Constituent models

available at *http://www.ipnl.in2p3.fr/perso/richard/SemConf/Talks.html*

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Exotics with a mixing of light and heavy quarks: Rather old idea

ON THE POSSIBLE EXISTENCE OF STABLE FOUR-QUARK SCALAR MESONS WITH CHARM AND STRANGENESS ☆

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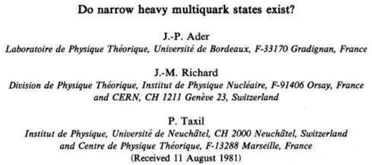
Received 4 September 1980



Do narrow heavy multiquark states exist?

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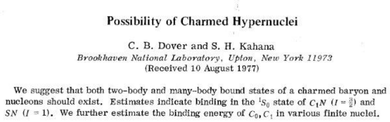
(Received 11 August 1981)



Possibility of Charmed Hypernuclei

C. B. Dover and S. H. Kahana
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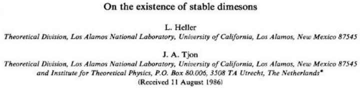
(Received 10 August 1977)

We suggest that both two-body and many-body bound states of a charmed baryon and nucleons should exist. Estimates indicate binding in the $^1S_0$ state of C_1N ($I=\frac{3}{2}$) and SN ($I=1$). We further estimate the binding energy of C_0, C_1 in various finite nuclei.



On the existence of stable dimesons

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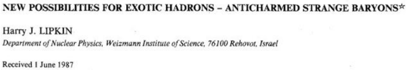
(Received 11 August 1986)



NEW POSSIBILITIES FOR EXOTIC HADRONS – ANTICHARMED STRANGE BARYONS ☆

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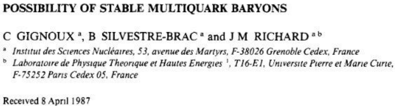
Received 1 June 1987



POSSIBILITY OF STABLE MULTIQUARK BARYONS

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Received 8 April 1987

Mesons

- Assume $V(r)$ (central) + spin-spin + . . .
- Assume constituent masses
- Solve radial equation for energy and radial function
- For ${}^3\mathrm{S}_1\,{}^3\mathrm{D}_1$ two coupled equations: for instance S-wave component of ψ'' without assuming it is mainly 2S.
- More interesting if flavor independence is assumed
- Some mathematical developments (level order, etc.)
- Empirical $V(r)$ supported by lattice QCD

Baryons in the quark model

- Some technicalities: solving the 3-body problem
  - Faddeev equations
  - Hyperspherical expansion $\tilde{X} = \{\boldsymbol{x}, \boldsymbol{y}\} = [r, \Omega_5]$
  - Variational methods
- Some minor math. developments, expansion around HO, or around hyperscalar
- Pairwise interaction? (see last section for alternatives)
- If so, link meson–baryon?

$$V = \sum_{i<j} (1/2) V(r_{ij})$$

as given by color-octet exchange ?

Baryons Results and problems

- Baryons usually heavier per quarks than mesons

$$\mathcal{M}(qqq)/3 > \mathcal{M}(q\bar{q})/2$$

- or $(\bar{q}\bar{q}\bar{q}) + (qqq) > 3(\bar{q}q)$
  (annihilation via quark rearrangement allowed)
- But masses can change the pattern

$$\mathcal{M}(\bar{Q}\bar{Q}\bar{Q}) + \mathcal{M}(qqq) < 3\mathcal{M}(\bar{Q}q)$$

  if M/m large enough. $\bar{\Omega}_{bbb}$ does not annihilate on matter!
- Too many states with $\boldsymbol{x}$ and $\boldsymbol{y}$ excitations.
- Diquark model (Lichtenberg) perhaps too restrictive

The simple prototype

$$H = \sum_i m_i + \frac{\boldsymbol{p}_i^2}{2\, m_i} - \frac{3}{16} \sum_{i<j} \tilde{\lambda}_i.\tilde{\lambda}_j v_c(r_{ij}) - \frac{3}{16} \sum \tilde{\lambda}_i.\tilde{\lambda}_j\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j \frac{v_{ss}(r_{ij})}{m_i\, m_j}$$

contains

- constituent masses
- kinetic energy
- chromoelectric energy (or say, central potential)
- chromomagnetic energy (or say, spin-spin)

We consider electric and magnetic parts separately, and then try to combine.
Before: lessons from atomic physics

Lessons from atomic physics

with flavor independence!

