Charm spectroscopy from lattice QCD

Mike Peardon
School of Mathematics

Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

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Charm spectroscopy – collaborators:

Using Hadron Spectrum Collaboration datasets

Trinity College Dublin:

Sinéad Ryan, Pol Vilaseca

Cambridge University:

Christopher Thomas, Graham Moir

Universität Bonn:

Liuming Liu

JLab:

Jo Dudek, Robert Edwards, Bálint Joó, David Richards

Details in:

JHEP 1207 (2012) 126 [arXiv:1204.5425]
JHEP 1305 (2013) 021 [arXiv:1301.7670]

Overview

1. Anatomy of a lattice calculation
   1. Technicalities: Euclidean space-time, quark fields, ...
   2. Variational methods and the spectrum in a box.
   3. Technology: distillation and operator construction.
2. Results
   1. Charmonium.
   2. Charmonium hybrids.
   3. D and D_s.
3. Challenges
   1. Current work, technical and physics limitations.
   2. Opportunities.

Field theory on a Euclidean lattice

- Monte Carlo simulations are only practical using importance sampling
- Need a non-negative weight for each field configuration on the lattice

Minkowski → Euclidean

- Benefit: can isolate lightest states in the spectrum.
- Problem: direct information on scattering is lost and must be inferred indirectly.
- For excitations and resonances, must use a variational method.

Quarks on the computer

- Most computer time spent handling quark dynamics
- Calculation of two-point correlator between isovector quark bilinears:

$$C(t) = \frac{\int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi \; \bar{\psi}_u\Gamma^a\psi_d(t) \; \bar{\psi}_d\Gamma^b\psi_u(0) \;\; e^{-S_G[U]+\bar{\psi}_f M_f[U]\psi_f}}{\int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi \; e^{-S_G[U]+\bar{\psi}_f M_f[U]\psi_f}}$$

$$= \frac{\int \mathcal{D}U \;\; \mathbf{Tr}\, \Gamma^a M_d^{-1}(t,0)\Gamma^b M_u^{-1}(0,t) \;\; \det M^2[U] \; e^{-S_G[U]}}{\int \mathcal{D}U \;\; \det M^2[U] \; e^{-S_G[U]}}$$

- Quarks in lagrangian → determinant
- Quarks in measurement → propagators

Both present their own specific problems

Variational method in Euclidean QFT

- Ground-state energies found from $t \to \infty$ limit of:

Euclidean-time correlation function

$$
\begin{aligned}
C(t) &= \langle 0|\, \Phi(t) \quad \Phi^\dagger(0)\, |0\rangle \\
&= \sum_{k,k'} \langle 0|\, \Phi |k\rangle \langle k| e^{-\hat{H}t} |k'\rangle \langle k'| \Phi^\dagger\, |0\rangle \\
&= \sum_{k} |\langle 0|\, \Phi |k\rangle|^2 \quad e^{-E_k t}
\end{aligned}
$$

- So $\lim_{t\to\infty} C(t) = Z e^{-E_0 t}$
- Variational idea: find operator Φ to maximise $C(t)/C(t_0)$ from sum of basis operators $\Phi = \sum_a v_a \phi_a$

[C. Michael and I. Teasdale. NPB215 (1983) 433]
[M. Lüscher and U. Wolff. NPB339 (1990) 222]

Excitations

Variational method

If we can measure $C_{ab}(t) = \langle 0|\phi_a(t)\phi_b^\dagger(0)|0\rangle$ for all a, b and solve generalised eigenvalue problem:

$$\mathbf{C}(t)\,\underline{v} = \lambda \mathbf{C}(t_0)\,\underline{v}$$

then

$$\lim_{t-t_0\to\infty} \lambda_k = e^{-E_k t}$$

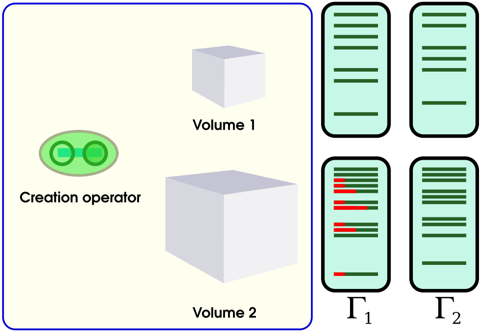
For this to be practical, we need:

- a 'good' basis set that resembles the states of interest
- all elements of this correlation matrix measured

[see Blossier et.al. JHEP 0904 (2009) 094]

Inputs and outputs...

