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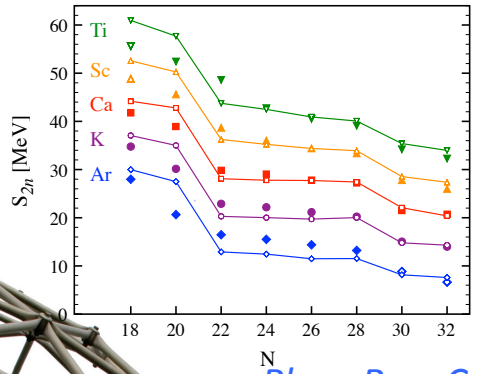
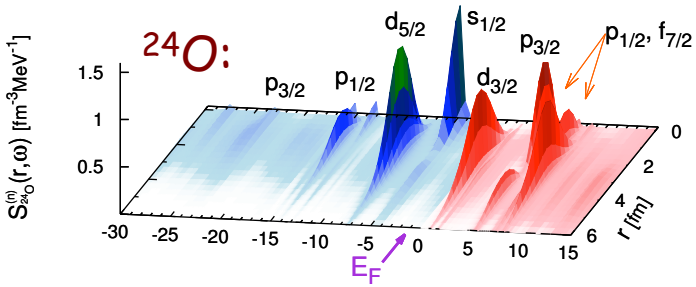
Medium-mass isotopes from chiral and lattice QCD interactions

Carlo Barbieri — University of Surrey

May 12, 2016



*Phys. Rev. C **89**, 061301R (2014)*

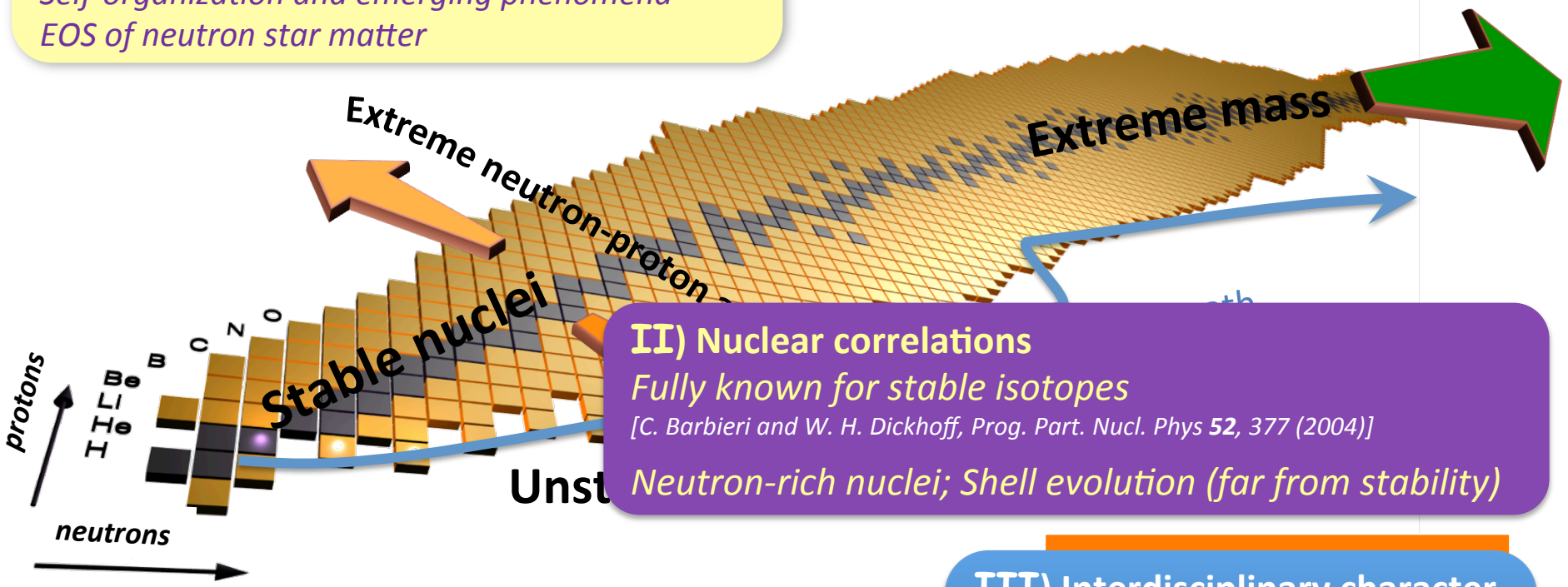


*Phys. Rev. Lett. **111**, 062501 (2013)*
*Phys. Rev. C **92**, 014306 (2015)*

Current Status of low-energy nuclear physics

Composite system of interacting fermions
Binding and limits of stability
Coexistence of individual and collective behaviors
Self-organization and emerging phenomena
EOS of neutron star matter

Experimental programs
RIKEN, FAIR, FRIB



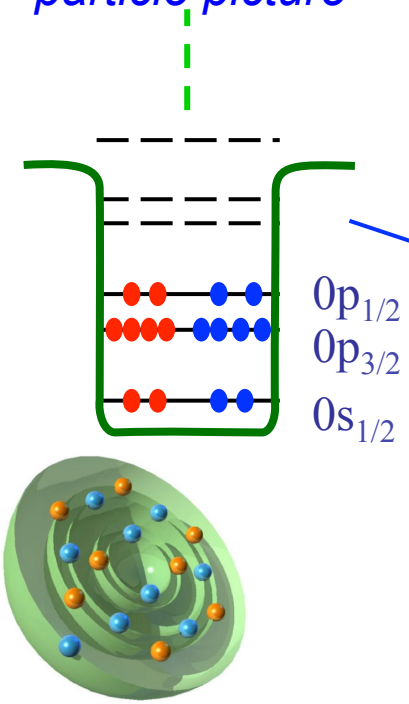
II) Nuclear correlations
Fully known for stable isotopes
*[C. Barbieri and W. H. Dickhoff, Prog. Part. Nucl. Phys **52**, 377 (2004)]*
Neutron-rich nuclei; Shell evolution (far from stability)

I) Understanding the nuclear force
QCD-derived; 3-nucleon forces (3NFs)
First principle (ab-initio) predictions

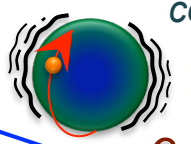
III) Interdisciplinary character
Astrophysics
Tests of the standard model
Other fermionic systems:
ultracold gasses; molecules;

Concept of correlations

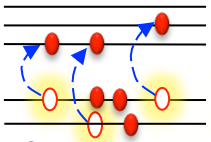
independent particle picture



Particle-vibration coupling (PV)



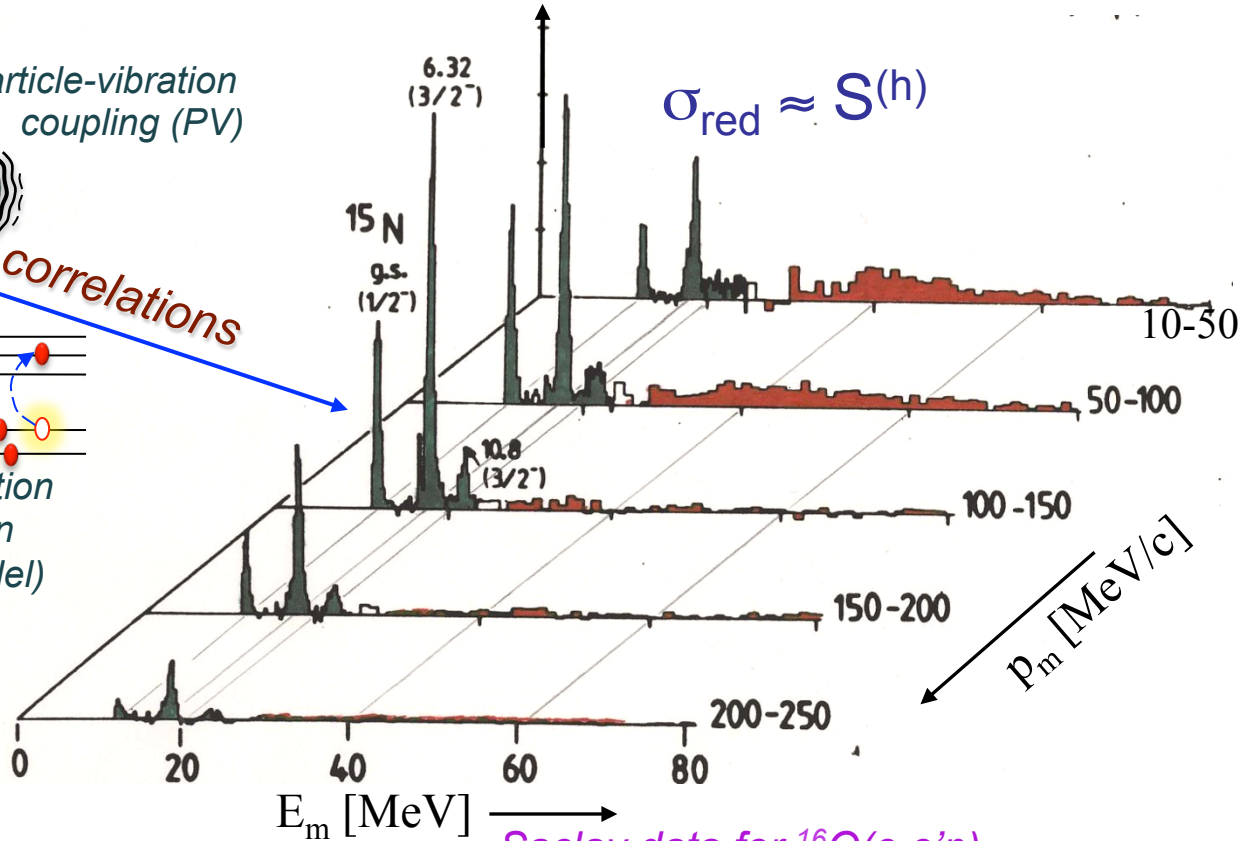
correlations



Configuration interaction (shell model)

Spectral function: distribution of momentum (p_m) and energies (E_m)

$\sigma_{red} \approx S^{(h)}$



Saclay data for $^{16}O(e,e'p)$

[Mougey et al., Nucl. Phys. A335, 35 (1980)]

Understood for a few stable closed shells:

*[CB and W. H. Dickhoff, Prog. Part. Nucl. Phys **52**, 377 (2004)]*

Concept of correlations

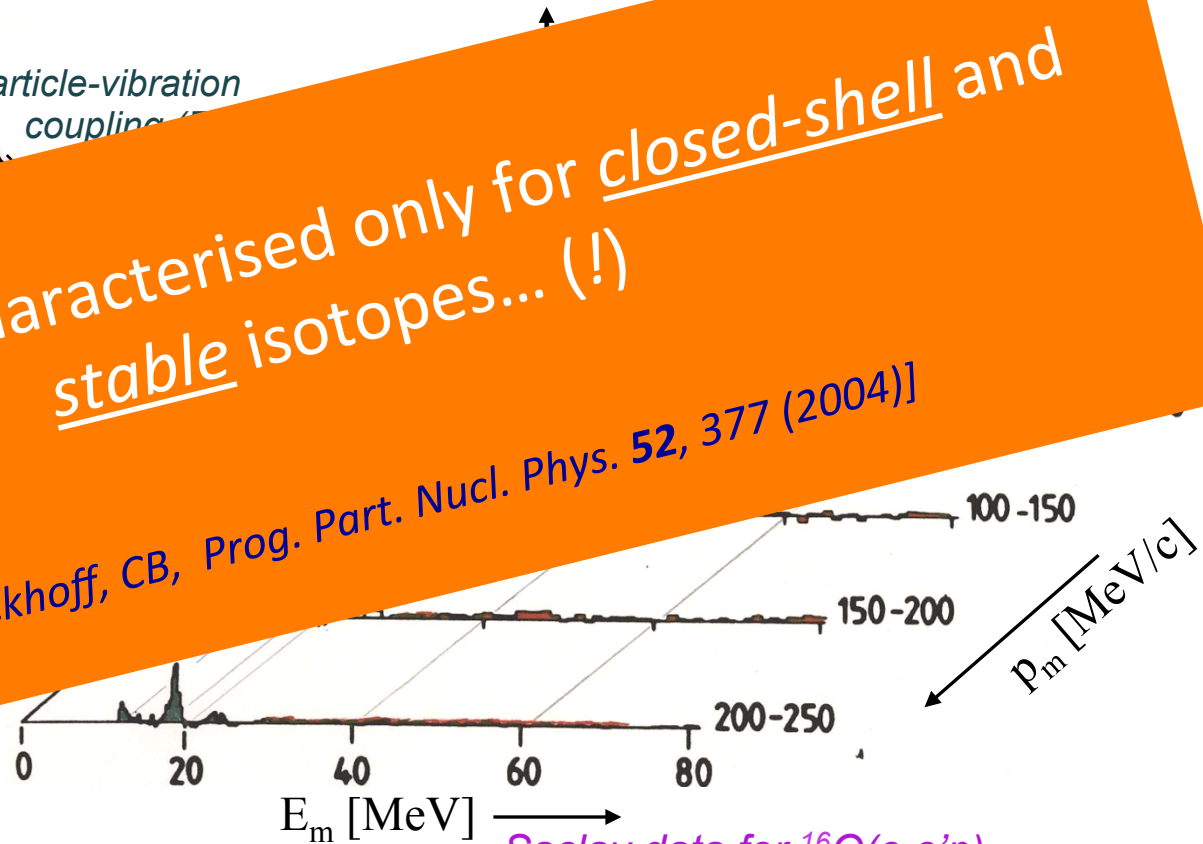
independent particle picture

Spectral function: distribution of momentum (p_m) and

Particle-vibration

So far, fully characterised only for closed-shell and stable isotopes… (!)

[W. Dickhoff, CB, Prog. Part. Nucl. Phys. **52**, 377 (2004)]



Saclay data for ^{16}O(e,e'p)
[Mougey et al., Nucl. Phys. A335, 35 (1980)]

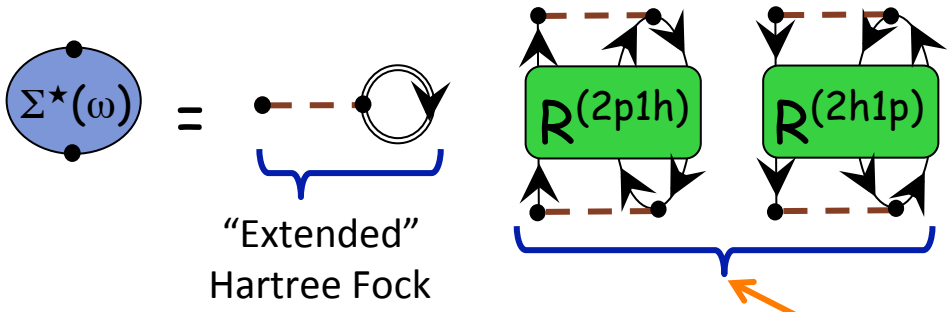
Understood for a few stable closed shells:
*[CB and W. H. Dickhoff, Prog. Part. Nucl. Phys **52**, 377 (2004)]*

Ab-Initio SCGF approaches

The FRPA Method in Two Words

Particle vibration coupling is the main cause driving the distribution of particle strength—on both sides of the Fermi surface…

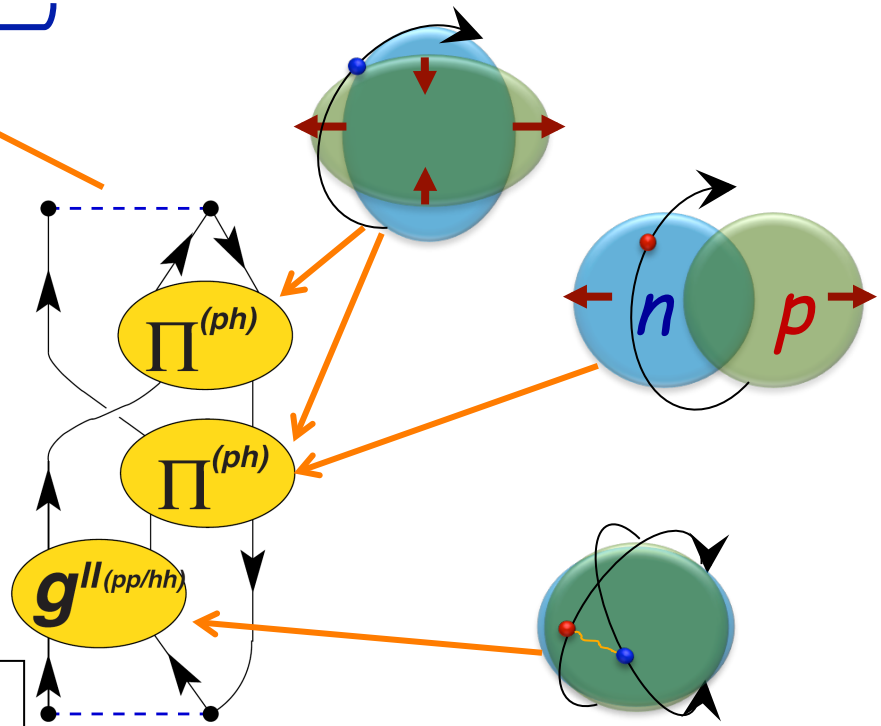
CB et al.,
*Phys. Rev. C**63**, 034313 (2001)*
*Phys. Rev. A**76**, 052503 (2007)*
*Phys. Rev. C**79**, 064313 (2009)*



- *A complete expansion requires <u>all types</u>* of particle-vibration coupling

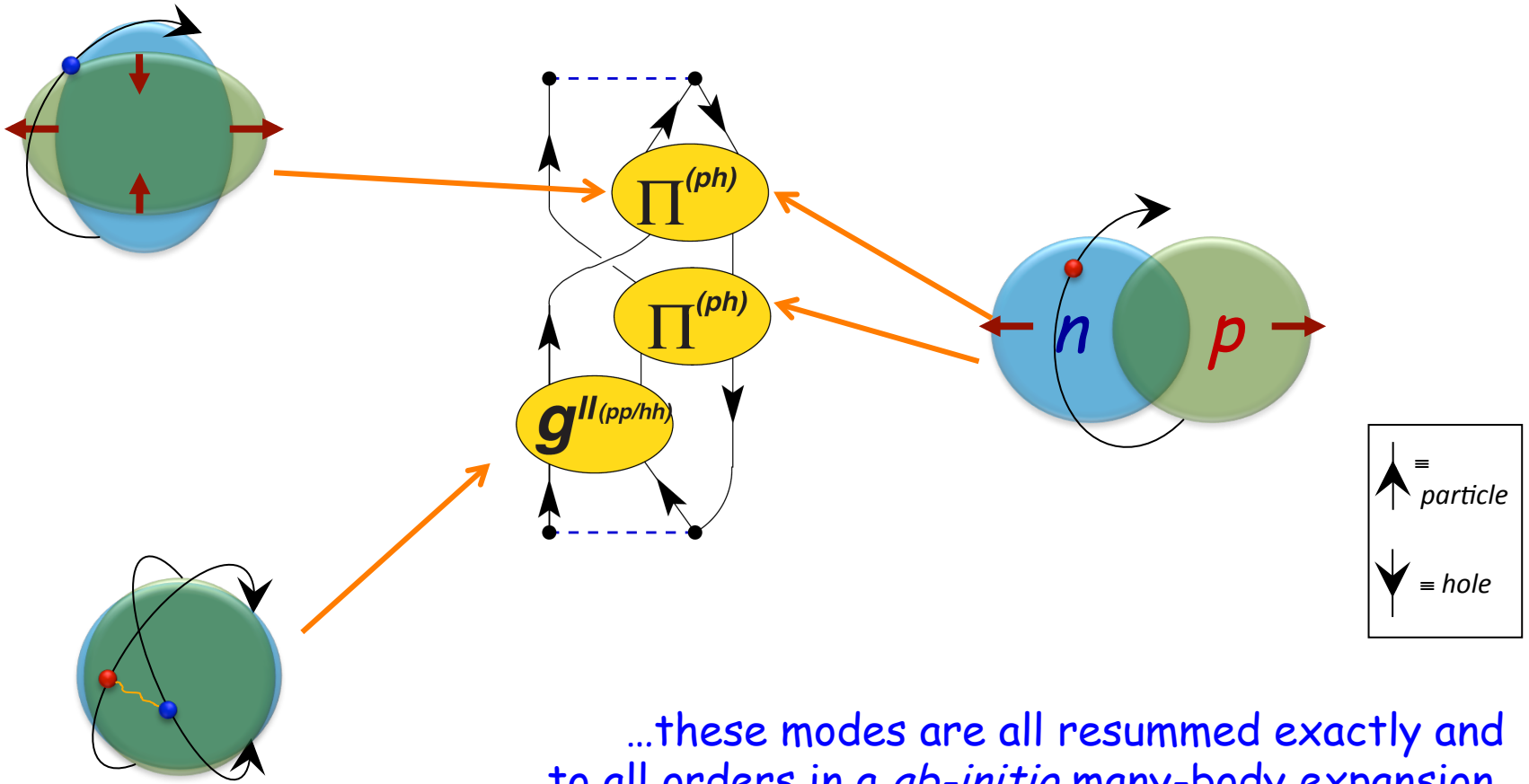
  …these modes are all resummed exactly and to all orders in a *ab-initio* many-body expansion.

- The Self-energy $\Sigma^{\star}(\omega)$ yields *both* single-particle states and scattering



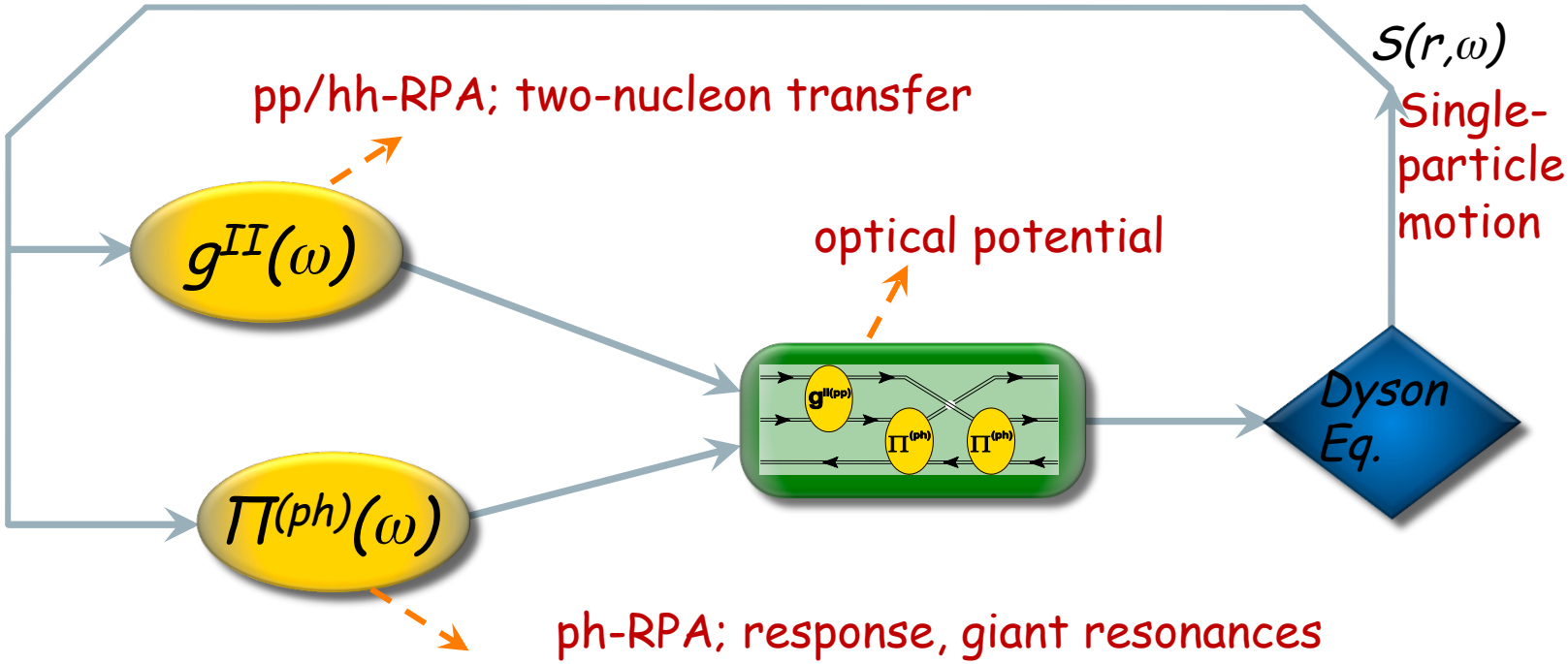
The FRPA Method in Two Words

Particle vibration coupling is the main cause driving the distribution of particle strength—a least close to the Fermi surface…



…these modes are all resummed exactly and to all orders in a *ab-initio* many-body expansion.

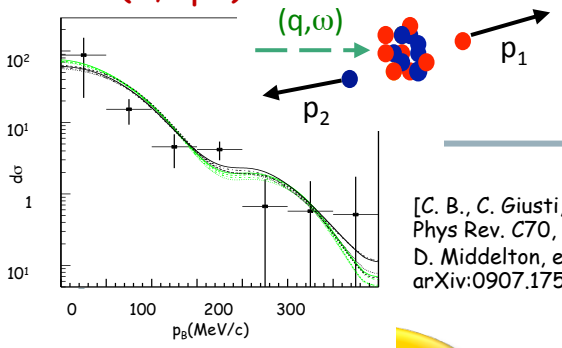
Self-Consistent Green's Function Approach



- Global picture of nuclear dynamics
- Reciprocal correlations among effective modes
- Guaranties *macroscopic conservation laws*

Self-Consistent Green's Function Approach

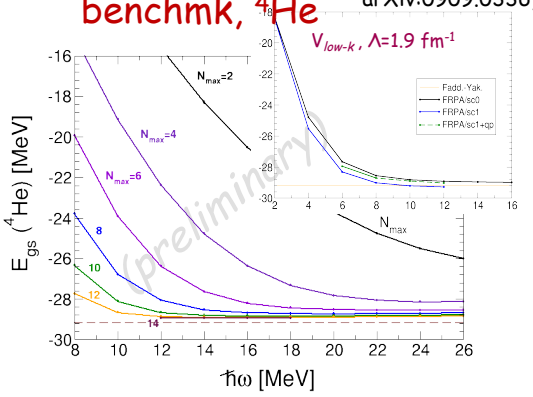
^{16}O(e,e'pn)^{14}N @ MAINZ



[C. B., C. Giusti, et al. Phys Rev. C70, 014606 (2004)
D. Middelton, et al. arXiv:0907.1758; EPJA in print]

Binding energy benchmk, ^{4}He

[C. B., arXiv:0909.0336]



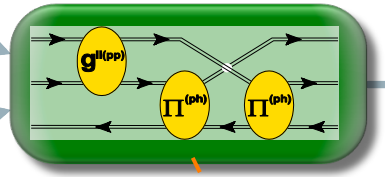
Ionization energies/ affinities, in atoms

[CB, D. Van Neck, AIP Conf.Proc.1120,104 ('09) & in prep]

		Hartree-Fock	FRPAc	Experiment [16, 17]
He:	1s	0.918 (+14)	0.9008 (-2.9)	0.9037
Be^{2+}:	1s	5.6672 (+116)	5.6551 (-0.5)	5.6556
Be:	2s	0.3093 (-34)	0.3224 (-20.2)	0.3426
	1s	4.733 (+200)	4.5405 (+8)	4.533
Ne:	2p	0.852 (+57)	0.8037 (+11)	0.793
	1s	1.931 (+149)	1.7967 (+15)	1.782
Mg^{2+}:	2p	3.0068 (+56.9)	2.9537 (+3.8)	2.9499
	1s	4.4827	4.3589	
Mg:	3s	0.253 (-28)	0.280 (-1)	0.281
	2p	2.282 (+162)	2.137 (+17)	2.12
Ar:	3p	0.591 (+12)	0.579 (≈0)	0.579
	3s	1.277 (+202)	1.065 (-10)	1.075
	3s		1.544	
	2p	9.571 (+411)	9.219 (+59)	9.160

$g^{II}(\omega)$

$\Pi^{(ph)}(\omega)$

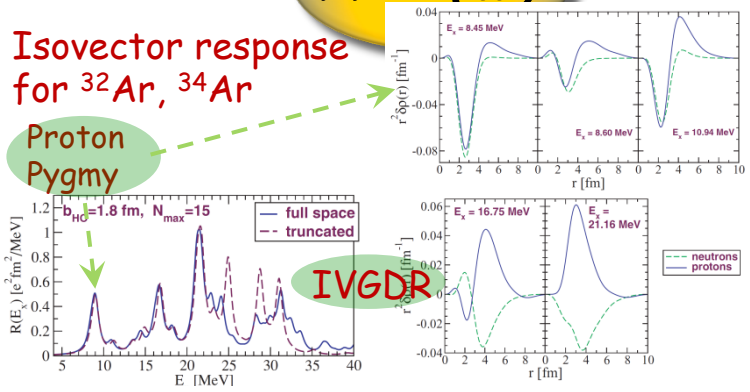


Dyson Eq.

