# Cusps, Resonances, and Exotic States

Eric Swanson

# Multi-electron States

1946: Wheeler suggests that $Ps_2$ might be bound

Wheeler, J. A. Polyelectrons. Ann. NY Acad. Sci. 48, 219–238 (1946).

1946: Ore proves it is unbound

1947: Hylleraas & Ore prove it is bound

Hylleraas, E. A. & Ore, A. Binding energy of the positronium molecule. Phys. Rev.71, 493–496 (1947).

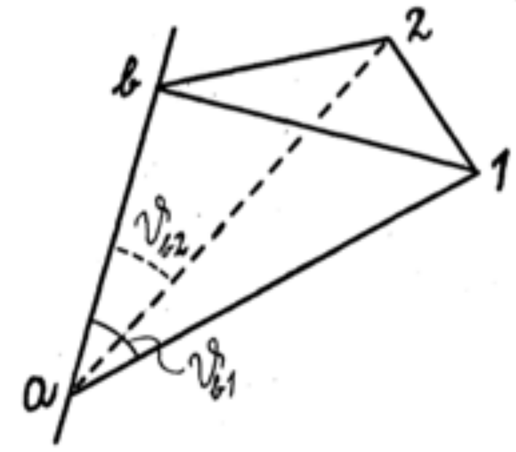



FIG. 1. Coordinate system for the positronium molecule.

2007: $Ps_2$ is observed

Cassidy, D.B.; Mills, A.P. (Jr.) (2007). "The production of molecular positronium". Nature 449 (7159): 195–197

# Multi-quark States

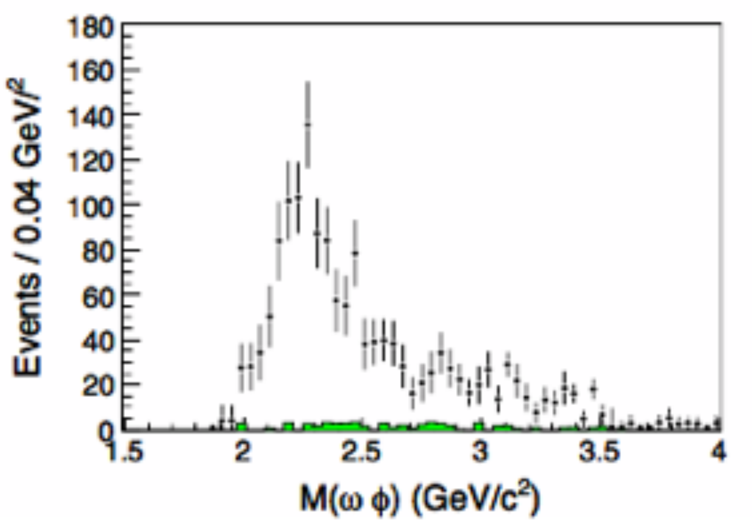
$$\mathcal{L}_{QED} \longrightarrow$$

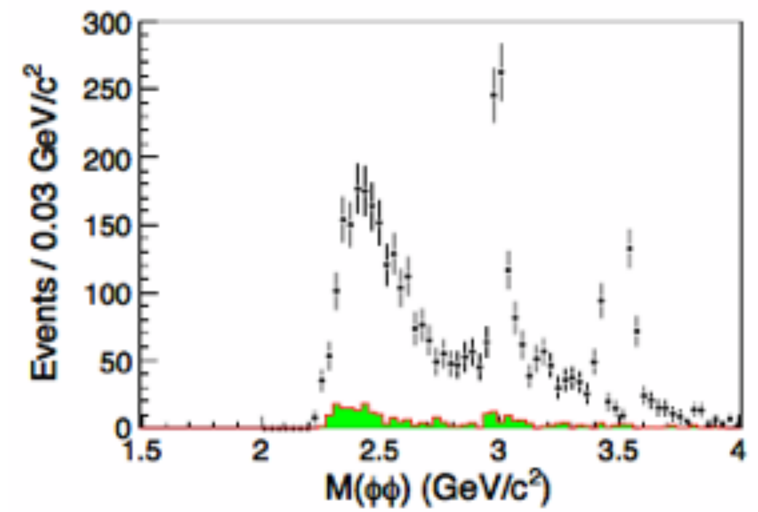
$$\{\tfrac{1}{2}[\nabla_1{}^2+\nabla_2{}^2+\nabla_a{}^2+\nabla_b{}^2]+E/4+V\}\Psi=0,$$

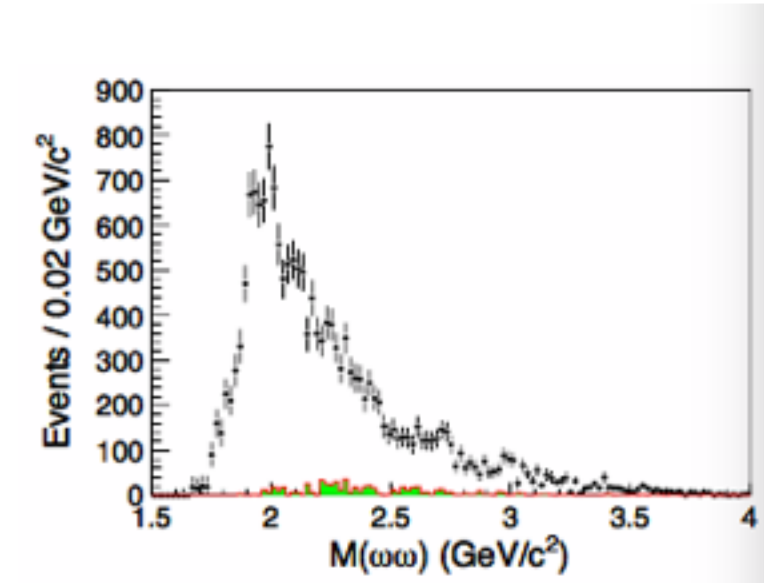
$$V=\frac{1}{r_{1a}}+\frac{1}{r_{2b}}+\frac{1}{r_{1b}}+\frac{1}{r_{2a}}-\frac{1}{r_{12}}-\frac{1}{r_{ab}}.$$

$$\mathcal{L}_{QCD} = \bar{\psi}_i\left(i(\gamma^\mu D_\mu)_{ij} - m\,\delta_{ij}\right)\psi_j - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a \qquad \longrightarrow$$

# Thresholds



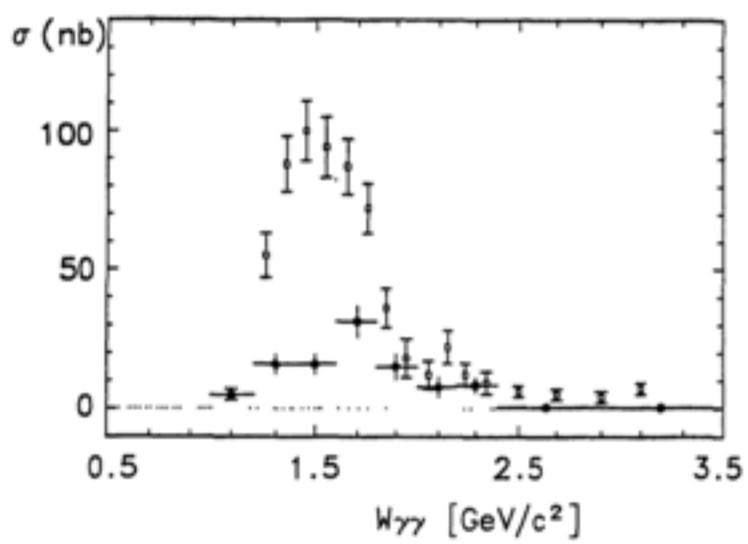
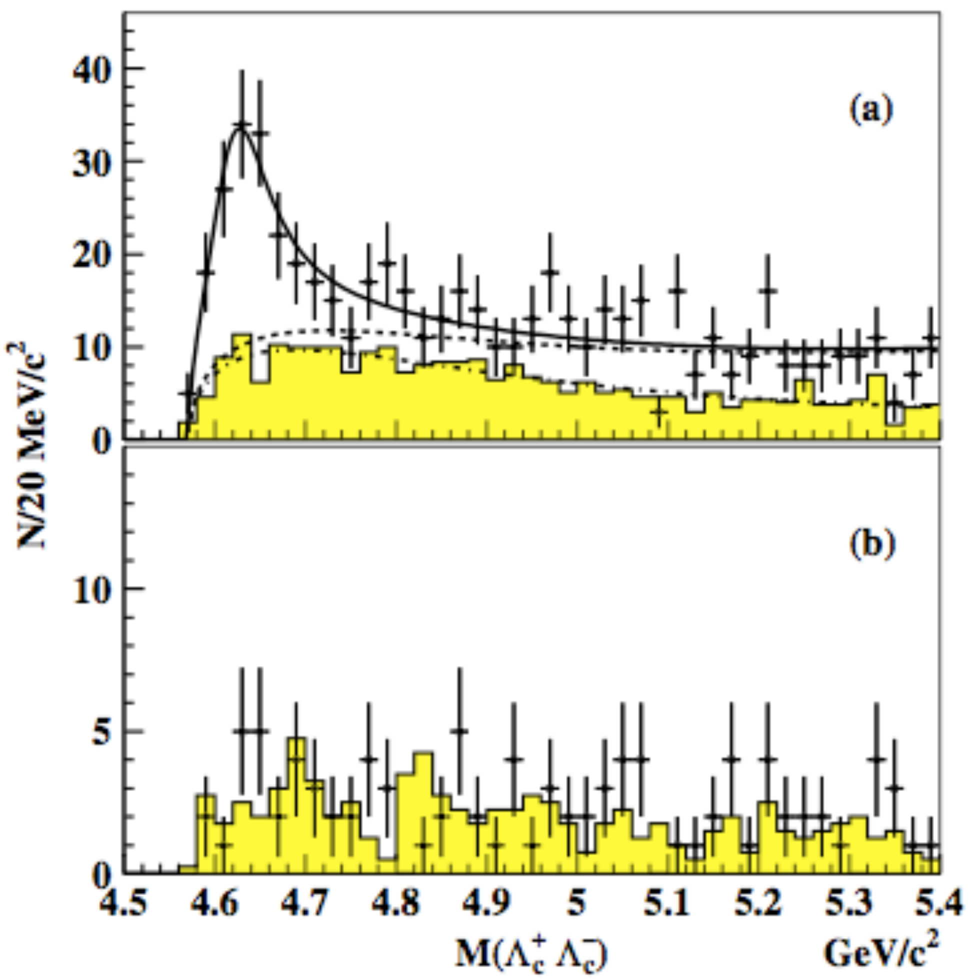
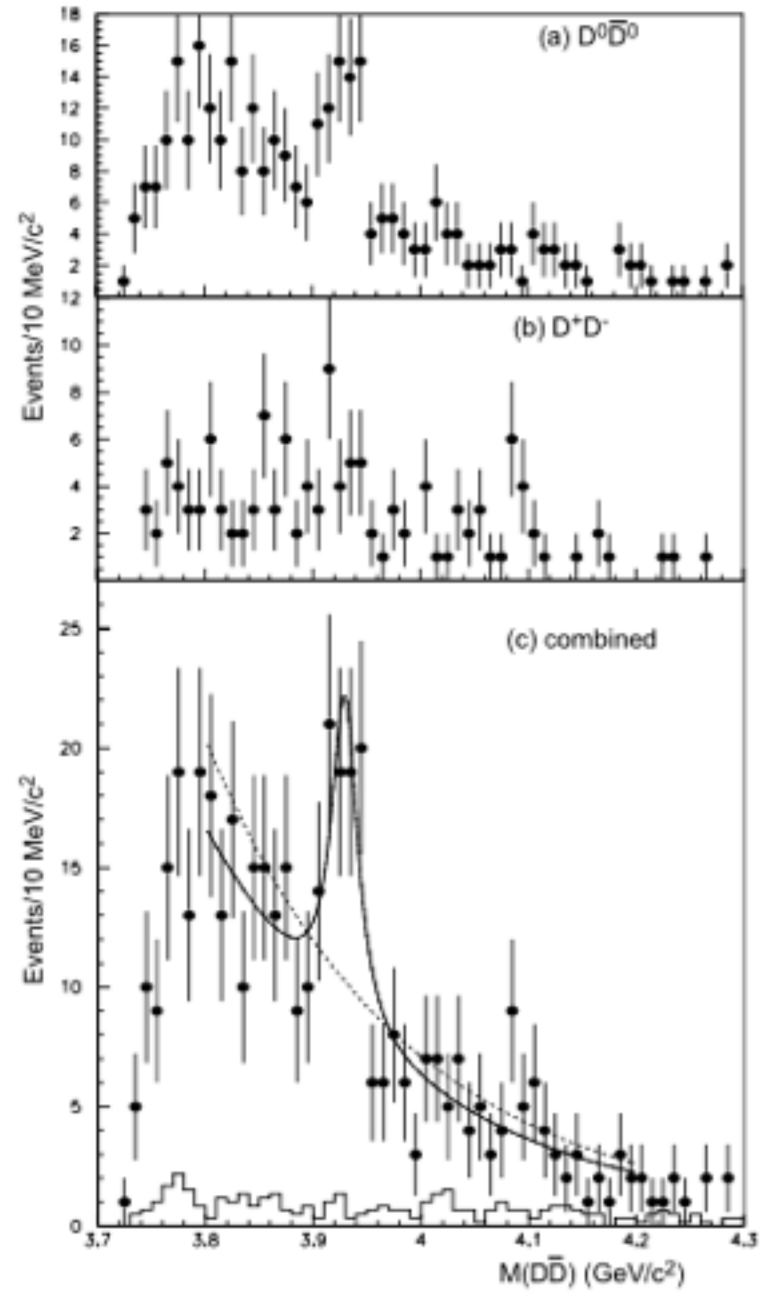




Figure 5.2: Comparison of the $\gamma\gamma \to \rho\rho$ measured cross sections. The reaction $\gamma\gamma \to \rho^0\rho^0$ is presented as squares and is the measurement by PLUTO [11] and the reaction $\gamma\gamma \to \rho^+\rho^-$ as full dots.

# $X(4630)$

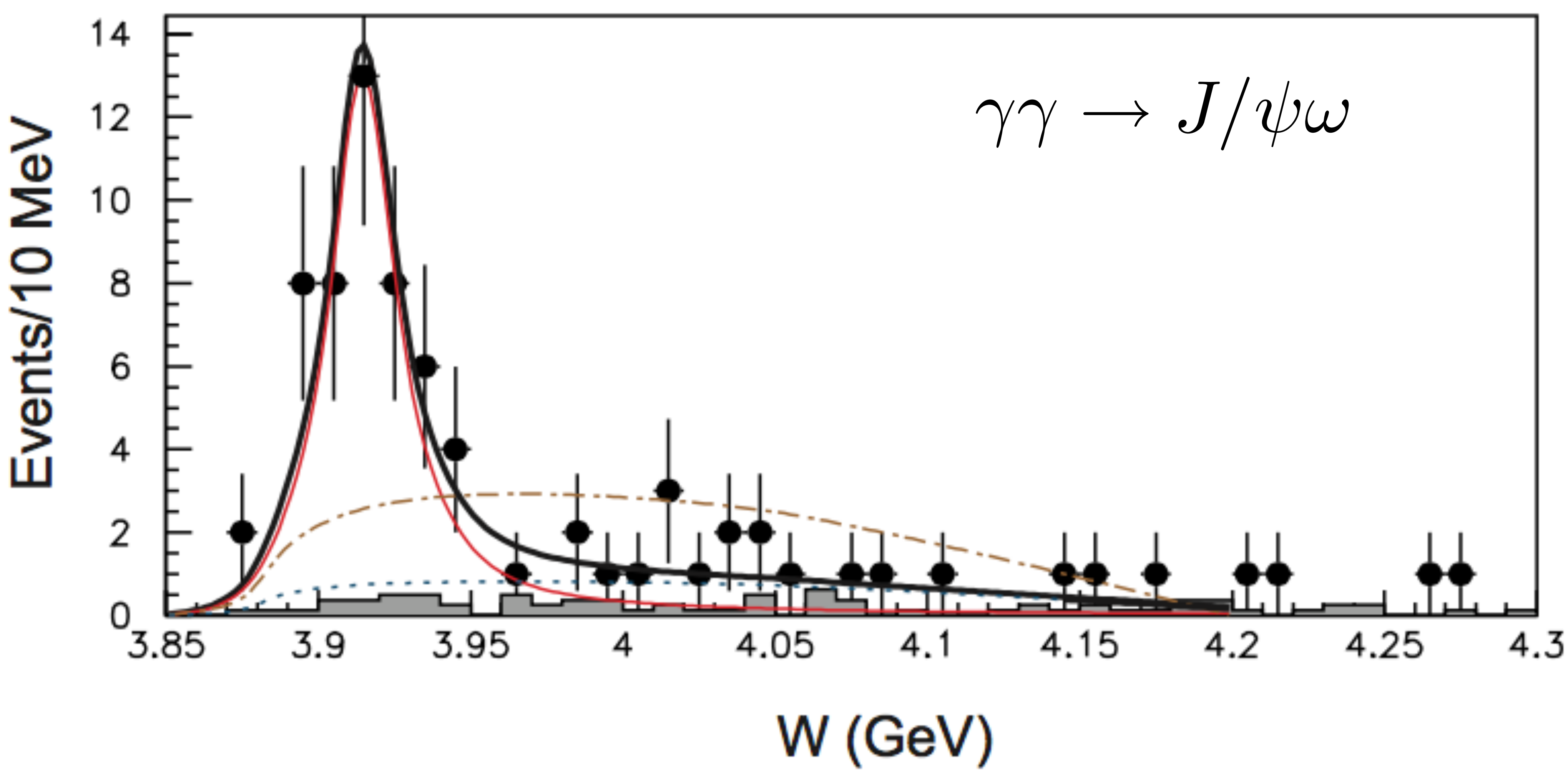


(a)
(b)
N/20 MeV/c²
M(Λc+ Λc−)
GeV/c²

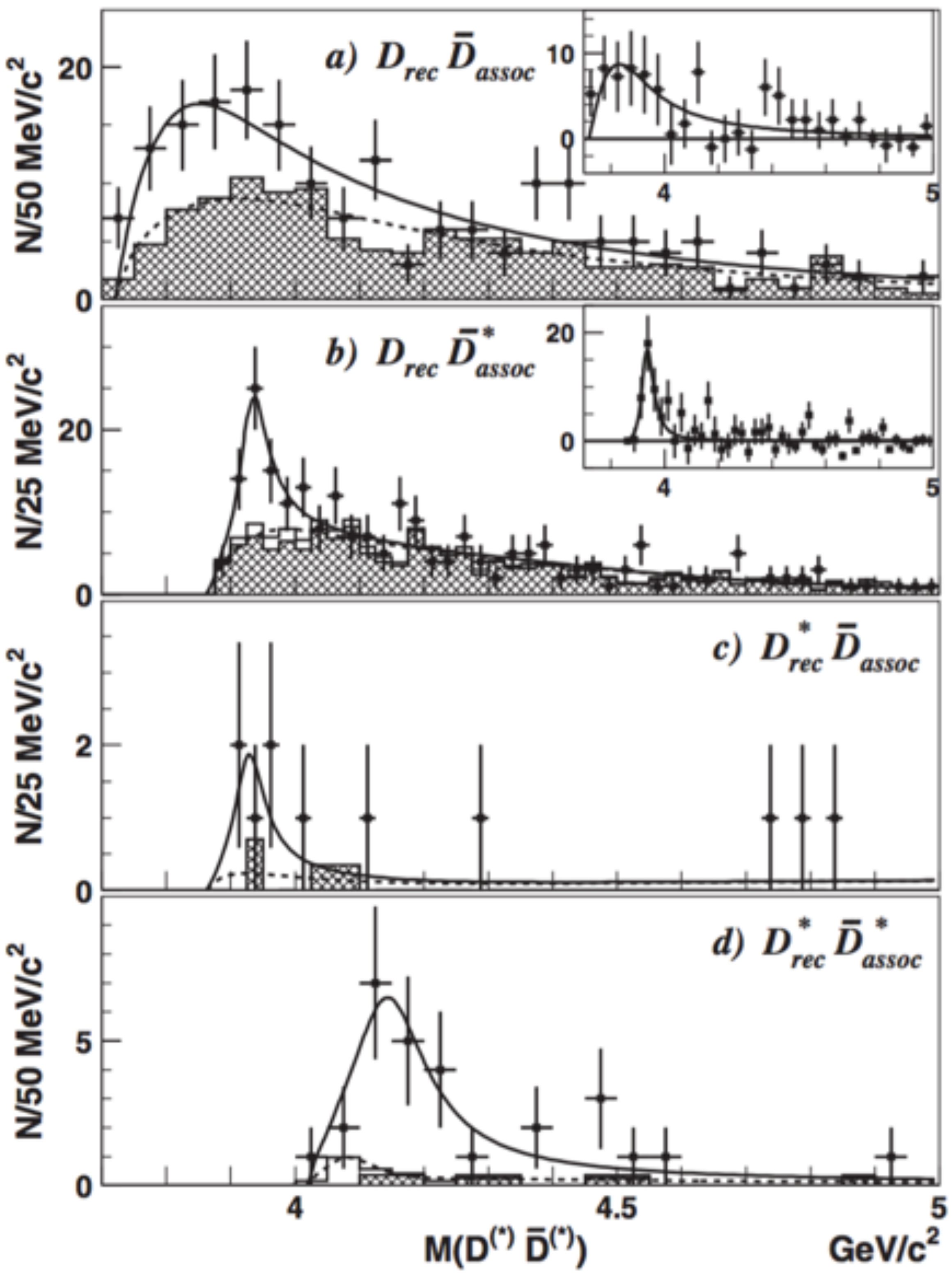



(a) D⁰D̄⁰
(b) D⁺D⁻
(c) combined
Events/10 MeV/c²
Events/10 MeV/c²
M(DD̄) (GeV/c²)

*not* a threshold enhancement (?)

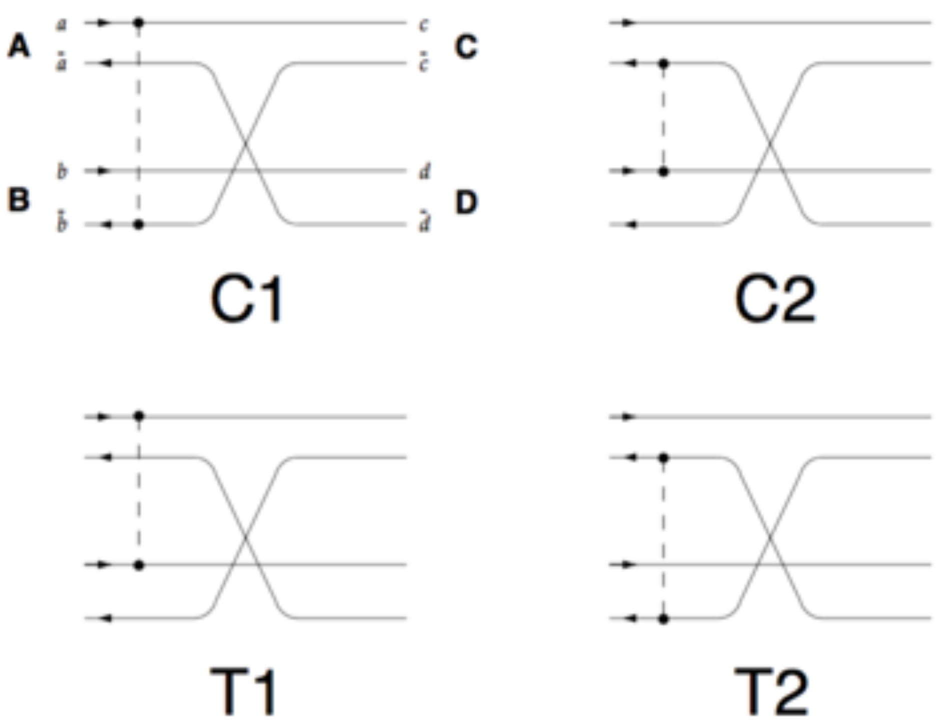


$$e^+e^- \to J/\psi D^{(*)}\bar{D}^{(*)}$$

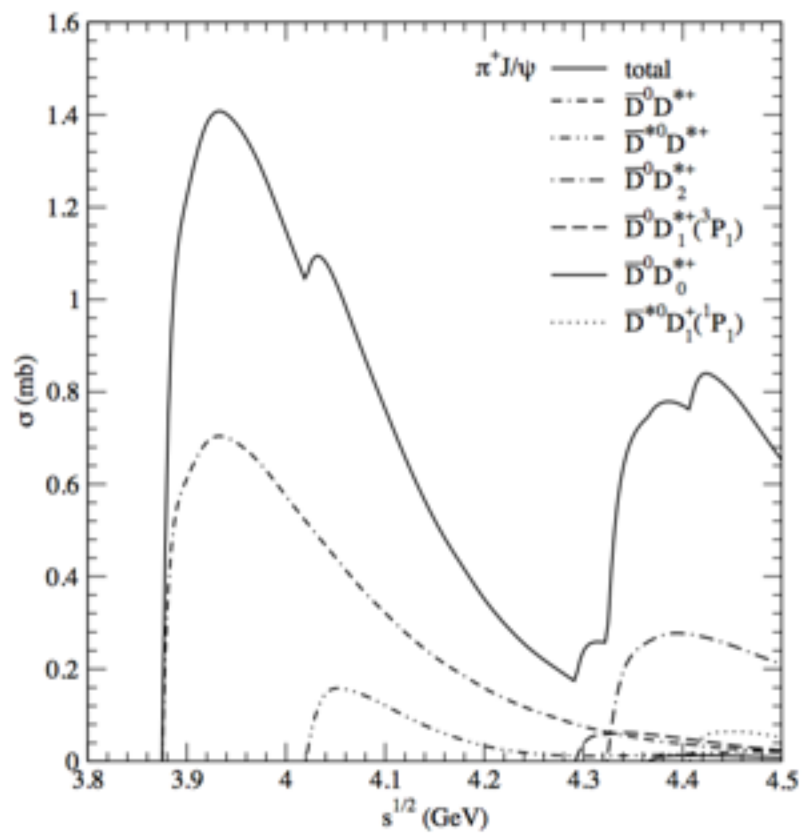



[Belle] PRL100, 202001 (08)

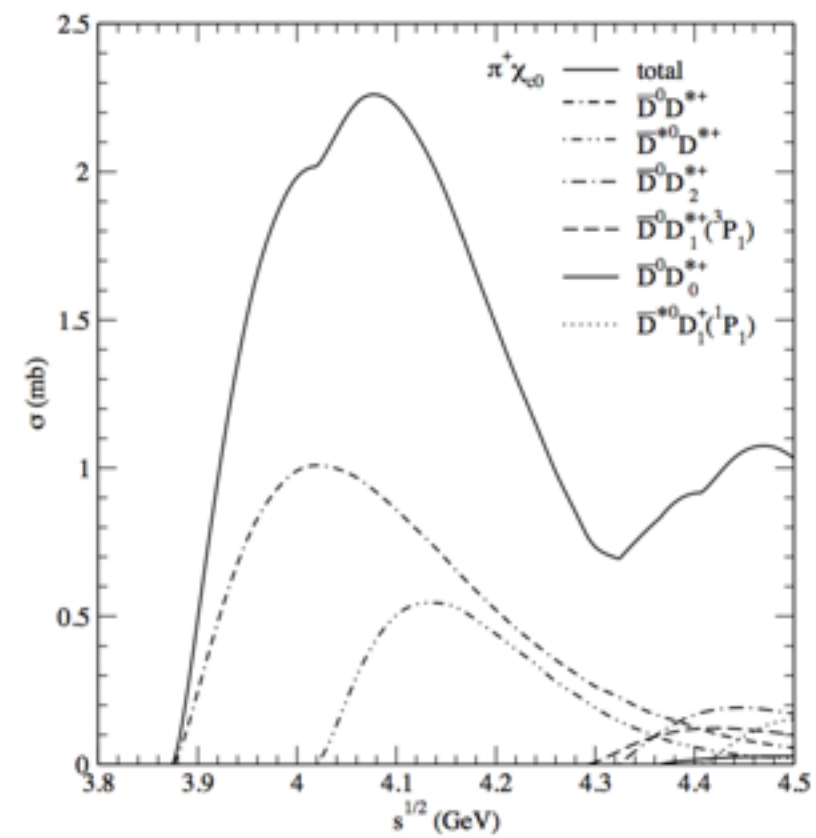
# A Quark Model Example




S-wave




P-wave



# A Quark Model Example

$$\sigma(s) = \sigma_{max}\left(\frac{\epsilon}{\epsilon_{max}}\right)^p \exp[p(1-\epsilon/\epsilon_{max})]$$

