Higher Partial Waves of NN Scattering from LQCD at m_π=800 MeV

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CalLat

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Motivation

- Lattice can ultimately provide input to NP
- Two-nucleon (NN) matrix elements largely require NN scattering
- Can study dependence of NP on parameters of Standard Model
  - Inaccessible to experiment
  - Nice for EFTs
- ...

Outline

- Brief review of Lüscher formalism
- HPC
- Considerations for cubic volume
    - Sinks and Sources
- Cubic Irreps
- Results

Lüscher Formalism

$$\det\left[(\mathcal{M}^{\infty})^{-1} + \delta\mathcal{G}^{V}\right] = 0$$

Infinite volume scattering amplitudes

finite volume spectrum + boundary conditions

Lattice calculation

Two-Nucleon Spectrum

- Spectrum given by effective mass of (schematic) NN correlator:

$$\left\langle \Omega \middle| \mathcal{O}_{I m_I}^{J' m'_J S' m'_S}(t) \bar{\mathcal{O}}_{I m_I}^{J m_J S m_S}(0) \middle| \Omega \right\rangle$$

- Sink

$$\mathcal{O}_{J m_J I m_I; S\ell}(t, |\boldsymbol{k}|) = \sum \text{Clebsh-Gordans} \sum_{R \in \mathcal{O}_h} Y_{\ell m_\ell}\left(\widehat{R\boldsymbol{k}}\right) N_{m_{s_1}}^{m_{I_1}}(t, R\boldsymbol{k}) \, N_{m_{s_2}}^{m_{I_2}}(t, -R\boldsymbol{k})$$

- Source

$$\mathcal{O}_{J m_J I m_I; S\ell}(t, \boldsymbol{x}, \boldsymbol{\Delta x}) = \sum \text{Clebsch-Gordans} \sum_{R \in \mathcal{O}_h} Y_{\ell m_\ell}\left(\widehat{R\boldsymbol{\Delta x}}\right) N_{m_{s_1}}^{m_{I_1}}(t, \boldsymbol{x}) \, N_{m_{s_2}}^{m_{I_2}}(t, x + R\boldsymbol{\Delta x})$$

Two-Nucleon Spectrum

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$$\left\langle \Omega \middle| \mathcal{O}^{[J'\ell' S']}_{\Lambda'\mu', I m_I}(t) \bar{\mathcal{O}}^{[J\ell S]}_{\Lambda\mu, I m_I}(0) \middle| \Omega \right\rangle$$

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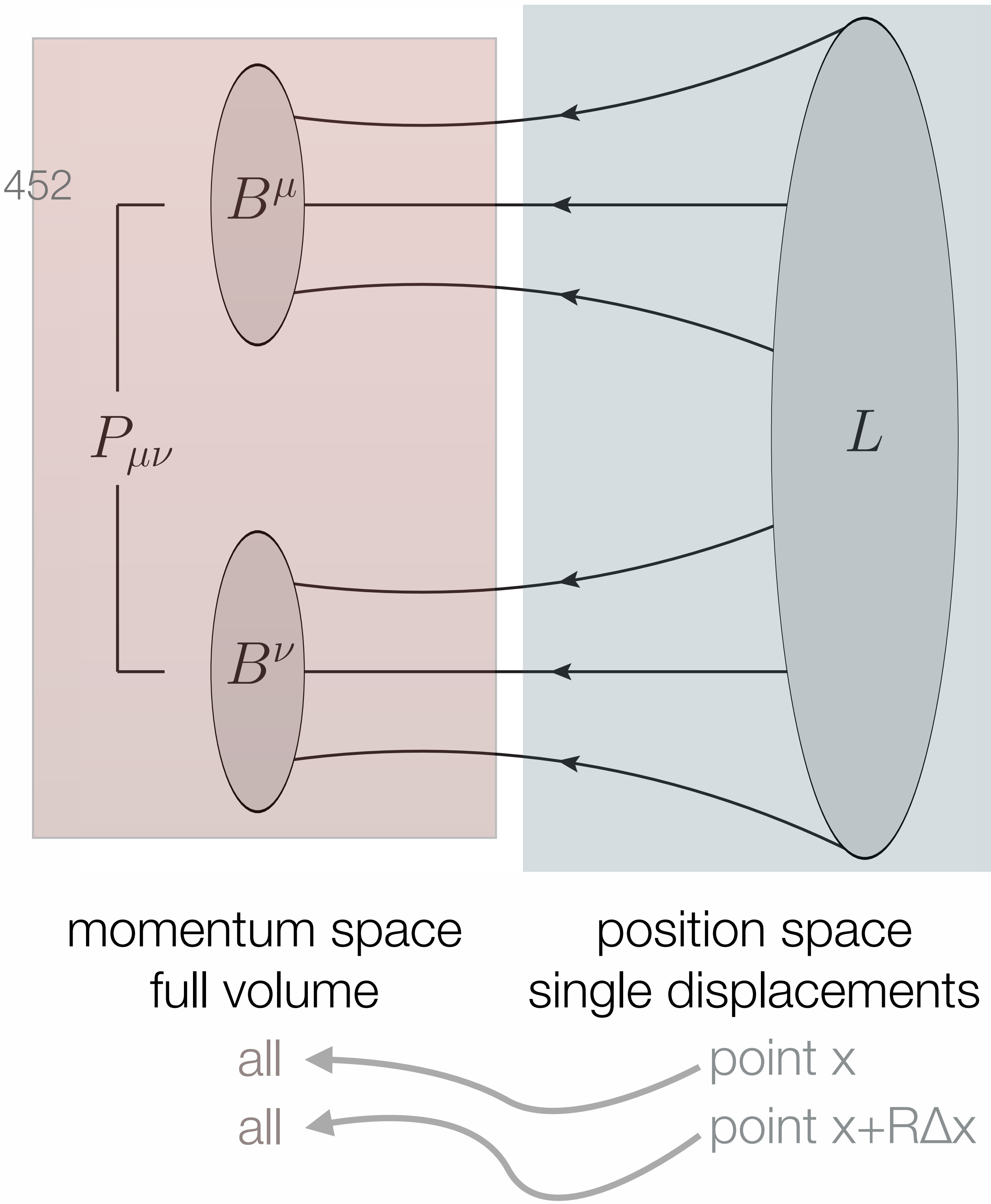
- Box breaks rotational symmetry → spectrum falls into irreps of $\mathcal{O}_h$, not SO(3).
    - Subduction

$$\mathcal{O}^{[J\ell S]}_{\Lambda\mu, I m_I}(t, |\boldsymbol{k}|) = \sum_{m_J} [\text{CG}^J_\Lambda]_{\mu, m_J} \mathcal{O}_{J m_J I m_I; S\ell}(t, |\boldsymbol{k}|)$$

HPC

Doi & Endres 1205.0585, Detmold & Orginos 1207.1452

- Use baryon blocks
- Use sparse tensor contraction to take advantage of sparsity of L
- For each source displacement RΔx, store (sink-side) full-volume correlator for each S'm'$_S$ Sm$_S$ Im$_I$



$$C_{Im_I}^{S'm'_S Sm_S}(\boldsymbol{k}', t'-t, R\boldsymbol{\Delta x}) =$$

$$\sum_{\boldsymbol{x}} \left\langle \Omega \middle| \left(N_{i'}^{\mu'}(t', \boldsymbol{k}') P_{\mu'\nu'}^{S'm'_S} T_{Im_I}^{i'j'} N_{j'}^{\nu'}(t', -\boldsymbol{k}') \right) \left(\bar{N}_i^{\mu}(t, \boldsymbol{x}) P_{\mu\nu}^{Sm_S} T_{Im_I}^{ij} \bar{N}_j^{\nu}(t, \boldsymbol{x} + R\boldsymbol{\Delta x}) \right) \middle| \Omega \right\rangle$$

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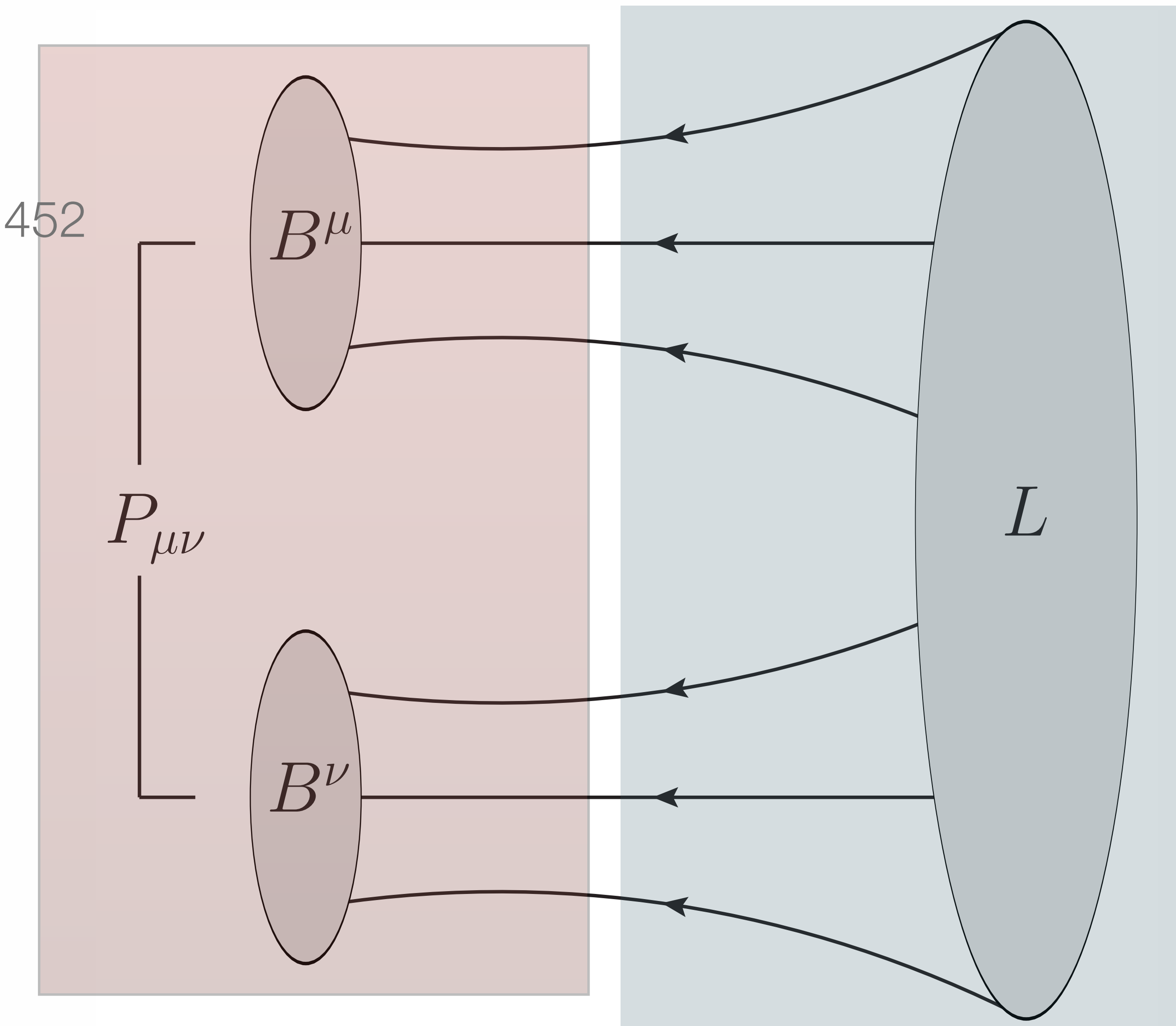


momentum space
full volume

position space
single displacements

all ← point x

all ← point x+RΔx

$$C^{S'm'_S\,Sm_S}_{Im_I}(\boldsymbol{k}', t'-t, R\boldsymbol{\Delta x}) =$$

$$\sum_{\boldsymbol{x}} \Big\langle \Omega \Big| \Big(N^{\mu'}_{i'}(t',\boldsymbol{k}') P^{S'm'_S}_{\mu'\nu'} T^{i'j'}_{Im_I} N^{\nu'}_{j'}(t',\text{-}\boldsymbol{k}') \Big) \Big(\bar{N}^{\mu}_{i}(t,\boldsymbol{x}) P^{Sm_S}_{\mu\nu} T^{ij}_{Im_I} \bar{N}^{\nu}_{j}(t,\boldsymbol{x}+R\boldsymbol{\Delta x}) \Big) \Big| \Omega \Big\rangle$$

Sample with optimal Sobol sequence

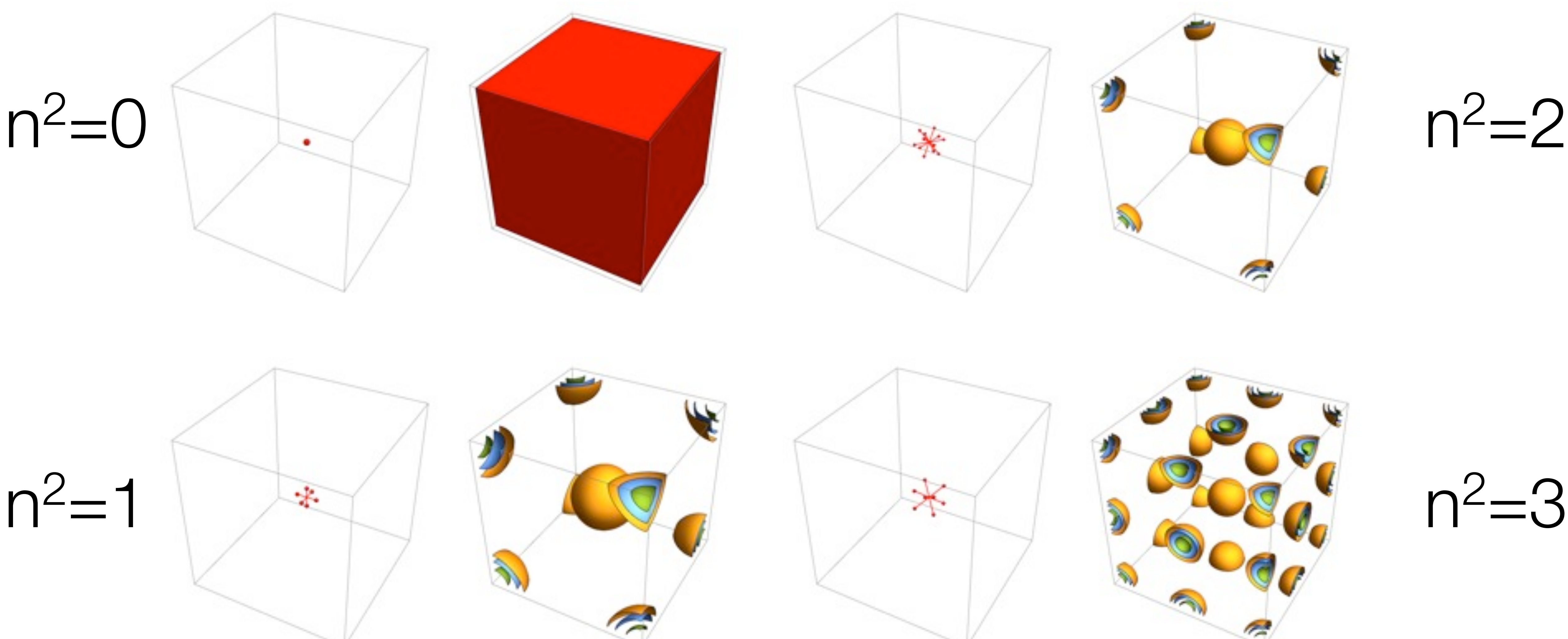
Projectors

Sinks

Luu & Savage 1101.3347

- Project to eigenstates of a noninteracting theory in a box.
- Full volume information → exactly project to any desired irrep