- Three body
    - Obvious binding of $(\alpha^{++}e^-\mu^-)$
    - Not so obvious binding of $\mathrm{H}^- = (p, e^-, e^-)$ as any factorized wf $\Psi = f(\boldsymbol{r}_1)\,f(\boldsymbol{r}_2)$ fails. Solved by Hylleraas, Chandrasekhar, …
    - Every (M^+, m^-, m^-) stable $\forall M/m$ (Hill)
    - $(\mu^+\mu^-e^-)$ unstable !
- Four body
    - $(e^+e^+e^-e^-)$ weakly bound
    - $(\mu^+\mu^+e^-e^-)$ is more stable, with many excitations
    - $(\mu^+e^+\mu^-e^-)$ is unstable
    - Any $(m_1^+m_2^+m^-m^-)$ is stable (Varga, Fleck, R.)
    - $(M^+m^-M^-m^-)$ stable and Borromean near $M/m = 2$, as none of the 3-body subsystems is stable (R., 2003)
- Stability of (M^+, m^-, m^-) and $(m_1^+m_2^+m^-m^-)$: degenerate thresholds

H_2 vs. Ps_2

- $$H(\mathrm{H}_2) = \left(\frac{1}{4M} + \frac{1}{4m}\right) \sum \boldsymbol{p}_i^2 + V + \left(\frac{1}{4M} - \frac{1}{4m}\right) (\boldsymbol{p}_1^2 + \boldsymbol{p}_2^2 - \boldsymbol{p}_3^2 - \boldsymbol{p}_4^2)$$
- $$H(\mathrm{H}_2) = H_0 + H_A$$
- where $H(\mathrm{H}_2)$ and H_0 of the rescaled Ps_2 have the same threshold
- as the systems with inverse masses $\{1/M, 1/m\}$ and $\{1/(2M) + 1/(2m), 1/(2M) + 1/(2m)\}$ have the same reduced mass.
- H_A breaks C conjugation, and lowers $H(\mathrm{H}_2)$ as compared to H_0

Atomic physics vs. quark model

No obvious excess of attraction

- Consider Ps$_2$ with $V = \sum g_{ij}(-1/r_{ij})$
- The threshold Ps + Ps has $g_{13} = g_{24} = +1$, others $g_{ij} = 0$, hence $\sum g_{ij} = +2$.
- The molecule has (after renumbering) $g_{12} = g_{34} = -1$, others $g_{ij} = +1$, hence the same $\sum g_{ij} = +2$. Weak binding. Not so obvious.
- In quark models with a pure spin-independent interaction assumed to be pairwise and colour-dependent, $V \propto \sum \tilde\lambda_i^{(c)}.\tilde\lambda_j^{(c)}\, v(r_{ij})$, color neutrality imposes something similar:

$$\left\langle \sum \tilde\lambda_i^{(c)}.\tilde\lambda_j^{(c)} \right\rangle (qq\bar q\bar q) = 2 \left\langle \sum \tilde\lambda_i^{(c)}.\tilde\lambda_j^{(c)} \right\rangle (q\bar q) ,$$

hence binding, again, is not obvious

Ps_2 vs. tetraquark

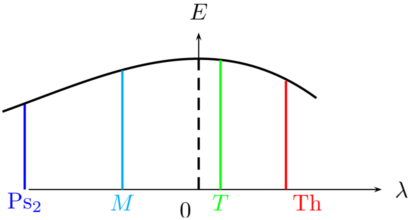
Meson-meson, atom-atom, Ps_2, tetraquark of frozen color given by

$$H = \sum \boldsymbol{p}_i^2/(2m) + \sum g_{ij} v(r_{ij}) \; .$$

After suitable renumbering:

$$H = \sum \frac{\boldsymbol{p}_i^2}{2m} + \left(\frac{1}{3} - \lambda\right)[v_{12} + v_{23}] + \left(\frac{1}{3} + \frac{\lambda}{2}\right)(v_{13} + v_{14} + v_{23} + v_{24}] \; .$$

- Atomic physics Ps_2 vs. Threshold
- Quark model with frozen color $T = (\bar{3}, 3)$ or $M = (6, \bar{6})$



Tetraquarks penalized by the non-Abelian algebra!!!