$p = 1/2 + L_{min}^{CD}$ endothermic

$p = -1/2 + L_{min}^{CD}$ exothermic

$$\epsilon = \sqrt{s} - M_C - M_D$$

$\epsilon_{max} \sim (0.2 - 0.5)\,\Lambda_{\rm QCD}$ scale

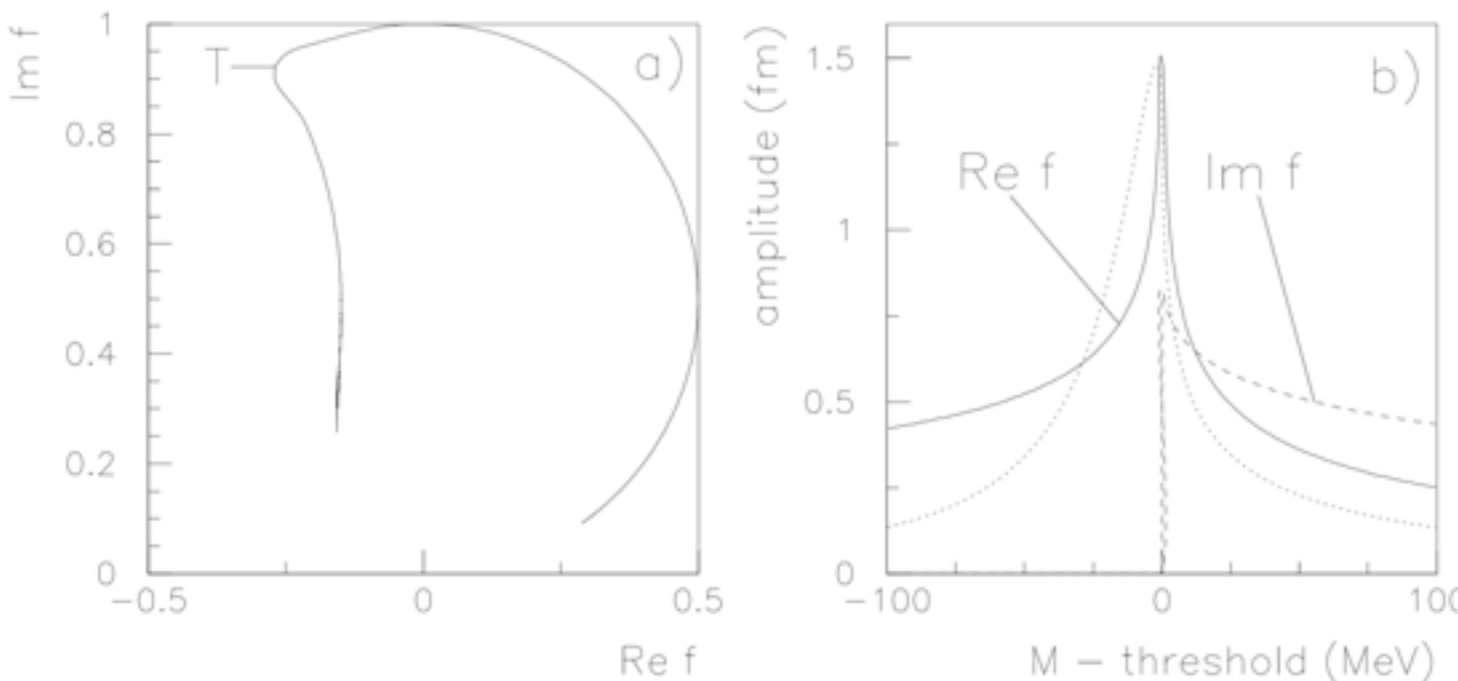
# Cusps

E.P. Wigner, Phys. Rev. 73 (1948) 1002

D. V. Bugg, Europhys. Lett. 96, 11002 (2011)

D. V. Bugg, Int. J. Mod. Phys. A 24, 394 (2009)

# f0(980) example with effective range parameterization of the amplitude range [Bugg]



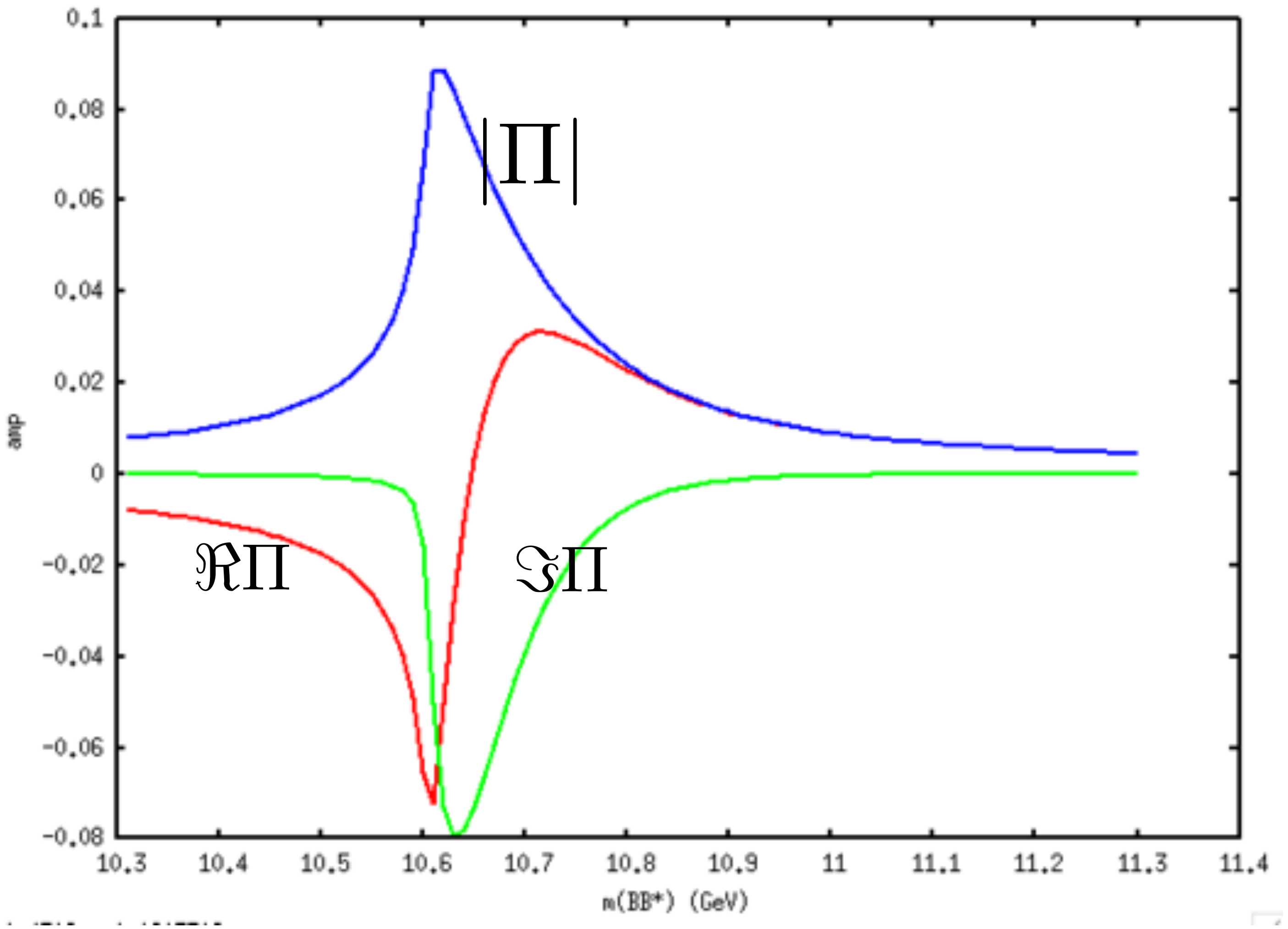

## Other examples

$$K^- d \to \pi^-(\Lambda p) \qquad \bar{p}p \to \bar{\Lambda}\Lambda \qquad J/\psi \to \gamma\bar{p}p$$

# Example

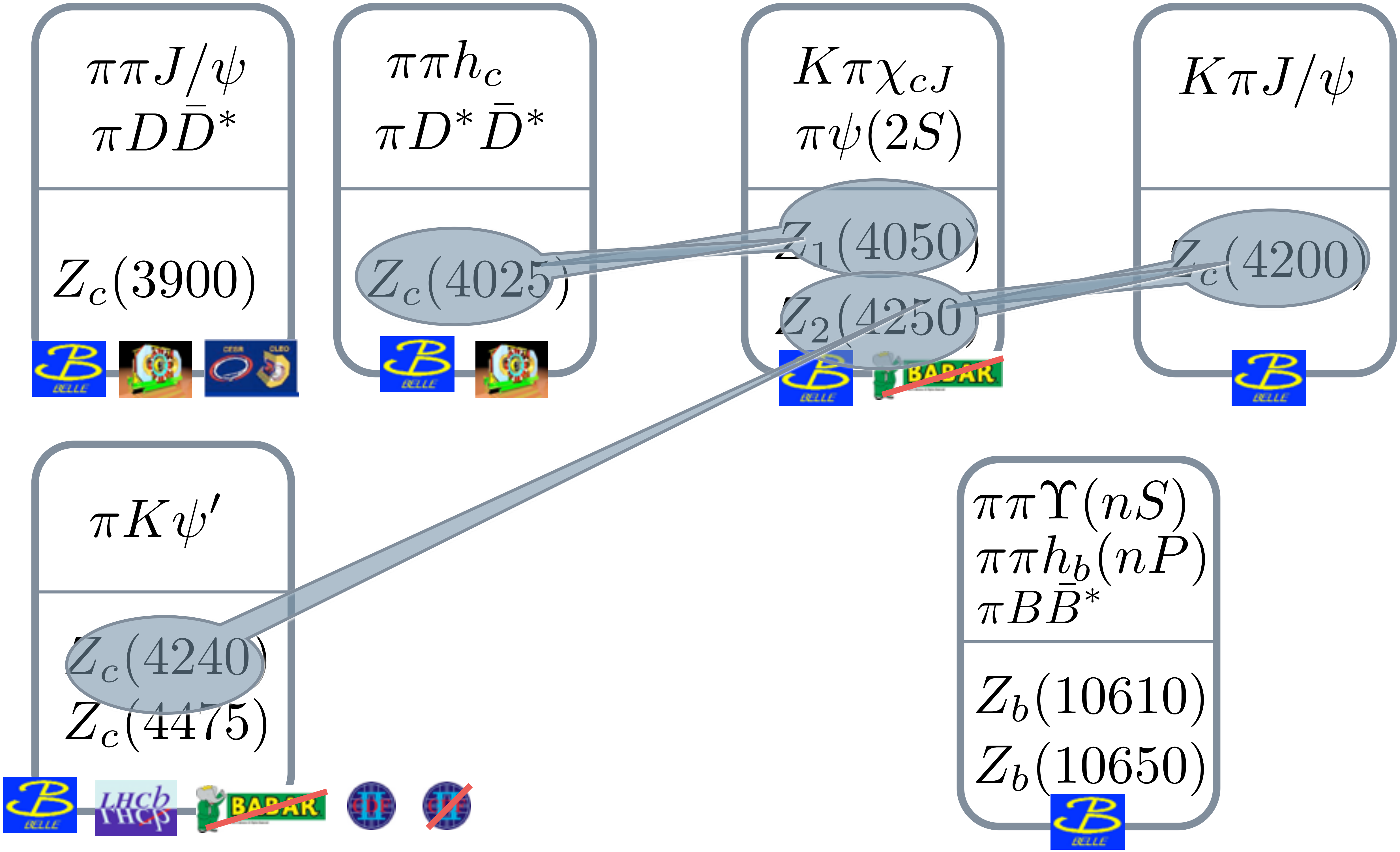
$$\Pi = \int \frac{d^3q}{(2\pi)^3} \frac{\mathrm{e}^{-q^2/\Lambda^2}}{E - m_B - m_{B^*} - q^2/2\mu + i\epsilon}$$

$\Lambda = 0.5$ GeV



# Exotic Experiment

# four-quark states(?)

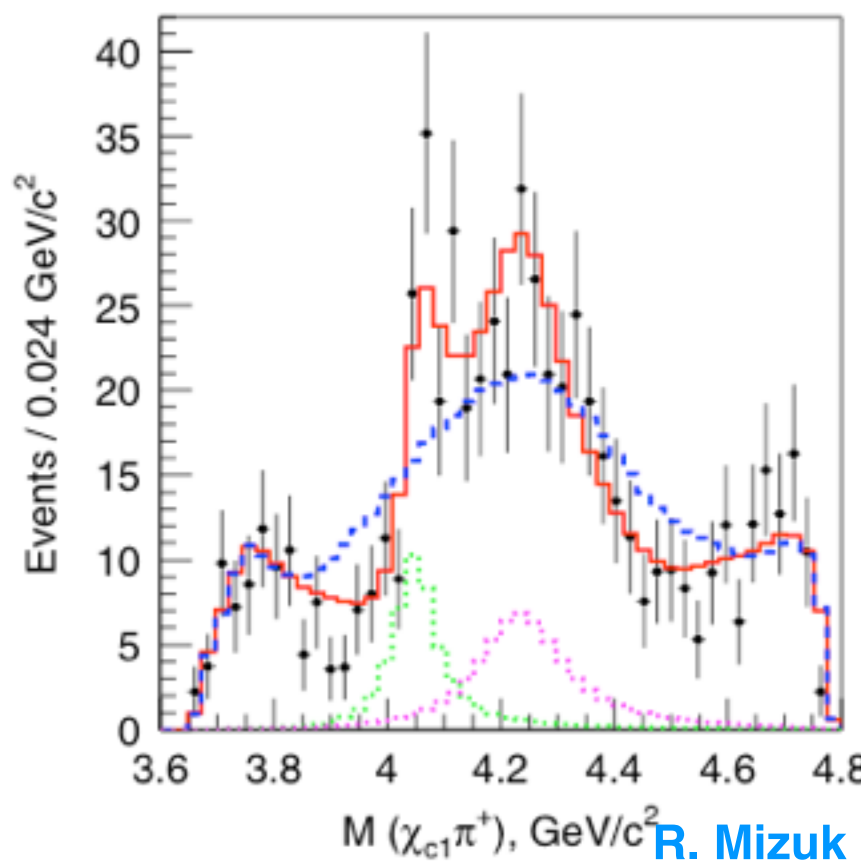


$Z_1(4050)$
$Z_2(4250)$

$B \to ZK \to \chi_{c1}\pi K$

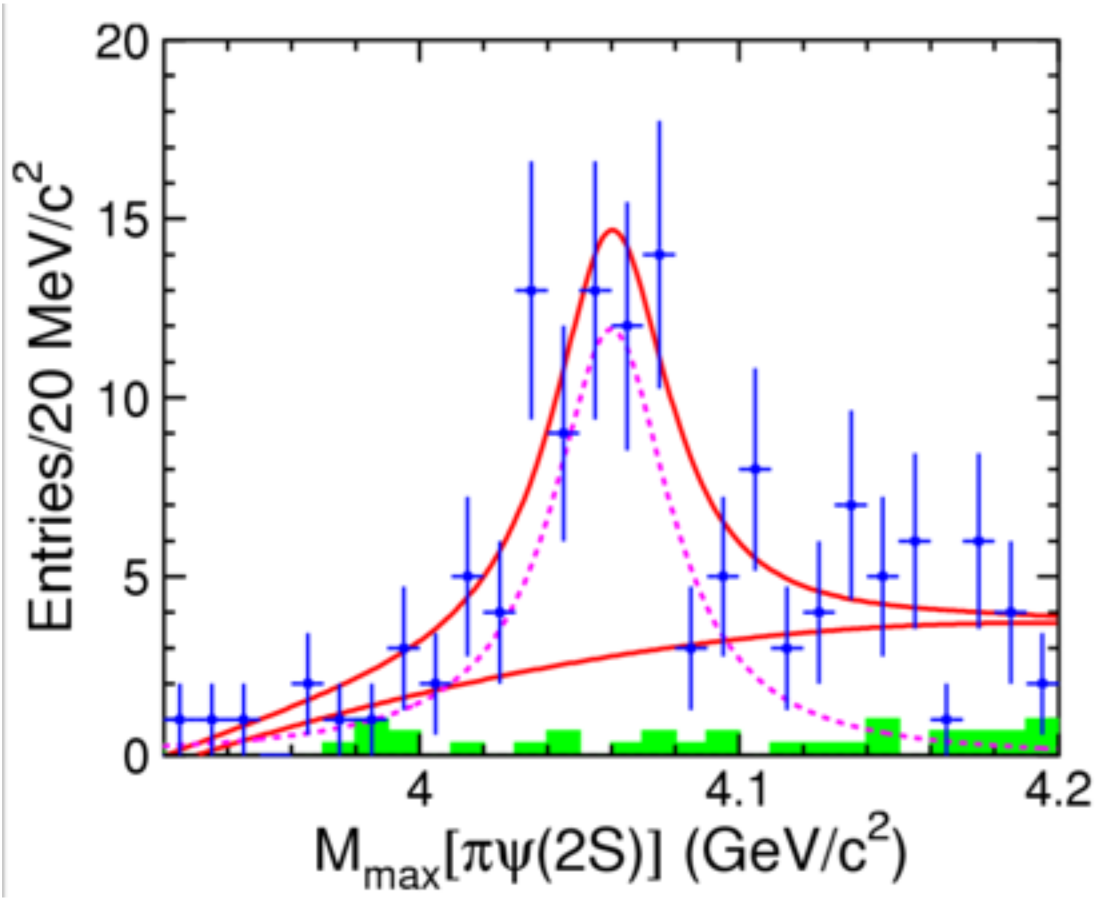
$M = 4051(30)$
$\Gamma = 82(51)$
$M = 4248^{+185}_{-45}$
$\Gamma = 177^{+113}_{-72}$
$J^{PC} = ?$

.manifestly exotic
.dubious




R. Mizuk et al. [Belle], PRD76, 072004 (08)

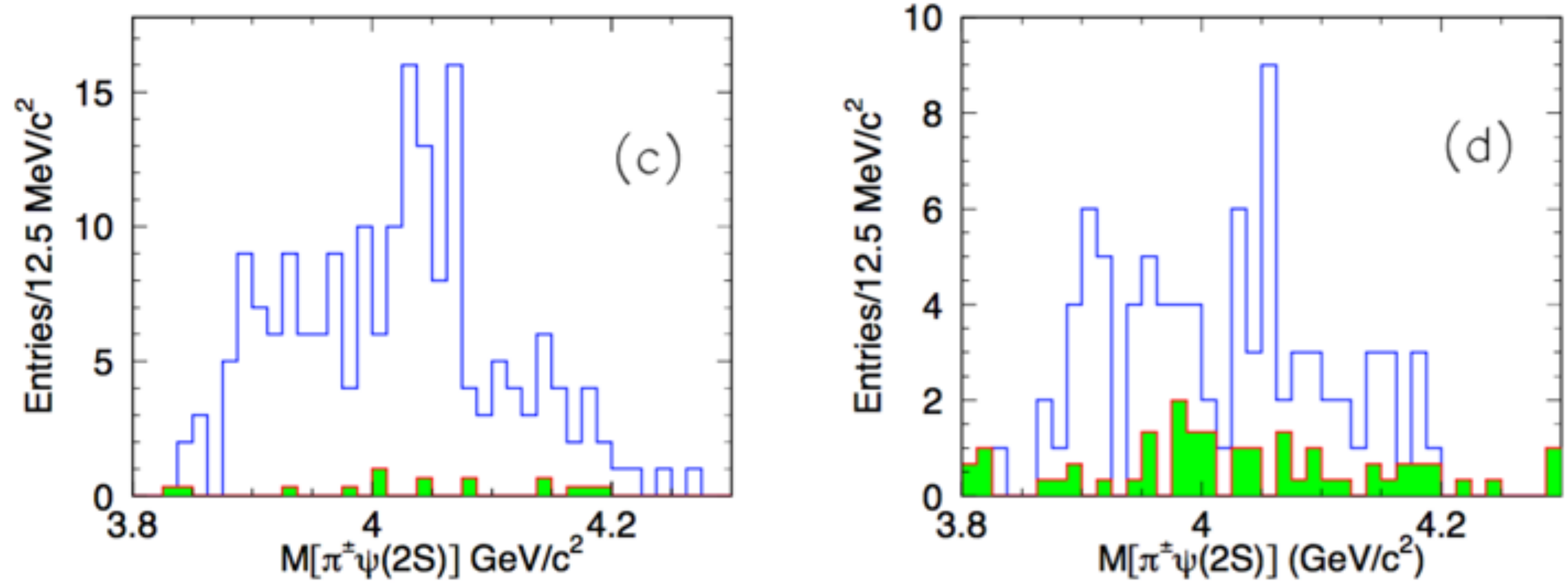
$Z_1(4050)$



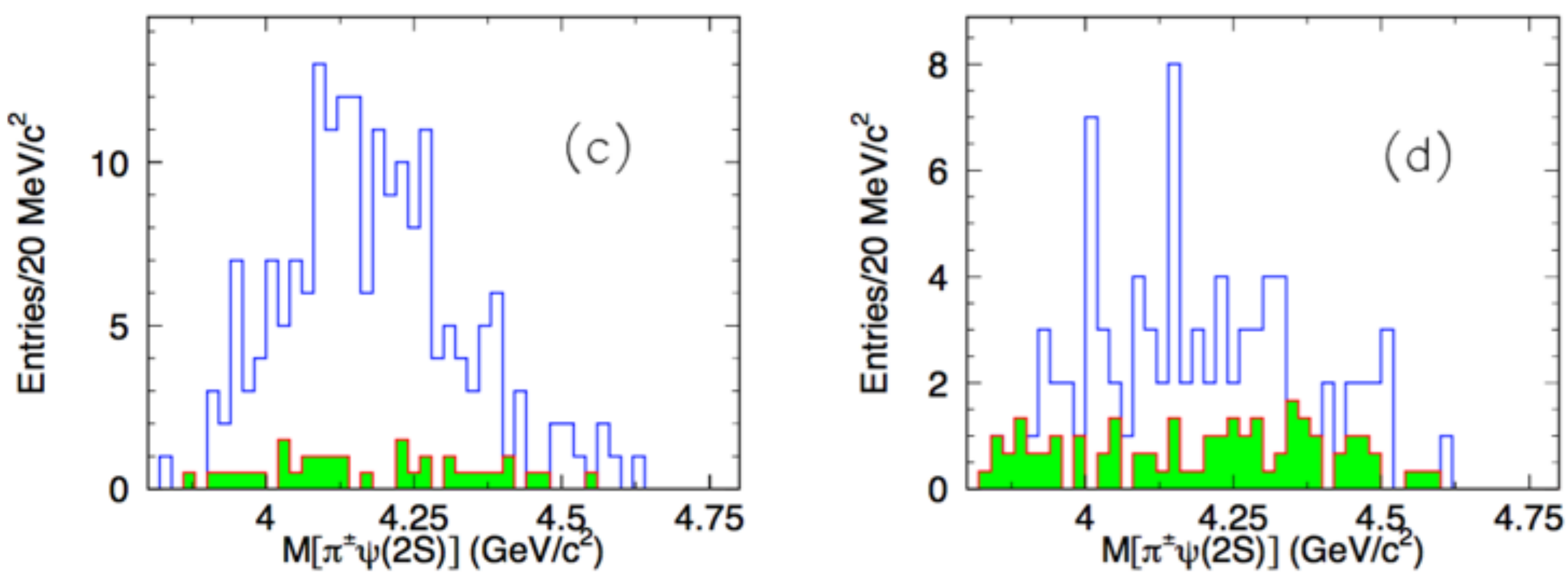

from Y(4360)

[Belle] 1410.7641

# $Z_1(4050)$




Y(4360)




Y(4660)

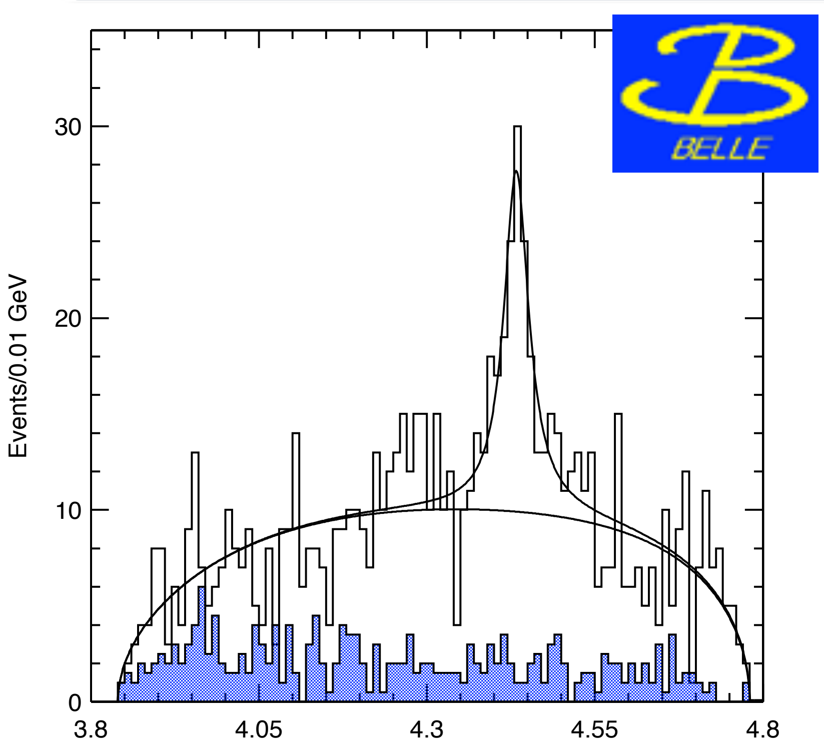
[Belle] 1410.7641

# $Z^+(4430)$

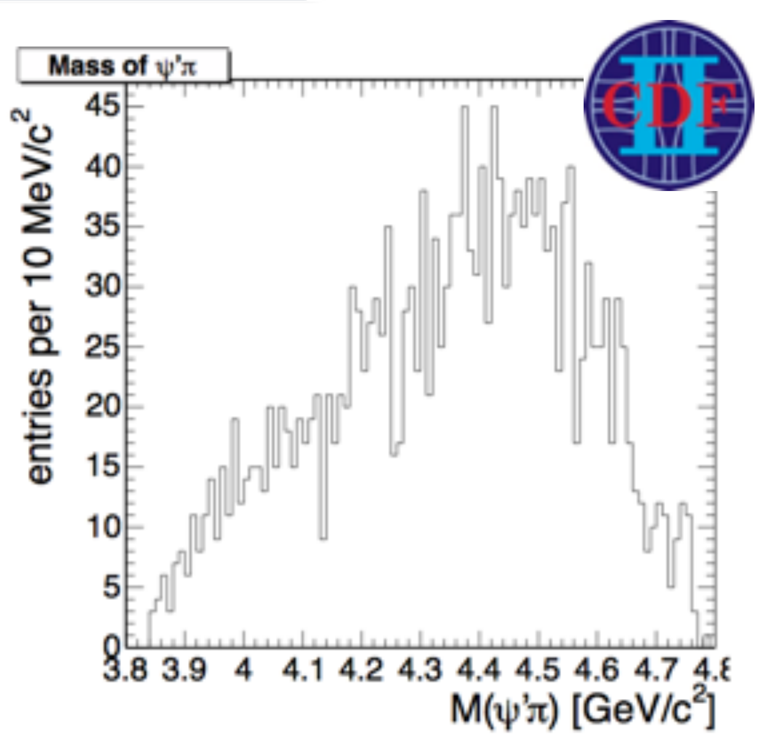
$B \to K\pi^+\psi'$

$$M = 4443^{+24}_{-18}$$
$$\Gamma = 107^{+113}_{-71}$$
$$J^{PC} = ?$$

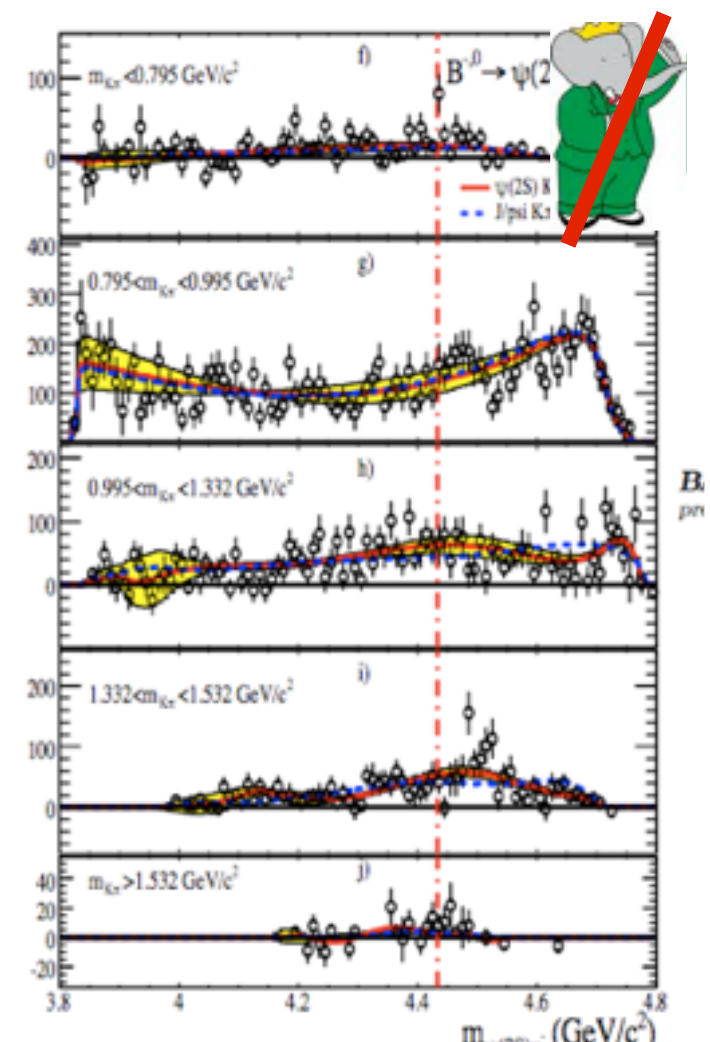
.manifestly exotic
.not confirmed by BaBar