A_1^+



Sinks

Luu & Savage 1101.3347

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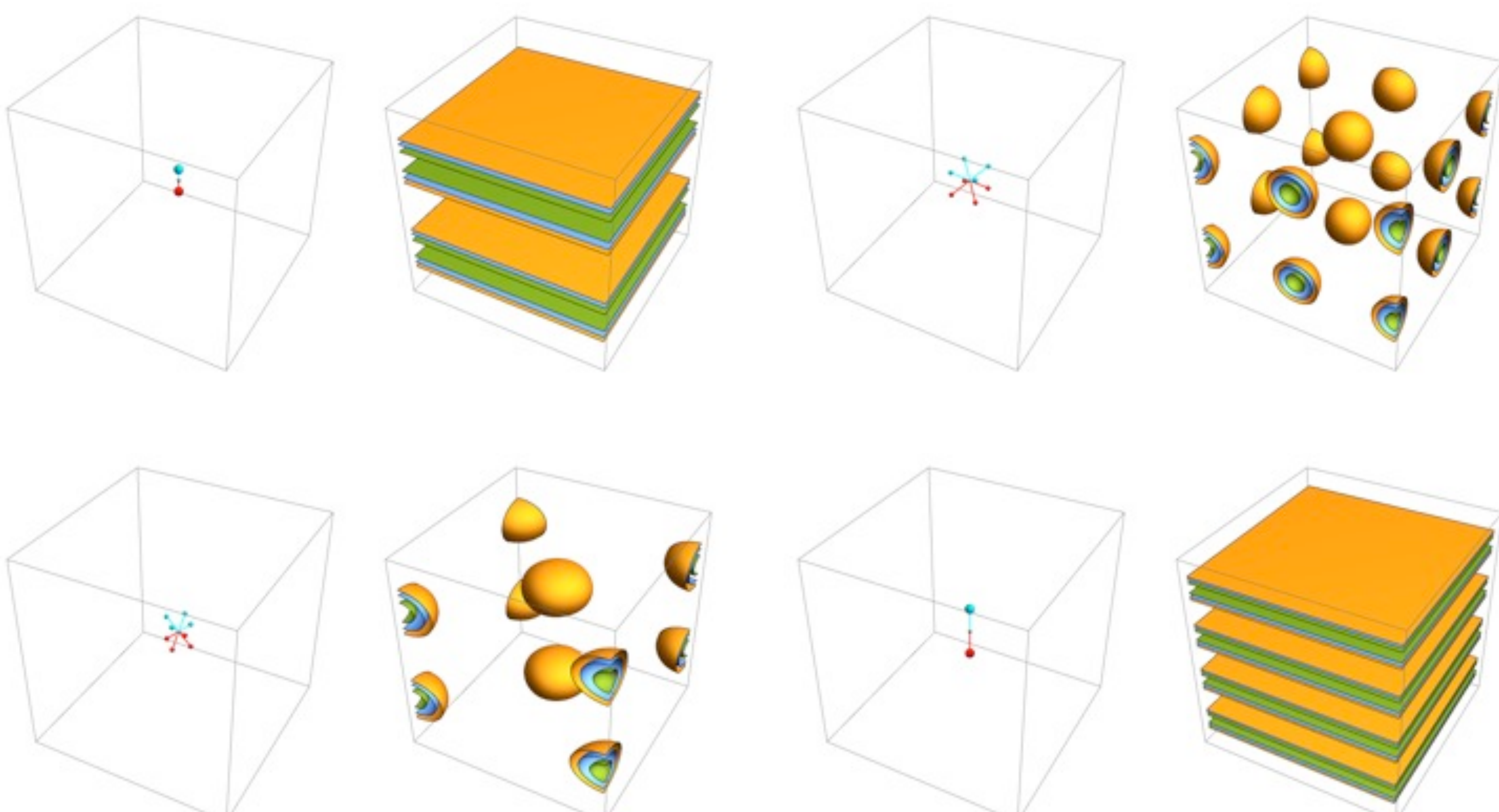
- Full volume information → exactly project to any desired irrep

T_1^-

$n^2=1$ $n^2=3$

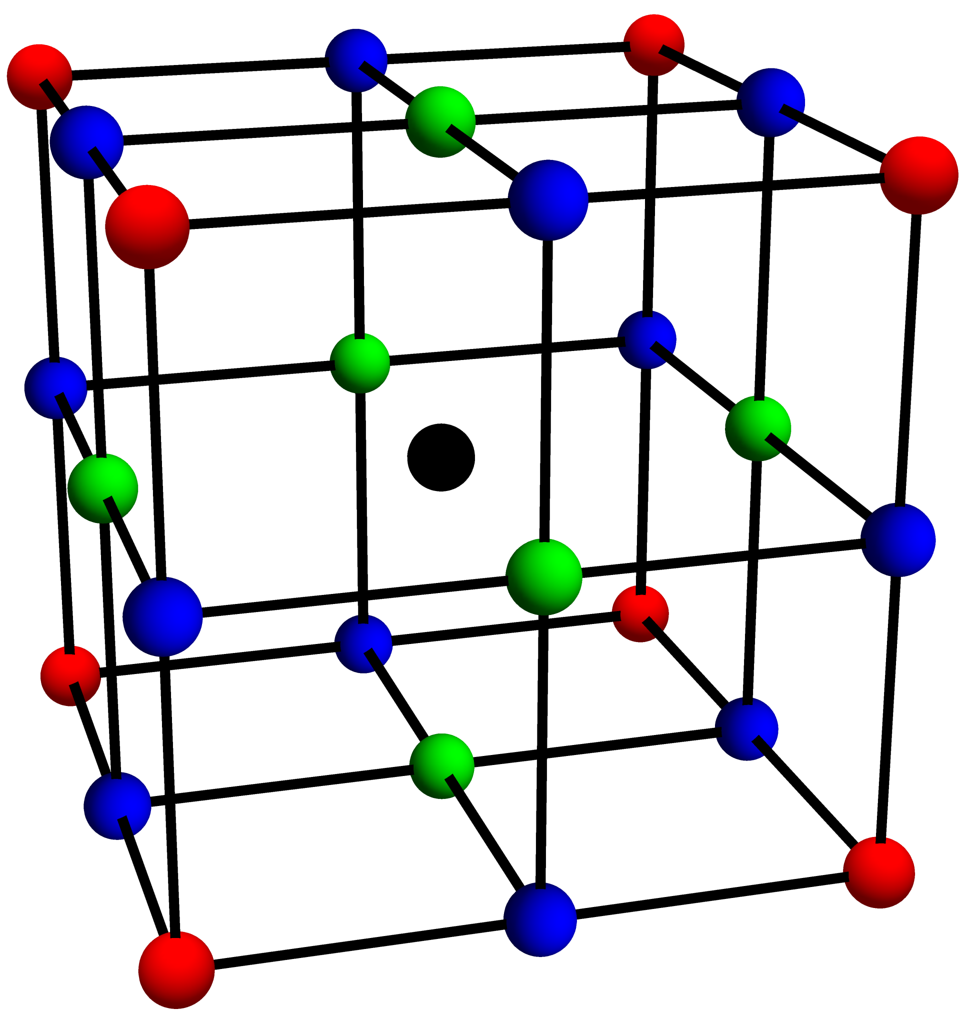
$n^2=2$ $n^2=4$



Sources

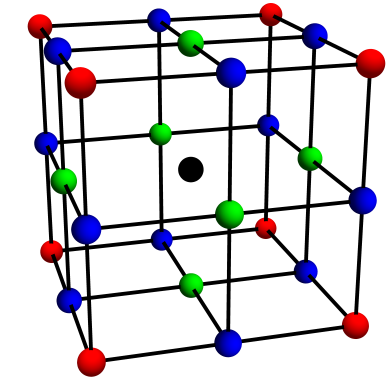
- Exact projection source-side requires spatial-volume-to-all propagators.

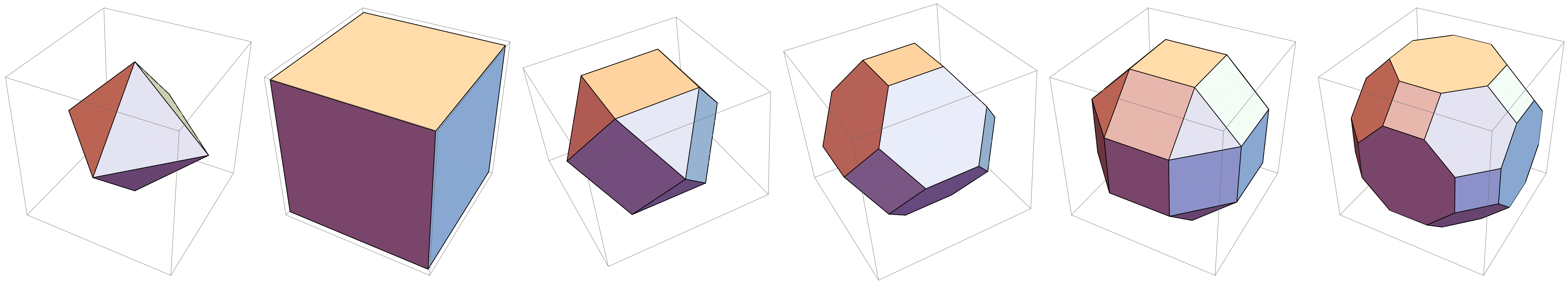
- Pick displacements



description	$\Delta x \propto$	count
local	(0,0,0)	1
face	(0,0,1)	6
edge	(0,1,1)	12
corner	(1,1,1)	8

Sources





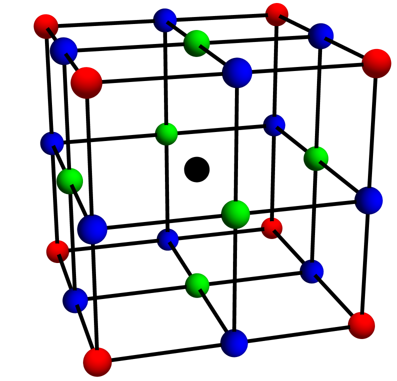
(0,0,1) (1,1,1) (0,1,1) (0,1,2) (1,1,2) (1,2,3)

Solids generated by $\mathcal{O}_h$ ↔ Irreps of $\mathcal{O}_h$

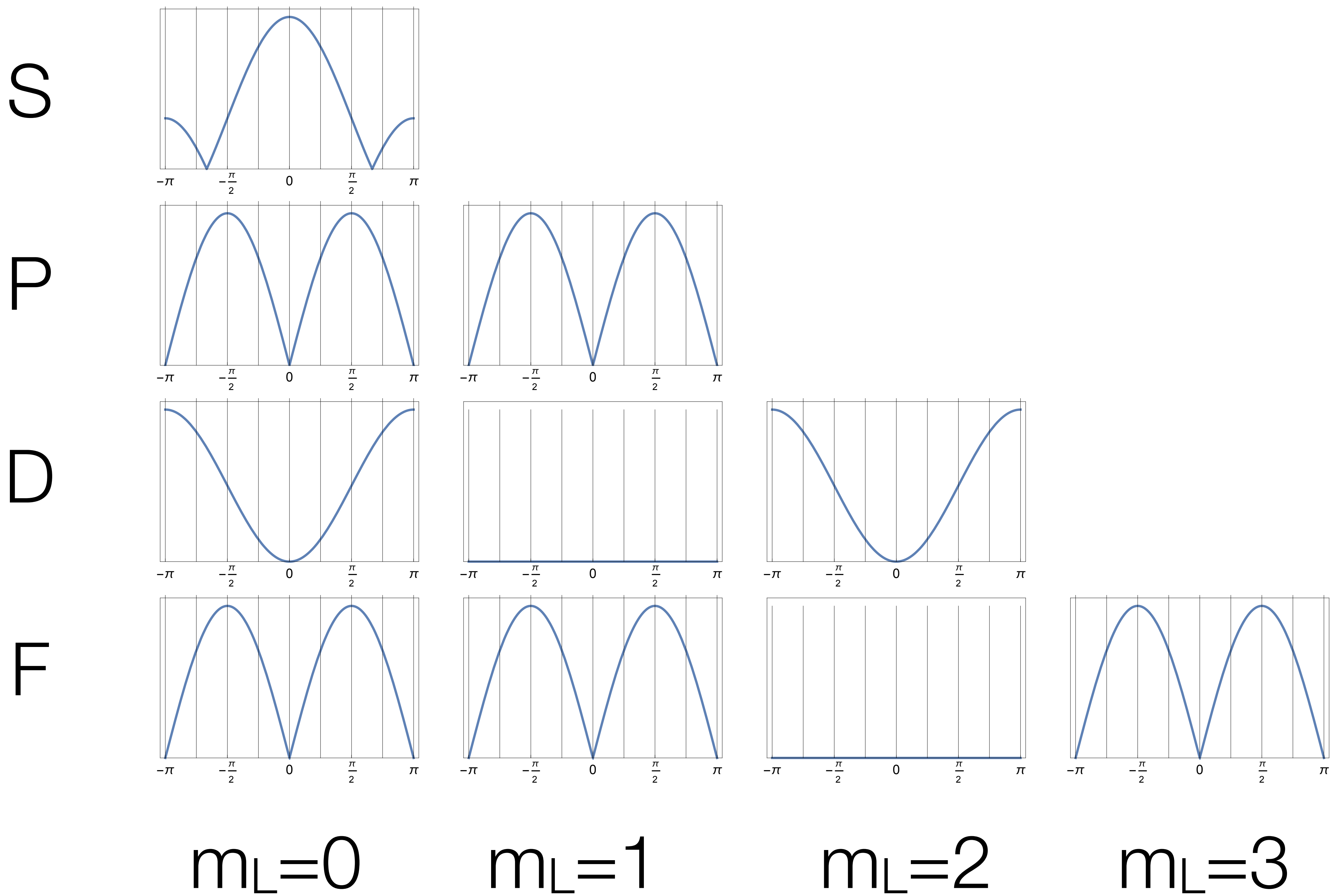
face	octahedron	6
edge	cuboctahedron	12
corner	cube	8
knight's move	trucated octahedron	24
more complicated	small rhombicutobctahedron	24
more complicated	great rhombicuboctahedron	48

Too expensive.

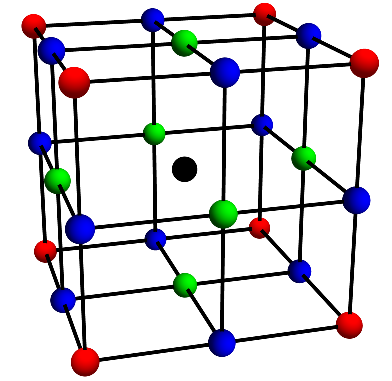
Source Overlap



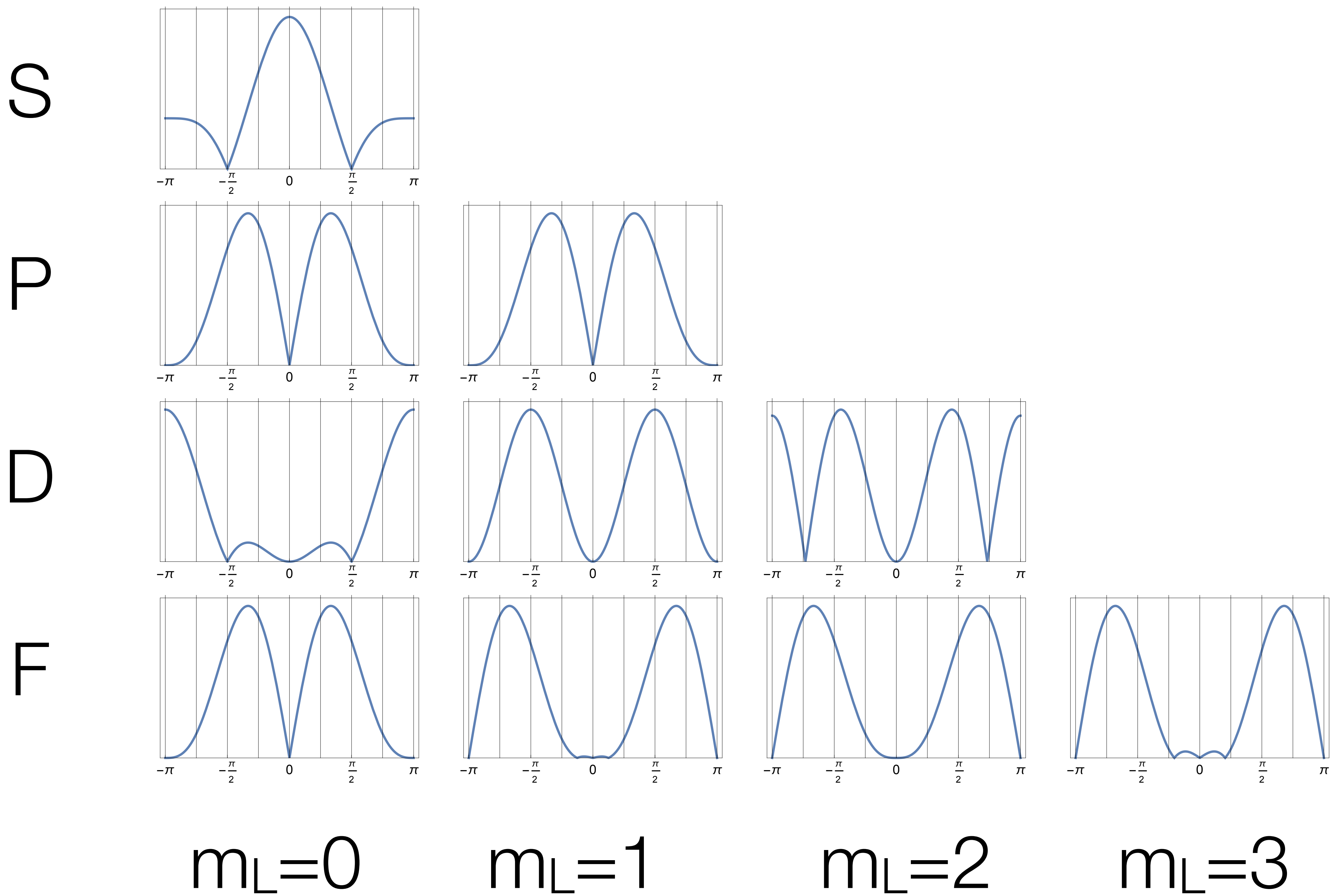
Project Luu & Savage momentum sources to faces as a function of $\pi\Delta x/L$

