Pionless EFT for Few-Body Systems

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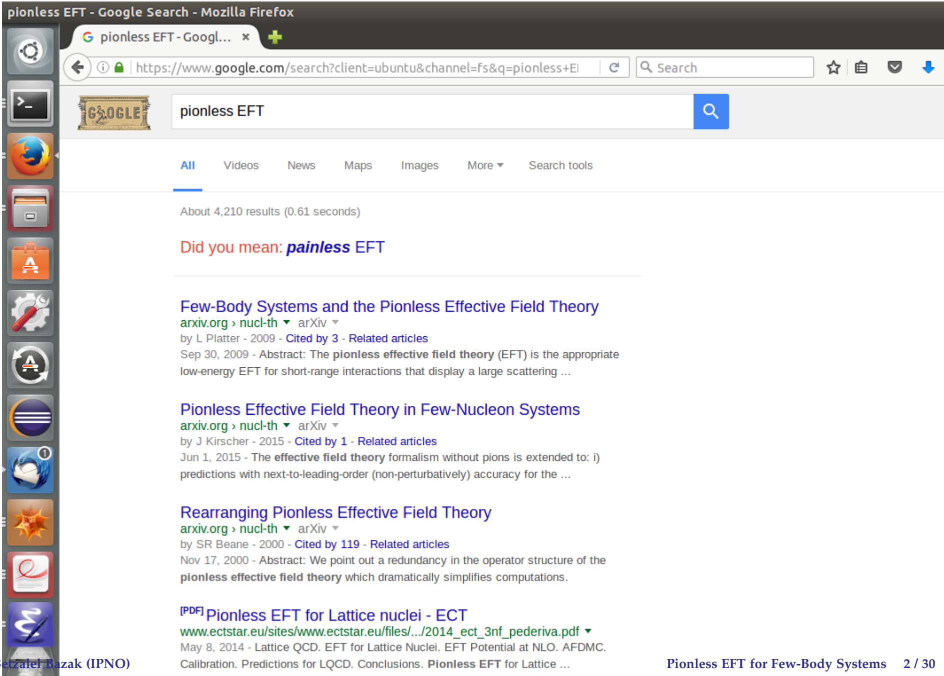
Nuclear Physics from Lattice QCD

Insitute for Nuclear Theory

April 7, 2016

Seattle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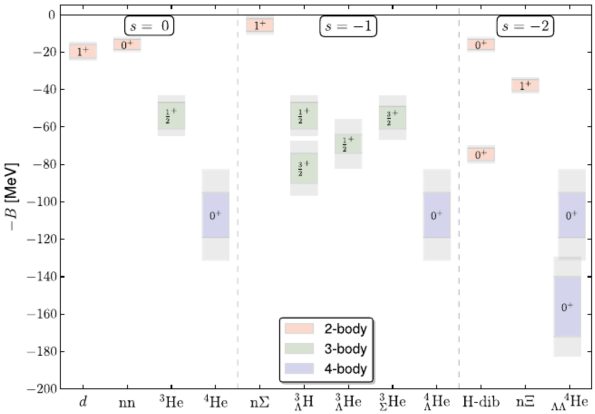
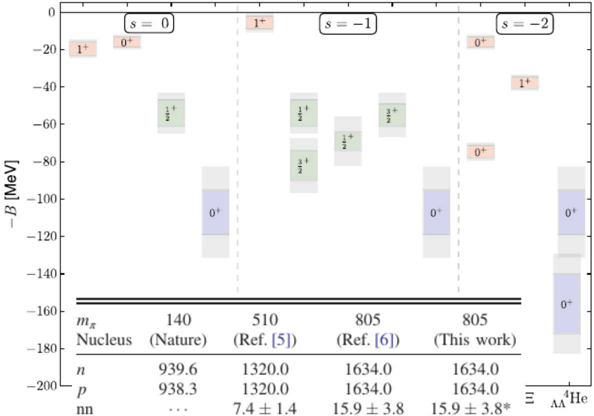
www.ectstar.eu/sites/www.ectstar.eu/files/.../2014_ect_3nf_pederiva.pdf
May 8, 2014 - Lattice QCD. EFT for Lattice Nuclei. EFT Potential at NLO. AFDMC. Calibration. Predictions for LQCD. Conclusions. **Pionless EFT** for Lattice ...



