Overview of the HAL QCD potential method and recent results

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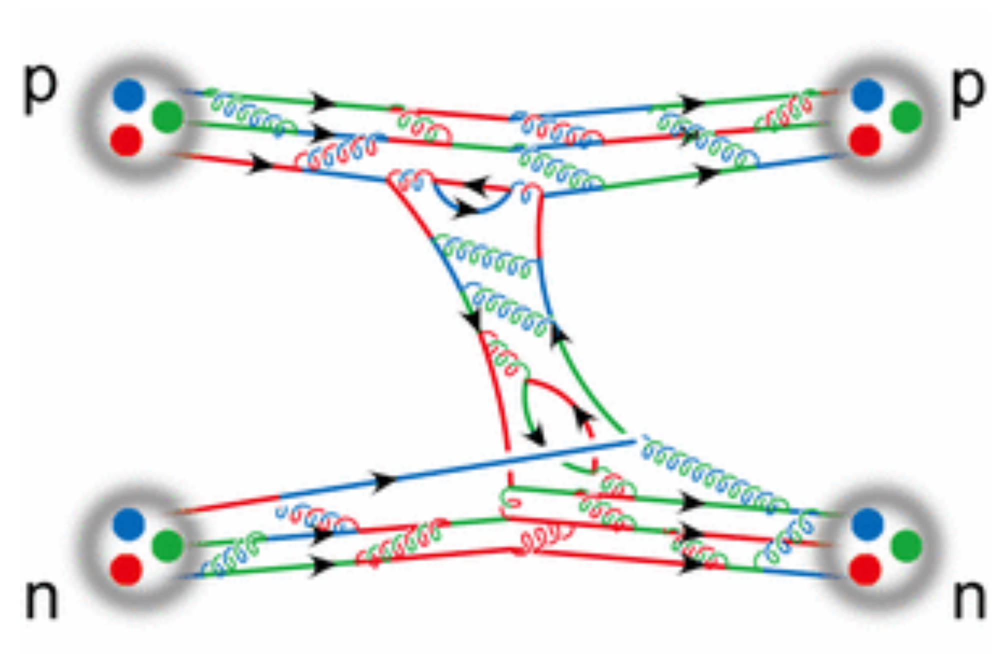
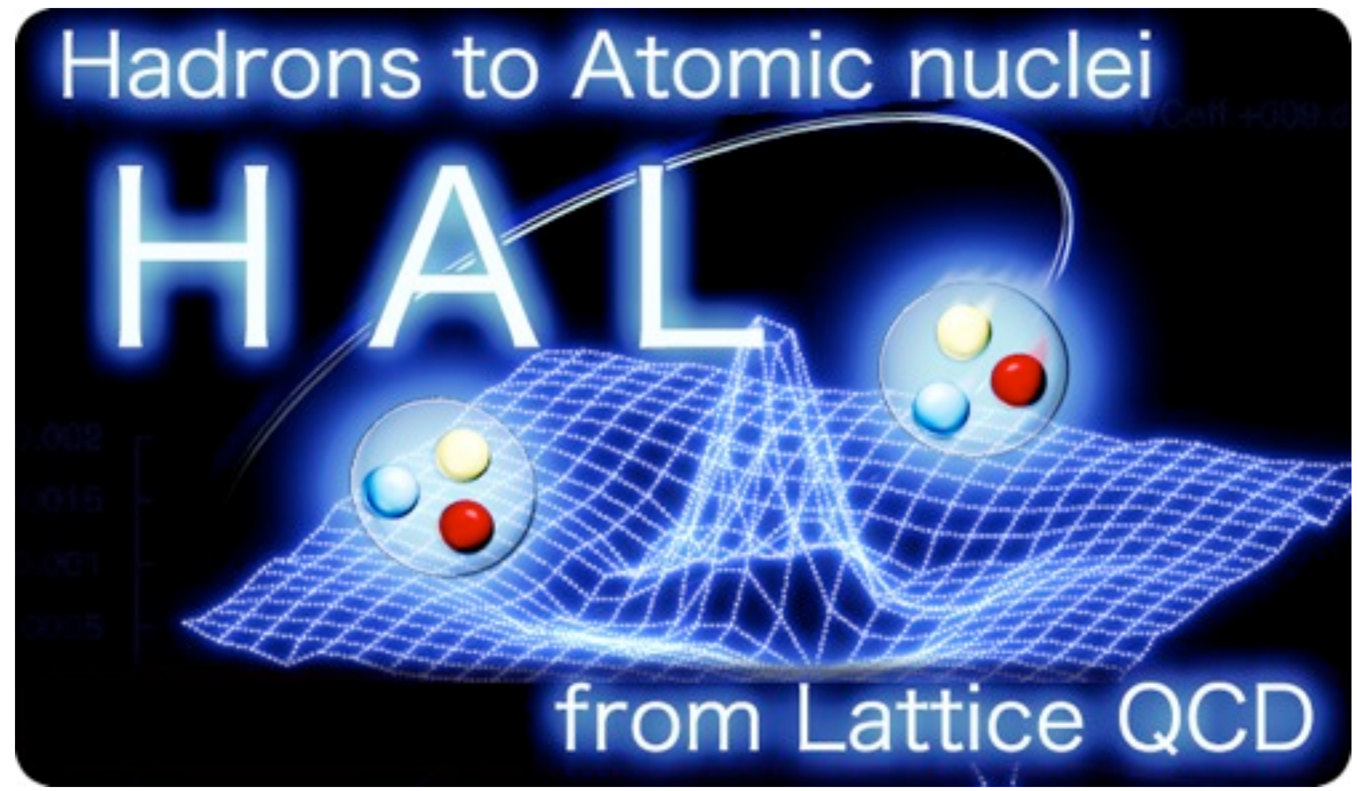


INT Program INT-16-1

Nuclear Physics from Lattice QCD

March 21 - May 27, 2016, INT, University of Washington, Seattle, USA

For
HAL QCD Collaboration



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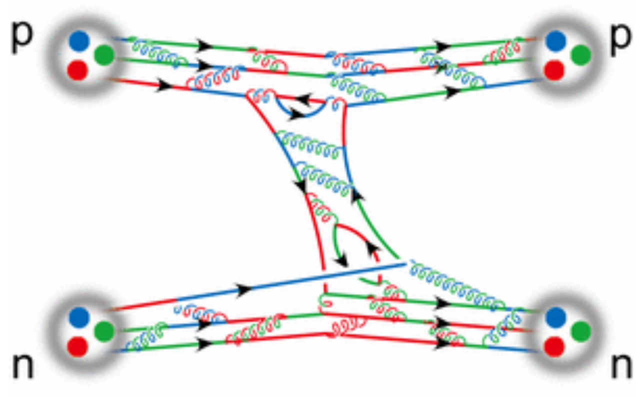
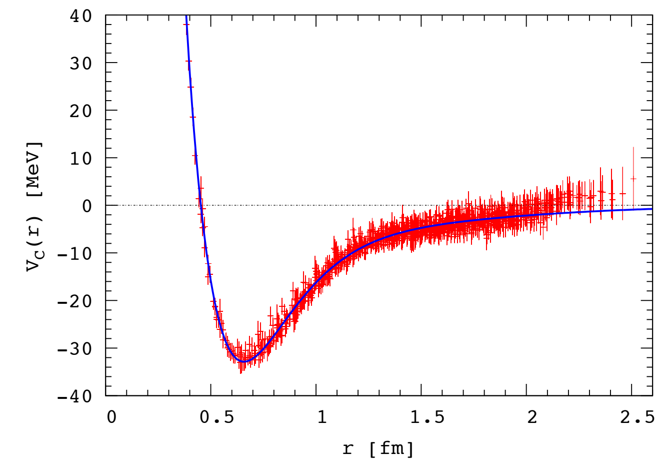
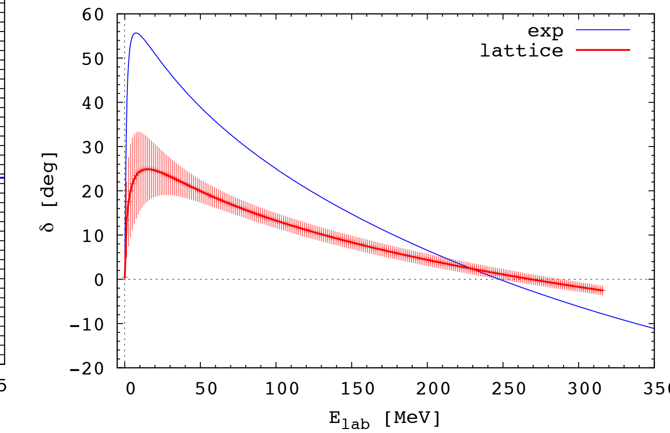
Our strategy to Nuclear/Astro physics from QCD

Potentials from lattice QCD

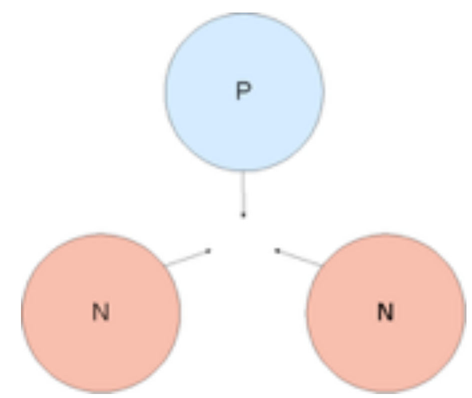




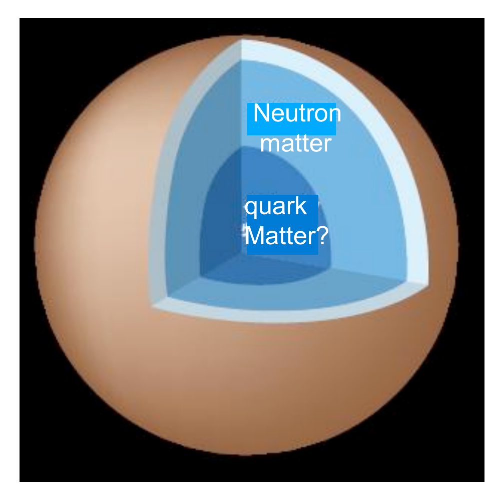


Nuclear Physics
with these potentials

parameters of EFT



Neutron stars
Supernova explosion



1. HAL QCD method

--Overview--

HAL QCD strategy

$NN \to NN$ ~~$NN \to NN + \text{others}$~~ Aoki, Hatsuda & Ishii, PTP123(2010)89.

Define a non-local but energy-independent potential below inelastic threshold in QCD

$$[\epsilon_k - H_0]\,\varphi_{\mathbf{k}}(\mathbf{x}) = \int d^3y\, \underline{U(\mathbf{x},\mathbf{y})}\varphi_{\mathbf{k}}(\mathbf{y})$$

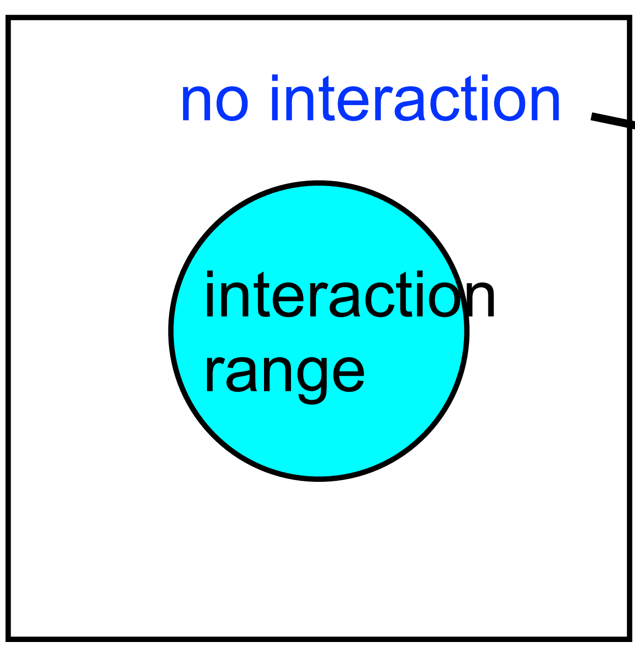
$$\epsilon_k = \frac{\mathbf{k}^2}{2\mu} \qquad H_0 = \frac{-\nabla^2}{2\mu}$$

general $U_{\mathbf{k}}(\mathbf{x},\mathbf{y}) \to U(\mathbf{x},\mathbf{y})$ ⟺ $V_{\mathbf{k}}(\mathbf{x})$

from Nambu-Bethe-Salpeter (NBS) wave function

energy $W_k = 2\sqrt{\mathbf{k}^2 + m_N^2}$

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0|N(\mathbf{x}+\mathbf{r},0)N(\mathbf{x},0)|NN, W_k\rangle$$



$r = |\mathbf{r}| \to \infty$

$$\varphi_{\mathbf{k}}(\mathbf{r}) \simeq \sum_{l,m} C_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr} Y_{ml}(\Omega_{\mathbf{r}})$$

$\delta_l(k)$ scattering phase shift (phase of the S-matrix by unitarity) in QCD.

⇒ potential $U(x,y)$ is faithful to QCD phase shift $\delta_l(k)$.

A non-local but energy-independent potential exists.

Proof

$$U(\mathbf{x},\mathbf{y}) = \sum_{\mathbf{k},\mathbf{k}'}^{W_k, W_{k'} \le W_{\rm th}} \left[\epsilon_k - H_0\right] \varphi_{\mathbf{k}}(\mathbf{x}) \eta^{-1}_{\mathbf{k},\mathbf{k}'} \varphi^\dagger_{\mathbf{k}'}(\mathbf{y})$$

inner product

$\eta^{-1}_{\mathbf{k},\mathbf{k}'}$: inverse of $\eta_{\mathbf{k},\mathbf{k}'} = (\varphi_{\mathbf{k}}, \varphi_{\mathbf{k}'})$

For $^\forall W_{\mathbf{p}} < W_{\rm th} = 2m_N + m_\pi$ (threshold energy)

$$\int d^3y\, U(\mathbf{x},\mathbf{y}) \phi_{\mathbf{p}}(\mathbf{y}) = \sum_{\mathbf{k},\mathbf{k}'} \left[\epsilon_k - H_0\right] \varphi_{\mathbf{k}}(x) \eta^{-1}_{\mathbf{k},\mathbf{k}'} \eta_{\mathbf{k}',\mathbf{p}} = \left[\epsilon_p - H_0\right] \varphi_{\mathbf{p}}(x)$$

Derivative (velocity) expansion

$$U(\mathbf{x},\mathbf{y}) = V(\mathbf{x},\nabla)\delta^3(\mathbf{x}-\mathbf{y})$$

$$V(\mathbf{x},\nabla) = V_0(r) + V_\sigma(r)(\boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2}) + V_T(r)S_{12} + V_{\rm LS}(r)\mathbf{L}\cdot\mathbf{S} + O(\nabla^2)$$

LO LO LO NLO NNLO

tensor operator $S_{12} = \frac{3}{r^2}(\boldsymbol{\sigma_1}\cdot\mathbf{x})(\boldsymbol{\sigma_2}\cdot\mathbf{x}) - (\boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2})$

spins

At LO, for example, we simply have

$$V_{\rm LO}(\mathbf{x}) = \frac{[\epsilon_k - H_0]\varphi_{\mathbf{k}}(\mathbf{x})}{\varphi_{\mathbf{k}}(\mathbf{x})}$$

→ phase shifts and binding energy below inelastic threshold

Note truncation of the derivative expansion introduces some systematics, which fortunately can be estimated explicitly.

Advantages

- milder finite volume corrections

- extension to coupled channel problems is easy. Inelastic scattering can be treated.

$$A + B \to C + D$$

- extensions to 3-body potential /particle production are possible with non-relativistic approximation, though numerically demanding.

$$A + B \to A + B + C$$

- ground state saturation is not required to extract the potential. (See later.)

Disadvantages

- higher numerical cost than the standard method in both Flops and Bytes.

2. Previous results

Extraction of potentials

Standard method

NBS wave function

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0|N(\mathbf{x}+\mathbf{r},0)N(\mathbf{x},0)|NN,W_k\rangle$$

Potential

$$[\epsilon_k - H_0]\varphi_{\mathbf{k}}(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_{\mathbf{k}}(\mathbf{y})$$

4-pt Correlation function

source for NN

$$F(\mathbf{r},t-t_0) = \langle 0|T\{N(\mathbf{x}+\mathbf{r},t)N(\mathbf{x},t)\}\overline{\mathcal{J}}(t_0)|0\rangle$$

complete set for NN

$$\begin{aligned} F(\mathbf{r},t-t_0) &= \langle 0|T\{N(\mathbf{x}+\mathbf{r},t)N(\mathbf{x},t)\}\sum_{n,s_1,s_2}|2N,W_n,s_1,s_2\rangle\langle 2N,W_n,s_1,s_2|\overline{\mathcal{J}}(t_0)|0\rangle + \cdots \\ &= \sum_{n,s_1,s_2} A_{n,s_1,s_2}\varphi^{W_n}(\mathbf{r})e^{-W_n(t-t_0)}, \quad A_{n,s_1,s_2} = \langle 2N,W_n,s_1,s_2|\overline{\mathcal{J}}(0)|0\rangle. \end{aligned}$$

ground state saturation at large t

$$\lim_{(t-t_0)\to\infty} F(\mathbf{r},t-t_0) = A_0\varphi^{W_0}(\mathbf{r})e^{-W_0(t-t_0)} + O(e^{-W_{n\neq 0}(t-t_0)})$$

NBS wave function

This is a standard method in lattice QCD and was employed for our first calculation.

Improved method (time-dependent method)

Ishii et al. (HALQCD), PLB712(2012) 437

normalized 4-pt function

$$R(\mathbf{r},t) \equiv F(\mathbf{r},t)/(e^{-m_N t})^2 = \sum_n A_n \varphi^{W_n}(\mathbf{r}) e^{-\Delta W_n t}$$

$$\Delta W_n = W_n - 2m_N = \frac{\mathbf{k}_n^2}{m_N} - \frac{(\Delta W_n)^2}{4m_N}$$

$$-\frac{\partial}{\partial t} R(\mathbf{r},t) = \left\{ H_0 + U - \frac{1}{4m_N}\frac{\partial^2}{\partial t^2} \right\} R(\mathbf{r},t)$$

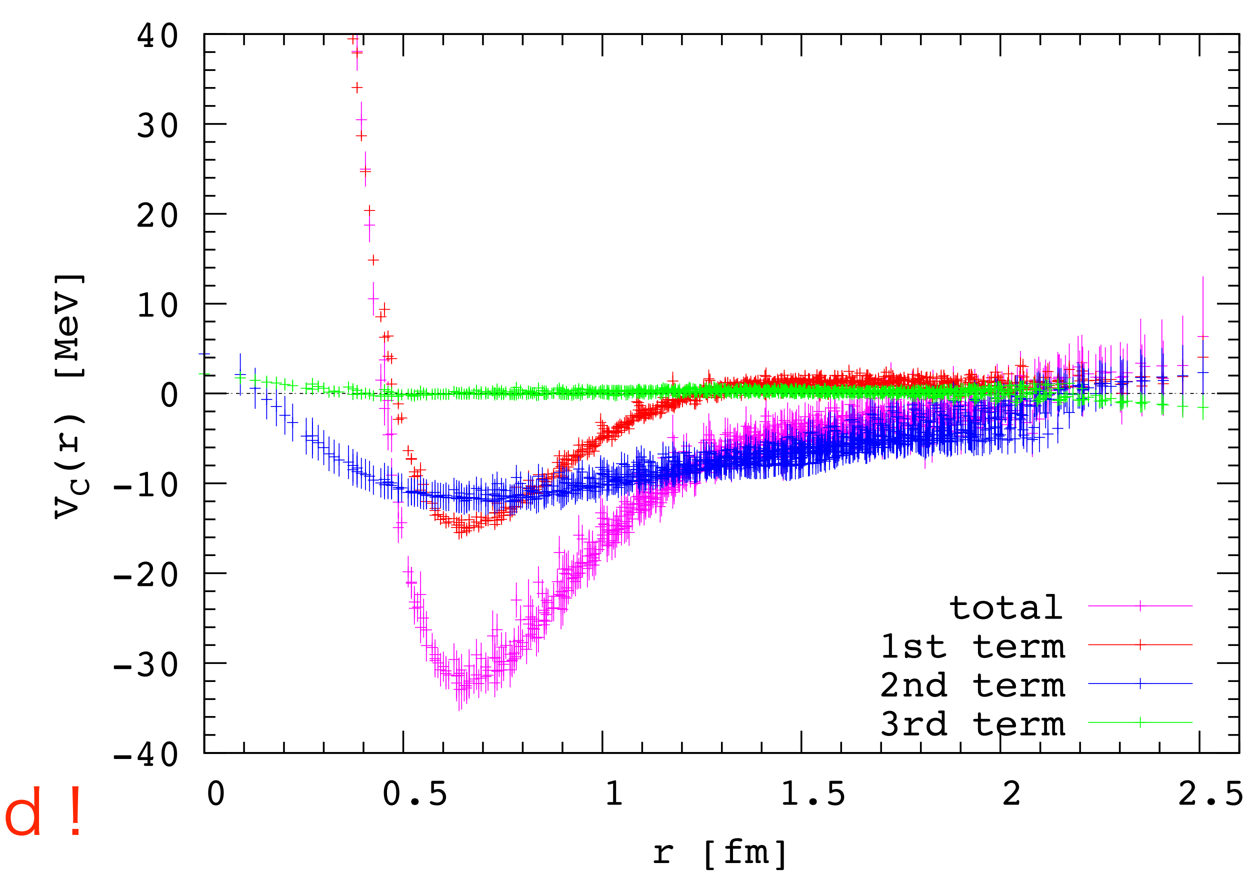
potential

Leading Order

$$\left\{ -H_0 - \frac{\partial}{\partial t} + \frac{1}{4m_N}\frac{\partial^2}{\partial t^2} \right\} R(\mathbf{r},t) = \int d^3r'\, U(\mathbf{r},\mathbf{r}') R(\mathbf{r}',t) = V_C(\mathbf{r}) R(\mathbf{r},t) + \cdots$$

1st 2nd 3rd total

3rd term(relativistic correction) is negligible.

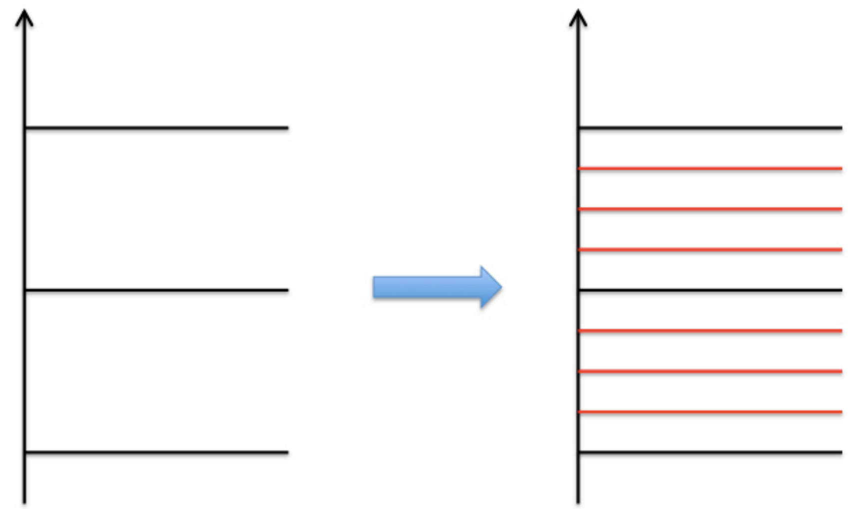


Ground state saturation is no more required !
(advantage over the finite volume method.)