Isovector response for ^{32}Ar, ^{34}Ar

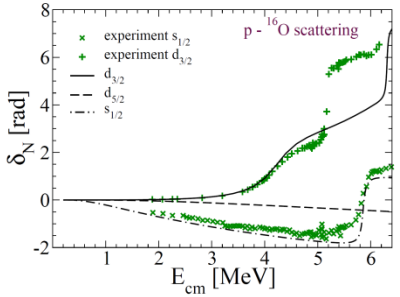
Proton Pygmy

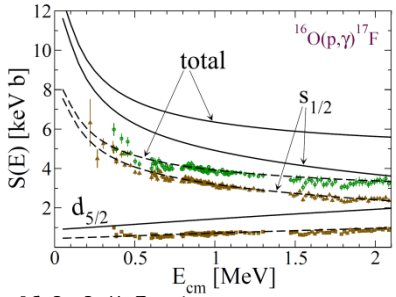
IVGDR



[C. B., K. Langanke, et al., Phys Rev. C77, 024304 (2008)]

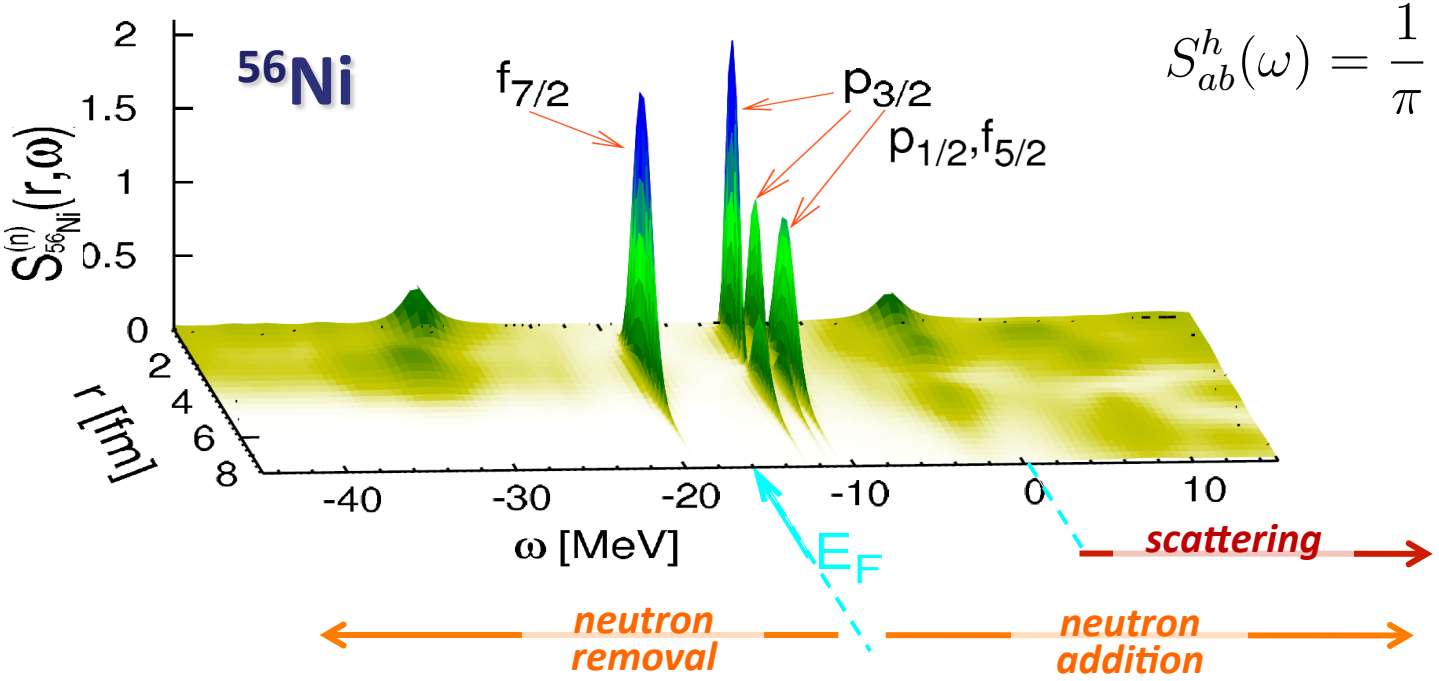
^{16}O(p,γ)





[C. B., B. K. Jennings Nucl. Phys A758, 395c (2005) Phys Rev. C72, 014613 (2005)]

^{56}Ni neutron spectral function



$$S^h_{ab}(\omega) = \frac{1}{\pi}\, Im\, g_{ab}(\omega)$$

W. Dickhoff, CB, Prog. Part. Nucl. Phys. 53, 377 (2004)
CB, M.Hjorth-Jensen, Pys. Rev. C**79**, 064313 (2009)

Approaches in GF theory

Truncation scheme:	Dyson formulation (closed shells)	Gorkov formulation (semi-/doubly-magic)
1st order:	Hartree-Fock	HF-Bogolioubov
2nd order:	2nd order	2nd order (w/ pairing)
. . .	. . .	
3rd and all-orders sums, P-V coupling:	ADC(3) FRPA etc…	G-ADC(3) …work in progress

Approaches in GF theory

Truncation scheme:

Dyson formulation (closed shells)

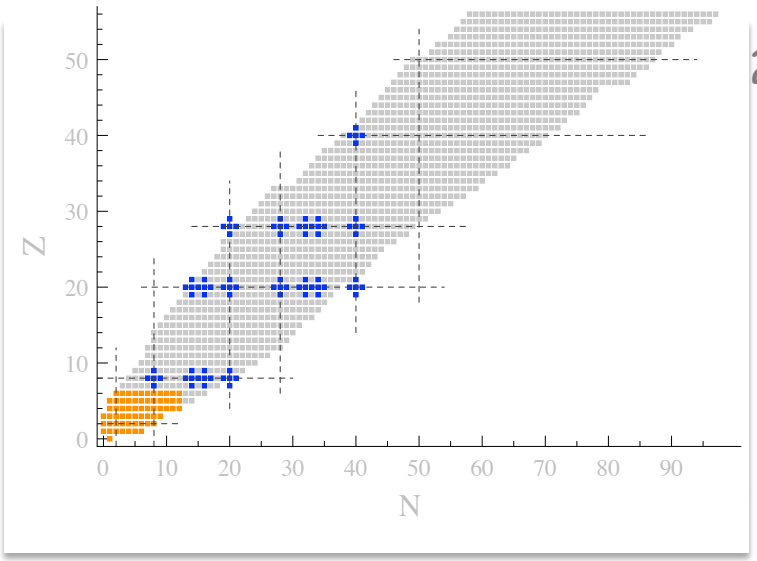
Gorkov formulation (semi-/doubly-magic)

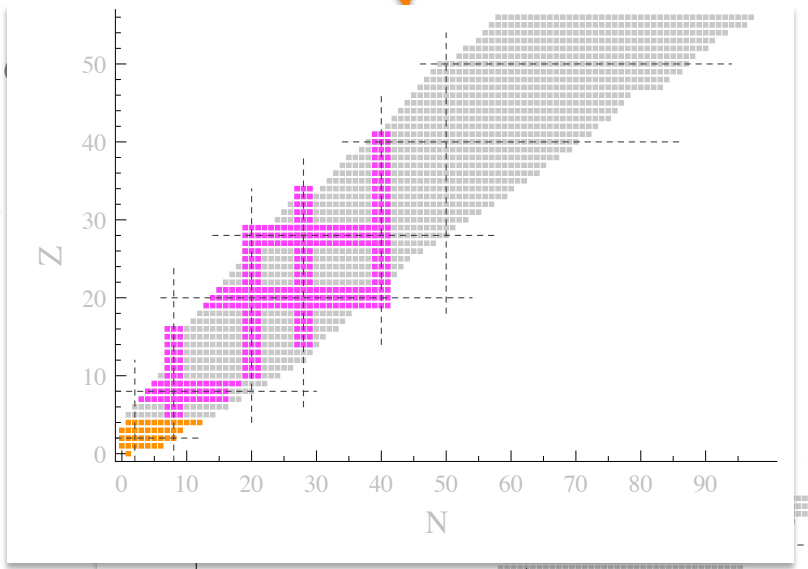
1st order: Hartree-Fo

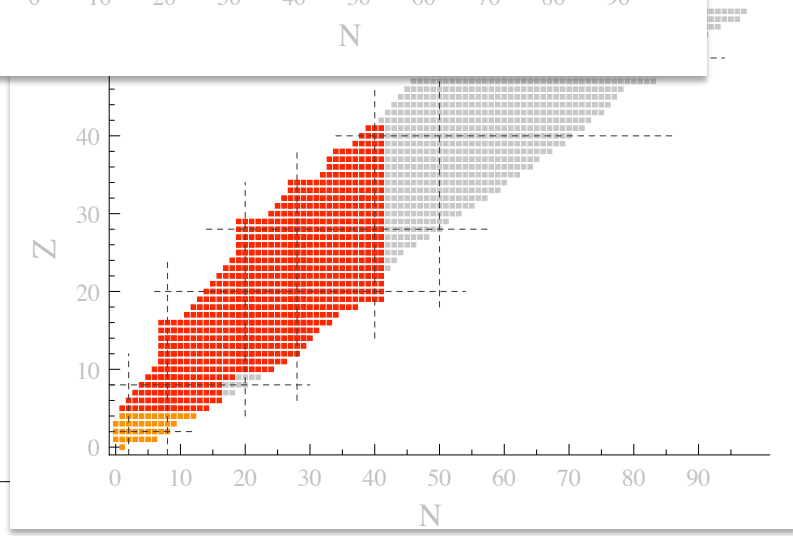
2nd order

. . .

ADC(3)
FRPA
etc...







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pics. credits: V. Somà

Gorkov and symmetry breaking approaches

V. Somà, CB, T. Duguet, , Phys. Rev. C **89**, 024323 (2014)
V. Somà, CB, T. Duguet, Phys. Rev. C **87**, 011303R (2013)
V. Somà, T. Duguet, CB, Phys. Rev. C **84**, 064317 (2011)

- Ansatz $\quad \ldots \approx E_0^{N+2} - E_0^N \approx E_0^N - E_0^{N-2} \approx \ldots \approx 2\mu$

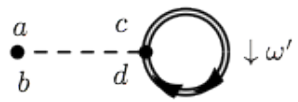
- Auxiliary many-body state $\quad |\Psi_0\rangle \equiv \sum_N^{\text{even}} c_N \, |\psi_0^N\rangle$

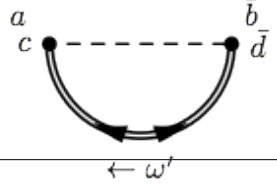
  → Mixes various particle numbers

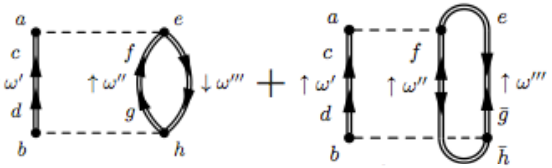
  → Introduce a "grand-canonical" potential $\quad \Omega = H - \mu N$

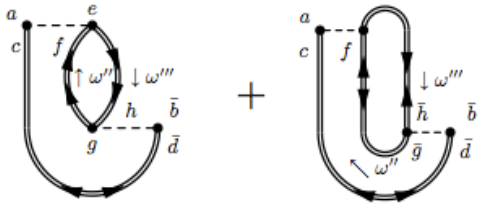
→ $|\Psi_0\rangle$ minimizes $\Omega_0 = \langle\Psi_0|\Omega|\Psi_0\rangle$ under the constraint $N = \langle\Psi_0|N|\Psi_0\rangle$

- This approach leads to the following Feynman diagrams:

$\Sigma^{11\,(1)}_{ab} =$ 

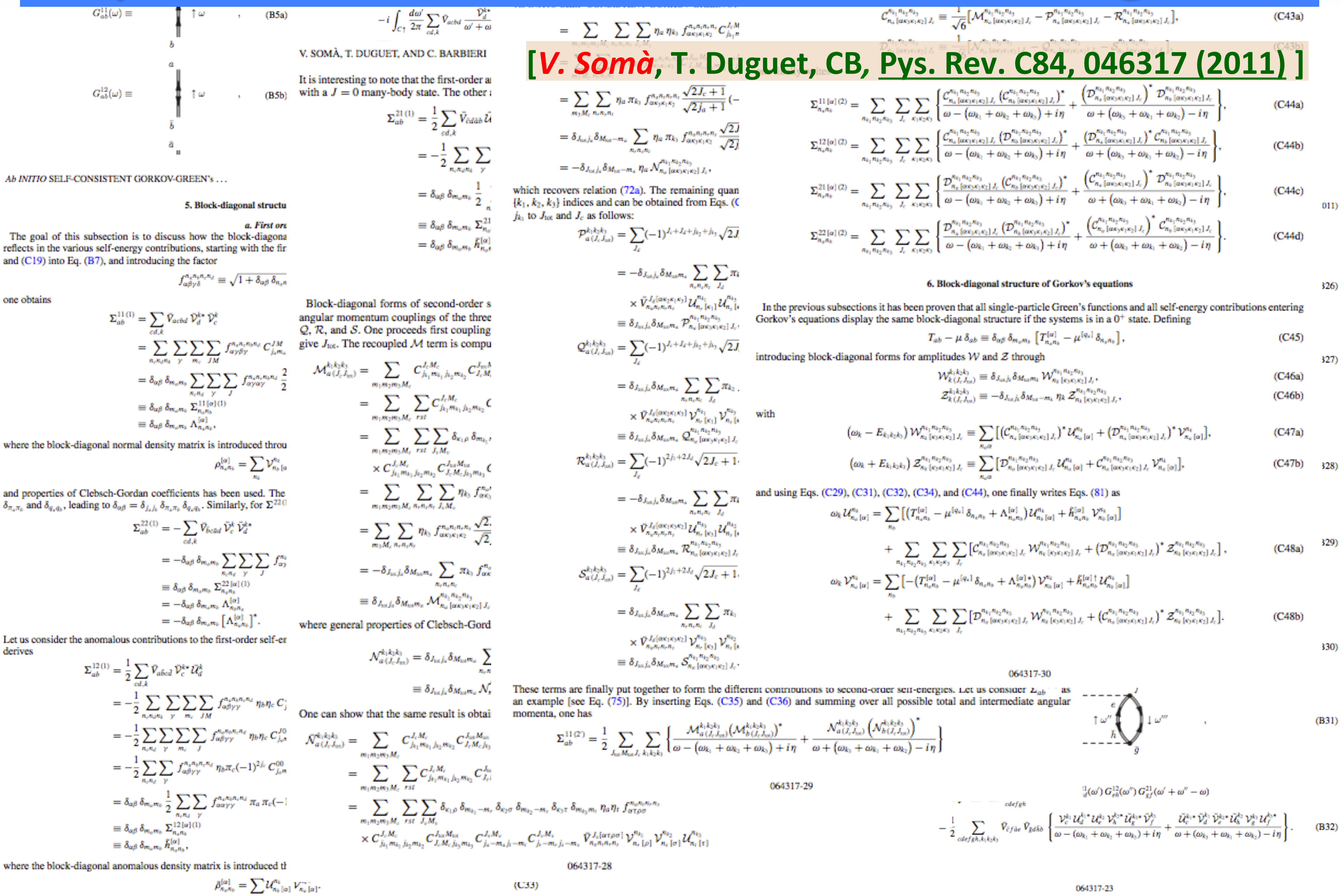
$\Sigma^{12\,(1)}_{ab} =$ 

$\Sigma^{11\,(2)}_{ab}(\omega) =$ 

$\Sigma^{12\,(2)}_{ab}(\omega) =$ 

Espressions for 1st & 2nd order diagrams

[*V. Somà*, T. Duguet, CB, Pys. Rev. C84, 046317 (2011)]

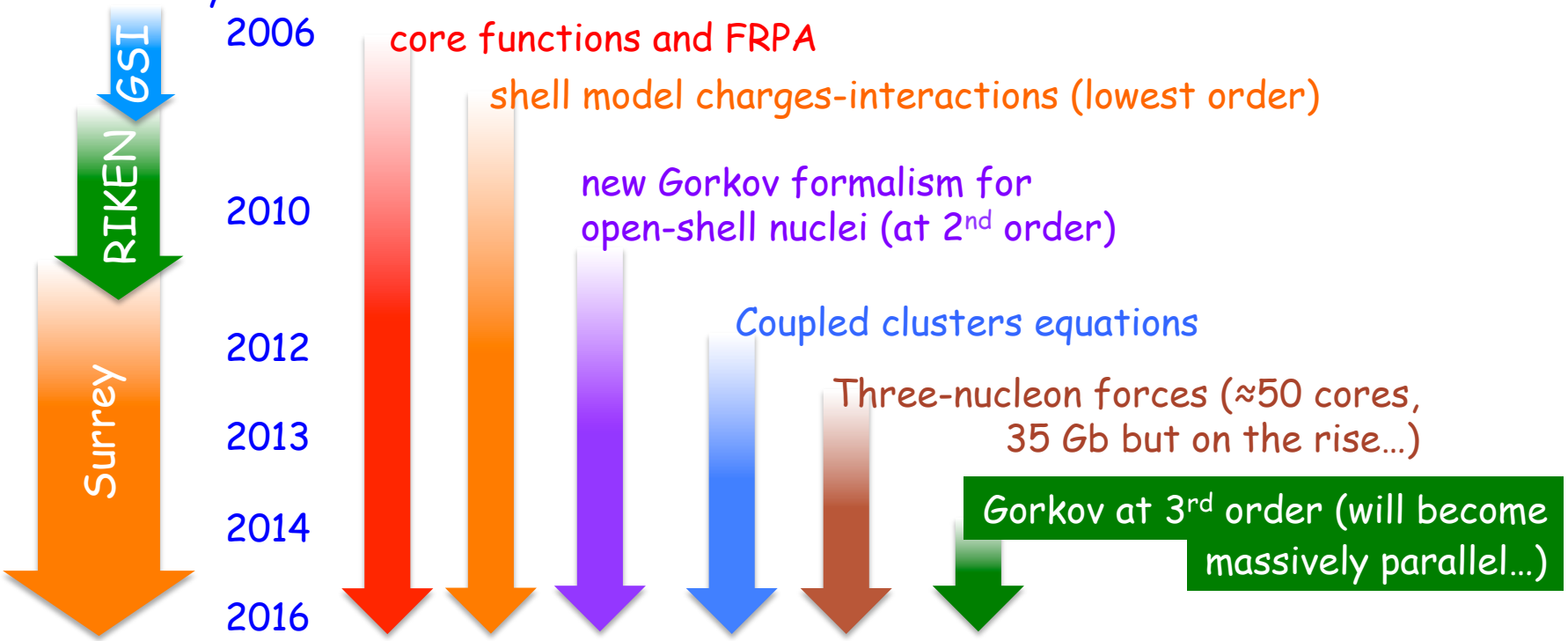


Ab-initio Nuclear Computation & BcDor code

BoccaDorata code:
(*C. Barbieri* 2006-14
V. Somà 2011-14
A. Cipollone 2012-13)

- Provides a *C++ class library* for handling many-body propagators (≈40,000 lines, MPI&OpenMP based).
- Allows to solve for nuclear spectral functions, many-body propagators, RPA responses, coupled cluster equations and effective interaction/charges for the shell model.

Code history:

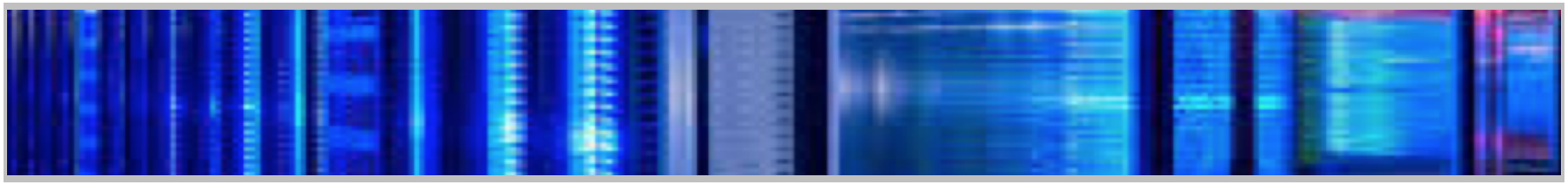


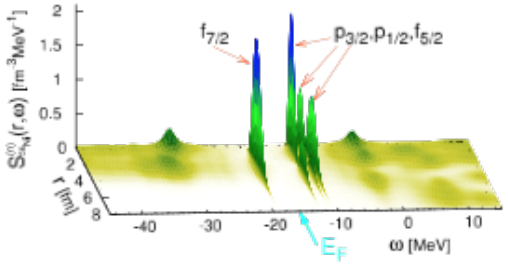
… applications …

Ab-initio Nuclear Computation & BcDor code

http://personal.ph.surrey.ac.uk/~cb0023/bcdor/

Computational Many-Body Physics





Download

Documentation

Welcome

From here you can download a public version of my self-consistent Green's function (SCGF) code for nuclear physics. This is a code in J-coupled scheme that allows the calculation of the single particle propagators (a.k.a. one-body Green's functions) and other many-body properties of spherical nuclei.

This version allows to:

- Perform Hartree-Fock calculations.
- Calculate the the correlation energy at second order in perturbation theory (MBPT2).
- Solve the Dyson equation for propagators (self consistently) up to second order in the self-energy.
- Solve coupled cluster CCD (doubles only!) equations.

When using this code you are kindly invited to follow the creative commons license agreement, as detailed at the weblinks below. In particular, we kindly ask you to refer to the publications that led the development of this software.

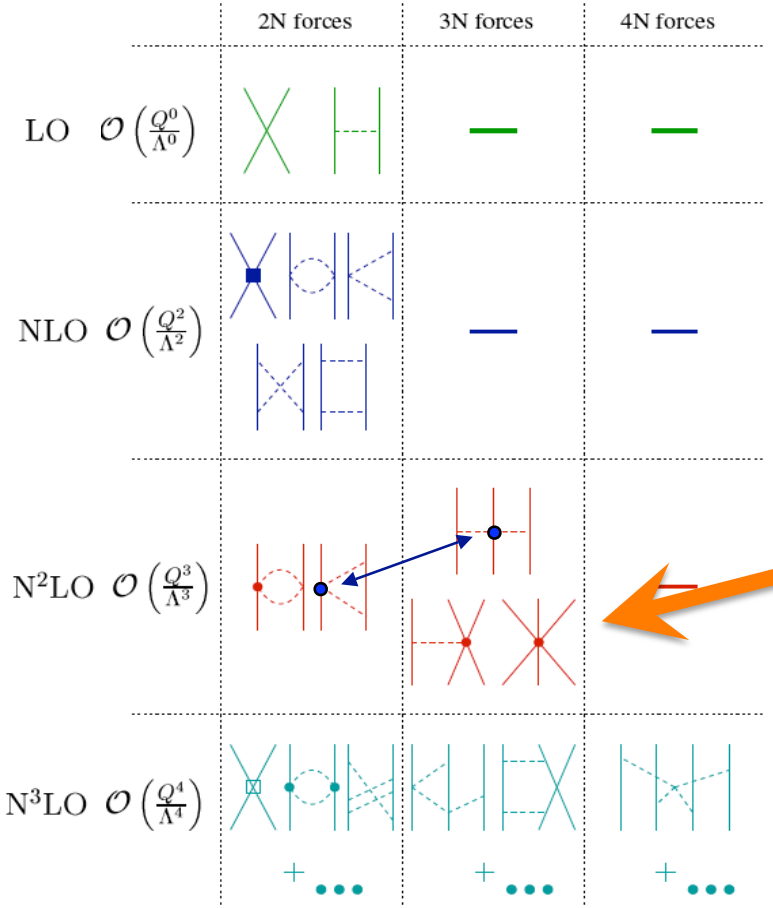
Relevant references (which can also help in using this code) are:

Prog. Part. Nucl. Phys. 52, p. 377 (2004),
Phys. Rev. A76, 052503 (2007),
Phys. Rev. C79, 064313 (2009),
Phys. Rev. C89, 024323 (2014)

Medium-mass isotopes from chiral interactions

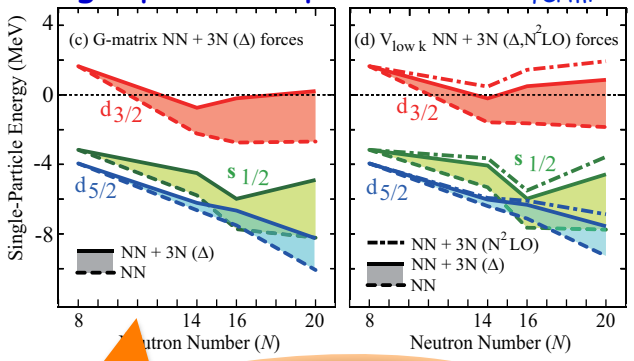
Modern realistic nuclear forces

Chiral EFT for nuclear forces:



(3NFs arise naturally at N2LO)

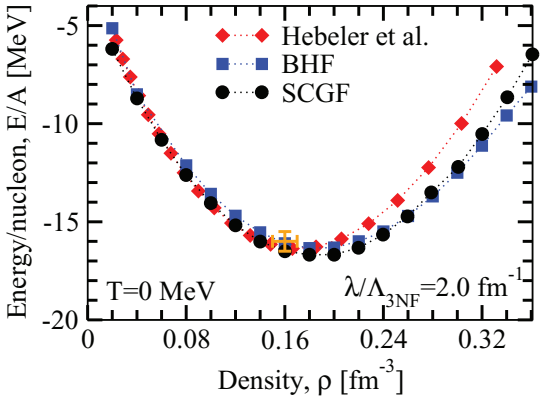
Single particle spectrum at E_{fermi}:



[T. Otsuka et al., Phys Rev. Lett **105**, 032501 (2010)]

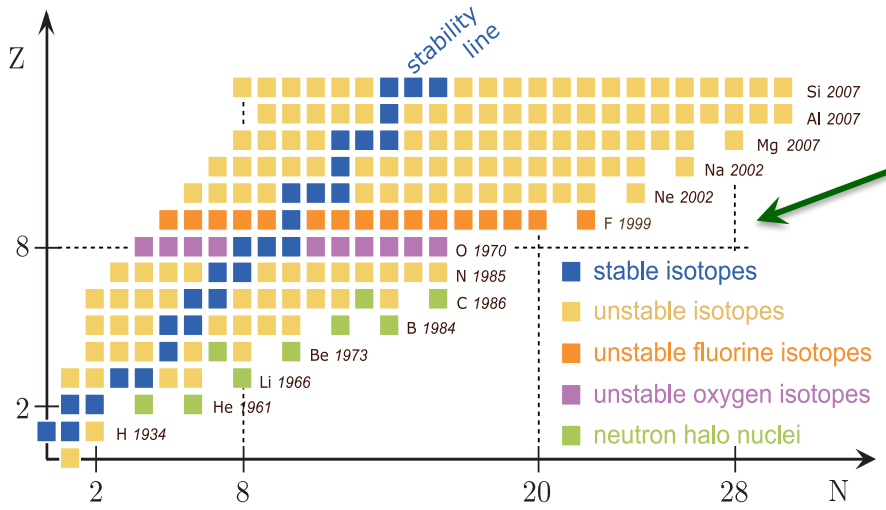
Need at LEAST 3NF!!!
("cannot" do RNB physics without…)

Saturation of nuclear matter:



[A. Carbone et al., Phys Rev. C **88**, 044302 (2013)]

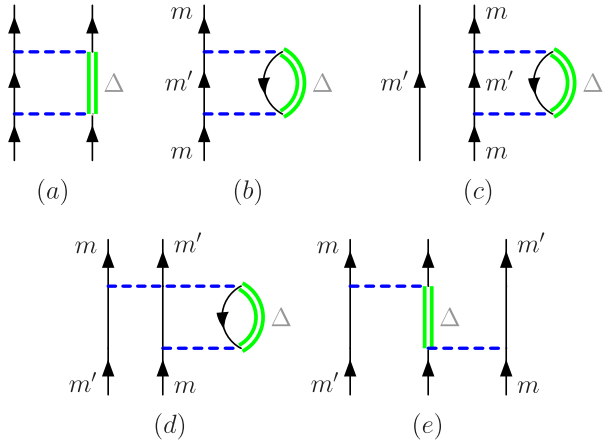
Oxygen puzzle…

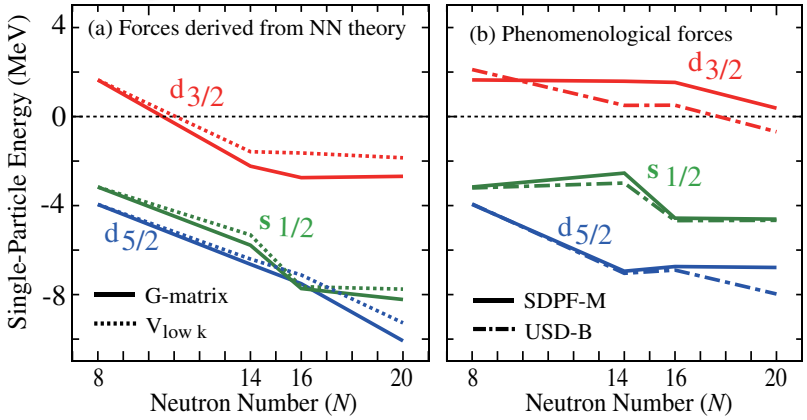


The oxygen dripline is at ^{24}O, at odds with other neighbor isotope chains.