S. R. Beane *et al.*(NPLQCD Collaboration), Phys. Rev. D **87**, 034506 (2013)

N. Barnea *et al.*, Phys. Rev. Lett. **114**, 052501 (2015);
J. Kirscher *et al.*, Phys. Rev. C **92**, 054002 (2015)



m_π	140	510	805	805
Nucleus	(Nature)	(Ref. [5])	(Ref. [6])	(This work)
n	939.6	1320.0	1634.0	1634.0
p	938.3	1320.0	1634.0	1634.0
nn	$\cdots$	7.4 ± 1.4	15.9 ± 3.8	15.9 ± 3.8*
D	2.224	11.5 ± 1.3	19.5 ± 4.8	19.5 ± 4.8*
^3n	$\cdots$			$\cdots$
^{3}H	8.482	20.3 ± 4.5	53.9 ± 10.7	53.9 ± 10.7*
^{3}He	7.718	20.3 ± 4.5	53.9 ± 10.7	53.9 ± 10.7
^{4}He	28.30	43.0 ± 14.4	107.0 ± 24.2	89 ± 36
^{5}He	27.50			98 ± 39
^{5}Li	26.61			98 ± 39
^{6}Li	32.00			122 ± 50

S. R. Beane *et* 34506 (2013)

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Universality

- Consider particles interacting through 2-body potential with range R.
- Classically, the particles ‘feel’ each other only within the potential range.
- But, in the case of resonant interaction, the wave function can have much larger extent.
- At low energies, the 2-body physics is completely govern by the scattering length, a.

$$\lim_{k\to 0} k \cot \delta(k) = -\frac{1}{a} + \frac{1}{2} r_{\mathrm{eff}} k^2$$

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- When $|a| \gg R$ the potential details has no influence: *Universality*.

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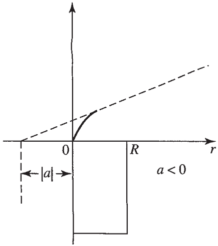

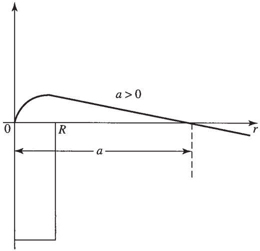

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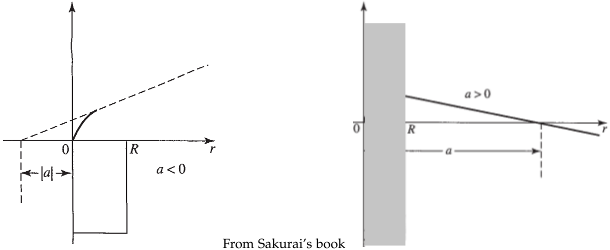

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- Corrections to universal theory are of order of r_{eff}/a and R/a.
- For $a > 0$, we have universal dimer with energy $E = -\hbar^2/ma^2$.
- ^{4}He Atoms: $a \approx 170.9a_0$, (a_0 = the Bohr radius), is much larger than its van der Waals radius, $r_{vdW} \approx 9.5a_0$.
- Nucleus: $a_s \approx -23.4$ fm, $a_t \approx 5.42$ fm, $R = \hbar/m_\pi c \approx 1.4$ fm.
  Deuteron binding energy, 2.22 MeV, is close to $\hbar/ma_t{}^2 \approx 1.4$ MeV.
- Ultracold atoms near a Feshbach resonance,

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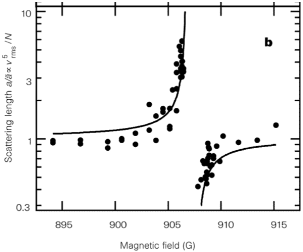
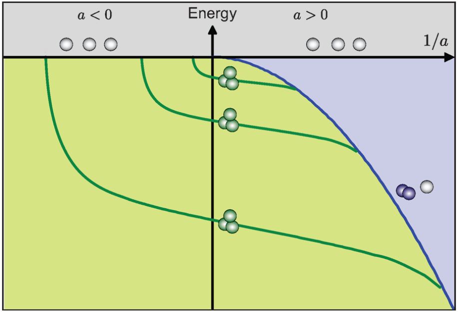