Remarks

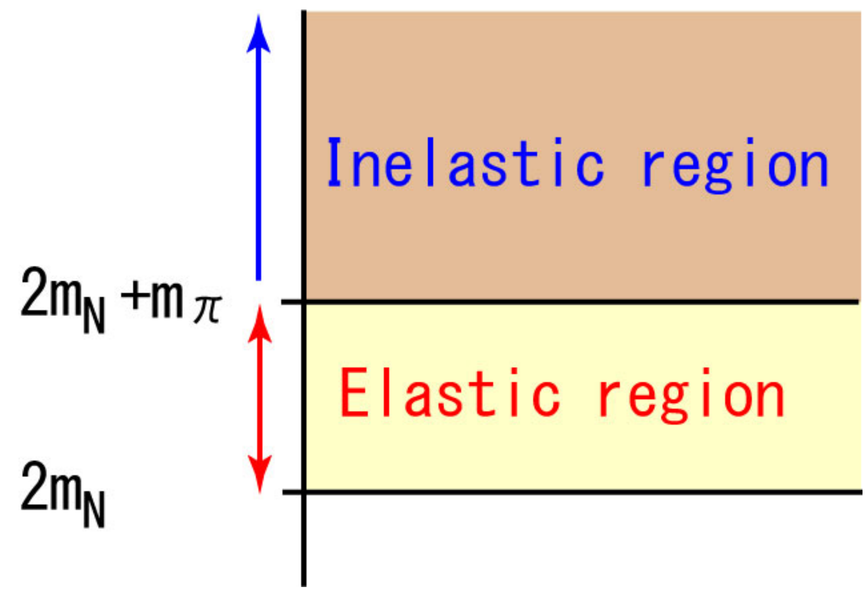
excited state contributions become bigger
in the larger volume

$$\Delta E \propto \frac{1}{L^2}$$



time-dependent HAL QCD method makes
this difficulty milder

$$\Delta E \simeq m_\pi$$



remaining t-dependence of the potential

1. Inelastic contributions (including excited states of one baryon)

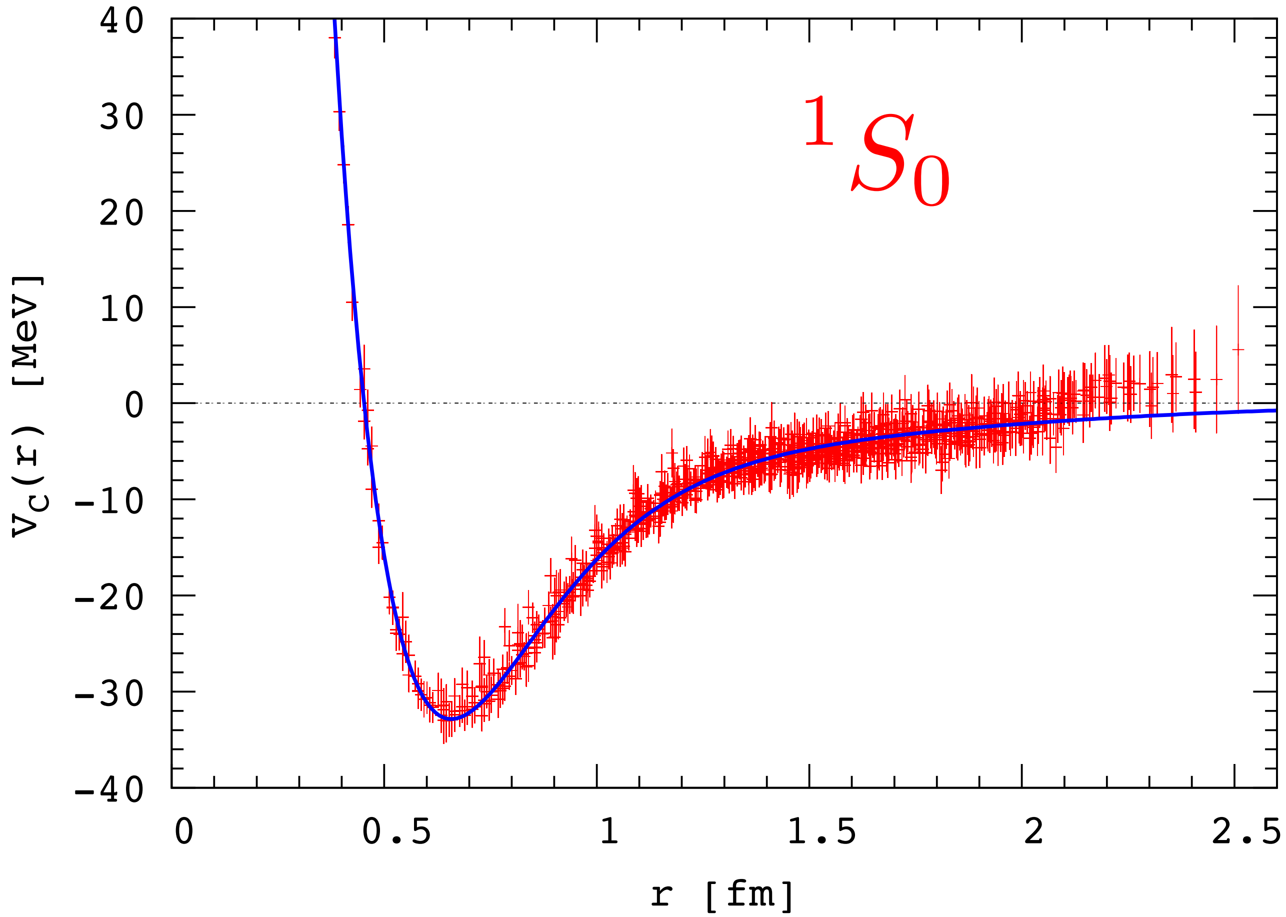
$$R(\mathbf{r},t) = F(\mathbf{r},t)/G_N(t)^2$$

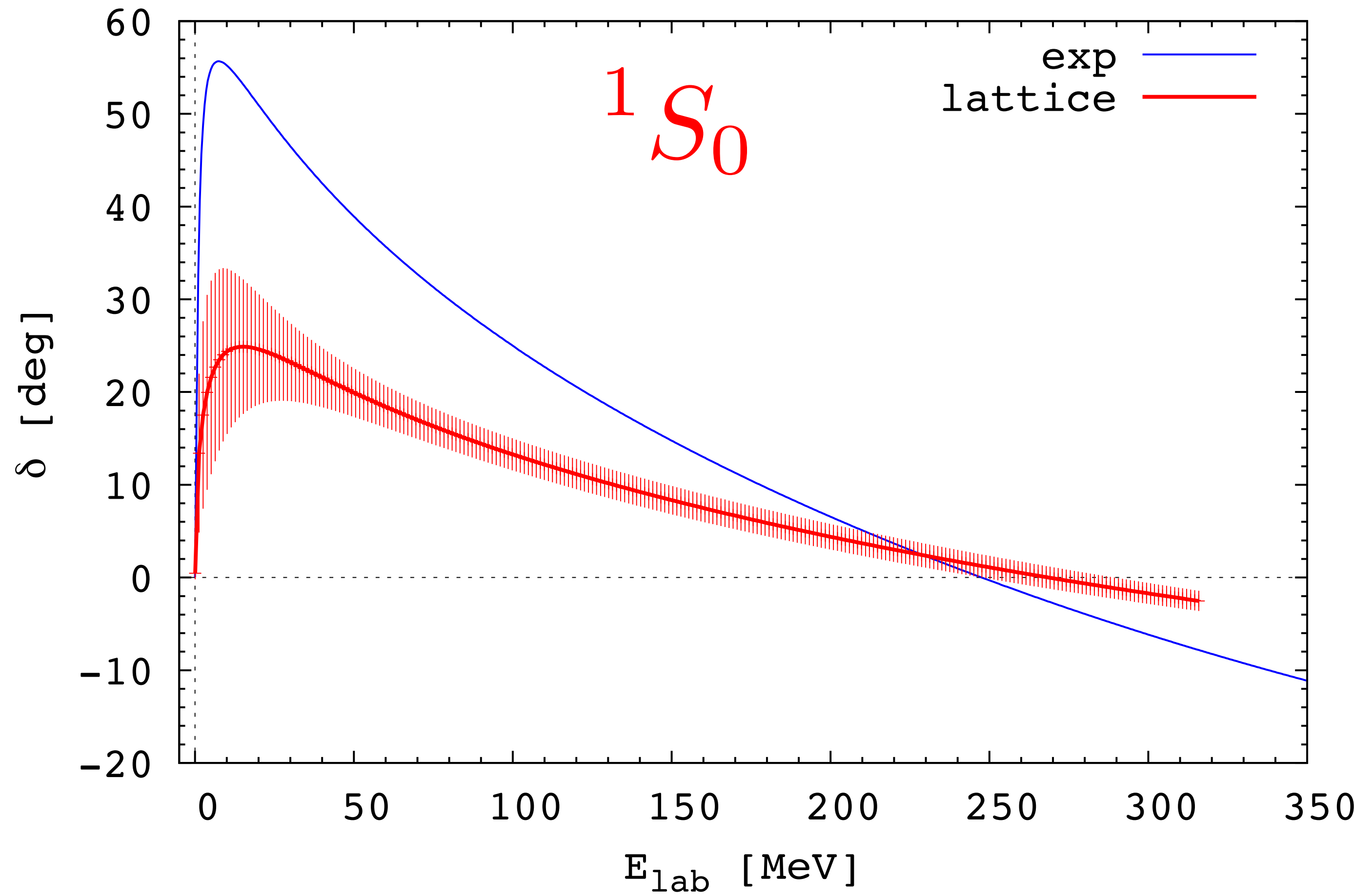
2. Higher order terms in the derivative expansion

2+1 flavor QCD a=0.09fm, L=2.9fm $m_\pi \simeq 700$ MeV

Ishii *et al.* (HALQCD), PLB712(2012) 437.

NN potential → **phase shift**





Qualitative features of NN potential are reproduced.

→ It has a reasonable shape. The strength is weaker due to the heavier quark mass.

Need calculations at physical quark mass on "K" computer.

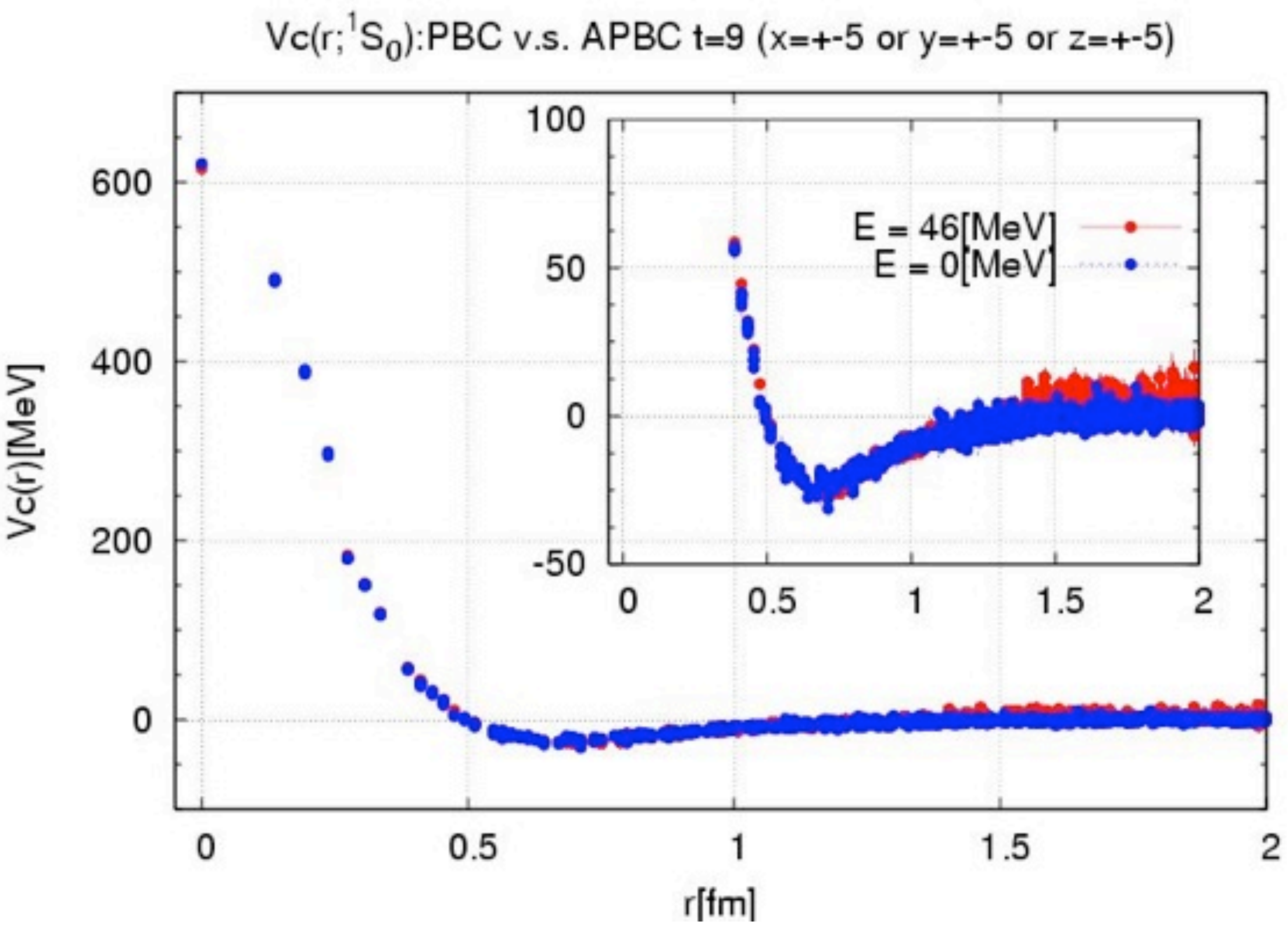
→ **Doi's Talk.**

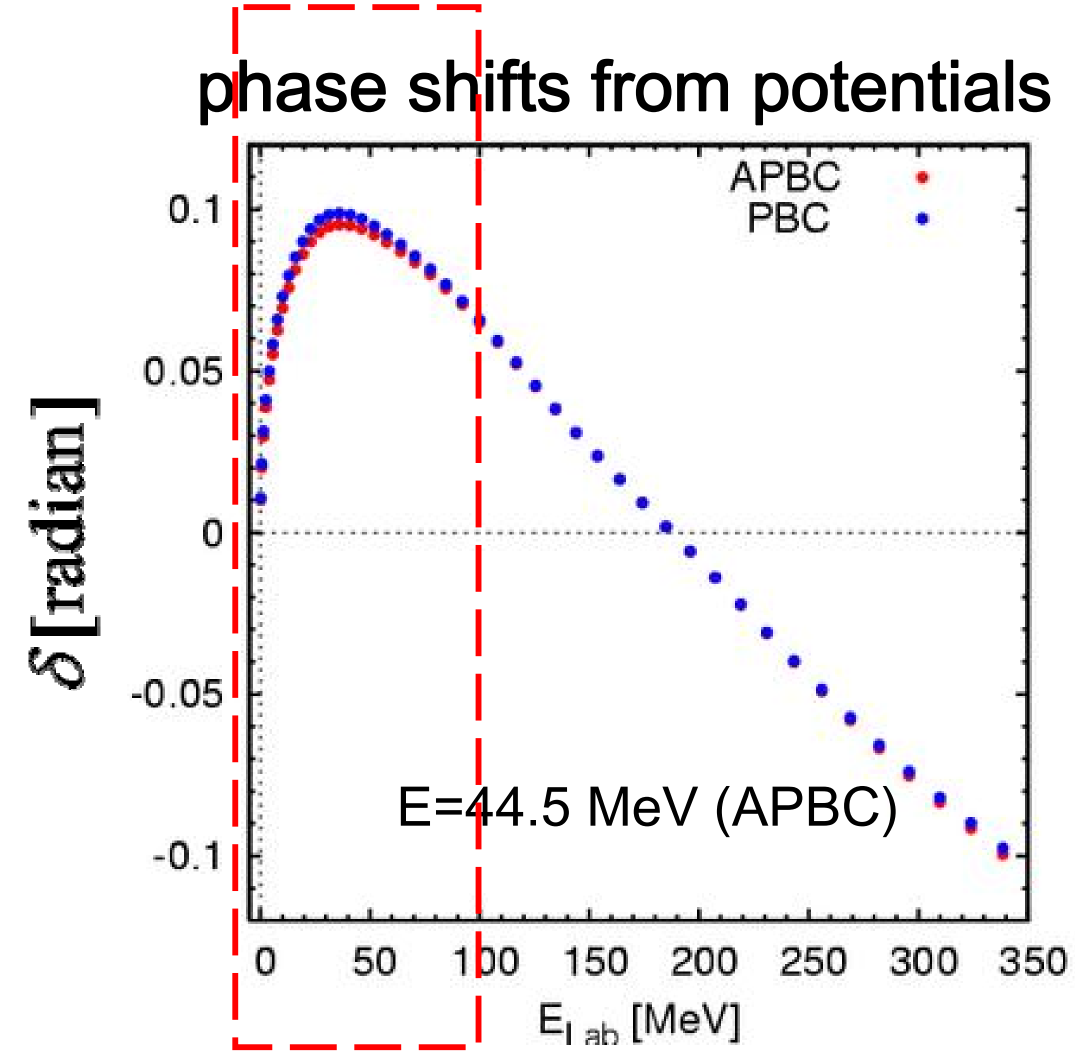
Convergence of velocity expansion

If the higher order terms are large, LO potentials determined from NBS wave functions at different energy become different.(cf. LOC of ChPT).

Numerical check in quenched QCD

K. Murano, N. Ishii, S. Aoki, T. Hatsuda PTP 125 (2011)1225.





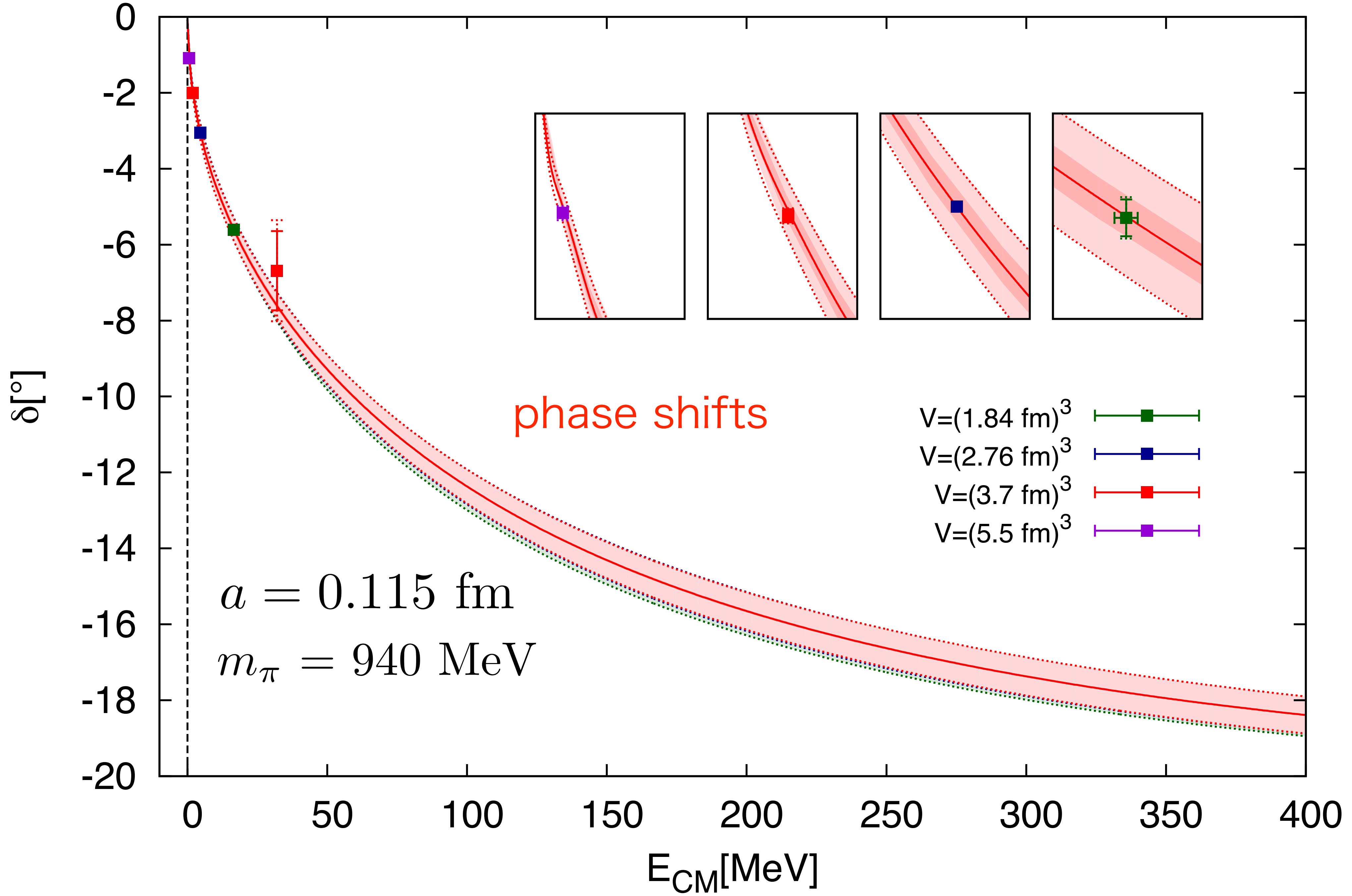
Higher order terms turn out to be very small at low energy in our scheme.

Recent comparison: Iritani's talk

Potential vs. Finite volume

Kurth, Ishii, Doi, Aoki & Hatsuda, JHEP 1312(2013)015

$I = 2$ $\pi\pi$ scattering in quenched QCD



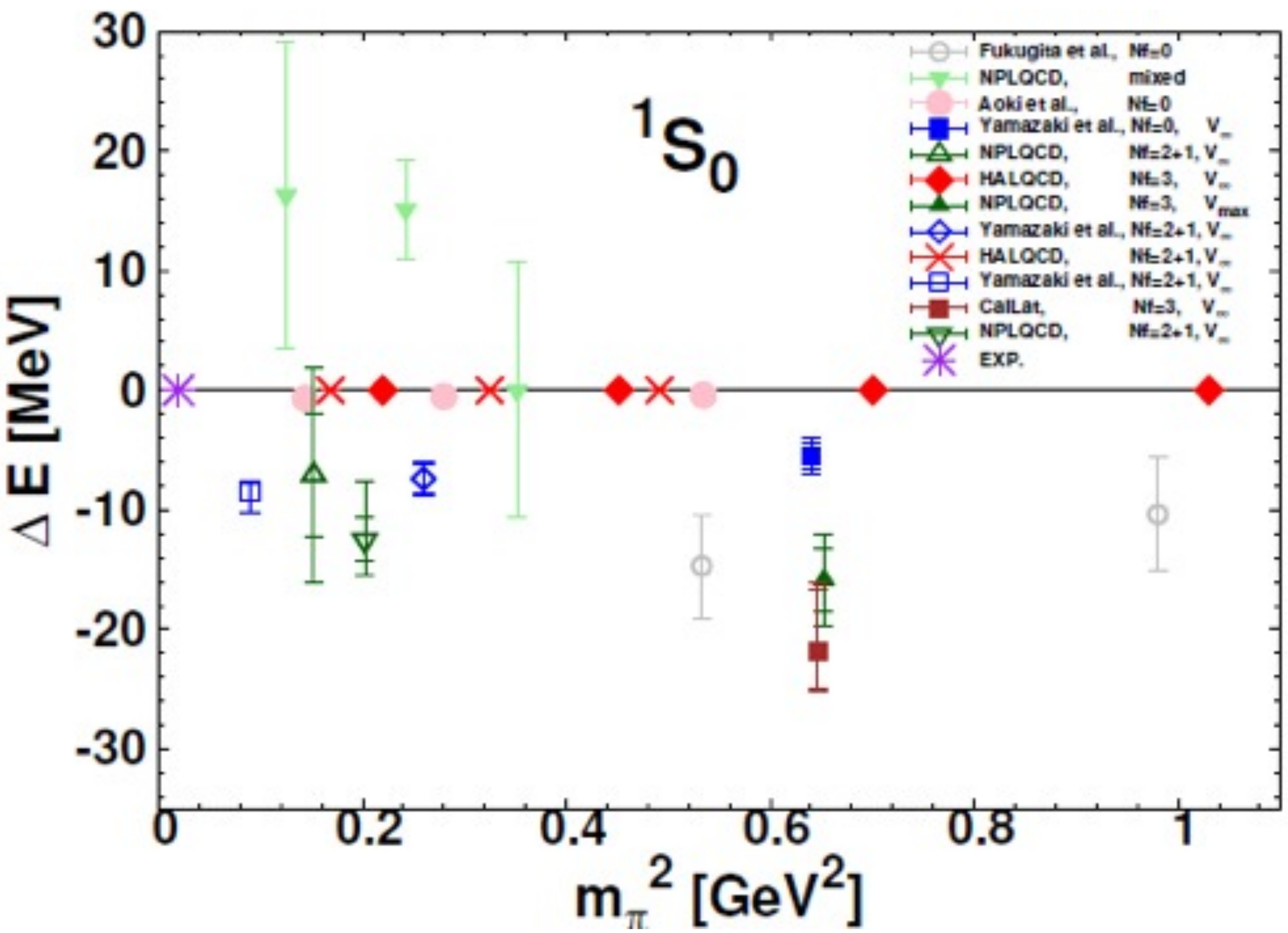
both methods agree very well.