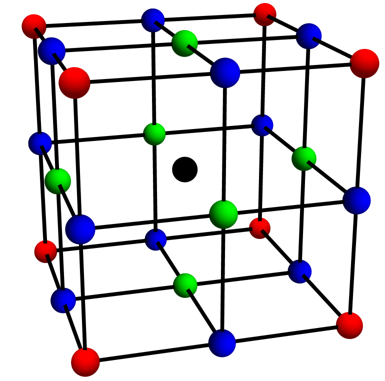


Source Overlap



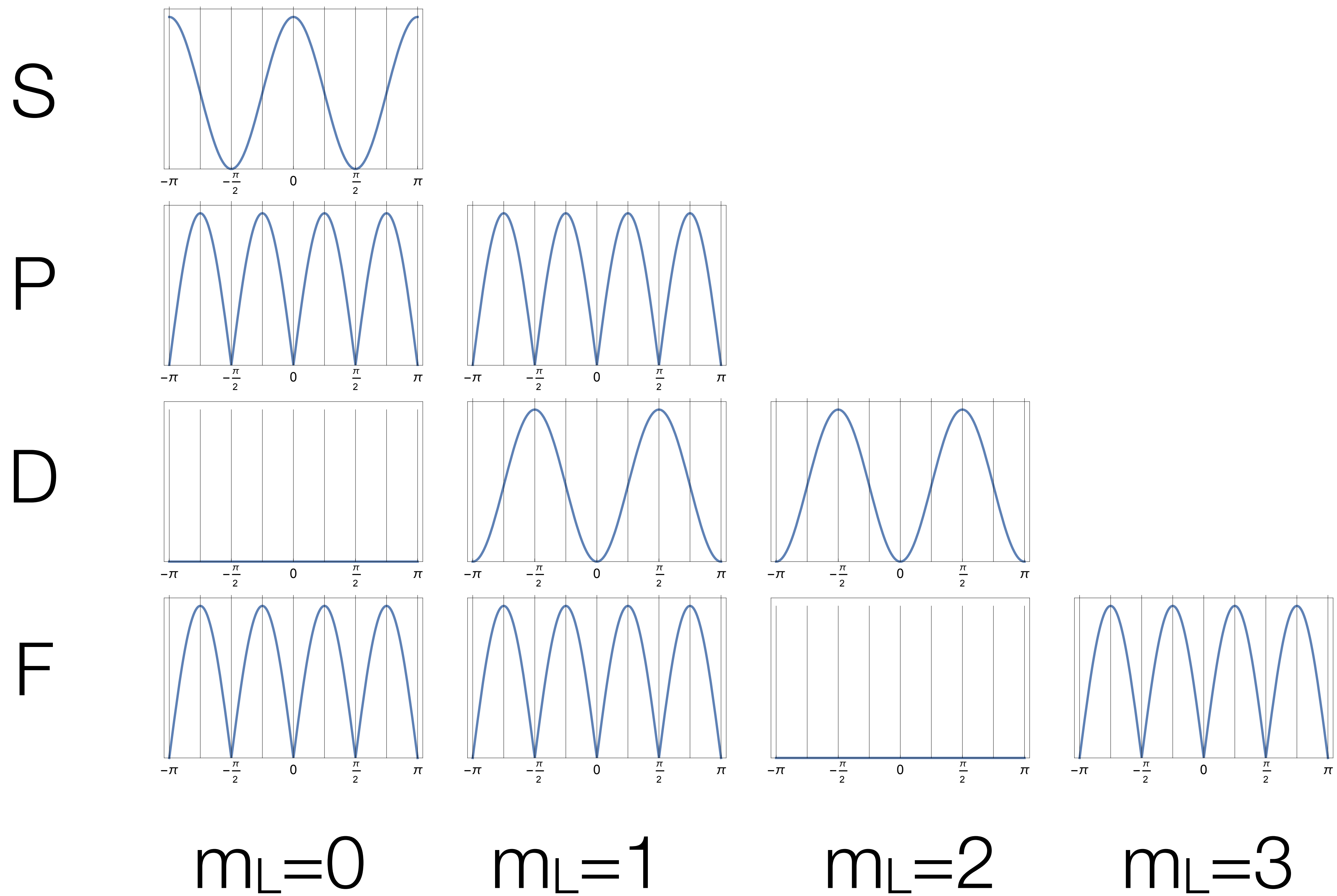
Project Luu & Savage momentum sources to edges as a function of πΔx/L

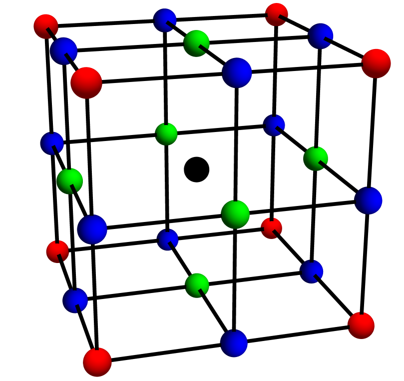




Source Overlap

Project Luu & Savage momentum sources to corner as a function of πΔx/L



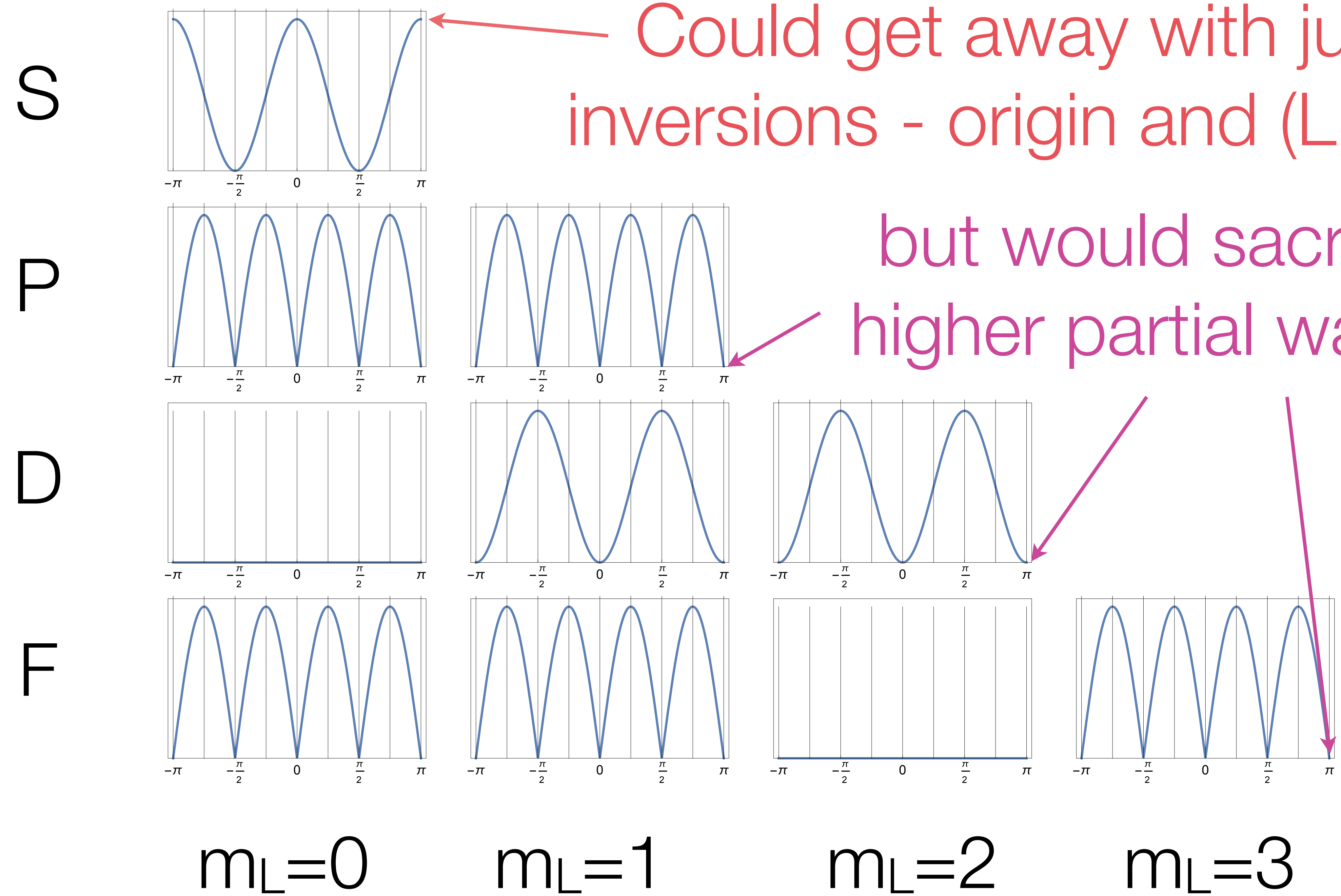


Source Overlap

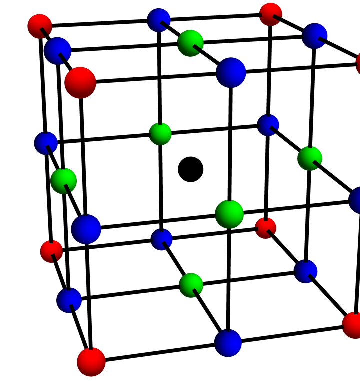
Project Luu & Savage momentum sources to corner as a function of πΔx/L

Could get away with just 2 inversions - origin and (L,L,L)/2

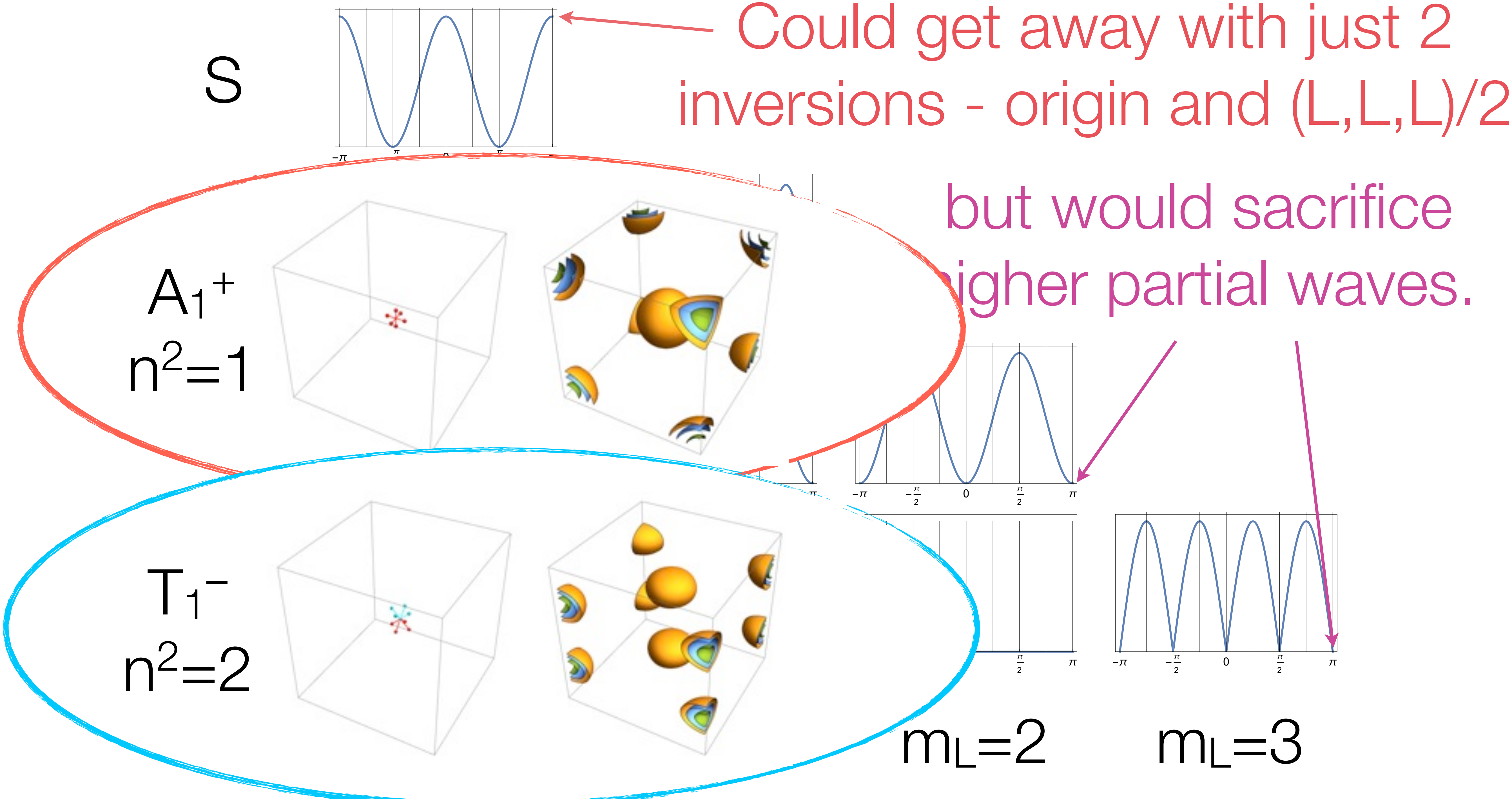
but would sacrifice higher partial waves.



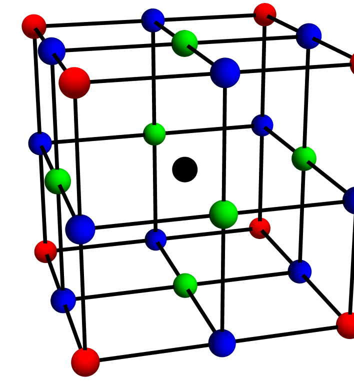
Source Overlap



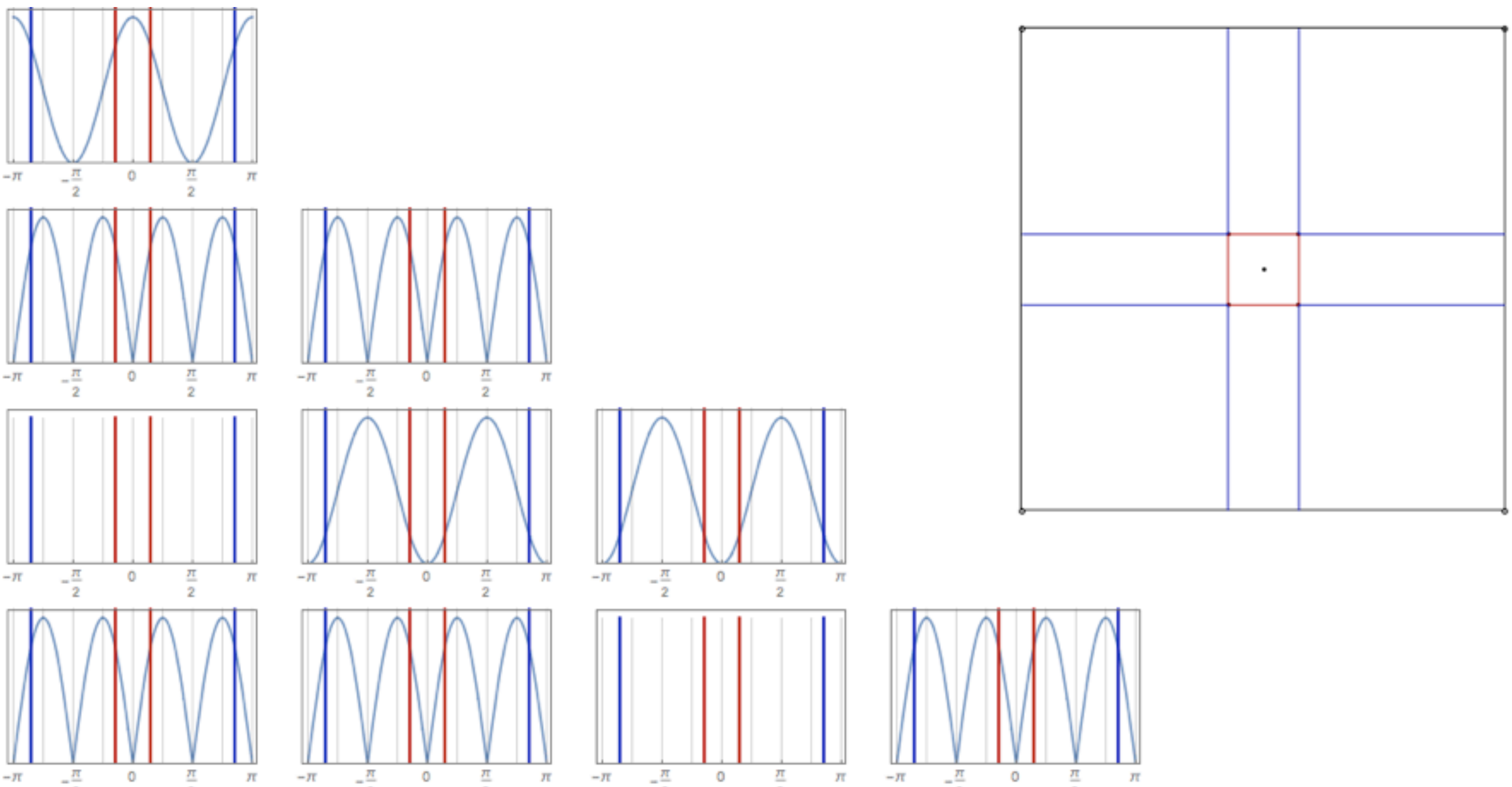
Project Luu & Savage momentum sources to corner as a function of πΔx/L



Future Sources: Take advantage of periodicity



- Take advantage of periodicity.
- Add 1 more inversion, get a cubic irrep source with different Δx



Subduction

HadSpec 1004.4930

Isospin 0		Isospin 1	
Partial wave	Irreps	Partial wave	Irreps
$^1P_1$	T_1^-	$^1S_0$	A_1^+
$^3S_1, {}^3D_1$	T_1^+	$^3P_0$	A_1^-
$^3D_2$	$E^+ \oplus T_2^+$	$^3P_1$	T_1^-
$^3D_3$	$A_2^+ \oplus T_1^+ \oplus T_2^+$	$^3P_2, {}^3F_2$	$E^- \oplus T_2^-$
$^1F_3$	$A_2^- \oplus T_1^- \oplus T_2^-$	$^1D_2$	$E^+ \oplus T_2^+$
		$^3F_3$	$A_2^- \oplus T_1^- \oplus T_2^-$
		$^3F_4$	$A_1^- \oplus E^- \oplus T_1^- \oplus T_2^-$

Some states only couple to particular sources.