Phenomenological shell model interaction reflect this in the s.p. energies but no realistic NN interaction alone is capable of reproducing this…

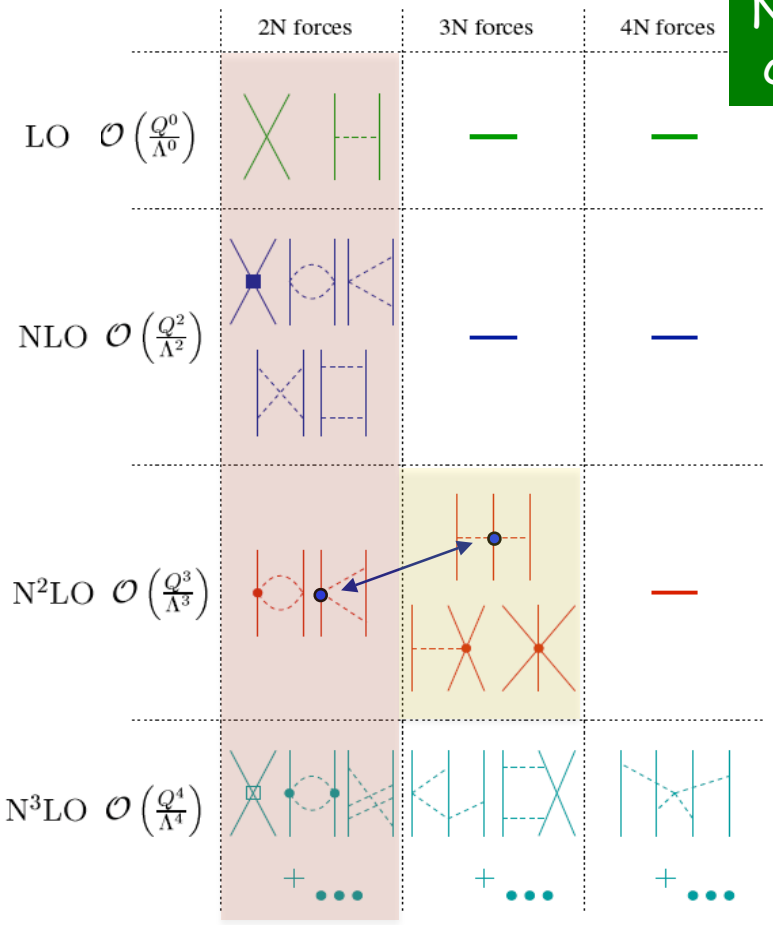
The fujita-Miyazawa 3NF provides repulsion through Pauli screening of other 2NF terms:





[T. Otsuka et al., Phys Rev. Lett **105**, 32501 (2010)]

Chiral Nuclear forces - SRG evolved



(3NFs arise naturally at N2LO)

[Jurgenson, Navrátil, Furnstahl, Phys. Rev. Lett. **103**, 082501 (2009); Hebeler, Phys. Rev. C 85, 021002 (2012)]

N3LO (Λ = 500Mev/c) chiral NN interaction

N2LO (Λ = 400Mev/c) chiral 3N interaction

SRG evolution to λ=2.0 fm^{-1}

V_{NN}

V_{3N} induced

V_{3N} full

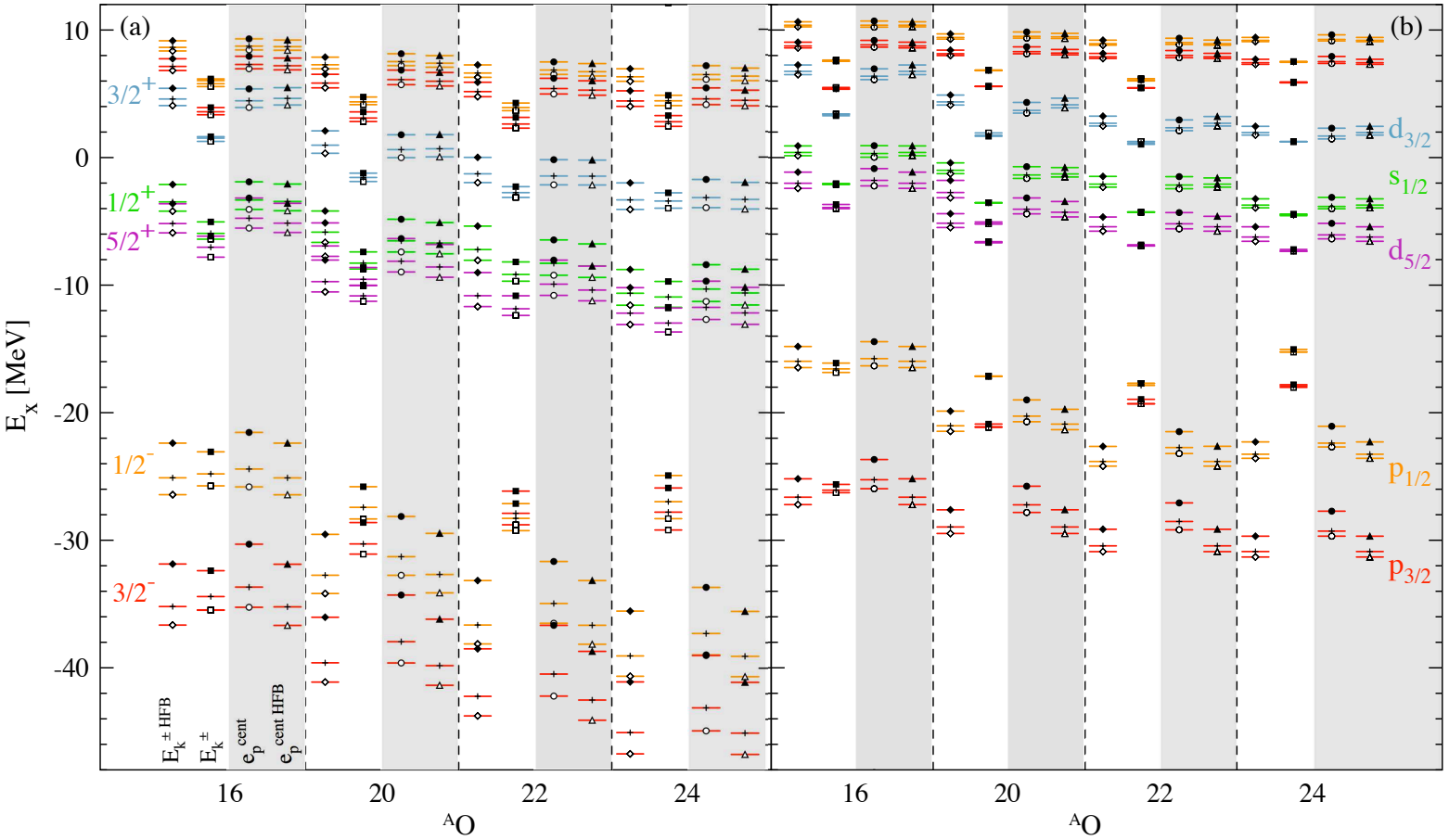
"induced" Hamiltoninan

"full" Hamiltoninan

Convergence of s.p. spectra w.r.t. SRG

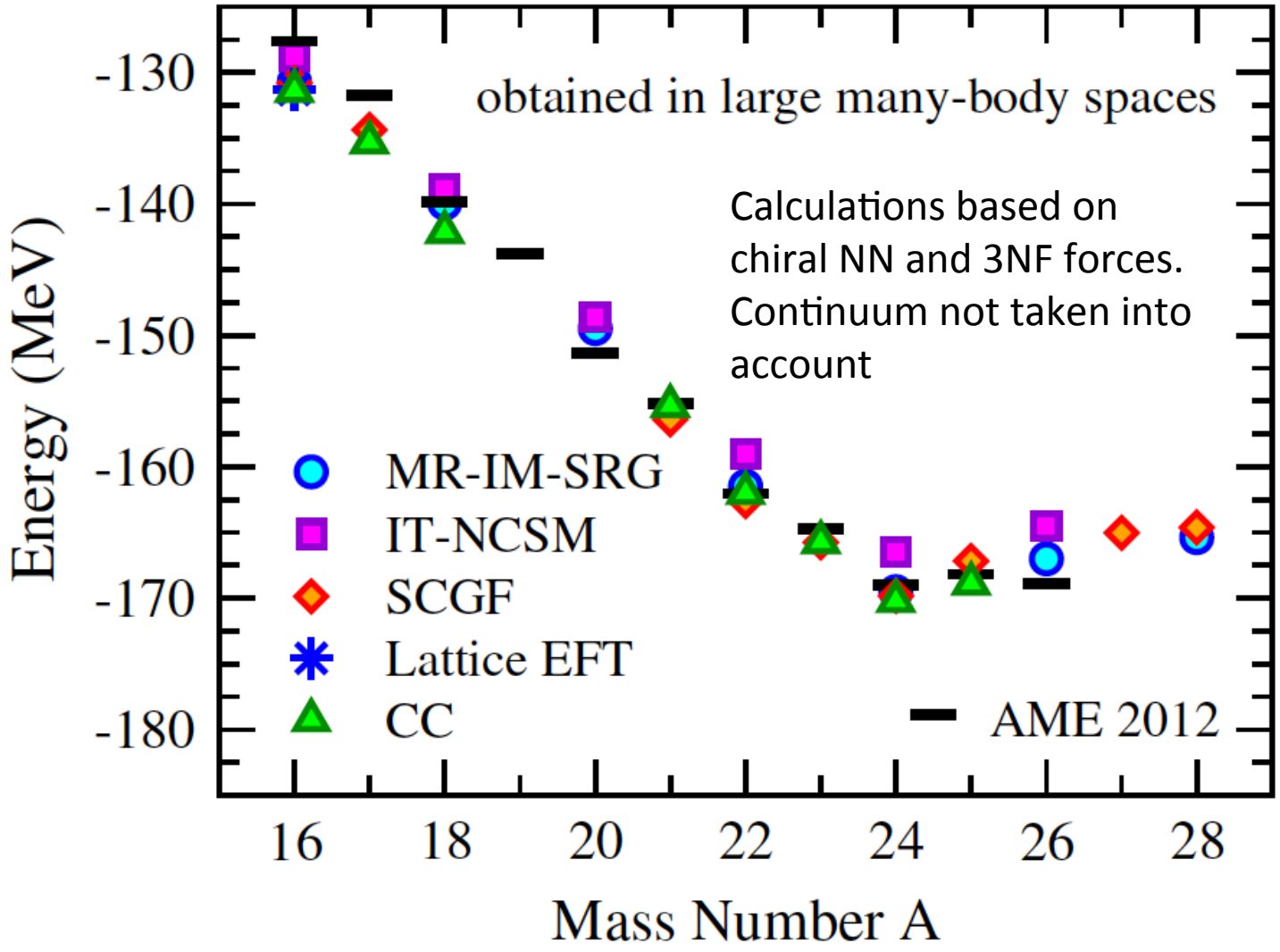
Cutoff dependence is reduces, indicating good convergence of many-body truncation and many-body forces

PhysRevC**92**, 034313 (2015)
✔only *dominant s.p.* peaks shown



NN terms (no induced 3NF) ← → NN+3NF fully included

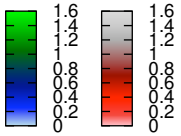
Benchmark of ab-initio methods in the oxygen isotopic chain

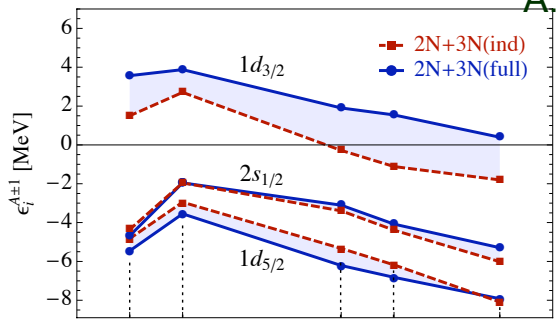


Hebeler, Holt, Menendez, Schwenk, Ann. Rev. Nucl. Part. Sci. in press (2015)

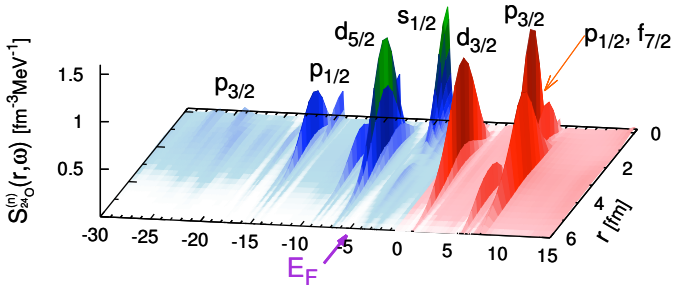
Neutron spectral function of Oxygens

A. Cipollone, CB, P. Navrátil, *Phys. Rev. C* **92**, 014306 (2015)

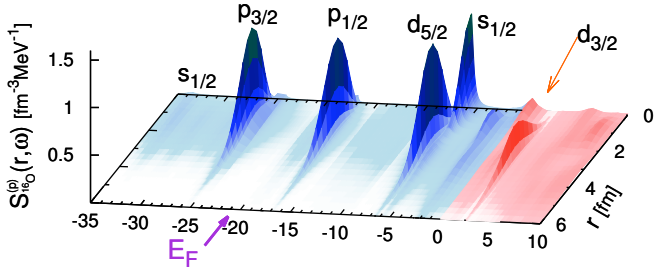




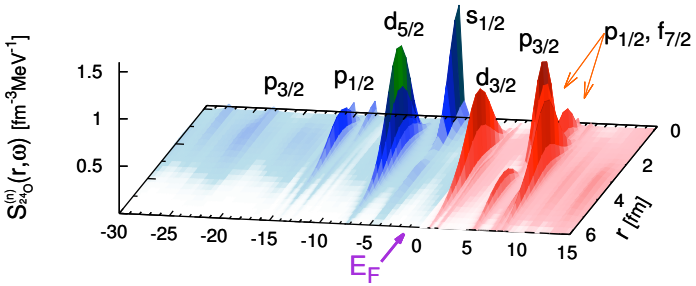
^{22}O:



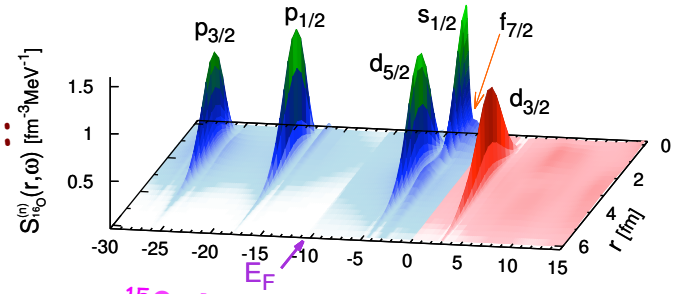
^{14}O:



^{24}O:

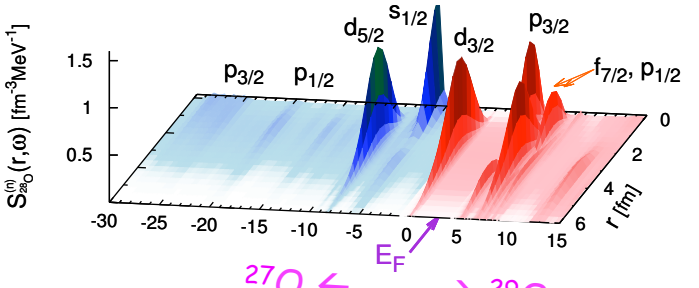


^{16}O:



$^{15}O \leftarrow \quad \rightarrow {}^{17}O$

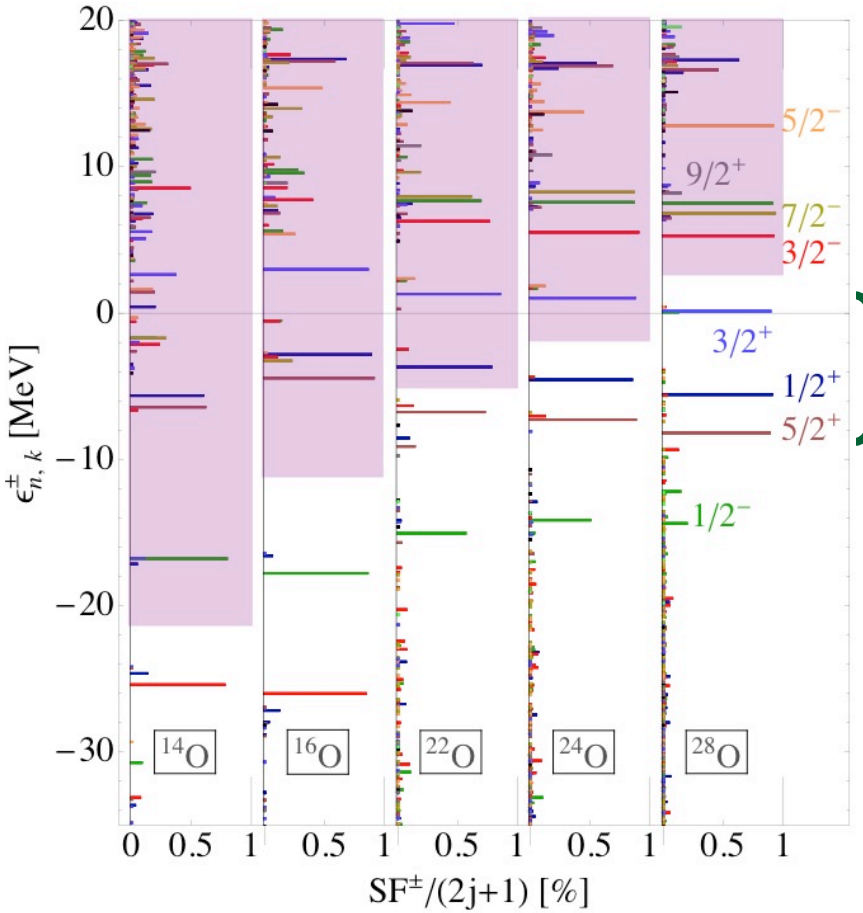
^{28}O:

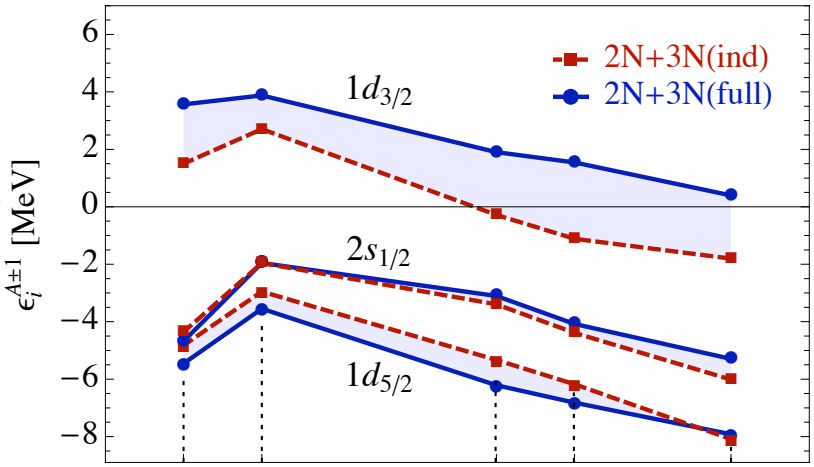


$^{27}O \leftarrow \quad \rightarrow {}^{29}O$

Results for the N-O-F chains

A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)
and Phys. Rev. C **92**, 014306 (2015)





→ $d_{3/2}$ *raised* by genuine 3NF

→ cf. microscopic shell model [Otsuka et al, PRL**105**, 032501 (2010).]

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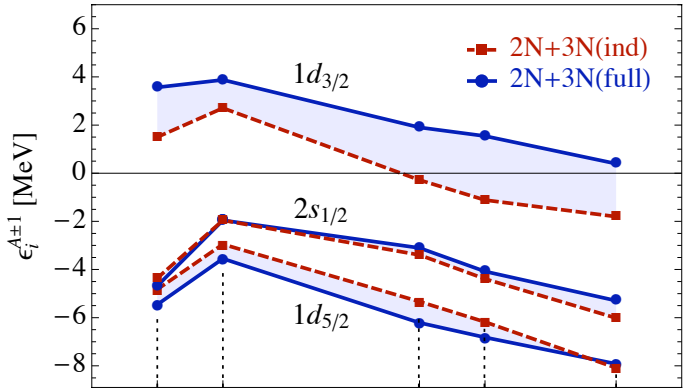
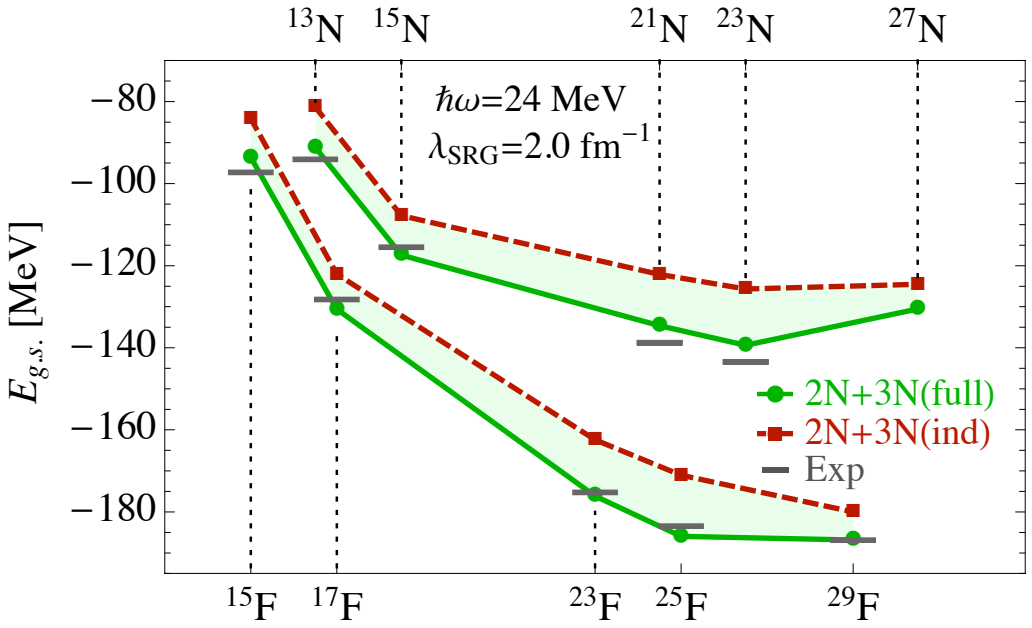
A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)





→ 3NF crucial for reproducing binding energies and driplines around oxygen

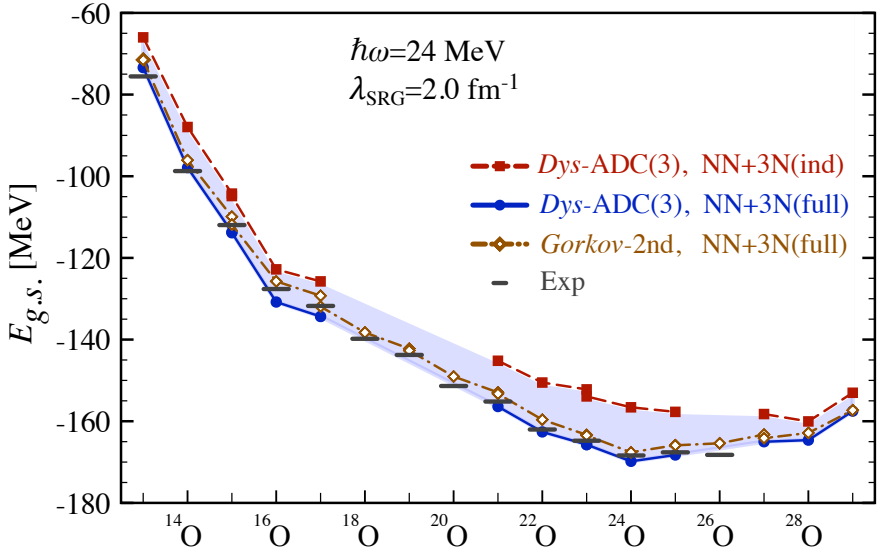
→ *$d_{3/2}$ raised* by genuine 3NF

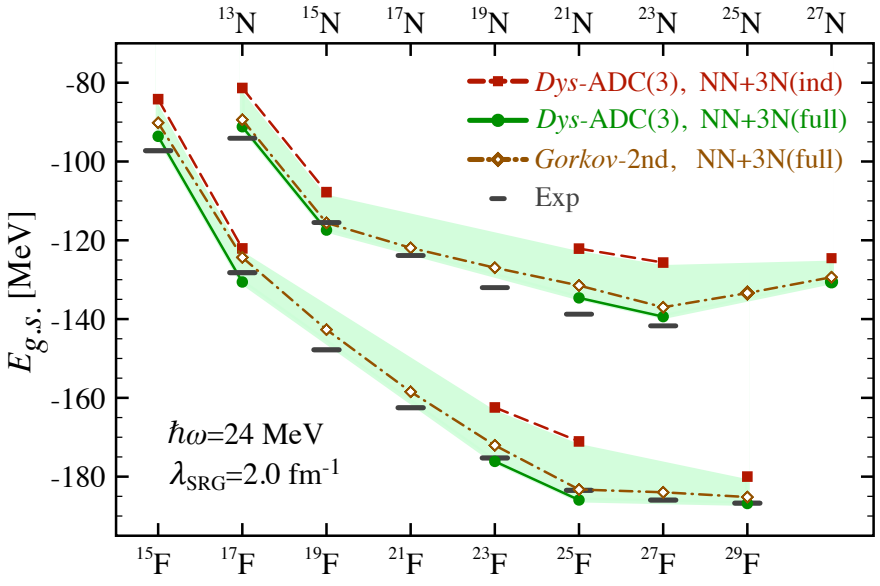
→ cf. microscopic shell model [Otsuka et al, PRL**105**, 032501 (2010).]

N3LO (Λ = 500Mev/c) chiral NN interaction evolved to 2N + 3N forces (2.0fm^{-1})
N2LO (Λ = 400Mev/c) chiral 3N interaction evolved (2.0fm^{-1})

Results for the N-O-F chains

A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)
and Phys. Rev. C **92**, 014306 (2015)





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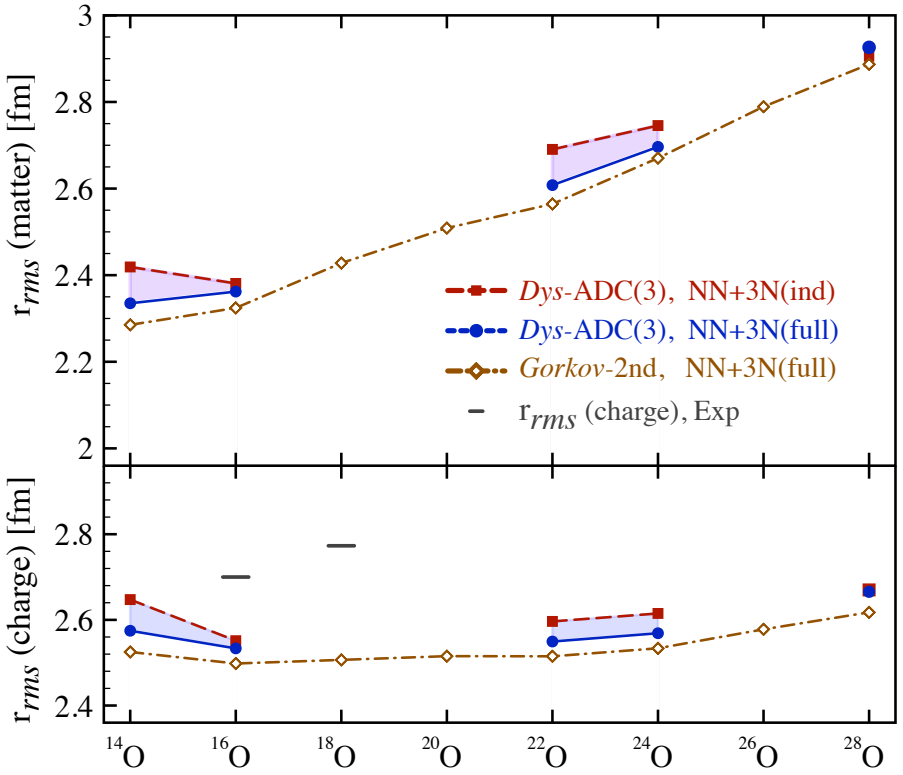
Results for the oxygen chain

A. Cipollone, CB, P. Navrátil, Phys. Rev. C **92**, 014306 (2015)



→ Single particle spectra slightly to spread and

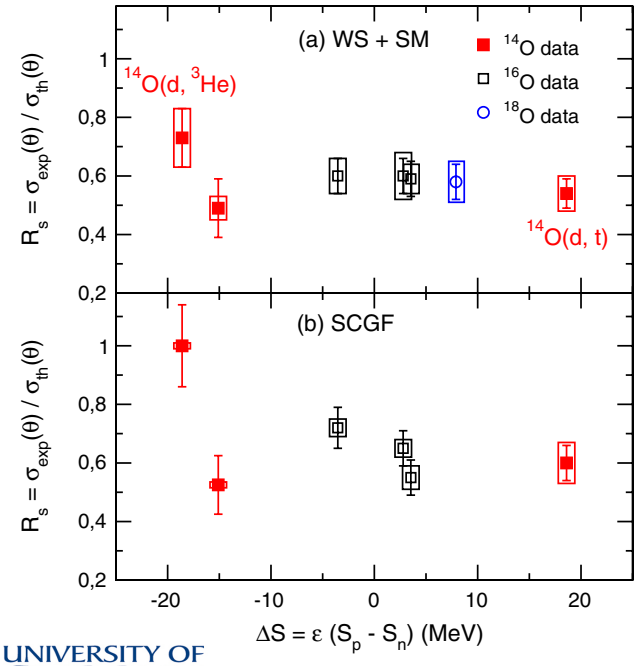
→ systematic underestimation of radii

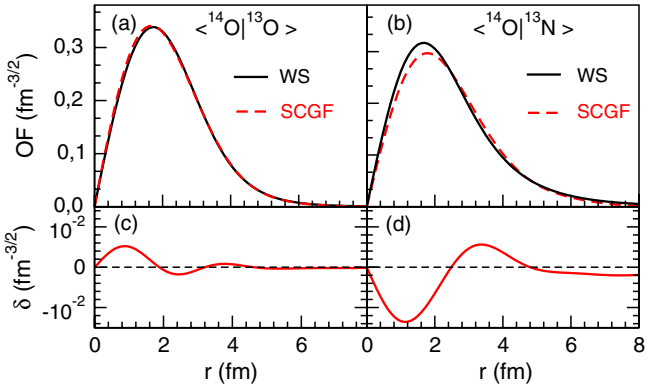
Single nucleon transfer in the oxygen chain

[F. Flavigny et al, PRL**110**, 122503 (2013)]

→ Analysis of $^{14}O(d,t)^{13}O$ and $^{14}O(d,^3He)^{13}N$ transfer reactions @ SPIRAL

Reaction	E^* (MeV)	J^π	R_{rms}^{HFB} (fm)	r_0 (fm)	C^2S_{exp} (WS)	C^2S_{th} $0p + 2\hbar\omega$	R_s (WS)	C^2S_{exp} (SCGF)	C^2S_{th} (SCGF)	R_s (SCGF)
^{14}O (d, t) ^{13}O	0.00	$3/2^-$	2.69	1.40	1.69 (17)(20)	3.15	0.54(5)(6)	1.89(19)(22)	3.17	0.60(6)(7)
^{14}O $(d, ^3He)$ ^{13}N	0.00	$1/2^-$	3.03	1.23	1.14(16)(15)	1.55	0.73(10)(10)	1.58(22)(2)	1.58	1.00(14)(1)
	3.50	$3/2^-$	2.77	1.12	0.94(19)(7)	1.90	0.49(10)(4)	1.00(20)(1)	1.90	0.53(10)(1)
^{16}O (d, t) ^{15}O	0.00	$1/2^-$	2.91	1.46	0.91(9)(8)	1.54	0.59(6)(5)	0.96(10)(7)	1.73	0.55(6)(4)
^{16}O $(d, ^3He)$ ^{15}N [19,20]	0.00	$1/2^-$	2.95	1.46	0.93(9)(9)	1.54	0.60(6)(6)	1.25(12)(5)	1.74	0.72(7)(3)
	6.32	$3/2^-$	2.80	1.31	1.83(18)(24)	3.07	0.60(6)(8)	2.24(22)(10)	3.45	0.65(6)(3)
^{18}O $(d, ^3He)$ ^{17}N [21]	0.00	$1/2^-$	2.91	1.46	0.92(9)(12)	1.58	0.58(6)(10)			





- Overlap functions and strengths from GF
- Rs independent of asymmetry

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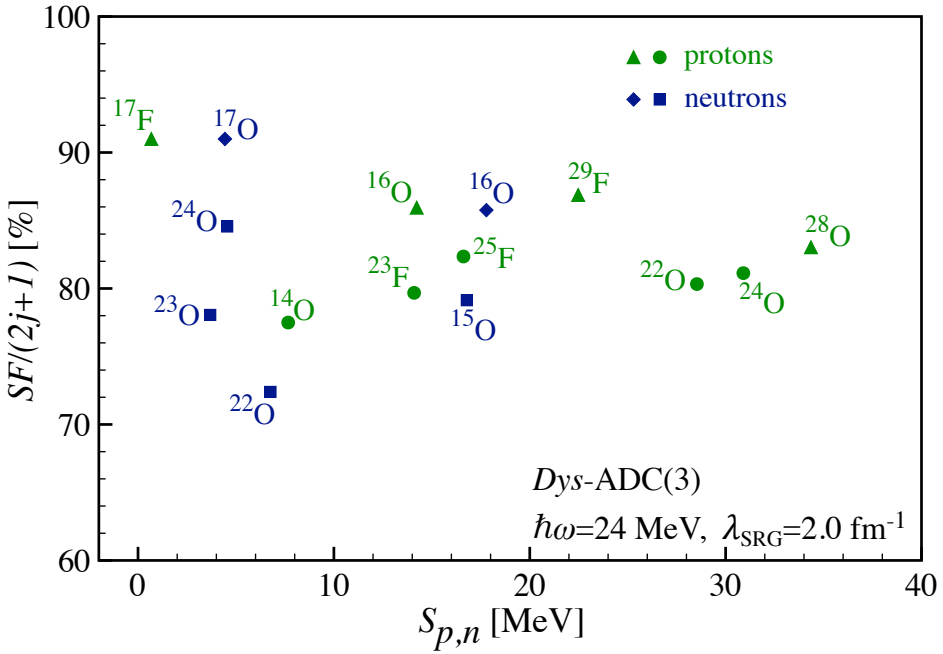
Z/N asymmetry dependence of SFs - Theory

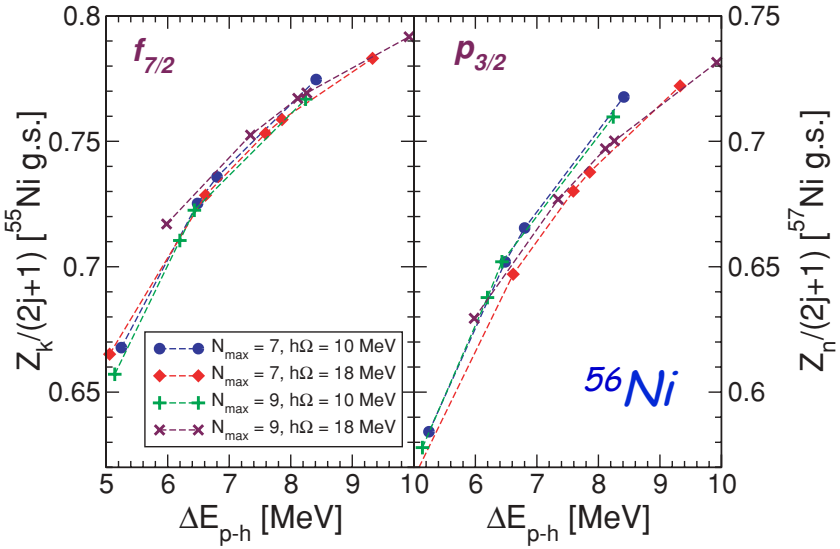
Ab-initio calculations explain (a very weak) the Z/N dependence but the effect is much lower than suggested by direct knockout

Rather the quenchng is high correlated to the gap at the Femi surface.