This establishes a validity of the potential method and shows a good convergence of the velocity expansion.

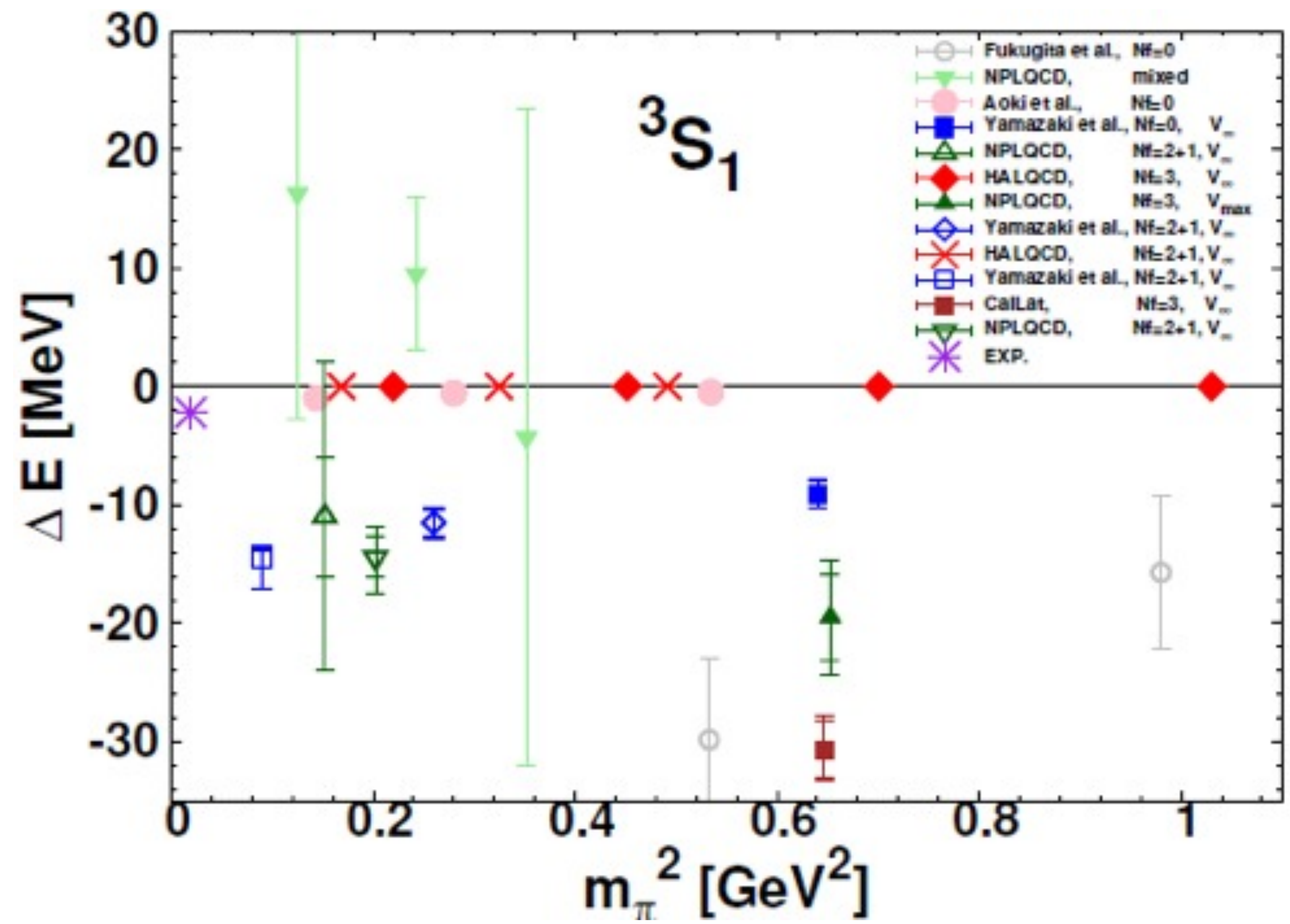
Potential vs. Direct

Reviewed in T.Doi, PoS LAT2012,009 (+ updates)

“di-neutron”



“deuteron”



HAL (potential) method (HAL) : unbound

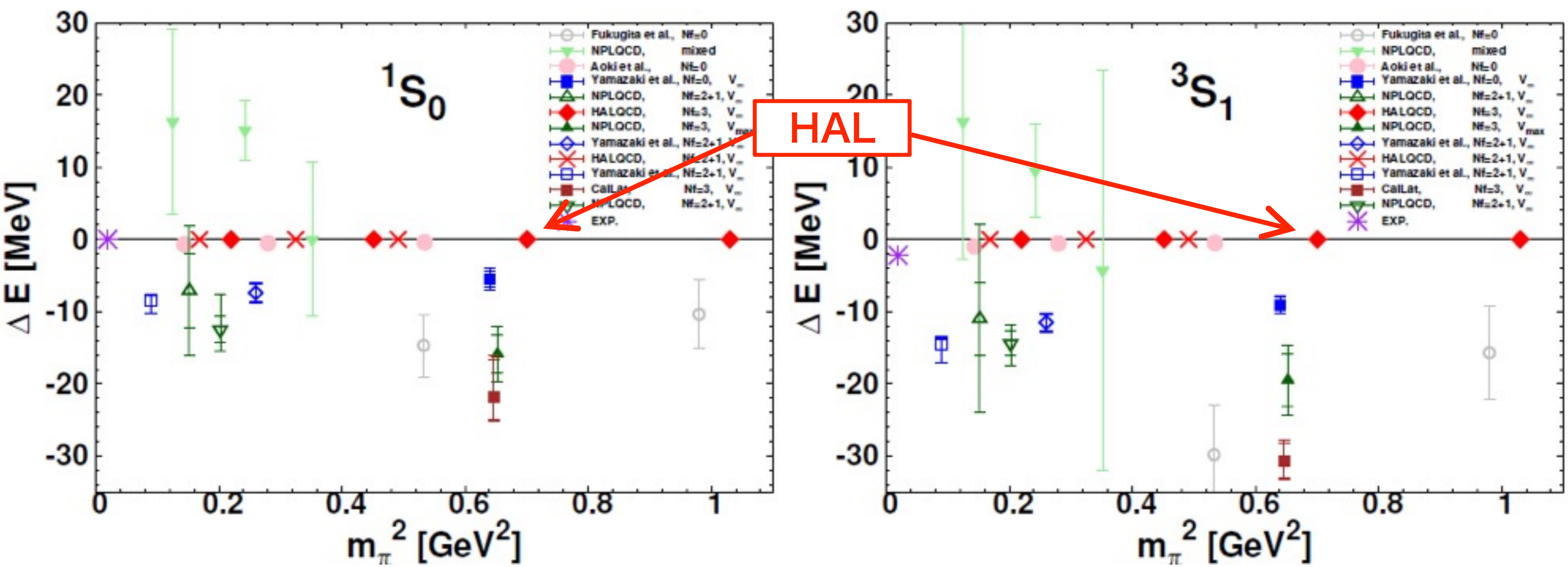
Direct method (PACS-CS (Yamazaki et al.)/NPL/CalLat): bound

Which is correct ? → **Iritani’s talk**

Potential vs. Direct

Reviewed in T.Doi, PoS LAT2012,009 (+ updates)

“di-neutron” **“deuteron”**



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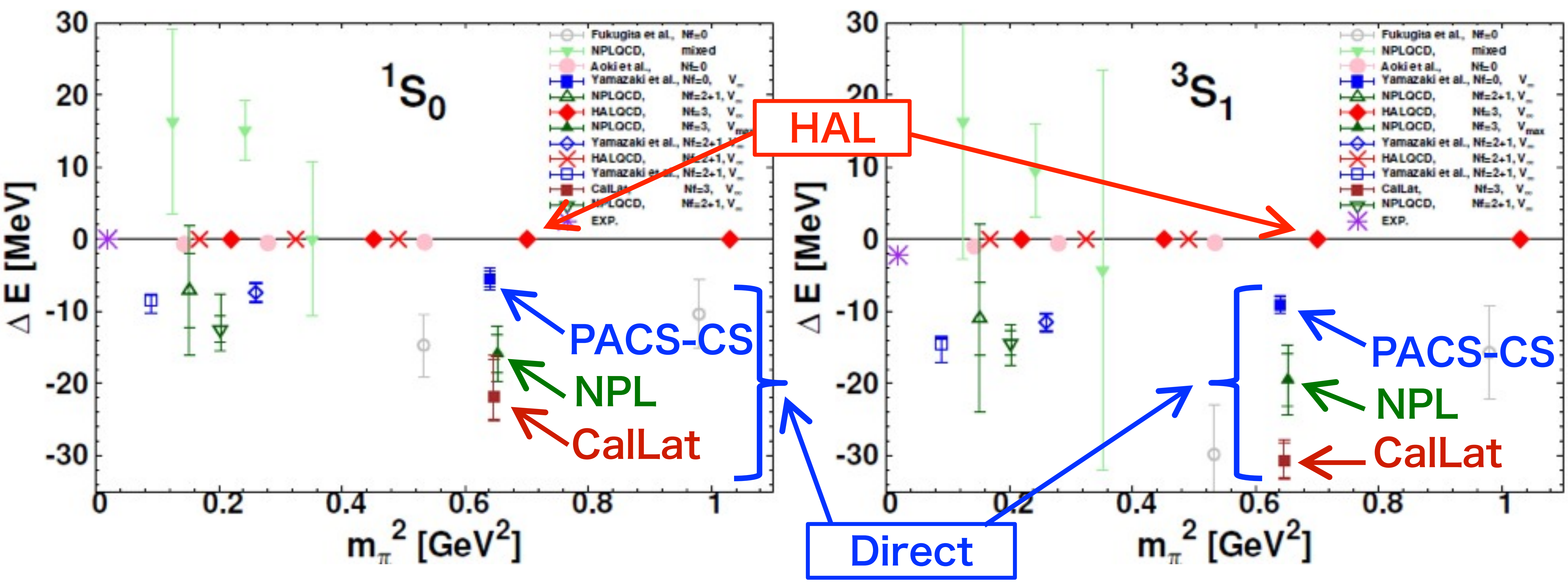
“di-neutron” **“deuteron”**



HAL (potential) method (HAL) : unbound

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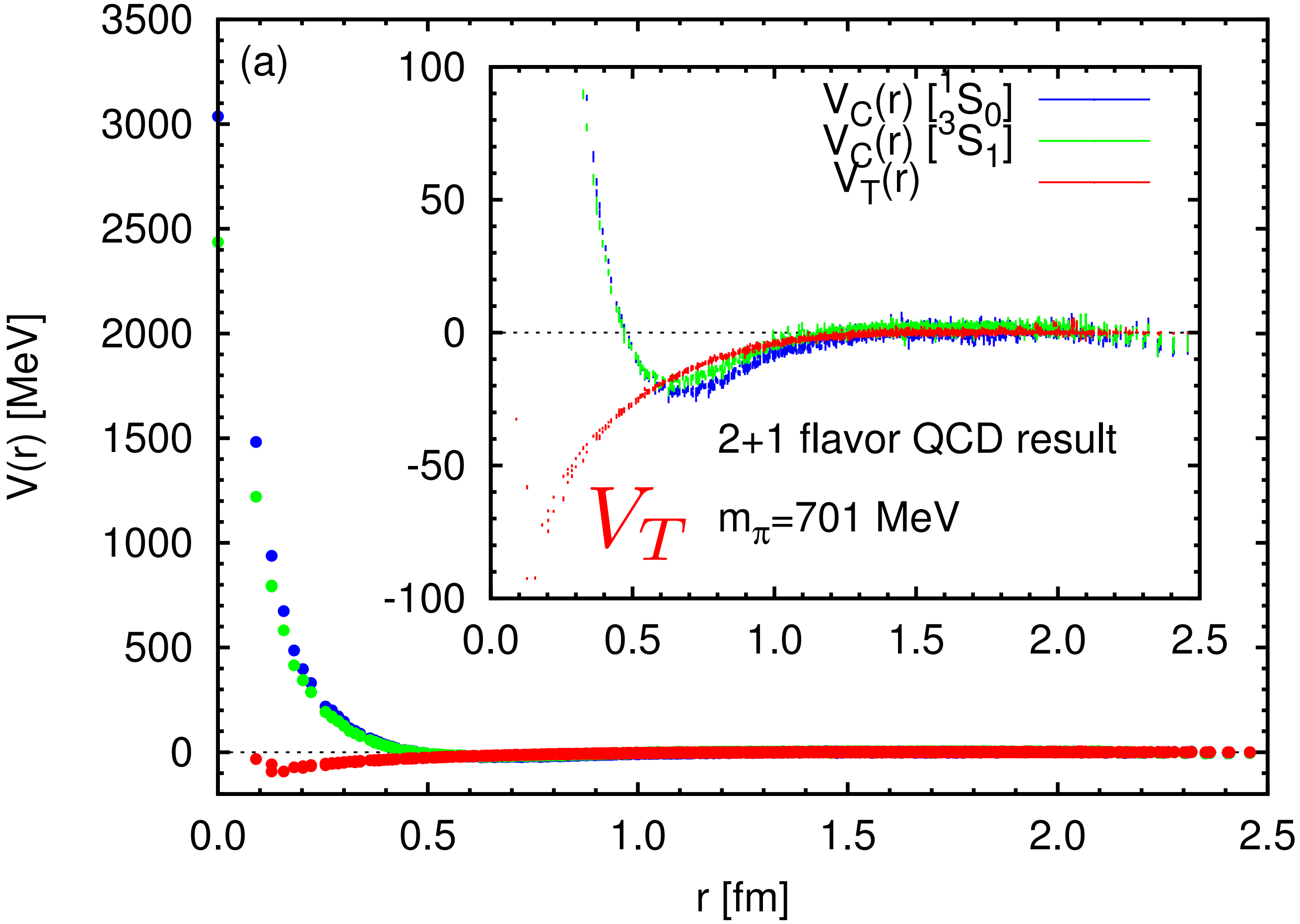
Which is correct ? → **Iritani’s talk**

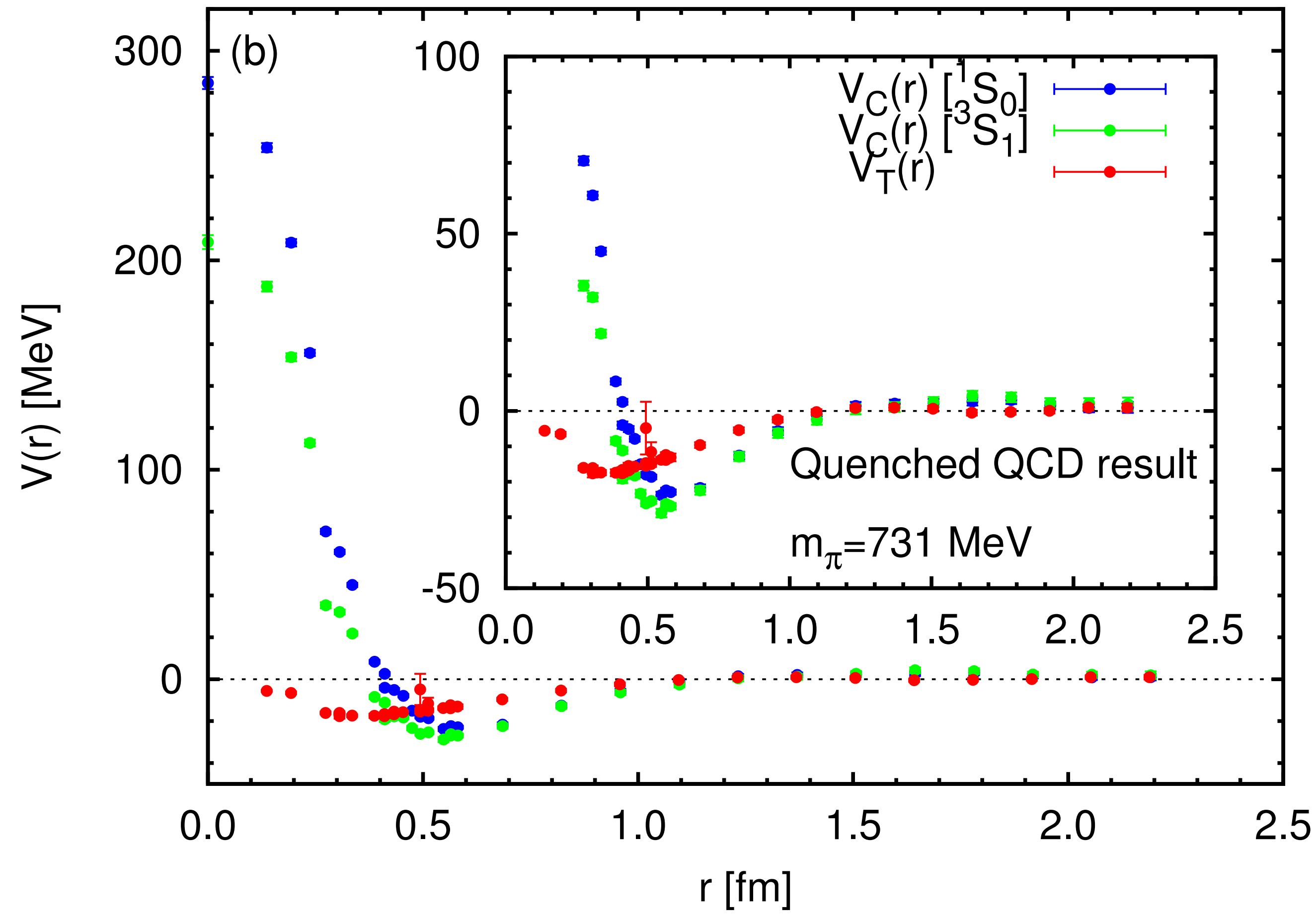
LO Tensor potential

full QCD



$a \simeq 0.091$ fm $\quad L \simeq 2.9$ fm

quenched QCD

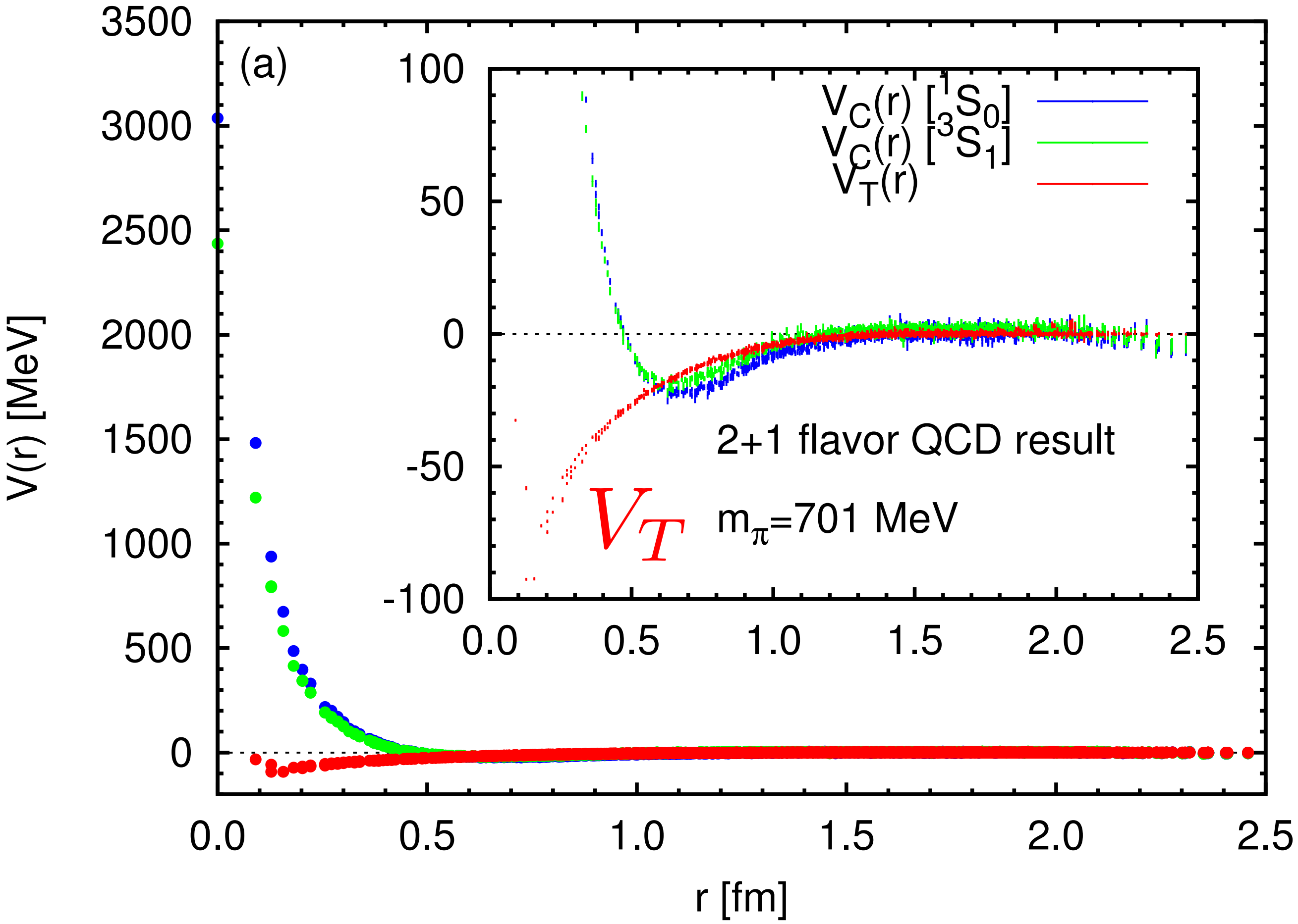


$a \simeq 0.137$ fm $\quad L \simeq 4.4$ fm

- no repulsive core in the tensor potential.
- the tensor potential is enhanced in full QCD

