Resonances & QCD

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March 2016

Composite particles & QCD

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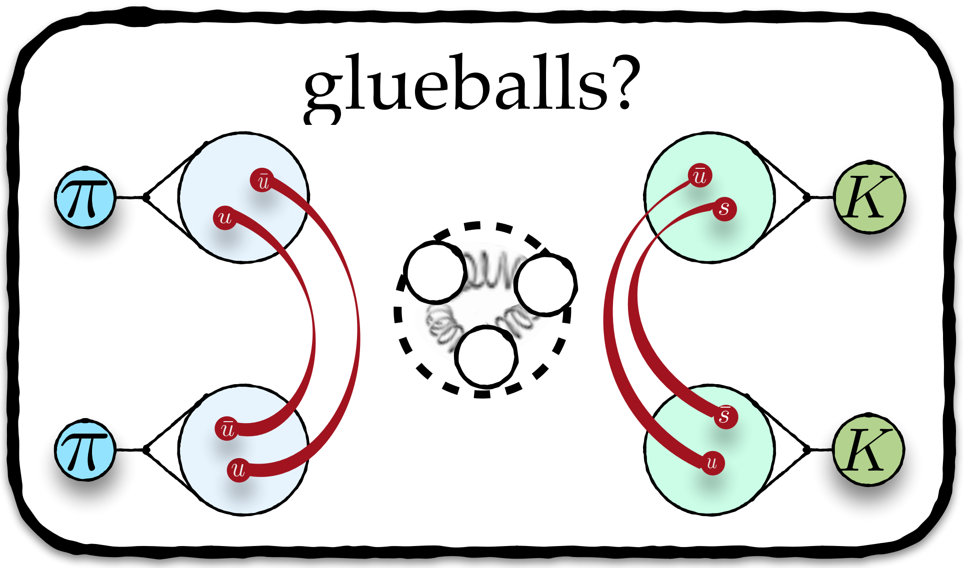
Resonances / Bound states



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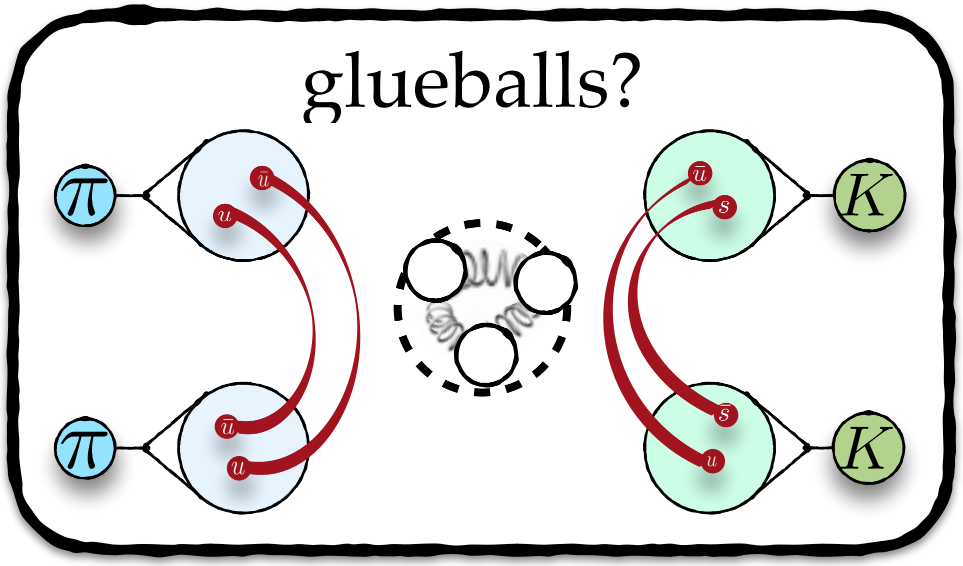

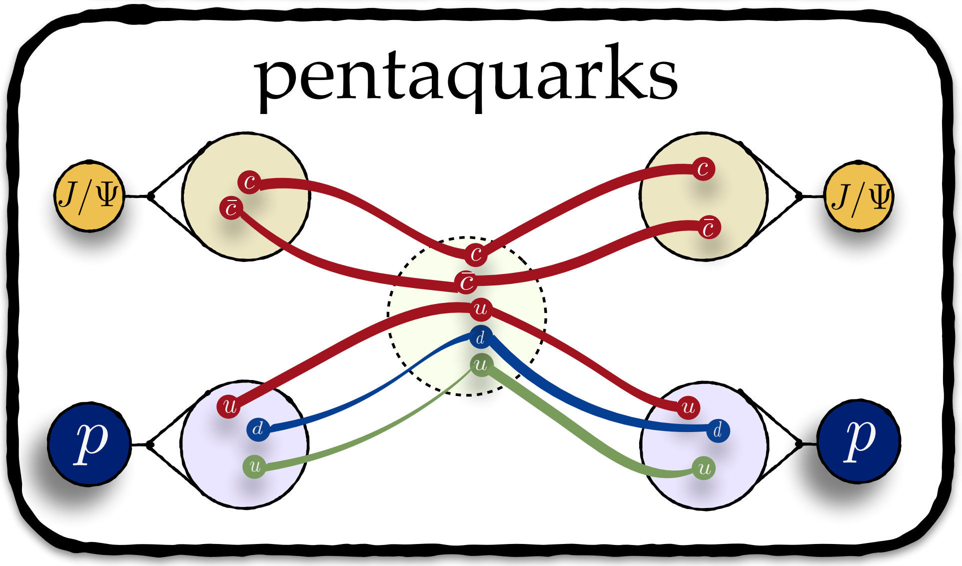

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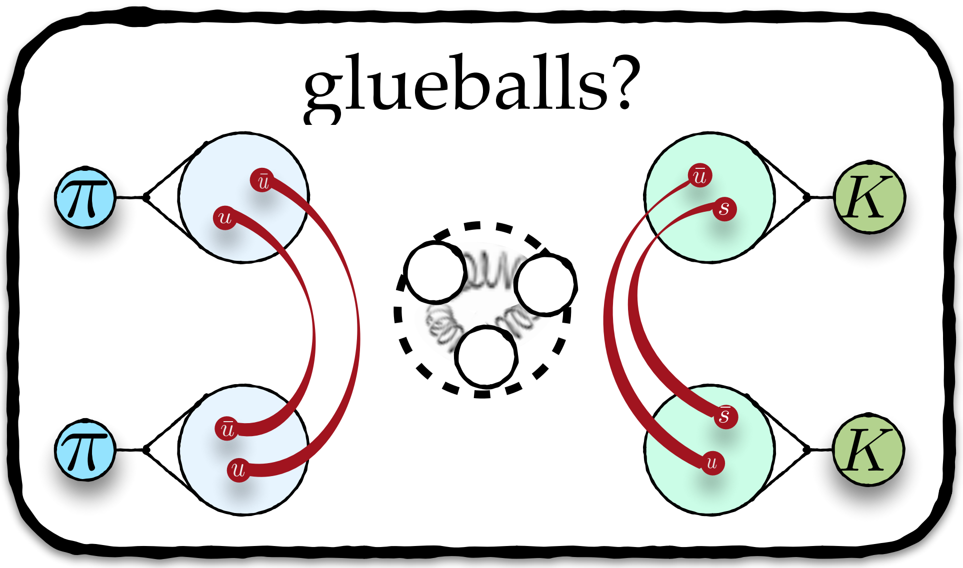

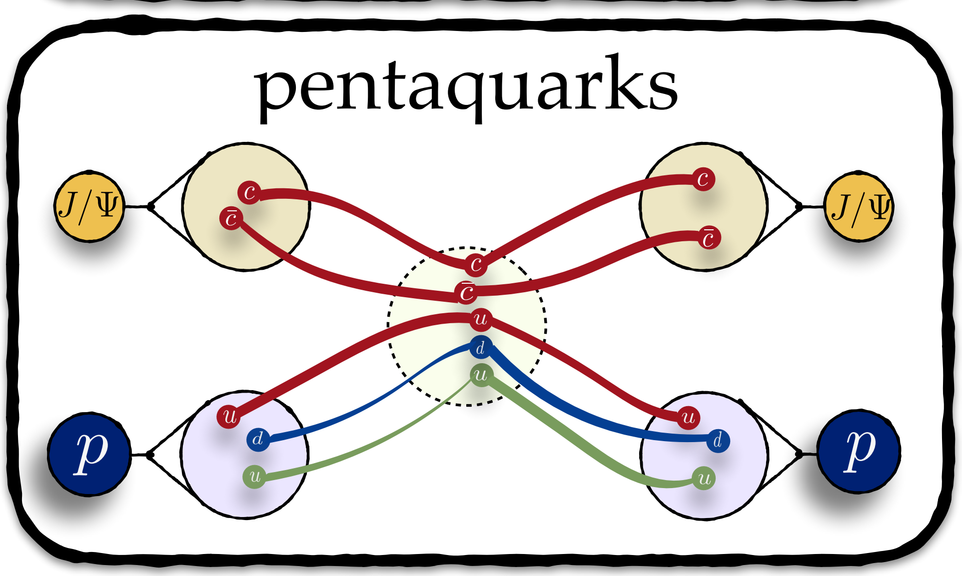

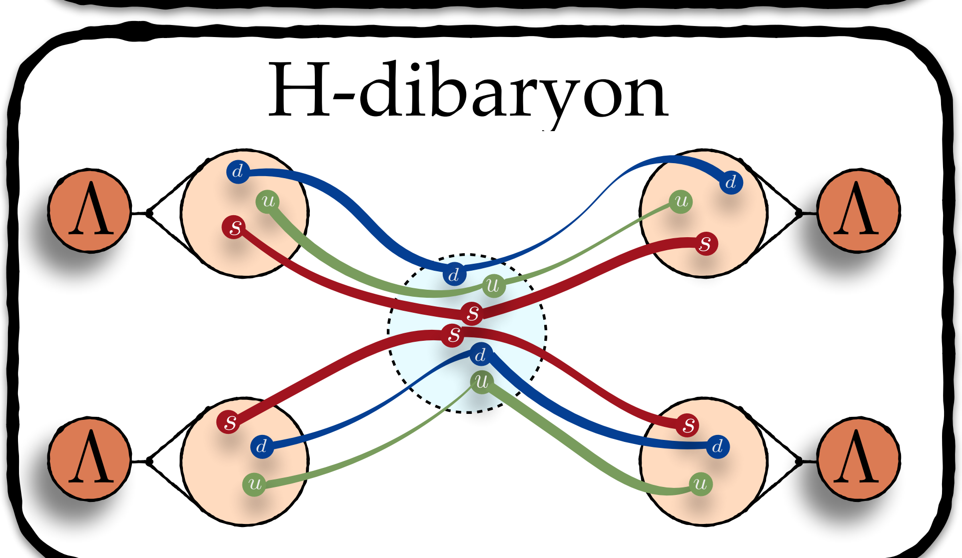

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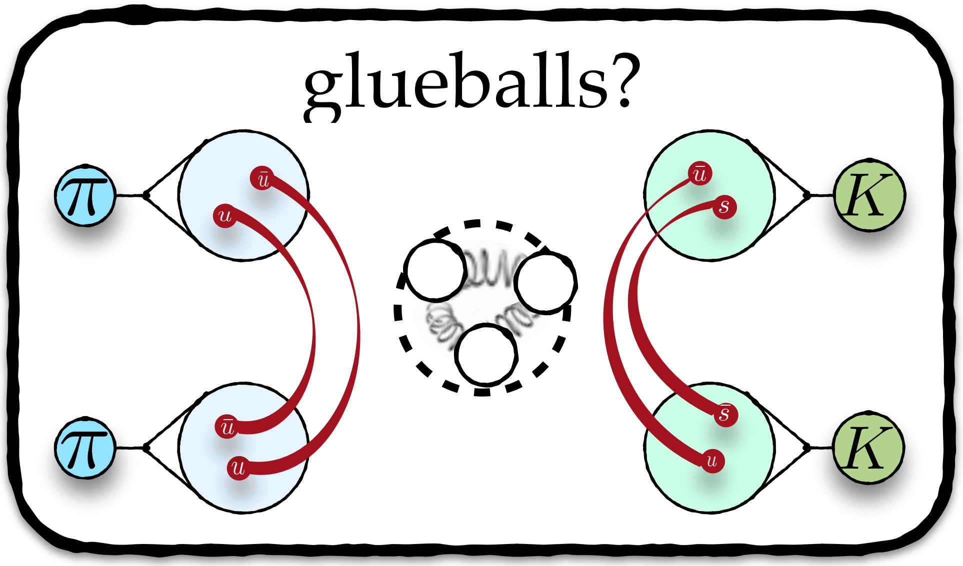

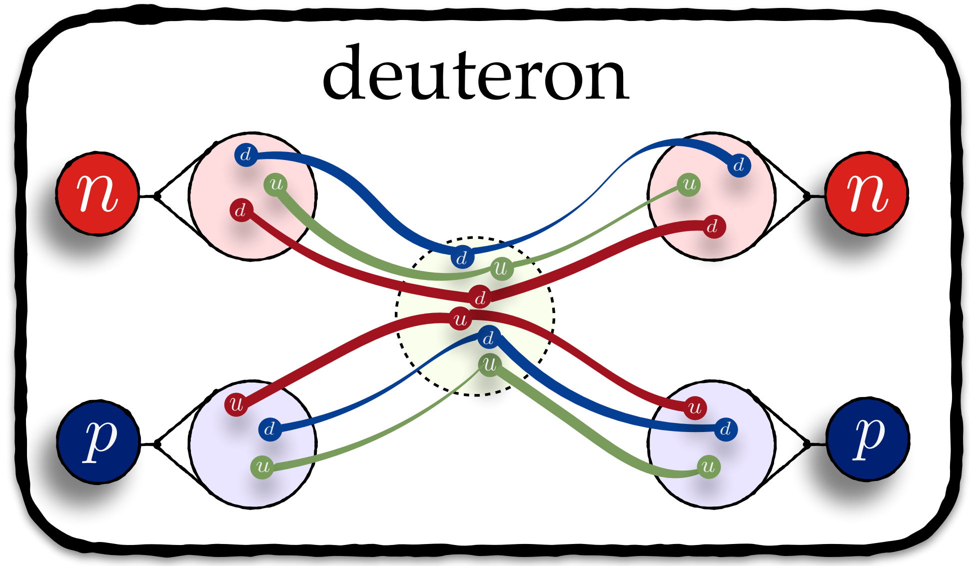

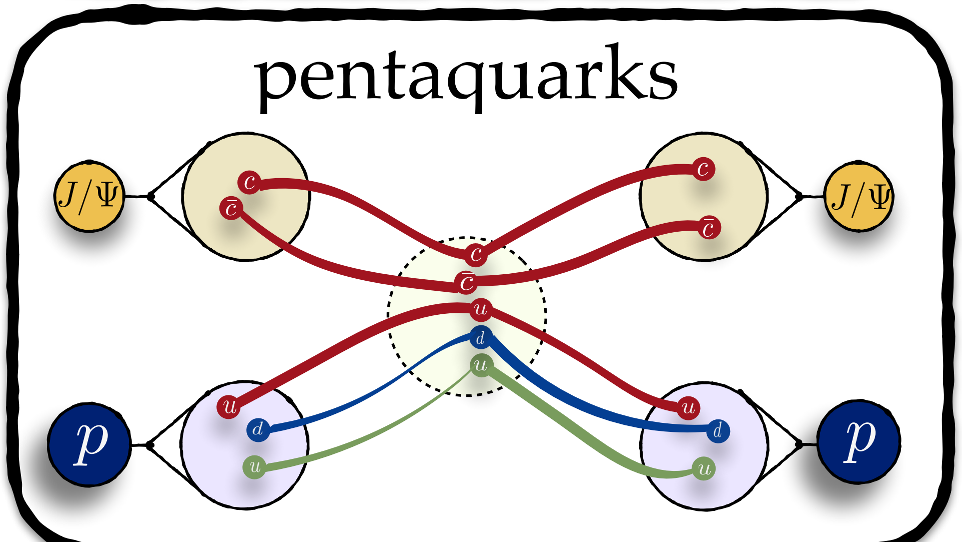

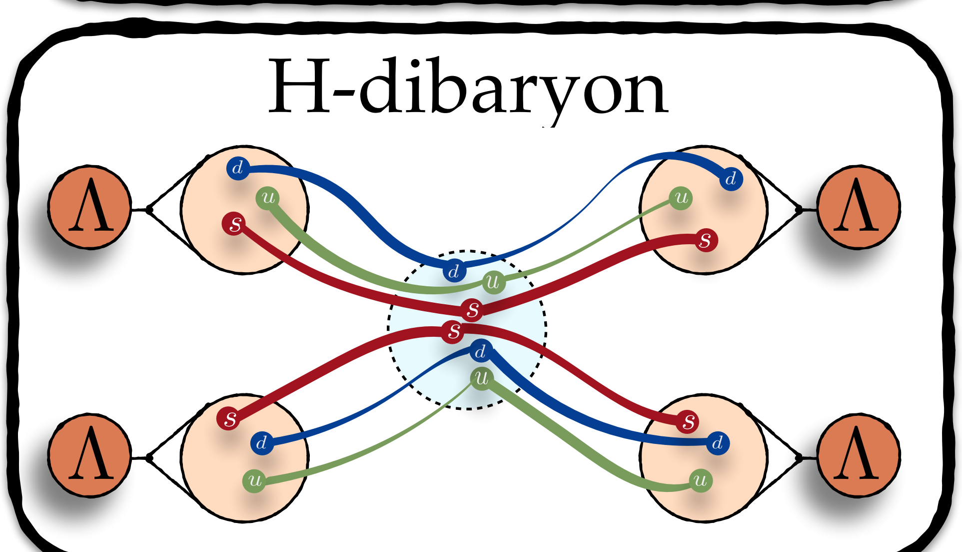

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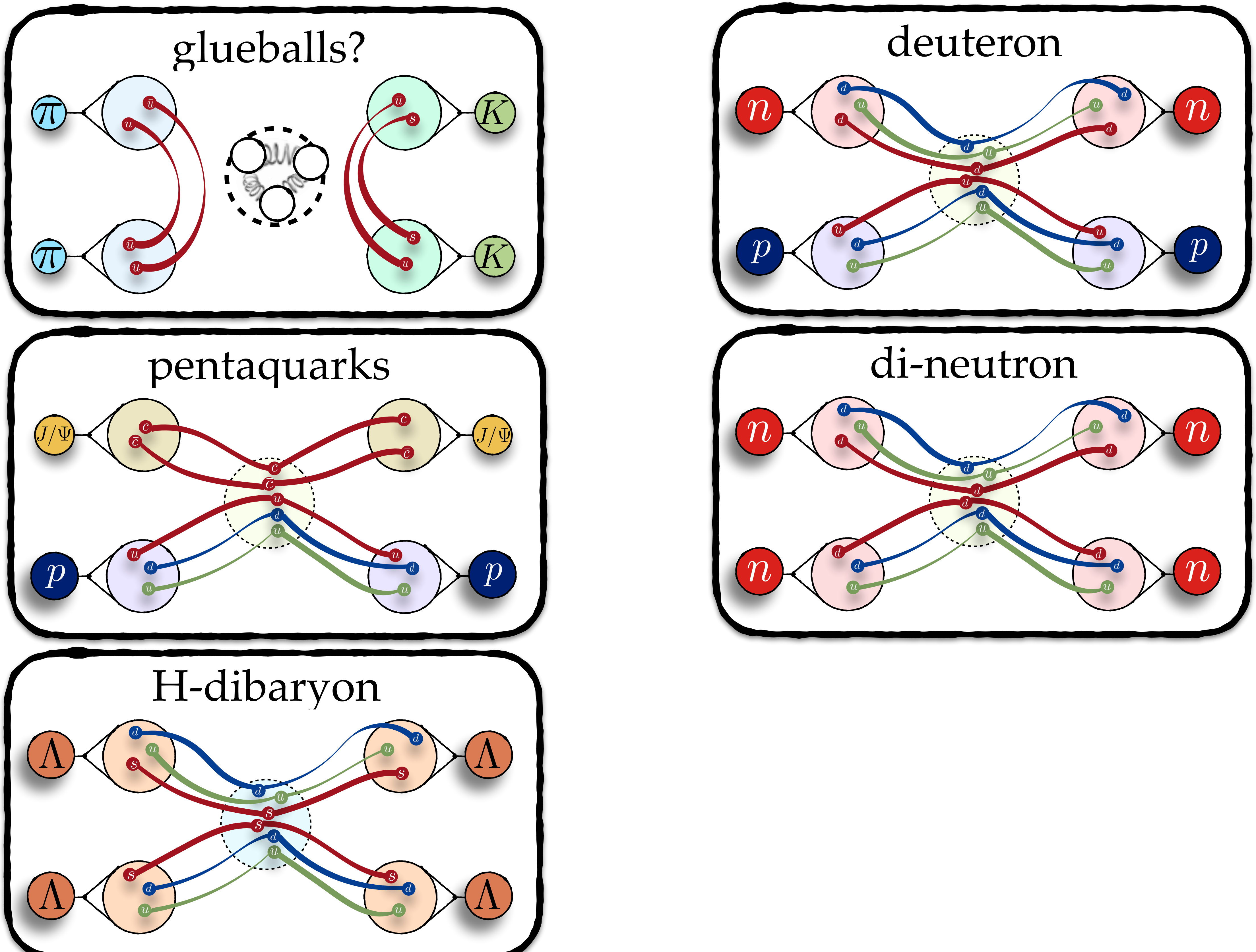

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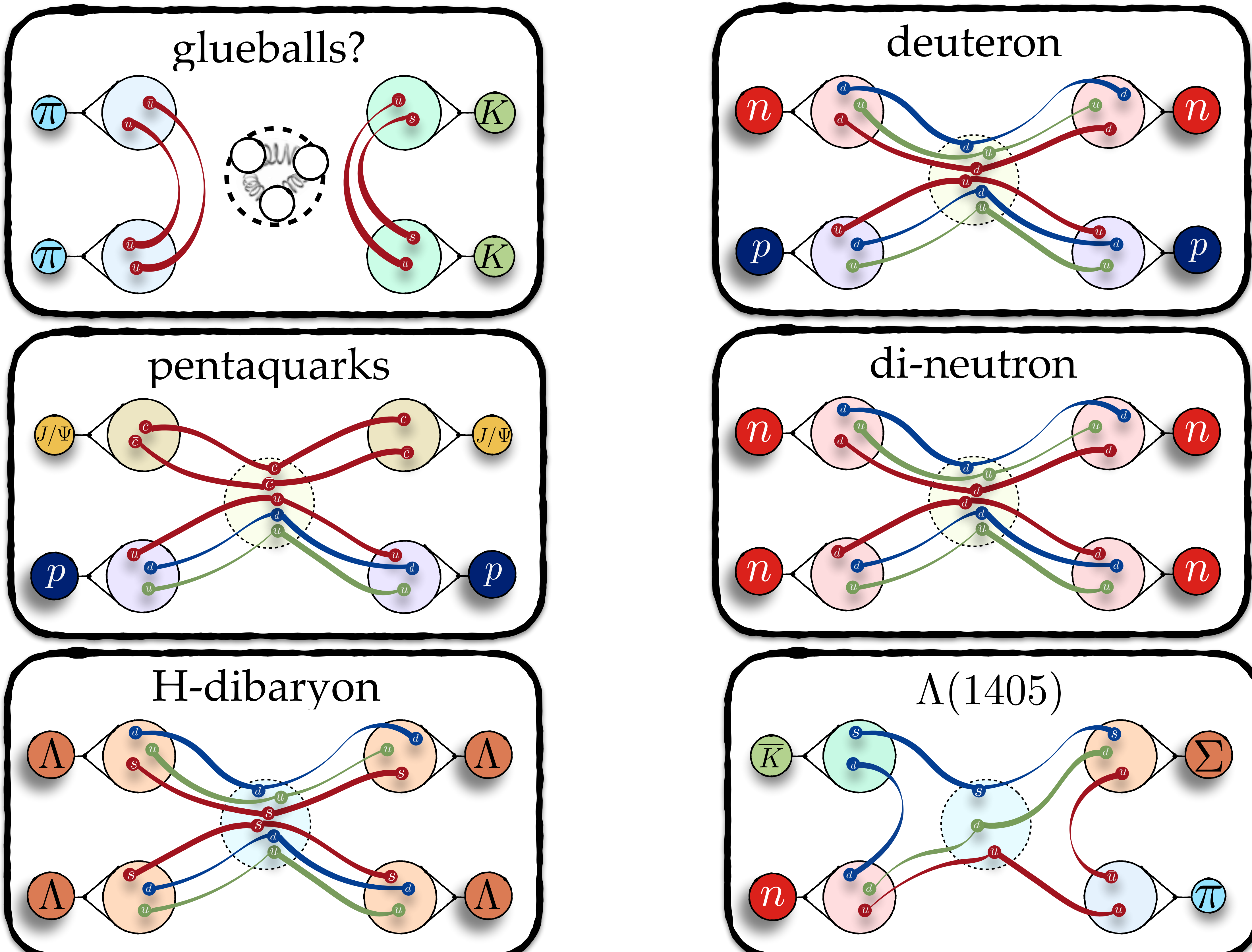
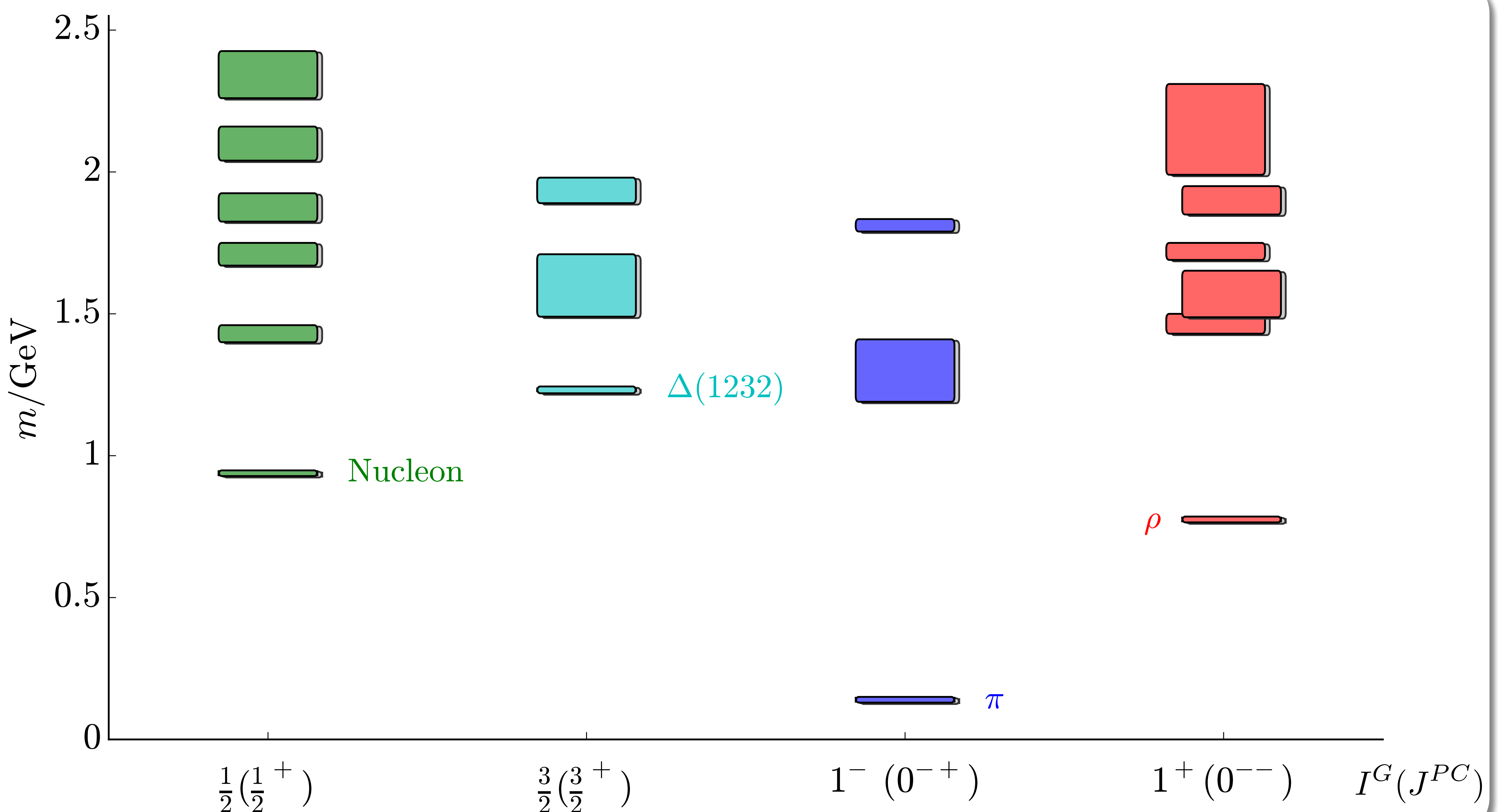
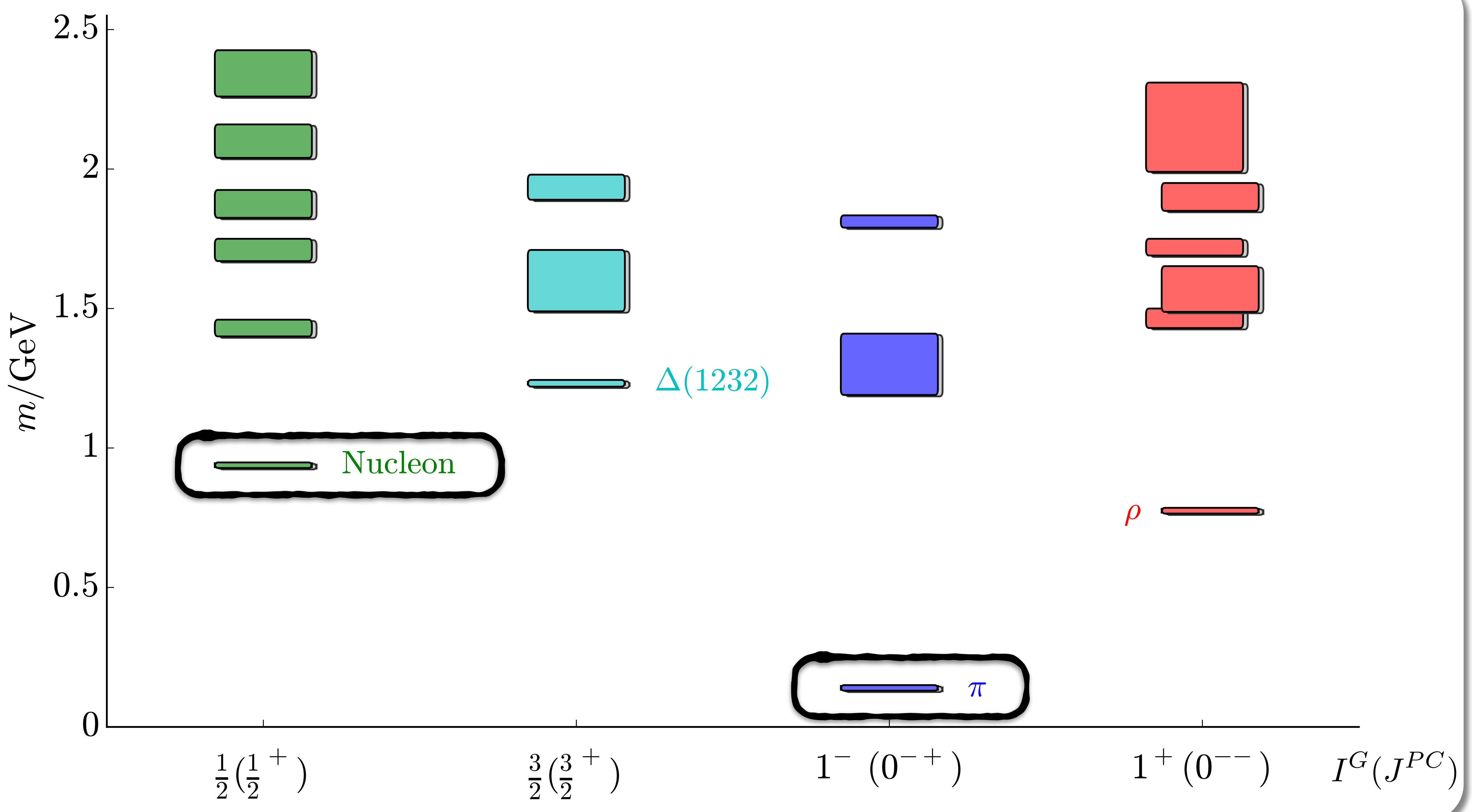


Why are resonances important?



Why are resonances important?

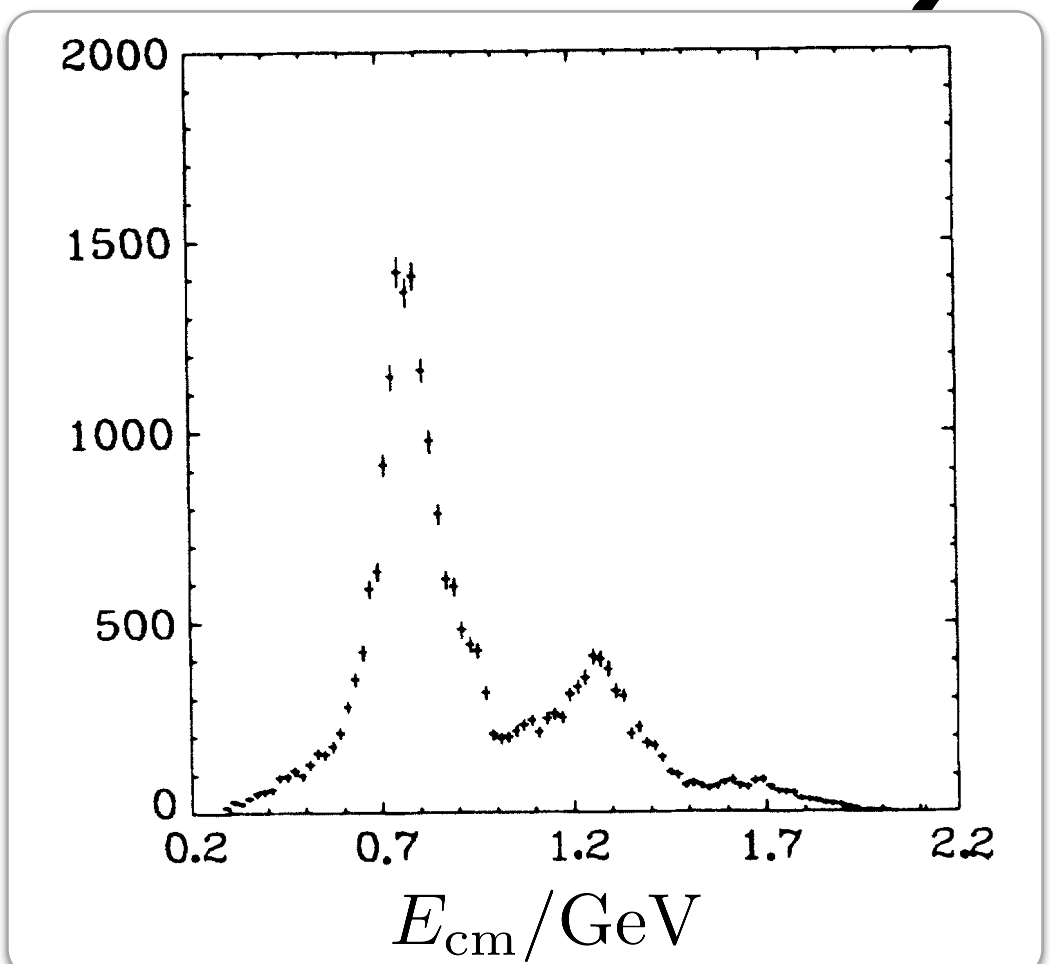


A pseudo-quantitative definition

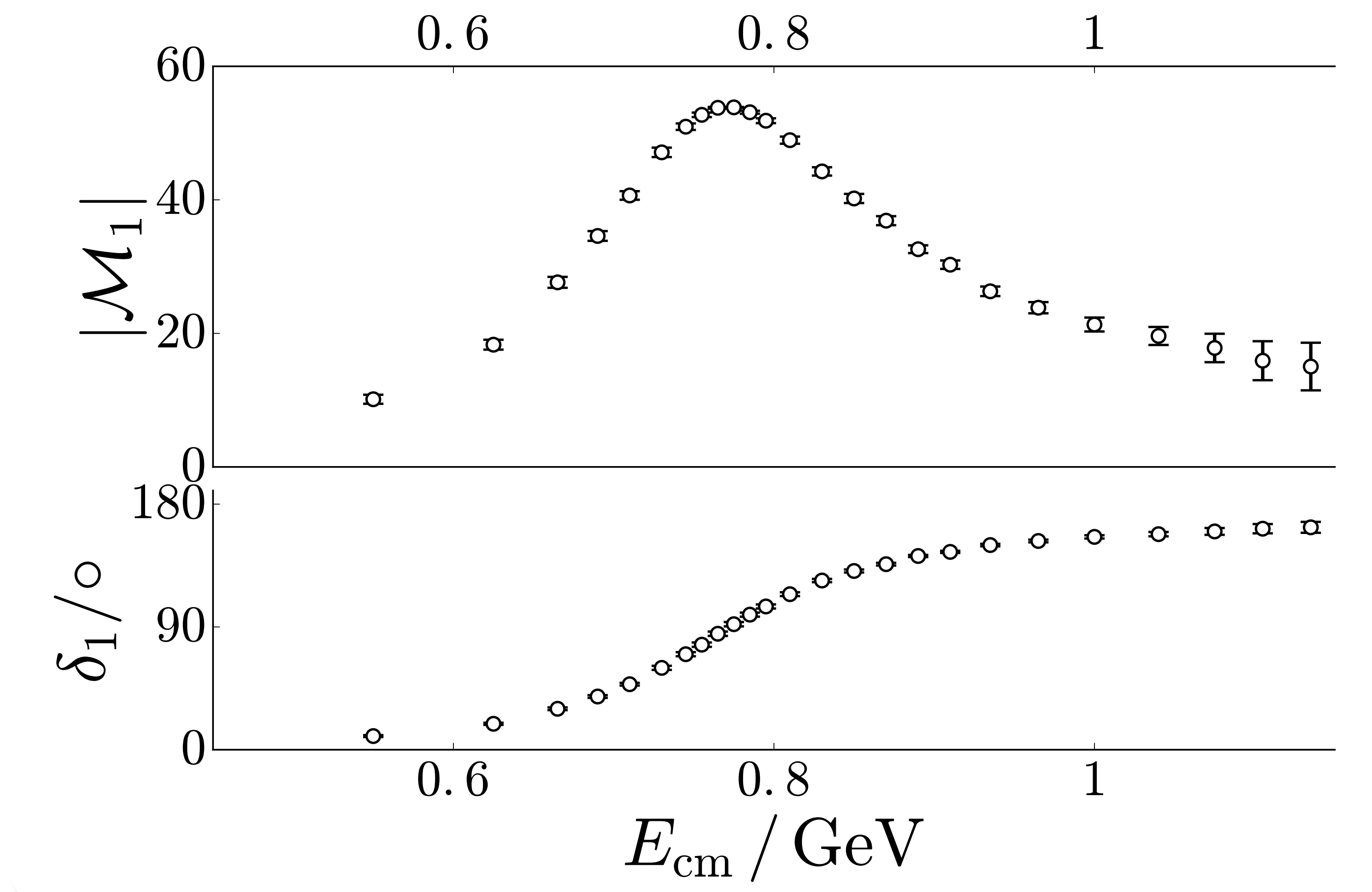
(bump in an amplitude - e.g., $\pi\pi$ scattering in ϱ-channel)

amplitude analysis

$$\mathcal{M}_1 = \frac{8\pi E_{\rm cm}}{p} \frac{1}{\cot \delta_1 - i}$$



scattering data

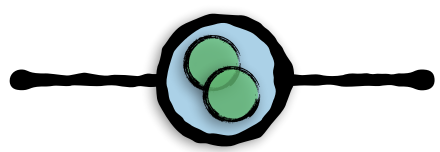


partial wave amplitudes

Protopopescu et al. (1972)

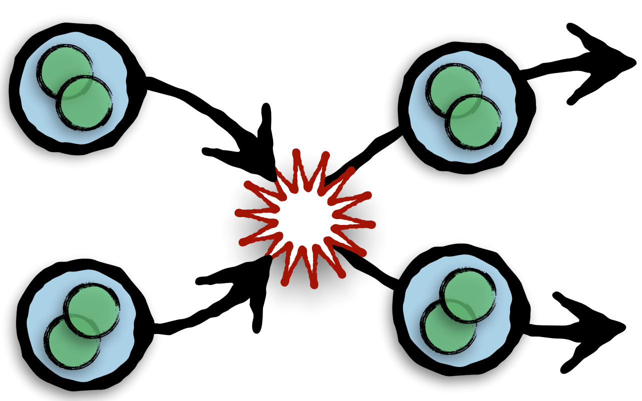
A quantitative definition

(poles in the complex plane)



$\sim \dfrac{i}{p^2 - m^2}$ [1-body propagator]