S.-K Choi et al. [Belle] 0708.1790




F. Rubbo, Torino thesis

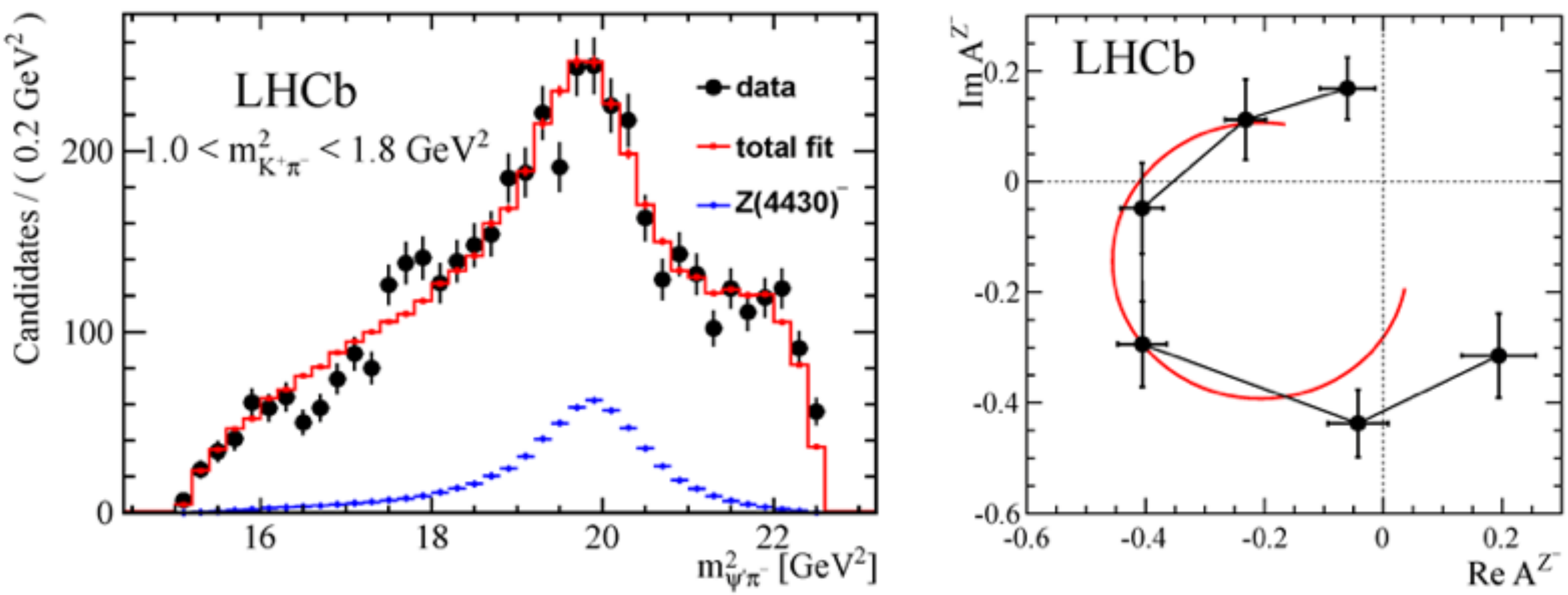



Mokhtar. 0810.1073

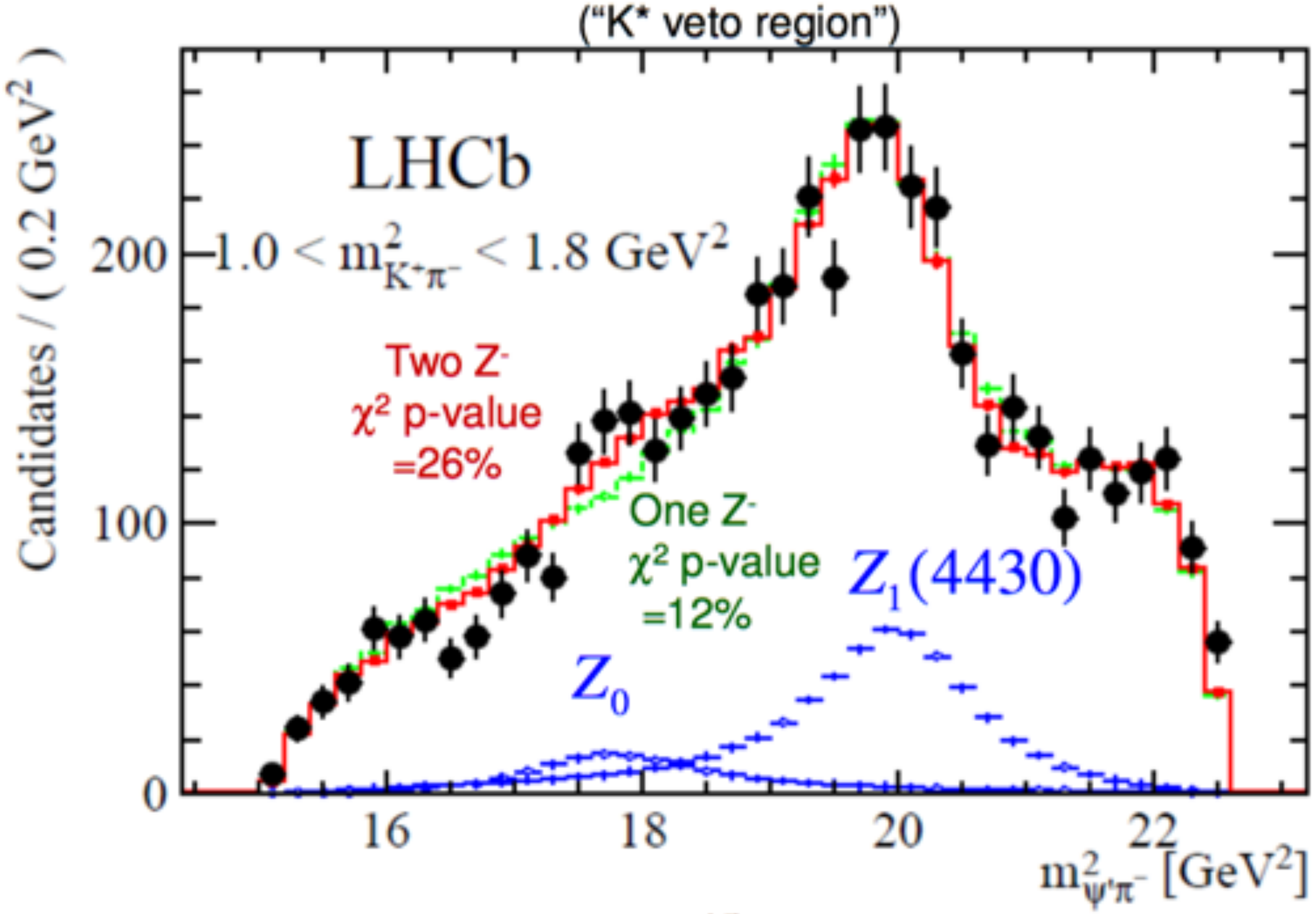
$Z^+(4430)$

.confirmed by LHCb

$J^P = 1^+$



# Z(4240) [?]




$$M(Z_0) = 4239 \pm 18^{+45}_{-10}\ \text{MeV}$$

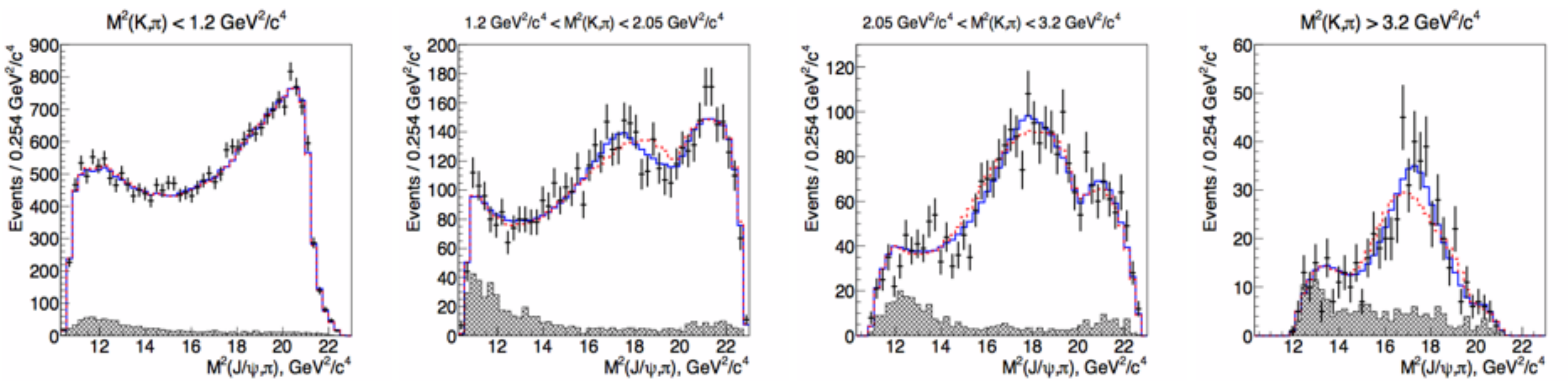
$$\Gamma(Z_0) = 220 \pm 47^{+108}_{-74}\ \text{MeV}$$

$J^P(Z$

over

# $Z_c(4200)$

$$B \to K\pi J/\psi$$



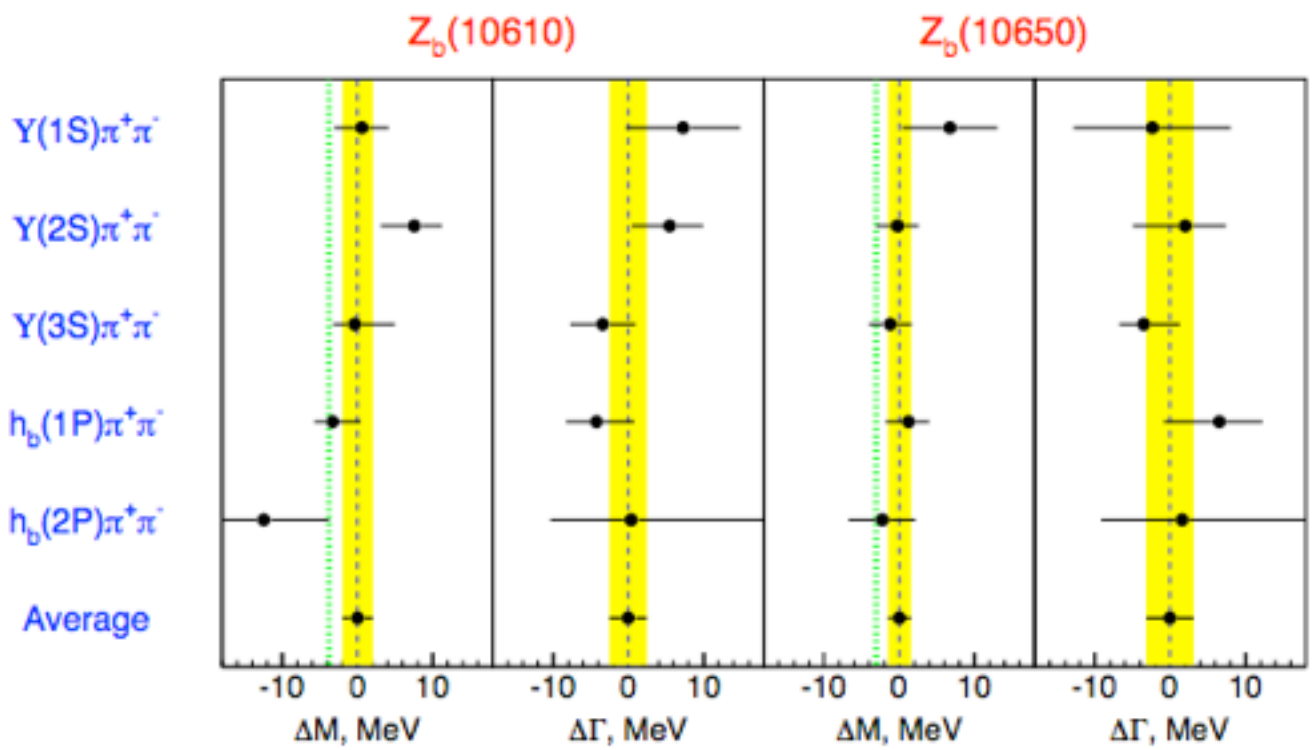

dotted: without Zc(4200)

K. Chilikin et al. [Belle] 1408.6457

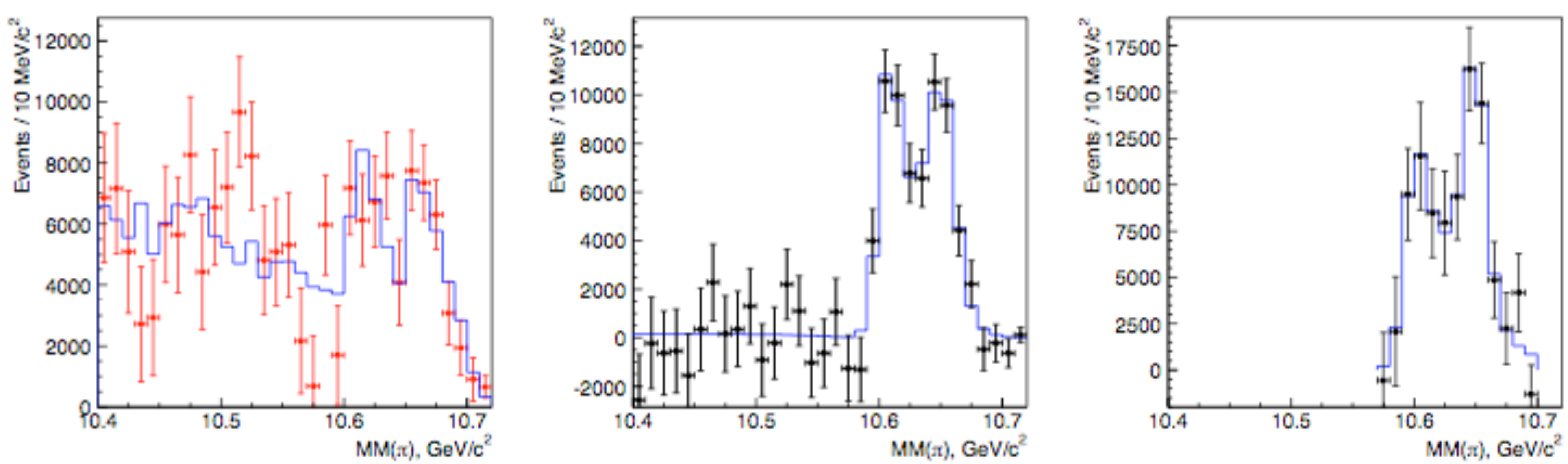
$Z_b^+(10610) \quad Z_b^+(10650)$

Adachi et al. [Belle] 1105.4583

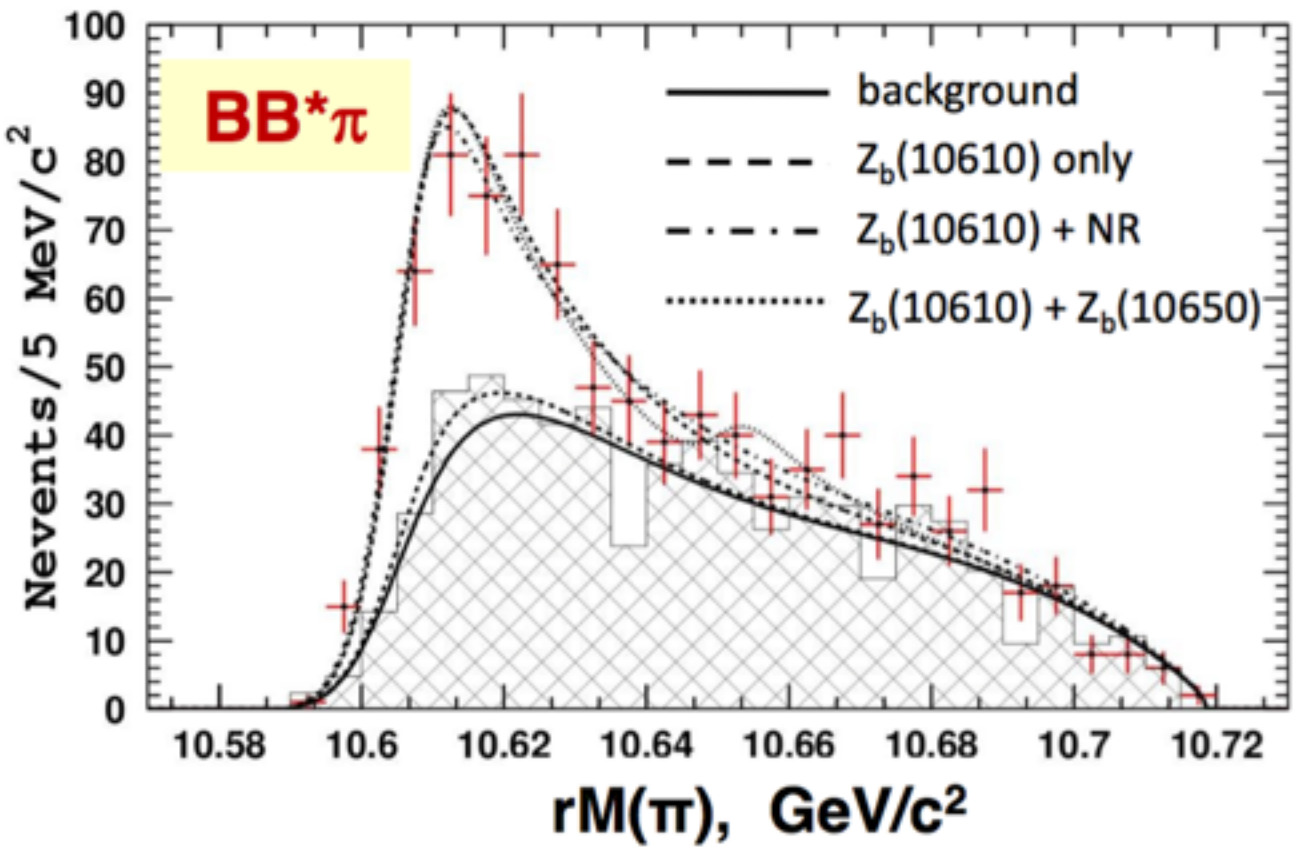
$$I^G J^P = 1^+ 1^+$$

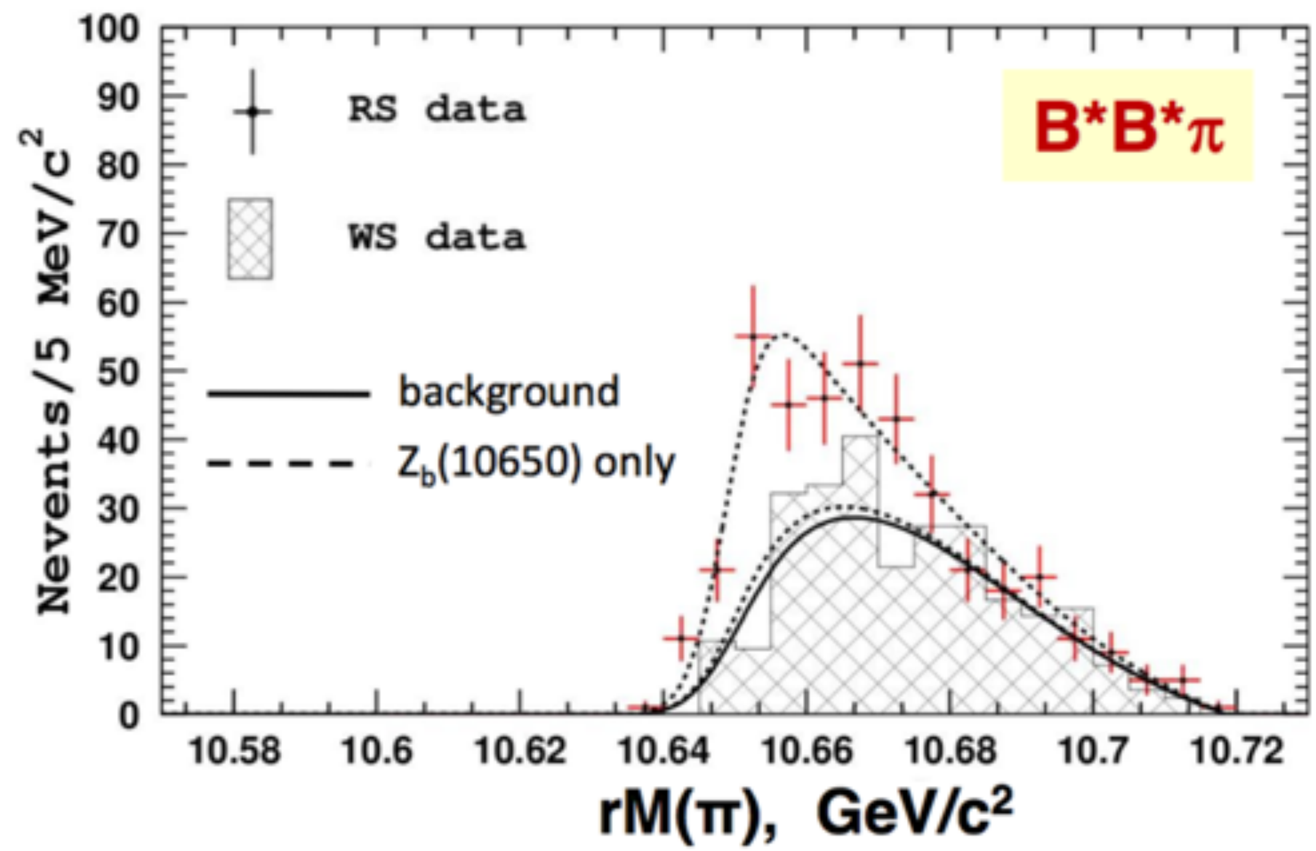



$\Upsilon(2S)$ $h_b(1P)$ $h_b(2P)$



$Z_b^+(10610) \quad Z_b^+(10650)$

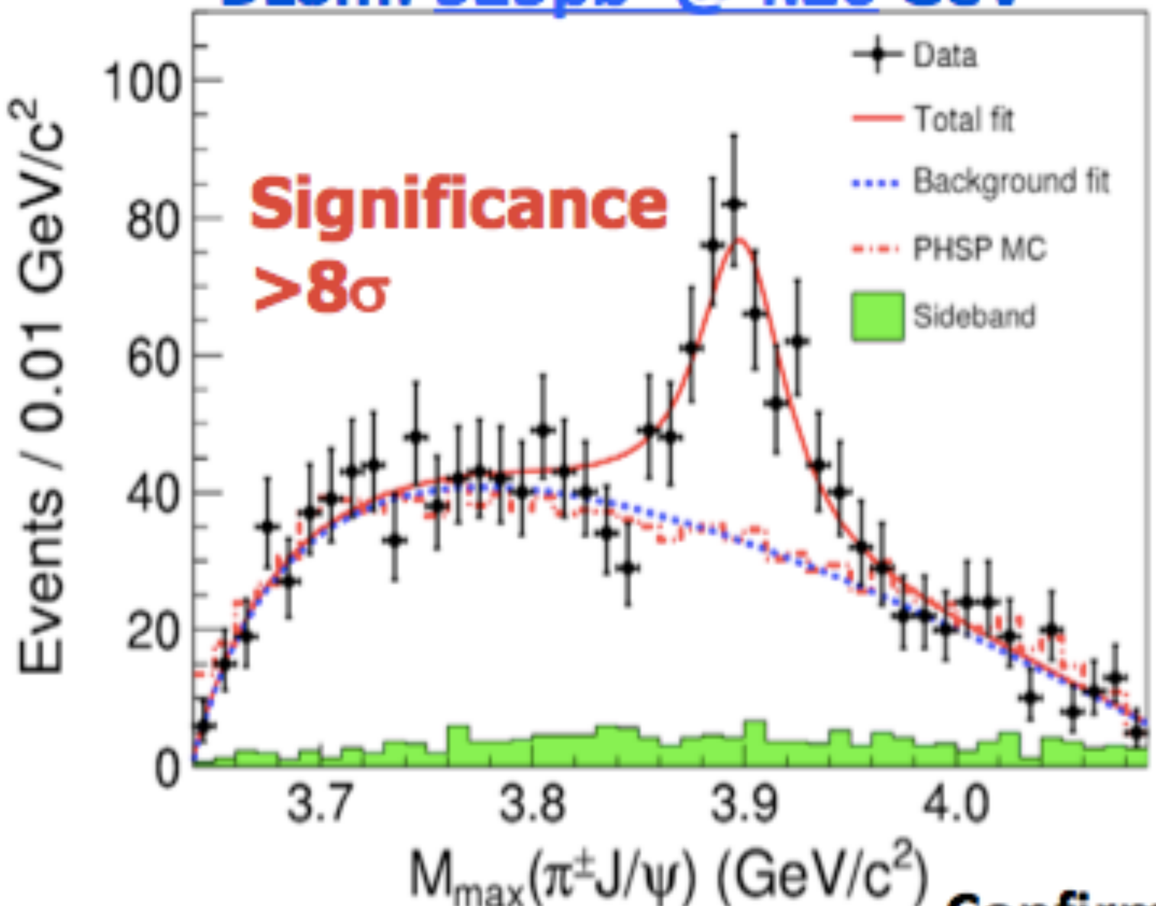



[Belle] preliminary [C.-Z. Yuan, INT Nov 2015]

# Observation of Zc(3900) at BESIII

**BESIII: 525pb$^{-1}$@4.26 GeV**

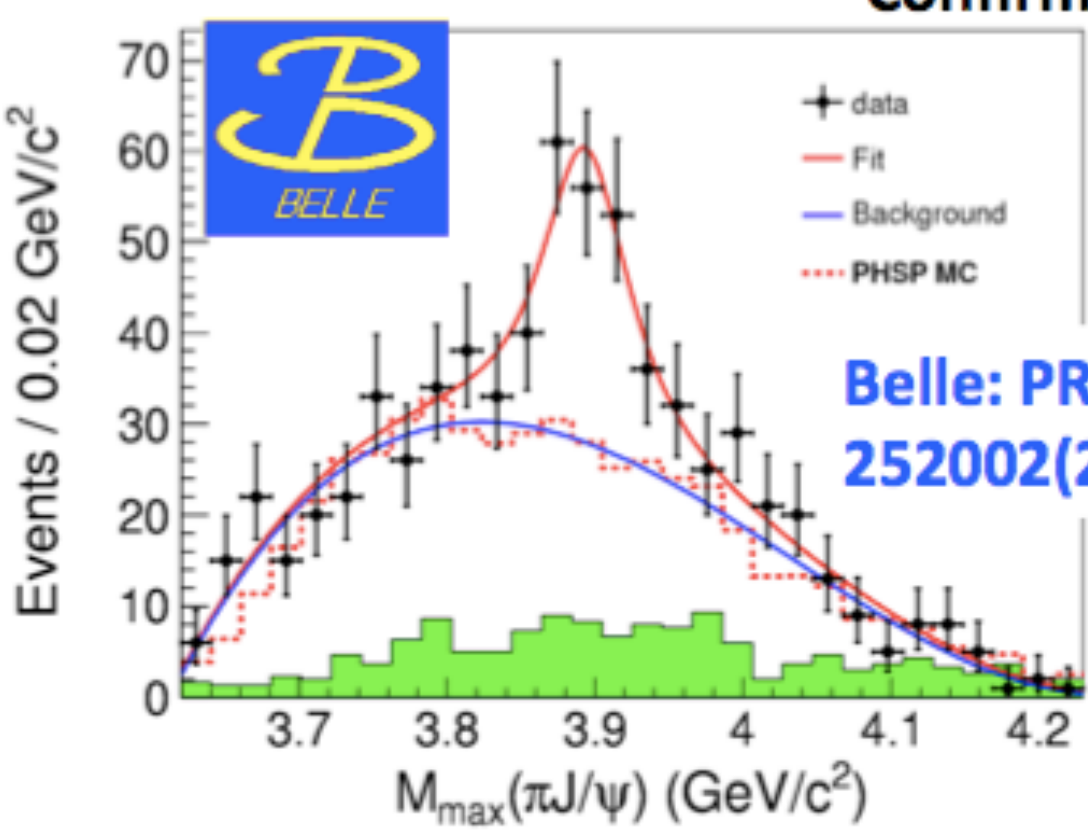



BESIII: PRL110, 252001 (2013)

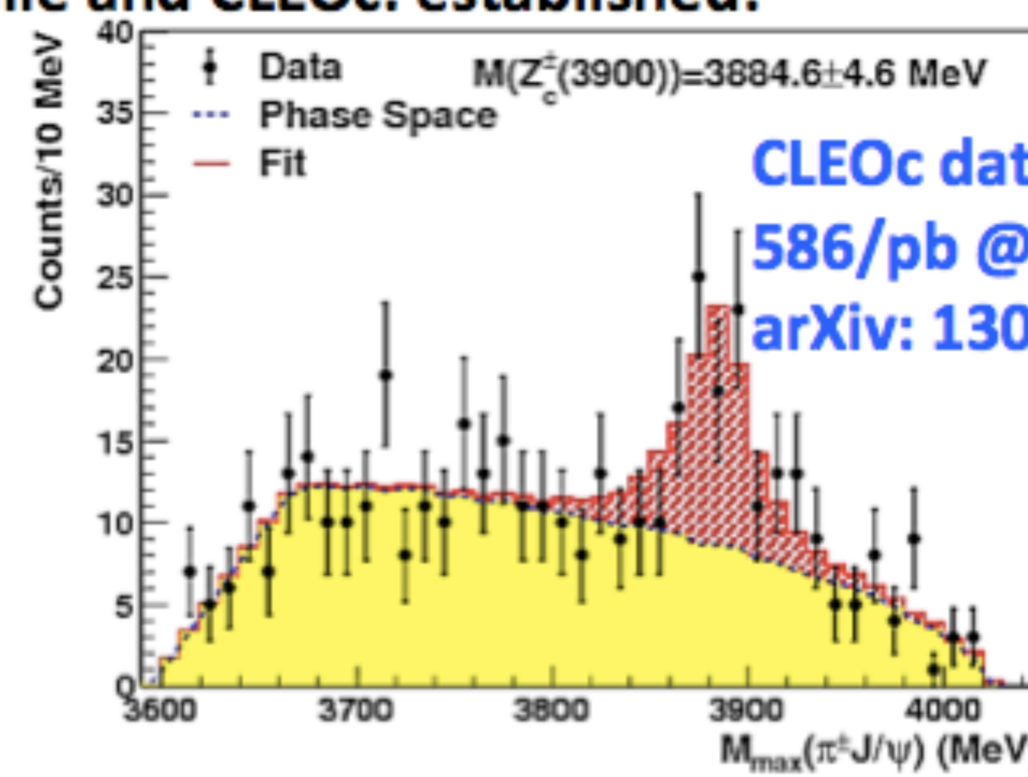
- **M = 3899.0±3.6±4.9 MeV**
- **Γ = 46±10±20 MeV**
- **307 ± 48 events**

**The mass position is 24 MeV away from DD* threshold!**
**A Partial wave analysis is on going!**

**Confirmed by Belle and CLEOc: established!**




**Belle: PRL 110, 252002(2013)**



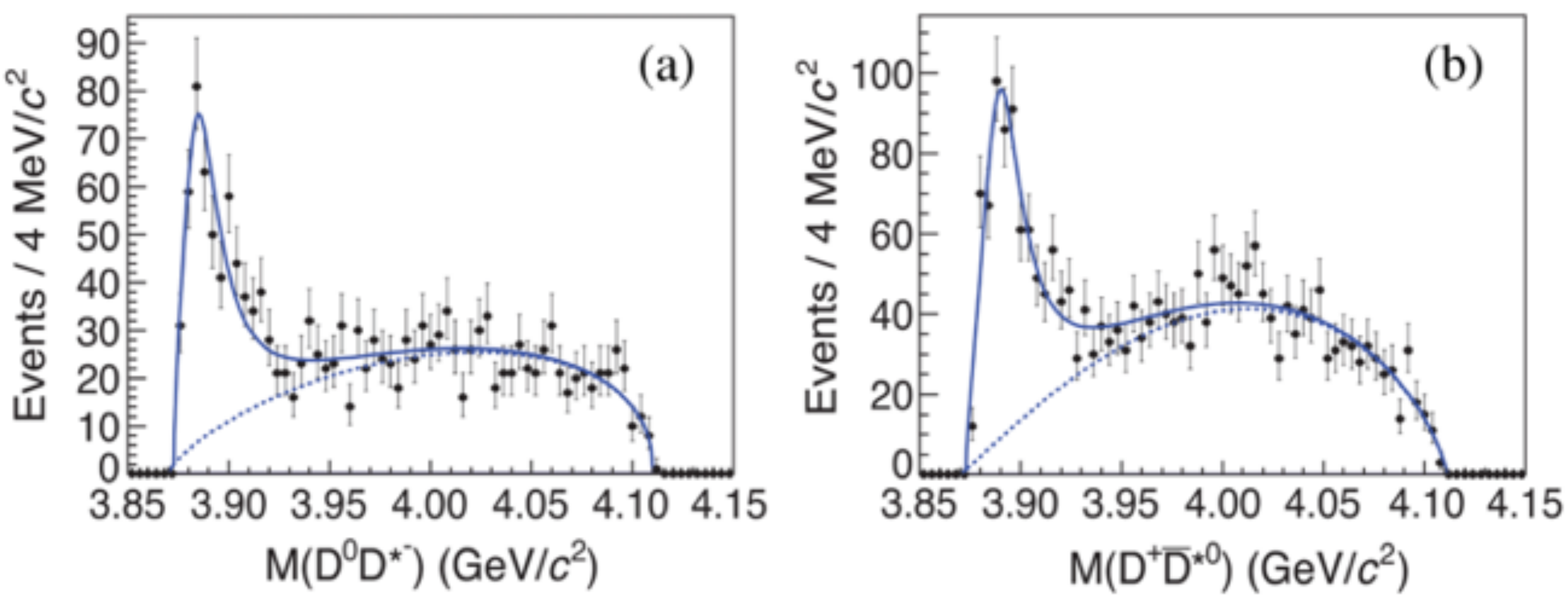

**CLEOc data**
**586/pb @4.17GeV**
**arXiv: 1304.3036**

# Zc(3900)

$e^+e^- \to \pi D\bar{D}^*$ $\sqrt{s} = 4.26$

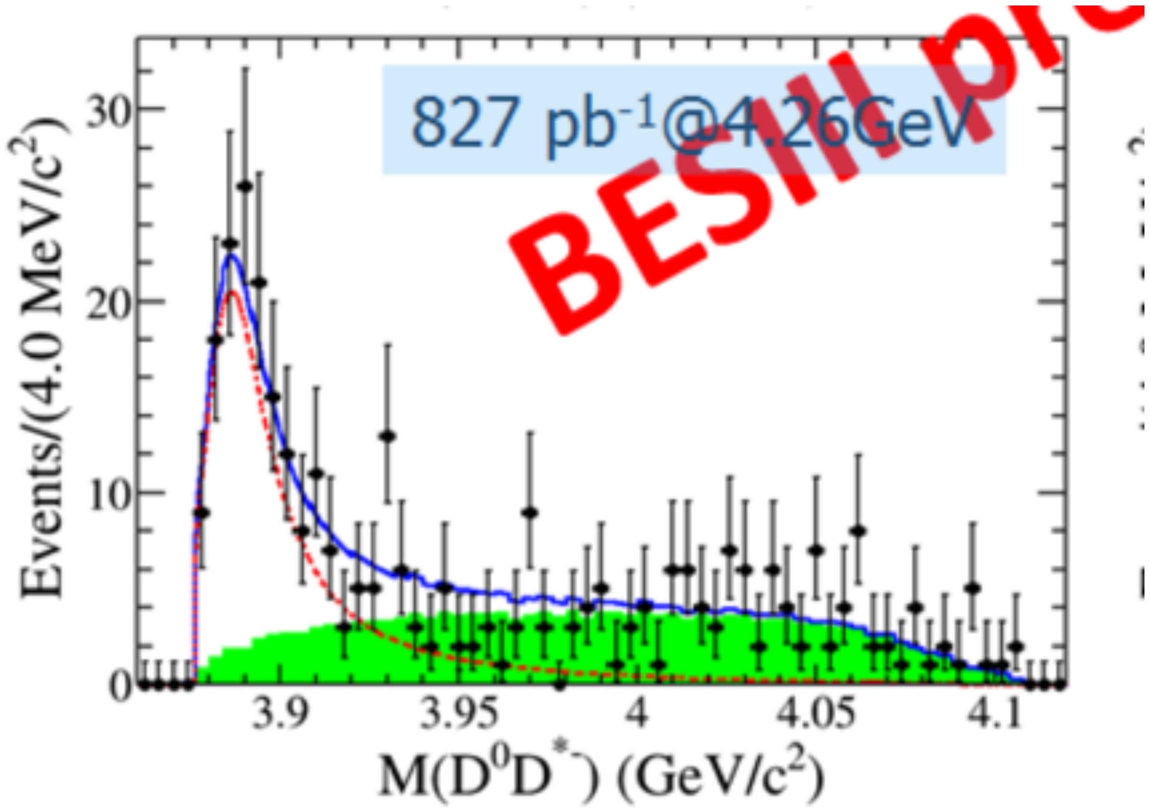
$M = 3883.9 \pm 1.5 \pm 4.2$

$\Gamma = 24.8 \pm 3.3 \pm 11.0$




BESIII PRL112 022001 (14)