Chromoelectricity and multiquarks

$$H = \sum_i \frac{\boldsymbol{p}_i^2}{2\,m_i} - \frac{3}{16}\sum_{i<j} \tilde{\lambda}_i.\tilde{\lambda}_j v_c(r_{ij})$$

- does not bind $(qq\bar{q}\bar{q})$ with equal masses
- binds $(QQ\bar{q}\bar{q})$ if M/m large enough
- the result is confirmed in *QCD* sum rule (Nielsen et al.), lattice QCD (Michael et al., Bicudo et al., etc.) and in the molecular approach (Manohar et al, . . .)
- same favorable symmetry breaking that benefits to $\mathrm{H}_2(ppe^-e^-)$ as compared to $\mathrm{Ps}_2(e^+e^+e^-e^-)$
- hence the spin-independent quark model can produce stable multiquarks in extreme circumstances

Chromomagnetism and multiquarks: *H*

- Jaffe (1977) studied (*uuddss*) with

$$\tilde{H} = \sum_i \tilde{m}_i - C \sum_{i<j} \tilde{\lambda}_i.\tilde{\lambda}_j\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j$$

  - SU$(3)_f$ symmetry
  - Same short-range correlation C as for baryons,
- the colour-spin algebra reveals a good surprise,
- $\delta M = -8C$ for N, Λ, Ξ, …
- Thus $\delta M = -16\,C$ for the degenerate thresholds $N\Xi$, $\Lambda\Lambda$
- And $\delta M = -24\,C$ for (*uuddss*), i.e., 150 MeV below threshold
- But removing the approximations (Oka, Yazaki, Rosner, Karl et al., …) reduces the attraction and eventually spoils the binding

Chromomagnetism and multiquarks: *H*

Full calculation

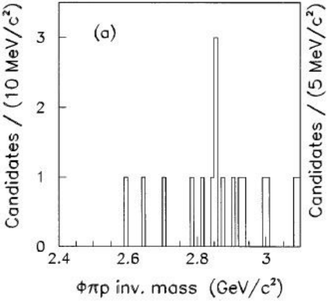
- *H* with kinetic energy, central potentil, and spin-spin potential
- Oka et al., Carbonell et al. . . . unbound
- Maltman & Wolfe: bound
- If long-range meson exchange added (σ): bound (Zhang et al., Maltman et al.)
- See review by Valcarce
- Will be revisited shortly with improved few-body techniques (Hiyama, Oka, Valcarce, Vijande, Sorba, R.)

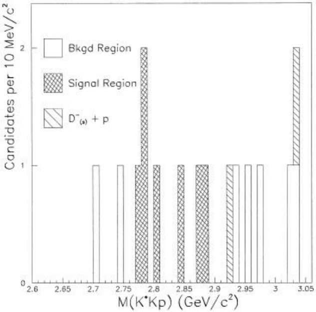
Chromomagnetism and multiquarks: Pentaquark

- Same exercise repeated in 1987 for $P = (\bar{Q}uuds)$ (or $ddus$ or $ssud$)
- Same assumptions: SU(3)$_f$, C borrowed from baryons
- and $m_Q \to \infty$ in the chromomagnetic operator
- $\delta M = -16\,C$ vs. $\delta M = -8\,C$ for $(\bar{Q}q) + (qqq)$
- Again, any correction reduces the binding
- Silvestre-Brac and Leandri (Grenoble) and Yuan et al. (China) extended the estimate to other configurations, including $(\bar{c}cqqq)$, and found interesting candidates, especially for $J^P = (1/2)^-$
- Not followed by a more detailed 5-body calculation

The 1987-vintage pentaquark: Search at Fermilab

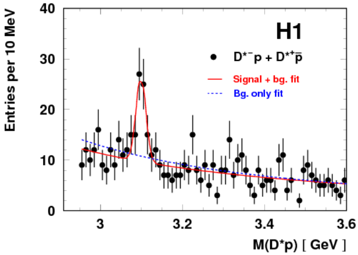
Aitala et al. searched for $P_c^0 = (\bar{c}suud) \to K^{*,0} K^- p$ and $\phi\pi^- p$. Not conclusive.

