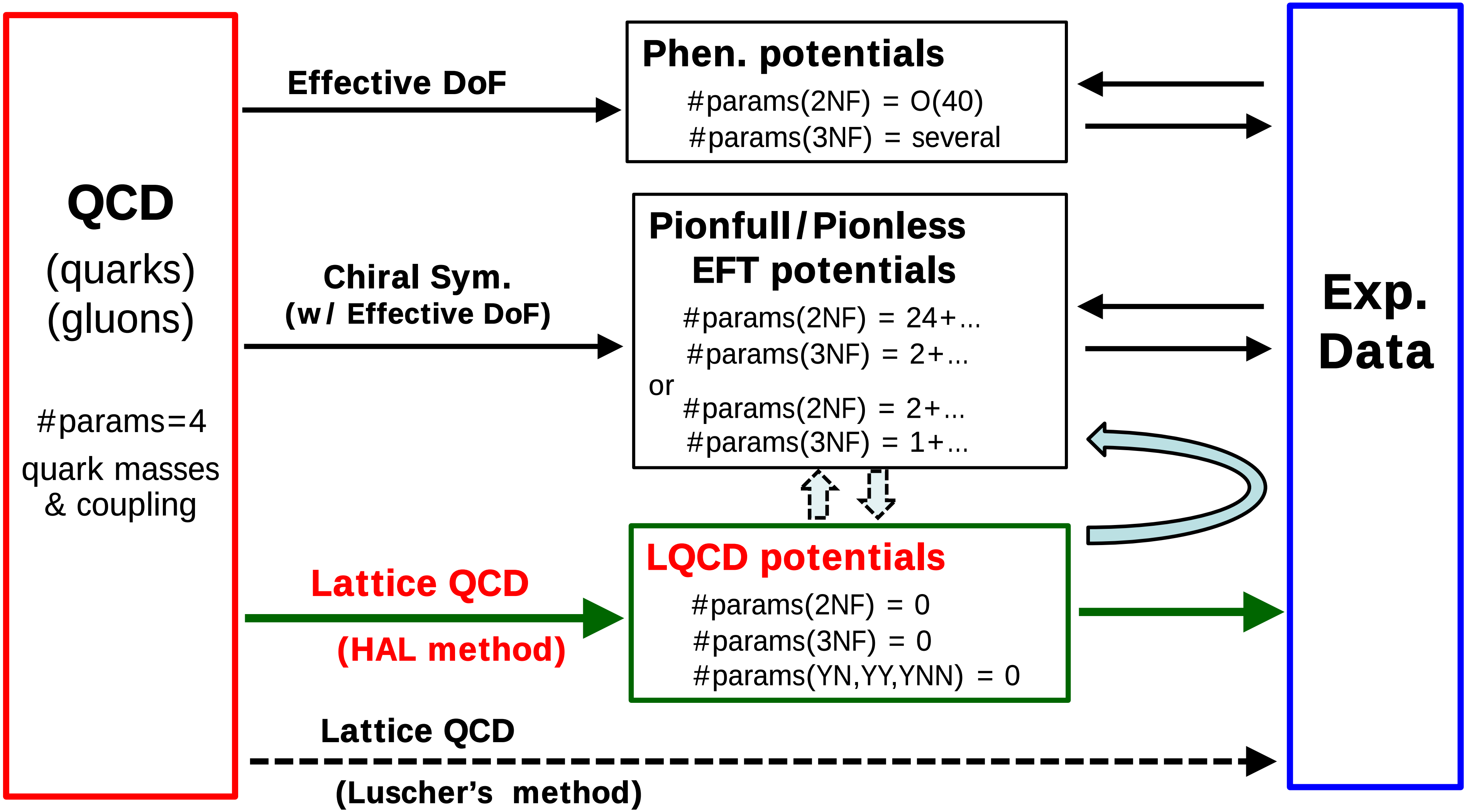
Phase shifts



Phen. Potential



Various Theoretical methods



(Comparison between 2 LQCD methods → T. Iritani's talk)

- **Outline**
  - ~~Introduction~~
  - (Theoretical framework)
  - Challenges at physical quark masses
    - Signal/Noise Issue [🡐 T. Iritani's talk]
    - Coupled Channel Systems
    - Computational Challenge
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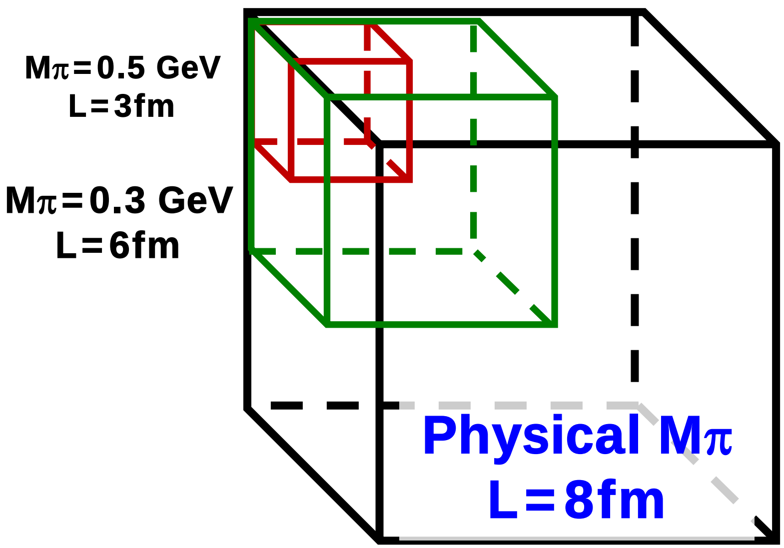
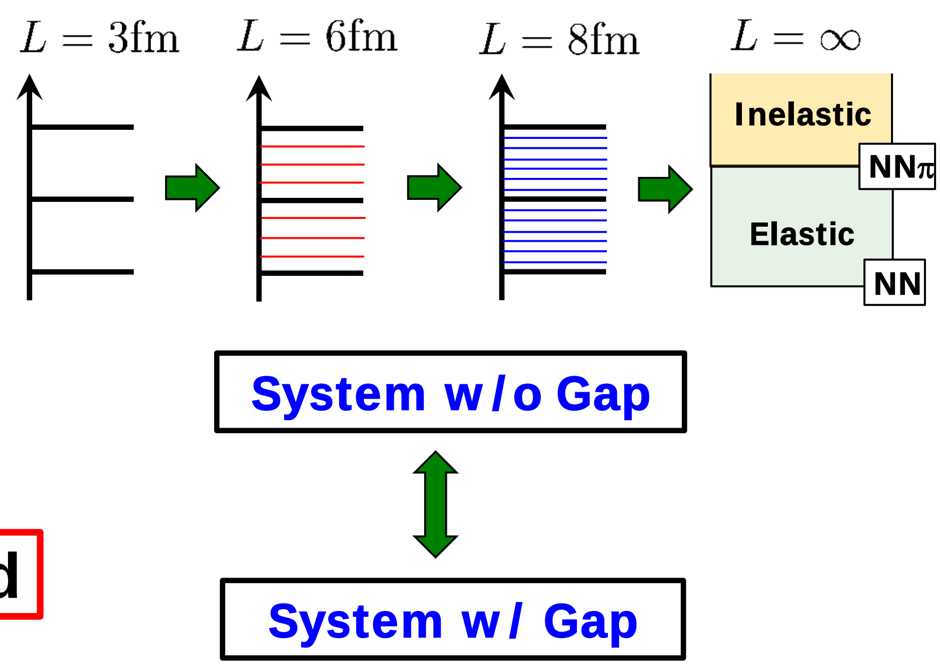
Signal/Noise issue w/ ~continuum on Lat

- Challenge in Luscher's method : ground state saturation

$$S/N \sim \exp[-\mathbf{A} \times (\mathbf{m_N} - \mathbf{3/2m_\pi}) \times \mathbf{t}] \qquad 1/t \simeq \Delta E \simeq \frac{1}{m_N}\frac{(2\pi)^2}{L^2}$$



(simple)
$S/N \propto$ 10^{-4} 10^{-13} 10^{-25}



Our solution:

- **Time-dependent HAL method**
  - E-indep potential
  - ➔ "Signal" from (elastic) excited states
  - ~~G.S. saturation~~ ➔ Elastic states saturation [Exponential Improvement]

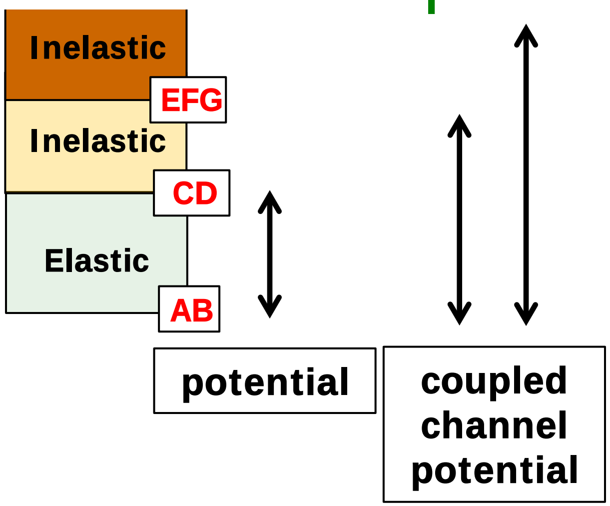
N.Ishii et al. PLB712(2012)437

Coupled Channel systems

(beyond inelastic threshold)

- Essential in many interesting physics
  - Hyperon Forces (e.g., H-dibaryon (ΛΛ-NΞ-ΣΣ))
  - Exotic mesons, Resonances, etc. (e.g., Zc(3900))

➔ Coupled channel potentials in HAL method



$$\psi_{AB}(\boldsymbol{r},\boldsymbol{k}) = 1/\sqrt{Z_A Z_B}\cdot\langle 0|\phi_A(\boldsymbol{x}+\boldsymbol{r})\phi_B(\boldsymbol{x})|W\rangle$$

$$\psi_{CD}(\boldsymbol{r},\boldsymbol{q}) = 1/\sqrt{Z_C Z_D}\cdot\langle 0|\phi_C(\boldsymbol{x}+\boldsymbol{r})\phi_D(\boldsymbol{x})|W\rangle$$

$$W = \sqrt{m_A^2+k^2}+\sqrt{m_B^2+k^2} = \sqrt{m_C^2+q^2}+\sqrt{m_D^2+q^2}$$

$$(E_{k_i}^{AB}-H_0^{AB})\psi_{AB}(\boldsymbol{r},k_i) = \int d\boldsymbol{r}' U_{AB,AB}(\boldsymbol{r},\boldsymbol{r}')\psi_{AB}(\boldsymbol{r}',k_i) + \int d\boldsymbol{r}' U_{AB,CD}(\boldsymbol{r},\boldsymbol{r}')\psi_{CD}(\boldsymbol{r}',q_i)$$

$$(E_{q_i}^{CD}-H_0^{CD})\psi_{CD}(\boldsymbol{r},q_i) = \int d\boldsymbol{r}' U_{CD,AB}(\boldsymbol{r},\boldsymbol{r}')\psi_{AB}(\boldsymbol{r}',k_i) + \int d\boldsymbol{r}' U_{CD,CD}(\boldsymbol{r},\boldsymbol{r}')\psi_{CD}(\boldsymbol{r}',q_i)$$

S.Aoki et al. (HAL Coll.), Proc. Jpn. Acad. Ser. B87(2011)

S.Aoki et al. (HAL Coll.), PRD87(2013)034512

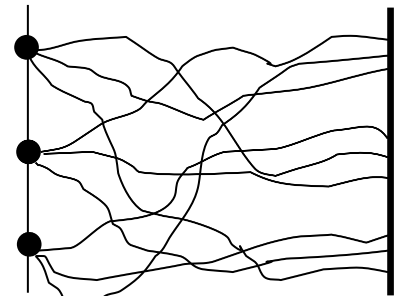
Computational Challenge

- **Enormous comput. cost for multi-baryon correlators**
  - Wick contraction (permutations)

    $\sim [\,(\frac{3}{2}A)!\,]^2$ (A: mass number)

  - color/spinor contractions

    $\sim 6^A \cdot 4^A \;\;\text{or}\;\; 6^A \cdot 2^A$



t=t(sink) t=t(src)

  - **Unified Contraction Algorithm (UCA)** TD, M.Endres, CPC184(2013)117
    - A novel method which unifies two contractions

Permuted Sum

$$\Pi^{2N} \simeq \langle qqqqqq(t)\bar{q}(\xi'_1)\bar{q}(\xi'_2)\bar{q}(\xi'_3)\bar{q}(\xi'_4)\bar{q}(\xi'_5)\bar{q}(\xi'_6)(t_0)\rangle \times \text{Coeff}^{2N}(\xi'_1,\cdots,\xi'_6)$$

Sum over color/spinor unified list

Drastic Speedup

$\times 192$ for ${}^3\text{H}/{}^3\text{He}$, $\times 20736$ for ${}^4\text{He}$, $\times 10^{11}$ for ${}^8\text{Be}$ (x add'l. speedup)

See also subsequent works: Detmold et al., PRD87(2013)114512
Gunther et al., PRD87(2013)094513

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    - Computational Challenge ➔ Unified Contraction Algorithm (UCA)
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Simulations w/ ~ physical masses

HPCI Strategic Program Field 5
"The origin of matter and the universe"
FY2010-15

Gauge Config Generation

K-computer(RIKEN/AICS)

Baryon Forces

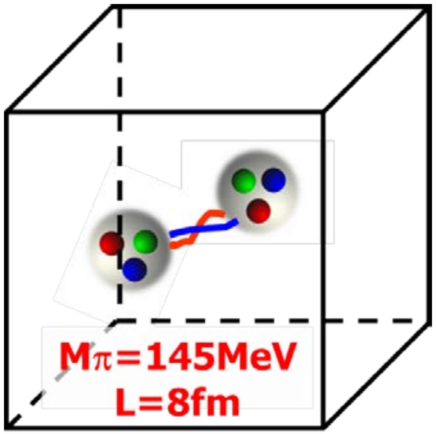
Calc of NBS
➔ HAL QCD method

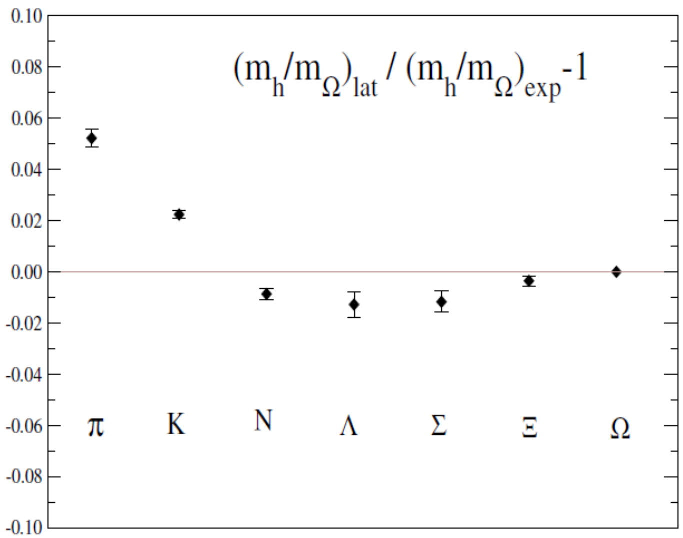
HA-PACS (Tsukuba U.)