# Zc(3900)




Wolfgang Gradl, "Bound States in QCD", St Goar, Mar 24-27, 2015

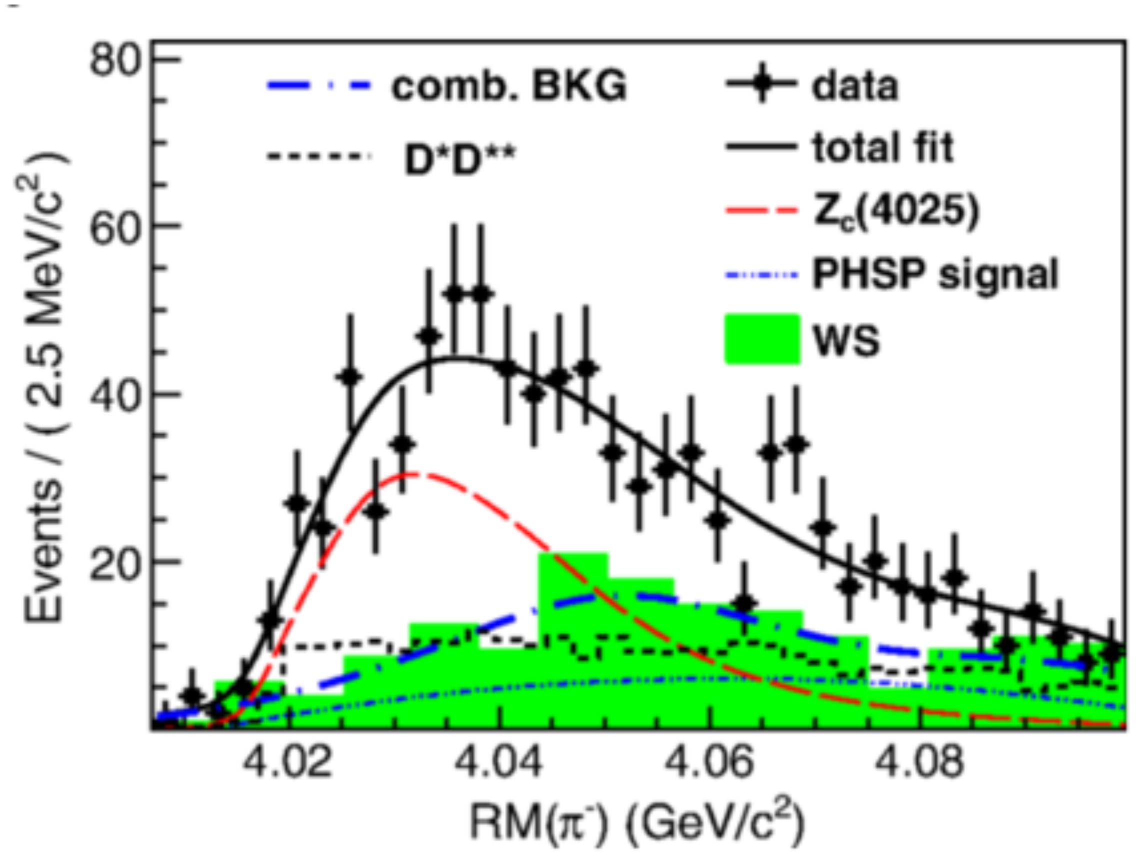
New BESIII result with all three particles identified.
Much smaller background.

$$Z_c(4025)$$

$$e^+e^- \rightarrow (D^*\bar{D}^*)^{\pm}\pi^{\mp}$$

$$M = 4026.3 \pm 2.6 \pm 3.7$$

$$\Gamma = 24.8 \pm 5.6 \pm 7.7$$



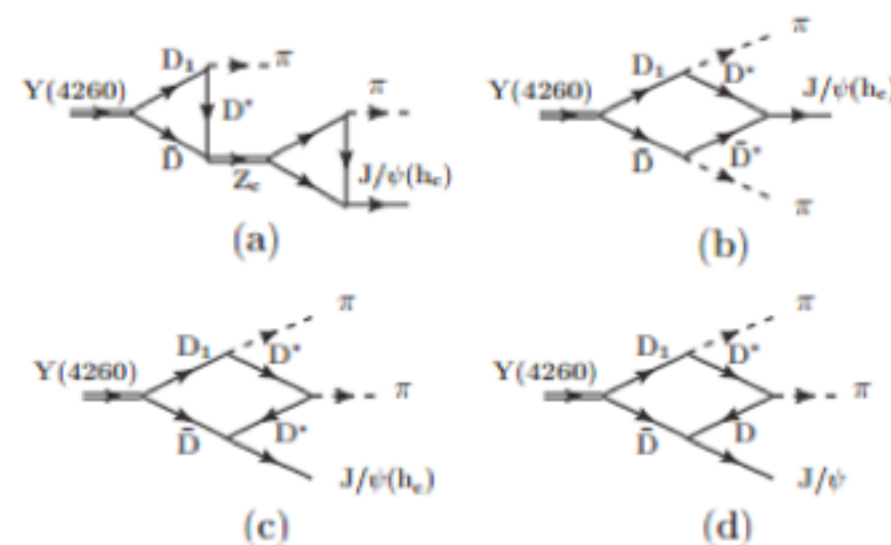

BESIII Phys. Rev. Lett. 112, 132001 (2014)

Zc(3900)
Zc(4025)

# Theory

From SPIRE HEP Database (21st, Apr):

1. Tetraquarks
   - arXiv:1110.1333, 1303.6857
   - arXiv:1304.0345, 1304.1301
2. Hadronic molecules
   - arXiv:1303.6608, 1304.2882, 1304.1850
3. Four quark state (1 or 2)
   - arXiv:1304.0380
4. Meson loop
   - arXiv:1303.6355
   - arXiv:1304.4458
5. ISPE model
   - arXiv:1303.6842
6. ...

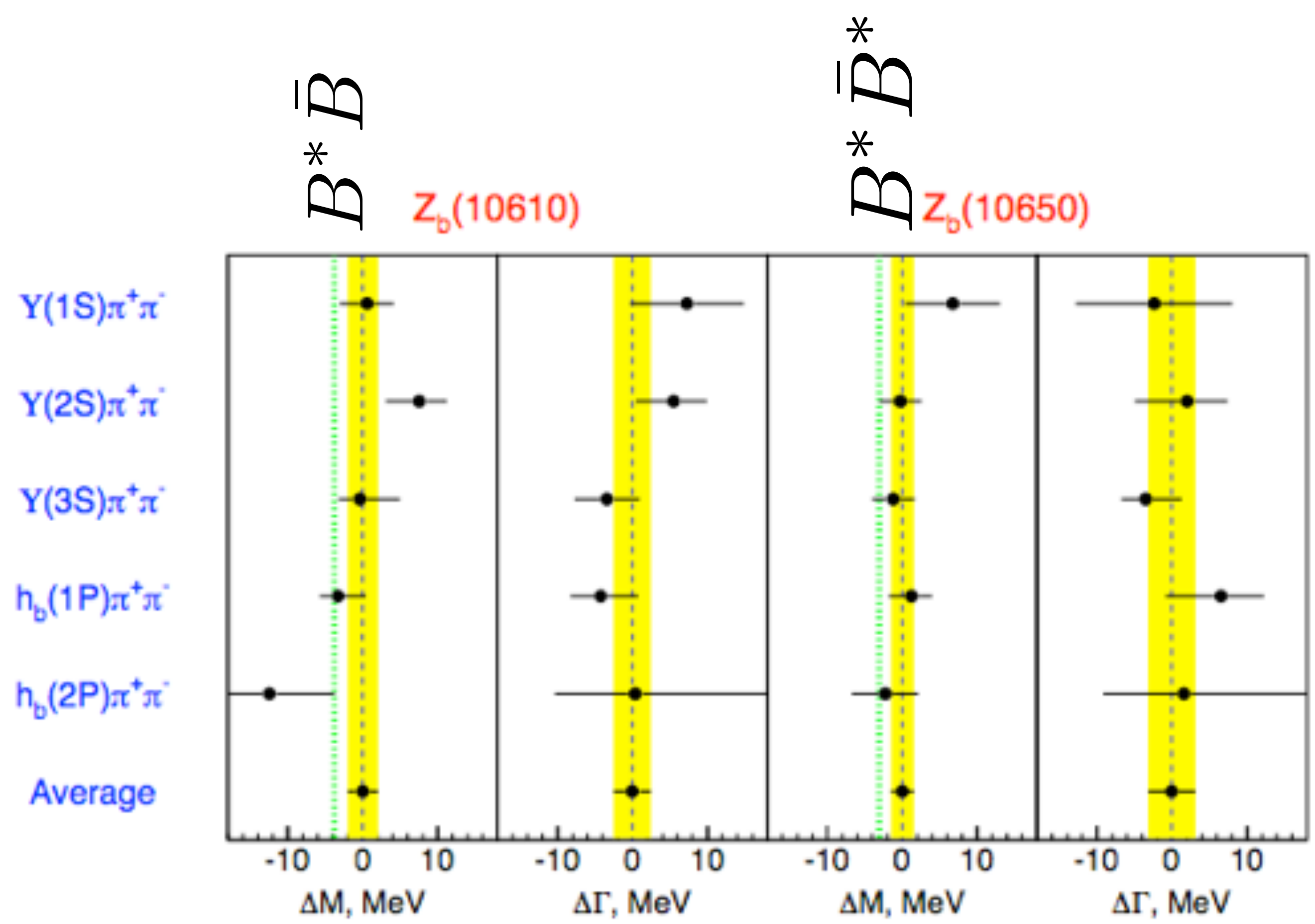


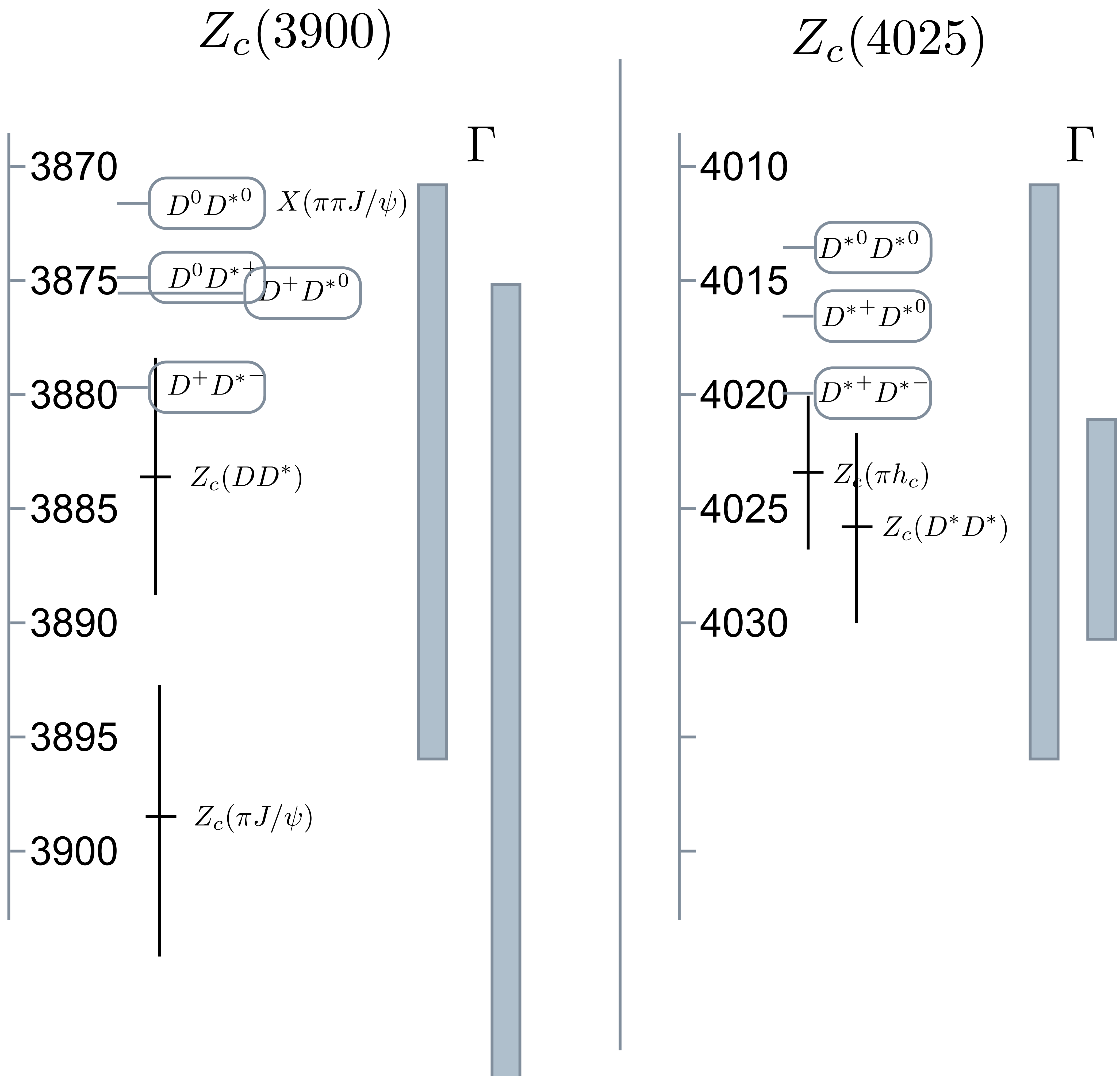

Meson loop

# Exotic Phenomenology

# Charged Exotics as Threshold Cusps

It seems foolish to ignore that many of these states are just above open charm/bottom thresholds.



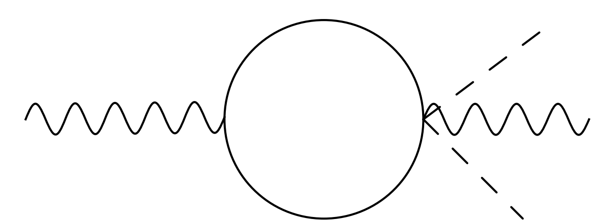


$Z_c(3900)$
$Z_c(4025)$
Γ
Γ
3870
3875
3880
3885
3890
3895
3900
$D^0 D^{*0}$
$X(\pi\pi J/\psi)$
$D^0 D^{*+}$
$D^+ D^{*0}$
$D^+ D^{*-}$
$Z_c(DD^*)$
$Z_c(\pi J/\psi)$
4010
4015
4020
4025
4030
$D^{*0} D^{*0}$
$D^{*+} D^{*0}$
$D^{*+} D^{*-}$
$Z_c(\pi h_c)$
$Z_c(D^* D^*)$

# Cusp Model

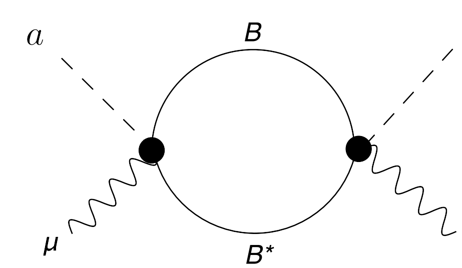
Q: how does Y(5S) couple to Yππ?

$$\Upsilon(5S) \to \text{ hidden bottom} = 3.8\%$$



$$\Upsilon(5S) \to B^{(*)}\bar{B}^{(*)} = 57.3\%$$

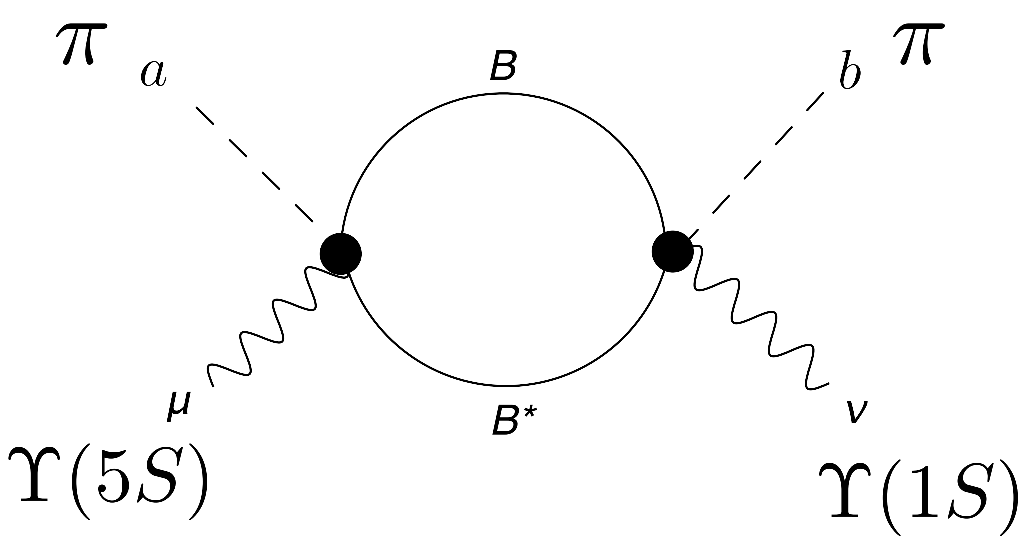
$$\Upsilon(5S) \to B^{(*)}\bar{B}^{(*)}\pi = 8.3\%$$

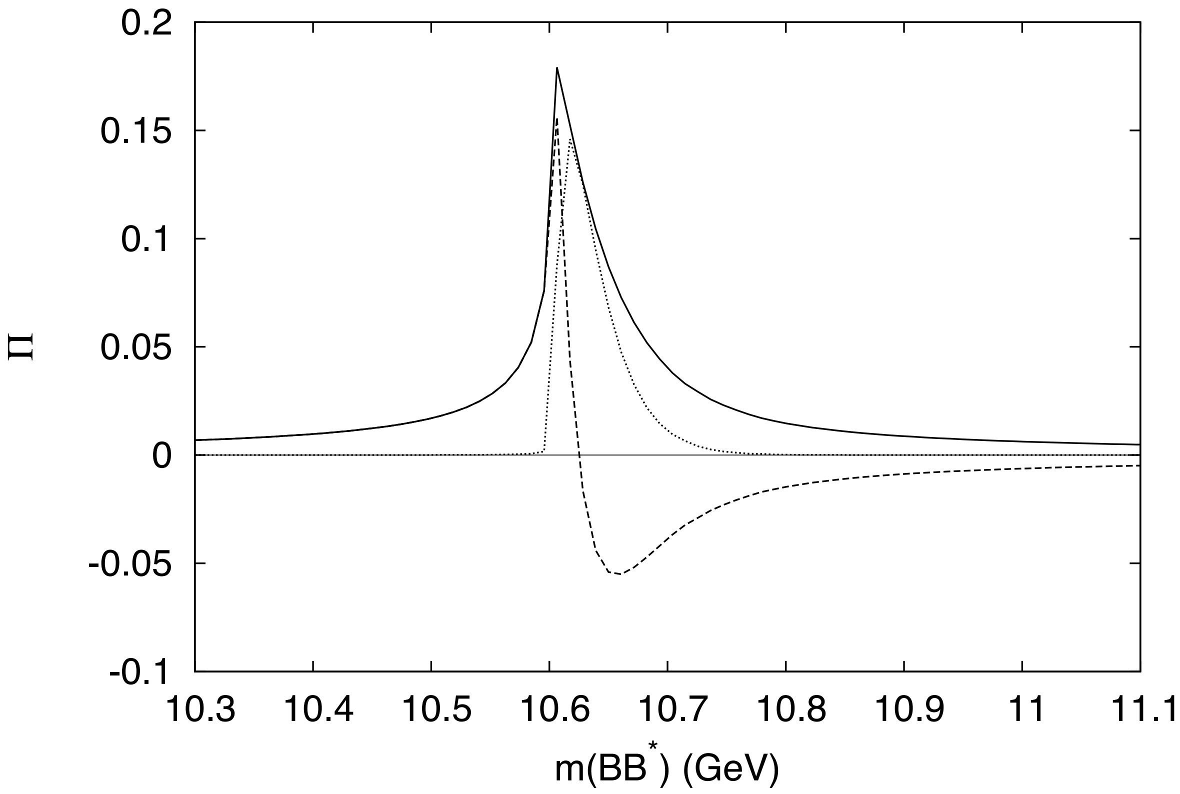



$$\Upsilon(5S) \to \Upsilon(nS)\pi\pi < 7.8 \cdot 10^{-3}$$

E.S. Swanson, arXiv:1409.3291

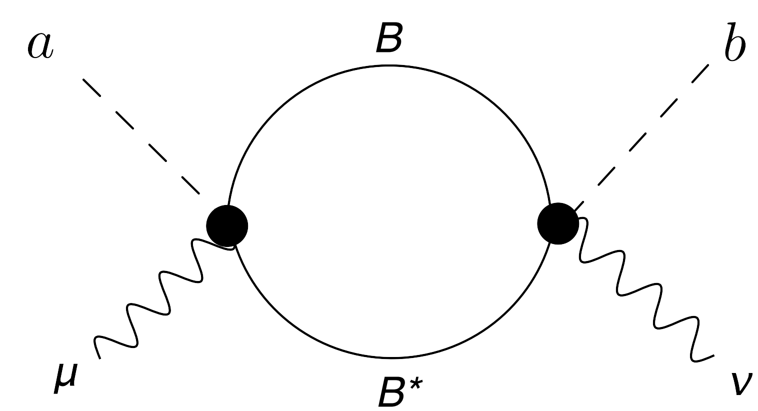
# Cusp Model




[NB: this exhibits phase motion!]

# Cusp Model




$$\mathrm{Im}\Pi_{\alpha\beta}(s) = \sum_i k_i^{1+\ell_{\alpha i}+\ell_{\beta i}} F_{\alpha i}(s) F_{\beta i}(s)$$

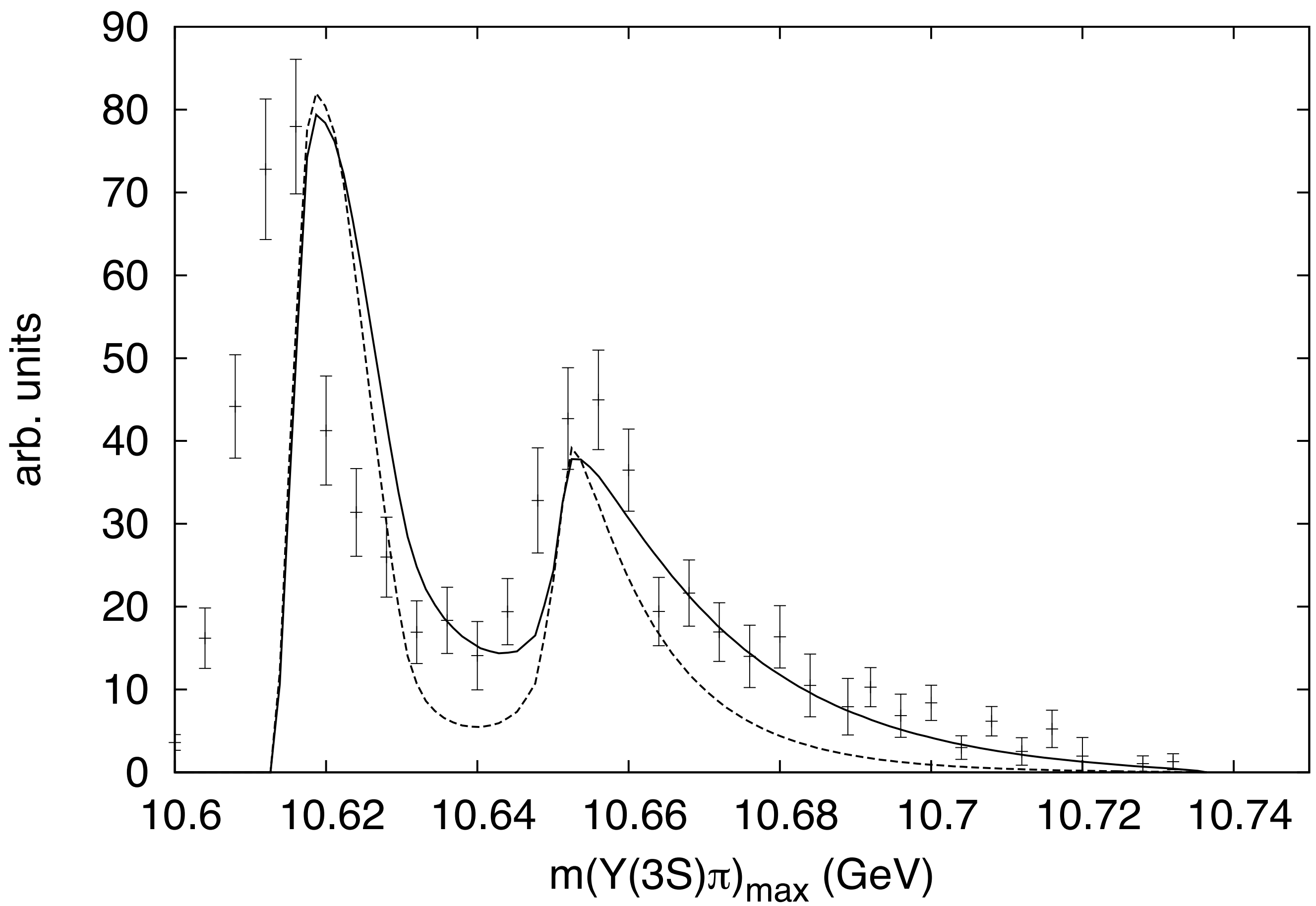
$$F_{\alpha i} = g_{\alpha i} \exp(-s/2\beta_{\alpha i}^2)$$

$$k_i^2 = \frac{\left(s - (m_{1i} + m_{2i})^2\right)\left(s - (m_{1i} - m_{2i})^2\right)}{4s}$$

$$\Pi_{\alpha\beta}(s) = \frac{1}{\pi} \int_{s_{th}}^{\infty} ds' \, \frac{\mathrm{Im}\Pi_{\alpha\beta}(s')}{s' - s - i\epsilon}$$

# Cusp Model $\Upsilon(5S) \to \Upsilon(nS)\pi\pi$

Zb(10610), Zb(10650)




$$\beta_{\alpha i} = 0.7 \text{ GeV}$$

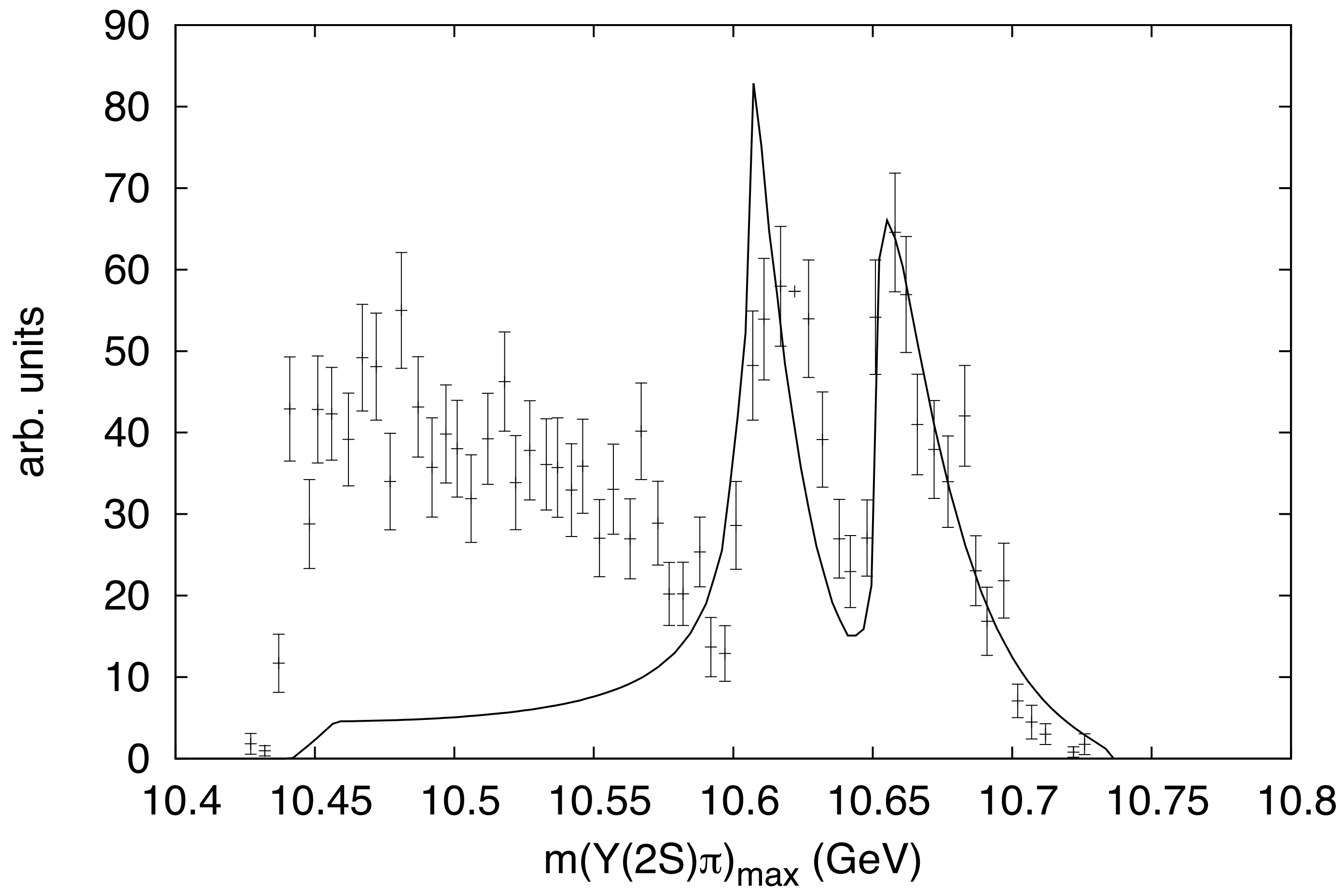
$$g^2_{\Upsilon(nS)BB^*} = 0.9 \cdot g^2_{\Upsilon(nS)B^*B^*}$$

Adachi et al. [Belle Collaboration], arXiv:1105.4583 [hep-ex];

Garmash et al. [Belle Collaboration], arXiv:1403.0992 [hep-ex].