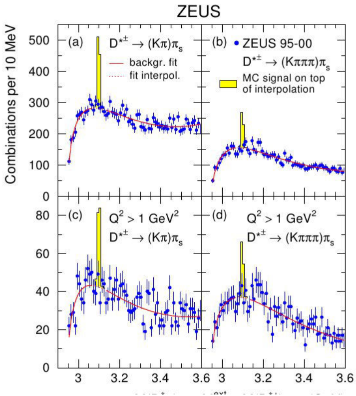




A fraction of this collaboration was interested in doing some search at CERN or at some hadron factories, but this was never approved as a priority.

The 1987-vintage pentaquark: Search at Hera





Chromomagnetism and multiquarks: $X(3872)$

- Høgaasen et al. have analysed $X(3872)$ using
$$\tilde{H} = \sum_i \tilde{m}_i - \sum_{i<j} C_{ij}\, \tilde{\lambda}_i.\tilde{\lambda}_j\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j$$
with the C_{ij} deduced from ordinary hadrons,
e.g., $C_{\bar{c}q}$ from $D^* - D$.
- Good surprise that one state, with $J^{PC} = 1^{++}$ has almost exactly the mass and the properties of $X(3872)$, in particular, in its $(c\bar{c}) - (q\bar{q})$ projection, is pure octet-octet and vector-vector. Hence the decay
$$\mathrm{X} \to \mathrm{charmonium} + \mathrm{light\ meson}$$
is suppressed.
- The model predicts an $I = 1$ partner slightly above, with $J^{PC} = 1^{++}$, unlike the $X(3900)^{\pm}$ seen at BESIII, Belle, and CLEOc.
- (Preliminary) the conclusions are supported by a more detailed 4-body calculation
- See also Stancu et al.

Combining chromo-electric and -magnetic effects

- For instance, Rosina et al., Yasui et al. $(cc\bar{u}\bar{d})$ with $J^P = 1^+$ has favorable chromoelectric binding and favorable chromomagnetic interaction as compared to $D + D^*$
- this requires an accurate solution of the 4-body problem, or 5-body for Pentaquark
- a method is based on correlated Gaussians

$$\Psi = \sum_{cs} \sum_{i} \gamma_{i,cs} \left[\exp(-\sum_{j<k} a_{jk}\, r_{jk}^2) \pm \cdots \right]$$

where $\cdots$ means terms deduced by permutation

- For given $\{a_{jk}\}$, variational energy E and coefficients γ_i given by a generalized eigenvalue equation
- Non linear parameters $\{a_{jk}\}$ deduced by astute methods (Kamimura et al., Suzuki et al., . . .)
- $\sum_{cs}$ spin-color states, ≥ 15 for pentaquark with spin $1/2$

Combining CE & CM: Heavy dibaryons

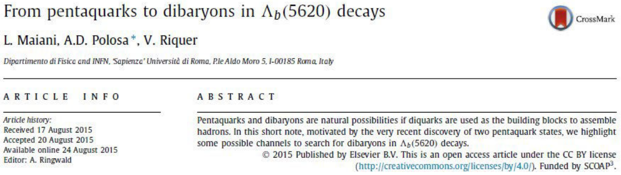
- Dibaryons ($QQ'qqqq$) tentatively combine
  - The chromoelectric interaction QQ'
  - The chromomagnetic interaction in $qqqq$ (triplet of SU(3)$_f$) as in the 1987-vintage pentaquark
- while the thresholds such as $(QQq) + (qqq)$ and $(Qqq) + (Qqq)$ get only one effect.
- If its works, it means that one has many discoveries awaiting in the double-charm sector
  - double-charm baryons
  - double-charm mesons
  - double-charm dibaryons

Variants and approximations: 1. Born-Oppenheimer

- Very useful
- $V(r)$ is already the BO potential after integrating our the gluons and light quarks
- (QQq) BO tested on explicit models (note: first excitation is between QQ, so the diquark model is not very useful here)
- Suggestion to treat all XYZ, and even P as levels in various BO potentials, with explicit g, $q\bar{q}$ or qqq additions

Variants and approximations: 2. Diquarks

- The model was invented to minimize the number of baryon excitations, and a re very useful, for instance, for multiparticle production
- It is regularly rediscovered, e.g., for pentaquarks, and for the X, Y, Z mesons,
- It is at last realised that if it leads to pentaquarks, it also lead to dibaryons, etc.