S. Inouye *et al.*, Nature 392, 151 (1998)

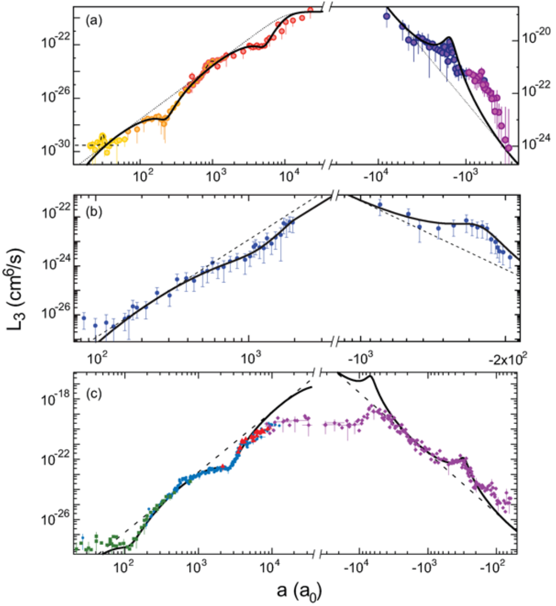
Efimov Physics

- *The unitary limit*: $E_2 = 0$, $a \longrightarrow \infty$.
- In 1970 V. Efimov found out that if $E_2 = 0$ the 3-body system will have an **infinite** number of bound states.
- The 3-body spectrum is $E_n = E_0 e^{-2\pi n/s_0}$ with $s_0 = 1.00624$.



F. Ferlaino and R. Grimm, Physics **3**, 9 (2010)

Efimov Physics in Ultracold Atoms

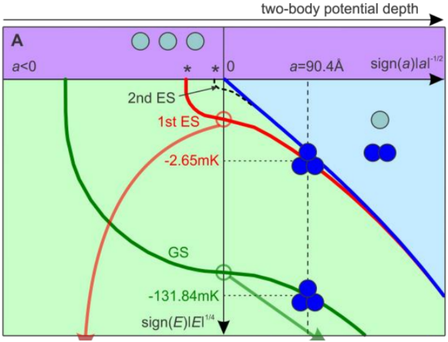


F. Ferlaino and R. Grimm, Physics **3**, 9 (2010)

- ^{39}K
  M. Zaccanti *et al.*,
  Nature Phys. **5**, 586 (2009).
- ^{7}Li
  N. Gross, Z. Shotan, S. Kokkelmans, and L. Khaykovich,
  Phys. Rev. Lett. **103**, 163202 (2009).
- ^{7}Li
  S.E. Pollack, D. Dries, and R.G. Hulet,
  Science **326**, 1683 (2009)

^{4}He Atoms

- The system of ^{4}He atoms is known to be a natural candidate for universal physics, since $a \approx 170.9 a_0 \gg r_{vdW} \approx 9.5 a_0$.
- The ^{4}He dimer is bound by 1.62 mK ($\hbar^2/ma^2 \approx 1.48$ mK).
- $E_3 \approx 131.84$ mK, $E_3^* \approx 2.6502$ mK, giving a ratio of $E_3^*/E_3 \approx 49.7$.
- Recently this excited state was observed experimentally.



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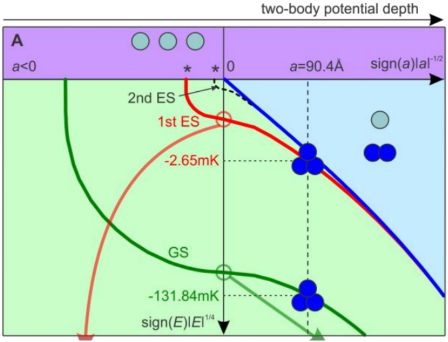

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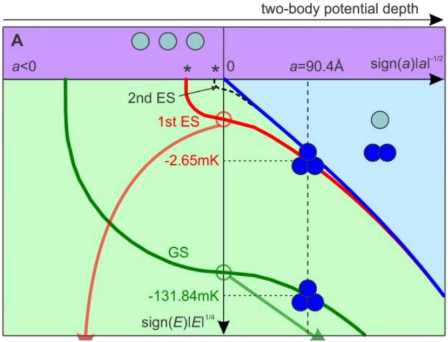

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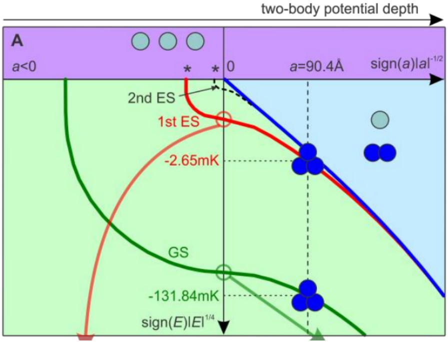

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Effective Field Theory (EFT)

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- For example, nuclear structure involves energies that are much smaller than the typical QCD mass scale, $M_{QCD} \approx 1$ GeV.
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Pionless EFT