unphysical mixing

Building Cubic Irrep Correlators

Gauge Configuration

HPC

$C^{S'm'_S Sm_S}_{Im_I}(\boldsymbol{k}', t'-t, \boldsymbol{\Delta x})$ Full-volume correlator for single Δx sources

Combine different Δx with different weights, phases

Full-volume correlator for ℓm_ℓ sources

More Clebsch-Gordans, projection to momentum shells

time correlator of operators labeled by J

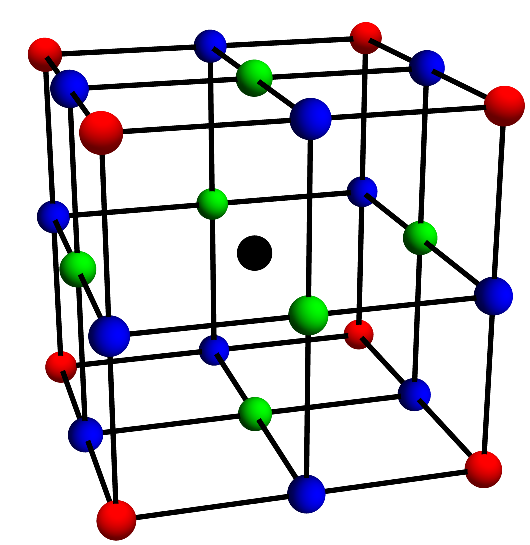
Subduction

time correlator of operator of cubic irrep

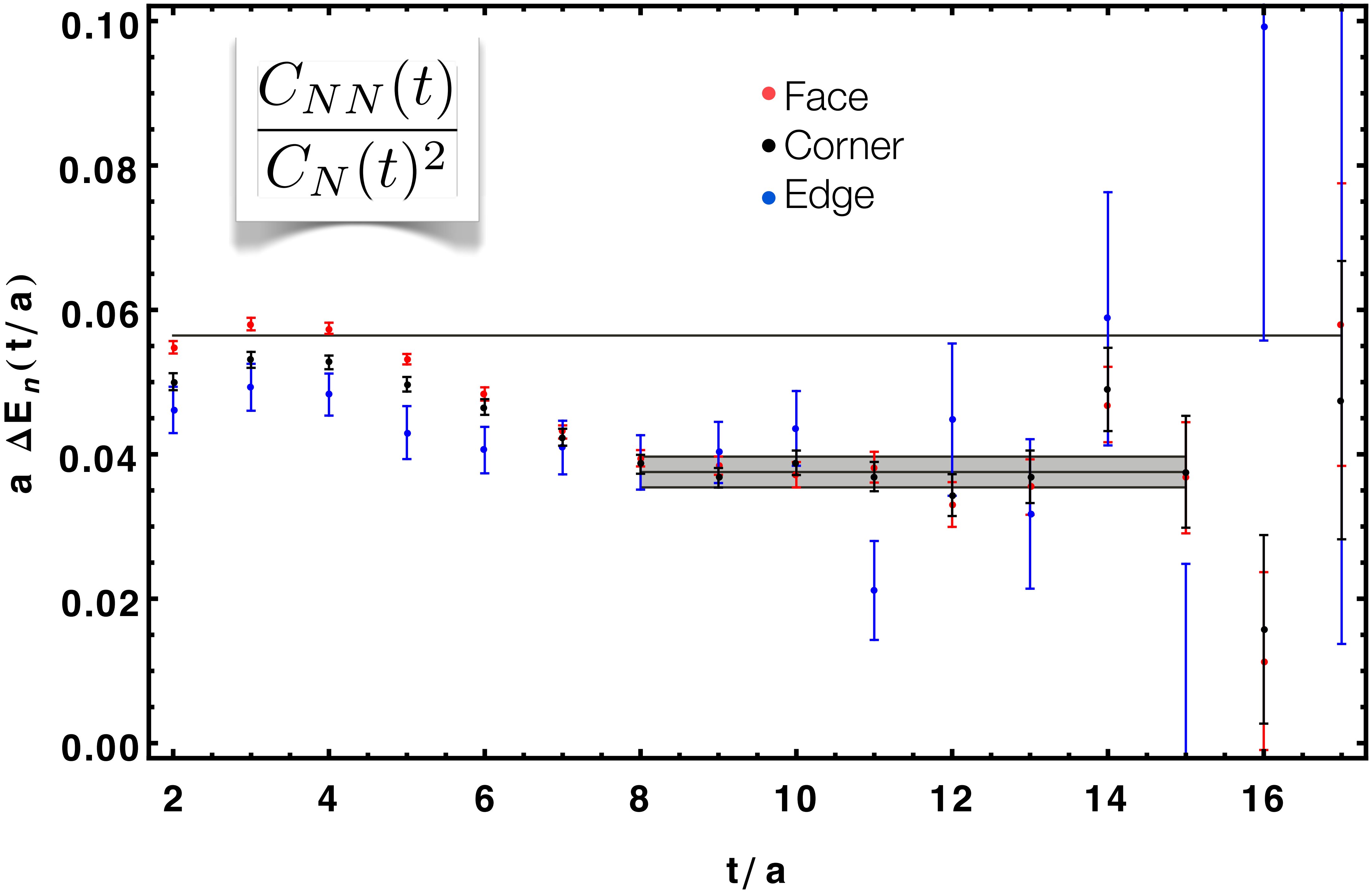
$$\left\langle \Omega \middle| \mathcal{O}^{[J'\ell'S']}_{\Lambda'\mu',Im_I}(t)\,\mathcal{O}^{[J\ \ell\ S\ ]}_{\Lambda\mu,Im_I}(0) \middle| \Omega \right\rangle \rightarrow \delta\mathcal{G}^V$$

Lattices

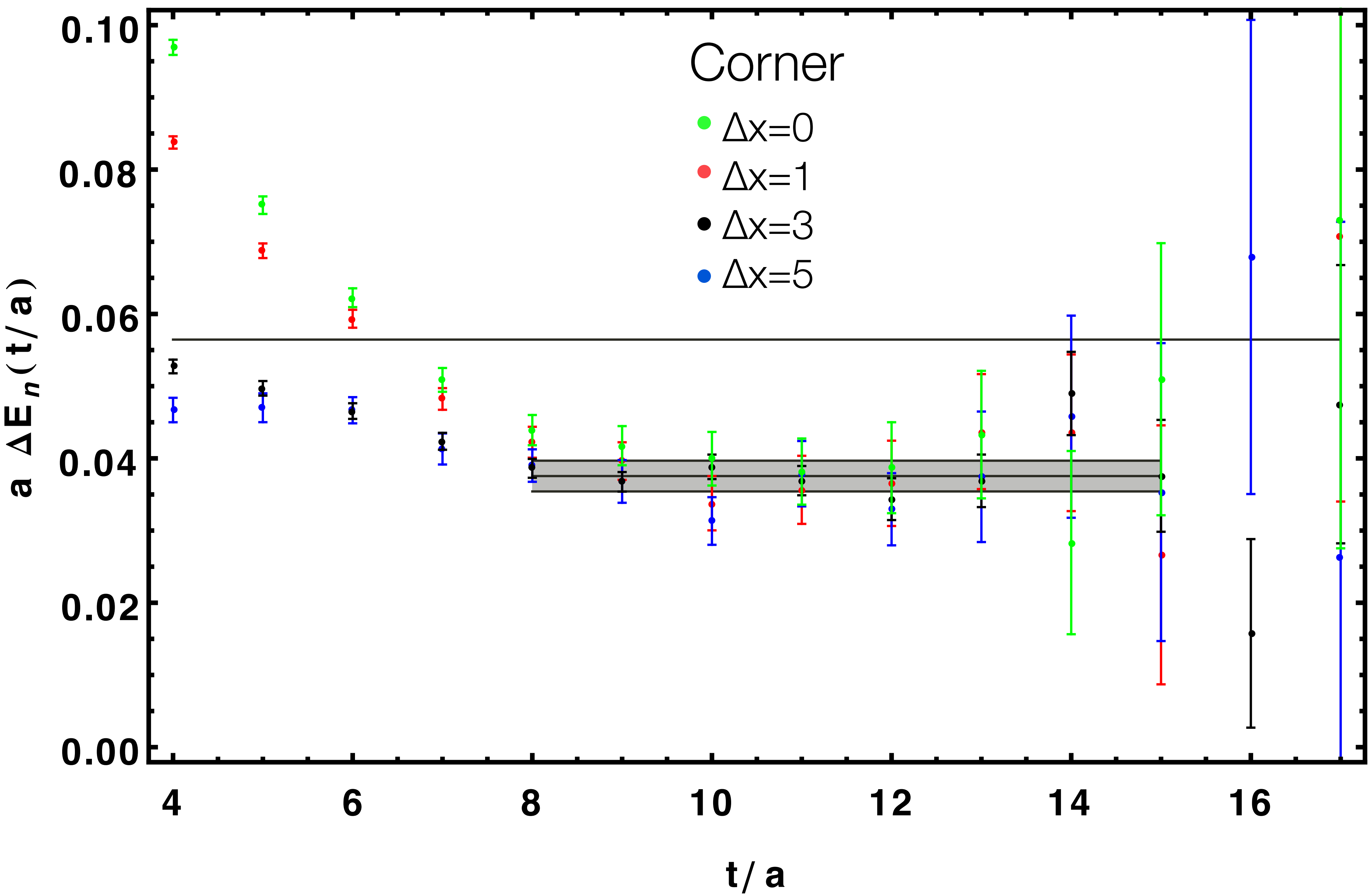
- W&M / JLab isotropic clover with a≈0.145 fm
  - Same lattices as NPLQCD S-wave calculation 1301.5790
- SU(3) limit: $m_K = m_\pi \approx 807$ MeV
- Volumes:
  - $24^3 \times 48 \approx (3.5\text{fm})^3 \text{x} (7\text{fm})$. $m_\pi L \approx 14$
  - $32^3 \times 48 \approx (4.7\text{fm})^3 \text{x} (7\text{fm})$. $m_\pi L \approx 18.5$
- Measurements:
  - 20 × (4000 configurations) = 80K measurements
  - 75 × (1000 configurations) = 75K measurements
- Sources:
  - × (8 corners)
  - × (12 edges)



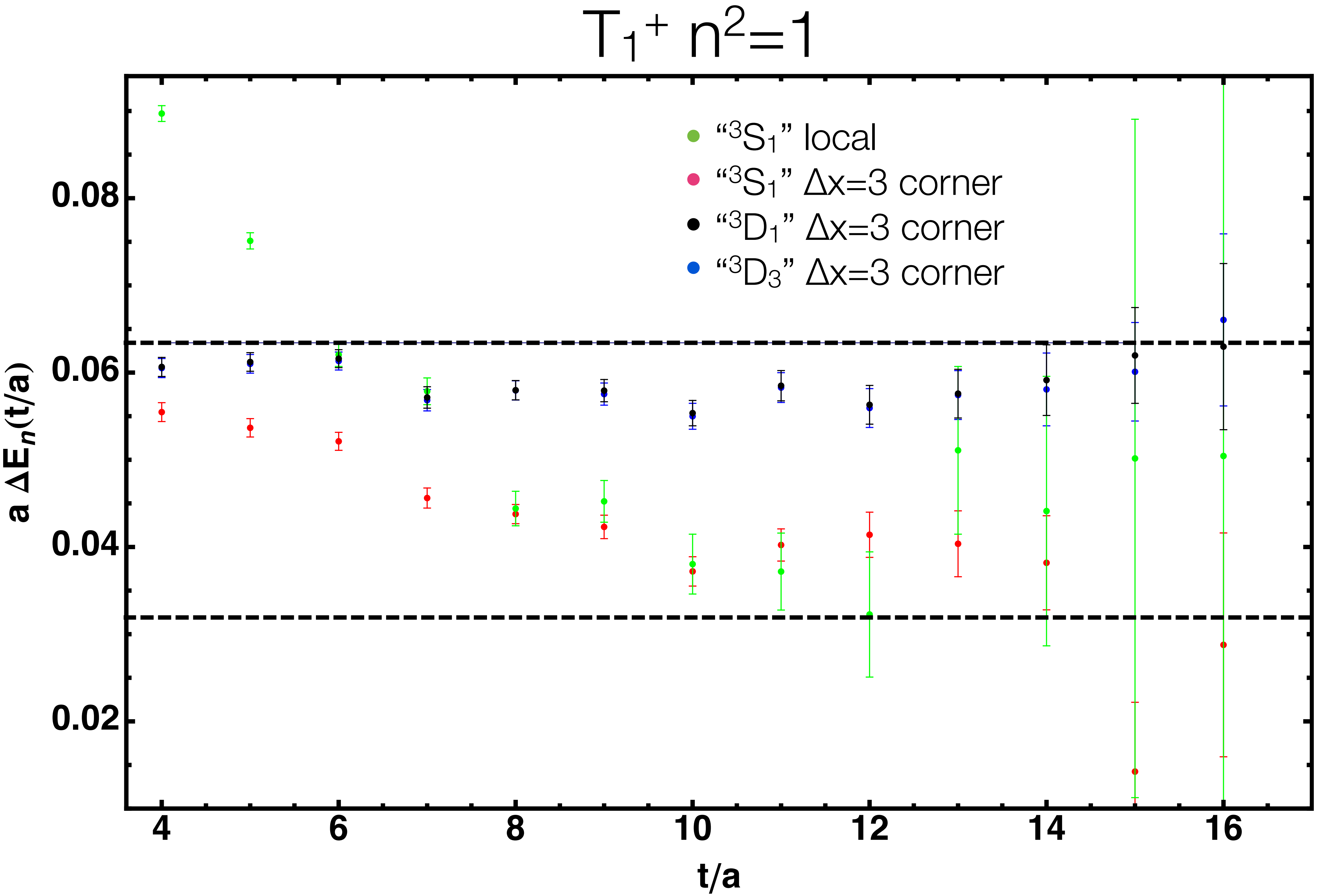
Different sources give same plateau $A_1{}^+$, $\Delta x=3$



Different displacements give same plateau A_1^+



Different sources can separate different states



Lüscher

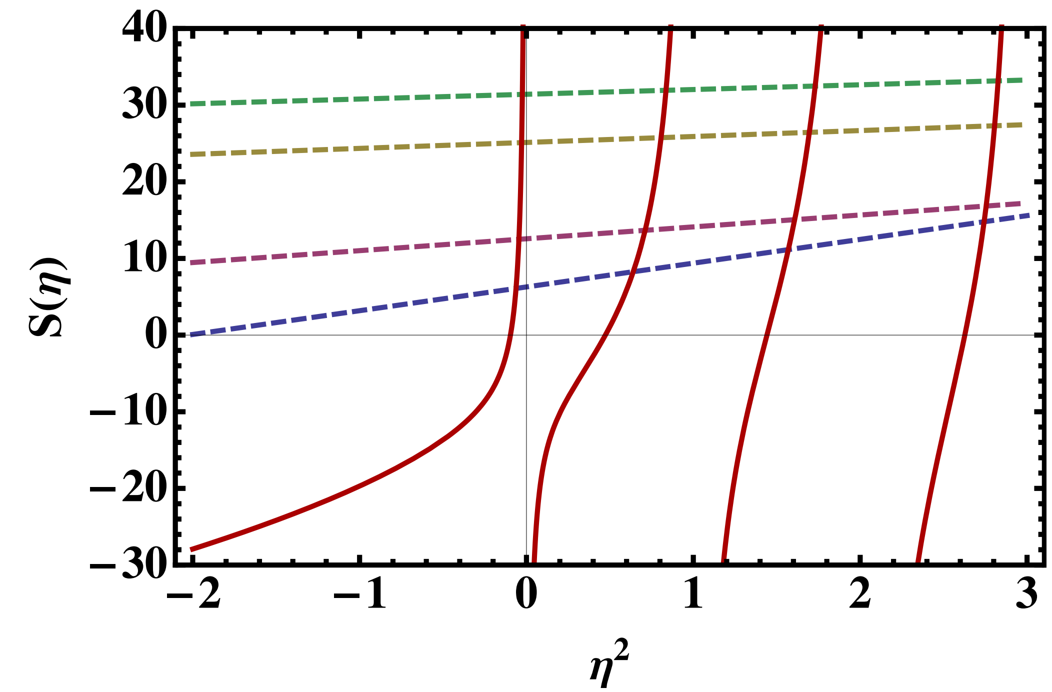
$$\det\left[\left(\mathcal{M}^{\infty}\right)^{-1} + \delta\mathcal{G}^{V}\right] = 0$$

Briceño, Davoudi & Luu 1305.4903

- Errors can get blown up

$$p\cot\delta(p) = \frac{1}{\pi L} S\left(\left(\frac{pL}{2\pi}\right)^2\right) \qquad S(\eta) = \lim_{\Lambda\to\infty}\left[\sum_{\mathbf{j}}^{|\mathbf{j}|<\Lambda} \frac{1}{|\mathbf{j}|^2 - \eta^2} - 4\pi\Lambda\right]$$

Drut and Nicholson 1208.6556



- Partial waves mix
  - Physical effect ← tensor force
  - UNphysical effect ← SO(3) is broken to $\mathcal{O}_h$
  - Solving the coupled matrix equation is tough