LO Tensor potential

full QCD



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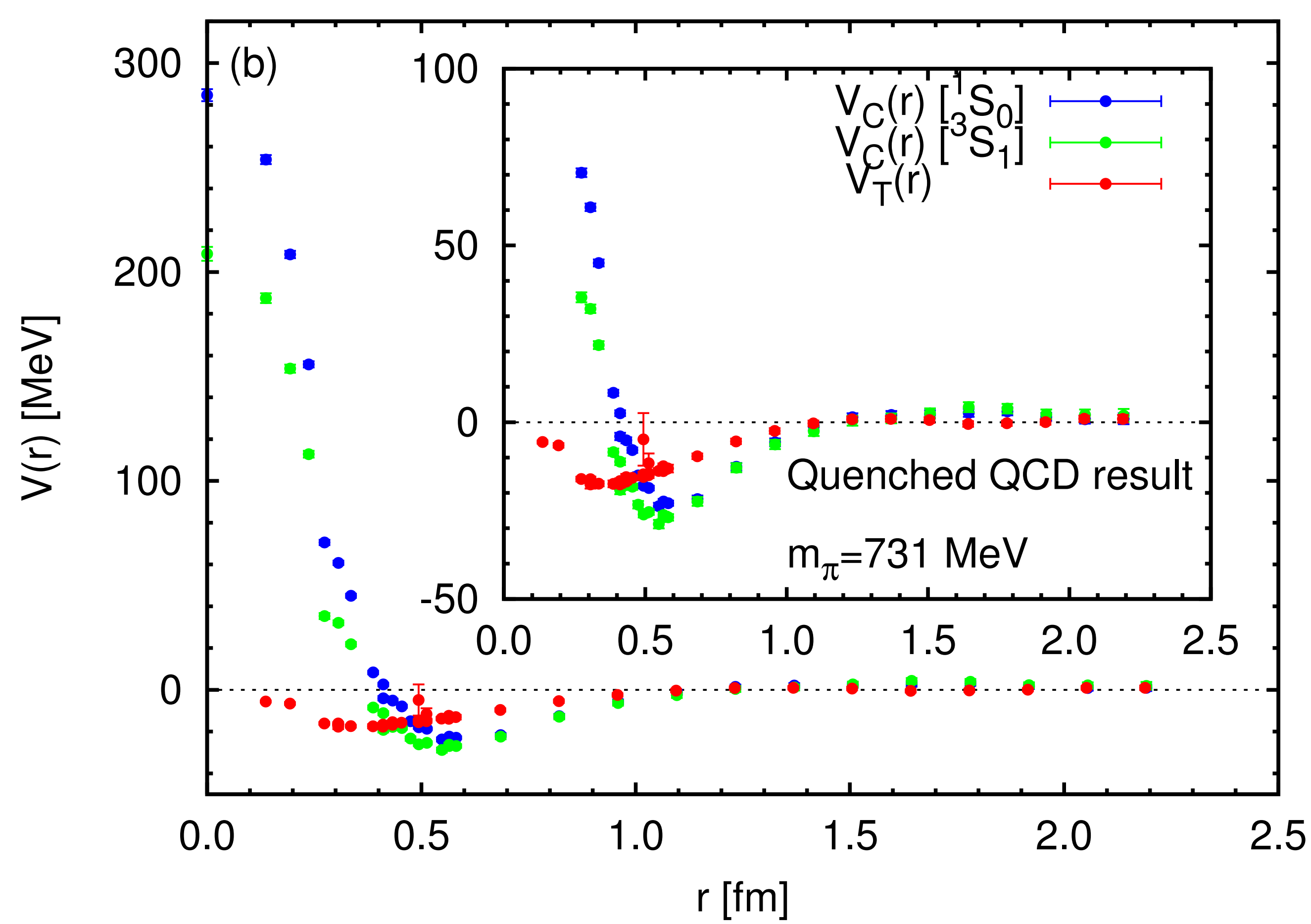

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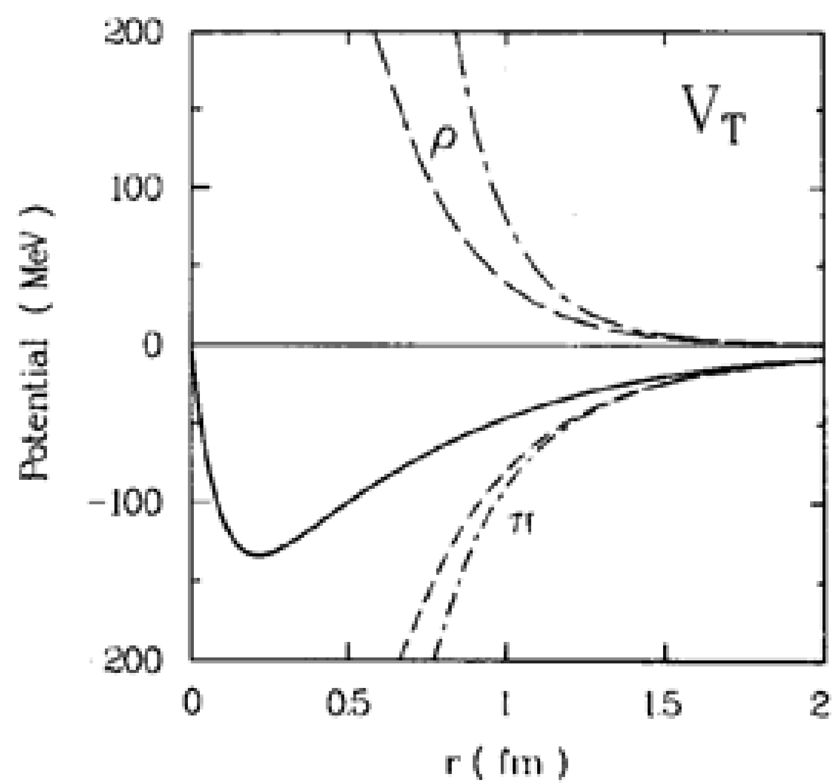
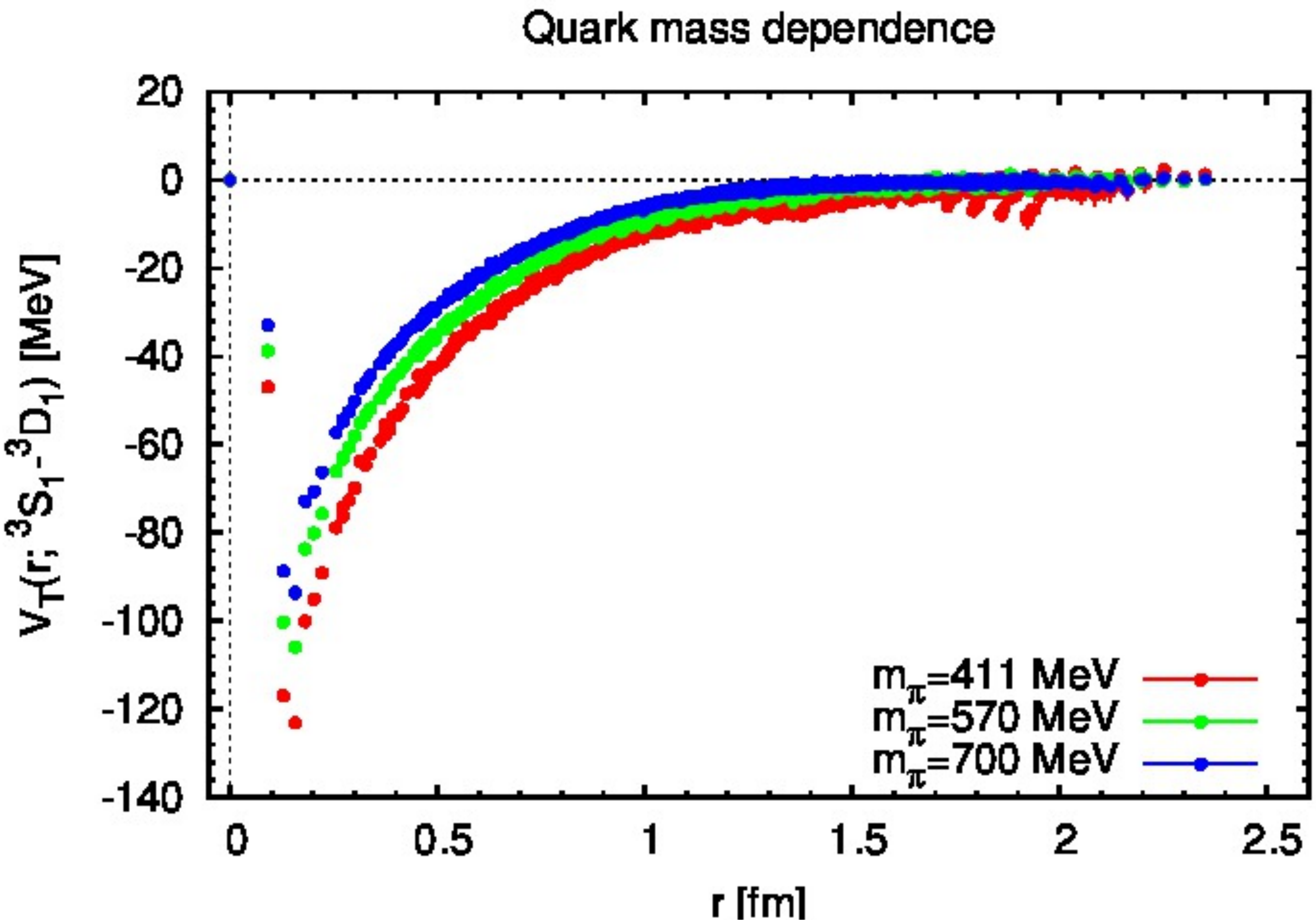


from R.Machleidt, Adv.Nucl.Phys.**19**

Fig. 3.7. The contributions from π and ρ (dashed) to the $T = 0$ tensor potential. The solid line is the full potential. The dash-dot lines are obtained when the cutoff is omitted.

Quark mass dependence (full QCD)

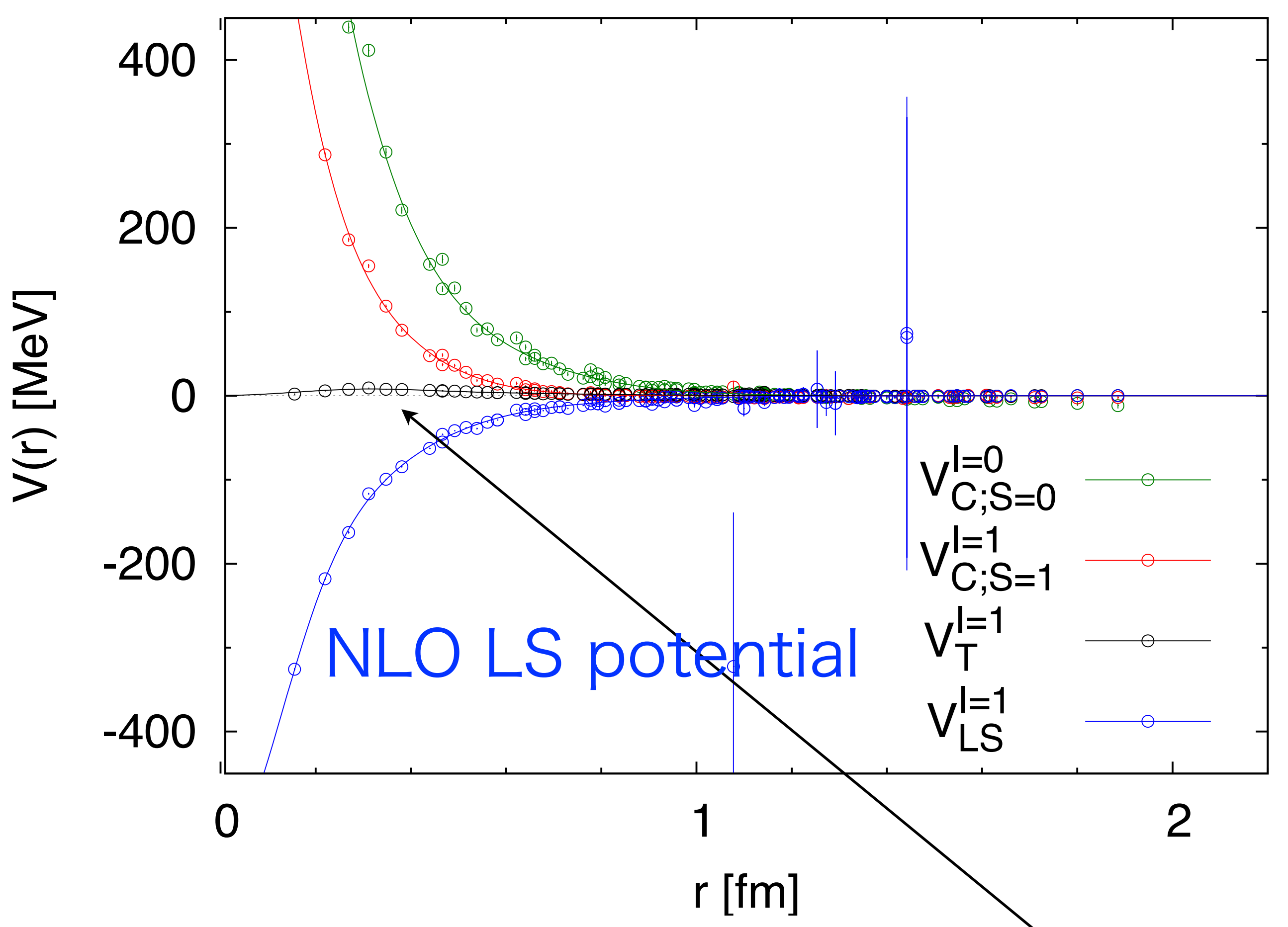


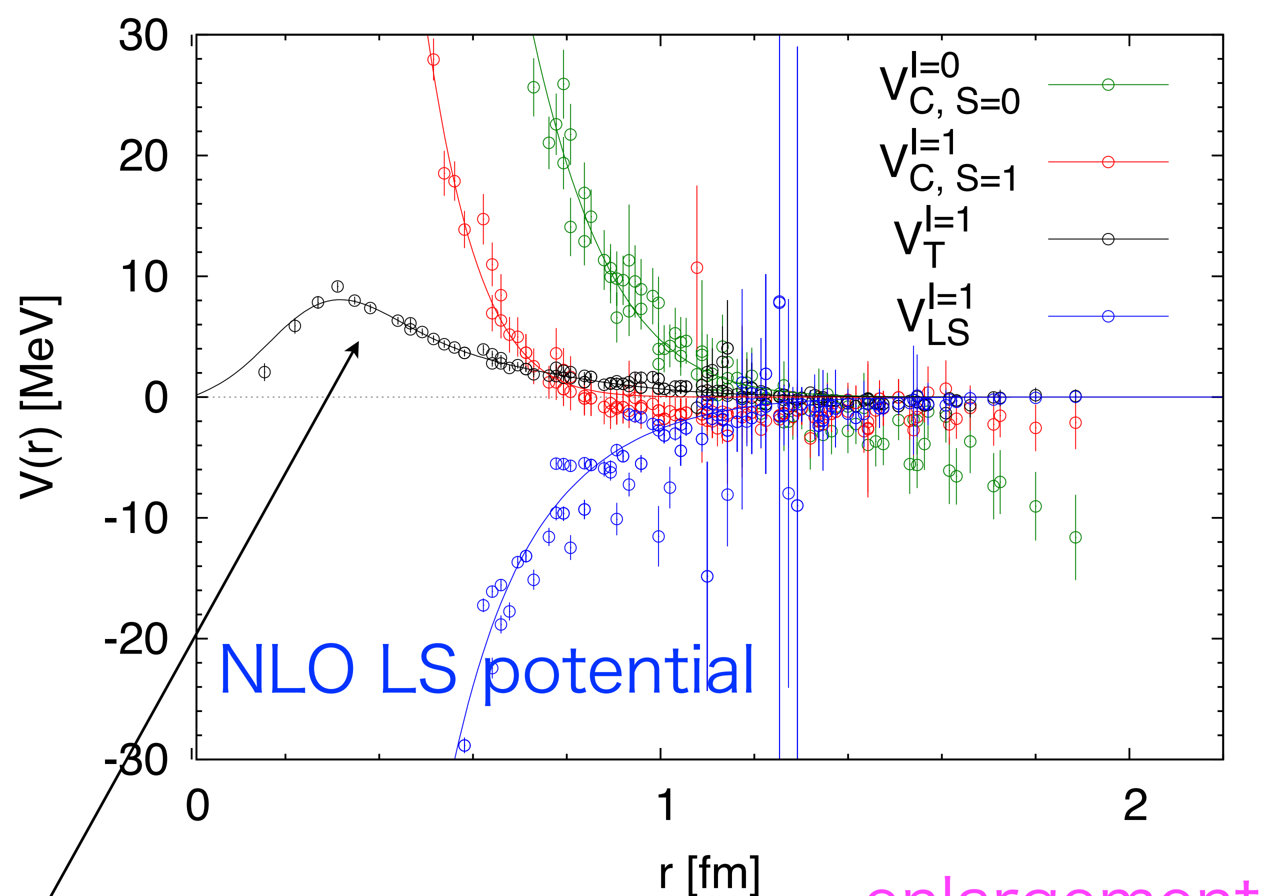
- the tensor potential increases as the pion mass decreases.
  - manifestation of one-pion-exchange ?

Negative parity potentials

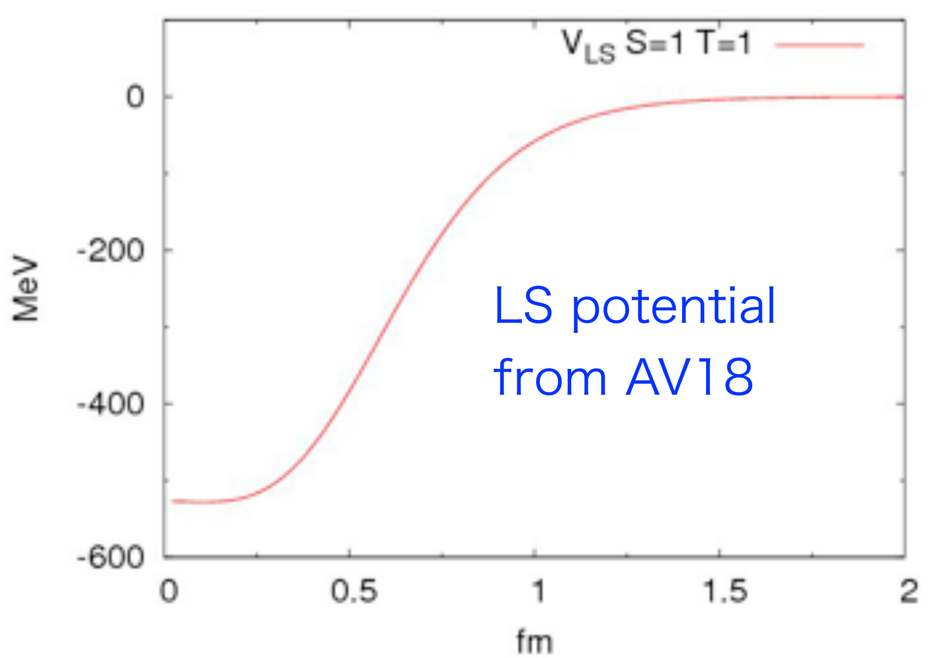
$a = 0.16$ fm, $L = 2.5$ fm, $m_\pi = 1100$ MeV

Murano et al. (HAL QCD), PLB735(2014)19





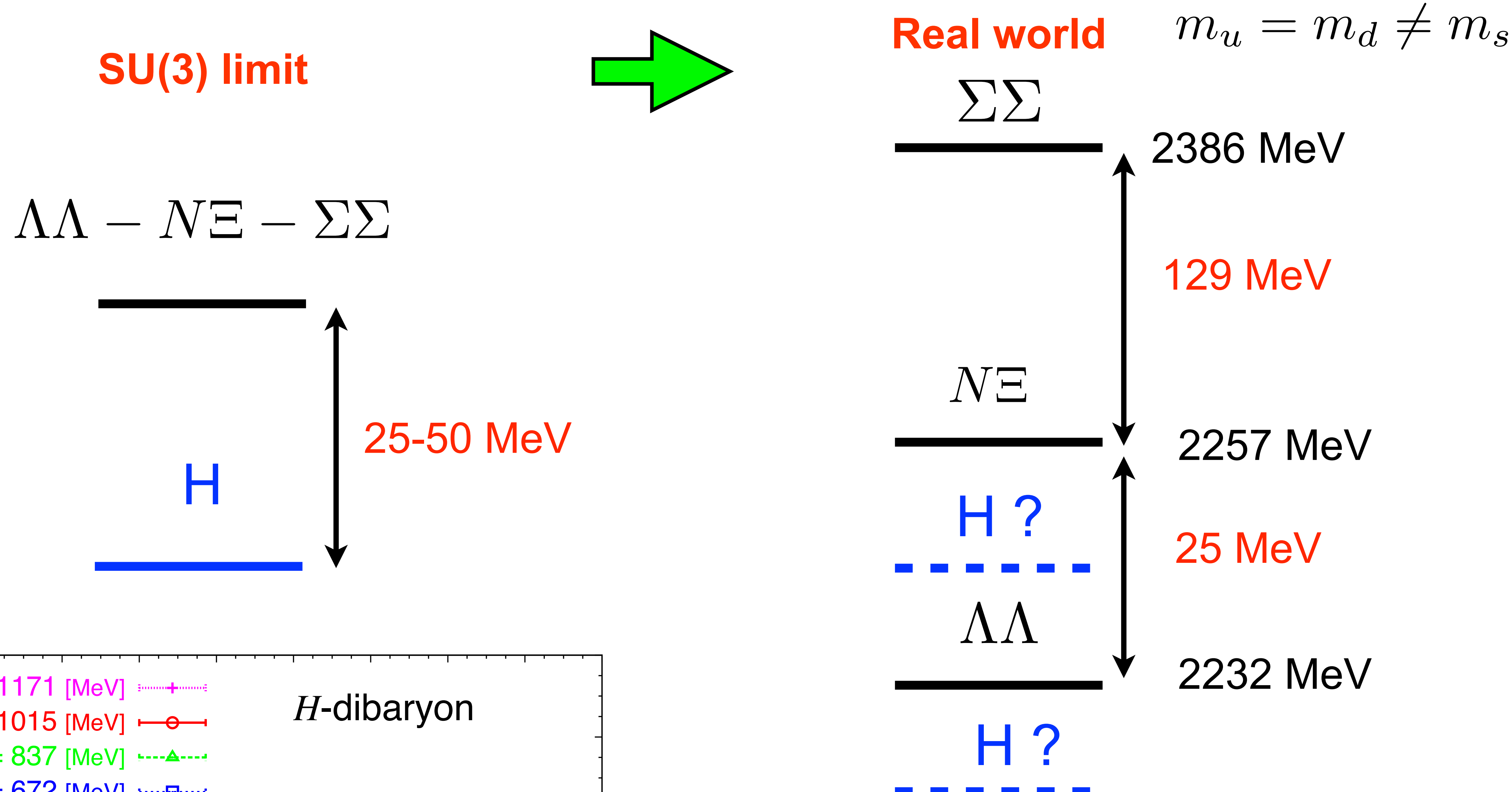
enlargement

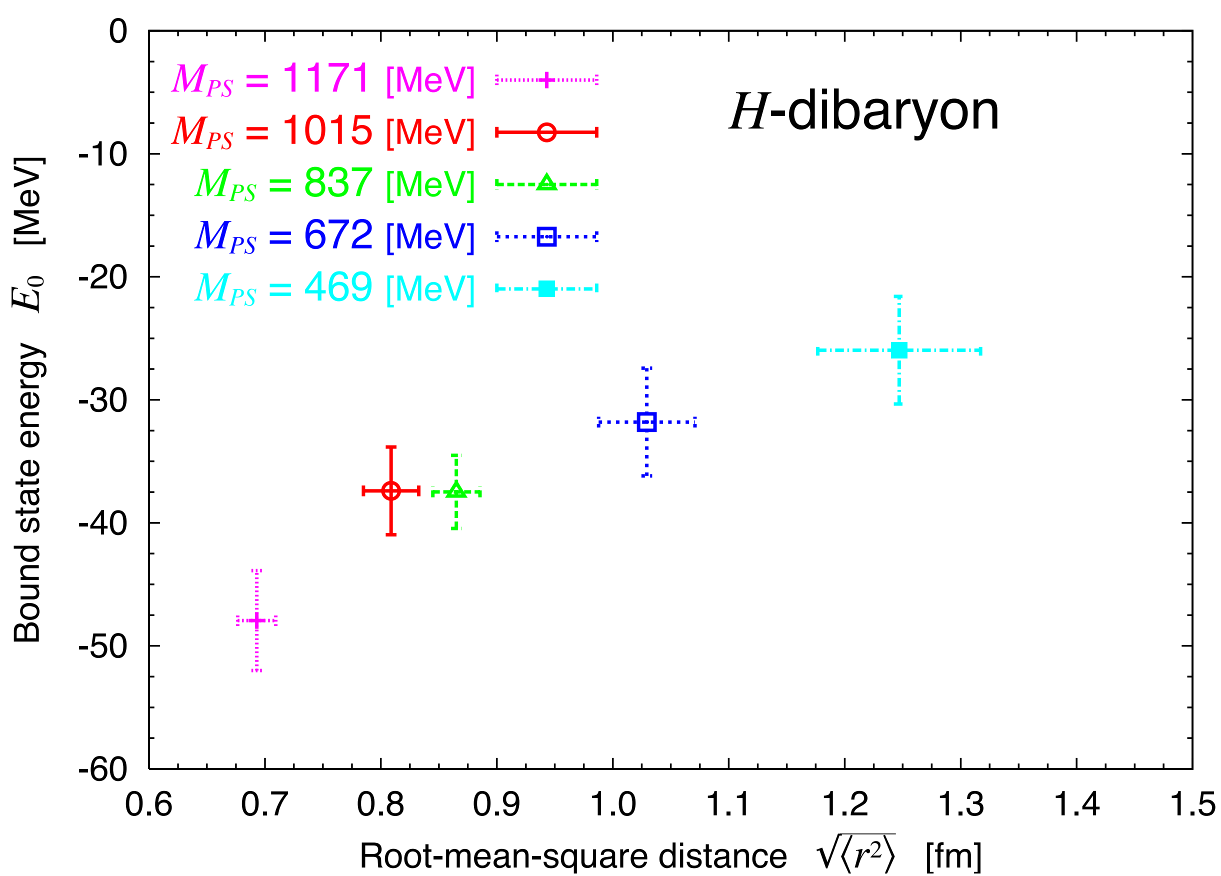


tensor potential is very weak.