From pentaquarks to dibaryons in $\Lambda_b(5620)$ decays

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ABSTRACT

Pentaquarks and dibaryons are natural possibilities if diquarks are used as the building blocks to assemble hadrons. In this short note, motivated by the very recent discovery of two pentaquark states, we highlight some possible channels to search for dibaryons in $\Lambda_b(5620)$ decays.

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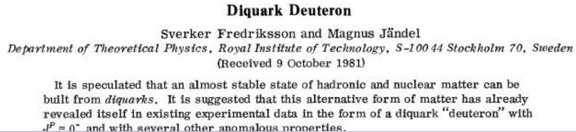

Diquark Deuteron

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(Received 9 October 1981)

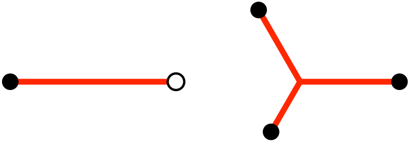
It is speculated that an almost stable state of hadronic and nuclear matter can be built from *diquarks*. It is suggested that this alternative form of matter has already revealed itself in existing experimental data in the form of a diquark "deuteron" with $J^P = 0^-$ and with several other anomalous properties.

Pairwise or multibody interaction?

Steiner tree: baryons-1

- For baryons, the linear confinement is described by a Y-shape interaction (Artru, Merkuriev, Dosch, Kuti et al., Kogut et al., etc.)

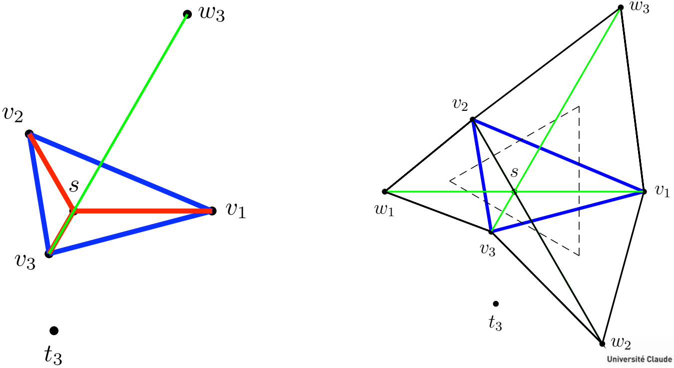
$$v = \sigma\, r_{12}\ , \qquad V_Y = \sigma \min_J \sum_{i=1}^{3} r_{iJ}\ .$$



-
- No dramatic change for baryon spectroscopy, as compared to the $1/2$ rule.
- Except for solving the 3-body problem (Taxil et al., Semay et al., etc.)

Steiner tree: baryons-2

- This baryon potential is the solution of the famous Fermat-Torricelli problem of the minimal path linking three points, with an interesting symmetry restoration, intimately related to a theorem by Napoleon.



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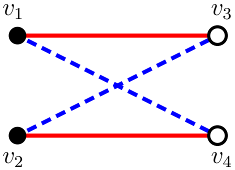
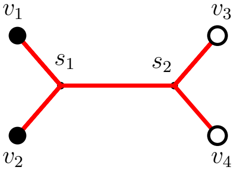
Steiner tree: tetraquarks-1

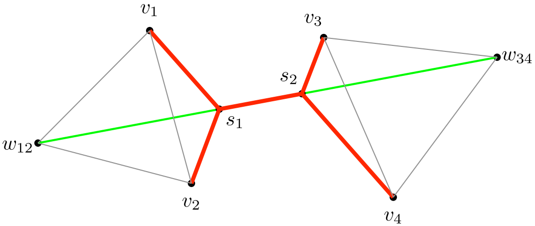
$$\begin{aligned} U &= \min\{V_{\text{flip-flop}}, V_{\text{Steiner}}\} \\ V_{\text{flip-flop}} &= \min\{d_{13}+d_{24}, d_{14}+d_{23}\}\ , \\ V_{\text{Steiner}} &= \min_{s_1,s_2}\Big(\|v_1 s_1\| + \|v_2 s_1\| + \|s_1 s_2\| \\ &\quad + \|s_2 v_3\| + \|s_2 v_4\|\Big)\ , \end{aligned}$$

U dominated by the flip-flop term,





Steiner tree: tetraquarks-2



In the planar case, very simple construction of the connected term of the potential (this speeds up the computation).

$$V_4 = \sigma \left\| w_{12} w_{34} \right\| ,$$

maximal distance between the two Melznak points.