# Cusp Model $\Upsilon(5S) \to \Upsilon(nS)\pi\pi$

Zb(10610), Zb(10650)



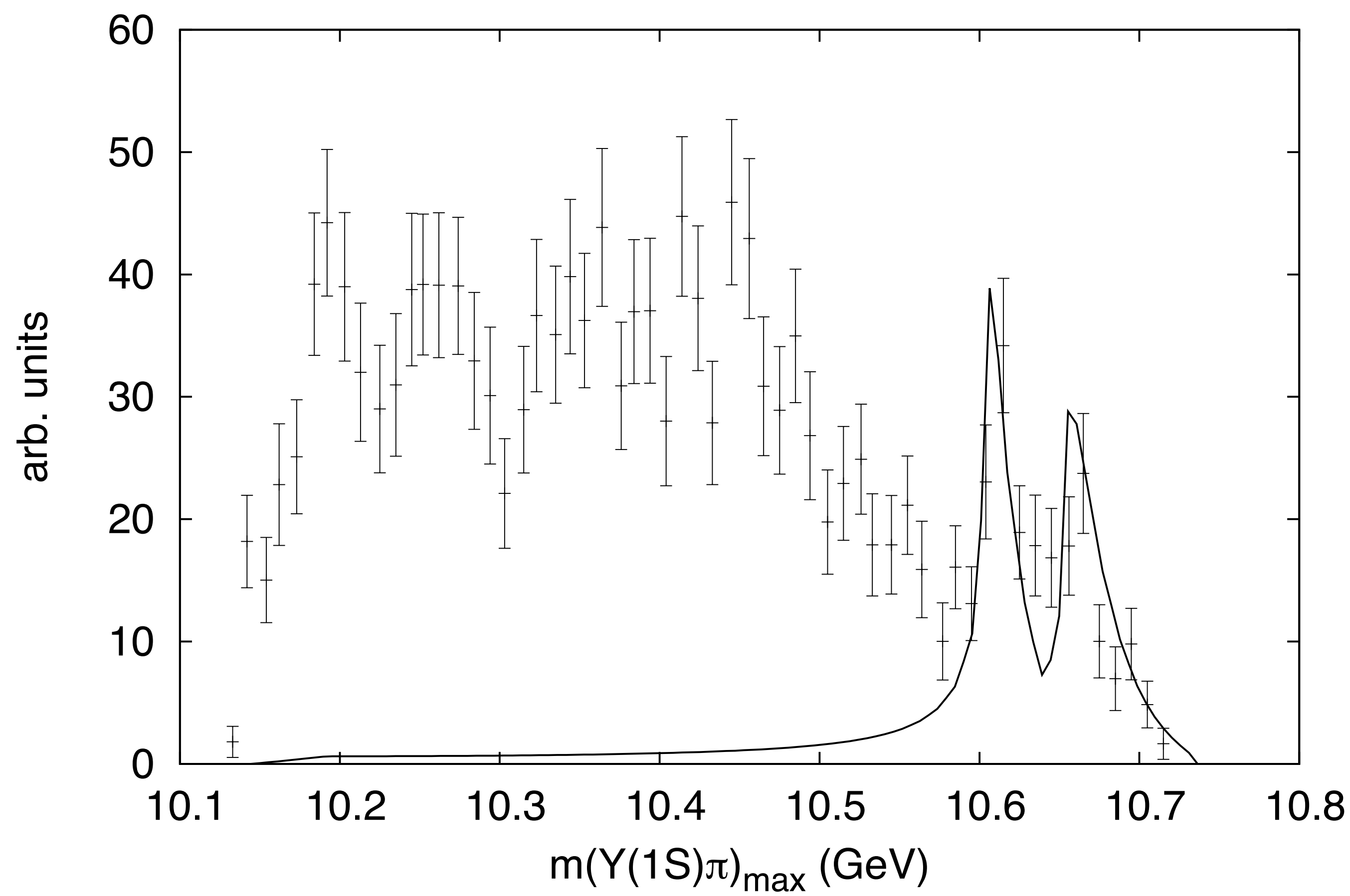

same couplings used!

Adachi et al. [Belle Collaboration], arXiv:1105.4583 [hep-ex];

Garmash et al. [Belle Collaboration], arXiv:1403.0992 [hep-ex].

# Cusp Model $\Upsilon(5S) \to \Upsilon(nS)\pi\pi$

Zb(10610), Zb(10650)



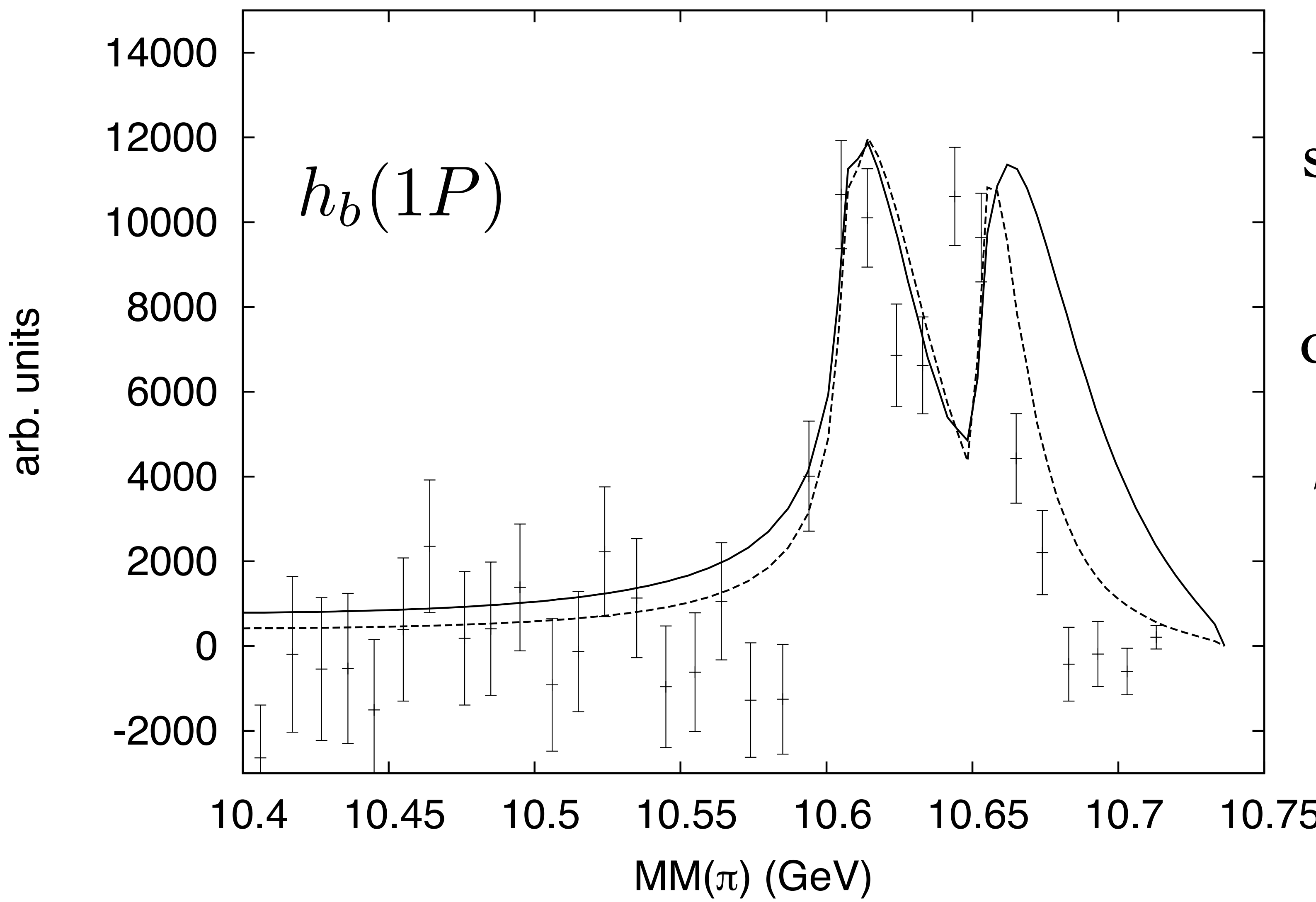

30% smaller coupling required

Adachi et al. [Belle Collaboration], arXiv:1105.4583 [hep-ex];

Garmash et al. [Belle Collaboration], arXiv:1403.0992 [hep-ex

# Cusp Model $\Upsilon(5S) \to h_b(nP)\pi\pi$

Zb(10610), Zb(10650)




solid line: same as above

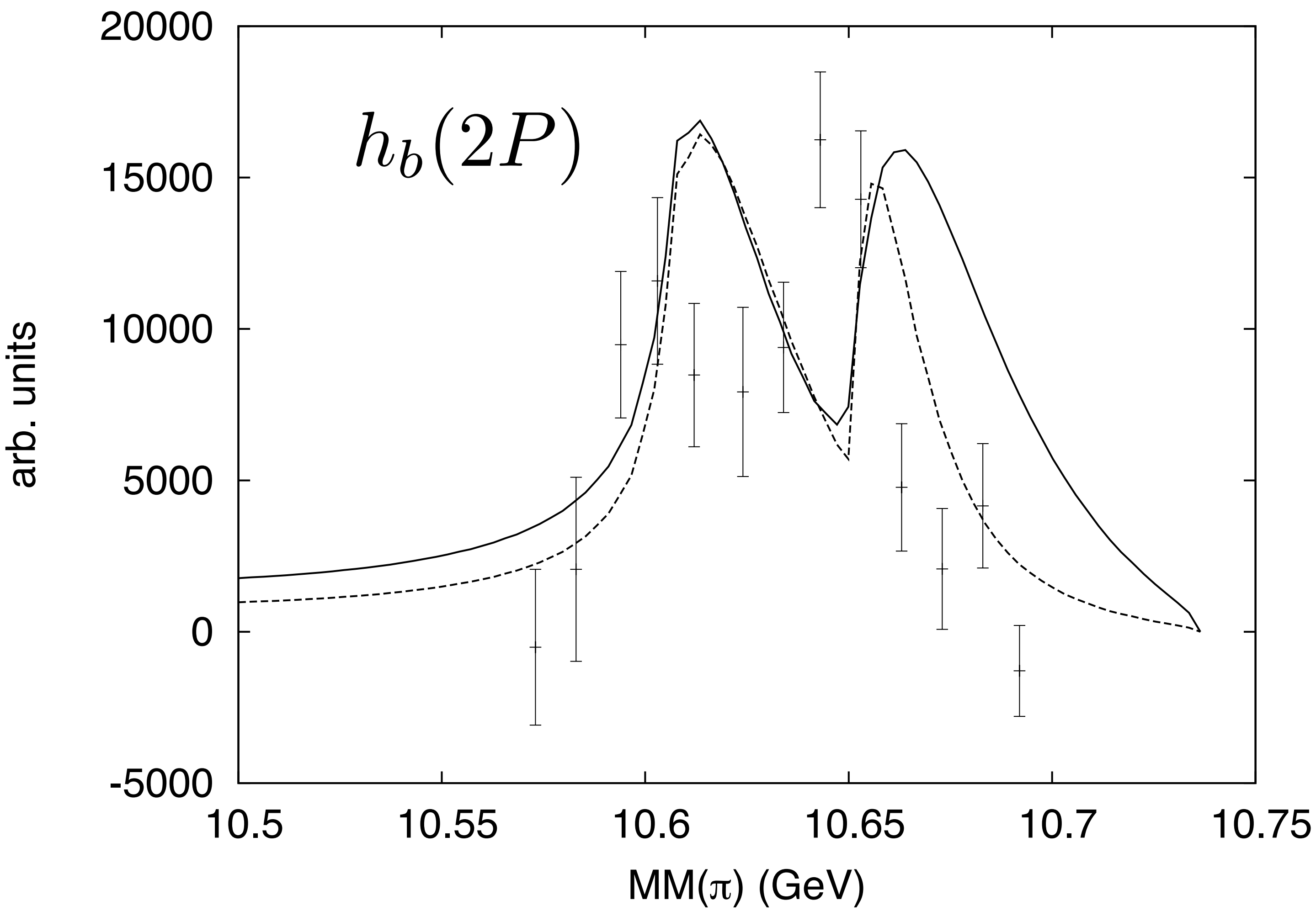
dashed line:

$\beta_{BB^*} = 0.7$ GeV, $\beta_{B^*B^*} = 0.4$ GeV

$g^2_{BB^*} = 0.5\, g^2_{B^*B^*}$

# Cusp Model $\Upsilon(5S) \to h_b(nP)\pi\pi$

Zb(10610), Zb(10650)




solid line: same as above

dashed line:

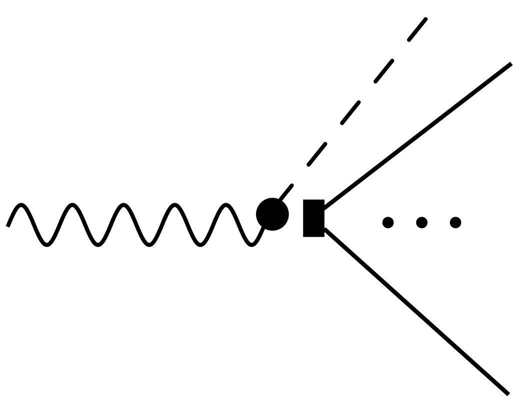
$\beta_{BB^*} = 0.7$ GeV, $\beta_{B^*B^*} = 0.4$ GeV

$g^2_{BB^*} = 0.5\, g^2_{B^*B^*}$

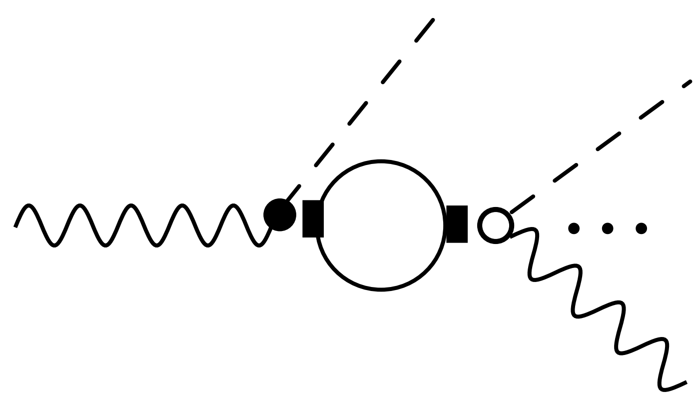
E.S. Swanson, arXiv:1504.07952

# Cusp Model-II

Attempt a "microscopic" cusp model
[separable nonrelativistic model; solve exactly]
[iterate all bubbles]



$Y(4260) \to \pi D\bar{D}^*$

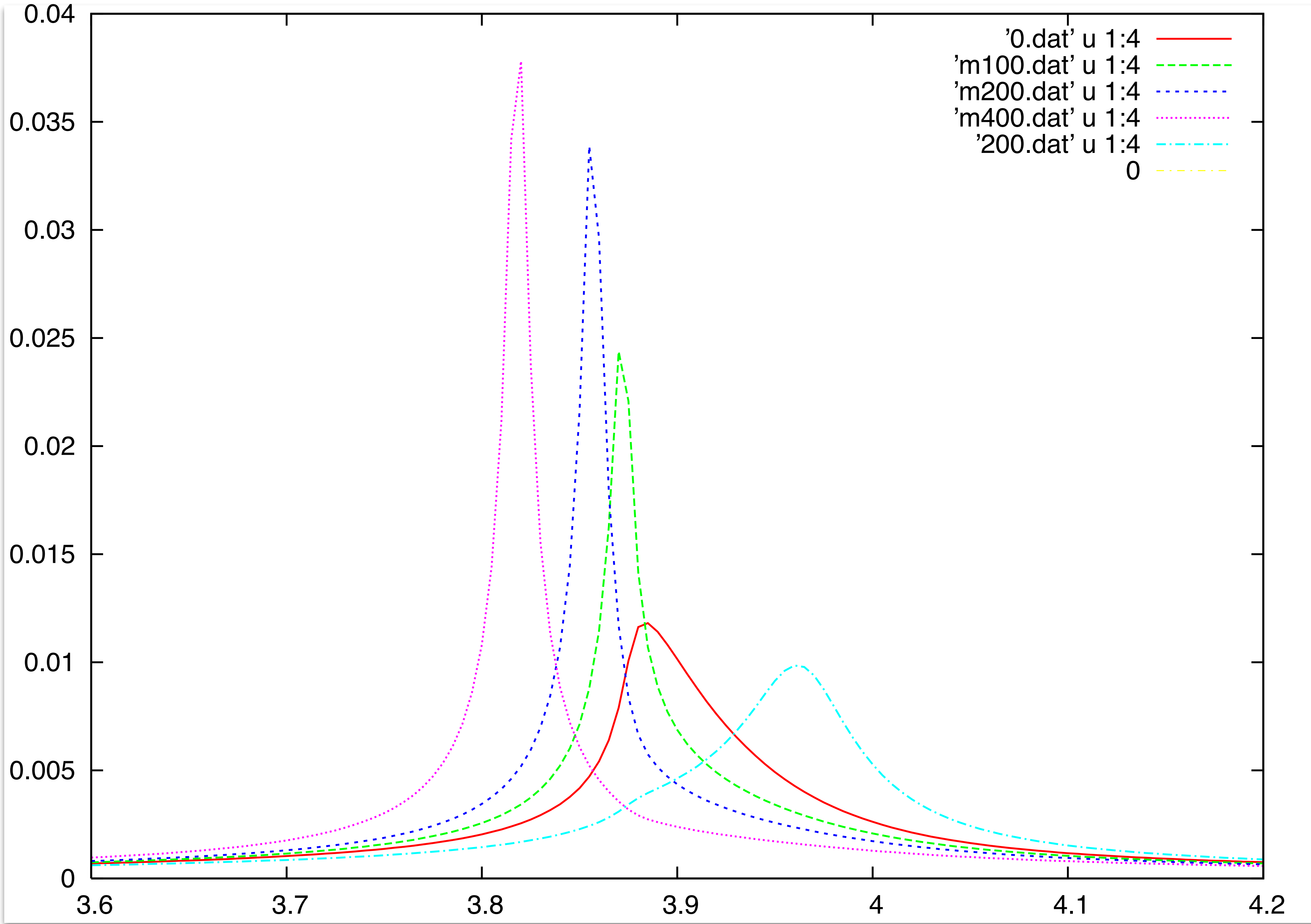


$Y(4260) \to \pi\pi J/\psi$

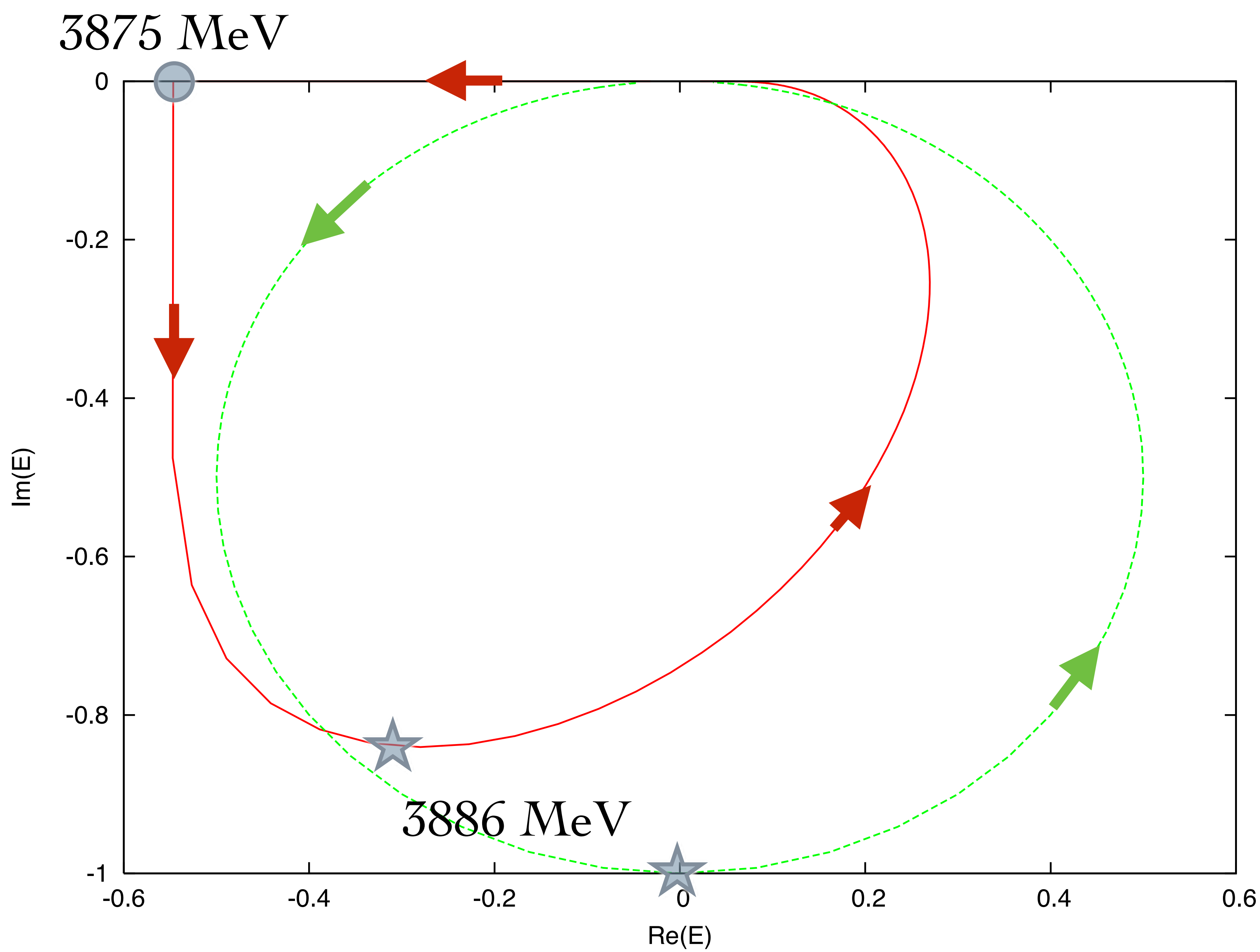
$$g_{DD^*} \cdot \exp(-\lambda(s_{\pi Y})/\beta^2_{\pi Y}) \exp(-\lambda(s_{DD^*})/\beta^2_{DD^*})$$