FX100 (RIKEN/Wako)

Simulation Setup

- **Nf = 2+1 clover fermion + Iwasaki gauge action**
  - APE-Stout smearing (α=0.1, n_{stout}=6)
  - Non-perturbatively O(a)-improved
  - 1/a ~= 2.3 GeV (a ~= 0.085 fm)
  - Volume: 96^4 ~= $(8\text{ fm})^4$
  - m(pi) ~= 145 MeV, m(K) ~= 525 MeV
  - #traj ~= 2000 generated
    - DDHMC (ud) + UVPHMC (s) w/ preconditioning

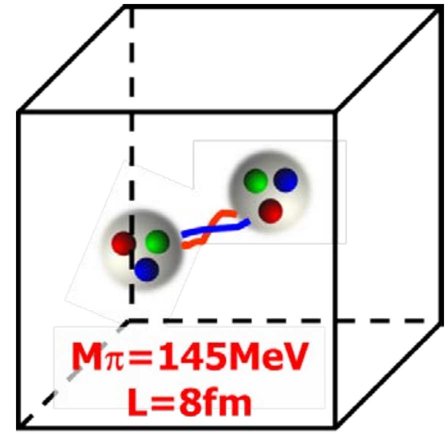




deviation from the Exp.:
$\delta m_\pi \sim +5\%, \;\; \delta m_K \sim +2\%.$

K.I. Ishikawa et al., PoS LAT2015, 075

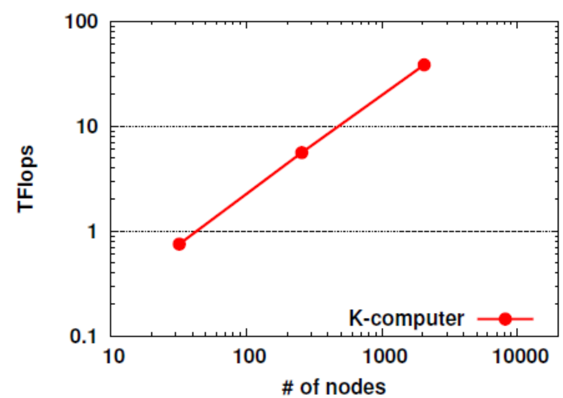
Simulation Setup



- **Measurements**
  - ud, s mass = sea mass (unitary point)
  - Wall source
    - Coulomb gauge fixing after smearing
    - Spacial PBC & Temporal DBC w/ forward/backward average
  - #stat = 200 configs x 4 rotation x 20-72 src in this talk
    - #stat ➔ x1.3-4 in FY2015 (& add'l x2 in FY2016)
    - (Relativistic term omitted in this preliminary analyses)

- **Code development**
  - Efficient implementation of UCA
  - Many channels w/ L^3 dof in NBS
  - Block solver for multiple RHS
  - K-computer @ 2048 node (x 8core/node)
    - ~25% efficiency (~65 TFlops sustained)

Weak scaling
(NBS calc part, w/o solver, w/o IO)



Strategy for phys point BB-forces calc

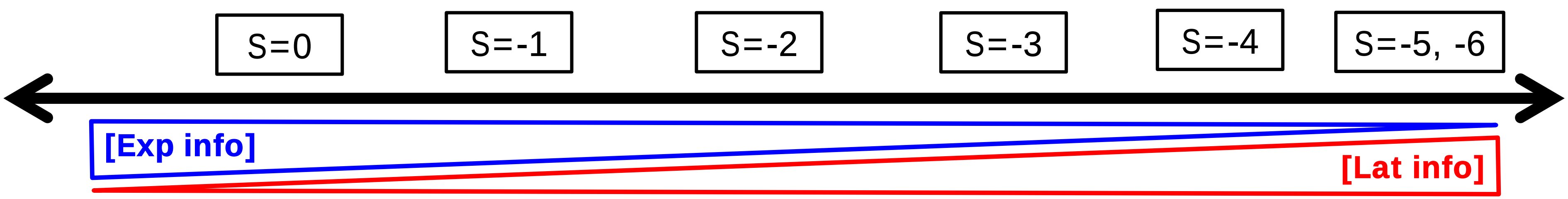
- Focus on the most important forces:
  - Central/tensor forces for all NN/YN/YY in P=(+) (S, D-waves)

Central **Tensor**

$$U(\vec{r},\vec{r'}) = V_c(r) + S_{12}V_T(r) + \vec{L}\cdot\vec{S}V_{LS}(r) + \mathcal{O}(\nabla^2)$$

LO **LO** **NLO** **NNLO** (derivative expansion)

- **Hyperon forces provide precious "predictions"**



NN **ΛN, ΣN** **ΛΛ, ΛΣ, ΣΣ, NΞ** **ΛΞ, ΣΞ** **ΞΞ** **ΩΩ**

"milestone-postdiction"

Hypernuclear phys @ J-PARC

H-dibaryon ?, Ξ-hypernuclei

New bound state(s) ?

Λ appearance in NS & EoS ?

$\Omega\Omega$ system (S= –6)

$^1S_0$: Pauli allowed channel, candidate for exotic bound state

Model varies from bound state to repulsive interactions

HAL study @ m(pi)=0.7GeV: nearly bound (Unitary Region)

M. Yamada et al., PTEP2015, 071B01

See also S. Aoki's talk

c.f. Luscher's method @ m(pi)=0.39GeV: weak repulsion

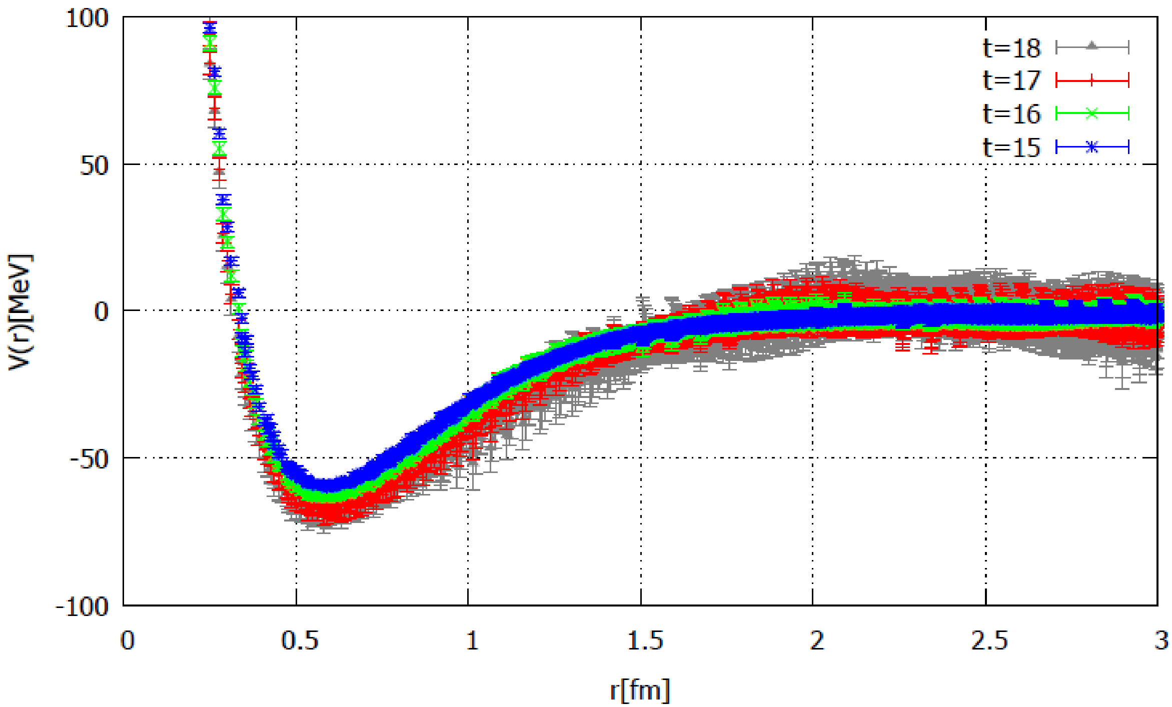
a = -0.16(22)fm

M. Buchoff et al, PRD85(2012)094511

$\Omega\Omega$ system in $^1S_0$

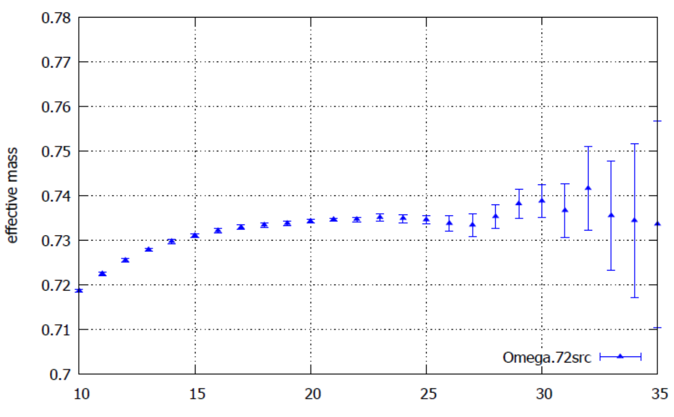
Potential

Preliminary



(200conf x 4rot x 72src)

m(eff) for single Ω



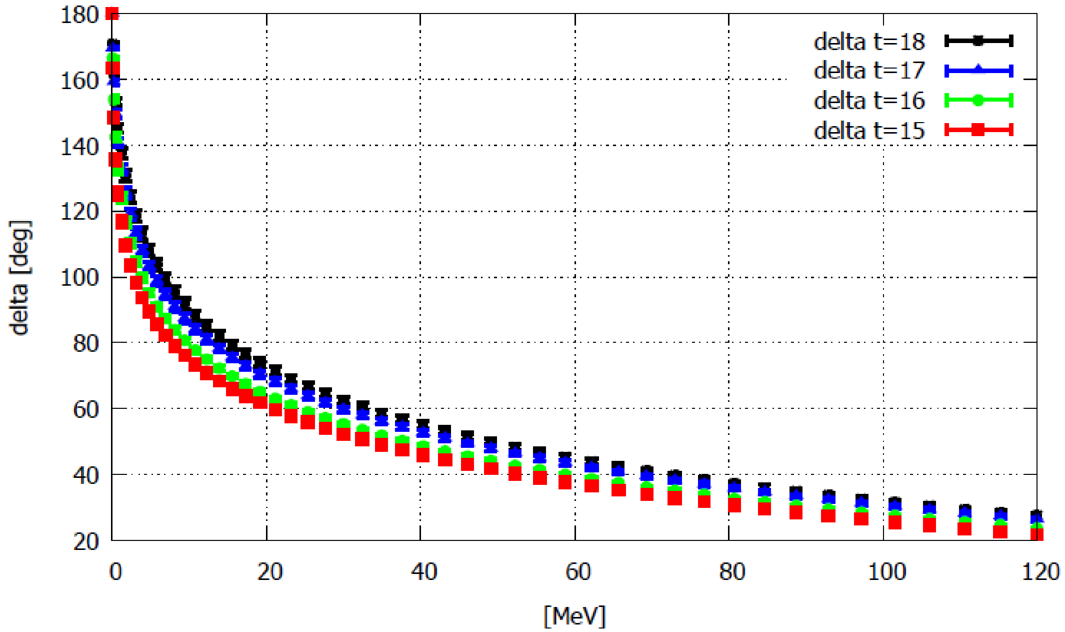
t = 18 : ~0.2-0.3% sys error

[S. Gongyo / K. Sasaki]

ΩΩ system in $^1S_0$

Preliminary

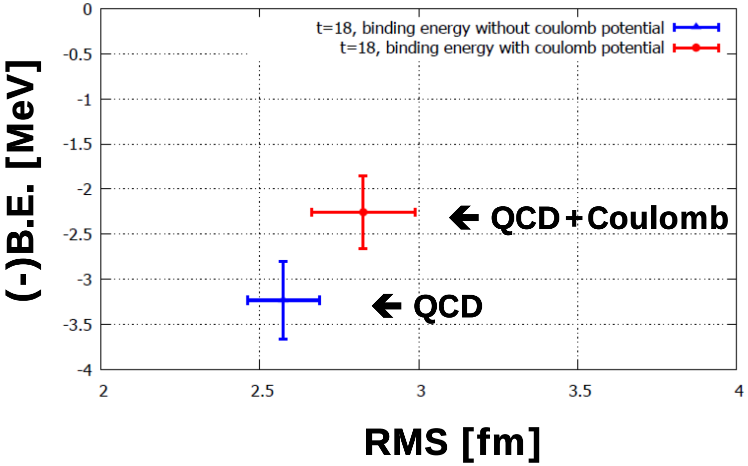
Phase Shifts



Scatt. Length

a = -3.53(18)(54) fm

(@ t=18 (&17))



The Most Strange Dibaryon

➔ HIC experiments ?