A. Cipollone, CB, P Navrátil
Phys. Rev. C**92**, 014306 (2015)

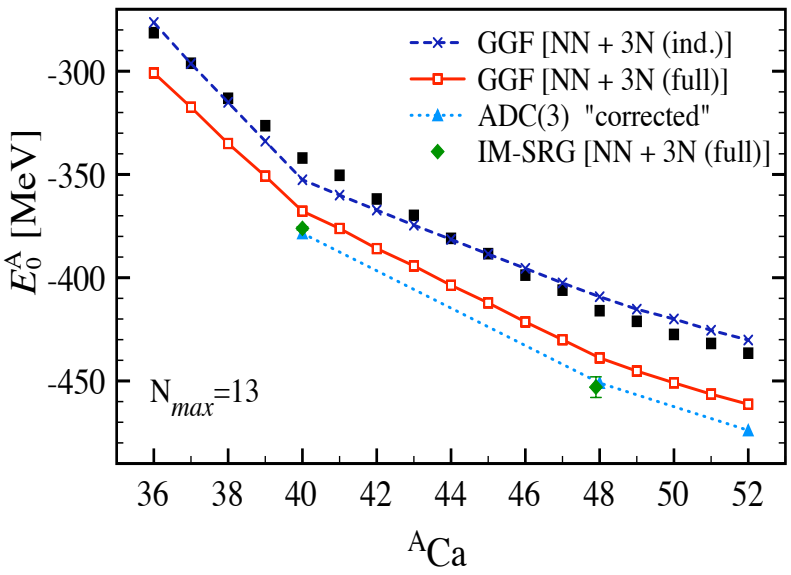
Spectroscopic factor are strongly correlated to p-h gaps:

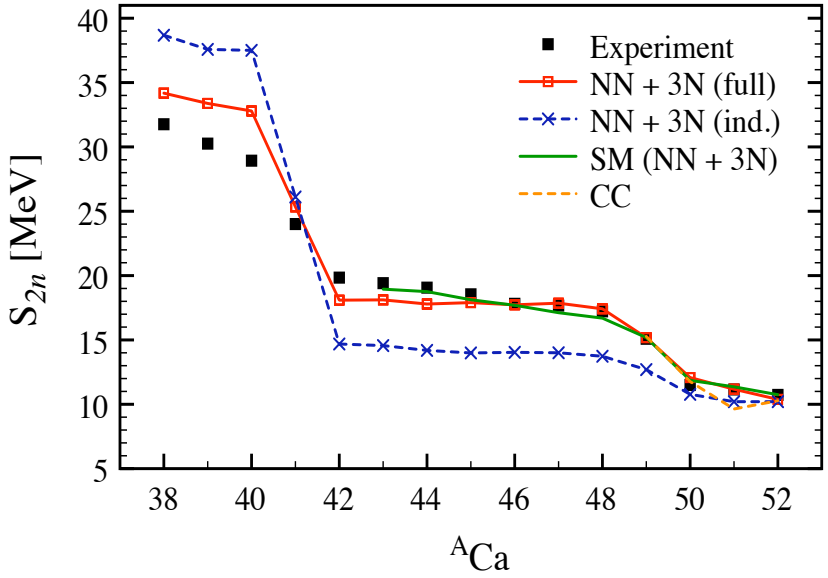


CB, M. Hjorth-Jensen,
Phys. Rev. C **79**, 064313 (2009)

Calcium isotopic chain

Ab-initio calculation of the whole Ca: *induced* and full 3NF investigated





→ *induced* and full 3NF investigated

→ genuine (N2LO) 3NF needed to reproduce the energy curvature and S_{2n}

→ N=20 and Z=20 gaps overestimated!

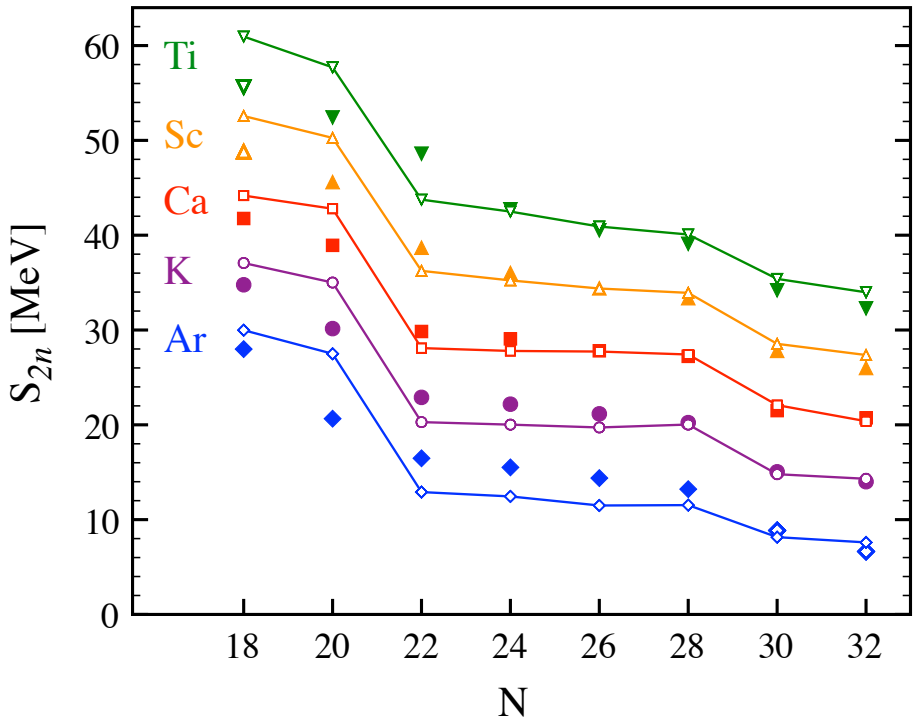
→ Full 3NF give a correct trend but over bind!

V. Somà, CB *et al.* Phys. Rev. C**89**, 061301R (2014)

Neighbouring Ar, K, Ca, Sc, and Ti chains

V. Somà, CB *et al.* Phys. Rev. C**89**, 061301R (2014)

Two-neutron separation energies predicted by chiral NN+3NF forces:



→ First *ab-initio* calculation over a contiguous portion of the nuclear chart—open shells are now possible through the Gorkov-GF formalism

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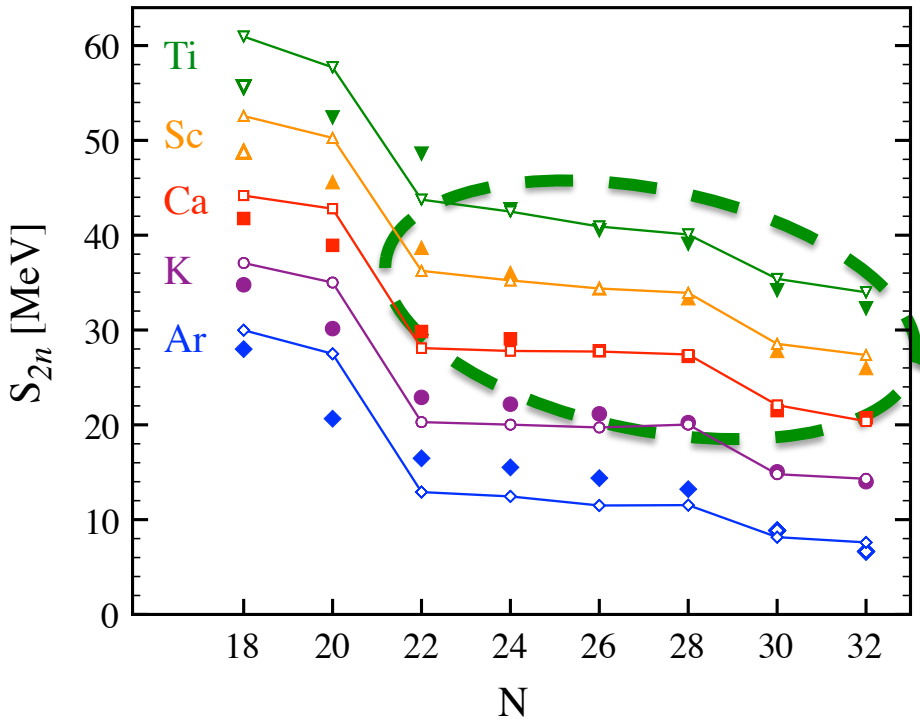

Works well in the *pf* shell

→ First *ab-initio* calculation over a contiguous portion of the nuclear chart—open shells are now possible through the Gorkov-GF formalism

Neighbouring Ar, K, Ca, Sc, and Ti chains

V. Somà, CB *et al.* Phys. Rev. C**89**, 061301R (2014)

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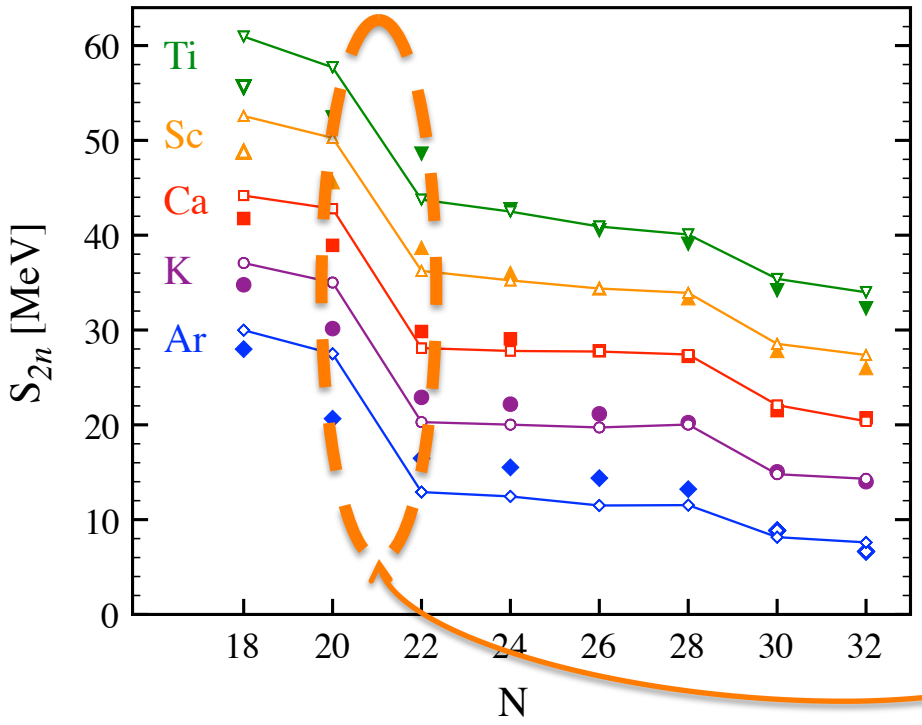

Over estimated
N=20 and Z=20 gaps

→ First *ab-initio* calculation over a contiguous portion of the nuclear chart—open shells are now possible through the Gorkov-GF formalism

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V. Somà, CB *et al.* Phys. Rev. C**89**, 061301R (2014)

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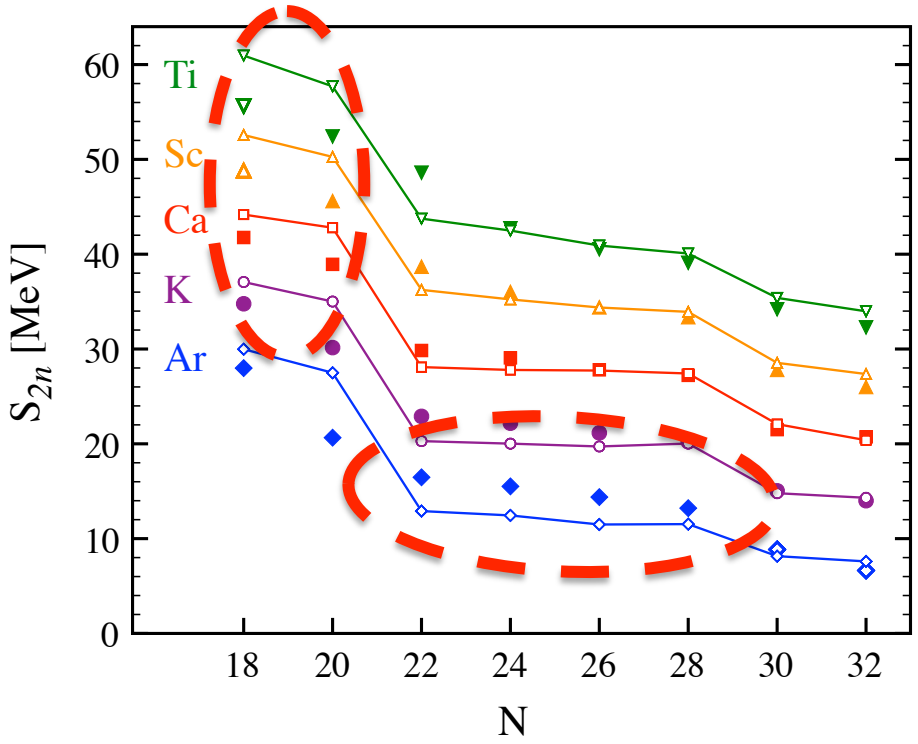


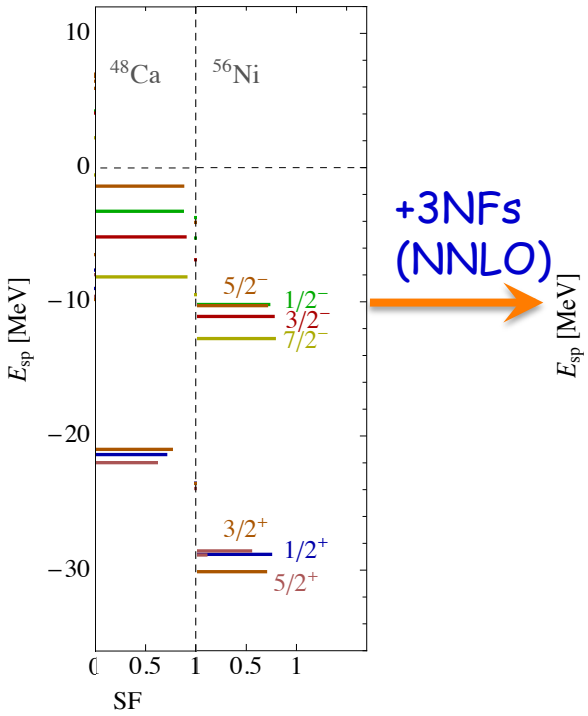
Lack of deformation due to quenched cross-shell quadrupole excitations

→ First *ab-initio* calculation over a contiguous portion of the nuclear chart—open shells are now possible through the Gorkov-GF formalism

The sd-pf shell gap

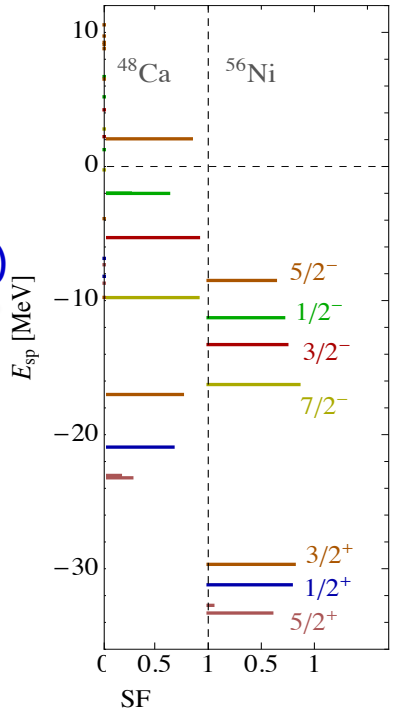
Neutron spectral distributions for ^{48}Ca and ^{56}Ni:

2N + 3NF (induced)



+3NFs (NNLO)

2N + 3NF (FULL)



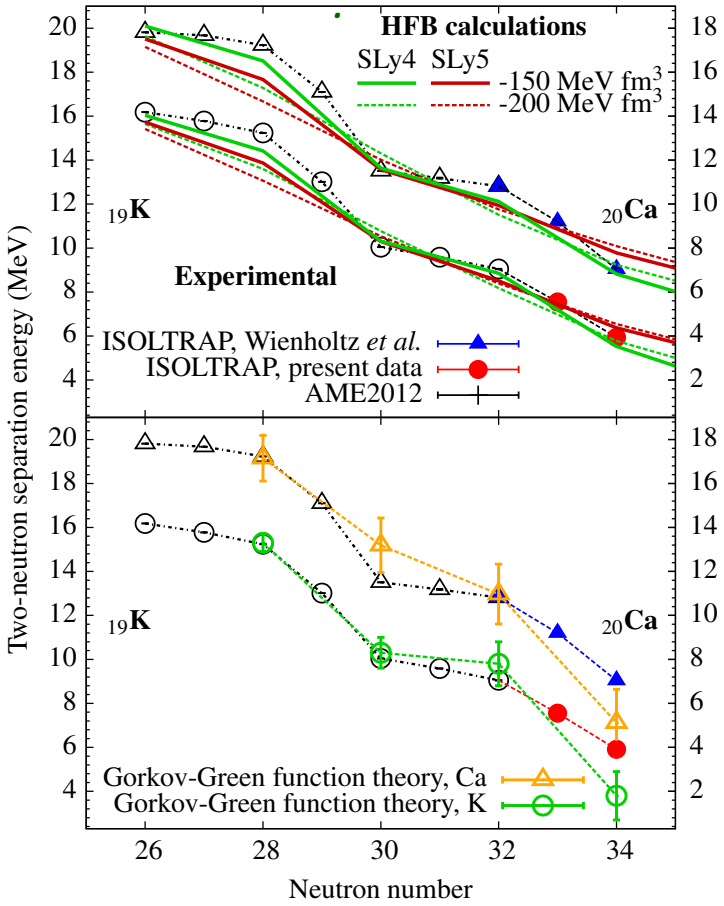
- *sd-pf separation is overestimated* even with leading order N2LO 3NF
- Correct increase of $p_{3/2}$-$f_{7/2}$ splitting (see Zuker 2003)

	2NF only	2+3NF(ind.)	2+3NF(full)	Experiment
^{16}O:	2.10	2. 41	2.38	2.718±0.210 [19]
^{44}Ca:	2.48	2.93	2.94	3.520±0.005 [20]

CB *et al.*, arXiv:1211.3315 [nucl-th]

Two-neutron separation energies for neutron rich K isotopes

*M. Rosenbusch, et al., PRL**114**, 202501 (2015)*



Measurements @ ISOLTRAP

Theory tend to overestimate the gap at N=34, but overall good

→ *Error bar in predictions* are from extrapolating the many-body expansion to convergence of the model space.

Inversion of $d_{3/2}$—$s_{1/2}$ at N=28

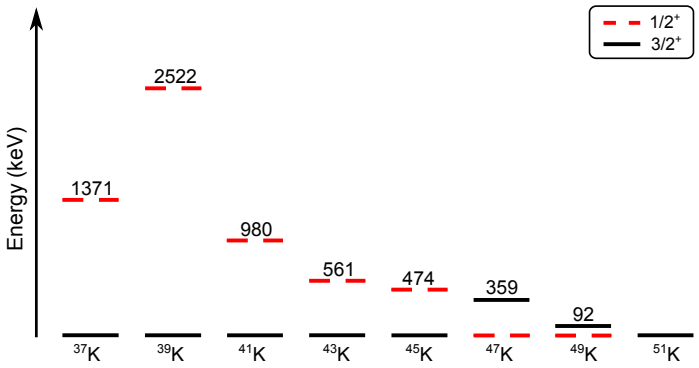


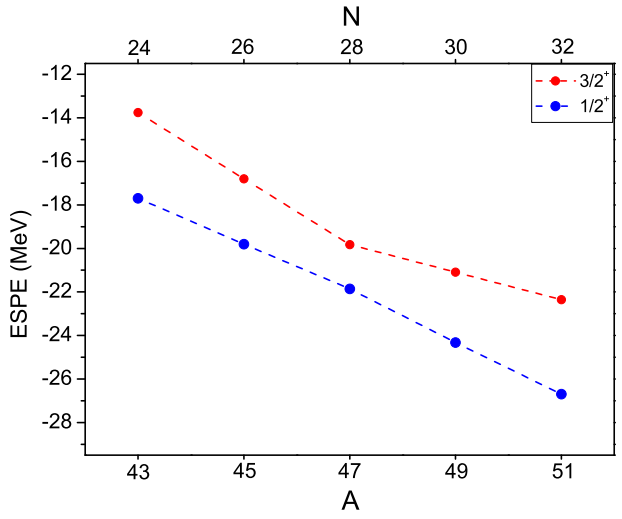
FIG. 1. (color online) Experimental energies for $1/2^+$ and $3/2^+$ states in odd-A K isotopes. Inversion of the nuclear spin is obtained in ^{47,49}K and reinversion back in ^{51}K. Results are

J. Papuga, et al., Phys. Rev. Lett. **110**, 172503 (2013);
Phys. Rev. C **90**, 034321 (2014)

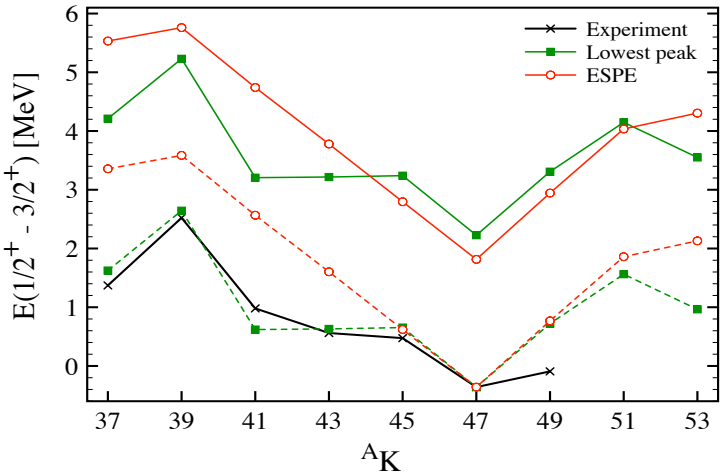
^{A}K isotopes

Laser spectroscopy @ ISOLDE

Change in separation described by chiral NN+3NF:



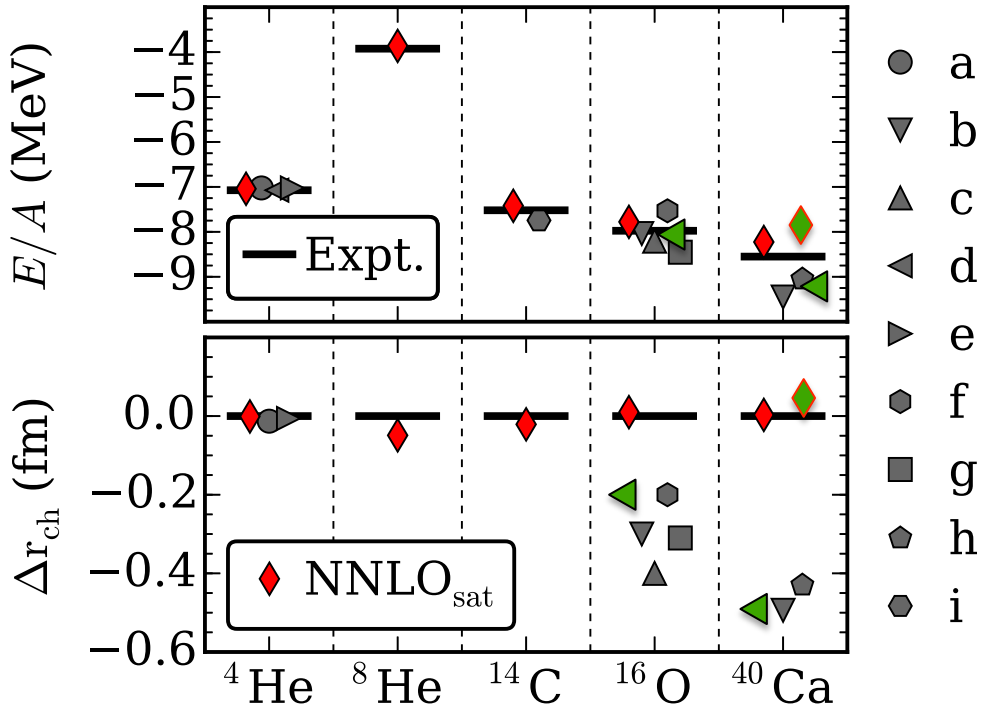
ESPE: "centroid" energies



(Gorkov calculations at 2nd order)

NNLO-sat : a global fit up to A≈24

A. Ekström *et al.* Phys. Rev. C**91**, 051301(R) (2015)



- Constrain NN phase shifts

- Constrain radii and energies up to A≲24

➔ Provides saturation up to large masses!