- The degrees of freedom in pionless EFT are the nucleons.
- We have to include all terms conserving our theory symmetries, order by order.
- For nucleons, the Leading Order (LO) is,

$$V_{LO} = a_1 + a_2 \sigma_i \cdot \sigma_j + a_3 \tau_i \cdot \tau_j + a_4 (\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j)$$

  where due to symmetry, only 2 are independent, corresponding to the two scattering lengths.
- The Next to Leading Order (NLO) is,

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$$\mathbf{q} = \mathbf{p}' - \mathbf{p}, \quad \mathbf{k} = \mathbf{p} + \mathbf{p}'$$

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- p-wave enters at N^3LO!

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- At NLO,

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- Terms proportional to $\boldsymbol{k} \cdot \boldsymbol{q}$ are omitted due to time reversal symmetry.
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Stochastic Variational Method I

- To solve the N-body Schrodinger equation, we use correlated Gaussian basis,

$$\psi(\eta_1, \eta_2 ... \eta_{A-1}) = \sum_i c_i \mathcal{A} \exp(-\eta^T A_i \eta)$$

$\eta =$ Jacobi coordinates, $A_i =$ matrix of $(N-1) \times (N-1)$ numbers.

- Since $a \gg \Lambda^{-1}$, we need large spread of basis functions.
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$$\langle A|A'\rangle = \left(\frac{(2\pi)^{N-1}}{\det B}\right)^{3/2} \quad ; \qquad B = A + A'$$

$$\langle A|T_{int}|A'\rangle = 3\langle A|A'\rangle \mathrm{Tr}[AB^{-1}A'\Pi]; \qquad \Pi_{ij} = (2\mu_i)^{-1}\delta_{ij}$$

$$\langle A|V^{2b}|A'\rangle = \frac{\Lambda^2 C^{(0)}}{2m\sqrt{\pi}}\langle A|A'\rangle \sum_{i<j}\left(1 + f_{ij}\Lambda^2/2\right)^{-3/2} ; \; f_{ij} = \omega_{ij}^T B^{-1}\omega_{ij}, \; \mathbf{r}_{ij} = \omega_{ij}^T \boldsymbol{\eta}$$

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- We add basis function one by one, or try to replace an exist basis function by a new one.
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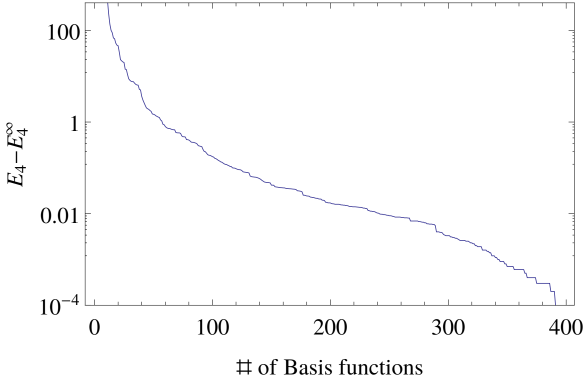

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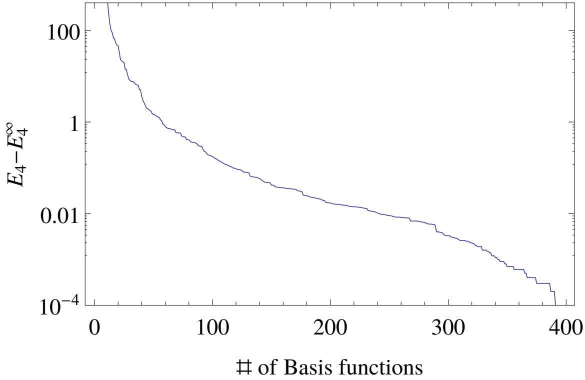

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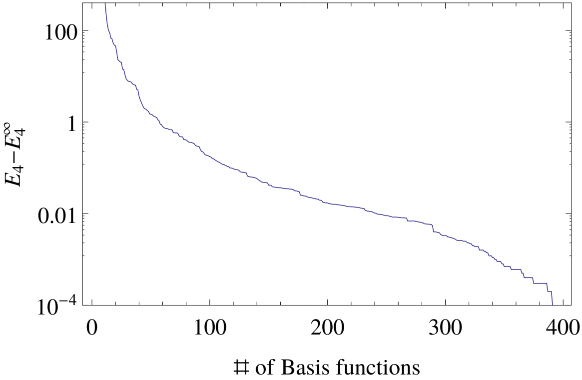