[S. Gongyo / K. Sasaki]

$\Xi\Xi$ system (S= –4)

- **$^1S_0$ ～ 27-plet**
  ⇔ NN($^1S_0$) + SU(3) breaking

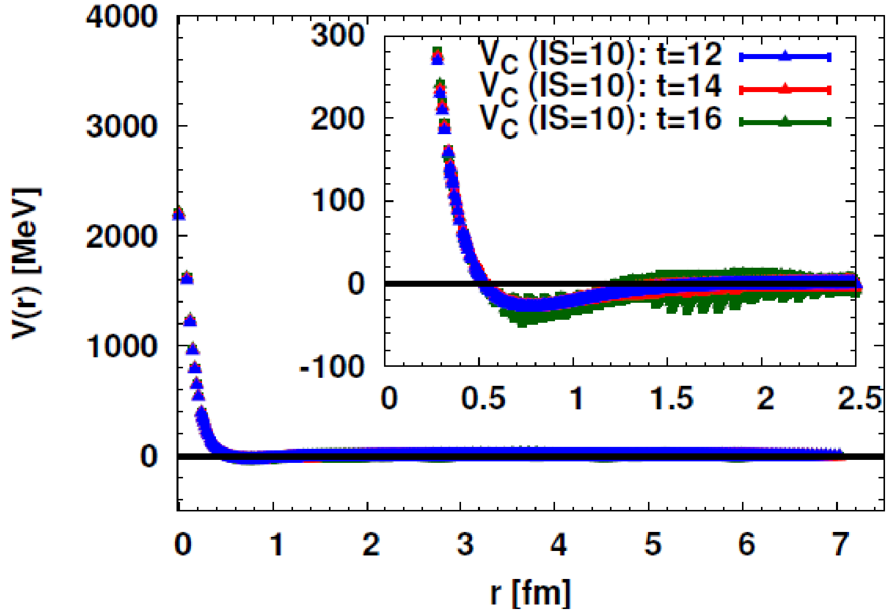
  Phen. model (Nijmegen) : possibly bound

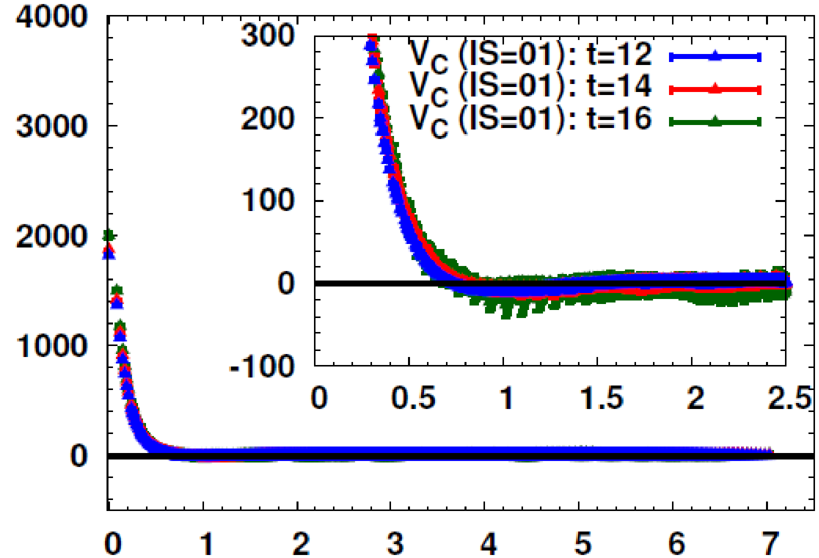
  EFT (Haidenbauer et al. ’14) : unbound favored

- **$^3S_1$-$^3D_1$ ～ 10-plet**
  ⇔ unique w/ hyperon DoF
  ⇔ Σ^- in neutron star

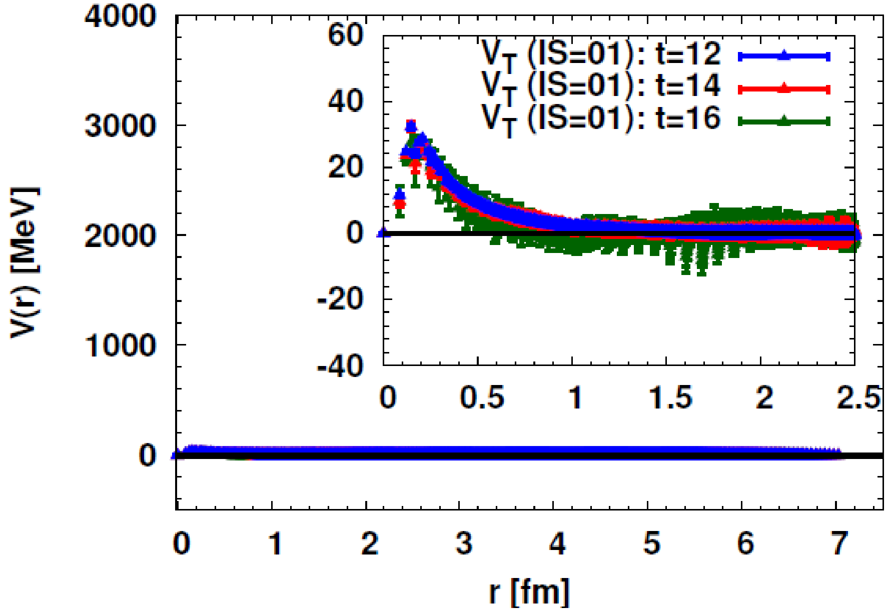
ΞΞ-Potentials

$^1S_0$ / $^3S_1$-$^3D_1$





Central



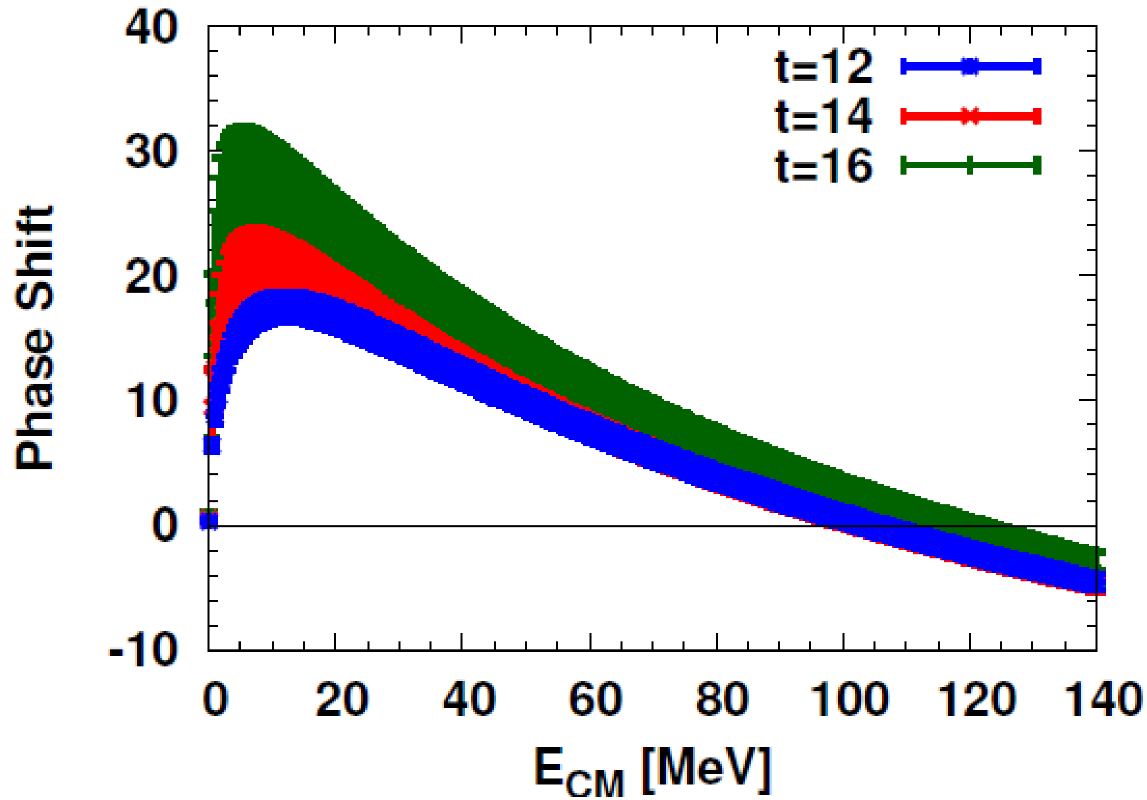
Tensor

- **$^1S_0$ ∼ 27-plet**
  ⇔ attractive Vc
- **$^3S_1$-$^3D_1$ ∼ 10-plet**
  ⇔ repulsive Vc, weak Vt

(200conf x 4rot x 44src)

ΞΞ phase shifts ($^1S_0$)

Preliminary

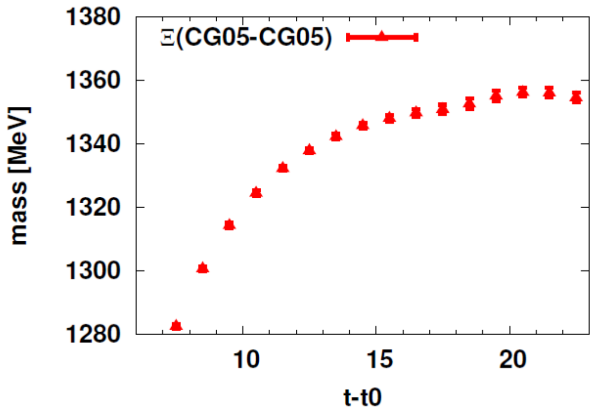


Scatt. Length

a = 1.35(047) fm (t=14)

a = 1.97(113) fm (t=16)

m(eff) for single Ξ



t = 14-18 : ~0.3-1% sys error

ΞΞ ($^1S_0$) is unbound

(t-dependence will be checked again w/ larger #stat)

(2-gauss + 2-OBEP fit)

(200conf x 4rot x 44src)

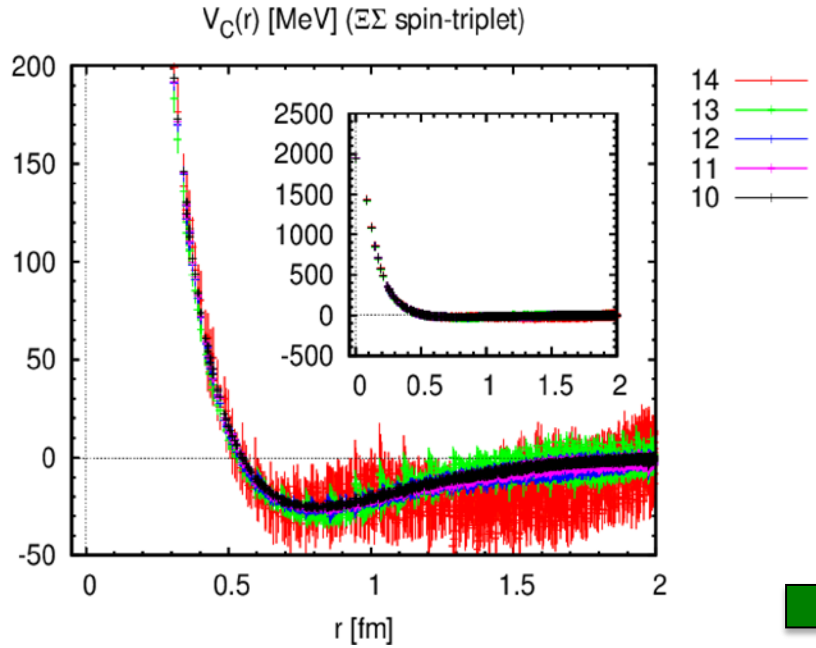
➔ HIC experiments ?

S= –3 systems

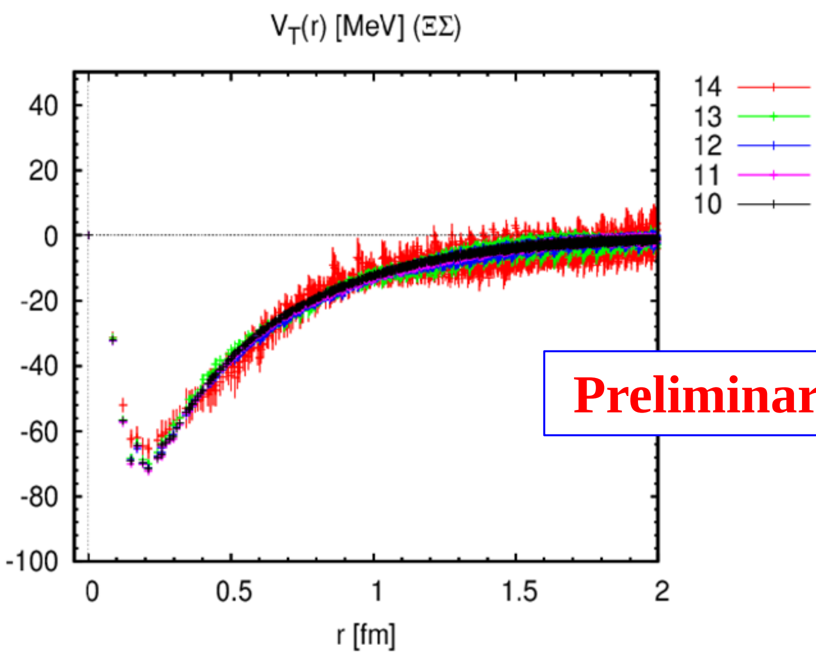
- $\Xi\Sigma$ (I=3/2)
  - $^1S_0$ ∼ 27-plet
    ⇔ NN($^1S_0$) + SU(3) breaking
  - $^3S_1$-$^3D_1$ ∼10*-plet
    ⇔ NN($^3S_1$-$^3D_1$) + SU(3) breaking
- $\Xi\Lambda$–$\Xi\Sigma$ (I=1/2) : coupled channel
  - $^1S_0$ ∼ 27-plet & 8s-plet
  - $^3S_1$-$^3D_1$ ∼10-plet & 8a-plet

ΞΣ(I=3/2, spin triplet)

Central

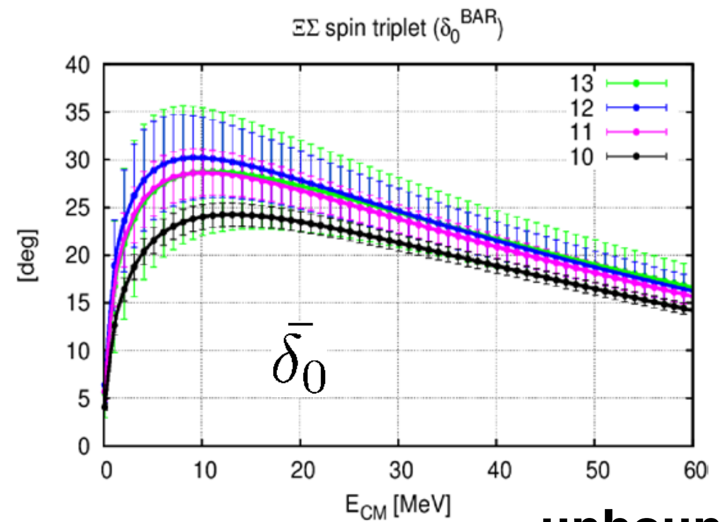


Tensor

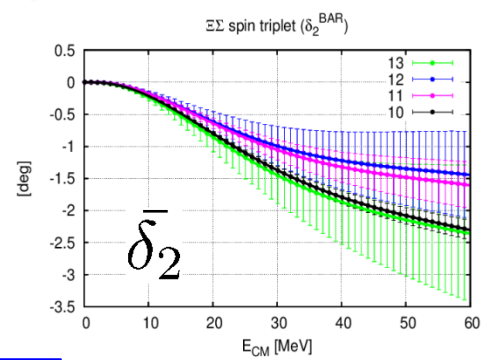


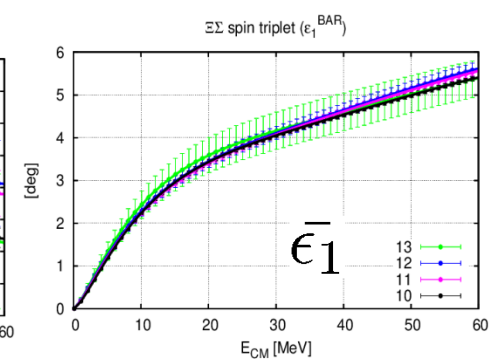
Preliminary

(bar) phase shifts & mixing



unbound





N.B. t-dep should be checked;
single m_B has ~0.3-3% sys @ t=10-14

(200conf x 4rot x 20src)