# Cusp Model-II

## effect of the bubble sum

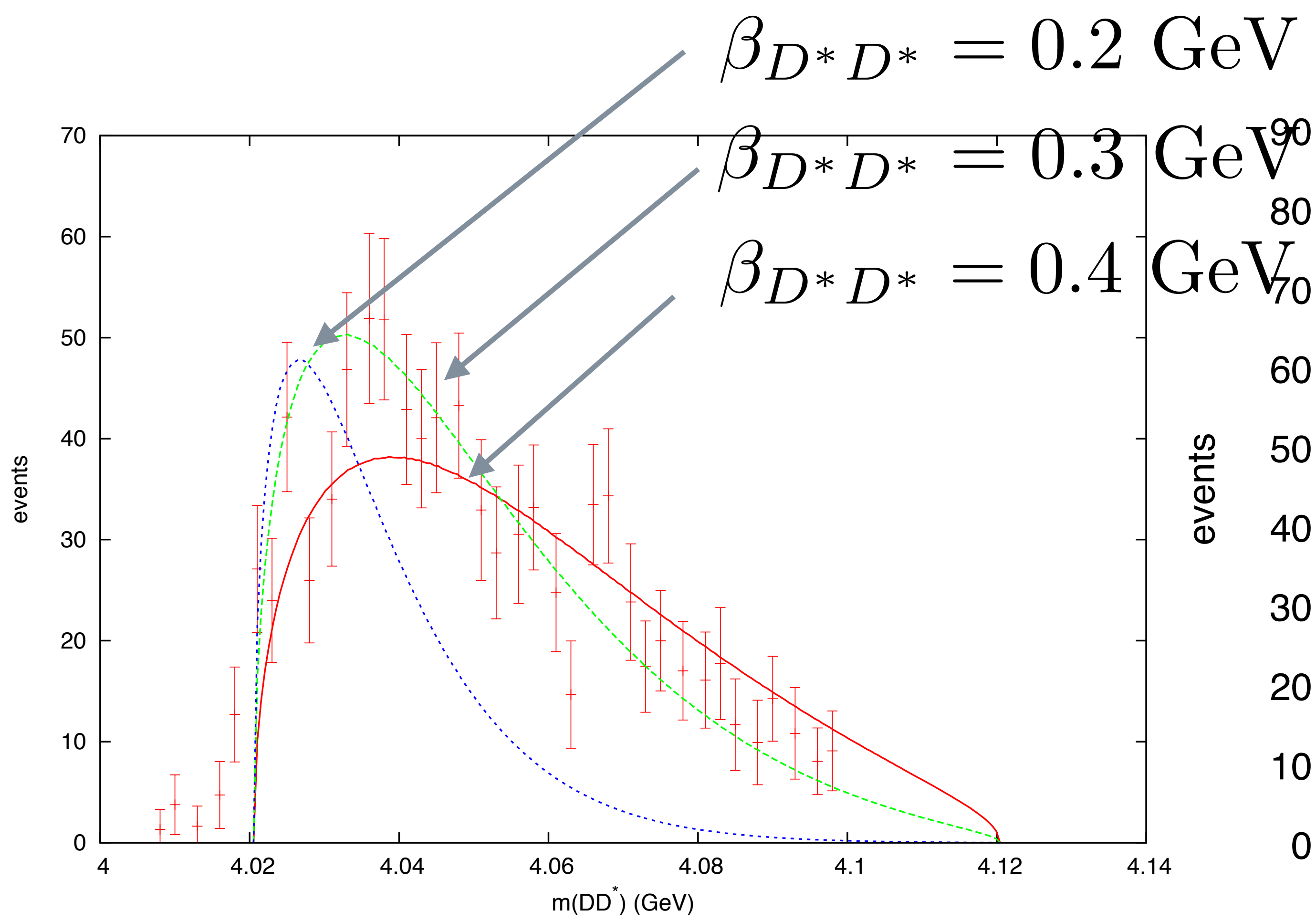


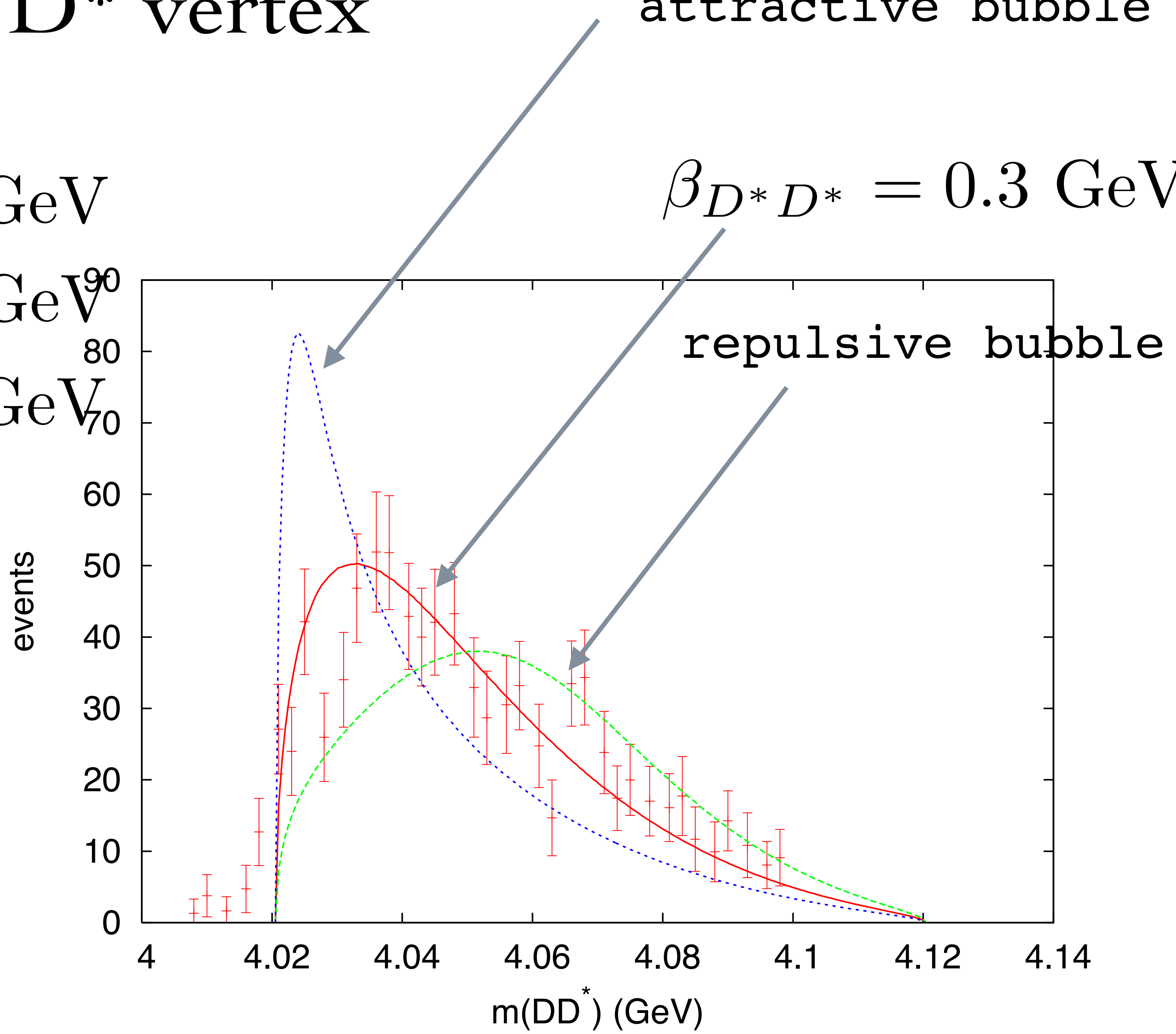
# Cusp Model-II



# Cusp Model-II

fit the pi Y: D*D* vertex

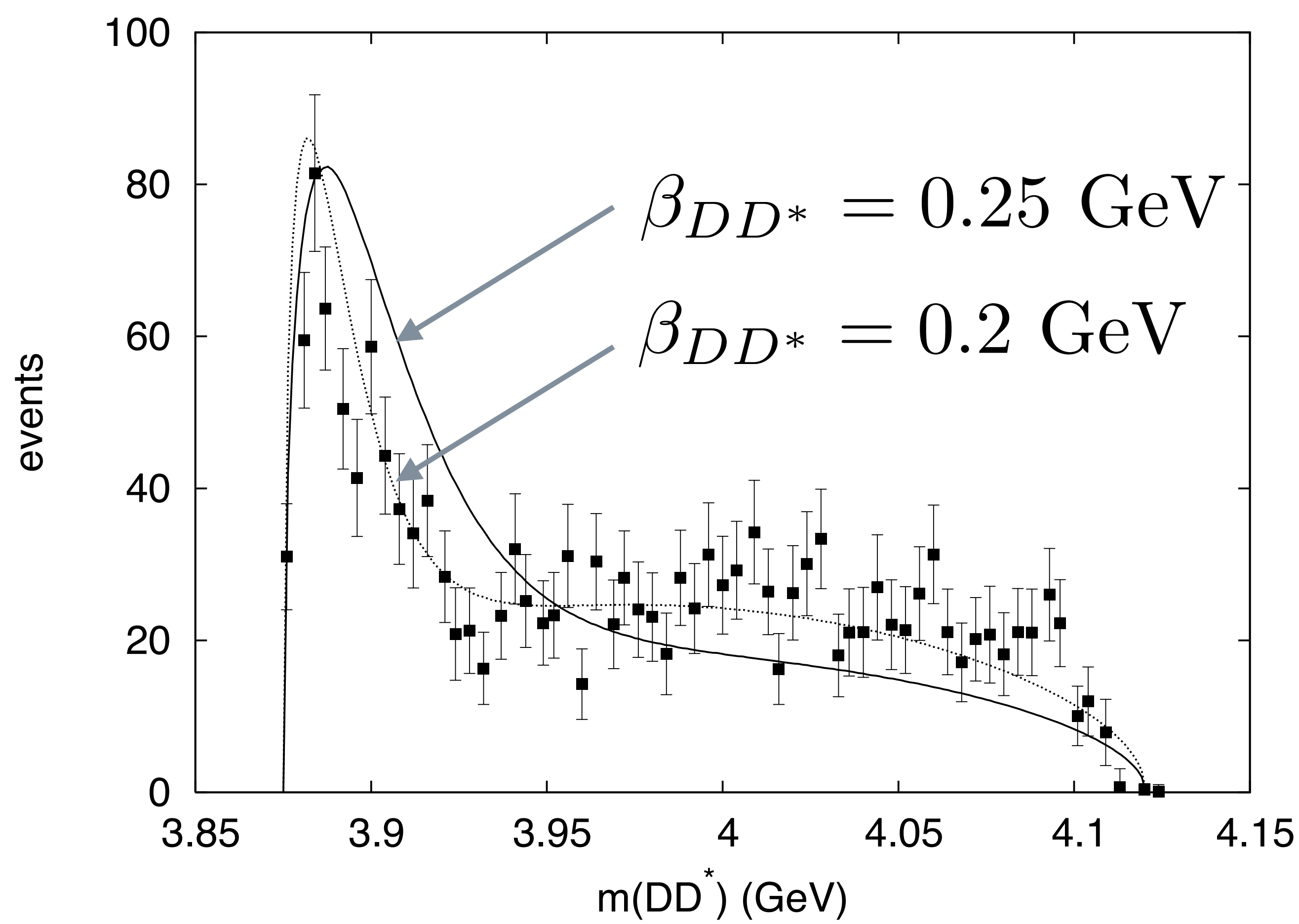


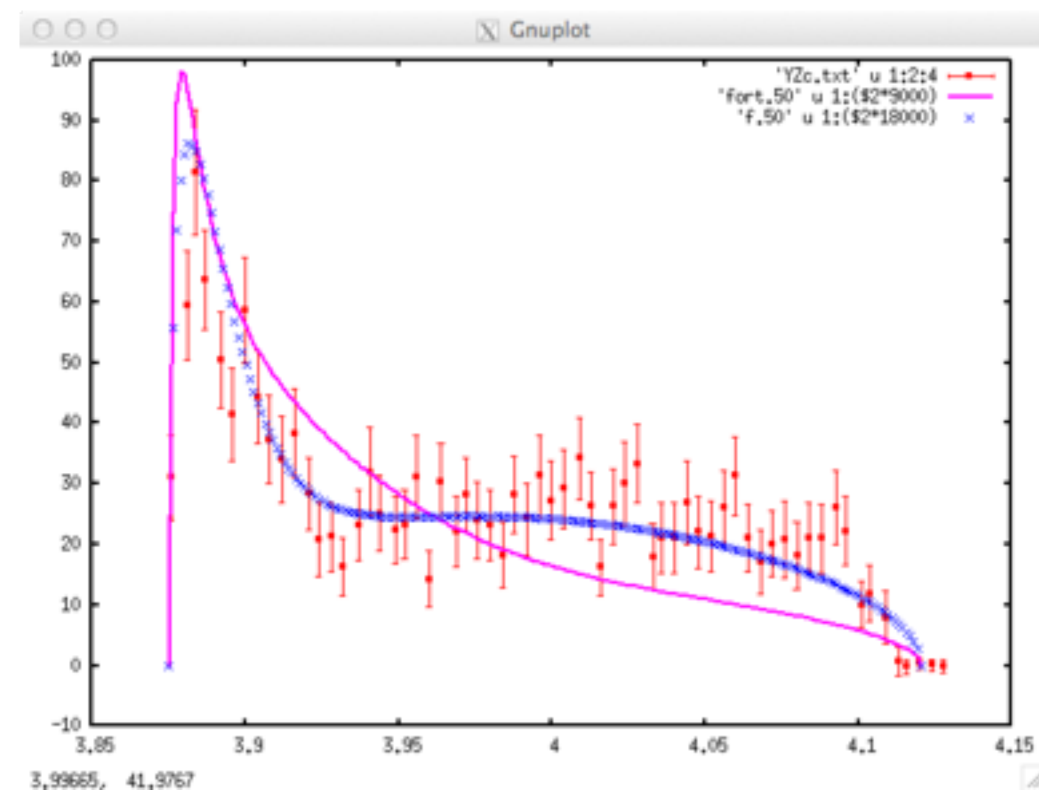




no evidence for π D* dynamics, background, or bubble

# Cusp Model-II

fit the pi Y: DD* vertex



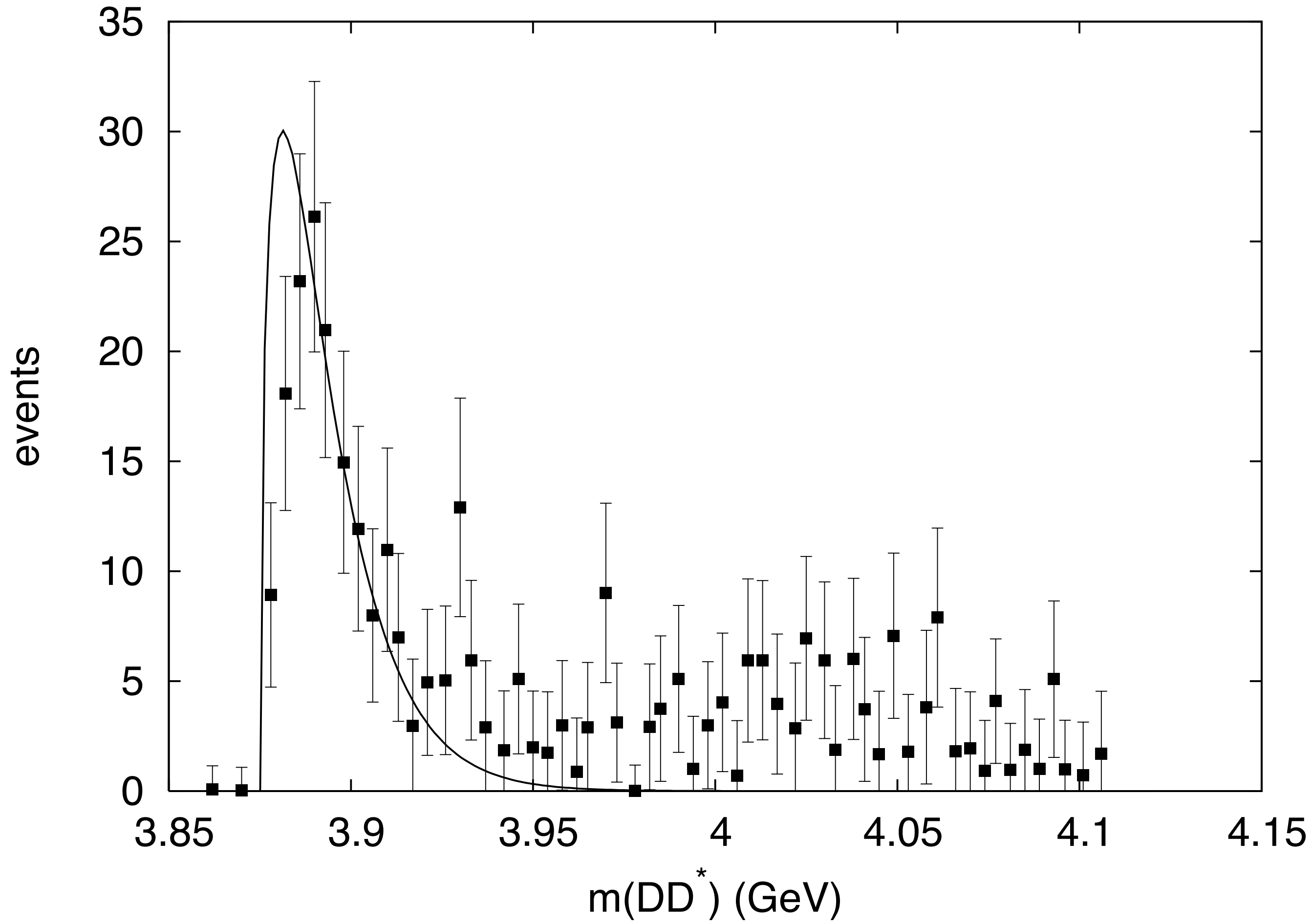




no evidence for bubble

evidence for incoherent background

# Cusp Model-II

fit the pi Y: DD* vertex

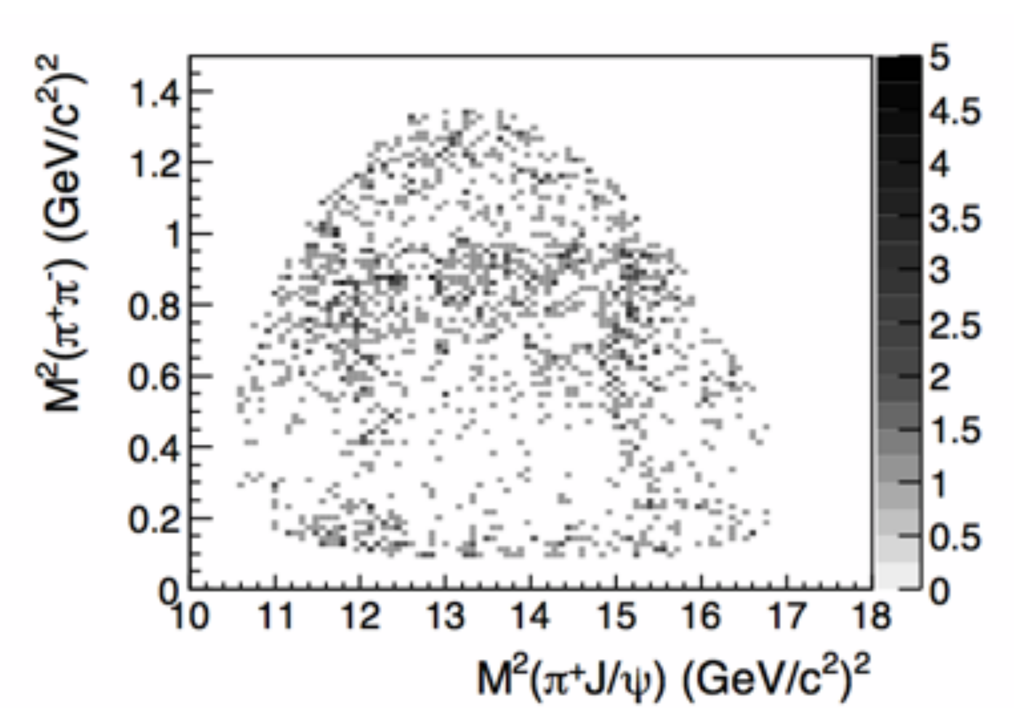


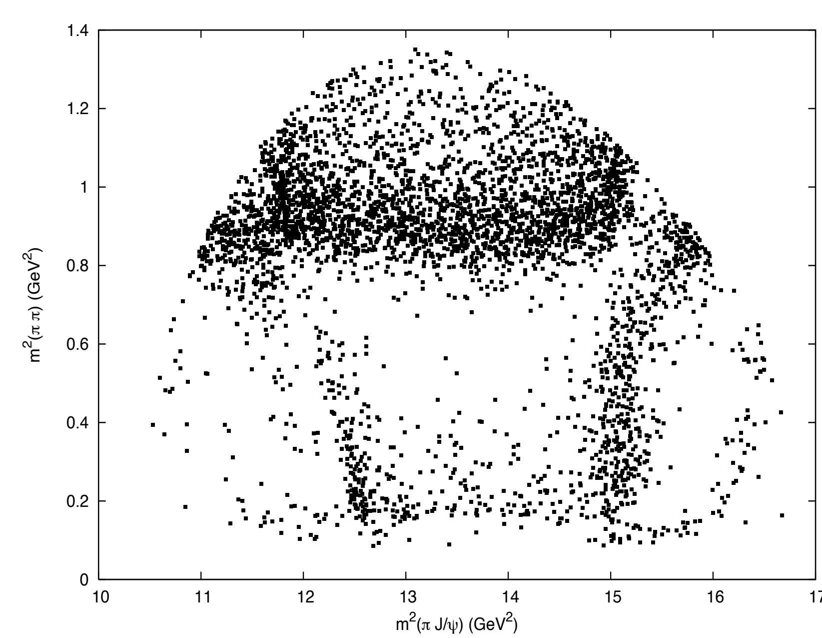

Wolfgang Gradl, "Bound States in QCD", St Goar, Mar 24-27, 2015

# Cusp Model-II

continue to Y: pi pi J/psi

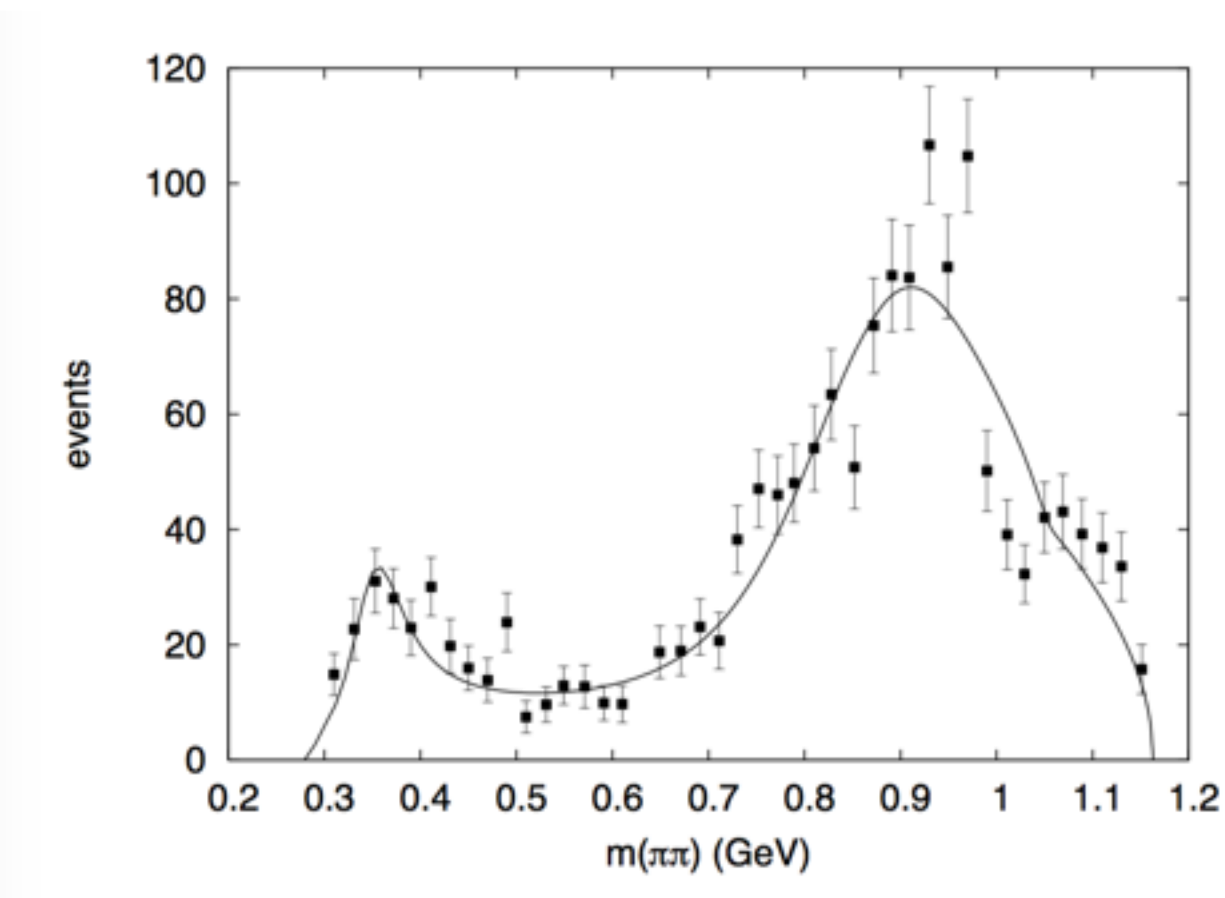
Now pi pi dynamics is important

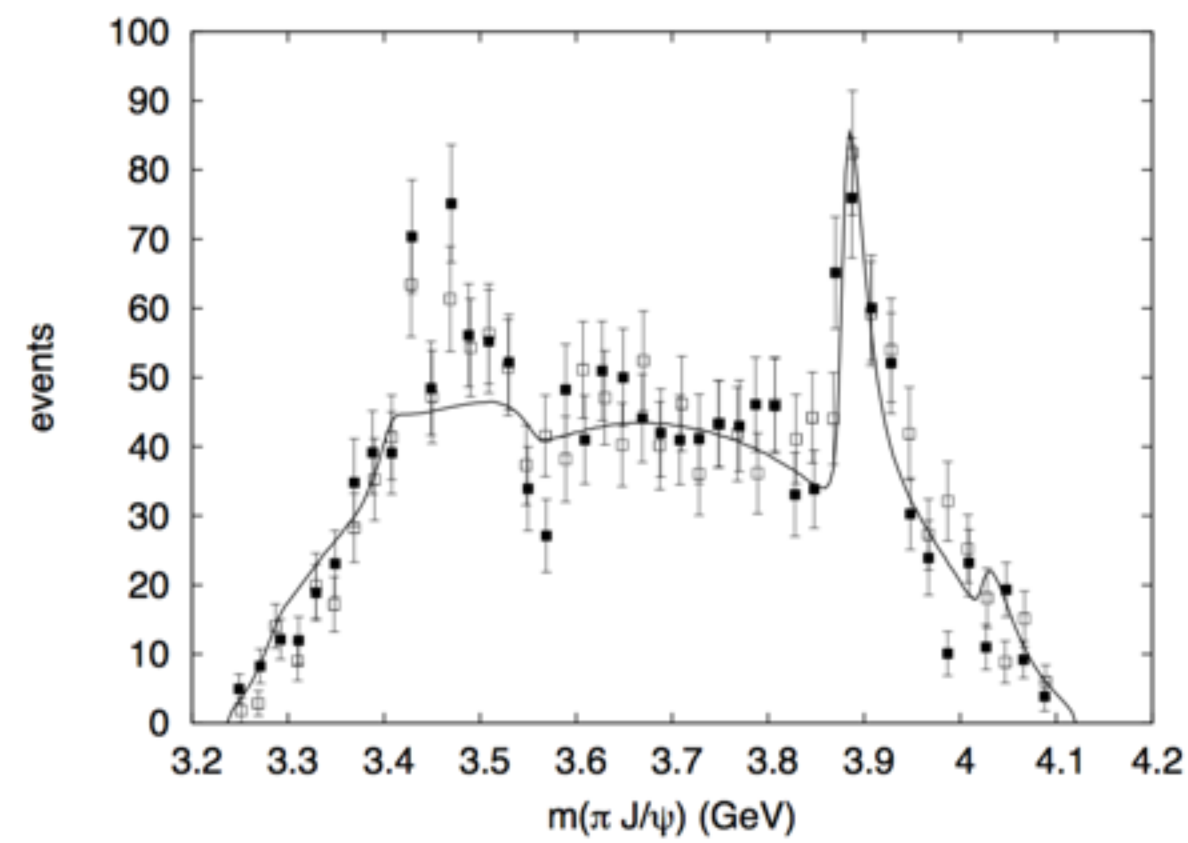


# Cusp Model-II

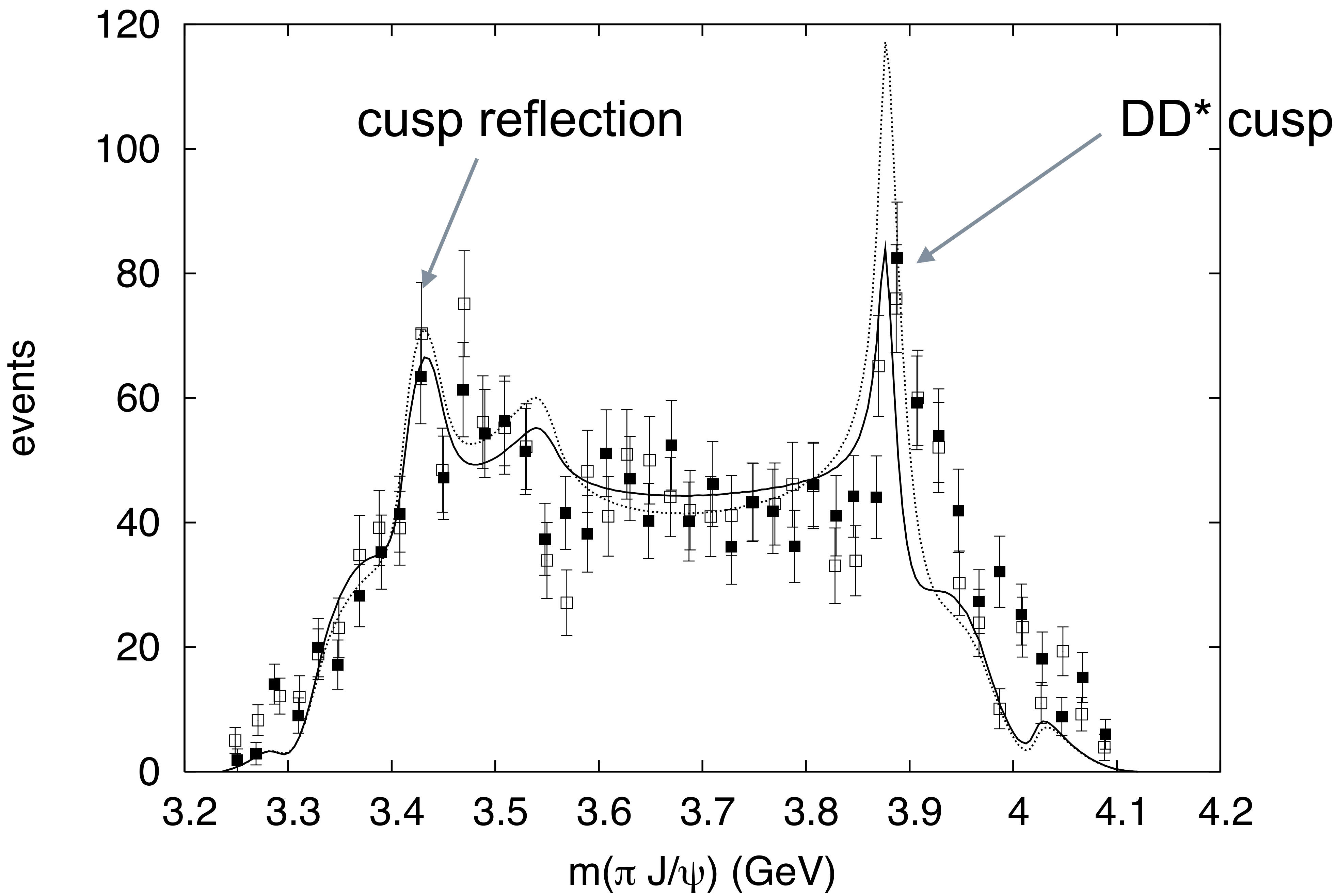
Y: pi pi J/psi



# Cusp Model-II

Y: pi pi J/psi




BESIII

# Cusp Model-II

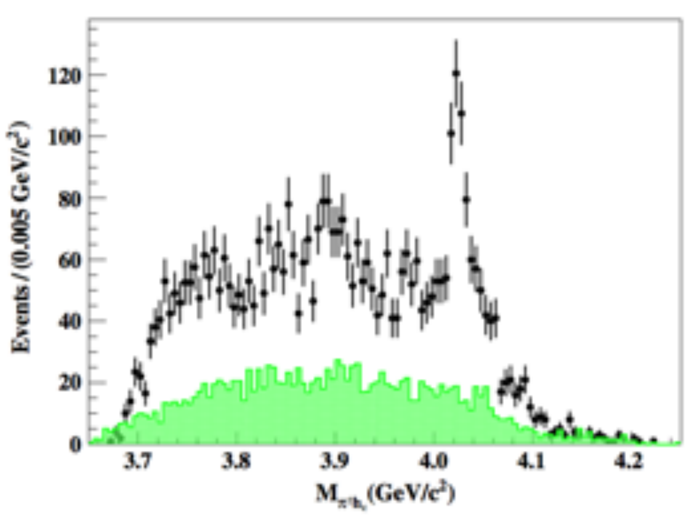
M. Ablikim et al. [BESIII Collaboration], Phys. Rev. Lett. 111, 242001 (2013).

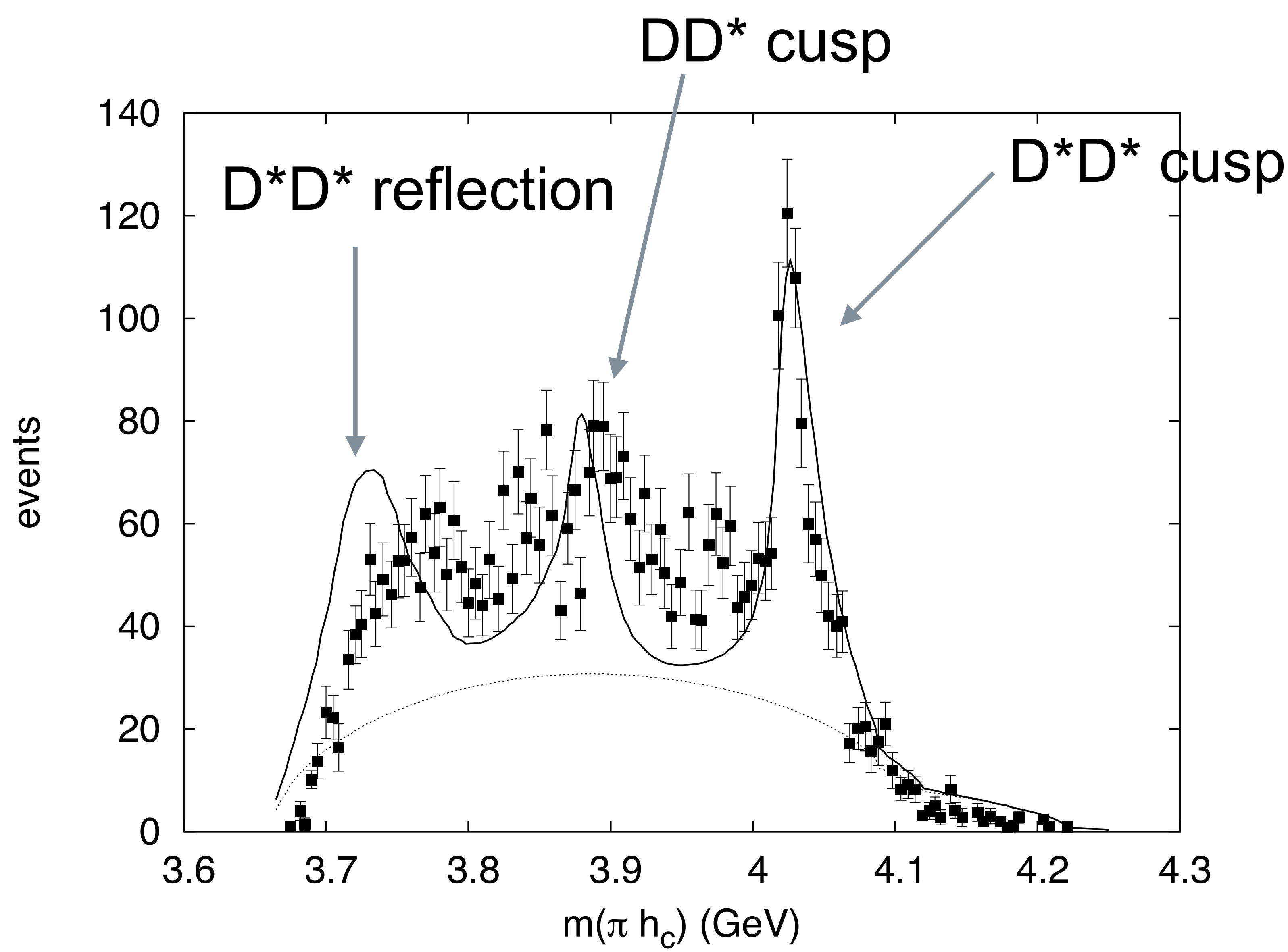
$$e^+e^- \to \pi^+\pi^-h_c$$

[incoherent background only]

sums 13 different ee energy values

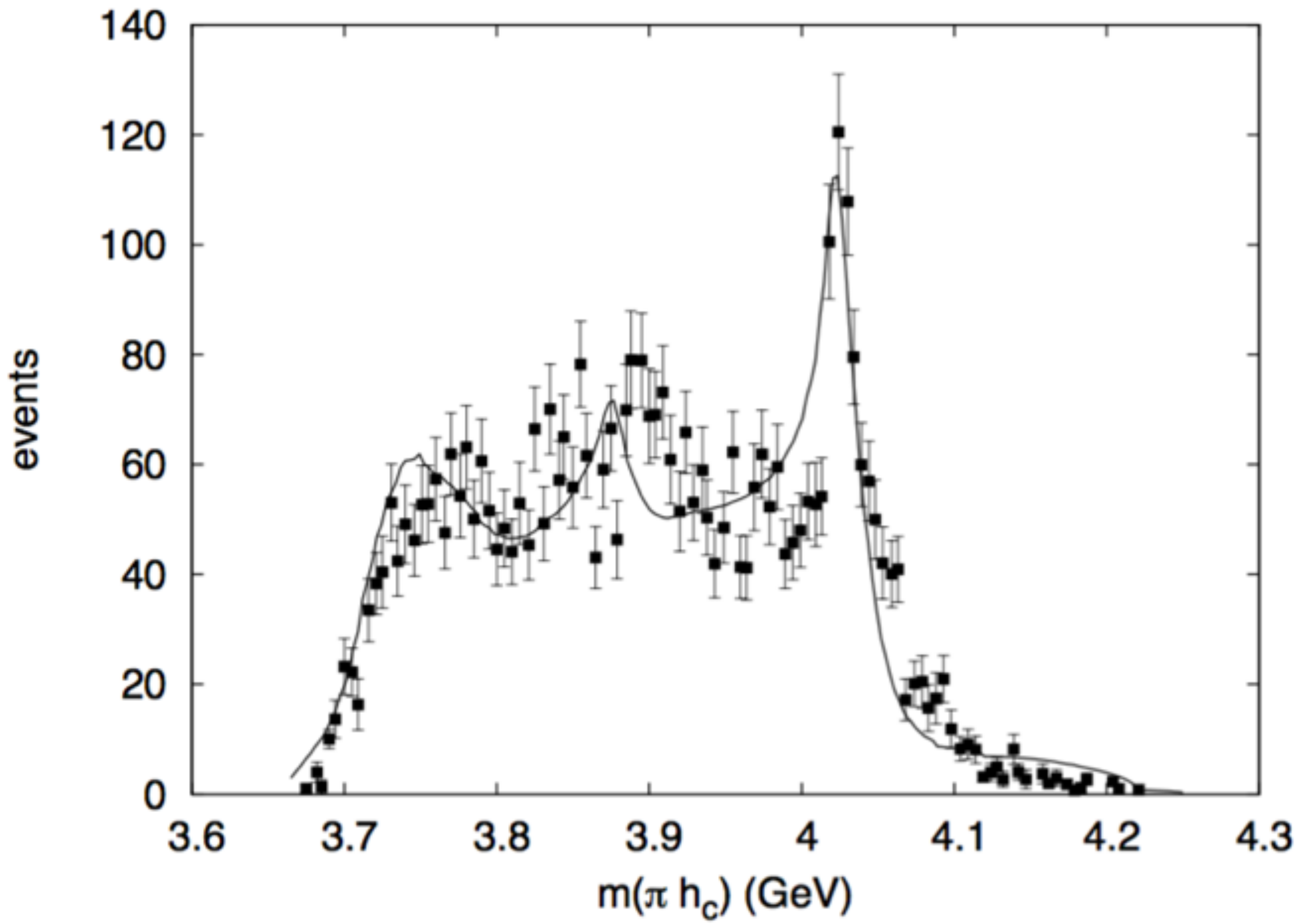
“no significant Zc(3900) observed”