NNLOsat (V2 + W3) -- *Grkv 2nd ord.*

From **SCGF:**

V2-N3LO(500) + W3-NNLO(400MeV/c) w/ SRG at 2.0 fm^{-1}

A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)

V. Somà, CB *et al.* Phys. Rev. C**89**, 061301R (2014)

BE and radii for Oxygens

- New fits of chiral interactions (NNLOsat) highly improve comparison to data

- Deficiencies remain for neutron rich isotopes

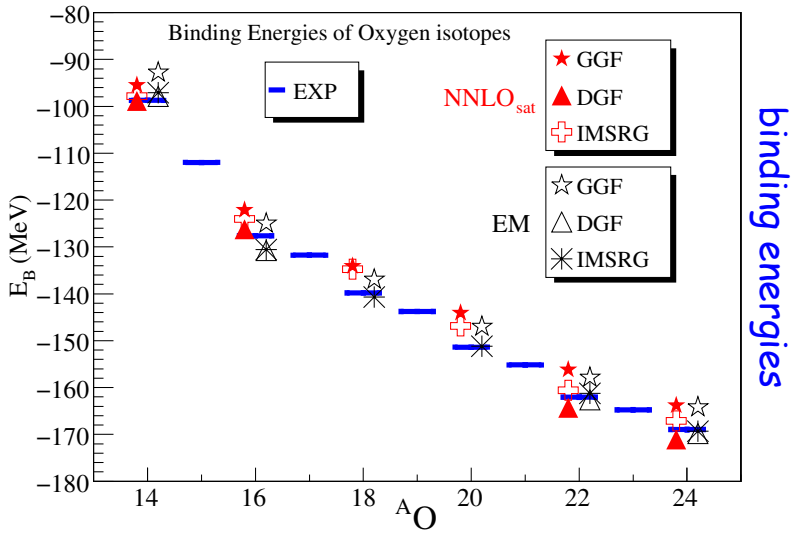
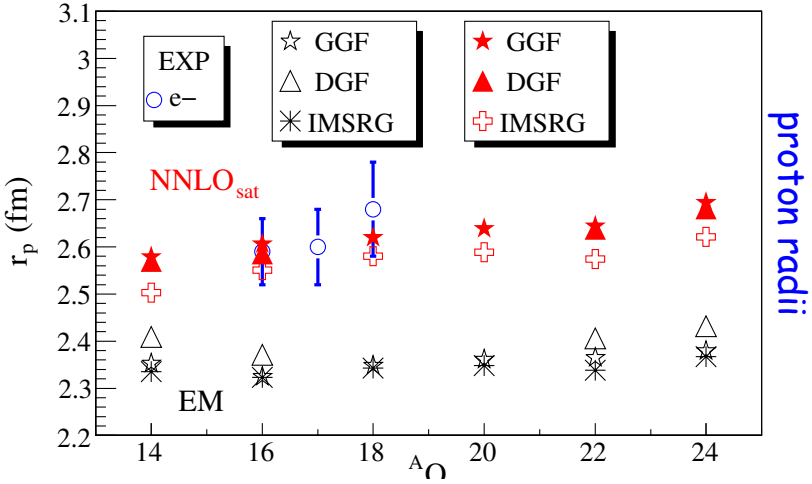
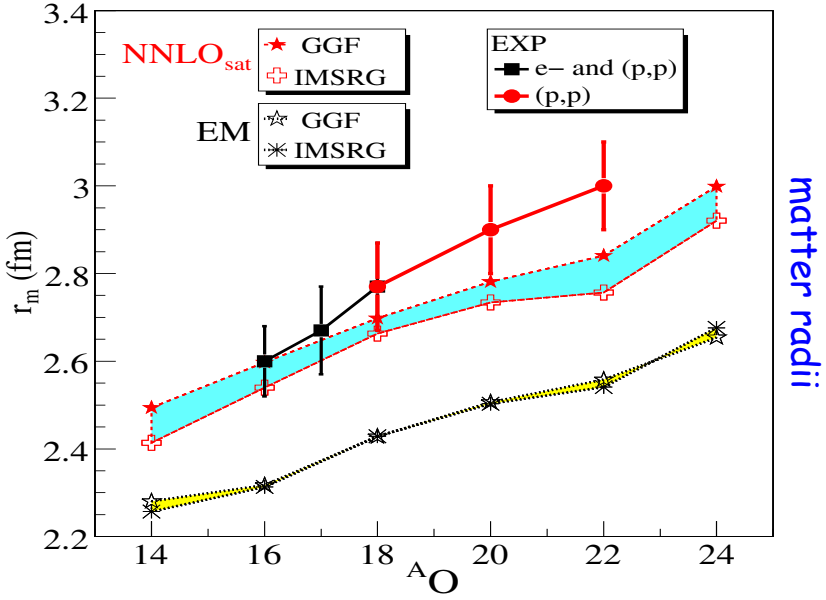


FIG. 1. Oxygen binding energies. Results from SCGF and IMSRG calculations performed with EM [20–22] and $NNLO_{sat}$ [26] interactions are displayed along with available experimental data.



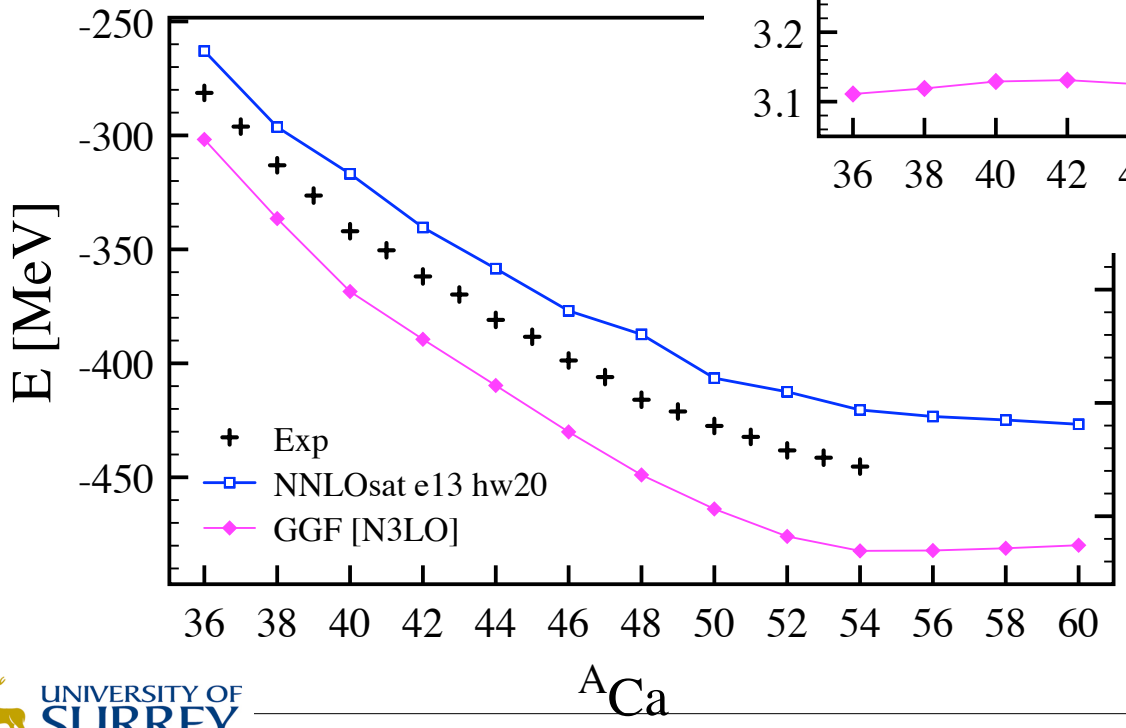


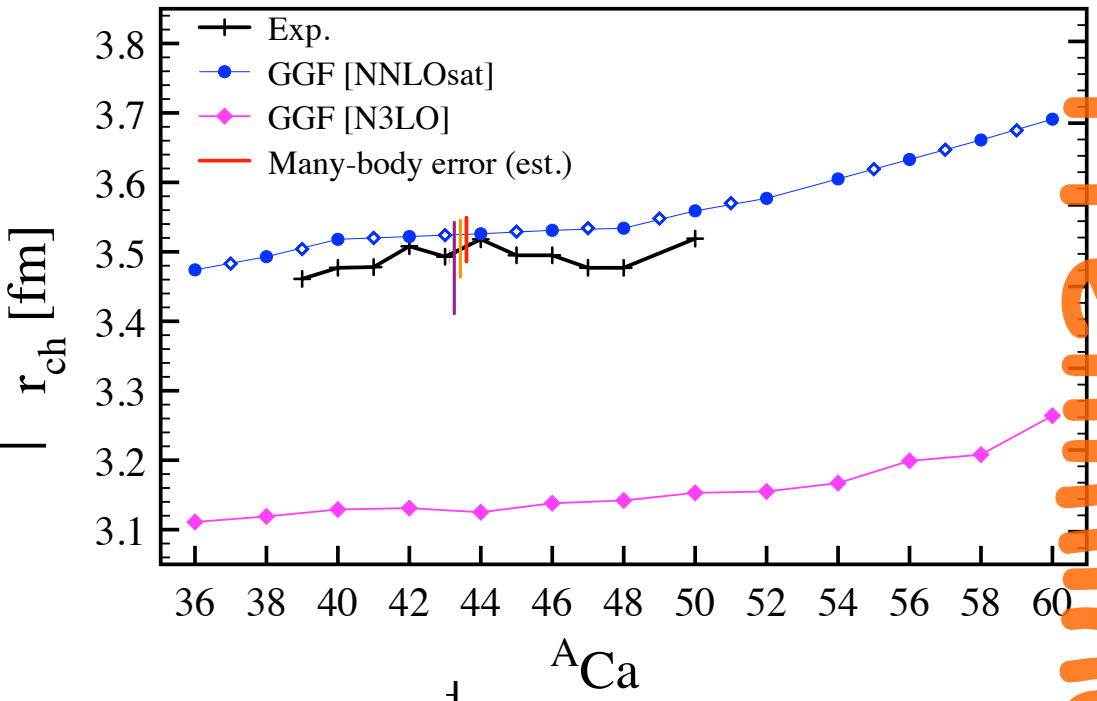
V. Somà, V. Lapuox, CB, et al., *in preparation*

BE and charge radii in ^{A}Ca

2nd order GGF 'correct' to give a slight under binding and larger radii

Radii of even-odd are possible



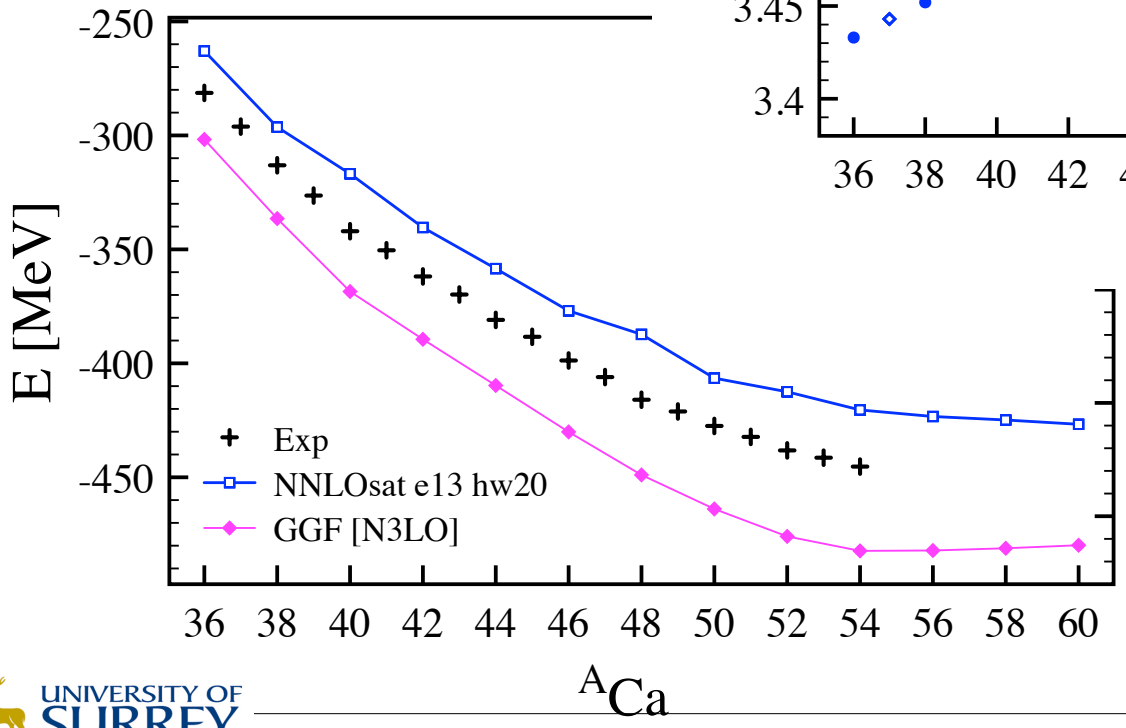


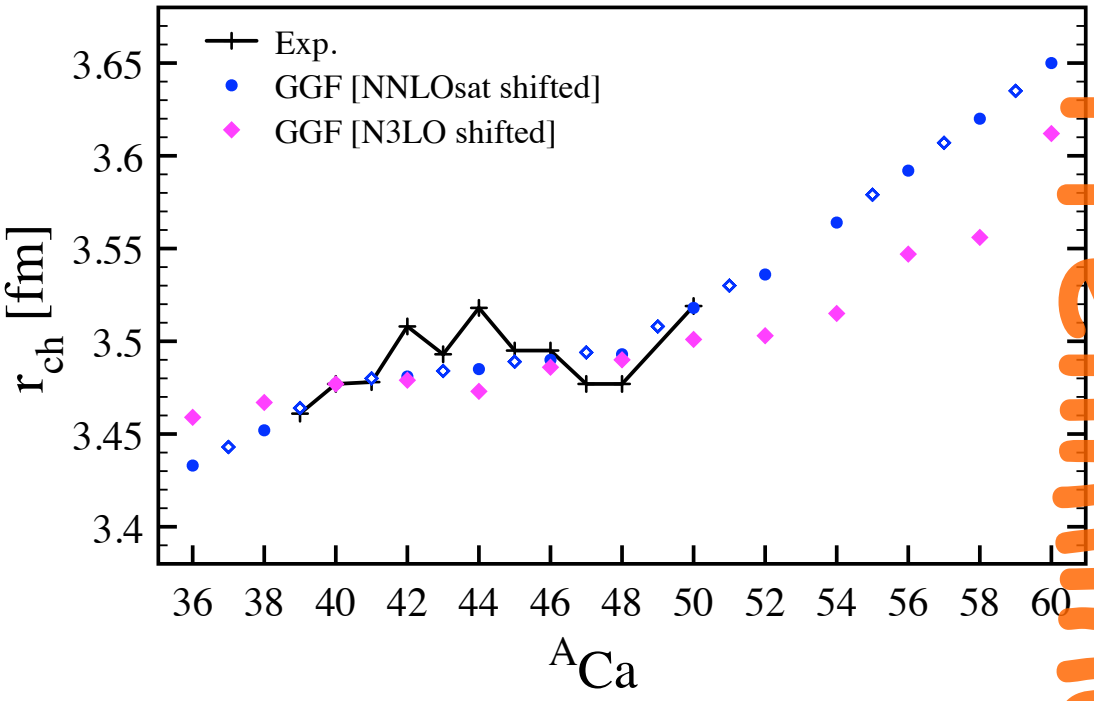
Preliminary

BE and charge radii in ^{A}Ca

2nd order GGF 'correct' to give a slight under binding and larger radii

Radii of even-odd are possible





Preliminary

NNLO sat improves trend of radii

radii od $^{42-46}$Ca require shell model...

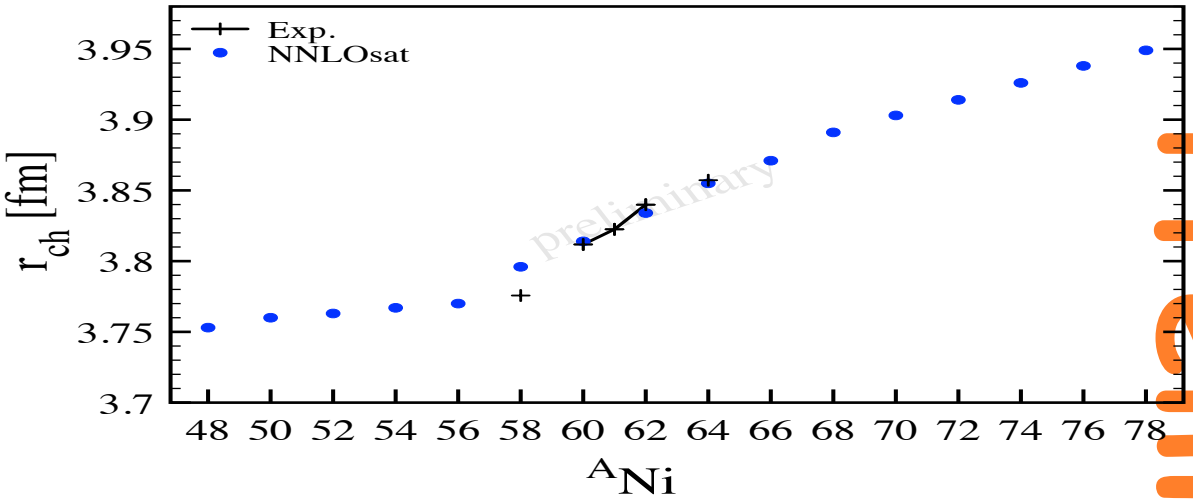
charge radii in the pf shell

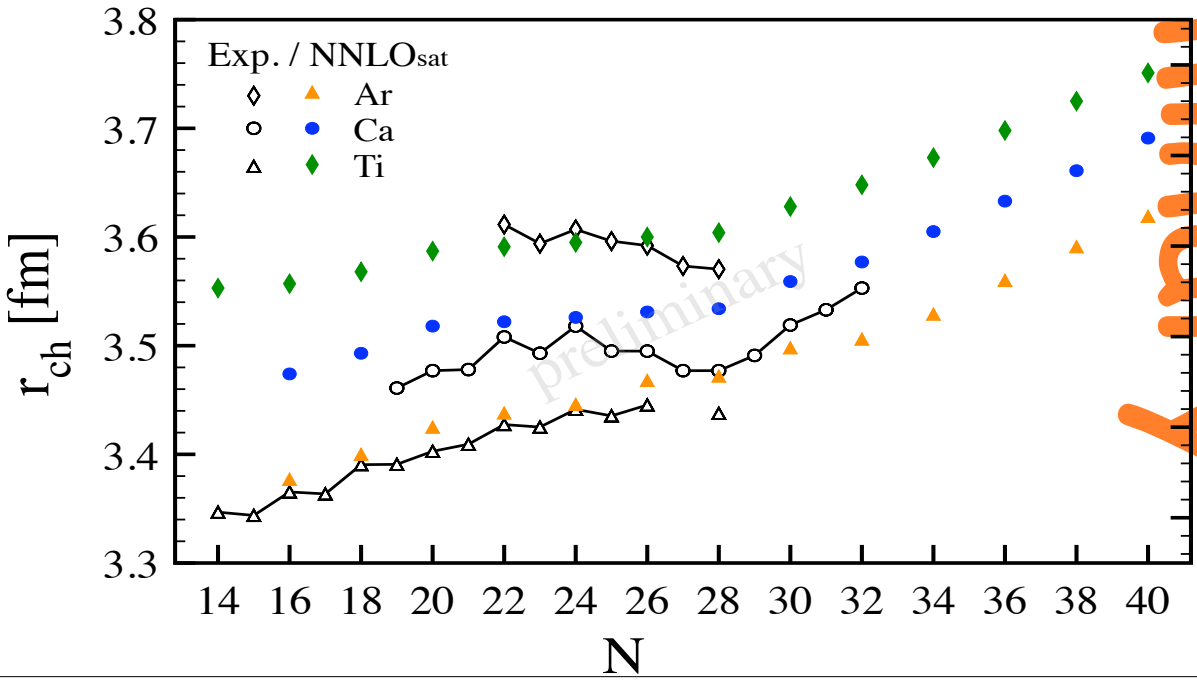
Size of radii not prefect but remains overall correct throughout the ***pf*** shell with NNLO-sat.

This suggests that saturation is indeed under control.

➔ Improvements of many-body truncations beyond 2nd order Gorkov will also be relevant.

(work in progress!)

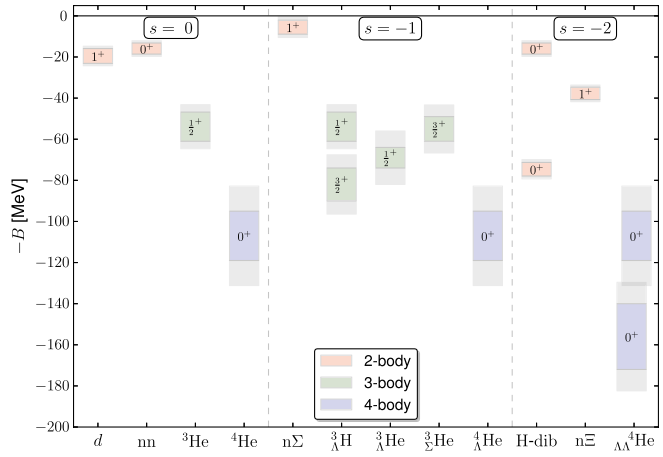




Preliminary

Study of nuclear interactions from Lattice QCD

Other paths in LQCD, see:



NPLQCD, PRD**87**, 034506 (2013)

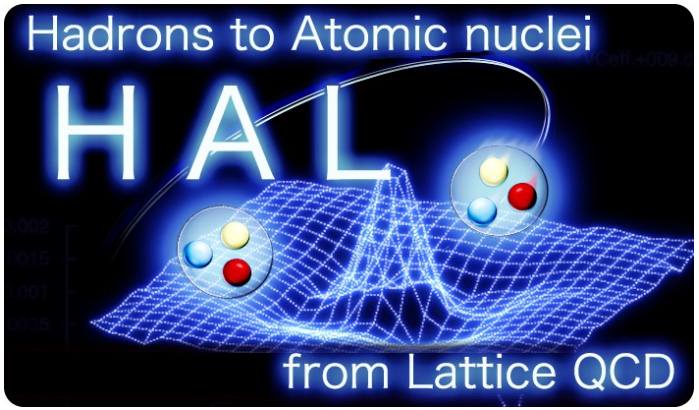
nuclei, the deuteron and the dineutron in $2 + 1$ flavor QCD with $m_\pi = 0.51$ GeV and $m_N = 1.32$ GeV. The bound states are distinguished from the attractive scattering states by investigating the spatial volume dependence of the energy shift ΔE_L. In the infinite spatial volume limit we obtain

$$-\Delta E_\infty = \begin{cases} 43(12)(8) & \text{MeV for } ^4\text{He}, \\ 20.3(4.0)(2.0) & \text{MeV for } ^3\text{He}, \\ 11.5(1.1)(0.6) & \text{MeV for } ^3\text{S}_1, \\ 7.4(1.3)(0.6) & \text{MeV for } ^1\text{S}_0. \end{cases} \quad (17)$$

PACS-CS PRD **86**, 074514 (2012)

Study of nuclear interactions from Lattice QCD

In collaboration with:



Why should we investigate LQCD interactions?

- It gives complimentary insight to the EFT approach:
- Allows to approach physical interaction from heavy quark masses (opposite direction than the chiral limit).
- Can study implications of SU(3) limit.
- No need to fit to experiment. No LEC constants.
- Provides consistent interactions in the Hyperon sector.

- It is very fundamental approach (QCD), and an alternative to Chiral-EFT.

Challenges and limitations:

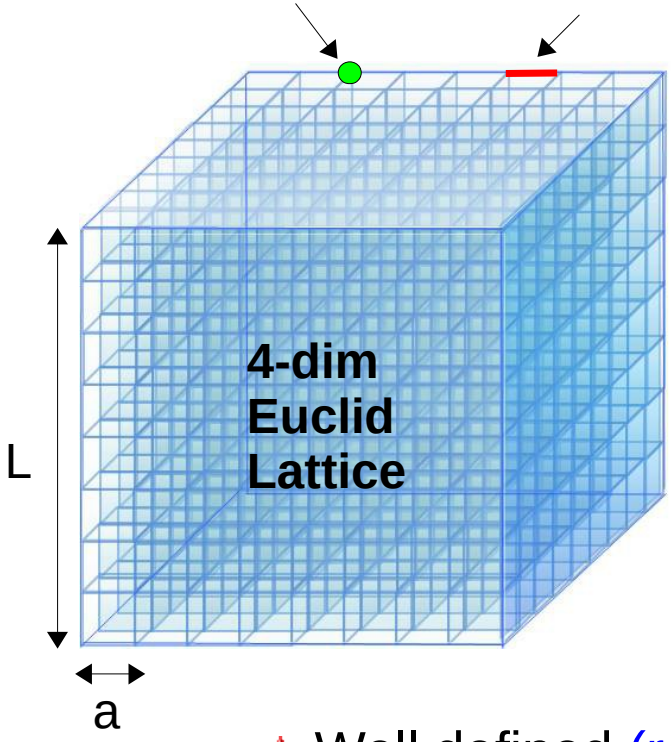
- Mostly LO terms of the NN force exploited so far (but being improved).
- Physical pion mass limit requires efforts (but underway).
- NNN only barely addressed.
- Strong short-range repulsion is a challenge to ab-initio approaches.

Lattice QCD

$$L = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \bar{q}\,\gamma^\mu \left(i\partial_\mu - g\,t^a A^a_\mu \right) q - m\,\bar{q}\,q$$

quarks q
on the sites

gluons $U = e^{ia A_\mu}$
on the links



Vacuum expectation value

$$\langle O(\bar{q}, q, U) \rangle$$

path integral

$$= \int dU\, d\bar{q}\, dq\, e^{-S(\bar{q},q,U)}\, O(\bar{q}, q, U)$$

$$= \int dU \det D(U) e^{-S_U(U)}\, O\left(D^{-1}(U)\right)$$

quark propagator

$$= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} O\left(D^{-1}(U_i)\right)$$

{ U_i } : ensemble of gauge conf. U
generated w/ probability $\det D(U)\, e^{-S_U(U)}$

- Well defined (reguralized)
- Manifest gauge invariance
- Fully non-perturvative
- Highly predictive

UNIVERSITY OF SURREY

Slide, courtesy of T. Inoue (YITP talk, Oct. 8th 2015)

The HAL-QCD Method

Define a general potential U(***r***,***r***') which is and non-local but energy independent up to inelastic threshold, such that:

$$\frac{-\nabla^2}{2\mu}\varphi_{\vec{k}}(\vec{r}) + \int d\vec{r}\,' U(\vec{r},\vec{r}\,')\varphi_{\vec{k}}(\vec{r}\,') = E_{\vec{k}}\varphi_{\vec{k}}(\vec{r})$$

for the Nambu-Bethe-Salpeter (NBS) wave function,

$$\varphi_{\vec{k}}(\vec{r}) = \sum \langle 0|B_i(\vec{x}+\vec{r},t)B_j(\vec{x},t)|B=2,\vec{k}\rangle$$

Operationally, measure the 4-pt function on the QCD Lattice

$$\psi(\vec{r},t) = \sum_{\vec{x}} \langle 0|B_i(\vec{x}+\vec{r},t)B_j(\vec{x},t)\,J(t_0)|0\rangle = \sum_{\vec{k}} A_{\vec{k}}\varphi_{\vec{k}}(\vec{r})e^{-W_{\vec{k}}(t-t_0)} + \ldots$$

and extract U(***r***,***r***') from:

$$\left\{2M_B - \frac{\nabla^2}{2\mu}\right\}\psi(\vec{r},t) + \int d\vec{r}\,' U(\vec{r},\vec{r}\,')\psi(\vec{r}\,',t) = -\frac{\partial}{\partial t}\psi(\vec{r},t)$$

A *local potential* V(***r***) is then obtained through a derivative expansion of U(***r***,***r***'), which *must give the same observables* of the LQCD simulation:

$$U(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,')V(\vec{r},\nabla) = \delta(\vec{r}-\vec{r}\,')\left\{V(\vec{r}) + \mathcal{O}(\nabla) + \mathcal{O}(\nabla^2) + \ldots\right\}$$

$$\rightarrow\ V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

Tensor/Yukawa force in S-D

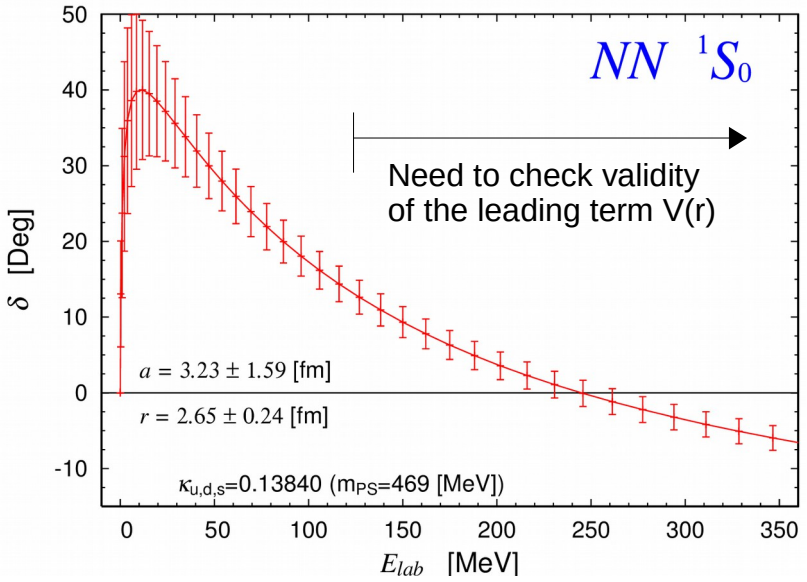
Spin-orbit force, P waves

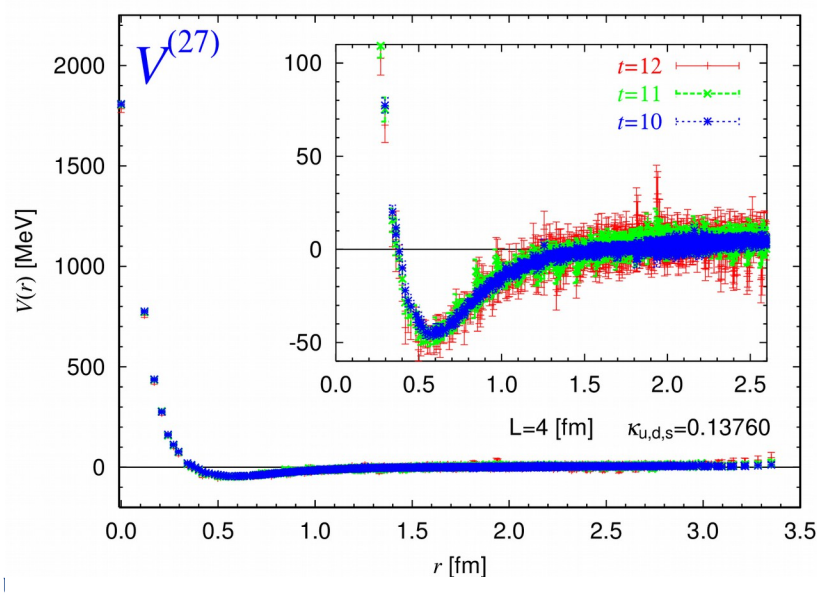
Prog. Theor. Phys. 123 89 (2010); Phys. Lett. B712 , 437 (2012); Prog. Theor. Exp. Phys. 01A105 (2012)

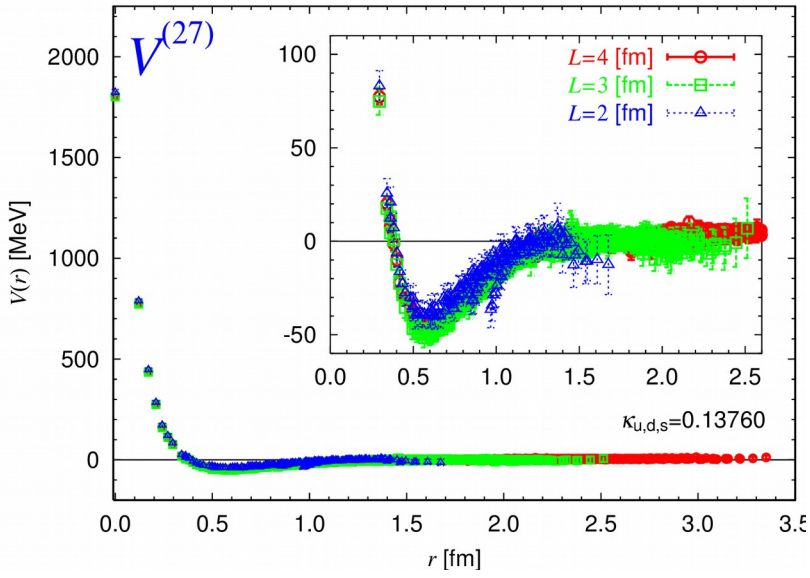
The HAL-QCD Method

Advantages:

- ✓ No need to separate E eigenstate. Just need to measure $\psi(\vec{r},t)$
- ✓ Then, potential can be extracted.
- ✓ Demand a minimal lattice volume. No need to extrapolate to V=∞.
- ✓ Can output more observables.
- ✓ One can address *large nuclei* too!!