Lüscher

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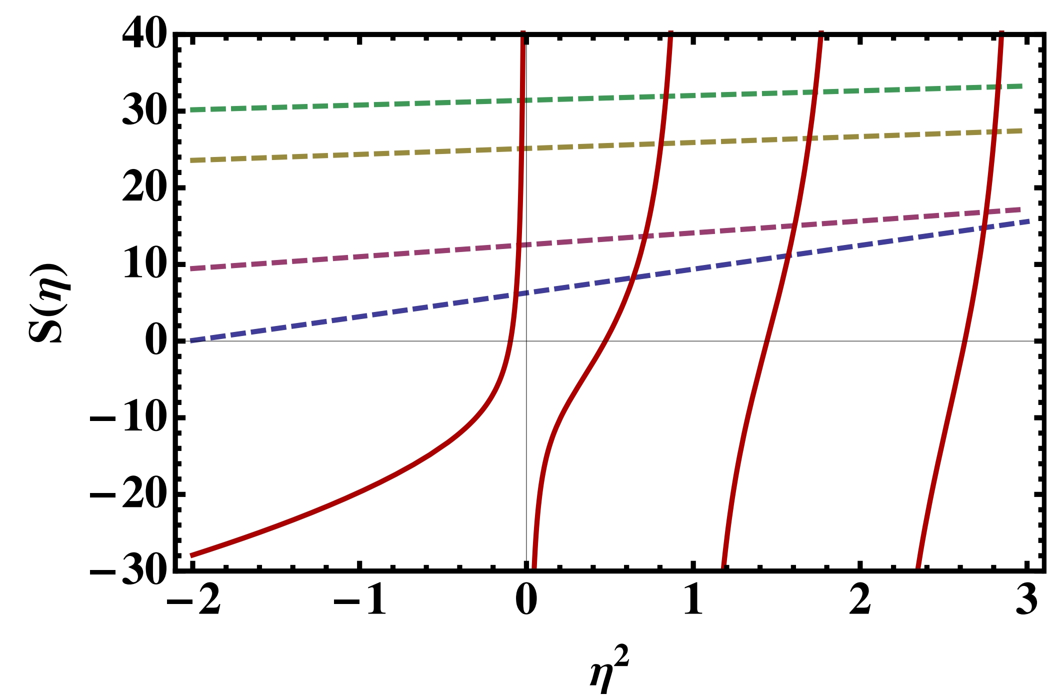

- Partial waves mix
  - Phy ← tensor force
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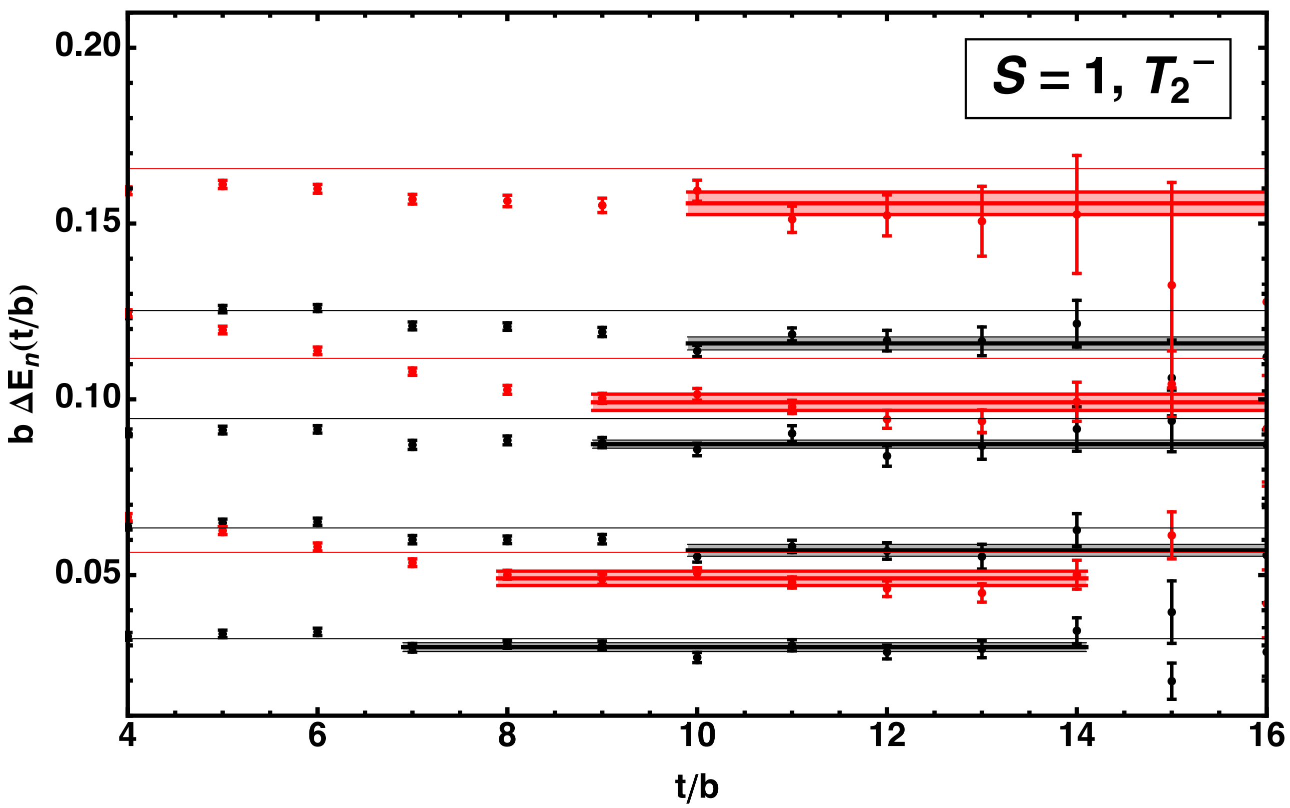
Neglect ☹

Clean separation of momentum shells

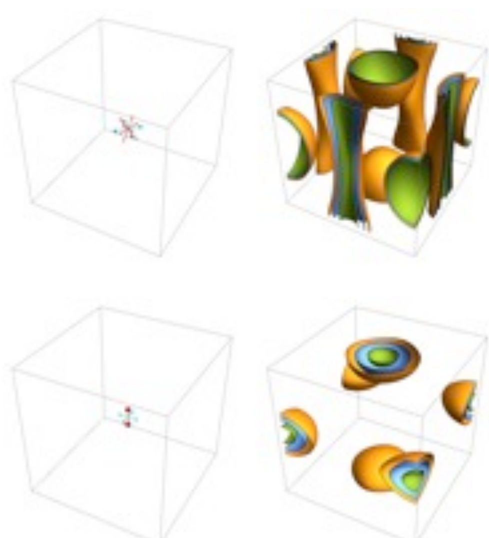
• L=24
• L=32

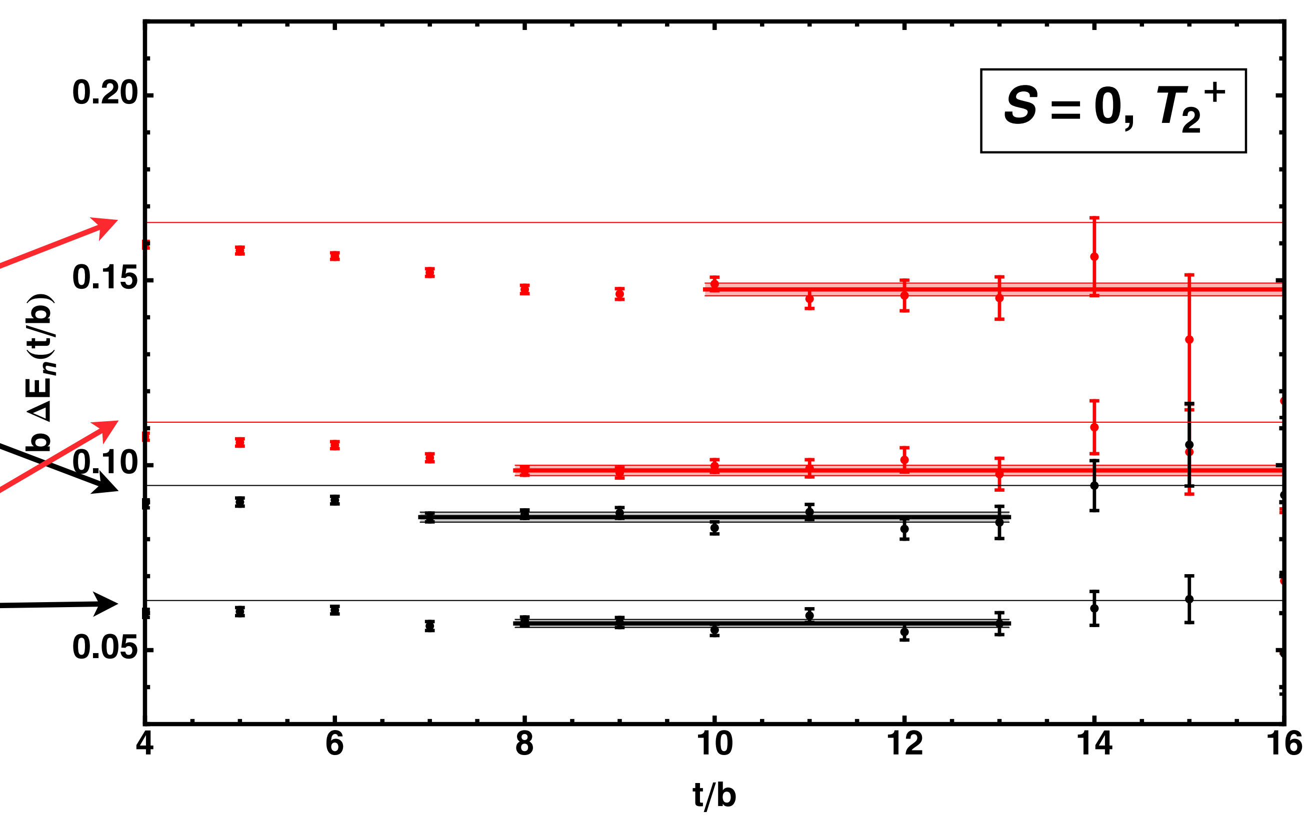


⇐ ΔE → Lüscher

n^2=2

n^2=1

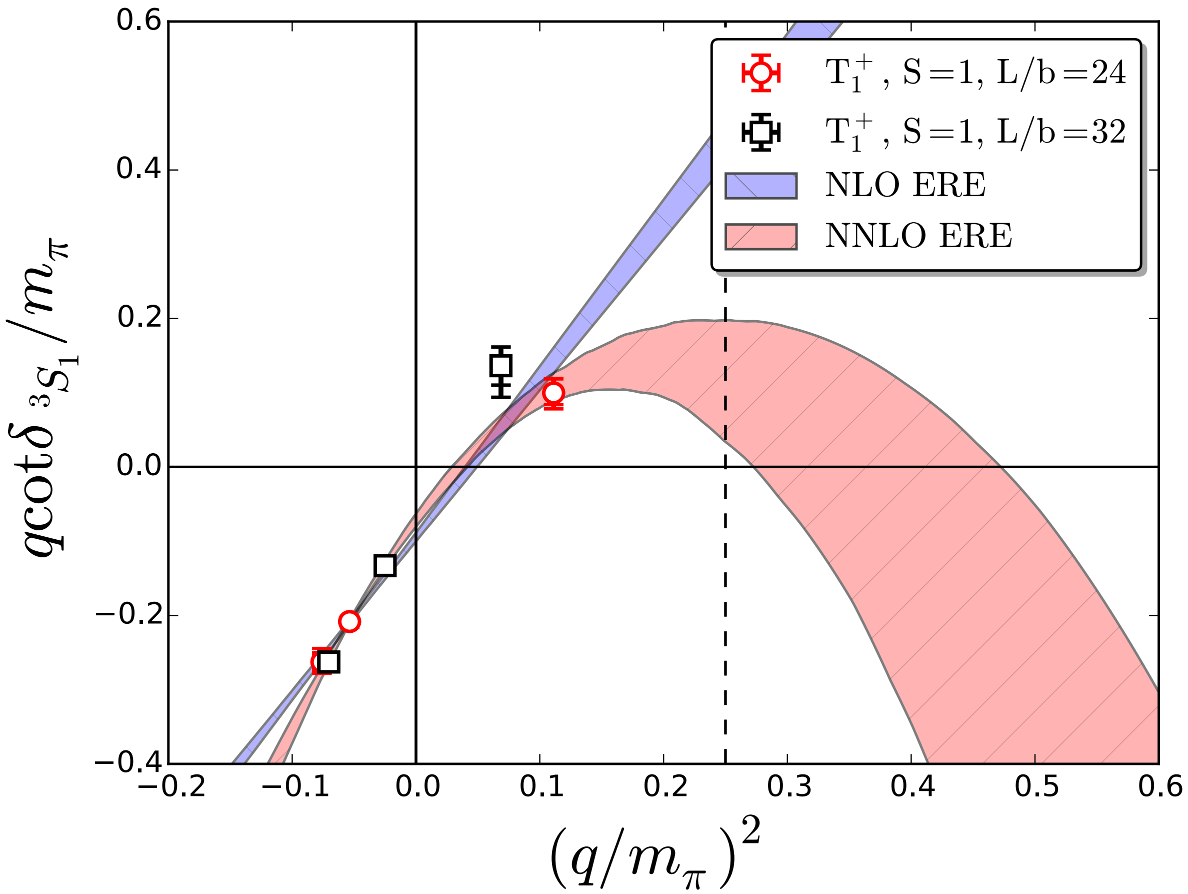




S wave

NPLQCD 1301.5790

$$q^{2\ell+1}\cot(\delta_\ell) = -\frac{1}{a_\ell} + \frac{1}{2}r_\ell q^2 + \frac{1}{4!}P_\ell q^4 + \cdots$$

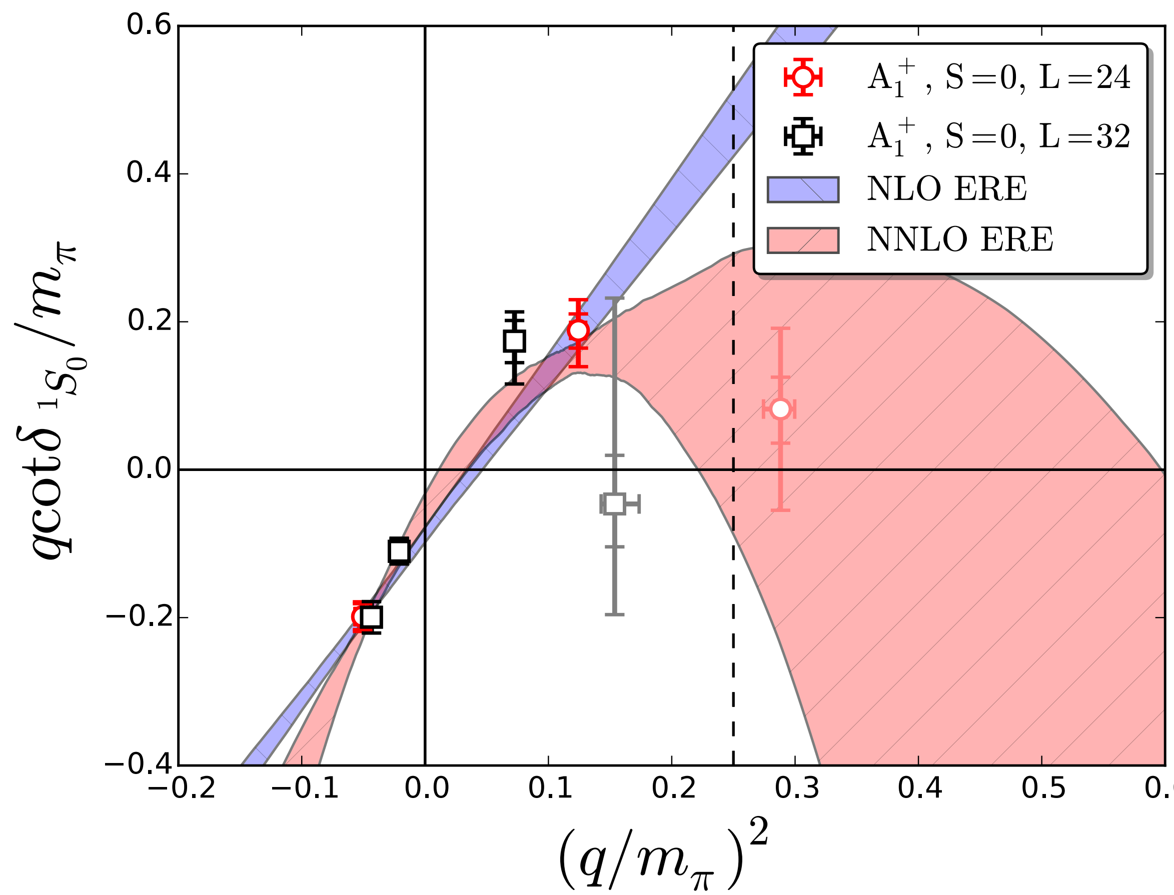


$$q^{2\ell+1}\cot(\delta_\ell) = iq$$

$$^1S_0 = 21.8(^{+3.2}_{-5.1})(^{+0.8}_{-2.8})\mathrm{MeV}$$

$$^3S_1 = 30.7(^{+2.4}_{-2.5})(^{+0.5}_{-1.6})\mathrm{MeV}$$

$$^3S_1 = 3.3(^{+1.0}_{-0.9})(^{+0.6}_{-0.2})\mathrm{MeV}$$??