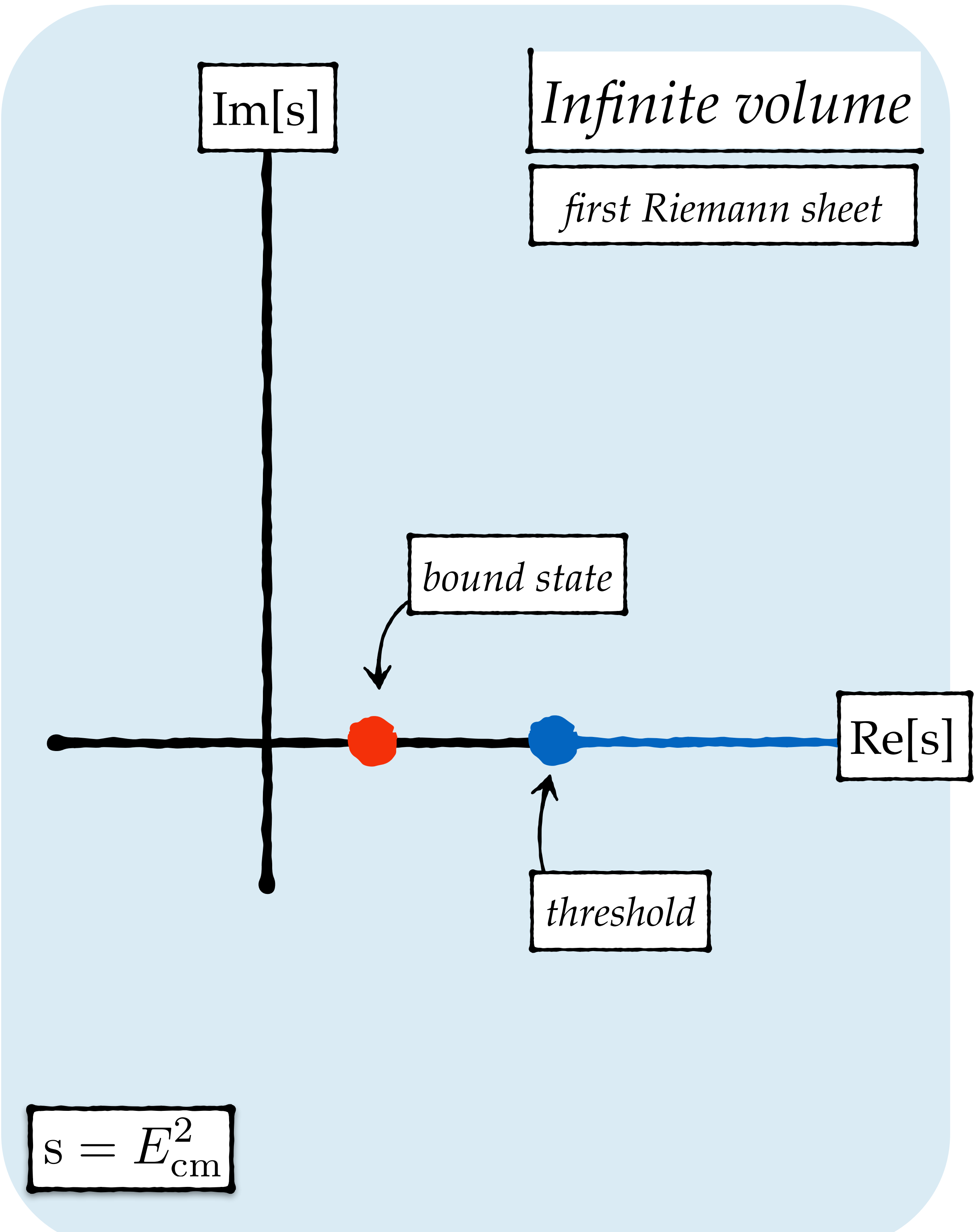
"poles correspond to particles"



$\sim i\mathcal{M}$ [scattering amplitudes]

"poles correspond to either bound states, virtual bound states or resonances"

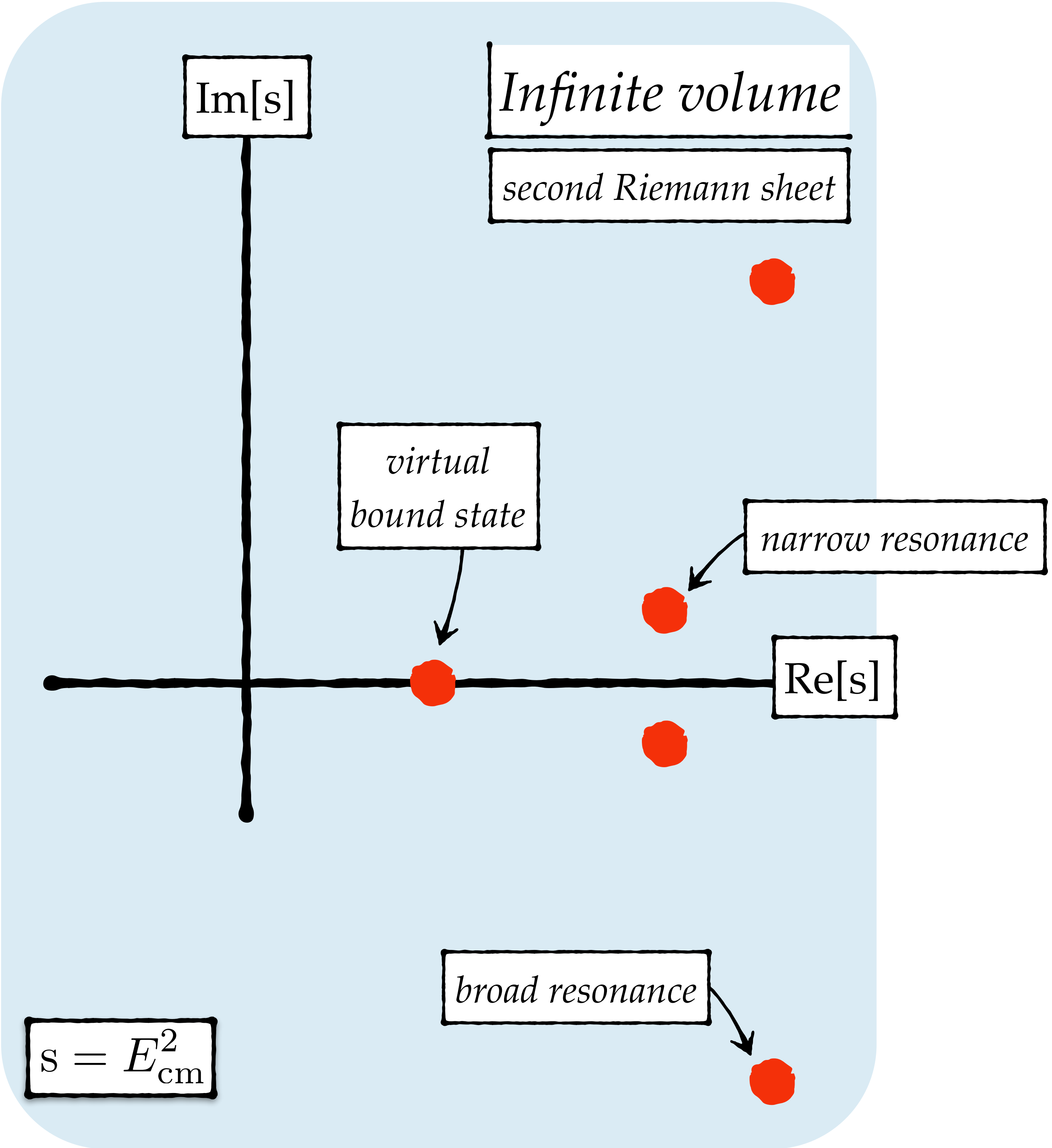
Infinite volume spectrum



Consider any QCD channel with definite quantum numbers:

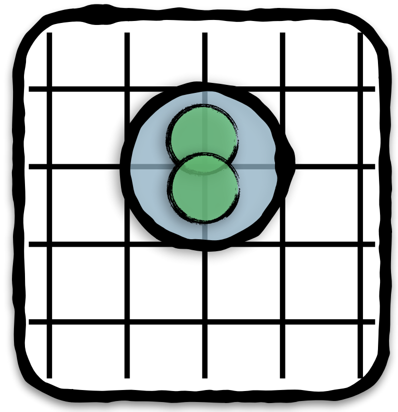
- angular momentum - J
- parity - P
- isospin - I
- ...

Infinite volume spectrum



Lattice QCD

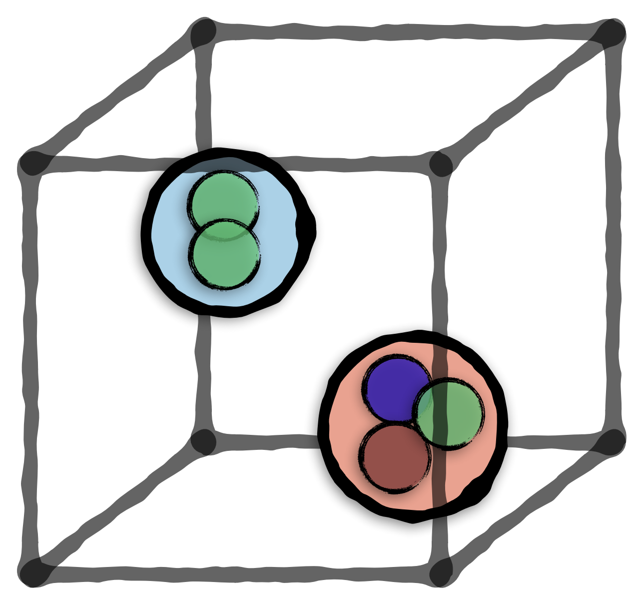
- Lattice spacing:



$\} \, a \sim 0.1 \text{ fm}$

- Wick rotation [Euclidean spacetime]: $t_M \rightarrow -it_E$

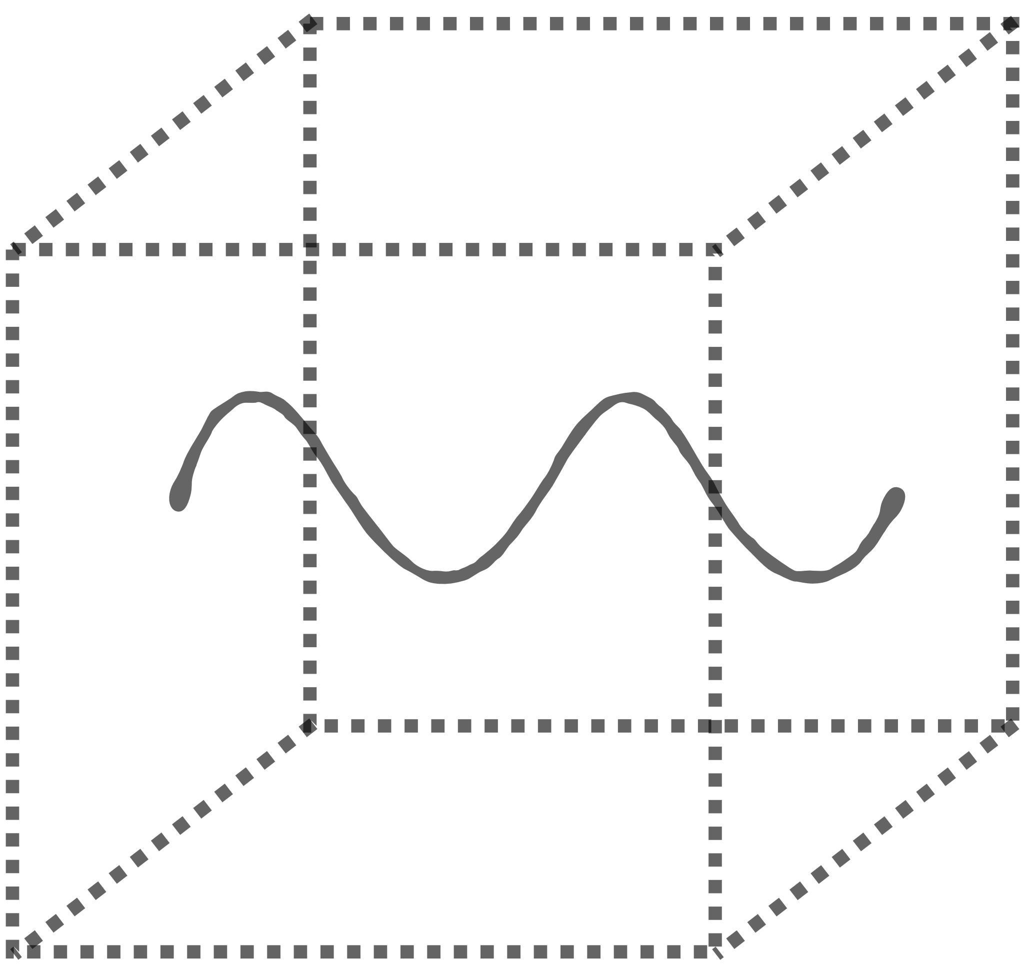
- Finite volume:



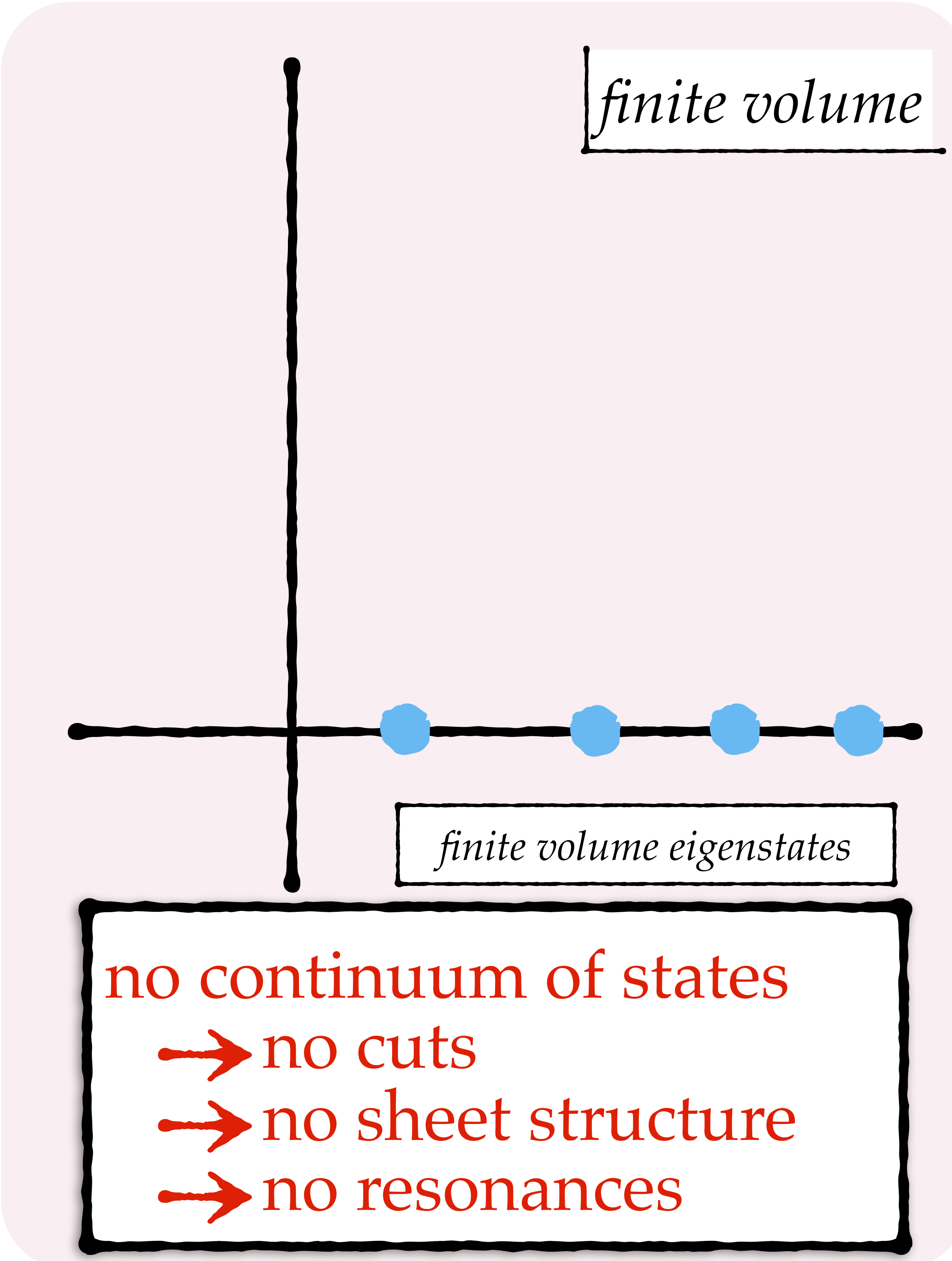
- Quark masses: $m_q \rightarrow m_q^{\text{phys.}}$

Have we '*mangled*' QCD too much?

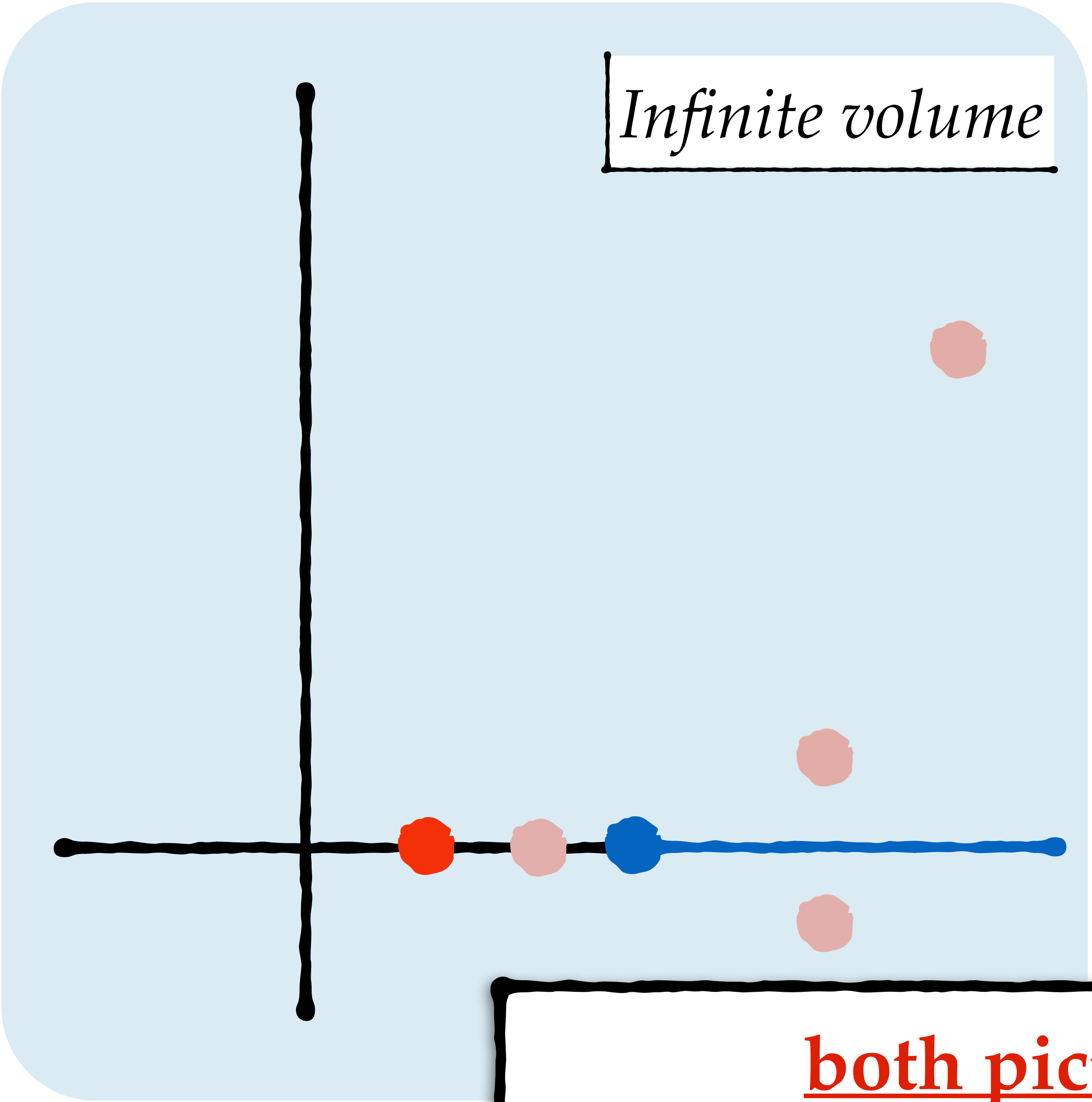
Finite vs. infinite volume spectrum

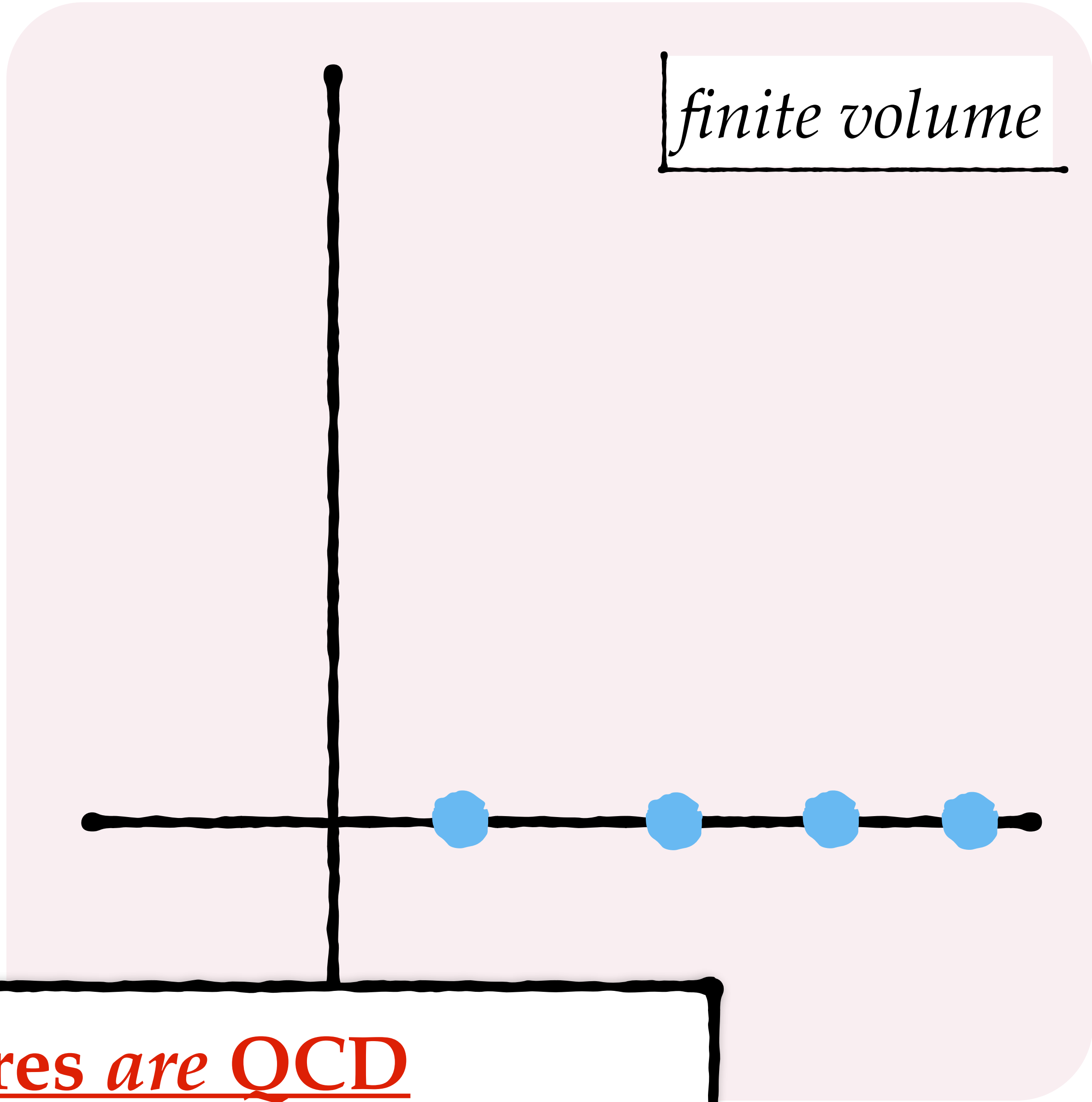


"only a finite number of modes can exist in a finite volume"



Finite vs. infinite volume spectrum

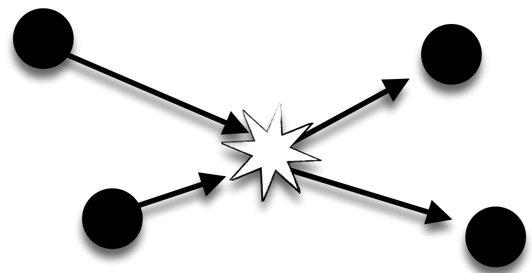




both pictures *are* QCD

the connection is perhaps not obvious since we have historically been "confined" to thinking about infinite volume physics

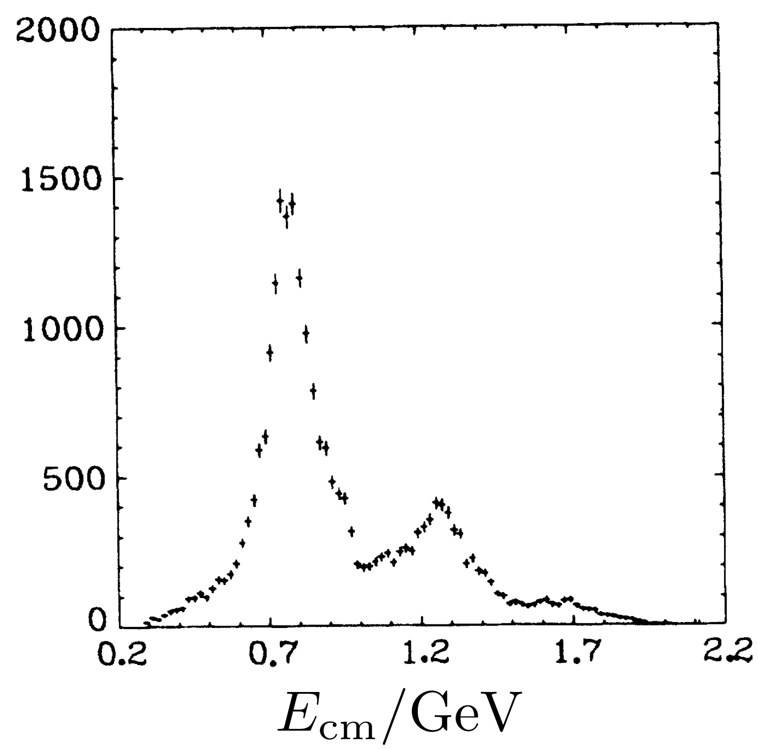
Experiment

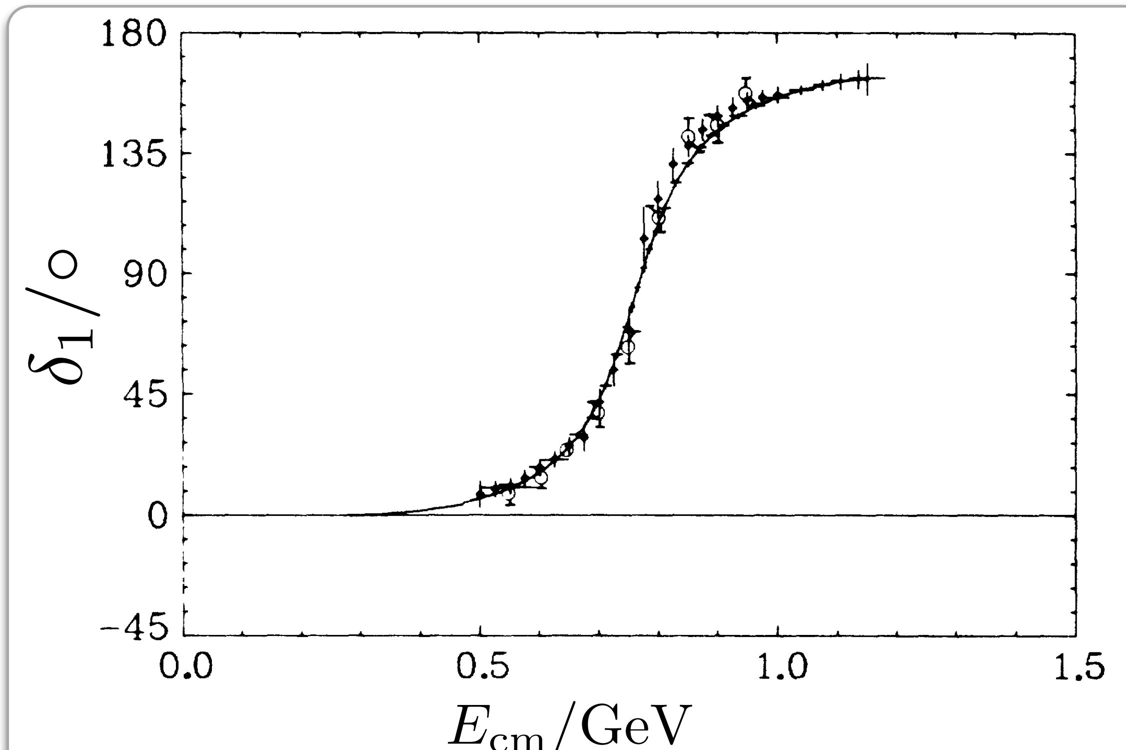


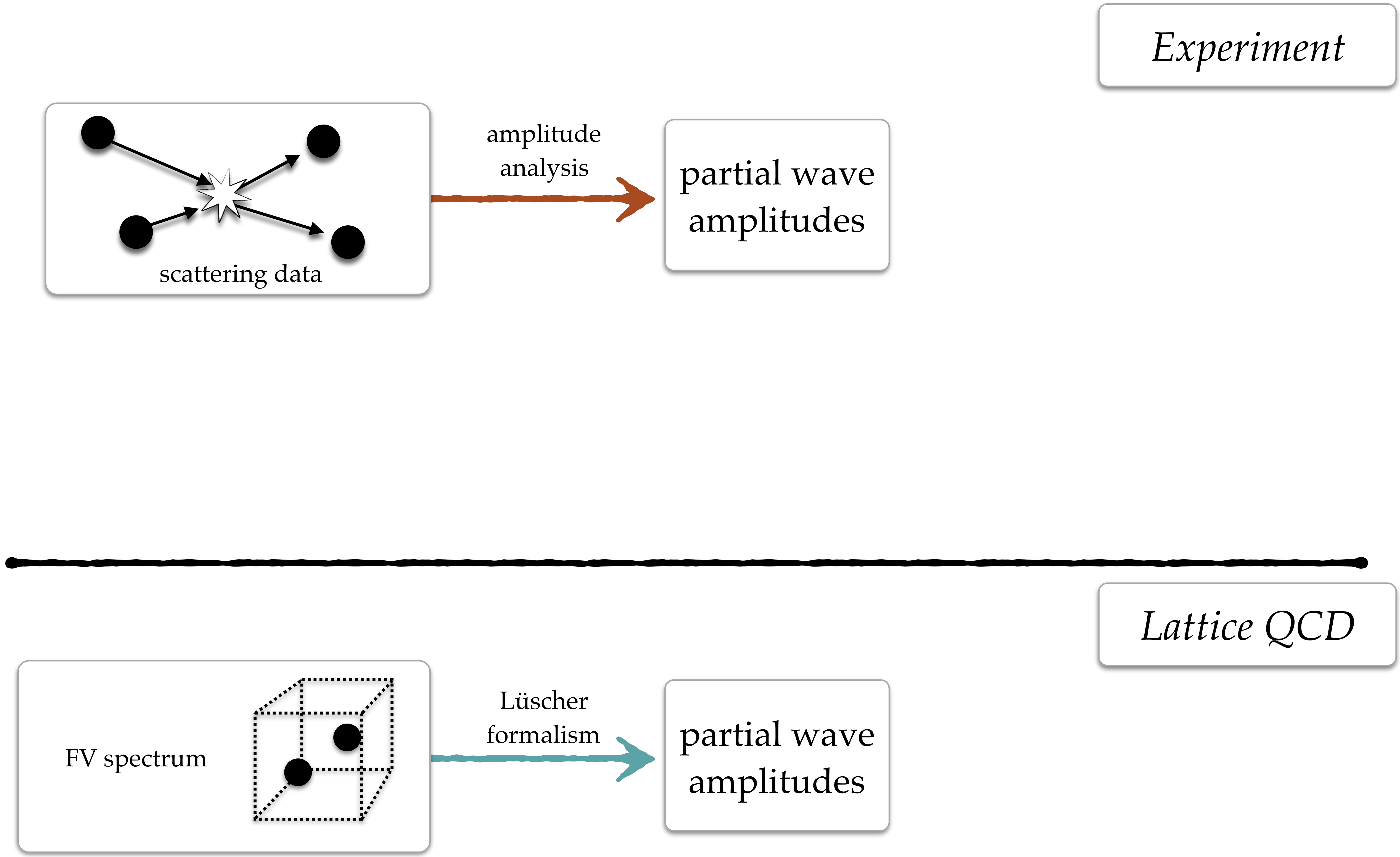
scattering data

amplitude analysis

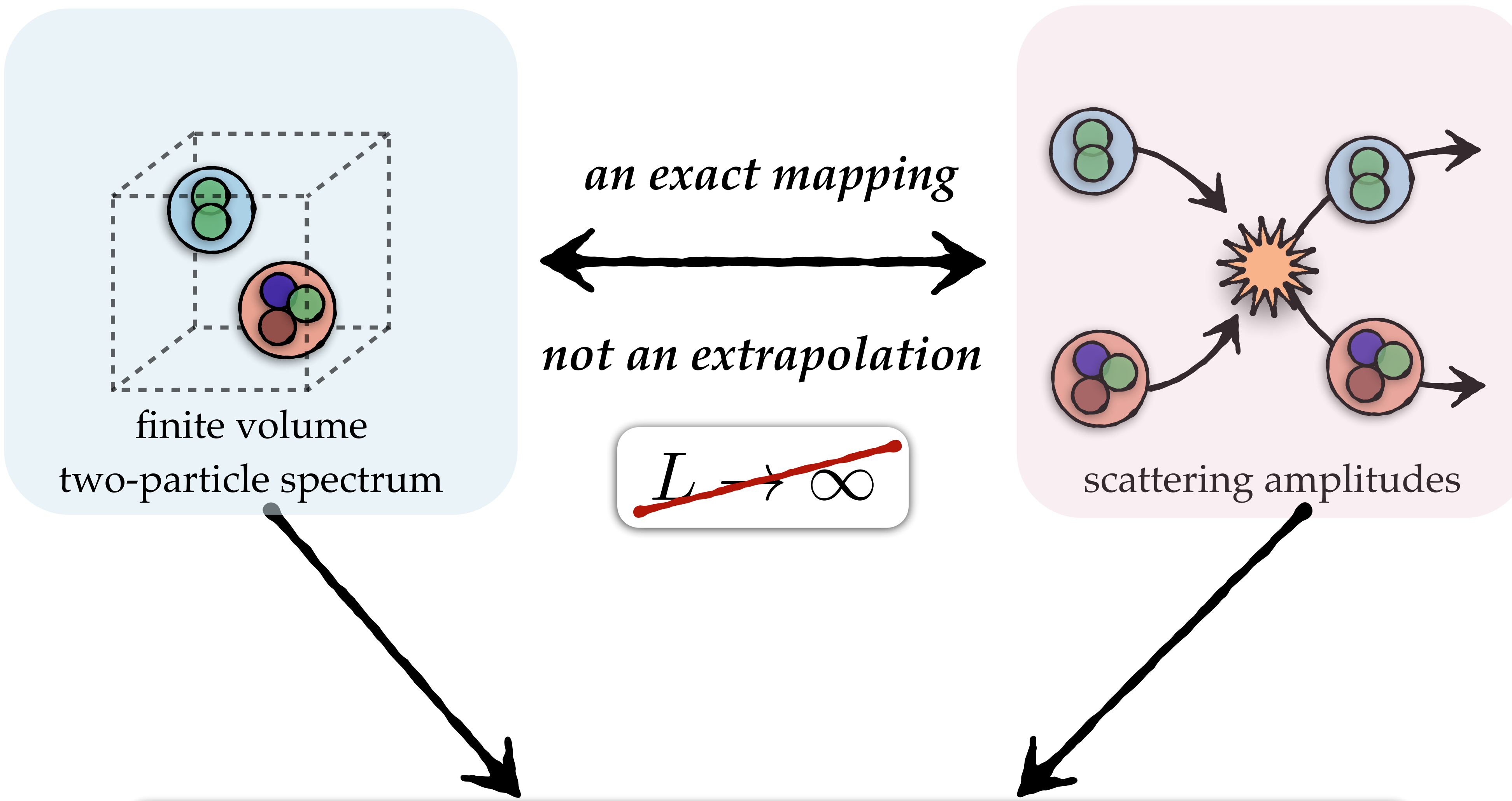
partial wave amplitudes






Experiment
scattering data
amplitude analysis
partial wave amplitudes
Lattice QCD
FV spectrum
Lüscher formalism
partial wave amplitudes

Lüscher formalism



poles satisfy: $\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$

Lüscher formalism

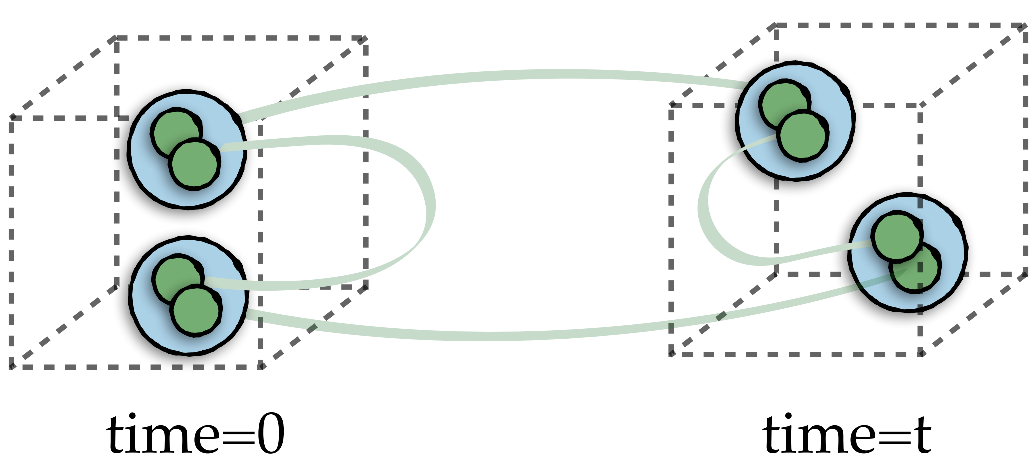
- Lüscher (1986, 1991) [elastic scalar bosons]
- Rummukainen & Gottlieb (1995) [moving elastic scalar bosons]
- Kim, Sachrajda, & **Sharpe** / Christ, Kim & Yamazaki (2005) [QFT derivation]
- Bernard, Lage, Meissner & **Rusetsky** (2008) [Nπ systems]
- RB, **Davoudi**, Luu & **Savage** (2013) [generic spinning systems]
- Feng, Li, & Liu (2004) [inelastic scalar bosons]
- **Hansen** & **Sharpe** / RB & **Davoudi** (2012) [moving inelastic scalar bosons]
- RB (2014) [moving inelastic spinning particles]

poles satisfy: $\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$

Extracting the spectrum

Two-point correlation functions:

$$C_{ab}^{2pt.}(t,\mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle$$



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Two-point correlation functions:

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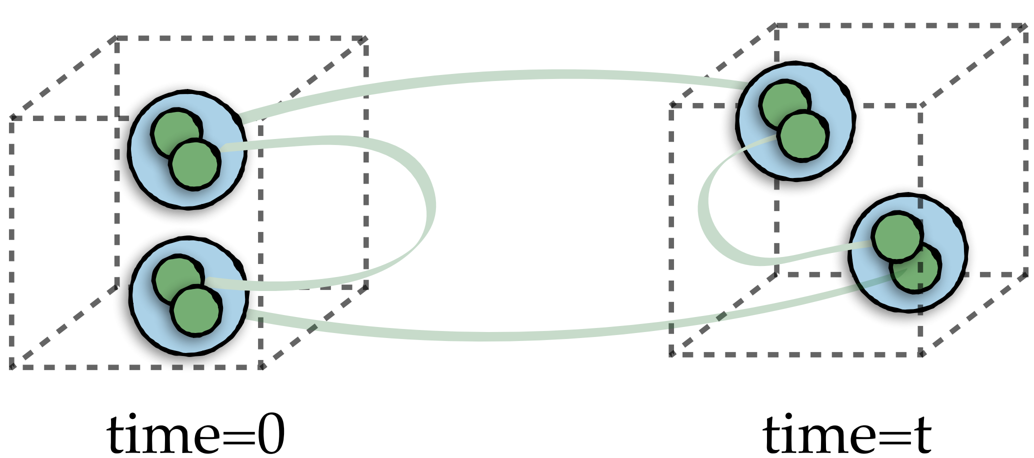
$$= \sum_n \langle 0|\mathcal{O}_b(t,\mathbf{P})|n,L\rangle\langle n,L|\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle$$

insert complete set of states

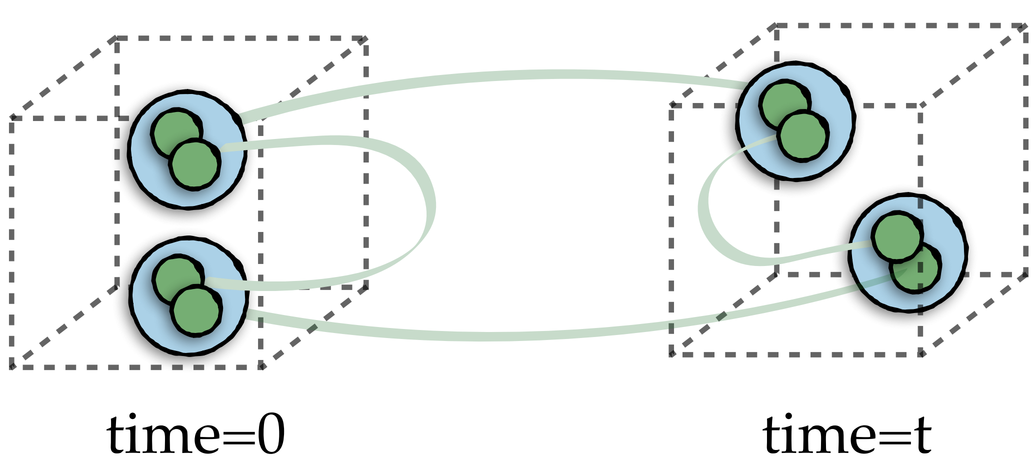


Extracting the spectrum

Two-point correlation functions:

$$
\begin{aligned}
C_{ab}^{2pt.}(t,\mathbf{P}) &\equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle \\
&= \sum_n \langle 0|\mathcal{O}_b(t,\mathbf{P})|n,L\rangle\langle n,L|\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle \\
&= \sum_n \langle 0|e^{t\hat{H}_{QCD}}\mathcal{O}_b(0,\mathbf{P})e^{-t\hat{H}_{QCD}}|n,L\rangle\langle n,L|\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle
\end{aligned}
$$

remember Heisenberg operators? in Euclidean spacetime?