[N. Ishii]

H-dibaryon channel (S= –2)

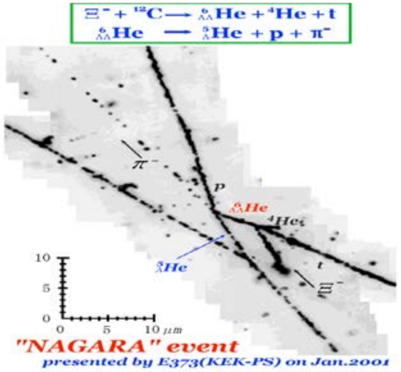
($^1S_0$, ΛΛ–NΞ–ΣΣ, Coupled Channel)

R. Jaffe (1977), **"Perhaps a Stable Dihyperon"**

NAGARA-event (2001)

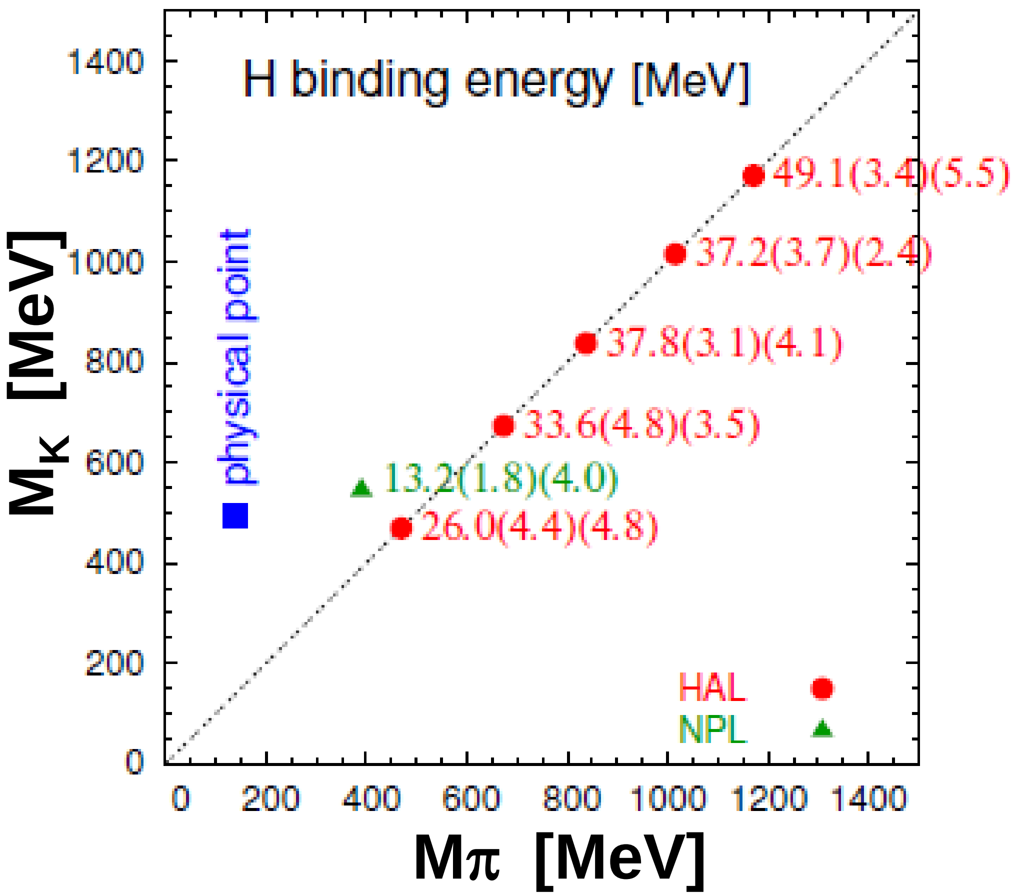
$$\Xi^- + {}^{12}\mathrm{C} \to {}_{\Lambda\Lambda}{}^{6}\mathrm{He} + {}^{4}\mathrm{He} + t$$

- ΛΛ weak attraction
- No deeply bound H-dibaryon

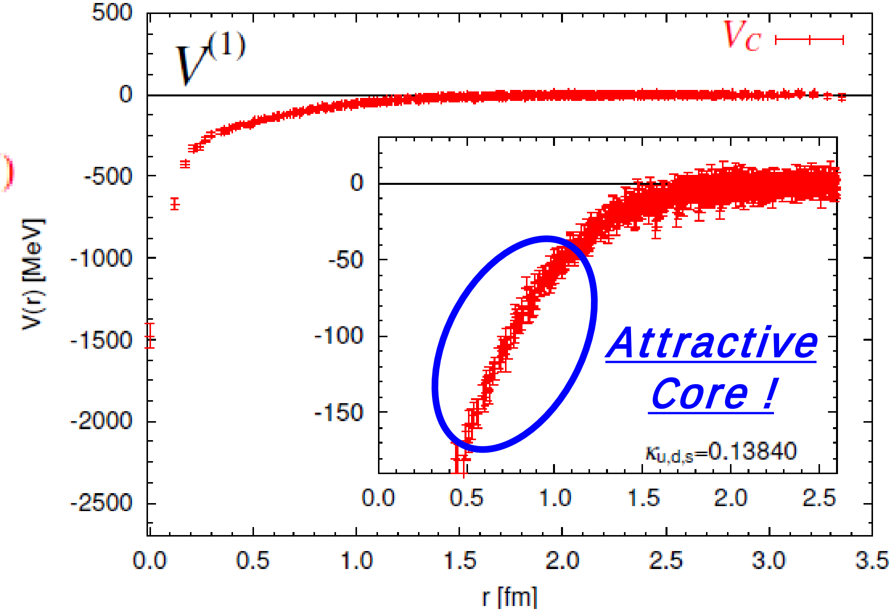


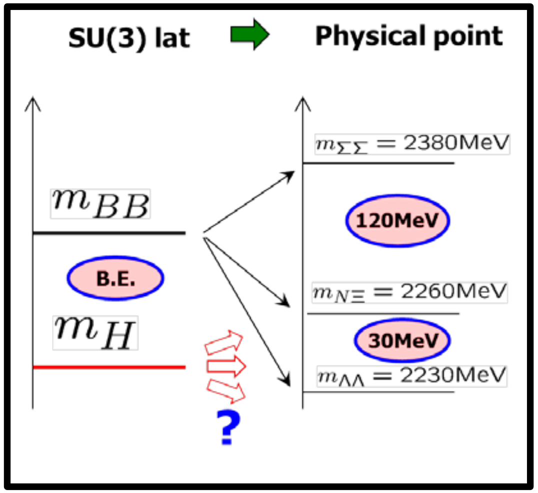
H-dibaryon @ Nf=3, heavy masses

[T. Inoue]



Potential in Flavor-singlet channel





Inoue et al. (HAL QCD Coll.) PRL106(2011)162002

Beane et al. (NPLQCD Coll.) PRL106(2011)162001

c.f. B.E. = 74.6(3.3)(3.4) MeV @ m_π=0.8GeV by NPL ('12)

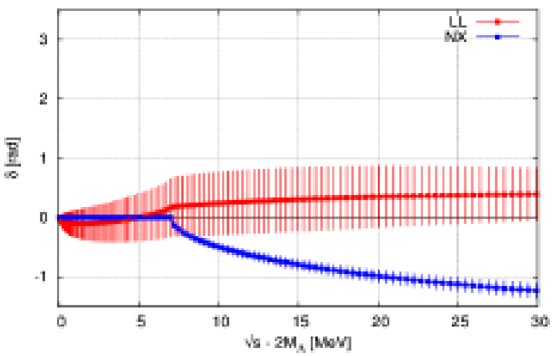
H-dibaryon @ Nf=2+1, heavy masses [K. Sasaki]

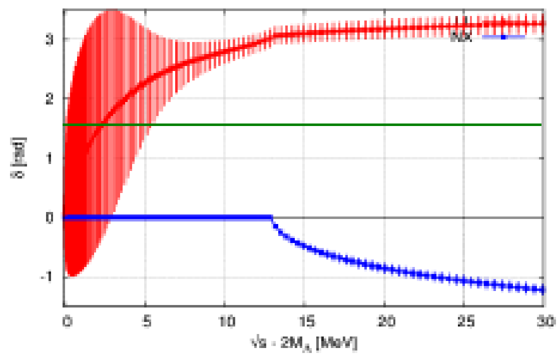
ΛΛ and NΞ phase shifts

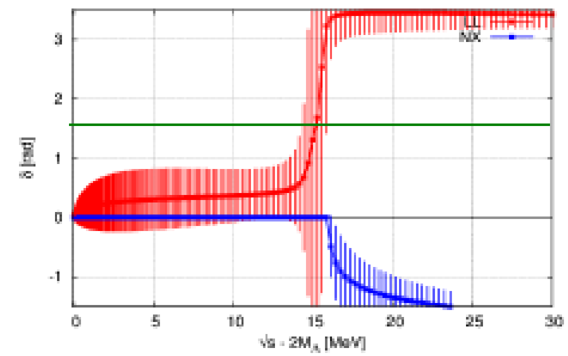
▶ N_f = 2+1 full QCD with L = 2.9fm

Preliminary!

mπ = 700 MeV	*mπ = 570 MeV*	*mπ = 410 MeV*

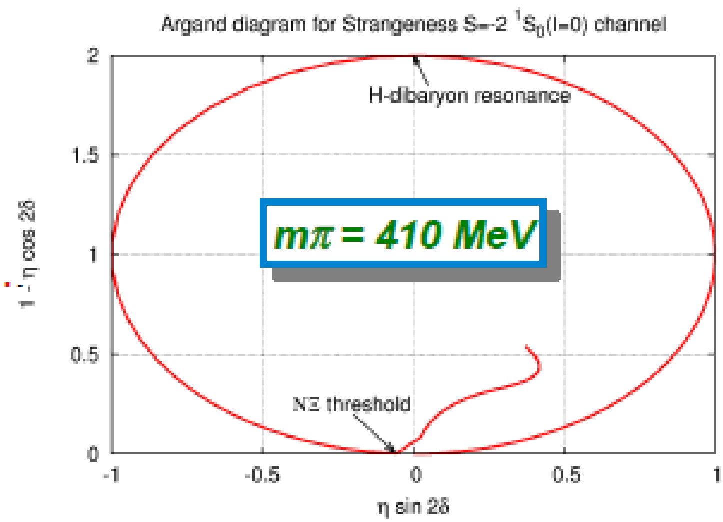






- *mπ = 700 MeV :* bound state
- *mπ = 570 MeV* : resonance near ΛΛ threshold
- *mπ = 410 MeV* : resonance near NΞ threshold.

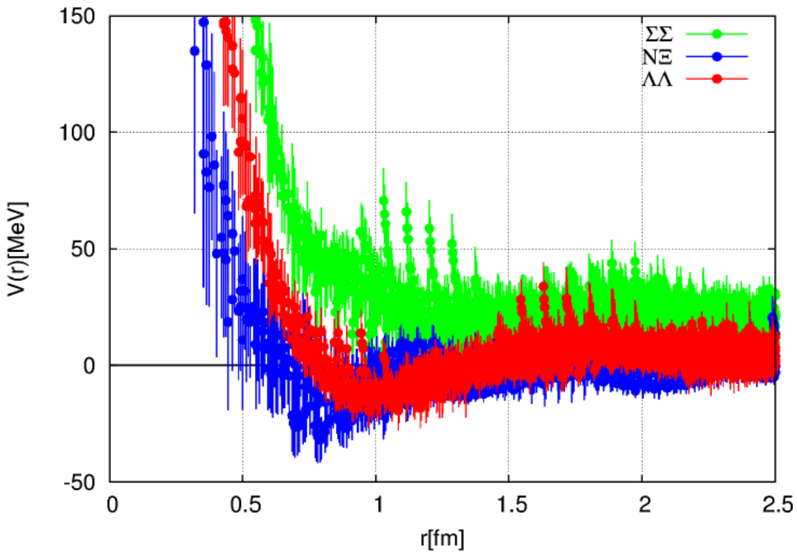
H-dibaryon is unlikely bound state



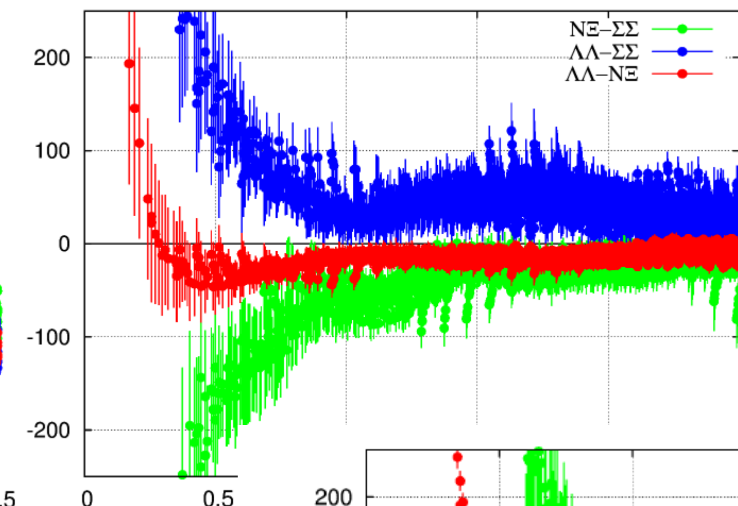
H-dibaryon @ Nf=2+1, m_π=145 MeV

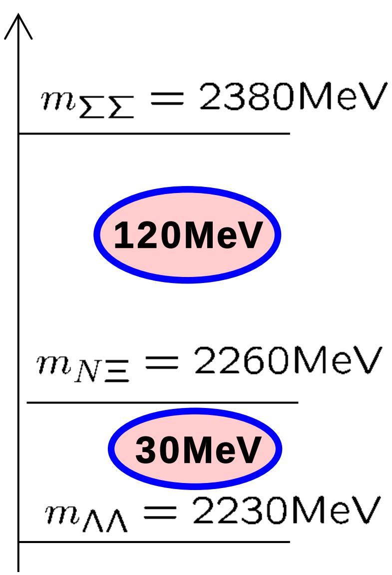
[K. Sasaki]