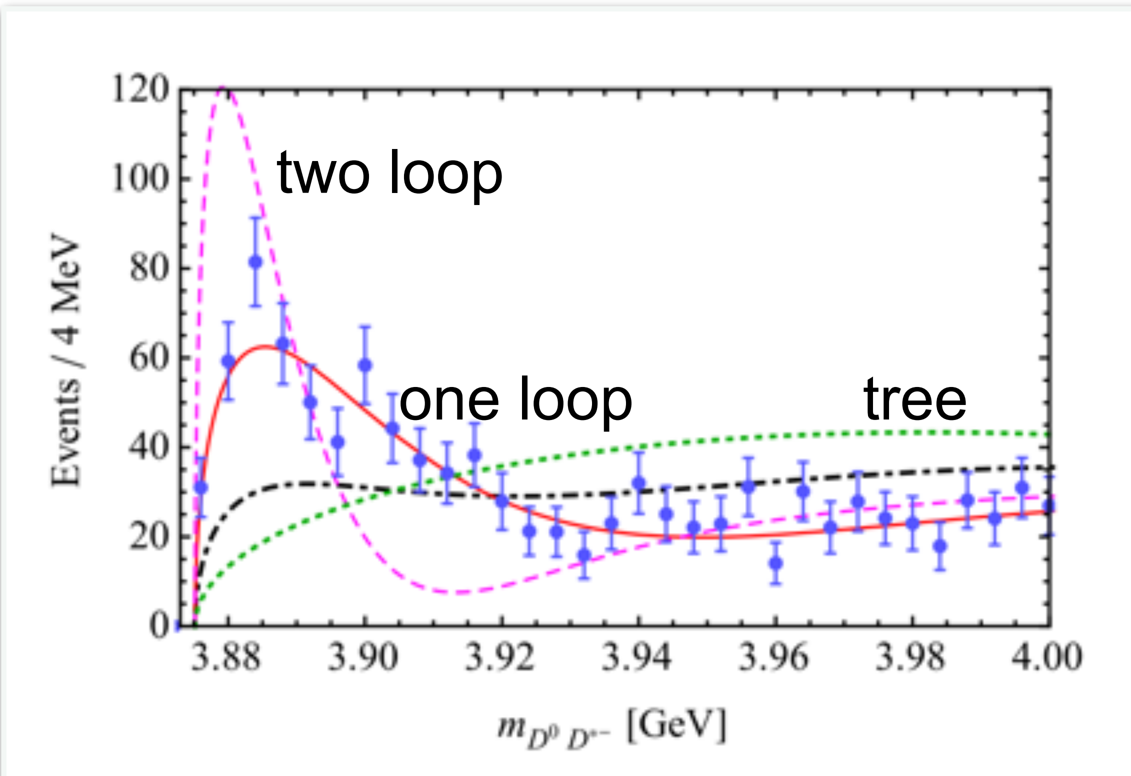
# Cusp Model-II

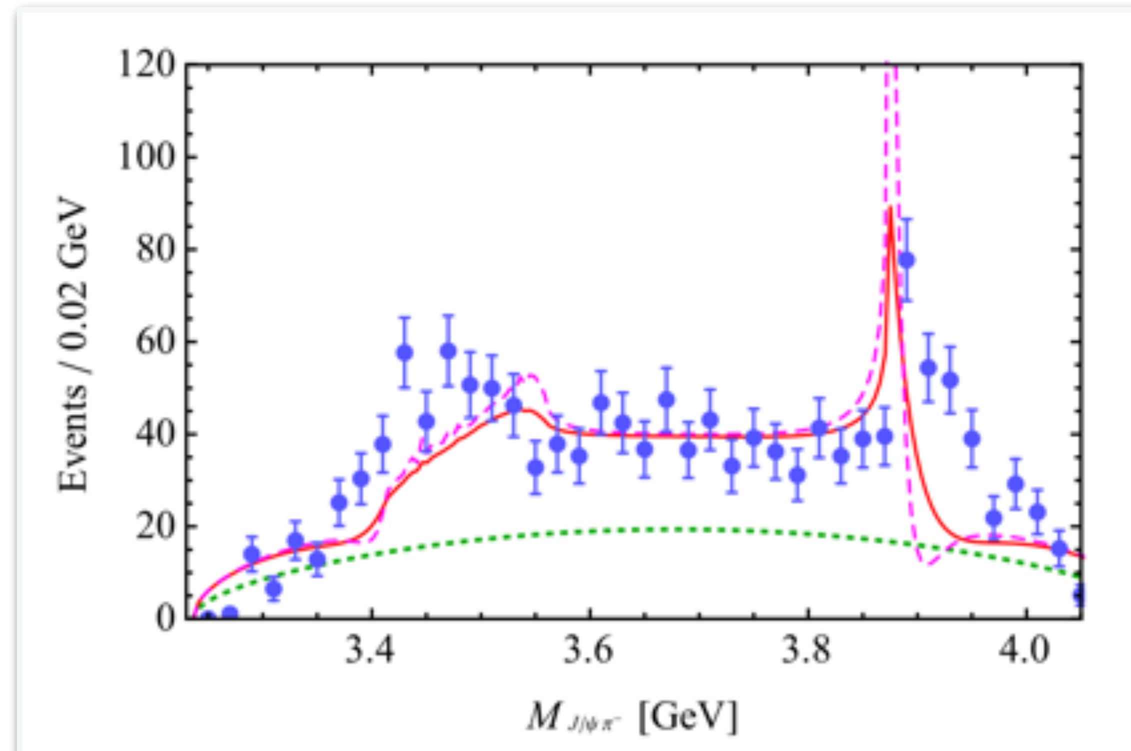
$e^+e^- \to \pi^+\pi^- h_c$



# Cusp Model-III

F.-K. Guo et al. arXiv: 1411.5584

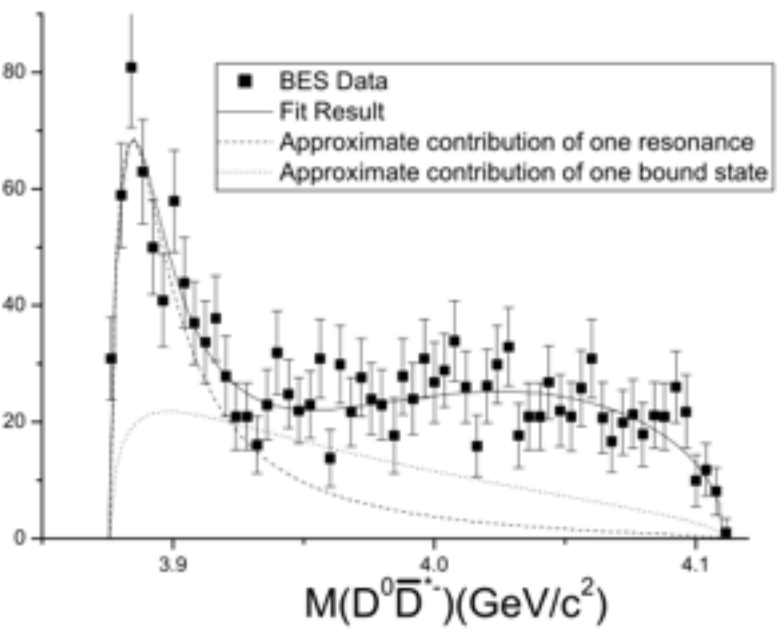


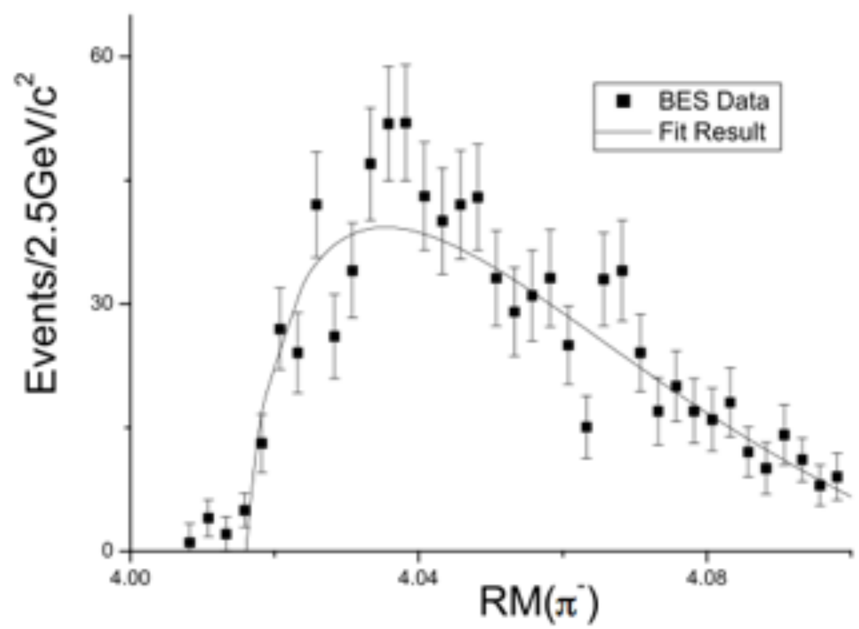



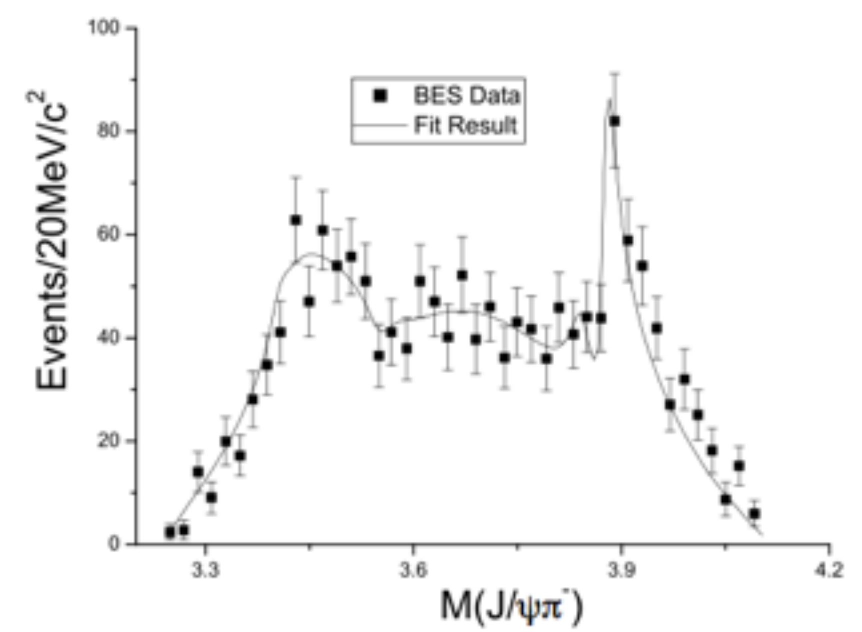

Hanhart *et al.* claim that the strength of the vertex requires bubble summation, which generates a pole.

# Cusp Model-IV

Z.Y. Zhou and Z. Xiao, ``Distinguishing cusp effects and near-threshold-pole effects,'' arXiv:1505.05761 [hep-ph].



# Exotic Phenomenology

## Additional Aspects

# other cusp channels

- $\Upsilon(5S) \to K\bar{K}\Upsilon(nS)$

| | |
|---|---|
| $B\bar{B}_s^* + B^*\bar{B}_s$ | 10695 |
| $B^*\bar{B}_s^*$ | 10745 |

- $e^+e^- \to K\bar{K}J/\psi$

| | |
|---|---|
| $D\bar{D}_s^* + D^*\bar{D}_s$ | 3980 |
| $D^*\bar{D}_s^*$ | 4120 |

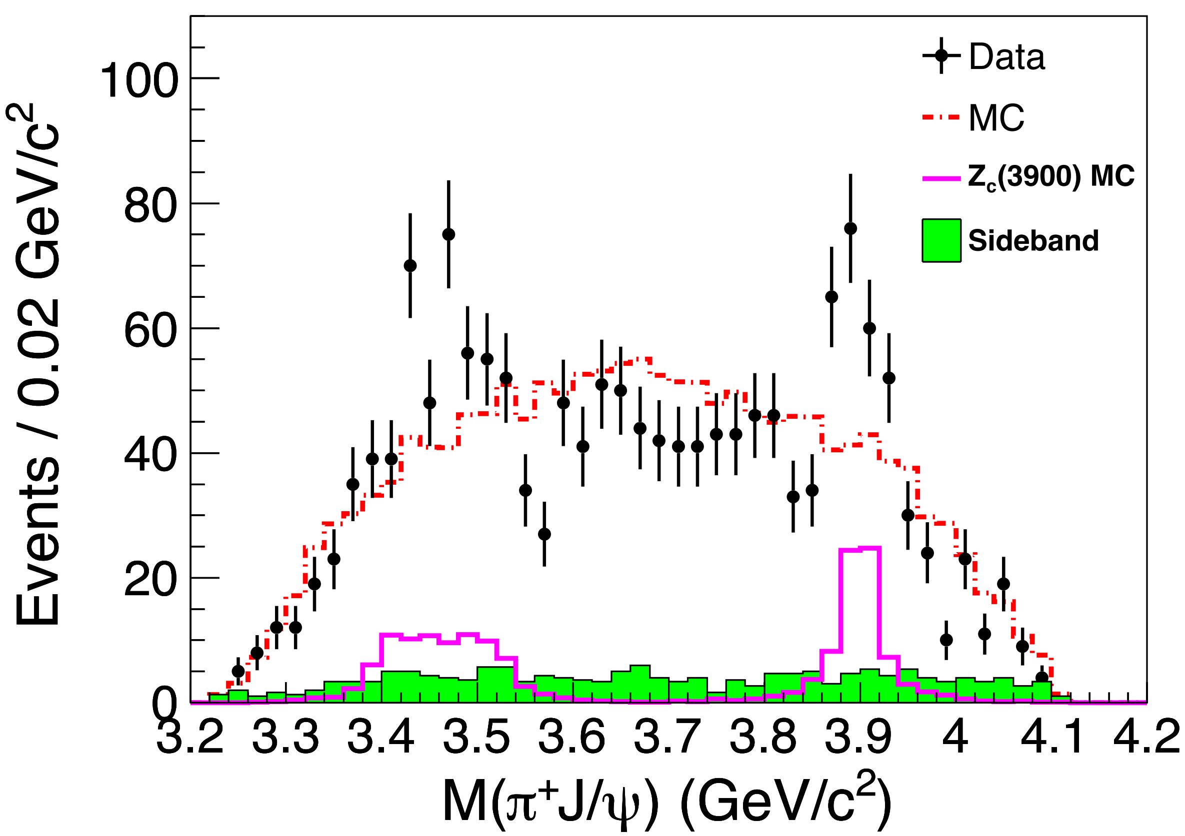
# will now argue for

- $\bar{B}^0 \to J/\psi\pi^0\pi^0$
- $B^\pm \to J/\psi\pi^\pm\pi^0$

# Cusp Model

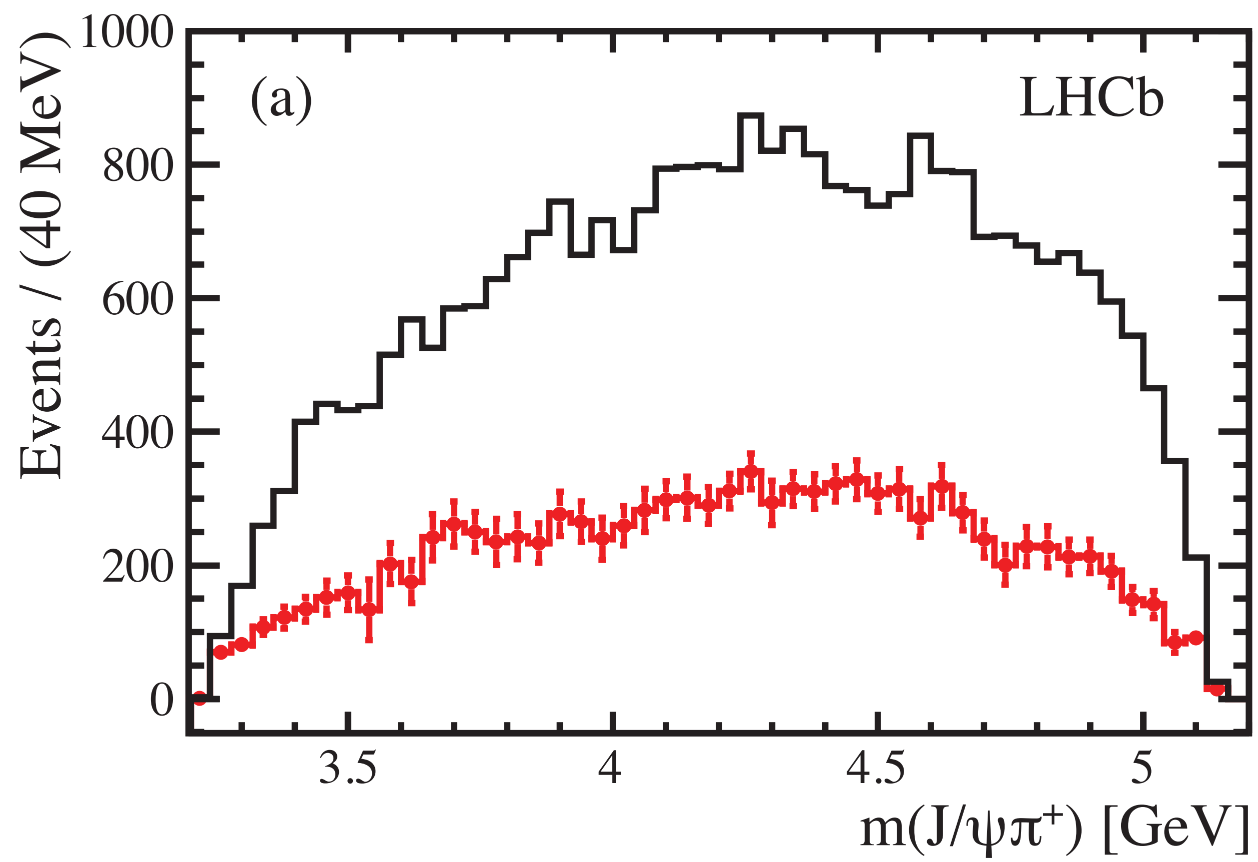
missing exotics…

$Y(4260) \rightarrow \pi^+\pi^- J/\psi$




BESIII

$B_0 \rightarrow \pi^+\pi^- J/\psi$



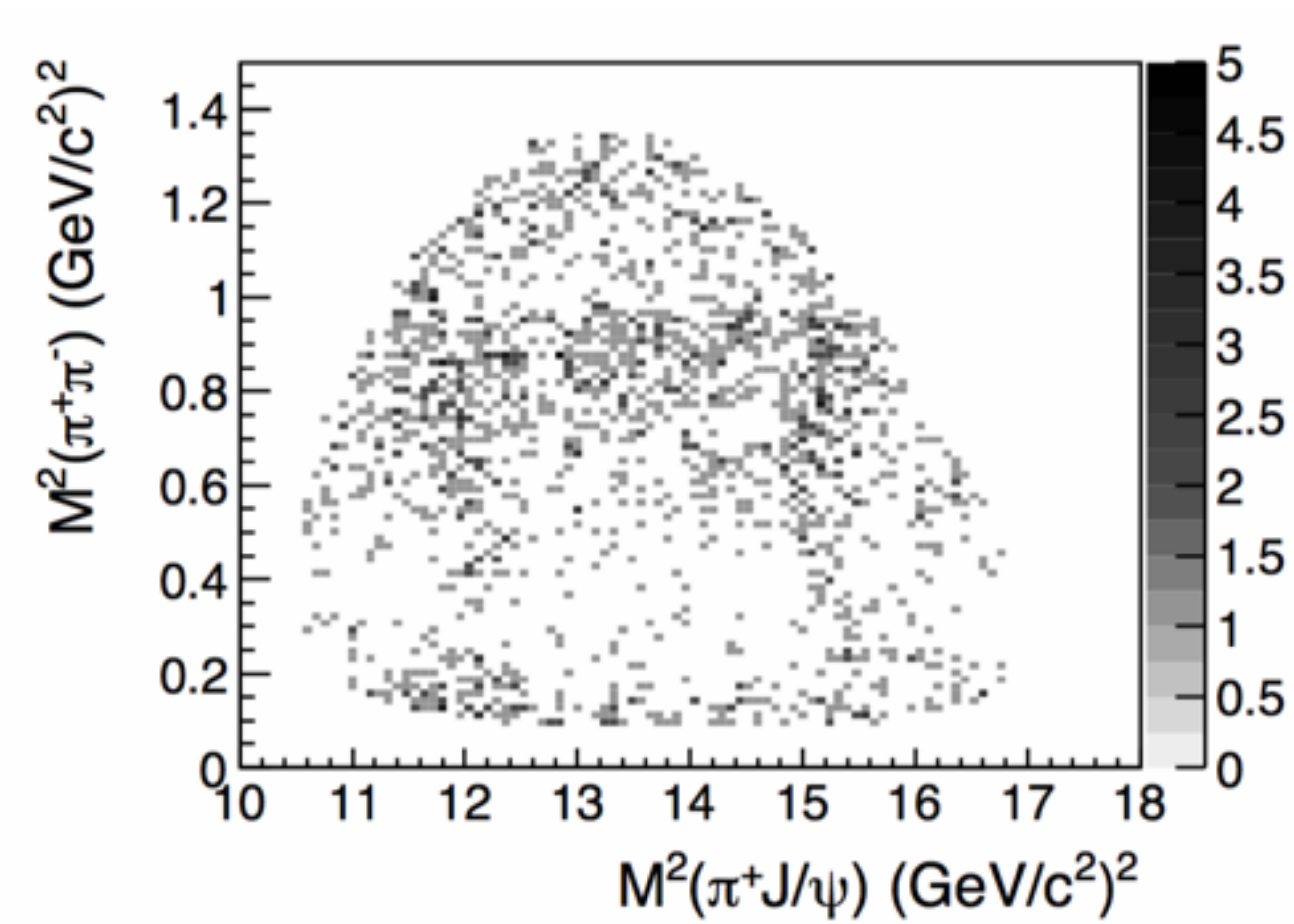

LHCb

R. Aaij et al. [LHCb Collaboration], Phys. Rev. D 90, 012003 (2014).

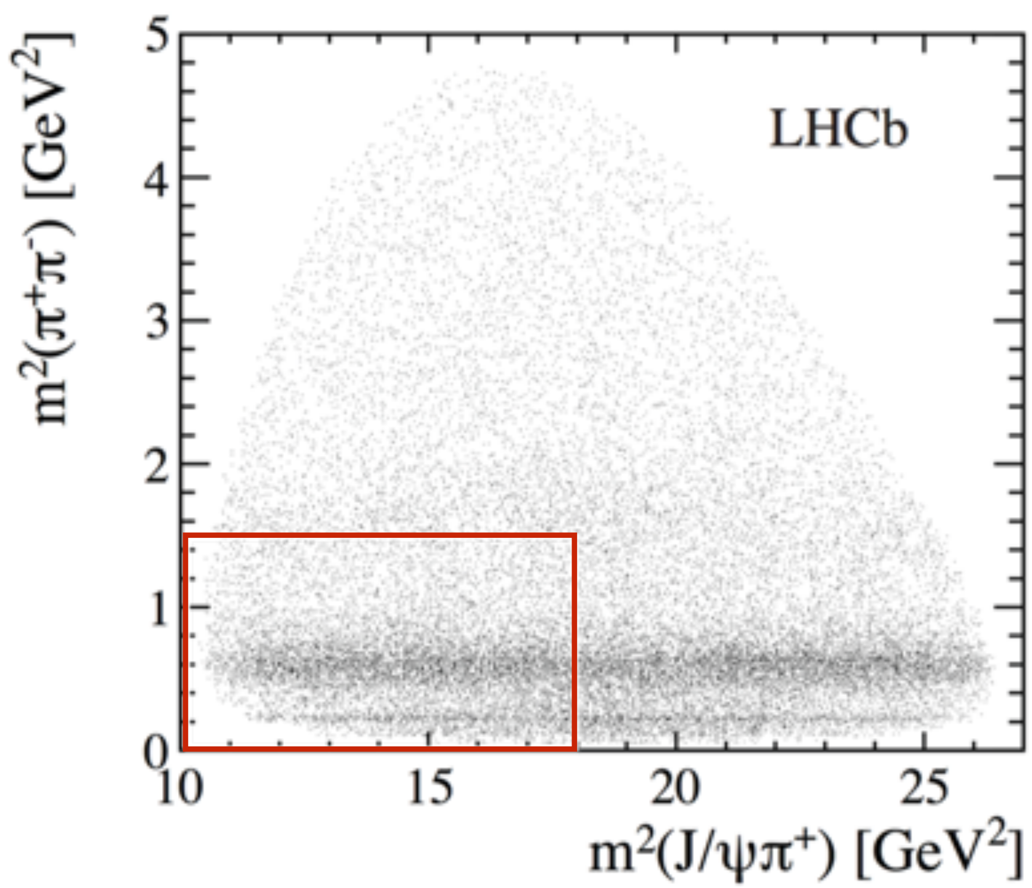
# Cusp Model

missing exotics…

$Y(4260) \to \pi^+\pi^- J/\psi$




$B_0 \to \pi^+\pi^- J/\psi$

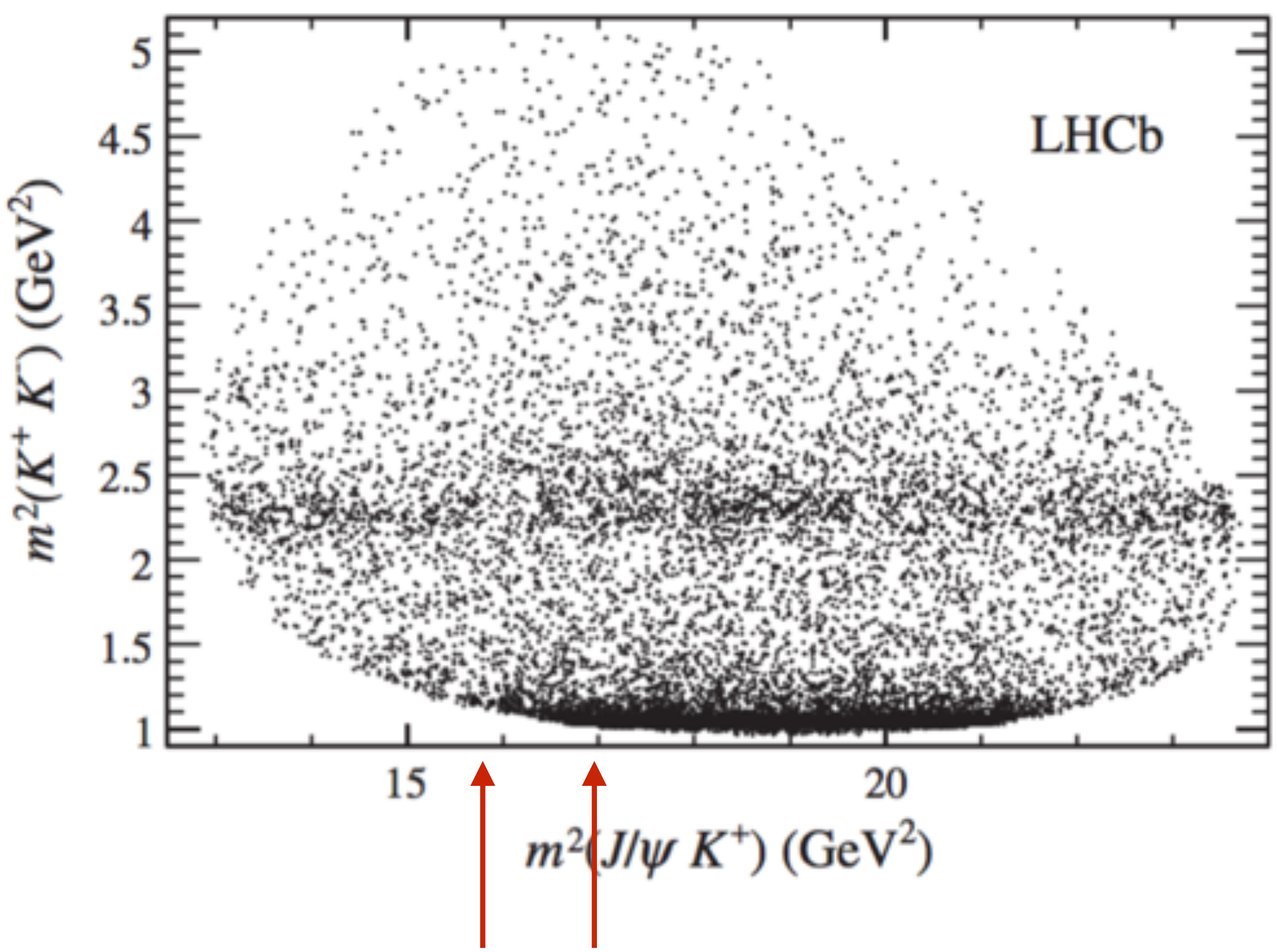


# Cusp Model

missing exotics…

$$\bar{B}_s^0 \to K^+ K^- J/\psi$$

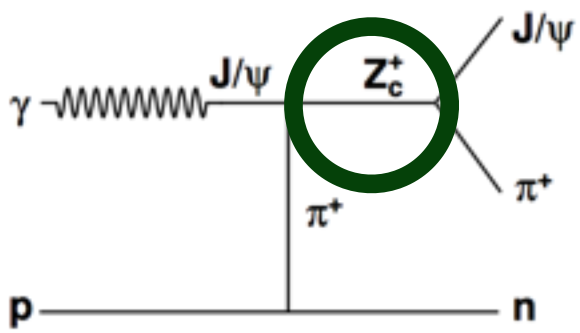
PHYSICAL REVIEW D **87,** 072004 (2013)