3. Recent results

H-dibaryon





coupled channel analysis is needed.

S. Aoki, *et al.* , Proc. Jpn. Acad. Ser. B, 87 (2011) 509.

Results from HAL QCD Collaboration

Sasaki for HAL QCD Collaboration

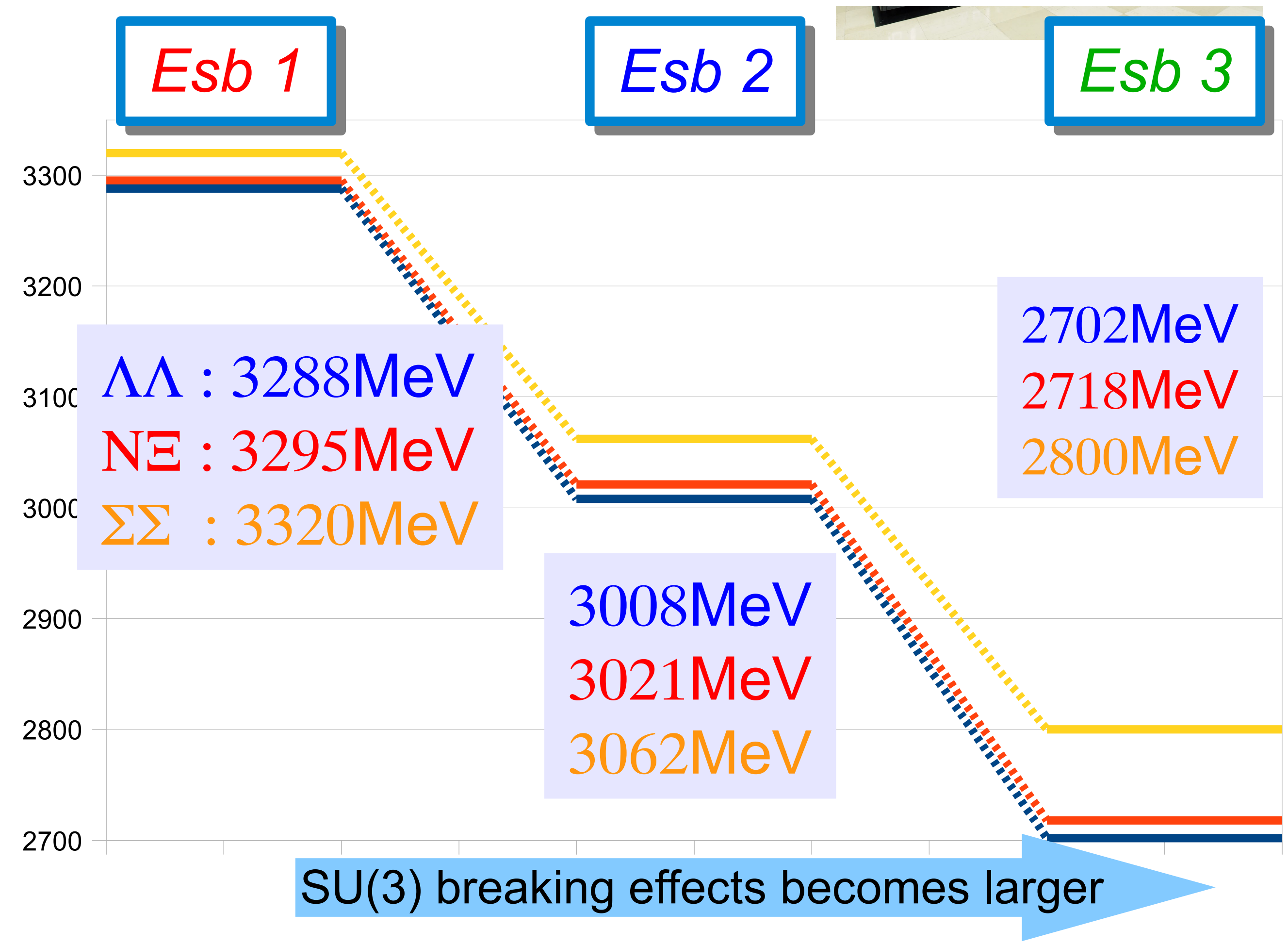
$$N_f = 2 + 1 \text{ full QCD with } L = 2.9 \text{ fm}$$

Gauge ensembles

In unit of MeV	Esb 1	Esb 2	Esb 3
π	701±1	570±2	411±2
K	789±1	713±2	635±2
m_π/m_K	0.89	0.80	0.65
N	1585±5	1411±12	1215±12
Λ	1644±5	1504±10	1351± 8
Σ	1660±4	1531±11	1400±10
Ξ	1710±5	1610± 9	1503± 7

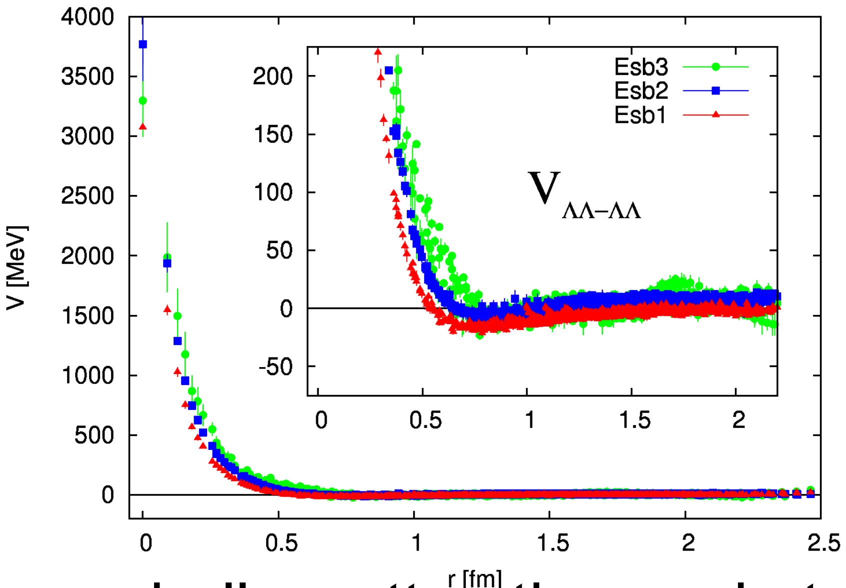
u,d quark masses lighter

thresholds

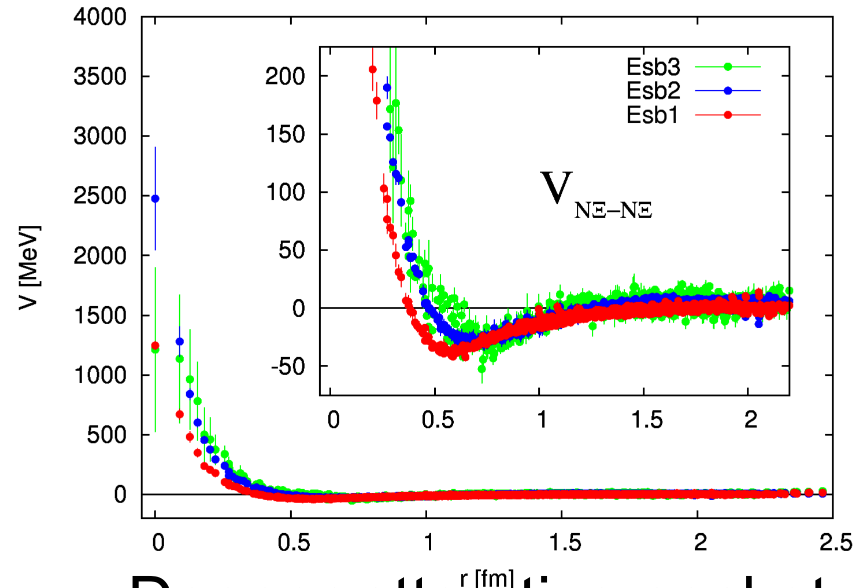


coupled channel 3x3 potentials

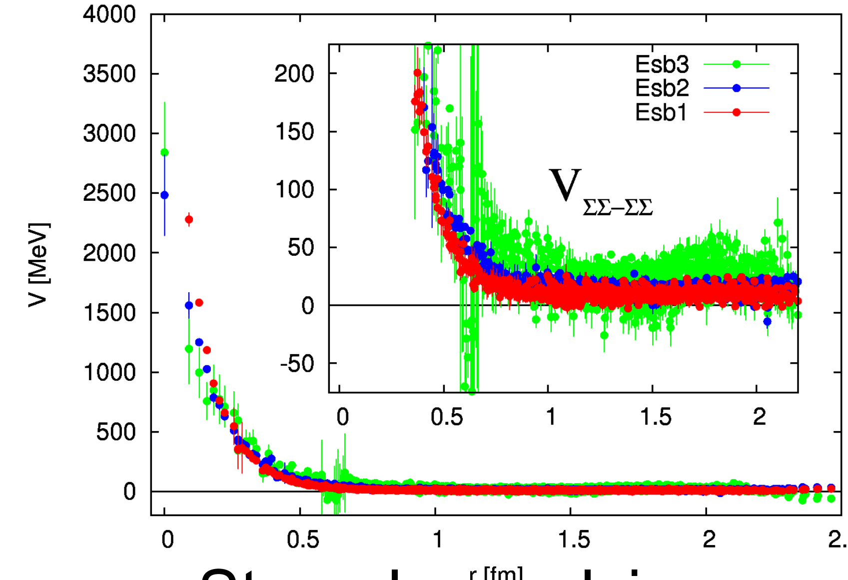
Diagonal elements



shallow attractive pocket



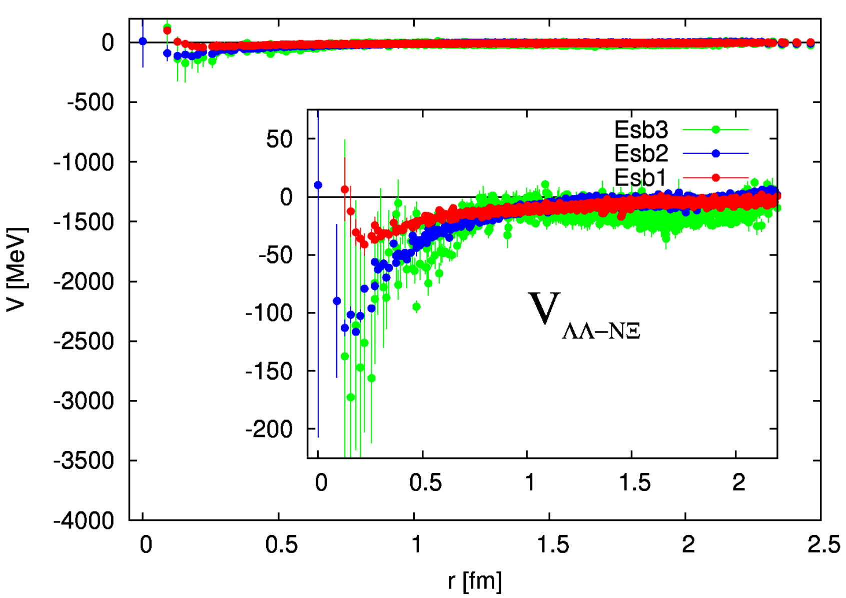
Deeper attractive pocket

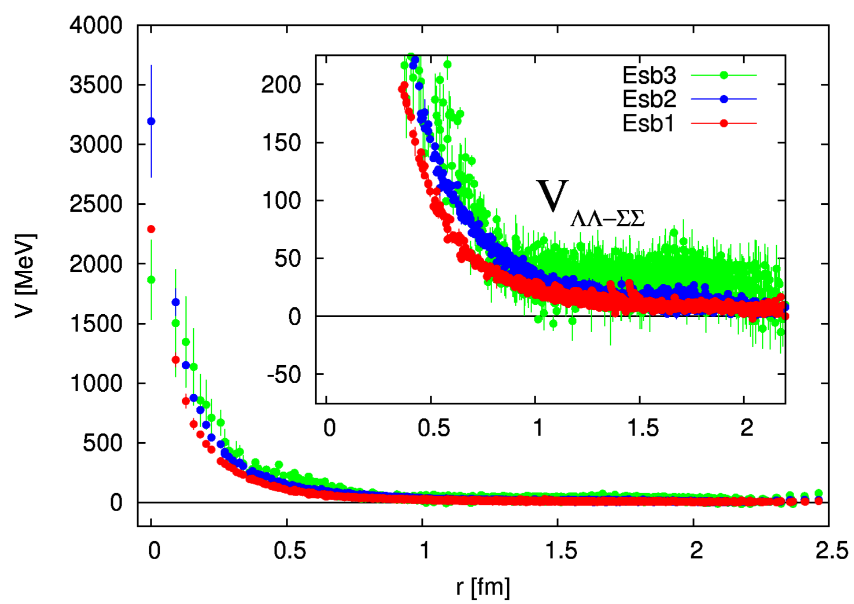


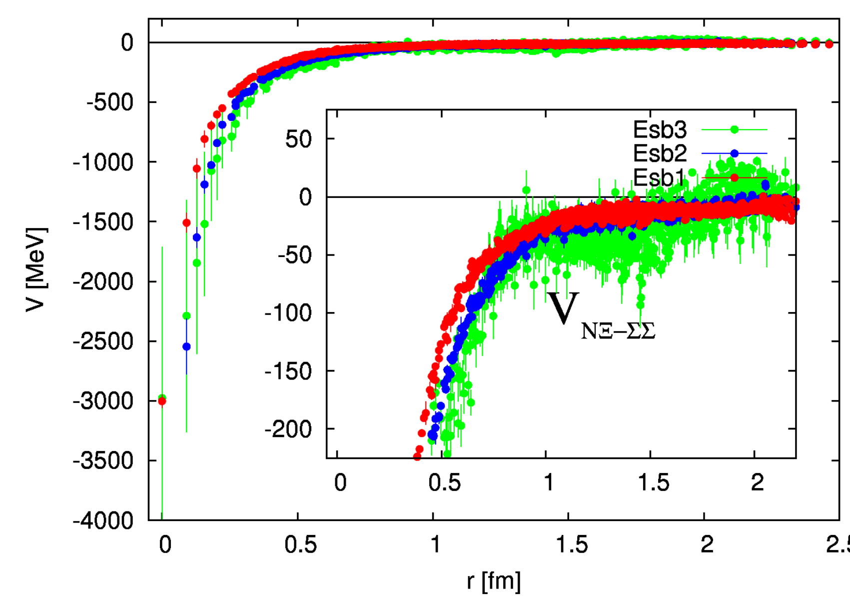
Strongly repulsive

All channels have repulsive core

Off-diagonal elements



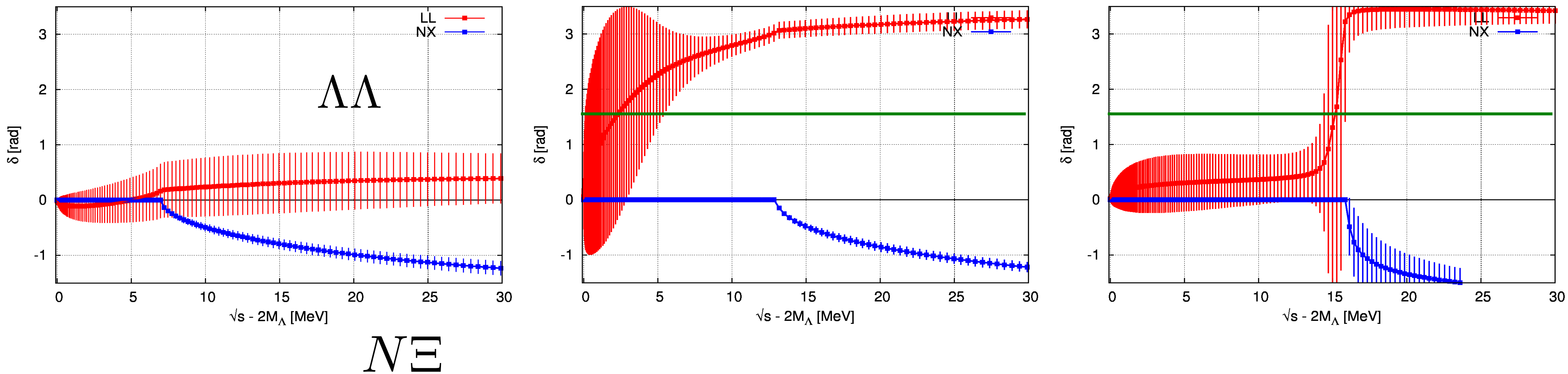




$\Lambda\Lambda$ and $N\Xi$ phase shift

Preliminary !

$m\pi$ = 700 MeV	**$m\pi$ = 570 MeV**	**$m\pi$ = 410 MeV**



Bound H-dibaryon coupled to NΞ	H as resonance near ΛΛ threshold (H as bound NΞ)	H as resonance near NΞ threshold (H as bound NΞ)

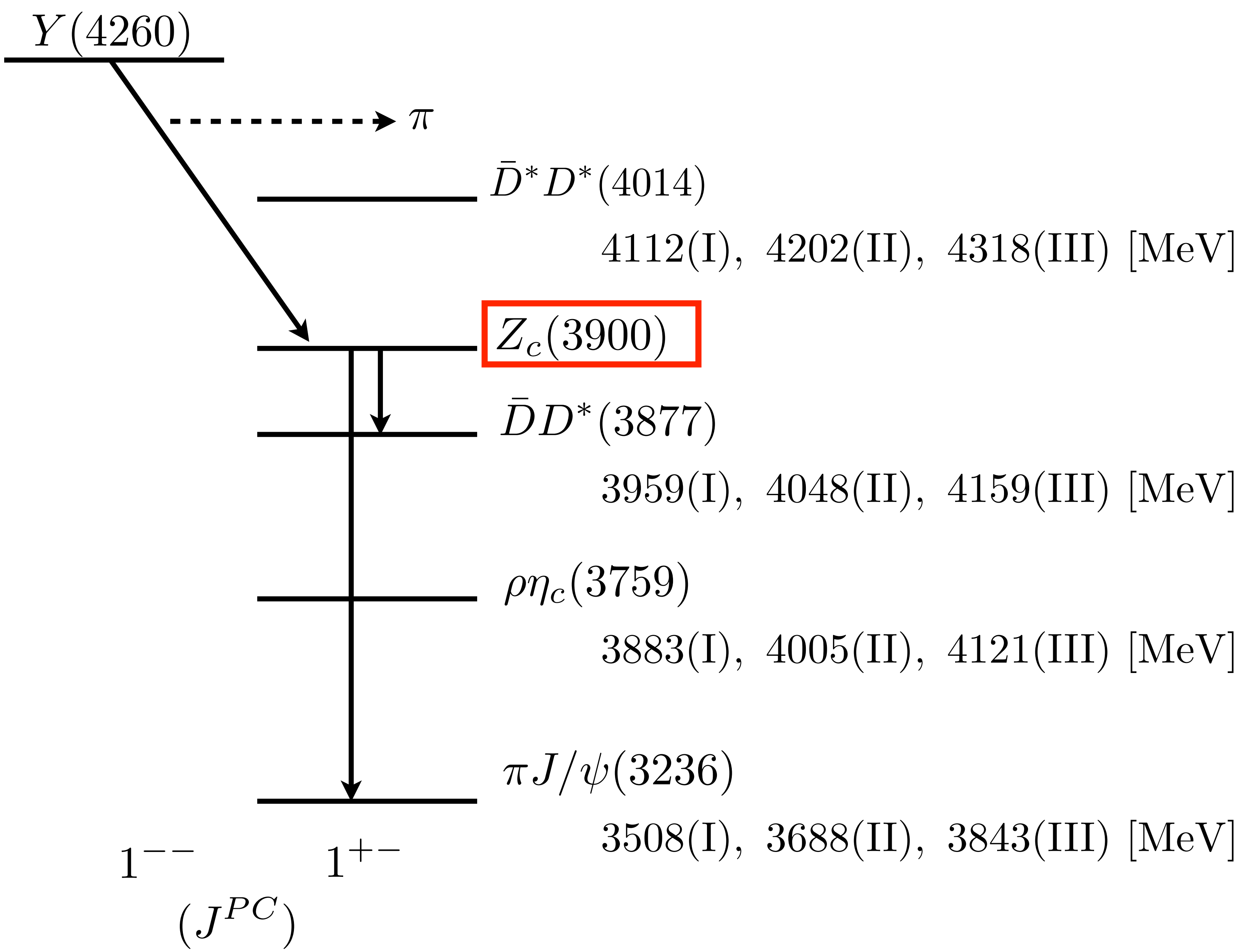
This suggests that H-dibaryon becomes resonance at physical point.
Below or above NΞ ? Need simulation at physical point.