diagonal

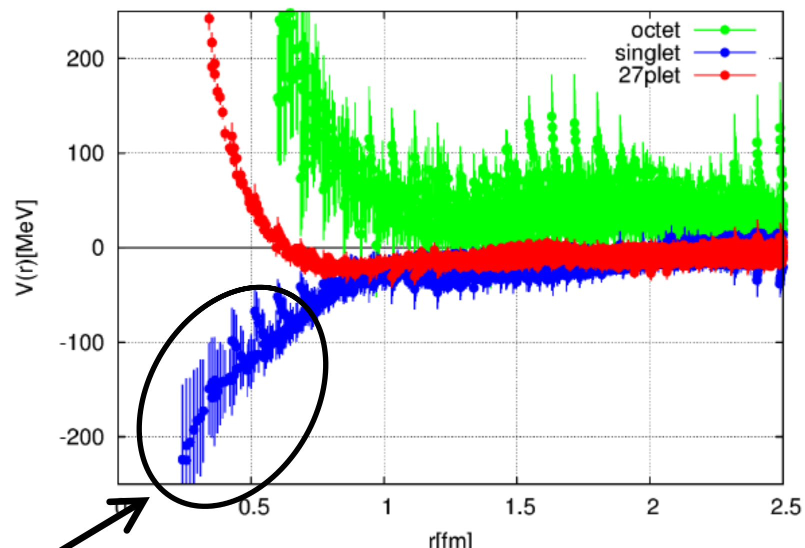


off-diagonal





diagonal in SU(3)-irrep base



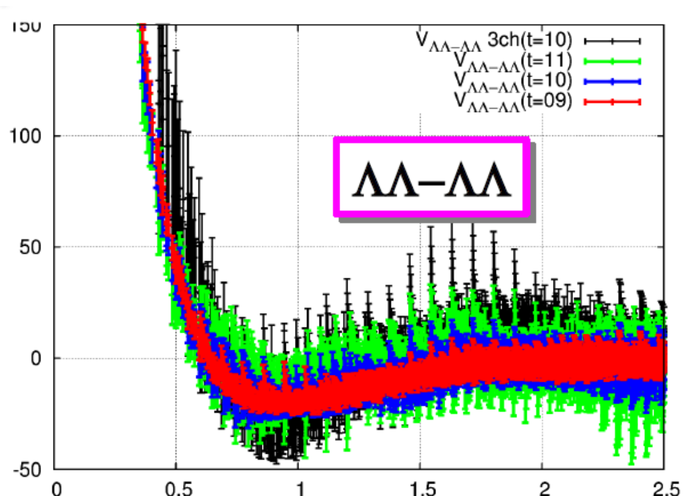
Strong Attraction in flavor-singlet channel

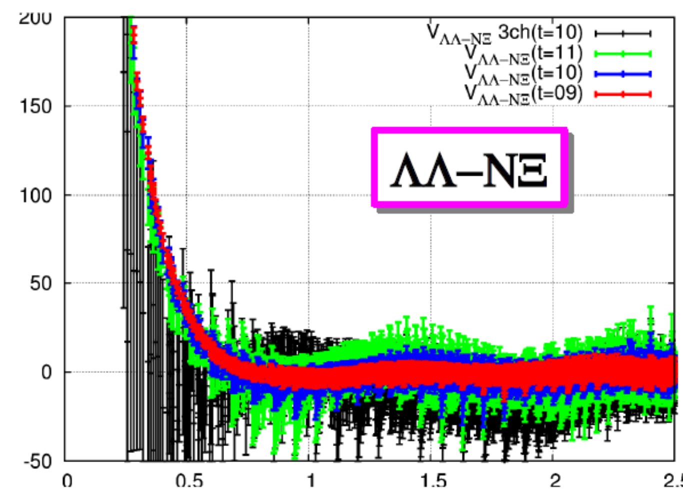
(200conf x 4rot x 20src, t=10)

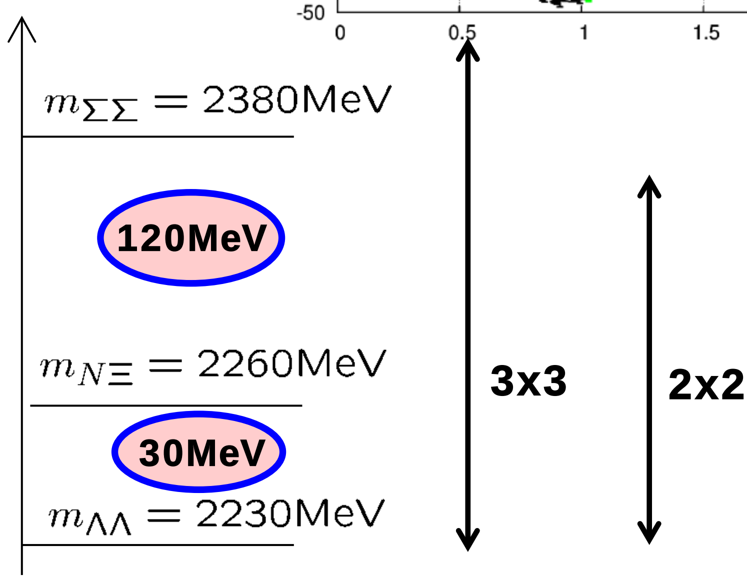
ΛΛ, NΞ (effective) 2x2 coupled channel analysis

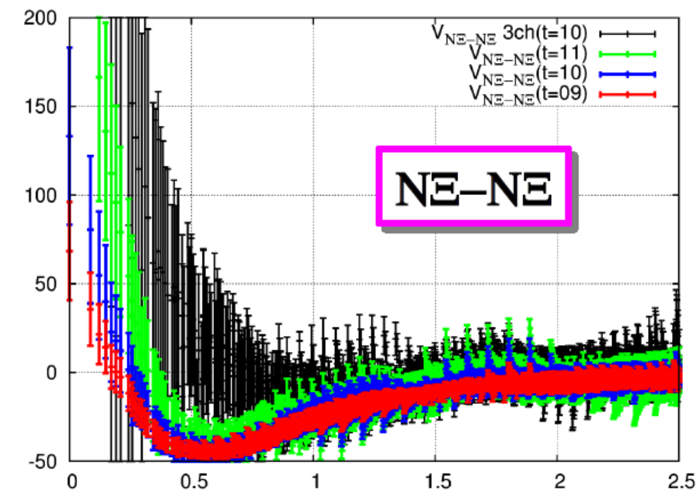
ΣΣ channel ←→ couples strongly to flavor octet channel
←→ noisy because they are quark-Pauli forbidden

→ Improve the S/N by considering only ΛΛ, NΞ dof at low energies





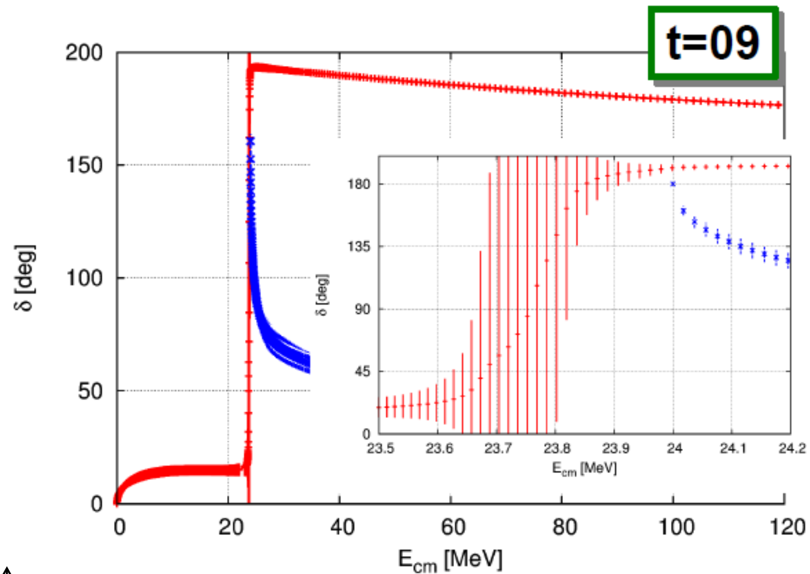


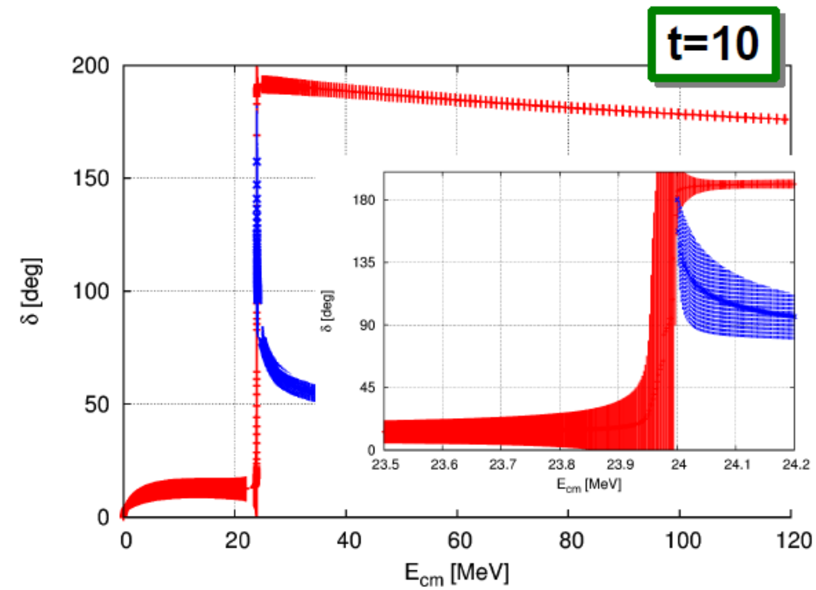


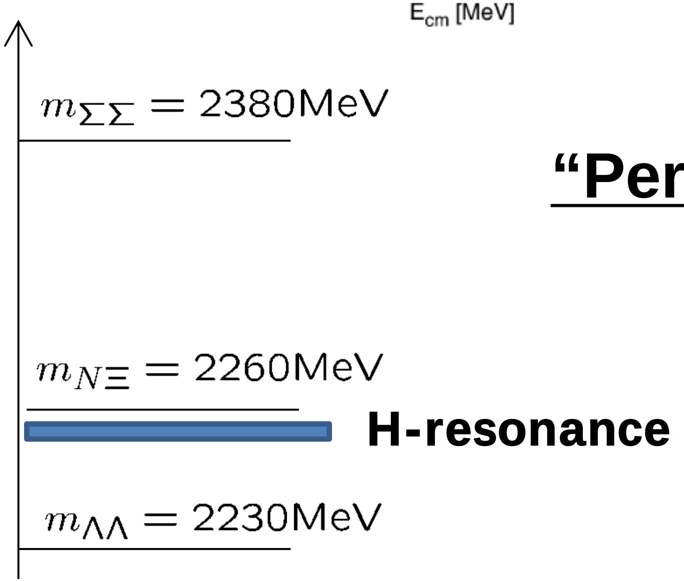
ΛΛ, NΞ (effective) 2x2 coupled channel analysis

ΛΛ, NΞ phase shifts

Preliminary







"Perhaps a Resonant Dihyperon"

➔ **J-PARC experiment (E42)**

N.B. t-dep should be checked;
single m_B has ~3% sys @ t=10

[K. Sasaki]

$N\Xi$–interactions (S= –2)

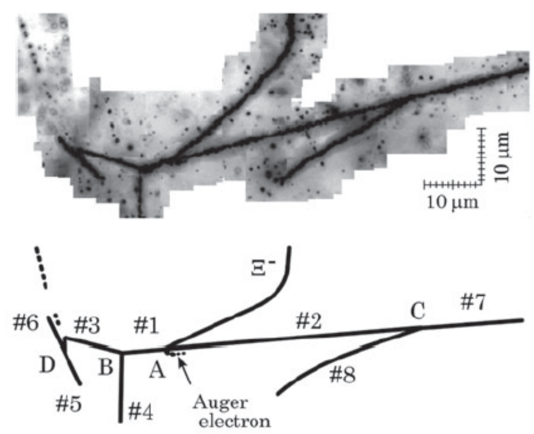
Ξ^- could appear in the core of Neutron Star

e.g., J. Schaffner-Bielich, NPA804(2008)309

KISO-event (2014)

$$\Xi^- + {}^{14}\mathrm{N} \rightarrow {}_{\Lambda}{}^{10}\mathrm{Be} + {}_{\Lambda}{}^{5}\mathrm{He}$$

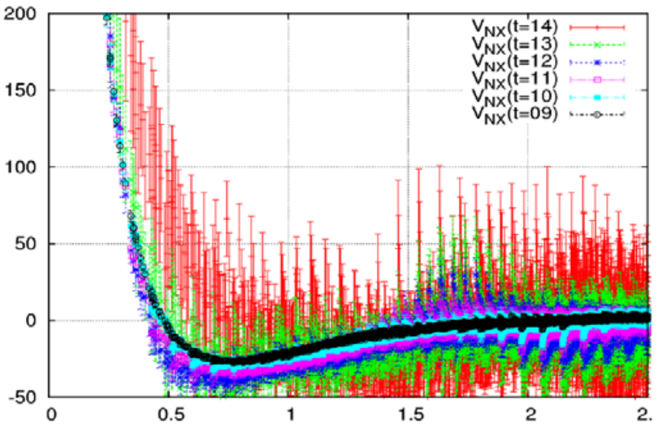
- First observation of Ξ-nuclei
- B.E. = 4.38(25) MeV
  (or 1.11(25) MeV)



NΞ-Potentials

[K. Sasaki]

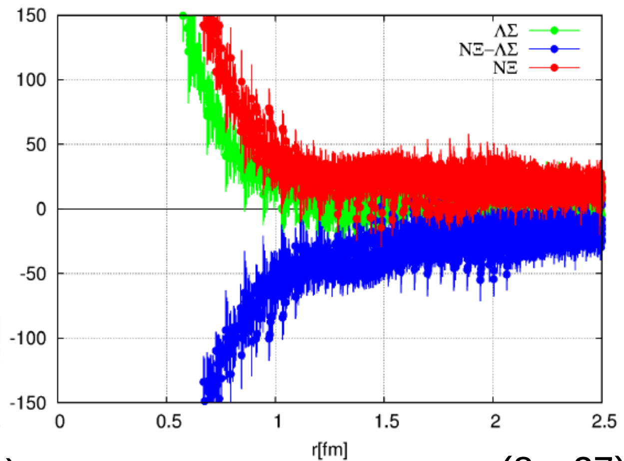
NΞ (I=0, $^3S_1$)



(8a)

Attractive

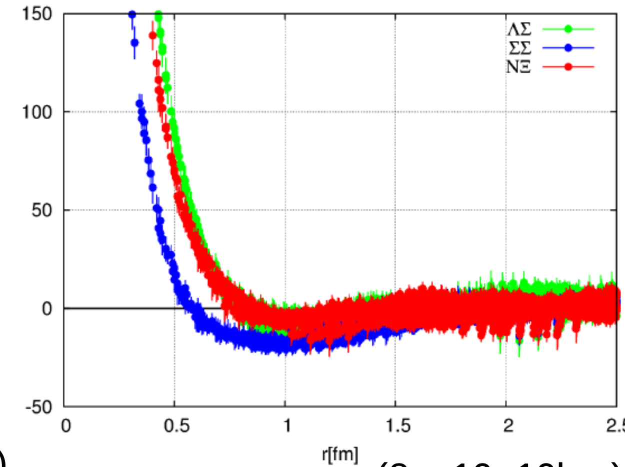
NΞ–ΛΣ (I=1, $^1S_0$)



(8s, 27)

Repulsive

NΞ–ΛΣ–ΣΣ (I=1, $^3S_1$)



(8a, 10, 10bar)

Attractive

(ΛΛ–NΞ–ΣΣ (I=0, $^1S_0$))

Is interaction net attractive ? Stay tuned !