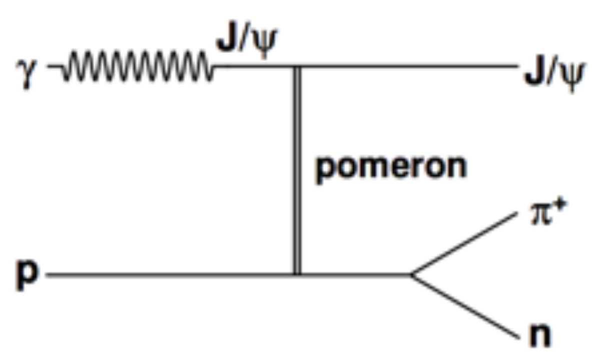


# Cusp Model

missing exotics…

## COMPASS



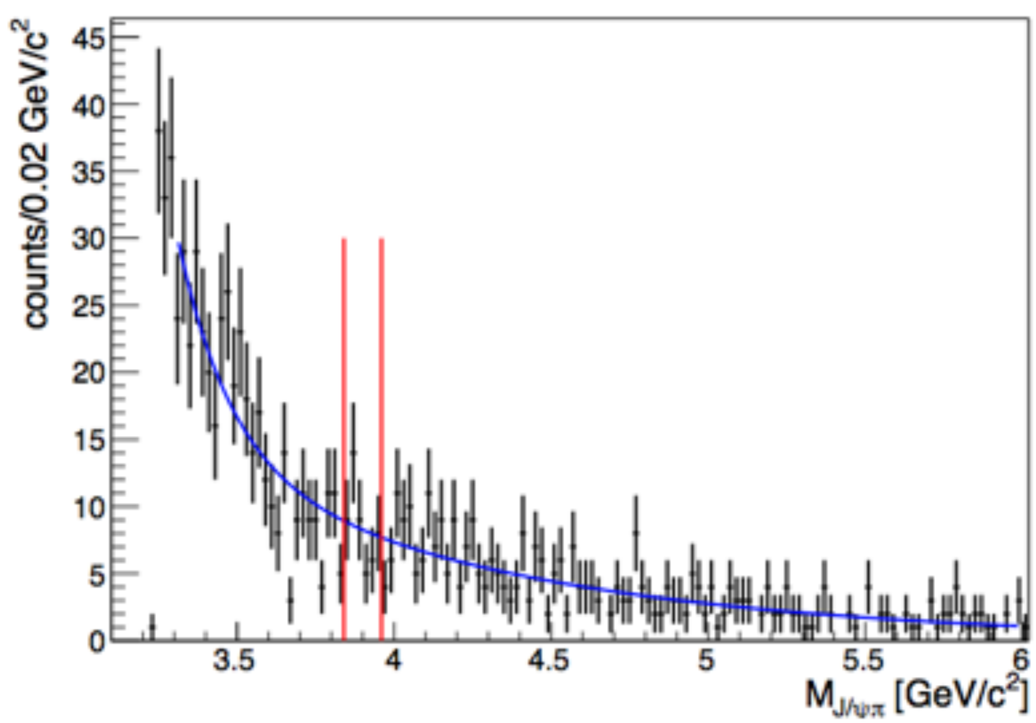




$$\sqrt{s_{\gamma N}} = 7\ \mathrm{GeV}$$

$$\exp(-\lambda(s_{\gamma N}, m_\psi^2, m_\pi^2)/(4s_{\gamma N}\beta^2) \approx$$

$$\exp(-(s_{\gamma N} - m_\psi^2)^2/(4s_{\gamma N}\beta^2) \approx$$

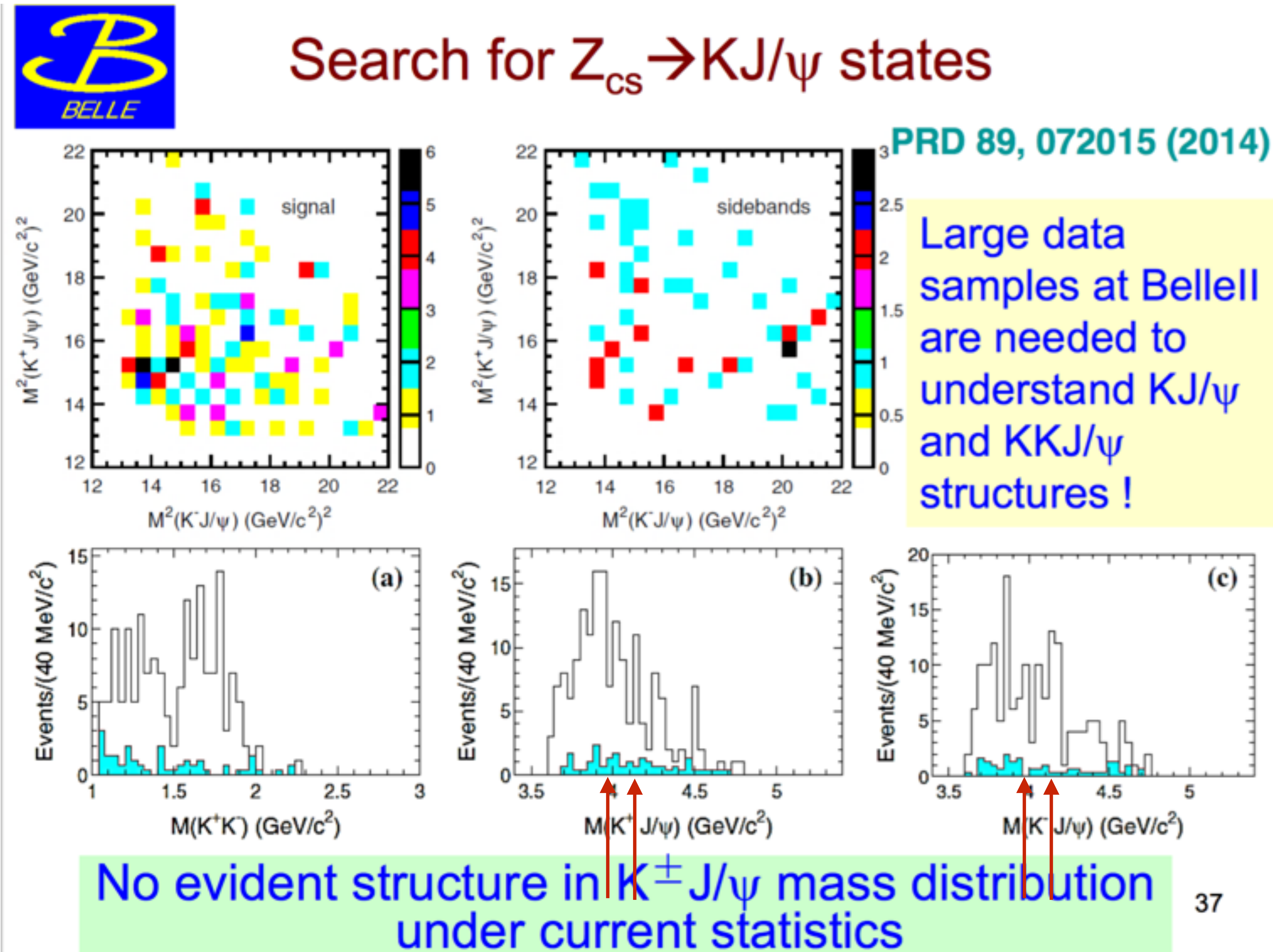
$$\exp(-88)$$




C. Adolph et al. [COMPASS] arXiv:1407.6186v1

# Cusp Model

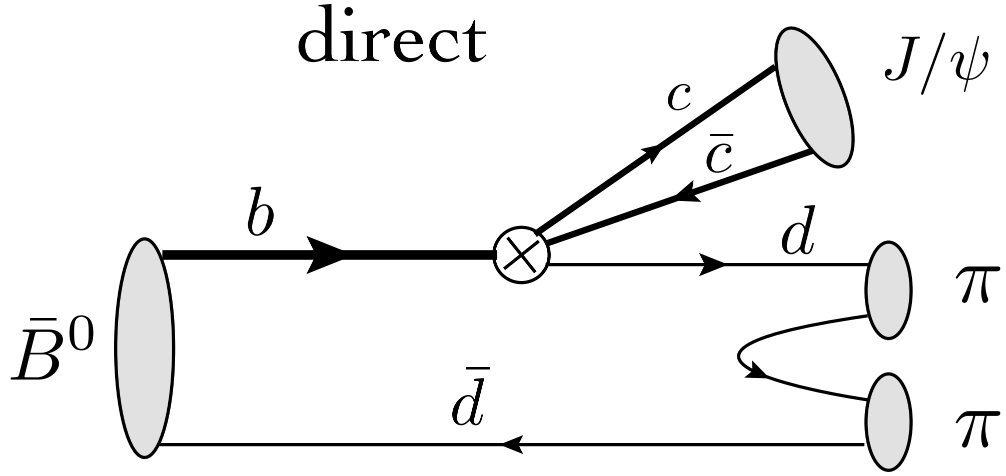
missing exotics…



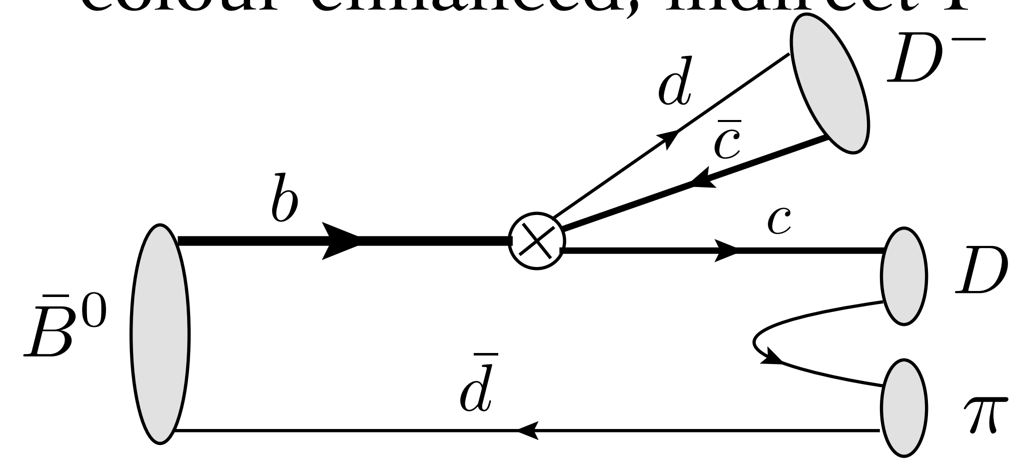
# Cusp Model

missing exotics…

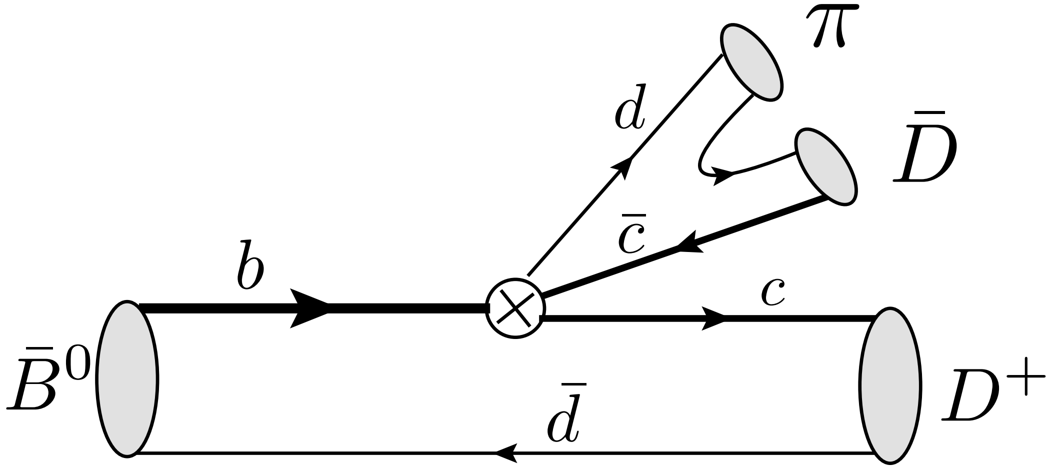
direct



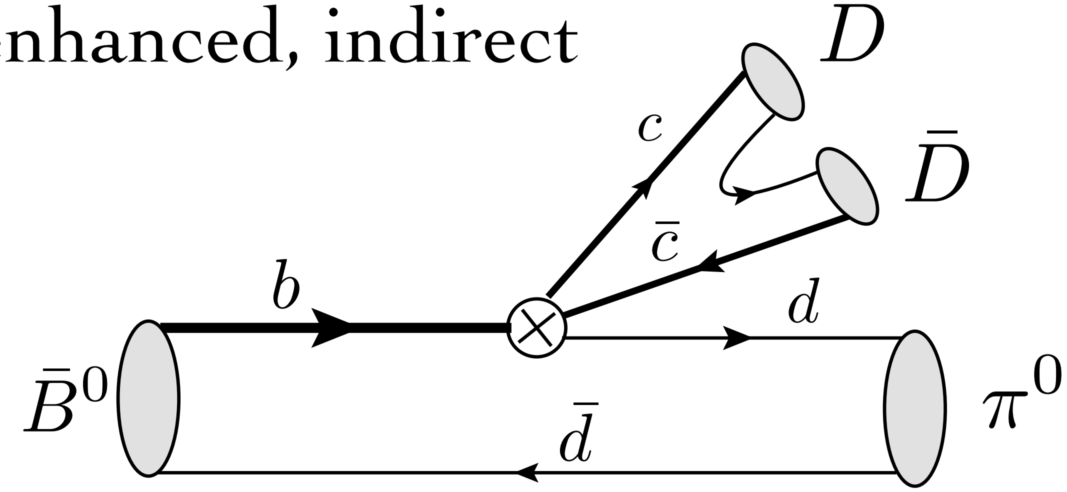

colour enhanced, indirect I




colour enhanced, indirect II



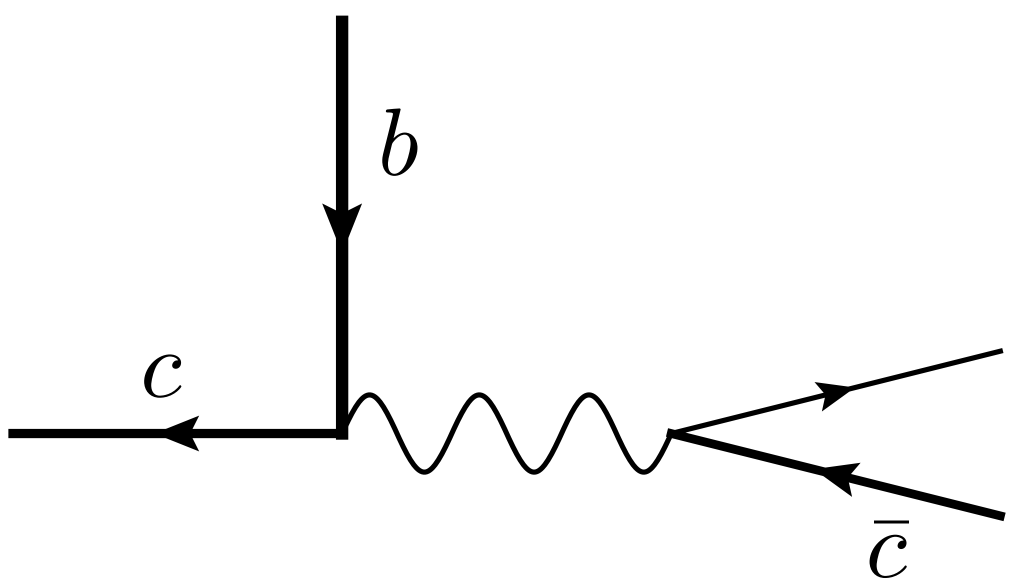

colour suppressed, wavefunction enhanced, indirect



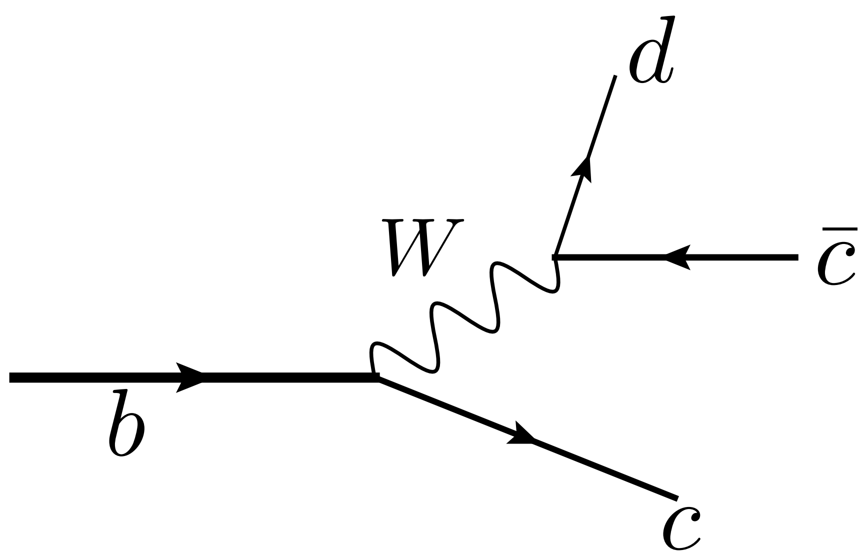
# Cusp Model

missing exotics…

the direct process is suppressed due to the small odds of back to back charm quarks making a J/psi



# Cusp Model




missing exotics…

in more detail…

$$\rho(m_{c\bar{c}}) = \int \overline{|\mathcal{M}|^2}(m_{c\bar{c}}, m_{d\bar{c}})\, dm^2_{d\bar{c}}$$

$$\bar{p} = \frac{\int_0^{\sqrt{m_b^2/4 - m_c^2}} \rho(p) p\, dp}{\int_0^{\sqrt{m_b^2/4 - m_c^2}} \rho(p)\, dp}$$

$$\mathcal{P}(\bar{p}) \doteq \int_{\bar{p}}^{\infty} d^3q\, |\psi(q)|^2$$

$$\mathcal{P}(0.92) = 25\%$$

# Cusp Model

missing exotics...

the wavefunction penalty is confirmed in the data

| $B \to X$ | Bf |
|---|---|
| $D^*D^*$ | $8 \cdot 10^{-4}$ |
| $DD^*$ | $4 \cdot 10^{-4}$ |
| $DD$ | $4 \cdot 10^{-4}$ |
| $\psi\pi$ | $4 \cdot 10^{-5}$ |
| $\psi\rho$ | $5 \cdot 10^{-5}$ |
| $\psi\pi\pi$ | $4 \cdot 10^{-5}$ |

# Cusp Model

missing exotics...

no penalty for extra light quarks

| $B \to X$ | Bf |
|---|---|
| $D\pi^+$ | $2.7 \cdot 10^{-3}$ |
| $D^0\pi^+\pi^-$ | $8 \cdot 10^{-4}$ |
| $D^-\pi^+\pi^+\pi^-$ | $6 \cdot 10^{-3}$ |
| $\psi K$ | $8.2 \cdot 10^{-4}$ |
| $\psi K\pi$ | $1.2 \cdot 10^{-3}$ |
| $\psi\pi^0$ | $1.7 \cdot 10^{-5}$ |
| $\psi\pi^+\pi^-$ | $4 \cdot 10^{-5}$ |

# Cusp Model    missing exotics…

direct => wavefunction suppressed
colour enhanced, indirect I, II => rescattering suppressed
colour suppressed, wavefunction enhanced => < rescattering suppressed

The first three must be weak since the Zc is not seen by LHCb in B -> psi pi+ pi- .

The same happens in Bs -> psi K+ K- , which 'should' see a 3980 (DsD* + DDs*) and a 4215 (DsDs*).

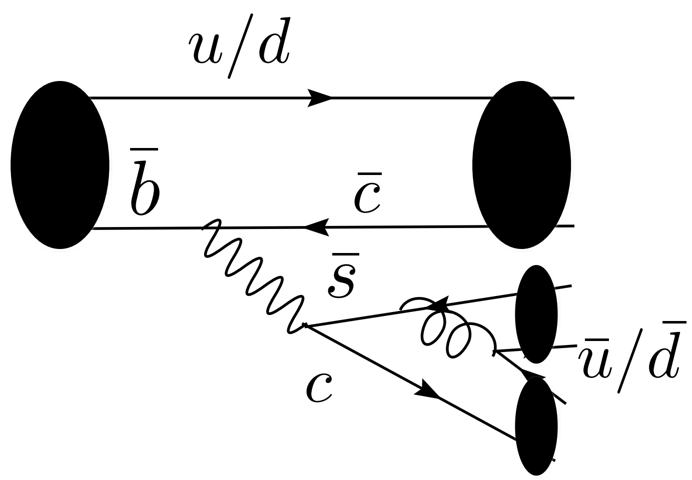
We conclude that either the direct diagram or the rescattering wavefunction enhanced diagram dominates.

If the latter dominates then cusp states should be visible in

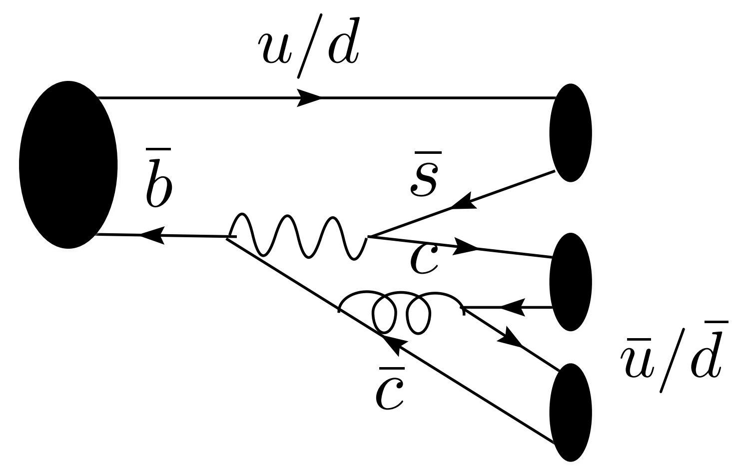
$$B^0 \to \pi^0\pi^0 J/\psi \qquad B^\pm \to \pi^\pm\pi^0 J/\psi \qquad B_s \to \pi\varphi J/\psi$$

# Cusp Model

Application to X(3872)




colour enhanced, II
rescattering suppressed




colour suppressed
rescattering enhanced

$$B^+ \to K^+ D^0 \bar{D}^0$$

$$B^+ \to K^0 D^+ D^0$$

$$B^0 \to K^0 D^- D^+$$

$$B^0 \to K^+ D^- D^0$$

$$B^+ \to K^+ D^0 \bar{D}^0$$

$$B^+ \to K^+ D^+ D^-$$

$$B^0 \to K^0 D^+ D^-$$

$$B^0 \to K^0 D^0 \bar{D}^0$$

# Cusp Model

## Application to X(3872)

colour enhanced
rescattering suppressed

$$B^+ \to K^+ D^0 \bar{D}^0$$

$$B^+ \to K^0 D^+ D^0$$

$$B^0 \to K^0 D^- D^+$$

$$B^0 \to K^+ D^- D^0$$

colour suppressed
rescattering enhanced

$$B^+ \to K^+ D^0 \bar{D}^0$$

$$B^+ \to K^+ D^+ D^-$$

$$B^0 \to K^0 D^+ D^-$$

$$B^0 \to K^0 D^0 \bar{D}^0$$

$$\frac{Br(B^0 \to K^0 X)}{Br(B^+ \to K^+ X)} = \left| \frac{N_c Z_{+-} + \gamma Z_{00} + \gamma Z_{+-}}{N_c Z_{00} + \gamma Z_{00} + \gamma Z_{+-}} \right|^2 \approx \left| \frac{\gamma}{N_c + \gamma} \right|^2$$

# Cusp Model

Application to X(3872)

$$\frac{Br(B^0 \to K^0 X)}{Br(B^+ \to K^+ X)} = \left| \frac{N_c Z_{+-} + \gamma Z_{00} + \gamma Z_{+-}}{N_c Z_{00} + \gamma Z_{00} + \gamma Z_{+-}} \right|^2 \approx \left| \frac{\gamma}{N_c + \gamma} \right|^2$$

$$\frac{Br(B^0 \to K^0 X)}{Br(B^+ \to K^+ X)} = \ 0.50 \pm 0.30 \pm 0.05$$

Thus $\gamma \approx 7^{+17}_{-4.6}$

Now: 0.82 +- 0.22 +- 0.05 arXiv:0809.1224

# Cusp Model

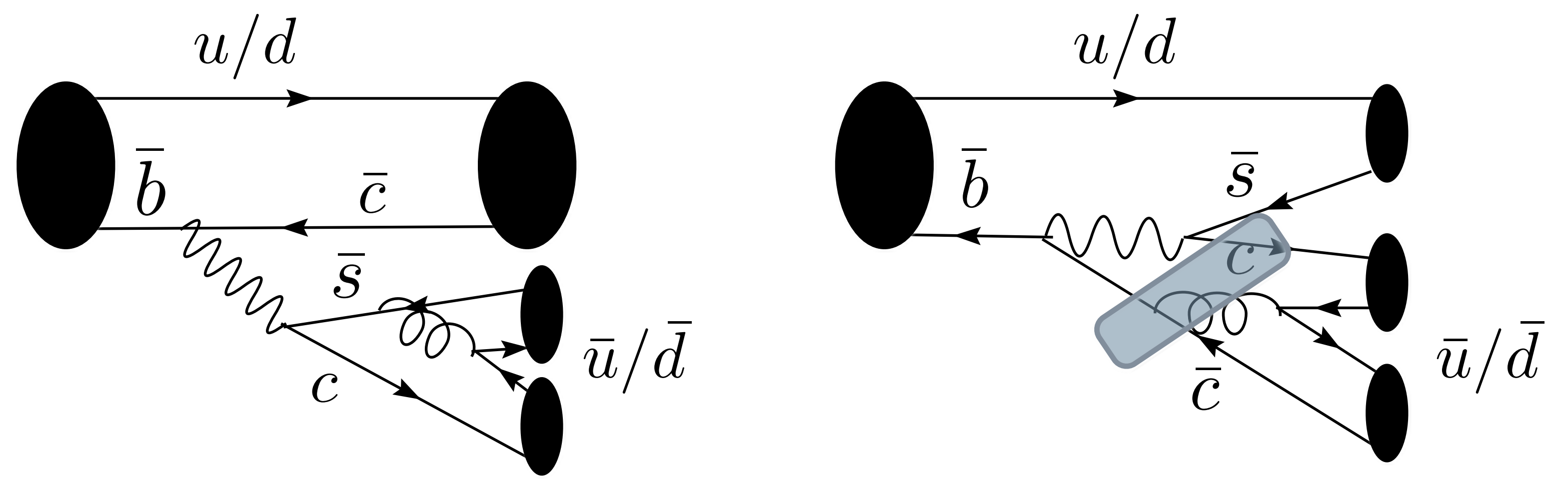
Application to X(3872)

| colour enhanced rescattering suppressed | colour suppressed rescattering enhanced |
| --- | --- |
| $B^+ \to K^+ D^0 \bar{D}^0$ | $B^+ \to K^+ D^0 \bar{D}^0$ |
| $B^+ \to K^0 D^+ D^0$ | $B^+ \to K^+ D^+ D^-$ |
| $B^0 \to K^0 D^- D^+$ | $B^0 \to K^0 D^+ D^-$ |
| $B^0 \to K^+ D^- D^0$ | $B^0 \to K^0 D^0 \bar{D}^0$ |

An $X^+$ or $X^-$ should be made with approximately the same strength as the X. These modes are not seen ⇒ X has no charge-partners, and X is not a cusp effect.

# Cusp Model

Application to X(3872)




Note that the rescattering enhanced diagram goes through a $\chi'_{c1}$ explaining the large production seen, if this state has a large overlap with the X.

# Cusp Model

## Application to X(3872)

## X-χ mixing

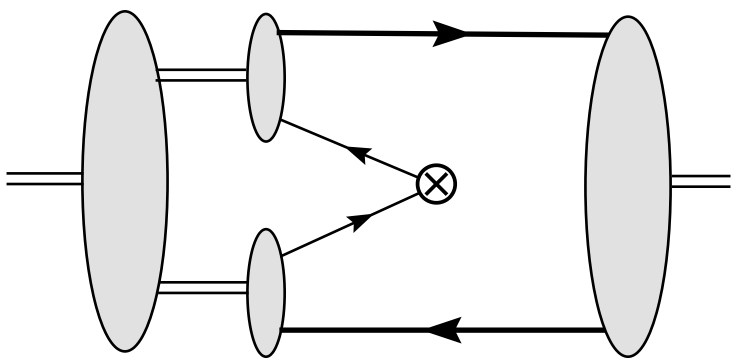


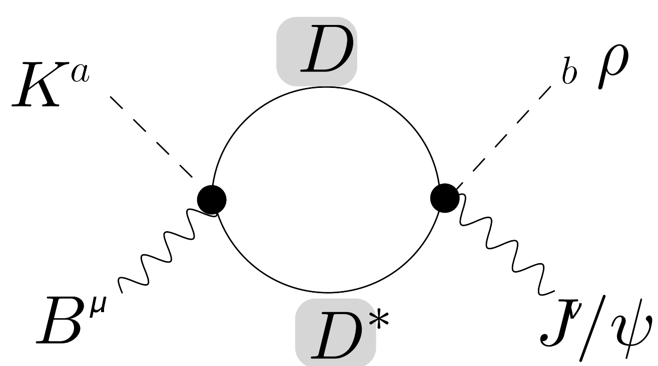
Table 1: $X - \chi_{c1}$ Mixing.

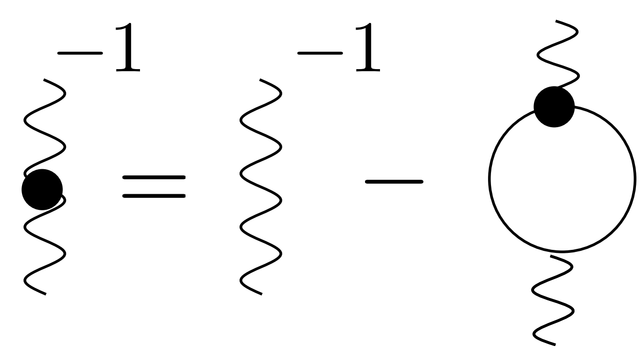
| state | $E_B$ (MeV) | $a$ (fm) | $Z_{00}$ | $a_\chi$ (MeV) | prob |
|---|---|---|---|---|---|
| $\chi_{c1}$ | 0.1 | 14.4 | 93% | 94 | 5% |
| | 0.5 | 6.4 | 83% | 120 | 10% |
| $\chi'_{c1}$ | 0.1 | 14.4 | 93% | 60 | 100% |
| | 0.5 | 6.4 | 83% | 80 | $> 100\%$ |

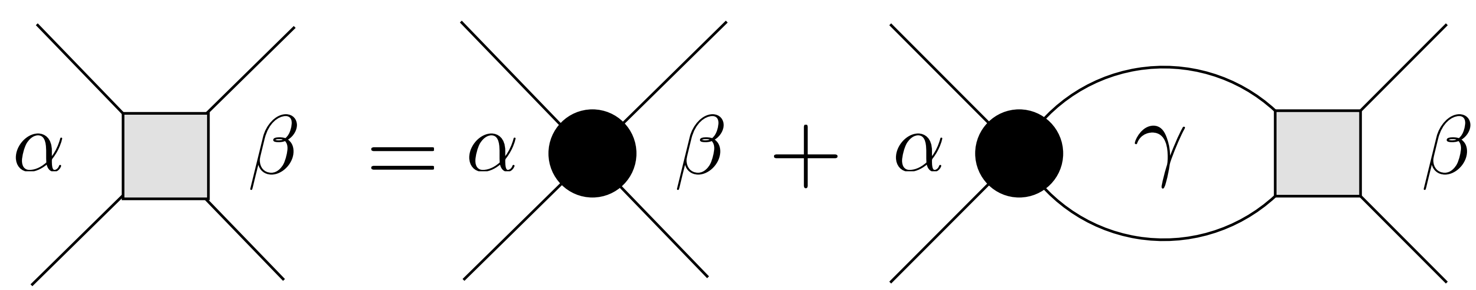
# Cusp Model

to do

- examine the X(3872): interplay of cusp, possible bound state dynamics, and mixing with cc states



# Cusp Diagnostics

- lie just above thresholds
- S-wave quantum numbers
- asymmetric lineshapes
- partner states of similar width – widths will depend on channel
- the reaction $\Upsilon(5S) \to K\bar{K}\Upsilon(nS)$ should reveal "states" at 10695 ($B\bar{B}_s^* + B^*\bar{B}_s$) and 10745 ($B^*\bar{B}_s^*$)

$$e^+e^- \to K\bar{K}J/\psi$$

$$\bar{B}^0 \to J/\psi\pi^0\pi^0$$
$$B^\pm \to J/\psi\pi^\pm\pi^0$$

(if the wavefunction enhanced rescattering diagram contributes)