Doi's talk

$Z_c(3900)$

A tetraquark candidate

Y. Ikeda, *et al.* , arXiv:1602.03465[hep-lat]



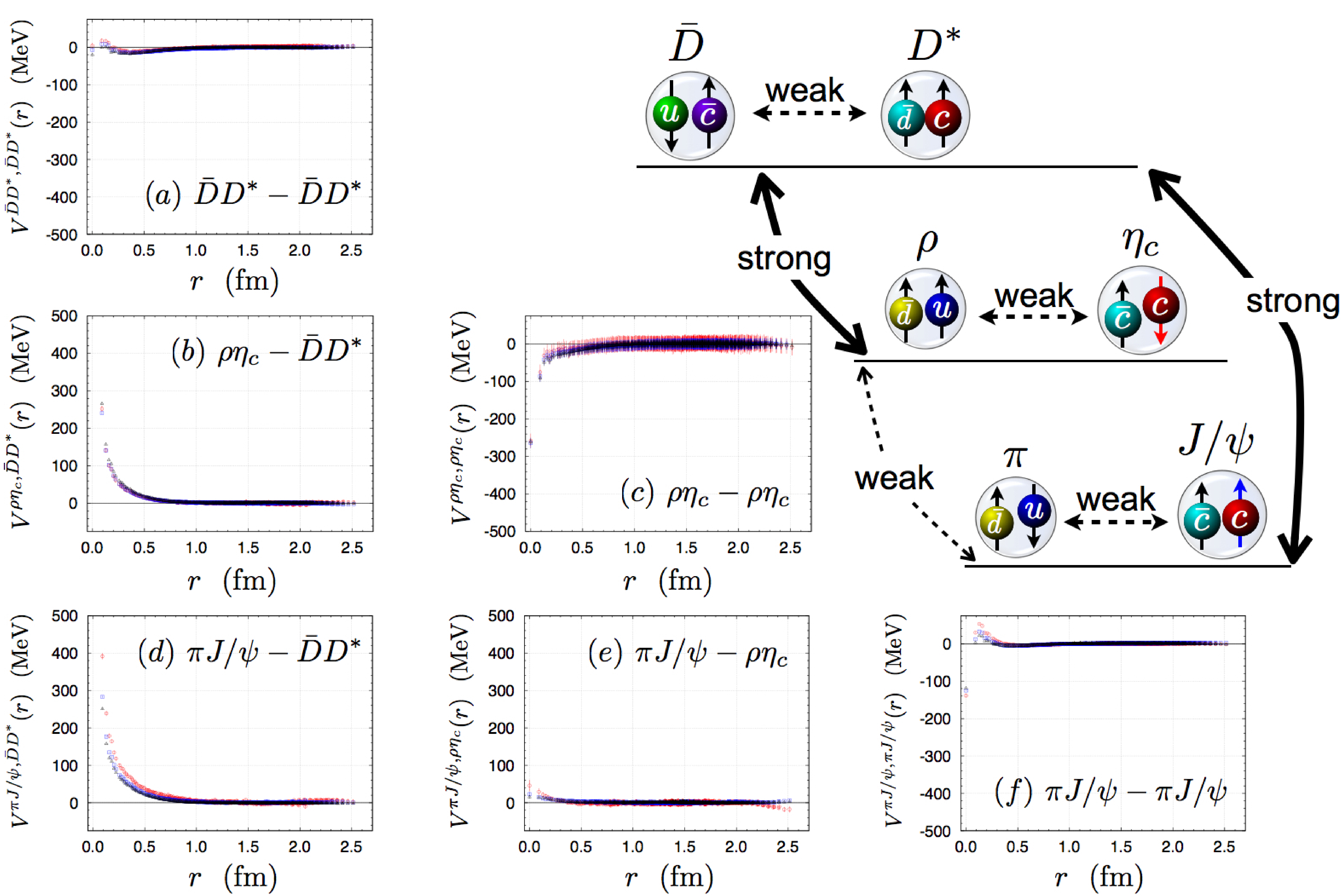
coupled channel analysis is needed.

$m_\pi \simeq 410\ (\text{Case I}),\ 570\ (\text{Case II}),\ 700\ (\text{Case III})\ \text{MeV}$

$L \simeq 2.9\ \text{fm},\ a \simeq 0.09\ \text{fm}$

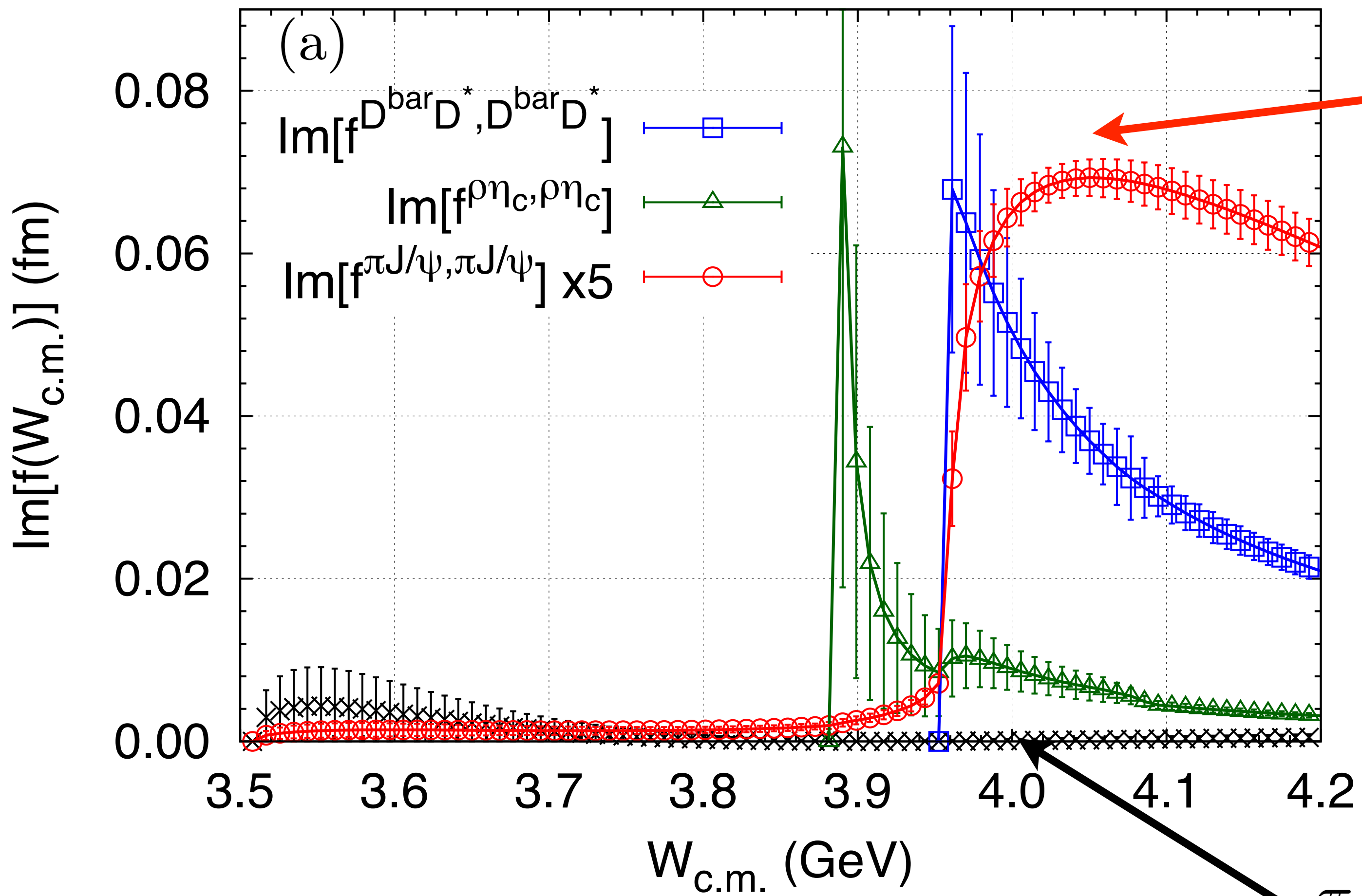
coupled channel 3x3 potentials

Case I



invariant mass spectra

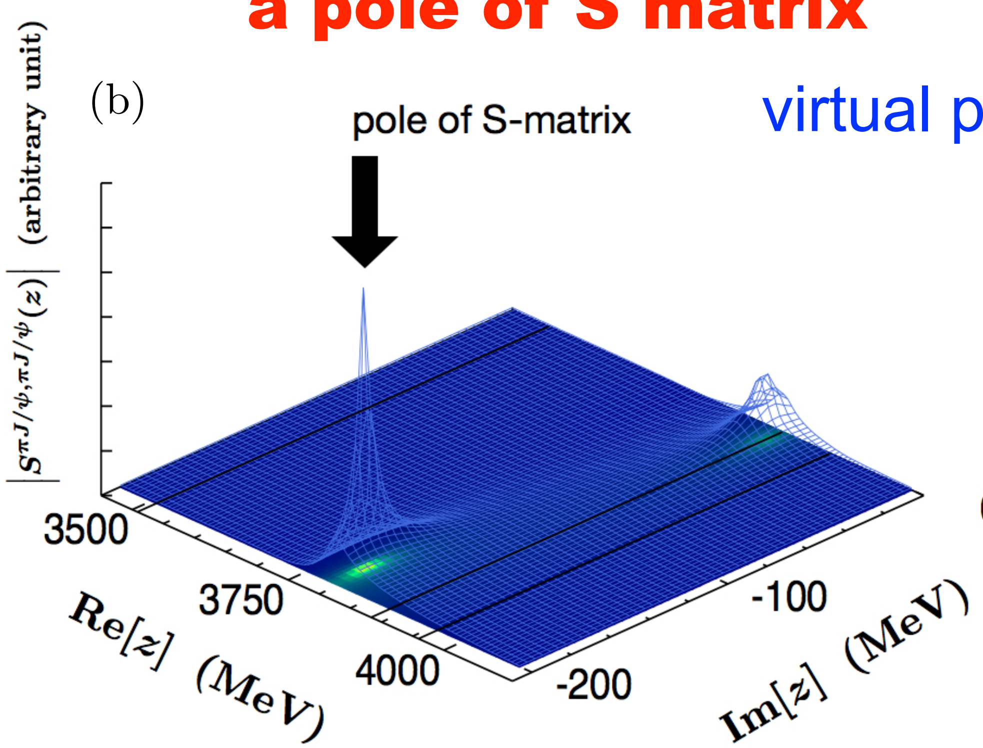
Case I



the peak structure in the $\pi J/\psi$ spectrum ($Z_c(3900)$) seems a "threshold cusp".

$\pi J/\psi$ mass spectrum w/o off-diagonal parts

a pole of S matrix



virtual pole in the second Riemann sheets of all channels

$$z_{\text{pole}} - (m_D + m_{D^*}) = -167(94) - 183(46)i$$

its contribution to the two-body amplitudes is highly suppressed.

fits to experimental data

Case I

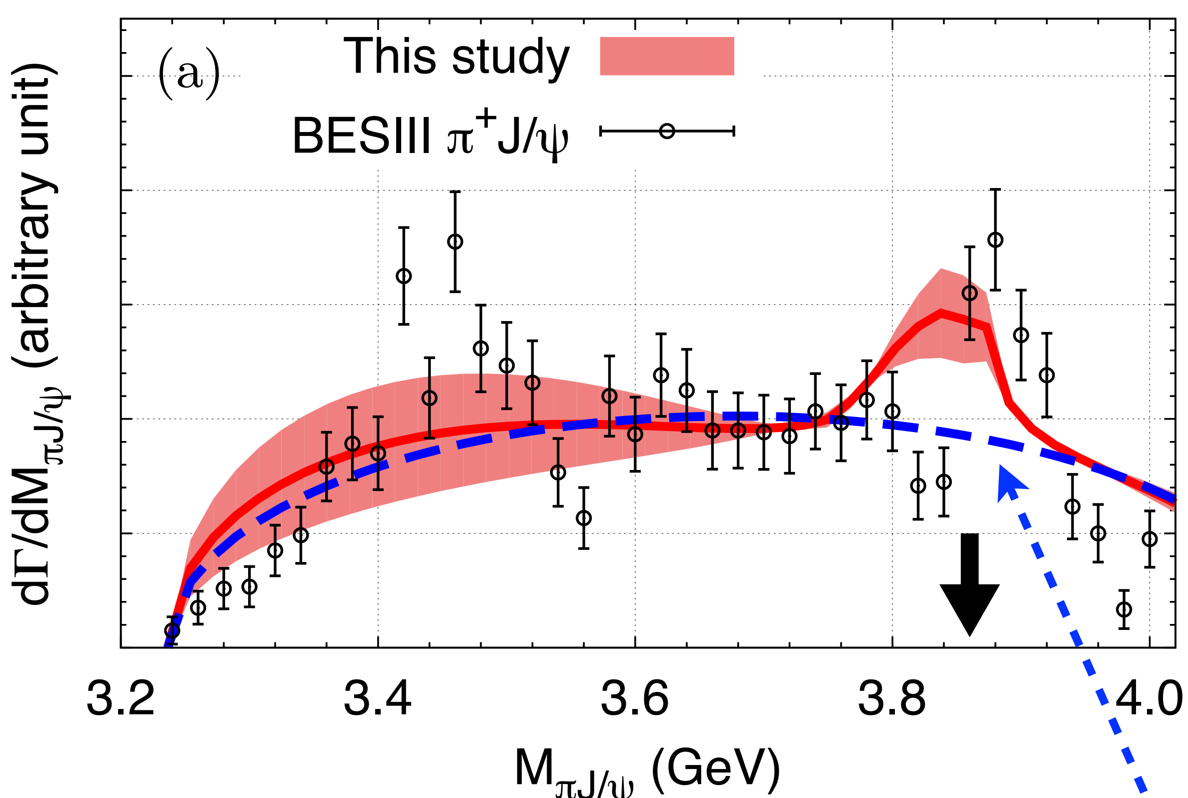
model $Y(4260) \to \pi + \pi J/\psi, \pi + \bar{D}D^{\star}$

$$T^{Y\to\pi+\beta}(\vec{p},\vec{q}_\beta;W_3) = \sum_{\alpha=\pi J/\psi,\bar{D}D^*} C^{Y\to\pi+\alpha} \times \left[\delta_{\alpha\beta} + \int d\vec{q}_\alpha \frac{T^{\alpha\beta}(\vec{q}_\alpha,\vec{q}_\beta,\vec{p};W_3)}{W_3 - E_\pi(\vec{p}) - E_\alpha(\vec{p},\vec{q}_\alpha) + i\epsilon}\right]$$

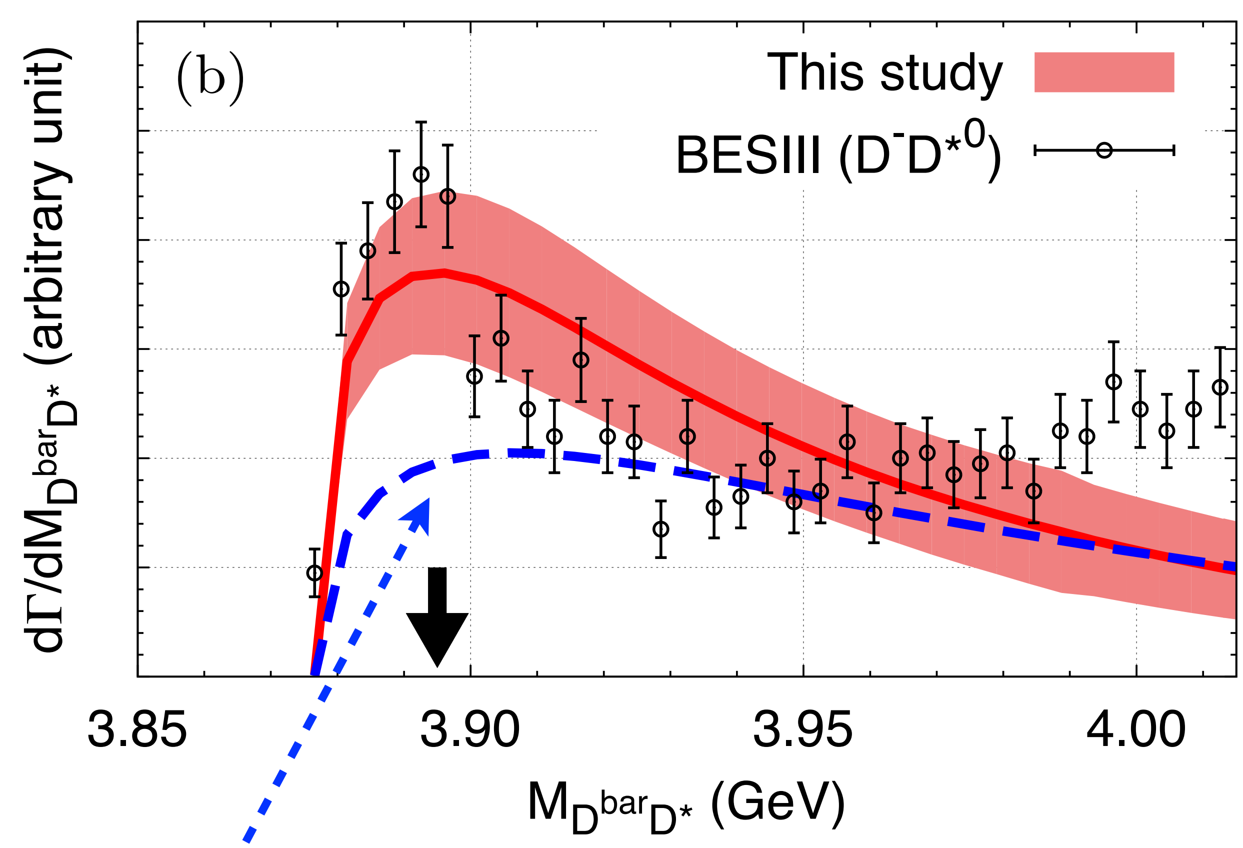
fit parameters ($C^{Y\to\pi+\alpha}$)

lattice data ($T^{\alpha\beta}$)

$Y(4260) \to \pi + \pi J/\psi$



$Y(4260) \to \pi + \bar{D}D^{\star}$



off-diagonals switched-off

4. Summary

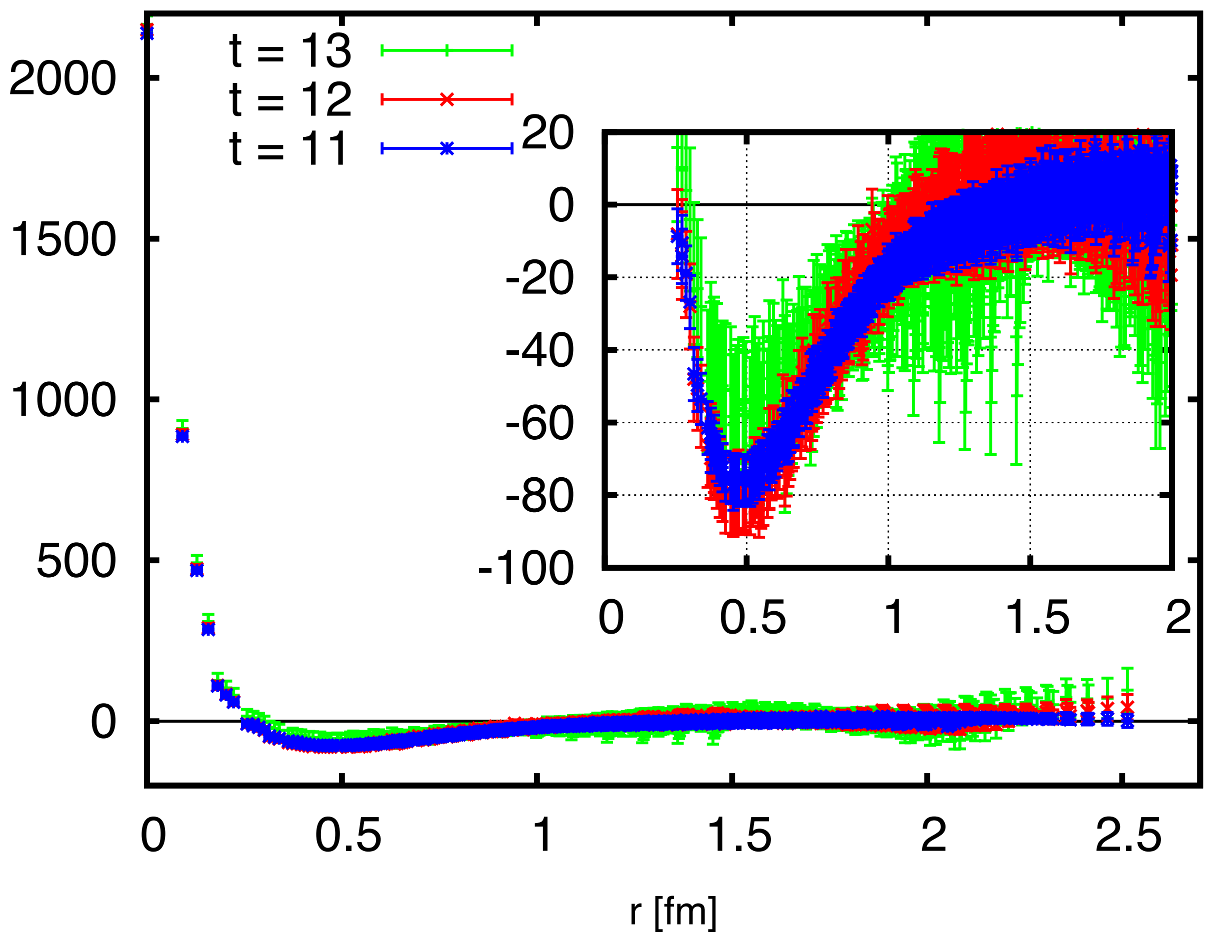
- HAL QCD strategy is a very powerful one for “nuclear physics from lattice QCD”.
- work well also for multi-channel scatterings
- More results

ΩΩ

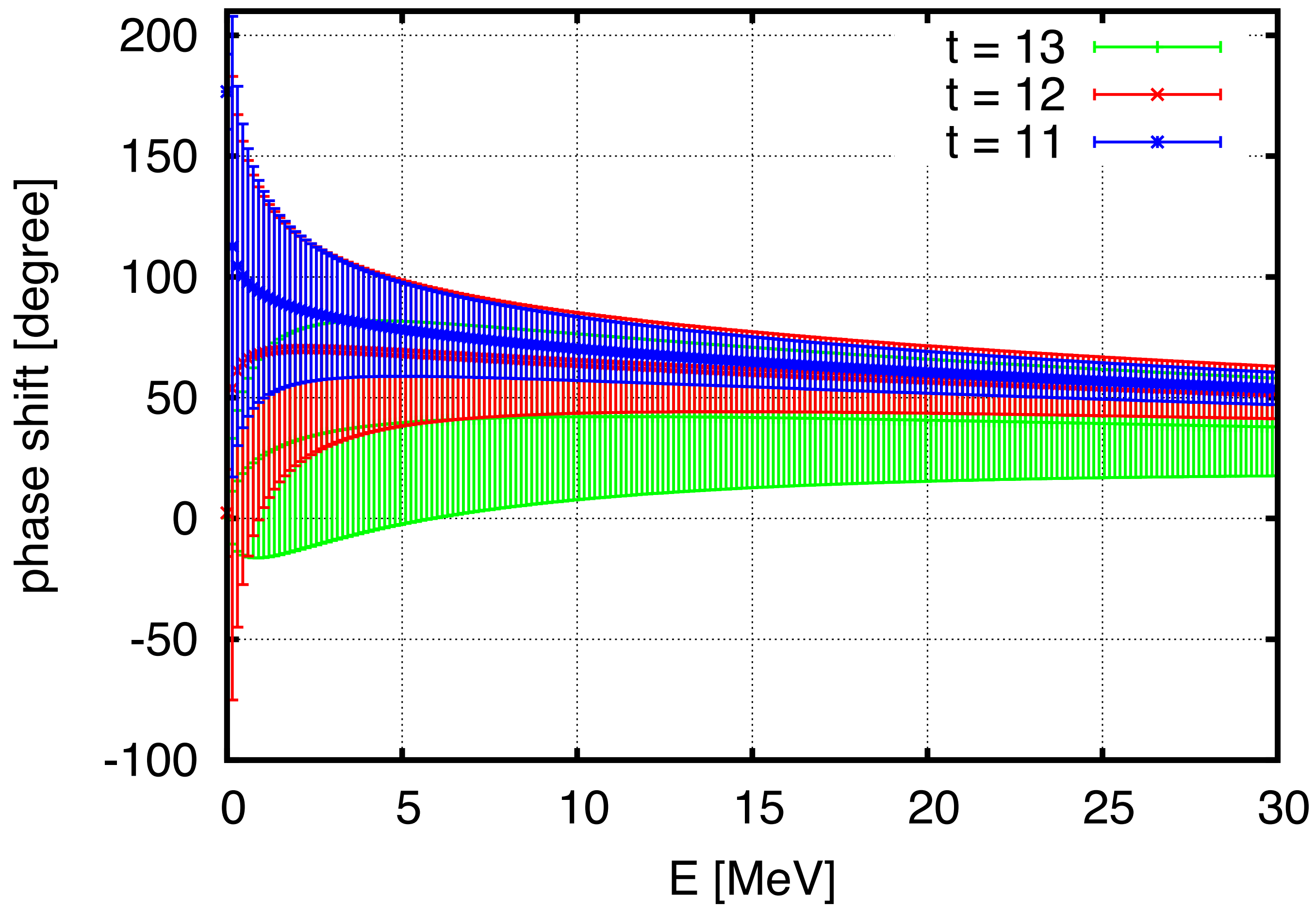
M. Yamada, *et al.*, PTEP2015(2015)071B01

$m_\pi \simeq 700$ MeV, $L \simeq 2.9$ fm, $a \simeq 0.09$ fm

potential



phase shift



NN-like structure, strong attraction → **about to be bound**
"Unitary region"