- Choose the lattice volume and quark masses, which fixes the spectrum in each symmetry channel $\Gamma_1, \Gamma_2, \ldots$
- Choose a basis set of creation operators.

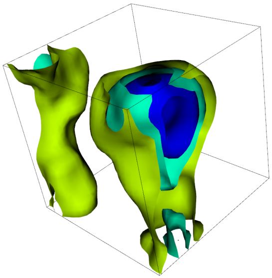


- Compute the two-point correlation matrix and solve the generalised eigenvalue problem. If overlap onto some states is small, they will be missing. This is particularly seen in two-meson continuum, which is usually not observed when just single-meson-like operators are used.

Distillation

- We can ameliorate the problems coming from measuring quark propagation by looking more carefully at how hadrons can be constructed most efficiently.
- Smeared fields: determine $\tilde{\psi}$ from the "raw" field in the path-integral, ψ:

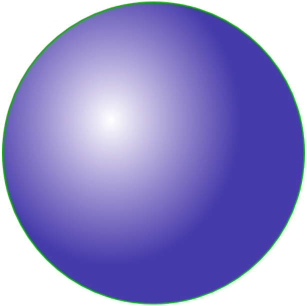
$$\tilde{\psi}(t) = \Box[U(t)]\psi(t)$$



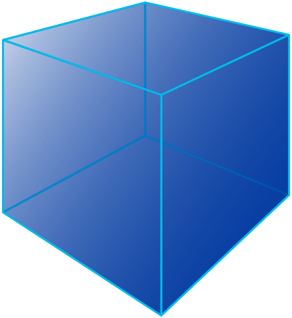
- Extract confinement-scale degrees of freedom while preserving symmetries.
- Build creation operators on smeared fields.
- Re-define smearing to be a projection operator into a small vector space smooth fields: distillation.

A tale of two symmetries

- **Continuum: states classified by J^P irreducible representations of $O(3)$.**



$O(3)$



O_h

- **Lattice regulator breaks $O(3) \rightarrow O_h$**
- **Lattice: states classified by R^P "quantum letter" labelling irrep of O_h**

Irreps of O_h

- O has 5 conjugacy classes (so O_h has 10)
- Number of conjugacy classes = number of irreps
- Schur: $24 = 1^2 + 1^2 + 2^2 + 3^2 + 3^2$
- These irreps are labelled A_1, A_2, E, T_1, T_2

	E	$8C_3$	$6C_2$	$6C_4$	$3C_2$
A_1	1	1	1	1	1
A_2	1	1	-1	-1	1
E	2	-1	0	0	2
T_1	3	0	-1	1	-1
T_2	3	0	1	-1	-1

Spin on the lattice

- O_h has 10 irreps: $\{A_1^{g,u}, A_2^{g,u}, E^{g,u}, T_1^{g,u}, T_2^{g,u}, \}$, where $\{g, u\}$ label even/odd parity.
- Link to continuum: subduce representations of $O(3)$ into O_h

	A_1	A_2	E	T_1	T_2
$J = 0$	1				
$J = 1$				1	
$J = 2$			1		1
$J = 3$		1		1	1
$J = 4$	1		1	1	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

- Enough to search for degeneracy patterns in the spectrum?

$$4 \equiv 0 \oplus 1 \oplus 2$$

Operator basis – derivative construction

- **A closer link to the continuum is needed**
- **Start with continuum operators, built from n derivatives:**

$$\Phi = \bar{\psi}\, \Gamma \left(D_{i_1} D_{i_2} D_{i_3} \ldots D_{i_n} \right) \psi$$

- **Construct irreps of $SO(3)$, then subduce these representations to O_h**
- **Now replace the derivatives with lattice finite differences:**

$$D_j \psi(x) \rightarrow \frac{1}{a} \left(U_j(x) \psi(x + \hat{j}) - U_j^\dagger(x - \hat{j}) \psi(x - \hat{j}) \right)$$

Example: $J^{PC} = 2^{++}$ meson creation operator

- **Need more information to discriminate spins. Consider continuum operator that creates a 2^{++} meson:**

$$\Phi_{ij} = \bar{\psi}\left(\gamma_i D_j + \gamma_j D_i - \frac{2}{3}\delta_{ij}\gamma\cdot D\right)\psi$$