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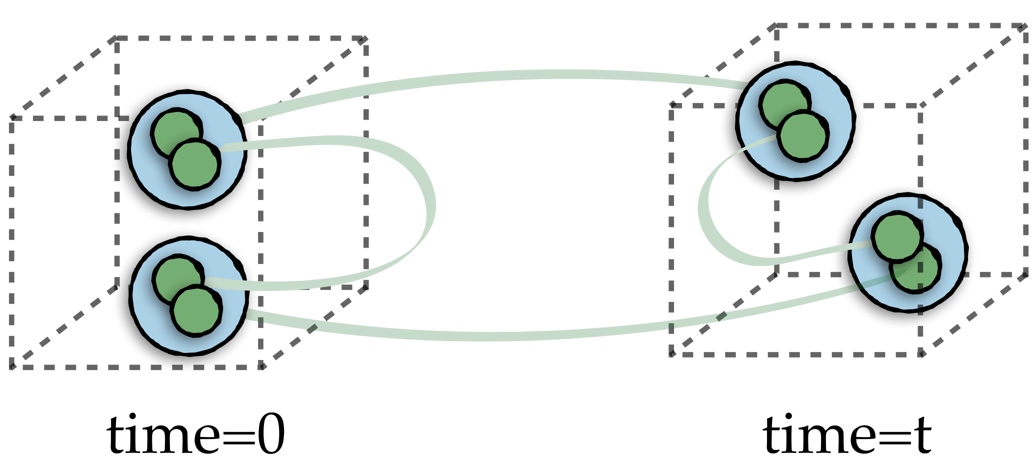
&= \sum_n \langle 0|e^{t\hat{H}_{QCD}}\mathcal{O}_b(0,\mathbf{P})e^{-t\hat{H}_{QCD}}|n,L\rangle\langle n,L|\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle \\
&= \sum_n Z_{b,n}Z_{a,n}^\dagger e^{-E_n t}
\end{aligned}
$$

spectrum

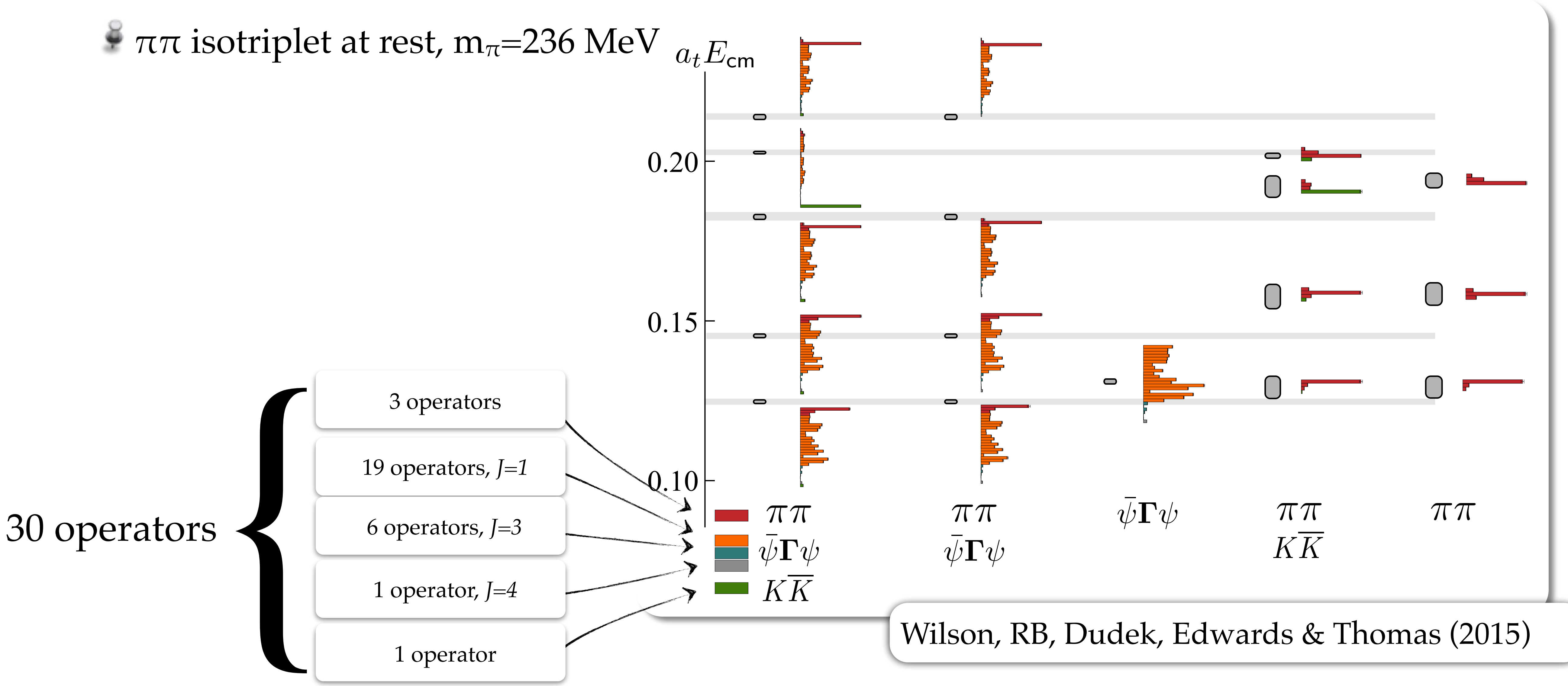


Extracting the spectrum

Two-point correlation functions:

$$C_{ab}^{2pt.}(t,\mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle = \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t}$$

- Use a large basis of operators with the same quantum numbers
- 'Diagonalize' correlation function
- $\pi\pi$ isotriplet at rest, m_π=236 MeV



Wilson, RB, Dudek, Edwards & Thomas (2015)

Extracting the spectrum

Two-point correlation functions:

$$C_{ab}^{2pt.}(t, \mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t, \mathbf{P})\mathcal{O}_a^\dagger(0, -\mathbf{P})|0\rangle = \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t}$$

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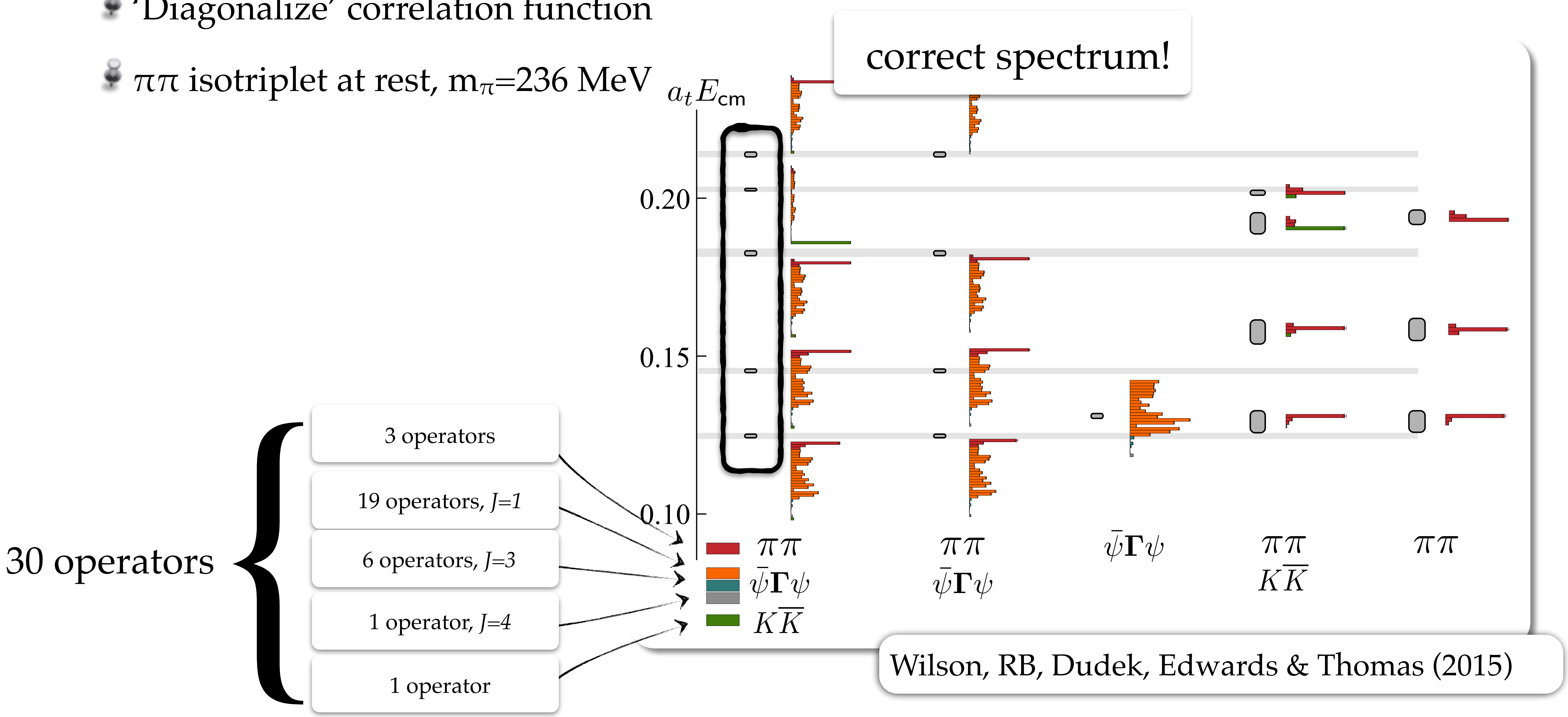
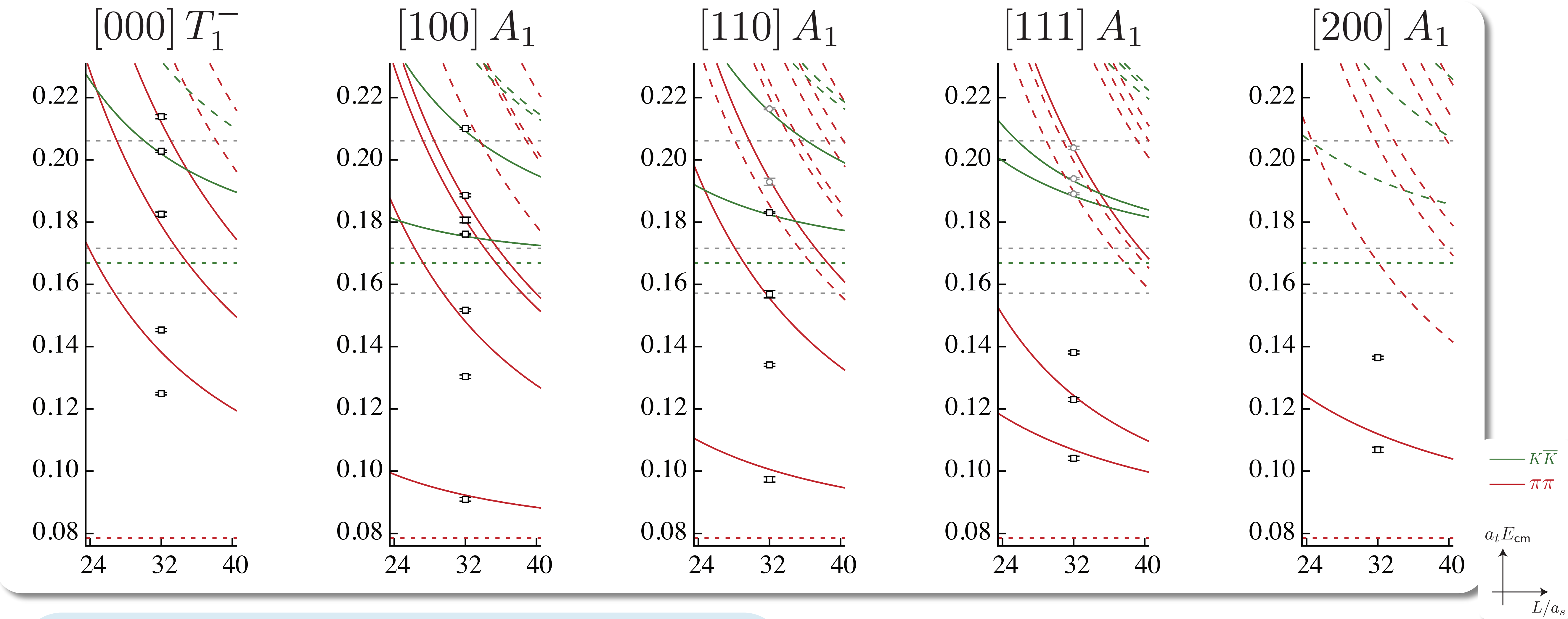


Wilson, RB, Dudek, Edwards & Thomas (2015)

$\pi\pi$ scattering

(I=1 channel)

A subset of the spectrum:



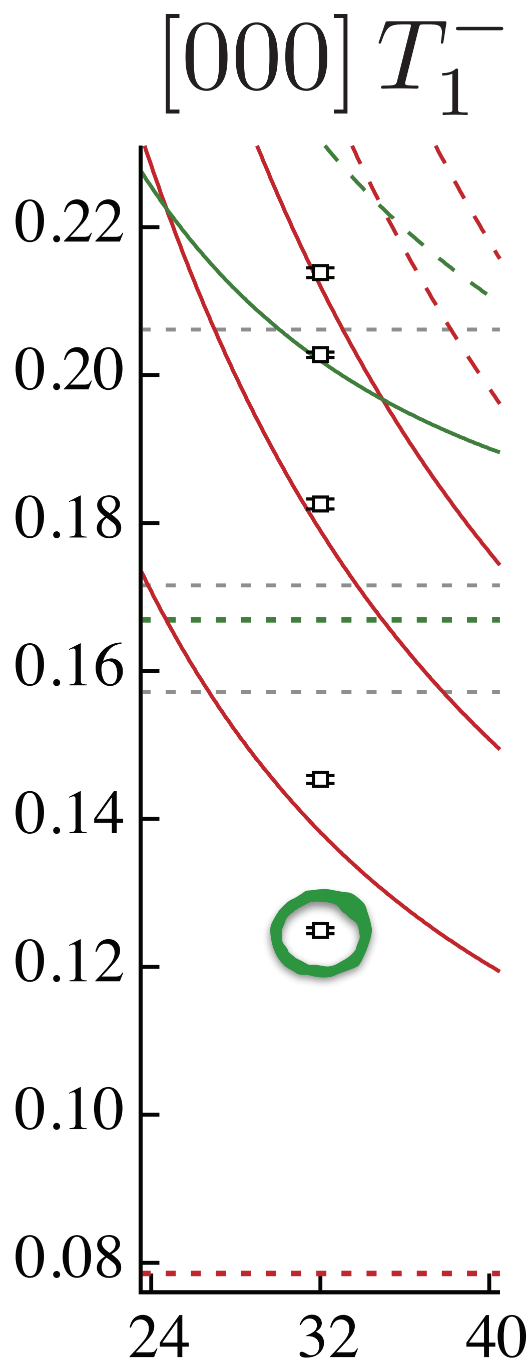
Wilson Thomas Dudek Edwards

Hadron Spectrum Collaboration

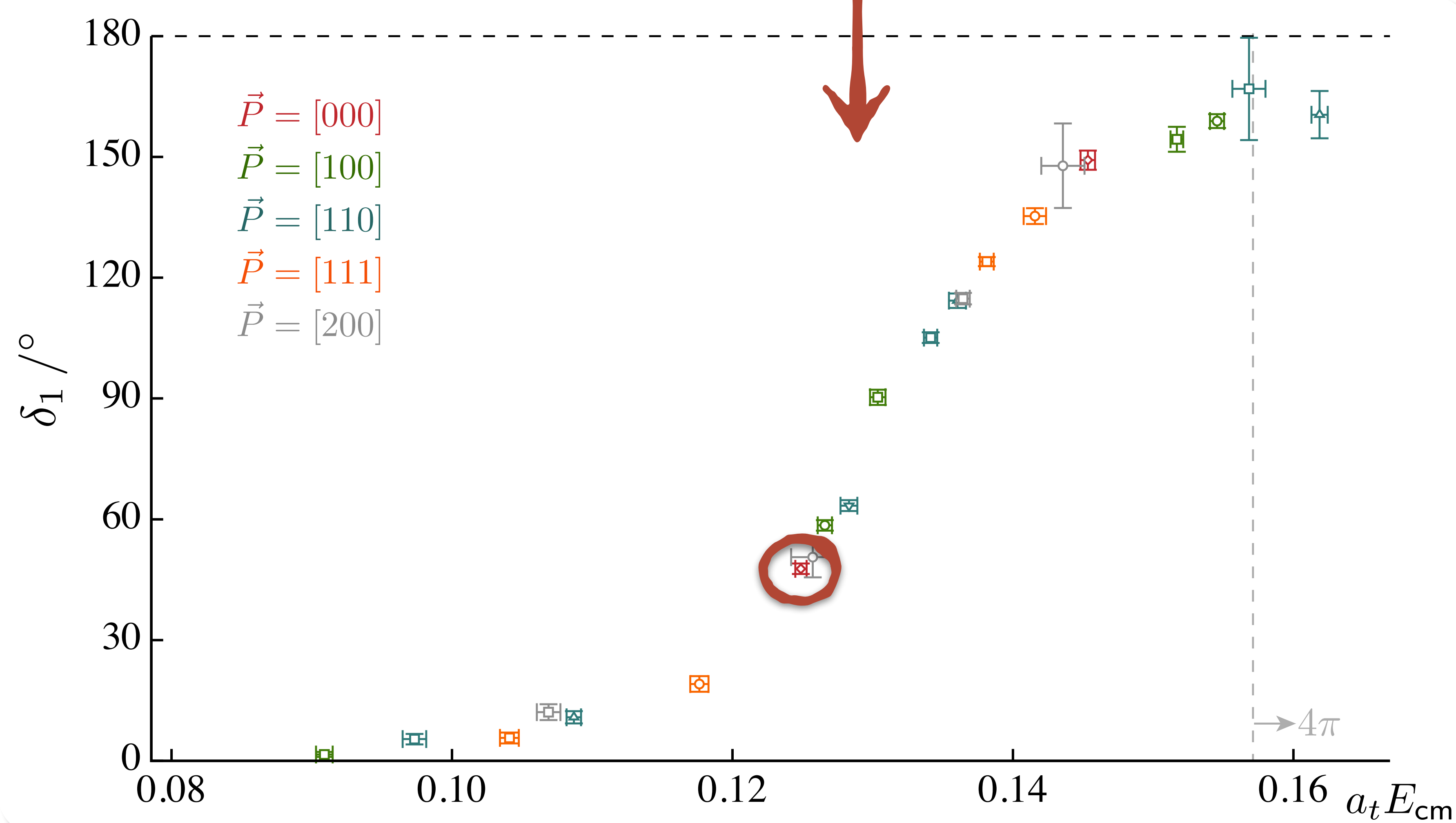
Wilson, RB, Dudek, Edwards & Thomas (2015)

$\pi\pi$ scattering

(I=1 channel)



$$\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$$

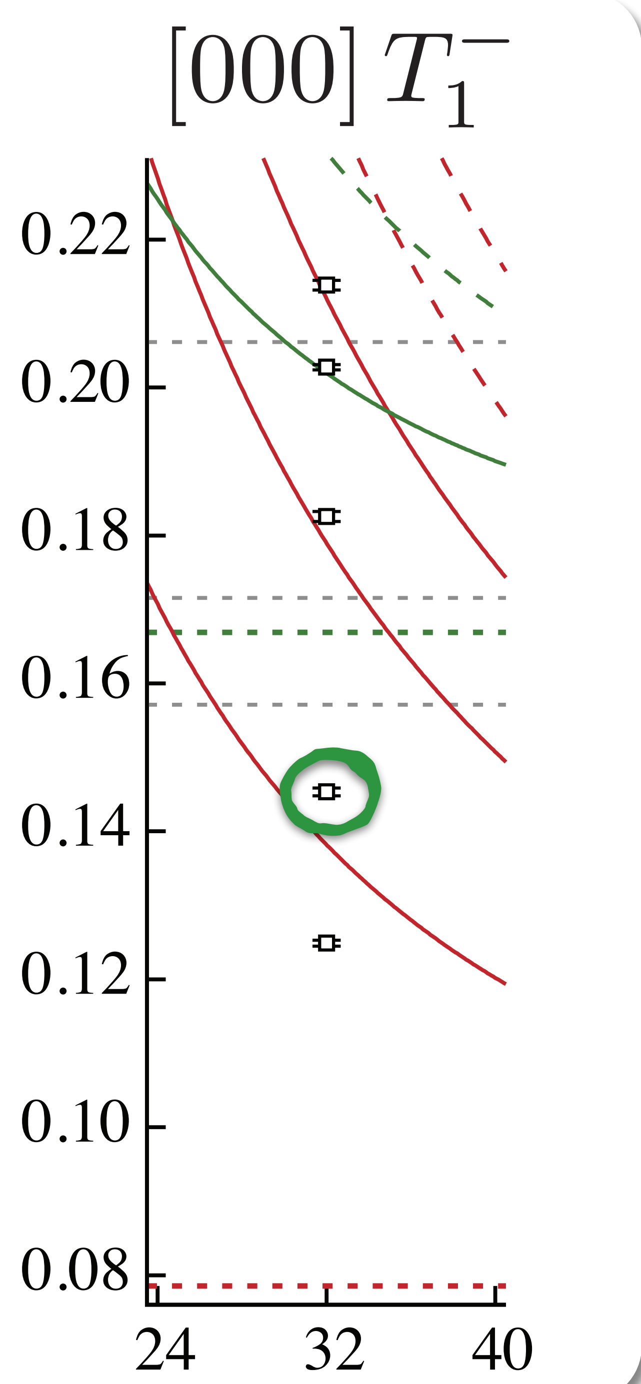


$$\mathcal{M} \propto \frac{1}{\cot \delta_1 - i}$$

Wilson, RB, Dudek, Edwards & Thomas (2015)

$\pi\pi$ scattering

(I=1 channel)



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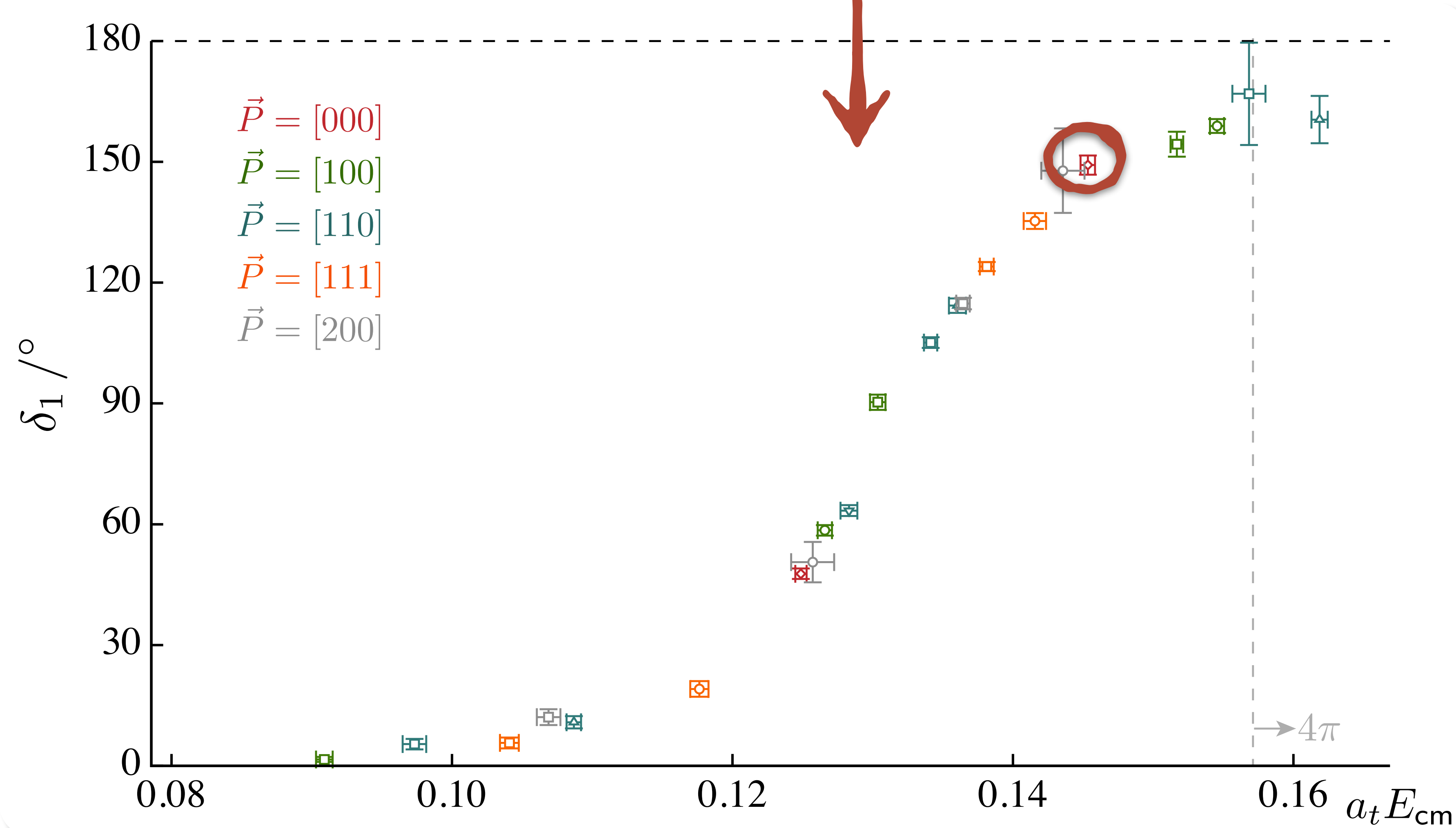

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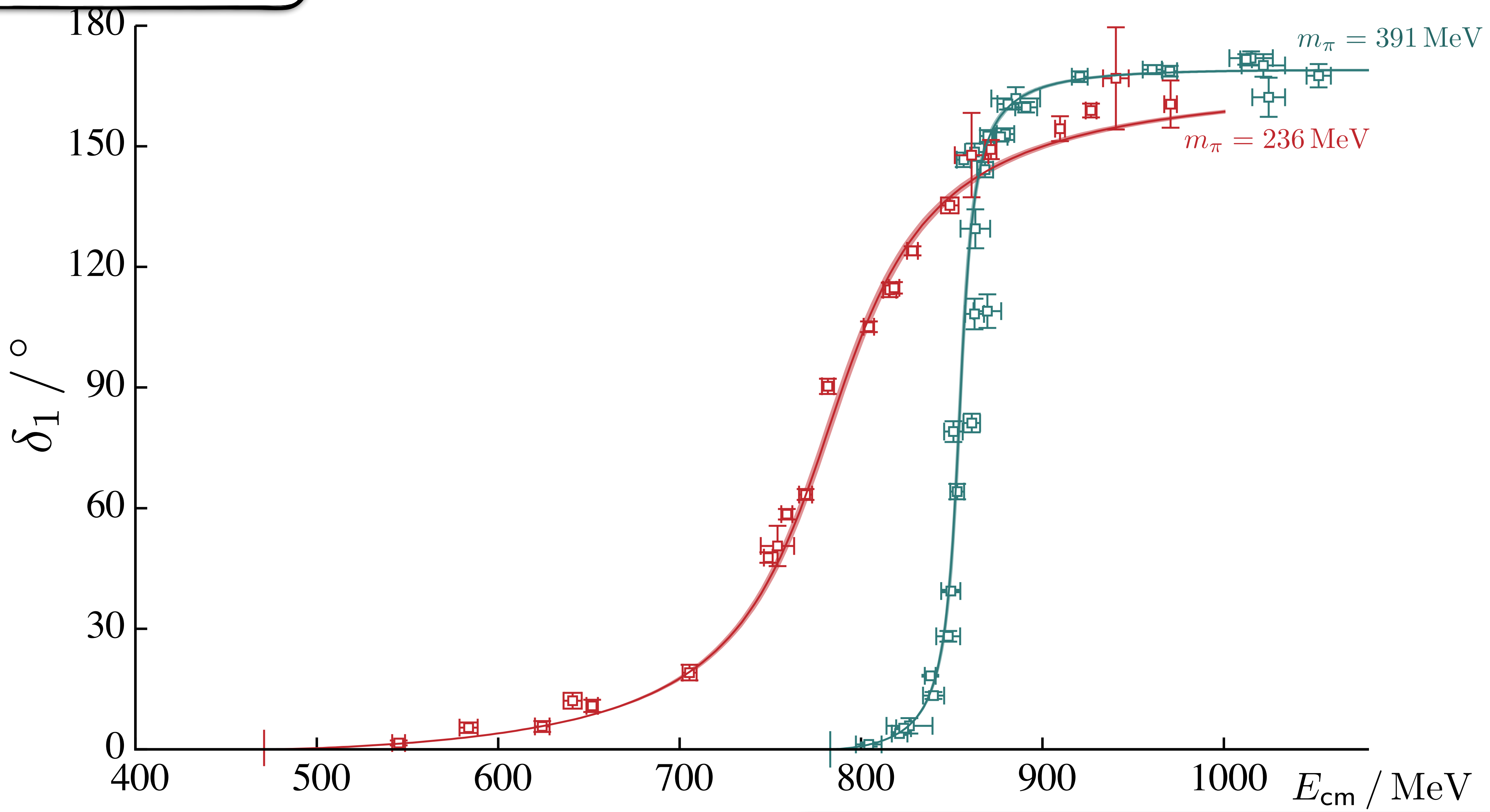
HadSpec Collaboration



Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)

Inelastic scattering

$$\det\left[\begin{pmatrix} F_{\pi\pi} & \\ & F_{K\overline{K}} \end{pmatrix}^{-1} + \begin{pmatrix} \mathcal{M}_{\pi\pi,\pi\pi} & \mathcal{M}_{\pi\pi,K\overline{K}} \\ \mathcal{M}_{\pi\pi,K\overline{K}} & \mathcal{M}_{K\overline{K},K\overline{K}} \end{pmatrix}\right] = 0$$

Hansen & Sharpe / RB & Davoudi (2012)

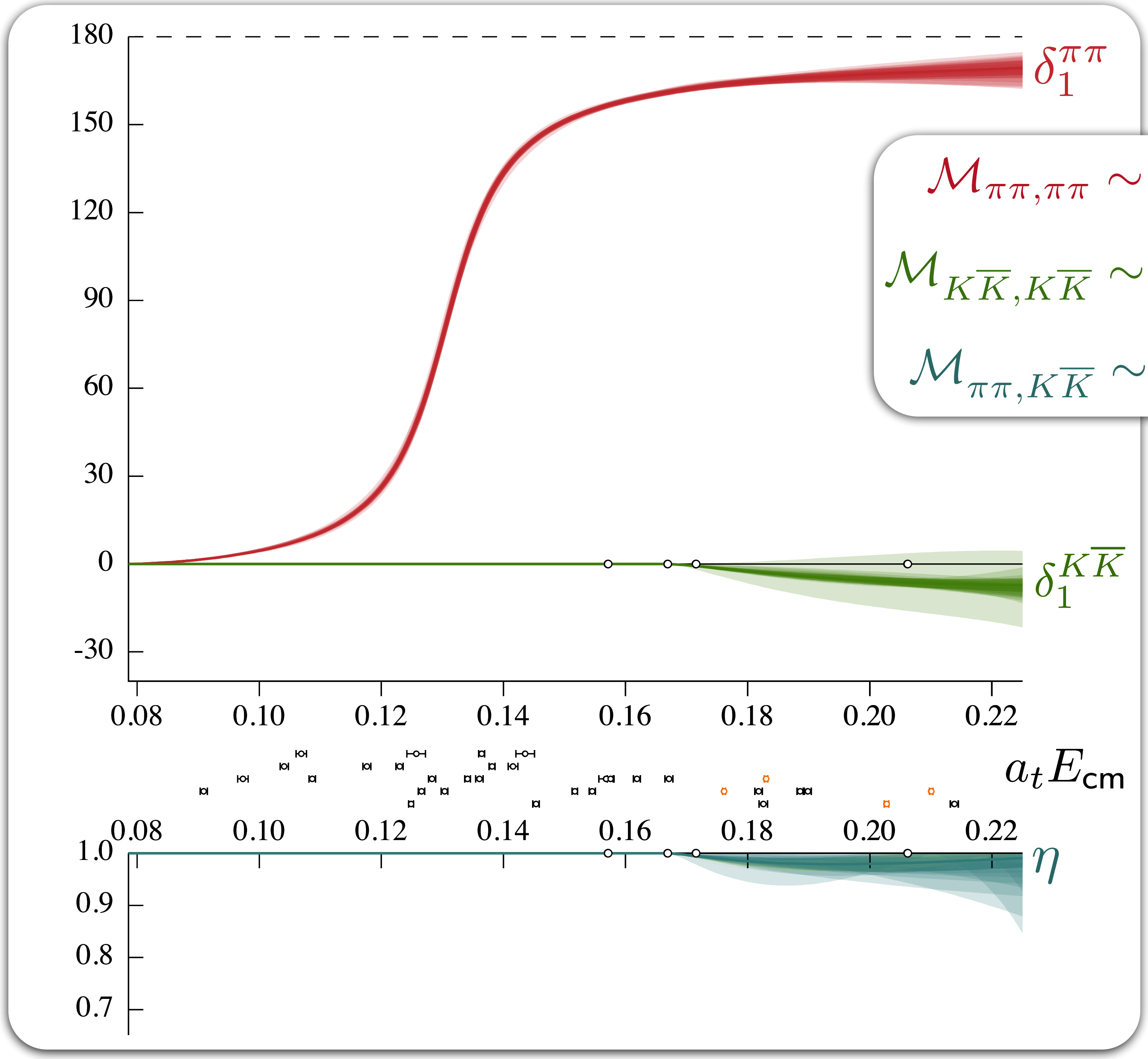
Above inelastic threshold, spectrum depends on three functions:

- two phase shifts and one inelasticity / mixing angle
- no longer one-to-one mapping

Pragmatic solution:

- parametrize scattering amplitude
- fit energy-independent parameters
- test parametrization-dependence of results

Inelastic scattering

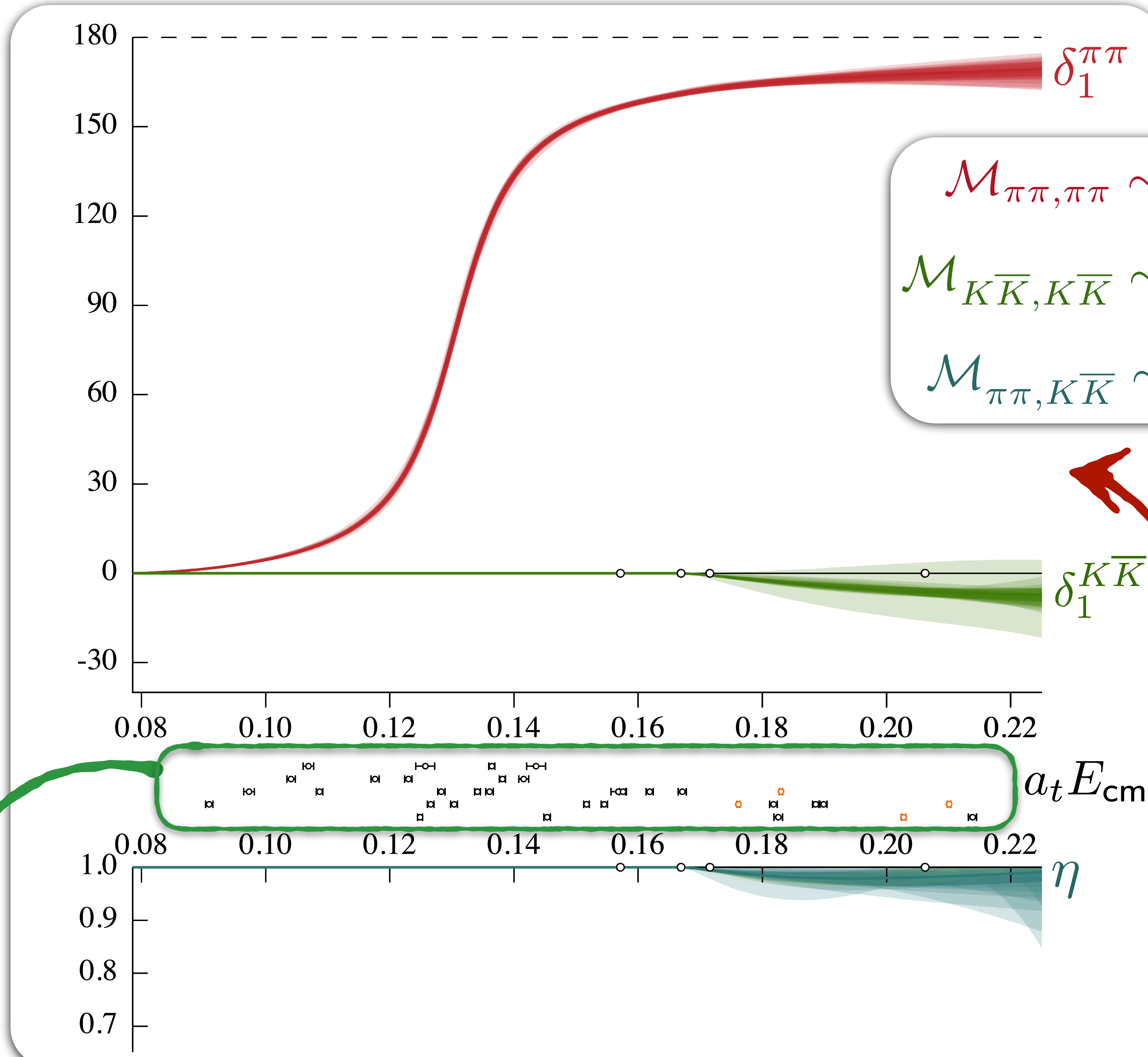


$$\mathcal{M}_{\pi\pi,\pi\pi} \sim \eta\, e^{2i\delta_1^{\pi\pi}} - 1$$

$$\mathcal{M}_{K\overline{K},K\overline{K}} \sim \eta\, e^{2i\delta_1^{K\overline{K}}} - 1$$

$$\mathcal{M}_{\pi\pi,K\overline{K}} \sim \sqrt{1-\eta^2}\; e^{i(\delta_1^{\pi\pi}+\delta_1^{K\overline{K}})}$$

Inelastic scattering



$$\mathcal{M}_{\pi\pi,\pi\pi} \sim \eta\, e^{2i\delta_1^{\pi\pi}} - 1$$

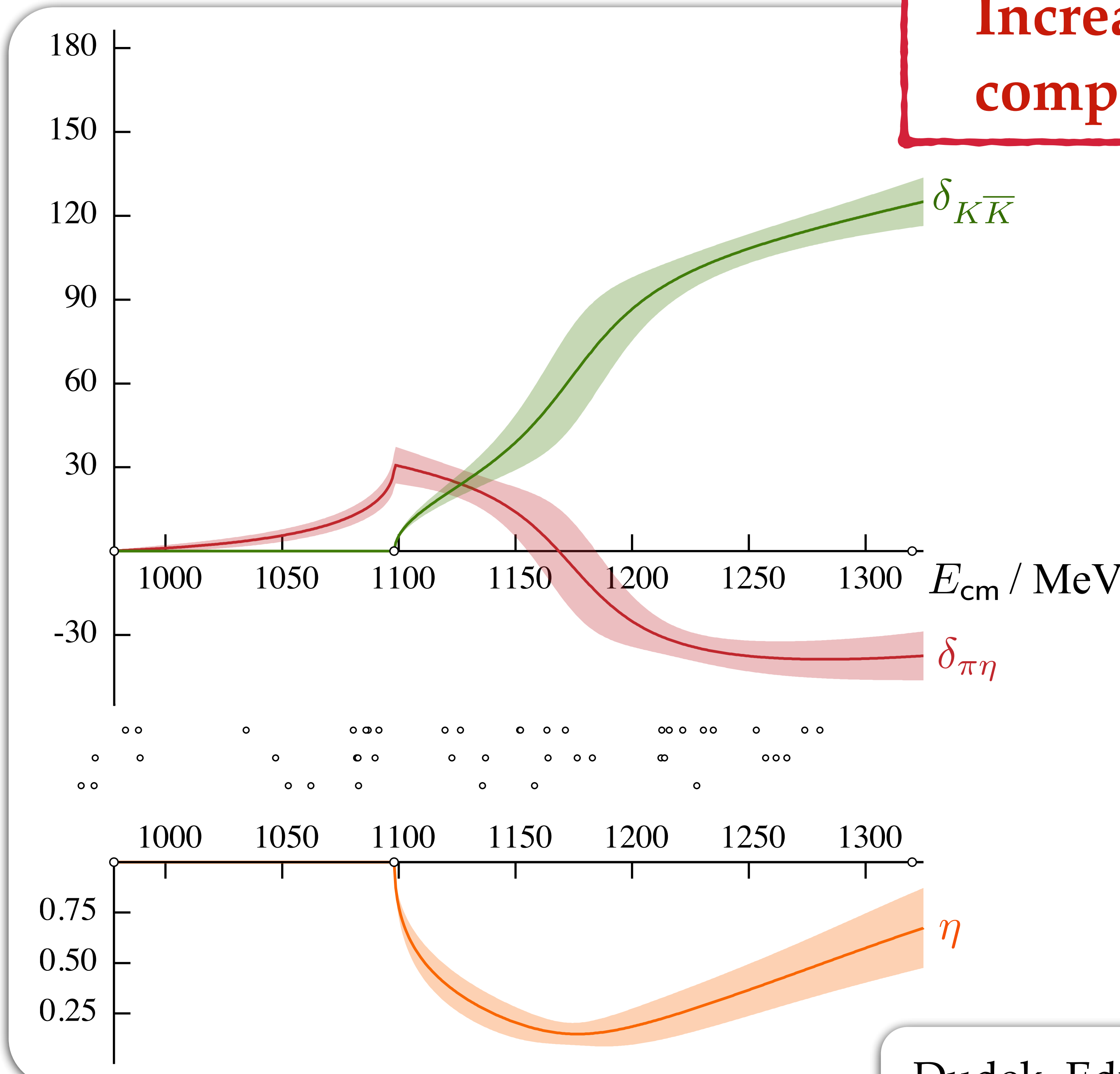
$$\mathcal{M}_{K\overline{K},K\overline{K}} \sim \eta\, e^{2i\delta_1^{K\overline{K}}} - 1$$

$$\mathcal{M}_{\pi\pi,K\overline{K}} \sim \sqrt{1-\eta^2}\, e^{i(\delta_1^{\pi\pi}+\delta_1^{K\overline{K}})}$$

$$\det\left[\begin{pmatrix} F_{\pi\pi} & \\ & F_{K\overline{K}} \end{pmatrix}^{-1} + \begin{pmatrix} \mathcal{M}_{\pi\pi,\pi\pi} & \mathcal{M}_{\pi\pi,K\overline{K}} \\ \mathcal{M}_{\pi\pi,K\overline{K}} & \mathcal{M}_{K\overline{K},K\overline{K}} \end{pmatrix}\right] = 0$$

$\pi\eta$–$K\overline{K}$ scattering

(I=1 channel)