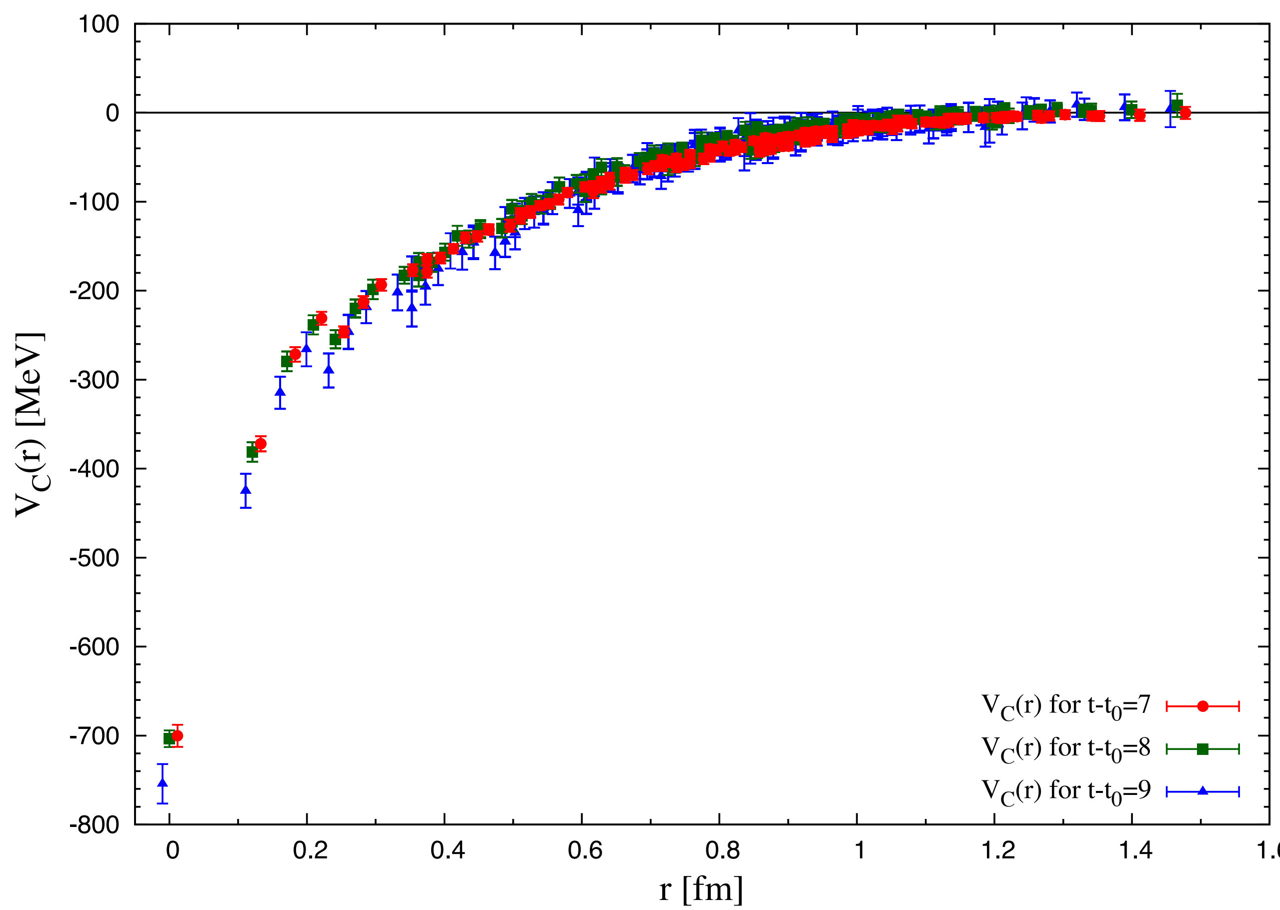
physical point ? → **Doi's talk**

$N\Omega$

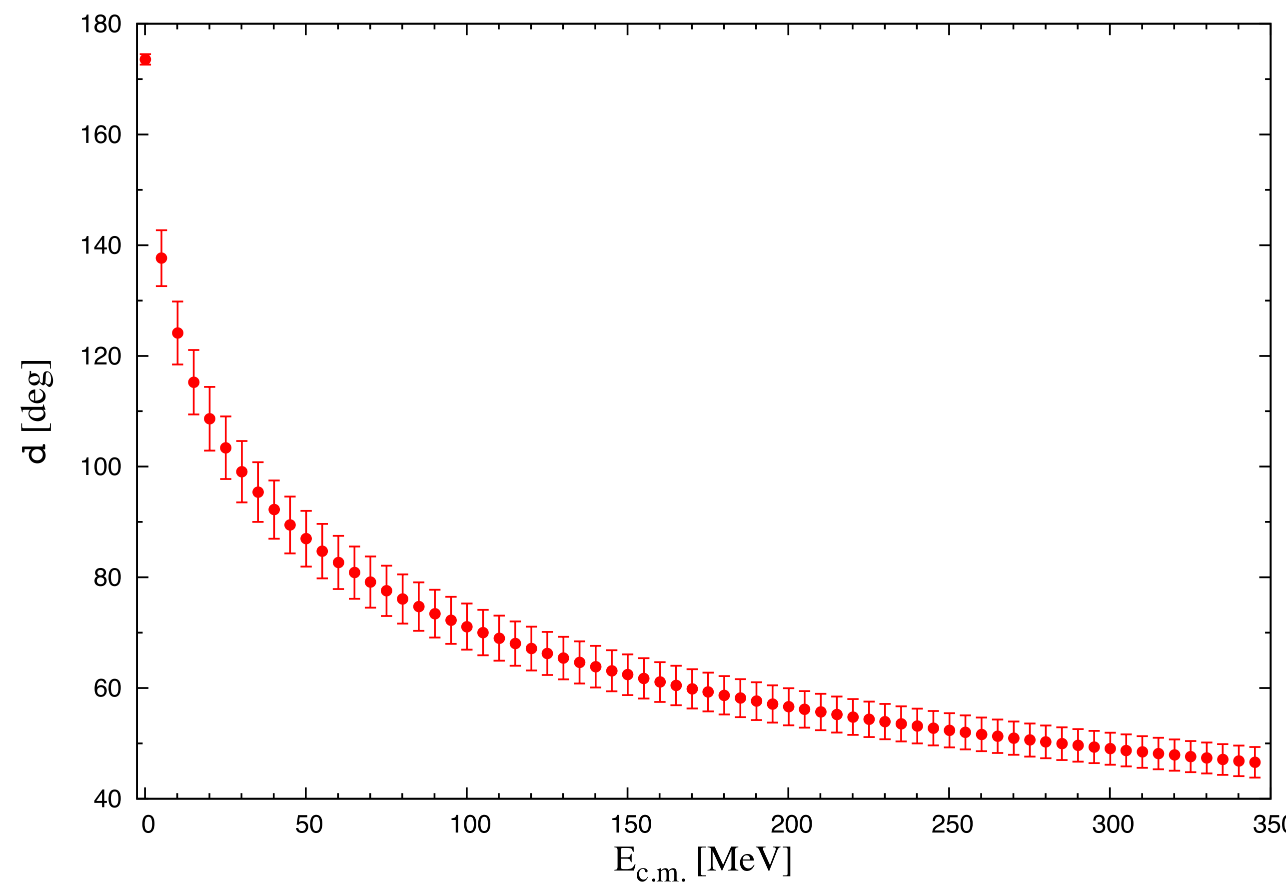
F. Etminan, *et al.*, NPA928(2014)89

$m_\pi \simeq 875$ MeV, $L \simeq 1.9$ fm, $a \simeq 0.12$ fm

potential



phase shift



attractive potential
without repulsive core

a bound state

$$
\begin{aligned}
B_{N\Omega} &= 18.9(5.0)(^{+12.1}_{-1.8}) \text{ MeV}, \\
a_{N\Omega} &= -1.28(0.13)(^{+0.14}_{-0.15}) \text{ fm}, \\
(r_e)_{N\Omega} &= 0.499(0.026)(^{+0.029}_{-0.048}) \text{ fm}.
\end{aligned}
$$

Back-up:
Some applications to nuclear physics

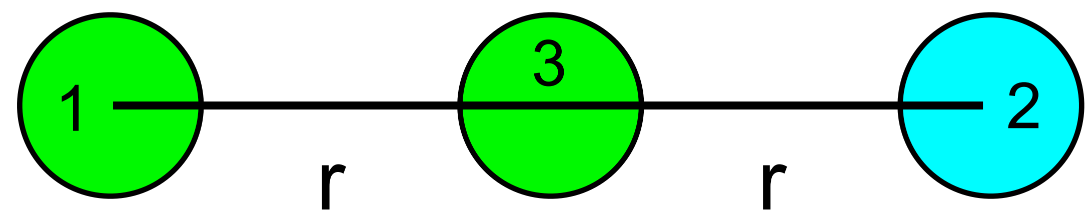
Three nuclear force (3NF)

Doi et al. (HAL QCD), PTP 127 (2012) 723

$$V_{3N}(\mathbf{x}_1-\mathbf{x}_3,\mathbf{x}_2-\mathbf{x}_3)=\sum_{i<j}V_{2N}(\mathbf{x}_i-\mathbf{x}_j)+V_{3NF}(\mathbf{x}_1-\mathbf{x}_3,\mathbf{x}_2-\mathbf{x}_3)$$

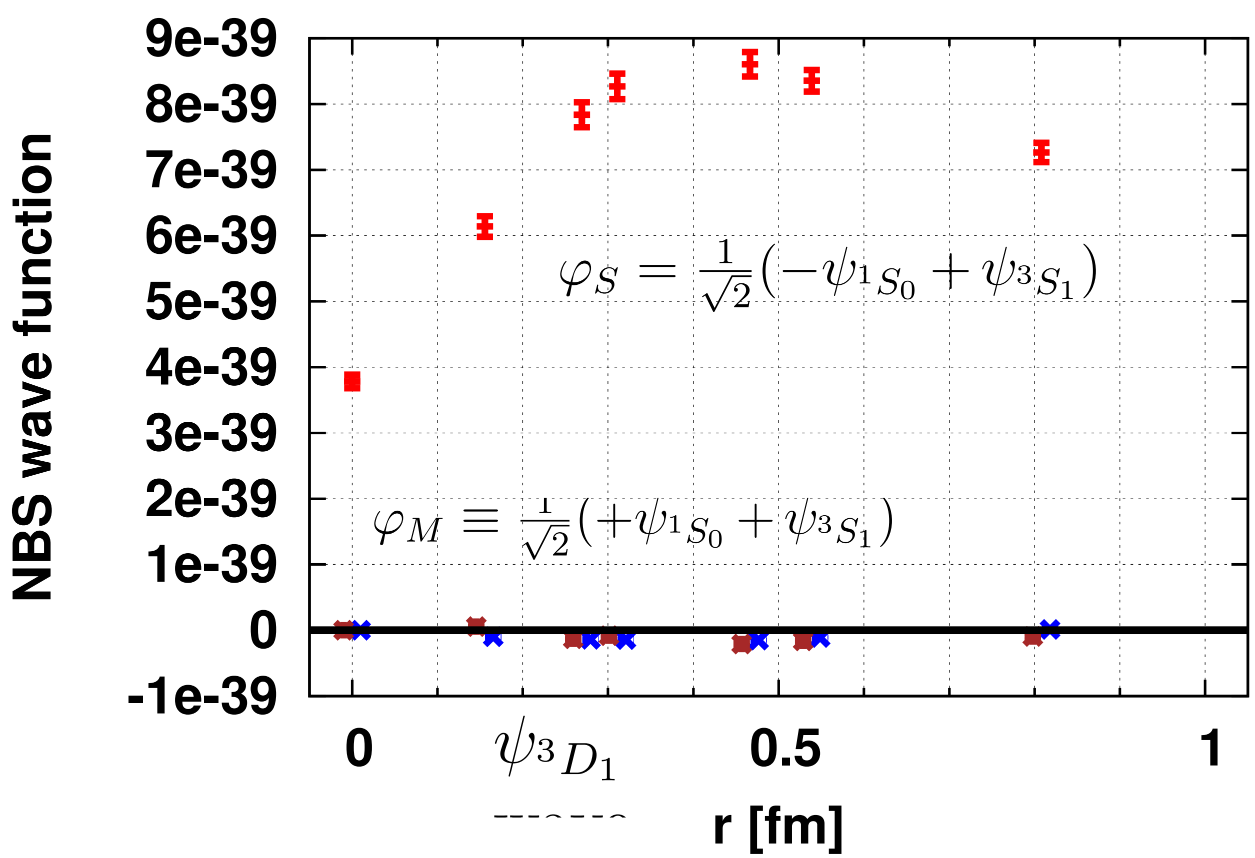
2-body potential   3-body potential

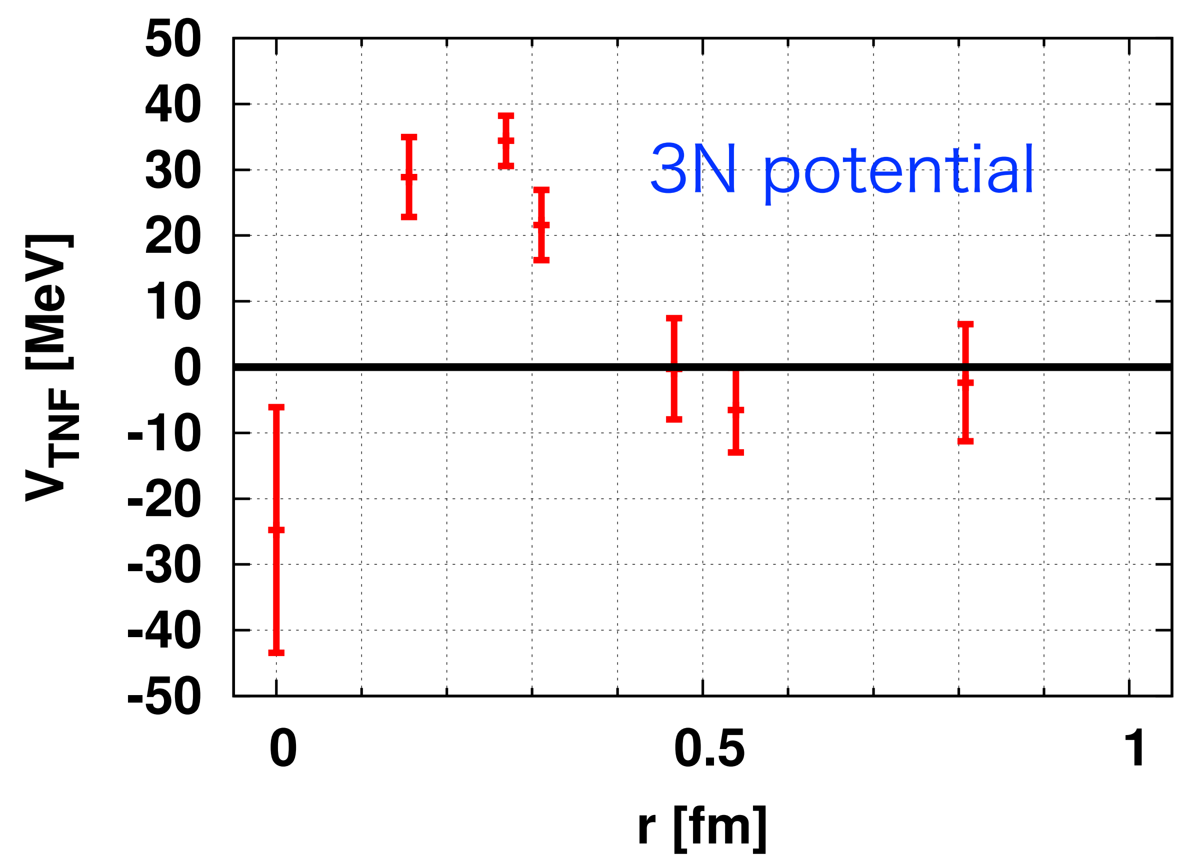
(1,2) pair ${}^1S_0,\ {}^3S_1,\ {}^3D_1$



Linear setup

wave functions



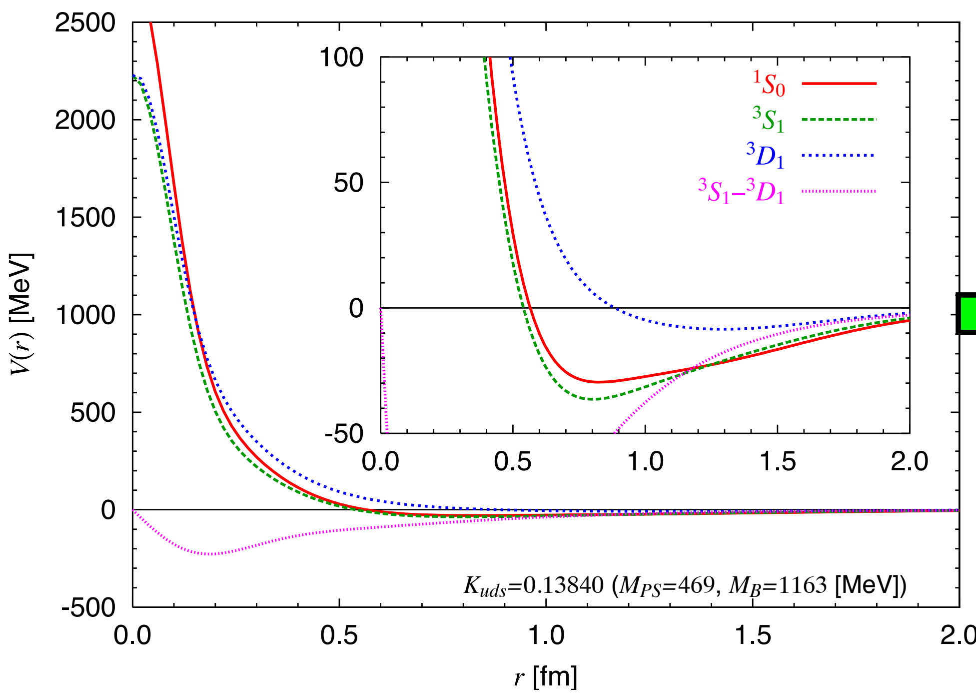


scalar/isoscalar 3NF is seen at short distance.

Equation of State of nuclear matter

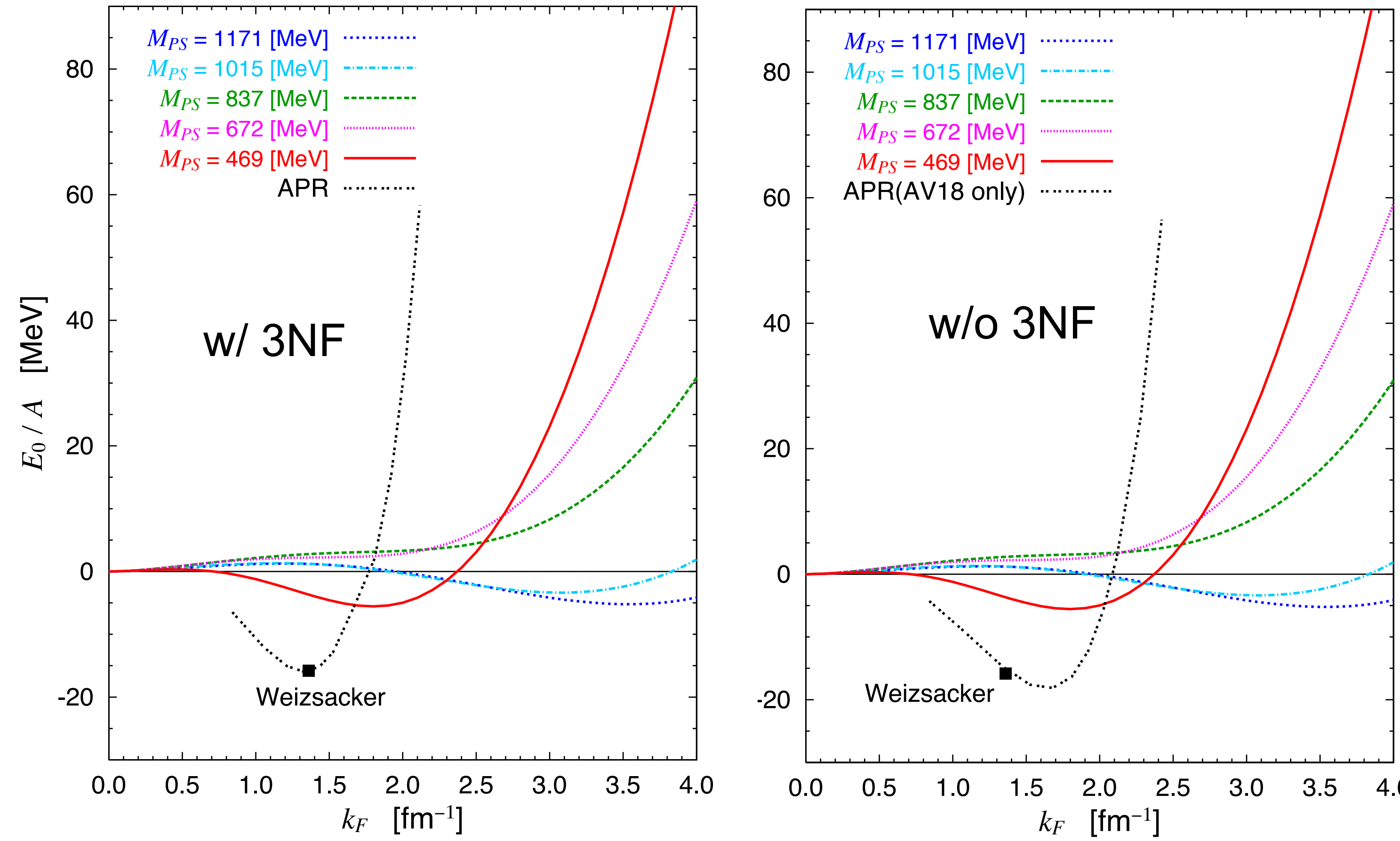
Inoue et al. (HAL QCD Coll.), PRL111(2013)112503