- **Lattice: Substitute gauge-covariant lattice finite-difference $D_{\rm latt}$ for D**
- **A reducible representation:**

$$\Phi^{T_2} = \{\Phi_{12}, \Phi_{23}, \Phi_{31}\}$$

$$\Phi^{E} = \left\{\frac{1}{\sqrt{2}}(\Phi_{11} - \Phi_{22}), \frac{1}{\sqrt{6}}(\Phi_{11} + \Phi_{22} - 2\Phi_{33})\right\}$$

- **Look for signature of continuum symmetry:**

$$\langle 0|\Phi^{(T_2)}|2^{++(T_2)}\rangle = \langle 0|\Phi^{(E)}|2^{++(E)}\rangle$$

Results

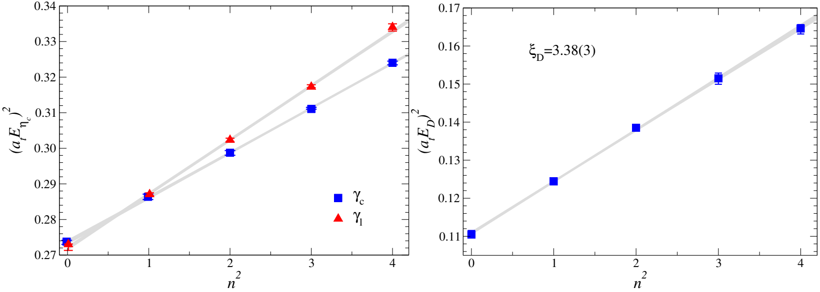
$c\bar{c}$, $c\bar{l}$ and $c\bar{s}$

CAVEAT EMPTOR

- $m_\pi \approx 400 MeV$
- **No two-meson operators in basis**
- **No disconnected charm contributions**
- **No** $a \to 0$ **extrapolation**

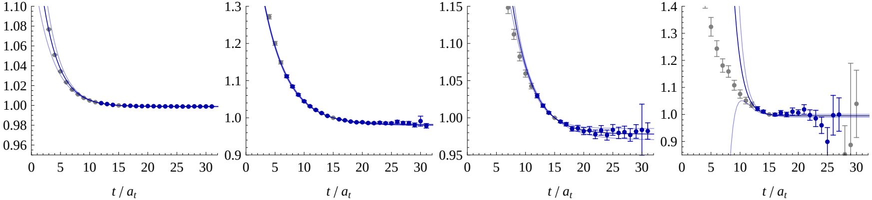
Dispersion relations - η_c and D mesons

- **Action parameters for charm quark tuned to ensure dispersion relation for η_c is relativistic**
- **Using these tuned parameters, D meson also has relativistic dispersion relation**



Fits to $\lambda_k(t)$

- Variational basis, so can access excited states
- Fit $\lambda_k(t)$ to one or two exponentials
- Second exponential to stabilise some fits - value not used
- Plots show $\lambda_k(t) \times e^{E_k(t-t_0)}$



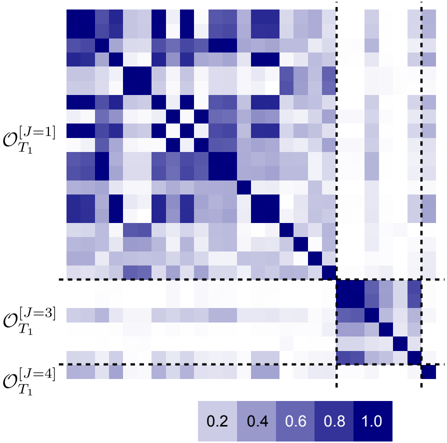
- Data from T_1^{--} channel ($J = 1, 3, 4, \ldots$)

Subduction of derivative-based operators

- T_1^{--} variational basis
- 26 operators, up to $D_i D_j D_k$
- Correlation matrix at $t/a_t = 5$, normalised:

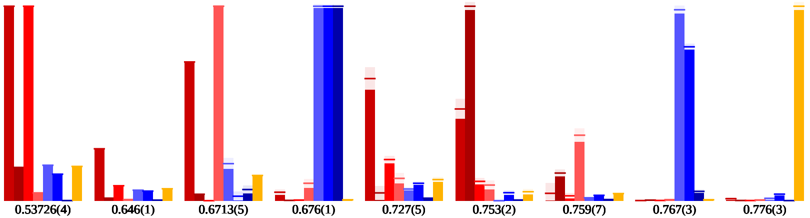
$$Q_{ij} = \frac{C_{ij}}{\sqrt{C_{ii} C_{jj}}}$$