Regularization I: non local potential

- At LO, we have only contact interaction,

$$V(r_{ij}) = g\delta(r_{ij}).$$

- This interaction needs regularization and renormalization.
- The bound state of two identical bosons (here $\hbar = m = 1$),

$$-\nabla^2\psi(r) + g\delta(r)\psi(r) = -B_2\psi(r)$$

and in momentum space,

$$p^2\phi(p) + g\int \frac{d^3p'}{(2\pi)^3}\phi(p') = -B_2\phi(p)$$

- Therefore,

$$\frac{1}{g} = \int \frac{d^3p'}{(2\pi)^3}\frac{1}{p'^2 + B_2}$$

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- Doing so for the incoming and outcoming momenta we have,

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$$g = \frac{8\sqrt{2}\pi^{3/2}}{\Lambda}\left(1 + \sqrt{\pi}\frac{Q}{\Lambda} + \ldots\right).$$

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Regularization II: local potential

- We can cut the momentum transfer $q = p - p'$, to get a local potential,

$$\langle r|V|r'\rangle = g\delta_\Lambda(r)\delta(r - r')$$

  but now the two-body equation is to be solved numerically.
- The LEC is renormalized by fixing one observable, like the dimer binding energy or the scattering length, to its physical value.
- Using dimension less LEC,

$$V_{LO}(r) = \frac{4\pi\hbar^2}{m\Lambda}C^{(0)}(\Lambda)\delta_\Lambda(\boldsymbol{r}), \;\; C^{(0)}(\Lambda) = 2.38\left(1 + \frac{2.25}{a\Lambda} - \frac{4.68}{(a\Lambda)^2} + \ldots\right)$$

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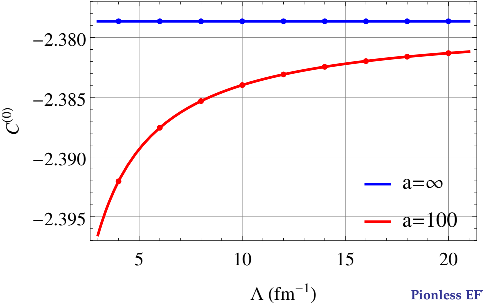

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- NLO term is to be taken in perturbative way.
- For energies,

$$\Delta E = \langle \psi_{LO} | V_{NLO} | \psi_{LO} \rangle$$

- For scattering amplitude - distorted wave Born approximation,

$$\Delta f_k = -\frac{m}{k^2}\int dr \psi_{LO}^2 V_{NLO}$$

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Two-boson system

- From TTY potential we have $a = 189a_0$ and $B_2 = 1.31$ mK. LO is fitted to $a = 189a_0$, NLO to $r_{eff} = 14.2a_0$.
- $r_{eff}/a \approx 8\%$; $(r_{eff}/a)^2 \approx 0.5\%$
- Fitting to powers of Q_2/Λ, ($B_A = AQ_A^2/2m$) we extract $B_2^{LO} = 1.21$ mK, $B_2^{NLO} = 1.30$ mK, $B_2^{TTY} = 1.31$ mK

Tang, Toennies & Yiu, PRL **74**, 1546 (1995)

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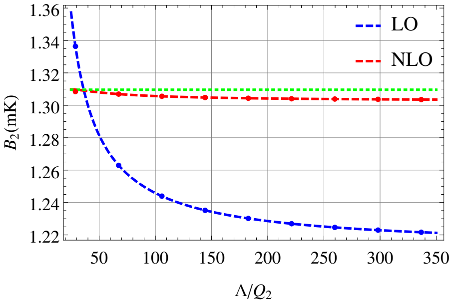

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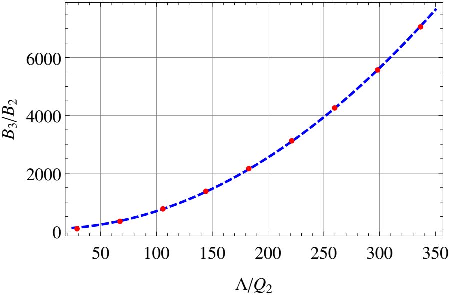
Three-boson system