NN potentials $m_\pi = 470$ MeV



BHF

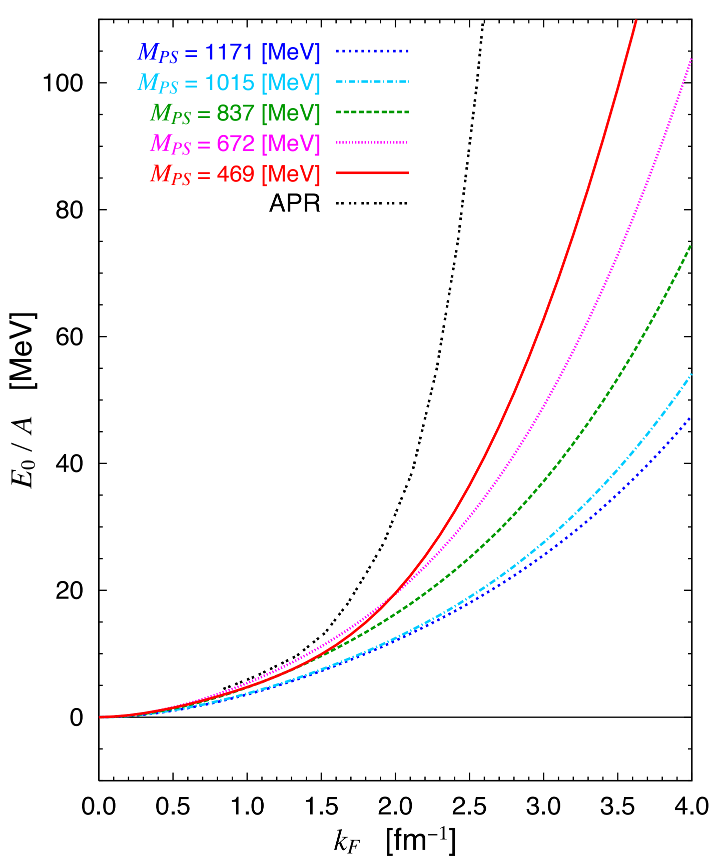
Energy density of Nuclear matter



Fermi momentum

A. Akmal, V.R. Pandharipande, G.G. Ravenhall, Phys. Rev. C58 1804 (1998)

Neutron matter

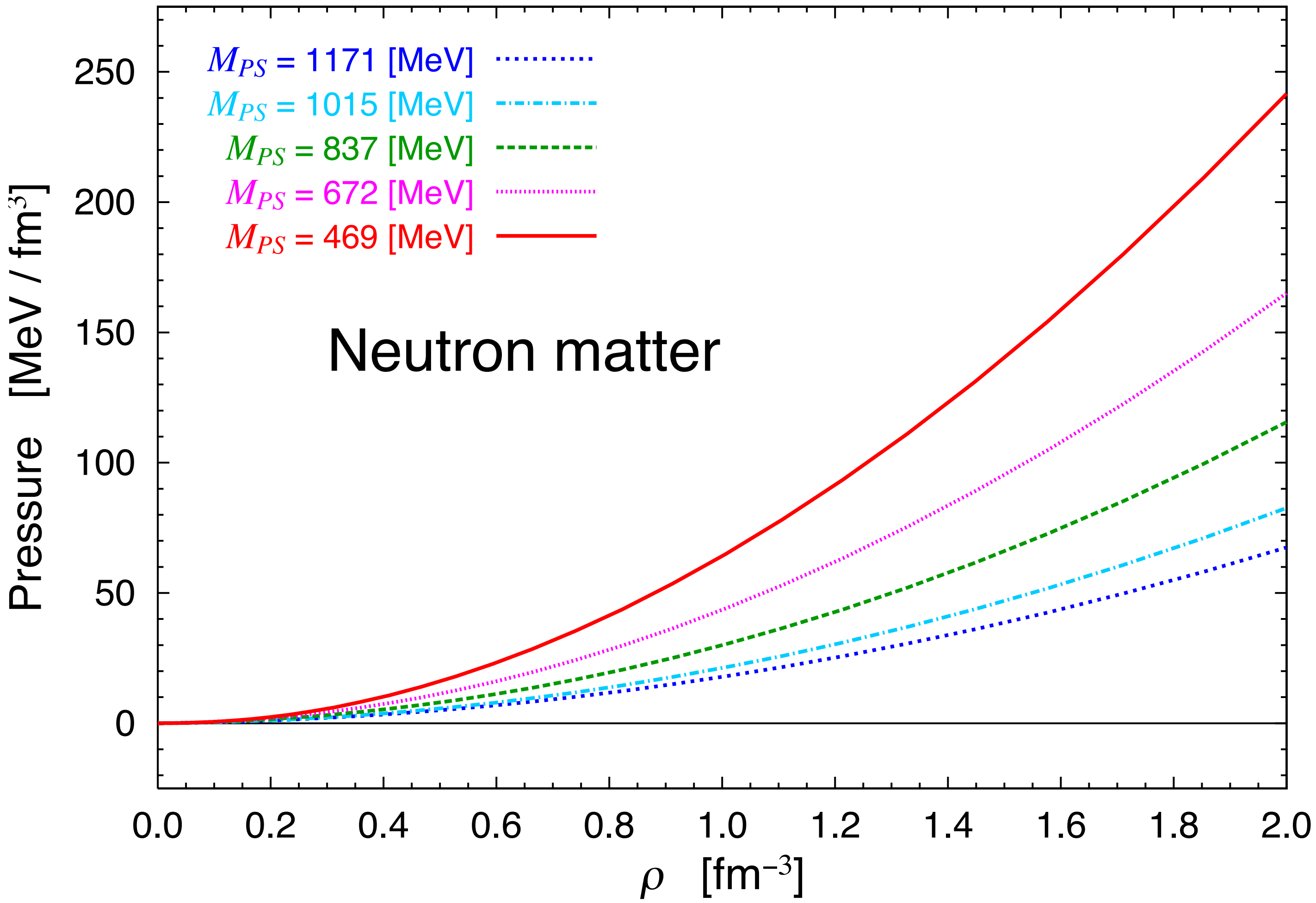


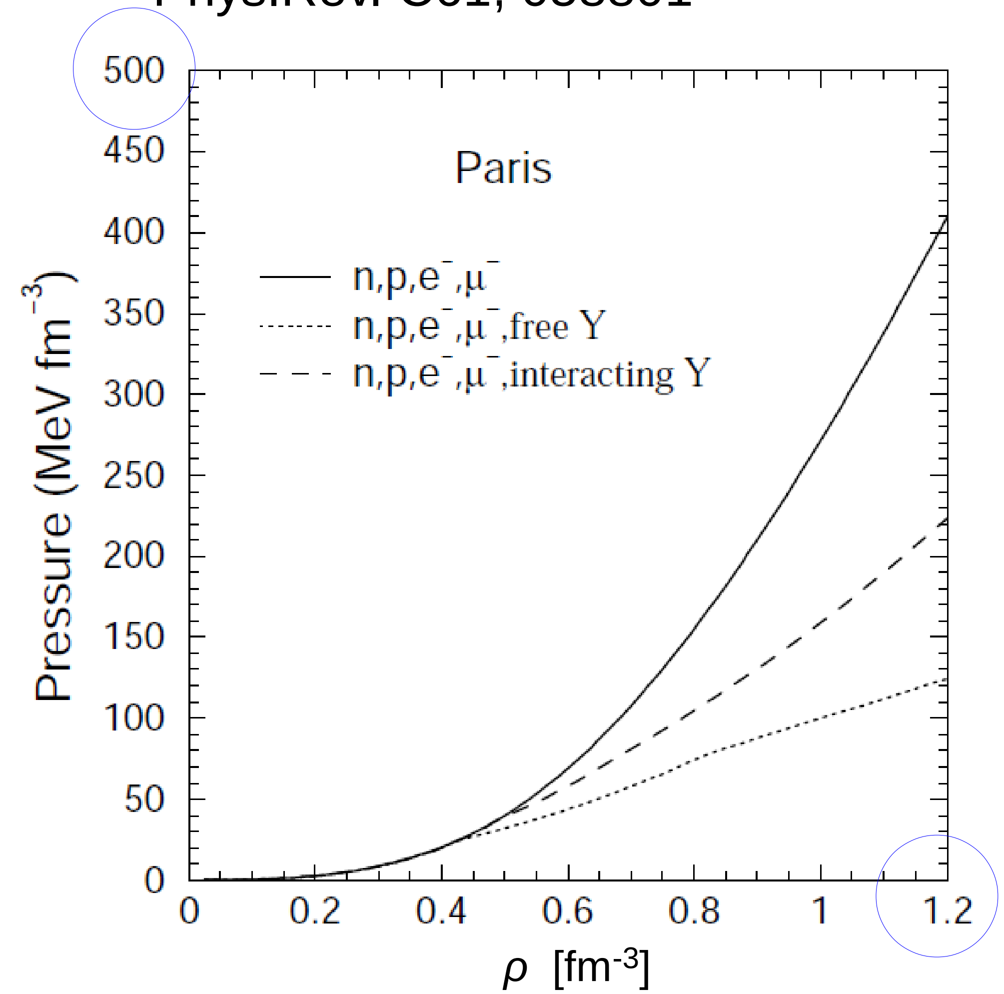
Nuclear matter shows the saturation at the lightest pion mass, but the saturation point deviates from the empirical one obtained by Weizsacker mass formula.

No saturation for Neutron matter.

Pressure of Neutron matter

M. Baldo, F. Burgio, H.-J.Schulze,
Phys.Rev. C61, 058801



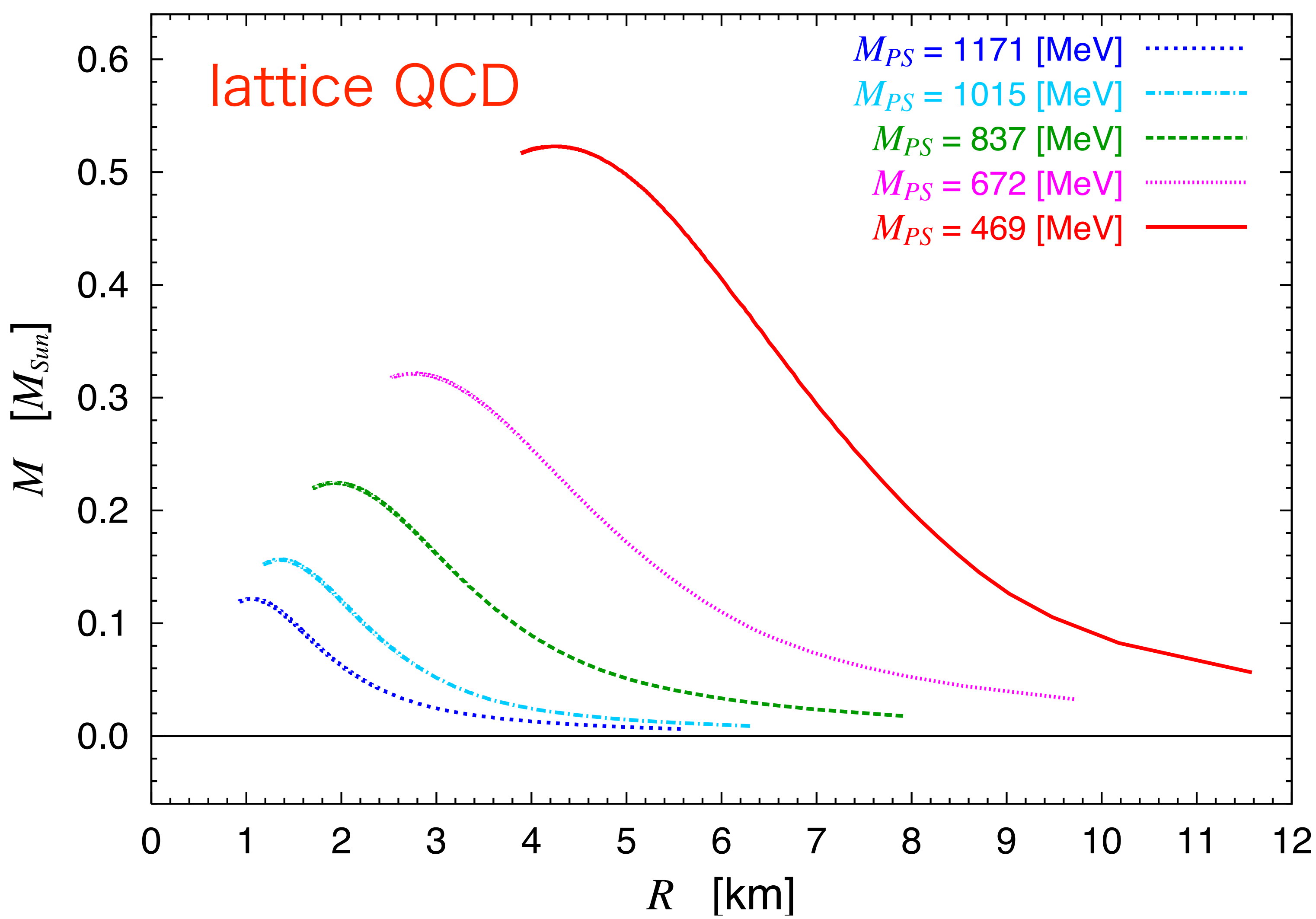


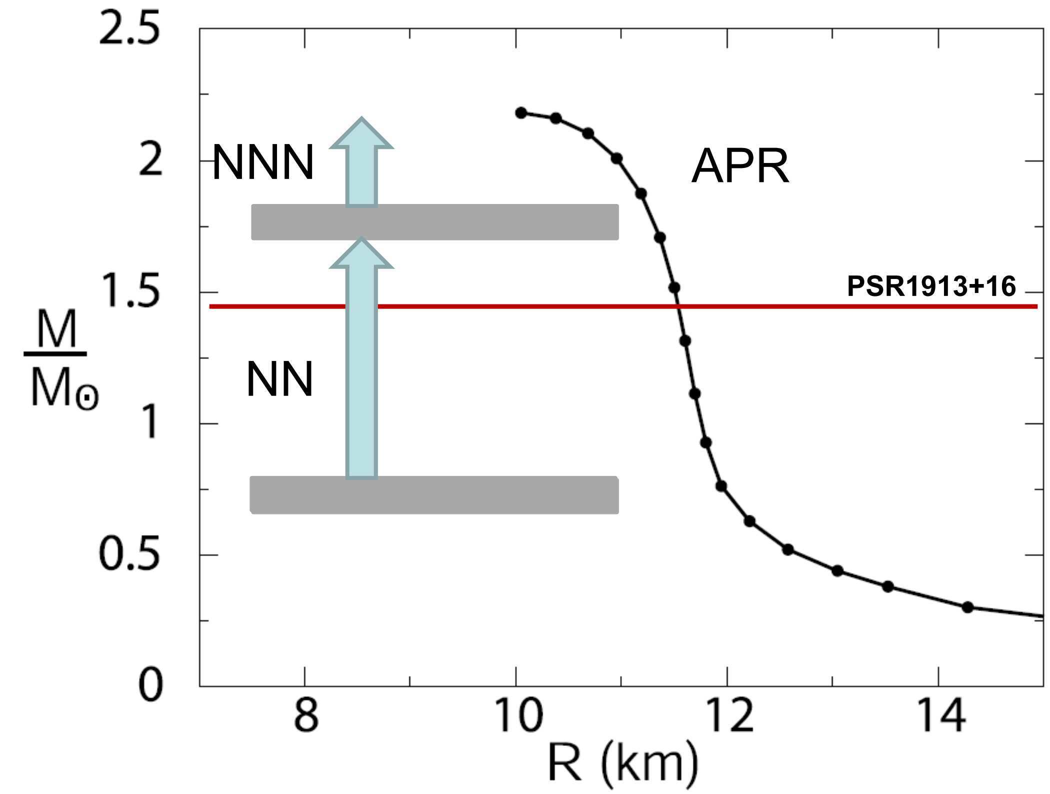
pressure $$P = \rho^2 \frac{d(E_0/A)}{d\rho} = \frac{\gamma k_F^4}{18\pi^2}\frac{d(E_0/A)}{dk_F}$$

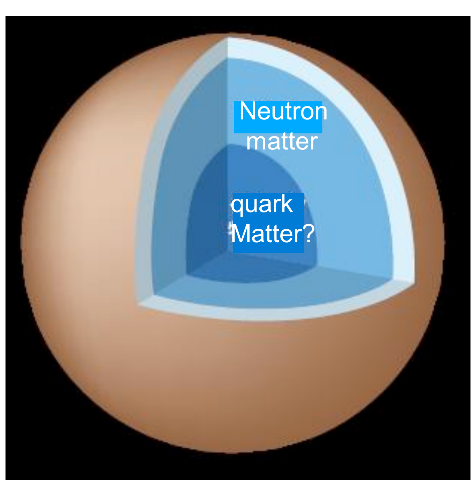
density $$\rho = \frac{\gamma k_F^3}{6\pi^2}$$

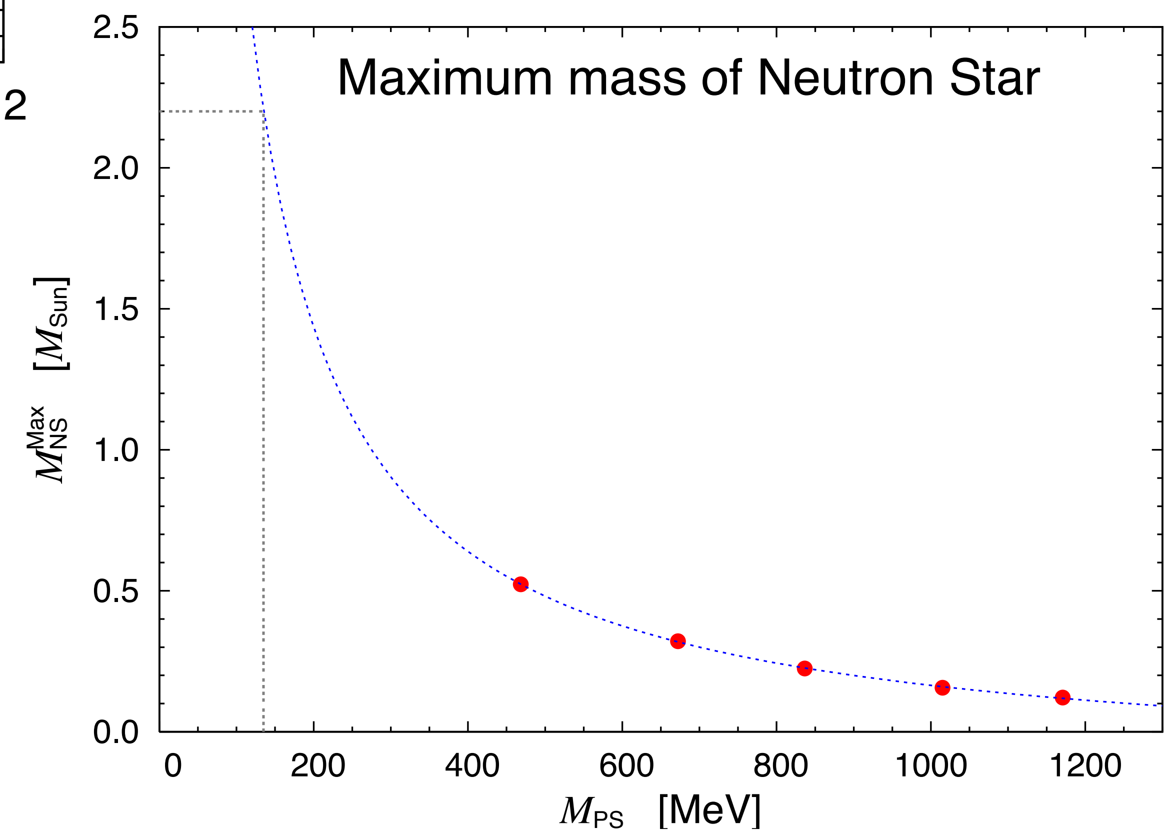
Our Neutron matter becomes harder as the pion mass decreases, but it is still softer than phenomenological models.

Neutron star M-R relation









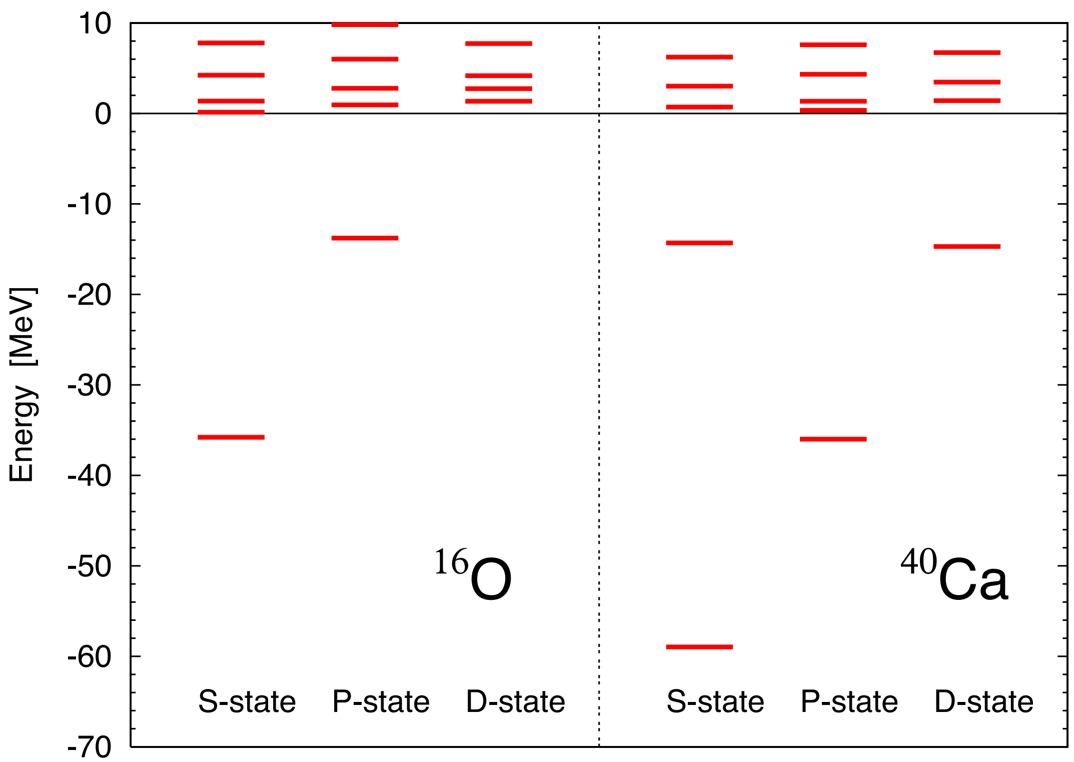
Maximum mass of Neutron vs. pion mass

Medium-Heavy Nuclei

Inoue et al. (HAL QCD Coll.), arXiv:1408.4829[hep-lat]

$m_\pi = 470$ MeV

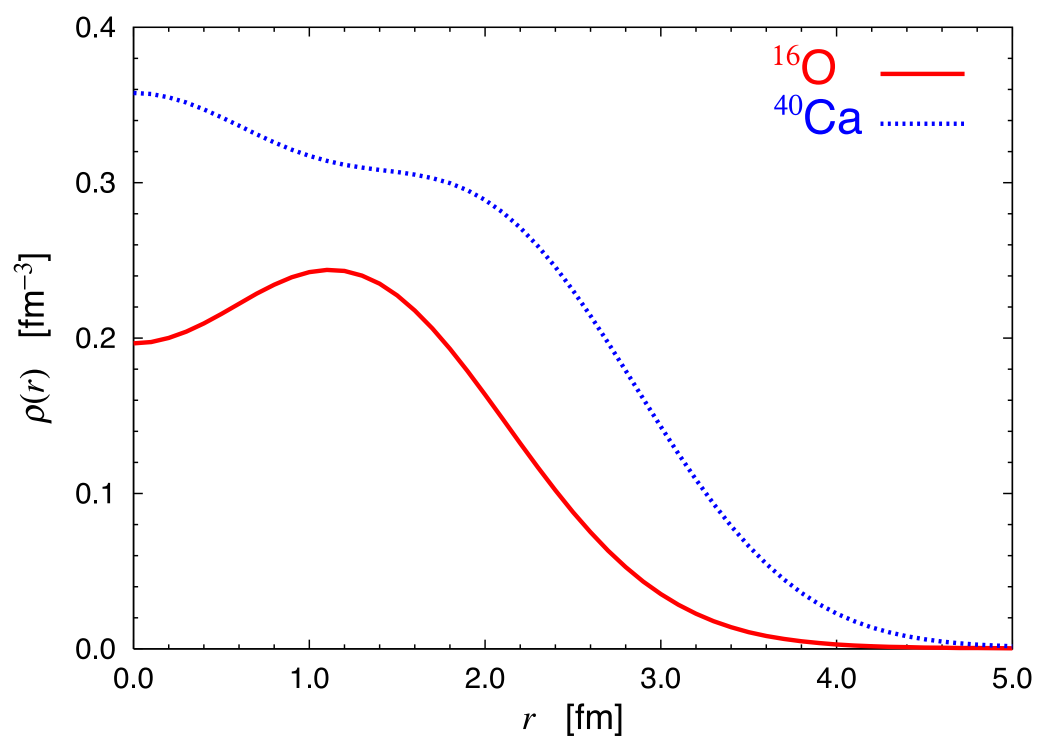
single particle level



	Single particle level				Total energy		Radius
	$1S$	$1P$	$2S$	$1D$	E_0	E_0/A	$\sqrt{\langle r^2 \rangle}$
^{16}O	-35.8	-13.8			-34.7	-2.17	2.35
^{40}Ca	-59.0	-36.0	-14.7	-14.3	-112.7	-2.82	2.78

^{16}O(exp)	-127 MeV	2.73 fm
^{40}Ca(exp)	-342 MeV	3.48 fm

Nucleon number distribution



E_0/A vs $A^{-1/3}$ form lattice QCD