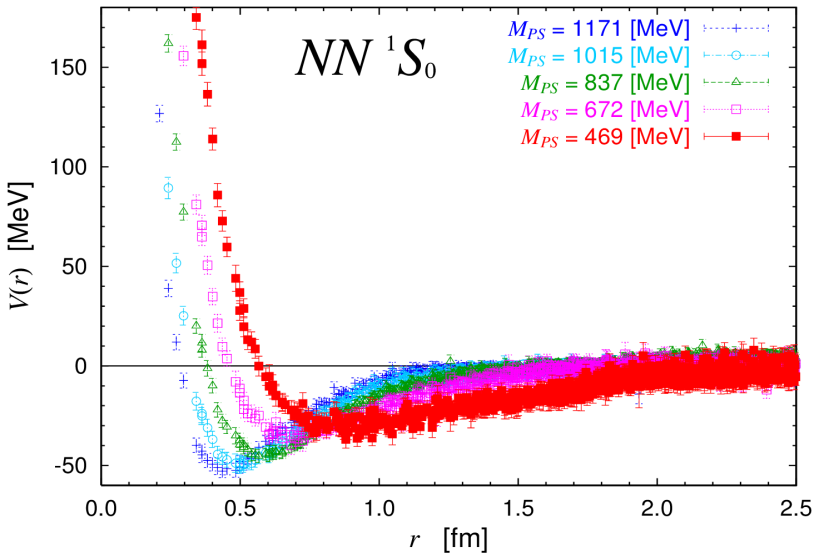
SURREY

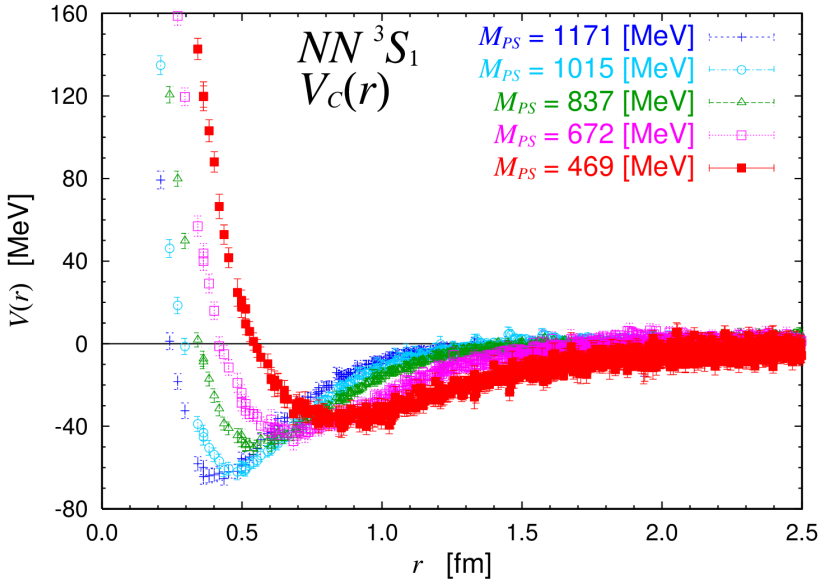
Prog. Theor. Exp. Phys. 01A105 (2012)

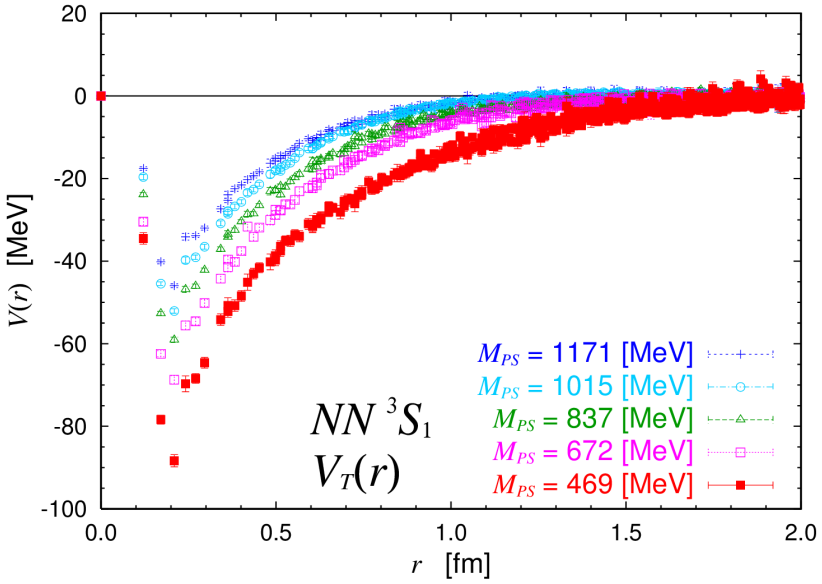
Two-Nucleon HAL potentials

Quark mass dependence of *V(r)* for NN partial wave ($^1S_0$, $^3S_1$, $^3S_1$-$^3D_1$)

➔ Potentials become stronger m_π as decreases.







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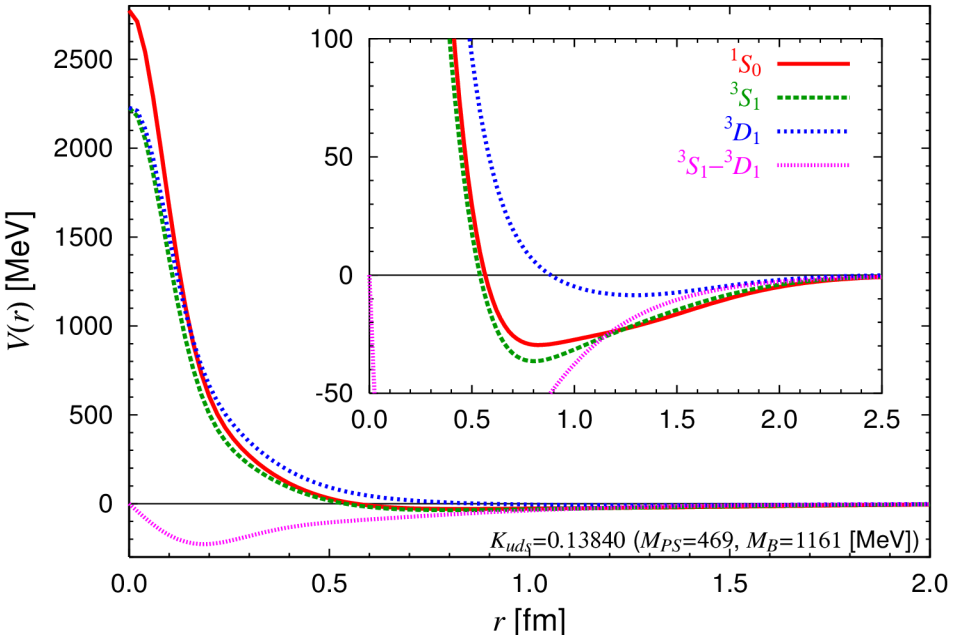
Prog. Theor. Exp. Phys. 01A105 (2012)

Two-Nucleon HAL potentials



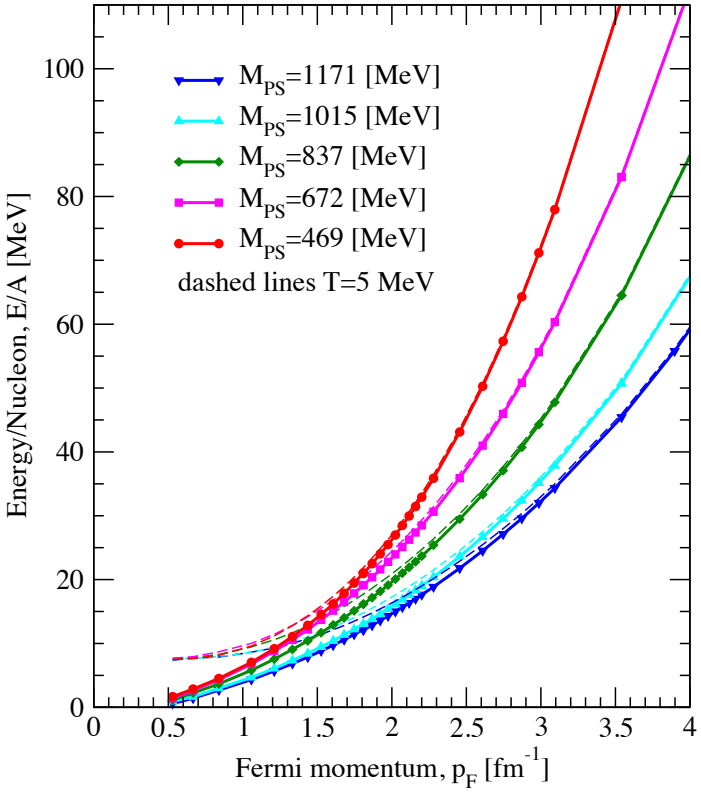
Potential in partial waves at the lightest m_π=469 MeV:

- ✓ Repulsive core, attractive pocket, and strong tensor components
- ✓ Similar to phenomenological potentials (e.q. AV18).
- ✓ Central value obtained from least χ^2 fit to data.
- ✓ Higher orders in the velocity expansion will also be available soon…

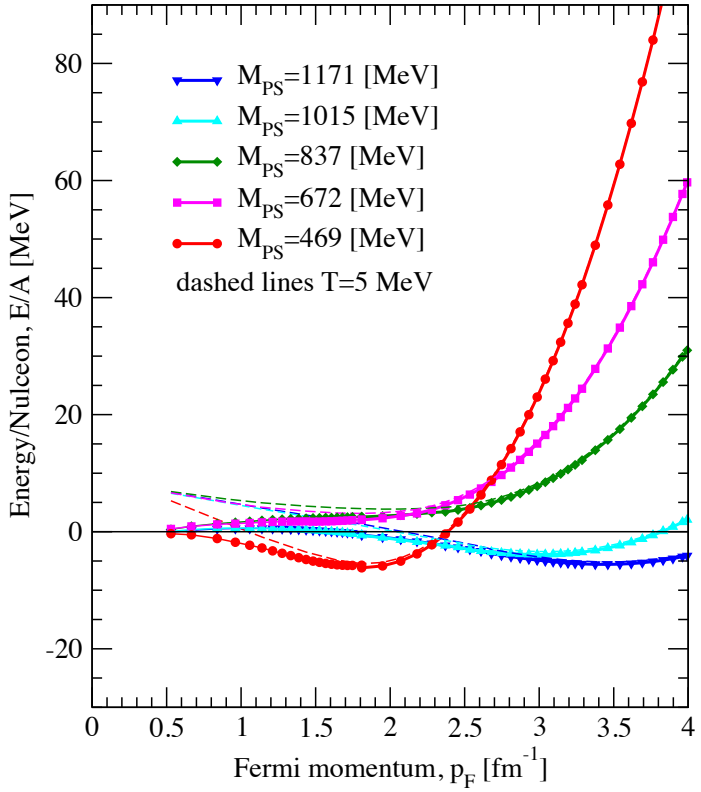
Prog. Theor. Exp. Phys. 01A105 (2012)

Infinite matter

Pure Neutron Matter



Symmetric Nuclear Matter



PNM unbound as usual, but less stiff

SNM saturates at m_π= 469 MeV but under bound and at higher densities that physical.

T. Inoue *et al.*, Phys. Rev. Lett. **111** 112503 (2013).

Finite-T results by A. Carbone, priv. comm.

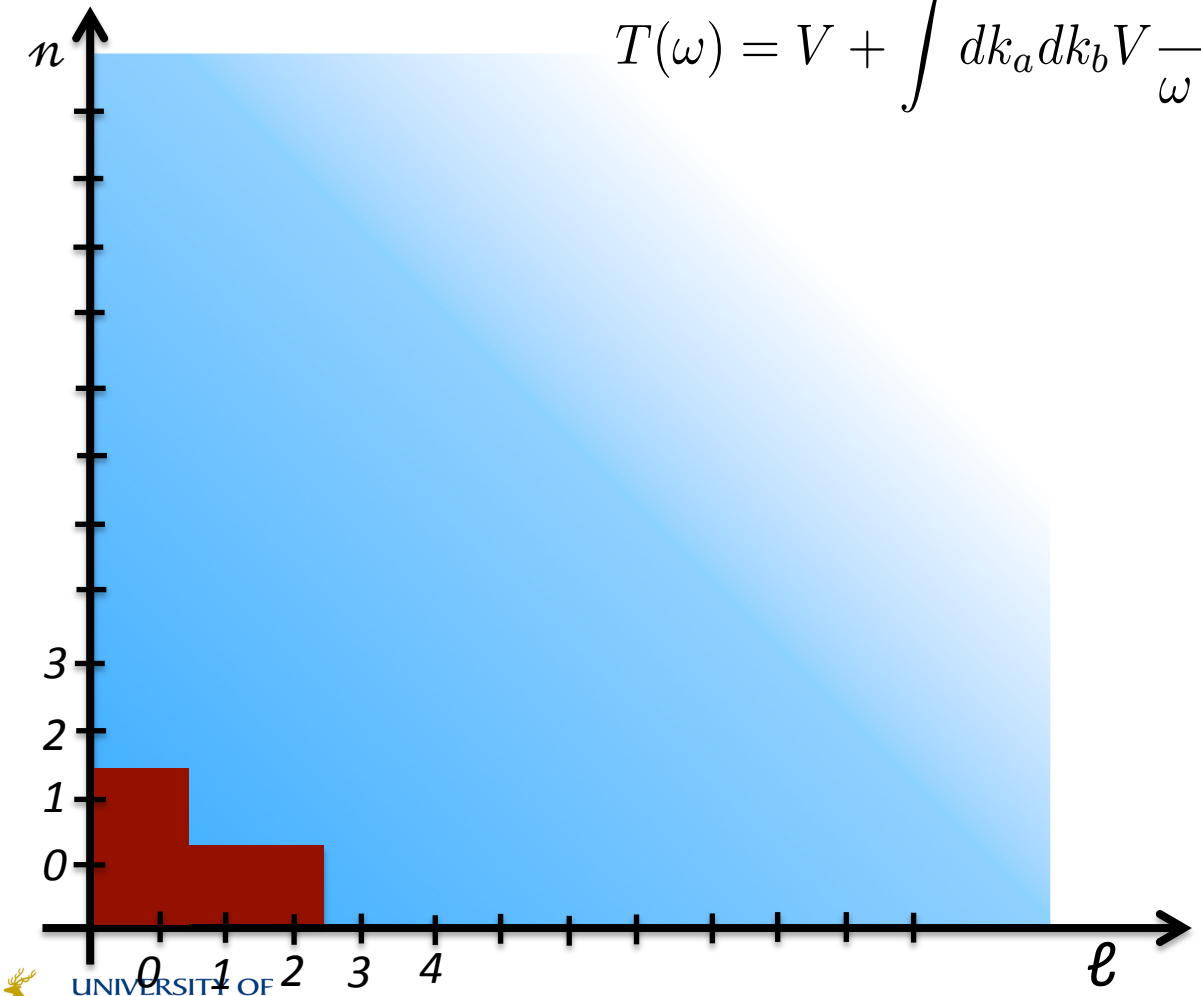
Application of microscopic (Ab-Initio) SCGF to potentials with hard cores.

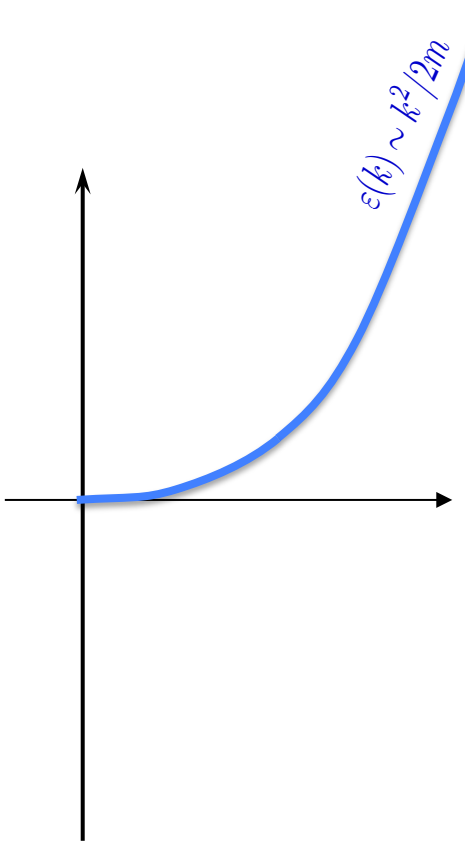
How do we do it?? → *With a G-matrix!*

Analysis of Brueckner HF

Scattering of two nucleon in free space:

$$T(\omega) = V + \int dk_a dk_b V \frac{1}{\omega - k_a^2/2m - k_b^2/2m + i\eta} T(\omega)$$

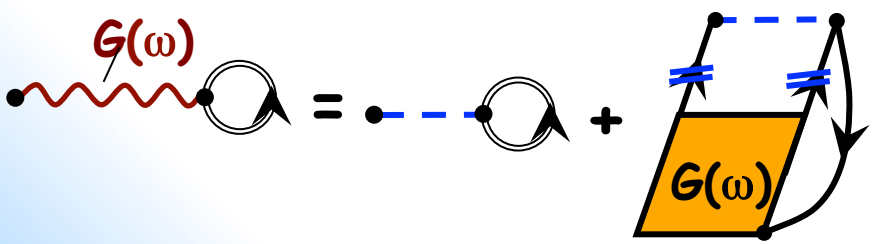


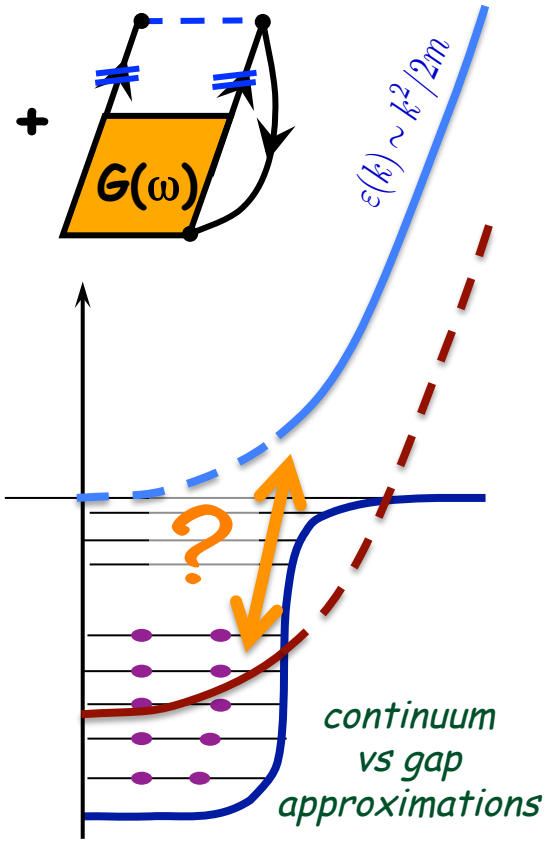


Analysis of Brueckner HF

Scattering of two nucleons outside the Fermi sea (➔BHF):

$$G(\omega) = V + \int dk_a dk_b V \frac{\hat{Q}}{\omega - \varepsilon(k_a) - \varepsilon(k_b) + i\eta} G(\omega)$$

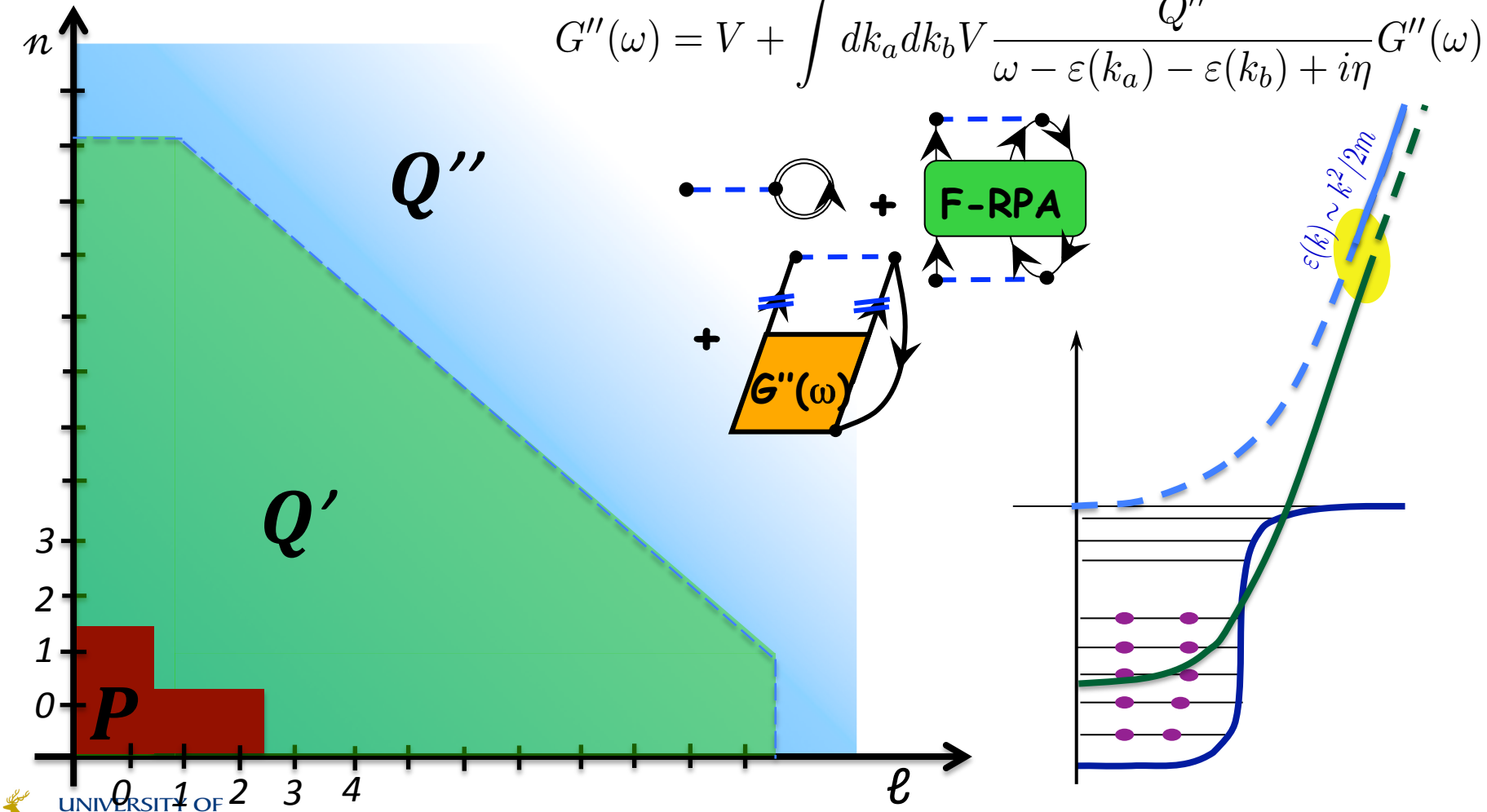




Mixed SCGF-Brueckner approach

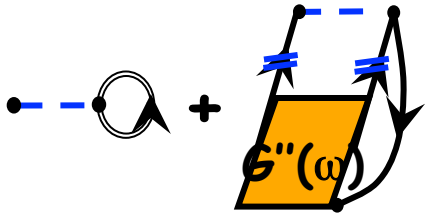
Solve full many-body dynamics in model space (P+Q') and the Goldstone's ladders outside it (i.e. in Q'' only):

$$G''(\omega) = V + \int dk_a dk_b V \frac{\hat{Q}''}{\omega - \varepsilon(k_a) - \varepsilon(k_b) + i\eta} G''(\omega)$$

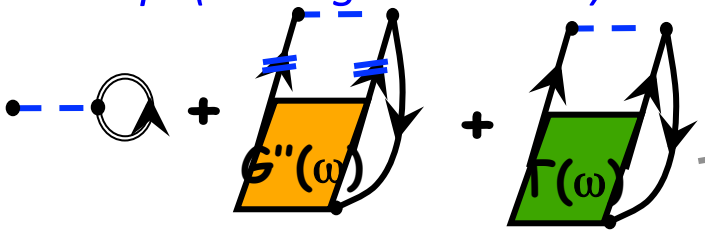


Different levels of approximation:

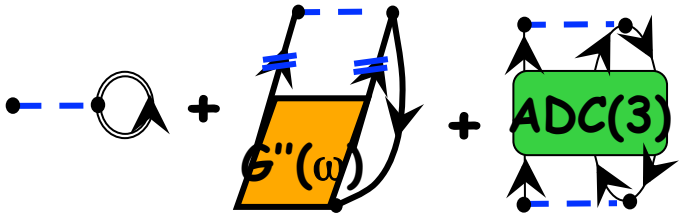
Goldstone's ladders outside the model space (i.e. in Q'' only):

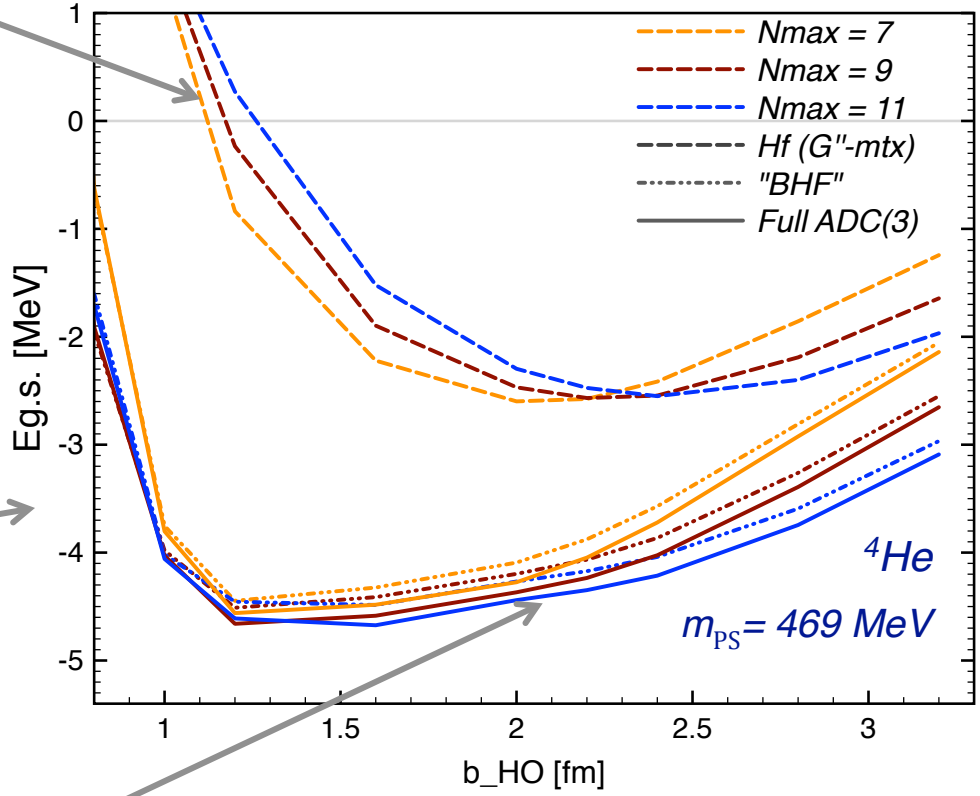


All ladders inside and outside the mod. sp. (analogous to BHF):



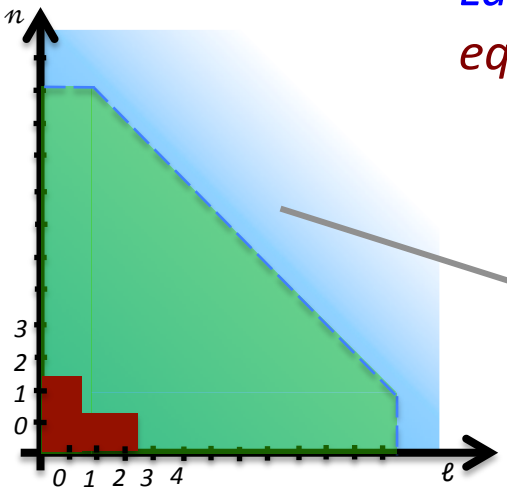
Full many-body dynamics [at ADC(3)]