Increasingly complex systems

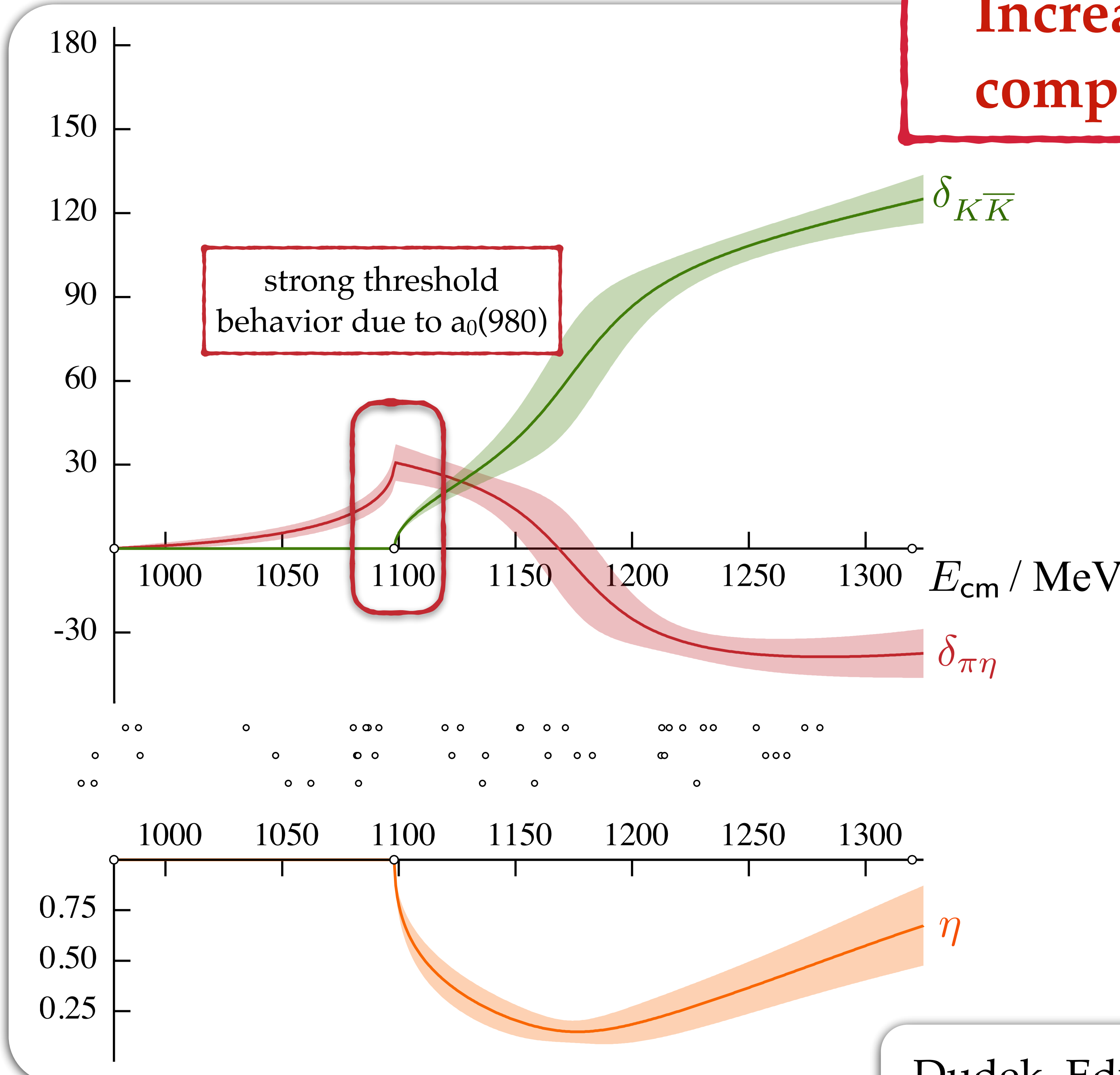
Dudek, Edwards & Wilson (2016)

RB

$\pi\eta$–$K\overline{K}$ scattering

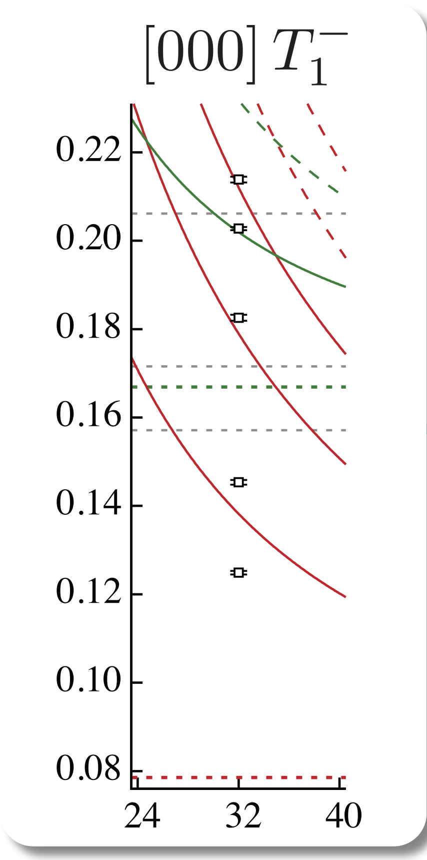
(I=1 channel)

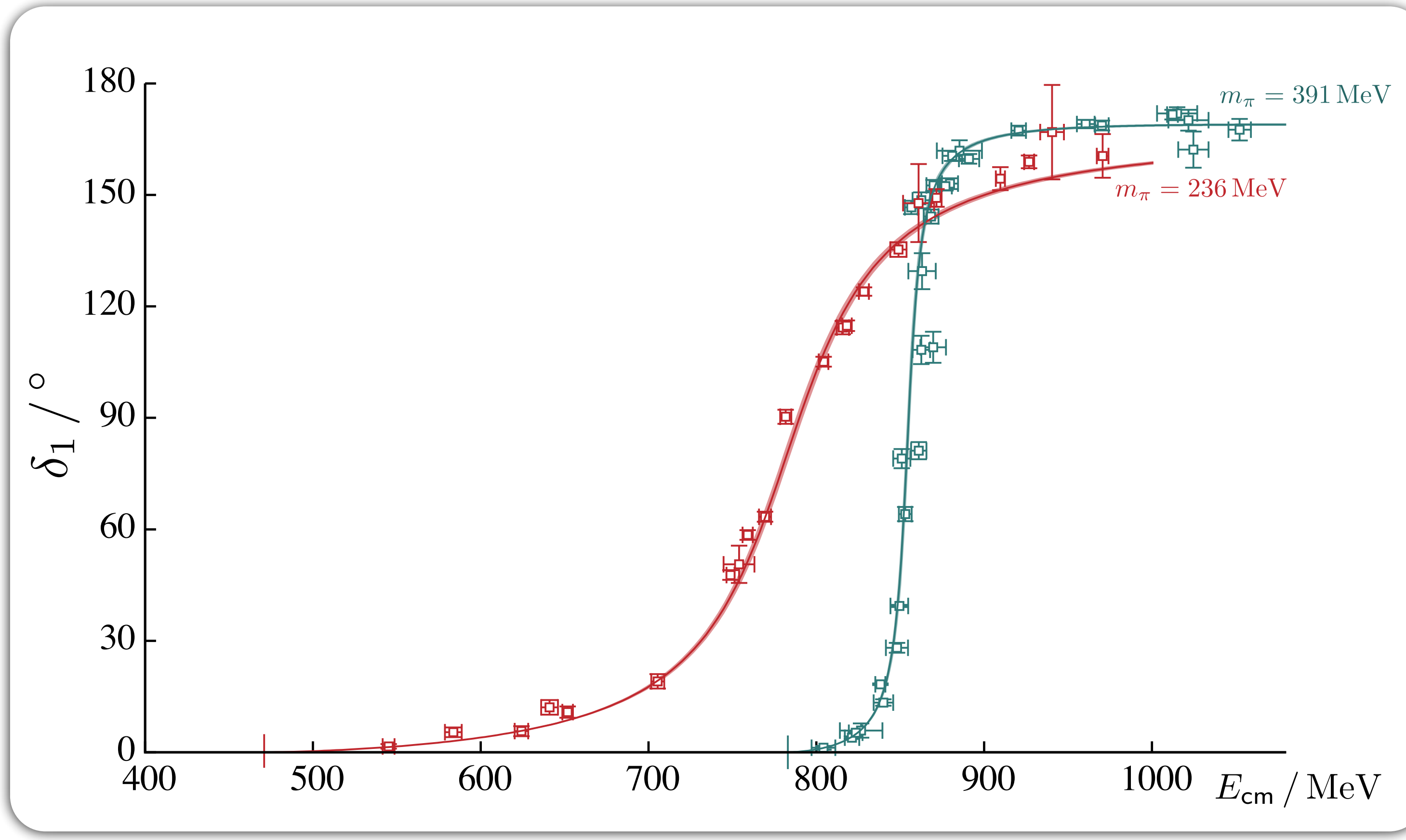
Increasingly complex systems



Dudek, Edwards & Wilson (2016)

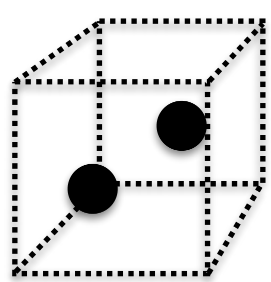
~~RB~~





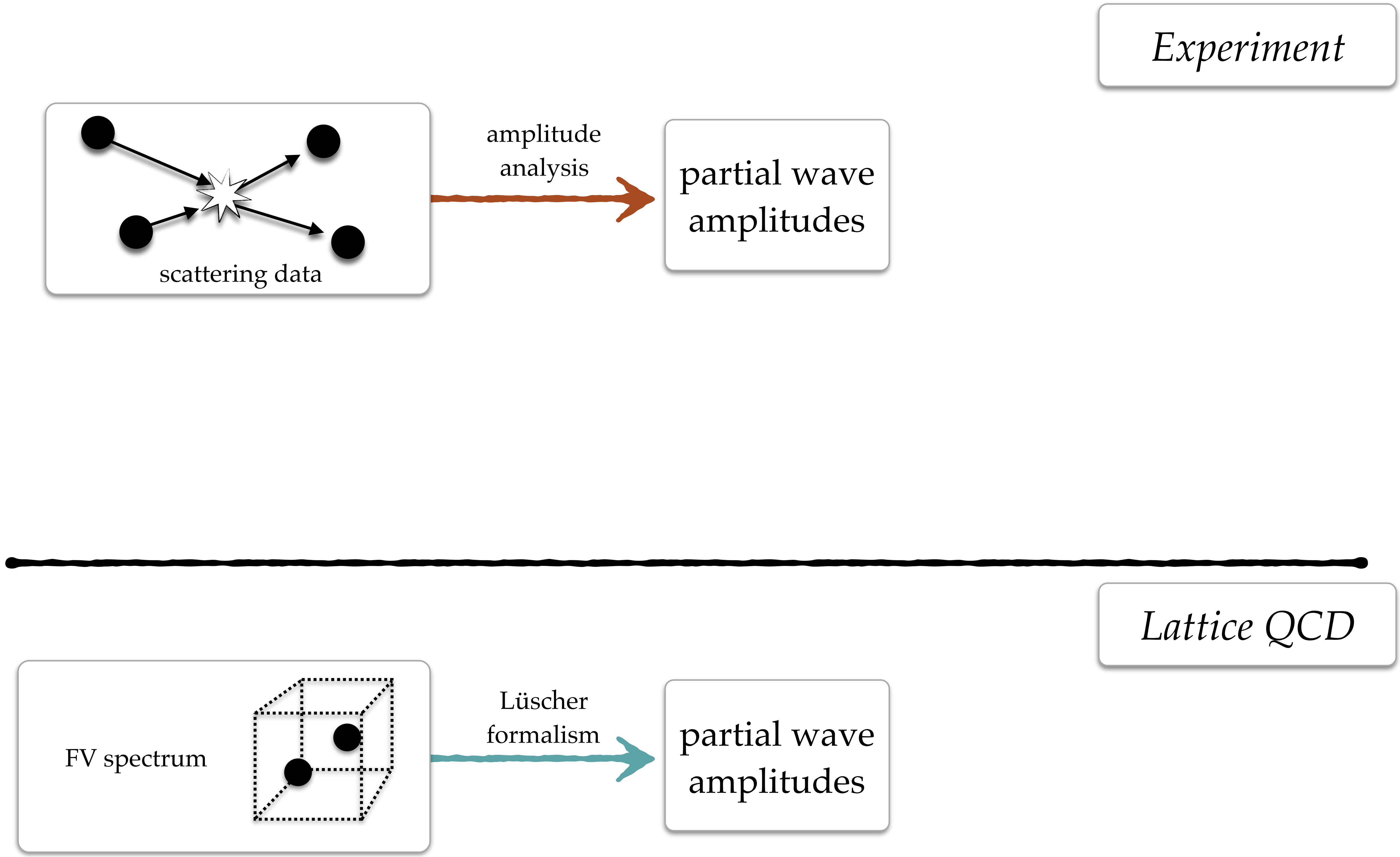
Lattice QCD

FV spectrum



Lüscher formalism

partial wave amplitudes


Experiment
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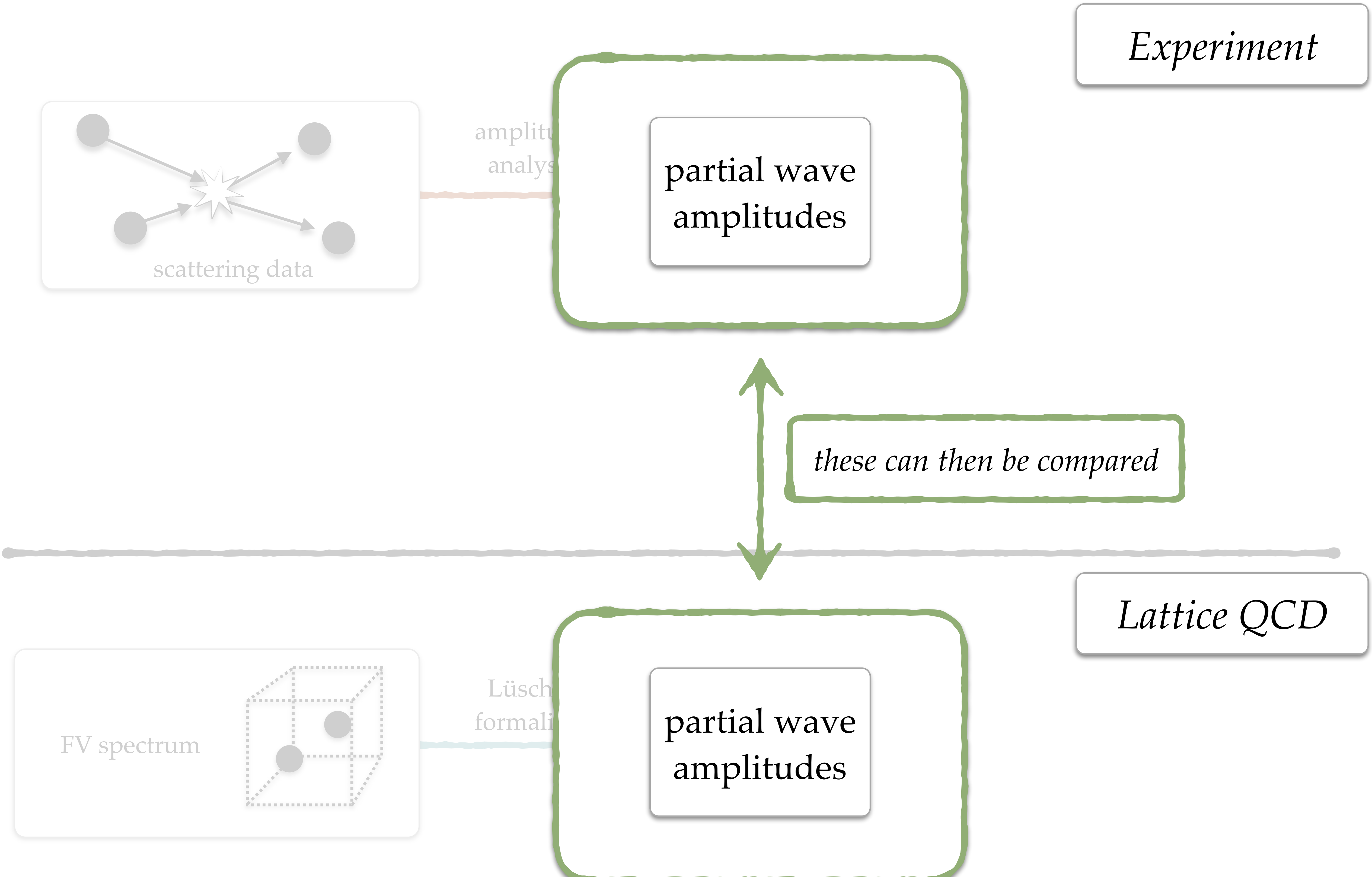
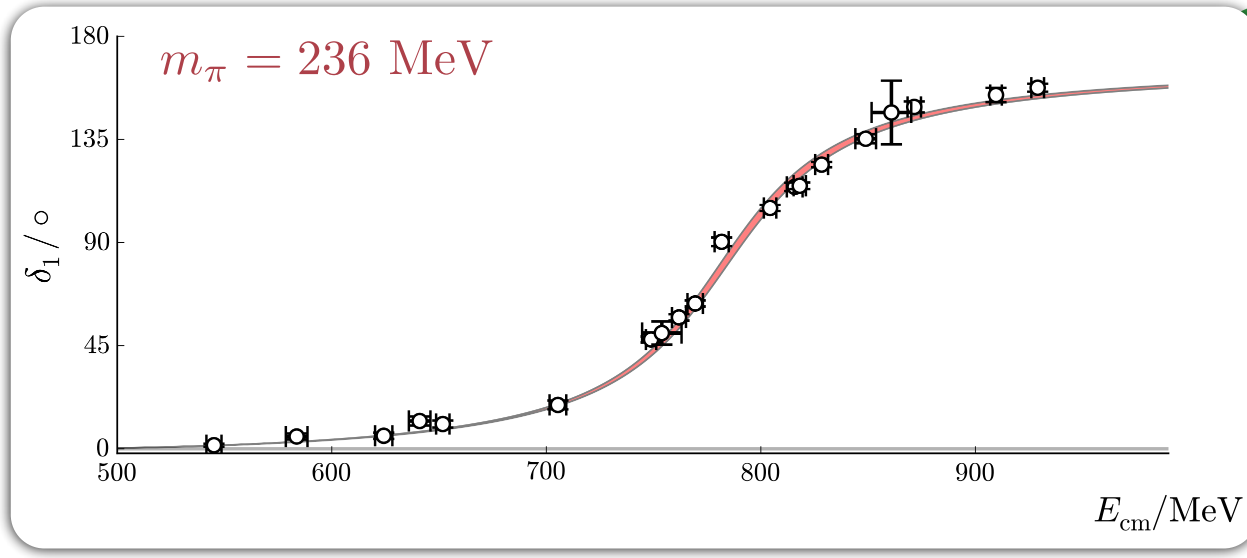
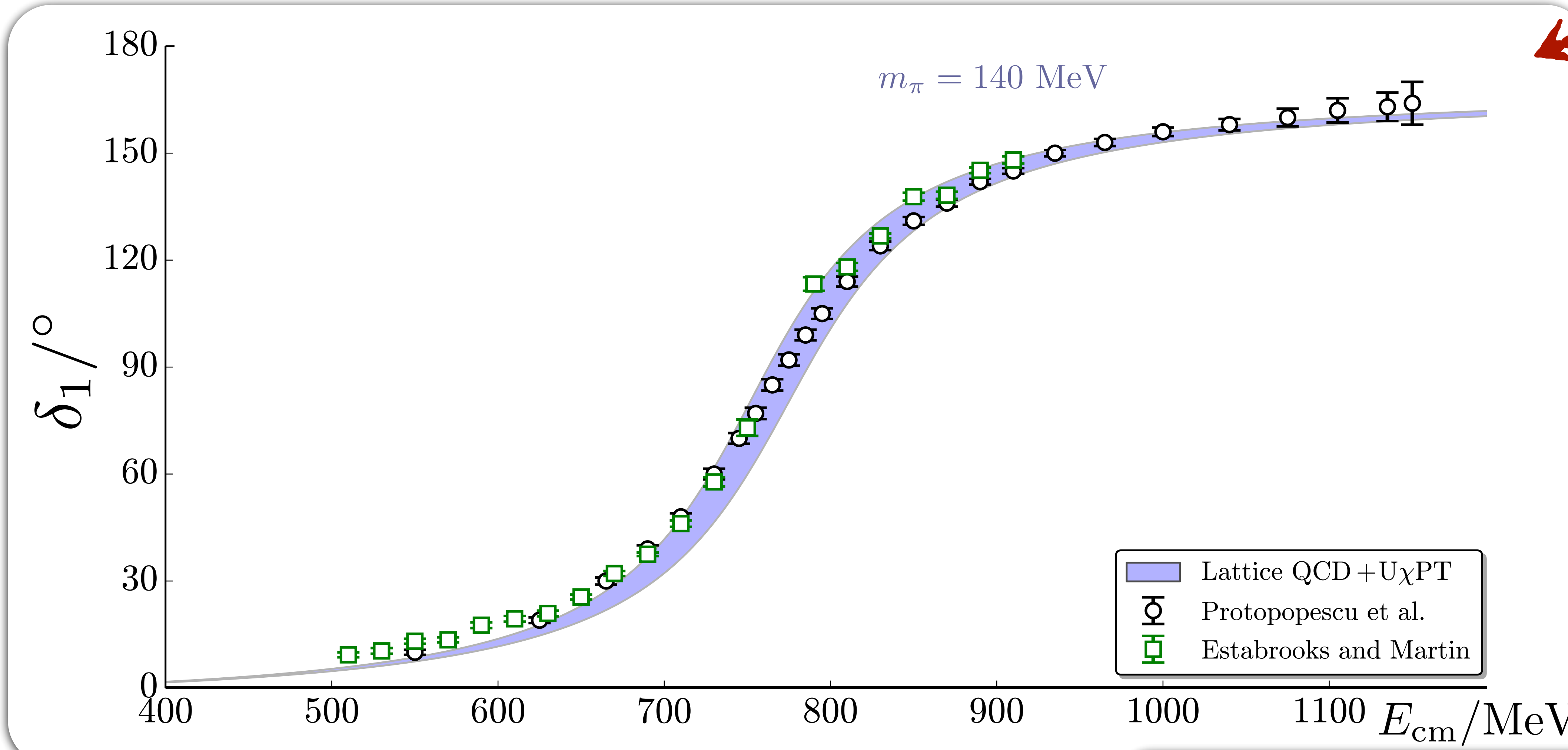
partial wave amplitudes


Experiment
scattering data
partial wave amplitudes
these can then be compared
Lattice QCD
FV spectrum
partial wave amplitudes

Comparing with experiment

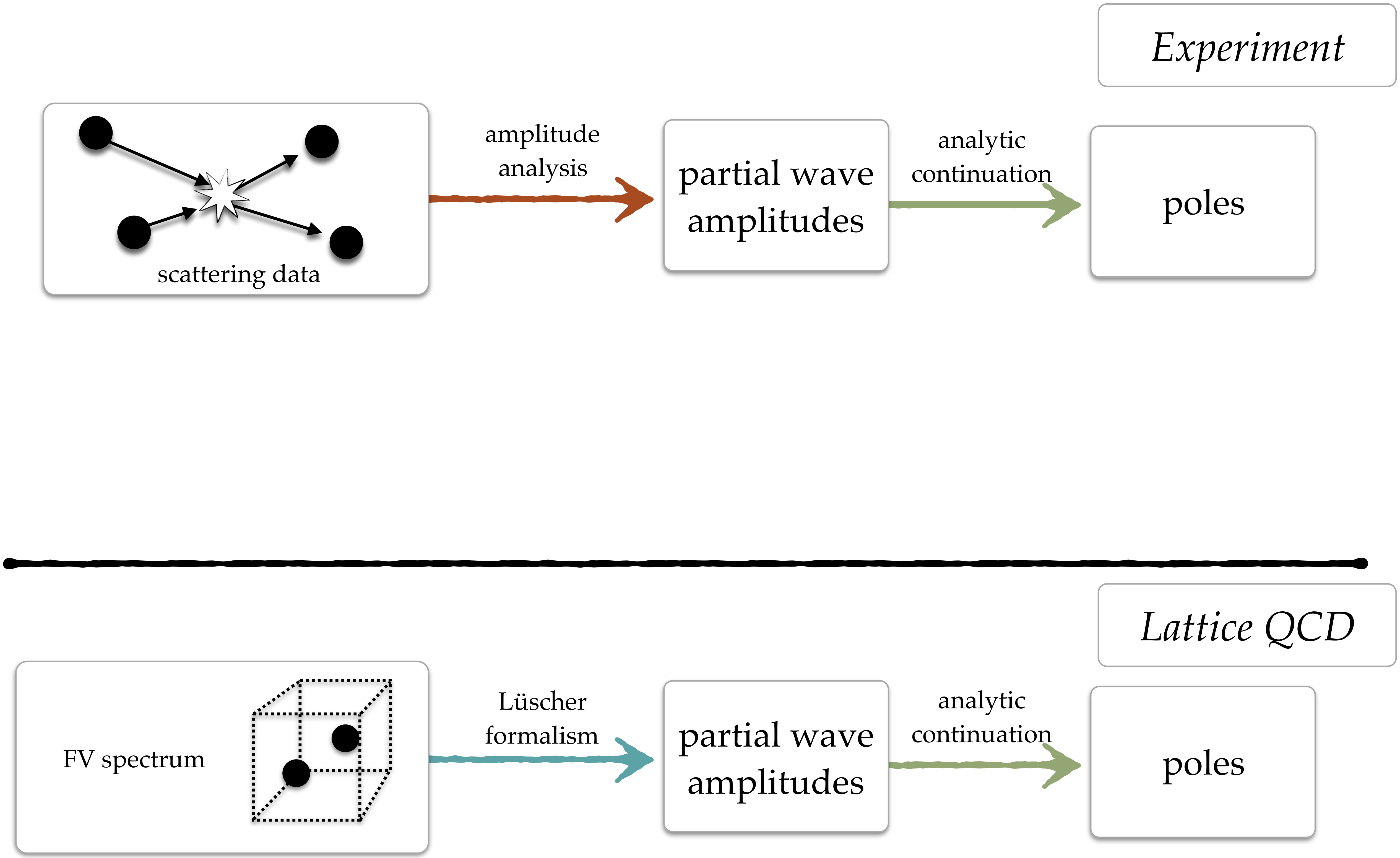


$$\det\left[F^{-1} + \mathcal{M}_{U\chi PT}\left(m_\pi, \{\alpha\}\right)\right] = 0$$

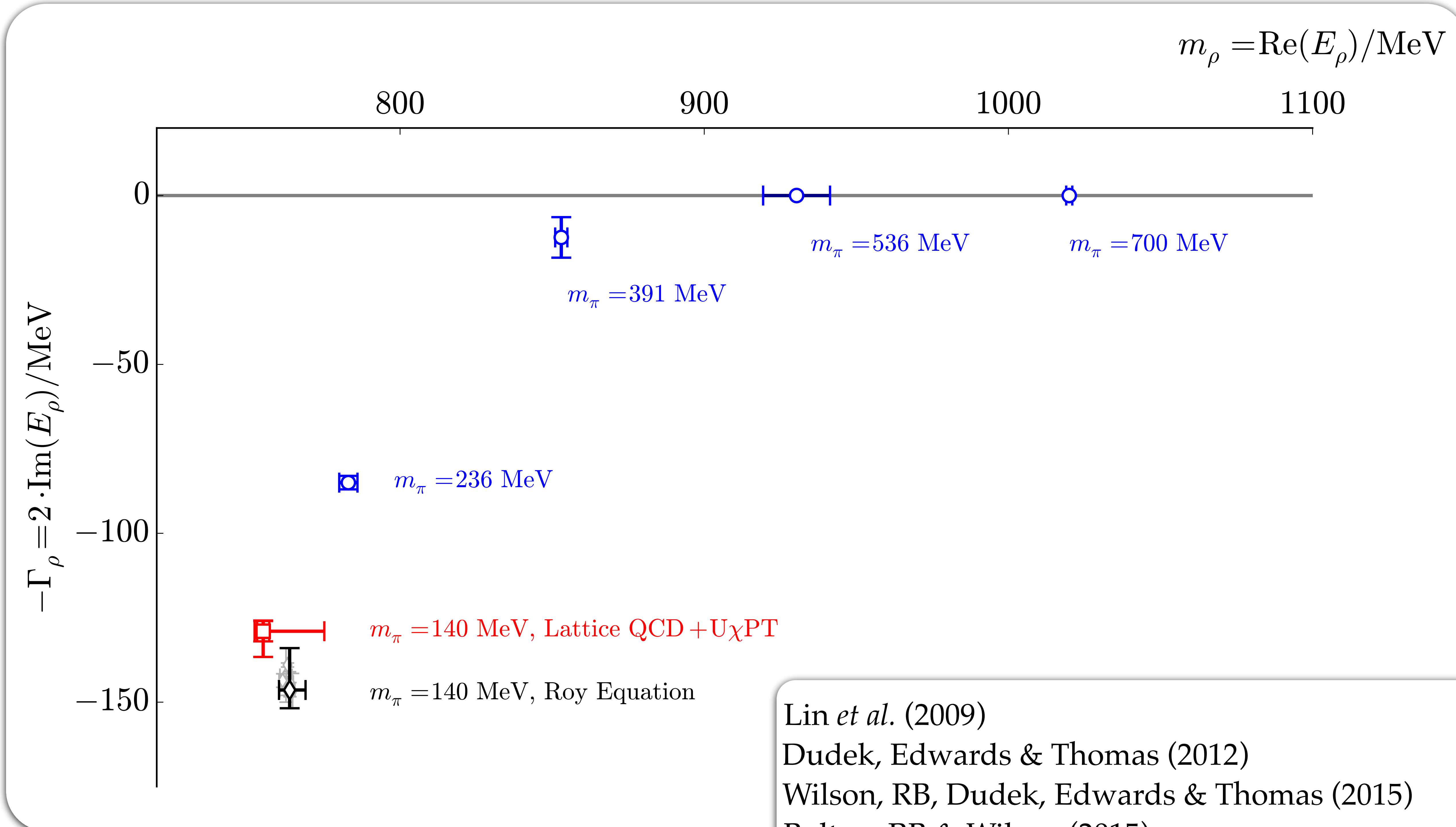


First chiral extrapolation of a resonant amplitude

Bolton, RB & Wilson (2015)
UχPT - Dobado and Pelaez (1997)

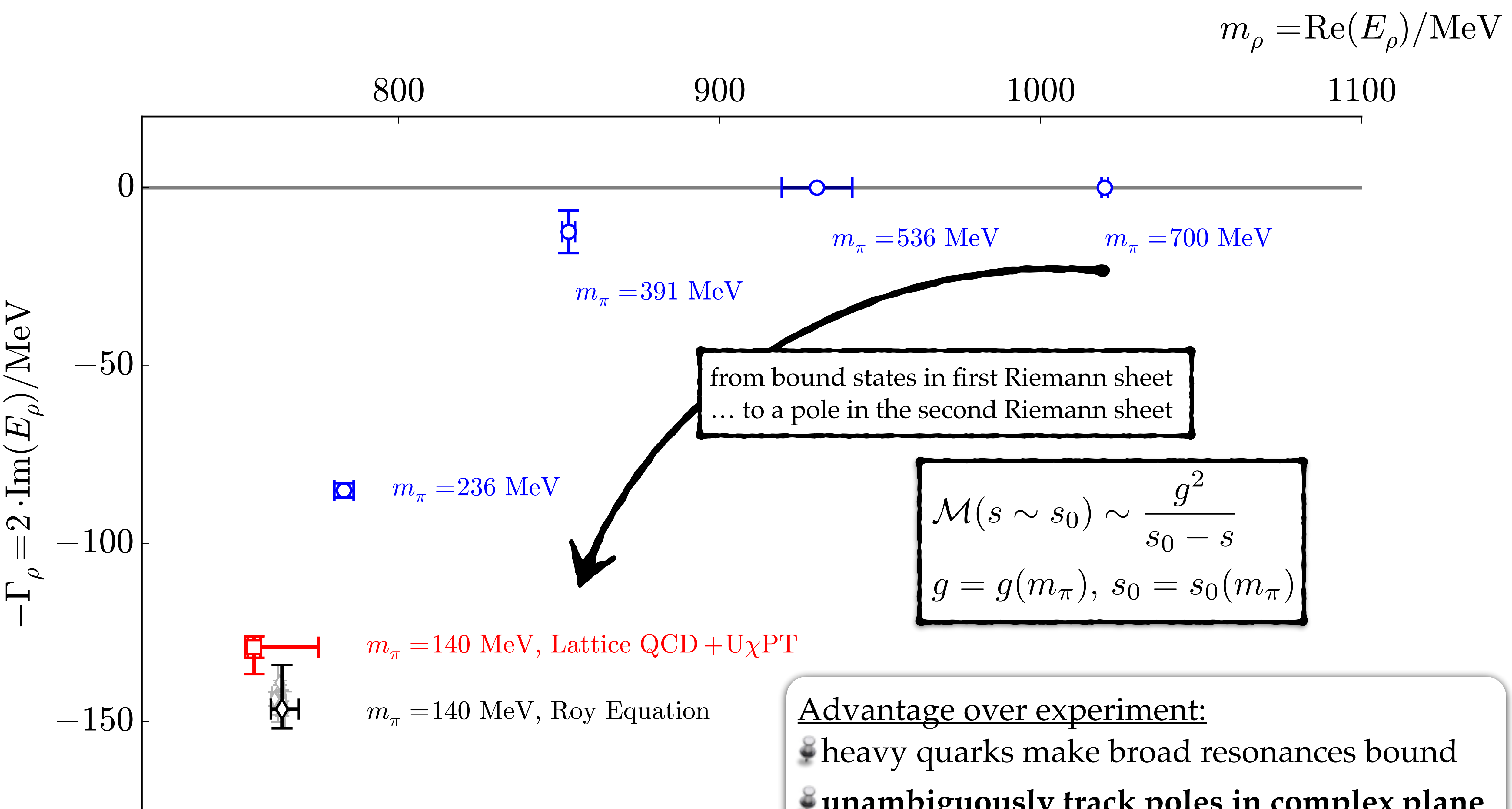

Experiment
scattering data
amplitude
analysis
partial wave
amplitudes
analytic
continuation
poles
Lattice QCD
FV spectrum
Lüscher
formalism
partial wave
amplitudes
analytic
continuation
poles

Quark-mass dependence of poles



Lin *et al.* (2009)
Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)
Bolton, RB & Wilson (2015)

Quark-mass dependence of poles



$$\mathcal{M}(s \sim s_0) \sim \frac{g^2}{s_0 - s}$$

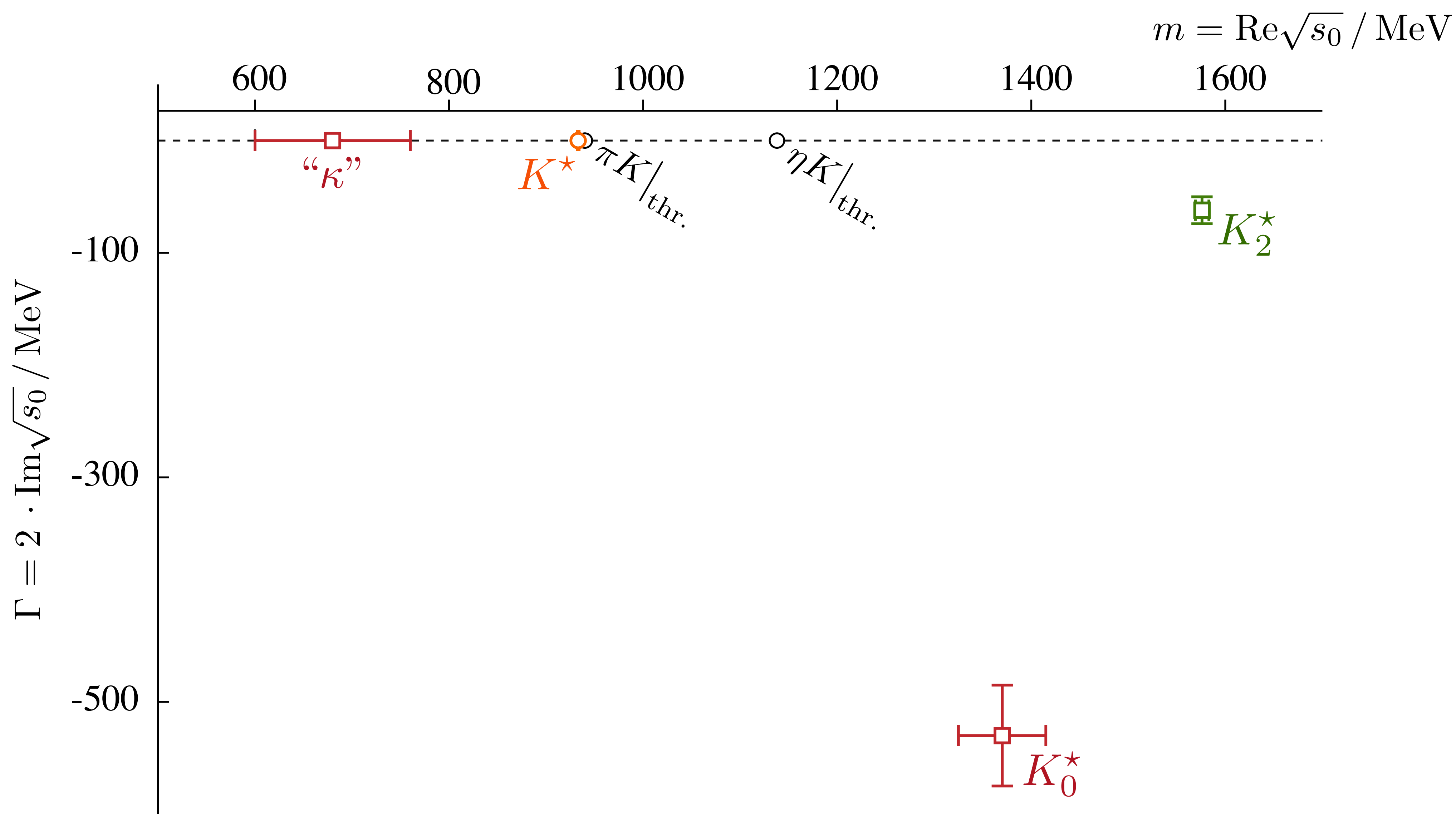
$$g = g(m_\pi),\ s_0 = s_0(m_\pi)$$

Advantage over experiment:

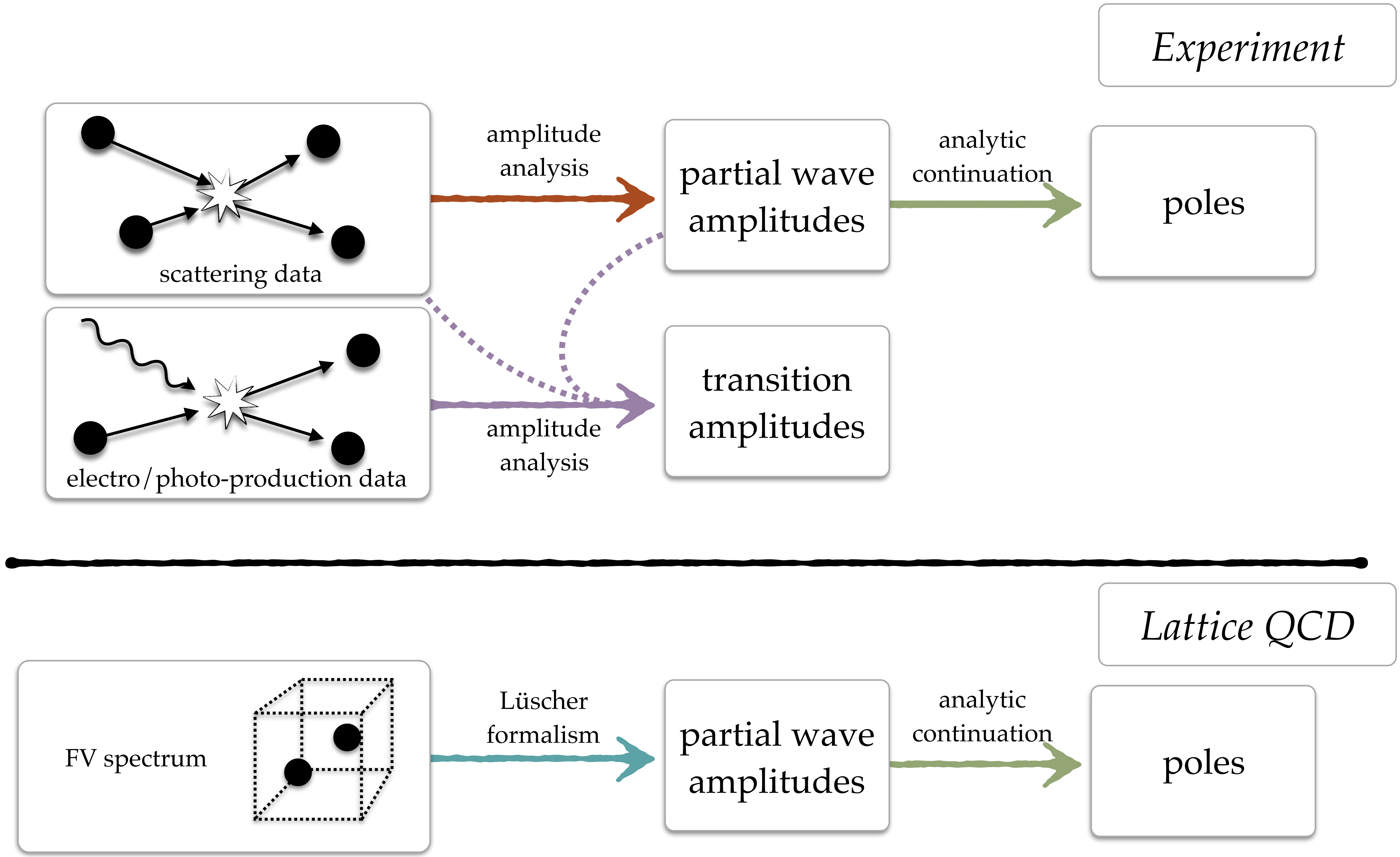
- heavy quarks make broad resonances bound
- **unambiguously track poles in complex plane**