Trying to calculate the trimer binding energy we get the Thomas collapse:

$$B_3 \propto \frac{\hbar\Lambda^2}{m}$$



- To stabilize the system, a 3-body counter term must be introduced at LO,

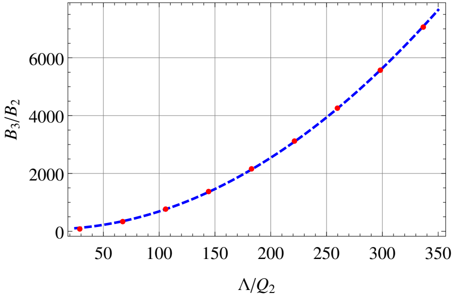
$$V_{LO}^{3N} = \frac{4\pi\hbar^2}{m\Lambda^4} D^{(0)} \sum_{i<j<k} \sum_{cyc} \delta_\Lambda(\boldsymbol{r}_{ij})\delta_\Lambda(\boldsymbol{r}_{jk}),$$

  Λ_* is a new momentum scale, $D^{(0)} = f(a\Lambda, \Lambda/\Lambda_*)$
- $D^{(0)}$ is fixed by another observable.

Three-boson system

Trying to calculate the trimer binding energy we get the Thomas collapse:

$$B_3 \propto \frac{\hbar\Lambda^2}{m}$$



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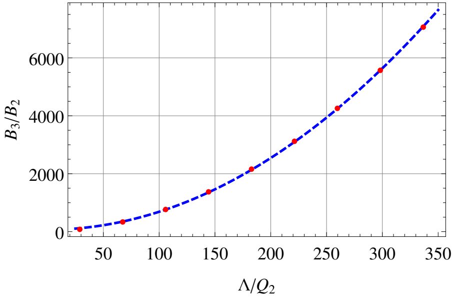

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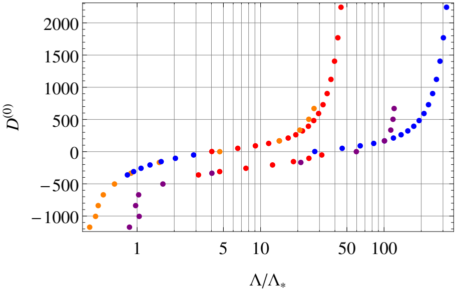
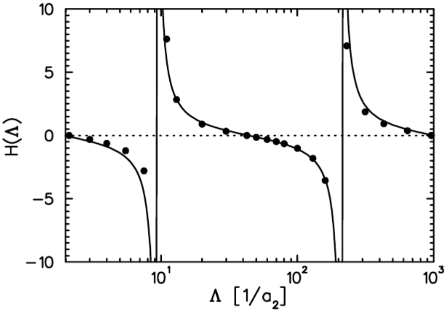


$\Lambda_3 = \Lambda_2$, local, smooth cutoff



$\Lambda_3 \ll \Lambda_2$, non-local, sharp cutoff

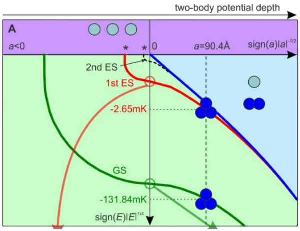
P. F. Bedaque, H.W. Hammer, and U. van Kolck

Phys. Rev. Lett. **82** 463 (1999).

$\Lambda_3 = \Lambda_2$, non-local, smooth cutoff

R.F. Mohr *et al.*, Ann. Phys. **321**, 225 (2006).

Atom-dimer scattering

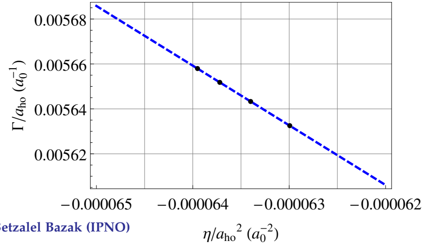


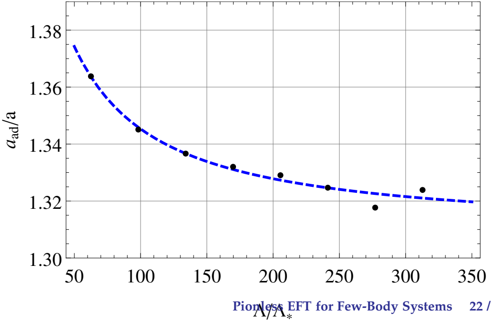
LO and NLO are fitted to $B_3^* = 1.74 B_2$.
The atom-dimer scattering length is calculated in a trap, using

$$\frac{1}{\sqrt{2}} \frac{\Gamma[(1-\eta)/4]}{\Gamma[(3-\eta)/4]} = \frac{a_{ad}/a_{ho}}{1 - a_{ad} r_{ad} \eta / (4 a_{ho}^2)}$$