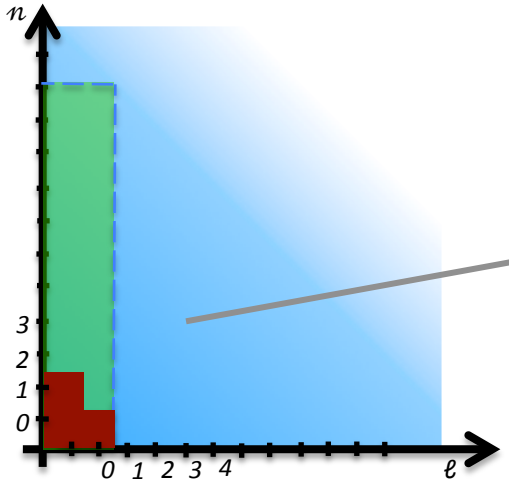


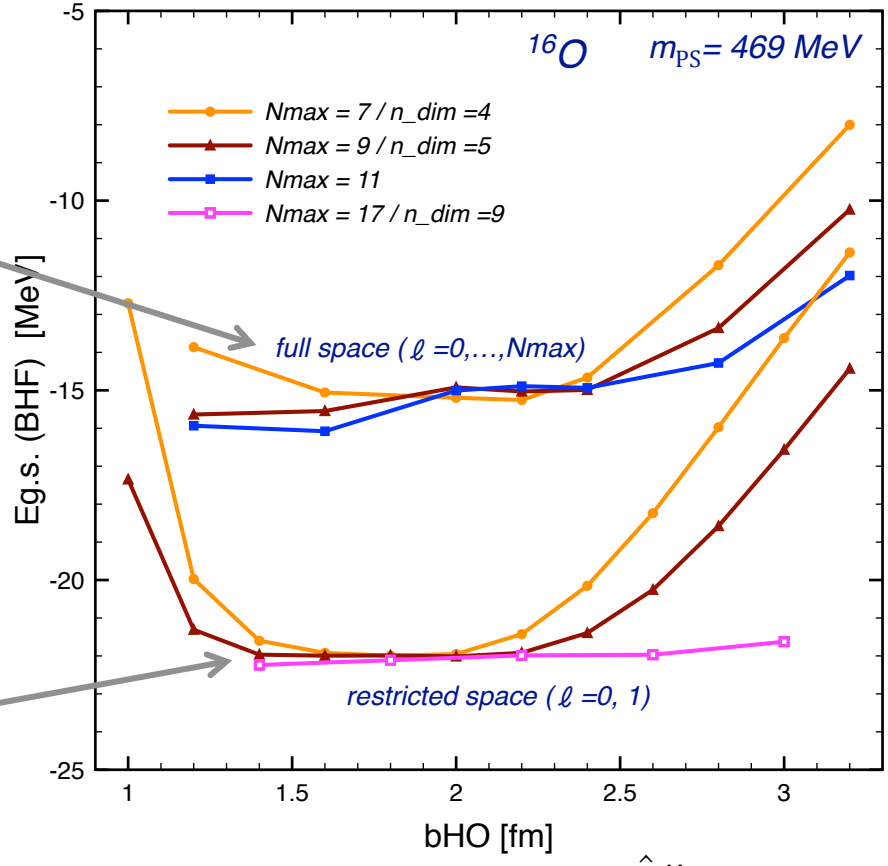


Sensitivity of BHF of the ε(k) spectrum

Ladders calculated inside and outside the model are NOT equivalent because of the different ε(k) spectrum:







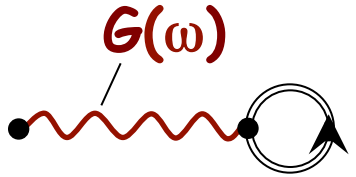
$$G''(\omega) = V + \int dk_a dk_b V \frac{\hat{Q}''}{\omega - \varepsilon(k_a) - \varepsilon(k_b) + i\eta} G''(\omega)$$

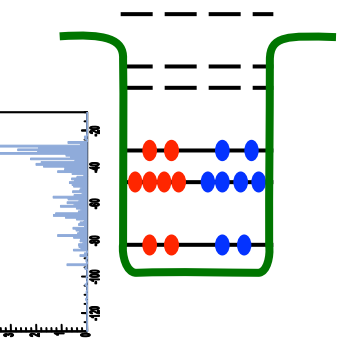
Treating short-range corr. with a G-matrix

- The short-range core can be treated by summing ladders outside the model space:

$$\Sigma^{\mathrm{MF}}_{\alpha\beta}(\omega) = i\sum_{\gamma\delta}\int \frac{d\omega'}{2\pi} G_{\alpha\gamma,\delta\beta}(\omega+\omega')g_{\delta\gamma}(\omega')$$

=





$$\Sigma^{\star}(\mathbf{r},\mathbf{r}';\omega) = \Sigma^{MF}(\mathbf{r},\mathbf{r}';\omega) + \tilde{\Sigma}(\mathbf{r},\mathbf{r}';\omega)\ .$$

$$Z_\alpha = \int d\mathbf{r}\ |\psi^{A\pm1}_\alpha(\mathbf{r})|^2 = \left.\frac{1}{1-\dfrac{\partial\Sigma^{\star}_{\hat{\alpha}\hat{\alpha}}(\omega)}{\partial\omega}}\right|_{\omega=\pm(E^{A\pm1}_\alpha - E^A_0)}$$

Two contributions to the derivative:

- $\Sigma^{\mathrm{MF}}_{\alpha\beta}(\omega)$ is due to scattering to (high-k) states in the Q space
- $\Sigma(\mathbf{r},\mathbf{r}';\omega)$ accounts for low-energy (long range) correlations

(Galitskii-Migdal-Boffi-) Koltun sumrule

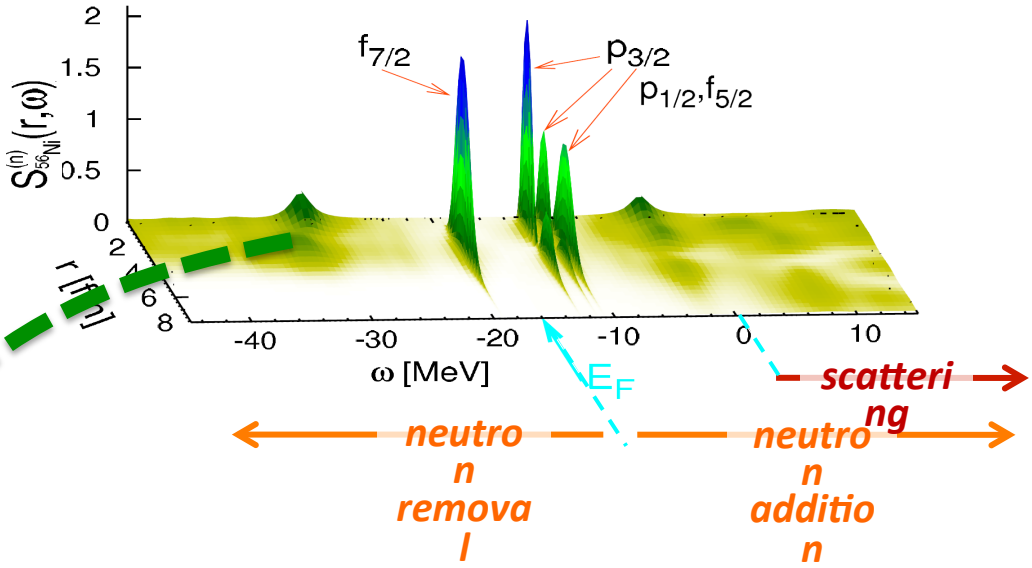
- Koltun sum rule (with NNN interactions):

$$\sum_\alpha \frac{1}{\pi}\int_{-\infty}^{\epsilon_F} d\omega\,\omega\,\mathrm{Im}\,G_{\alpha\alpha}(\omega) = \langle\Psi_0^N|\hat{T}|\Psi_0^N\rangle + 2\langle\Psi_0^N|\hat{V}|\Psi_0^N\rangle + 3\langle\Psi_0^N|\hat{W}|\Psi_0^N\rangle$$

two-body

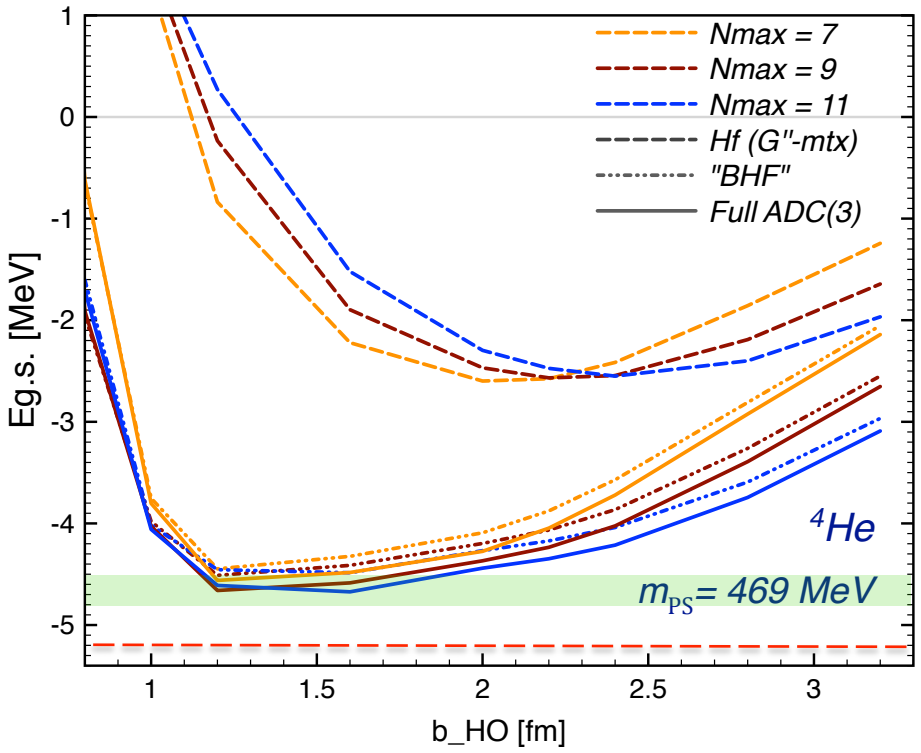
three-body

$$E_0^N = \frac{1}{2\pi}\int_{-\infty}^{\epsilon_F} \mathrm{d}\omega \sum_{\alpha\beta}(T_{\alpha\beta}+\omega\delta_{\alpha\beta})\mathrm{Im}\,G_{\beta\alpha}(\omega) - \frac{1}{2}\langle\Psi_0^N|\widehat{W}|\Psi_0^N\rangle$$



High-k and missing energy tail from SRC… (currently neglected in calculating Koltun SR)

Benchmark on ^{4}He



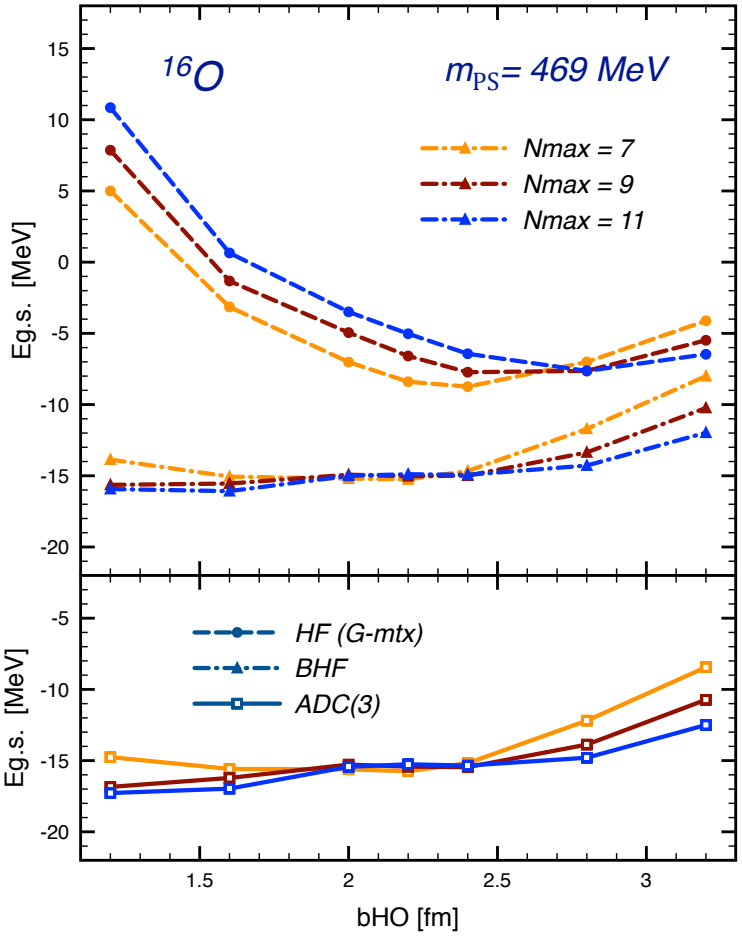
Can benchmark the Gmtx+ADC(3) method on light 4He, where exact solutions are possible:

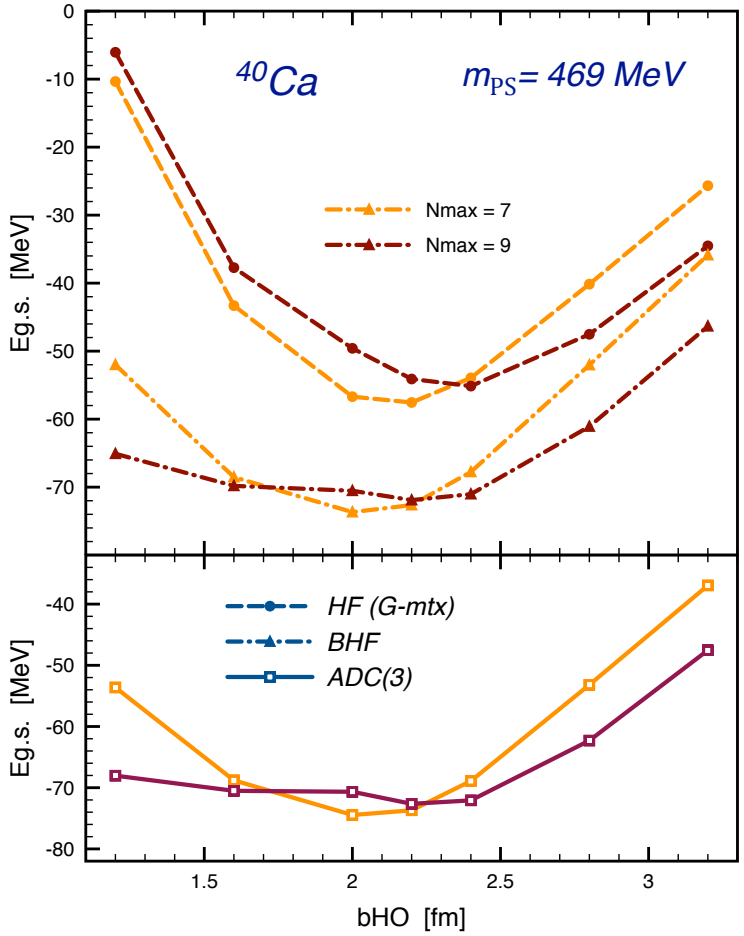
	G(ω) + ADC(3)	Exact
HALQCD @ m_π=469MeV	4.7(2) MeV	5.09 MeV[1]

➔ *Can expect accuracy on binding energies at about 10%*

[1]H. Nemura *et al.*, Int. J. Mod. Phys. E **23**, 1461006 (2014)

Binding of ^{16}O and ^{40}Ca:

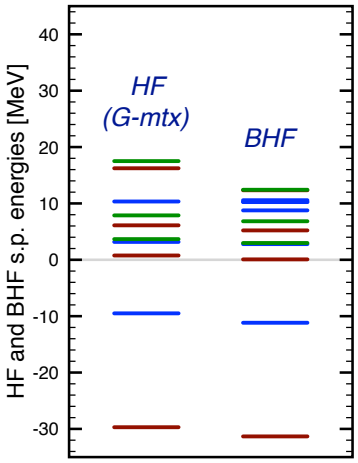


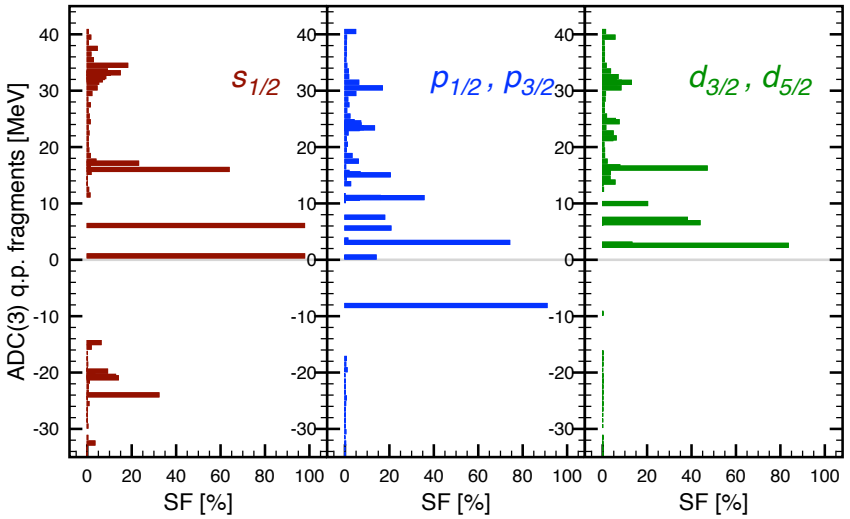


Binding energies are ~15 MeV ^{16}O and 70-75MeV for ^{40}Ca. Possibly being underestimated by 10%

➔ 16O at m_π= 469 MeV is unstable toward 4-α breakup!

Spectral strength in ^{16}O and ^{40}Ca:





Particle-hole gaps

^{16}O

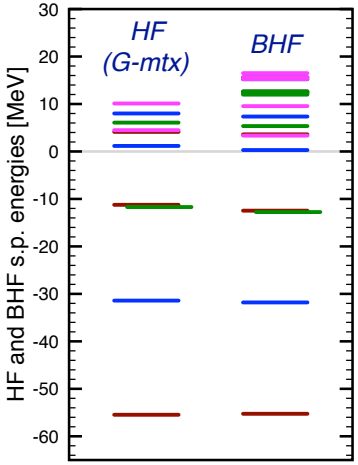
m_π= 469 MeV: ~8 MeV

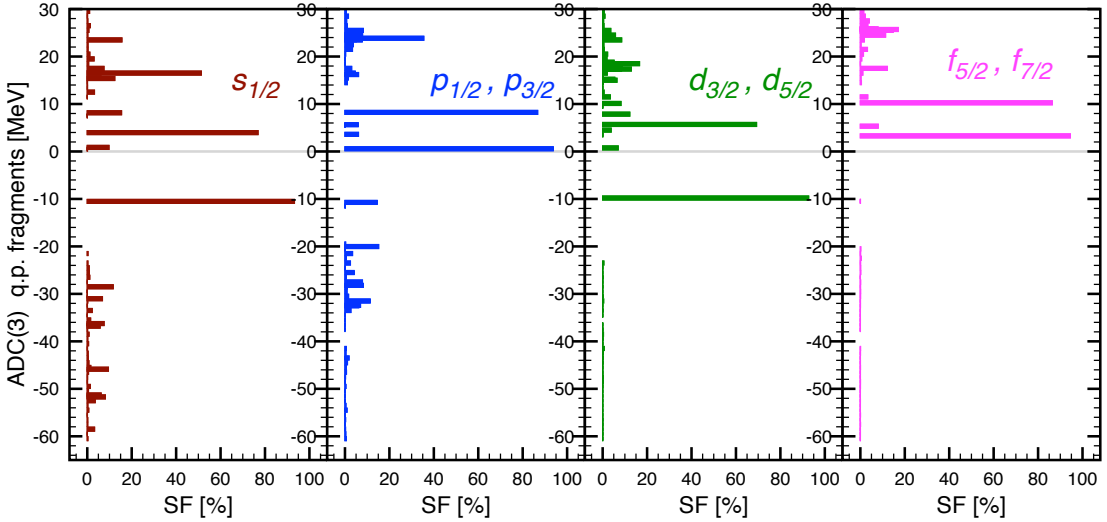
Expt (phys m_π): 11.5 MeV

^{40}Ca

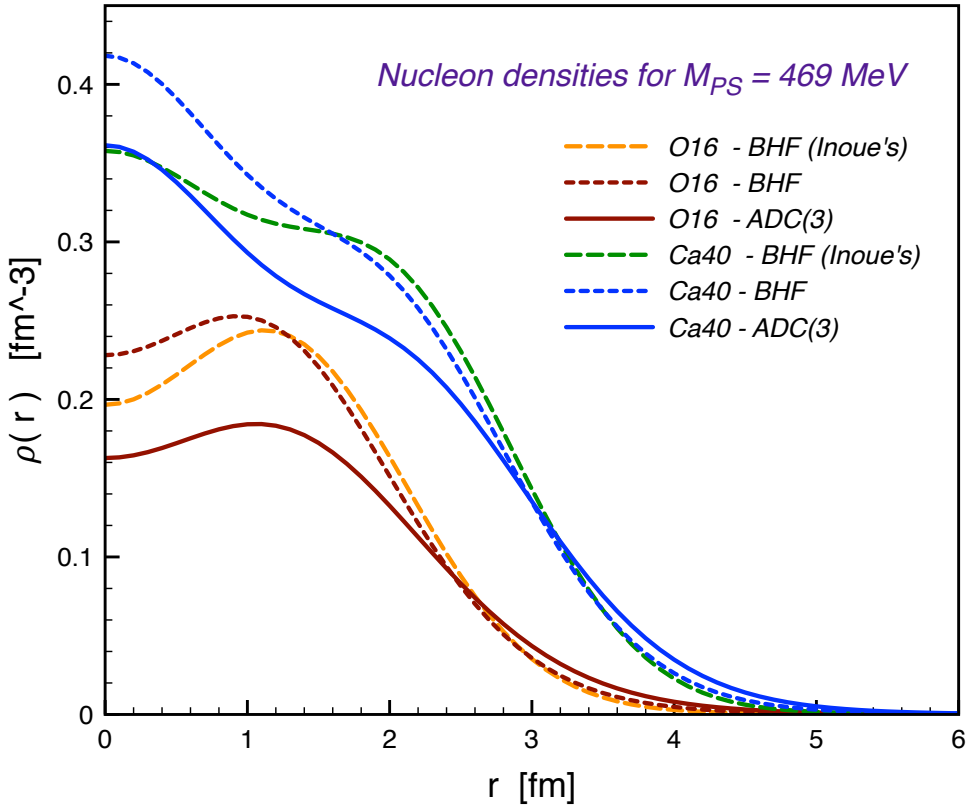
m_π= 469 MeV: ~10 MeV

Expt (phys m_π): 7.5 MeV





Matter distribution of ^{16}O and ^{40}Ca:



Calculated matter radii at m_π= 469 MeV are:

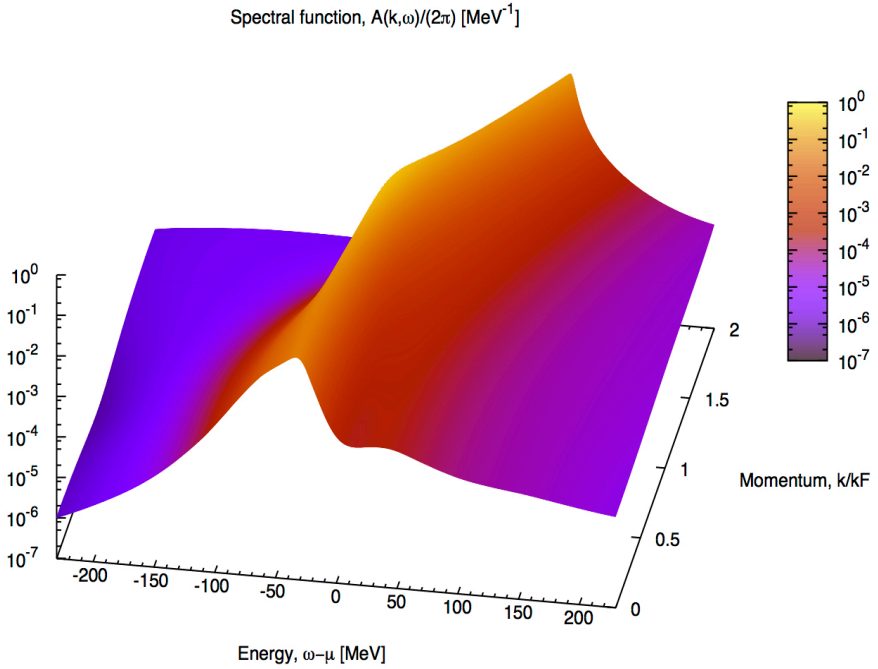
	^{16}O	^{40}Ca
BHF (Inoue)	2.35 fm	2.78 fm
"BHF"	2.33 fm	2.78 fm
ADC(3)	2.60 fm	2.97 fm
r_{charge} (expt.)	2.73 fm	3.48 fm

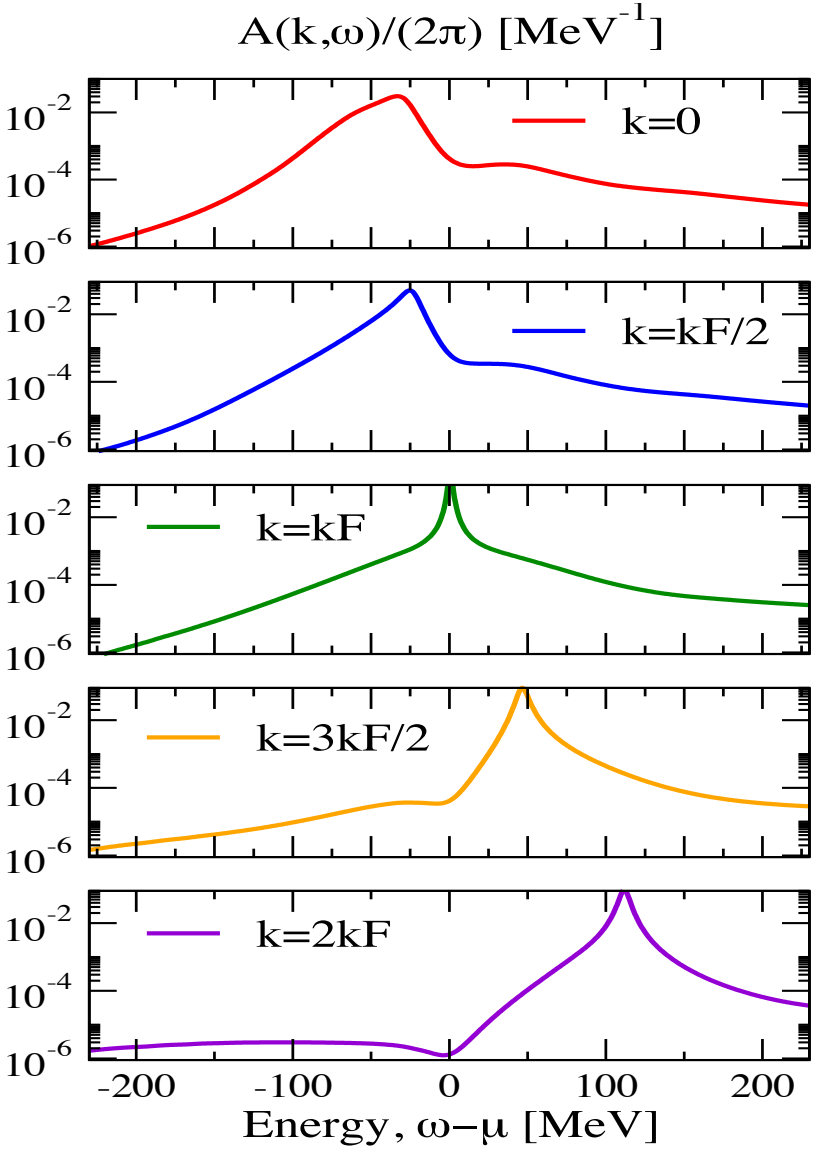
➔ Radii discrepancy worsens with increasing A

SCGF in infinite SNM @ m_π=469MeV

Single particle spectral distribution behaves as usual.

BHF results and binding remain confirmed in SCGF calculations.





Results by A. Carbone, priv. comm.

Summary

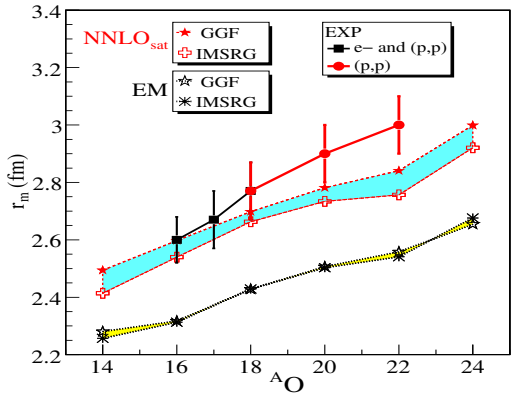
Thank you for your attention!!!