(net attractive @ m(pi)=0.66-88GeV)

$S = -1$ systems

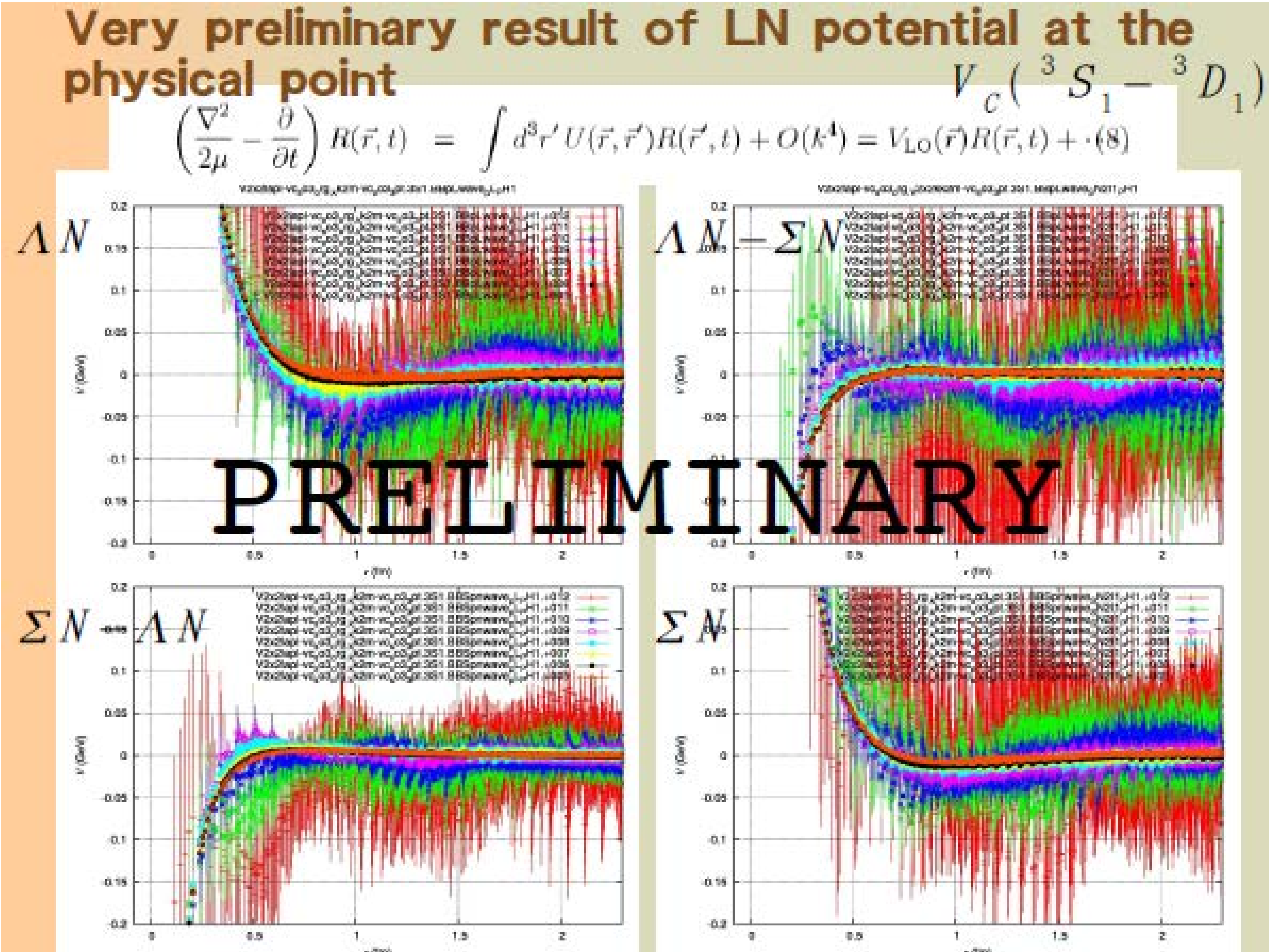
←→ strangeness nuclear physics (Λ-hypernuclei @ J-PARC)

Λ should (?) appear in the core of Neutron Star

←→ Huge impact on EoS of high dense matter

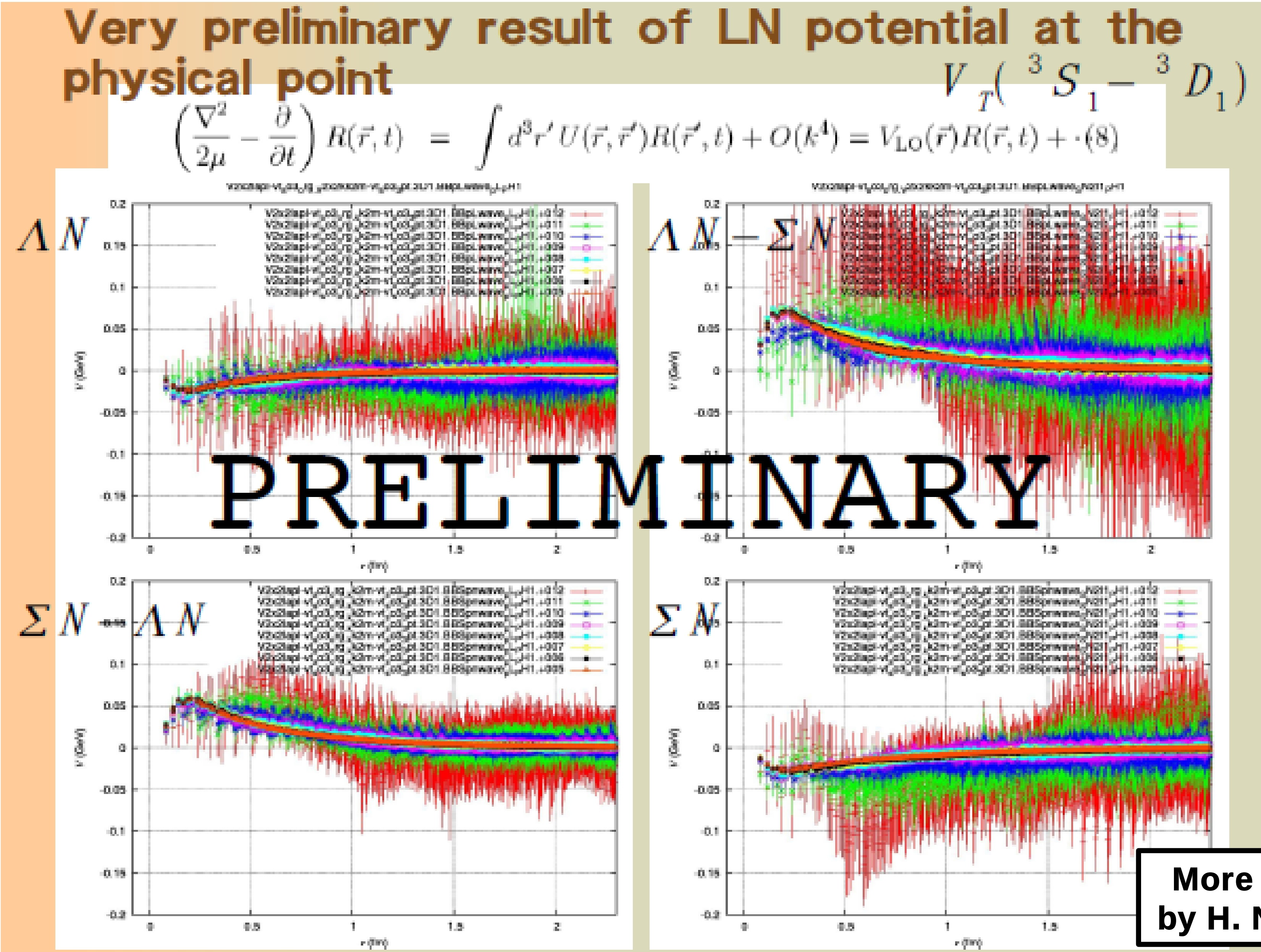
- **$\Lambda N - \Sigma N$ (I=1/2) : coupled channel**
  - **$^1S_0$ ∼ 27-plet & 8s-plet**
  - **$^3S_1$-$^3D_1$ ∼10*-plet & 8a-plet**
- **ΣN (I=3/2)**
  - **$^1S_0$ ∼ 27-plet**
    ⇔ NN($^1S_0$) + SU(3) breaking
  - **$^3S_1$-$^3D_1$ ∼10-plet**

ΛN–ΣN Vc potential in $^3S_1$–$^3D_1$ [H. Nemura]



ΛN–ΣN Vt potential in $^3S_1$–$^3D_1$

[H. Nemura]