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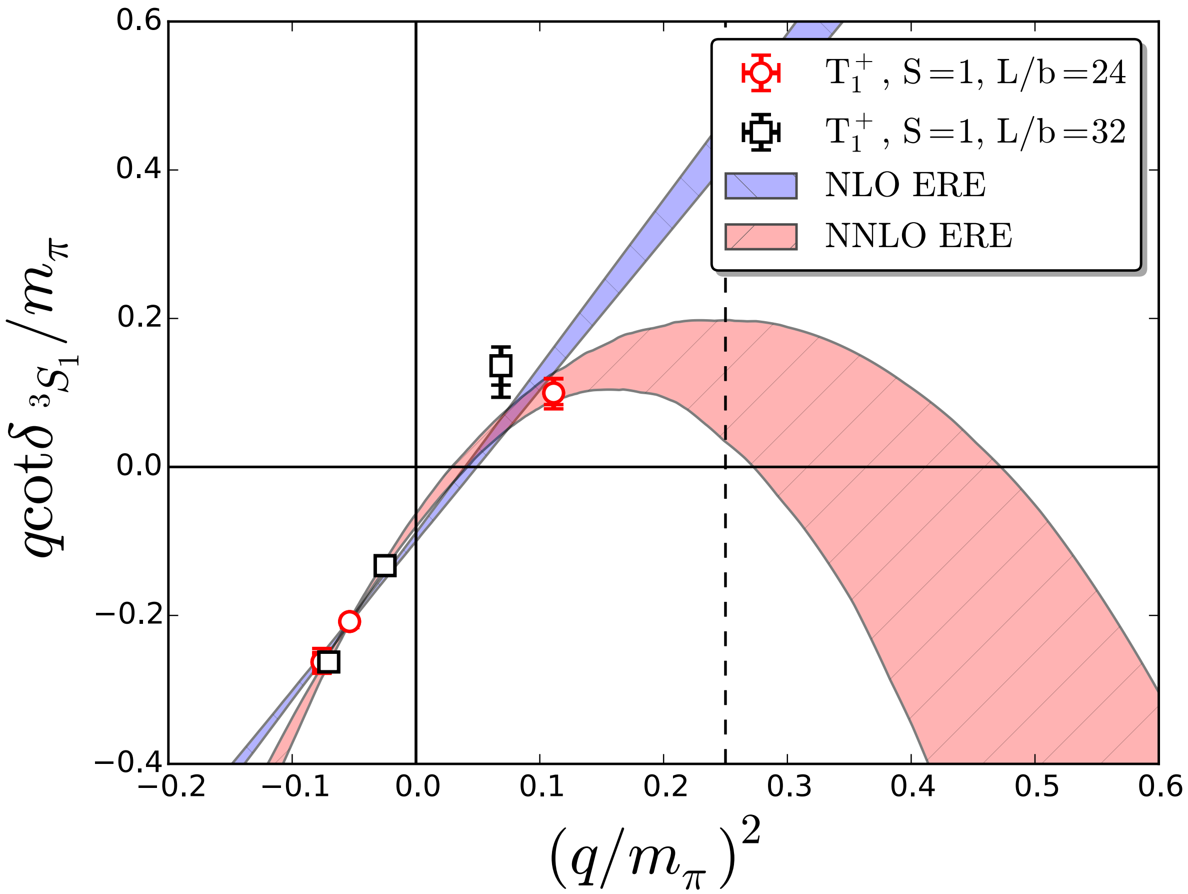

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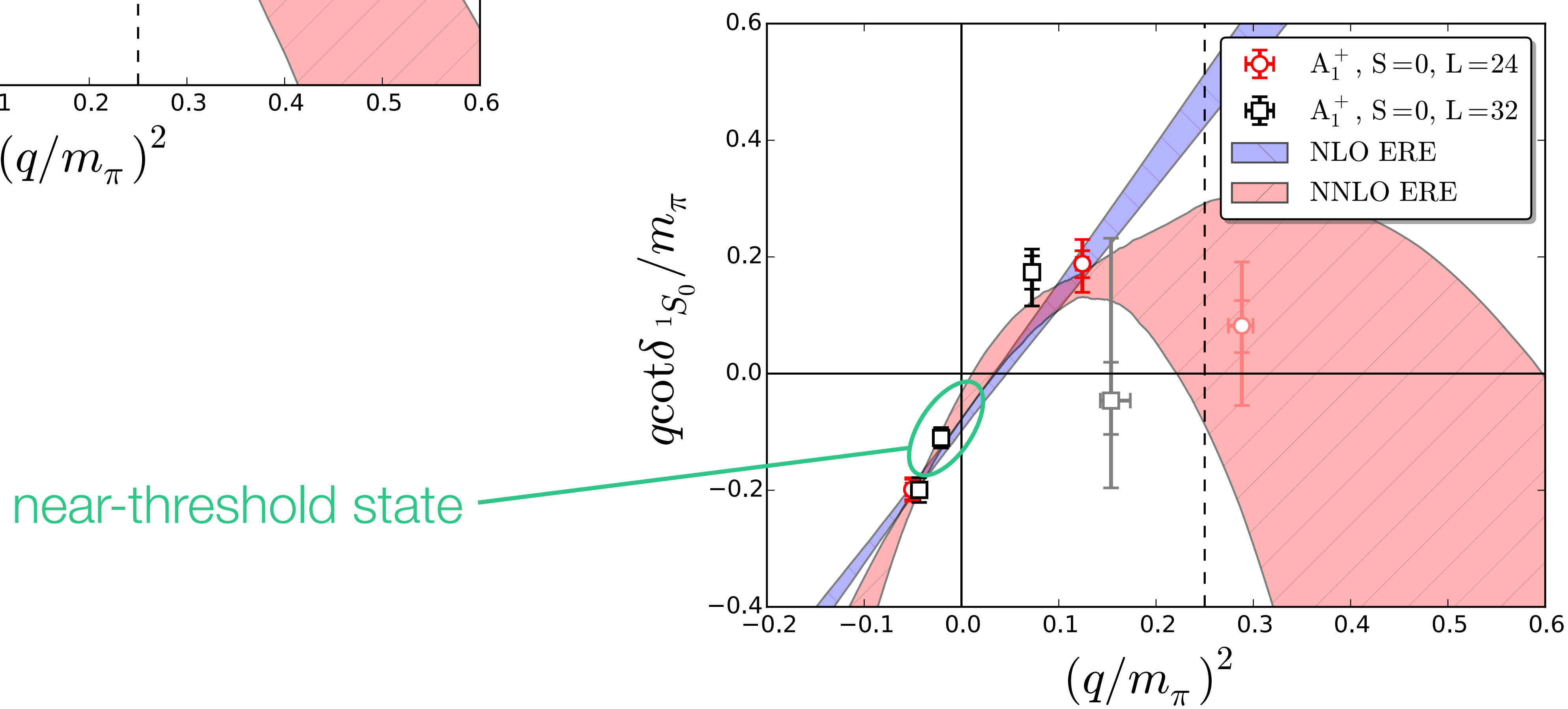

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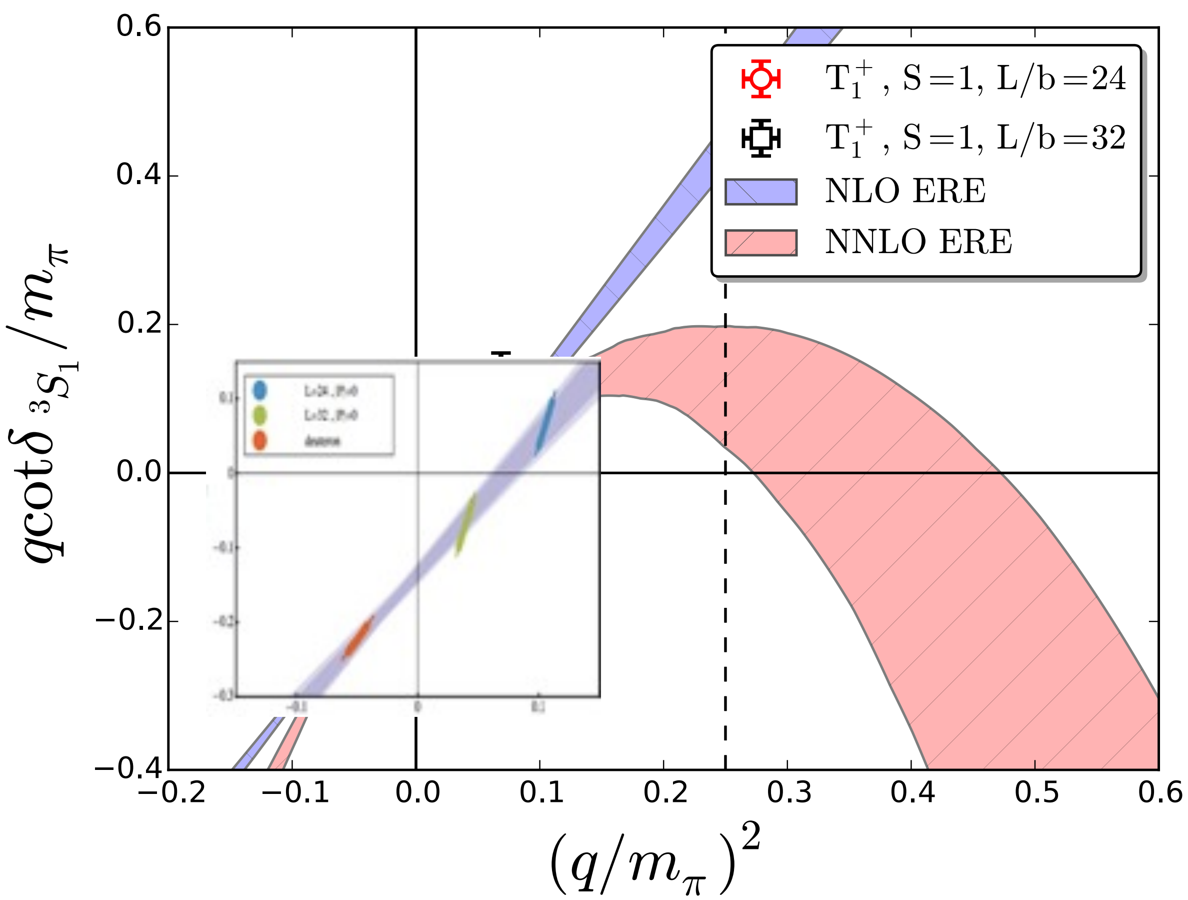

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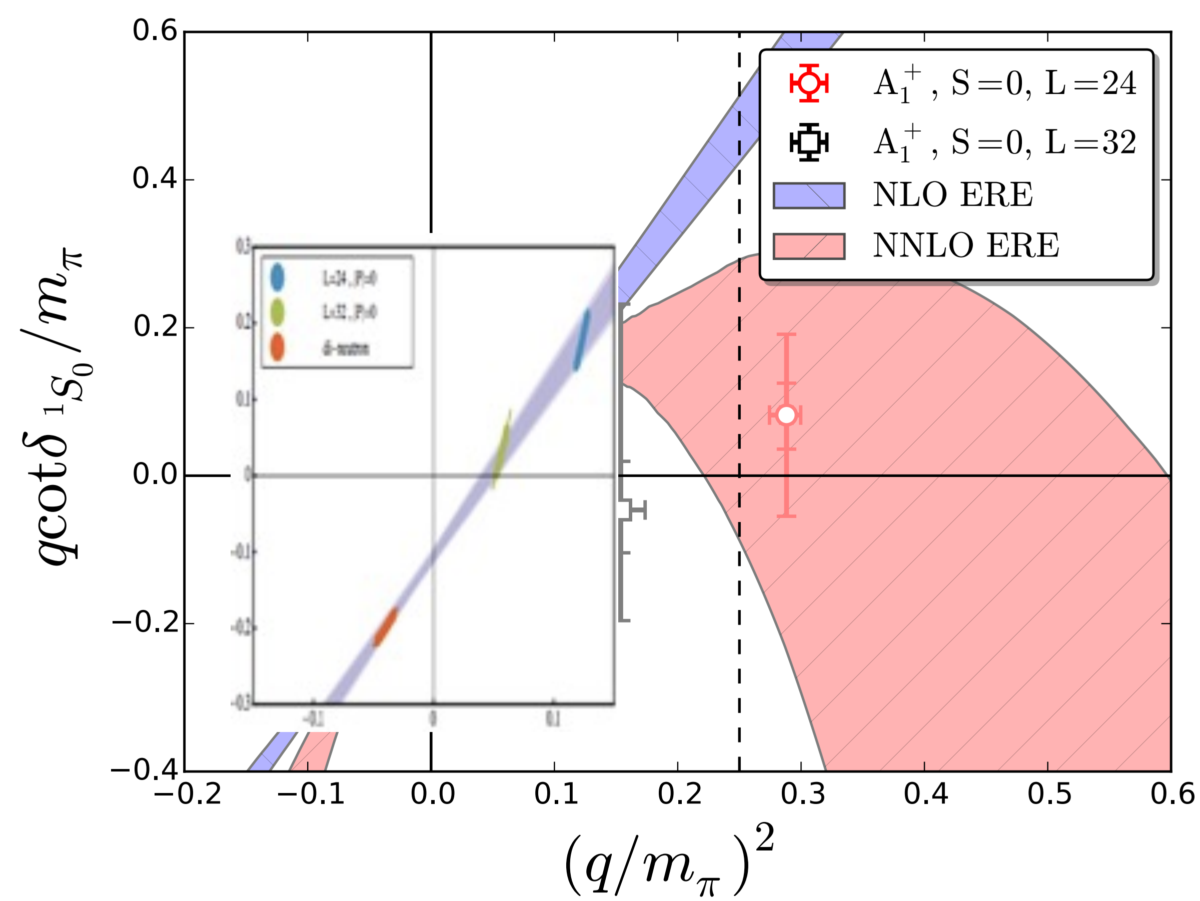
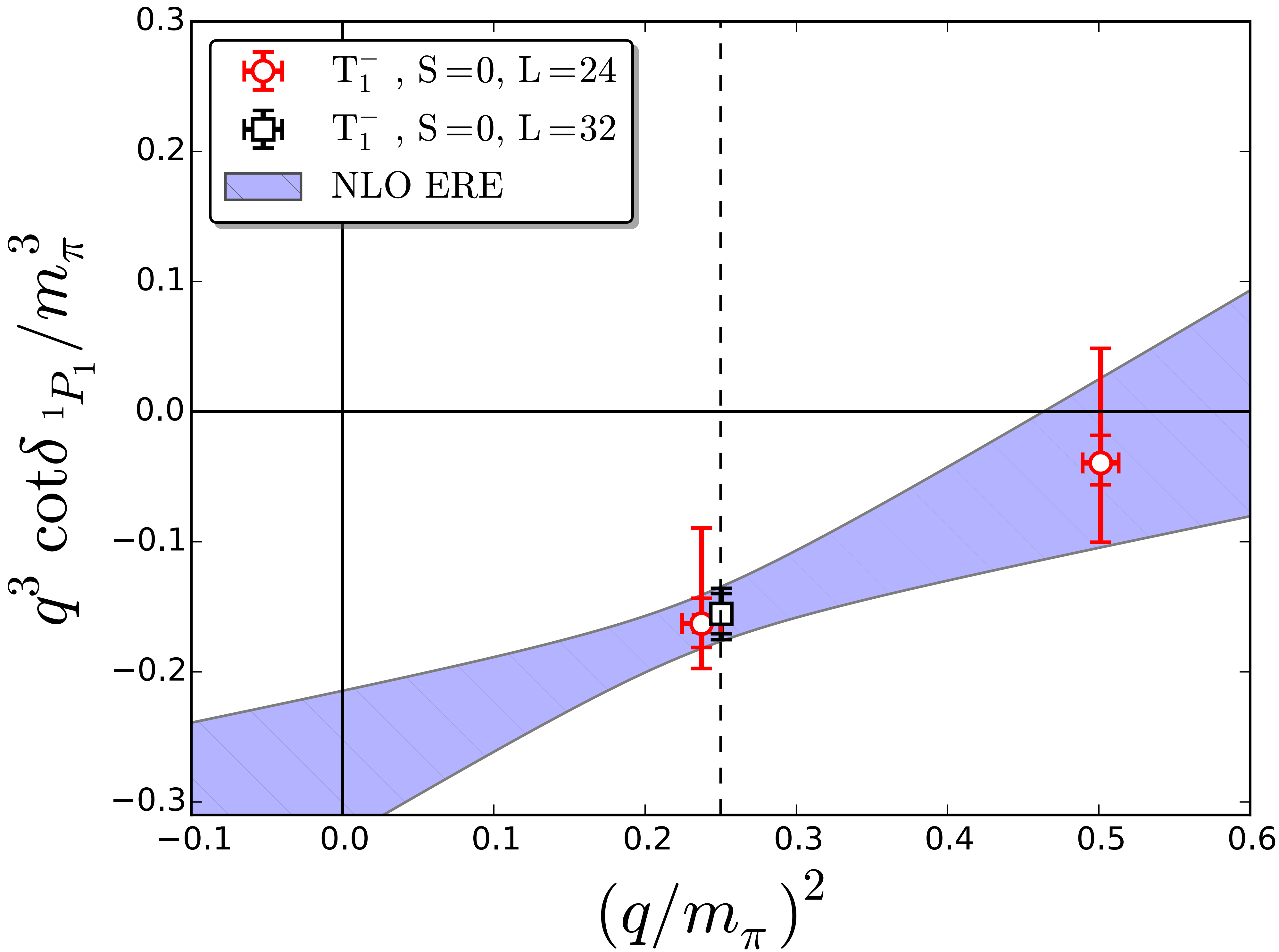
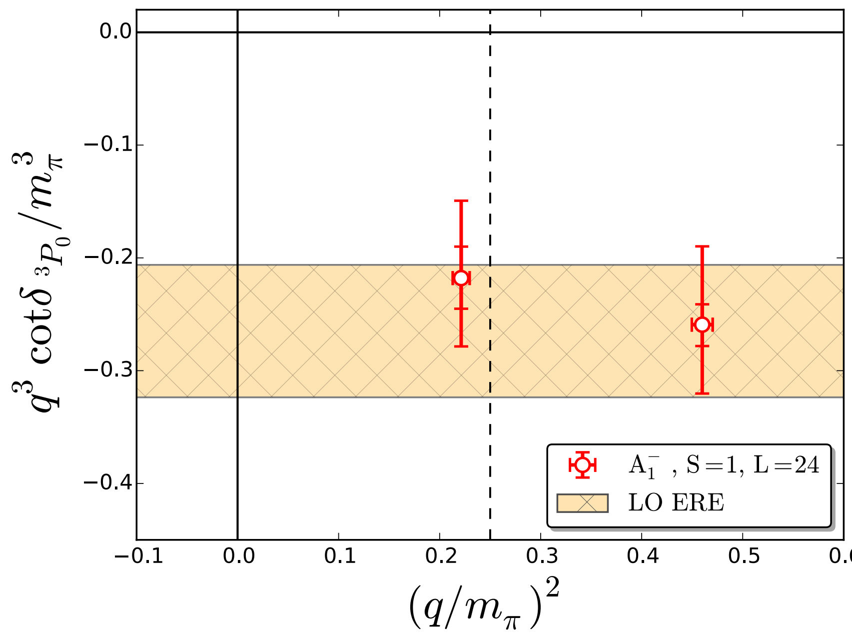