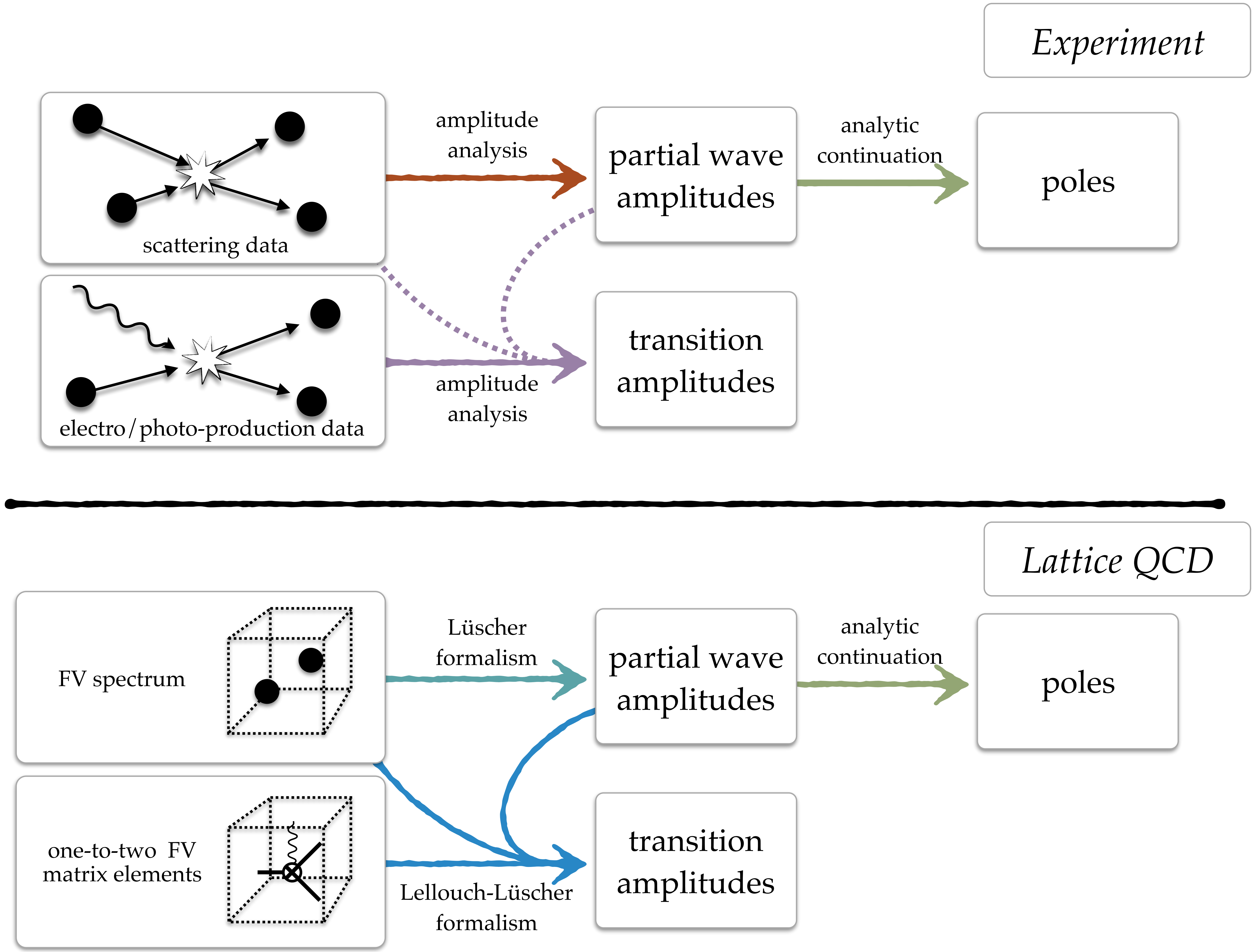
K* poles

πK-Kη in I=1/2, m_π=391MeV



Wilson, Dudek, Edwards & Thomas (2014)


Experiment
scattering data
amplitude analysis
partial wave amplitudes
analytic continuation
poles
electro/photo-production data
amplitude analysis
transition amplitudes
Lattice QCD
FV spectrum
Lüscher formalism
partial wave amplitudes
analytic continuation
poles



Experiment
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formalism
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analytic
continuation
poles
FV spectrum
one-to-two FV
matrix elements
transition
amplitudes
Lellouch-Lüscher
formalism

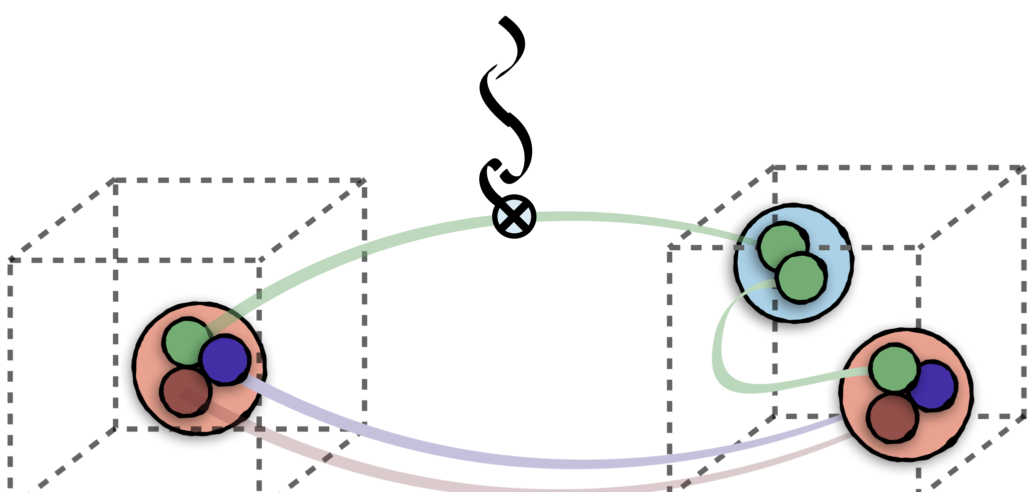
Matrix elements

1) Access matrix elements:

$$C^{3pt.}_{\mathbf{2}\to\mathbf{1}\mathcal{J}} = \langle \mathcal{O}_1(\delta t)\mathcal{J}(t)\mathcal{O}_2^\dagger(0)\rangle \longrightarrow \langle \mathbf{1}|\mathcal{J}|\mathbf{2}\rangle_L Z_1 Z_2^* e^{-(\delta t - t)E_1} e^{-tE_2} + \cdots$$

2) Interpret matrix elements:

$$\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{1}\rangle_L\right|^2 = \mathcal{H}\ \mathcal{R}\ \mathcal{H}$$



RB, Hansen & Walker-Loud (2014)
RB & Hansen (2015)
RB & Hansen (2015)

Hansen Walker-Loud

Matrix elements

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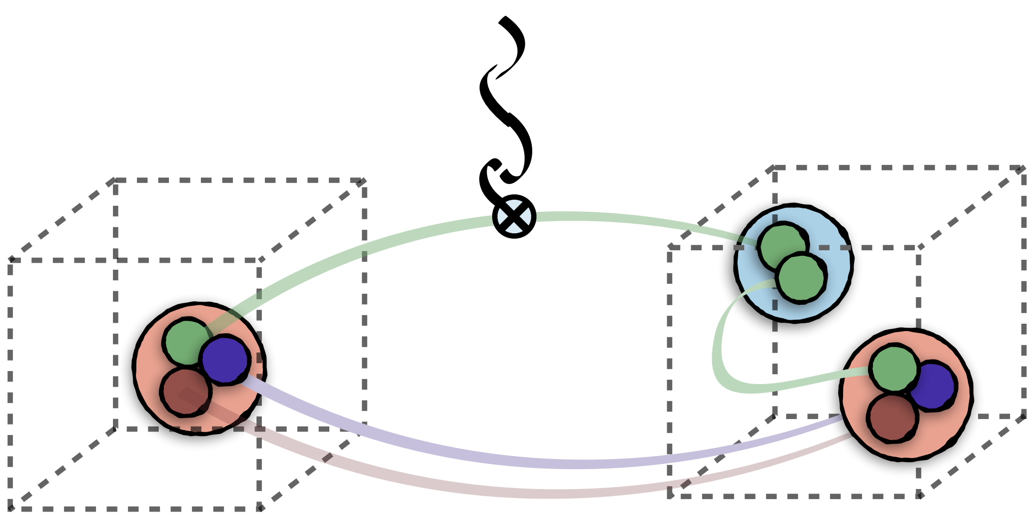
2) Interpret matrix elements:

$$\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{1}\rangle_L\right|^2 = \mathcal{H}\,\mathcal{R}\,\mathcal{H}$$

known finite volume function

$$\mathcal{R}\left(E_{\mathbf{2}}, L, \delta, \frac{\partial\delta}{\partial E_{\mathbf{2}}}\right)$$



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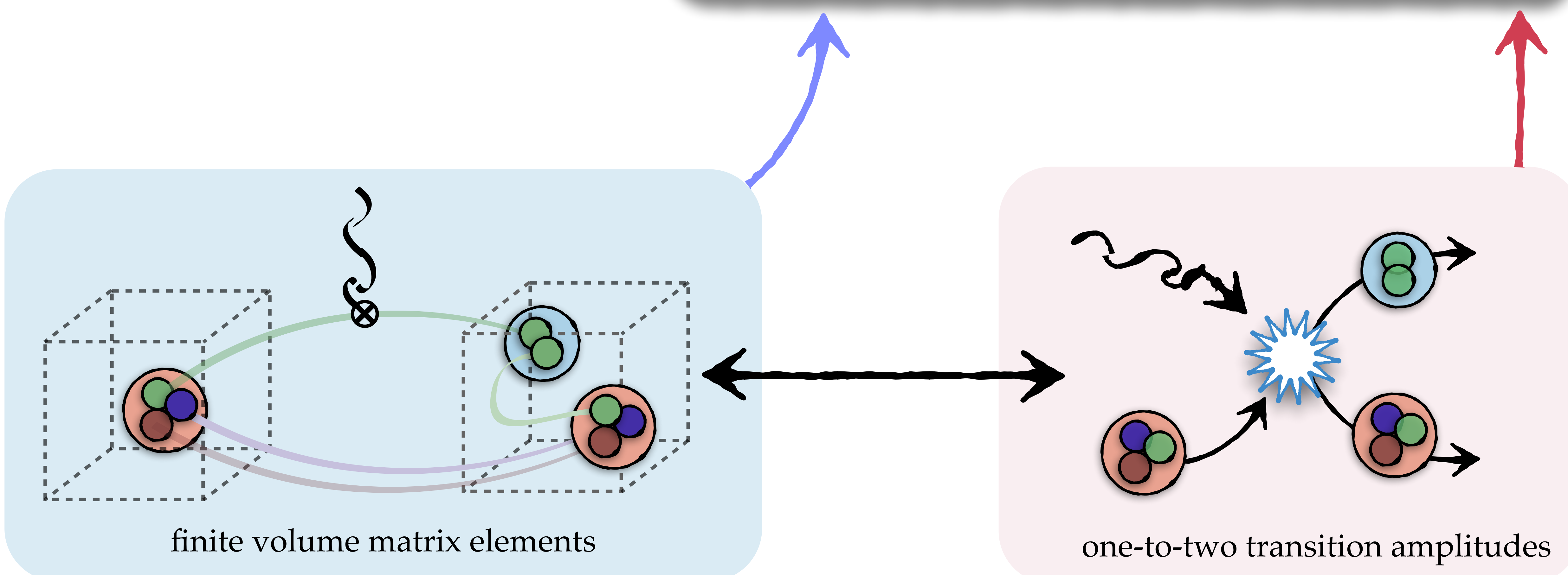

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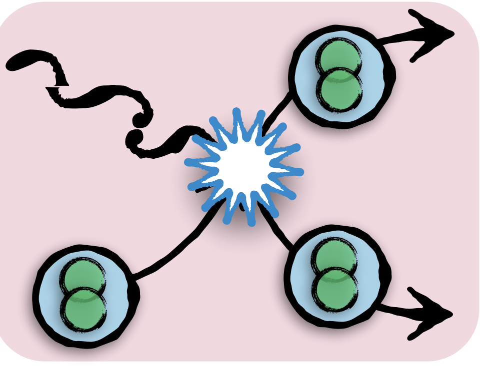
summarizes everything previously done and more!

Lellouch-Lüscher formalism

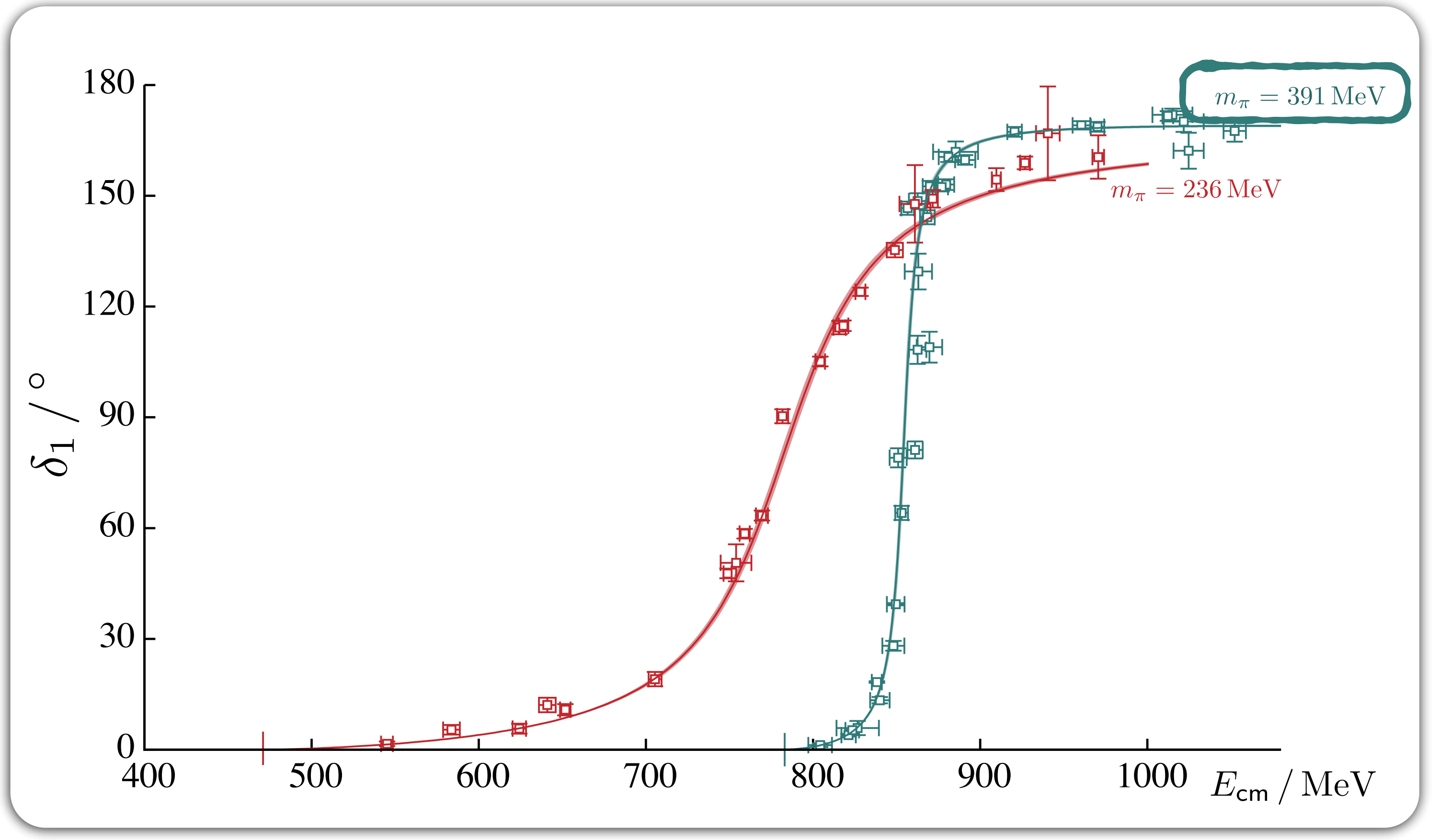
- Lellouch & Lüscher (2000) [K-to-ππ at rest]
- Christ, Kim & Yamazaki / Kim, Sachrajda & **Sharpe** (2005) [moving K-to-ππ]
- Meyer [Bγ-to-BB] (2011)
- **Hansen & Sharpe** [moving D-to-ππ/KK] (2012)
- Agadjanov, V. Bernard, Meissner & **Rusetsky** [Nγ-to-Nπ] (2013)

$\pi\gamma^*$-to-$\pi\pi$



Exploratory $\pi\gamma^*$-to-$\pi\pi$ / $\pi\gamma^*$-to-ϱ calculation:

- $m_\pi = 391$ MeV

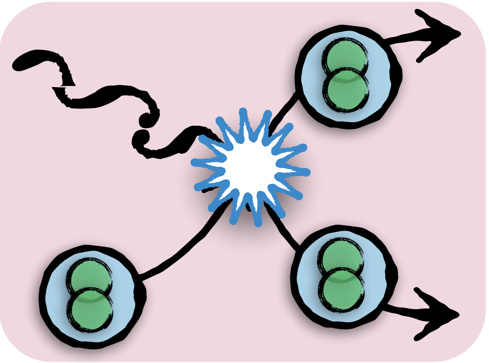


- Matrix element determined in **42** kinematic point: $(E_{\pi\pi}, Q^2)$
- Lorentz decomposition:

$$\mathcal{H}^{\mu}_{\pi\pi,\pi\gamma^\star} = \epsilon^{\mu\nu\alpha\beta}\, P_{\pi,\nu}\, P_{\pi\pi,\alpha}\, \epsilon_\beta(\lambda_{\pi\pi}, \mathbf{P}_{\pi\pi})\, \frac{2}{m_\pi} \mathcal{A}_{\pi\pi,\pi\gamma^\star}$$

$\pi\pi/\rho$ polarization — *$\pi\pi/\rho$ helicity* — *Lorentz scalar*

$\pi\gamma^*$-to-$\pi\pi$



1. Building block of $N\gamma^*$-to-$N\pi$

2. Hadronic light-by-light contribution to g_μ-2

3. ϱ-to-$\pi\gamma^*$ decay

4. chiral anomaly

⋮

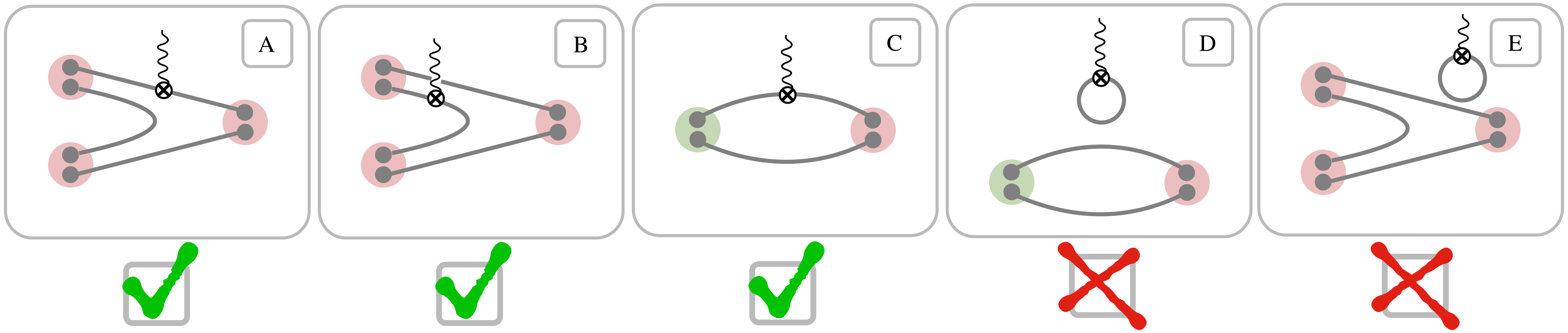
5a. First resonating 1-to-2 calculation

5b. First resonance form factor

5c. Testing ground for more challenging processes

Correlation functions

Contractions:



Operators and matrix elements:

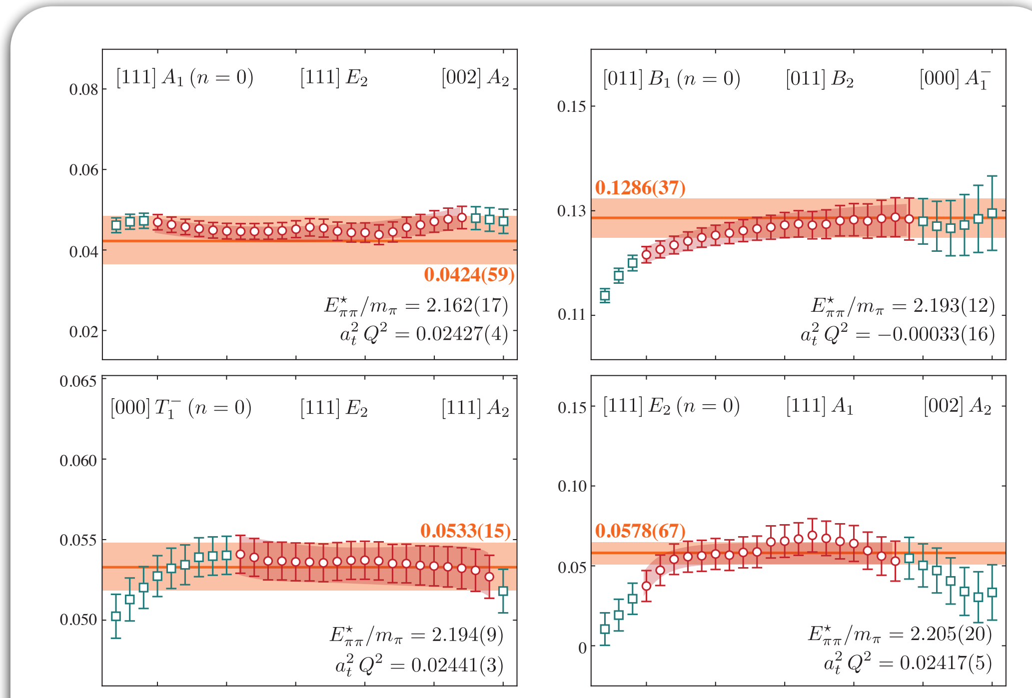
$$C^{(3)}_{\pi\pi_n,\mu,\pi}(\mathbf{P}_\pi, \mathbf{P}_{\pi\pi}; \Delta t, t) = \big\langle 0 \big| \Omega_\pi(\Delta t, \mathbf{P}_\pi)\ \widetilde{\mathcal{J}}_\mu(t, \mathbf{P}_\pi - \mathbf{P}_{\pi\pi})\ \Omega^\dagger_{\pi\pi}(0, \mathbf{P}_{\pi\pi}) \big| 0 \big\rangle$$

$$= e^{-(E_{\pi\pi}-E_\pi)t}\, e^{-E_\pi \Delta t} \big\langle \pi; L \big| \widetilde{\mathcal{J}}_\mu \big| \pi\pi; L \big\rangle + \dots$$

Ω_π =optimized 'π' operator,
linear combo. of $\sim$ 10 ops.

$\Omega_{\pi\pi}$ =optimized '$\pi\pi$' operator,
linear combo. of $\sim$ 20-30 ops.

$\widetilde{\mathcal{J}}_\mu$ =electromagnetic current



Correlation functions

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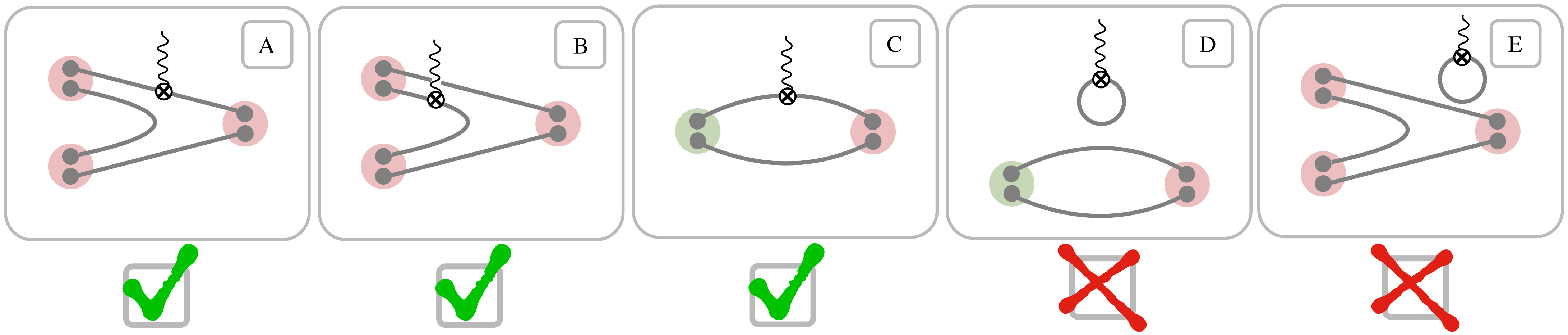
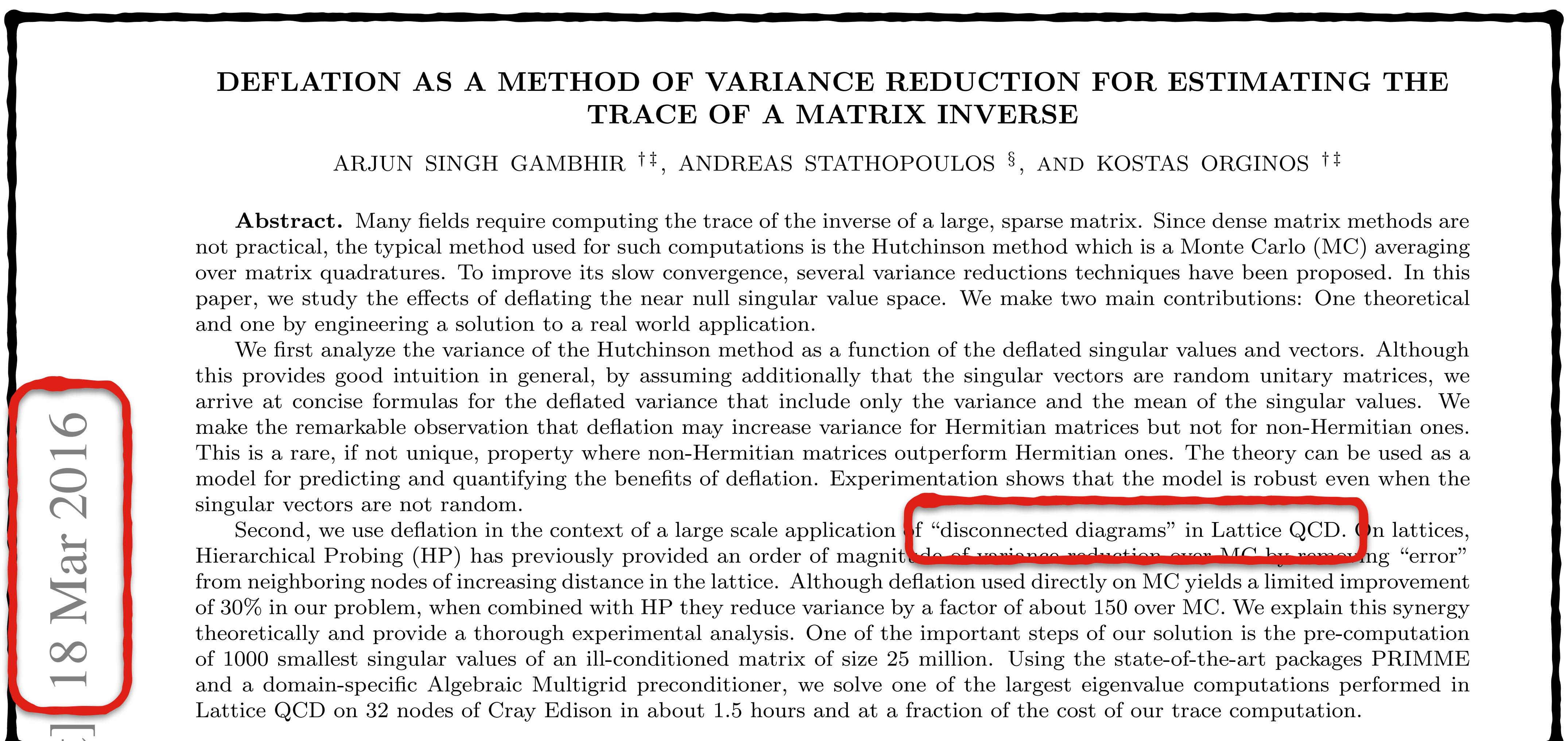




DEFLATION AS A METHOD OF VARIANCE REDUCTION FOR ESTIMATING THE TRACE OF A MATRIX INVERSE

ARJUN SINGH GAMBHIR [†‡], ANDREAS STATHOPOULOS [§], AND KOSTAS ORGINOS [†‡]

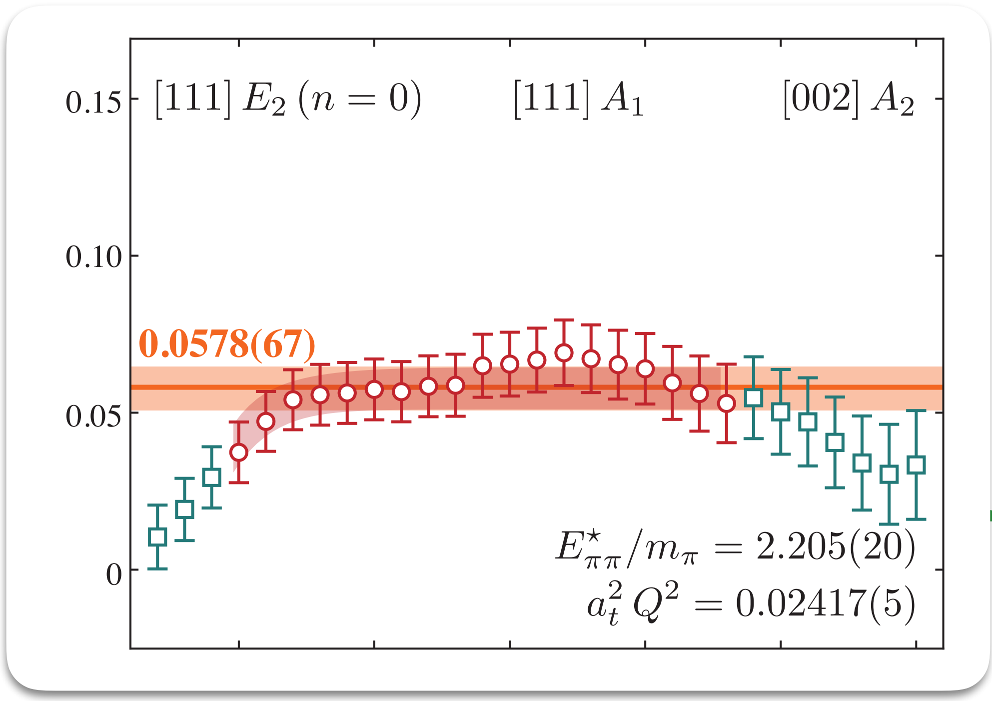
Abstract. Many fields require computing the trace of the inverse of a large, sparse matrix. Since dense matrix methods are not practical, the typical method used for such computations is the Hutchinson method which is a Monte Carlo (MC) averaging over matrix quadratures. To improve its slow convergence, several variance reductions techniques have been proposed. In this paper, we study the effects of deflating the near null singular value space. We make two main contributions: One theoretical and one by engineering a solution to a real world application.

We first analyze the variance of the Hutchinson method as a function of the deflated singular values and vectors. Although this provides good intuition in general, by assuming additionally that the singular vectors are random unitary matrices, we arrive at concise formulas for the deflated variance that include only the variance and the mean of the singular values. We make the remarkable observation that deflation may increase variance for Hermitian matrices but not for non-Hermitian ones. This is a rare, if not unique, property where non-Hermitian matrices outperform Hermitian ones. The theory can be used as a model for predicting and quantifying the benefits of deflation. Experimentation shows that the model is robust even when the singular vectors are not random.

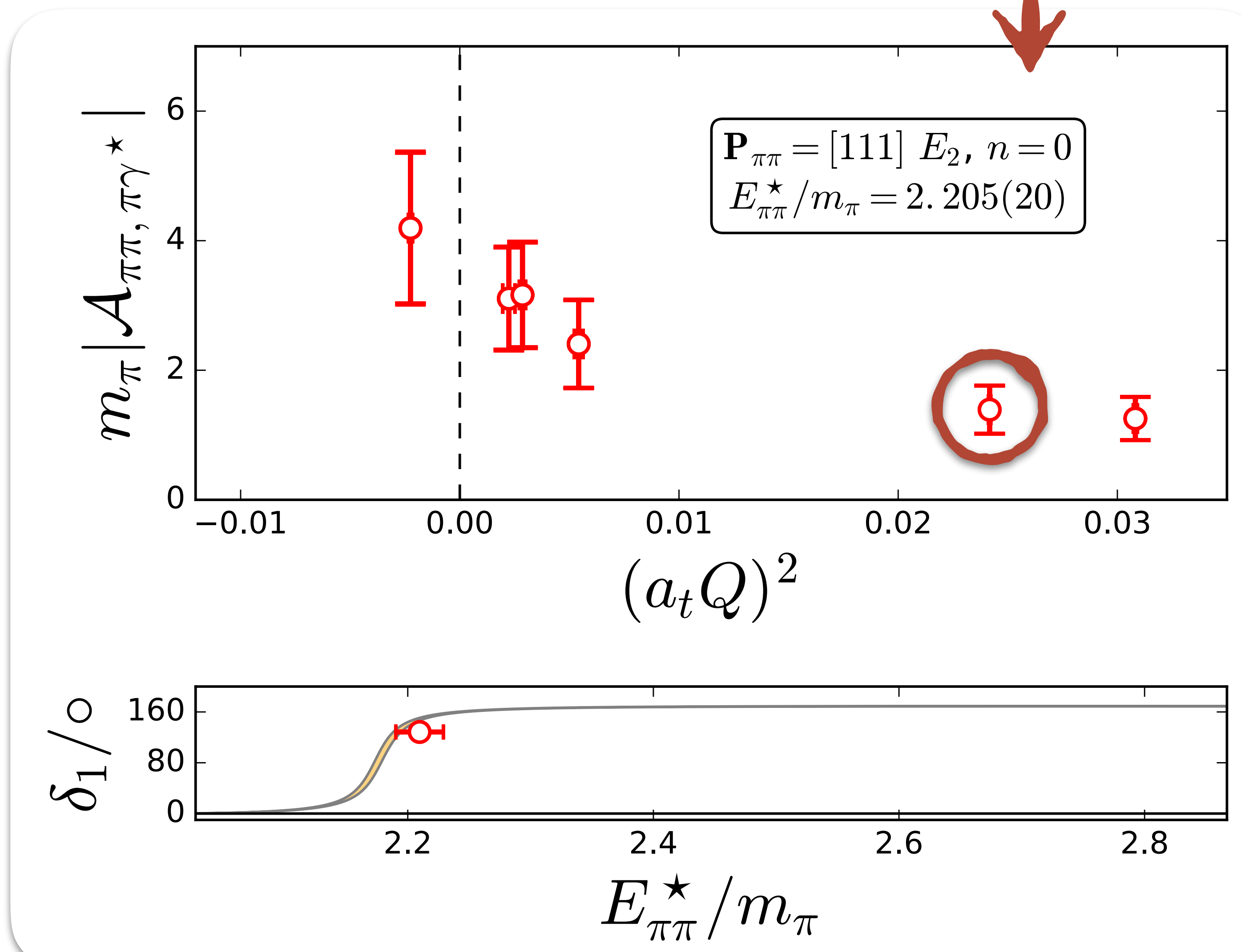
Second, we use deflation in the context of a large scale application of "disconnected diagrams" in Lattice QCD. On lattices, Hierarchical Probing (HP) has previously provided an order of magnitude of variance reduction over MC by removing "error" from neighboring nodes of increasing distance in the lattice. Although deflation used directly on MC yields a limited improvement of 30% in our problem, when combined with HP they reduce variance by a factor of about 150 over MC. We explain this synergy theoretically and provide a thorough experimental analysis. One of the important steps of our solution is the pre-computation of 1000 smallest singular values of an ill-conditioned matrix of size 25 million. Using the state-of-the-art packages PRIMME and a domain-specific Algebraic Multigrid preconditioner, we solve one of the largest eigenvalue computations performed in Lattice QCD on 32 nodes of Cray Edison in about 1.5 hours and at a fraction of the cost of our trace computation.

18 Mar 2016

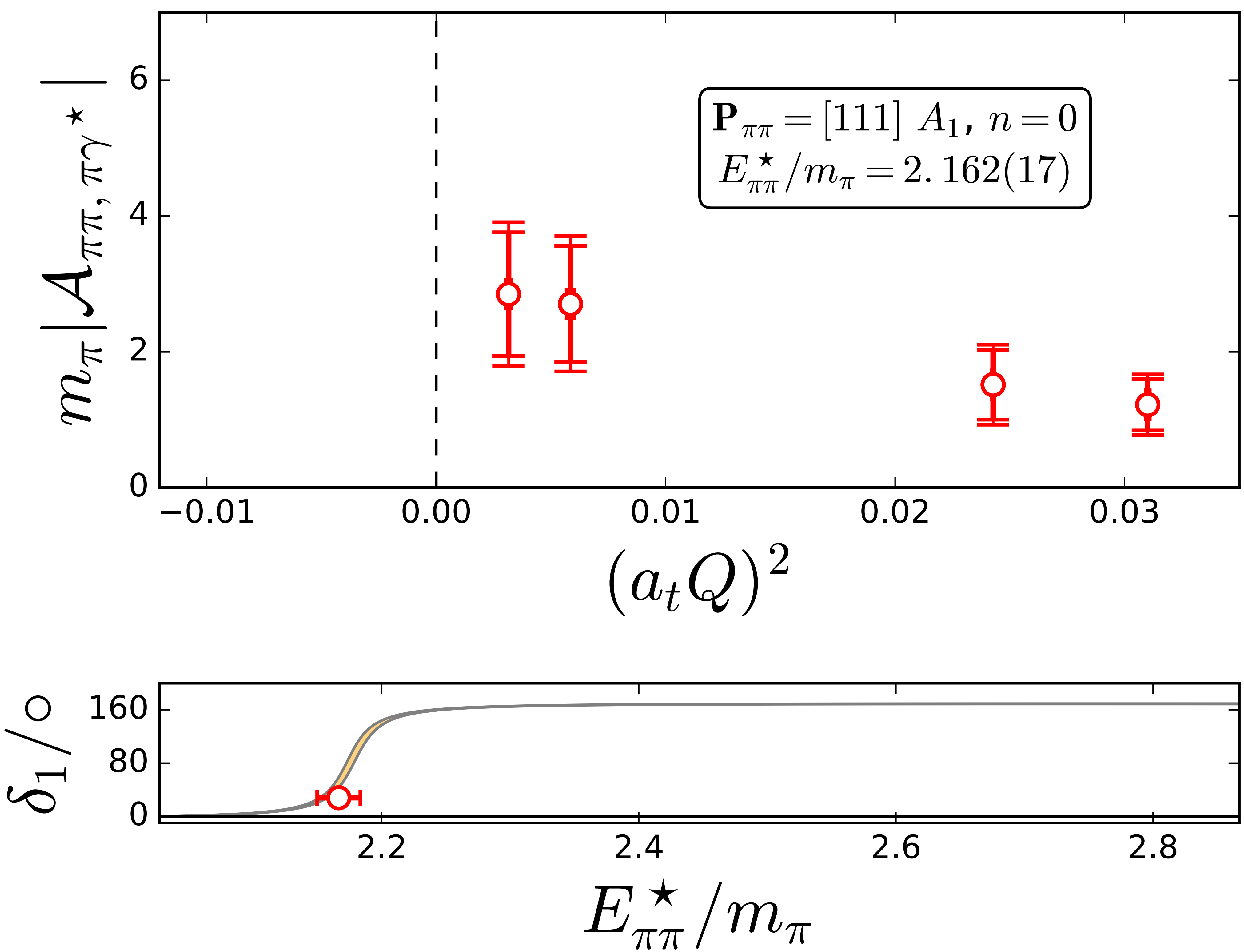
$\pi\gamma^*$-to-$\pi\pi$ amplitude



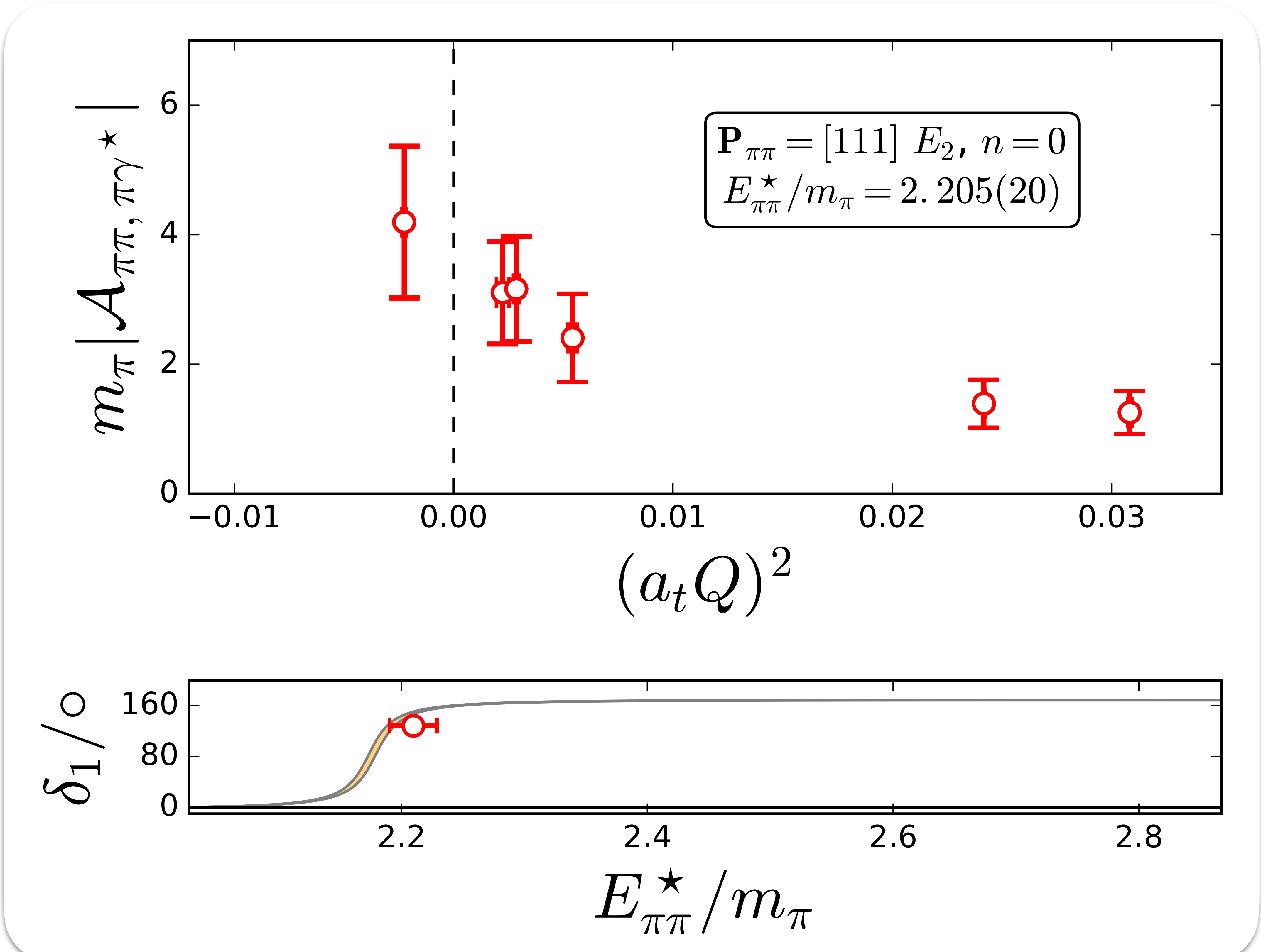
$$\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{1}\rangle_L\right|^2 = \mathcal{H}\,\mathcal{R}\,\mathcal{H}$$