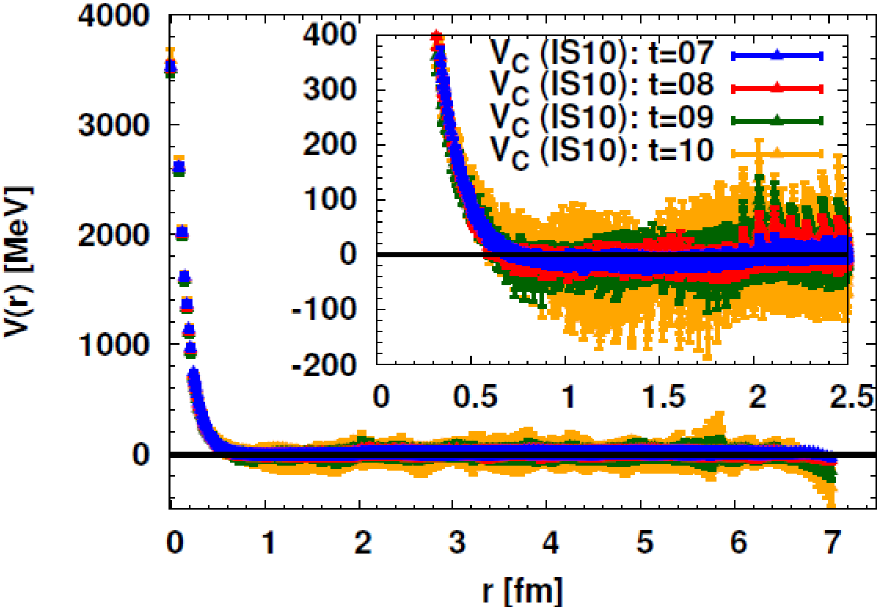


More in talk
by H. Nemura

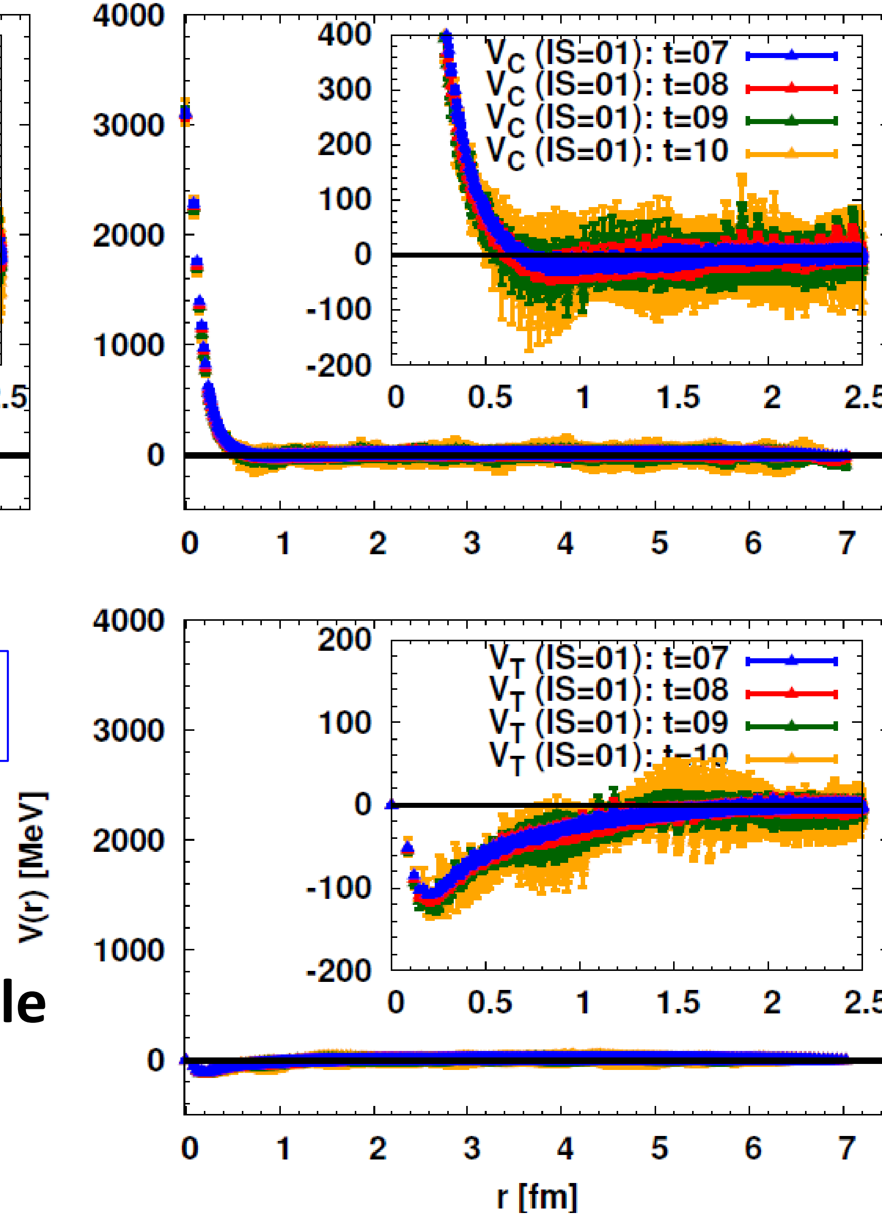
NN system $(S = 0)$

NN-Potentials

$^1S_0$



$^3S_1$-$^3D_1$



Central

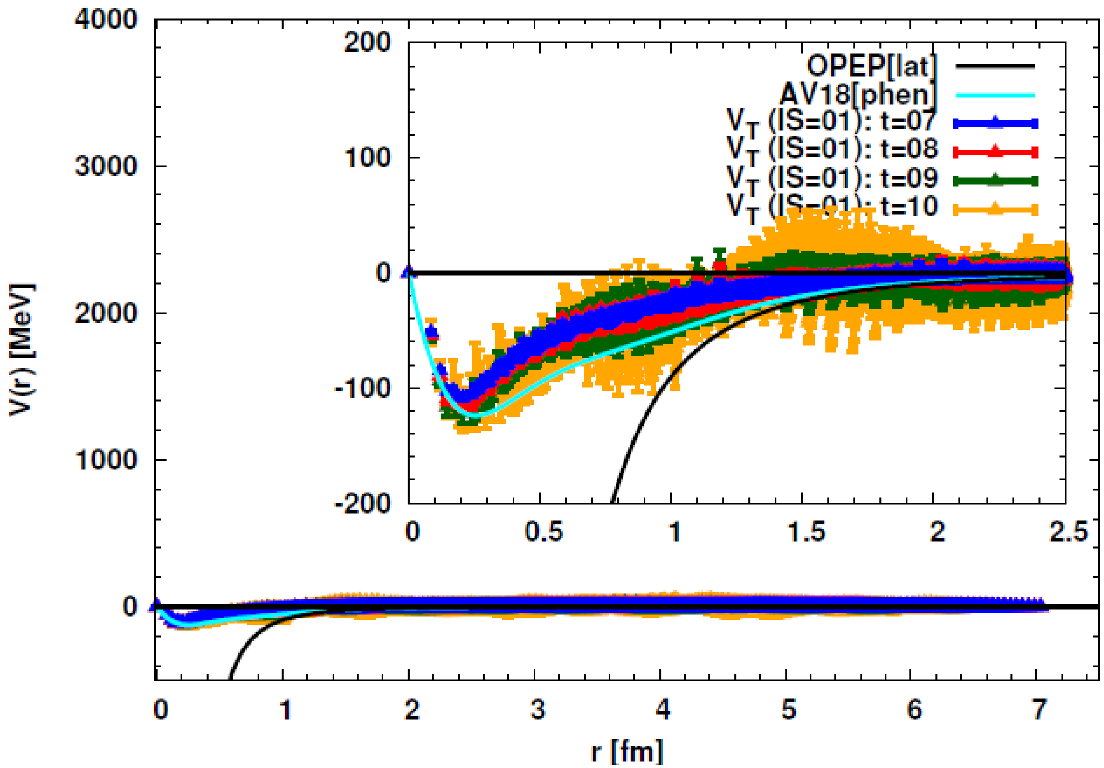
Tensor

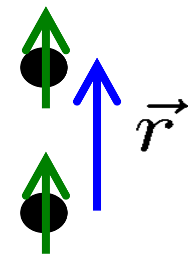
Preliminary

- **Vc: repulsive core + long-range attraction**
- **Vt: tensor force clearly visible**

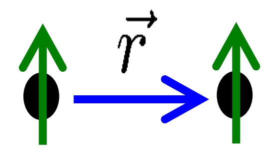
(200conf x 4rot x 44src)

NN-Potentials (tensor)





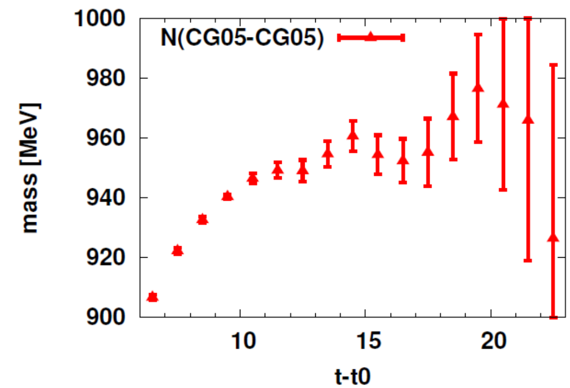
(attractive)



(repulsive)

- **Qualitatively similar tail as OPEP force**
- **Larger t w/ larger #stat is desirable**

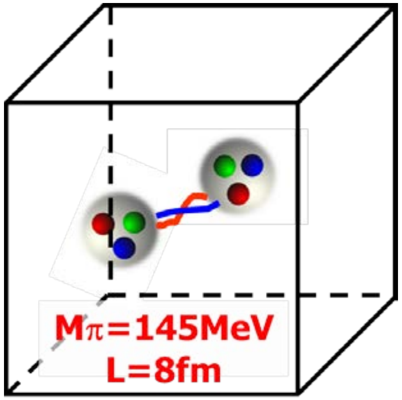
m(eff) for single N



t = 8-10 : ~2-4% sys error

Summary

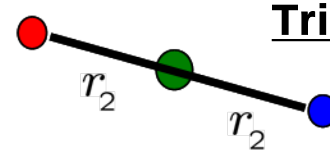
- The 1st LQCD for Baryon Interactions at ~ phys. point
  - m(pi) ~= 145 MeV, L ~= 8fm, 1/a ~= 2.3GeV
  - Central & Tensor forces calculated for all NN/YN/YY in P=(+) channel
  - Key formula / algorithm
    - t-dep HAL QCD method
    - Coupled channel formalism
    - Unified contraction algorithm (UCA)
  - Various exciting results
    - $\Omega\Omega$ ($^1S_0$) : a new exotic dibaryon state
    - $\Xi\Xi$ ($^1S_0$) : most likely an unbound state
    - H-dibaryon : indication of a resonance
    - NN : tensor force is clearly visible



- Prospects
  - #stat will be ~ x3 – x8 from today's figs
  - New techniques to improve S/N are under R&D
  - [Exascale-Era] LS-forces, P=(-) channel, 3-baryon forces, etc., & EoS

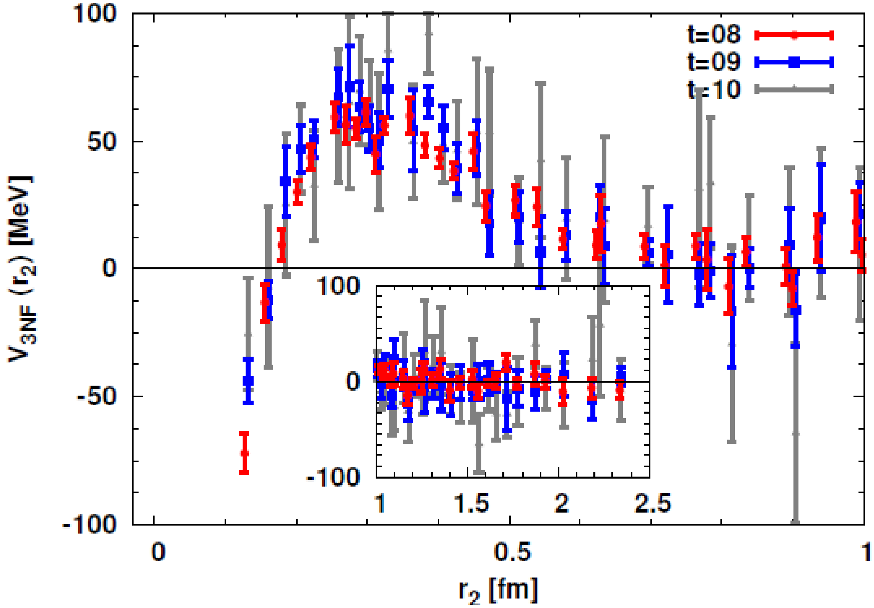
3N-forces (3NF)

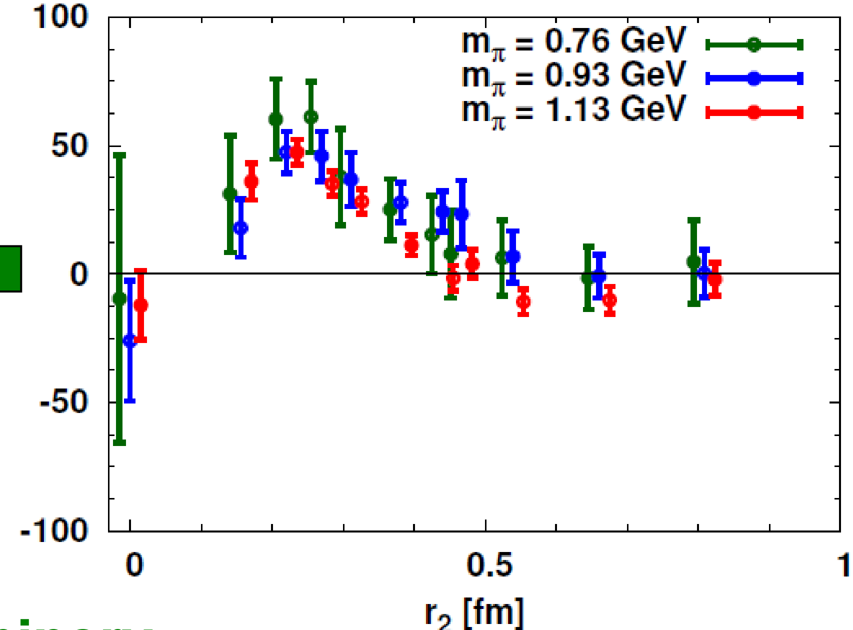
Triton channel



Nf=2+1, mπ=0.51 GeV

Nf=2, mπ=0.76-1.1 GeV





Preliminary

Short-range repulsive 3NF

Kernel: ~50% efficiency achieved !

T.D. et al. (HAL Coll.) PTP127(2012)723
\+ t-dep method updates etc.

Backup Slides

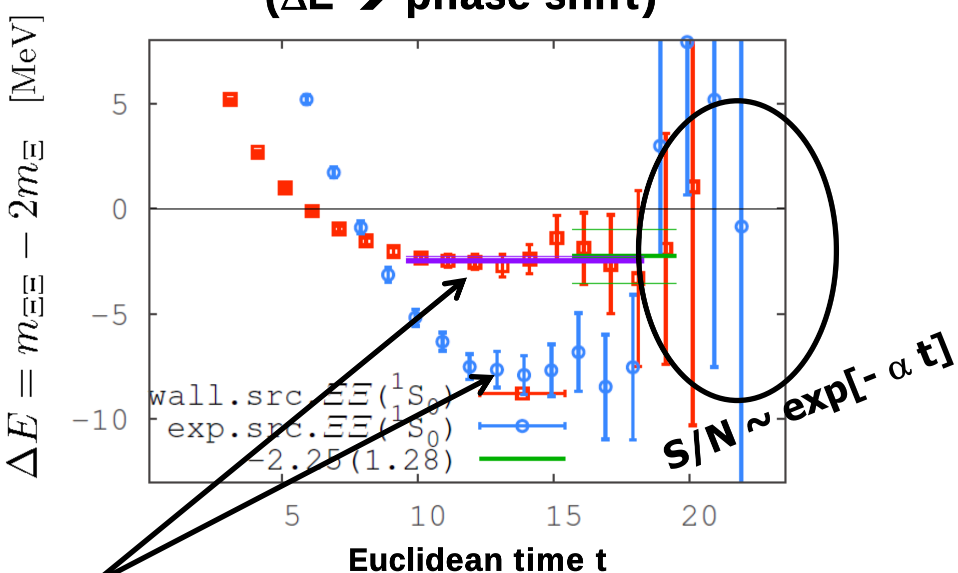
Reliability Test of LQCD methods

- High-stat study for BB-system (@m(pi)=0.5GeV)   T. Iritani et al. (HAL Coll.)
  - Benchmark w/ two LQCD setup (wall & smeared src)

➔ Physical outputs should NOT depend on these setup

Luscher's method (traditional)

(ΔE ➔ phase shift)

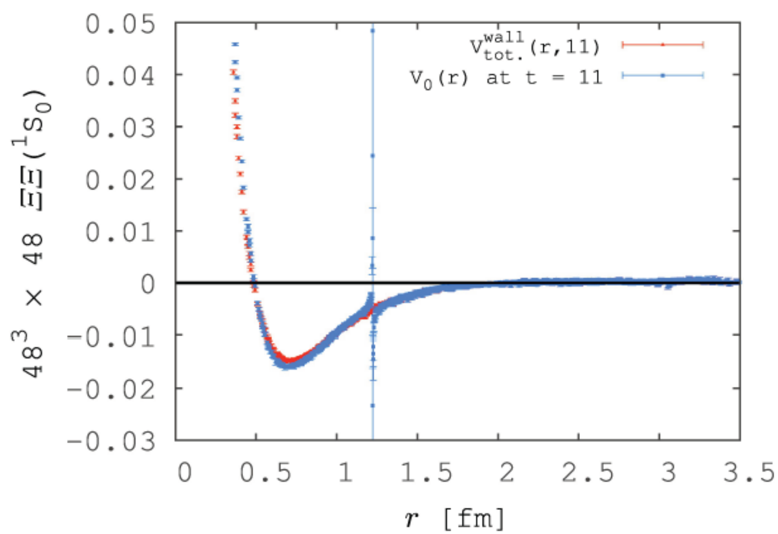


Inconsistent "signal" (red (wall) vs blue (smeared))
➔ cannot judge which (or neither) is reliable

FAILED

HAL method (new !)