- Reasonable spin separation seen



Spin identification

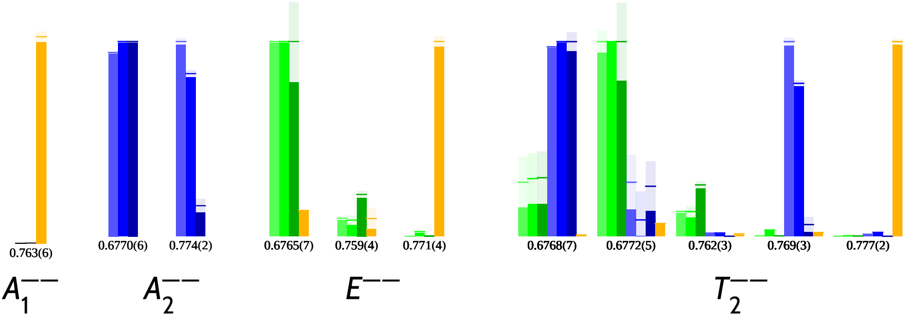
- Using $Z = \langle 0|\Phi|k\rangle$, helps to identify continuum spins
- For high spins, can look for agreement between irreps
- Data below for T_1^{--} irrep, colour-coding is Spin 1, Spin 3 and Spin 4.



- Can help identify glue-rich states, using operators with $[D_i, D_j]$

. . . the rest of the spin-4 state

- All polarisations of the spin-4 state are seen
- Spin labelling: Spin 2, Spin 3 and Spin 4.

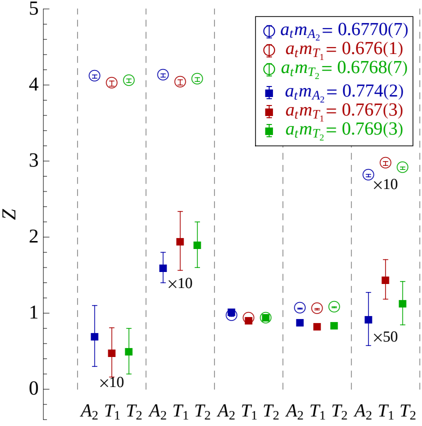


Identifying spin - operator overlaps

- **Example – 3^{--} continuum**
- **Look for remnant of continuum symmetry:**

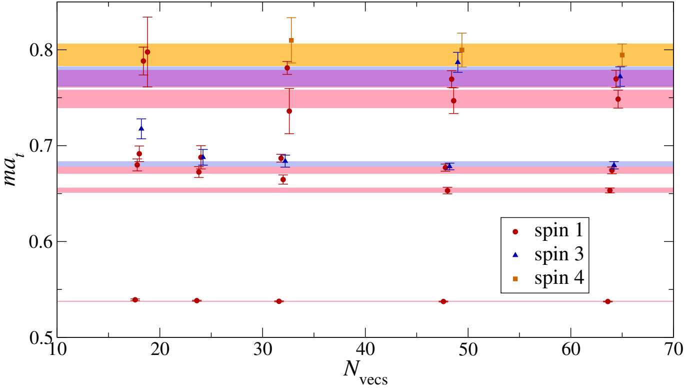
$$\langle 0|\Phi^{[J=3]}_{A_2^{--}}|k\rangle = \langle 0|\Phi^{[J=3]}_{T_1^{--}}|k\rangle = \langle 0|\Phi^{[J=}_{T_2^{--}}$$

- **Can identify two spin-3 states.**



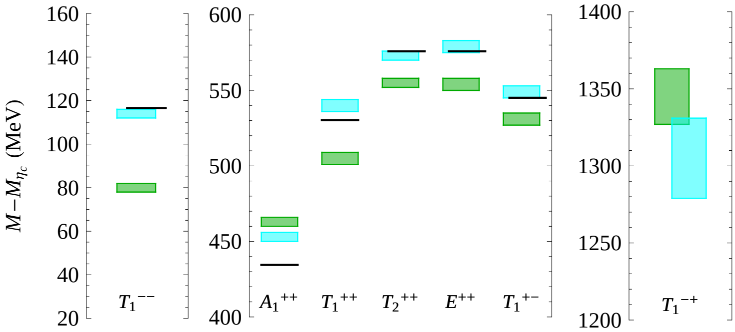
Spectrum - dependence on distillation basis