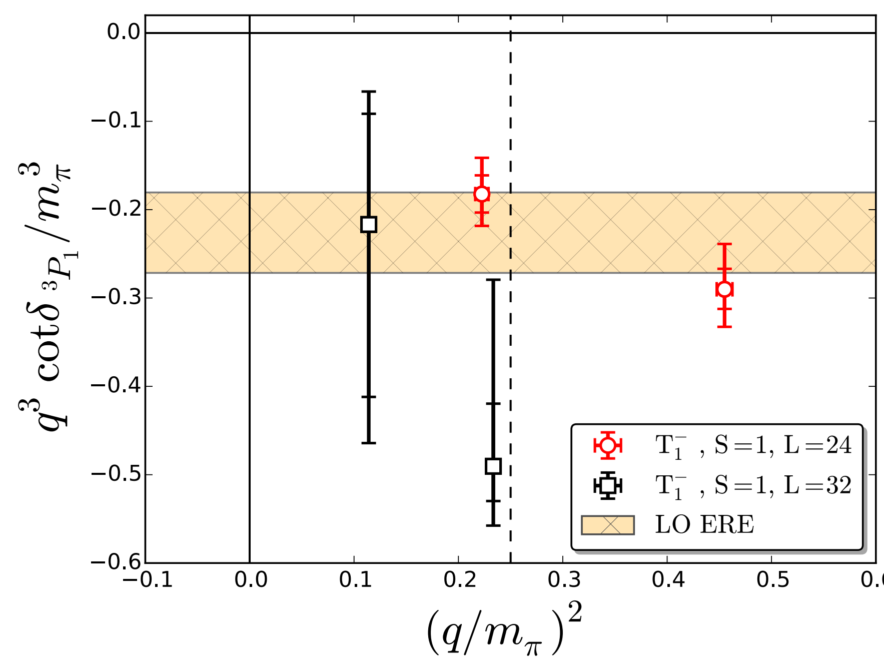
I=0 P-wave

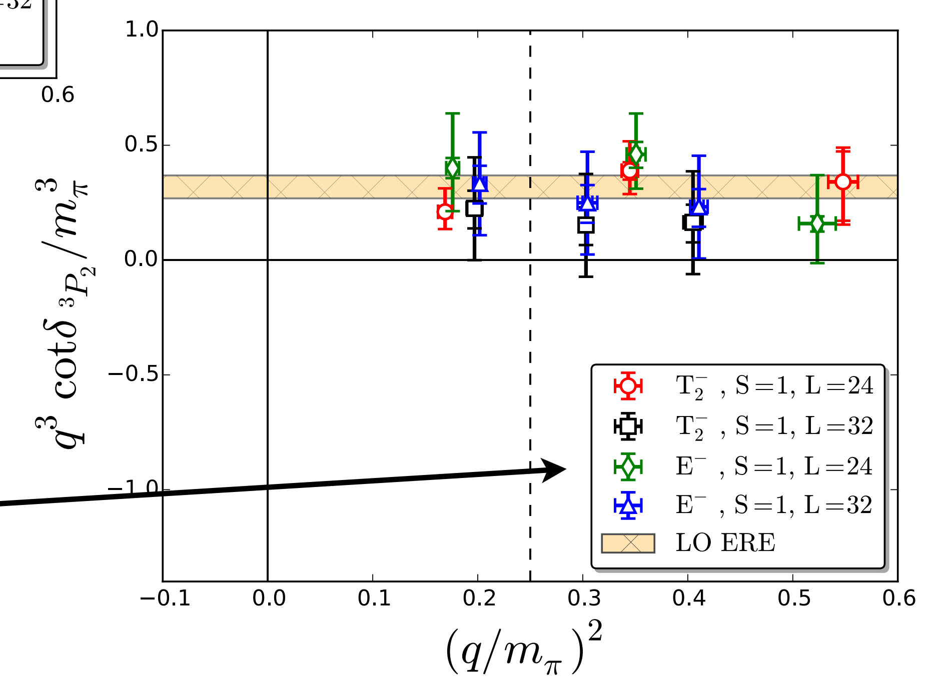


T_1^- , S=0, L=24
T_1^- , S=0, L=32
NLO ERE
$q^3 \cot\delta_{^1P_1}/m_\pi^3$
$(q/m_\pi)^2$
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.1
0.0
0.1
0.2
0.3
0.4
0.5
0.6

I=1 P-wave

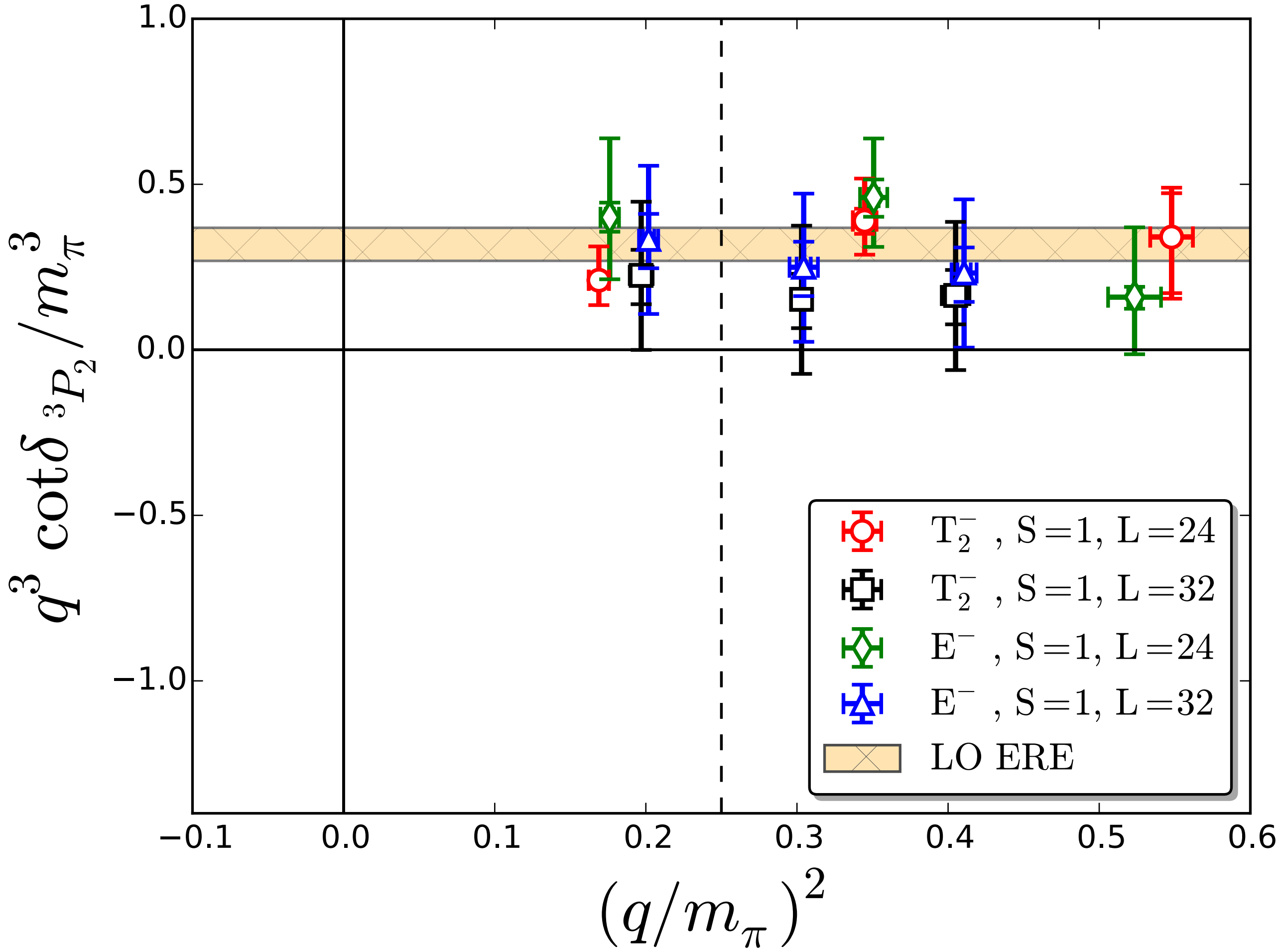




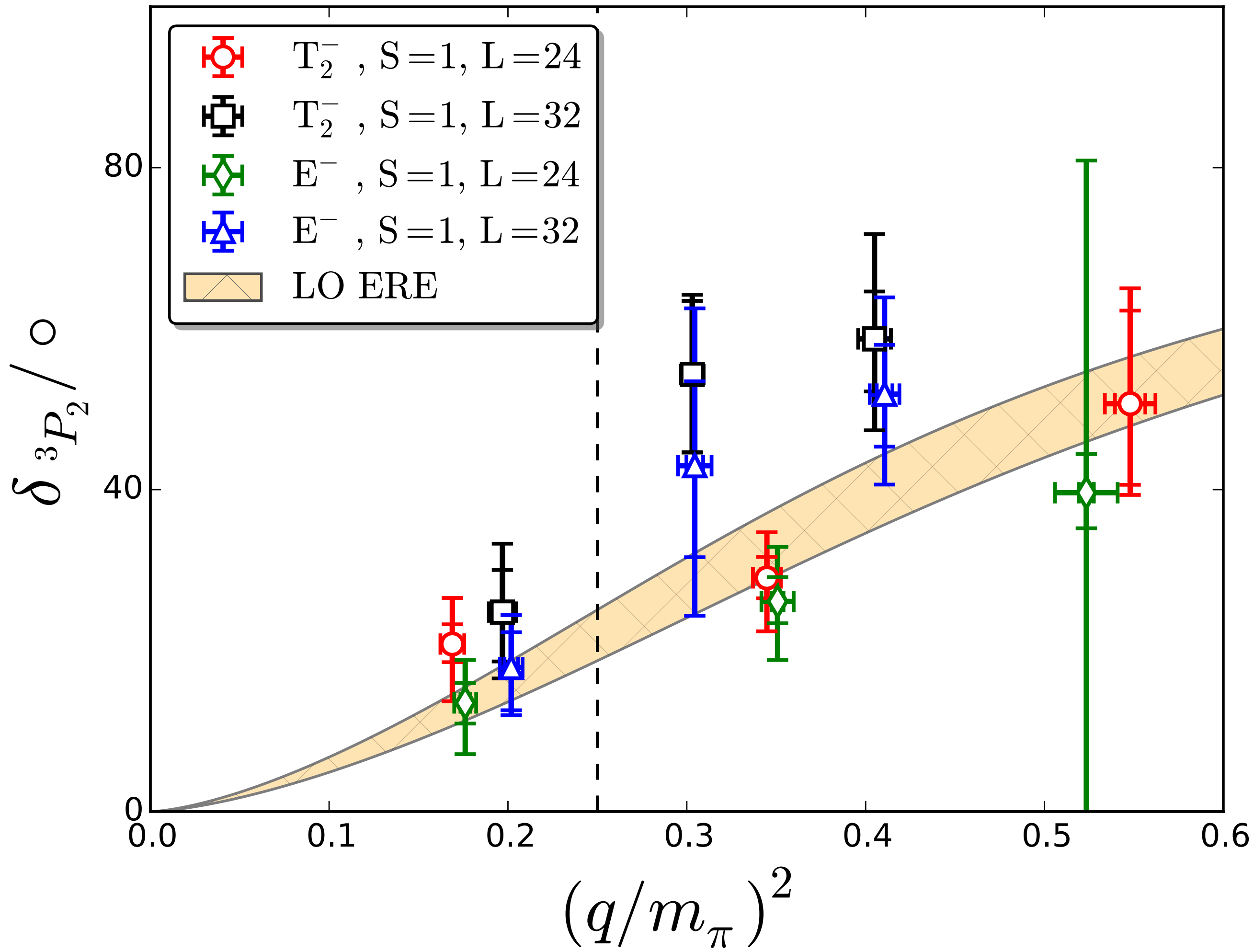


two cubic irreps!