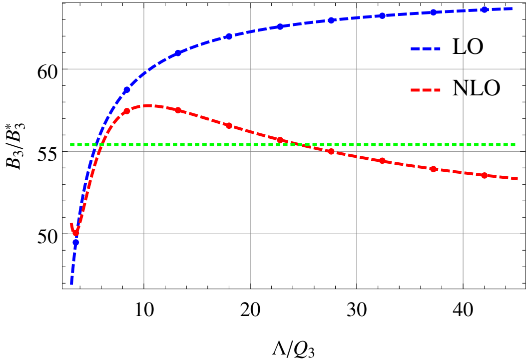
$$a_{ho} = \sqrt{\hbar/(2\mu\omega)} \quad \mu = 2m/3 \quad \eta = 2(E_3 - E_2)/(\hbar\omega)$$

Suzuki *et al.*, PRA **80**, 033601 (2009); Stetcu *et al.*, Ann. Phys. **325**, 1644 (2010).





Trimer ground state



- Fitting to powers of Q_3/Λ , $B_3(\Lambda) = B_{3\infty}(1 + \alpha\frac{Q_3}{\Lambda} + \beta\left(\frac{Q_3}{\Lambda}\right)^2 + \gamma\left(\frac{Q_3}{\Lambda}\right)^3$

$B_{3\infty}/B_3^*$	α	β	γ
64.83	-0.78	–	–
64.80	-0.78	-0.05	–
64.80	-0.80	0.36	-1.95

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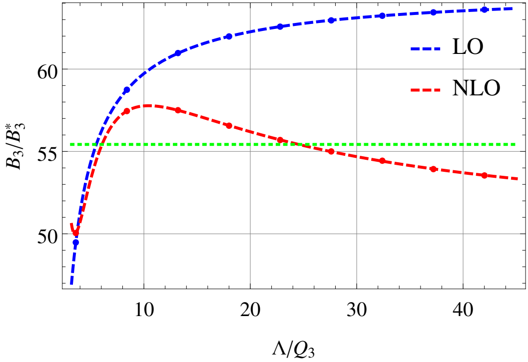

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  we extract

$$B_3^{LO}/B_3^* = 64.8, \quad B_3^{NLO}/B_3^* = 49.8, \quad B_3^{TTY}/B_3^* = 55.4$$

$$Q_3 r_{eff} \approx 0.6 \gg Q_2 r_{eff} \approx 0.08$$

Four-boson system

Are more terms needed to stabilize heavier systems?

$B_4^{LO}/B_3 = 4.2(1), B_4^{TTY}/B_3 = 4.43$

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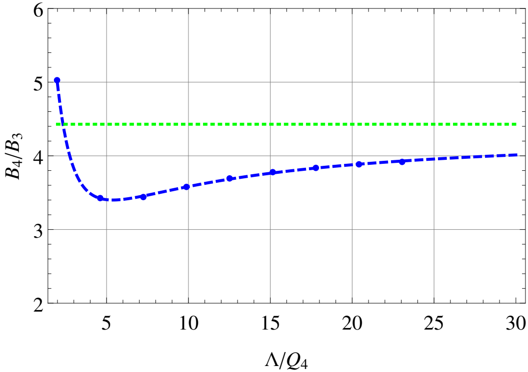
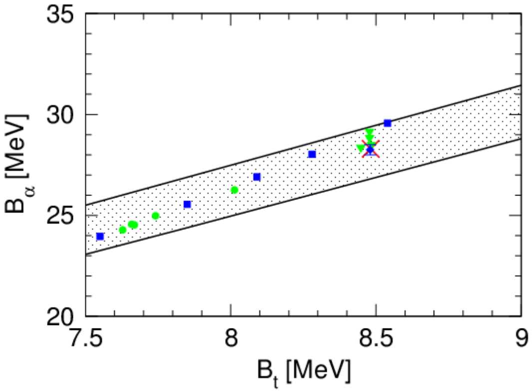


$$B_4^{LO}/B_3 = 4.2(1), B_4^{TTY}/B_3 = 4.43$$

Tjon line

Another evidence is the Tjon line, the correlation between the binding energies of the triton and the α-particle.

J.A. Tjon, Phys. Lett. B **56**, 217 (1975).