$\pi\gamma^*$-to-$\pi\pi$ amplitude



$\pi\gamma^*$-to-$\pi\pi$ amplitude



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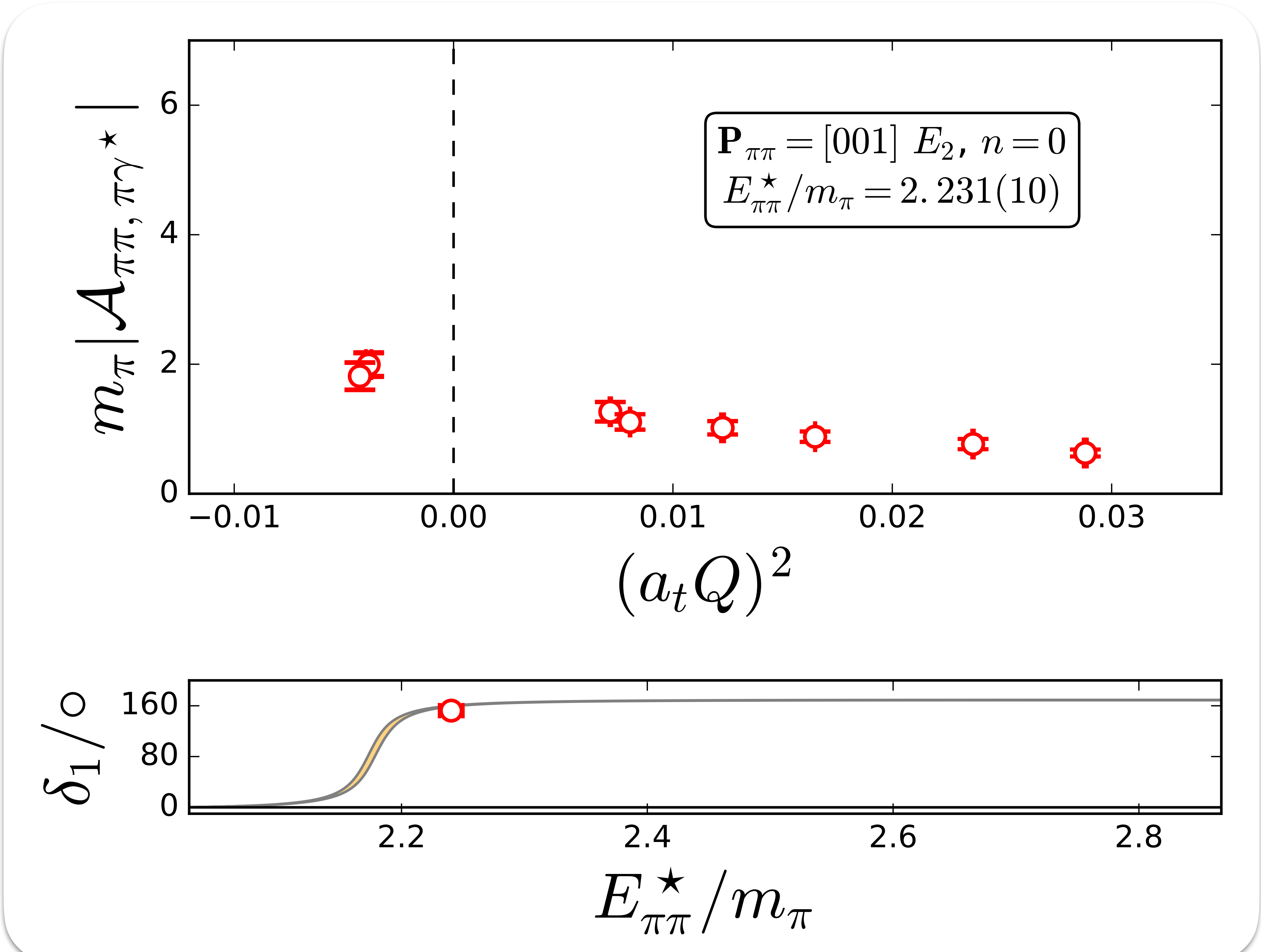
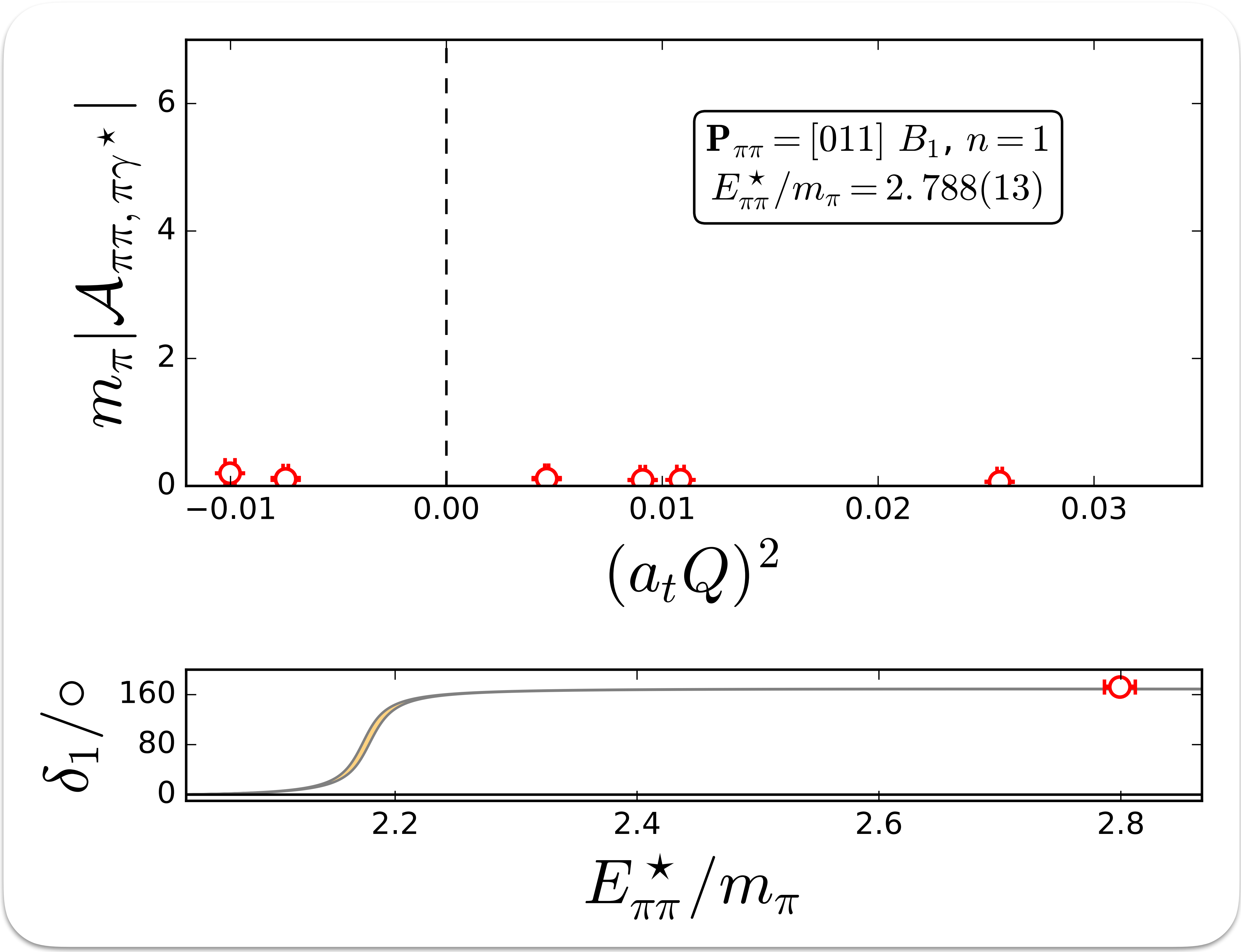


$\pi\gamma^*$-to-$\pi\pi$ amplitude



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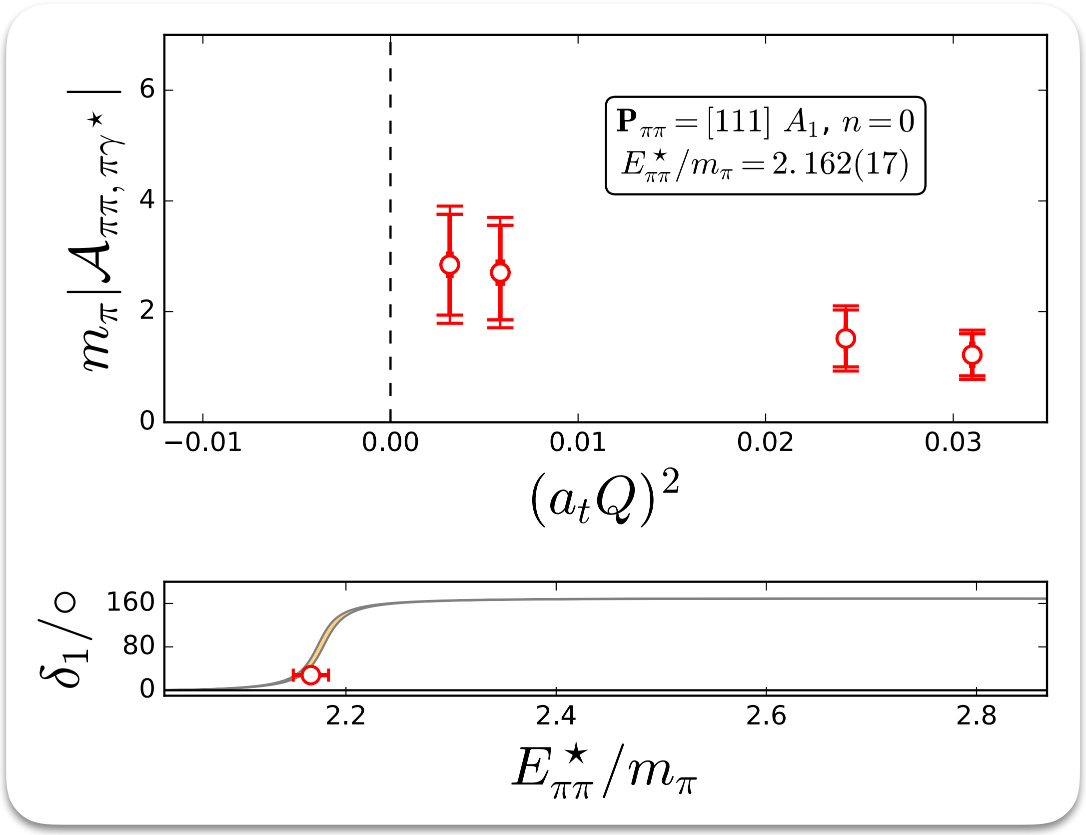
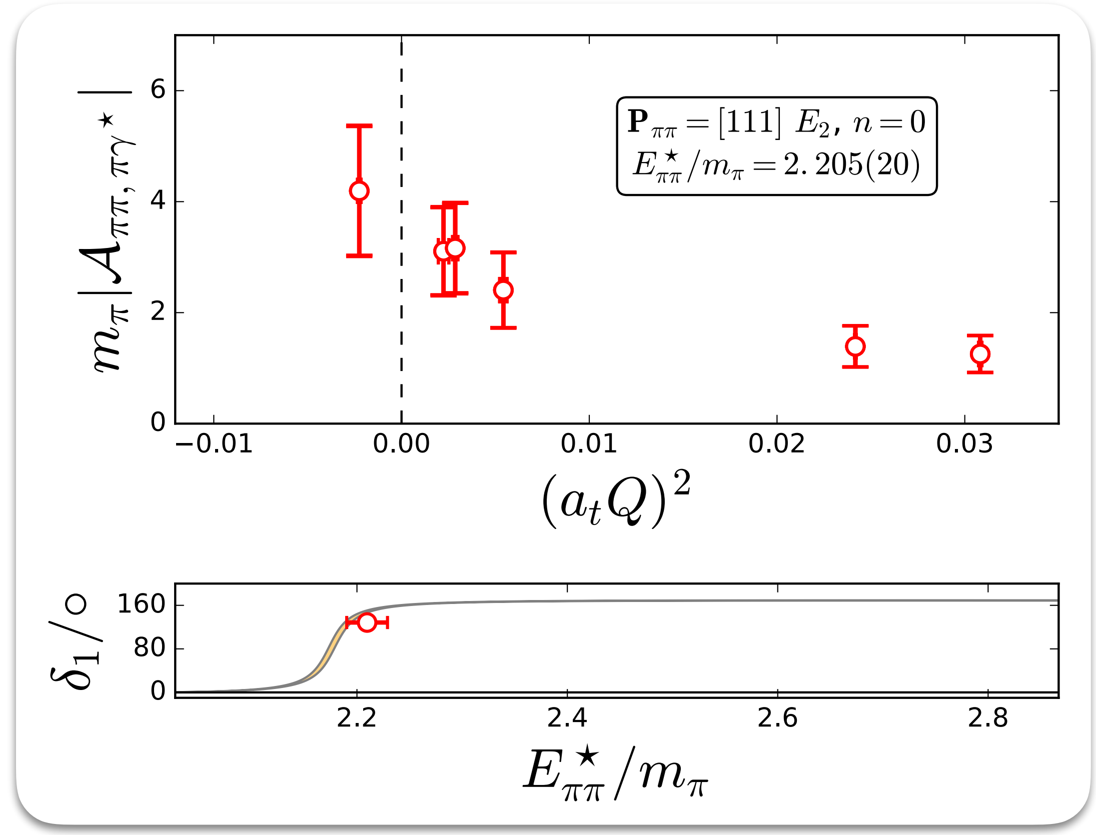
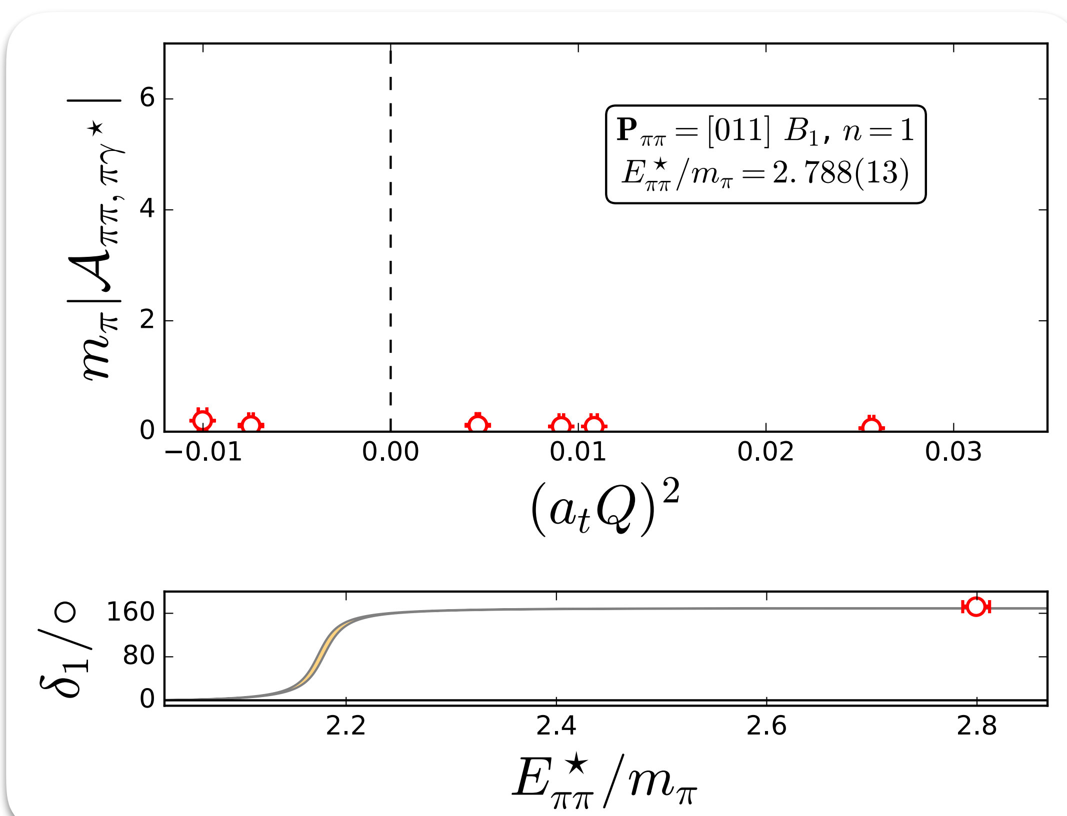
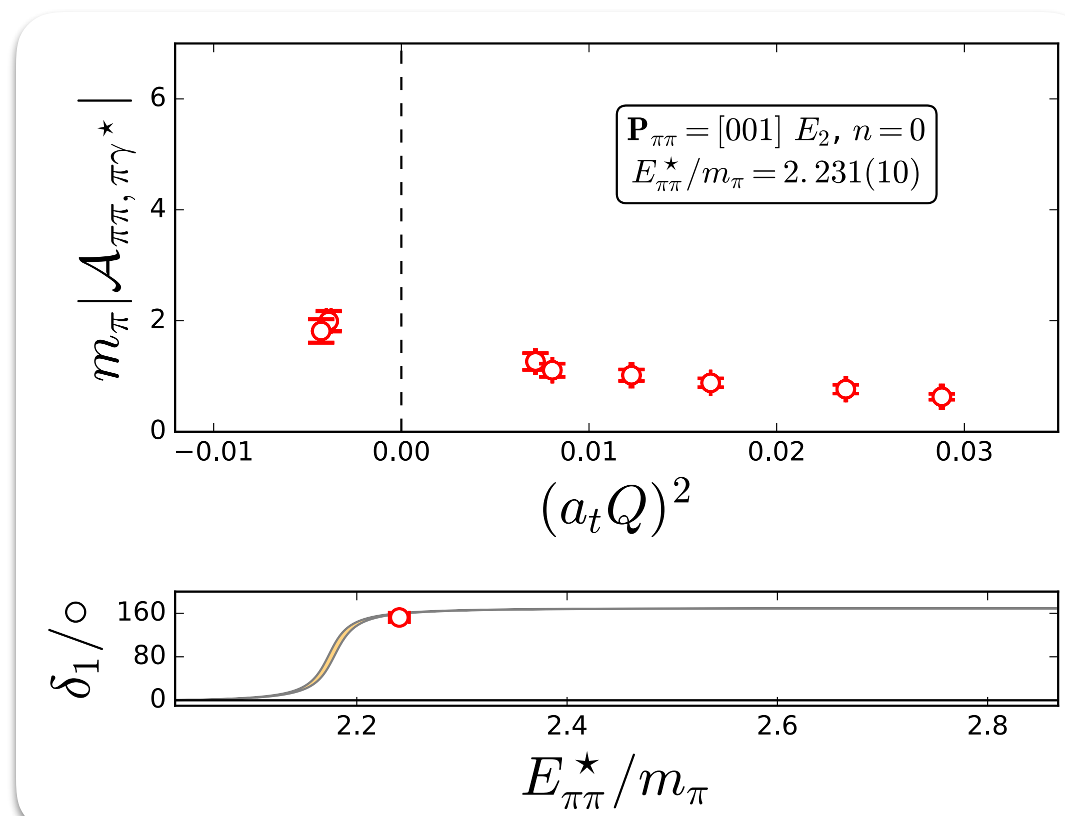
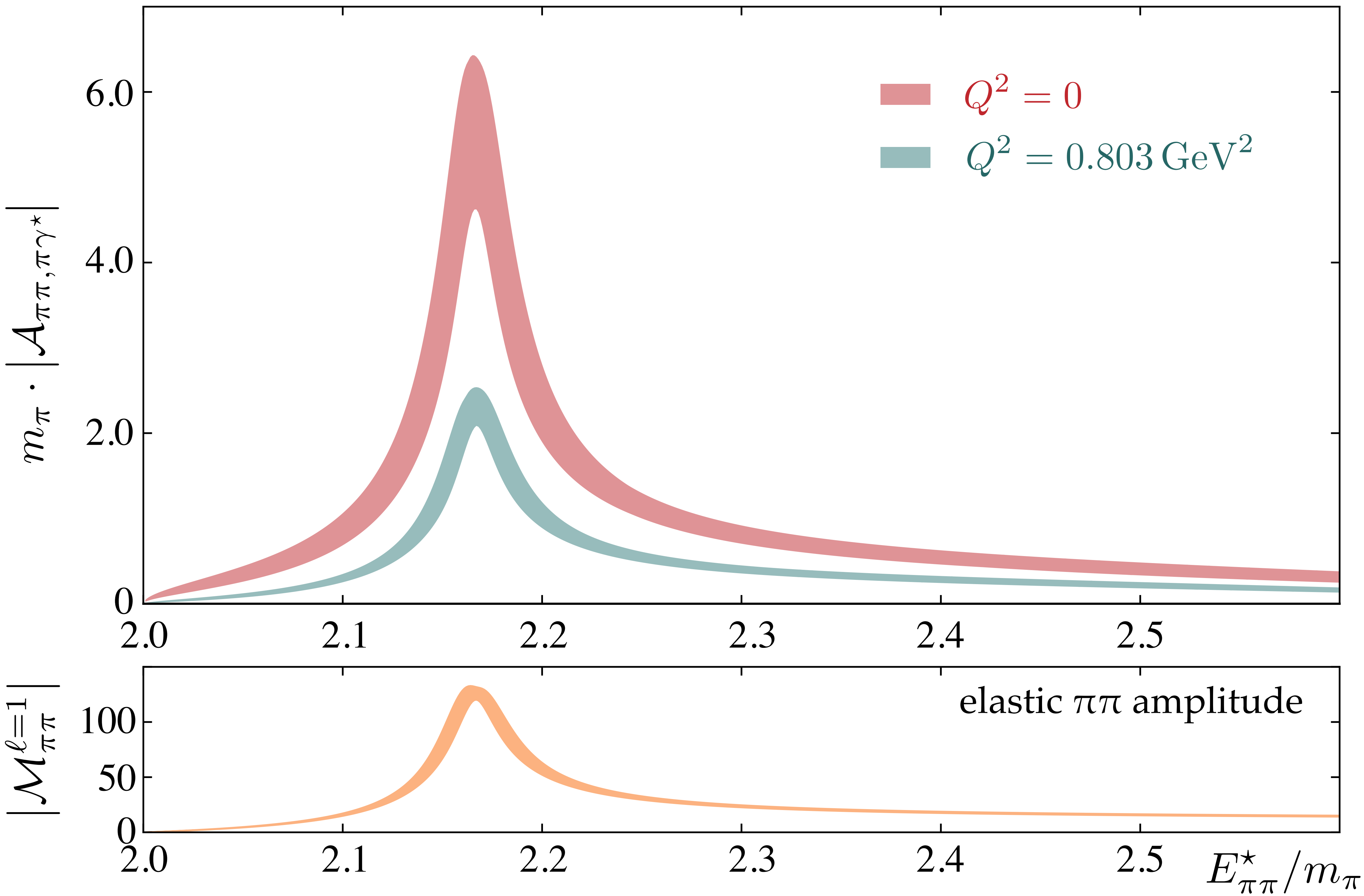


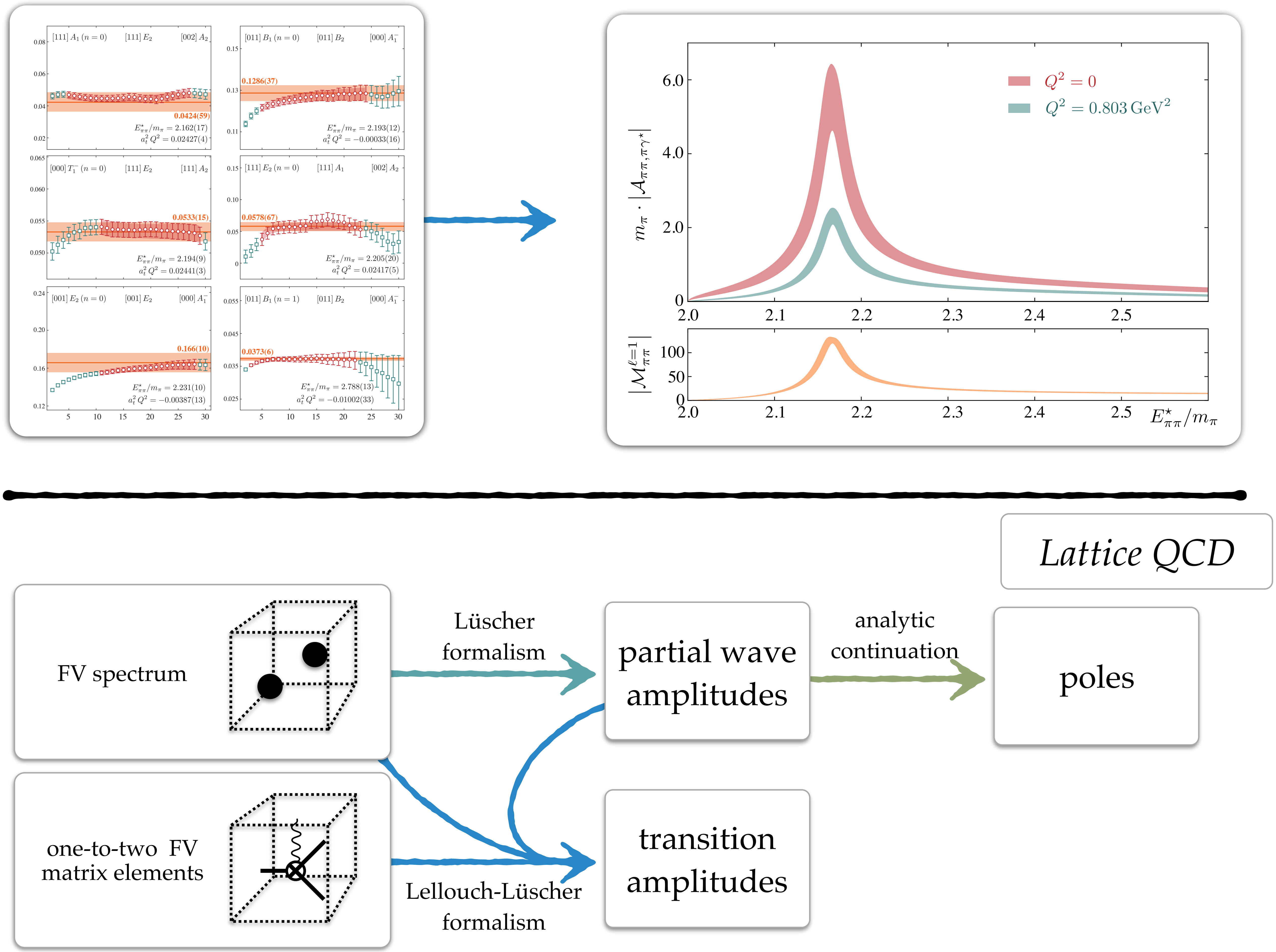




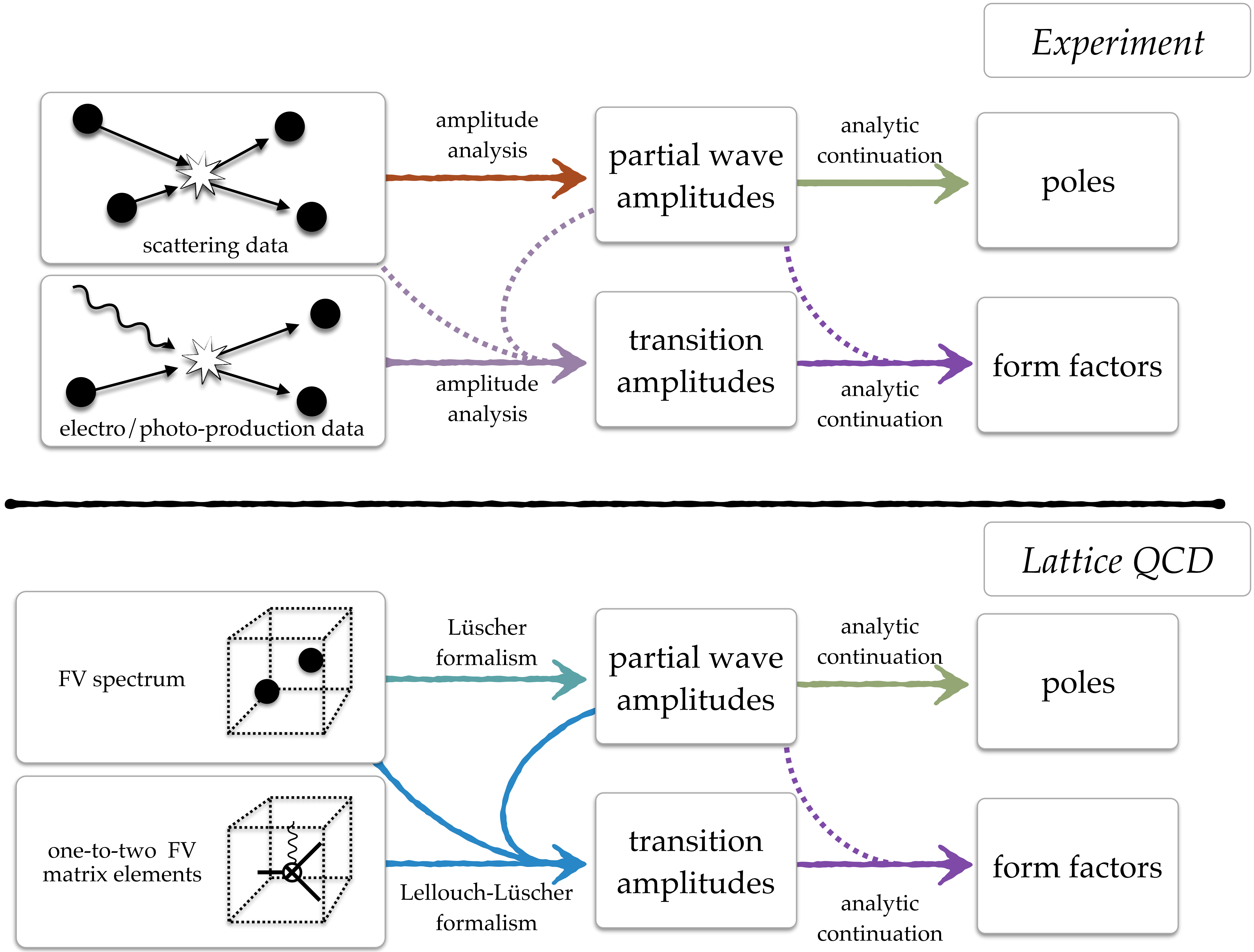


$\pi\gamma$*-to-$\pi\pi$ amplitude





$Q^2 = 0$
$Q^2 = 0.803\,\mathrm{GeV}^2$
$m_\pi \cdot |\mathcal{A}_{\pi\pi,\pi\gamma^\star}|$
$|\mathcal{M}^{\ell=1}_{\pi\pi}|$
$E^\star_{\pi\pi}/m_\pi$
Lattice QCD
FV spectrum
one-to-two FV matrix elements
Lüscher formalism
Lellouch-Lüscher formalism
partial wave amplitudes
analytic continuation
poles
transition amplitudes


Experiment
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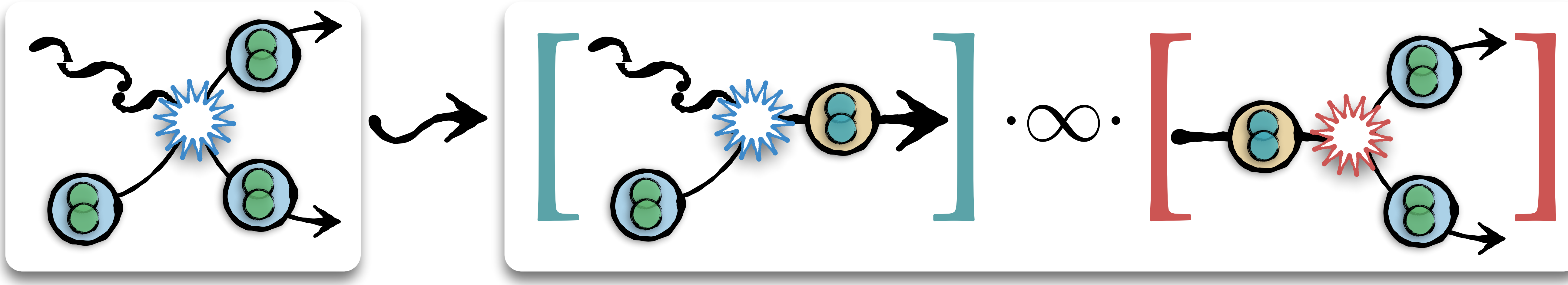
Form factor at ϱ pole

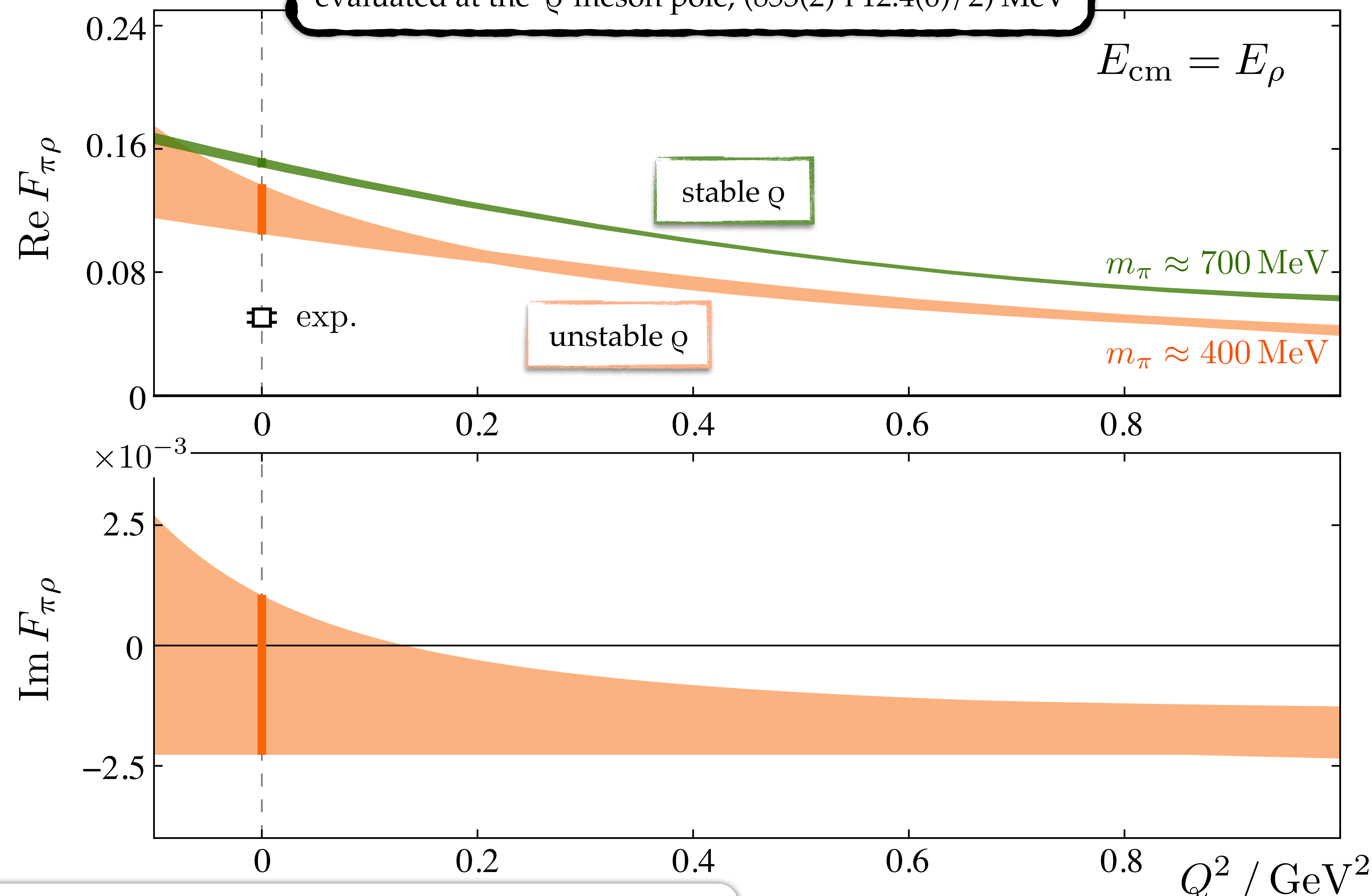
- Near the ϱ-pole, the $\pi\gamma^*$-to-$\pi\pi$ diverges
- The residue encodes the $\pi\gamma^*$-to-ϱ form factor



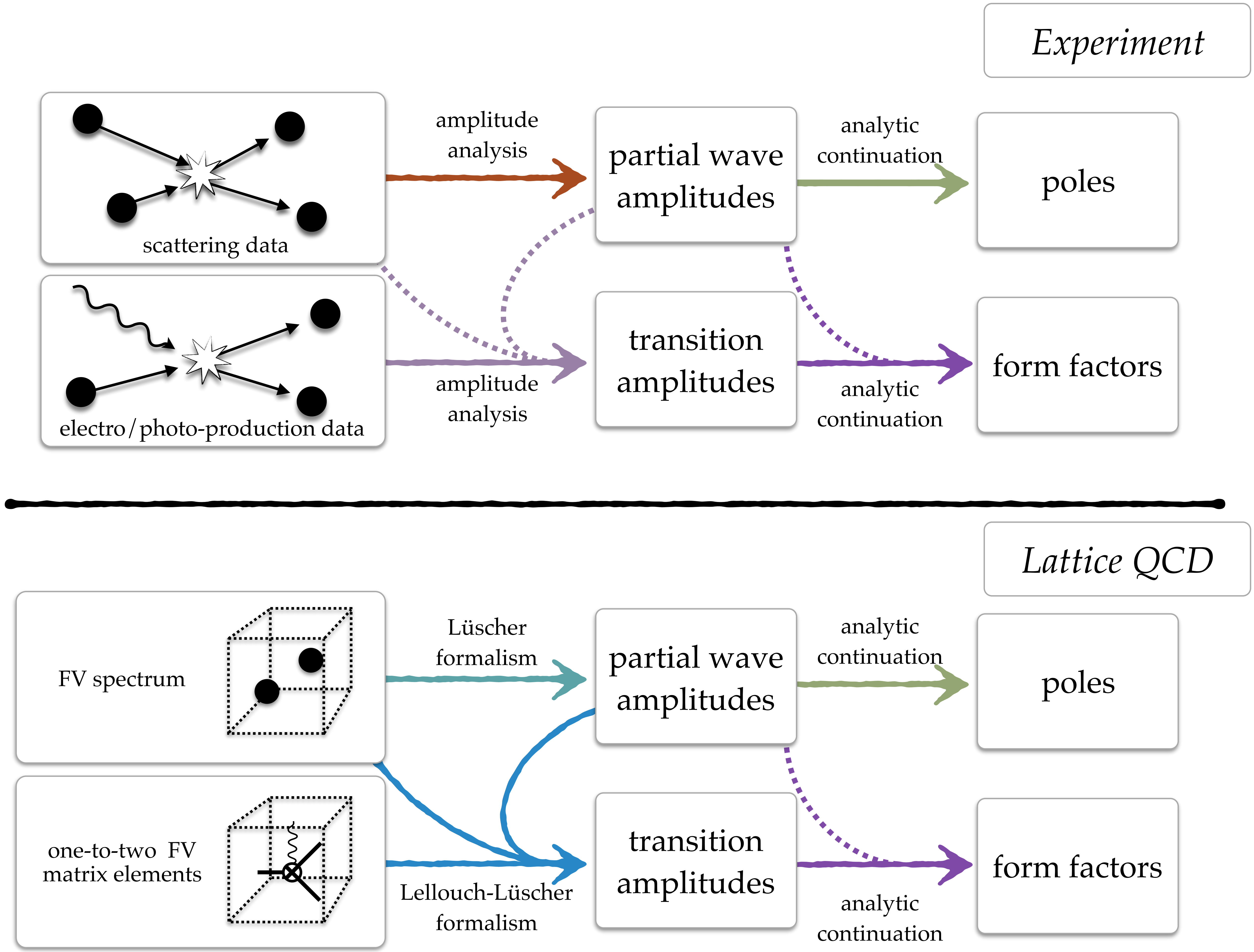
$$\mathcal{A}_{\pi\pi,\pi\gamma^\star}(E_{\pi\pi},Q^2) = F(E_{\pi\pi},Q^2) \times \left[\frac{1}{\cot\delta_1(E_{\pi\pi}) - i}\right] \times \sqrt{\frac{16\pi}{q_{\pi\pi}\,\Gamma(E_{\pi\pi})}}$$

Form factor at ϱ pole

evaluated at the ϱ-meson pole, (853(2)-i 12.4(6)/2) MeV



Shultz, Dudek, & Edwards (2014)
RB, Dudek, Edwards, Shultz, Thomas & Wilson (2015)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Outlook for the future

- Necessity for formalism
- Lattice can do much more than experiment
  - track poles as a function of m_π ✓ TOP SECRET
  - Form factors of unstable particles
  - three-particle scattering

Outlook for the future

- Necessity for formalism
- Lattice can do much more than experiment
  - track poles as a function of m_π ✓ TOP SECRET
  - [Form factors of unstable particles]
  - three-particle scattering