I=1 P-wave

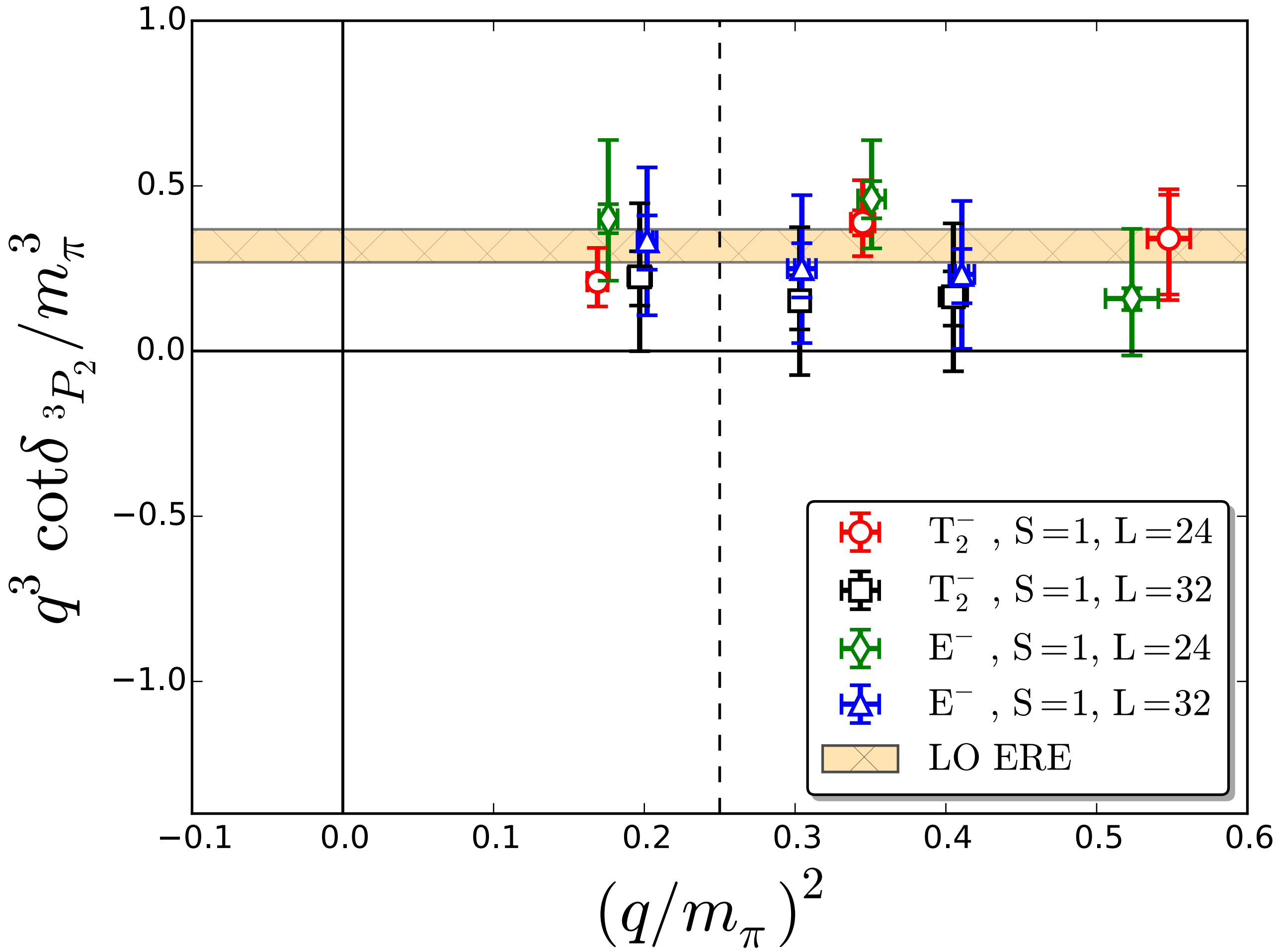


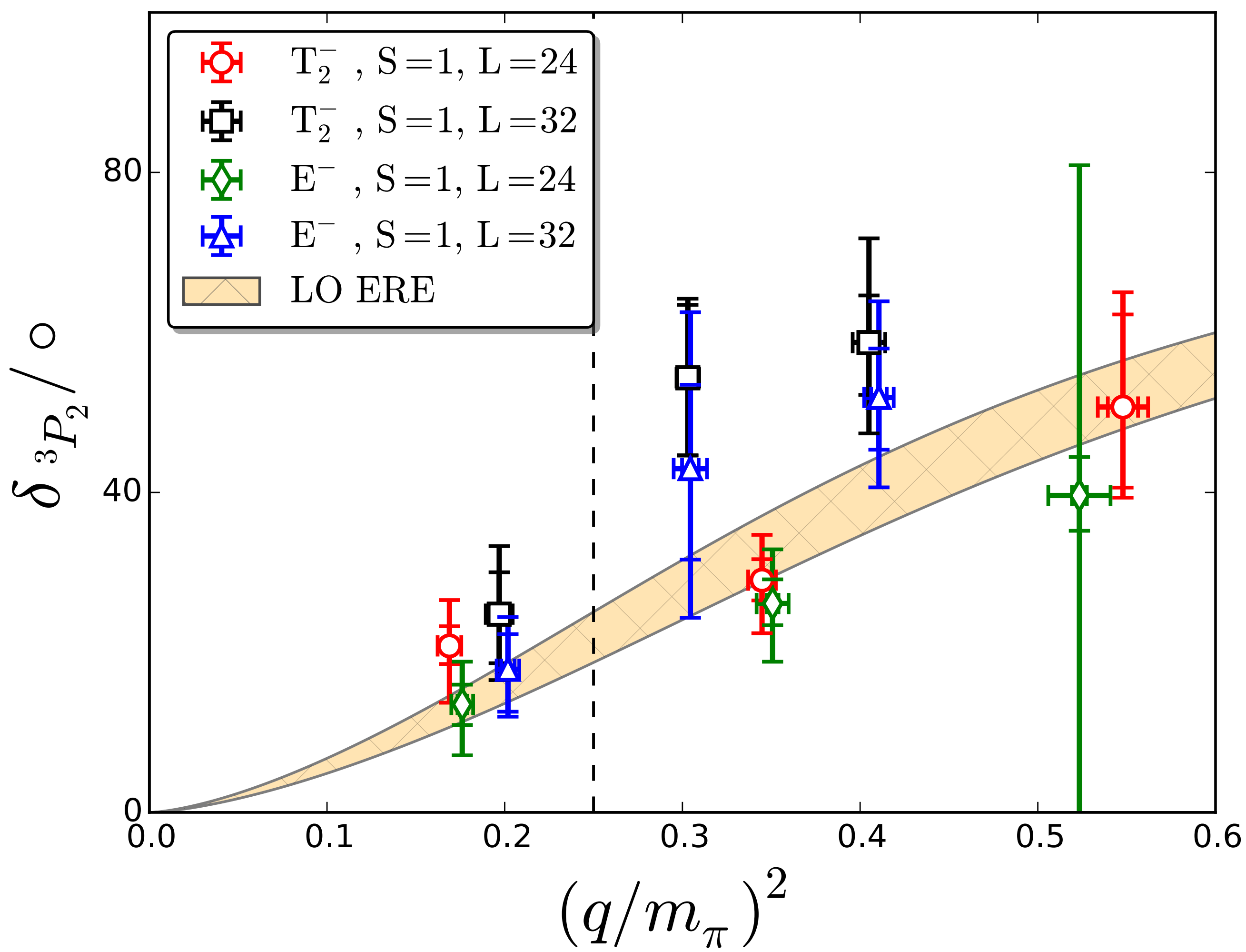
$q^3 \cot\delta_{^3P_2}/m_\pi^3$
$(q/m_\pi)^2$
T_2^- , S=1, L=24
T_2^- , S=1, L=32
E^- , S=1, L=24
E^- , S=1, L=32
LO ERE



$\delta_{^3P_2}/\circ$
$(q/m_\pi)^2$
T_2^- , S=1, L=24
T_2^- , S=1, L=32
E^- , S=1, L=24
E^- , S=1, L=32
LO ERE

I=1 P-wave





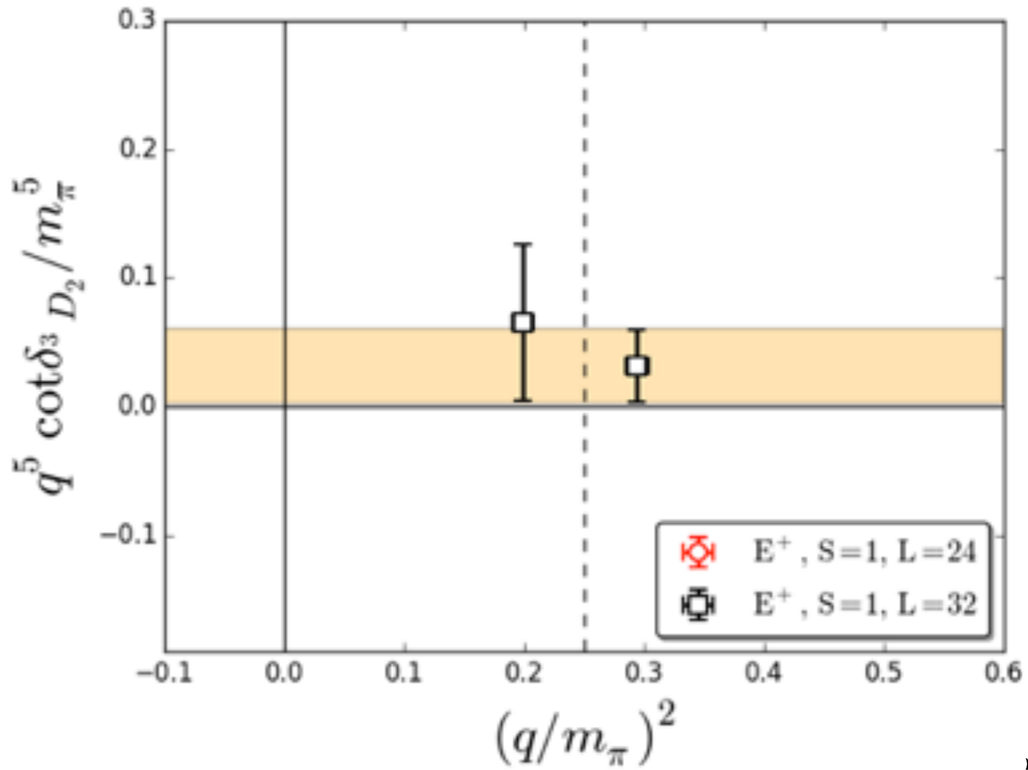
Concordance between different irreps

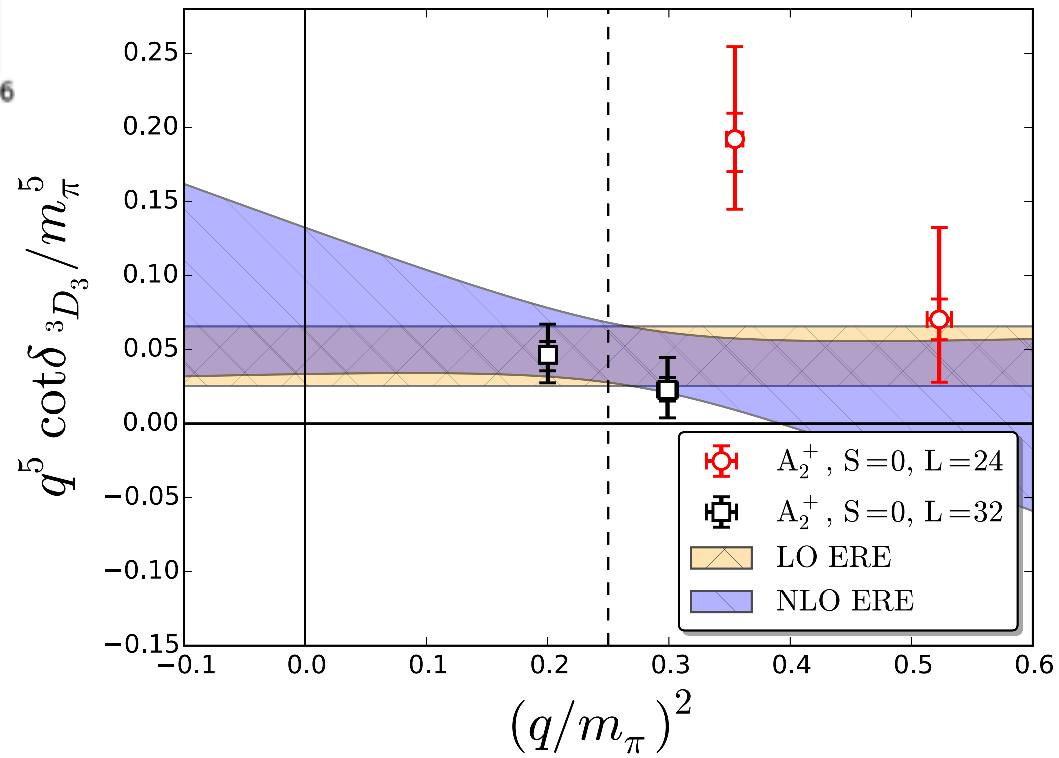
↓

small mixing?

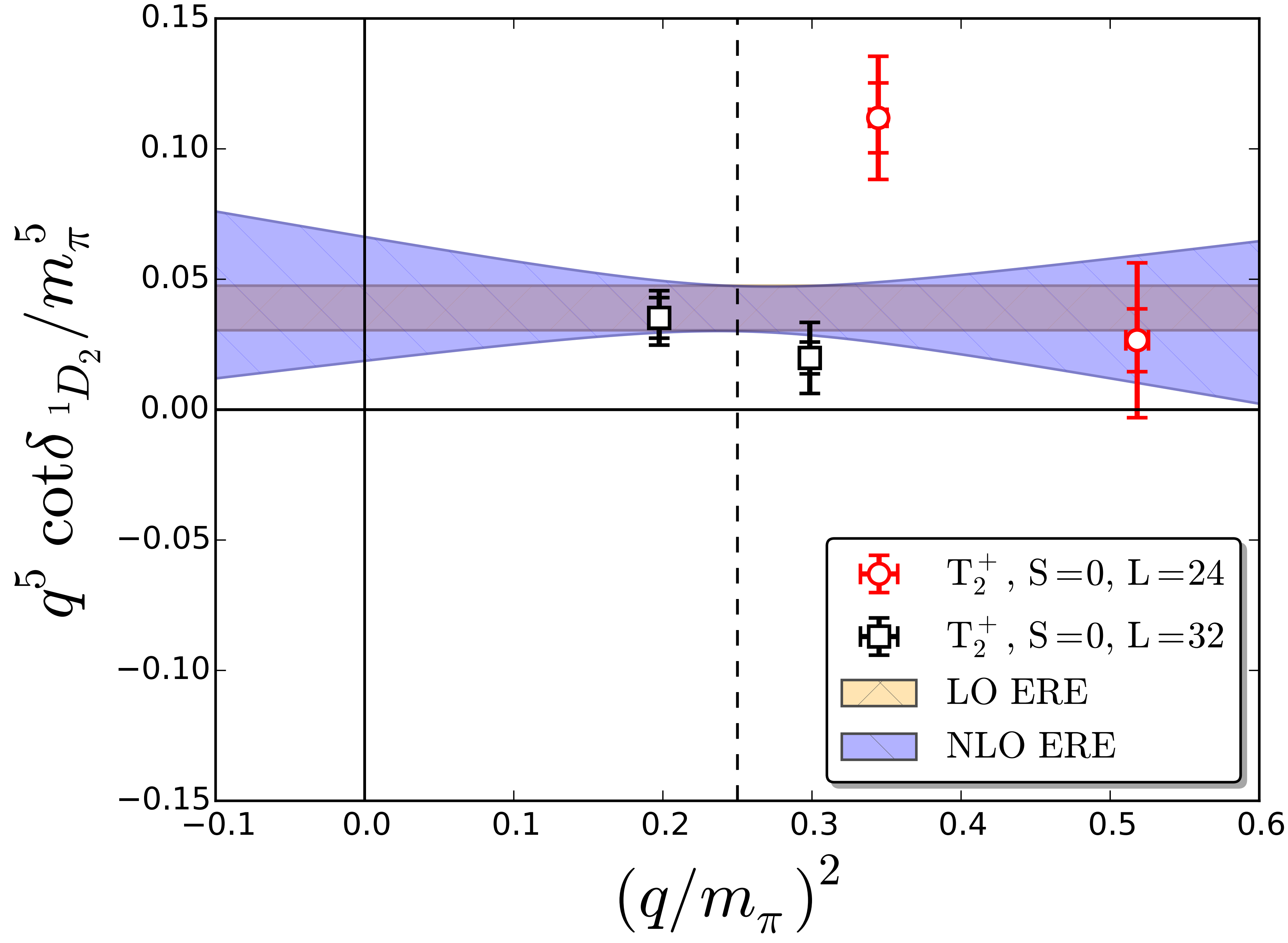
I=0 D-wave

☹ No $^3D_1$ without disentangling partial waves



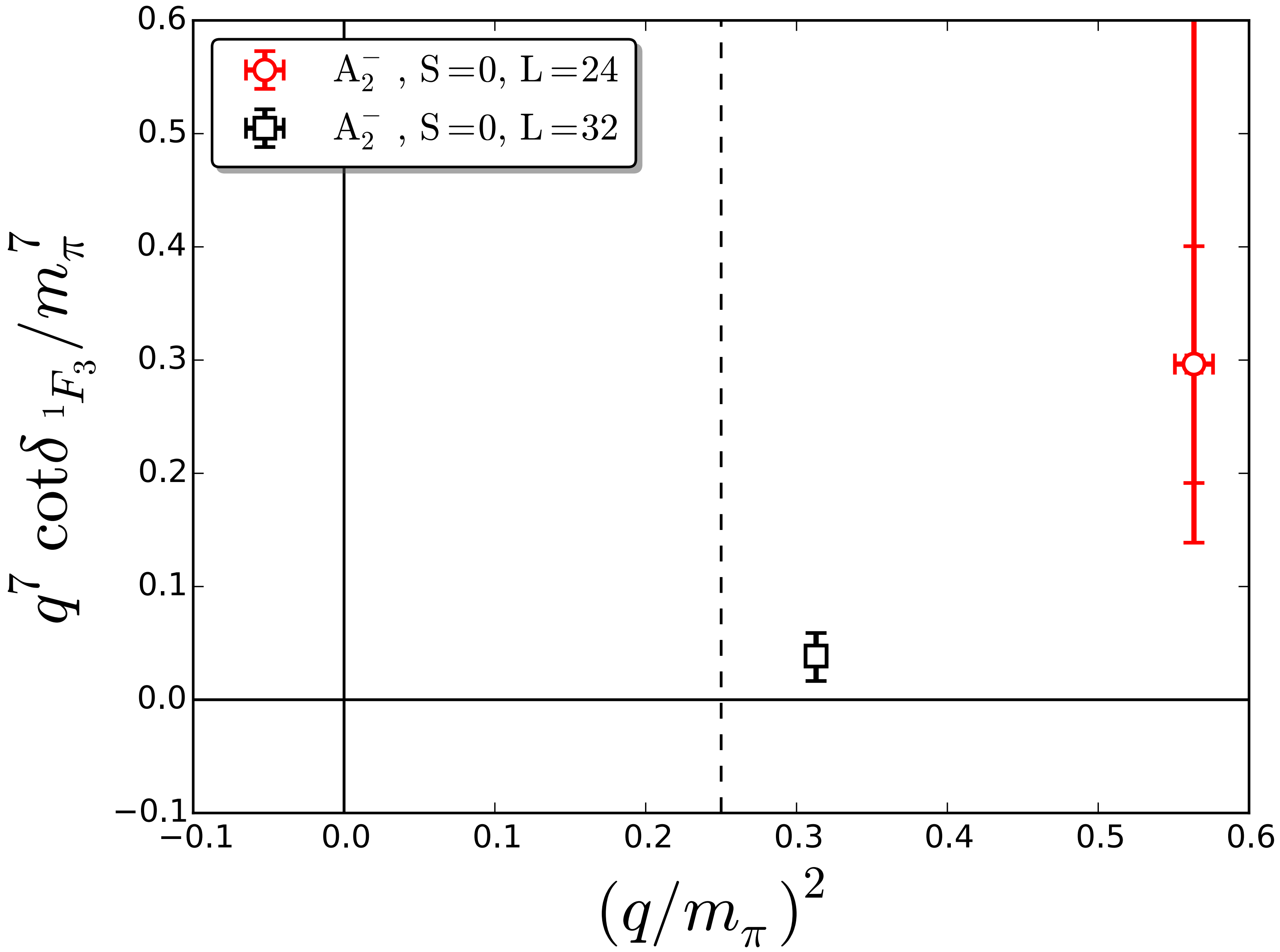


I=1 D-wave



0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.1
0.0
0.1
0.2
0.3
0.4
0.5
0.6
$q^5 \cot\delta_{^1D_2}/m_\pi^5$
$(q/m_\pi)^2$
T_2^+, S=0, L=24
T_2^+, S=0, L=32
LO ERE
NLO ERE

F Waves



☹ Other

F-waves require disentangling from P-waves

Summary

- Subduction to cubic irreps
  - Inform the choice of sources and sinks
  - Different sources give same plateau after subduction ☺
- Momentum projection separates different n^2, gives multiple points per irrep
- Displaced sources essential for parity-odd and D-waves
- One pole (and a near-threshold state) in $^1S_0$, two poles in $^3S_1$ (?)
- Different irreps give nice signal for $^3P_2$.
- Achieved signals up to F!

The Future

- Disentangle unphysical partial wave mixing.
- Get more physics/cycle with better sources.
- Lower the pion mass / head towards continuum.
- Improve NN correlators via better operators.
- Add boosts, more volumes?
- Match to nuclear physics (πEFT, $\not{\pi}$EFT, HOBET, ...)