Mid-masses and chiral interactions:

- → *Leading order 3NF are crucial to predict many important features that are observed experimentally (drip lines, saturation, orbit evolution, etc…)*
- → *Experimental binding is predicted accurately up to the lower sd shell (A≈30) but deteriorates for medium mass isotopes (Ca and above) with roughly 1 MeV/A over binding.*
- → *New fits of chiral interaction are promising for low-energy observables*

HALQCD Nuclear forces:

- → *Strong short range behavior calls for new ideas in ab-initio many-body methods. Diagram resummation through G-matrix is starting point(to be extended)*



- → *At m_π=469MeV, closed shell 4He, 16O and 40Ca are bound. But oxygen is unstable toward 4-α break up, calcium stays bound. Underestimation of radii increases with A do to large saturation density (as for EM(500)+NLO3NF).*

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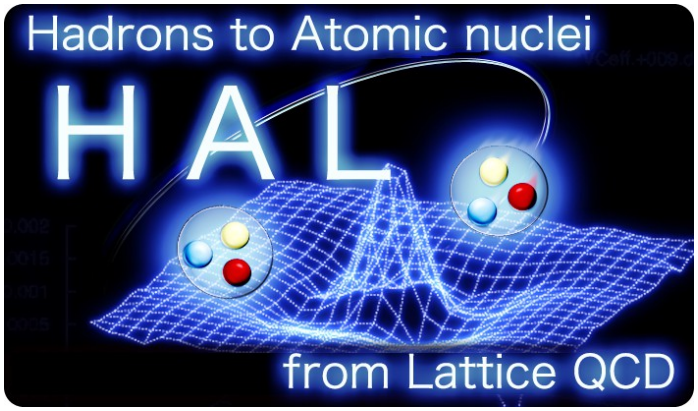

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H. Nemura, K. Sasaki	Univ. Tsukuba
F. Etminan	Univ. Birjand
T. Miyamoto,	Univ. Tsukuba
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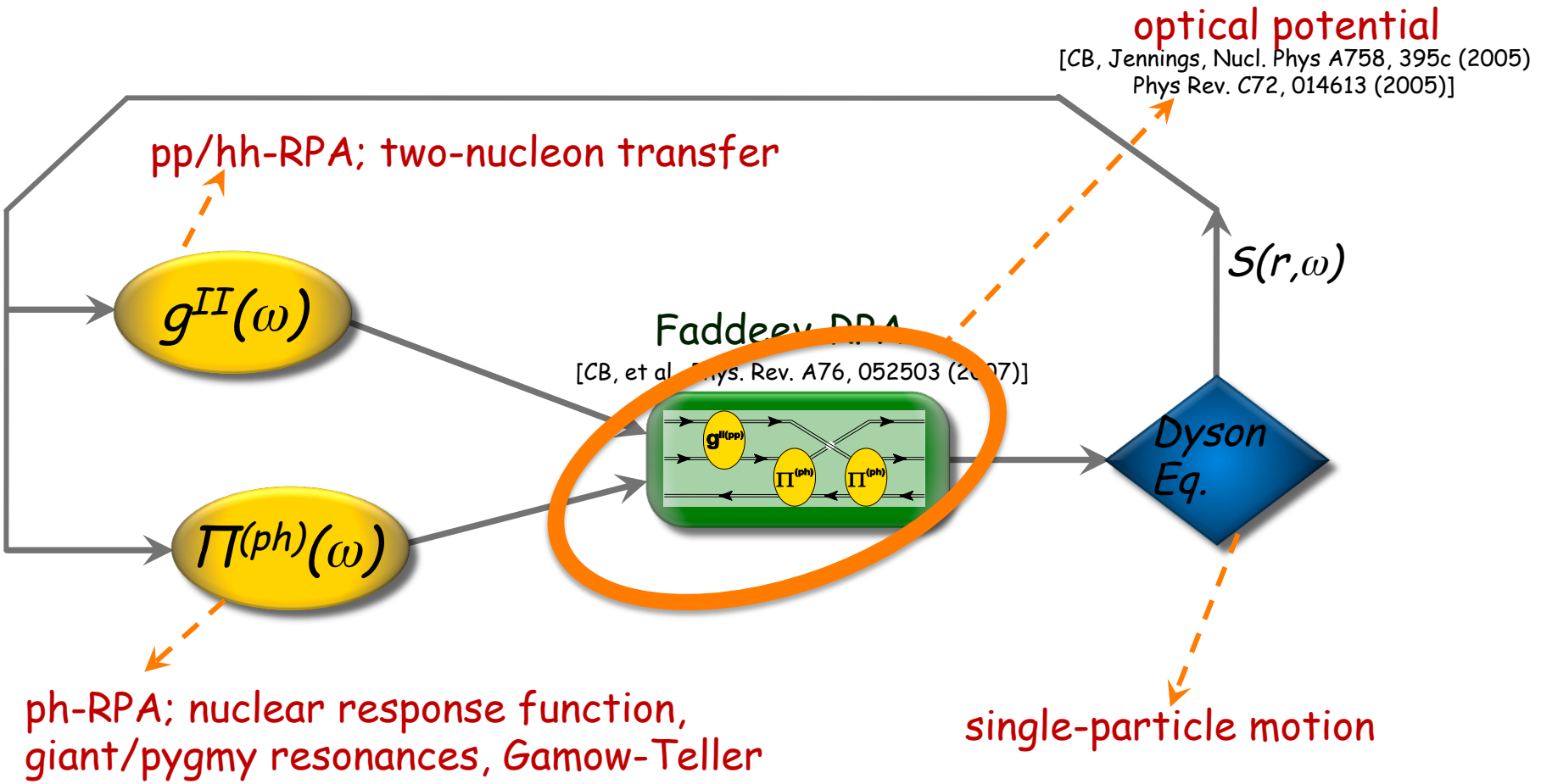
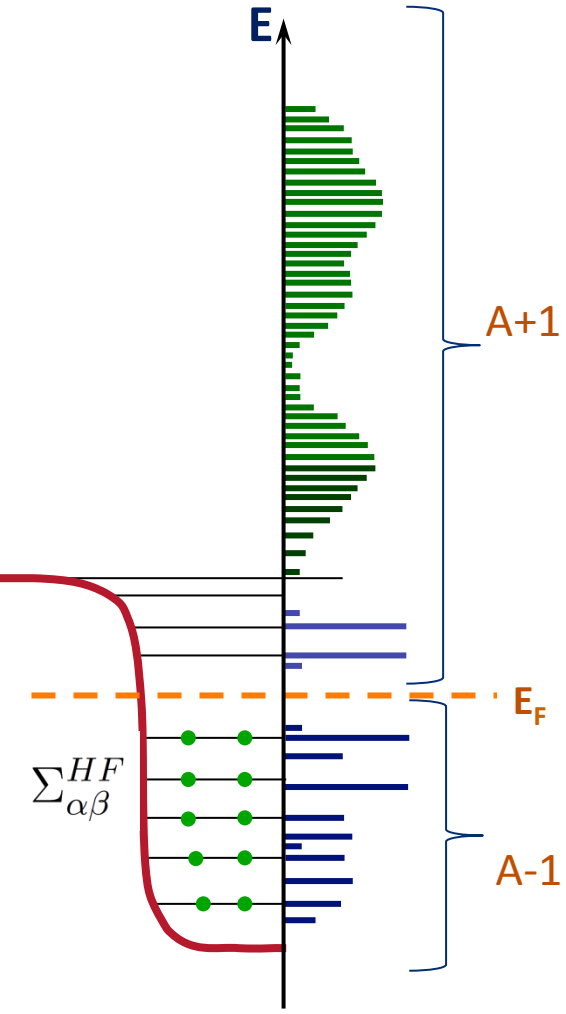
Optical Potentials Based on the Nuclear Self-energy

Self-Consistent Green's Function Approach



Nucleon elastic scattering



The irreducible self-energy is a nucleon-nucleus optical potential [see e.g. Mahaux and Sartor, Adv. Nucl. Phys. 20, (1991)]

$$\Sigma^{\star}(\mathbf{r},\mathbf{r}';\varepsilon) = \Sigma^{HF}_{\alpha\beta} - \frac{1}{\pi}\int_{\varepsilon_T^{>}}^{\infty} dE' \, \frac{Im\,\Sigma^{\star}(\mathbf{r},\mathbf{r}';E')}{\varepsilon - E' + i\eta} + \frac{1}{\pi}\int_{-\infty}^{\varepsilon_T^{<}} dE' \, \frac{Im\,\Sigma^{\star}(\mathbf{r},\mathbf{r}';E')}{\varepsilon - E' - i\eta}$$

mean-field

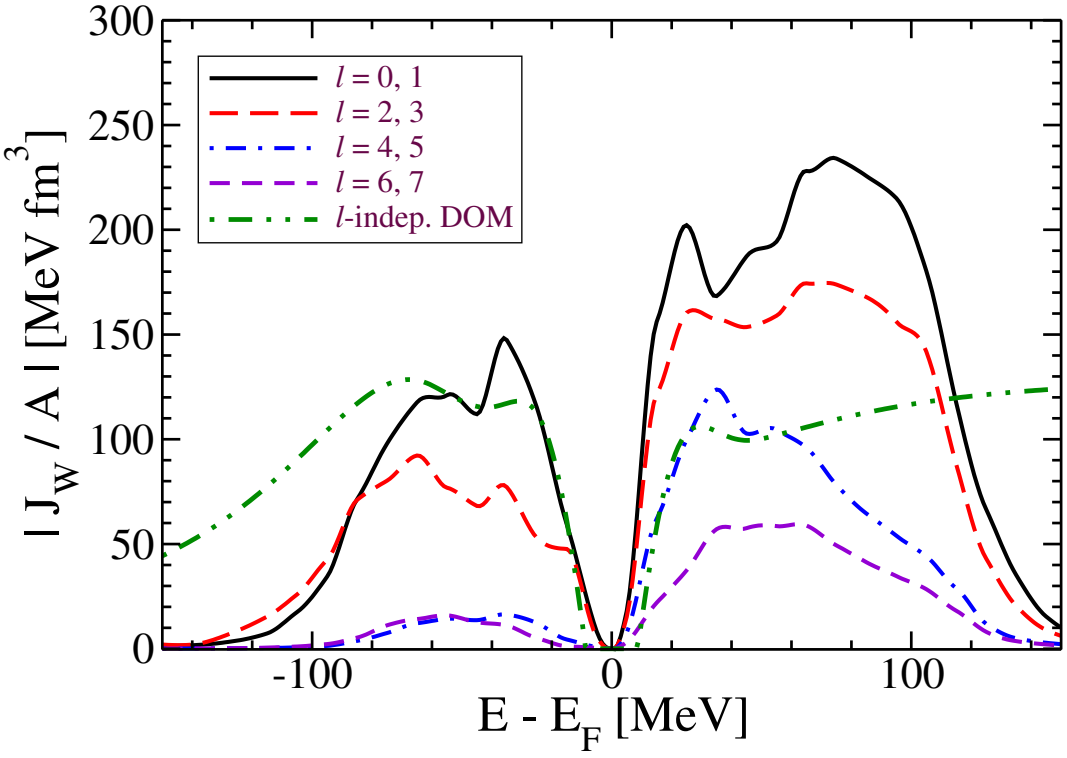
resonances beyond mean-field

➔ This provides *consistent* overlaps and scattering wave functions

Convergence of Ab-Initio Calculated Optical Potentials

- J_w: integral over the imaginary optical potential (overall absorption)

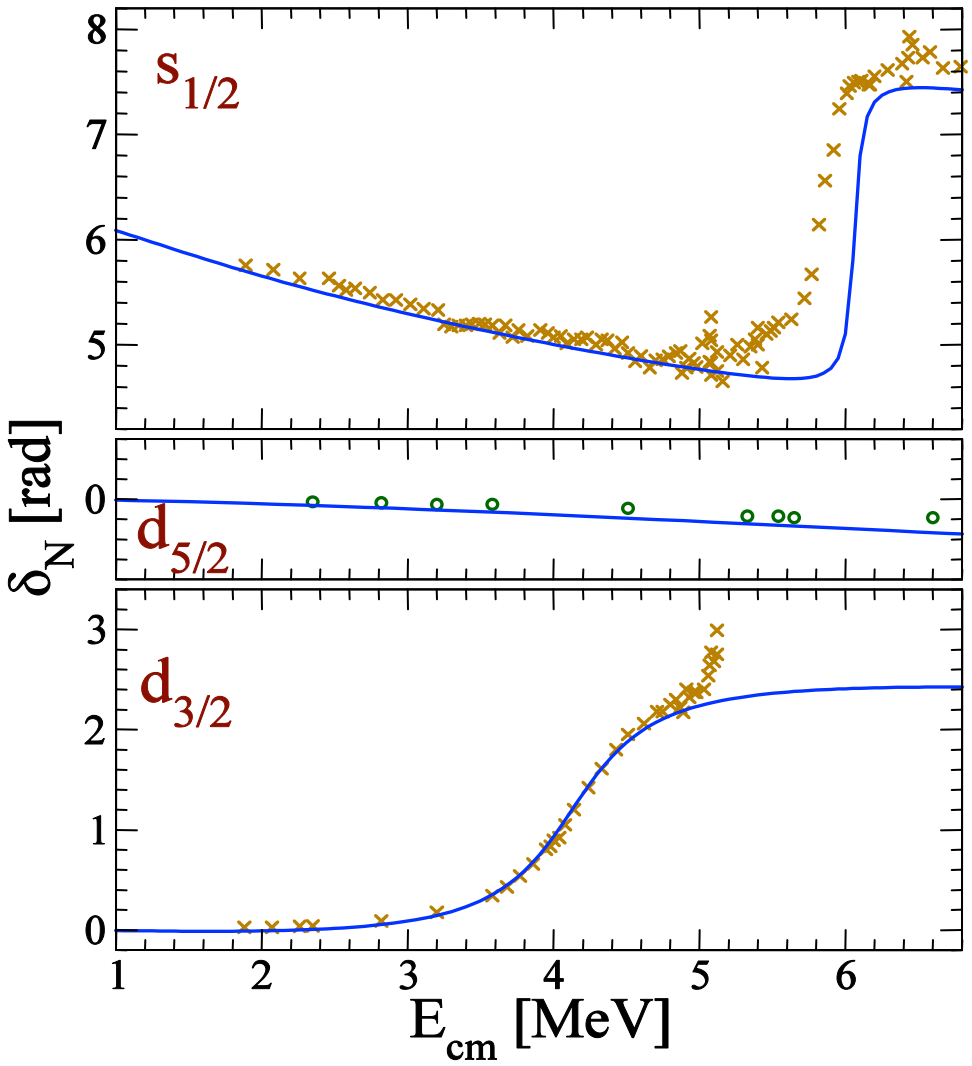
- angular momentum dependence (non locality) not negligible!
  → in particular below E_F



S. Waldecker, CB, W.Dickhoff – Phys. Rev. C84, 034616 (2011)

p-^{16}O phase shifts – positive parity waves

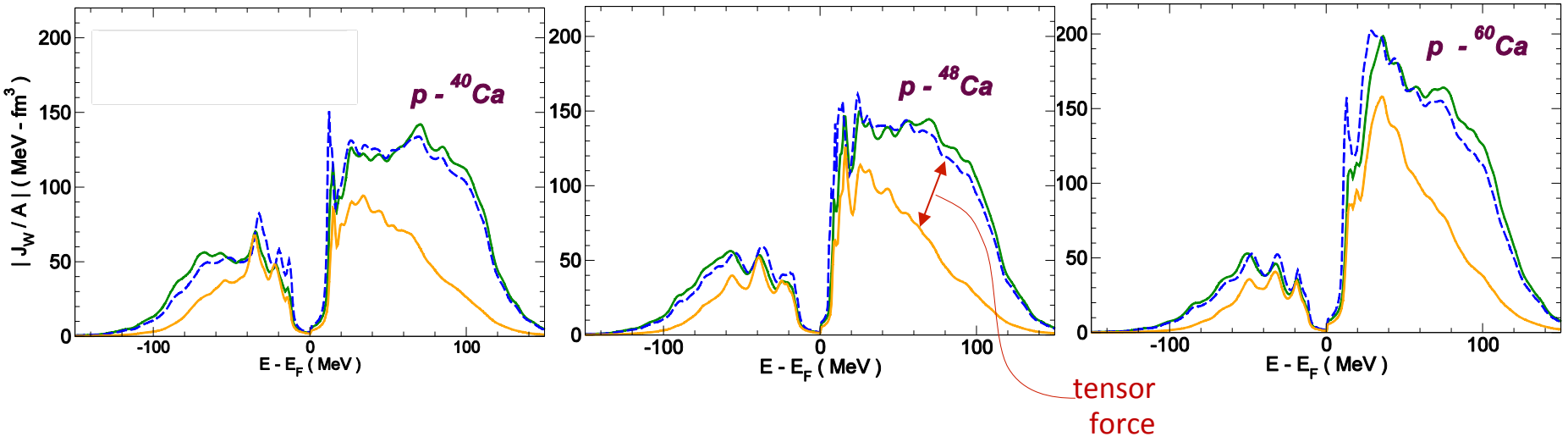
[C.B., B.Jennings,
Phys. Rev. C**72**, 014613 (2005)]



- AV18 interaction
- The phase shift are in agreement with the experiment!
- BUT does not reproduce phase shifts and bound state energies at the same time
→ need for improved H / 3NF
- Non-MF resonances "OK"

Microscopic Optical Potential from FRPA

- absorption away from E_F is enhanced by the tensor force
- little effects from charge exchange (e.g. p-^{48}Ca <-> n-^{48}Sc)



J_W: integral over the imaginary opt. pot (overall absorption)

Full FRPA result (w/ av18)
Charge-exchange d.o.f. suppressed
Tensor force suppressed

S. Waldecker, CB, W.Dickhoff – Phys. Rev. C84, 034616 (2011)

UNIVERSITY OF SURREY

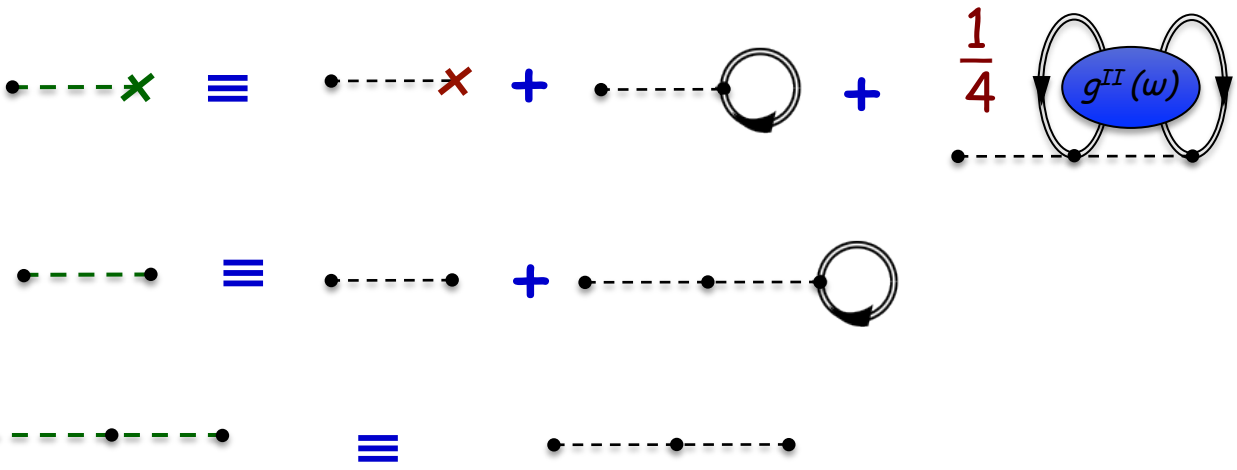
Adding 3-nucleon forces

Inclusion of NNN forces

A. Carbone, CB, et al., Phys. Rev. C**88**, 054326 (2013)

- NNN forces can enter diagrams in three different ways:

→ Define new 1- and 2-body interactions and use only interaction-irreducible diagrams

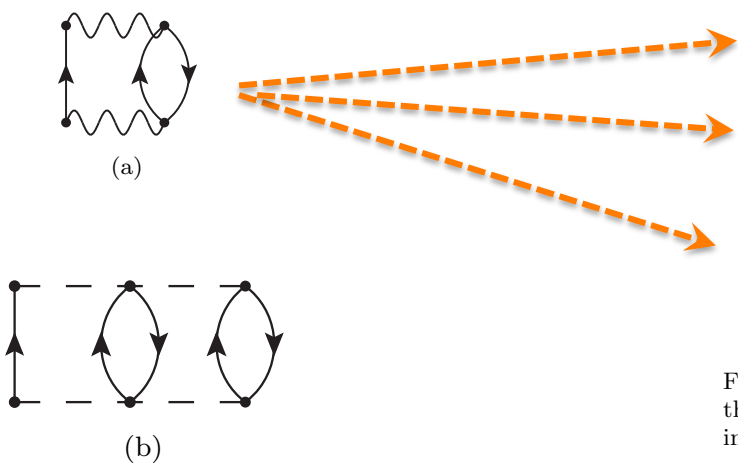


- Contractions are with *fully correlated density matrices* (BEYOND a normal ordering…)

Inclusion of NNN forces

A. Carbone, CB, et al., Phys. Rev. C**88**, 054326 (2013)

- Second order PT diagrams with 3BFs:



effectively:

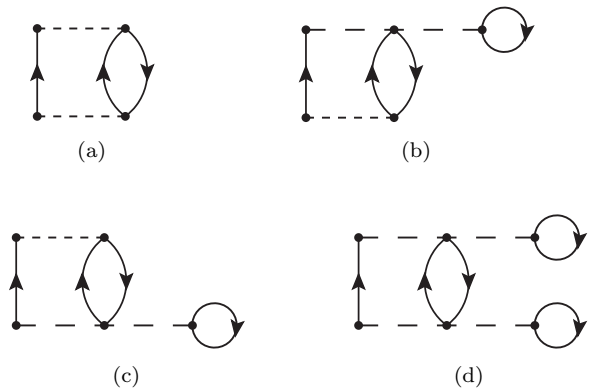


FIG. 4. The one *interaction irreducible* diagrams (a) and the three *interaction reducible* ones (b, c and d) that are contained in Fig. 3a.

Inclusion of NNN forces

A. Carbone, CB, et al., Phys. Rev. C**88**, 054326 (2013)

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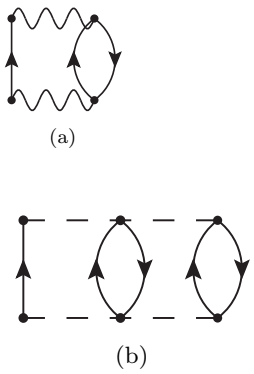

- *Third order* PT diagrams with 3BFs:

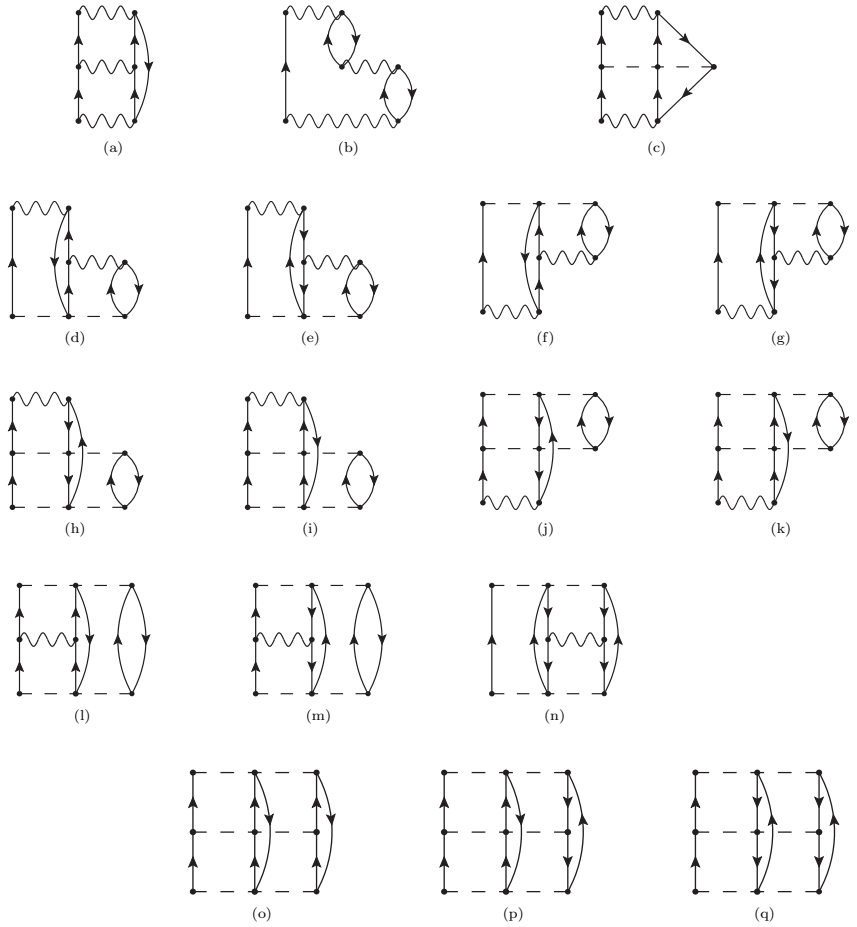

FIG. 5. 1PI, *skeleton* and *interaction irreducible* self-energy diagrams appearing at 3^{rd}-order in perturbative expansion (7), making use of the effective hamiltonian of Eq. (9).

➔ Use if effective interactions

➔ Need to correct the Koltun sum rule (for energy)

Inclusion of NNN forces

A. Carbone, CB, et al., Phys. Rev. C**88**, 054326 (2013)

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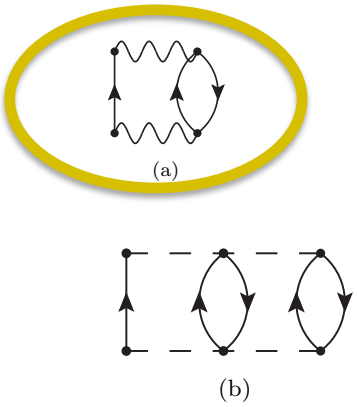

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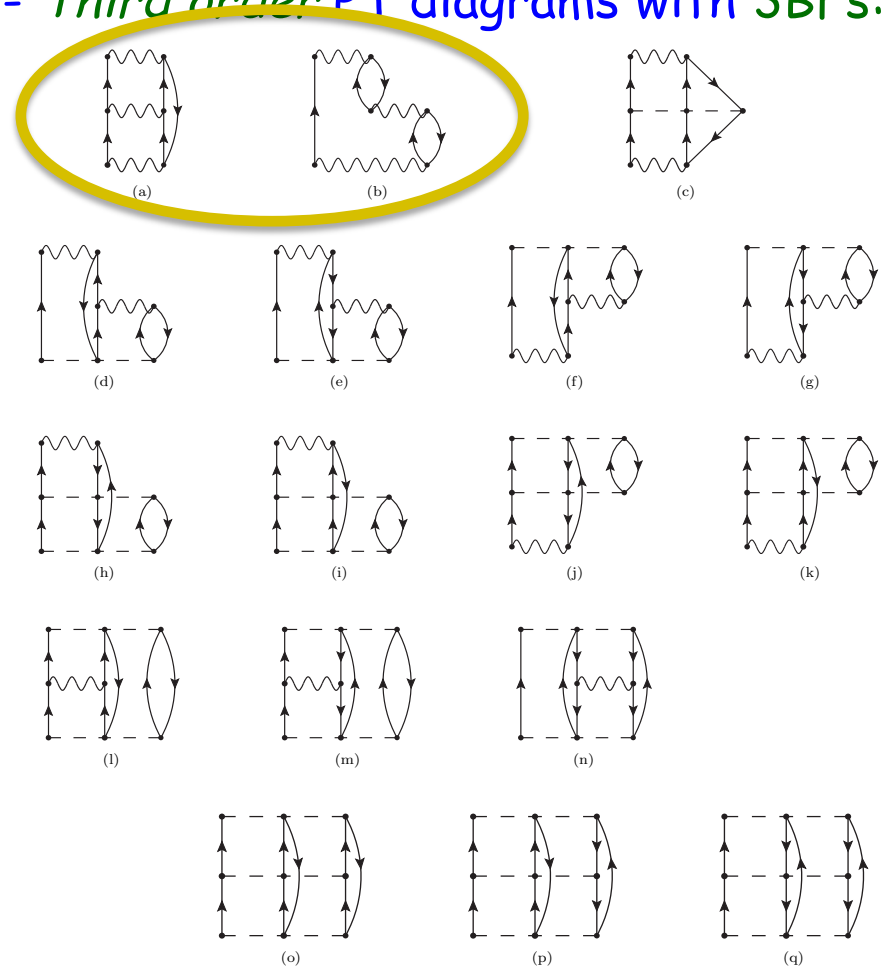

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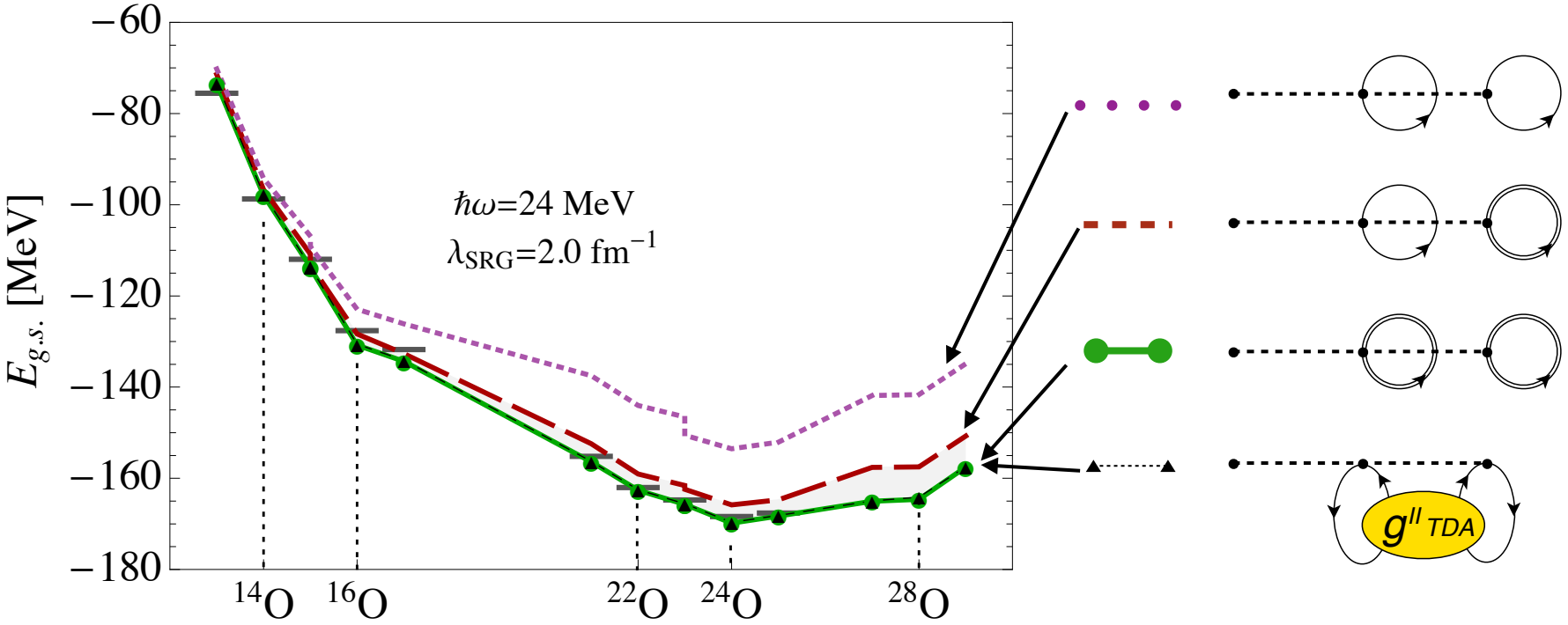
3N forces in FRPA/FTDA formalism

→ Ladder contributions to static self-energy are negligible (in oxygen)



CB, *arXiv:1405.3002v2 [nucl-th]* (2014)

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