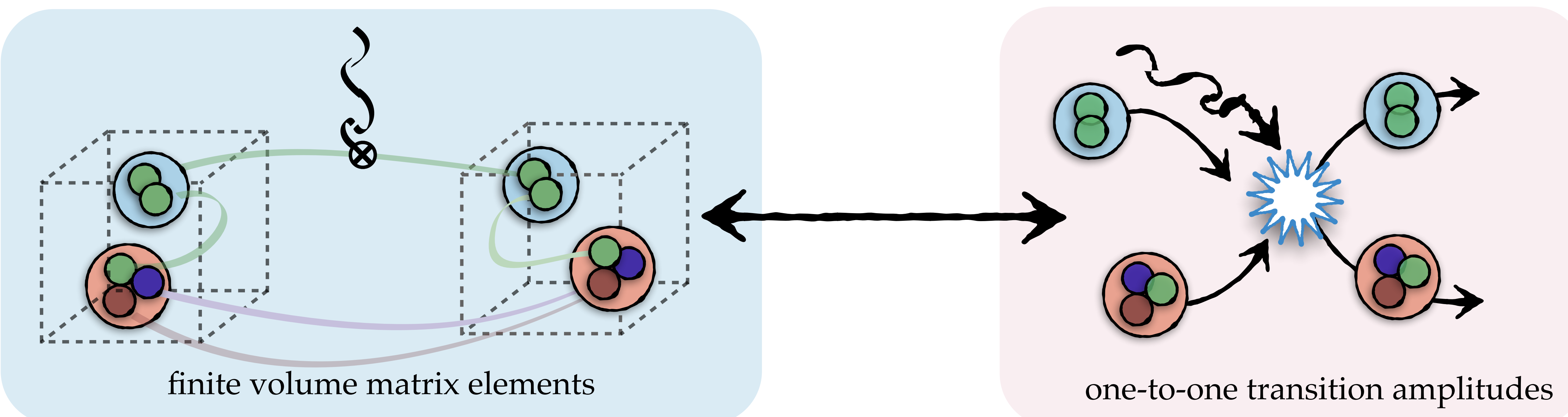
- electroweak, scalar,…, form factors:
  - resonances
  - NN, N-Hyperon,…

RB & Hansen (2015)

2-to-2 Matrix elements

$$|\langle \mathbf{2}|\mathcal{J}|\mathbf{2}\rangle_L| = \frac{1}{\sqrt{L^3}}\sqrt{\mathrm{Tr}\left[\mathcal{R}\ \mathcal{W}_{L,\mathrm{df}}\ \mathcal{R}\ \mathcal{W}_{L,\mathrm{df}}\right]}$$

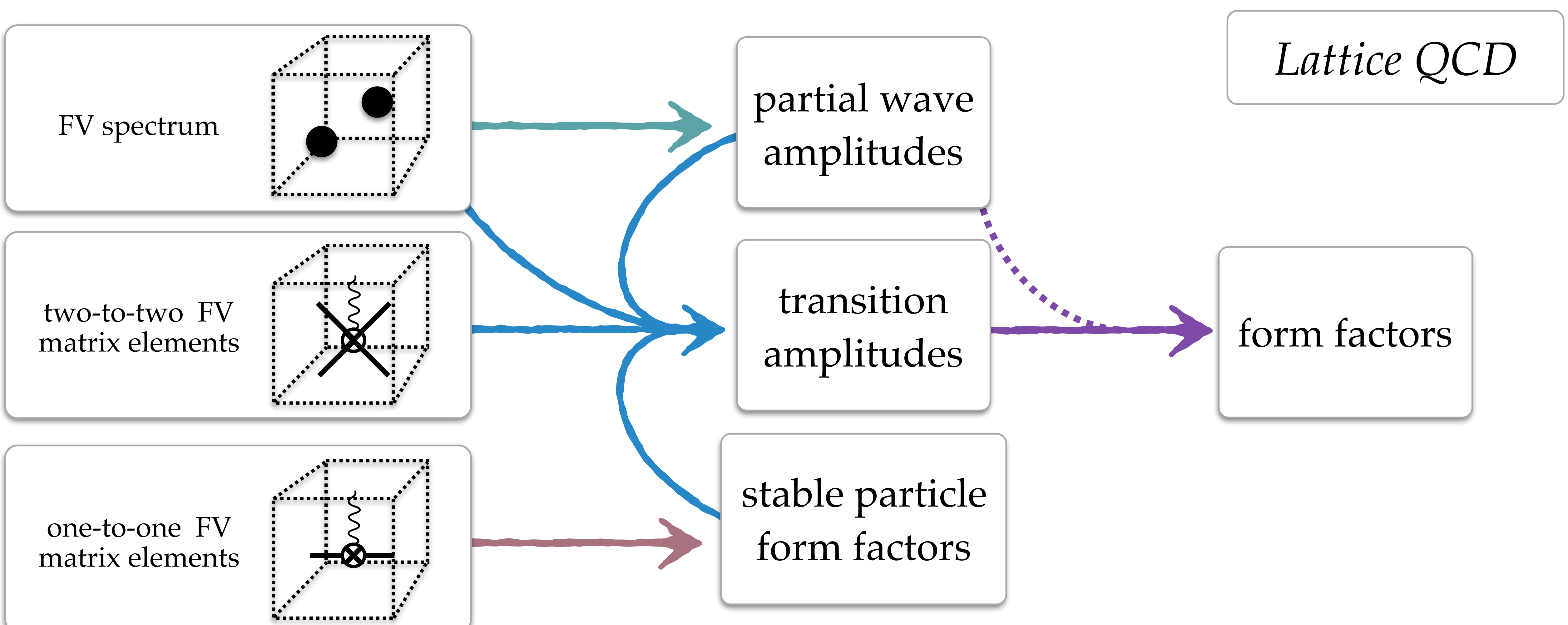
RB & Hansen (2015)



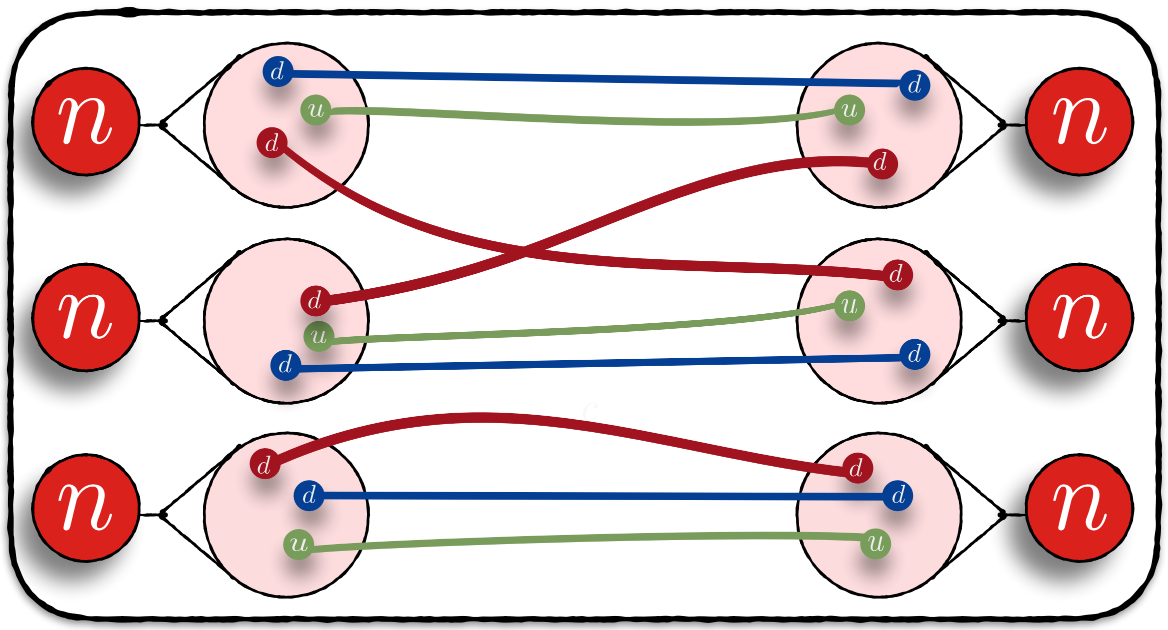
see also RB & Davoudi (2012), Bernard, Hoja, Meißner, Rusetsky (2012)

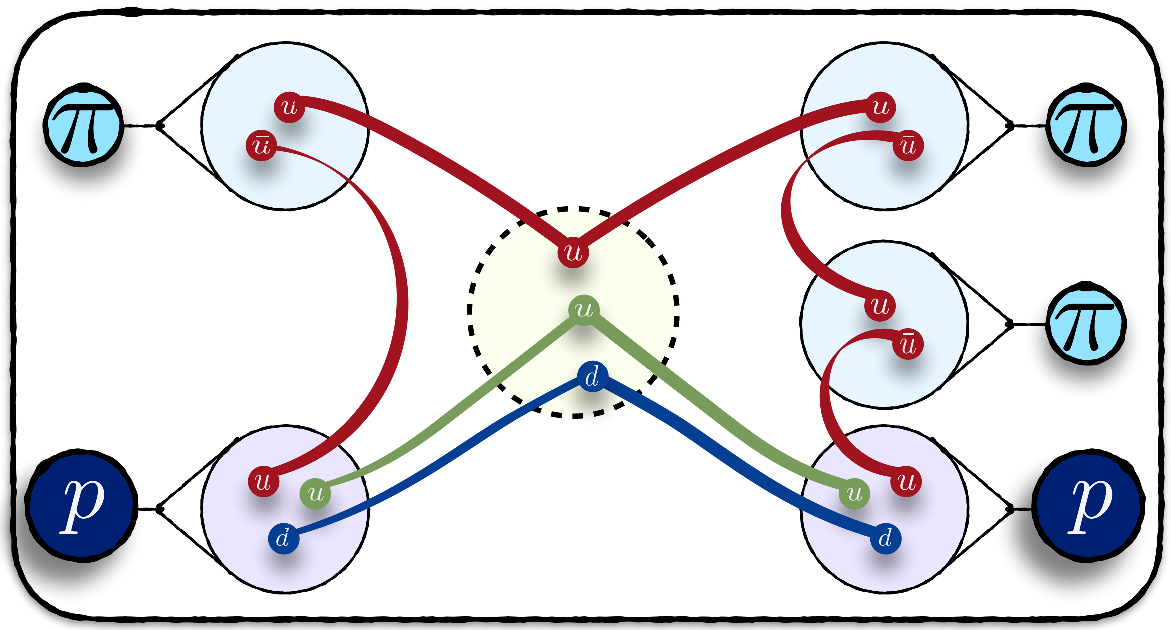
Outlook for the future

- Necessity for formalism
- Lattice can do much more than experiment
  - track poles as a function of m_π ✓
  - Form factors of unstable particles ✓
  - three-particle scattering



On-going challenge - three-body

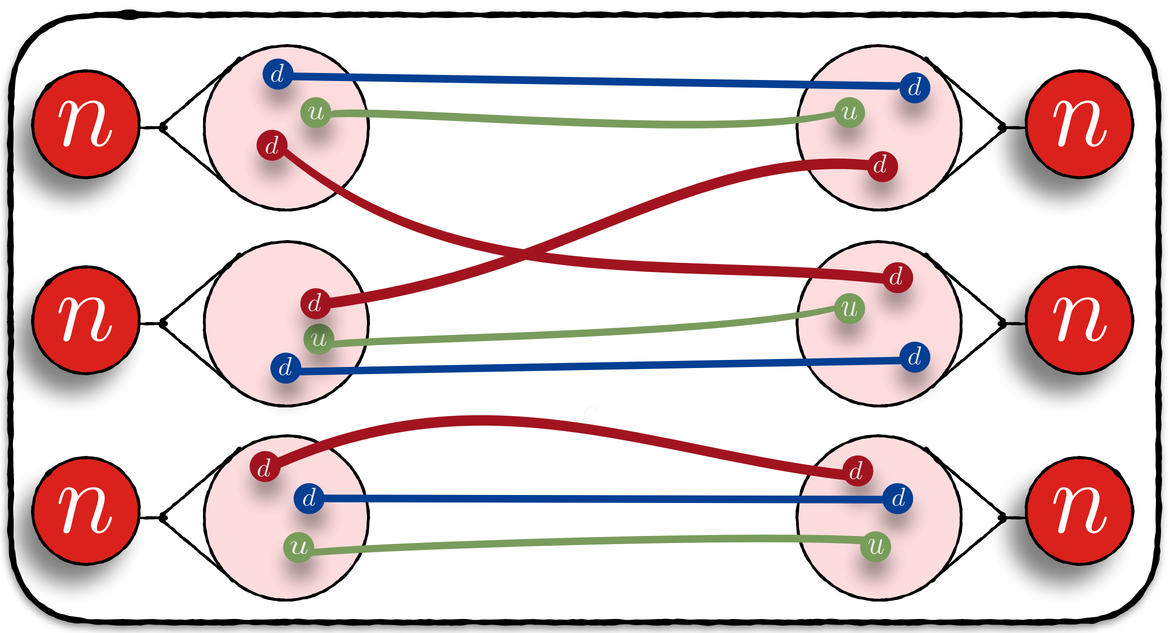


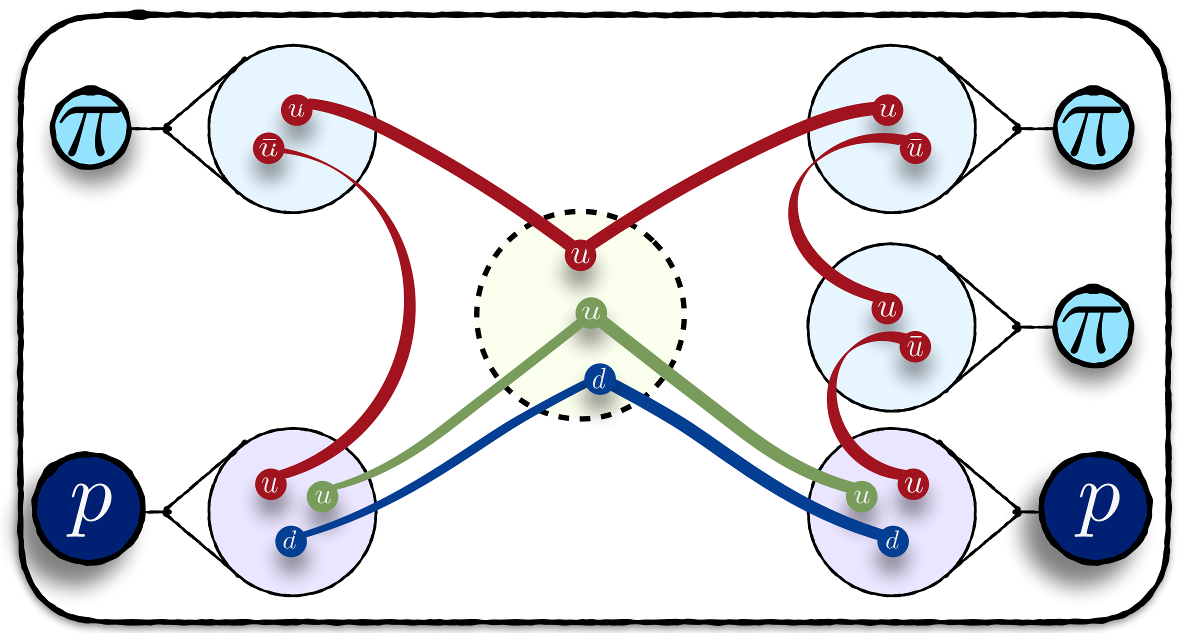


More contractions,
more channels, etc.

Formal open question,
Harder to analyze

On-going challenge - three-body



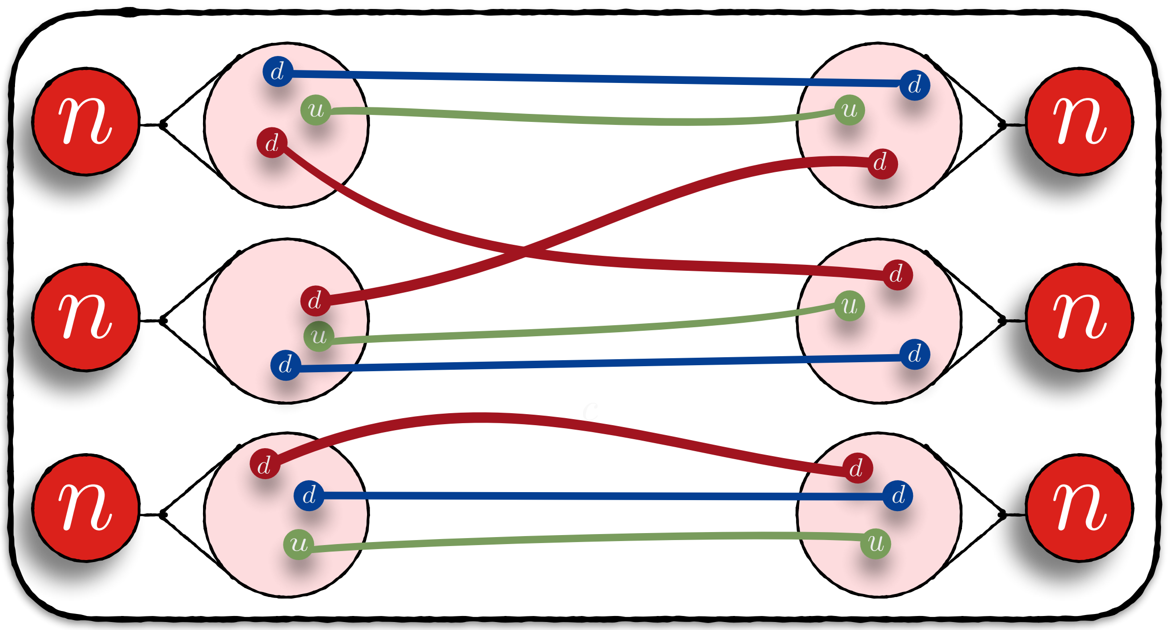


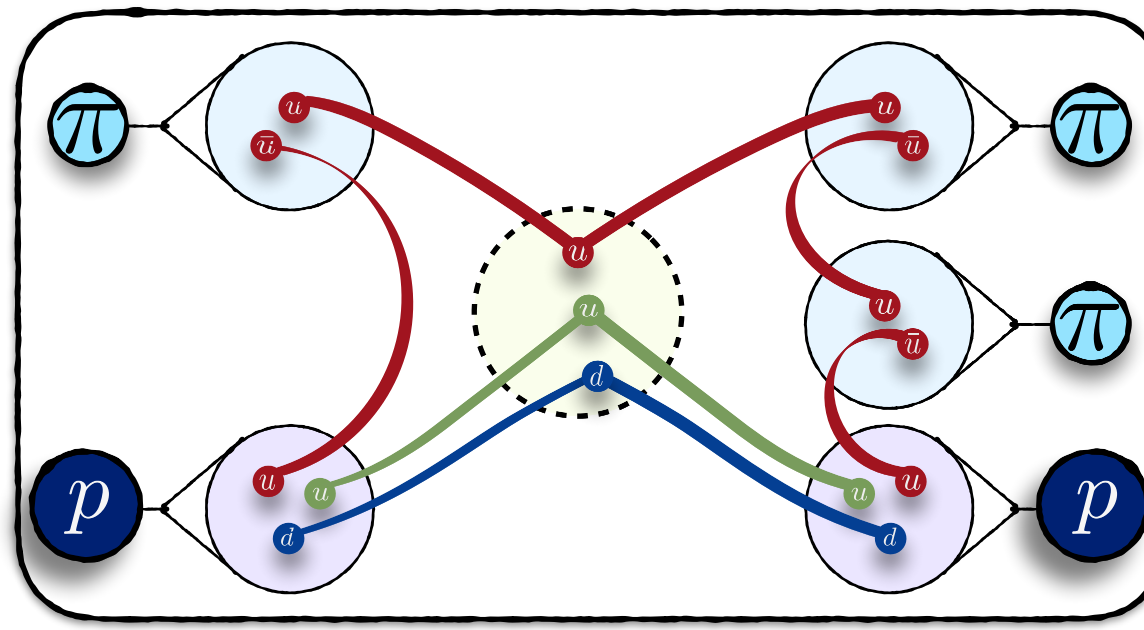
obtaining FV spectrum is harder, but doable

[More contractions, more channels, etc.]

[Formal open question, Harder to analyze]

On-going challenge - three-body





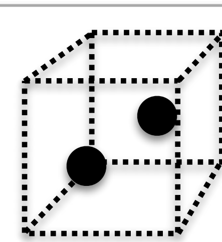
obtaining FV spectrum is harder, but doable

[More contractions, more channels, etc.]

[Formal open question, Harder to analyze]

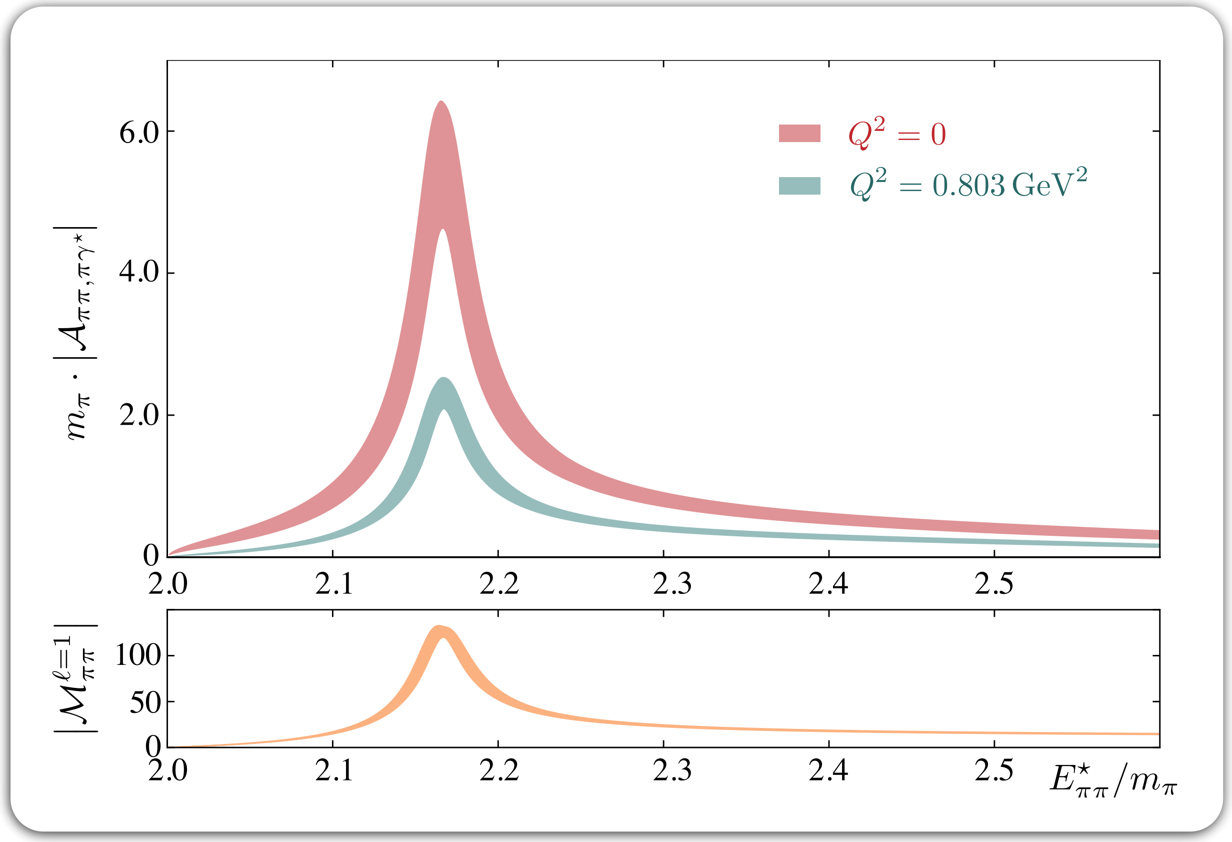
had we had the spectrum, we don't know in general what to do with it

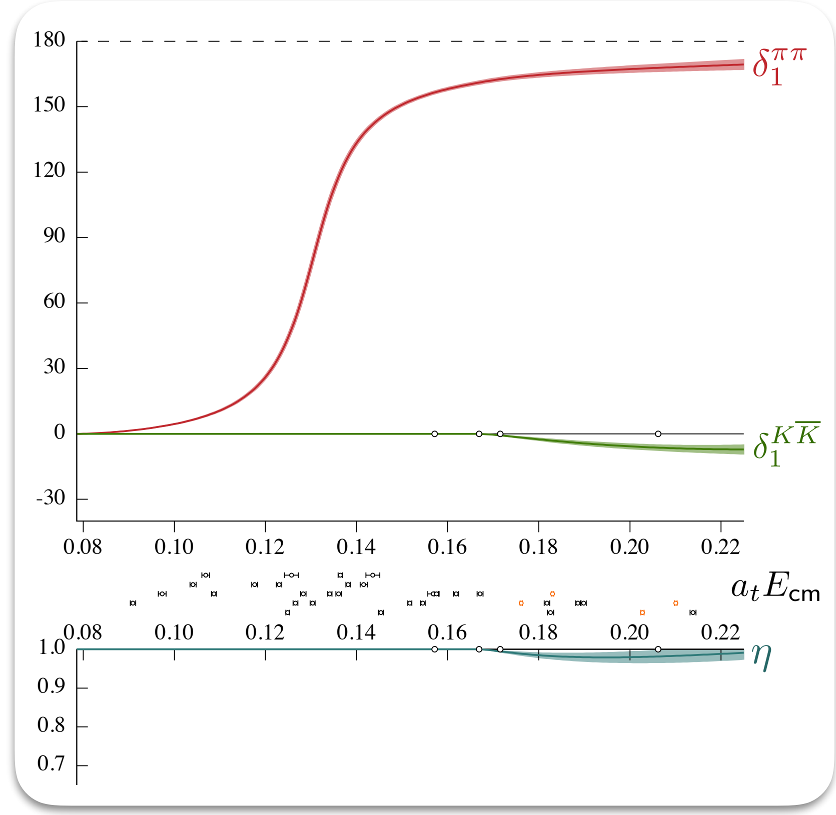
most promising progress by Hansen & Sharpe ['14-'15]

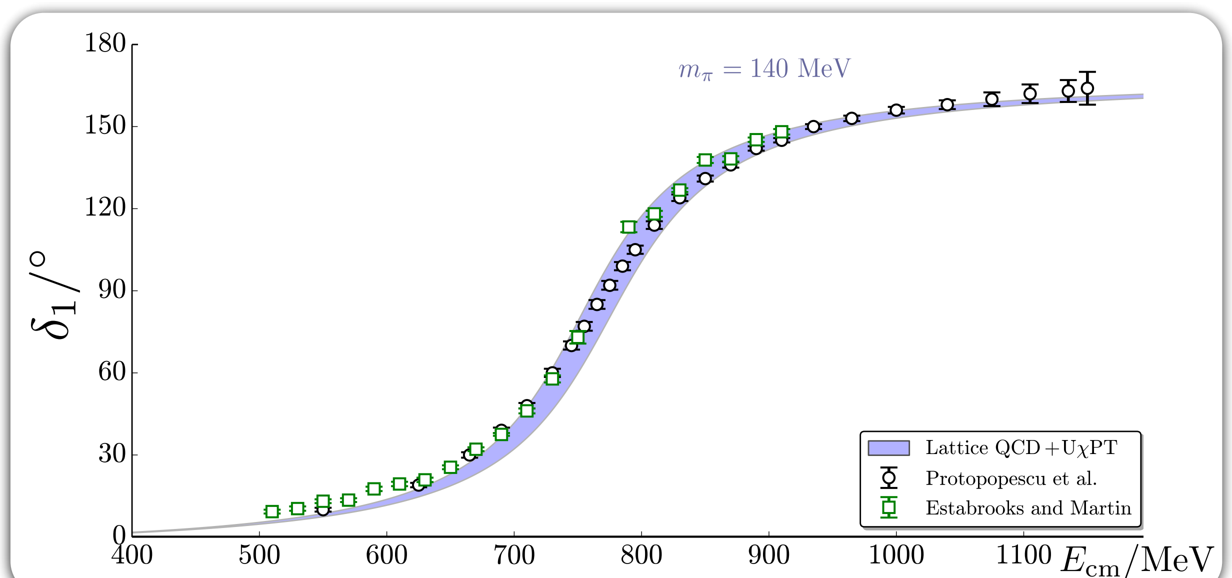
FV spectrum  **???** → **partial wave amplitudes**

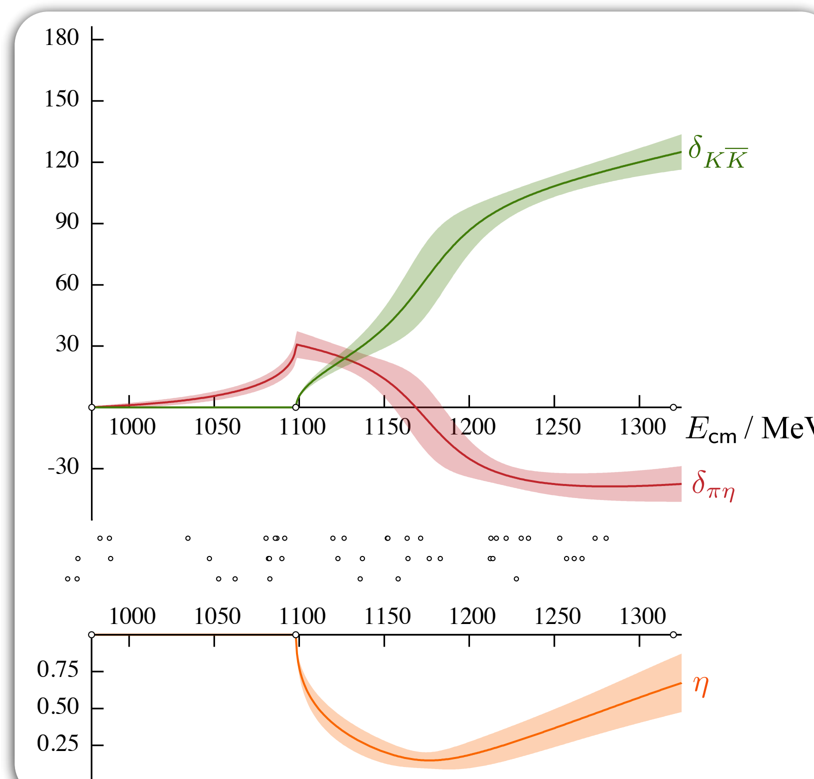
see Max's talk

The big picture!

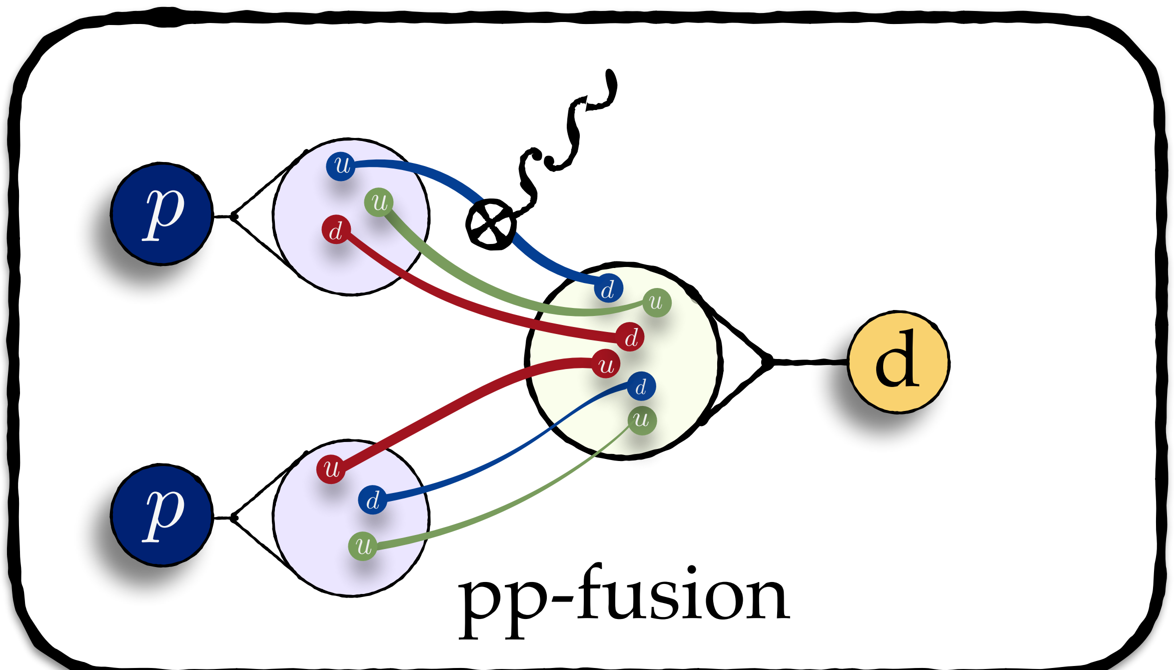


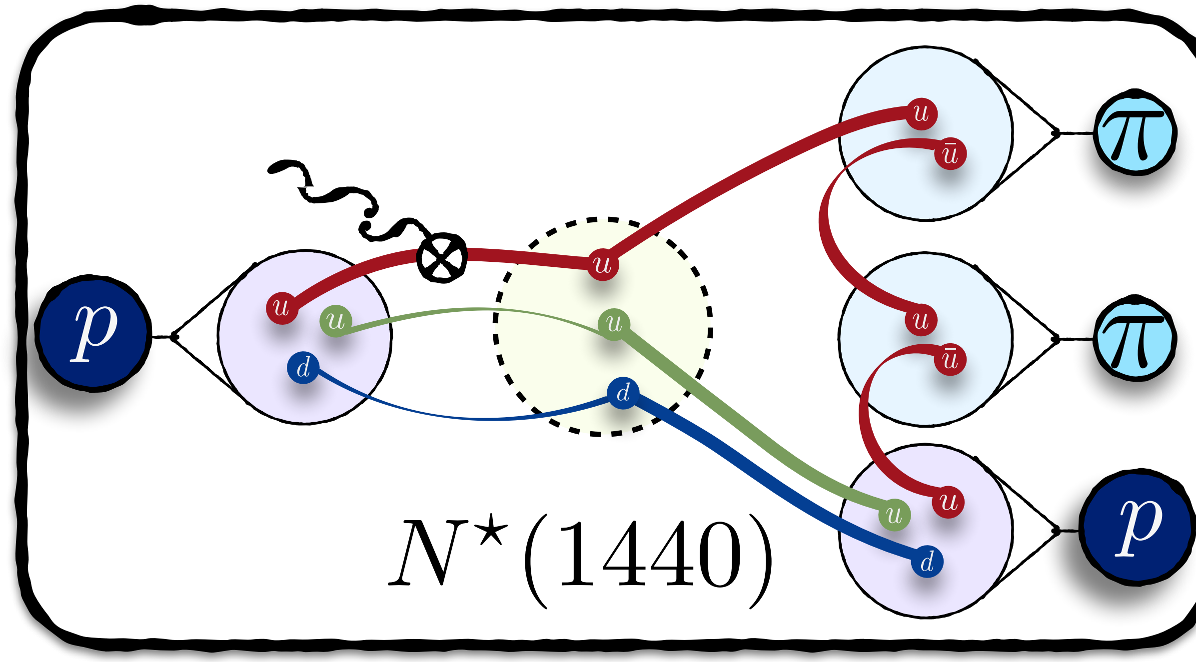


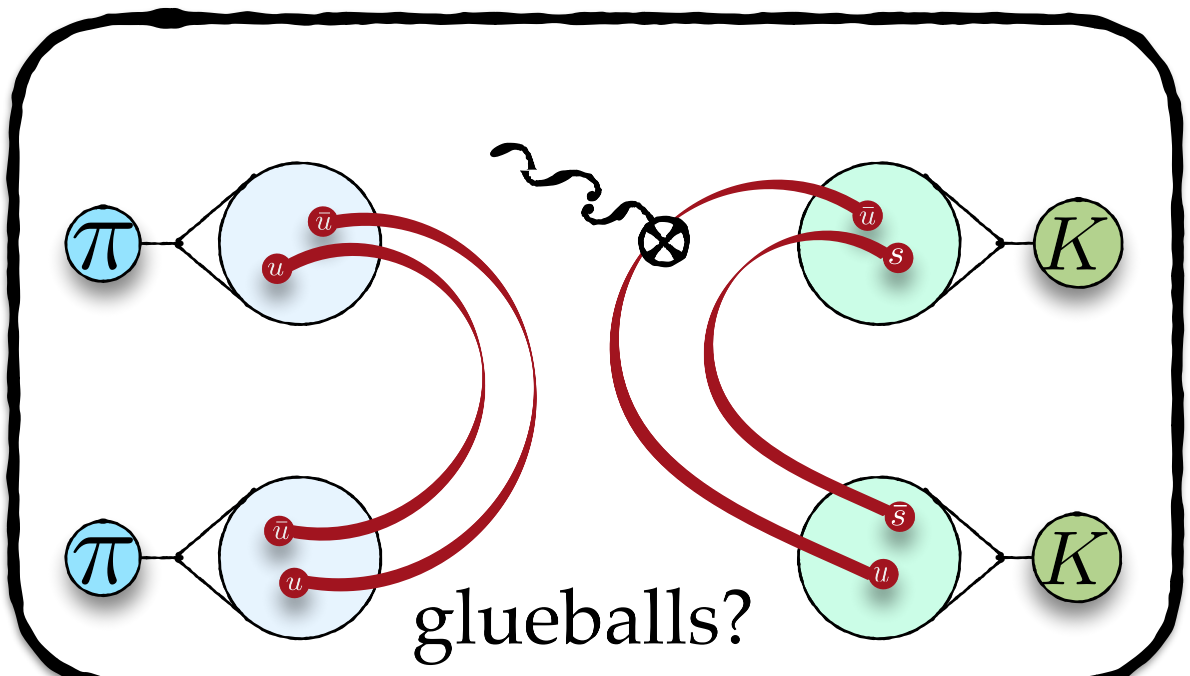


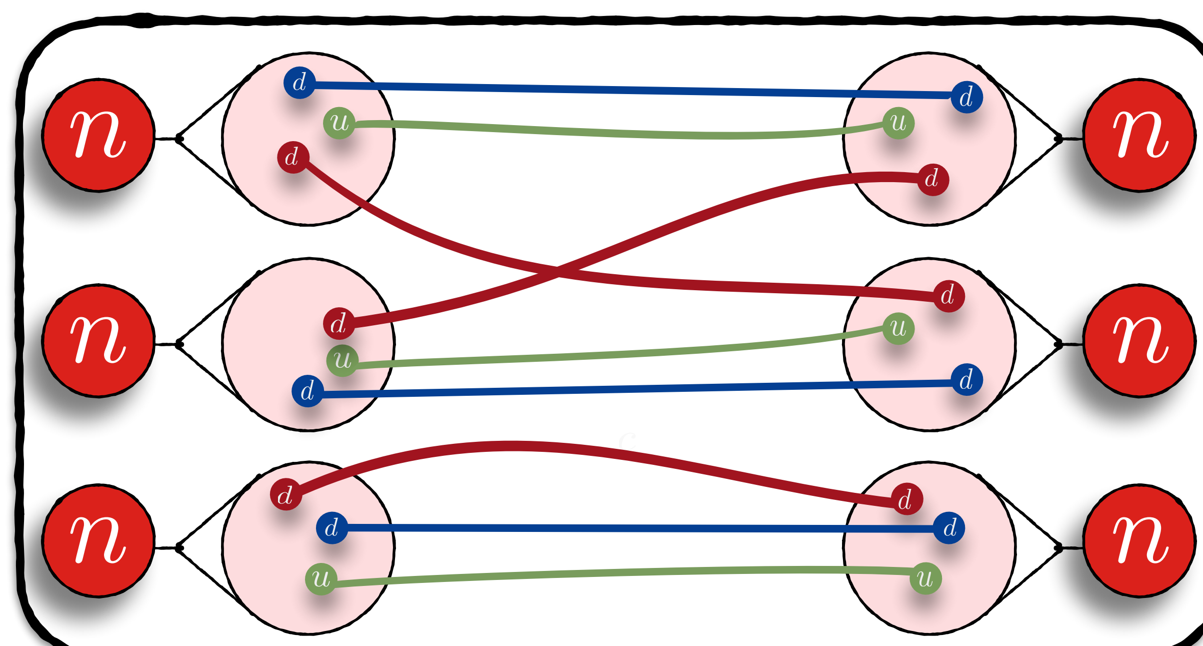


The big picture!