- 16^3 lattice - vary $N_{\mathcal{D}}$
- Calculation done on smaller ensemble



- Stable spectrum for $N_{\mathcal{D}} > 48$

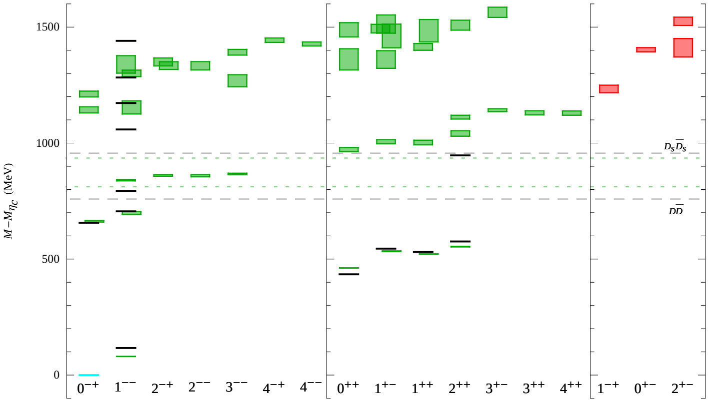
Lattice artefacts in charmonium



- **Hyperfine structure sensitive to lattice artefacts. Boost co-efficient of action term to suppress these.**
- **green → light blue. Shifts are ≈ 40 MeV.**

[Liu et.al. arXiv:1204.5425]

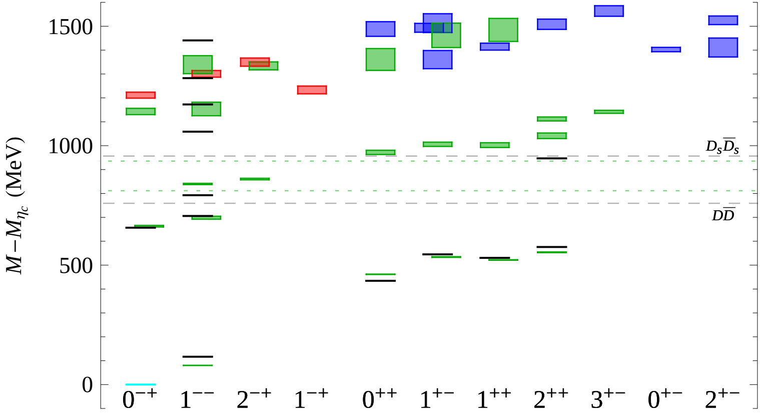
Excitation spectrum of charmonium



- **Quark model:** 1*S*, 1*P*, 2*S*, 1*D*, 2*P*, 1*F*, 2*D*, . . . **all seen.**
- **Not all fit quark model: spin-exotic (and non-exotic) hybrids seen**

[Liu et.al. arXiv:1204.5425]

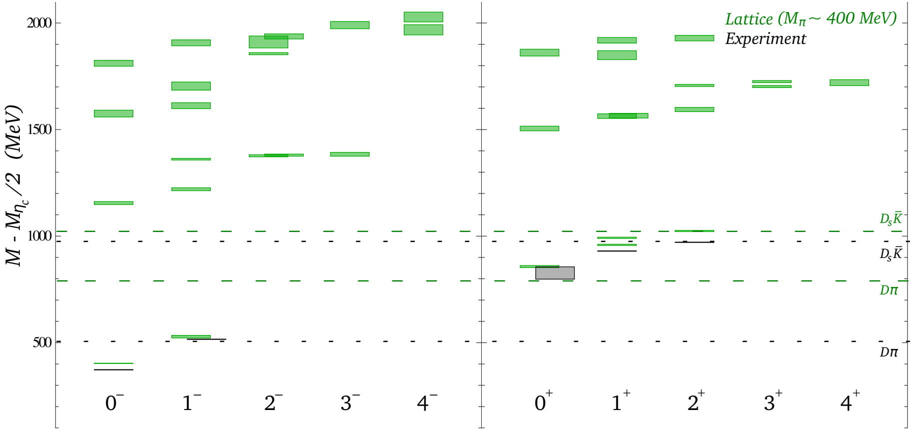
Gluonic excitations in charmonium?



- See states created by operators that excite intrinsic gluons
- two- and three-derivatives create states in the open-charm region.