Steiner tree: tetraquarks-3

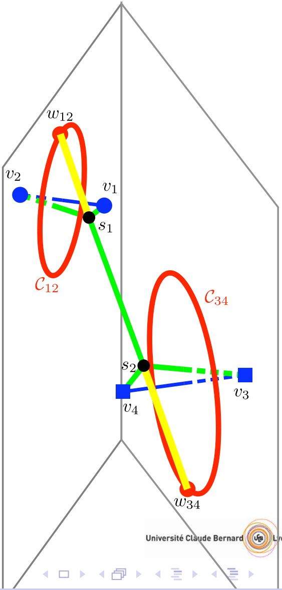
$$V_4 = \sigma \left\| w_{12} w_{34} \right\| \ ,$$

maximal distance between the two Melznak circles.

$$V_4 \le \sigma \left\{ \frac{\sqrt{3}}{2} \left[\|\boldsymbol{x}\| + \|\boldsymbol{y}\| \right] + \|\boldsymbol{z}\| \right\} \ ,$$

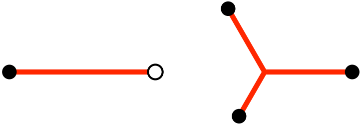
which is exactly solvable. The Jacobi var.

$$\boldsymbol{x} = v_1 v_2 ,$$
$$\boldsymbol{y} = v_3 v_4 ,$$
$$\boldsymbol{z} = (v_1 + v_2)/2 - (v_3 + v_4)/2 \ ,$$

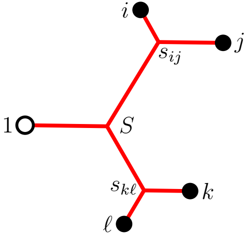


Steiner tree: pentaquark

- $U = \min\{$flip-flop, Steiner$\}$,



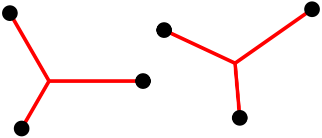
- Flip-flop
- Connected Steiner tree

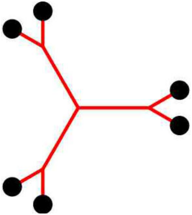


  - ($\bar{q}qqqq$), as well as ($\overline{Q}qqqq$), ($\bar{q}qqqQ$) for $M \gg m$, and probably many other configurations bound vs. spontaneous dissociation. (hyperscalar approx. with flip-flop alone sufficient to prove binding)
  - But short-range forces and antisymmetrisation constraints not yet included.
  - ($\bar{c}uuds$) should survive, as spin effects might help.

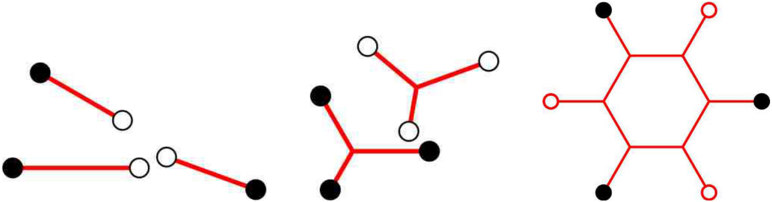
Steiner-tree: hexaquark

- Same scenario: flip-flop and connected diagrams,
- The latter, more interesting, but less important for the dynamics,
- Binding is obtained in most cases, where antisymmetrisation is neglected.
-





Steiner-tree: baryon-antibaryon

- Again: flip-flop and connected diagrams,
- Binding obtained in some cases.
- 

Outlook

- The constituent quark model suggests very few bound states
- It involves intricate few-body calculations
- We aim at providing some benchmark estimates
- Multiquarks require both chromoelectric and chromomagnetic effects
- Double-flavor sector very promising
  - Doubly-heavy baryons
  - Doubly-heavy tetraquarks
  - Perhaps doubly-heavy dibaryons