L. Platter, H.-W. Hammer, U.-G. Meissner, Phys. Lett. B **607**, 254 (2005).

5- and 6- boson system

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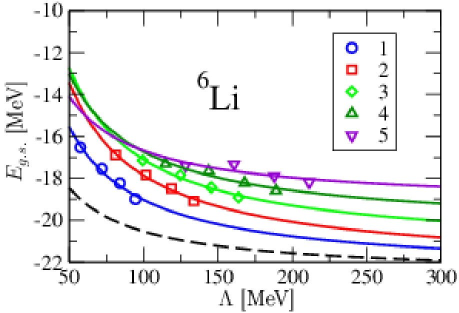
For nucleons,



I. Stetcu, B.R. Barrett, and U. van Kolck, Phys. Lett. B **653** , 358 (2007).

A	x	x^2	x^3	Ref. [1]
4	9.63	10.22	9.94	9.88
5	17.51	19.08	18.40	18.05
6	24.37	27.46	24.98	26.28

[1] J. von Stecher, J. Phys. B: At. Mol. Opt. Phys. 43, 101002 (2010)

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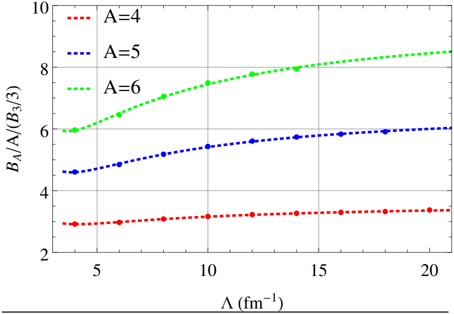

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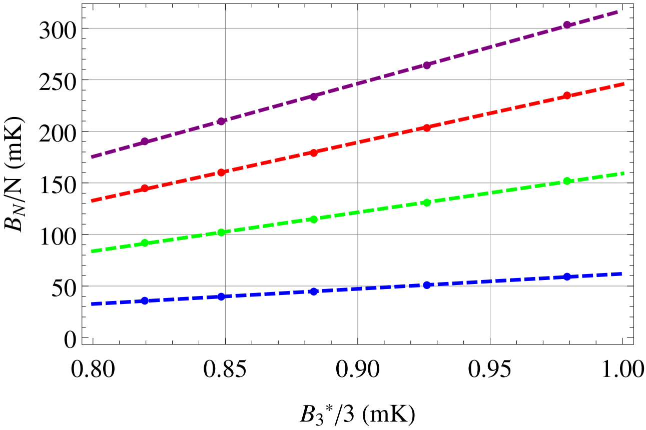
Generalized Tjon-lines

Correlation between B_3^* to B_3, B_4, B_5, and B_6:



...Therefore, no 4, 5 or 6-body terms are needed at LO.

Real Nuclei

- At LO, the 2-body potential reads,

$$V_{LO} = a_1 + a_2\sigma_i \cdot \sigma_j + a_3\tau_i \cdot \tau_j + a_4\sigma_i \cdot \sigma_j\tau_i \cdot \tau_j$$

- using the fermionic symmetry,

$$V_{LO} = C_S\hat{P}_S + C_T\hat{P}_T$$

  where $\hat{P}_\alpha$ is projection operator on channel α
- The 2-body LECs are fitted to the deuteron binding energy and the singlet $^1S_0$ np scattering length.
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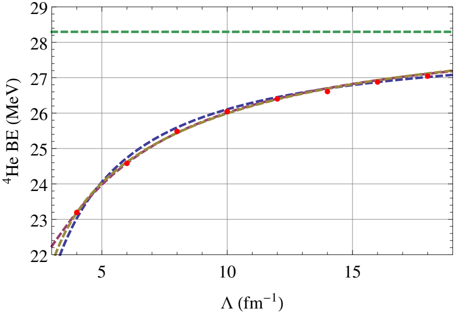
α - ^{4}He nuclei



$$B_4(\Lambda) = B_\infty \left[1 + \alpha \frac{Q_4}{\Lambda} + \beta \left(\frac{Q_4}{\Lambda} \right)^2 + \gamma \left(\frac{Q_4}{\Lambda} \right)^3 + ... \right]$$

B_∞	α	β	γ
28.16	-1.25	–	–
28.66	-1.77	3.17	–
28.88	-2.12	8.07	-19.21

Conclusion

- A pionless EFT was constructed for few-body systems.
- The ^{4}He atomic system was studied, and our EFT fits nicely the known results.
- The convergence of pionless EFT for $A = 4, 5$ and 6 was studied.
- Generalized Tjon-lines were introduced, showing that at LO no 4,5 or 6-body term is needed.
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