[Liu et.al. arXiv:1204.5425]

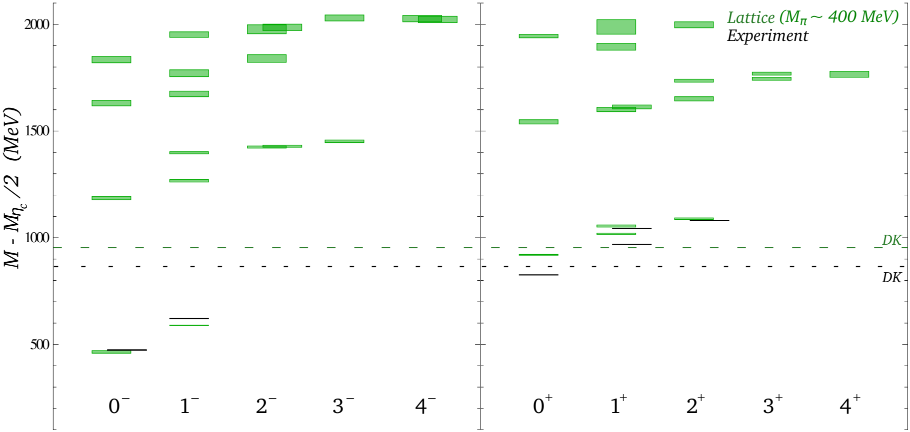
Excitation spectrum of D



- Subtract $\frac{1}{2}M_{\eta_c}$ to reduce systematic error
- Thresholds for both physical M_π and $M_\pi \approx 400$ MeV

[Moir et.al. JHEP 1305 (2013) 021]

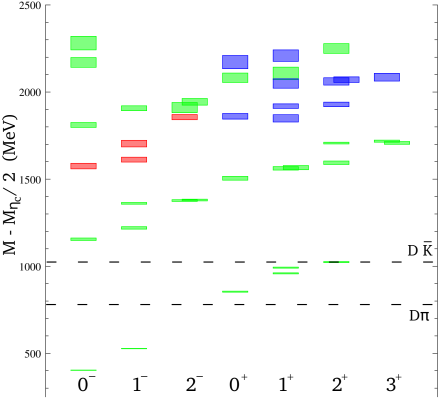
Excitation spectrum of D_s



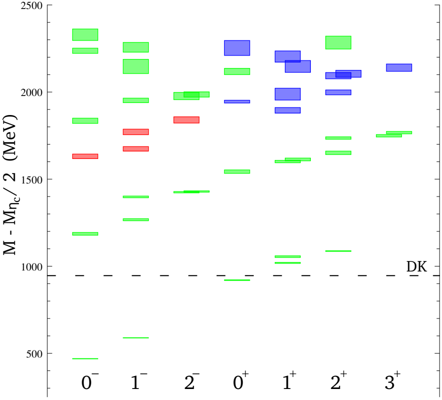
- **Subtract $\frac{1}{2}M_{\eta_c}$ to reduce systematic error**
- **Thresholds for both physical M_π and $M_\pi \approx 400$ MeV**

[Moir et.al. JHEP 1305 (2013) 021]

Hybrid excitations in D/D_s spectrum



D



D_s

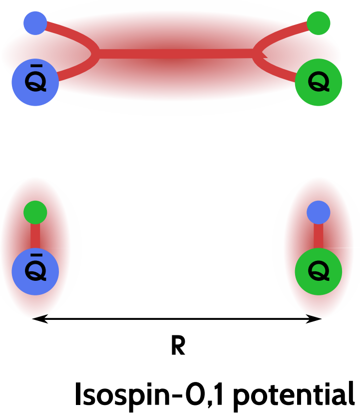
Challenges

Work in progress in HSC

- Scattering properties and matrix elements (talks tomorrow).
- DK, $D\pi$ soon...
- ... then $D\bar{D}$ (with tetraquarks?)
- Tetraquark computations are very (too?) expensive in current framework - working on technical improvements, which would have an impact on the cost of baryon (and pentaquark) calculations.
- Other suggestions?

Static sources - worth revisiting?

- Lattice calculations with static colour sources have a long history.
- Technology developed by Hadron Spectrum Collaboration makes these calculations accessible.
- Do they tell us anything about the tetraquarks/molecules?



Summary

- Technology for studying excited mesons works well in the charm sector.
- Next steps to look at scattering of charmed mesons underway.
- New technology needed to speed up calculations with tetraquarks.