Collaborators

formalism

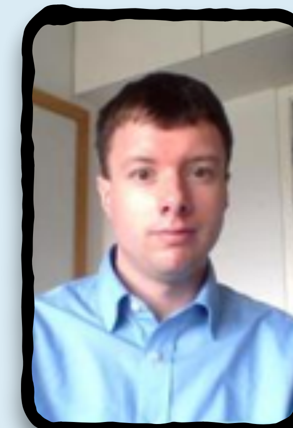
Hansen

Walker-Loud

numerical

Wilson

Shultz



Thomas

Dudek

Edwards

HadSpec Collaboration

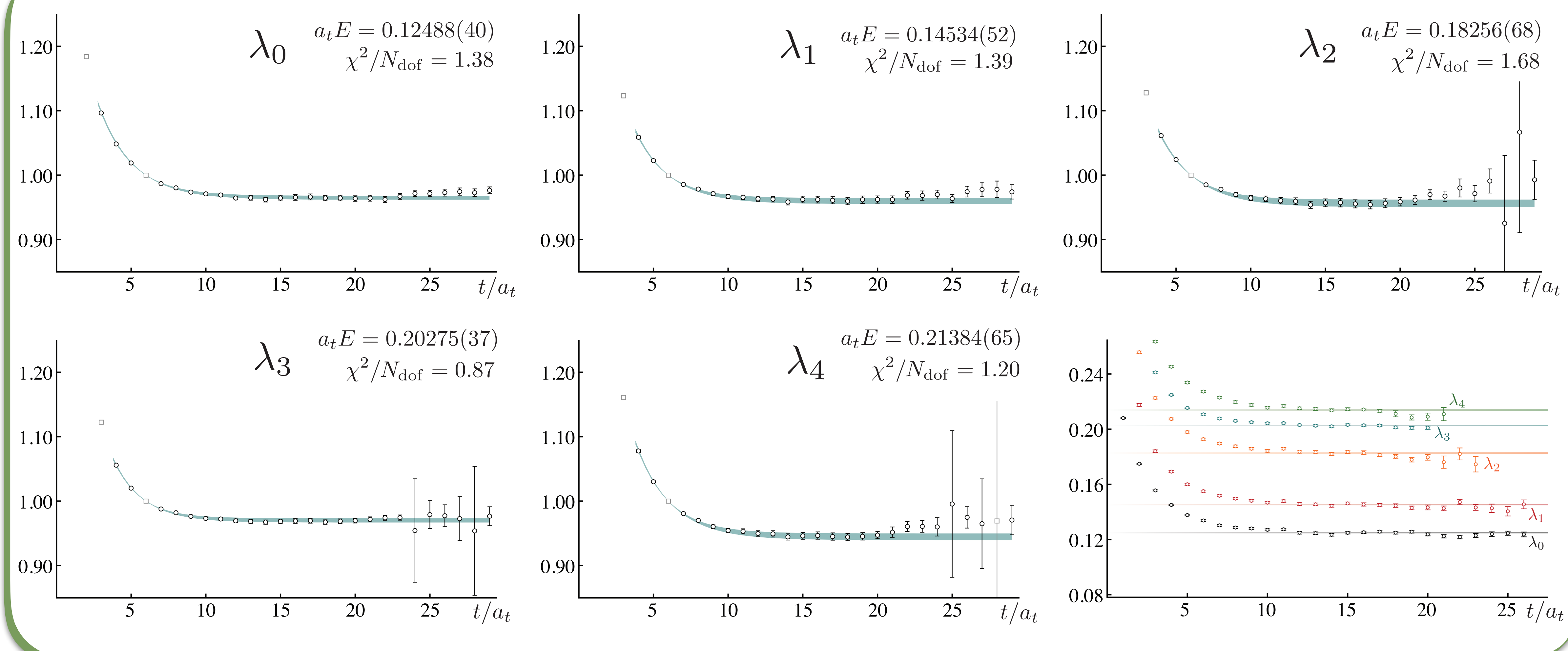
Bolton

Back-up slides

Determining spectrum

$$C(t)v_n(t) = \lambda_n(t)C(t_0)v_n(t),$$
$$\lambda_n(t) \sim e^{-E_n(t-t_0)}$$

$[000]\ T_1^-$



Parametrization

$$t(s) = \frac{1}{\rho(s)} \frac{\sqrt{s}\,\Gamma(s)}{m_R^2 - s - i\sqrt{s}\,\Gamma(s)},$$

$$\Gamma(s) = \frac{g_R^2}{6\pi} \frac{k^3}{s}$$

$$t_{ij}^{-1}(s) = \frac{1}{(2k_i)^\ell} K_{ij}^{-1}(s) \frac{1}{(2k_j)^\ell} + I_{ij}(s)\,,$$

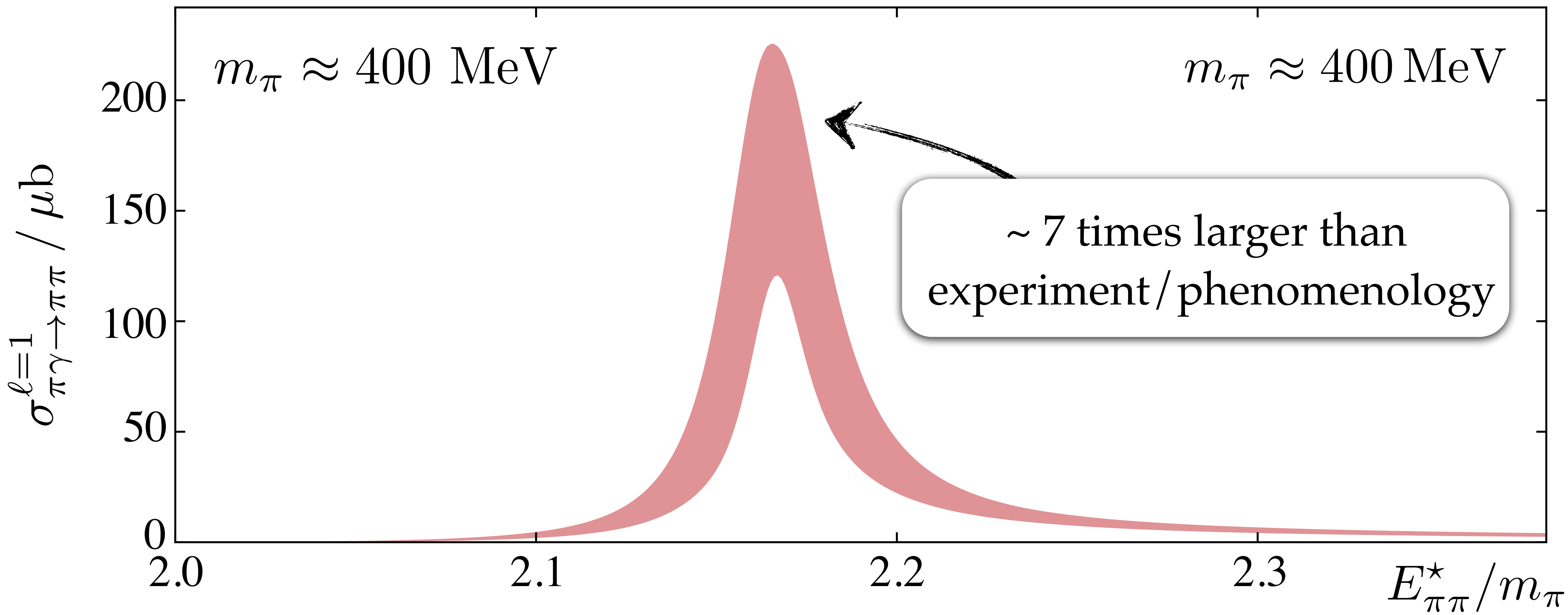
$$\mathrm{Im}\, I_{ij}(s) = -\delta_{ij}\, \rho_i(s)$$

$$K_{ij}(s) = \frac{g_i\, g_j}{m^2 - s} + \sum_{n=0}^{N} \gamma_{ij}^{(n)} \left(\frac{s}{s_0}\right)^n,$$

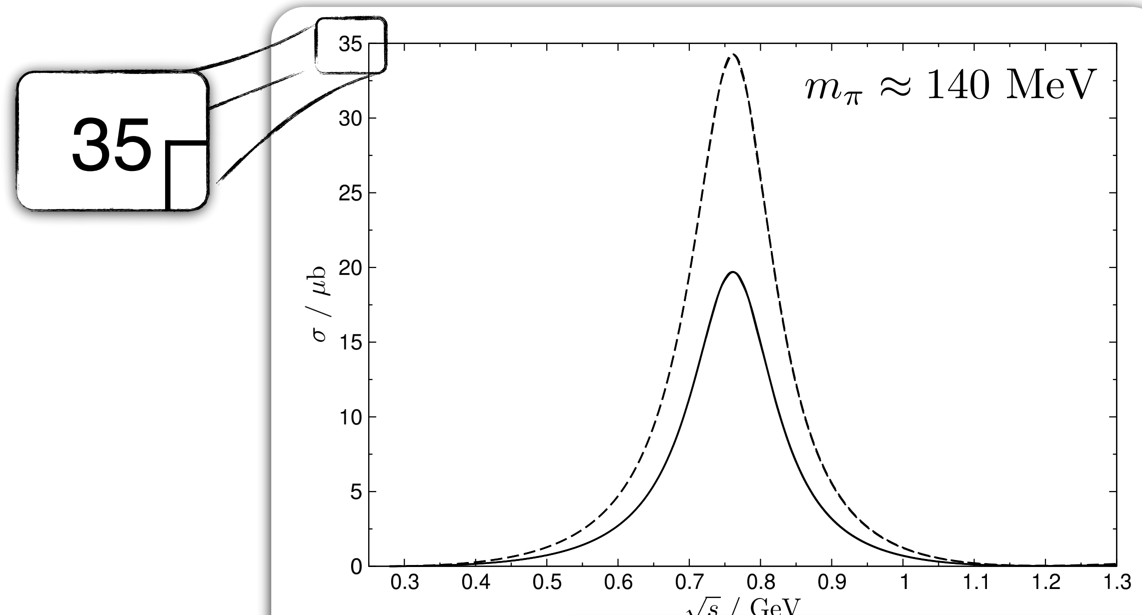
$$K_{ij}^{-1} = \sum_{m=0}^{M} c_{ij}^{(m)} s^m,$$

Comparison with phenomenology

$\pi\gamma$-to-$\pi\pi$ cross section



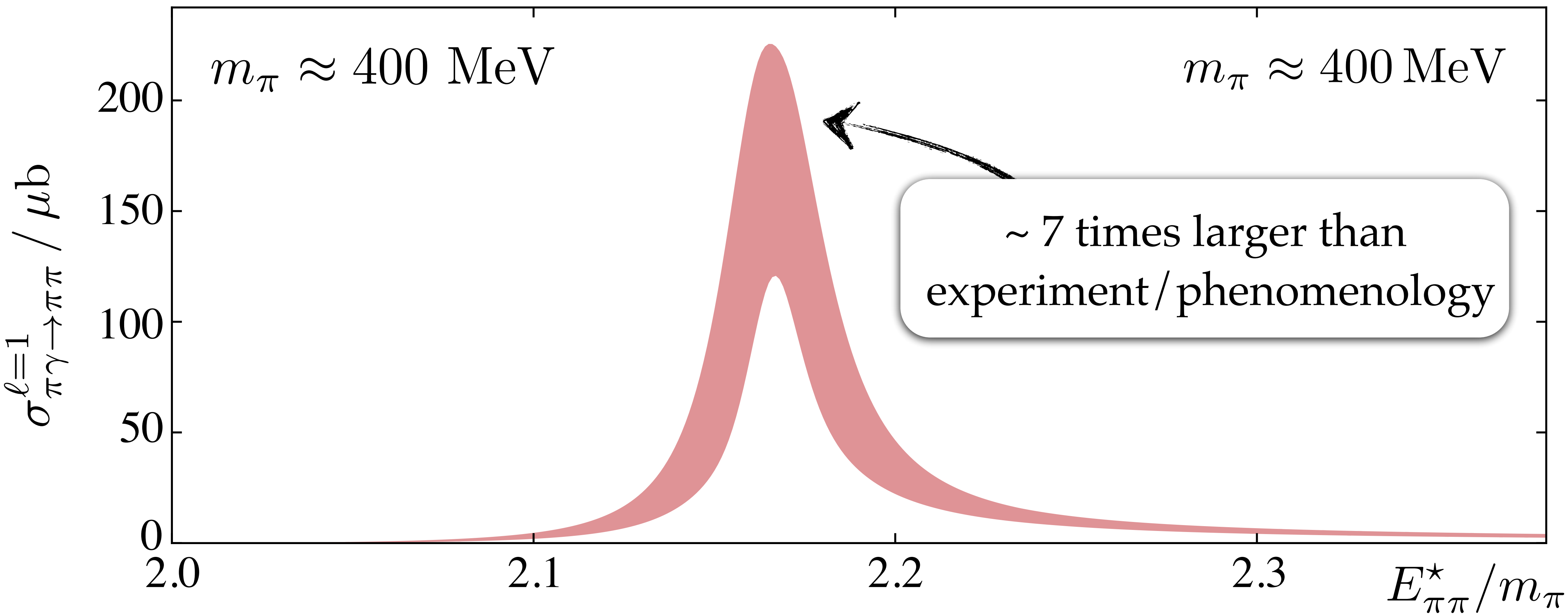
~ 7 times larger than experiment / phenomenology



non trivial quark-mass dependence!

Hoferichter, Kubis, & Sakkas (2012)

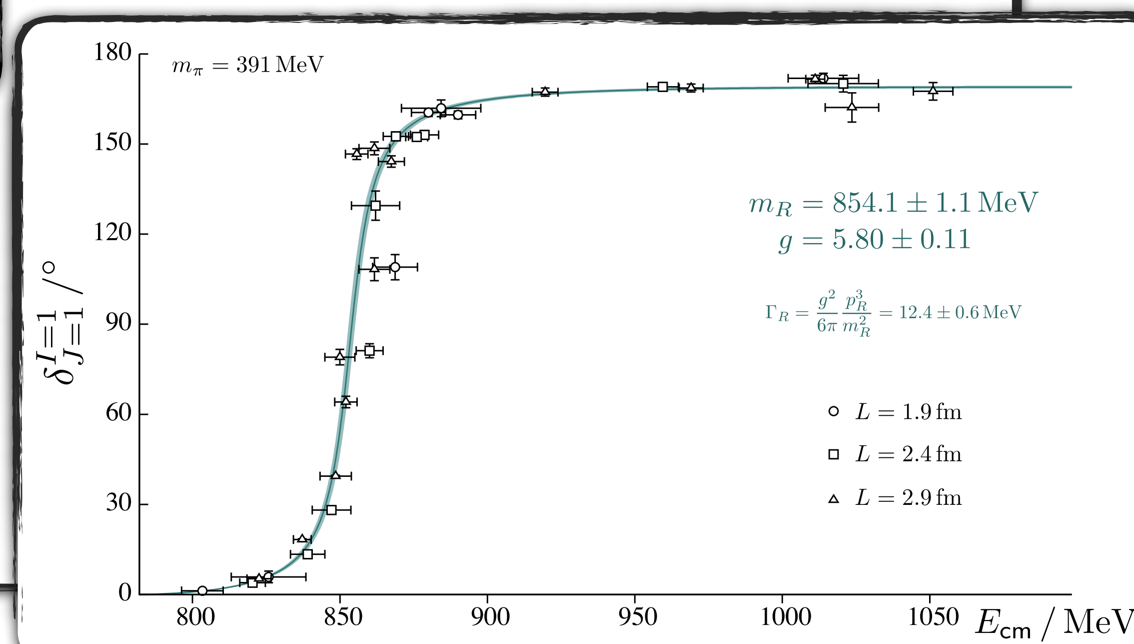
$\pi\gamma$-to-$\pi\pi$ cross section



$$\lim_{E^\star_{\pi\pi}\to m_\rho} \sigma(\pi^+\gamma \to \pi^+\pi^0) \propto \frac{q^\star_{\pi\gamma}\, F^2_{\pi\rho}(m_\rho, 0)}{m_\pi^2} \times \frac{1}{\Gamma_1(m_\rho)}$$

0.60 x (physical)

12 x (physical)



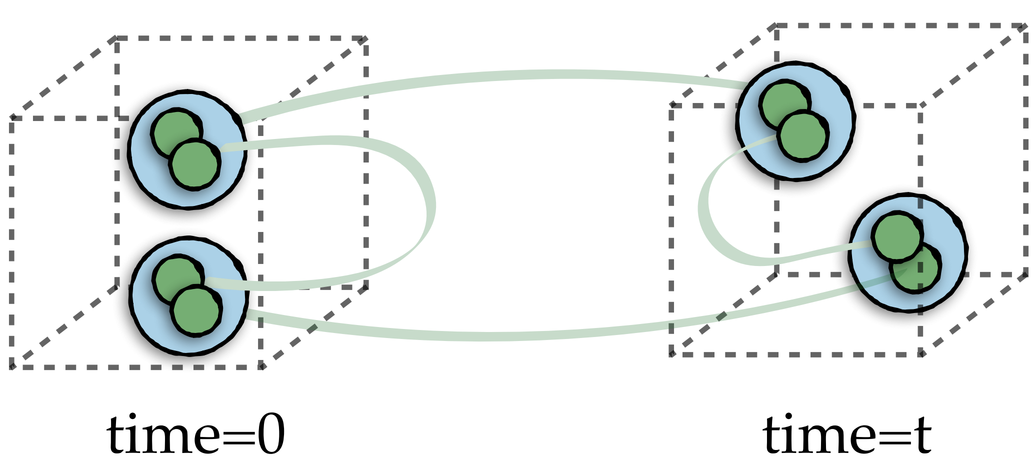
On determining correlation function using small basis of operators

Extracting the spectrum

Two-point correlation functions:

$$
\begin{aligned}
C_{ab}^{2pt.}(t,\mathbf{P}) &\equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle \\
&= \sum_n \langle 0|\mathcal{O}_b(t,\mathbf{P})|n,L\rangle\langle n,L|\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle \\
&= \sum_n \langle 0|e^{t\hat{H}_{QCD}}\mathcal{O}_b(0,\mathbf{P})e^{-t\hat{H}_{QCD}}|n,L\rangle\langle n,L|\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle \\
&= \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t}
\end{aligned}
$$

spectrum

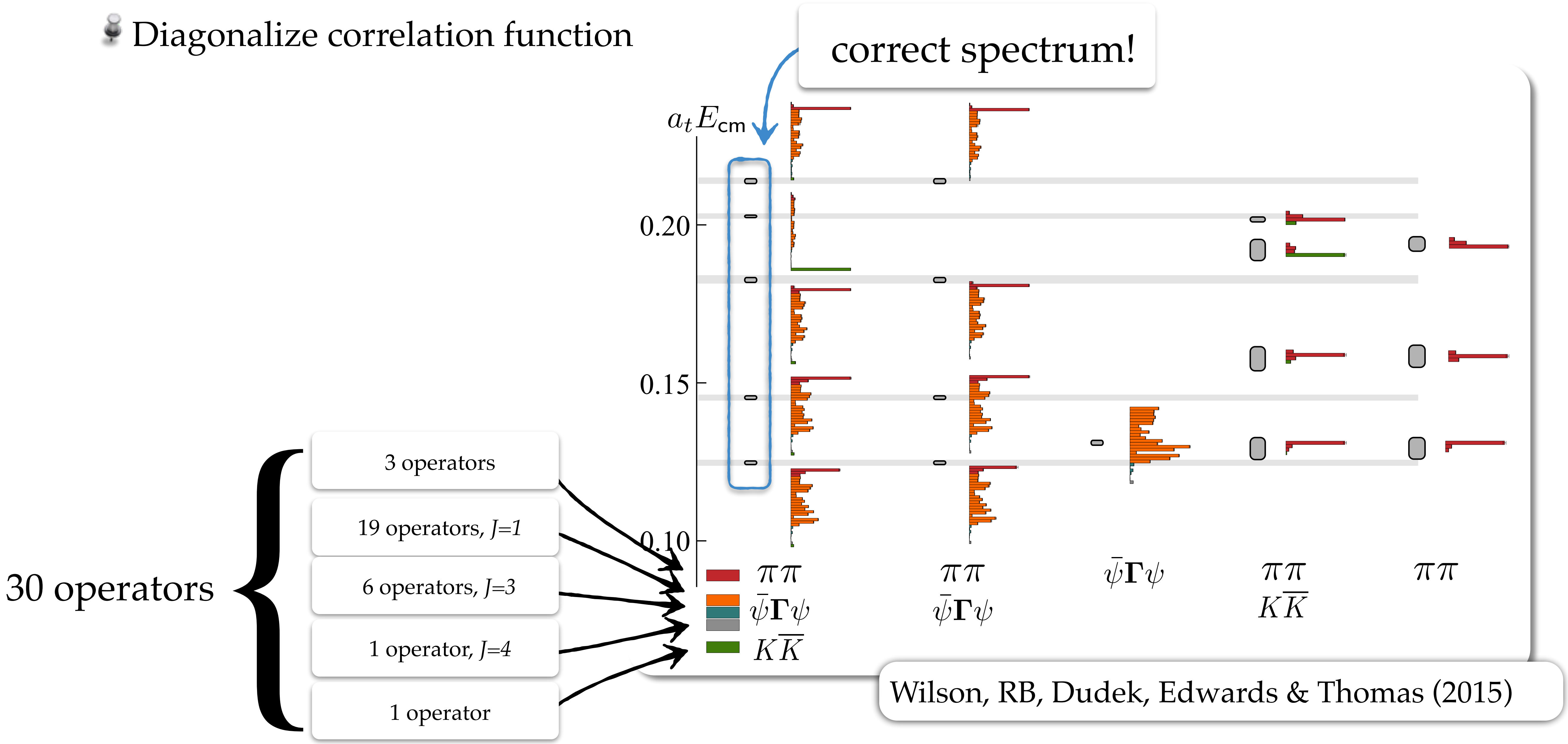


Extracting the spectrum

Two-point correlation functions:

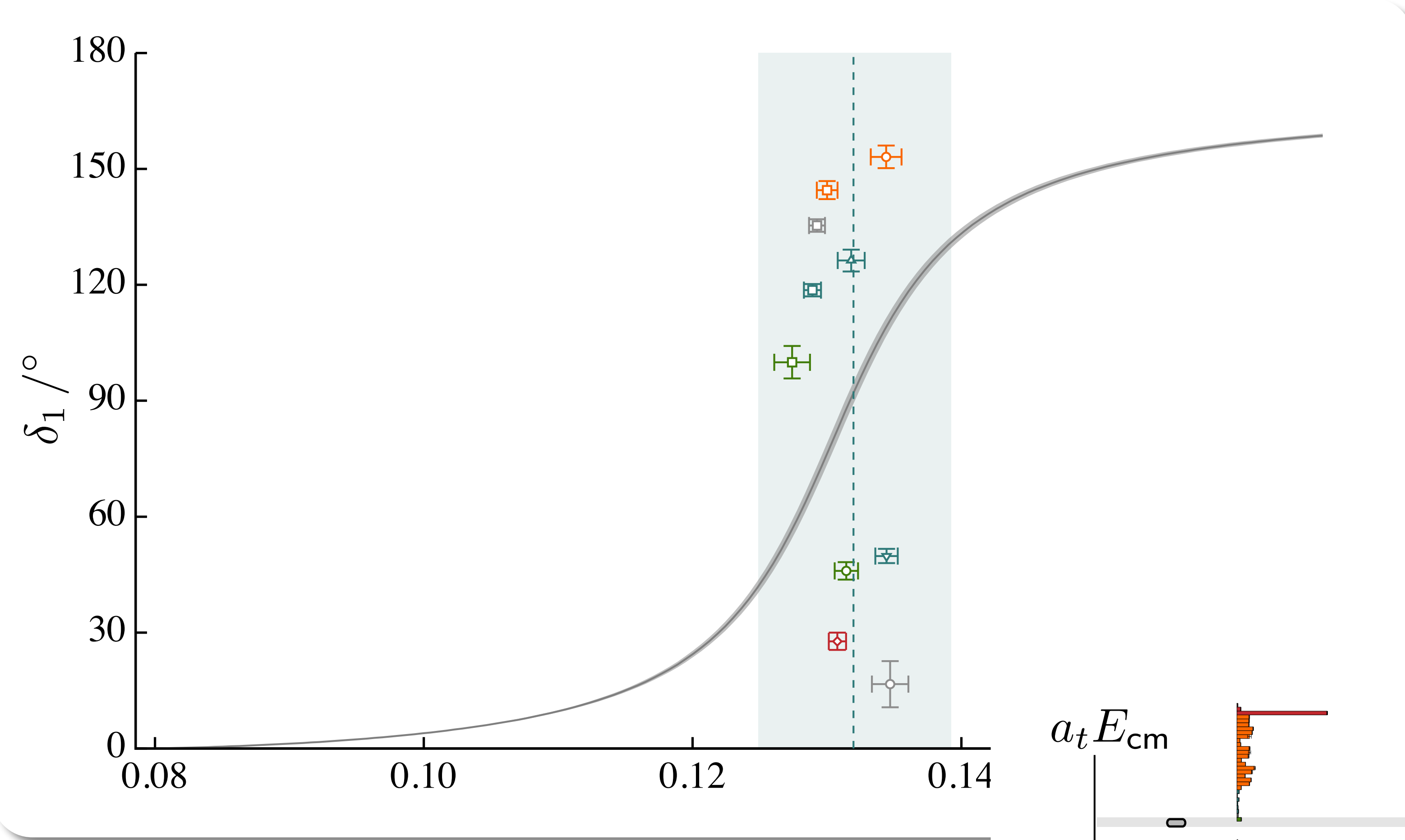
$$C_{ab}^{2pt.}(t, \mathbf{P}) \equiv \langle 0 | \mathcal{O}_b(t, \mathbf{P}) \mathcal{O}_a^\dagger(0, -\mathbf{P}) | 0 \rangle = \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t}$$

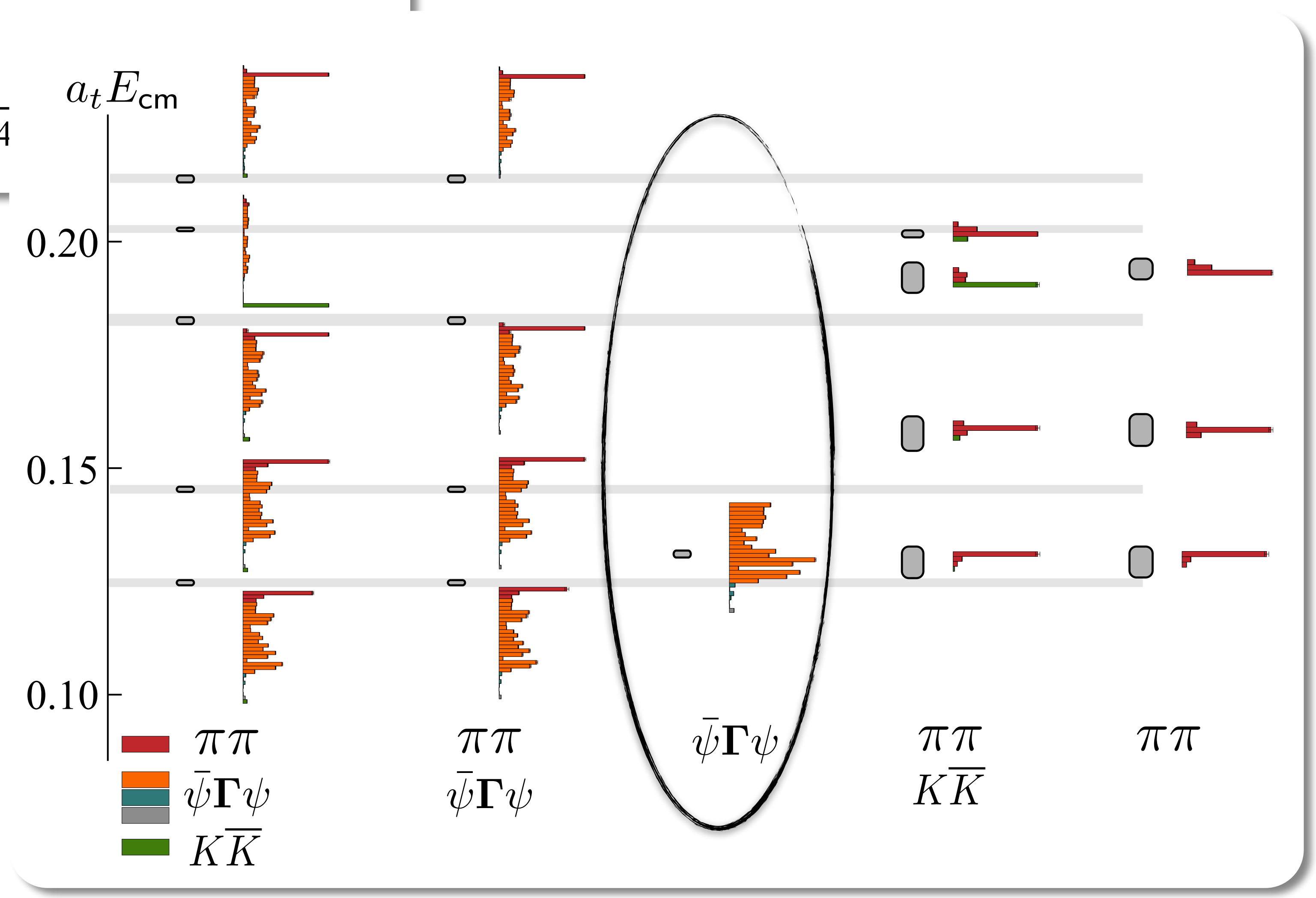
- Use a large basis of operators with the same quantum numbers
- Diagonalize correlation function



Wilson, RB, Dudek, Edwards & Thomas (2015)

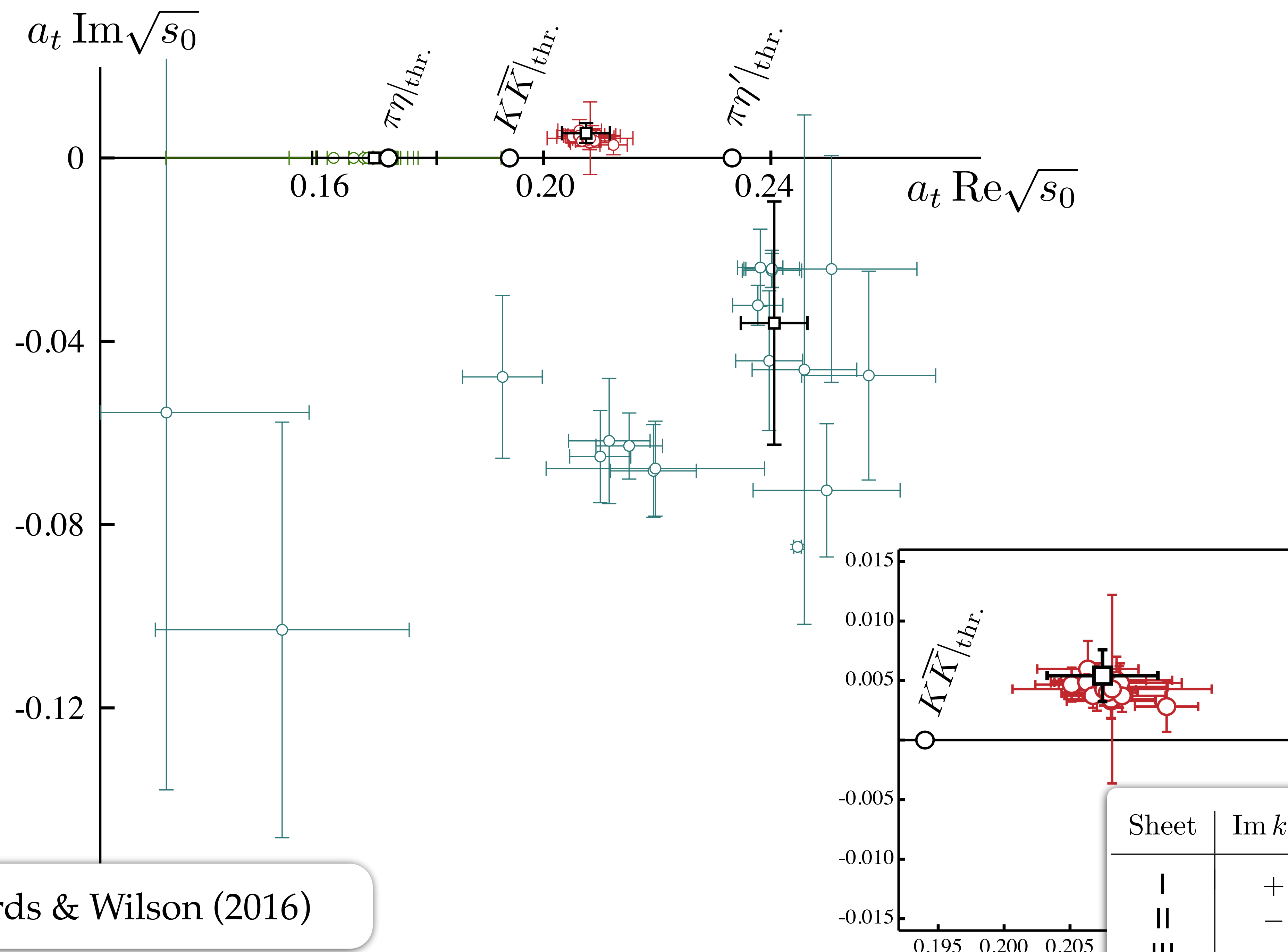
The incorrect answer





$a_0(980)$ poles

πη-KK-πη′ in I=1, m_π=391MeV



Sheet	Im $k_{\pi\eta}$	Im $k_{K\overline{K}}$
I	+	+
II	−	+
III	−	−
IV	+	−

[blue]
[red]

Dudek, Edwards & Wilson (2016)

RB

Unitarized χPT

$$\mathcal{M}_{\mathrm{U}\chi\mathrm{PT}} = \mathcal{M}_{\mathrm{LO}} \frac{1}{\mathcal{M}_{\mathrm{LO}} - \mathcal{M}_{\mathrm{NLO}}} \mathcal{M}_{\mathrm{LO}}$$

$$\begin{aligned}
S &= 1 + 2i\sigma\mathcal{M} \\
\mathcal{M} &= \left(\mathrm{Re}(\mathcal{M}^{-1}) - i\sigma\right)^{-1} \\
\mathcal{M}^{-1} &= \mathcal{M}_{\mathrm{LO}}^{-1} \frac{1}{1 + \mathcal{M}_{\mathrm{LO}}^{-1}\mathcal{M}_{\mathrm{NLO}} + \ldots} = \mathcal{M}_{\mathrm{LO}}^{-1}\left(1 - \mathcal{M}_{\mathrm{LO}}^{-1}\mathcal{M}_{\mathrm{NLO}} + \ldots\right) \\
\mathrm{Re}(\mathcal{M}^{-1}) &= \mathcal{M}_{\mathrm{LO}}^{-1}\left(1 - \mathcal{M}_{\mathrm{LO}}^{-1}\,\mathrm{Re}(\mathcal{M}_{\mathrm{NLO}}) + \ldots\right)
\end{aligned}$$

Dobado and Pelaez (1997)

Oller, Oset, and Pelaez (1998)

Oller, Oset, and Pelaez (1999)

LL-factor

Relationship between amplitude and "form factor":

$$\mathcal{A}_{\pi\pi,\pi\gamma^\star}(E^\star_{\pi\pi},Q^2) = \left(\frac{F(E^\star_{\pi\pi},Q^2)}{\cot\delta_1(E^\star_{\pi\pi}) - i}\right)\sqrt{\frac{16\pi}{q^\star_{\pi\pi}\,\Gamma(E^\star_{\pi\pi})}}$$

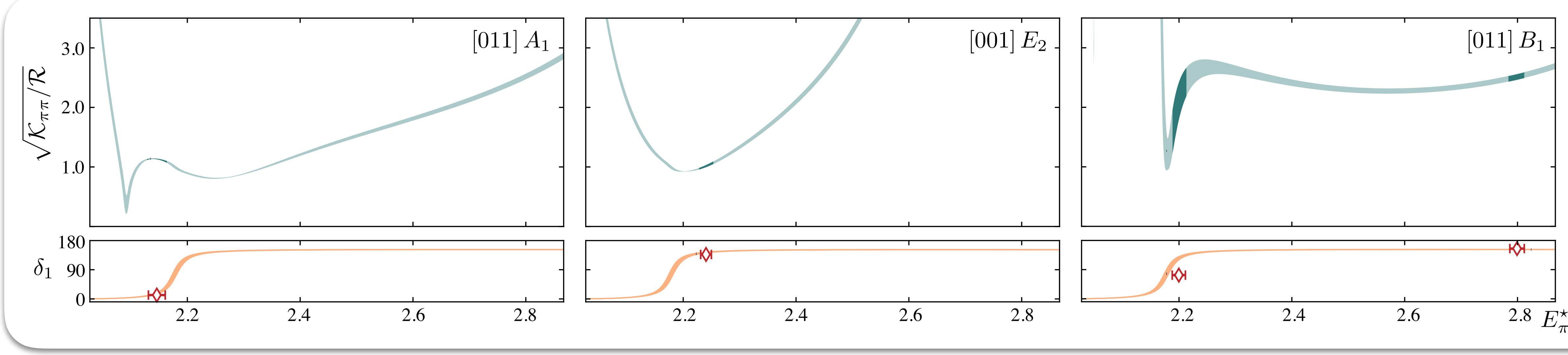
$$F(E^\star_{\pi\pi},Q^2) = \tilde{\mathcal{A}}(E^\star_{\pi\pi},Q^2;L)\sqrt{\frac{\mathcal{K}_{\pi\pi}}{\mathcal{R}}},$$

$$\frac{1}{\sqrt{2E^\star_{\pi\pi}\,\mathcal{K}_{\pi\pi}(E^\star_{\pi\pi})}} = \sin\delta_1(E^\star_{\pi\pi})\sqrt{\frac{16\pi}{q^\star_{\pi\pi}\,\Gamma(E^\star_{\pi\pi})}}$$

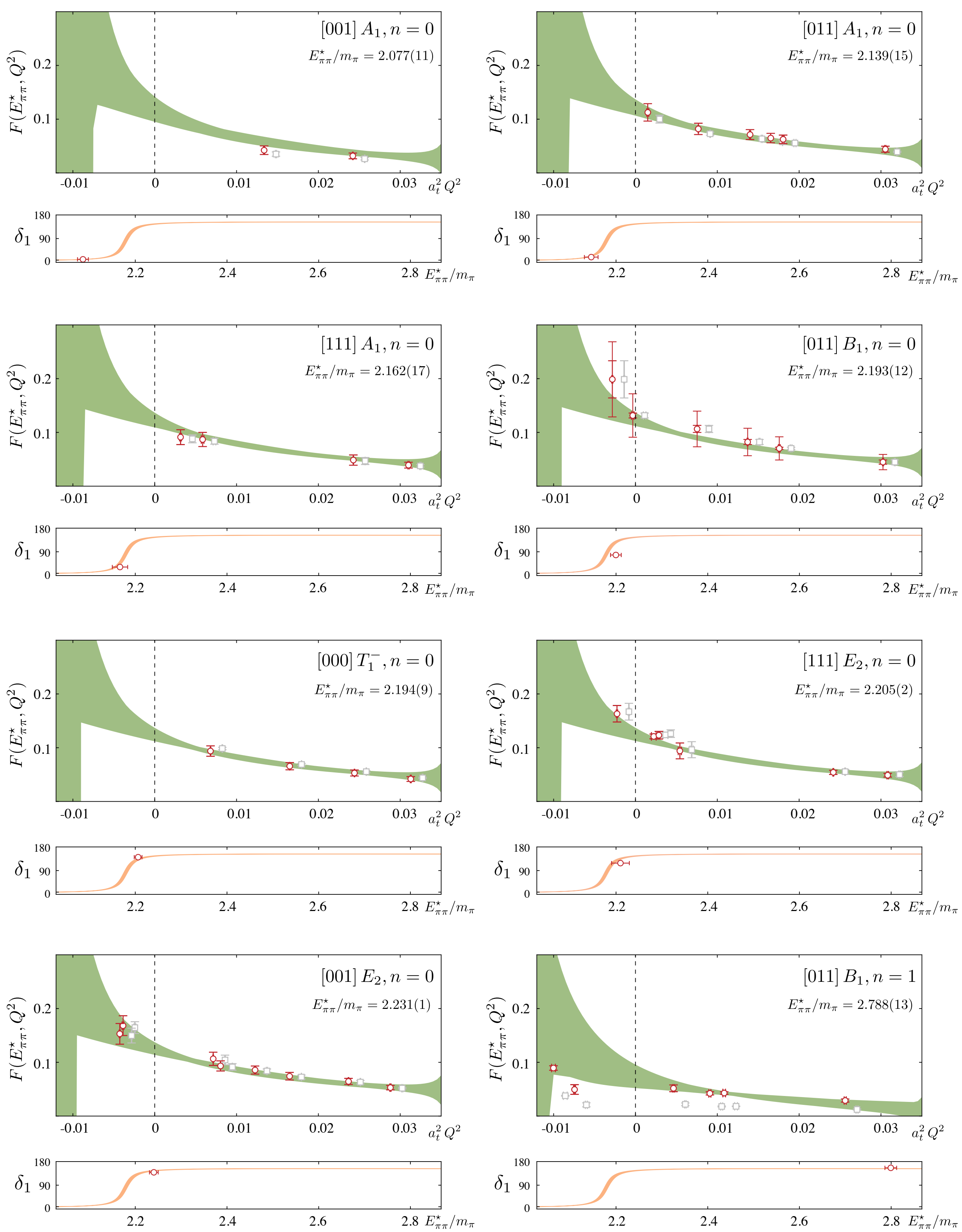
LL factor:

$$\frac{2E_\pi}{\mathcal{R}} = 32\pi\,\frac{E_\pi E_{\pi\pi}}{q^\star_{\pi\pi}}\,\cos^2\delta_1\,\frac{\partial}{\partial P^\star_{0,\pi\pi}}\left(\tan\delta_1 + \tan\phi^{\mathbf{P}_{\pi\pi},\Lambda_{\pi\pi}}\right)\Big|_{P^\star_{0,\pi\pi}=E^\star_{\pi\pi}}$$

$$= 32\pi\,\frac{E_\pi E_{\pi\pi}}{q^\star_{\pi\pi}}\left(\delta_1' + r\phi'\right),$$



"Form factor"

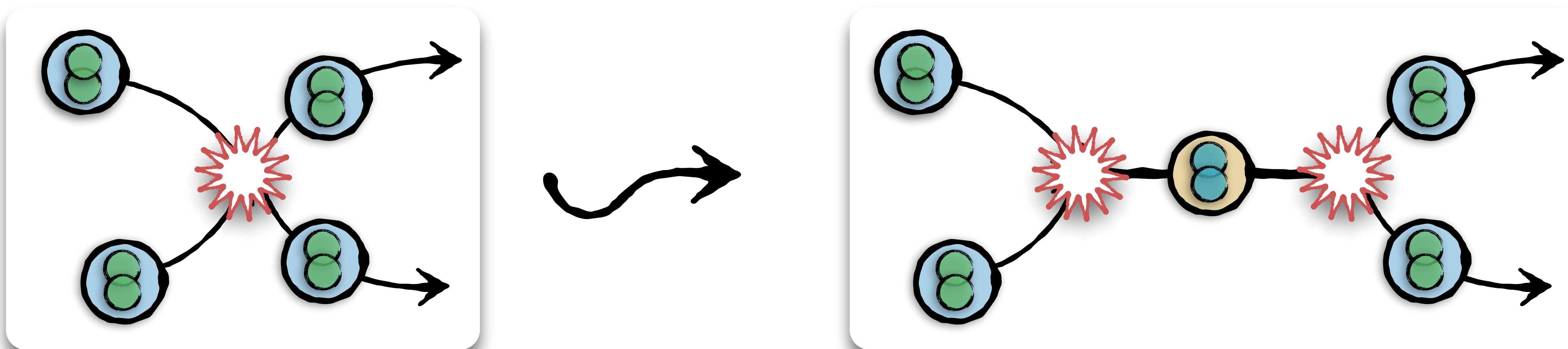


Fit parametrization:

$$h^{[\{\alpha,\beta\}]}(E^\star_{\pi\pi}, Q^2) = \frac{\alpha_1}{1 + \alpha_2 Q^2 + \beta_1(E^{\star 2}_{\pi\pi} - m_0^2)} + \alpha_3 Q^2 + \alpha_4 Q^4 + \alpha_5 \exp\left[-\alpha_6 Q^2 - \beta_2(E^{\star 2}_{\pi\pi} - m_0^2)\right] + \beta_3(E^{\star 2}_{\pi\pi} - m_0^2) + \beta_4(E^{\star 4}_{\pi\pi} - m_0^4),$$

Intuitive explanation

- the elastic $\pi\pi$ amplitude is dynamically enhanced by the presence of the ϱ-meson



- Similarly, the $\pi\gamma^*$-to-$\pi\pi$ amplitude is enhanced by the ϱ-meson