(V(r) ➔ phase shift)

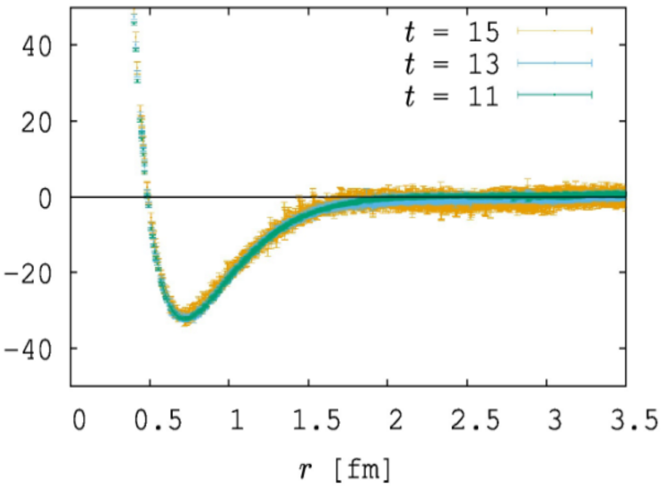


V^{eff}(r) from wall & V^{LO}(r) from wall+smeared are consistent

PASSED

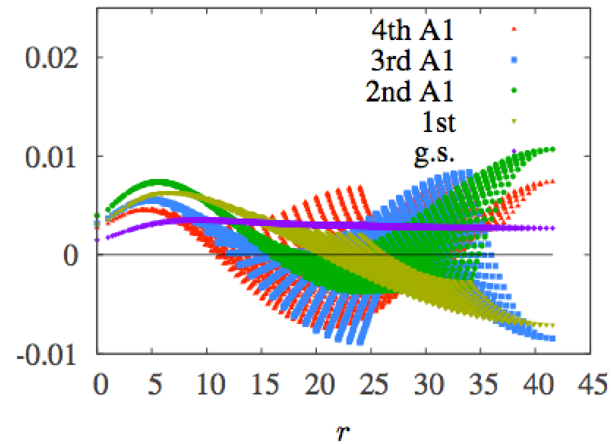
Understand the origin of "fake plateaux"

Potential



Solve Schrodinger eq. in Finite V

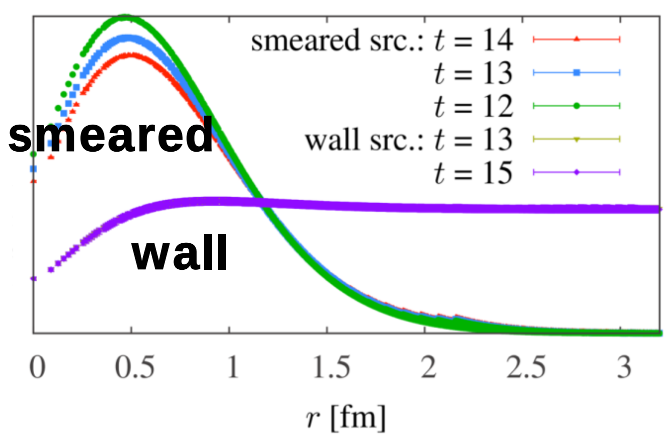
Eigen-wave functions



Eigen-energies

n-th A1	ΔE_n [MeV]
0	-2.58(1)
1	52.49(2)
2	112.08(2)
3	169.78(2)
4	224.73(1)

NBS correlator Ψ(r,t)



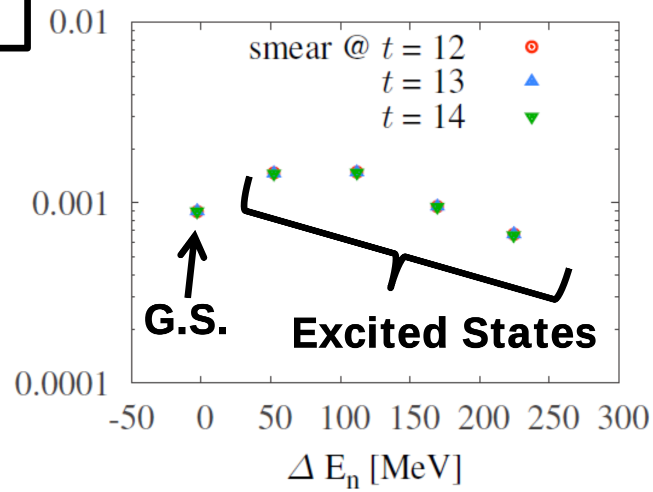
Decompose NBS correlator to each eigenstates

Decompose NBS correlator to each eigenstates

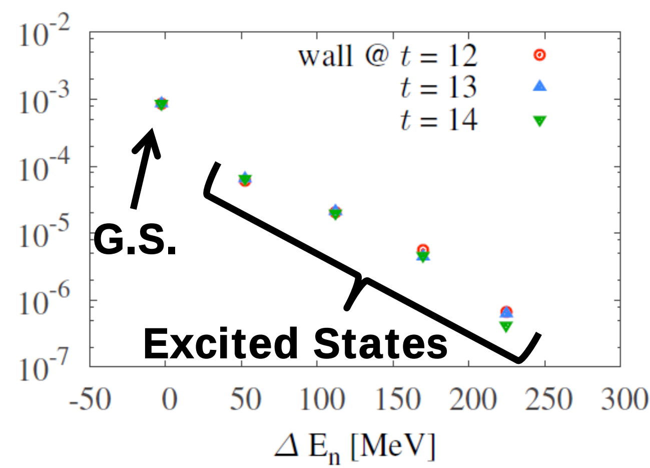
NBS correlator $\Psi(r,t)$

Contribution from each (excited) states (@ t=0)

smeared



wall

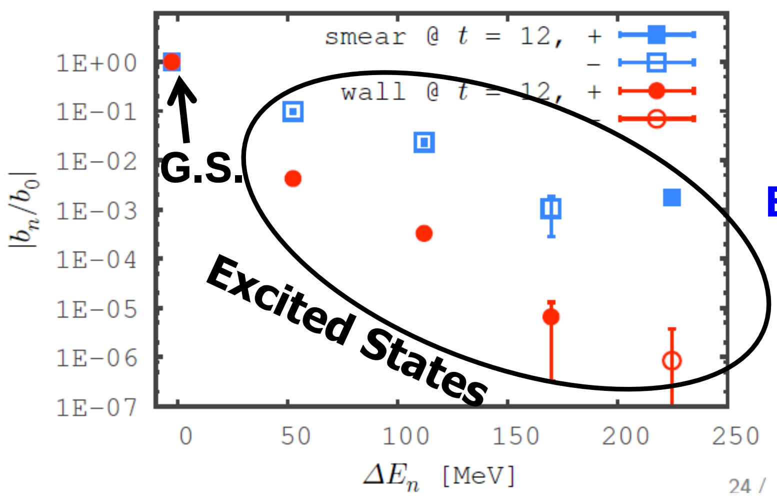


excited states NOT suppressed **excited states suppressed**

R-correlator
$R(t) = \Sigma_r \Psi(r,t)$

(R(t) w/ smeared has been used in Luscher's method)

Contribution from each (excited) states (@ t=0)

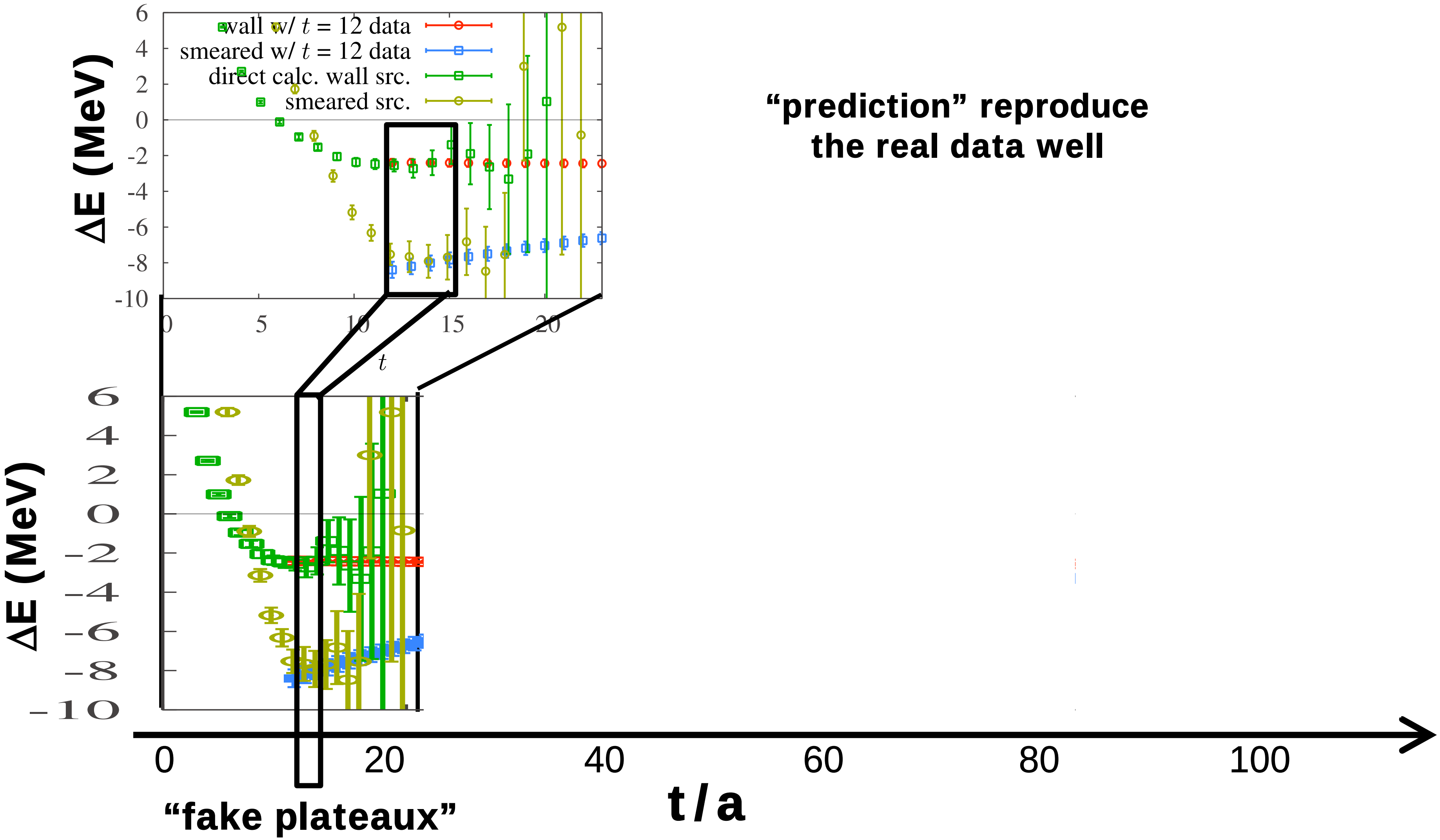


Blue: smeared

Red: wall

Understand the origin of "fake plateaux"

We are now ready to "predict" the behavior of m(eff) of ΔE at any "t"

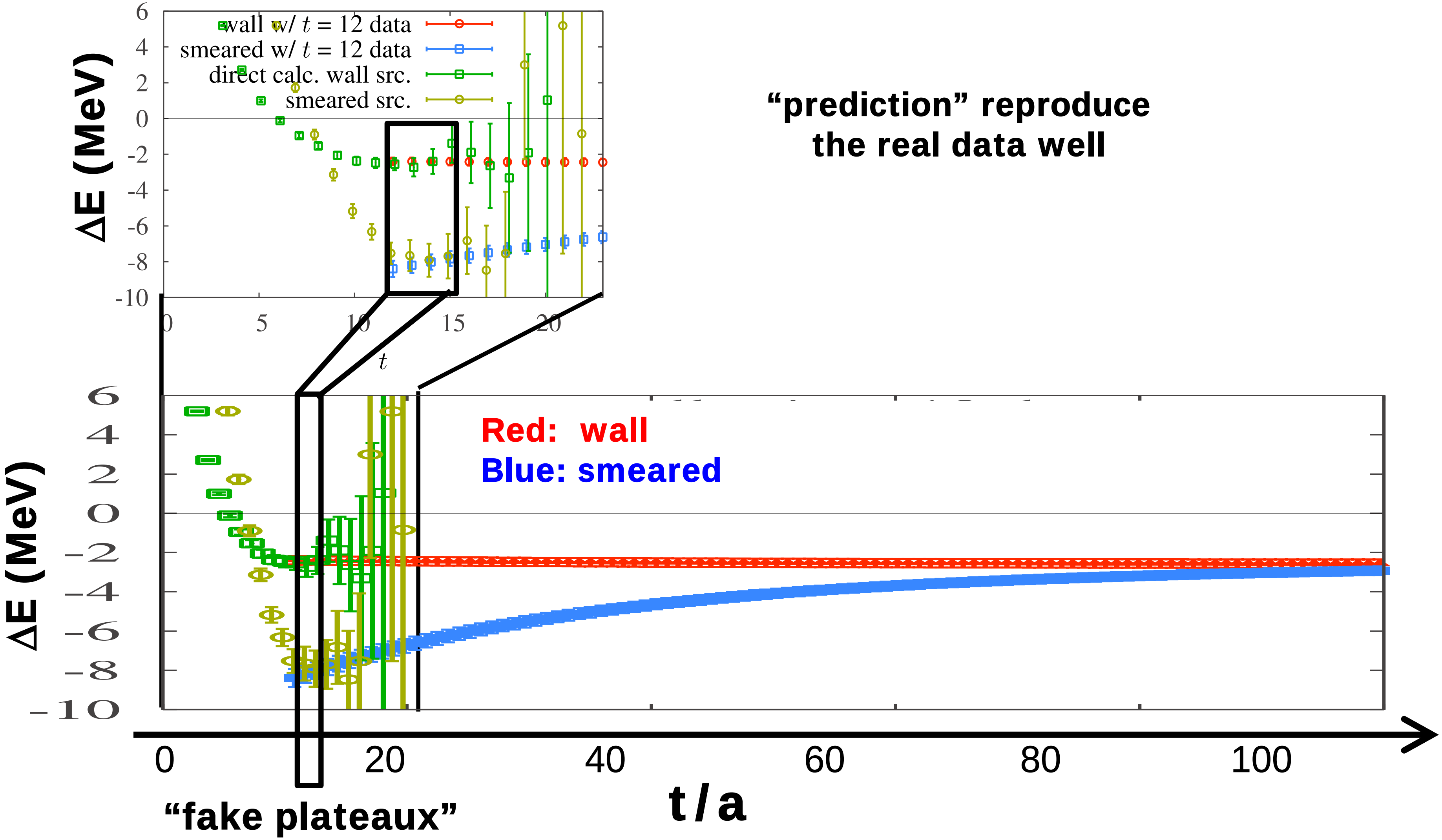


Extreme care is necessary for the results from the Luscher's method

To obtain a "real plateau", t/a >100 (t>10fm) is necessary

Understand the origin of "fake plateaux"

We are now ready to "predict" the behavior of m(eff) of ΔE at any "t"



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